Chapter 10

Circle

Contents for Discussion

• Circle • Chord and Arc of a circle • Diameter and Circumference • Theorems Related to Circle • Ratio of Circumference and Diameter of a Circle (π) • Area of a Circle • Cylinder.

Learning Outcomes : After studying this chapter, I will be able to–

- develop the concept of circle.
- explain the concept of Pi (π).
- find the circumference and the area of a circular region solving related problems.
- use theorems related to circle to solve problems and using measuring tape to measure the circumference and the area of a circular region.
- measure the area of the outer surfaces of a cylinder with the help of the area a of quadrilateral and a circle.

Practice

Solutions to Mathematical Problems following 100% accurate format for best prep.

Dear learners, mathematical problems of this chapter have been divided into exercise, multiple choice, short, creative and exercise-based activities in light of the learning outcomes. Practice the solutions well to ensure the best preparation in the exam.

Exercise 10.1 : Circle, Chord and Arc, Diameter and Circumference Theorems related to Circle

At a Glance Important Contents of Exercise

- **Circle :** If any point on a curve in a plane is always equidistant from a fixed point within the area bounded by that curve, is called a circle.
- **Chord :** A line joining any two points of a circle is called a chord of the circle.
- **Diameter :** If any chord of a circle passes through the center then the chord is called a diameter of the circle.
- **Radius :** Half the length of the diameter of a circle is called the radius. If the diameter is d the radius of the circle, $r = \frac{d}{2}$.
- The line connecting the midpoint of a chord with the center and diameter of a circle is perpendicular to that chord.

Solutions to Exercise Problems — Let's solve the textbook problems

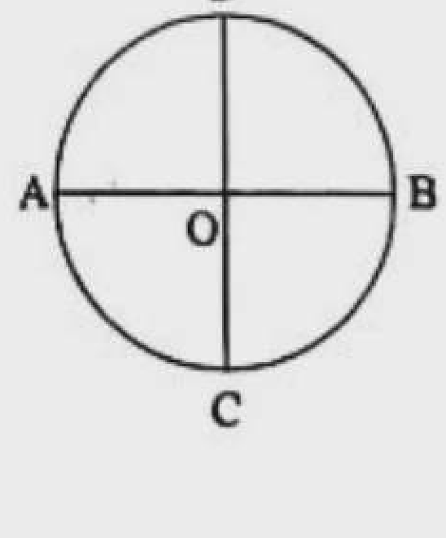
Solutions to Geometrical Problems

1. Prove that, if two chords of a circle bisect each other, their point of intersection will be the centre of the circle.

Solution :

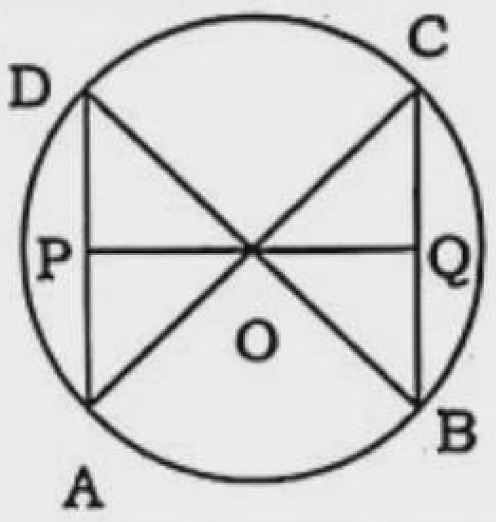
General Enunciation : If two chords of a circle mutually bisect, then the point of intersection of the chords is the centre of the circle.

Particular Enunciation : Let us suppose that ACBD is any circle with centre O. AB and CD are two chords which mutually bisect at O. We shall have to prove that O is the centre of the circle.



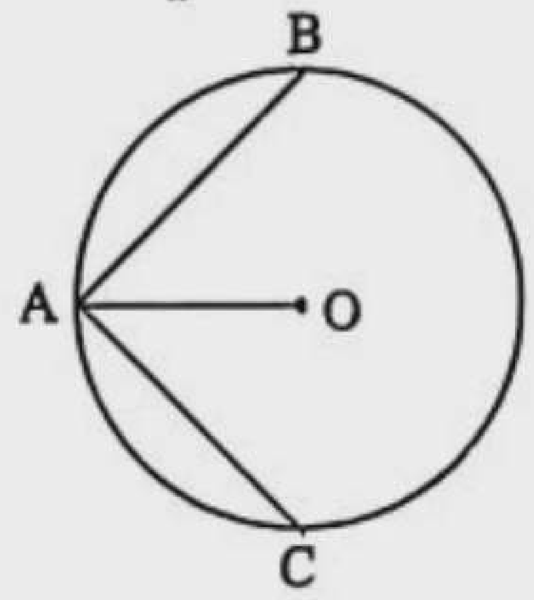
Proof : According to proposition,
AO = OC and BO = OD .... (i)
Again, (i) is possible if and only if AB = CD.
or, $\frac{AB}{2} = \frac{CD}{2}$
or, OB = OD .... (ii)
$\therefore$ From (i) and (ii), we have,
AO = OB = OD = OC.
That is, A, B, C, D lie on the circle ABCD with centre at O.
Thus it is proved that if two chords of a circle mutually bisect, their point of intersection is the centre of the circle.

2. **Prove that the line joining the midpoints of two parallel chords passes through the centre and is perpendicular to the two chords.**
Solution :
General Enunciation : The line joining the mid-points of two parallel chords of a circle passes through the centre of the circle and is perpendicular to both the chords.
Particular Enunciation :



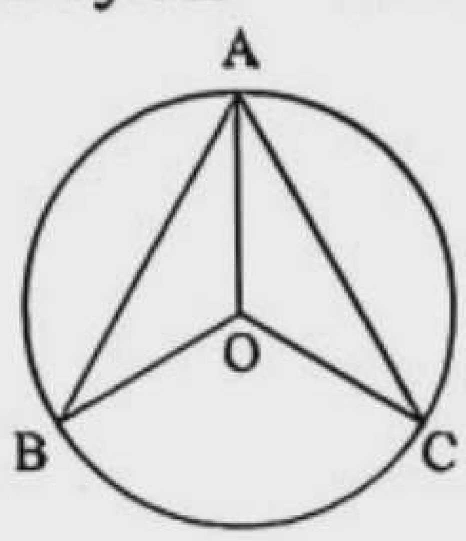
Let us suppose that ABCD is any circle with centre at O. AD and BC are its two parallel chords. P and Q are the mid-points of AD and BC respectively. We shall have to prove that P, O and Q are co-linear and POQ is perpendicular to both AD and BC.
Construction : The lines AOC and BOD are drawn.
Proof : From Δ AOD, we have,
AO = DO, since they are radii of the same circle.
$\therefore$ Δ AOD is an isosceles triangle.
Again, AP = PD, since P is the mid-point of AD according to proposition.
$\therefore$ OP $\perp$ AD.
Similarly, from Δ BOC, it can be found that OQ $\perp$ BC.
Since AD || BC, hence POQ must be collinear and P, O, Q, will lie on POQ, a straight line.
So, P, O and Q are Co-linear.
Thus the proposition has been proved.

3. **Two chords AB and AC of a circle make equal angles with the radius through A. Prove that AB = AC.**
Solution :
General Enunciation : If AB and CD are two chords of a circle and they make equal angles with the radius passing through A, then AB = AC.
Particular Enunciation : Let us suppose that ABC is any circle with centre at O. AB and AC are two chords of the circle ABC. AB and AC make equal angles with the radius OA ie. $\angle BAO = \angle CAO$.
We shall have to prove that AB = AC.



Construction : B, O and C, O are joined.
Proof : From ΔAOB and ΔAOC, we have,
OB = OC, since they are the radii of the same circle.
From ΔAOC, OA = OC (Radii of same circle)
$\therefore \angle OAC = \angle ACO$
Similarly, from ΔAOB,
$\angle OAB = \angle ABO = \angle OAC = \angle ACO$
($\because \angle OAB = \angle OAC$ as proposition)
Since, $\angle OAB = \angle OAC$ and $\angle OBA = \angle OCA$
$\therefore \angle AOB = \angle AOC$
Now, from ΔAOB and $\angle AOC$,
OA = OA, OB = OC
$\angle AOB = \angle AOC$ (included angle)
$\therefore \Delta AOB \cong \Delta AOC$.
So, AB = AC **(Proved)**

4. **In the figure, O is the centre of the circle and the chord AB = chord AC. Prove that $\angle BAO = \angle CAO$.**
Solution :
Proposition : Let us suppose, O is the centre of any circle ABC. AB and AC are its two chords related by AB = AC.

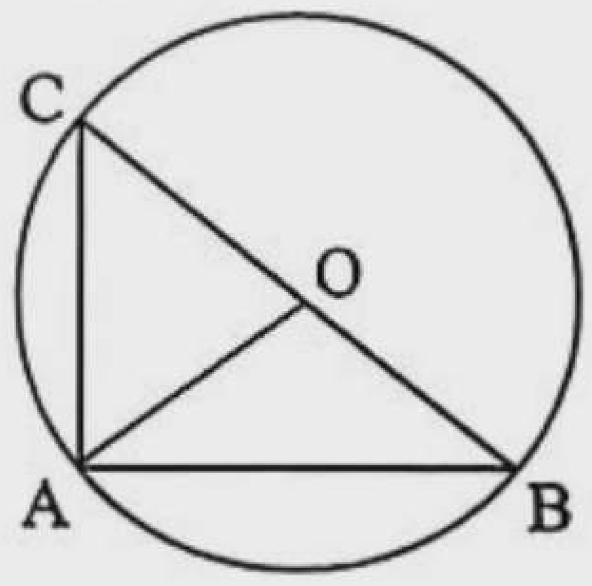


We shall have to prove, $\angle BAO = \angle CAO$.
Construction : B, O and C, O are joined. A, O are also joined.
Proof : From Δ AOB and Δ AOC, we have,
AB = AC, according to proposition,
OB = OC, since they are the radii of the same circle.
OA is the common side of both the triangles.
$\therefore \Delta AOB \cong \Delta AOC$.
So, $\angle BAO = \angle CAO$. **(Proved)**

5. A circle passes through the vertices of a right-angled triangle. Show that the centre lies on the midpoint of the hypotenuse.

Solution :

General Enunciation : If a circle passes through the vertices of a right angled triangle, then centre of the circle lies on the mid-point of the hypotenuse.



Particular Enunciation : Let us suppose, ABC circle with centre O passes through the vertices of the right triangle ABC where $\angle A = 90°$ and hypotenuse BC. It is to be proved that O is the mid-point of the hypotenuse BC.

Construction : A and O are joined.

Proof : From the ΔAOC, we have, $OA = OC$, since they are the radii of same circle.

Again from ΔAOB, we have, $OA = OB$, since they are radii of same circle.

So, $OB = OA = OC$.

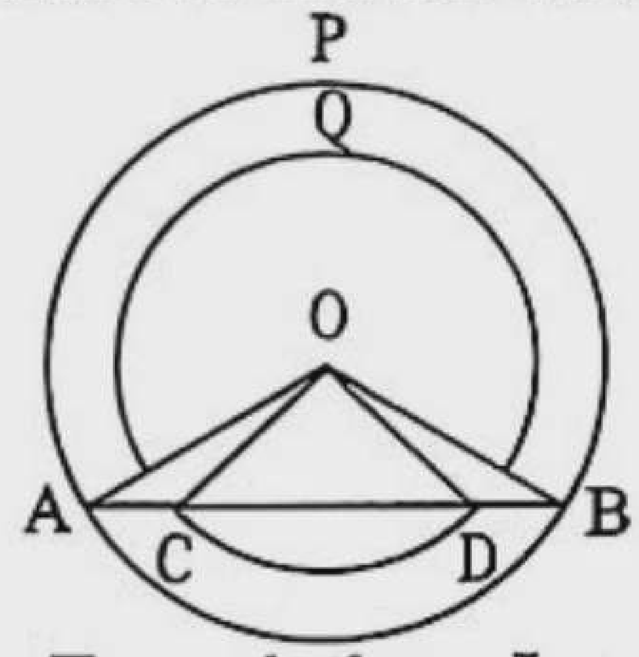
That is, $OB = OC$

$\therefore$ O is the mid-point of BC, the hypotenuse of the circle. **(Proved)**

6. A chord AB of one of two concentric circles intersects the other at C and D. Prove that AC = BD.

Solution :

General Enunciation : AB is any chord of circle PAB with centre O. If AB intersects another circle CDQ with the same centre O at C and D, then AC and BD will be equal.



Particular Enunciation : Let us suppose that ABP and CDQ are any two circles with the same centre O.

AB is any chord of the circle ABP and AB cuts the circle CDQ at C and D. We have to prove that $AC = BD$.

Construction : A, O; C, O; B, O and D, O are joined.

Proof : In ΔAOC and ΔBOD,

$OA = OB$, since they are radii of the same circle ABP.

Again $OC = OD$, the radii of the same circle CDQ.

$\therefore \angle OAB = \angle OBA$ .... (1)

Now, external $\angle OCB = \angle OAC$

$= \angle AOC + \angle OAC$ .... (1)

And external $\angle ODC = \angle BOD + \angle BDO$ .... (2)

Since in ΔCOD, $\angle OCD = \angle ODC$

$\therefore$ from (1), (2), we get,

$\angle AOC + \angle OAC = \angle BOD + \angle BDO$

$\angle AOC = \angle BOD$, since in (1) $\angle OAB$ or, $\angle OAC = \angle OBA$ or $\angle OBD$

$\therefore \Delta AOC \cong \Delta BOD$.

$\therefore AC = BD$. **(Proved)**

Multiple Choice Q/A Designed as per topic

10.1 Circle

Textbook Page 157

1. **What are the minimum requirements of points for drawing a circle?** *(Easy)*

c (a) 1 (b) 2 (c) 3 (d) 4

2. **A straight line intersects a circle at how many points?** *(Easy)*

b (a) 1 (b) 2 (c) 3 (d) 4

3. **What is the angle at the centre of a circle?** *(Easy)* *[DjB '17]*

a (a) 360° (b) 180° (c) 90° (d) 0°

4. **How much degrees is subtended at the centre of a circle?** *(Medium)* *[DB' 15]*

d (a) 90° (b) 180° (c) 270° (d) 360°

5. **How many parts does a chord divide a circle?** *(Easy)* *[JB' 15]*

b (a) One (b) Two (c) Three (d) Four

6. i. The distance between a point on a circle and its centre equals its radius.
ii. A tangent of a circle intersects the circle in two points.
iii. Any chord of a circle determines two arcs of the circle.

What is/are the correct answer/answers after the above statements? *(Medium)*

a (a) i & iii (b) ii & iii (c) i & ii (d) i, ii & iii

7. **In the case of circle —.** *[CtgB '16]*

i. the line joining the midpoints of two parallel chords passes through the centre.
ii. the line joining the midpoints of two parallel chords is perpendicular to the two chords
iii. each chord of a circle is equidistant from the center

Which one is correct? *(Easy)*

a (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

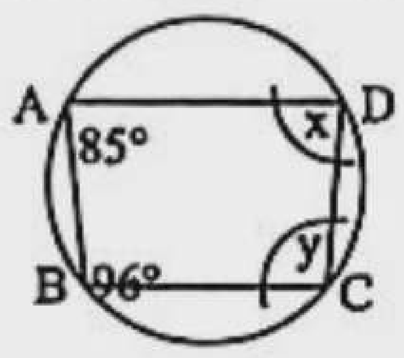
8. In a circle —. *[CB' 15]*
   i. the perpendicular from the centre to a chord bisects the chord
   ii. a straight line can intersect it at more than two points
   iii. the diameter is twice of its radius

   Which one of the following is true? *(Medium)*

ⓑ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

10.2 Chord and Arc of a circle ▸ Textbook Page 160

9. **In the adjoining figure, ∠x = — ?** *(Medium)*

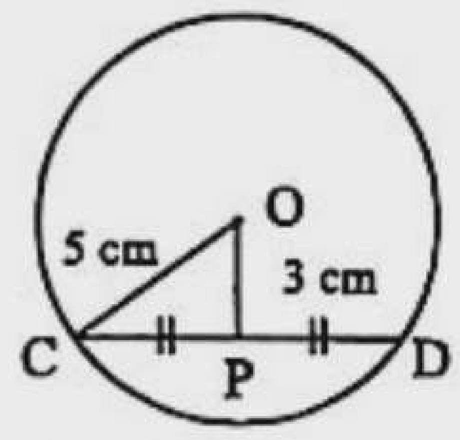


ⓐ 84° ⓑ 45°
ⓐ ⓒ 60° ⓓ 75°

10. **∠y = ?** *(Medium)*

ⓒ ⓐ 70° ⓑ 85° ⓒ 95° ⓓ 75°

11.



In the figure, O is centre of the circle. Which one is the length of CD? *(Medium) [RB '19]*

ⓒ ⓐ 4cm ⓑ 6cm ⓒ 8cm ⓓ 10cm

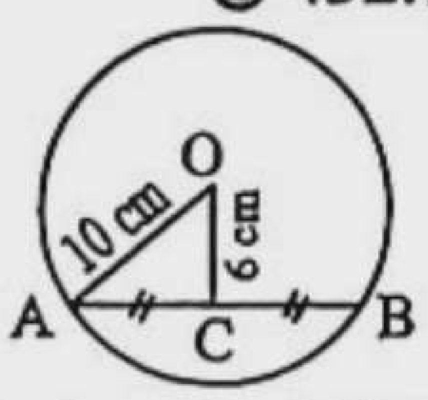
12. **What is subtended angle at the centre of a circle?** *(Hard)* *[RB '19]*

ⓓ ⓐ 0° ⓑ 90° ⓒ 180° ⓓ 360°

13. **The diametre of a circle is 12cm. What is the circumference of the circle? (use $\pi = 3.14$)** *(Medium)* *[CB '19]*

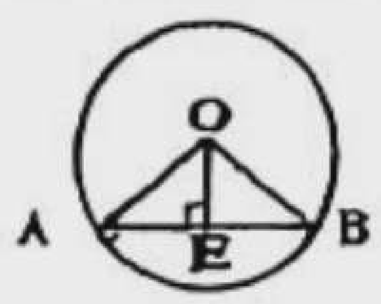
ⓐ 18.84 cm ⓑ 37.68 cm
ⓑ ⓒ 113.76 cm ⓓ 452.16 cm

14.



In figure what is the length of AB in cm? *(Medium)* *[SB '18]*

ⓒ ⓐ 8 cm ⓑ 12 cm ⓒ 16 cm ⓓ 20 cm



O is the centre of the circle and OE ⊥ AB.

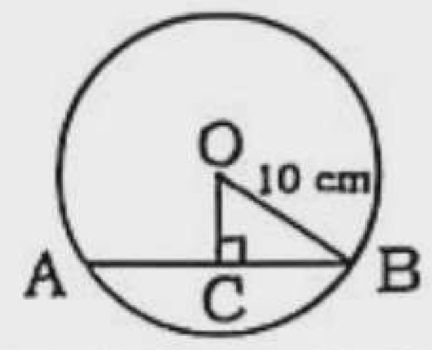
15. **How much cm is the circumference of the circle with the diameter of 10 cm?** *(Medium)* *[CtgB '17]*

ⓐ 15.70 (approx) ⓑ 157.0 (approx)
ⓒ ⓒ 31.4 (approx) ⓓ 314.1 (approx)

16. **What is the angle subtended at the centre of a circle?** *(Medium)* *[DB '16]*

ⓓ ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

17.



OC ⊥ AB and AB = 16 cm, OC = ? *(Easy) [SB '16]*

ⓒ ⓐ 2 cm ⓑ 5 cm ⓒ 6 cm ⓓ 8 cm

18. **How many arcs exist when a circle is divided by a chord?** *(Easy)* *[RB' 15]*

ⓑ ⓐ One ⓑ Two ⓒ Three ⓓ Four

19. **Each chord divides a circle into how many arcs?** *(Easy)* *[DjB' 15]*

ⓑ ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

20. **In a circle with centre O, AB and CD are two equal chords. E and F are the middle points of AB and CD respectively if OE = 3 unit then OF = what?** *(Hard)*
[Viqarunnisa Noon School and College, Dhaka]

ⓑ ⓐ 6 unit ⓑ 3 unit ⓒ 4 unit ⓓ 2 unit

21. i. The major arc and minor arc of a circle are never equally long.
    ii. A diameter of a circle is longer than any chord other than a diameter.
    iii. A line intersecting a circle at exactly one point is a chord.

    What is the correct answer after the above statements? *(Medium)*

ⓐ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

22. i. If two arcs of a circle are congruent, their chords are congruent.
    ii. All chords of a circle are shorter than a diameter of the circle.
    iii. All chords of a circle have only two points with the circle.

    What is the correct answer after the above statements? *(Medium)*

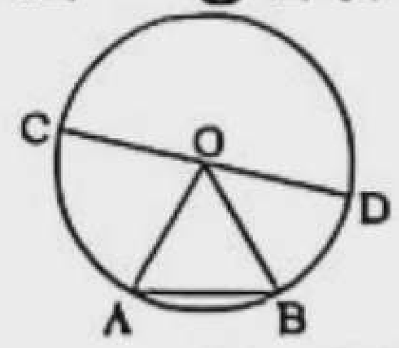
ⓓ ⓐ i & iii ⓑ ii & iii ⓒ iii ⓓ i, ii & iii

23. **Of a circle —.** *[BB '17]*
    i. the perpendicular bisector of any chord passes through the centre
    ii. the radius is half of the diameter
    iii. no more than two points can be intersected in a circle by any intersector

    Which one is correct? *(Medium)*

ⓓ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

24.

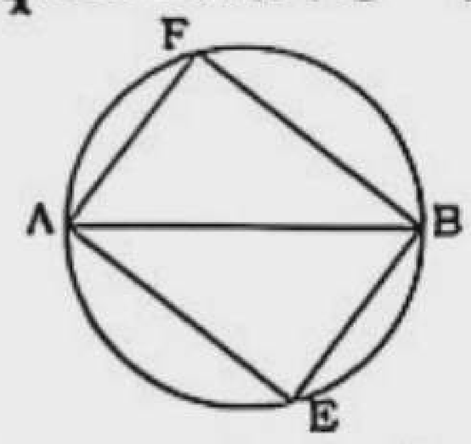


In a circle with centre O of the figure —. *[CB '16]*
   i. AB = diameter
   ii. OA = OD
   iii. CD > AB

   Which one is correct? *(Medium)*

ⓒ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

■ **Observe the adjacent figure carefully and answer the questions 25 – 28 :**



25. If AB is the greatest chord of the circle, ∠AEB = ∠AFB = What? *(Easy)*
d ⓐ 30° ⓑ 45° ⓒ 60° ⓓ 90°

26. If ∠A = 90°, ∠B = What?
d ⓐ 30° ⓑ 45° ⓒ 60° ⓓ 90°

27. If AE = FB and AF ≠ AE, quadrilateral AFBE is —. *(Easy)*
ⓐ a square ⓑ a rectangle
b ⓒ a rhombus ⓓ a trapezium

28. i. ΔAEB is a right triangle
ii. ΔAEB and ΔAFB are congruent
iii. ΔAEB is an obtuse-angled triangle
What is/are the correct answer/answers after the above statements? *(Hard)*
d ⓐ i & iii ⓑ ii & iii ⓒ iii ⓓ i & ii

■ **Answer to the questions 29 and 30 with the help of given information :**

In the figure MN = 12 cm and OP = 8 cm. *[CtgB '18]*

29. What is the value of PN? *(Easy)*
b ⓐ 4 cm ⓑ 6 cm ⓒ 8 cm ⓓ 10 cm

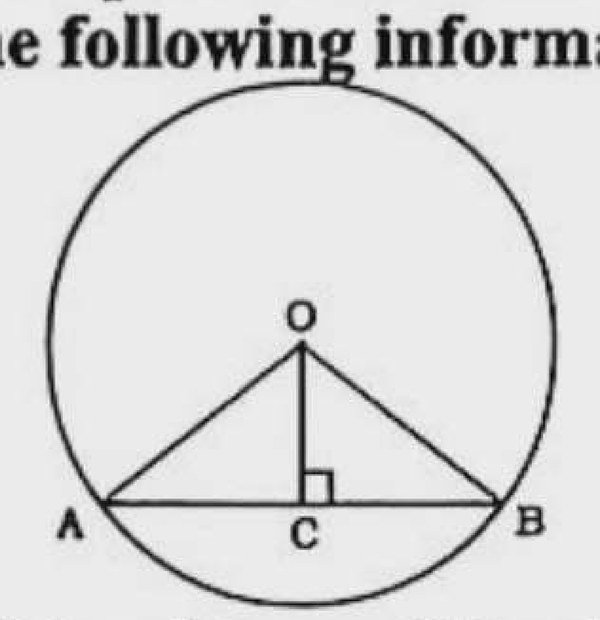
30. What is the area of triangle OPM? *(Medium)*
ⓐ 20 sq cm ⓑ 24 sq cm
b ⓒ 48 sq cm ⓓ 96 sq cm

■ **Answer the questions No. 31 and 32 using the above information.**

31. ∠OEA = What? *(Easy)* *[RB '17]*
c ⓐ 0° ⓑ 45° ⓒ 90° ⓓ 180°

32. The radius of the circle is— *[RB '17]*
i. AO
ii. BO
iii. AB
Which one is correct? *(Medium)*
a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

■ **Answer the question No. 33 and 34 on the basis of the following information :**



In figure, OA = 13 cm, OC = 5 cm. *[DB' 15]*

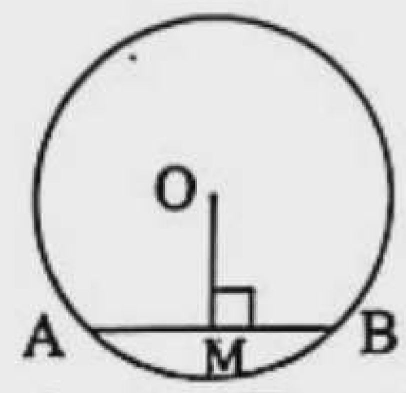
33. What is the value of AB? *(Easy)*
b ⓐ 12 ⓑ 24 ⓒ 65 ⓓ 194

34. If ∠OAB = 60°, what type of triangle is ABC? *(Medium)*
ⓐ Equilateral ⓑ Scalene
a ⓒ Right angled ⓓ Acute angled

10.3 Diameter and Circumference ▸ Textbook Page 160

35. If the circumference of the wheel of a car is 5.15 metres, what is the diametre of the wheel? *(Medium)* *[DB '19]*
ⓐ 0.82 metre ⓑ 0.96 metre
d ⓒ 1.28 metre ⓓ 1.64 metre

36. *[MB '19]*



Here, OM = 6cm, AB = 16cm. What is the length of OA? *(Medium)*
a ⓐ 10 cm ⓑ 14 cm ⓒ 96 cm ⓓ 100cm

37. AB and CD be two equal chords of a circle with the centre 'O'. If OM ⊥ AB, ON ⊥ CD then which one is correct? *(Hard)* *[MB '19]*
ⓐ OM < ON ⓑ OM = ON
b ⓒ OM > ON ⓓ AM = ON

38. What is the circumference of the circular garden with diameter 6 cm? *(Medium)[RB '16]*
d ⓐ 36 π cm ⓑ 12 π ⓒ 9 π ⓓ 6 π

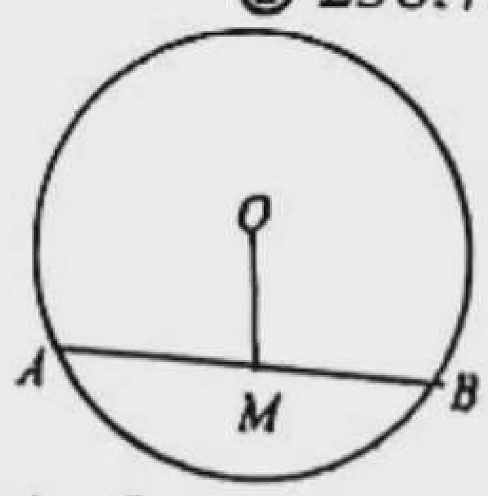
39. If the radius is 5 cm of a circle, what is the circumference in cm? *(Medium)* *[DjB '16]*
a ⓐ 31.42 ⓑ 39.33 ⓒ 78.65 ⓓ 157.08

40. What is called the length of circle? *(Hard) [DB' 15]*
ⓐ Chord ⓑ Arc
d ⓒ Diameter ⓓ Circumference

41. What is the circumference of the circle of 5 cm diameter? *(Medium)* *[RB' 15]*
ⓐ 15 cm ⓑ 15.71 cm
b ⓒ 17.7 cm ⓓ 18.7 cm

42. The diametre of a wheel of a vehicle is 38 cm. What will be the distance covered by two complete round? *(Medium)*
[Ideal School & College, Dhaka]
ⓐ 59.69 cm ⓑ 76 cm
d ⓒ 119.38 cm ⓓ 238.76 cm

43.

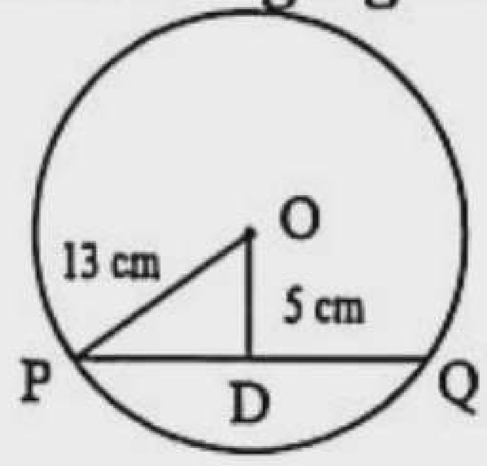


If, in figure, O is the centre of a circle with OM ⊥ AB, then— *[CB '17]*
i. diameter of the circle is AB
ii. ∠OMA = ∠OMB = 1 right angle
iii. AM = BM
Which one is correct? *(Medium)*
c ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

44. **A diameter in a circle is—** *[SB '17]*
i. the biggest chord
ii. 2 times of radius
iii. a chord which goes through the centre
Which one is correct? *(Easy)*
(d) (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

45. **The diameter of a circle is —.** *[DB' 15]*
i. the greatest chord of the circle
ii. 2 times of radius
iii. not pass through the centre
Which one of the following is true? *(Hard)*
(a) (a) i & ii (b) ii & iii (c) i & iii (d) i, ii & iii

Answer the questions No. 46 and 47 on the basis of the following figure :



[JB '19]

46. **What is the circumference of the circle?** *(Medium)*
(d) (a) 5π (b) 10π (c) 13π (d) 26π

47. **What is the length of chord PQ ?** *(Hard)*
(a) (a) 24 cm (b) 18 cm (c) 16 cm (d) 12 cm

10.4 Theorems Related to Circle ▸ Textbook Page 161

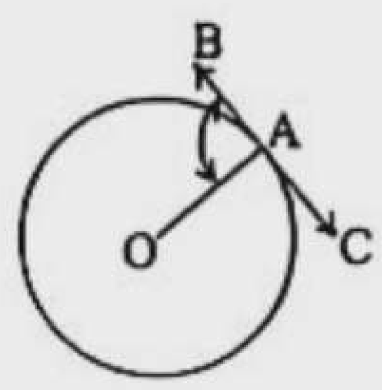
48. **The measure of a semicircular angle is — ?** *(Easy)*
(d) (a) 60° (b) 70° (c) 80° (d) 90°

49. **How many tangents can be drawn from a point outside of a circle?** *(Easy)*
(a) (a) 1 (b) 2 (c) 3 (d) 4

50. **How many tangents can be drawn from a point inside circle?** *(Medium)*
(d) (a) 1 (b) 2 (c) 3 (d) No one

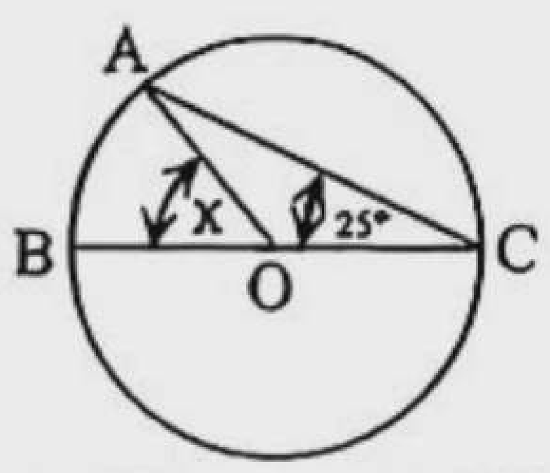
51. **At the point A of a circle with centre O, a tangent AB subtending an angle $\angle AOB$ = 60°. Then $\angle ABO$ = ?** *(Hard)*
(a) (a) 30° (b) 45° (c) 60° (d) 75°

52. **If O is the centre of the circle and BC is the tangent. Then $\angle OAB$ = ?** *(Hard)*



(a) 70° (b) 85°
(c) (c) 90° (d) 75°

53. **If O is the centre of the circle ABC. Then $\angle x$ = ?** *(Medium)*

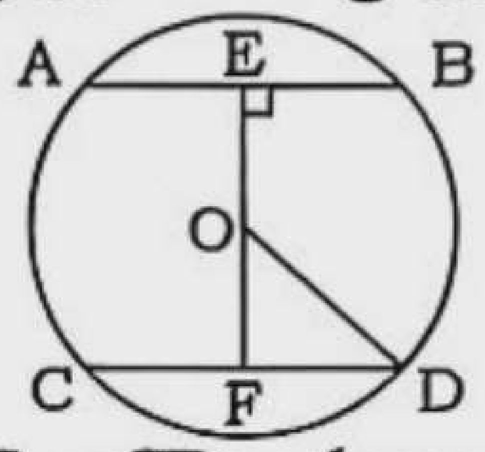


(a) (a) 50° (b) 85° (c) 65° (d) 75°

54. **How many vertices of a cyclic quadrilateral lie on the circle?** *(Easy)*
(c) (a) 2 (b) 3 (c) 4 (d) 1

55. **What is the measure of a semi-circular angle?** *(Easy)* *[DB '16]*
(d) (a) 180° (b) 120° (c) 100° (d) 90°

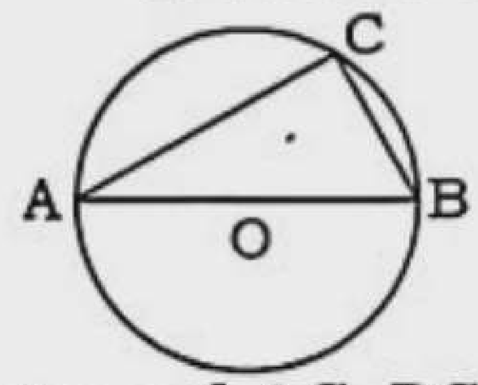
56. **In ΔABC, if AB = AC and $\angle A$ = 80° then, what is the value of $\angle B$?** *(Medium)* *[RB '16]*
(b) (a) 40° (b) 50° (c) 60° (d) 100°



In figure, AB = CD = 6 cm, OE = 4 cm and F is the mid-point of CD. O is the centre of the circle.

57. **The diameter of a circle is 6 cm. What is the circumference of the circle? (π = 3.14)** *(Hard)* *[SB '16]*
(a) 9.42 cm (b) 28.26 cm
(c) (c) 18.84 cm (d) 113.04 cm

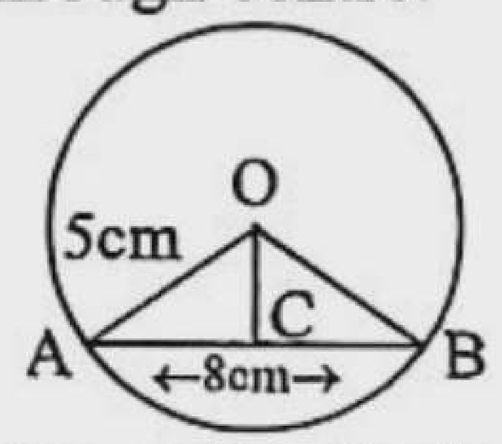
58.



If AB is a diameter and AC, BC are chords of O centered circle then which one is correct? *(Medium)* *[DjB '16]*
(a) AB > AC + BC (b) AB < AC
(d) (c) AB < BC (d) AB > AC

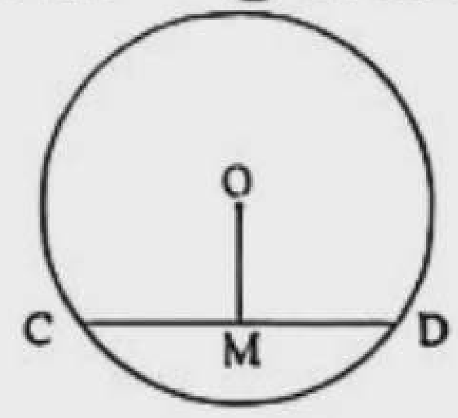
59. **In the circle with centre O, chord AB > CD then which one is correct?** *(Medium)*
[Viqarunnisa Noon School and College, Dhaka]
(a) AB is nearer to centre
(b) CD is nearer to centre.
(c) chords are equidistant from centre
(a) (d) AB passes through centre.

60.



If O is the centre of the circle and OC ⊥ AB then—. *[CB '18]*
i. $\angle OAC + \angle AOC = 90°$
ii. AC = BC
iii. Area of the ΔAOB is 12sq cm.
Which one of the following is correct? *(Hard)*
(a) (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

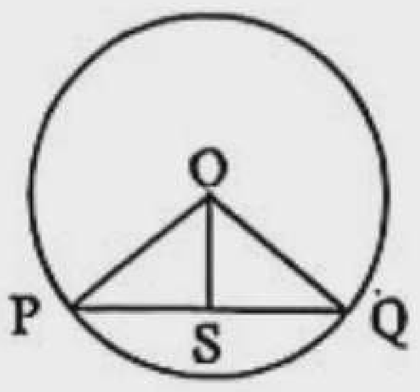
61.



[RB' 15]

In the figure, if OM ⊥ CD in the circle with centre O is —.
i. the diameter of the circle is CD
ii. $\angle OMC = \angle OMD$ = 1 right angle
iii. CM = DM
Which one of the following is true? *(Easy)*
(b) (a) i & ii (b) ii & iii (c) i & iii (d) i, ii & iii

62.



[CtgB '15]

In figure, OS ⊥ PQ

i. $PS = SQ$

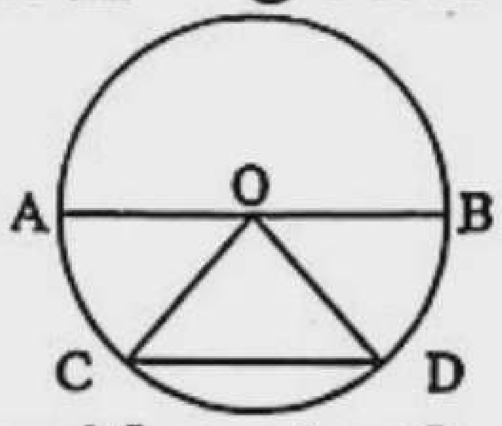
ii. $\angle OSQ = \angle OSP$

iii. $PQ \neq OQ$

Which one of the following is true? *(Medium)*

ⓓ ⓐ ii & iii ⓑ i & iii ⓒ i & ii ⓓ i, ii & iii

63.



In above circle with centre O. [Ctg.B' 15]

i. $AB > CD$

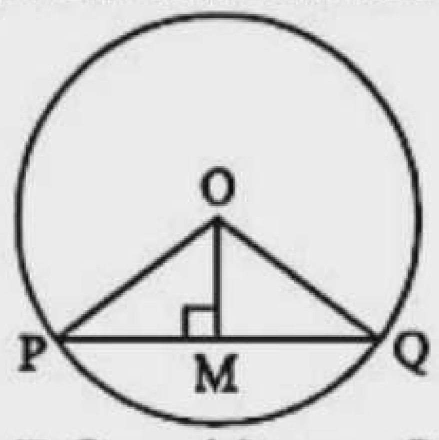
ii. $OC = OD$

iii. $AB = CD$

Which one of the following is true? *(Easy)*

ⓐ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

■ **Answer the questions no. 64 and 65 with the help of given information :**



In the figure, PQ = 10cm, OM = 6cm. [BB '19]

64. What is the value of MQ? *(Medium)*

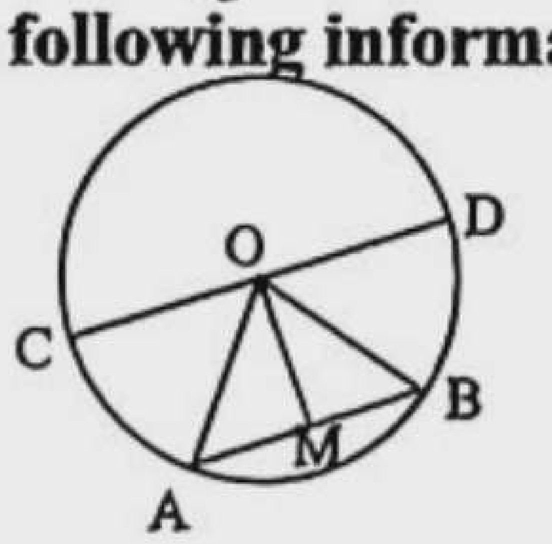
ⓑ ⓐ 3cm ⓑ 5cm ⓒ 6cm ⓓ 8cm

65. What is the area of triangle OPM? *(Hard)*

ⓐ 15 sq cm ⓑ 30 sq cm

ⓐ ⓒ 60 sq cm ⓓ 120 sq cm

■ **Answer the question no. 66 and 67 based on the following information :—.**



[BB '18]

66. In the figure, it is a circle with centre O in which —.

i. AB = diameter

ii. $OA = OC$

iii. $AB < CD$

Which one of the following is correct? *(Easy)*

ⓒ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

67. If AB||CD and OM = MB, which one of the value of ∠AOC? *(Hard)*

ⓑ ⓐ 90° ⓑ 45° ⓒ 30° ⓓ 0°

■ **Answer to the questions number 68 – 70 using the above figure :** [RB '16]

68. What is the length of BE? *(Easy)*

ⓒ ⓐ 6 cm ⓑ 4 cm ⓒ 3 cm ⓓ 2 cm

69. What is the area of ΔOFD? *(Medium)*

ⓐ 6 sq. cm ⓑ 12 sq. cm

ⓐ ⓒ 20 sq. cm ⓓ 24 sq. cm

70. What is the area of the circle? *(Hard)*

ⓐ 28.27 sq. cm ⓑ 50.27 sq. cm

ⓒ ⓒ 78.54 sq. cm ⓓ 113.10 sq. cm

O is the centre of the circle and OE ⊥ AB.

■ **Answer the questions No. 71 and 72 using the above information :** [SB '16]

71. ∠OEA = ? *(Easy)*

ⓒ ⓐ 0° ⓑ 45° ⓒ 90° ⓓ 180°

72. The radius of the circle is —.

i. AO

ii. OB

iii. AB

Which one is correct? *(Medium)*

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Mathematics

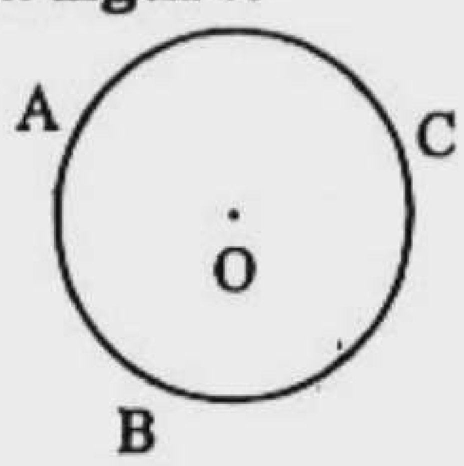
Short Q/A Designed as per topic

10.1 Circle

▸ Textbook Page 159

Question 1. Define a circle with a figure.

Solution : If any point on a curve lying in a plane is always equidistant from a fixed point within the area bounded by that curve, then the curve is called a circle.



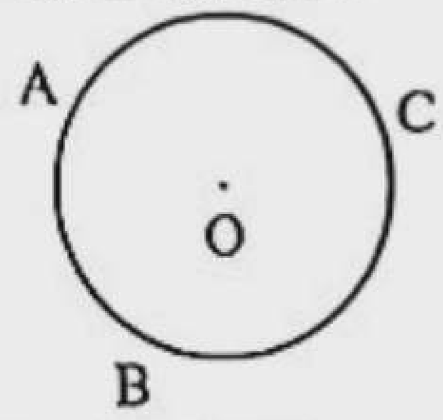
In the figure, ABC is a circle with center O.

Question 2. What is used to draw a circle perfectly?

Solution : A pencil compass is used to draw a circle perfectly. The pointed leg of the compass is pressed on the paper and the pencil attached to the other leg is rotated around to draw the circle.

Question 3. What is the center of a circle?

Solution : The point inside a circle from which all points on the circumference are equidistant is called the center of the circle.

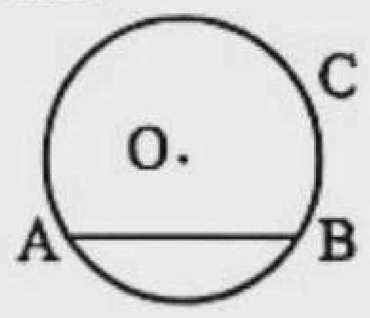


In the figure, O is the center of circle ABC.

10.2 Chord and Arc of a Circle ▸ Textbook Page 160

Question 4. What is a chord of a circle?

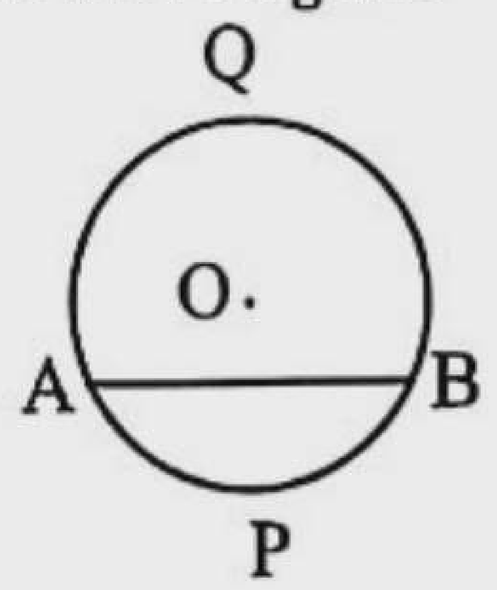
Solution : The joining line segment of any two points of a circle is the chord of the circle.



In the figure, AB is a chord of the circle ABC with center O.

Question 5. Define an arc of a circle with a figure.

Solution : Each part of a circle divided by a chord is called an arc of the circle, or arc in short.

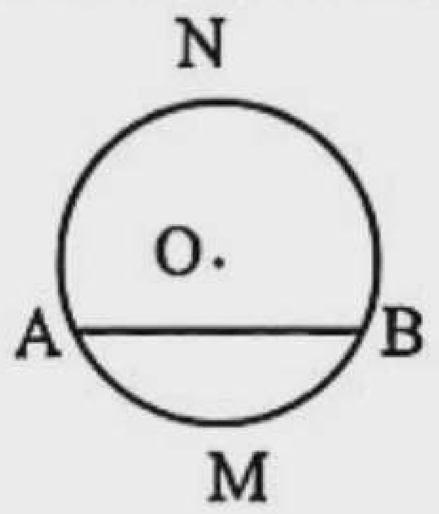


In the figure, APBQ is a circle with center O.

The arcs made by chord AB are APB and AQB.

Question 6. Into how many arcs does each chord divide a circle? Draw a figure to show it.

Solution : Each chord divides a circle into two arcs.

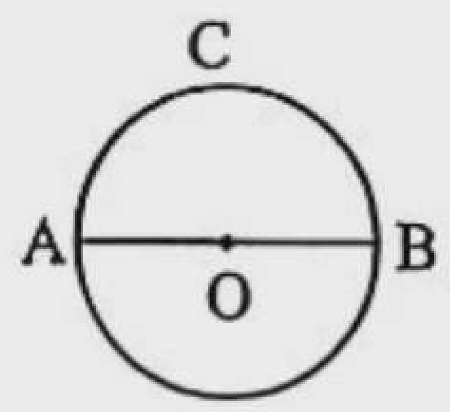


In the figure, chord AB of circle AMBN with center O has divided the circle into arcs AMB and ANB.

10.3 Diameter and Circumference ▸ Textbook Page 160

Question 7. What is a diameter?

Solution : A chord that passes through the center of a circle is called a diameter.



The diameter is the largest chord of the circle.

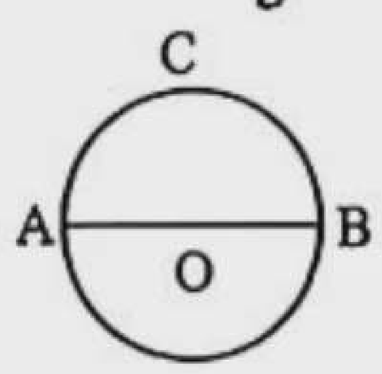
In the figure, AB is a diameter of the circle ABC with center O.

Question 8. What are the number and shape of arcs made by a diameter of a circle?

Solution : A diameter of a circle makes two arcs. The two arcs are equal to each other and each of them is a semicircle.

Question 9. Define the radius of a circle with a figure.

Solution : The fixed distance from the center of a circle to any point on the circumference is called the radius of the circle.



The radius is half of the diameter.

In the figure, OA and OB are radii of the circle ABC with center O.

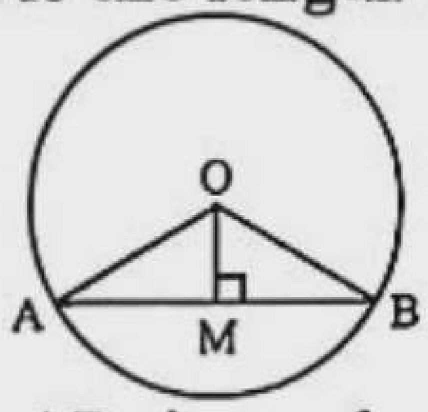
Question 10. What is the circumference of a circle?

Solution : The complete length of the circle is called its circumference.

In other words, the distance covered from any point on the circle, moving along the circle, and back to that point is the circumference.

10.4 Theorems Related to Circle ▸ Textbook Page 161

Question 11. In the figure, if AB = 24 cm and OM = 5 cm, what is the length of OA in cm?



Solution : Here, AB is a chord other than the diameter in the circle with center O, and OM and $OM \perp AB$.

AB = 24 cm, OM = 5 cm

$\therefore \quad AM = \frac{1}{2}AB = \frac{1}{2} \times 24 = 12$ cm

In triangle Δ AOM, $\angle OMA$ = is a right angle.

$\therefore \quad OA^2 = OM^2 + AM^2$

or, $OA^2 = 5^2 + (12)^2 = 25 + 144 = 169$

or, $OA = \sqrt{169} = 13$ cm

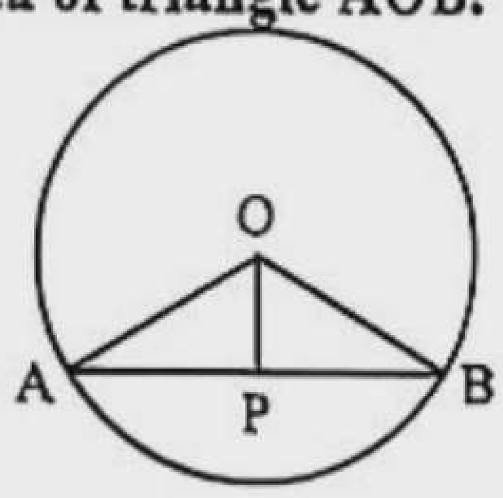
$\therefore \quad OA = 13$ cm

Question 12. In the circle with center O, AB is a chord other than the diameter and $OP \perp AB$. If AB = 16 cm and OP = 6 cm, find the area of triangle AOB.

Solution : Here, in the circle with center O, AB is a chord other than the diameter and $OP \perp AB$.

AB = 16 cm and OP = 6 cm

Join O, A and O, B



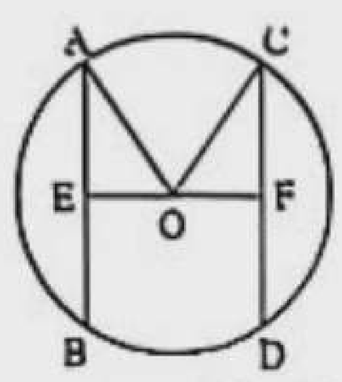
$\therefore$ Area of triangle Δ AOB

$$= \frac{1}{2} \times AB \times OP$$

$$= \frac{1}{2} \times 16 \times 6 = 48 \text{ sq. cm}$$

The area of triangle Δ AOB sq. cm

Question 13. In the figure, if AB = CD = 8 cm and EF = 6 cm, what is the length of OC in cm?



Solution : Here, in the circle with center O, AB = CD = 8 cm and EF = 6 cm.

Since AB = CD. $OE = OF = \frac{1}{2}EF = \frac{1}{2} \times 6 = 3$ cm

and $CF = \frac{1}{2}CD$ [$\therefore$ Since, the perpendicular drawn from the center of the circle to a chord other than the diameter bisects that chord]

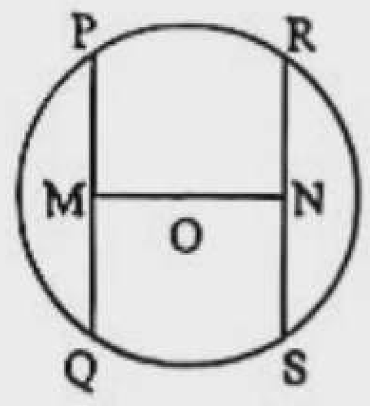
$$= \frac{1}{2} \times 8 \text{ cm} = 4 \text{ cm}$$

In triangle Δ COF, $OC^2 = OF^2 + CF^2 = 3^2 + 4^2 = 9 + 16 = 25$

$\therefore \quad OC = \sqrt{25} = 5$ cm

Therefore, OC = 5 cm

Question 14. In the figure, if OM = ON and PQ = 12 cm, what is the length of SN?



Solution : Since, OM = ON, PQ = RS

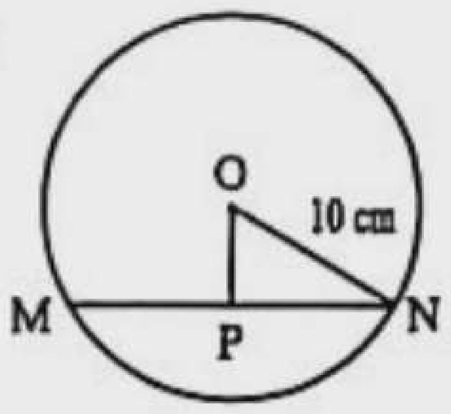
$\therefore$ RS = PQ = 12 cm

Again, the perpendicular drawn from the center of the circle to a chord other than the diameter bisects that chord. That is, RN = SN

$\therefore$ $SN = \frac{1}{2}RS = \frac{1}{2} \times 12 = 6$ cm

Therefore, SN = 6 cm

Question 15.



Since, OP $\perp$ MN and MN = 16 cm, OP = ?

Solution : Since OP $\perp$ MN, PM = PN

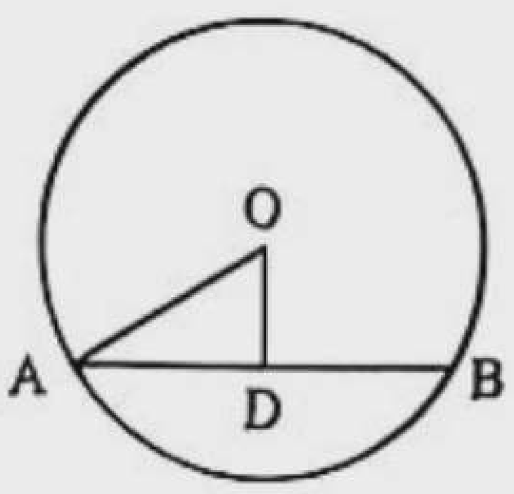
$\therefore$ $PM = PN = \frac{1}{2}MN = \frac{1}{2} \times 16 = 8$ cm

Now, in triangle, Δ PON, $OP^2 + PN^2 = ON^2$

or, $OP^2 = ON^2 - PN^2 = 10^2 - 8^2 = 100 - 64 = 36$

$\therefore$ $OP = \sqrt{36} = 6$ cm

Question 16.



In the figuure, if OA = 8 cm, OD = 6 cm, what is the length of AB?

Solution : Here, OD $\perp$ AB

$\therefore$ AD = BD

In triangle Δ AOD, $OD^2 + AD^2 = OA^2$

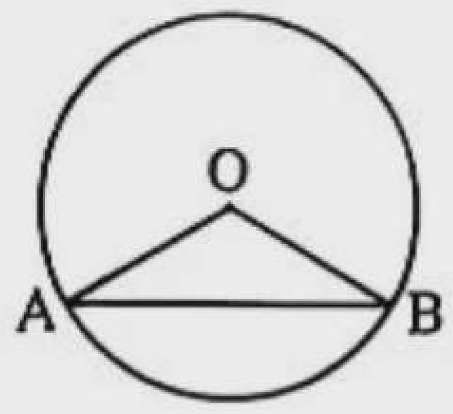
or, $AD^2 = OA^2 - OD^2 = 8^2 - 6^2 = 64 - 36 = 28$

$\therefore$ $AD = \sqrt{28} = 2\sqrt{7}$ cm

Now, $AB = 2AD = 2 \times 2\sqrt{7} = 4\sqrt{7}$ cm

Therefore, $AB = 4\sqrt{7}$ cm

Question 17. In the figure, if O is the center and $\angle AOB = 120^\circ$, what is the measure of $\angle OAB$?



Solution : Here, OA = OB [radius of the same circle]

$\therefore$ $\angle OAB = \angle OBA$

[Angles opposite to equal sides of a triangle are equal]

Now, in triangle Δ AOB, $\angle OAB + \angle OBA + \angle AOB = 180^\circ$ [Sum of the angles of a triangle is 180°]

or, $\angle OAB + \angle OAB + 120^\circ = 180^\circ$

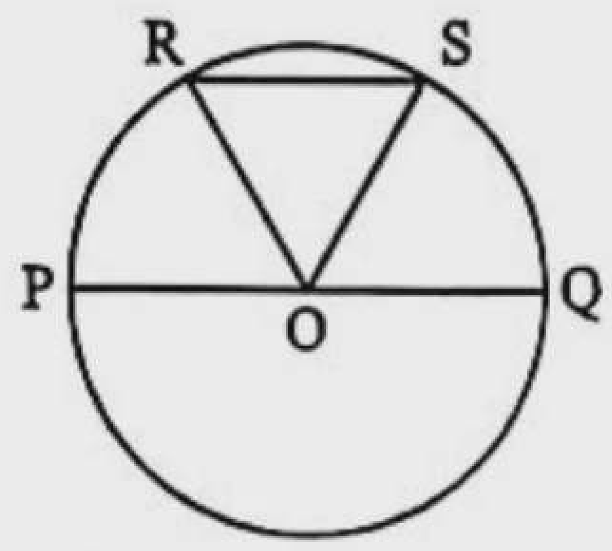
[$\because \angle OBA = \angle OAB$ and $\angle AOB = 120^\circ$]

or, $2\angle OAB = 180^\circ - 120^\circ = 60^\circ$

$\therefore$ $\angle OAB = \frac{60^\circ}{2} = 30^\circ$

Therefore, $\angle OAB = 30^\circ$.

Question 18.



In the figure, O is the center of the circle, OR = OS = RS, and PQ || RS. Find the yalue of $\angle POR$.

Solution : In triangle Δ ORS, OR = OS = RS

$\therefore$ Triangle Δ ORS is an equilateral triangle.

$\therefore$ $\angle ORS = 60^\circ$

Again, PQ || RS and OR is a transversal

$\therefore$ $\angle POR$ = alternate $\angle ORS = 60^\circ$

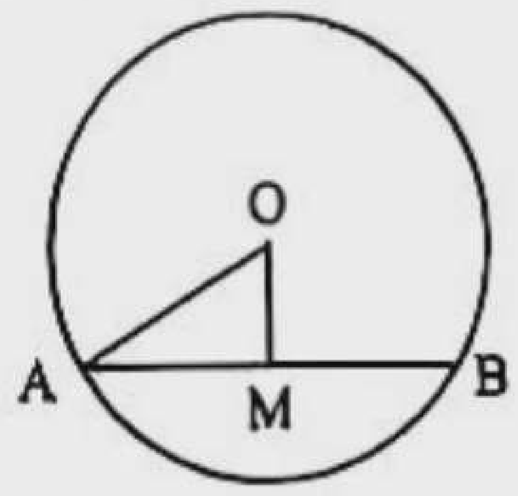
The value of $\angle POR$ is 60°.

Question 19. If the length of a chord of a circle is 6 cm and its radius is 4 cm, what is the length of the perpendicular drawn from the center to the chord in cm?

Solution : Let the chord AB in the circle with center O be 6 cm and the radius OA = 4 cm.

So, OM $\perp$ AB

So, $AM = BM = \frac{1}{2}AB = \frac{1}{2} \times 6$ cm = 3 cm



$\therefore$ $OM^2 + AM^2 = OA^2$

or, $OM^2 = OA^2 - AM^2 = 4^2 - 3^2 = 16 - 9 = 7$

$\therefore$ $OM = \sqrt{7}$

$\therefore$ The length of the perpendicular drawn from the center to the chord is $\sqrt{7}$ cm.

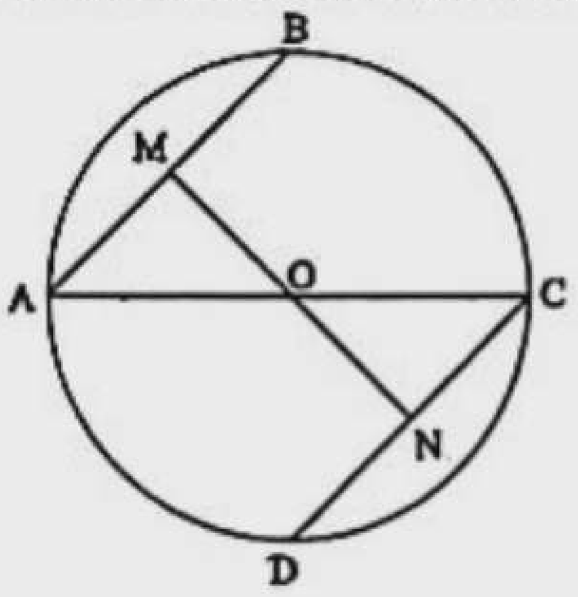
Creative Q/A — Designed as per learning outcomes

Ques. 01 "O is the centre of the circle ABCD and AC is a diameter. AB and CD are two parallel chords situated opposite sides of AC. Two perpendiculars OM and ON to the chords AB and CD respectively are drawn."

a. Now according to the given data draw geometric figure and give a brief description. *[Easy]* 2
b. Prove that, AB = CD. *[Medium]* 4
c. If AB > CD, prove that OM < ON. *[Hard]* 4

Solution to Question No. 01 :

a According to given data, a circle is drawn below where O is the centre of the circle ABCD and AC is a diameter. AB and CD are two parallel chords situated on opposite sides of AC. Two perpendiculars OM and ON to the chords AB and CD respectively are drawn from O.

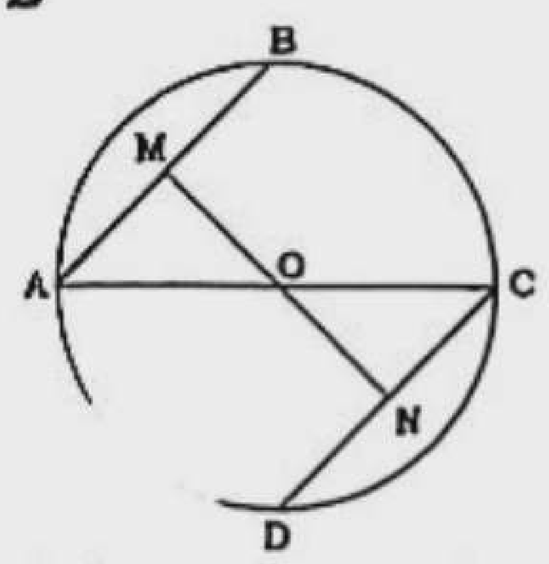


b Let us prove that, AB = CD.

Proof : Since O is the centre of the circle and OM $\perp$ AB, then we get,

$AM = \frac{1}{2} AB$.

Similarly, $CN = \frac{1}{2} CD$.



Now in right angled ΔOAM and ΔOCN, we get,

$\angle OMA = \angle ONC$ [$\because$ each of them is one right angle]

$\angle OAM = \angle OCN$ [$\because$ alternate angles where AB||BC]

and, $\angle AOM = \angle CON$ (vertically opposite angles)

$\therefore$ Both the triangles are similar

That is why, $\frac{AM}{CN} = \frac{OA}{OC}$

$\Rightarrow \frac{AM}{CN} = 1$

(OA = OC, as the radii of same circle)

$\Rightarrow AM = CN$.

$\therefore \frac{1}{2} AB = \frac{1}{2} CD$

i.e. AB = CD. **(Proved)**

c In the figure shown in (b), O is the centre of the circle ABCD. AB and CD are two chords where AB > CD. OM and ON are the perpendiculars from centre O to the chords AB and CD respectively. (O, A) and (O, C) are joined. Let us prove that, OM < ON.

Proof : From (b) we get, $AM = \frac{1}{2} AB$. and $CN = \frac{1}{2} CD$.

Now in right angled ΔOAM, OA is the hypotenuse,

$\therefore OA^2 = OM^2 + AM^2$ ... ... (i)

Again, in right angled ΔONC, OC is the hypotenuse,

$\therefore OC^2 = ON^2 + CN^2$ ... ... (ii)

But, OA and OC are the radii of the same circle. So, OA = OC. Then from equations (i) and (ii) we get,

$OM^2 + AM^2 = ON^2 + CN^2$.

or, $AM^2 - CN^2 = ON^2 - OM^2$ ... ... (iii)

But given that AB > CD

$\therefore \frac{1}{2} AB > \frac{1}{2} CD$

or, $AM > CN$

or, $AM^2 > CN^2$

or, $AM^2 - CN^2 > 0$

$\therefore ON^2 - OM^2 > 0$ [From equation (iii)]

or, $ON^2 > OM^2$

$\therefore ON > OM$

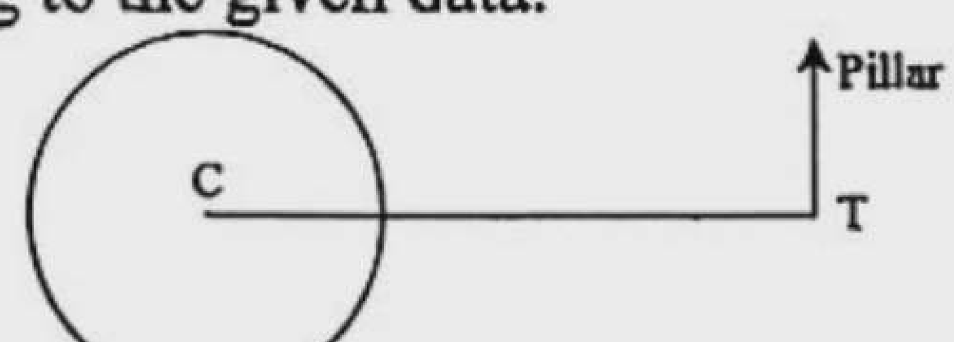
i.e. OM < ON. **(Proved)**

Ques. 02 C is the centre of a circle with radius 3 cm. T is the foot-point of a pillar 10 cm away from the centre.

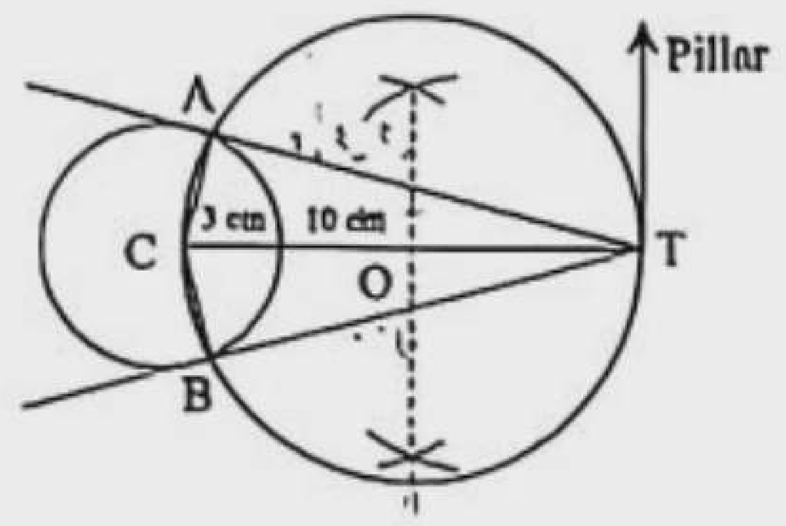
a. Now draw the geometric figure according to the given information. *[Easy]* 2
b. Draw two tangents from the foot-point of the pillar to the circle and show that the points of contact are at equal distance from the foot-point of the pillar. *[Medium]* 4
c. By considering the chord of contact of the tangents as the side of an equilateral triangle, prove that the tangents at the vertices of the triangle form an equilateral triangle. *[Hard]* 4

Solution to Question No. 02 :

a C is the centre of the circle with radius 3 cm. T is the foot-point of a pillar 10 cm away from the centre C. The geometric figure is drawn below according to the given data.



b We have to draw two tangents from T to the circles with centre C and we have to prove that these two tangents are equal i.e. AT = BT.



Construction : (C, T) are joined. Let us determine the mid–point O of CT. By taking radius OC or OT and with centre O let us draw a circle which passes through the centre C and interests the circle with centre C at A and B. Then, A, T and B, T are joined. AT and BT are the required tangents. (A, C) and (B, C) are joined.

Proof : Since, CA and CB are the radii passing through the points of contact,

$\therefore \angle CAT = 1$ right angle $= \angle CBT$.

Now, in right ΔCTA and ΔCTB we get,

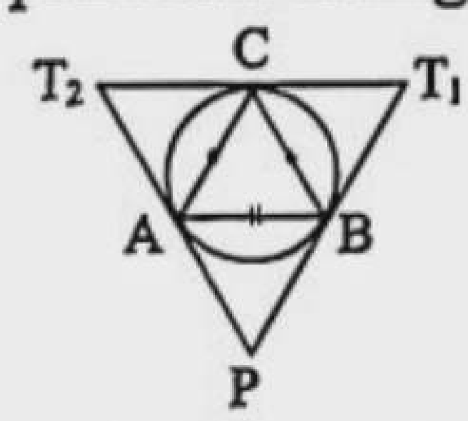
CA = CB [Radii of the same circle]

Hypotenuse CT = Hypotenuse CT.

Since the hypotenuse and one side of the right triangles are equal, hence the two triangles are congruent.

$\therefore$ AT = BT. **(Proved)**

c Let us consider that AB is the chord of contact. An equilateral ΔABC is inscribed the circle with centre O. Three tangents PQ, QR and PR are drawn at the vertices A, C and B of ΔABC respectively. Let us prove that, the tangents at the vertices of the ΔABC form an equilateral triangle i.e. ΔPQR is an equilateral triangle.



Proof : Since ΔABC is an equilateral triangle then we get, $\angle A = \angle B = \angle C = 60°$.

Now $\angle PAB$ = the angle $\angle B$, the alternate segment = 60°.

And $\angle ABP$ = the angle $\angle A$, the alternate segment = 60°.

In ΔABP, $\angle ABP = \angle PAB = 60°$.

$\therefore \angle P = 60°$.

Similarly we can show that, $\angle Q = \angle R = 60°$.

Since the three angles of the ΔPQR are 60° then it is an equilateral triangle.

Ques. 03 PQRS is a circle with centre O, AB is the diametre. PQ and RS are two chords other than diametre of the circle where PQ > RS.

a. Determine the area of circle of which radius is 20 cm. *[Easy]* 2

b. Prove that, PM = QM. *[Medium]* 4

c. Prove that, the chord PQ is nearer to the centre O than the chord RS. *[Hard]* 4

● *Dinajpur Board 2019*

Solution to Question No. 03 :

a We know that,

area of a circle having radius 'r' unit $= \pi r^2$ sq. unit

Here, radius of the circle = 20 cm

$\therefore$ Area of the circle $= \pi \times 20^2$ sq. cm

$= 1256$ sq. cm.

$\therefore$ The determined area of the circle is 1256 sq. cm.

b Let, PQ is a chord of a circle with centre O. OM ⊥ PQ. We have to prove that, PM = QM.

Construction : O, P and O, Q are joined .

Proof : Since, OM ⊥ PQ,

so ΔOPM and ΔOQM are right angled triangles.

Now, in right angled ΔOPM and ΔOQM,

hypotenuse OP = hypotenuse OQ [radius of same circle]

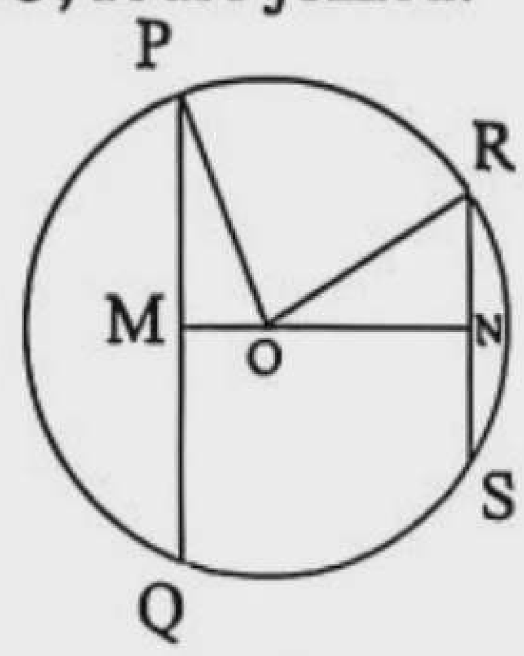
OM = OM [common side]

$\therefore \Delta OPM \cong \Delta OQM$

$\therefore$ PM = QM **[Proved]**

c Let us have a circle PQRS with centre O and PQ and RS are two chords other than diametre such that, PQ > RS. We have to prove that, the chord PQ is nearer to the centre O than the other chord RS.

Construction : OM ⊥ PQ and ON ⊥ RS are drawn. O, P and O, R are joined.



Proof : In right angled triangle POM, $\angle OMP = 90°$

$\therefore OP^2 = PM^2 + OM^2$ ···· (i) [By Pythagoras's theorem]

Again, in right angled triangle RON, $\angle ONR = 90°$

$\therefore OR^2 = RN^2 + ON^2$ ····· (ii) [By Pythagoras's theorem]

Since, OP and OR are the radii of the same circle,

hence, OP = OR

or, $OP^2 = OR^2$

or, $PM^2 + OM^2 = RN^2 + ON^2$ [From equation (i) and (ii)]

or, $\left(\frac{1}{2}PQ\right)^2 + OM^2 = \left(\frac{1}{2}RS\right)^2 + ON^2$ [$\because$ OM ⊥ PQ $\therefore$ $PM = \frac{1}{2}PQ$ and $\because$ ON ⊥ RS $\therefore$ $RN = \frac{1}{2}RS$]

or, $\frac{1}{4}PQ^2 + OM^2 = \frac{1}{4}RS^2 + ON^2$

or, $PQ^2 + 4\,OM^2 = RS^2 + 4\,ON^2$

or, $PQ^2 - RS^2 = 4\,ON^2 - 4\,OM^2$ ················ (iii)

Mathematics

Here, $PQ^2 - RS^2 > 0$, Since $PQ - RS > 0$ (Given)

$\therefore$ From (iii), we get,

$4\,ON^2 - 4\,OM^2 > 0$

or, $ON^2 - OM^2 > 0$

or, $ON^2 > OM^2$

or, $ON > OM$ .... (iv)

The inequality (iv) refers to that RS is farther than PQ from O. That is, the chord PQ is nearer to the centre O than the chord RS.

Ques. 04 In a circle with centre O, CM and PS are two chords other than diametre and their mid-points are X and Y respectively.

a. Find the length of OC, when CM = 16 cm. OX = 6 cm. *[Easy]* 2

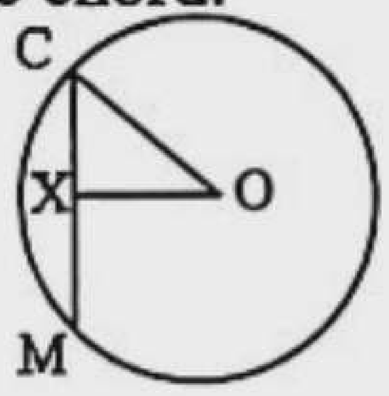
b. Prove that, $OX \perp CM$. *[Medium]* 4

c. If $CM > PS$, then prove that, $OX < OY$. *[Hard]* 4

● *Sylhet Board 2019*

Solution to Question No. 04 :

a In a circle with centre O, CM is any chord. X is the midpoint of CM. O, X and O, C are joined. Since the line segment joining the centre of a circle to the midpoint of a chord other than diametre, is perpendicular to the chord.



So, $OX \perp CM$.

Given, CM = 16 cm and OX = 6 cm

$\therefore CX = \frac{16}{2}$ cm = 8 cm[$\because$ X is the midpoint of CM]

Now, in right angled triangle OCX, we get,

$OC^2 = CX^2 + OX^2$ [By Pythagoras's theorem]

or, $OC = \sqrt{CX^2 + OX^2}$

or, $OC = \sqrt{8^2 + 6^2}$ [$\because$ CX = 8, OX = 6]

$\therefore$ OC = 10

$\therefore$ Determined length of OC is 10 cm.

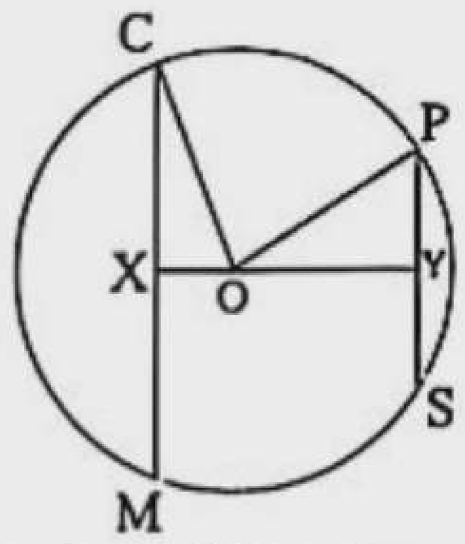
b Let CM be a chord other than diametre of a circle with centre O and O is joined to the midpoint X of CM. It is to be proved that, OX is perpendicular to CM.

Construction : Join O, C and O, M.

Proof :

1. In ΔOCX and ΔOMX, CX = MX OC = OM and OX = OX Therefore, $\Delta OCX \cong \Delta OMX$ $\therefore \angle OXC = \angle OXM$	[X is the midpoint of CM] [radius of same circle] [common side]
2. Since the two angles are equal and make a straight angle $\angle OXC = \angle OXM$ = 1 right angle. Therefore, $OX \perp CM$. **(Proved)**	[SSS theorem]

c Let, CMSP is a circle with centre O and CM, PS are two chords of the circle such that $CM > PS$. X and Y are the midpoints of CM and PS. O, X and O, Y are joined. It is required to prove that, $OX < OY$.



Construction : C, O and P, O are joined.

Proof : In ΔOCX and ΔOPY, we get,

OC = OP, since they are the radii of the same circle

$CX > PY$, since $CM > PS \Rightarrow \frac{CM}{2} > \frac{PS}{2}$

Now from right triangle COX, we get,

$OC^2 = OX^2 + CX^2$ ---------------- (i)

Again from right triangle POY, we get,

$OP^2 = OY^2 + PY^2$ -------------------- (ii)

since OC = OP, hence

$OX^2 + CX^2 = OY^2 + PY^2$

or, $CX^2 - PY^2 = OY^2 - OX^2$

Here, $CX^2 - PY^2 > 0$ hence $CX > PY$

So, $OY^2 - OX^2 > 0$

or, $OY^2 > OX^2$

or, $OY > OX$.

$\therefore OX < OY$ **(Proved)**

Ques. 05

In the figure, EF = GH

a. The perimeter of a circular garden is 94.2m. Find its radius. *[Easy]* 2

b. In the light of the stem, prove that OM = ON. *[Medium]* 4

c. If EF and GH chords intersect at the point P inside the circle, then prove that, EP = HP and GP = FP. *[Hard]* 4

● *Cumilla Board 2018*

Solution to Question No. 05 :

a We know, perimeter of a circle with radius r unit is $2\pi r$ unit.

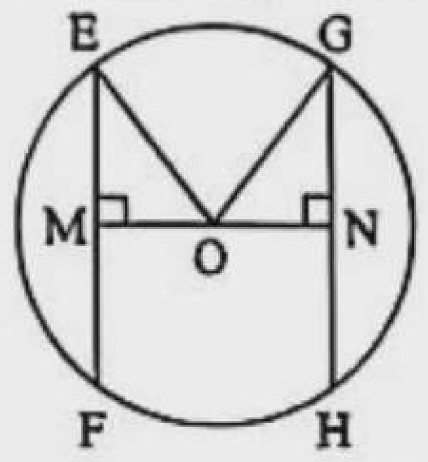
Here, $2\pi r = 94.2 \Rightarrow r = \frac{94.2}{2\pi} \Rightarrow r = 15$

$\therefore$ The radius of the circle is 15 m.

b According to the stem, let EFGH is a circle with centre O. EF and GH are two chords and OM, ON are perpendiculars to EF and GH respectively from O. Besides, EF = GH.

Now, it is required to prove that OM = ON.

Construction : O, E and O, G are joined.



Proof : $EM = \frac{1}{2}EF$ and $GN = \frac{1}{2}GH$, since $OM \perp EF$ and $ON \perp GH$.

Again, Since EF = GH (given),

EM = GN.

$\angle EMO = \angle GNO$ (given)

OE = OG, since they are the radii of the same circle.

$\therefore \Delta EOM \cong \Delta GON$

$\therefore$ OM = ON **(Proved)**

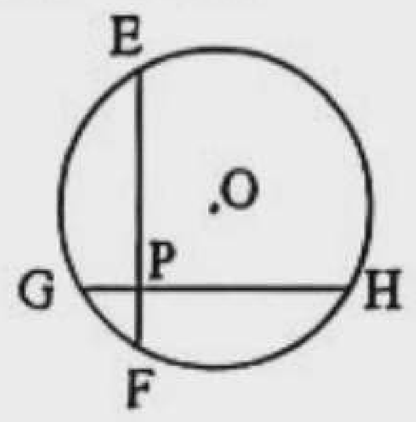
c Let, EF and GH are two equal chords of a circle which mutually interseet at P, a point inside the circle. Now, it is required to prove that EP = HP and GP = FP.

Proof : Chord EF = Chord GH (given)

$\therefore$ Arc EGF = Arc GFH,

Since the two chords EF, GH are equal and as such they stand on equal arcs of the same circle.

Now, Arc EGF = Arc GFH



$\Rightarrow$ Arc EG + Arc GF = Arc GF + Arc FH

$\Rightarrow$ Arc EG = Arc FH $\Rightarrow$ EP = PH .... (1)

Again, We have,

EF = GH

$\Rightarrow$ EP + PF = GP + PH

$\Rightarrow$ EP + PF = GP + EP, from (1),

$\Rightarrow$ PF = GP .... (2)

Thus from (1) and (2), EP = PH and GP = FP **(Proved).**

Ques. 06 AB is diameter and CD is chord other than diameter of circle with centre O.

a. Find out the area of a circular field with diameter 6.4 metre. *[Easy]* 2

b. Prove that AB > CD. *[Medium]* 4

c. If E is the midpoint of CD, then prove that $OE \perp CD$. *[Hard]* 4

● *Cumilla Board 2017*

Solution to Question No. 06 :

a We know that, area of a circle

$= \pi r^2$, where r = radius of the circle

$= \pi\left(\frac{d}{2}\right)^2$, where d = diameter of the circle

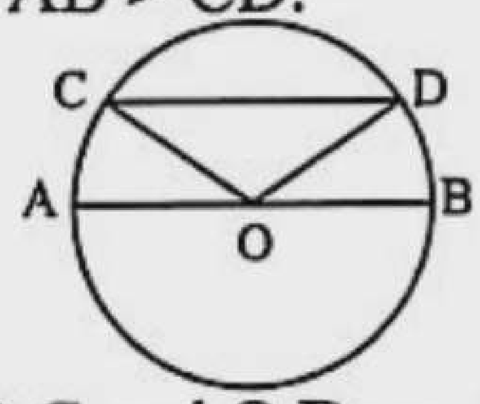
$= \pi\left(\frac{6.4}{2}\right)^2$ sq. m, putting d = 6.4 m

$= 3.14 \times (3.2)^2$ sq. m

= 32.15 sq. m (approx.)

$\therefore$ The area of the circular field with diameter 6.4 m is 32.15 sq. m.

b Let AB is a diameter and CD is a chord other than a diameter of a circle with centre O. Now it is to be proved that AB > CD.



Construction : O,C and O,D are joined.

Proof : In the figure,

OA = OC = OD = OB, since they are the radii of the same circle with centre O.

Now from ΔCOD, we get,

OC + OD > CD

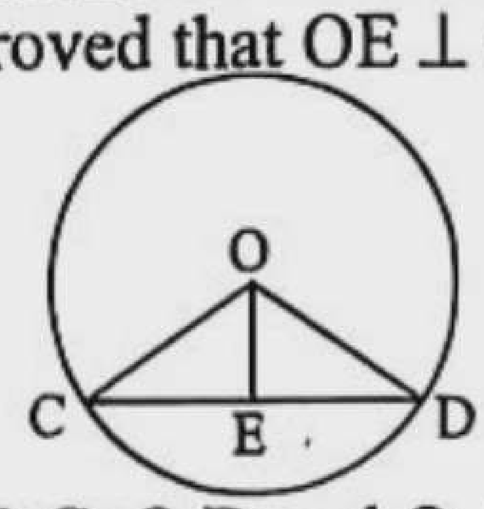
or, OA + OB > CD, since OC = OA and OD = OB

or, AB > CD

$\therefore$ AB > CD. **(Proved)**

c Let, CD is a chord of a circle with centre O. E is the mid-point of the chord CD.

Now it is to be proved that $OE \perp CD$.



Construction : O,C; O,D and O, E are joined.

Proof : In ΔCOE and ΔDOE, OC = OD, since they are the radii of the same circle with centre O.

CE = DE, since E is the mid-point of CD. OE is common to both the triangles.

$\therefore \Delta COE \cong \Delta DOE$

$\therefore \angle CEO = \angle DEO$

But the straight angle CED = 180°

$\therefore \angle CEO + \angle DEO = 180°$

or, $2\angle CEO = 180°$

or, $\angle CEO = 90°$

$\therefore OE \perp CE$ at E

$\therefore OE \perp CD$. **(Proved)**

Ques. 07 O is the centre and AB and CD are two chords of the circle ABCD.

a. If chord AB is diameter of the circle ABCD, show that AB > CD. *[Easy]* 2

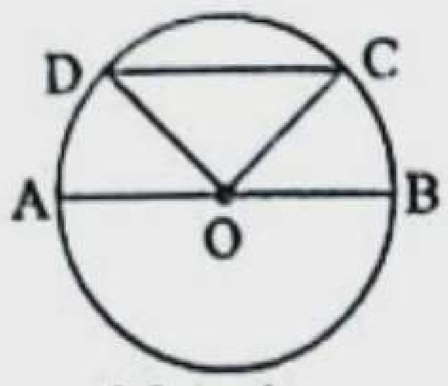
b. If AB = CD, show that the distances of O from AB and CD are equal. *[Medium]* 4

c. If AB bisects CD, prove that two parts of chords AB are equal to the two parts of CD. *[Hard]* 4

▸ *Rajuk Uttara Model College, Dhaka*

Solution to Question No. 07 :

a Let, O be the centre of the circle ABCD. Let, AB be the diameter and CD be a chord other than diameter of the circle. It is required to prove that AB > CD.



[∵ the sum of two sides is greater than the third side of a triangle.]

Construction : Join O, C and O, D.

Proof : OA = OB = OC [radius of the same circle]

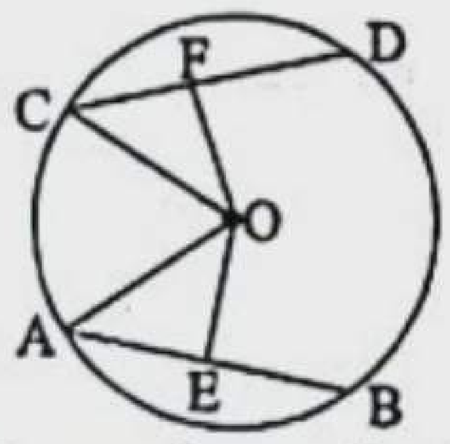
Now, in ΔOCD,

$OC + OD > CD$

or, $OA + OB > CD$

Therefore, $AB > CD$

b **Proposition :** Let AB and CD be two equal chords of a circle with the centre O. It is to be proved that the chords AB and CD are equidistant from the centre.



Construction : Draw from O, the perpendiculars OE and OF to the chords AB and CD respectively. Join 0,A and 0,C.

Steps	Justification
(1) $OE \perp AB$ and $OF \perp CD$ Therefore, AE = BE and CF = DF.	[Perpendicular from the centre bisects the chord]
$\therefore AE = \frac{1}{2}AB$ and $CF = \frac{1}{2}CD$	[supposition]
(2) But AB = CD	[radius of same circle] [Step 2]
$\therefore \frac{1}{2}AB = \frac{1}{2}CD$ $\therefore AE = CF$.	[RHS theorem]
(3) Now between the right-angled ΔOAE and ΔOCF hypotenuse OA = hypotenuse OC and AE = CF. $\Delta OAE = \Delta OCF$ OE = OF.	
(4) But OE and OF are the distances from O to the chords AB and CD respectively. Therefore, the chords AB and CD are equidistant from the centre of the circle. (Proved)	

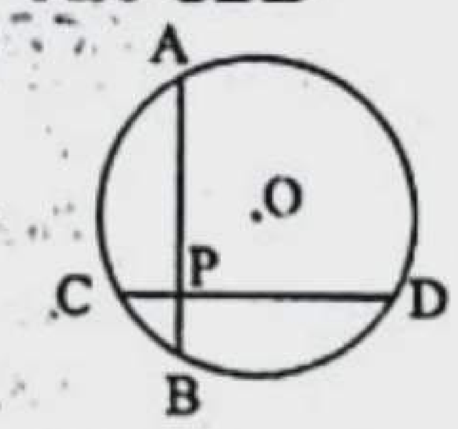
c Let, AB and CD are two equal chords of a circle which mutually intersect at P, a point inside the circle. Now, it is required to prove that AP = DP and CP = BP.

Proof : Chord AB = Chord GH (given)

∴ Arc ACB = Arc CBD,

Since the two chords EF, GH are equal and as such they stand on equal arcs of the same circle.

Now, Arc ACB = Arc CBD



⇒ Arc AC + Arc CB = Arc CB + Arc BD

⇒ Arc AC = Arc BD ⇒ AP = PD .... (1)

Again, We have,

AB = CD ⇒ AP + PB = CP + PD

⇒ AP + PB = CP + AP, from (1),

⇒ PB = CP .... (2)

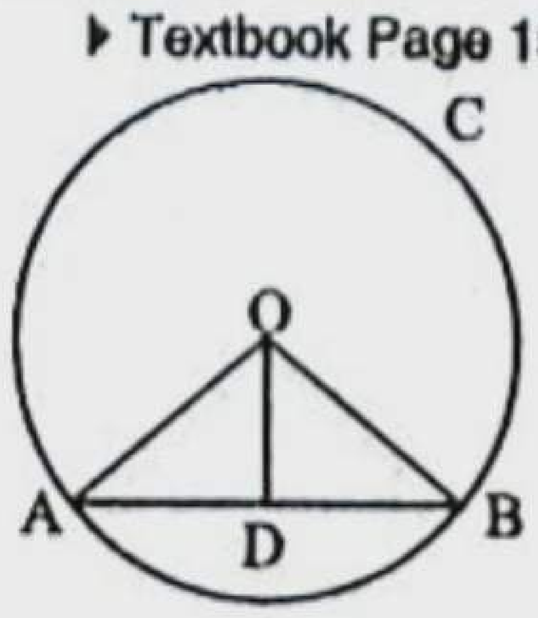
Thus from (1) and (2), AP = PD and CP = BP **(Proved).**

Solutions to Textual Activities — Along with textual reference

Activity 01 **Prove that the perpendicular from the centre circle to a chord bisects the chord.**

▸ Textbook Page 150

Solution : Proposition : Let, AB is a chod in the circle ABC with centre O and OD is the perpendicular drawn on the chord AB from centre O. It is to be proved that, OD bisects AB chord at point D i.e., AD = BD.



Construction : Join O, A and O, B.

Proof :

Steps	Justification
(1) $OD \perp AB$ So, $\angle ODA = \angle ODB$ Therefore, Δ ODA and Δ ODB both are right angled triangle.	[Both are right angle]
(2) Now, In right angled Δ ODA and Δ ODB, Hypotenuse OA = Hypotenuse OB	[Radius of same circle]
and OD = OD	[Common side]
So, $\Delta ODA \cong \Delta ODB$	[RHS theorem]
Therefore, AD = BD. **(Proved)**	

Exercise 10.2 : Theorems Related to Circle

At a Glance Important Contents of Exercise

- **Chord and diameter of a circle :** A straight line joining any two points on a circle is called a chord of a circle. What is the largest centroidal chord of the circle?
- The diameter of the circle is called The center of the circle divides the diameter of the circle into two equal parts. That is, the diameter of a circle is twice the radius of that circle.
- The line connecting the midpoint of a chord with the center and diameter of a circle is perpendicular to that chord.
- A perpendicular bisector of any chord of a circle is the centroid.
- Any straight line cannot intersect a circle in more than two points.
- If two chords of a circle bisect each other, their point of intersection is the center of the circle.
- The straight line joining the midpoints of two parallel chords is centripetal and perpendicular to the chords.
- All equal chords of a circle are equidistant from the center.
- All chords equidistant from the center of a circle are equal to each other.
- The diameter of the circle is the largest chord.
- Midpoints of all chords of a circle are congruent.
- Of the two chords of the circle, the larger chord is closer to the center than the smaller chord.

Solutions to Exercise Problems — Let's solve the textbook problems

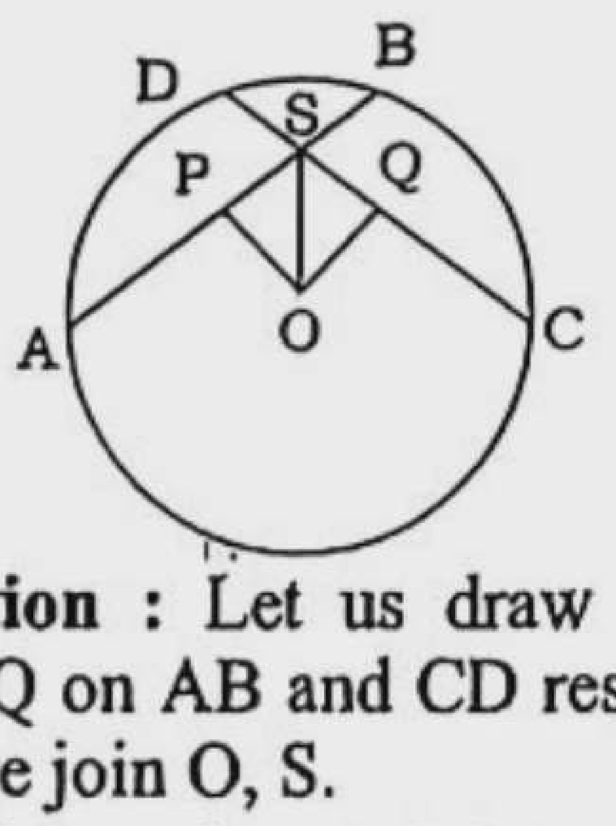
Solutions to Geometrical Problems

1. **If two equal chords of a circle intersect within the circle, prove that the segments of one chord are equal to corresponding segments of the other chord.**

Solution :

General Enunciation : If two equal chords of a circle mutually intersect, then the two parts of one chord are equal to the two corresponding parts of the other chord.

Particular Enunciation : Let us suppose, ACBD is any circle with centre O and AB and CD are two equal chords of the circle. Let us also suppose, AB and CD mutually intersect at S. We shall have to prove that AS = CS and SB = SD.



Construction : Let us draw perpendiculars OP and OQ on AB and CD respectively from O. Then we join O, S.

Proof : Comparing Δ POS and ΔQOS, we have, OP = OQ, since the distance of two equal chords from the centre is equal. OS is common to both the triangles.

And $\angle$OPS = 90° = $\angle$OQS, according to construction.

$\therefore$ Δ POS and Δ QOS are congruent.

So, PS = QS .... (i)

Again, AP = PB, since the perpendicular from the centre on any chord of a circle bisects the chord.

Similarly, CQ = QD.

So, AP = BP = CQ = QD, since AB = CD according to proposition.

Thus we have,

AP = CQ.

Or, AP + PS = CQ + PS, adding PS on both sides.

Or, AP + PS = CQ + QS, since from (i) PS = QS.

Or. AS = CS.

Further we have,

PB = QD

or, PS + SB = QS + SD.

or, PS + SB = PS + SD, since from (i) PS = QS.

or, SB = SD, eliminating PS from both sides.

So, it is proved that AS = CS and SB = SD.

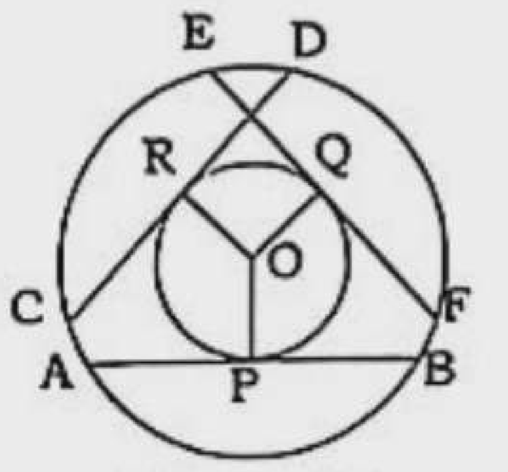
2. **Prove that the bisecting points of equal chords lie on a circle.**

Solution :

General Enunciation : The bisecting-points of equal chords lie on a circle.

Particular Enunciation : Let us suppose that ABFDEC is any circle with centre at O. AB, CD and EF are three chords of the circle so that AB = CD = EF. We shall have to prove that the mid-points of AB, CD and EF lie on the circumference of a circle.

Mathematics



Construction : From O, perpendiculars OP, OQ and OR on AB, EF and CD respectively are drawn. Now, P, Q and R are respectively mid-points of AB, EF and CD.

Proof : According to proposition, AB = CD = EF.

∴ The perpendiculars from O to AB, CD and EF will be equal to one another.

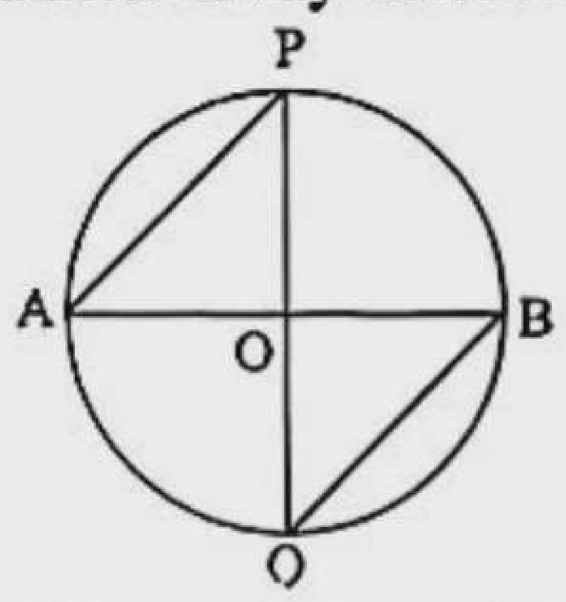
That is, Perpendicular OP = Perpendicular OQ = Perpendicular OR.

Again, since OP = OQ = OR, hence the circumference of the circle with centre O and radius OP or OQ or OR will contain P, Q and R. Thus it is proved that the mid-points of the equal chords of a circle lie on the same circle.

3. Show that equal chords drawn from the end points on opposite sides of a diameter are parallel.

Solution :

General Enunciation : The equal chords drawn in opposite directions from the end points of a diameter of any circle are mutually parallel.



Particular Enunciation : Let us suppose that APBQ is any circle with centre at O. AP and BQ are two equal chords of the circle drawn in opposite directions from the end points A and B of the diameter AB. We shall have to show that AP || BQ.

Construction : Let us join O, P and O, Q.

Proof : From ΔAOP and ΔBOQ, we have, OA = OB and OP = OQ, since OA, OB, OP and OQ are the radii of the same circle APBQ. And AP = BQ, according to proposition.

∴ ΔAOP ≅ Δ BOQ.

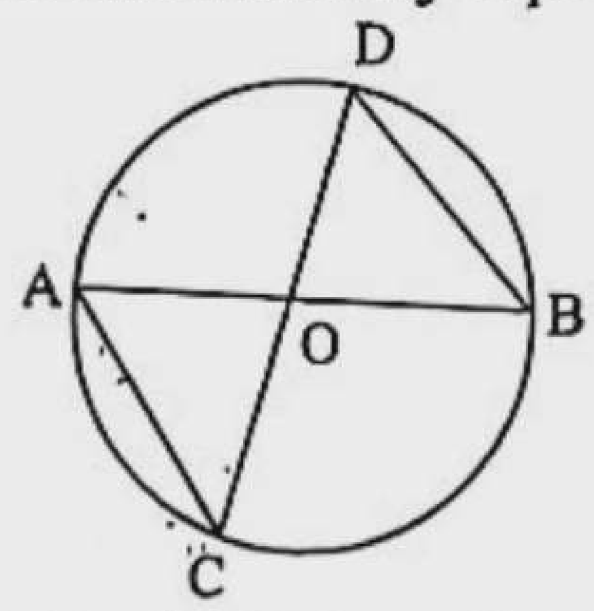
So, ∠PAO = ∠QBO.

Again, since ∠PAO = ∠QBO and AB is the transversal of AP and BQ, hence ∠PAO and ∠QBO are alternate angles to each other. Therefore shown that AP and BQ are mutually parallel. **(Shown)**

4. Show that parallel chords drawn from the end points of a diameter are equal.

Solution :

General Enunciation : The two chords drawn parallel to each other and in opposite directions from the end points of a diameter of any circle are mutually equal.



Particular Enunciation: Let us suppose that ACBD is a circle with the centre at O and AB is a diameter. AC and BD are two parallel chords of the circle drawn in opposite directions from A and B respectively.

We shall have to show that chord AC = chord BD.

Construction : O, C and O, D are joined.

Proof : From ΔAOC and ΔBOD, we have, OA = OB and OC = OD, since OA, OB, OC. and OD are radii of the same circle.

Again, AC || BD according to proposition, and AB and CD are their transversals.

∴ ∠OAC = ∠OBD and ∠OCA = ∠ODB, since they are alternate angles.

∴ ∠AOC = ∠BOD.

∴ ΔAOC and ΔBOD are similar triangles.

So, $\frac{AC}{BD} = \frac{OA}{OB}$

∴ $\frac{AC}{BD} = 1 (OA = OB)$

Therefore, AC = BD.

Thus showed that AC = BD.

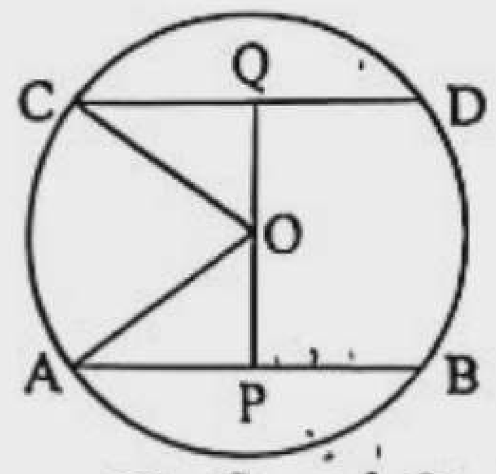
5. Prove that between the two chords the larger one is nearer to the centre than the smaller one.

Solution :

General Enunciation : Between two unequal chords of a circle, the larger chord is nearer to the centre than the smaller one.

Particular Enunciation : Let us, Suppose that ABDC is any circle with centre at O. AB and CD are two chords of the circle as such that AB > CD.

Let, OP and OQ are the perpendicular distances of AB and CD respectively from O. We shall have to show that OQ > OP.



Construction : O, C and O, A are joined.

Proof : According to proposition, AB > CD.

Or, $\frac{AB}{2} > \frac{CD}{2}$

Or, AP > CQ, since OP bisects AB and OQ bisects CD.

Now from right angled triangle AOP, we have, $OA^2 = OP^2 + AP^2$ .... (i)

Again, from right angled triangle COQ, we have, $OC^2 = OQ^2 + CQ^2$ .... (ii)

Since OA and OC are mutually equal for they are the radii of the same circle.

$\therefore$ From (i) and (ii) we have,

$OP^2 + AP^2 = OQ^2 + CQ^2$.

But AP > CQ as per proposition mentioned earlier.

$\therefore$ $OP^2 < OQ^2$

Or, OP < OQ.

Or, OQ > OP. **(Proved)**

Creative Questions with Solutions

Ques. 06 In a circle with centre 'O', PQ and RS are two equal chords and their mid-points are M and N respectively.

a. Find out the radius of the circle with area 314 sq. cm. 2

b. Prove that, OM = ON. 4

c. If the chords PQ and RS bisect each other, prove that the two parts of one chord are equal to the two parts of the other. 4

Solution to Question No. 06 :

a Let the radius of the circle be r cm.

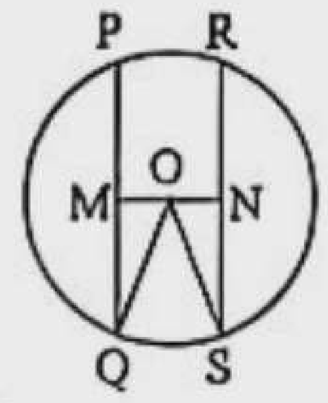
$\therefore$ $\pi r^2 = 314$ (given)

or, $r^2 = \frac{314}{\pi} = 100$

or, $r = \sqrt{100} = 10$

So, the required radius is 10 cm.

b Let PQSR is a circle with centre O and M, N are the mid-points of the chords, PQ and RS respectively. Now it is required to prove that OM = ON.



Construction : O, Q and O, S are joined.

Proof : In ΔMOQ and ΔNOS,

OQ = OS, since they are the radii of the same circle.

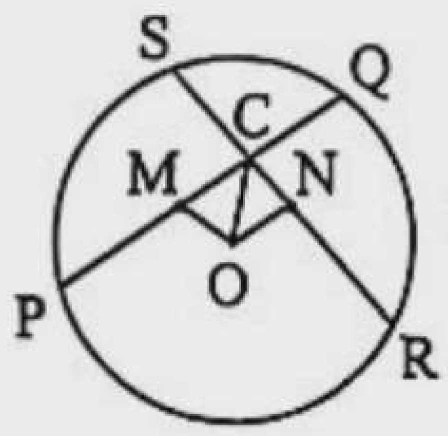
QM = SN, since PQ = RS and M is mid point of PQ and N is mid point of RS.

$\angle QMO = \angle SNO$, since $OM \perp PQ$ and $ON \perp RS$.

$\therefore$ $\Delta MOQ \cong \Delta NOS$.

$\therefore$ OM = ON **(Proved)**

c Let PRQS is a circle with centre O where chord PQ = chord RS and they mutually intersect at C, an interior point of the circle. Now it is required to prove that CS = CQ and PC = RC.



Construction : $OM \perp PQ$ and $ON \perp RS$ are drawn from O. O, C are joined.

Proof : PM = QM, since perpendicular to any chord from centre bisects the chord.

RN = NS, for the same reason stated above.

But PQ = RS and M, N are mid points of PQ and RS respectively.

$\therefore$ PM = MQ = RN = NS ....(i)

Again in ΔMOC and ΔNOC, OM = ON, OC is common to both of then and $\angle OMC = \angle ONC = 90°$.

$\therefore$ ΔMOC and ΔNOC are congruent.

$\therefore$ MC = NC ....(ii)

Now PM = RN from (i)

$\Rightarrow$ PM + MC = RN + MC

$\Rightarrow$ PM + MC = RN + NC, since MC = NC from (ii)

$\Rightarrow$ PC = RC ....(1)

Again, MQ = NS from (i)

$\Rightarrow$ MQ – MC = NS – MC

$\Rightarrow$ MQ – MC = NS – NS, since MC = NC from (ii)

$\Rightarrow$ CQ = CS ....(2)

$\therefore$ From (1) and (2) it is proved that PC = RC and CS = CQ.

Multiple Choice Q/A Designed as per topic

10.4 Theorems Related to Circle ▸ Textbook Page 161

1. **What is called the angle whose vertex is at the centre of the circle?** *(Easy)*

ⓐ Central angle ⓑ Semi-circular angle

ⓒ Inscribed angle ⓓ None of the above

ⓐ

2. **In the case of semi-circle, what is called the angle formed at the centre?** *(Easy)*

ⓐ Right angle ⓑ Straight angle

ⓒ Radian angle ⓓ None of the above

ⓑ

3. **If the measure of an inscribed angle at the circumference be 35°, what will be the measure of the central angle against it?** *(Medium)*

ⓐ 90° ⓑ 70° ⓒ 80° ⓓ 60°

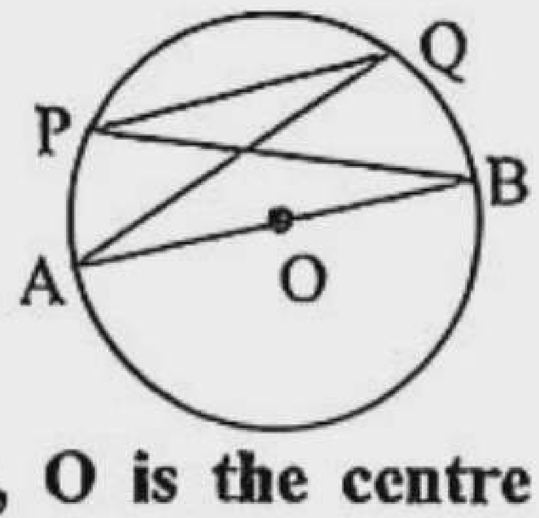
ⓑ

4. **What type of angle is subtended on the major arc of a circle?** *(Easy)*

ⓐ Obtuse angle ⓑ Acute angle

ⓒ Right angle ⓓ None of the above

ⓐ

5. 

In the figure, O is the centre of the circle. Which one is the greatest chord? *(Medium)* *[DjB '19]*

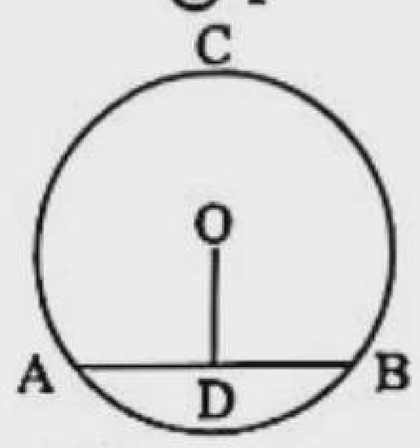
ⓓ ⓐ PQ ⓑ BP ⓒ AQ ⓓ AB

6. **Which one of the following is the angle subtended at the centre of a circle?** *(Medium) [DjB '18]*

ⓓ ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

7. **What is the ratio of the circumference and diameter of a circle?** *(Easy) [DB '16]*

ⓓ ⓐ $2\pi r$ ⓑ $2r$ ⓒ r ⓓ π

8. 

ABC is a circle with centre O. OD ⊥ AB. AB = 16 cm and OD = 6 cm. Then what is the radius of the circle in cm? *(Medium) [CB' 15]*

ⓐ ⓐ 10 ⓑ 14 ⓒ 17 ⓓ 22

9. **If an acute angle is 30° of a right angled triangle, Which one is the value of other acute angle?** *(Hard) [Ctg.B' 15]*

ⓒ ⓐ 45° ⓑ 90° ⓒ 60° ⓓ 30°

10. **How many maximum points does a straight line intersect a circle?** *(Hard) [SB' 15]*

ⓒ ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4

11. **Maximum in how many points a straight line can intersects a circle?** *(Easy) [DjB' 15]*

ⓑ ⓐ 1 ⓑ 2 ⓒ 4 ⓓ infinite

12. i. Any two circles having congruent diameters have congruent radii.
ii. A diameter divides a circle into two arcs of equal length.
iii. If a point lies inside a circle, its distance from the centre is less than radius.

What is the correct answer after the above statements? *(Easy)*

ⓓ ⓐ i & iii ⓑ ii & iii ⓒ i & ii ⓓ i, ii & iii

13. i. Concentric circles are circles with the same centre.
ii. A line segment which has both endpoints on a circle is called a chord of the circle.
iii. If two chords of a circle are congruent, their minor arcs are congruent.

What is the correct answer after the above statements? *(Medium)*

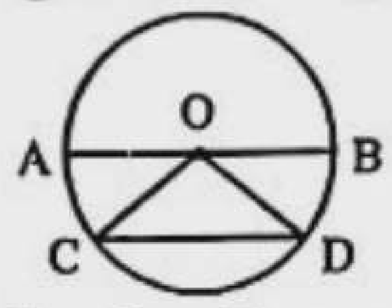
ⓓ ⓐ i & iii ⓑ ii & iii ⓒ iii ⓓ i, ii & iii

14. 

In the figure, O is the centre and OD ⊥ QR. If OP = 5 cm and OD = 3 cm, then— *[DjB '19]*
i. PQ = 10 cm
ii. QR = 8 cm
iii. OR = 4 cm

Which one is correct? *(Medium)*

ⓐ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

15. 

In the figure, circle with centre O–
i. AB diameter
ii. OC = OD
iii. AB > CD

What is the correct answer after the above statements? *(Medium)*

ⓓ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

16. 

In the figure, O is centre and AB = CD then–
i. AB and CD are two equal chord
ii. Radius AO = CO
iii. OP = OQ

What is the correct answer after the above statements? *(Medium)*

ⓑ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

Using the following information answer the question No. 17 and 18 :

In figure, OM = ON = 6 cm, RS = 16 cm and N is the midpoint of RS. Centre of the circle is 0. *[Rajuk Uttara Model College, Dhaka]*

17. **What is the area of ΔONS?** *(Medium)*

ⓐ 6 sq. cm. ⓑ 12 sq. cm.

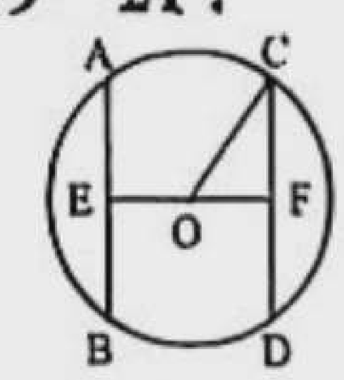
ⓒ ⓒ 24 sq. cm. ⓓ 48 sq. cm.

18. **What is the area of the circle?** *(Hard)*

ⓐ 31.416 sq. cm. ⓑ 113.097 sq. cm

ⓓ ⓒ 201.06 sq. cm ⓓ 314.16 sq. cm.

Using the following information answer the question No. 19 – 21 :



In the figure AB = CD and OE ⊥ AB, OF ⊥ CD. *[Viqarunnisa Noon School & College, Dhaka]*

19. **Which one is correct?** *(Easy)*

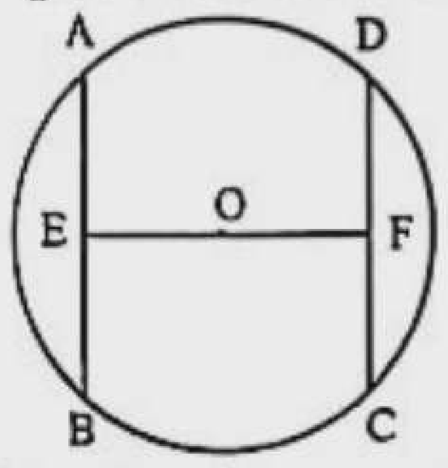
ⓐ OE = AE ⓑ OE = AB

ⓒ ⓒ AE = BE ⓓ OF > OF

20. In CF = 3 cm, OF = 4 cm then, OC = ? *(Hard)*
(c) ⓐ 3.5 cm ⓑ 4.5 cm ⓒ 5 cm ⓓ 5.5 cm

21. Which one is correct? *(Medium)*
ⓐ OC = OF ⓒ OE = OF
(b) ⓒ OC = OE ⓓ OF = CF

■ **Using the following information answer the question No. 22 and 23 :**



In the figure, AB chord, OE ⊥ AB and OF ⊥ CD. *[Rajuk Uttara Model College, Dhaka]*

22. Which one is correct? *(Easy)*
(a) ⓐ OE = OF ⓑ OC = OF
ⓒ OF = CF ⓓ OC = OE

23. If CF = 3 cm the, is how much cm? *(Medium)*
(c) ⓐ 5 ⓑ 4 ⓒ 6 ⓓ 3

■ AB and CD are two equal chords which intersect at E of the circle with centre O.

Using the following information answer the question No. 24 and 25 :

24. If AE = 5 cm, then CE = how much cm? *(Easy)*
(d) ⓐ 4.5 ⓑ 5.5 ⓒ 6 ⓓ 5

25. If AB = 7 cm and AE = 5 cm then, DE = ? *(Easy)*
(a) ⓐ 2 ⓑ 1.5 ⓒ 1 ⓓ 2.5

Short Q/A Designed as per topic

10.4 Theorems Related to Circle ▸ Textbook Page 161

Question 1. In a circle, AB and CD are two equal chords, and the distance from the center to AB is 5 cm. Find the distance from the center to CD.

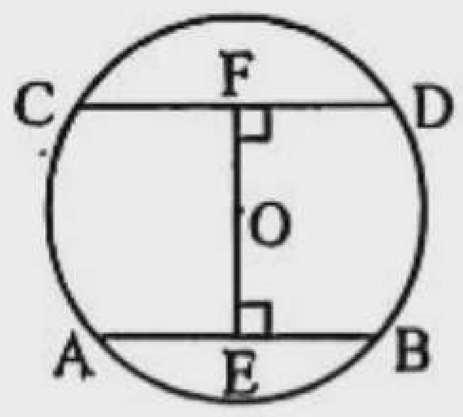
Solution : Here, the chords AB and CD of the circle are equal to each other; and the distance from the center to AB is 5 cm.

Since all equal chords of a circle are equidistant from the center

So, the distance from the center to CD = the distance from the center to AB = 5 cm.

∴ The distance from the center to CD is 5 cm.

Question 2. In the figure, AB and CD are two equal chords other than the diameter in the circle with center O. The perpendicular distances from the center O to the chords AB and CD are OE = 3 cm and OF, respectively. Find the length of OF.



Solution : Here, in the circle with center O, AB and CD are two equal chords other than the diameter. The perpendicular distance from the center O to AB is OE = 3 cm, and the perpendicular distance from the center O to CD is OF.

Proof : Chords (other than the diameter) AB and CD are equal.

∴ OE = OF = 3 cm. [Since all equal chords of a circle are equidistant from the center]

∴ The length of OF is 3 cm.

Question 3.

In the figure, AB and CD are two equal chords other than the diameter in the circle with center O. If AM = 5 cm and ON = 4 cm, what is the length of OA in cm?

Solution : Here, in the circle with center O, AM = 5 cm and ON = 4 cm.

Since AB and CD are equal chords, the perpendicular distances from the center to the chords are equal.

∴ OM = ON

or, OM = 4 cm [∵ ON = 4 cm]

Now, in right-angled triangle AOM,

$$OA^2 = AM^2 + OM^2$$
$$= 5^2 + 4^2$$
$$= 25 + 16 = 41$$

∴ $OA = \sqrt{41} = 6.4$ cm (approx.)

Therefore, OA = 6.4 cm (approx.).

Question 4. In a circle, PQ and RS are two chords other than the diameter. The distances from the center to the chords are equal. If PQ = 15 cm, find the length of RS.

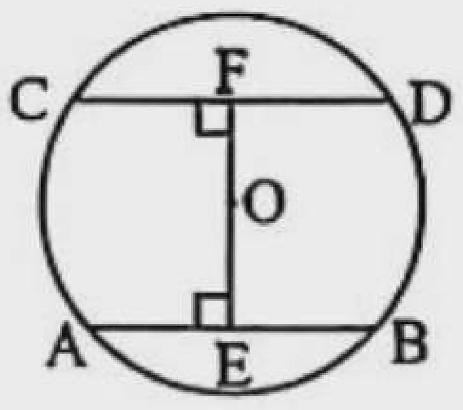
Solution : Here, the distances from the center of the circle to the chords PQ and RS, which are other than the diameter, are equal, and PQ = 15 cm.

Since all chords equidistant from the center of a circle are equal to each other

∴ RS = 15 cm. [Since PQ = 15 cm]

∴ The length of RS is 15 cm.

Question 5. In the figure, AB and CD are two chords other than the diameter in the circle with center O. OE ⊥ AB and OF ⊥ CD. If OE = OF and CD = 12 cm, what is the length of AB in cm?



Solution : Here, in the circle with center O, the perpendicular distances from the center to the chords AB and CD, which are other than the diameter, are OE and OF, respectively, and OE = OF.

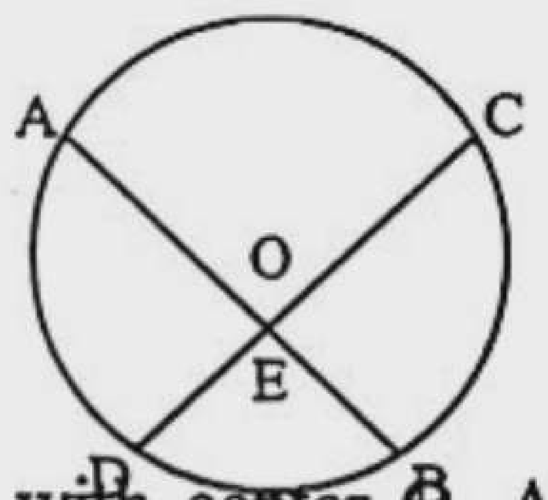
∴ AB = CD [∴ Since all chords equidistant from the center of a circle are equal to each other]

or, AB = 12 cm [∵ CD = 12 cm]

The length of AB is 12 cm.

Question 6. In the circle with center O, AB and CD are two equal chords that intersect each other at point E. If AB = 7 cm and AE = 5 cm, what is the length of DE?

Solution :



In the circle with center O, AB and CD are two equal chords that intersect each other at point E.

∴ The segments of one chord are equal to the corresponding segments of the other chord.

∴ AE = CE and BE = DE

Here, AB = 7

or, AE + BE = 7 [∵ AB = AE + BE]

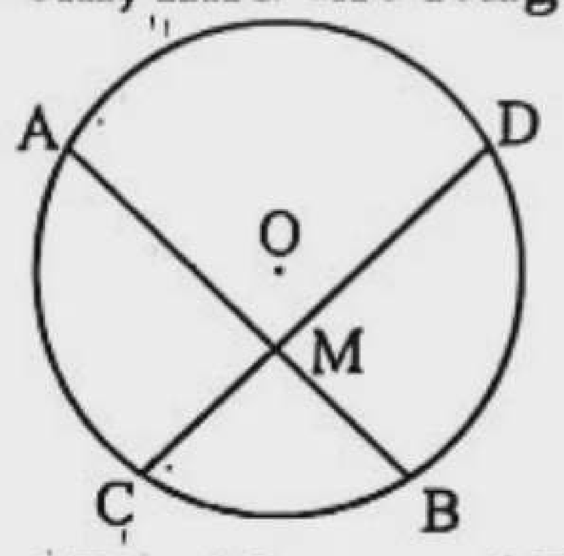
or, 5 + BE = 7

or, BE = 7 – 5 = 2

∴ DE = BE = 2 cm

Therefore, DE = 2 cm

Question 7. In the figure, AB and CD are two equal chords in the circle with center O, intersecting each other at point M. If AB = 10 cm and DM = 6 cm, find the length of BM.



Solution : Here, AB = 10 cm and DM = 6 cm.

The chords AB and CD intersect each other at point M.

∴ AM = DM (since the segments of one chord are equal to the corresponding segments of the other chord when two equal chords of a circle intersect each other).

or, AM = 6 cm

Now, BM = AB – AM

= (10 – 6) cm = 4 cm

The length of BM is 4 cm.

Question 8. In a circle, two parallel chords AB and CD are drawn from the two ends of a diameter on opposite sides. If CD = 7 cm, what is the length of AB?

Solution : Here, two parallel chords AB and CD are drawn from the two ends of a diameter on opposite sides of the circle, and CD = 7 cm.

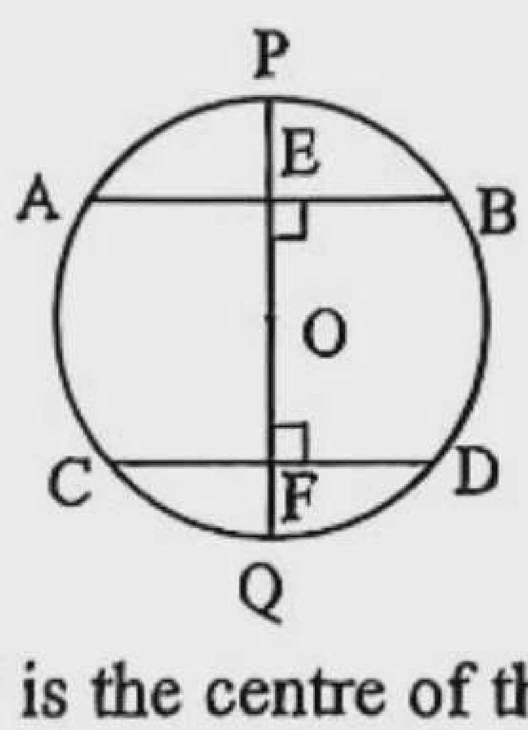
Since parallel chords drawn from the two ends of a diameter on opposite sides of a circle are equal to each other.

Then, the length of chord AB = the length of chord CD = 7 cm.

Therefore, AB = 7 cm.

Creative Q/A — Designed as per learning outcomes

Ques. 01



In the figure 'O' is the centre of the circle and PQ is the diametre.

a. If the radius of the circle is 4 cm, then find its circumference. *[Easy]* 2

b. Prove that, PQ > CD. *[Medium]* 4

c. If AB > CD, then prove that, OE < OF. *[Hard]* 4

● *Dhaka Board 2019*

Solution to Question No. 01 :

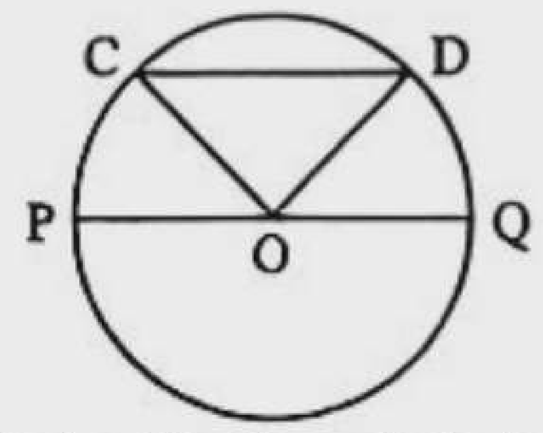
a Given that; radius of the circle = 4 cm.

We know, circumference of a circle having radius, r = 2πr unit.

∴ Circumference of the circle = 2.π.4 cm.

= 8π cm.

b Let O be the centre of the circle PQDC. Let PQ be the diameter and CD be a chord other than diameter of the circle. It is required to prvoe that, PQ > CD.



Construction : Join O, C and O, D.

Proof : $OP = OQ = OC = OD$

[radius of the same circle]

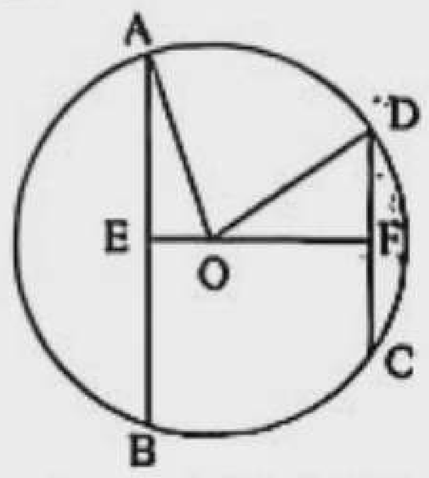
Now, in ΔOCD,

$OC + OD > CD$ [$\because$ the sum of two sides is greater than the third side of a triangle]

or, $OP + OQ > CD$

Therefore, $PQ > CD$. **(Proved)**

c Let ABCD is a circle with centre O where AB and CD are two chords such that $AB > CD$. OE and OF are two perpendiculars from O to the chords AB and CD respectively. It is required to prove that $OE < OF$.



Construction : O, A and O, D are joined.

Proof : Since OE and OF are two perpendiculars from O to AB and CD respectively,

ΔAOE and ΔDOF are two right angles where $\angle AEO$ and $\angle DFO$ are both right angles.

Now from right ΔAOE,

$OA^2 = OE^2 + AE^2$ ------ (i), according to the theorem of Pythagoras

Again, from right ΔDOF,

$OD^2 = OF^2 + DF^2$ ----- (ii), according to the theorem of Pythagoras

Since OA and OD are the radii of the same circle, hence,

$OE^2 + AE^2 = OF^2 + DF^2$

or, $AE^2 - DF^2 = OF^2 - OE^2$

But $AE^2 - DF^2 > 0$, since $AB > CD \Rightarrow AE > DF$

$\therefore$ $OF^2 - OE^2 > 0$

or, $OF^2 > OE^2$

or, $OF > OE$

or, $OE < OF$. **(Proved)**

Ques. 02 AB and CD are two chords other than diametre of circle ABCD with centre O. P and Q are two mid-points of AB and CD respectively.

a. Determine the radius of a circle of which the circumference is 44 cm. *[Easy]* 2
b. If AB is a diametre, prove that, $AB > CD$. *[Medium]* 4
c. If $AB = CD$, prove that, $PO = QO$. *[Hard]* 4

• *Cumilla Board 2019*

Solution to Question No. 02 :

a Let, radius of the circle = r cm

$\therefore$ circumference of the circle = $2\pi r$ am.

According to question,

$2\pi r = 44$

or, $r = \frac{44}{2\pi}$

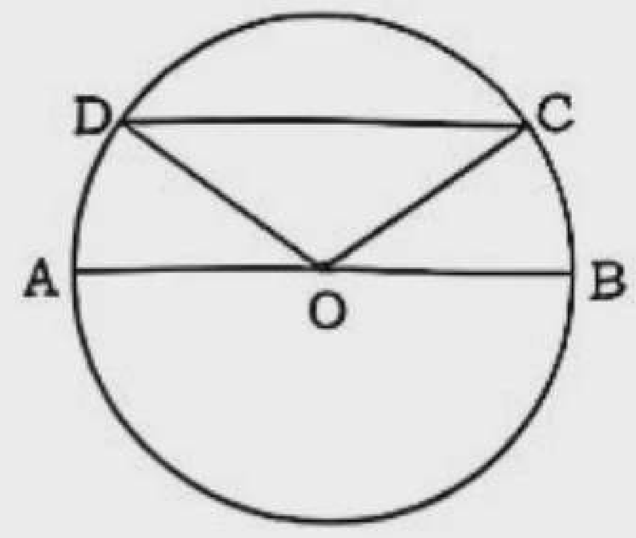
or, $r = \frac{44}{3 \times 3.14}$

$\therefore$ $r = 7.006$

$\therefore$ Radius of the circle = 7,006 cm **(Ans.)**

b Let O be the centre of the circle ABCD. Let AB be the diameter and CD be a chord other than diameter of the circle.

It is required to prove that, $AB > CD$.



Construction: O, C and O, D are joined.

Proof: $OA = OB = OC = OD$

[radius of the same circle]

Now, in ΔOCD,

$OC + OD > CD$

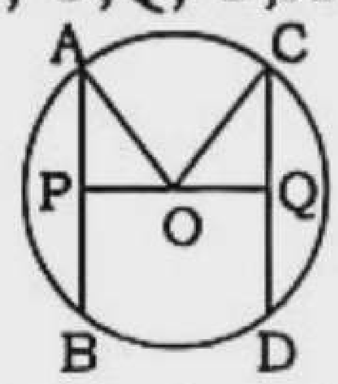
or, $OA + OB > CD$

Therefore, $AB > CD$. **(Proved)**

c According to the given stem, let ABDC is a circle with centre O, P and Q are the mid-point of the two equal chords AB and CD.

Now, it is required to prove that $PO = QO$.

Construction : O,P; O,Q; O,A and O,C are joined.



Proof : Here $OA = OC$, since they are the radivs of the same circle.

$AP = CQ$, since $AB = CD$ and P, Q are the mid-points of AB and CD.

$\angle APO = \angle CQO = 90°$, since chord AB and CD = chord CD and the line joining the mid-points P and Q ABC with the centre are perpendiculars to the chords.

Now, in ΔAOP and ΔCOQ,

$OA = OC$, $AP = CQ$, $\angle APO = \angle CQO$.

$\therefore \Delta AOP \cong \Delta COQ$ $\therefore$ $PO = QO$. **(Proved)**

Ques. 03 CD and EF be two chords other than diametre of a circle with centre O. OP and OQ are the perpendicular from O to the chords CD and EF respectively.

a. If OC = 5cm, then find the area of the circle. *[Easy]* 2
b. If $OP = OQ$, then prove that, $CD = EF$. *[Medium]* 4
c. If $CD > EF$, then prove that, $OP < OQ$. *[Hard]* 4

• *Barishal Board 2019*

Solution to Question No. 03 :

a Given, radius of the circle, $r = OC = 5$ cm

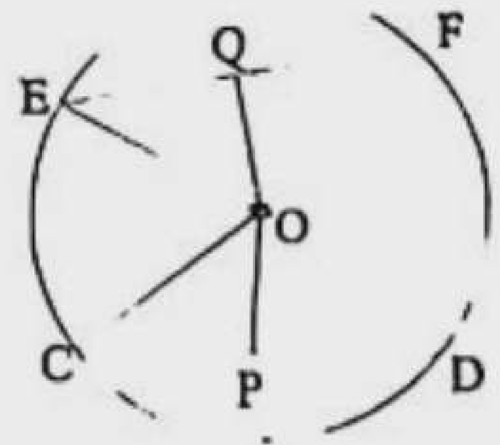
$\therefore$ Area of circle = πr^2 sq. unit

$= \pi (5)^2$

$= 25 \times 3.14$ sq. cm [$\because \pi = 3.14$]

$= 78.5$ sq cm **(Ans.)**

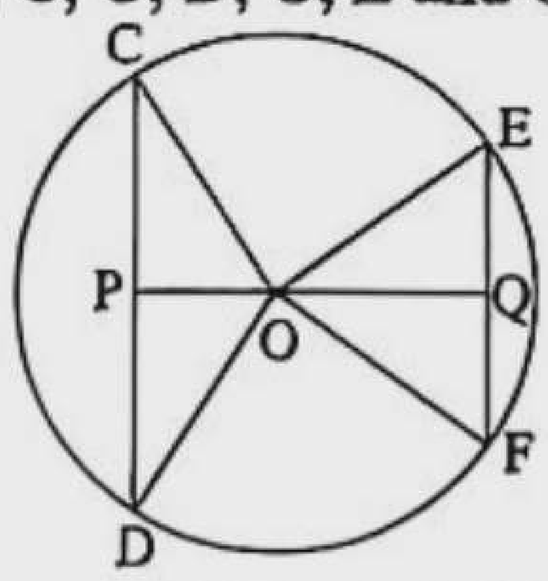
b **Proposition:** Let, CD and EF be two chords of a circle with centre O. OP and OQ are the perpendiculars from O to the chords CD and EF respectively.



Then OP and OQ represent the distances from centre to the chords CD and EF respectively.
If OP = OQ , it is to be proved that CD=EF.
Construction : O,E and O,C are joined.
Proof :

Steps	Justification
(1) Since $OP \perp CD$ and $EF \perp CD$. Therefore, $\angle OPC = \angle OQE$ = 1 right angle	[right angles]
(2) Now, between the right-angled ΔOCP and ΔOEQ hypotenuse OC = hypotenuse OE and OP = OQ $\therefore \Delta OCP \cong \Delta OEQ$ $\therefore CP = EQ$.	[radius of same circle] supposition] [RHS theorem]
(3) $CP = \frac{1}{2}CD$ and $EQ = \frac{1}{2}EF$	[Perpendicular from the centre bisects the chord]
(4) Therefore, $\frac{1}{2}CD = \frac{1}{2}EF$ i.e., CD = EF	

c Let, O is the centre. CD and EF are two chords of a circle CDEF such that CD > EF. Besides OP and OQ are perpendiculars from O to CD and EF. Now it is required to prove that, OP < OQ.
Construction : O, C; O, D; O, E and O, F are joined.



Proof : In the figure, $\angle OPC = 90° = \angle OQE$, since
Now, In ΔOCP, $OC^2 = OP^2 + PC^2$ .... (1)
and In ΔOEQ, $OE^2 = OQ^2 + QE^2$ .... (2)
Since OE and OC are the radius of the same circle
$\therefore OC = OE$
or, $OC^2 = OE^2$
or, $OP^2 + PC^2 = OQ^2 + QE^2$ [From (1) and (2)]
$\therefore PC^2 - QE^2 = OQ^2 - OP^2$ .... (3)
Given, CD > EF
or, $\frac{1}{2}CD > \frac{1}{2}EF$
or, PC > QE
or, $PC^2 > QE^2$
$\therefore PC^2 - QE^2 > O$

From (3) we get,
$OQ^2 - OP^2 > O$
or, $OQ^2 > OP^2$
or, OQ > OP
$\therefore$ OP < OQ **[Proved]**

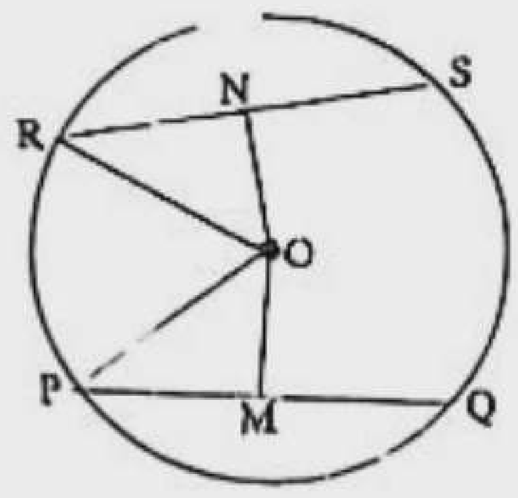
Ques. 04 In a circle with centre 'O', PQ and RS are two equal chords and their mid points are M and N respectively.
a. Find the radius of the circle with area of 314 sq. cm. *[Easy]* 2
b. Prove that chords are equidistant from the centre. *[Medium]* 4
c. If the chords PQ and RS bisect each other, prove that two parts of one chord are equal to the two parts of the other. *[Hard]* 4

▸ *Ideal School & College, Dhaka*

Solution to Question No. 04 :

a Given,
Area of the circle = 314 sq. cm
Let, radius of the circle = r cm
We know,
Area of the circle = πr^2
According to the question,
$\pi r^2 = 314$
or, $r^2 = \frac{314}{\pi}$
or, $r^2 = 99.91$
or, $r^2 = 9.995$
$\therefore$ r = 10 (approx)
$\therefore$ Radius of the circle = 10 cm (approx).

b **Proposition:** Let, PQ and RS be two equal chords of a circle with the centre O. It is to be proved that the chords AB and RS are equidistant from the centre.



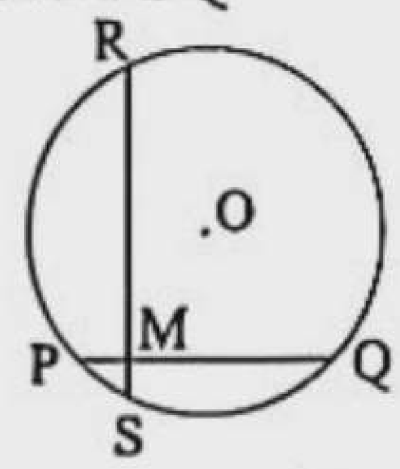
Construction : From O, perpendiculars OM and ON to the chords PQ and RS respectively are drawn. O, P and O, R are joined.
Proof :

Steps	Justification
(1) $OM \perp PQ$ and $ON \perp RS$. Therefore, PM = QM and RN = SN. $\therefore PM = \frac{1}{2}PQ$ and $RN = \frac{1}{2}RS$.	[Perpendicular from the centre bisects the chord]
(2) But PQ = SR $\Rightarrow \frac{1}{2}PQ = \frac{1}{2}RS$ $\therefore$ PM = RN.	[supposition]

Steps	Justification
(3) Now between the right-angled ΔOPM and ΔORN hypotenuse OP = hypotenuse OR and PM = RN. $\therefore \Delta OPM \cong \Delta ORN$ $\therefore$ OM = ON.	[radius of same circle] [Step 2] [RHS theorem]
(4) But OM and ON are the distances from O to the chords PQ and RS respectively. Therefore, the chords PQ and RS are equidistant from the centre of the circle. **(Proved)**	

C Let, RS and PQ are two equal chords of a circle which mutually interest at M, a point inside the circle. Now, it is required to prove that RM = QM and MS = PM.

Proof : Chord RS = Chord PQ (given)
$\therefore$ Arc RPS = Arc PSQ,
Since the two chords RS, PQ are equal and as such they stand on equal arcs of the same circle.
Now, Arc RPS = Arc PSQ



$\Rightarrow$ Arc RP + Arc PS = Arc PS + Arc SQ
$\Rightarrow$ Arc RP = Arc SQ $\Rightarrow$ RM = MQ
RS = PQ
$\Rightarrow$ RM + MS = PM + MQ
$\Rightarrow$ RM + MS = PM + RM, from (1),
$\Rightarrow$ MS = PM ....(2)
Thus from (1) and (2), RM = MQ and PM = MS **(Proved)**

Exercise 10.3 : Ratio of Circumference and Diameter of a Circle and Area of a Circle

At a Glance Important Contents of Exercise

- **Circumference :** The length of the circle is called circumference.
- **Arc of circle :** Any part on the curved line of circle is called arc of circle.
- If radius of the circle is r unit– (i) Circumference = $2\pi r$ unit; (ii) Area = πr^2 sq. unit
- Let, r be the radis of a right circular cylinder and h be its height– (i) Area of curred surface = $2\pi rh$ sq. unit. (ii) Area of the whole surface = $2\pi r(r + h)$ sq. unit

Solutions to Exercise Problems — Let's solve the textbook problems

MCQs with Answers

1. On a plane —.
i. Innumerable circles can be drawn with two particular points.
ii. The three points of a circle are not on one staright line. Hence, only one circle can be drawn.
iii. A straight line can intersect at more than two points in a circle.
Which one of the following is correct?
a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

2. In a circle with radius 2r —.
i. Circumference is $4\pi r$ unit
ii. Diameter is 4r unit
iii. Area is $2\pi^2 r^2$ sq unit
Which one of the following is correct?
a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

3. For a circle with radius 3 cm, what will be the length of the chord of 6 cm from the centre in cm?
d ⓐ 6 ⓑ 3 ⓒ 2 ⓓ 0

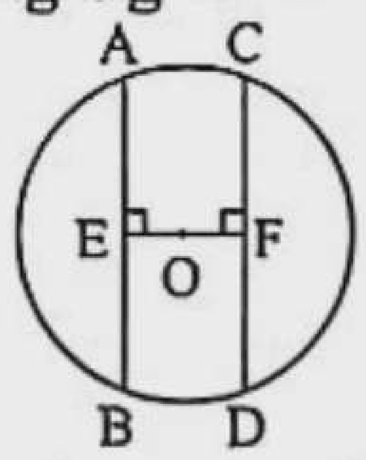
4. What will be the area of a circle with unit radius?
ⓐ 1 sq. unit ⓑ 2 sq. unit
c ⓒ π sq. unit ⓓ π^2 sq. unit

5. What will be the length of a radius of a circle with circumference 23 cm?
ⓐ 2.33 cm (approx.) ⓑ 3.66 cm (approx.)
b ⓒ 7.32 cm (approx.) ⓓ 11.5 cm (approx.)

6. What will be the area in between the space of the two uni centered circles with radii 3 cm and 2 cm?
ⓐ π sq. cm ⓑ 3π sq. cm
d ⓒ 4π sq. cm ⓓ 5π sq. cm

7. The diametre of a wheel of a vehicle is 38 cm. What will be the distance covered by two complete round?
ⓐ 59.69 cm ⓑ 76 cm
d ⓒ 119.38 cm ⓓ 238.76 cm

▣ **Answer questions 8, 9 and 10 on the basis of the following figure :**



In the figure, O is the centre of the circle, BE = 4 cm.

8. **If OE = OF, what will be the lengths of CD in cm?**
ⓓ ⓐ 3 cm ⓑ 4 cm ⓒ 6 cm ⓓ 8 cm

9. **AB = CD, OE = 3 cm. What will be the radius of the circle in cm?**
ⓒ ⓐ 3 ⓑ 4 ⓒ 5 ⓓ 6

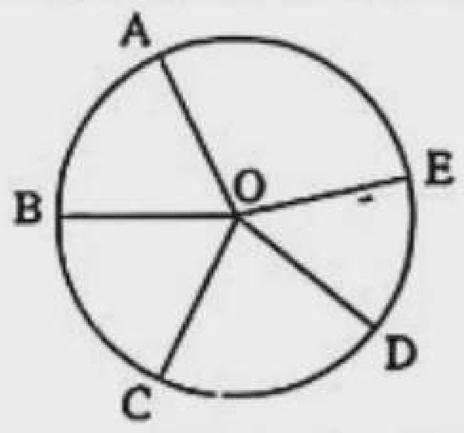
10. **If AB > CD, which of the following will be correct?**
ⓐ CF < BE ⓑ OE > OF
ⓒ ⓒ OE < OF ⓓ OE = OF

Mathematics

Solutions to Geometrical Problems

11. **Construct with a pencil compass a circle with a suitable centre and a radius. Draw a few radius in the circle and measure them to see if they all are of equal length or not.**

Solution : According to the need, let us draw a circle with centre at O and with radius OB = 2 cm.



Then four other radii OA, OC, OD and OE are drawn. The lengths of the radii are measured and is found as under :

OA = 2 cm
OB = 2 cm
OC = 2cm
OD = 2cm
and OE = 2 cm.

12. **Find the circumference of the circles with the following radius :**
(a) 10 cm; (b) 14 cm; (c) 21 cm.

Solution :

(a) We know that the circumference of a circle with radius r = $2\pi r$ unit of length.

Here, r = 10 cm, $\pi = \frac{22}{7}$.

∴ The required circumference

$$= 2 \times \frac{22}{7} \times 10 \text{ cm}$$

$$= \frac{440}{7} \text{ cm} = 62{\cdot}86 \text{ cm (approx.)}$$

(b) We know that the circumference of a circle with radius r = $2\pi r$ unit of length.

Here, r = 14 cm, $\pi = \frac{22}{7}$.

∴ The required circumference

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{2}{\cancel{14}} \text{ cm} = 88 \text{ cm.}$$

(c) We know that the circumference of a circle with radius r = $2\pi r$ unit of length.

Here, r = 21 cm, $\pi = \frac{22}{7}$.

∴ The required circumference

$$= 2 \times \frac{22}{\cancel{7}} \times \overset{3}{\cancel{21}} \text{ cm.} = 132 \text{ cm.}$$

13. **Find the area of the circles given below :**
(a) radius = 12 cm; (b) diameter = 34 cm; (c) Radius = 21 cm.

Solution :

(a) We know that the area of a circle with radius r = πr^2 sq. unit.

Here, $\pi = \frac{22}{7}$, r = 12 cm.

∴ The required area $= \frac{22}{7} \times (12)^2$ sq. cm.

$$= \frac{22}{7} \times 144 \text{ sq. cm.}$$

$$= \frac{3168}{7} \text{ sq. cm.}$$

$$= 452{\cdot}57 \text{ sq. cm. (approx.)}$$

(b) We know that the area of a circle with radius r = πr^2 sq. unit.

Here, $\pi = \frac{22}{7}$, d = 34 cm ⇒ r = $\frac{34}{2}$ cm = 17 cm.

∴ The required area $= \frac{22}{7} \times (17)^2$ sq. cm.

$$= \frac{22 \times 289}{7} \text{ sq. cm.}$$

$$= 908{\cdot}29 \text{ sq. cm. (approx)}$$

(c) We know that the area of the circle with radius r = πr^2 sq. unit.

Here, $\pi = \frac{22}{7}$, r = 21 cm.

∴ The required area $= \frac{22}{7} \times (21)^2$ sq. cm.

$$= \frac{22 \times 441}{7} \text{ sq. cm.}$$

$$= 1386 \text{ sq. cm.}$$

14. **If the circumference of a circular sheet is 154 cm, find its radius. Also find the area of the sheet.**

Solution :

We know that the circumference of a circle with radius r = $2\pi r$ unit of length.

Here, $2\pi r = 154$ cm

$$\therefore r = \frac{154}{2\pi} \text{ cm} = \frac{154}{2 \times \frac{22}{7}} \text{ cm}$$

$$= \frac{154 \times 7}{2 \times 22} \text{ cm}$$

$$= 24{\cdot}5 \text{ cm.}$$

Again, we know that the area of a circle with radius $r = \pi r^2$ unit of area.

Here, $r = 24{\cdot}5$ cm.

$$\therefore \text{ The required area} = \frac{22}{7} \times (24{\cdot}5)^2 \text{ sq. cm.}$$

$$= 1886{\cdot}5 \text{ sq. cm.}$$

15. A gardener wants to fence a circular garden of diameter 21m. Find the length of the rope he needs to purchase if he makes 2 rounds of the fence. Also find the cost of the rope if it costs Tk.18 per metre.

Solution :

Here the radius of the circular garden,

$$r = (21 \div 2) \text{ m.}$$

We know the circumference of a circle with radius $r = 2\pi r$ unit of length.

$\therefore$ The circumference of the garden

$$= 2 \times \frac{22}{7} \times \frac{21}{2} \text{ m}$$

$$= 66 \text{ m.}$$

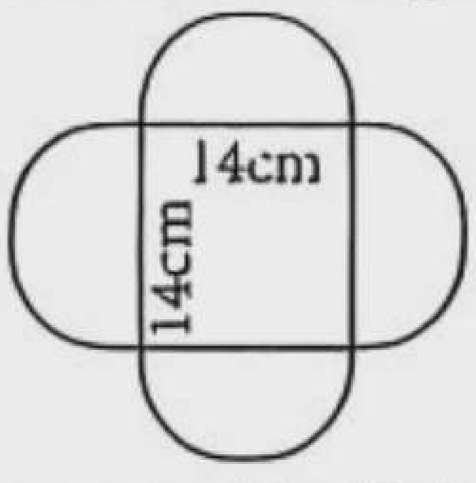
So, the length of the rope for fencing around the garden turning twice $= 2 \times 66$ m $= 132$ m.

We have, price per metre of rope $= 18$ taka.

$\therefore$ The price of 132 m of rope $= 18 \times 132$ Taka $= 2376$ Taka.

That is, the gardener should buy rope for 2376 taka.

16. Find the perimeter of the given shape.



Solution : There are 4 half circle here.

Diameter of half circle, $d = 14$ cm

$$\therefore \text{ Radius, } r = \frac{14}{2} \text{ cm} = 7 \text{ cm}$$

Perimeter of 1 half circle $= \frac{1}{2} \times 2\pi r = \pi r$

$\therefore$ Perimeter of 4 half circle $= 4 \times \pi r$

$$= 4 \times \frac{22}{7} \times 7 \text{ cm} = 88 \text{ cm}$$

Required perimeter of the given shape is 88 cm

17. From a circular board sheet of radius 14 cm, two circular parts of radius 1.5 cm and a rectangle of length 3 cm and breadth 1cm are removed. Find the area of the remaining board.

Solution :

Here the area of the circular board sheet with radius 14 cm

$$= \pi \times (14)^2 \text{ sq. cm.}$$

$$= \frac{22 \times 14 \times 14}{7} \text{ sq. cm.}$$

$$= 616 \text{ sq. cm.}$$

Again, the area of 2 circular parts each of radius $1{\cdot}5$ cm $= 2 \times \pi \cdot (1{\cdot}5)^2$ sq. cm.

$$= \frac{2 \times 22 \times 1{\cdot}5 \times 1{\cdot}5}{7} \text{ sq. cm.}$$

$$= 14{\cdot}14 \text{ sq. cm.}$$

And the area of the rectangle with length 3 cm and breadth 1 cm. $= 3 \times 1$ sq. cm $= 3$ sq. cm.

$\therefore$ Total area of 2 circular parts with radius $1{\cdot}5$ cm each and a rectangular area of length 3 cm and breadth 1 cm $= (14{\cdot}14 + 3)$ sq. cm. $= 17{\cdot}14$ sq. cm.

So, the rest part of the board after separation of two circular and a rectangular parts.

$$= (616 - 17{\cdot}14) \text{ sq. cm}$$

$$= 598{\cdot}86 \text{ sq. cm.}$$

18. The height of a right circular cylinder of radius 5.5 cm is 8 cm. Find the area of the whole surfaces of the cylinder ($\pi = 3.14$).

Solution : We know that,

the area of whole surface of a right circular cylinder $= 2\pi r^2 + 2\pi rh$, where

r = radius of base and h = height of the cylinder.

Here given that $r = 5.5$ cm, $h = 8$ cm

$\therefore$ The area of the whole surface

$= (2 \times 3.14 \times 5.5^2 + 2 \times 3.14 \times 5.5 \times 8)$ sq. cm

$= 466.29$ sq. cm

So, the required area of the whole surface of the cylinder is 466.29 sq. m.

Mathematics

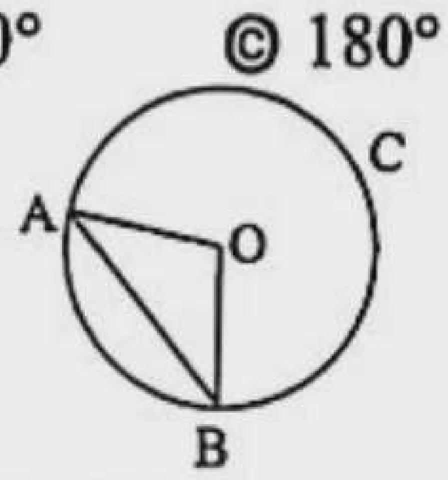
Multiple Choice Q/A Designed as per topic

10.5 Ratio of Circumference and Diameter of a Circle (π) ▸ Textbook Page 164

1. **What is called the parallelogram inscribed in a circle?** *(Easy)*
ⓐ Square ⓑ Rhombus
ⓒ Trapezium ⓓ Rectangle
Ans: ⓓ

2. **How many degree is found in the centre of pie-chart?** *(Easy)* *[JB '19]*
ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°
Ans: ⓓ

3. **What is the angle subtended at the centre of a circle?** *(Easy)* *[DB '18]*
ⓐ 0° ⓑ 90° ⓒ 180° ⓓ 360°
Ans: ⓓ

4.


In the figure, if O is the centre and ∠AOB = 100°, then ∠OAB = ? *(Medium)* *[RB '18]*
ⓐ 80° ⓑ 60° ⓒ 50° ⓓ 40°
Ans: ⓓ

5. **Which one is the circumference of a circle with radius 6 cm?** *(Medium)* *[DjB '18]*
ⓐ 18.84 cm ⓑ 37.69 cm
ⓒ 113.09 cm ⓓ 226.19 cm
Ans: ⓑ

6. **If the diameter of a circle is 10 cm, what is the circumference of the circle in cm.?** *(Hard)* *[DB '17]*
ⓐ 3.14 ⓑ 31.4 ⓒ 62.8 ⓓ 314
Ans: ⓒ

7. **The diameter of a circle is 6 cm. What is the circumference of the circle?** *(Medium)* *[DjB '17]*
ⓐ 9 π ⓑ 6 π ⓒ 3 π ⓓ 2 π
Ans: ⓑ

8. **The symbol π(pi) is a letter symbol derived from language of —.** *(Easy)* *[JB '16]*
ⓐ Latin ⓑ Japani
ⓒ Greek ⓓ Sanskrit
Ans: ⓒ

9. **Which one of the following is true for π(pi)?** *(Medium)* *[CtgB '16]*
ⓐ Rational number ⓑ Irrational number
ⓒ Natural number ⓓ Whole number
Ans: ⓑ

10. **If the area of a circle is A and circumference is c, which one of the following is the value of A?** *(Medium)[CtgB '16]*
ⓐ cr ⓑ $\frac{2}{cr}$ ⓒ $\frac{c^2}{4\pi}$ ⓓ $\frac{cr}{4}$
Ans: ⓒ

11. **What is the circumference in cm of a circle with radius 14 cm?** *(Medium)* *[DjB' 15]*
ⓐ 14π ⓑ $14\pi^2$ ⓒ 28π ⓓ 196π
Ans: ⓒ

12. **Ratio of circumference and diameter of circle is expressed by π. In fact π is an —.** *(Medium)* *[DjB' 15]*
ⓐ irrational number ⓑ rational number
ⓒ real number ⓓ integer
Ans: ⓐ

13. **Who estimated π as $\frac{62832}{20000}$?** *(Easy)*
[Ideal School & College, Dhaka]
ⓐ Arya Bhatta ⓑ Sreenibash Ramanujan
ⓒ Newton ⓓ Both (a) and (b)
Ans: ⓐ

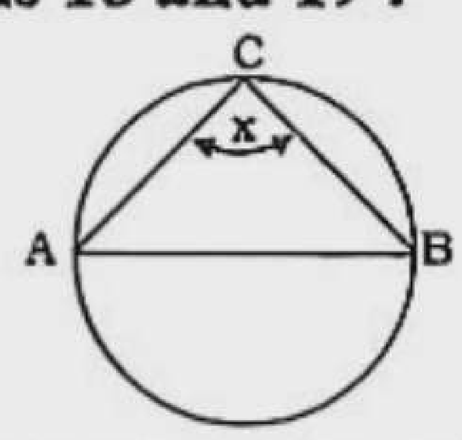
14. **The diametre of a circle is 10 cm. What is the circumference of the circle?** *(Medium)*
[Ideal School & College, Dhaka]
ⓐ 5.34 cm ⓑ 22.7 cm
ⓒ 31.4 cm ⓓ 40 cm
Ans: ⓒ

15. i. If r is the radius of a circle then perimeter is 2πr unit.
ii. If r is the radius of a circle then area is πr^2 square unit.
iii. If r is the radius of a circle then diameter is 2r unit.
What is the correct answer after the above statements? *(Hard)*
ⓐ i & iii ⓑ ii & iii ⓒ iii ⓓ i, ii & iii
Ans: ⓓ

16. i. The diameter of a circle divides the circle into two semi-circles.
ii. A chord of a circle divides the circle into a minor and a major arc.
iii. The diameter of a circle is the biggest chord and passes through the centre.
What is the correct answer after the above statements? *(Medium)*
ⓐ i & iii ⓑ ii & iii ⓒ iii ⓓ i, ii & iii
Ans: ⓓ

17. **In circle—** *[DjB '17]*
i. Each chord divides a circle into two arcs
ii. The perpendicular bisector of any chord passes through the centre of the circle
iii. The ratio of the circumference and the diameter of a circle is constant.
Which one is correct? *(Medium)*
ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii
Ans: ⓓ

Consider the adjoining figure and answer the questions 18 and 19 :



18. **If AB is the diameter of the circle ABC, then ∠x = What?** *(Easy)*
ⓐ 60° ⓑ 70° ⓒ 80° ⓓ 90°
Ans: ⓓ

19. i. $AB^2 = BC^2 + AC^2$
ii. $AC^2 = BC^2 + AB^2$
iii. $BC^2 = AB^2 + AC^2$
What is/are the correct answer/answers after the above statements? *(Medium)*
ⓐ i ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii
Ans: ⓐ

■ The radius of a wheel of a car is 4 centimetres. **Answer the questions No. 20 and 21 in respect of the above information :** *[JB '16]*

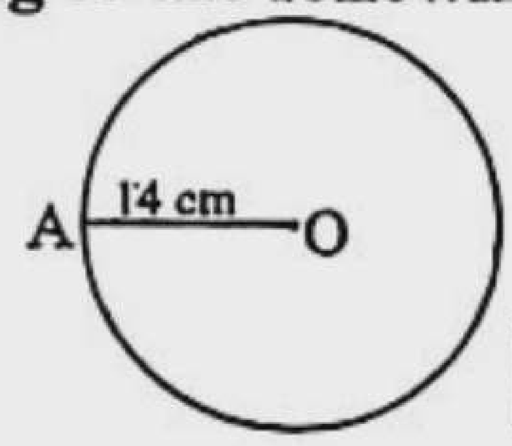
20. What is the perimeter of a wheel of the car in cm? *(Easy)*

ⓑ ⓐ 24.13 ⓑ 25.13 ⓒ 26.13 ⓓ 27.13

21. What is the area of a wheel of the car in sq. cm? *(Medium)*

ⓐ ⓐ 16π ⓑ 8π ⓒ 4π ⓓ 2π

■ **Answer the questions No. 22 and 23 according to the following information :**



$\left[\pi = \frac{22}{7}\right]$ *[JB' 15]*

22. How many centimetres is the circumference of the circle? *(Medium)*

ⓑ ⓐ 44 ⓑ 88 ⓒ 176 ⓓ 616

23. What is the area of the circle? *(Hard)*

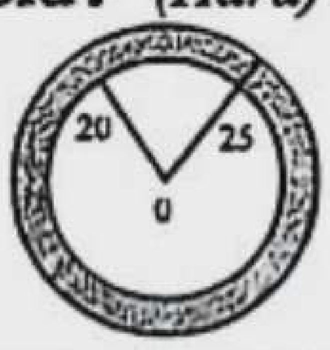
ⓐ 616 cm^2 ⓑ 176 cm^2

ⓐ ⓒ 88 cm^2 ⓓ 44 cm^2

10.6 Area of a Circle

▸ Textbook Page 165

24. What is the area of the shaded portion of the circular field? *(Hard)*



ⓑ ⓐ 200π ⓑ 225π ⓒ 425π ⓓ 625π

25. The radius of a right circular cylinder is 4 cm and the height is 6 cm. What is the area of the curved surface of the cylinder? (π = 3.14) *(Hard)* *[DB '19]*

ⓐ 75.36 sq cm ⓑ 150.72 sq cm

ⓑ ⓒ 226.08 sq cm ⓓ 301.44 sq cm

26. What will be the area of a circle of diametre 6 cm? *(Hard)* *[BB '19]*

ⓐ 6π sq cm ⓑ 9π sq cm

ⓑ ⓒ 12π sq cm ⓓ 36π sq cm

27. Which one is the area of a circular region with the diameter 17.60 cm? *(Medium)* *[RB '18]*

ⓐ 112.7 sq metre ⓑ 155.26 sq metre

ⓓ ⓒ 160.79 sq metre ⓓ 243.16 sq metre

28. If radius of a circle 2r, what is the area of that circle? *(Hard)* *[JB '18]*

ⓐ 2 πr sq unit ⓑ 4 πr sq unit

ⓓ ⓒ πr^2 sq unit ⓓ 4 πr^2 sq unit

29. If the diameter of a circle is 2r, what is the area of the circle? *(Easy)* *[BB '18]*

ⓐ ⓐ πr^2 ⓑ $2\pi r^2$ ⓒ $\frac{\pi r^2}{2}$ ⓓ 2πr

30. What is the formulae of the area of a circle? *(Medium)* *[RB '17]*

ⓐ πr^2 sq. unit ⓑ $\frac{1}{3}\pi r^2 h$ sq. unit

ⓐ ⓒ 2 πrh sq. unit ⓓ $\pi r l$ sq. unit

31. The diameter of a circular garden is 20 feet. There is a road with 3 metres wide outside around the garden. What is the area of the road in sq. feet? *(Hard)* *[BB '17]*

ⓓ ⓐ 9π ⓑ 51π ⓒ 60π ⓓ 69π

32. What is the area of a circle in square centimetre whose diametre is 9 cm? (π = 3.14) *(Hard)* *[CtgB '16]*

ⓒ ⓐ 18.84 ⓑ 28.26 ⓒ 63.585 ⓓ 254.34

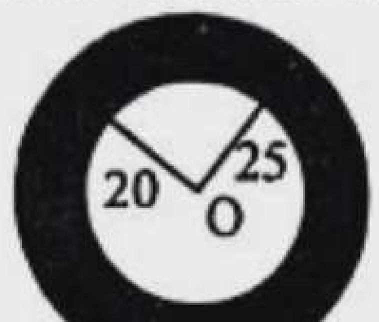
33. What is the value of the angle which is subtended at the centre of a circle? *(Hard)* *[CtgB '16]*

ⓓ ⓐ 90° ⓑ 120° ⓒ 180° ⓓ 360°

34. Which one of the following is the formula for determining area of circle? *(Hard)* *[Ctg.B' 15]*

ⓒ ⓐ $\frac{4}{3}\pi r^3$ ⓑ $4\pi r^2$ ⓒ πr^2 ⓓ $\frac{3}{4}\pi r^3$

35. What is the area of shaded region? *(Medium)*



[Rajuk Uttara Model College, Dhaka]

ⓐ ⓐ 255π ⓑ 5π ⓒ 25π ⓓ $5\pi^2$

36. If the circumference is c and radius is r of a circle —. *[SB' 15]*

i. $\frac{c}{2r} = \pi$

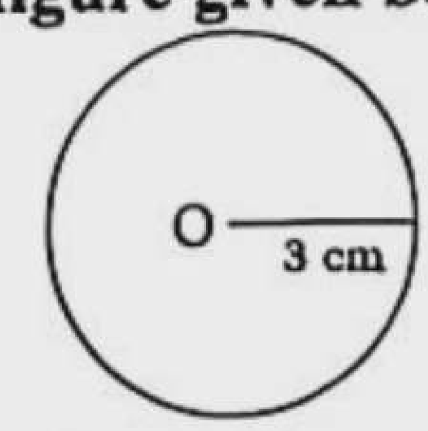
ii. $c = \frac{\pi}{2r}$

iii. $\frac{r}{c} = \pi$

Which one of the following is true? *(Easy)*

ⓐ ⓐ i ⓑ ii ⓒ ii & iii ⓓ i, ii & iii

■ **Answer the question No. 37 and 38 in the light of the figure given below :**



37. What is the circumference of the circle in cm? *(Medium)* *[JB '17]*

ⓓ ⓐ 6 ⓑ 7.5 ⓒ 6.28 ⓓ 18.84

38. What is the area of the circle in sq. cm? *(Hard)* *[JB '17]*

ⓓ ⓐ 9 ⓑ 18.84 ⓒ 14.13 ⓓ 28.26

Answer the questions No. 39 and 40 in the light of the following information —
Circumference of a circle is 44 cm. *[CB' 15]*

39. Radius of a circle is what cm? *(Easy)*
a ⓐ 7 ⓑ 14 ⓒ 22 ⓓ 44

40. What is the area of the circle in sq. cm. (approx.) *(Hard)*
a ⓐ 154 ⓑ 616 ⓒ 308 ⓓ 49

10.7 Cylinder

▸ Textbook Page 168

41. The radius of a right circular cylinder is 5cm and its height is 7cm. What is the area of the whole surfaces of the cylinder? *(Medium)* *[MB '19]*
d ⓐ 25π ⓑ 50π ⓒ 70π ⓓ 120π

42. If the edge of a cube is 5 unit, what is the area of the entire surface of the cube in square unit? *(Medium)* *[DB '18]*
c ⓐ 30 ⓑ 125 ⓒ 150 ⓓ 750

43. The radius and the height of a right circular cylinder are 5 cm and 7 cm respectively. Which one is the area of the curved surface of the cylinder? *(Hard)* *[RB '18]*
ⓐ 109.9 sq cm. ⓑ 159.08 sq cm
c ⓒ 219.9 sq cm ⓓ 549.5 sq cm

44. What is the area of the whole surface of a cube whose each side is $\sqrt{5}$ metres? *(Hard)* *[BB '18]*
ⓐ 5 sq m ⓑ 20 sq m
c ⓒ 30 sq m ⓓ 150 sq m

45. If the side of a cube is 5 cm, then which one of the following is the area of the whole surfaces of the cube? *(Medium)* *[DjB '18]*
ⓐ 25 sq cm ⓑ 100 sq cm
d ⓒ 125 sq cm ⓓ 150 sq cm

46. The radius of a right circular cylinder is 2 cm. and its height is 5 cm. Which is the area of the curved surface of the cylinder? *(Medium)* *[DB '17]*
ⓐ 10.2 sq cm. ⓑ 31.4 sq cm.
d ⓒ 40.3 sq cm. ⓓ 62.8 sq cm.

47. How many surfaces of a rectangular solid are there? *(Medium)* *[JB '17]*
c ⓐ 2 ⓑ 4 ⓒ 6 ⓓ 8

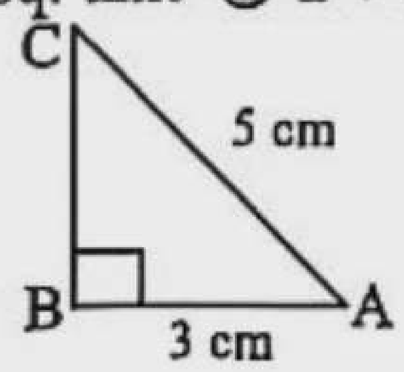
48. If 3 cm is the radius of the base and 6 cm. is the height of a right circular cylinder, then what is the area of its whole surface? *(Hard)* *[CB '16]*
ⓐ 54π sq. cm ⓑ 6π sq. cm
a ⓒ 27π sq. cm ⓓ 18π sq. cm

49. The radius of a right-circular cylinder is 4.5 cm and height is 6 cm then what is the area of its curved surface in sq. cm? *(Medium)* *[CB' 15]*
ⓐ 169.56 ⓑ 84.78
a ⓒ 296.73 ⓓ 127.17

50. How much unit is the diameter of the cylinder? *(Medium)* *[SB' 15]*
d ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 6

51. If length of a rectangular solid is a unit, breadth is b unit and height is c unit then which is the area of the whole face of the solid? *(Easy)* *[DjB' 15]*
ⓐ abc sq. unit ⓑ 2(ab + bc + ca) sq. unit
b ⓒ (ab + bc + ca) sq. unit ⓓ a + b + c sq. unit

52.



In the above figure— *[BB '19]*
i. the length of BC is 4cm
ii. the area of triangle ABC is 12 sq cm
iii. $\angle BAC + \angle BCA = 90°$
Which one is correct? *(Medium)*
b ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer the questions number 53 and 54 according to the following statement :
The diameter of a right circular cylinder is 8cm and height is 12 cm. ($\pi = 3.14$) *[CB '18]*

53. Which is the area of the base of the cylinder? *(Easy)*
ⓐ 25.12sq cm ⓑ 50.24sq cm
b ⓒ 64.00 sq cm ⓓ 200.96 sq cm

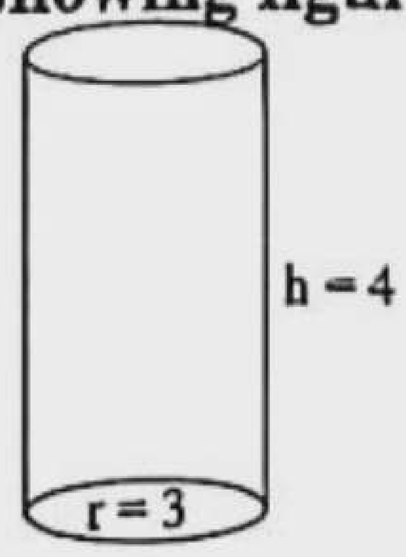
54. Which one is the area of the curved surface of the cylinder? *(Medium)*
ⓐ 96 sq cm ⓑ 192 sq cm
c ⓒ 301.44 sq cm ⓓ 602.88 sq cm

According to the following information answer the questions No. 55 and 56 :
The radius of a right circular cylinder is 3 cm and its height is 5 cm. *[SB '18]*

55. What is the area of the base of the cylinder? *(Easy)*
ⓐ 9π sq cm ⓑ 15π sq cm
a ⓒ 25π sq cm ⓓ 30π sq cm

56. What is the area of whole surfaces of cylinder in sq cm? *(Medium)*
ⓐ 56.56 sq com (Approx) ⓑ 94.20 sq cm (Approx)
c ⓒ 150.80 sq cm (Approx) ⓓ 251.20 sq cm (Approx)

Answer the questions No. 57 to 58 in the light of the following figure :



[SB' 15]

57. What is area in square unit of the curve surface of a cylinder? *(Easy)*
b ⓐ 76.814 ⓑ 75.39 ⓒ 74.39 ⓓ 70.75

58. What is the area in square unit of the two end surfaces? *(Medium)*
a ⓐ 56.55 ⓑ 55.55 ⓒ 54.55 ⓓ 52.56

Short Q/A — Designed as per topic

10.5 Ratio of Circumference and Diameter of a Circle (π)

Textbook Page 164

Question 1. What is the ratio of the circumference and diameter of a circle?

Solution : Let the radius of the circle be r.

$\therefore$ Then, the diameter is 2r and the circumference is $= 2\pi r$

$\therefore$ Circumference: Diameter $= 2\pi r : 2r = \frac{2\pi r}{2r} = \pi$

$\therefore$ Hence, the ratio of the circumference and diameter of a circle is π.

Question 2. What is the circumference of a circle with a diameter of 13 cm?

Solution : Given that the diameter of the circle, d = 13 cm

$\therefore$ Radius of the circle, $r = \frac{d}{2} = \frac{13}{2} = 6.5$ cm

$\therefore$ Circumference of the circle $= 2\pi r$

$= 2 \times 3.14 \times 6.5$ cm $= 40.82$ cm

The circumference of the circle is 40.82 cm.

Question 3. If the diameter of a wheel is 35 cm, how many cm will the wheel cover in two rotations?

Solution : Given that the diameter of the wheel, 2r = 35 cm

$\therefore$ Circumference of the wheel $= 2\pi r = 2r \times \pi$

$= 35 \times 3.14$ cm $= 109.9$ cm

In one rotation, the wheel covers 109.9 cm.

$\therefore$ " 2 " " " " (109.9×2) cm

$= 219.8$ cm (approx)

The required distance is 219.8 cm (approx)

Question 4. If the circumference of a wheel of a vehicle is 45 cm, find its radius.

Solution : Let the radius of the wheel be = r cm.

Then, the circumference of the wheel is $= 2\pi r$ cm

According to the question, $2\pi r = 45$

or, $r = \frac{45}{2\pi} = \frac{45}{2 \times 3.14} = 7.17$ cm (approx)

The radius of the wheel is 7.17 cm (approx.)

Question 5. If the diameter of a wheel of a vehicle is 38 cm, how many times will the wheel rotate to cover a distance of 119.3808 meters?

Solution : Given that the diameter of the wheel, 2r = 38 cm

$\therefore$ Circumference of the wheel $= 2\pi r = 2r \times \pi$

$= 38 \times 3.1416$ cm $= 119.3808$ cm

Here, 119.3808 cm $= 119.3808 \times 100$ cm

$\therefore$ The number of rotations required $= \frac{119.3808 \times 100}{119.3808}$

$= 100$

$\therefore$ The wheel will rotate 100 times.

10.6 Area of a Circle

Textbook Page 165

Question 6. If the circumference of a circle is π, what is its area?

Solution : Let the radius of the circle be r units.

$\therefore$ Then, the circumference of the circle is $2\pi r$ units.

According to the question, $2\pi r = \pi$

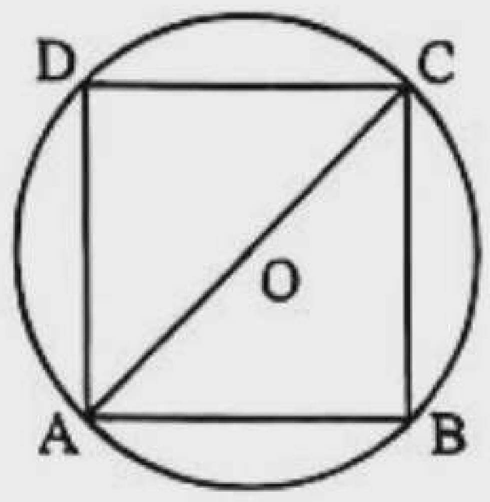
or, $r = \frac{\pi}{2\pi} = \frac{1}{2}$

$\therefore$ The area of the circle is $= \pi r^2 = \pi \left(\frac{1}{2}\right)^2$ square units

$= \frac{\pi}{4}$ square units

The area of the circle is $\frac{\pi}{4}$ square units

Question 7. If the length of the diagonal of a square located inside a circle is 14 cm, what is the area of the circle?



Solution : In the figure, the length of the diagonal of the square ABCD inside the circle with center O is AC, which is equal to the diameter of the circle.

Here, AC = 14 cm

That is, the diameter, 2r = 14 cm

$\therefore$ The radius $r = \frac{14}{2} = 7$ cm

$\therefore$ the area of the circle $= \pi r^2 = 3.14 \times 7^2$ sq. cm

$= 3.14 \times 49$ sq. cm

$= 153.86$ sq. cm (approx.)

The area of the circle is 153.86 sq. cm (approx.)

Question 8. If the area of a circle is π square units, what is the diameter in units?

Solution : Let the radius of the circle be r units.

$\therefore$ Then, the area of the circle is $= \pi r^2$ sq. units.

According to the question, $\pi r^2 = \pi$

or, $r^2 = 1$

$\therefore$ $r = \sqrt{1} = 1$

$\therefore$ The diameter of the circle $= 2r = 2 \times 1 = 2$ unit

The diameter is 2 units

Question 9. What is the diameter of a circle with an area of 1962.5 square cm?

Solution : Let the radius of the circle be r cm.

∴ the area of the circle is πr^2 square cm

According to the question, $\pi r^2 = 1962.5$

or, $r^2 = \frac{1962.5}{\pi} = \frac{1962.5}{3.14} = 625$

∴ $r = \sqrt{625} = 25$

∴ The diameter of the circle $= 2r = 2 \times 25 = 50$ cm

The diameter of the circle is 50 cm

Question 10. The diameter of a circular garden is 30 feet. There is a road 5 feet wide outside the garden. What is the area of the road in square feet?

Let the radius of the circular garden be r feet.

Then, the diameter is 2r feet.

According to the question, $2r = 30$

∴ $r = \frac{30}{2} = 15$

The radius of the garden including the road, R = (15 + 5) feet = 20 feet

∴ The area of the road $= \pi R^2 - \pi r^2$

$= (R^2 - r^2)\pi = (20^2 - 15^2)\pi$ square feet

$= (400 - 22.5)\pi$ square feet

$= 175\pi$ square feet

The area of the road is 175π square feet

Question 11. What is the area of a circular garden with a diameter of 12.8 meters?

Solution The diameter of the circular garden, d = 12.8 m

∴ The radius of the garden, $r = \frac{12.8}{2} = 6.4$ m

∴ The area of the circular garden

$= \pi r^2 = 3.14 \times (6.4)^2$ sq. m

$= 3.14 \times 40.96$ eMÆ wg. = 128.61 sq. m (approx)

The area of the garden is 128.61 sq. m (approx.)

10.7 Cyllnder

▸ Textbook Page 168

Question 12. The radius of a right circular cylinder is 7 cm and its height is 9 cm. Find the area of the curved surface of the cylinder.

Solution : Given that the radius of the cylinder, r = 7 cm, and the height, h = 9 cm

∴ The area of the curved surface of the cylinder $= 2\pi rh$

$= 2 \times 3.14 \times 7 \times 9$ square cm

$= 395.64$ square cm (approx.)

The area of the curved surface of the cylinder is 395.64 square cm (approx.)

Question 13. The diameter of a right circular cylinder is 8 cm and its height is 12 cm. Find the total surface area of the cylinder ($\pi = 3.14$).

Solution : Given that the height of the cylinder, h = 12 cm, and the diameter of the cylinder, d = 8 cm

∴ Radius of the cylinder, $r = \frac{8}{2}$ cm = 4 cm

∴ The total surface area of the cylinder $= 2\pi r(r + h)$

$= 2 \times 3.14 \times 4(4 + 12)$ square cm

$= 2 \times 3.14 \times 4 \times 16$ square cm

$= 401.92$ square cm

The total surface area of the cylinder is 401.92 square cm.

Question 14. The height of a right circular cylinder is 21 cm and the area of its curved surface is 504 square cm. Find the radius of the cylinder.

Solution : Given that the height of the right circular cylinder, h = 21 cm

Let the radius of the right circular cylinder be r cm.

∴ The area of the curved surface of the right circular cylinder is $2\pi rh$ square cm.

According to the question, $2\pi rh = 504\pi$

or, $2\pi r \times 21 = 504\pi$

or, $r = \frac{504\pi}{2\pi \times 21}$

∴ r = 12 cm

The radius of the right circular cylinder is 12 cm.

Question 15. The radius of the base of a cylinder is 12 cm and the total surface area is 792 square cm. Find the height of the cylinder.

Solution : Given that the radius of the cylinder, r = 12 cm

Let the height be h cm

∴ The total surface area of the cylinder is $2\pi r(r + h)$ square cm.

According to the question, $2\pi r(r + h) = 792\pi$

or, $2\pi \times 12(12 + h) = 792\pi$

or, $12 + h = \frac{792\pi}{2\pi \times 12}$

or, $12 + h = 33$

∴ $h = 33 - 12 = 21$

The height of the cylinder is 21 cm.

Question 16. If the area of one end face of a right circular cylinder is 78.5 square cm, find its radius.

Solution : Let the radius of the base of the right circular cylinder be r cm.

Then, the area of one end face of the right circular cylinder is πr^2 square cm.

According to the question, $\pi r^2 = 78.5$

or, $r^2 = \frac{78.5}{\pi} = \frac{78.5}{3.14} = 25$

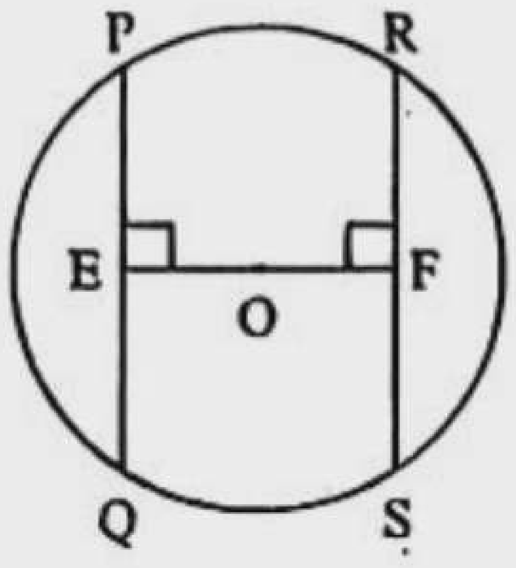
∴ $r = \sqrt{25} = 5$ cm

The radius is 5 cm

Mathematics

Creative Q/A — Designed as per learning outcomes

Ques. 01



In the figure, PQ = RS and O is the centre of the circle.

a. If the area of a circle is $75.39m^2$, find the radius of the circle. *[Easy]* 2

b. Prove that, OE = OF. *[Medium]* 4

c. If PQ > RS, prove that, OE < OF. *[Hard]* 4

• *Rajshahi Board 2019*

Solution to Question No. 01 :

a We know that,

area of a circle with radius 'r' unit = πr^2 sq. unit

Given, area of a circle is 75.39 m^2

$\therefore \pi r^2 = 75.39$

or, $r^2 = \frac{75.39}{\pi}$

or, $r^2 = \frac{75.39}{3.14}$ $[\because \pi = 3.14]$

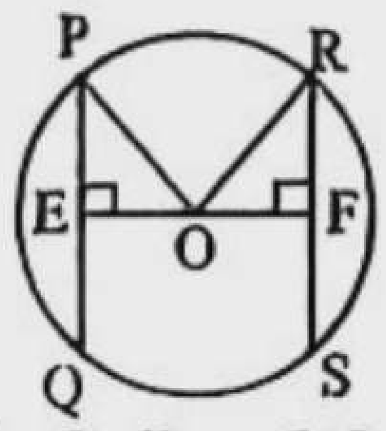
or, $r^2 = 24.009$

or, $r = \sqrt{24.009}$

$\therefore r = 4.9$

$\therefore$ Radius of the circle is 4.9 m (approx).

b Let, PQ and RS are two equal chord of a circle with centre O. OE and OF are two perpendicular on PQ and RS respectively. We have to prove that, OE = OF.



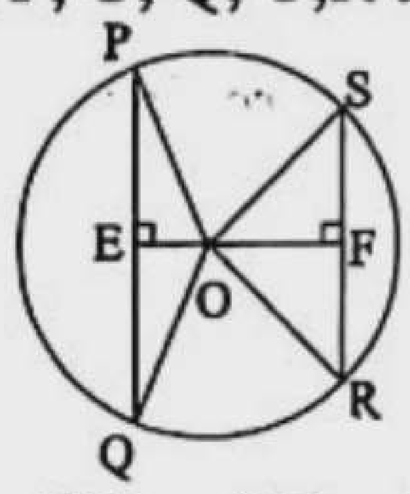
Construction : Join O, P and O, R.

Proof :

Steps	Justification
1. $OE \perp PQ$ and $OF \perp RS$ Therefore, $EP = \frac{1}{2}PQ$ and $FR = \frac{1}{2}RS$	[Perpendicular from the centre of a circle to a chord bisects the chord]
2. PQ = RS or, $\frac{1}{2}PQ = \frac{1}{2}RS$ $\therefore$ EP = FR	[Given]
3. In ΔOPE and ΔORF OP = OR EP = FR and $\angle OEP = \angle OFR$ $\therefore \Delta OPE \cong \Delta ORF$ $\therefore$ OE = OF **[Proved]**	 [radius of same circle] [From (2)] [both are right angle]

c Let, O is the centre, PQ and RS are two chords of a circle PQRS such that PQ > RS. Besides, OE and OF are perpendiculars to PQ and RS. Now it is to be proved that, OE < OF.

Construction : O, P; O, Q; O,R and O,S are joined.



Proof : Here, OP = OQ = OR = OS, since they are the radii of the same circle PQRS.

Now in right Δ POE, $OP^2 = PE^2 + OE^2$.............(1),

Again, in right ΔFOS,

$OS^2 = SF^2 + OF^2$..............(2)

Now from (1) and (2), we get,

$PE^2 + OE^2 = SF^2 + OF^2$, since OP = OS, the radius of the same circle.

or, $PE^2 - SF^2 = OF^2 - OE^2$..................(3)

Here, PE – SF > O, since $PQ > RS \Rightarrow \frac{1}{2}PQ > \frac{1}{2}RS$

$\Rightarrow PE > SF$.

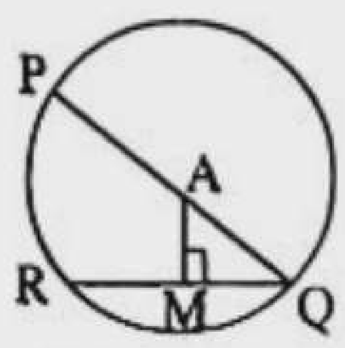
$\therefore$ From (3), $OF^2 - OE^2 > O$

or, $OF^2 > OE^2$

or, OF > OE

$\therefore$ OE < OF **(proved)**

Ques. 02



In figure, a circle with centre A whose diameter is PQ = 6 cm.

a. Determine the area of a circle with centre A. *[Easy]* 2

b. Prove that M is the mid-point of RQ. *[Medium]* 4

c. Prove that PQ > RQ. *[Hard]* 4

• *Dhaka Board 2018*

Solution to Question No. 02 :

a Here the diameter of a circle is PQ = 6cm.

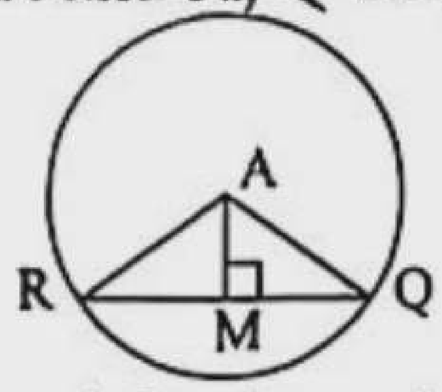
$\therefore$ Radius of the circle = $\frac{6}{2}$ or, 3cm.

$\therefore$ Area of the circle = $\pi \times 3^2$ sq cm = 28.26 sq cm.

Mathematics

b According to the given information, RQ is a chord of a circle with centre A and $AM \perp RQ$. Now, it is to be proved that M is the mid-point of RQ.

Construction : A, R and A, Q are joined.



Proof : Here RA = AQ, since they are the radii of the same circle. $AM \perp RQ$, according to stem.

$\therefore \angle AMR = \angle AMQ = 90°$.

Now, in ΔRAM and ΔQAM,

AM is common to both the triangles.

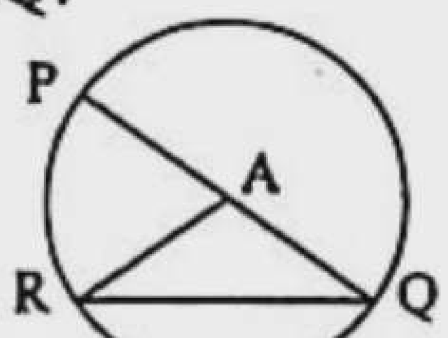
$RA = AQ$, $\angle AMR = \angle AMQ$.

$\therefore \Delta RAQ \cong \Delta QAM$.

$\therefore RM = MQ$.

$\therefore$ M is the mid-point of RQ. **(Proved)**

c Here in the adjoining figure, A is the centre of a circle PRQ with radius PQ and a chord RQ, which does not pass through A. Now, it is required to prove that $PQ > RQ$.



Construction : A, R are joined.

Proof : Here PA = AQ, since they are the radii of the same circle.

and RA = AQ.

Now, in Δ RAQ,

$RA + AQ > RQ$, since sum of any two sides of a triangle is greater than the 3rd angle.

or, $PA + AQ > RQ$, putting RA = PA.

or, $PQ > RQ$.

$\therefore PQ > RQ$. **(Proved)**

Ques. 03 PQ and RS are two chords other than diameter in a circle with centre O. OE and OF are the perpendiculars from O to the chords PQ and RS respectively. MN is the diameter.

a. Determine the peremeter of the circle, if MN = 8 cm. *[Easy]* 2

b. Prove that, MN is the greatest chord of that circle. *[Medium]* 4

c. Prove that, OE < OF, if PQ > RS. *[Hard]* 4

● *Rajshahi Board 2017*

Solution to Question No. 03 :

a Here diameter of the circle d = 8 cm.

$\therefore$ Perimeter of the circle = $2\pi r$ unit, where r = radius

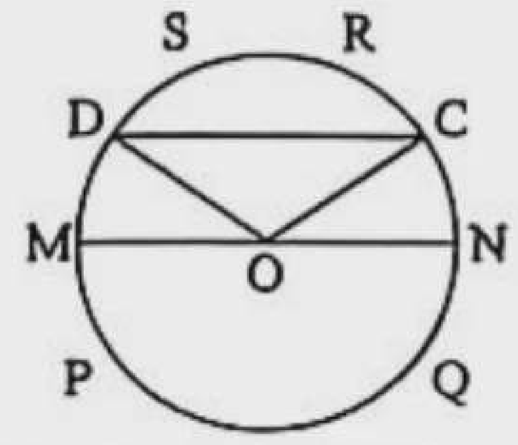
= $2r\pi$ unit, where r = radius

= $d\pi$ unit, where d = diameter.

= 8π cm.

So, perimeter of the circle = 8π cm.

b Let, PQRS is a circle with centre O. MN is a diameter and DC is any chord which does not pass through the centre O of the circle. Now it is required to prove that MN > CD that is, MN is the greatest chord of the circle



Construction : O, C and O, D are joined.

Proof : Here OM = ON = OC = OD, since they are the radii of the same circle ABCD.

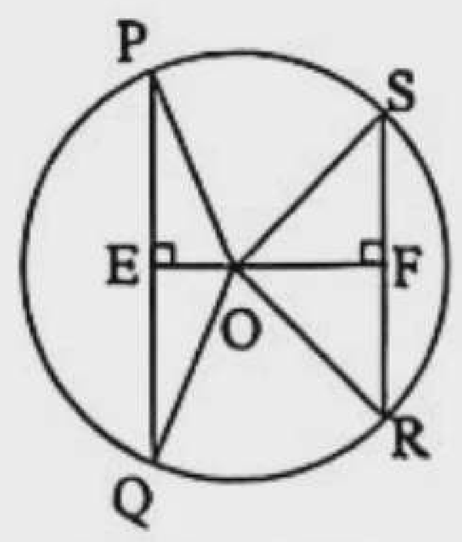
Now in ΔOCD, $OC + OD > DC$

or, $OM + ON > CD$

or, $MN > CD$ **(Proved)**

c Let, O is the centre, MN and RS are two chords of a circle PQRS such that PQ > RS. Besides, OE and OF are perpendiculars to PQ and RS. Now it is to be proved that OE < OF.

Construction : O, P; O, Q; O,R and O,S are joined.



Proof : Here, OP = OQ = OR = OS, since they are the radii of the same circle PQRS. $OE \perp PQ$ and $OF \perp RS$ are drawn.

Now in right Δ POE, $OP^2 = PE^2 + OE^2$.............(1),

Again, in right ΔFOS,

$OS^2 = SF^2 + OF^2$..............(2)

Now from (1) and (2), we get,

$PE^2 + OE^2 = SF^2 + OF^2$, since OP = OS, the radius of the same circle.

or, $PE^2 - SF^2 = OF^2 - OE^2$..................(3)

Here, $PE - SF > O$,

since PQ > RS

$\Rightarrow \frac{1}{2} PQ > \frac{1}{2} RS$

$\Rightarrow PE > SF$.

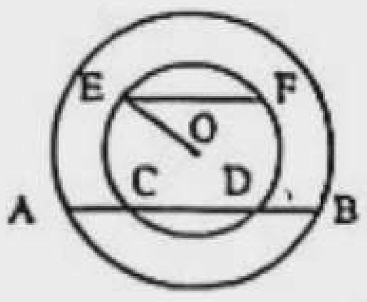
$\therefore$ from (3), $OF^2 - OE^2 > O$

or, $OF^2 > OE^2$

or, $OF > OE$

that is OE < OF **(proved)**

Ques. 04



In the figure, two co-centric circle are shown. O is the centre of the two circles and OE = 8 cm.

a. The area of the larger circle 254.34 sq. metre. What is the area between two circumference of the two circles? *[Easy]* 2

b. Prove that AC = BD. *[Medium]* 4

c. In the figure EF > CD, prove that the chord EF is nearer than the chord CD to the centre. *[Hard]* 4

● *Chattogram Board 2017*

Solution to Question No. 04 :

a Here area of smaller circle with radius 8 cm

$= \pi r^2$ sq unit

$= 3.14 \times 8^2$ sq. cm

$= 200.96$ sq. cm

And area of larger circle = 254.34 sq. cm (given)

$\therefore$ Area of the region between two circumference of the two circles = 254.34 sq. cm – 200.96 sq. cm

= 53.38 sq. cm

b Let us have two co-centric circles with centre at O. ACDB is a chord of the larger circle and CD is a chord of the smaller circle. Now it is required to prove that AC = BD.

Construction : O, A; O, B; O, C; O, D are joined. $OP \perp ACDB$ is drawn.

Proof : In ΔAOP and ΔBOP, OA = OB, since they are the radii of the same circle. OP is common to both the triangles.

$\angle APO = 90° = \angle BPO$, according to construction.

$\therefore\ \Delta AOP \cong \Delta BOP \Rightarrow AP = BP$ -------------- (1)

Again, in ΔCOP and ΔDOP,

OC = OD, since they are the radii of the same circle OP is common to both of them.

$\angle OPC = 90° = \angle OPD$, according to construction.

$\therefore \Delta COP \cong \Delta DOP \Rightarrow CD = DP$ -------------- (2)

Now we have from (1) that,

AP = BP

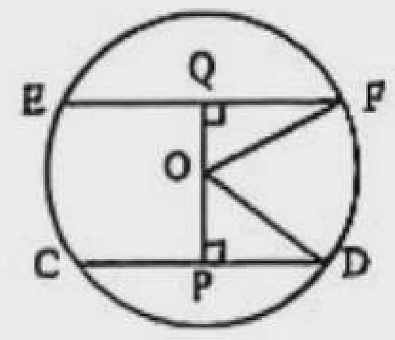
or, AC + CP = PD + DB

or, AC + CP = CP + DB, putting PD = CP from (2)

or, AC = DB, subtracting CP from both sides

$\therefore$ AC = BD. **(Proved)**

c Let us have a circle with centre O and CD, EF are two chords of the circle such that EF > CD. Now it is required to prove that EF is nearer to the centre than CD.



Construction : $OP \perp CD$ and $OQ \perp EF$ are drawn. O, D and O, F are joined.

Proof : In right triangle POD, where $\angle OPD = 90°$, $OD^2 = OP^2 + PD^2$, according to the theorem of Pythagoras ---------------------- (1)

Again, in right ΔQOF, where $\angle OQF = 90°$ $OF^2 = OQ^2 + QF^2$, according to the theorem of Pythagoras ---------------------- (2)

Since OD and OF are the radii of the same circle, hence $OD^2 = OF^2$.

$\therefore$ From (1) and (2) we get,

$OP^2 + PD^2 = OQ^2 + QF^2$

or, $OP^2 + \left(\frac{1}{2}CD\right)^2 = OQ^2 + \left(\frac{1}{2}EF\right)^2$

or, $OP^2 + \frac{1}{4}CD^2 = OQ^2 + \frac{1}{4}EF^2$

or, $4\,OP^2 + CD^2 = 4OQ^2 + EF^2$

or, $EF^2 - CD^2 = 4OP^2 - 4OQ^2$ ------------ (3)

Here, $EF^2 - CD^2 > O$, since EF > CD (given)

$\therefore$ From (3), we have,

$4OP^2 - 4OQ^2 > O$

or, $OP^2 - OQ^2 > O$

or, $OP^2 > OQ^2$

or, OP > OQ ------------------ (4)

The inequality (4) refers to that CD is farther to EF from O. That is, EF is nearer to the centre of the circle compared to CD. **(Proved)**

Ques. 05 A circle with centre O. PQ is the diameter and AB and CD are the two chords other than diameter. Where AB > CD.

a. The perimeter of a circular sheet is 157 cm. Find its radius and area. *[Easy]* 2

b. Prove that, the chord AB is nearer to the centre than the chord CD. *[Medium]* 4

c. Prove that, PQ is the greatest chord of the circle. *[Hard]* 4

● *Barisal Board 2017*

Solution to Question No. 05 :

a Let the r cm is the radius of a circular sheet whose perimeter is 157 cm.

$\therefore$ Radius of the circle, $2\pi r = 157$

$$\Rightarrow r = \frac{157}{2\pi} = 25$$

Again, we know,

area of a circle with radius r unit $= \pi r^2$ sq. unit

$\therefore$ Area of the circle with radius,

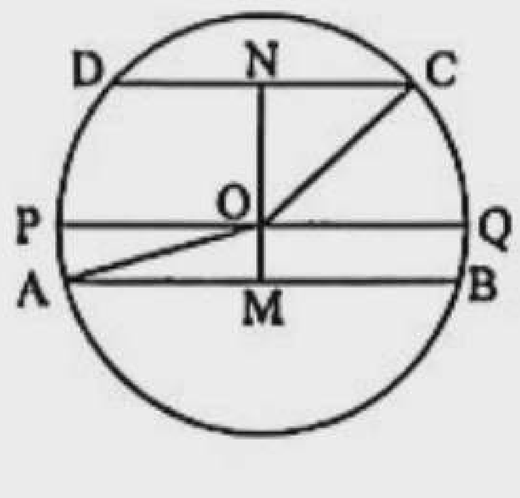
r = 25 cm is $3.14 \times (25)^2$ sq. cm

= 1962.5 sq. cm

So, the radius of the circle is 25 cm and area of the circle is 1962.5 sq. cm.

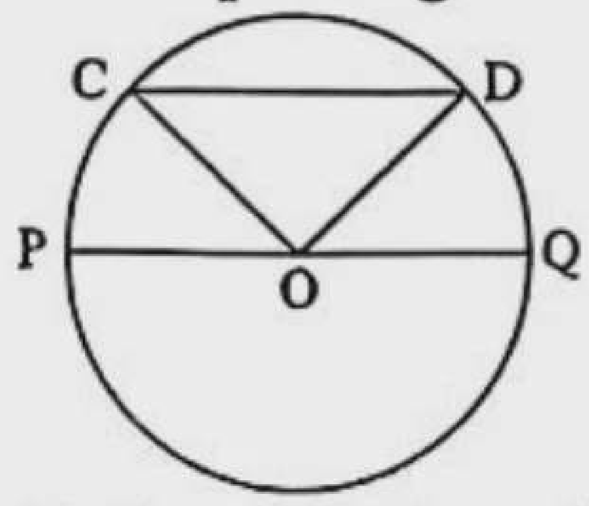
b Let there is a circle with centre O and PQ is one of its diameter. Its chord AB is greater than its chord CD i.e. AB > CD where AB and CD are other than diameter.
Now, it is required to prove that AB is nearer to the centre than CD.

Construction : OM ⊥ AB and ON ⊥ DC are drawn from the centre O of the circle. O, A and O, C are joined.



Proof : In right ΔAOM, $OA^2 = OM^2 + AM^2$, according to theorem of Pythagoras ----- (1)
Again, in right ΔCON,
$OC^2 = NC^2 + ON^2$, according to theorem of Pythagoras -------------------- (2)
Since OA and OC are the radii of the same circle ABCD, OA = OC.
∴ From (1) and (2), we can have,
$OM^2 + AM^2 = ON^2 + NC^2$
or, $AM^2 - NC^2 = ON^2 - OM^2$
Now, $AM^2 - NC^2 > O$, since AM > NC
∴ $ON^2 - OM^2 > O$ or, ON > OM
∴ Proved the proposition.

c Let there is a circle with centre O and PQ is one of its diameter and CD is any chord other than a diameter. Now it is required to prove that PQ is the greatest chord and in this case it will be sufficient to prove, PQ > CD for proving the hypothesis.



Construction : O, C and O, D are joined.
Proof : Here, OP = OQ = OC = OD, since they are the radii of the same circle PQDC with centre O.
Now in ΔCOD,
OC + OD > CD
or, OP + OQ > CD
or, PQ > CD
So, PQ is greater than CD, that is, diameter is greater than any chord of a circle. **(Proved)**

Solutions to Textual Activities Along with textual reference

Activity 01 Draw three circles of different radius of your choice and complete thae table below by measuring diameter and circumference. Are the ratios of circumference and radius approximately the same? ▸ Textbook Page 165

Circle	Radius	Circumference	Diameter	Circumference /Diameter
1	3.5 cm	22 cm	7.0 cm	22/7 = 3.142

Solution :

Circle	Radius	Circumference	Diameter	Circumference/ Diameter
1	3.5 cm	22 cm	7.0 cm	22/7 = 3.142
2	4.0 cm	25.136 cm	8.0 cm	25.136/8 = 3.142
3	4.5 cm	28.278 cm	9.0 cm	28.278/9 = 3.142
4	5.0 cm	31.42 cm	10 cm	31.42/10 = 3.142

From the above table, it is seen that, circumference/ Diameter are same in every casse. So, we can say that, the ratio of circumference and diameter is constatn.

Super Suggestions

Super Suggestions with 100% preparatory questions selected by the Master Trainer Panel

Dear learners, important multiple choice, short and creative questions of this chapter selected by Master Trainer Panel for Half-Yearly and Annual Exams are presented below. Learn the answers to the mentioned questions well to ensure 100% preparation.

Question Pattern		7★	5★	3★
MCQs with Answers	Learn each MCQs in this chapter thoroughly.			
Short Q/A	Exercise 10.1	1, 4, 5, 8, 9, 15	2, 3, 6, 10, 11	7, 12, 13, 17
	Exercise 10.2	1, 4, 5	2, 3	6
	Exercise 10.3	2, 5, 6, 10, 11, 13	1, 3, 7, 8, 14	4, 9, 12, 15, 16
Creative Q/A	Exercise 10.1	1, 4, 5	2, 3	6
	Exercise 10.2	1, 4	3	2
	Exercise 10.3	1, 4	2	3

Assessment & Evaluation

A question bank presented in the form of a class test to assess the preparation

Class Test Time : 3 hours **Mathematics** Full marks : 100

Class : Eight

Multiple Choice Questions (Each question carries 1 mark) $1 \times 30 = 30$

[N.B. : Answer all the questions. Each question carries one mark. Block fully, with a ball-point pen, the circle of the letter that stands for the correct/best answer in the "Answer Sheet" for Multiple Choice Question Type Examination.]

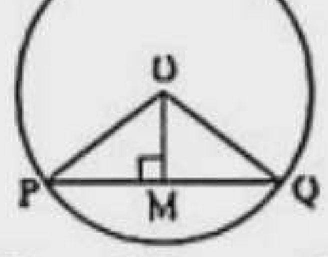
1. **A straight line intersects a circle at how many points?**
ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4
2. **How many parts does a chord divide a circle?**
ⓐ One ⓑ Two ⓒ Three ⓓ Four
3. **Each chord divides a circle into how many arcs?**
ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4
4. **In a circle—.**
i. the perpendicular from the centre to a chord bisects the chord
ii. a straight line can intersect it at more than two points
iii. the diameter is twice of its radius
Which one of the following is true?
ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii
5. **What is subtended angle at the centre of a circle?**
ⓐ 0° ⓑ 90° ⓒ 180° ⓓ 360°
6. **What is the angle subtended at the centre of a circle?**
ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°

■ **Answer to the questions 7 and 8 with the help of given information :**

In the figure MN = 12 cm and OP = 8 cm.

7. **What is the value of PN?**
ⓐ 4 cm ⓑ 6 cm ⓒ 8 cm ⓓ 10 cm
8. **What is the area of triangle OPM?**
ⓐ 20 sq cm ⓑ 24 sq cm
ⓒ 48 sq cm ⓓ 96 sq cm
9. **What is the circumference of the circular garden with diameter 6 cm?**
ⓐ 36π cm ⓑ 12π ⓒ 9π ⓓ 6π
10. **What is the circumference of the circle of 5 cm diameter?**
ⓐ 15 cm ⓑ 15.71 cm ⓒ 17.7 cm ⓓ 18.7 cm
11. **The measure of a semicircular angle is —?**
ⓐ 60° ⓑ 70° ⓒ 80° ⓓ 90°
12. **How many tangents can be drawn from a point inside circle?**
ⓐ 1 ⓑ 2 ⓒ 3 ⓓ No one
13. **How many vertices of a cyclic quadrilateral lie on the circle?**
ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 1

■ **Answer the questions no. 14 and 15 with the help of given information :**



In the figure, PQ = 10cm, OM = 6cm.

14. **What is the value of MQ?**
ⓐ 3cm ⓑ 5cm ⓒ 6cm ⓓ 8cm
15. **What is the area of triangle OPM?**
ⓐ 15 sq cm ⓑ 30 sq cm ⓒ 60 sq cm ⓓ 120 sq cm
16. **In $\triangle ABC$, if AB = AC and $\angle A = 80°$ then, what is the value of $\angle B$?**
ⓐ 40° ⓑ 50° ⓒ 60° ⓓ 100°
17. **At the point A of a circle with centre O, a tangent AB subtending an angle $\angle AOB = 60°$. Then $\angle ABO$ = ?**
ⓐ 30° ⓑ 45° ⓒ 60° ⓓ 75°
18. **How many arcs exist when a circle is divided by a chord?**
ⓐ One ⓑ Two ⓒ Three ⓓ Four
19. **How much degrees is subtended at the centre of a circle?**
ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°
20. **If the circumference of the wheel of a car is 5.15 metres, what is the diameter of the wheel?**
ⓐ 0.82 metre ⓑ 0.96 metre
ⓒ 1.28 metre ⓓ 1.64 metre
21. **If the measure of an inscribed angle at the circumference be 35°, what will be the measure of the central angle against it?**
ⓐ 90° ⓑ 70° ⓒ 80° ⓓ 60°
22. **Which one of the following is the angle subtended at the centre of a circle?**
ⓐ 90° ⓑ 180° ⓒ 270° ⓓ 360°
23. **How many maximum points does a straight line intersect a circle?**
ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4
24. **What will be the length of a radius of a circle with circumference 23 cm?**
ⓐ 2.33 cm (approx.) ⓑ 3.66 cm (approx.)
ⓒ 7.32 cm (approx.) ⓓ 11.5 cm (approx.)
25. **The diametre of a wheel of a vehicle is 38 cm. What will be the distance covered by two complete round?**
ⓐ 59.69 cm ⓑ 76 cm
ⓒ 119.38 cm ⓓ 238.76 cm
26. **hat is the circumference in cm of a circle with radius 14 cm?**
ⓐ 14π ⓑ $14\pi^2$ ⓒ 28π ⓓ 196π
27. **The symbol π(pi) is a letter symbol derived from language of—.**
ⓐ Latin ⓑ Japani ⓒ Greek ⓓ Sanskrit
28. **If radius of a circle 2r, what is the area of that circle?**
ⓐ $2\pi r$ sq unit ⓑ $4\pi r$ sq unit
ⓒ πr^2 sq unit ⓓ $4\pi r^2$ sq unit
29. **Which one of the following is the formula for determining area of circle?**
ⓐ $\frac{4}{3}\pi r^3$ ⓑ $4\pi r^2$ ⓒ πr^2 ⓓ $\frac{3}{4}\pi r^3$
30. **How much unit is the diameter of the cylinder?**
ⓐ 2 ⓑ 3 ⓒ 4 ⓓ 6

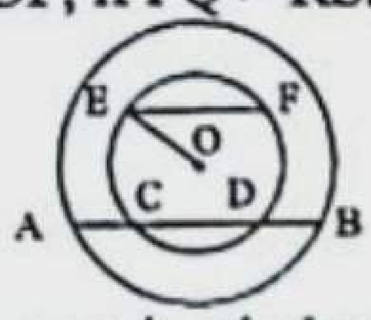
Short-Answer Question (Each question carries 2 marks)

Answer any 10 of the following questions : $2 \times 10 = 20$

1. What is the center of a circle?
2. Into how many arcs does each chord divide a circle? Draw a figure to show it.
3. What is the circumference of a circle?
4. If the length of a chord of a circle is 6 cm and its radius is 4 cm, what is the length of the perpendicular drawn from the center to the chord in cm?
5. In a circle, AB and CD are two equal chords, and the distance from the center to AB is 5 cm. Find the distance from the center to CD.
6. In a circle, PQ and RS are two chords other than the diameter. The distances from the center to the chords are equal. If PQ = 15 cm, find the length of RS.
7. In the circle with center O, AB and CD are two equal chords that intersect each other at point E. If AB = 7 cm and AE = 5 cm, what is the length of DE?
8. In a circle, two parallel chords AB and CD are drawn from the two ends of a diameter on opposite sides. If CD = 7 cm, what is the length of AB?
9. What is the ratio of the circumference and diameter of a circle?
10. If the diameter of a wheel is 35 cm, how many cm will the wheel cover in two rotations?
11. If the diameter of a wheel of a vehicle is 38 cm, how many times will the wheel rotate to cover a distance of 119.3808 meters?
12. If the length of the diagonal of a square located inside a circle is 14 cm, what is the area of the circle?
13. What is the diameter of a circle with an area of 1962.5 square cm?
14. What is the area of a circular garden with a diameter of 12.8 meters?
15. The diameter of a right circular cylinder is 8 cm and its height is 12 cm. Find the total surface area of the cylinder ($\pi = 3.14$).

Creative Question (Each question carries 10 marks)

Answer any 5 of the following questions : $10 \times 5 = 50$

1. C is the centre of a circle with radius 3 cm. T is the foot-point of a pillar 10 cm away from the centre.
   a. Now draw the geometric figure according to the given information. 2
   b. Draw two tangents from the foot-point of the pillar to the circle and show that the points of contact are at equal distance from the foot-point of the pillar. 4
   c. By considering the chord of contact of the tangents as the side of an equilateral triangle, prove that the tangents at the vertices of the triangle form an equilateral triangle. 4
2. In a circle with centre O, CM and PS are two chords other than diametre and their mid-points are X and Y respectively.
   a. Find the length of OC, when CM = 16 cm. OX = 6 cm. 2
   b. Prove that, $OX \perp CM$. 4
   c. If CM > PS, then prove that, OX < OY. 4
3. AB is diameter and CD is chord other than diameter of circle with centre O.
   a. Find out the area of a circular field with diameter 6.4 metre. 2
   b. Prove that AB > CD. 4
   c. If E is the midpoint of CD, then prove that $OE \perp CD$. 4
4. AB and CD are two chords other than diametre of circle ABCD with centre O. P and Q are two mid-points of AB and CD respectively.
   a. Determine the radius of a circle of which the circumference is 44 cm. 2
   b. If AB is a diametre, prove that, AB > CD. 4
   c. If AB = CD, prove that, PO = QO. 4
5. In a circle with centre 'O', PQ and RS are two equal chords and their mid points are M and N respectively.
   a. Find the radius of the circle with area of 314 sq. cm. 2
   b. Prove that chords are equidistant from the centre. 4
   c. If the chords PQ and RS bisect each other, prove that two parts of one chord are equal to the two parts of the other. 4
6. PQ and RS are two chords other than diameter in a circle with centre O. OE and OF are the perpendiculars from O to the chords PQ and RS respectively. MN is the diameter.
   a. Determine the peremeter of the circle, if MN = 8 cm. 2
   b. Prove that, MN is the greatest chord of that circle. 4
   c. Prove that, OE < OF, if PQ > RS. 4
7. 

   In the figure, two co-centric circle are shown. O is the centre of the two circles and OE = 8 cm.
   a. The area of the larger circle 254.34 sq. metre. What is the area between two circumference of the two circles? 2
   b. Prove that AC = BD. 4
   c. In the figure EF > CD, prove that the chord EF is nearer than the chord CD to the centre. 4
8. A circle with centre O. PQ is the diameter and AB and CD are the two chords other than diameter. Where AB > CD.
   a. The perimeter of a circular sheet is 157 cm. Find its radius and area. 2
   b. Prove that, the chord AB is nearer to the centre than the chord CD. 4
   c. Prove that, PQ is the greatest chord of the circle. 4

Answer Sheet ▸ Multiple Choice Questions

1	ⓑ	2	ⓑ	3	ⓑ	4	ⓑ	5	ⓓ	6	ⓓ	7	ⓑ	8	ⓑ	9	ⓓ	10	ⓑ	11	ⓓ	12	ⓓ	13	ⓒ	14	ⓑ	15	ⓐ
16	ⓑ	17	ⓐ	18	ⓑ	19	ⓓ	20	ⓓ	21	ⓑ	22	ⓓ	23	ⓒ	24	ⓑ	25	ⓓ	26	ⓒ	27	ⓒ	28	ⓓ	29	ⓒ	30	ⓓ

Solving Reference ▸ Short-Answer Questions

1 ▸ See Page 348; Ques. 03
2 ▸ See Page 348; Ques. 06
3 ▸ See Page 348; Ques. 10
4 ▸ See Page 349; Ques. 19
5 ▸ See Page 359; Ques. 01
6 ▸ See Page 359; Ques. 04
7 ▸ See Page 360; Ques. 06
8 ▸ See Page 360; Ques. 08
9 ▸ See Page 369; Ques. 01
10 ▸ See Page 369; Ques. 03
11 ▸ See Page 369; Ques. 05
12 ▸ See Page 369; Ques. 07
13 ▸ See Page 370; Ques. 09
14 ▸ See Page 370; Ques. 11
15 ▸ See Page 370; Ques. 13

Solving Reference ▸ Creative Questions

1 ▸ See Page 350; Ques. 02
2 ▸ See Page 352; Ques. 04
3 ▸ See Page 353; Ques. 06
4 ▸ See Page 361; Ques. 02
5 ▸ See Page 362; Ques. 04
6 ▸ See Page 372; Ques. 03
7 ▸ See Page 373; Ques. 04
8 ▸ See Page 373; Ques. 05