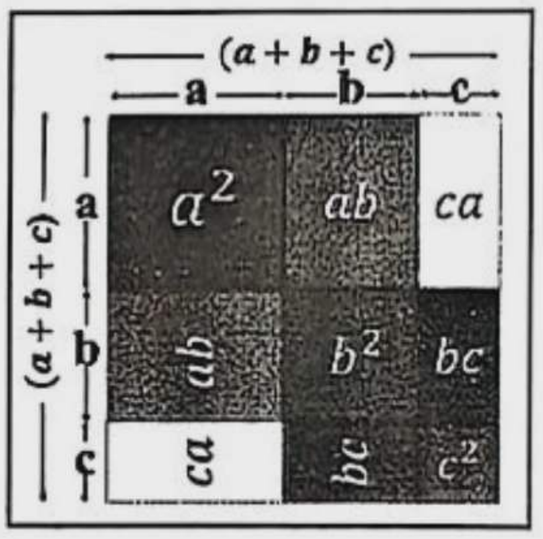
Chapter 04

Algebraic Formulae and Applications

Contents for Discussion

• Algebraic Formulae • Formulae of cubes and corollaries • Two more Formulae related to Cubes • Resolving into Factors • Factors of the expression of the form $x^2 + px + q$ • Factors of the expression in the form of $ax^2 + bx + c$ • H.C.F and L.C.M of Algebraic Expressions • Highest Common Factor (H.C.F.) • Least Common Multiple (L.C.M.).



Learning Outcomes : After studying this chapter I will be able to–

- Find the square of binomial and trinomial expressions, simplify and evaluate by applying algebraic formulae
- Find the cube of binomial and trinomial expressions, simplify and evaluate by applying algebraic formulae
- Factorize the expressions with the help of middle term distribution
- Find H.C.F. and L.C.M. of algebraic expressions.

Practice

Solutions to Mathematical Problems following 100% accurate format for best prep.

Dear learners, mathematical problems of this chapter have been divided into exercise, multiple choice, short, creative and exercise-based activities in light of the learning outcomes. Practice the solutions well to ensure the best preparation in the exam.

Exercise 4.1 : Algebraic Formulae

At a Glance Important Contents of Exercise

- **Prove of the algebraic formulae $(a + b)^2 = a^2 + 2ab + b^2$:**

Proof : The meaning of $(a + b)^2$ is multiplying $(a + b)$ with $(a + b)$.

$\therefore (a + b)^2 = (a + b)(a + b)$

$= a(a + b) + b(a + b)$

[Multiplying polynomial expresseon with polynomial expression]

$= a^2 + ab + ba + b^2$

$= a^2 + ab + ab + b^2$

$\therefore (a + b)^2 = a^2 + 2ab + b^2$

That is, the square of sum of two expression = Square of 1st quantity + 2 × 1st quantity × 2nd quantity + square of 2nd quantity.

- **Necessary formula :**

Formula 1 : $(a + b)^2 = a^2 + 2ab + b^2$

Formula 2 : $(a - b)^2 = a^2 - 2ab + b^2$

Formula 3 : $a^2 - b^2 = (a + b)(a - b)$

Formula 4 : $(x + a)(x + b) = x^2 + (a + b)x + ab$

Formula 5 : $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

- **Necessary Corollary :**

Corollary 1 : $a^2 + b^2 = (a + b)^2 - 2ab$

Corollary 2 : $a^2 + b^2 = (a - b)^2 + 2ab$

Corollary 3 : $(a + b)^2 = (a - b)^2 + 4ab$

Corollary 4 : $(a - b)^2 = (a + b)^2 - 4ab$

Corollary 5 : $2(a^2 + b^2) = (a + b)^2 + (a - b)^2$

Corollary 6 : $4ab = (a + b)^2 - (a - b)^2$

or, $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

Corollary 7 : $a^2 + b^2 + c^2 = (a + b + c)^2 - 2(ab + bc + ac)$

Corollary 8 : $2(ab + bc + ac) = (a + b + c)^2 - (a^2 + b^2 + c^2)$

Solutions to Exercise Problems — Let's solve the textbook problems

Solutions to Mathematical Problems

1. Find the square of the following expressions with the help of formulae :

(a) $5a + 7b$ (b) $6x + 3$
(c) $7p - 2q$ (d) $ax - by$
(e) $x^3 + xy$ (f) $11a - 12b$
(g) $6x^2y - 5xy^2$ (h) $-x - y$
(i) $-xyz - abc$ (j) $a^2x^3 - b^2y^4$
(k) 108 (l) 606
(m) 597 (n) $a - b + c$
(o) $ax + b + 2$ (p) $xy + yz - zx$
(q) $3p + 2q - 5r$ (r) $x^2 - y^2 - z^2$
(s) $7a^2 + 8b^2 - 5c^2$

Solution :

(a) Square of $(5a + 7b) = (5a + 7b)^2$
$= (5a)^2 + 2.5a.7b + (7b)^2$
$= 25a^2 + 70ab + 49b^2$

(b) Square of $(6x + 3) = (6x + 3)^2$
$= (6x)^2 + 2.6x.3 + (3)^2$
$= 36x^2 + 36x + 9$

(c) Square of $(7p - 2q) = (7p - 2q)^2$
$= (7p)^2 - 2.7p.2q + (2q)^2$
$= 49p^2 - 28pq + 4q^2$

(d) Square of $(ax - by) = (ax - by)^2$
$= (ax)^2 - 2.ax.by + (by)^2$
$= a^2x^2 - 2abxy + b^2y^2$

(e) Square of $(x^3 + xy) = (x^3 + xy)^2$
$= (x^3)^2 + 2.x^3.xy + (xy)^2$
$= x^6 + 2x^4y + x^2y^2$

(f) Square of $(11a - 12b)$
$= (11a - 12b)^2$
$= (11a)^2 - 2.11a.12b + (12b)^2$
$= 121a^2 - 264ab + 144b^2$

(g) Square of $(6x^2y - 5xy^2)$
$= (6x^2y - 5xy^2)^2$
$= (6x^2y)^2 - 2.6x^2y.5xy^2 + (5xy^2)^2$
$= 36x^4y^2 - 60x^3y^3 + 25x^2y^4$

(h) Square of $(-x - y)$
$= (-x - y)^2$
$= (-x)^2 - 2.(-x).y + (y)^2$
$= x^2 + 2xy + y^2$

(i) Square of $-xyz - abc$
$= (-xyz - abc)^2$
$= \{(-xyz) - (abc)\}^2$
$= (-xyz)^2 - 2(-xyz)(abc) + (abc)^2$
$= x^2y^2z^2 + 2abc\,xyz + a^2b^2c^2$

(j) Square of $a^2x^3 - b^2y^4$
$= (a^2x^3 - b^2y^4)^2$
$= (a^2x^3)^2 - 2.a^2x^3.b^2y^4 + (b^2y^4)^2$
$= a^4x^6 - 2a^2b^2x^3y^4 + b^4y^8$

(k) Square of 108
$= (108)^2 = (100 + 8)^2$
$= (100)^2 + 2.100.8 + (8)^2$
$= 10000 + 1600 + 64$
$= 11664$

(l) Square of 606
$= (606)^2 = (600 + 6)^2$
$= (600)^2 + 2.600.6 + (6)^2$
$= 360000 + 7200 + 36$
$= 367236$

(m) Square of 597
$= (597)^2$
$= (600 - 3)^2$
$= (600)^2 - 2.600.3 + 3^2$
$= 360000 - 3600 + 9 = 356409$

(n) Square of $(a - b + c)$
$= (a - b + c)^2$
$= \{(a - b) + c\}^2$
$= (a - b)^2 + 2.(a - b).c + c^2$
$= a^2 - 2ab + b^2 + 2ac - 2bc + c^2$
$= a^2 + b^2 + c^2 - 2ab - 2bc + 2ca$

(o) Square of $(ax + b + 2)$
$= (ax + b + 2)^2$
$= \{ax + (b + 2)\}^2$
$= (ax)^2 + 2.ax\,(b + 2) + (b + 2)^2$
$= a^2x^2 + 2abx + 4ax + b^2 + 4b + 4$

(p) Square of $(xy + yz - zx)$
$= (xy + yz - zx)^2$
$= \{(xy + yz) - zx\}^2$
$= (xy + yz)^2 - 2.(xy + yz).zx + (zx)^2$
$= (xy)^2 + 2xy.yz + (yz)^2 - 2zx.(xy + yz) + z^2x^2$
$= x^2y^2 + 2xy^2z + y^2z^2 - 2x^2yz - 2xyz^2 + z^2x^2$
$= x^2y^2 + y^2z^2 + z^2x^2 + 2xy^2z - 2x^2yz - 2xyz^2$

(q) Square of $(3p + 2q - 5r)$
$= (3p + 2q - 5r)^2$
$= \{(3p + 2q) - 5r\}^2$
$= (3p + 2q)^2 - 2(3p + 2q).5r + (5r)^2$
$= (3p)^2 + 2.3p.2q + (2q)^2 - 10r(3p + 2q) + 25r^2$
$= 9p^2 + 12pq + 4q^2 - 30pr - 20qr + 25r^2$
$= 9p^2 + 4q^2 + 25r^2 - 30pr - 20qr + 12pq$

(r) Square of $(x^2 - y^2 - z^2)$
$= (x^2 - y^2 - z^2)^2$
$= \{(x^2 - y^2) - z^2\}^2$
$= \{(x^2 - y^2)^2 - 2 \cdot (x^2 - y^2) \cdot z^2 + (z^2)^2\}$
$= (x^2)^2 - 2x^2y^2 + (y^2)^2 - 2z^2x^2 + 2y^2z^2 + z^4$
$= x^4 + y^4 + z^4 - 2x^2y^2 + 2y^2z^2 - 2z^2x^2$

Mathematics

(s) Square of $(7a^2 + 8b^2 - 5c^2)$
$= (7a^2 + 8b^2 - 5c^2)^2$
$= \{(7a^2 + 8b^2) - 5c^2\}^2$
$= (7a^2 + 8b^2)^2 - 2.(7a^2 + 8b^2).5c^2 + (5c^2)^2$
$= (7a^2)^2 + 2.7a^2.8b^2 + (8b^2)^2 - 10c^2(7a^2 + 8b^2) + 25c^4$
$= 49a^4 + 112a^2b^2 + 64b^4 - 70c^2a^2 - 80b^2c^2 + 25c^4$
$= 49a^4 + 64b^4 + 25c^4 + 112a^2b^2 - 80b^2c^2 - 70c^2a^2$

2. Simplify :

(a) $(x + y)^2 + 2(x + y)(x - y) + (x - y)^2$
(b) $(2a + 3b)^2 - 2(2a + 3b)(3b - a) + (3b - a)^2$
(c) $(3x^2 + 7y^2)^2 + 2(3x^2 + 7y^2)(3x^2 - 7y^2) + (3x^2 - 7y^2)^2$
(d) $(8x + y)^2 - (16x + 2y)(5x + y) + (5x + y)^2$
(e) $(5x^2 - 3x - 2)^2 + (2 + 5x^2 - 3x)^2 - 2(5x^2 - 3x + 2)(2 + 5x^2 - 3x)$

Solution :

(a) $(x + y)^2 + 2(x + y)(x - y) + (x - y)^2$
$= \{(x + y) + (x - y)\}^2$
$= (x + y + x - y)^2$
$= (2x)^2 = 4x^2$

(b) $(2a + 3b)^2 - 2(2a + 3b)(3b - a) + (3b - a)^2$
$= \{(2a + 3b) - (3b - a)\}^2$
$= (2a + 3\cancel{b} - 3\cancel{b} + a)^2 = (3a)^2 = 9a^2$

(c) $(3x^2 + 7y^2)^2 + 2(3x^2 + 7y^2)(3x^2 - 7y^2) + (3x^2 - 7y^2)^2$
$= (3x^2 + 7y^2 + 3x^2 - 7y^2)^2$
$= (6x^2)^2 = 36x^4$

(d) $(8x + y)^2 - (16x + 2y)(5x + y) + (5x + y)^2$
$= (8x + y)^2 - 2(8x + y)(5x + y) + (5x + y)^2$
$= \{8x + y - (5x + y)\}^2$
$= (8x + \cancel{y} - 5x - \cancel{y})^2 = (3x)^2$
$= 9x^2$

(e) $(5x^2 - 3x - 2)^2 + (2 + 5x^2 - 3x)^2 - 2(5x^2 - 3x - 2)(2 + 5x^2 - 3x)$
$= (5x^2 - 3x - 2)^2 - 2(5x^2 - 3x - 2)(5x^2 - 3x + 2) + (5x^2 - 3x + 2)^2$
$= \{5x^2 - 3x - 2 - (5x^2 - 3x + 2)\}^2$
$= (5\cancel{x}^2 - 3\cancel{x} - 2 - 5\cancel{x}^2 + 3\cancel{x} - 2)^2$
$= (-4)^2 = 16$

3. Find the product by applying formulae :

(a) $(x + 7)(x - 7)$
(b) $(5x + 13)(5x - 13)$
(c) $(xy + yz)(xy - yz)$
(d) $(ax + b)(ax - b)$
(e) $(a + 3)(a + 4)$
(f) $(ax + 3)(ax + 4)$
(g) $(6x + 17)(6x - 13)$
(h) $(a^2 + b^2)(a^2 - b^2)(a^4 + b^4)$
(i) $(ax - by + cz)(ax + by - cz)$
(j) $(3a - 10)(3a - 5)$
(k) $(5a + 2b - 3c)(5a + 2b + 3c)$
(l) $(ax + by + 5)(ax + by + 3)$

Solution :

(a) $(x + 7)(x - 7)$
$= (x)^2 - (7)^2$
$= x^2 - 49$

(b) $(5x + 13)(5x - 13)$
$= (5x)^2 - (13)^2$
$= 25x^2 - 169$

(c) $(xy + yz)(xy - yz)$
$= (xy)^2 - (yz)^2$
$= x^2y^2 - y^2z^2$

(d) $(ax + b)(ax - b)$
$= (ax)^2 - (b)^2 = a^2x^2 - b^2$

(e) $(a + 3)(a + 4)$
$= (a)^2 + (3 + 4)a + 3 \times 4$
$= a^2 + 7a + 12$

(f) $(ax + 3)(ax + 4)$
$= (ax)^2 + (3 + 4)ax + 3 \times 4$
$= a^2x^2 + 7ax + 12$

(g) $(6x + 17)(6x - 13)$
$= (6x)^2 + (17 - 13) \times 6x + 17 \times (-13)$
$= 36x^2 + 4 \times 6x - 221$
$= 36x^2 + 24x - 221$

(h) $(a^2 + b^2)(a^2 - b^2)(a^4 + b^4)$
$= \{(a^2)^2 - (b^2)^2\}(a^4 + b^4)$
$= (a^4 - b^4)(a^4 + b^4)$
$= (a^4)^2 - (b^4)^2 = a^8 - b^8$

(i) $(ax - by + cz)(ax + by - cz)$
$= \{(ax) - (by - cz)\}\{(ax) + (by - cz)\}$
$= (ax)^2 - (by - cz)^2$
$= a^2x^2 - (b^2y^2 - 2bcyz + c^2z^2)$
$= a^2x^2 - b^2y^2 + 2bcyz - c^2z^2$
$= a^2x^2 - b^2y^2 - c^2z^2 + 2bcyz$

(j) $(3a - 10)(3a - 5)$
$= (3a)^2 + (-10 - 5) \times 3a + (-10)(-5)$
$= 9a^2 + (-15) \times 3a + 50$
$= 9a^2 - 45a + 50$

(k) $(5a + 2b - 3c)(5a + 2b + 3c)$
$= \{(5a + 2b) - 3c\}\{(5a + 2b) + 3c\}$
$= (5a + 2b)^2 - (3c)^2$
$= 25a^2 + 20ab + 4b^2 - 9c^2$
$= 25a^2 + 4b^2 - 9c^2 + 20ab$

(l) $(ax + by + 5)(ax + by + 3)$
$= \{(ax + by) + 5\}\{(ax + by) + 3\}$
$= (ax + by)^2 + (5 + 3)(ax + by) + 5 \times 3$
$= a^2x^2 + b^2y^2 + 2abxy + 8ax + 8by + 15$

4. If $a = 4$, $b = 6$ and $c = 3$, find the value of $4a^2b^2 - 16ab^2c + 16b^2c^2$.

Solution :

Here, $a = 4$, $b = 6$ and $c = 3$

$\therefore$ The given expression
$= 4a^2b^2 - 16ab^2c + 16b^2c^2$
$= (2ab)^2 - 2.2ab.4bc + (4bc)^2$

$= (2ab - 4bc)^2$

$= (2 \times 4 \times 6 - 4 \times 6 \times 3)^2$

[Putting the value of a, b, c]

$= (48 - 72)^2$

$= (-24)^2 = 576.$

5. If $x - \frac{1}{x} = 3$, find the value of $x^2 + \frac{1}{x^2}$.

Solution :

Here, $x - \frac{1}{x} = 3$

$\therefore$ The given expression

$= x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2.x.\frac{1}{x}$

$= (3)^2 + 2$ [Putting $x - \frac{1}{x} = 3$]

$= 9 + 2 = 11$

6. If $a + \frac{1}{a} = 4$, what is the value of $a^4 + \frac{1}{a^4}$?

Solution :

Here, we have, $a + \frac{1}{a} = 4$

$\therefore$ The given expression, $a^4 + \frac{1}{a^4}$

$= (a^2)^2 + \left(\frac{1}{a^2}\right)^2 = \left(a^2 + \frac{1}{a^2}\right)^2 - 2.a^2.\frac{1}{a^2}$

$= \left\{\left(a + \frac{1}{a}\right)^2 - 2.a.\frac{1}{a}\right\}^2 - 2$

$= \{(4)^2 - 2\}^2 - 2$ [Putting $a + \frac{1}{a} = 4$]

$= (16 - 2)^2 - 2 = (14)^2 - 2$

$= 196 - 2 = 194.$

$\therefore$ The determined value is 194.

7. If $m = 6$, $n = 7$, find the value of $16(m^2 + n^2)^2 + 56(m^2 + n^2)(3m^2 - 2n^2) + 49(3m^2 - 2n^2)^2$.

Solution :

$16(m^2 + n^2)^2 + 56(m^2 + n^2)(3m^2 - 2n^2) + 49(3m^2 - 2n^2)^2$

$= \{4(m^2 + n^2)\}^2 + 2.4(m^2 + n^2)\,7(3m^2 - 2n^2) + \{7(3m^2 - 2n^2)\}^2$

$= (4m^2 + 4n^2)^2 + 2.(4m^2 + 4n^2)(21m^2 - 14n^2) + (21m^2 - 14n^2)^2$

$= (4m^2 + 4n^2 + 21m^2 - 14n^2)^2$

$= (25m^2 - 10n^2)^2$

$= \{25 \times (6)^2 - 10 \times (7)^2\}^2$ [Putting the value of m and n]

$= (25 \times 36 - 10 \times 49)^2$

$= (900 - 490)^2 = (410)^2 = 168100$

$\therefore$ The determined value is 168100.

8. If $a - \frac{1}{a} = m$, Show that $a^4 + \frac{1}{a^4} = m^4 + 4m^2 + 2$.

Solution :

Here, we have, $a - \frac{1}{a} = m$,

Now,

L.H.S. $= a^4 + \frac{1}{a^4} = (a^2)^2 + \left(\frac{1}{a^2}\right)^2$

$= \left(a^2 + \frac{1}{a^2}\right)^2 - 2.a^2.\frac{1}{a^2}$

$= \left\{\left(a - \frac{1}{a}\right)^2 + 2a.\frac{1}{a}\right\}^2 - 2$

$= (m^2 + 2)^2 - 2$ [Putting $a - \frac{1}{a} = m$]

$= (m^2)^2 + 2.m^2.2 + 2^2 - 2$

$= m^4 + 4m^2 + 4 - 2$

$= m^4 + 4m^2 + 2 = \text{R.H.S.}$

$\therefore a^4 + \frac{1}{a^4} = m^4 + 4m^2 + 2$ **(Showed)**

9. If $x - \frac{1}{x} = 4$, prove that $x^2 + \frac{1}{x^2} = 18$.

Solution :

Here, we have, $x - \frac{1}{x} = 4$

Now, L.H.S. $= x^2 + \left(\frac{1}{x}\right)^2$

$= \left(x - \frac{1}{x}\right)^2 + 2.x.\frac{1}{x}$

$= (4)^2 + 2 = 16 + 2$

$= 18 = \text{R.H.S.}$

$\therefore x^2 + \left(\frac{1}{x}\right)^2 = 18.$ **(Proved)**

10. If $m + \frac{1}{m} = 2$, prove that $m^4 + \frac{1}{m^4} = 2$.

Solution :

Here, we have, $m + \frac{1}{m} = 2$

Now, L.H.S. $= m^4 + \frac{1}{m^4}$

$= (m^2)^2 + \left(\frac{1}{m^2}\right)^2$

$= \left(m^2 + \frac{1}{m^2}\right)^2 - 2.m^2.\frac{1}{m^2}$

$= \left\{\left(m + \frac{1}{m}\right)^2 - 2.m.\frac{1}{m}\right\}^2 - 2$

$= \{(2)^2 - 2\}^2 - 2$ [Putting $m + \frac{1}{m} = 2$]

$= (4 - 2)^2 - 2 = (2)^2 - 2 = 4 - 2$

$= 2 = \text{R.H.S.}$

$\therefore m^4 + \frac{1}{m^4} = 2$ **(Proved)**

11. If $x + y = 12$ and $xy = 27$, find the value of $(x - y)^2$ and $x^2 + y^2$.

Solution :

Here, we have, $x + y = 12$

and $xy = 27$.

$\therefore$ The 1st quantity $= (x - y)^2 = (x + y)^2 - 4xy$

$= (12)^2 - 4 \times 27$

[Putting the value of $x + y$ and xy]

$= 144 - 108 = 36$

Again, the 2nd quantity

$= x^2 + y^2 = (x + y)^2 - 2xy$

$= (12)^2 - 2 \times 27$

[Putting the value of $x + y$ and xy]

$= 144 - 54 = 90$

12. If $a + b = 13$ and $a - b = 3$, find the value of $2a^2 + 2b^2$ and ab.

Solution :

Here, we have, $a + b = 13$

and $a - b = 3$

Now, the 1st quantity $= 2a^2 + 2b^2$

$= (a + b)^2 + (a - b)^2$

$= (13)^2 + (3)^2$

[Putting the value of $a + b$ and $a - b$]

$= 169 + 9 = 178$

Again, the 2nd quantity $= ab$

$= \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$= \left(\frac{13}{2}\right)^2 - \left(\frac{3}{2}\right)^2$

[Putting the value of $a + b$ and $a - b$]

$= \frac{169}{4} - \frac{9}{4} = \frac{160}{4} = 40$

Ans : 178 and 40.

13. Express as the difference of the square of two expressions :

(a) $(5p - 3q)(p + 7q)$ (b) $(6a + 9b)(7b - 8a)$

(c) $(3x + 5y)(7x - 5y)$ (d) $(5x + 13)(5x - 13)$

Solution :

(a) $(5p - 3q)(p + 7q)$

$= \left(\frac{5p - 3q + p + 7q}{2}\right)^2 - \left(\frac{5p - 3q - p - 7q}{2}\right)^2$

$\left[\text{since, } ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right]$

$= \left(\frac{6p + 4q}{2}\right)^2 - \left(\frac{4p - 10q}{2}\right)^2$

$= \left\{\frac{2(3p + 2q)}{2}\right\}^2 - \left\{\frac{2(2p - 5q)}{2}\right\}^2$

$= (3p + 2q)^2 - (2p - 5q)^2$

(b) $(6a + 9b)(7b - 8a)$

$= \left(\frac{6a + 9b + 7b - 8a}{2}\right)^2 - \left(\frac{6a + 9b - 7b + 8a}{2}\right)^2$

$\left[\text{Since, } ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right]$

$= \left(\frac{16b - 2a}{2}\right)^2 - \left(\frac{14a + 2b}{2}\right)^2$

$= \left\{\frac{2(8b - a)}{2}\right\}^2 - \left\{\frac{2(7a + b)}{2}\right\}^2$

$= (8b - a)^2 - (7a + b)^2$

(c) $(3x + 5y)(7x - 5y)$

$= \left(\frac{3x + 5y + 7x - 5y}{2}\right)^2 - \left(\frac{3x + 5y - 7x + 5y}{2}\right)^2$

$\left[\text{Since, } ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right]$

$= \left(\frac{10x}{2}\right)^2 - \left\{\frac{2(5y - 2x)}{2}\right\}^2$

$= (5x)^2 - (5y - 2x)^2$

$= (5x)^2 - \{-(2x - 5y)\}^2$

$= (5x)^2 - (2x - 5y)^2$

(d) $(5x + 13)(5x - 13)$

$= (5x)^2 - (13)^2$ $\{\because (a + b)(a - b) = a^2 - b^2\}$

Creative Questions with Solutions

Ques. 14 The two numbers are a and b. Here $a > b$. The sum of two numbers is 12 and the product is 32.

A. Multiply with the help of formulae : $(2x + 3)(2x - 7)$. 2

B. Find out the value of $2a^2 + 2b^2$. 4

C. Prove that, $(a + 2b)^2 - 5b^2 = 176$. 4

Solution to Question No. 14 :

A $(2x + 3)(2x - 7)$

$= (2x)^2 + (3 - 7)2x + (3)(-7)$

$= 4x^2 + (-4)2x - 21$

$= 4x^2 - 8x - 21$

B According to the given conditions,

$a + b = 12$ ---------- (i)

and $ab = 32$ ---------- (ii)

Now,

$2a^2 + 2b^2 = 2(a^2 + b^2)$

$= 2\{(a + b)^2 - 2ab\}$

$= 2\{(12)^2 - 2 \times 32\}$ [Using (i) & (ii)]

$= 2\{144 - 64\}$

$= 2 \times 80$

$= 160.$

Mathematics

G According to the given conditions,

$a + b = 12$ ---------- (i)

and $ab = 32$ ---------- (ii)

$\therefore a - b = \sqrt{(a+b)^2 - 4ab}$

$= \sqrt{(12)^2 - 4 \times 32} = \sqrt{144 - 128}$

$= \sqrt{16} = \pm 4$

$\because a > b$

So, $a - b = 4$ ---------- (iii)

Now,

L.H.S. $= (a + 2b)^2 - 5b^2 = a^2 + 2a.2b + 4b^2 - 5b^2$

$= a^2 + 4ab + 4b^2 - 5b^2$

$= a^2 - b^2 + 4ab$

$= (a + b)(a - b) + 4ab$

$= 12 \times 4 + 4 \times 32$ [Using (i), (ii), (iii)]

$= 48 + 128 = 176 =$ R.H.S

$\therefore (a + 2b)^2 - 5b^2 = 176$ **(Proved)**

Multiple Choice Q/A Designed as per topic

4.1 Algebraic Formulae ▸ Textbook Page 48

Mathematics

1. What is called the general rule expressed by algebraic symbols? *(Easy)*

ⓐ Formula ⓑ Corollary

a ⓒ Solutions ⓓ Expression

2. What is the square of 95? *(Medium)*

a ⓐ 9025 ⓑ 9015 ⓒ 9035 ⓓ 9005

3. What is the square of $\left(a + \frac{1}{a}\right)$? *(Hard)*

ⓐ $a^2 + \frac{1}{a^2} + 2$ ⓑ $a^2 + \frac{1}{a^2} - 2$

a ⓒ $a^2 + \frac{1}{a^2}$ ⓓ $a^2 - \frac{1}{a^2}$

4. What is the square of $x - \frac{1}{x}$? *(Hard)*

ⓐ $x^2 + \frac{1}{x^2} + 2$ ⓑ $x^2 + \frac{1}{x^2}$

c ⓒ $x^2 + \frac{1}{x^2} - 2$ ⓓ $x^2 - \frac{1}{x^2}$

5. If $x + \frac{1}{x} = 3$, what is the value of $\left(x^2 + \frac{1}{x^2}\right)$? *(Medium)*

c ⓐ 9 ⓑ 5 ⓒ 7 ⓓ 11

6. If $x - \frac{1}{x} = 2$, what is the value of $\left(x^2 + \frac{1}{x^2}\right)$? *(Medium)*

d ⓐ 10 ⓑ 8 ⓒ 4 ⓓ 6

7. What is the value of $36x^2 - 48xy + 16y^2$, when $x = 3$ and $y = 2$? *(Medium)*

a ⓐ 100 ⓑ 110 ⓒ 120 ⓓ 140

8. If p and q are two algebraic quantities, what is the difference of the square of the two quantities? *(Easy)*

ⓐ $p^2 + 2pq + q^2$ ⓑ $p^2 - 2pq + q^2$

d ⓒ $p^2 + q^2$ ⓓ $p^2 - q^2$

9. If $x^2 - \sqrt{5}x + 1 = 0$, then $x + \frac{1}{x} = ?$ *(Easy) [DB '19]*

c ⓐ $-2\sqrt{5}$ ⓑ $-\sqrt{5}$ ⓒ $\sqrt{5}$ ⓓ $5\sqrt{5}$

10. If $x + y = 6$ and $x - y = 4$, what is the value of $5xy$? *(Easy) [DB '19]*

c ⓐ 16 ⓑ 24 ⓒ 25 ⓓ 36

11. If $m^2 - 6m - 1 = 0$, then $m^2 + \frac{1}{m^2} = ?$ *(Medium) [DB '19]*

d ⓐ 32 ⓑ 34 ⓒ 36 ⓓ 38

12. If $x^2 + \frac{1}{x^2} = 10$, then what is the value of $x + \frac{1}{x}$? *(Easy) [JB '19]*

b ⓐ $\sqrt{6}$ ⓑ $2\sqrt{2}$ ⓒ $2\sqrt{3}$ ⓓ $\sqrt{14}$

13. Which one is the product of $(x - 11)$ and $(x + 5)$? *(Medium) [JB '19]*

ⓐ $x^2 - 6x + 55$ ⓑ $x^2 + 6x - 55$

d ⓒ $x^2 + 6x + 55$ ⓓ $x^2 - 6x - 55$

14. If $x - \frac{1}{x} = 5$, which one is the value of $\left(x + \frac{1}{x}\right)$? *(Hard) [CB '19]*

d ⓐ $\sqrt{21}$ ⓑ $\sqrt{23}$ ⓒ $\sqrt{27}$ ⓓ $\sqrt{29}$

15. If $a = x + \frac{1}{x}$ and $b = x - \frac{1}{x}$, then $(a - b)^2 = ?$ *(Hard) [CtgB '19]*

c ⓐ $2x$ ⓑ $4x$ ⓒ $4x^2$ ⓓ 0

16. If $p + q = 7$ and $pq = 9$, what is the value of $p^2 + q^2$? *(Medium) [SB '19]*

c ⓐ 67 ⓑ 40 ⓒ 31 ⓓ 13

17. If $m - \frac{1}{m} = 4$, what is the value of $\left(m + \frac{1}{m}\right)^2$? *(Easy) [SB '19]*

a ⓐ 20 ⓑ 18 ⓒ 14 ⓓ 12

18. If $a + b = 5$ and $a - b = 2$, what is the value of $a^2 + b^2$? *(Hard) [BB '19]*

b ⓐ $\frac{21}{2}$ ⓑ $\frac{29}{2}$ ⓒ 21 ⓓ 29

19. If $x^2 - 4x - 1 = 0$, when $x \neq 0$, what is the value of $\left(x - \frac{1}{x}\right)^2$? *(Hard) [BB '19]*

d ⓐ -4 ⓑ -1 ⓒ 4 ⓓ 16

20. What is the square of $(-x^2 + y)$? *(Hard) [BB '19]*

ⓐ $x^2 + y^2$ ⓑ $x^2 + y^2 - 2xy$

b ⓒ $-x^2 - 2xy - y^2$ ⓓ $x^2 + 2xy + y^2$

21. If $a^2 - 3a - 1 = 0$, then $a^2 + \frac{1}{a^2} = ?$ *(Easy) [DjB '19]*

c ⓐ 5 ⓑ 7 ⓒ 11 ⓓ 13

22. If $x + y = 7$ and $xy = 12$, then $2(x - y)^2 = ?$ *(Easy)* *[DjB '19]*

ⓑ ⓐ 1 ⓑ 2 ⓒ 25 ⓓ 75

23. Which one of the following will be right if we express $(3x - 7)(7 + 3x)$ in the form of the difference between two squares? *(Easy)* *[DB '18]*

ⓐ $3x^2 - 49$ ⓑ $(3x)^2 - (49)^2$

ⓓ ⓒ $9x^2 - 7$ ⓓ $9x^2 - 49$

24. What will be added to $x^2 - 8x - 3$ to make a perfect square? *(Hard)* *[RB '18]*

ⓐ ⓐ 19 ⓑ 13 ⓒ 7 ⓓ 3

25. What is value of 4ab, if $a + b = 6$ and $a - b = 4$? *(Medium)* *[BB '18]*

ⓑ ⓐ 7 ⓑ 20 ⓒ 26 ⓓ 52

26. What is the value of $\left(p - \frac{1}{p}\right)^2$ if $p + \frac{1}{p} = 2$? *(Easy)* *[BB '18]*

ⓐ ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4

27. If $x^2 + \frac{1}{x^2} = 2$, then which one of the following is the value of $x - \frac{1}{x}$? *(Medium)* *[DjB '18]*

ⓑ ⓐ -2 ⓑ 0 ⓒ 2 ⓓ 4

28. If $x^2 + \frac{1}{x^2} = 1$, what is the value of $x + \frac{1}{x}$? *(Medium)* *[DB '17]*

ⓑ ⓐ $\sqrt{2}$ ⓑ $\sqrt{3}$ ⓒ 2 ⓓ 3

29. What is the value of $\left(a - \frac{1}{a}\right)^2$, if $a + \frac{1}{a} = 4$? *(Medium)* *[RB '17]*

ⓑ ⓐ 8 ⓑ 12 ⓒ 16 ⓓ 20

30. If $x = p + \frac{1}{p}$ and $y = p - \frac{1}{p}$ then $(x + y)^2 = ?$ *(Hard)* *[JB '17]*

ⓓ ⓐ $2p$ ⓑ $4p$ ⓒ $2p^2$ ⓓ $4p^2$

31. If $a^4 + \frac{1}{a^4} = 119$ then $a^2 + \frac{1}{a^2} = ?$ *(Hard)* *[JB '17]*

ⓐ ⓐ 11 ⓑ $\sqrt{119}$ ⓒ 13 ⓓ 19

32. If $x + \frac{1}{x} = 2$, then which one of the following is the value of $x - \frac{1}{x}$? *(Easy)* *[CB '17]*

ⓐ ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 3

33. If $a + b = 7$ and $a - b = 3$, then which one of the following is the value of $a^2 + b^2$? *(Easy)* *[CB '17]*

ⓑ ⓐ 20 ⓑ 29 ⓒ 40 ⓓ 58

34. Which one of the following is correct of expression of $(y + 4)(y + 2)$ as the difference of two squares? *(Medium)* *[CB '17]*

ⓐ ⓐ $(y + 3)^2 - 1$ ⓑ $(y + 4)^2 - 1$

ⓐ ⓒ $(y + 2)^2 - 1$ ⓓ $(x - 3)^2 - 1$

35. Which one is square of $-a - b$? *(Appendix)* *[CtgB '17]*

ⓐ $-a^2 - 2ab - b^2$ ⓑ $a^2 - 2ab + b^2$

ⓒ ⓒ $a^2 + 2ab + b^2$ ⓓ $a^2 - 2ab - b^2$

36. If $x - \frac{1}{x} = 6$, what is the value of $\left(x + \frac{1}{x}\right)^2$? *(Hard)* *[SB '17]*

ⓑ ⓐ 32 ⓑ 40 ⓒ 38 ⓓ 44

37. $(3x + 1)(3x - 4) = ?$ *(Easy)* *[SB '17]*

ⓐ $9x^2 - 9x - 4$ ⓑ $9x^2 + 9x - 4$

ⓐ ⓒ $9x^2 - 9x + 4$ ⓓ $9x^2 + 9x + 4$

38. Which one is square of $x + 2y$? *(Easy)* *[DB '16]*

ⓐ $x^2 + 2xy + y^2$ ⓑ $x^2 + 4xy + 4y^2$

ⓑ ⓒ $x^2 + 2xy + 4y^2$ ⓓ $x^2 + xy + y^2$

39. If $a + b = 5$, $a - b = 4$, $a^2 - b^2 = ?$ *(Easy)* *[DB '16]*

ⓓ ⓐ 9 ⓑ 10 ⓒ 15 ⓓ 20

40. If $a + \frac{1}{a} = 3$, then which is the value of $a^2 + \frac{1}{a^2}$? *(Easy)* *[RB '16]*

ⓑ ⓐ 5 ⓑ 7 ⓒ 11 ⓓ 13

41. Which one of the following is the lowest form of $\frac{x^2 + 4x + 4}{x^2 - 4}$? *(Medium)* *[JB '16]*

ⓐ $\frac{x + 2}{x - 2}$ ⓑ $\frac{x - 2}{x + 2}$

ⓐ ⓒ $\frac{x^2 + 4x + 4}{x - 4}$ ⓓ $\frac{x^2 + 4x + 4}{x + 2}$

42. If $m - \frac{1}{m} = 6$, which one of the following is the value of $m^2 + \frac{1}{m^2}$? *(Easy)* *[CB '16]*

ⓒ ⓐ 31 ⓑ 34 ⓒ 38 ⓓ 40

43. Which is the simplified value of $\left(1 + \frac{1}{x}\right) \div \left(1 - \frac{1}{x^2}\right)$? *(Medium)* *[CB '16]*

ⓐ $\frac{x}{x - 1}$ ⓑ $\frac{x}{x + 1}$

ⓐ ⓒ $\frac{x - 1}{x}$ ⓓ $\frac{x + 1}{x}$

44. If $a^2 - 1 = 5a$, what is the value of $a^2 + \frac{1}{a^2}$? *(Medium)* *[CtgB '16]*

ⓓ ⓐ 21 ⓑ 23 ⓒ 25 ⓓ 27

45. Which one is the square of the algebraic expression $a + b - c$? *(Easy)* *[CtgB '16]*

ⓐ $a^2 + b^2 + c^2$

ⓑ $a^2 + b^2 - c^2 + 2ab - 2bc - 2ca$

ⓒ $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

ⓒ ⓓ $a^2 + b^2 + c^2 + 2ab - 2bc - 2ca$

46. If $x + \frac{1}{x} = 2$, which one of the following is the value of $x^2 + \frac{1}{x^2}$? *(Easy)* *[SB '16]*

ⓐ ⓐ 2 ⓑ 4 ⓒ 6 ⓓ 8

47. If $a - b = 5$ and $ab = 3$ then what is the value of $a^2 + b^2$? *(Easy)* *[DjB '16]*

ⓒ ⓐ 13 ⓑ 19 ⓒ 31 ⓓ 37

48. Which one of the following is the square of $(x - 3y)$? *(Medium)* *[DjB '16]*

ⓐ $x^2 + 6xy + 9y^2$ ⓑ $x^2 - 6xy + 9y^2$

ⓑ ⓒ $x^2 + 6xy - 9y^2$ ⓓ $x^2 - 6xy - 9y^2$

49. Which is the difference of the square of $(x + 6)$ and $(x + 4)$? *(Easy)* *[DjB '16]*

ⓐ $x^2 - 6^2$ ⓑ $x^2 - 4^2$

ⓓ ⓒ $(x)^2 - (10)^2$ ⓓ $(x + 5)^2 - (1)^2$

50. What is to be subtracted from $(4x^2 + 4y^2)$ so that the difference will be a perfect square? *(Medium)* *[Rajuk Uttara Model College, Dhaka]*

ⓐ ⓐ 8xy ⓑ 12xy ⓒ 16xy ⓓ 20xy

51. If $x + y = 6$ and $x - y = 4$, then what is the value of $4xy$? *(Easy)* *[DB '15]*

ⓒ ⓐ 5 ⓑ 13 ⓒ 20 ⓓ 52

52. What is the value of $\frac{y}{y+1} - \frac{y}{1-y}$? *(Hard)* *[DjB '18]*

ⓓ ⓐ $\frac{2y}{1-y^2}$ ⓑ $\frac{2y}{y^2-1}$ ⓒ $\frac{-2y^2}{y^2-1}$ ⓓ $\frac{-2y^2}{1-y^2}$

53. i. Square of $x^2 + y^2 - z^2$ is $x^4 + y^4 + z^4 + 2x^2y^2 - 2y^2z^2 - 2z^2x^2$. *(Hard)*

ii. If $a + b = 8$ and $a - b = 4$, then the value of $4ab = 48$.

iii. Area of a square region is = $(\text{Length})^2$.

Which one of the following is correct?

ⓓ ⓐ i ⓑ i & ii ⓒ ii & iii ⓓ i, ii & iii

54. If $\left(3a + \frac{3}{a}\right)^2 = 9$, then— *(Hard)* *[CtgB '19]*

i. $\left(a + \frac{1}{a}\right)^2 = 3$ ii. $a + \frac{1}{a} = 1$

iii. $a^2 + \frac{1}{a^2} = -1$

Which one is correct?

ⓒ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

55. If $p + \frac{1}{p} = 2$ —. *(Hard)* *[SB '18]*

i. $p - \frac{1}{p} = 0$ ii. $p^3 + \frac{1}{p^3} = 2$

iii. $p^4 + \frac{1}{p^4} = 4$

Which one of the following is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

56. If $x^2 + y^2 = 4$ and $x + y = 2$ then —. *(Hard)* *[CtgB '17]*

i. $xy = 2$

ii. $xy = 0$

iii. $(x - y)^2 = 4$

Which one is correct?

ⓑ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

57. If $\left(\frac{a}{b} - 1\right)$ and $\left(1 - \frac{a}{b}\right)$ are the two expressions, the their —. *(Easy)* *[BB '17]*

i. sum = 0

ii. quotient = – 1

iii. product = $\frac{(a-b)^2}{b^2}$

Which one is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

58. If $x + y = 5$ and $x - y = 3$, then —. *(Easy)* *[DjB '16]*

i. $x^2 - y^2 = 15$

ii. $x^2 + y^2 = 17$

iii. $xy = 34$

Which one is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer to the questions No. 59 and 60 by using the following information :

$x + y = 5$ and $x - y = 3$. *[JB '19]*

59. Which one is the value of $x^2 - y^2$? *(Easy)*

ⓓ ⓐ 2 ⓑ 8 ⓒ 9 ⓓ 15

60. Which one is the value of $x^2 + y^2$? *(Easy)*

ⓓ ⓐ 1 ⓑ 4 ⓒ 8 ⓓ 17

According to the following information answer to the questions No. 61 and 62 :

$x^2 - 3x + 1 = 0$. *[DB '18]*

61. What is the value of $\left(x - \frac{1}{x}\right)$? *(Medium)*

ⓐ ⓐ $\sqrt{5}$ ⓑ $\sqrt{7}$ ⓒ 5 ⓓ 7

62. Which one of the following is the value of $x^2 + \frac{1}{x^2}$? *(Hard)*

ⓒ ⓐ 3 ⓑ 5 ⓒ 7 ⓓ 9

Answer the question Nos. 63 and 64 based on the following information :

$a^2 - 5a + 1 = 0$. *[RB '18]*

63. What is the value of $\left(a + \frac{1}{a}\right)$? *(Easy)*

ⓐ ⓐ 5 ⓑ 1 ⓒ – 1 ⓓ – 5

64. What is the value of $a - \frac{1}{a}$? *(Medium)*

Answer to the questions No. 65 and 66 based on the following information :

$3x + 2y = 8$, $3x - 2y = 4$. *[JB '18]*

65. What is the value of $9x^2 + 4y^2$? *(Medium)*

ⓒ ⓐ 16 ⓑ 24 ⓒ 40 ⓓ 100

Mathematics

66. **What is the value of xy?** *(Medium)*
a ⓐ 2 ⓑ 6 ⓒ 12 ⓓ 24

Answer to the questions No. 67 and 68 with the help of the given information :
If $x + y = 10$ and $x - y = 6$. *[CtgB '18]*

67. **What is the value of $2x^2 + 2y^2$?** *(Medium)*
c ⓐ 64 ⓑ 128 ⓒ 136 ⓓ 272

68. **What is the value of xy?** *(Easy)*
d ⓐ 136 ⓑ 64 ⓒ 36 ⓓ 16

Answer the questions No. 69 and 70 in base of the following informations :
$p + q = 3$, $pq = 1$

69. **Which one of the following is the value of $p^2 + q^2$?** *(Medium)* *[CB '17]*
b ⓐ 5 ⓑ 7 ⓒ 11 ⓓ 13

70. **Which one of the following is the value of $p - q$?** *(Easy)* *[CB '17]*
a ⓐ $\sqrt{5}$ ⓑ $\sqrt{7}$ ⓒ $\sqrt{13}$ ⓓ 5

Answer the questions no. 71 and 72 in the light of the following information :
$a + b = 4$, $a - b = 2$

71. **$4ab = ?$** *(Medium)* *[CtgB '17]*
c ⓐ 6 ⓑ 10 ⓒ 12 ⓓ 20

72. **$2(a^2 + b^2) = ?$** *(Medium)* *[CtgB '17]*
d ⓐ 6 ⓑ 10 ⓒ 12 ⓓ 20

Answer to the questions No. 73 and 74 in the light of the following information :
$x^2 - 4x + 1 = 0$ *[BB '17]*

73. **What is the value of $\left(x - \frac{1}{x}\right)^2$?** *(Medium)*
c ⓐ 0 ⓑ $2\sqrt{3}$ ⓒ 12 ⓓ 20

74. **$x^4 + \frac{1}{x^4} =$ what?** *(Medium)*
d ⓐ 7 ⓑ 9 ⓒ 34 ⓓ 194

$x - \frac{1}{x} = 5$, then answer the questions No. 75 and 76 : *[DjB '17]*

75. **$\left(x + \frac{1}{x}\right)^2 = ?$** *(Easy)*
d ⓐ 21 ⓑ 23 ⓒ 25 ⓓ 29

76. **$x^2 + \frac{1}{x^2} = ?$** *(Medium)*
d ⓐ 10 ⓑ 23 ⓒ 25 ⓓ 27

Answer to the questions number 77 and 78 using the following information :
$a + b = 4$ and $a - b = 2$. *[SB '16]*

77. **What is the value of $a^2 + b^2$?** *(Medium)*
c ⓐ 20 ⓑ 12 ⓒ 10 ⓓ 6

78. **What is the value of ab?** *(Hard)*
a ⓐ 3 ⓑ 10 ⓒ 16 ⓓ 20

Using the following information answer the question No. 79 and 80 :
$M = (a - 5)$, $N = (a + 2)^2$, $O = (a^2 + 2a)$ and $P = a^2 + 5a + 6$ are four algebraic expressions. *[Rajuk Uttara Model College, Dhaka]*

79. **Which one is the L.C.M. of N, O and P?** *(Medium)*
a ⓐ 4 ⓑ 3 ⓒ 5 ⓓ 6

80. **Which one of the following is a factor of the expression of $m^2 + 2mn - 2n - 1$?** *(Easy)*
d ⓐ $m - n$ ⓑ $n - 1$ ⓒ $m + n - 1$ ⓓ $m + 2n + 1$

Short Q/A Designed as per topic

4.1 Algebraic Formulae ▸ Textbook Page 48

Question 1. Find the square of $4a - 7b$.
Solution : Square of $4a - 7b$
$= (4a - 7b)^2$
$= (4a)^2 - 2 \times 4a \times 7b + (7b)^2$
$= 16a^2 - 56ab + 49b^2$.

Question 2. If $p + q = 7$ and $pq = 9$, Find the value of $p^2 + q^2$.
Solution : Given, $p + q = 7$ and $pq = 9$
Given expression $= p^2 + q^2$
$= (p + q)^2 - 2pq$
$= (7)^2 - 2 \times 9$
$= 49 - 18 = 31$
Required value 31.

Question 3. Multiply $(7x + 8)$ by $(7x - 8)$ with help of formula.
Solution : $(7x + 8)(7x - 8)$
$= (7x)^2 - (8)^2$ [$\because (a + b)(a - b) = a^2 - b^2$]
$= 49x^2 - 64$.

Question 4. Multiply $(5x + 17)$ by $(5x - 13)$ with the help of formula.
Solution : We know, $(x + a)(x + b) = x^2 + (a + b)x + ab$
$\therefore (5x + 17)(5x - 13) = (5x + 17)\{5x + (-13)\}$
$= (5x)^2 + \{17 + (-13)\} . 5x + 17(-13)$
$= 25x^2 + 20x - 221$.

Question 5. Simplify : $(3a^2 + 7b^2)^2 + 2(3a^2 + 7b^2)(3a^2 - 7b^2) + (3a^2 - 7b^2)^2$
Solution : Let, $3a^2 + 7b^2 = x$ and $3a^2 - 7b^2 = y$
Given expression $= x^2 + 2xy + y^2$
$= (x + y)^2$
$= (3a^2 + 7b^2 + 3a^2 - 7b^2)^2$ [Putting the value of x and y]
$= (6a^2)^2 = 36a^4$.

Question 6. Express $(4p - 3q)(6p + 5q)$ as the difference of two squares.
Solution : We know, $ab = \left(\frac{a + b}{2}\right)^2 - \left(\frac{a - b}{2}\right)^2$
$\therefore (4p - 3q)(6p + 5q)$

$$= \left(\frac{4p-3q+6p+5q}{2}\right)^2 - \left(\frac{4p-3q-6p-5q}{2}\right)^2$$
$$= \left(\frac{10p+2q}{2}\right)^2 - \left(\frac{-2p-8q}{2}\right)^2$$
$$= \left\{\frac{2(5p+q)}{2}\right\}^2 - \left\{\frac{-2(p+4q)}{2}\right\}^2$$
$$= (5p+q)^2 - (p+4q)^2.$$

Question 7. Determine the product (3x – 10) (3x – 5) by formula.

Solution : We know, $(x+a)(x+b) = x^2 + (a+b)x + ab$

$\therefore (3x-10)(3x-5) = (3x)^2 + \{(-10)+(-5)\}3x + (-10).(-5)$
$= 9x^2 + \{-10-5\}3x + 50$
$= 9x^2 - 45x + 50.$

Question 8. Find the squire of 3x + 2y + 5z.

Solution : Let, $3x = a$, $2y = b$ and $5z = c$

Square of given expression $= (a+b+c)^2$
$= a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
$= (3x)^2 + (2y)^2 + (5z)^2 + 2 \times 3x \times 2y + 2 \times 2y \times 5z + 2 \times 5z \times 3x$ [Putting value]
$= 9x^2 + 4y^2 + 25z^2 + 12xy + 20yz + 30zx$

$\therefore (3x + 2y + 5z)^2 = 9x^2 + 4y^2 + 25z^2 + 12xy + 20yz + 30zx.$

Question 9. Find the squire of 6a – 7b – 8c.

Solution : The squire of 6a – 7b – 8c.
$= (6a-7b-8c)^2$
$= \{6a-(7b+8c)\}^2$
$= (6a)^2 - 2 \times 6a \times (7b+8c) + (7b+8c)^2$
$= 36a^2 - 12a(7b+8c) + (7b)^2 + 2 \times 7b \times 8c + (8c)^2$
$= 36a^2 - 84ab - 96ac + 49b^2 + 112bc + 64c^2$
$= 36a^2 + 49b^2 + 64c^2 - 84ab - 96ac + 112bc.$

Question 10. If $m - \frac{1}{m} = 4$, Find the value of $m^2 + \frac{1}{m^2}$

Solution : Given, $m - \frac{1}{m} = 4$

Given expression $= m^2 + \frac{1}{m^2}$
$= \left(m - \frac{1}{m}\right)^2 + 2.m.\frac{1}{m}$ [$\because a^2 + b^2 = (a-b)^2 + 2ab$]
$= (4)^2 + 2$
$= 16 + 2 = 18$

Required value 18.

Question 11. If $x + \frac{1}{x} = 5$, Find the value of $x^4 + \frac{1}{x^4}$?

Solution : Given, $x + \frac{1}{x} = 5$

Given expression $= x^4 + \frac{1}{x^4}$
$= (x^2)^2 + \left(\frac{1}{x^2}\right)^2$
$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2.x^2.\frac{1}{x^2}$
$= \left\{\left(x + \frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right\}^2 - 2$
$= (5^2 - 2)^2 - 2$
$= (25-2)^2 - 2$
$= (23)^2 - 2 = 529 - 2 = 527$

Required value : 527.

Question 12. If $m^2 - 7m - 1 =, m^2 + \frac{1}{m^2} = ?$

Solution : Given,
$m^2 - 7m - 1 = 0$
or, $m^2 - 1 = 7m$
or, $\frac{m^2}{m} - \frac{1}{m} = \frac{7m}{m}$ [Dividing both sides by m]
or, $m - \frac{1}{m} = 7$
or, $\left(m - \frac{1}{m}\right)^2 = (7)^2$ [Squaring both sides]
or, $m^2 - 2 \times m \times \frac{1}{m} + \frac{1}{m^2} = 49$
or, $m^2 + \frac{1}{m^2} = 49 + 2$
$\therefore m^2 + \frac{1}{m^2} = 51$

Required value : 51.

Question 13. If a + b = 7 and a – b = 3, Find the value of $a^2 + b^2$?

Solution : Given, $a + b = 7$ and $a - b = 3$

Given expression $= a^2 + b^2$
$= \frac{(a+b)^2 + (a-b)^2}{2}$
$= \frac{(7)^2 + (3)^2}{2} = \frac{49+9}{2} = \frac{58}{2} = 29$

Required value : 29.

Question 14. If $x + \frac{1}{x} = \sqrt{5}$, Find the value of $\left(x - \frac{1}{x}\right)^2$

Solution : Given, $x + \frac{1}{x} = \sqrt{5}$

Given expression $= \left(x - \frac{1}{x}\right)^2$
$= \left(x + \frac{1}{x}\right)^2 - 4.x.\frac{1}{x}$ [$\because (a-b)^2 = (a+b)^2 - 4ab$]
$= (\sqrt{5})^2 - 4 = 5 - 4 = 1$

Required value : 1.

Question 15. If $a^2 - 4a + 1 = 0$, Find the value of $\left(a - \frac{1}{a}\right)^2$?

Solution : Given, $a^2 - 4a + 1 = 0$
or, $a^2 + 1 = 4a$
or, $a + \frac{1}{a} = 4$ [Dividing both sides by a]

Mathematics

Given expression $= \left(a-\frac{1}{a}\right)^2 = \left(a+\frac{1}{a}\right)^2 - 4.a.\frac{1}{a} = 4^2 -$

$4 = 16 - 4 = 12$

Required value : 12.

Question 16. If $x + y = 14$ and $x - y = 4$, Find the value of $2(x^2 + y^2)$

Solution : Given, $x + y = 14$ and $x - y = 4$

Given expression $= 2(x^2 + y^2)$

$= (x+y)^2 + (x-y)^2$

$= (14)^2 + (4)^2 = 196 + 16 = 212$

Required value : 212.

Question 17. If $a + b = 2\sqrt{3}$, $a - b = 2\sqrt{2}$, Find the value of 4ab?

Solution : Given, $a + b = 2\sqrt{3}$ and $a - b = 2\sqrt{2}$

Given expression $= 4ab$

$= (a+b)^2 - (a-b)^2$

$= (2\sqrt{3})^2 - (2\sqrt{2})^2 = 12 - 8 = 4$

Required value : 4.

Question 18. If $m^4 + \frac{1}{m^4} = 119$, $m^2 + \frac{1}{m^2} = ?$

Solution : Given, $m^4 + \frac{1}{m^4} = 119$

or, $(m^2)^2 + \left(\frac{1}{m^2}\right)^2 = 119$

or, $\left(m^2 + \frac{1}{m^2}\right)^2 - 2.m^2.\frac{1}{m^2} = 119$

or, $\left(m^2 + \frac{1}{m^2}\right)^2 = 119 + 2 = 121$

or, $m^2 + \frac{1}{m^2} = \sqrt{121} = 11$

Required value : 11.

Question 19. If $a + b = 9$ and $a - b = 5$, Find the value of ab?

Solution : Given, $a + b = 9$ and $a - b = 5$

We know, $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

$= \left(\frac{9}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \frac{81}{4} - \frac{25}{4}$

$= \frac{81-25}{4} = \frac{56}{4} = 14$

Required value : 14.

Question 20. If $\left(2p - \frac{2}{p}\right)^2 = 12$, Find the value of $p^2 + \frac{1}{p^2}$?

Solution : Given, $\left(2p - \frac{2}{p}\right)^2 = 12$

or, $\left\{2\left(p - \frac{1}{p}\right)\right\}^2 = 12$

or, $4\left(p - \frac{1}{p}\right)^2 = 12$

or, $\left(p - \frac{1}{p}\right)^2 = \frac{12}{4}$

or, $\left(p - \frac{1}{p}\right)^2 = 3$

$\therefore \quad p - \frac{1}{p} = \sqrt{3}$

Given expression $= p^2 + \frac{1}{p^2}$

$= \left(p - \frac{1}{p}\right)^2 + 2.p.\frac{1}{p}$

$= \left(\sqrt{3}\right)^2 + 2 = 3 + 2 = 5$

Required value : 5.

Question 21. If $(x + y)^2 = 16$ and $(x - y)^2 = 9$, Find the value of $2(x^2 - y^2)$?

Solution : Given, $(x + y)^2 = 16$

$\therefore x + y = \sqrt{16} = 4$

and $(x - y)^2 = 9$

$\therefore x - y = \sqrt{9} = 3$

Given expression $= 2(x^2 - y^2)$

$= 2(x+y)(x-y)$

$= 2 \times 4 \times 3 = 24$

Required value : 24

Question 22. If $a^2 + 1 = 4a$, Find the value of $\frac{a}{a^2 - 3a + 1}$?

Solution : Given, $a^2 + 1 = 4a$

Given expression $= \frac{a}{a^2 - 3a + 1} = \frac{a}{(a^2 + 1) - 3a}$

$= \frac{a}{4a - 3a} = \frac{a}{a} = 1$

Required value : 1

Question 23. If $x + \frac{1}{x} = 2$, Show that, $x^4 + \frac{1}{x^4} = 2$.

Solution : Given, $x + \frac{1}{x} = 2$

or, $\left(x + \frac{1}{x}\right)^2 = (2)^2$ [Squaring]

or, $x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2} = 4$

or, $x^2 + \frac{1}{x^2} = 4 - 2 = 2$

or, $\left(x^2 + \frac{1}{x^2}\right)^2 = (2)^2$ [Squaring again]

or, $x^4 + 2.x^2.\frac{1}{x^2} + \frac{1}{x^4} = 4$

or, $x^4 + \frac{1}{x^4} = 4 - 2$

$\therefore \quad x^4 + \frac{1}{x^4} = 2$. **(Shown)**

Creative Q/A Designed as per learning outcomes

Ques. 01 $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$ is an algebraic expression.

a. Simplify the given expression using formula. 2
b. Find the value of $(2x - 3y + 5z)^2$. 4
c. Determine the value of the expression when $x = 3$, $y = -1$ and $z = 1$. 4

Solution to Question No. 01 :

a $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$
$= \{(2x - 3y + 5z) + (x - 2y + 3z)\}\{(2x - 3y + 5z) - (x - 2y + 3z)\}$
$= (2x - 3y + 5z + x - 2y + 3z)(2x - 3y + 5z - x + 2y - 3z)$
$= (3x - 5y + 8z)(x - y + 2z)$. **(Ans.)**

b $(2x - 3y + 5z)^2 = (2x)^2 + (-3y)^2 + (5z)^2 + 2 \times 2x \times (-3y) + 2(-3y) \times 5z + 2 \times 5z \times 2x$
$= 4x^2 + 9y^2 + 25z^2 - 12xy - 30yz + 20zx$. **(Ans.)**

c $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$
$= (3x - 5y + 8z)(x - y + 2z)$, from (a)
$= (3 \times 3 - 5 \times (-1) + 8 \times 1)(3 - 1 \times (-1) + 2 \times 1)$
$= (9 + 5 + 8)(3 + 1 + 2) = 22 \times 6$
$= 132$ **(Ans.)**

Ques. 02 $a - \frac{1}{a} = 2$ and $x^2 + \frac{1}{x^2} = 3$

a. If $x + y = 5$ and $xy = 6$, then find the value of $x^3 - y^3$. 2
b. Show that, $\frac{a^8 + 1}{a^4} = 34$. 4
c. Find the value of $\left(\frac{x^6 + 1}{x^3}\right)^2$. 4

● *Chattogram Board 2019*

Solution to Question No. 02 :

a Given,
$x + y = 5$
or, $(x + y)^2 = 5^2$ [Squaring both sides]
or, $(x - y)^2 + 4xy = 25$ [$\because (a + b)^2 = (a - b)^2 + 4ab$]
or, $(x - y)^2 + 4.6 = 25$ [$\because xy = 6$]
or, $(x - y)^2 = 25 - 24$
or, $(x - y)^2 = 1$
or, $x - y = 1$
Now,
$x^3 - y^3 = (x - y)^3 + 3xy(x - y)$
$= (1)^3 + 3.6.1$ [$\because x - y = 1$ and $xy = 6$]
$= 1 + 18$
$= 19$
$\therefore$ Determined value of $x^3 - y^3$ is 19.

b Given,
$a - \frac{1}{a} = 2$
or, $a^2 - 2.a.\frac{1}{a} + \frac{1}{a^2} = 4$ [Squaring both sides]
or, $a^2 + \frac{1}{a^2} = 4 + 2$
or, $a^2 + \frac{1}{a^2} = 6$
or, $a^4 + 2.a^2.\frac{1}{a^2} + \frac{1}{a^4} = 36$ [Squaring again]
or, $a^4 + \frac{1}{a^4} = 36 - 2$
or, $\frac{a^8 + 1}{a^4} = 34$
$\therefore$ $\frac{a^8 + 1}{a^4} = 34$ **(Showed).**

c Given,
$x^2 + \frac{1}{x^2} = 3$
or, $\left(x + \frac{1}{x}\right)^2 - 2.x.\frac{1}{x} = 3$ [$\because a^2 + b^2 = (a + b)^2 - 2ab$]
or, $\left(x + \frac{1}{x}\right)^2 = 3 + 2$
or, $\left(x + \frac{1}{x}\right)^2 = 5$
or, $x + \frac{1}{x} = \sqrt{5}$
or, $x^3 + \frac{1}{x^3} + 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right) = 5\sqrt{5}$ [Cubing both sides]
or, $x^3 + \frac{1}{x^3} + 3.\sqrt{5} = 5\sqrt{5}$ $\left[\because + x + \frac{1}{x} = \sqrt{5}\right]$
or, $x^3 + \frac{1}{x^3} = 5\sqrt{5} - 3\sqrt{5}$
or, $\frac{x^6 + 1}{x^3} = 2\sqrt{5}$
or, $\left(\frac{x^6 + 1}{x^3}\right)^2 = (2\sqrt{5})^2$ [Squaring both sides]
$\therefore \left(\frac{x^6 + 1}{x^3}\right)^2 = 20$
$\therefore$ The determined value of $\left(\frac{x^6 + 1}{x^3}\right)^2$ is 20.

Mathematics

Ques. 03 $p+q+\sqrt{3}$, $p-q=\sqrt{2}$ and $A=x^4+\frac{1}{x^4}$.

a. Resolve into factors : $2x^7y-128xy^7$. 2

b. Prove that $p^3+q^3=\frac{9\sqrt{3}}{4}$. 4

c. If $A=m^4+4m^2+2$, prove that $x^2-1=mx$. 4

● *Rajuk Uttara Model College, Dhaka*

Solution to Question No. 03 :

a $2x^7y-128xy^7$

$= 2xy(x^6-y^6)$

$= 2xy\{(x^3)^2-(y^3)^2\}$

$= 2xy(x^3-y^3)(x^3+y^3)$

$= 2xy(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

$= 2xy(x-y)(x+y)(x^2+xy+y^2)(x^2-xy+y^2)$ **(Ans.)**

b Given,

$p+p=\sqrt{3}$

$p-q=\sqrt{2}$

We know,

$4pq=(p+q)^2-(p-q)^2$

or, $4pq=(\sqrt{3})^2-(\sqrt{2})^2$

or, $4pq=3-2$

or, $4pq=1$

$\therefore\ pq=\frac{1}{4}$

L.H.S $= p^3+q^3$

$=(p+q)^3-3pq(p+q)$

$=(\sqrt{3})^3-3\times\frac{1}{4}\times\sqrt{3}$

$=3\sqrt{3}-\frac{3\sqrt{3}}{4}$

$=\frac{12\sqrt{3}-3\sqrt{3}}{4}$

$=\frac{9\sqrt{3}}{4}$

$=$ R.H.S.

$\therefore\ p^3+q^3=\frac{9\sqrt{3}}{4}$. **(Proved)**

c Given,

$A=x^4+\frac{1}{x^4}$

$\therefore\ x^4+\frac{1}{x^4}=m^4+4m^2+2$

or, $(x^2)^2+\left(\frac{1}{x^2}\right)^2=(m^2)^2+2.2.m^2+(2)^2-2$

or, $\left(x^2+\frac{1}{x^2}\right)^2-2.x^2.\frac{1}{x^2}=(m^2+2)^2-2$

or, $\left(x^2+\frac{1}{x^2}\right)^2=(m^2+2)^2-2+2$

or, $\left(x^2+\frac{1}{x^2}\right)^2=(m^2+2)^2$

or, $x^2+\frac{1}{x^2}=m^2+2$

or, $\left(x-\frac{1}{x}\right)^2+2x\cdot\frac{1}{x}=m^2+2$

or, $\left(x-\frac{1}{x}\right)^2=m^2+2-2$

or, $\left(x-\frac{1}{x}\right)^2=m^2$

or, $x-\frac{1}{x}=m$; [By squaring root on both sides]

or, $\frac{x^2-1}{x}=m$

$\therefore\ x^2-1=mx$. **(Proved)**

Ques. 04 $x^2+\frac{1}{x^2}=4$ and $p^4+\frac{1}{p^4}=322$ are two algebraic equations.

a. Find the value of $x+\frac{1}{x}$ 2

b. Prove that $\frac{x^6-1}{x^3}=5\sqrt{2}$ 4

c. Show that, $P-\frac{1}{P}=4$ 4

● *Viqarunnisa Noon School and College, Dhaka*

Solution to Question No. 04 :

a Given,

$x^2+\frac{1}{x^2}=4$

or, $\left(x+\frac{1}{x}\right)^2-2.x.\frac{1}{x}=4$

or, $\left(x+\frac{1}{x}\right)^2-2=4$

or, $\left(x+\frac{1}{x}\right)^2=4+2$

or, $\left(x+\frac{1}{x}\right)^2=6$

$\therefore\ x+\frac{1}{x}=\sqrt{6}$ [By squaring root on both sides]

b From 'a' we get,

$x+\frac{1}{x}=\sqrt{6}$

or, $\left(x+\frac{1}{x}\right)^2=(\sqrt{6})^2$ [By squaring on both sides]

or, $\left(x-\frac{1}{x}\right)^2+4.x.\frac{1}{x}=6$

or, $\left(x-\frac{1}{x}\right)^2=6-4$

or, $\left(x-\frac{1}{x}\right)^2=2$

or, $x-\frac{1}{x}=\sqrt{2}$ [By squaring root on both sides]

or, $\left(x-\frac{1}{x}\right)^3=(\sqrt{2})^3$ [By cubing on both sides]

or, $x^3-\frac{1}{x^3}-3.x.\frac{1}{x}\left(x-\frac{1}{x}\right)=2\sqrt{2}$

or, $x^3-\frac{1}{x^3}-3\sqrt{2}=2\sqrt{2}$

or, $x^3-\frac{1}{x^3}=2\sqrt{2}+3\sqrt{2}$

or, $x^3-\frac{1}{x^3}=5\sqrt{2}$

$\therefore \frac{x^6-1}{x^3}=5\sqrt{2}$ **(Proved)**

c Given,

$P^4+\frac{1}{P^4}=322$

or, $(P^2)^2+\left(\frac{1}{P^2}\right)^2=322$

or, $\left(P^2+\frac{1}{P^2}\right)^2-2P^2.\frac{1}{P^2}=322$

or, $\left(P^2+\frac{1}{P^2}\right)^2=322+2$

or, $\left(P^2+\frac{1}{P^2}\right)^2=324$

or, $P^2+\frac{1}{P^2}=18$ [By squaring root on both sides]

or, $\left(P-\frac{1}{P}\right)^2+2P.\frac{1}{P}=18$

or, $\left(P-\frac{1}{P}\right)^2=18-2$

or, $\left(P-\frac{1}{P}\right)^2=16$

$\therefore P-\frac{1}{P}=4$ **(Shown)**

Mathematics

Solutions to Textual Activities Along with textual reference

Task 01 *[Textbook Page 51]*

1. **Find the square of 2a + 5b.**
2. **Find the square of 4x – 7.**
3. **If a + b = 7 and ab = 9, find the value of a^2+b^2.**
4. **If x – y = 5 and xy = 6, find the value of $(x+y)^2$.**

Solution :

1. The square of (2a + 5b) is $(2a+5b)^2$.
   Now, $(2a+5b)^2=(2a)^2+2.2a.5b+(5b)^2$
   $=4a^2+20ab+25b^2$.
2. The square of (4x – 7) is $(4x-7)^2$.
   Now, $(4x-7)^2=(4x)^2-2.4x.7+(7)^2$
   $=16x^2-56x+49$.
3. Given, a + b = 7 and ab = 9.
   Now, $a^2+b^2=(a+b)^2-2ab$
   $=(7)^2-2\times 9$, Putting the value of a + b and ab.
   $=49-18=31$
   $\therefore a^2+b^2=31$.
4. Given, x – y = 5 and xy = 6.
   Now, $(x+y)^2=(x-y)^2+4xy$
   $=(5)^2+4\times 6$, putting x – y = 5 and xy = 6.
   $=25+24=49$
   $\therefore (x+y)^2=49$.

Task 02 *[Textbook Page 53]*

1. **Find the product of (5x + 7y) and (5x – 7y) by an appropriate formula.**
2. **Find the product of (x + 10) and (x – 14) by an appropriate formula.**
3. **Express (4x – 3y) (6x + 5y) as the difference of two squares.**

Solution :

1. (5x + 7y) (5x – 7y)
   $=(5x)^2-(7y)^2$, Since $(a+b)(a-b)=a^2-b^2$
   $=25x^2-49y^2$. **(Ans.)**
2. (x + 10) (x – 14)
   $=(x)^2+(10-14)x+(10)(-14)$, since $(x+a)(x-b)=x^2+(a-b)x+(a)(-b)$
   $=x^2-4x-140$.
3. (4x – 3y) (6x + 5y)
   $=\left(\frac{4x-3y+6x+5y}{2}\right)^2-\left(\frac{4x-3y-6x-5y}{2}\right)^2$,
   since $ab=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$.
   $=\left(\frac{10x+2y}{2}\right)^2-\left(\frac{-2x-8y}{2}\right)^2$
   $=(5x+y)^2-(x+4y)^2$.

Task 03 **Find the square by appropriate formula :** *[Textbook Page 54]*

1. **ax + by + c**
2. **4x + 5y – 7z**

Solution :

1. The square of (ax + by + c) is $(ax+by+c)^2$.
   Now, $(ax+by+c)^2$
   $=(ax)^2+(by)^2+(c)^2+2.ax.by+2.by.c+2.c.ax.$
   $=a^2x^2+b^2y^2+c^2+2abxy+2bcy+2cax$. **(Ans.)**
2. **4x + 5y – 7z**
   We know, $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ca$
   Here, Let, 4x = a, 5y = b and – 7z = c,
   $(4x+5y-7z)^2=(4x)^2+(5y)^2+(-7z)^2+2\times(4x)\times(5y)+2\times(5y)\times(-7z)+2\times(4x)\times(-7z)$
   $=16x^2+25y^2+49z^2+40xy-70yz-56xz$.

Exercise 4.2 : Formulae of cubes and corollaries

At a Glance Important Contents of Exercise

Formula 01 : $(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + b^3 + 3ab(a+b)$

Proof : $(a+b)^3 = (a+b)(a+b)^2$
$= (a+b)(a^2+2ab+b^2)$
$= a(a^2+2ab+b^2) + b(a^2+2ab+b^2)$
$= a^3 + 2a^2b + ab^2 + (a^2b + 2ab^2 + b^3)$
$= a^3 + 3a^2b + 3ab^2 + b^3$
$= a^3 + 3ab(a+b) + b^3 = a^3 + b^3 + 3ab(a+b)$

Formula 02 : $(a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$
$= a^3 - b^3 - 3ab(a-b)$

Proof : $(a-b)^3 = (a-b)(a-b)^2$
$= (a-b)(a^2-2ab+b^2)$
$= a(a^2-2ab+b^2) - b(a^2-2ab+b^2)$
$= a^3 - 2a^2b + ab^2 - a^2b + 2ab^2 - b^3$
$= a^3 - 3a^2b + 3ab^2 - b^3 = a^3 - b^3 - 3ab(a-b)$

Formula 03 : $a^3 + b^3 = (a+b)(a^2-ab+b^2)$

Proof : $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
$= (a+b)\{(a+b)^2 - 3ab\}$
$= (a+b)(a^2+2ab+b^2-3ab)$
$= (a+b)(a^2-ab+b^2)$

Conversely, $(a+b)(a^2-ab+b^2)$
$= a(a^2-ab+b^2) + b(a^2-ab+b^2)$
$= a^3 - a^2b + ab^2 + a^2b - ab^2 + b^3 = a^3 + b^3$
$\therefore (a+b)(a^2-ab+b^2) = a^3 + b^3$.

Formula 04 : $a^3 - b^3 = (a-b)(a^2+ab+b^2)$

Proof : $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
$= (a-b)\{(a-b)^2 + 3ab\}$
$= (a-b)(a^2-2ab+b^2+3ab)$
$= (a-b)(a^2+ab+b^2)$

Conversely, $(a-b)(a^2+ab+b^2)$
$= a(a^2+ab+b^2) - b(a^2+ab+b^2)$
$= a^3 + a^2b + ab^2 - a^2b - ab^2 - b^3$
$= a^3 - b^3$
$\therefore (a-b)(a^2+ab+b^2) = a^3 - b^3$.

Solutions to Exercise Problems Let's solve the textbook problems

Solutions to Mathematical Problems

1. Find the cube of the following expressions with the help of formula :

(a) $3x+y$ (b) x^2+y
(c) $5p+2q$ (d) a^2b+c^2d
(e) $6p-7$ (f) $ax-by$
(g) $2p^2-3r^2$ (h) x^3+2
(i) $2m+3n-5p$ (j) $x^2-y^2+z^2$
(k) $a^2b^2-c^2d^2$ (l) a^2b-b^3c
(m) x^3-2y^3 (n) $11a-12b$
(o) x^3+y^3

Solution :

(a) Cube of $(3x+y)$
$= (3x+y)^3 = (3x)^3 + 3\cdot(3x)^2 y + 3\cdot 3x\cdot y^2 + (y)^3$
$= 27x^3 + 27x^2y + 9xy^2 + y^3$

(b) Cube of (x^2+y)
$= (x^2+y)^3 = (x^2)^3 + 3\cdot(x^2)^2 y + 3\cdot x^2\cdot(y)^2 + (y)^3$
$= x^6 + 3x^4y + 3x^2y^2 + y^3$

(c) Cube of $(5p+2q)$
$= (5p+2q)^3$
$= (5p)^3 + 3\cdot(5p)^2\cdot 2q + 3\cdot(5p)\cdot(2q)^2 + (2q)^3$
$= 125p^3 + 150p^2q + 60pq^2 + 8q^3$

(d) Cube of (a^2b+c^2d)
$= (a^2b+c^2d)^3$
$= (a^2b)^3 + 3\cdot(a^2b)^2\cdot c^2d + 3\cdot(a^2b)\cdot(c^2d)^2 + (c^2d)^3$
$= a^6b^3 + 3a^4b^2c^2d + 3a^2bc^4d^2 + c^6d^3$

(e) Cube of $(6p-7)$
$= (6p-7)^3$
$= (6p)^3 - 3\cdot(6p)^2\cdot 7 + 3\cdot(6p)\cdot(7)^2 - (7)^3$
$= 216p^3 - 756p^2 + 882p - 343$

(f) Cube of $(ax-by)$
$= (ax-by)^3$
$= (ax)^3 - 3\cdot(ax)^2\cdot by + 3\cdot(ax)\cdot(by)^2 - (by)^3$
$= a^3x^3 - 3a^2x^2by + 3axb^2y^2 - b^3y^3$

(g) Cube of $(2p^2-3r^2)$
$= (2p^2-3r^2)^3$
$= (2p^2)^3 - 3\cdot(2p^2)^2\cdot 3r^2 + 3(2p^2)\cdot(3r^2)^2 - (3r^2)^3$
$= 8p^6 - 36p^4r^2 + 54p^2r^4 - 27r^6$

(h) Cube of (x^3+2)
$= (x^3+2)^3 = (x^3)^3 + 3\cdot(x^3)^2\cdot 2 + 3\cdot x^3\cdot(2)^2 + (2)^3$
$= x^9 + 6x^6 + 12x^3 + 8$

(i) Cube of $(2m+3n-5p)$
$= (2m+3n-5p)^3 = \{2m+(3n-5p)\}^3$
$= (2m)^3 + 3\cdot(2m)^2\cdot(3n-5p) + 3\cdot 2m\cdot(3n-5p)^2 + (3n-5p)^3$
$= 8m^3 + 12m^2(3n-5p) + 6m(9n^2 - 30pn + 25p^2) + 27n^3 - 135n^2p + 225np^2 - 125p^3$
$= 8m^3 + 36m^2n - 60m^2p + 54mn^2 - 180mnp + 150p^2m + 27n^3 - 135n^2p + 225np^2 - 125p^3$
$= 8m^3 + 27n^3 - 125p^3 + 36m^2n - 60m^2p - 180mnp + 54mn^2 - 135n^2p + 150mp^2 + 225np^2$

(j) Cube of $(x^2 - y^2 + z^2)$
$= (x^2 - y^2 + z^2)^3 = \{(x^2 - y^2) + z^2\}^3$
$= (x^2 - y^2)^3 + 3 \cdot (x^2 - y^2)^2 \cdot z^2 + 3 \cdot (x^2 - y^2)(z^2)^2 + (z^2)^3$
$= (x^2)^3 - 3 \cdot (x^2)^2 y^2 + 3 \cdot (x^2) \cdot (y^2)^2 - (y^2)^3 + 3z^2 \cdot (x^4 - 2x^2y^2 + y^4) + 3z^4 \cdot (x^2 - y^2) + z^6$
$= x^6 - 3x^4y^2 + 3x^2y^4 - y^6 + 3x^4z^2 - 6x^2y^2z^2 + 3y^4z^2 + 3x^2z^4 - 3y^2z^4 + z^6$
$= x^6 - y^6 + z^6 - 3x^4y^2 + 3x^4z^2 + 3x^2y^4 - 6x^2y^2z^2 + 3x^2z^4 + 3y^4z^2 - 3y^2z^4$

(k) Cube of $(a^2b^2 - c^2d^2)$
$= (a^2b^2 - c^2d^2)^3$
$= (a^2b^2)^3 - 3 \cdot (a^2b^2)^2 \cdot (c^2d^2) + 3 \cdot (a^2b^2) \cdot (c^2d^2)^2 - (c^2d^2)^3$
$= a^6b^6 - 3a^4b^4c^2d^2 + 3a^2b^2c^4d^4 - c^6d^6$

(l) Cube of $(a^2b - b^3c)$
$= (a^2b - b^3c)^3$
$= (a^2b)^3 - 3 \cdot (a^2b)^2 \cdot (b^3c) + 3 \cdot (a^2b) \cdot (b^3c)^2 - (b^3c)^3$
$= a^6b^3 - 3a^4b^5c + 3a^2b^7c^2 - b^9c^3$

(m) Cube of $(x^3 - 2y^3)$
$= (x^3 - 2y^3)^3$
$= (x^3)^3 - 3 \cdot (x^3)^2 \cdot (2y^3) + 3 \cdot (x^3) \cdot (2y^3)^2 - (2y^3)^3$
$= x^9 - 6x^6y^3 + 12x^3y^6 - 8y^9$

(n) Cube of $(11a - 12b)$
$= (11a - 12b)^3$
$= (11a)^3 - 3 \cdot (11a)^2 \cdot (12b) + 3 \cdot (11a) \cdot (12b)^2 - (12b)^3$
$= 1331a^3 - 4356a^2b + 4752ab^2 - 1728b^3$

(o) Cube of $(x^3 + y^3)$
$= (x^3 + y^3)^3$
$= (x^3)^3 + 3 \cdot (x^3)^2 \cdot y^3 + 3 \cdot (x^3) \cdot (y^3)^2 + (y^3)^3$
$= x^9 + 3x^6y^3 + 3x^3y^6 + y^9$

2. Simplify :

(a) $(3x + y)^3 + 3(3x + y)^2 (3x - y) + 3(3x + y)(3x - y)^2 + (3x - y)^3$

(b) $(2p + 5q)^3 + 3(2p + 5q)^2 (5q - 2p) + 3(2p + 5q)(5q - 2p)^2 + (5q - 2p)^3$

(c) $(x + 2y)^3 - 3(x + 2y)^2 (x - 2y) + 3(x + 2y)(x - 2y)^2 - (x - 2y)^3$

(d) $(6m + 2)^3 - 3(6m + 2)^2 (6m - 4) + 3(6m + 2)(6m - 4)^2 - (6m - 4)^3$

(e) $(x - y)^3 + (x + y)^3 + 6x(x^2 - y^2)$

Solution :

(a) $(3x + y)^3 + 3(3x + y)^2 (3x - y) + 3(3x + y)(3x - y)^2 + (3x - y)^3$
Let,
$3x + y = a$ and $3x - y = b$
$\therefore\ a^3 + 3a^2b + 3ab^2 + b^3$
$= (a + b)^3$
$= (3x + y + 3x - y)^3$ [Putting the value of a and b]
$= (6x)^3 = 216x^3$

(b) $(2p + 5q)^3 + 3(2p + 5q)^2 (5q - 2p) + 3(2p + 5q)(5q - 2p)^2 + (5q - 2p)^3$
Let, $2p + 5q = x$ and $5q - 2p = y$
$\therefore\ x^3 + 3x^2y + 3xy^2 + y^3$
$= (x + y)^3$
$= (2p + 5q + 5q - 2p)^3$ [Putting the value of x and y]
$= (10q)^3 = 1000\ q^3$

(c) $(x + 2y)^3 - 3(x + 2y)^2 (x - 2y) + 3(x + 2y)(x - 2y)^2 - (x - 2y)^3$
Let, $x + 2y = p$ and $x - 2y = q$
$\therefore\ P^3 - 3p^2q + 3pq^2 - q^3$
$= (p - q)^3$
$= \{(x + 2y) - (x - 2y)\}^3$
$= (x + 2y - x + 2y)^3$
$= (4y)^3 = 64y^3$

(d) $(6m + 2)^3 - 3(6m + 2)^2 (6m - 4) + 3(6m + 2)(6m - 4)^2 - (6m - 4)^3$
Let, $6m + 2 = x$ and $6m - 4 = y$
$\therefore\ x^3 - 3x^2y + 3xy^2 - y^3$
$= (x - y)^3$
$= \{(6m + 2) - (6m - 4)\}^3$
[Putting the value of x and y]
$= (6m + 2 - 6m + 4)^3$
$= (6)^3 = 216$

(e) $(x - y)^3 + (x + y)^3 + 6x(x^2 - y^2)$.
$= (x - y)^3 + 6x(x^2 - y^2) + (x + y)^3$
$= (x - y)^3 + 3 \cdot 2x(x + y)(x - y) + (x + y)^3$
Let, $x - y = p$ and $x + y = q$
$\therefore\ p + q = 2x$
$\therefore\ p^3 + 3(p + q) \cdot pq + q^3$
$= (p + q)^3$
$= (x - y + x + y)^3$
[Putting the value of p and q]
$= (2x)^3 = 8x^3$

3. If $a + b = 8$ and $ab = 15$, what is the value of $a^3 + b^3$.

Solution : Here, $a + b = 8$ and $ab = 15$
$\therefore\ a^3 + b^3$
$= (a + b)^3 - 3ab(a + b)$
$= (8)^3 - 3.15.8$ [Putting the value of a + b and ab]
$= 512 - 360 = 152.$

Ans : 152.

4. If $x + y = 2$, show that $x^3 + y^3 + 6xy = 8$.

Solution : Here, $x + y = 2$
Now, L.H.S. $= x^3 + y^3 + 6xy$
$= (x + y)^3 - 3xy(x + y) + 6xy$
$= (2)^3 - 3xy(2) + 6xy$ [Putting $x + y = 2$]
$= 8 - 6xy + 6xy = 8$
$=$ R. H. S.
$\therefore\ x^3 + y^3 + 6xy = 8$ **(Showed)**

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5. If $2x + 3y = 13$ and $xy = 6$, find the value of $8x^3 + 27y^3$.

Solution : Here, $2x + 3y = 13$ and $xy = 6$

$\therefore \quad 8x^3 + 27y^3$

$= (2x)^3 + (3y)^3$

$= (2x + 3y)^3 - 3 \times 2x \times 3y\,(2x + 3y)$

$= (2x + 3y)^3 - 18xy\,(2x + 3y)$

$= (13)^3 - 18 \times 6\,(13)$ [Putting the value of $2x + 3y$ and xy]

$= 2197 - 1404 = 793.$

6. If $p - q = 5$, $pq = 3$, find the value of $p^3 - q^3$.

Solution : Given, $p - q = 5$ and $pq = 3$

$\therefore \quad p^3 - q^3$

$= (p - q)^3 + 3pq\,(p - q)$

$= (5)^3 + 3.3.\,(5)$ [Putting the value of $p + q$ and pq]

$= 125 + 45 = 170.$

Ans : 170.

7. If $x - 2y = 3$, find the value of $x^3 - 8y^3 - 18xy$.

Solution : Given, $x - 2y = 3$,

$\therefore \quad x^3 - 8y^3 - 18xy$

$= \{(x)^3 - (2y)^3\} - 18xy$

$= (x - 2y)^3 + 3x\,.\,2y\,(x - 2y) - 18xy$

$= (3)^3 + 6xy(3) - 18xy$ [Putting $x - 2y = 3$]

$= 27 + 18xy - 18xy = 27$

Ans : 27.

8. If $4x - 3 = 5$, prove that $64x^3 - 27 - 180x = 125$.

Solution : Given, $4x - 3 = 5$

L. H. S.

$= 64x^3 - 27 - 180x$

$= \{(4x)^3 - (3)^3\} - 180x$

$= (4x - 3)^3 + 3\,.\,4x\,.\,3\,(4x - 3) - 180x$

$= (5)^3 + 36x\,(5) - 180x$ [Putting $4x - 3 = 5$]

$= 125 + 180x - 180x$

$= 125$

$=$ R.H.S.

$\therefore\ 64x^3 - 27 - 180x = 125$ **(Proved)**

9. If $a = -3$ and $b = 2$, find the value of $8a^3 + 36a^2b + 54ab^2 + 27b^3$.

Solution : Here, $a = -3$, $b = 2$

$\therefore \quad 8a^3 + 36a^2b + 54ab^2 + 27b^3.$

$= (2a)^3 + 3\,.\,(2a)^2\,.\,3b + 3\,.\,(2a)\,(3b)^2 + (3b)^3$

$= (2a + 3b)^3 = \{2.(-3) + 3 \times 2\}^3$

$= (-6 + 6)^3 = (0)^3 = 0$

10. If $a = 7$, find the value of $a^3 + 6a^2 + 12a + 1$.

Solution : Here, $a = 7$,

$\therefore \quad a^3 + 6a^2 + 12a + 1$

$= (a)^3 + 3\,.\,a^2\,.\,2 + 3\,.\,a\,.\,(2)^2 + (2)^3 - 7$

$= (a + 2)^3 - 7$

$= (7 + 2)^3 - 7$ [Putting the value of a]

$= (9)^3 - 7$

$= 729 - 7 = 722$

11. If $x = 5$, what is the value of $x^3 - 12x^2 + 48x - 64$?

Solution : Here, $x = 5$,

$\therefore \quad x^3 - 12x^2 + 48x - 64$

$= (x)^3 - 3\,.\,x^2\,.\,4 + 3x\,.\,(4)^2 - (4)^3$

$= (x - 4)^3$

$= (5 - 4)^3$ [Putting $x = 5$]

$= (1)^3 = 1$

12. If $a^2 + b^2 = c^2$, prove that $a^6 + b^6 + 3a^2b^2c^2 = c^6$.

Solution :

Here, $a^2 + b^2 = c^2$

L.H.S.

$= a^6 + b^6 + 3a^2b^2c^2$

$= \{(a^2)^3 + (b^2)^3\} + 3a^2b^2c^2$

$= (a^2 + b^2)^3 - 3a^2b^2\,(a^2 + b^2) + 3a^2b^2c^2$

$= (c^2)^3 - 3a^2b^2\,(c^2) + 3a^2b^2c^2$ [Putting $a^2 + b^2 = c^2$]

$= c^6 - 3a^2b^2c^2 + 3a^2b^2c^2$

$= c^6 =$ R.H.S.

$\therefore\ a^6 + b^6 + 3a^2b^2c^2 = c^6$. **(Proved)**

13. If $x + \frac{1}{x} = 4$, prove that $x^3 + \frac{1}{x^3} = 52$.

Solution : Here, $x + \frac{1}{x} = 4$

Now, L.H.S. $= x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)^3 - 3x\,.\,\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (4)^3 - 3(4) = 64 - 12 = 52$

$=$ R.H.S.

$\therefore \quad x^3 + \frac{1}{x^3} = 52$ **(Proved)**

14. If $a - \frac{1}{a} = 5$, what is the value of $a^3 - \frac{1}{a^3}$?

Solution : Here, $a - \frac{1}{a} = 5$

$\therefore \quad a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3.a\,.\,\frac{1}{a}\left(a - \frac{1}{a}\right)$

$= (5)^3 + 3(5) = 125 + 15 = 140.$

15. Find the product with the help of formula :

(a) $(a^2 + b^2)\,(a^4 - a^2b^2 + b^4)$

(b) $(ax - by)\,(a^2x^2 + abxy + b^2y^2)$

(c) $(2ab^2 - 1)\,(4a^2b^4 + 2ab^2 + 1)$

(d) $(x^2 + a)\,(x^4 - ax^2 + a^2)$

(e) $(7a + 4b)\,(49a^2 - 28ab + 16b^2)$

(f) $(2a - 1)\,(4a^2 + 2a + 1)\,(8a^3 + 1)$

(g) $(x + a)\,(x^2 - ax + a^2)\,(x - a)\,(x^2 + ax + a^2)$

(h) $(5a + 3b)\,(25a^2 - 15ab + 9b^2)\,(125a^3 - 27b^3)$

Solution :

(a) $(a^2 + b^2)(a^4 - a^2b^2 + b^4)$
$= (a^2 + b^2)\{(a^2)^2 - a^2b^2 + (b^2)^2\}$
$= (a^2)^3 + (b^2)^3 = a^6 + b^6$

(b) $(ax - by)(a^2x^2 + abxy + b^2y^2)$
$= (ax - by)\{(ax)^2 + ax.by + (by)^2\}$
$= (ax)^3 - (by)^3 = a^3x^3 - b^3y^3$

(c) $(2ab^2 - 1)(4a^2b^4 + 2ab^2 + 1)$
$= (2ab^2 - 1)\{(2ab^2)^2 + 2ab^2 . 1 + (1)^2\}$
$= (2ab^2)^3 - (1)^3 = 8a^3b^6 - 1$

(d) $(x^2 + a)(x^4 - ax^2 + a^2)$
$= (x^2 + a)\{(x^2)^2 - x^2 . a + (a)^2\}$
$= (x^2)^3 + (a)^3 = x^6 + a^3$

(e) $(7a + 4b)(49a^2 - 28ab + 16b^2)$
$= (7a + 4b)\{(7a)^2 - 7a . 4b + (4b)^2\}$
$= (7a)^3 + (4b)^3 = 343a^3 + 64b^3$

(f) $(2a - 1)(4a^2 + 2a + 1)(8a^3 + 1)$
$= \{(2a)^3 - (1)^3\}(8a^3 + 1)$
$= (8a^3 - 1)(8a^3 + 1)$
$= (8a^3)^2 - (1)^2 = 64a^6 - 1$

(g) $(x + a)(x^2 - ax + a^2)(x - a)(x^2 + ax + a^2)$
$= \{(x + a)(x^2 - ax + a^2)\}\{(x - a)(x^2 + ax + a^2)\}$
$= (x^3 + a^3)(x^3 - a^3)$
$= (x^3)^2 - (a^3)^2 = x^6 - a^6$

(h) $(5a + 3b)(25a^2 - 15ab + 9b^2)(125a^3 - 27b^3)$
$= (5a + 3b)\{(5a)^2 - 5a . 3b + (3b)^2\}(125a^3 - 27b^3)$
$= \{(5a)^3 + (3b)^3\}(125a^3 - 27b^3)$
$= (125a^3 + 27b^3)(125a^3 - 27b^3)$
$= (125a^3)^2 - (27b^3)^2$
$= 15625a^6 - 729b^6$

Mathematics

Multiple Choice Q/A Designed as per topic

4.2 Formulae of cubes and corollaries › Textbook Page 56

1. If $x + \frac{1}{x} = 4$, which one is the value of $x^3 + \frac{1}{x^3}$. *(Medium)* *[JB '19]*

ⓐ ⓐ 52 ⓑ 56 ⓒ 72 ⓓ 76

2. If $4x - 3 = 5$, what is the value of $64x^3 - 27 - 180x$? *(Medium)* *[CB '19]*

ⓑ ⓐ 25 ⓑ 125 ⓒ 140 ⓓ 305

3. If $a + b = 4$ and $a - b = 2$, then $a^3 - b^3 = ?$ *(Hard)* *[CtgB '19]*

ⓑ ⓐ 27 ⓑ 26 ⓒ 8 ⓓ 1

4. Which one of the cubic value of $x^3 + 2$? *(Easy)* *[DB '17]*

ⓓ ⓐ $x^6 + 8$ ⓑ $x^9 + 8$
ⓒ $x^6 4x^3 + 4$ ⓓ $x^9 + 6x^6 + 12x^3 + 8$

5. If $x + \frac{1}{x} = 2$, which one is the value of $x^3 + \frac{1}{x^3}$? *(Medium)* *[RB '17]*

ⓑ ⓐ 0 ⓑ 2 ⓒ 12 ⓓ 14

6. If $x - \frac{1}{x} = 1$, which one of the following is the value of $x^3 - \frac{1}{x^3}$? *(Medium)* *[CB '17]*

ⓐ ⓐ 4 ⓑ 6 ⓒ 7 ⓓ 8

7. If $a^3 - b^3 = 36$, $a - b = 3$, then ab = what? *(Easy)* *[BB '17]*

ⓒ ⓐ -1 ⓑ 0 ⓒ 1 ⓓ 3

8. If $x + y = 2$, $x^3 + y^3 + 6xy$ = what? *(Hard)* *[DB '16]*

ⓒ ⓐ -8 ⓑ 0 ⓒ 8 ⓓ 10

9. If $x + \frac{1}{x} = 2$ then which one is the value of $\left(x^3 + \frac{1}{x^3}\right)$? *(Hard)* *[RB '16]*

ⓑ ⓐ 0 ⓑ 2 ⓒ 12 ⓓ 14

10. If $a + \frac{1}{a} = 2$. *(Easy)* *[MB '19]*

i. $a^2 + \frac{1}{a^2} = 2$ ii. $a^3 + \frac{1}{a^3} = 2$

iii. $a^4 + \frac{1}{a^4} = 5$

Which one is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

11. If $p + q = 7$ and $pq = 2$ then —. *(Medium)* *[DB '18]*

i. $p^2 - q^2 = 7\sqrt{41}$
ii. $p^2 + q^2 = 45$
iii. $p^3 - q^3 = 45\sqrt{41}$

Which one of the following is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

12. The formula of Algebra is —. *(Hard)* *[BB '18]*

i. $(a - b)^2 = (a + b)^2 - 2ab$
ii. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$
iii. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$

Which one of the following is correct?

ⓒ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

13. If $x + \frac{1}{x} = 2$— *(Easy)* *[DB '17]*

i. $x^2 + \frac{1}{x^2} = 2$

ii. $x^3 - \frac{1}{x^3} = 0$

iii. $x^4 + \frac{1}{x^4} = 4$

Which one is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

14. If $y^2 - 2y + 1 = 0$, then— *(Hard)* *[CB '17]*

i. $\left(y - \frac{1}{y}\right) = 0$

ii. $y^2 + \frac{1}{y^2} = 2$

iii. $y^3 - \frac{1}{y^3} = 3$

Which one is correct?

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

15. i. $x^3 + y^3 = (x - y)(x^2 + xy + y^2)$ *(Medium)*

ii. $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$

iii. $(x + y)^3 = x^3 + y^3 + 3xy(x + y)$

Which one is correct? *[DB '16]*

c ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

16. If $x^2 - 2x + 1 = 0$ then —. *(Medium)* *[RB '16]*

i. $\left(x - \frac{1}{x}\right)^2 = 0$

ii. $x^2 + \frac{1}{x^2} = 2$

iii. $x^3 - \frac{1}{x^3} = 3$

Which one is correct?

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

17. If $a + \frac{1}{a} = 3$ *(Medium)* *[JB '16]*

i. $a^3 + \frac{1}{a^3} = 18$ ii. $\left(a - \frac{1}{a}\right)^2 = 5$

iii. $\left(a + \frac{1}{a}\right)^3 = 27$

Which one is correct?

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer the questions no. 18 and 19 based on the following information :

$m^2 + 1 = 2m$. *[DjB '19]*

18. What is the value of $2m + \frac{3}{m} = ?$ *(Medium)*

c ⓐ 2 ⓑ 3 ⓒ 5 ⓓ 7

19. What is the value of $m^3 - \frac{1}{m^3} = ?$ *(Medium)*

a ⓐ 0 ⓑ 2 ⓒ 5 ⓓ 6

If $a + \frac{1}{a} = 3$, then answer the questions No. 20 and 21 : *[MB '19]*

20. $a^2 + \frac{1}{a^2} = ?$ *(Medium)*

b ⓐ 5 ⓑ 7 ⓒ 11 ⓓ 13

21. $a^3 + \frac{1}{a^3} = ?$ *(Medium)*

a ⓐ 18 ⓑ 36 ⓒ 24 ⓓ 0

Answer to the questions number 22 and 23 based on the following information :

$a + b = 8$ and $ab = 12$. *[CB '18]*

22. Which one is the value of $a^2 - b^2$? *(Easy)*

a ⓐ 32 ⓑ 40 ⓒ 88 ⓓ 128

23. Which one is the value of $a^3 + b^3$? *(Easy)*

a ⓐ 224 ⓑ 320 ⓒ 352 ⓓ 476

Answer the questions No. 24 and 25 on the basis of following information :

$2x + y = 3$ and $xy = 1$. *[SB '18]*

24. What is the value of $4x^2 + y^2$? *(Hard)*

a ⓐ 5 ⓑ 7 ⓒ 11 ⓓ 13

25. What is the value of $8x^3 + y^3$? *(Medium)*

a ⓐ 9 ⓑ 18 ⓒ 36 ⓓ 45

Answer the questions No. 26 and 27 in the light of the following information :

$x + y = 5$ and $x - y = 1$. *[DjB '18]*

26. What is the value of $2(x^2 + y^2)$? *(Medium)*

d ⓐ 12 ⓑ 13 ⓒ 24 ⓓ 26

27. What is the value of $x^3 - y^3$? *(Hard)*

c ⓐ − 35 ⓑ − 17 ⓒ 19 ⓓ 35

Answer the question No. 28 and 29 based on the following information :

$a + b = 12$, $a - b = 2$

28. Which one is the value of $a^2 + b^2$? *(Medium)* *[DB '17]*

a ⓐ 74 ⓑ 100 ⓒ 146 ⓓ 196

29. Which one is the value of $a^3 + b^3$? *(Hard)* *[DB '17]*

b ⓐ 8 ⓑ 468 ⓒ 1000 ⓓ 1728

Answer the questions No. 30 and 31 based on the following information :

$a + b = 7$, $ab = 10$ and $a > b$. *[CtgB '16]*

30. What is the value of $a^2 + b^2$? *(Hard)*

a ⓐ 29 ⓑ 39 ⓒ 69 ⓓ 89

31. Which one is the value of $a^3 - b^3$? *(Hard)*

c ⓐ 57 ⓑ 63 ⓒ 117 ⓓ 133

Answer to the questions No. 32 and 33 in the light of the following information :

$x + \frac{1}{x} = 3$. *[DjB '16]*

32. What is the value of $\left(x - \frac{1}{x}\right)^2$? *(Medium)*

d ⓐ 13 ⓑ 11 ⓒ 7 ⓓ 5

33. What is the value of $x^3 + \frac{1}{x^3}$? *(Medium)*

c ⓐ 36 ⓑ 33 ⓒ 18 ⓓ 0

If $a - \frac{1}{a} = 3$ then answer question No. 34, 35 and 36. *[Viqarunnisa Noon School and College, Dhaka]*

34. If $\left(a + \frac{1}{a}\right)^2 = ?$ *(Easy)*

ⓒ ⓐ 5 ⓑ $\sqrt{5}$ ⓒ 13 ⓓ $\sqrt{13}$

35. $a^2 - \frac{1}{a^2} = ?$ *(Hard)*

ⓓ ⓐ 39 ⓑ $3\sqrt{5}$ ⓒ 15 ⓓ $3\sqrt{13}$

36. What is the value of $a^3 - \frac{1}{a^3}$? *(Medium)*

ⓓ ⓐ 27 ⓑ 36 ⓒ 3 ⓓ $6\sqrt{3}$

4.3 Two more Formulae related to Cubes ▸ Textbook Page 60

37. If $p^3 - q^3 = 45$, $p - q = 3$, what is the value of pq? *(Medium)* *[SB '19]*

ⓑ ⓐ −2 ⓑ 2 ⓒ 4 ⓓ 6

38. $\frac{x^3 + y^3 + 3xy(x+y)}{(x+y)^2 - 4xy} \div \frac{(x+y)^2}{(x-y)^3} = ?$ *(Easy)* *[BB '18]*

ⓐ ⓐ $x^2 - y^2$ ⓑ $x^2 + y^2$
ⓒ ⓒ $(x+y)^3$ ⓓ $x^3 + y^3$

39. Which one of the following is the value of $(2x - 2)(4x^2 + 4x + 4)$ if $x = 4$? *(Hard)* *[CB '17]*

ⓑ ⓐ 448 ⓑ 504 ⓒ 550 ⓓ 848

40. Which one of the following is the value of $(x - 2y)(x^2 + 2xy + 4y^2)$ if $x = 4$ & $y = 1$? *(Hard)* *[CB '17]*

ⓓ ⓐ 72 ⓑ 64 ⓒ 53 ⓓ 56

41. Which one of the following is the value of $(x + 3y)(x^2 - 3xy + 9y^2)$ if $x = 4$ & $y = 1$? *(Medium)* *[CB '17]*

ⓐ ⓐ 91 ⓑ 47 ⓒ 37 ⓓ 27

42. Which one of the following is the factor of $a^6 - b^6$? *(Medium)* *[JB '16]*

ⓐ $(a - b)(a^5 - b^5)$
ⓑ $(a^2 + b^2)(a^4 + a^2b^2 + b^4)$
ⓒ $(a^2 - b^2)(a^4 + a^2b^2 + b^4)$
ⓒ ⓓ $(a^3 - b^3)(a^4 + a^2b^2 + b^4)$

43. If $p + q = 3$, $pq = 2$, what is the value of $p^3 + q^3 = ?$ *(Hard)* *[CB '16]*

ⓐ ⓐ 9 ⓑ 18 ⓒ 27 ⓓ 45

44. If $3p + 2q = 13$ and $pq = 6$, then what is the value of $27p^3 + 8q^3$? *(Easy)* *[SB '16]*

ⓐ ⓐ 793 ⓑ 2089 ⓒ 2305 ⓓ 2521

45. The factor of $x^6 - y^6$ is —. *(Medium)* *[DjB '18]*

i. $x^2 - xy + y^2$
ii. $x^2 + xy + y^2$
iii. $x^2 + y^2$

Which one of the following is correct?

ⓐ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

Answer to the questions number 46 and 47 by using the following informations :
$x - y = 5$ and $xy = 6$. *[RB '19]*

46. Which one is the value of $x^3 - y^3$? *(Medium)*

ⓓ ⓐ 35 ⓑ 65 ⓒ 185 ⓓ 215

47. Which one is the value of $x^2 - y^2$? *(Medium)*

ⓑ ⓐ 37 ⓑ 35 ⓒ 25 ⓓ 5

Short Q/A Designed as per topic

4.2 Formulae of cubes and corolaries ▸ Textbook Page 56

Question 1. Find the cube of $a^3 - 2$.

Solution : Cube of $a^3 - 2$

$= (a^3 - 2)^3$

$= (a^3)^3 - 3.(a^3)^2.2 + 3.a^3.2^2 - (2)^3$

$[\because (a-b)^3 = a^3 - 3a^2b + 3ab^2 - b^3]$

$= a^9 - 6a^6 + 12a^3 - 8.$

Question 2. Find the cube of $3m + 2n + 4p$.

Solution : Cube of $3m + 2n + 4p$

$= (3m + 2n + 4p)^3$

$= \{(3m + 2n) + 4p\}^3$

$= (3m + 2n)^3 + 3.(3m + 2n)^2.4p + 3.(3m + 2n).(4p)^2 + (4p)^3$

$= (3m)^3 + 3.(3m)^2.2n + 3.3m.(2n)^2 + (2n)^3$
$+ 12p(9m^2 + 12mn + 4n^2) + 48p^2(3m + 2n) + 64p^3$

$= 27m^3 + 54m^2n + 36mn^2 + 8n^3 + 108m^2p + 144mn$
$+ 48n^2p + 144mp^2 + 96np^2 + 64p^3$

$= 27m^3 + 8n^3 + 64p^3 + 54m^2n + 36mn^2 + 48n^2p$
$+ 108m^2p + 144mp^2 + 96np^2 + 144mn$

Question 3. Simplify : $(x + 3y)^3 - 3(x + 3y)^2(x - 3y) + 3(x + 3y)(x - 3y)^2 - (x - 3y)^3$

Solution : Let, $x + 3y = a$ and $x - 3y = b$

Given expression $= a^3 - 3a^2b + 3ab^2 - b^3$

$= (a - b)^3$

$= \{(x + 3y) - (x - 3y)\}^3$

$= (x + 3y - x + 3y)^3 = (6y)^3 = 216y^3$

Question 4. If $a + b = 4$ and $ab = 3$, Find the value of $a^3 + b^3$

Solution : Given, $a + b = 4$ and $ab = 3$

Given expression $= a^3 + b^3$

$= (a + b)^3 - 3ab(a + b)$

$= (4)^3 - 3.3.4 = 64 - 36 = 28$

Required value : 28.

Question 5. If $m - n = 8$ and $mn = 15$, Find the value of $m^3 - n^3$

Solution : Given expression $= m^3 - n^3$

$= (m - n)^3 + 3mn(m - n)$

$= (8)^3 + 3.15.8 = 512 + 360 = 872$

Required value 872.

Question 6. If $p + q = 7$, Find the value of $p^3 + q^3 + 21pq$

Solution : Given, $p + q = 7$

Given expression $= p^3 + q^3 + 21pq$

$= (p + q)^3 - 3pq\,(p + q) + 21pq$

$= (7)^3 - 3pq \times 7 + 21pq$

$= 343 - 21pq + 21pq = 343$

Required value : 343.

Question 7. If $4a - 3 = 5$, Find the value of $64a^3 - 27 - 180a$

Solution : Given expression $= 64a^3 - 27 - 180a$

$= (4a)^3 - (3)^3 - 180a$

$= (4a - 3)^3 + 3.4a.3\,(4a - 3) - 180a$

$= (5)^3 + 3.4a.3.5 - 180a$

$= 125 + 180a - 180a = 125$

Required value : 125.

Question 8. If $x + \frac{1}{x} = 8$, Find the value of $x^3 + \frac{1}{x^3}$

Solution : Given, $x + \frac{1}{x} = 8$

Given expression $= x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (8)^3 - 3.8 = 512 - 24 = 488$

Required value : 488.

Question 9. If $a^2 - 3a + 1 = 0$, Show that, $a^3 + \frac{1}{a^3} = 18$

Solution : Given,

$a^2 - 3a + 1 = 0$

or, $a^2 + 1 = 3a$

or, $a + \frac{1}{a} = 3$ [Dividing both sides by a]

L.H.S. $= a^3 + \frac{1}{a^3}$

$= \left(a + \frac{1}{a}\right)^3 - 3.a.\frac{1}{a}\left(a + \frac{1}{a}\right)$

$= 3^3 - 3.3 = 27 - 9 = 18 =$ R.H.S.

$\therefore \quad a^3 + \frac{1}{a^3} = 18$. **(Shown)**

Question 10. If $3m + 2n = 13$ and $mn = 6$, Find the value of $27m^3 + 8n^3$?

Solution : Given, $3m + 2n = 13$ and $mn = 6$

Given expression $= 27m^3 + 8n^3$

$= (3m)^3 + (2n)^3$

$= (3m + 2n)^3 - 3.3m.2n\,(3m + 2n)$

$= (3m + 2n)^3 - 18mn\,(3m + 2n)$

$= (13)^3 - 18 \times 6 \times 13 = 2197 - 1404 = 793$

Required value : 793.

Question 11. If $x = -7$, Find the value of $x^3 + 6x^2 + 12x + 1$

Solution : Given, $x = -7$

Given expression $= x^3 + 6x^2 + 12x + 1$

$= x^3 + 3.x^2.2 + 3.x.2^2 + 2^3 - 7$

$= (x + 2)^3 - 7$

$= (-7 + 2)^3 - 7 = (-5)^3 - 7 = -125 - 7 = -132$

Required value : -132.

Question 12. If $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3}$, Find the value of $x^3 + \frac{1}{x^3}$?

Solution : Given, $\sqrt{x} + \frac{1}{\sqrt{x}} = \sqrt{3}$

or, $\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 = (\sqrt{3})^2$ [Squaring]

or, $(\sqrt{x})^2 + 2.\sqrt{x}.\frac{1}{\sqrt{x}} + \left(\frac{1}{\sqrt{x}}\right)^2 = 3$

or, $x + 2 + \frac{1}{x} = 3$

$\therefore x + \frac{1}{x} = 3 - 2 = 1$

Given expression $= x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (1)^3 - 3.1 = 1 - 3 = -2$

Required value : -2.

Question 13. If $a + b + c = 0$, Prove that, $a^3 + b^3 + c^3 = 3abc$.

Solution : Given, $a + b + c = 0$

$\therefore a + b = -c$

L.H.S. $= a^3 + b^3 + c^3$

$= (a + b)^3 - 3ab(a + b) + c^3$

$= (-c)^3 - 3ab\,(-c) + c^3$

$= -c^3 + 3abc + c^3 = 3abc =$ R.H.S.

$\therefore a^3 + b^3 + c^3 = 3abc$. **(Proven)**

Question 14. If $x + \frac{1}{x} = \sqrt{3}$, Find the value of $x^3 + \frac{1}{x^3}$?

Solution : Given, $x + \frac{1}{x} = \sqrt{3}$

Given expression $= x^3 + \frac{1}{x^3}$

$= \left(x + \frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (\sqrt{3})^3 - 3\sqrt{3} = 3\sqrt{3} - 3\sqrt{3} = 0$

Required value 0.

Question 15. If $m - \frac{1}{m} = 5$, Find the value of $m^3 - \frac{1}{m^3}$?

Solution : Given, $m - \frac{1}{m} = 5$

Given expression $= m^3 - \frac{1}{m^3}$

$= \left(m - \frac{1}{m}\right)^3 + 3.m.\frac{1}{m}\left(m - \frac{1}{m}\right)$

$= (5)^3 + 3.5 = 125 + 15 = 140$

Required value : 140.

4.3 Two more Formulae related to cubes

▸ Textbook Page 60

Question 16. Find the product of $(a^2 + 3)$ and $(a^4 - 3a^2 + 9)$ with the help of formula.

Solution : $(a^2 + 3)(a^4 - 3a^2 + 9)$
$= (a^2 + 3)\{(a^2)^2 - a^2.3 + (3)^2\}$
$= (a^2)^3 + (3)^3 = a^6 + 27$
Required product : $a^6 + 27$.

Question 17. Find the product of $(2p - 3q)$ and $(4p^2 + 6pq + 9q^2)$ with the help of formula.

Solution : $(2p - 3q)(4p^2 + 6pq + 9q^2)$
$= (2p - 3q)\{(2p)^2 + 2p.3q + (3q)^2\}$
$= (2p)^3 - (3q)^3 = 8p^3 - 27q^3$
Required product : $8p^3 - 27q^3$.

Creative Q/A Designed as per learning outcomes

Ques. 01 $x - y = 3$, $xy = 4$ and $p^2 - 2\sqrt{3}p + 1 = 0$.

a. Resolve into factors : $a^4 + a^2b^2 + b^4$. 2
b. Find the value of $x^3 - y^3 - 2(x + y)^2$. 4
c. Prove that, $p^3 + \frac{1}{p^3} = 18\sqrt{3}$. 4

● *Dhaka Board 2019*

Solution to Question No. 01 :

a $a^4 + a^2b^2 + b^4 = (a^2)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2$
$= (a^2 + b^2)^2 - (ab)^2$
$= (a^2 + ab + b^2)(a^2 - ab + b^2)$.

b Given that,
$x - y = 3$
and $xy = 4$
Now,
$x^3 - y^3 - 2(x + y)^2 = (x - y)^3 + 3xy(x - y) - 2\{(x - y)^2 + 4xy\}$
$= 3^3 + 3.4.3 - 2\{(3)^2 + 4.4\}$ [$\because x - y = 3$ & $xy = 4$]
$= 27 + 36 - 2(9 + 16)$
$= 63 - 50$
$= 13$
$\therefore$ Determined value of $x^3 - y^3 - 2(x + y)^2$ is 13.

c Given,
$p^2 - 2\sqrt{3}p + 1 = 0$
or, $p^2 + 1 = 2\sqrt{3}p$
or, $\frac{p^2 + 1}{p} = 2\sqrt{3}$
or, $p + \frac{1}{p} = 2\sqrt{3}$
or, $\left(p + \frac{1}{p}\right)^3 = \left(2\sqrt{3}\right)^3$ [Cubing both sides]
or, $p^3 + \frac{1}{p^3} + 3.p.\frac{1}{p}\left(p + \frac{1}{p}\right) = 24\sqrt{3}$
or, $p^3 + \frac{1}{p^3} + 3.2\sqrt{3} = 24\sqrt{3}$ [$\because p + \frac{1}{p} = 2\sqrt{3}$]
or, $p^3 + \frac{1}{p^3} = 24\sqrt{3} - 6\sqrt{3}$
$\therefore p^3 + \frac{1}{p^3} = 18\sqrt{3}$ **(Proved)**

Ques. 02 $a - \frac{1}{a} = 7$ and $x^2 - y^2 - z^2 = p$

a. Multiply $3x - 2xy^2$ by $9x^2 + 6x^2y^2 + 4x^2y^4$ by an appropriate formula. 2
b. Find the value of $a^4 + \frac{1}{a^4}$. 4
c. If $p \neq 0$, then prove that, $x^6 - y^6 - z^6 = 3x^2y^2z^2$. 4

● *Jashore Board 2019*

Solution to Question No. 02 :

a $(3x - 2xy^2) \times (9x^2 + 6x^2y^2 + 4x^2y^4)$
$= (3x - 2xy^2) \times \{(3x)^2 + 3x.\ 2xy^2 + (2xy^2)^2\}$
$= (3x)^3 - (2xy^2)^3$
$= 27x^3 - 8x^3y^6$ **(Ans.)**

b Given,
$a - \frac{1}{a} = 7$
or, $\left(a^2 - \frac{1}{a}\right)^2 = (7)^2$ [Squaring on both sides]
or, $a^2 + \frac{1}{a^2} - 2.a.\frac{1}{a} = 49$
or, $a^2 + \frac{1}{a^2} = 49 + 2$
or, $\left(a^2 + \frac{1}{a^2}\right)^2 = (51)^2$
or, $(a^2)^2 + \left(\frac{1}{a^2}\right)^2 + 2a^2.\frac{1}{a^2} = 2601$
or, $a^4 + \frac{1}{a^4} + 2 = 2601$
or, $a^4 + \frac{1}{a^4} = 2601 - 2$
$\therefore a^4 + \frac{1}{a^4} = 2599$ **(Ans.)**

c Given,
$x^2 - y^2 - z^2 = p$ ..... (i)
If $p = 0$ then from (i) we get,
$x^2 - y^2 - z^2 = 0$
or, $x^2 - y^2 = z^2$

Mathematics

or, $(x^2 - y^2)^3 = (z^2)^3$
or, $(x^2)^3 - (y^2)^3 - 3x^2y^2(x^2 - y^2) = z^6$
or, $x^6 - y^6 - 3x^2y^2z^2 = z^6$
$\therefore$ $x^6 - y^6 - z^6 = 3x^2y^2z^2$ **(Proved)**

Ques. 03 $P = a^2 + b^2 - c^2$, $Q = x^2 - \sqrt{5}x - 1$.

a. Determine the square of $3x - 4y + 5z$. 2
b. If $P = 0$, prove that, $a^6 + b^6 + 3a^2b^2c^2 = c^6$ 4
c. If $Q = 0$, determine the value of $x^4 + \frac{1}{x^4}$. 4

● *Cumilla Board 2019*

Solution to Question No. 03 :

a Square of $3x - 4y + 5z$
$= (3x - 4y + 5z)^2$
$= (3x)^2 + (-4y)^2 + (5z)^2 + 2(3x(-4y) + (-4y).5z + 5z.3x)$
$= 9x^2 + 16y^2 + 25z^2 + 2(-12xy - 20yz + 15xz)$
$= 9x^2 + 16y^2 + 25z^2 - 24xy - 40yz + 30xz$ **(Ans.)**

b Given, $P = a^2 + b^2 - c^2$
Now, $P = 0$
$\therefore$ $a^2 + b^2 - c^2 = 0$
or, $a^2 + b^2 = c^2$
or, $(a^2 + b^2)^3 = (c^2)^3$ [Cubing both sides]
or, $a^6 + b^6 + 3a^2b^2(a^2 + b^2) = c^6$
or, $a^6 + b^6 + 3a^2b^2c^2 = c^6$ [$\because a^2 + b^2 = c^2$]
$\therefore$ $a^6 + b^6 + 3a^2b^2c^2 = c^6$ **[Proved]**

c Given, $Q = x^2 - \sqrt{5}x - 1$
Now, $Q = 0$
$\therefore$ $x^2 - \sqrt{5}x - 1 = 0$
or, $x^2 - 1 = \sqrt{5}x$
or, $\frac{x^2 - 1}{x} = \frac{\sqrt{5}x}{x}$
or, $x - \frac{1}{x} = \sqrt{5}$
or, $\left(x - \frac{1}{x}\right)^2 = (\sqrt{5})^2$ [Squaring both sides]
or, $x^2 + \frac{1}{x^2} - 2x.\frac{1}{x} = 5$
or, $x^2 + \frac{1}{x^2} = 5 + 2$
or, $\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$ [Squaring again]
or, $(x^2)^2 + \left(\frac{1}{x^2}\right)^2 + 2x^2.\frac{1}{x^2} = 49$
or, $x^4 + \frac{1}{x^4} + 2 = 49$
or, $x^4 + \frac{1}{x^4} = 47$

$\therefore$ Determined value of $x^4 + \frac{1}{x^4}$ is 47 **(Ans.)**

Ques. 04 $P = 5x - 3$, $Q = 2x - 1$ and $R = 3x^2 - 10x + 3$ are three algebraic expressions.

a. Express PQ in the form of difference of two squares. 2
b. If $P = \frac{5}{x}$, determine the value of $\left(x^2 - \frac{1}{x^2}\right)^2$. 4
c. If $R = 0$, show that, $x^3 + \frac{1}{x^3} = \frac{730}{27}$. 4

● *Dinajpur Board 2019*

Solution to Question No. 04 :

a Given, $P = 5x - 3$
and $Q = 2x - 1$
Now, $PQ = (5x - 3)(2x - 1)$
$= \left(\frac{5x - 3 + 2x - 1}{2}\right)^2 - \left(\frac{5x - 3 - 2x + 1}{2}\right)^2$ $\left[\because ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2\right]$
$= \left(\frac{7x - 4}{2}\right)^2 - \left(\frac{3x - 2}{2}\right)^2$; Which is in the required form of difference of two squares.

b Given,
$P = 5x - 3$
or, $\frac{5}{x} = 5x - 3$ $\left[\because P = \frac{5}{x}\right]$
or, $5x - \frac{5}{x} = 3$
or, $5\left(x - \frac{1}{x}\right) = 3$
or, $x - \frac{1}{x} = \frac{3}{5}$
or, $x^2 - 2 \cdot x \cdot \frac{1}{x} + \frac{1}{x^2} = \frac{9}{25}$ [Squaring both sides]
or, $x^2 + \frac{1}{x^2} = \frac{9}{25} + 2$
or, $x^2 + \frac{1}{x^2} = \frac{59}{25}$
or, $\left(x^2 + \frac{1}{x^2}\right)^2 = \frac{3481}{625}$ [Squaring again]
or, $\left(x^2 - \frac{1}{x^2}\right)^2 + 4 \cdot x^2 \cdot \frac{1}{x^2} = \frac{3481}{625}$ [$\because (a+b)^2 = (a-b)^2 + 4ab$]
or, $\left(x^2 - \frac{1}{x^2}\right)^2 = \frac{3481}{625} - 4$
or, $\left(x^2 - \frac{1}{x^2}\right)^2 = \frac{3481 - 2500}{625}$
$\therefore$ $\left(x^2 - \frac{1}{x^2}\right)^2 = \frac{981}{625}$

$\therefore$ Determined value of $\left(x^2 - \frac{1}{x^2}\right)^2$ is $\frac{981}{625}$

C Given,
$R = 3x^2 - 10x + 3$
or, $0 = 3x^2 - 10x + 3$ $[\because R = O]$
or, $3x^2 + 3 = 10x$
or, $\frac{3x^2+3}{x} = 10$
or, $3x + \frac{3}{x} = 10$
or, $3\left(x + \frac{1}{x}\right) = 10$
or, $x + \frac{1}{x} = \frac{10}{3}$
or, $x^3 + \frac{1}{x^3} + 3 \cdot x \cdot \frac{1}{x}\left(x + \frac{1}{x}\right) = \frac{1000}{27}$ [Cubing both sides]
or, $x^3 + \frac{1}{x^3} + 3 \cdot \frac{10}{3} = \frac{1000}{27}$ $\left[\because x + \frac{1}{x} = \frac{10}{3}\right]$
or, $x^3 + \frac{1}{x^3} + 10 = \frac{1000}{27}$
or, $x^3 + \frac{1}{x^3} = \frac{1000}{27} - 10$
or, $x^3 + \frac{1}{x^3} = \frac{1000 - 270}{27}$
$\therefore x^3 + \frac{1}{x^3} = \frac{730}{27}$ **[Showed]**

Ques. 05 $2x^2 - 3x + 2 = 0$ is a algebraic equation.

a. Find the value of $\left(x + \frac{1}{x}\right)$. 2
b. Find the value of $x^4 + \frac{1}{x^4}$. 4
c. Prove that, $8x^6 + 9x^3 + 8 = 0$. 4

Dhaka Board 2018

Solution to Question No. 05 :

a Given that, $2x^2 - 3x + 2 = 0$
or, $x - \frac{3}{2} + \frac{1}{x} = 0$, dividing by 2x.
or, $x + \frac{1}{x} = \frac{3}{2}$
$\therefore x + \frac{1}{x} = \frac{3}{2}$.

b Here, $x^4 + \frac{1}{x^4} = (x^2)^2 + \left(\frac{1}{x^2}\right)^2$
$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2.x^2.\frac{1}{x^2}$
$= \left\{\left(x + \frac{1}{x}\right)^2 - 2.x.\frac{1}{x}\right\}^2 - 2$
$= \left\{\left(\frac{3}{2}\right)^2 - 2\right\}^2$, putting $x + \frac{1}{x} = \frac{3}{2}$
from (a) above,
$= \left(\frac{9}{4} - 2\right)^2 - 2 = \frac{1}{16} - 2 = -\frac{31}{16}$.

C Given, $2x^2 - 3x + 2 = 0 \Rightarrow 2x^2 + 2 = 3x$.....(i)
Now, $8x^6 + 9x^3 + 8$
$= (2x^2)^3 + 2^3 + 9x^3$
$= (2x^2 + 2)^3 - 3.2x^2.2(2x^2 + 2) + 9x^3$
$= (3x)^3 - 12x^2(3x) + 9x^3$,
putting $2x^2 + 2 = 3x$ from (i).
$= 27x^3 - 36x^3 + 9x^3$
$= 36x^3 - 36x^3$
$= 0$.
$\therefore 8x^6 + 9x^3 + 8 = 0$. **(proved)**

Ques. 06 $x + \frac{1}{x} = \sqrt{5}$, where $x > 0$ and $A = a^2 + 4a + 4$, $B = a^2 - 4$, $C = a^3 - 8$.

a. Find the cube of $(x^2 - 2)$ 2
b. Prove that $x^6 - 4x^3 - 1 = 0$ 4
c. Determine the H.C.F. of A, B and C. 4

Sylhet Board 2018

Solution to Question No. 06 :

a Cube of $(x^2 - 2) = (x^2 - 2)^3$
$= (x^2)^3 - 3.(x^2)^2.2 + 3.x^2.(2)^2 - (2)^3$
$= x^6 - 6x^4 + 12x^2 - 8$.

b Given that, $x + \frac{1}{x} = \sqrt{5}$, where $x > 0$.
$\therefore \left(x - \frac{1}{x}\right) = \sqrt{\left(x + \frac{1}{x}\right)^2 - 4} = \sqrt{5 - 4} = 1$
Now, $x^6 - 4x^3 - 1 = 0$
or, $x^3 - 4 - \frac{1}{x^3} = 0$, dividing both sides by x^3,
or, $x^3 - \frac{1}{x^3} - 4 = 0$,.....(i)
In this stage, it will be sufficient to prove $x^6 - 4x^3 - 1 = 0$ if we can prove $x^3 - \frac{1}{x^3} - 4 = 0$,.....(2)
LHS of (2) $= x^3 - \frac{1}{x^3} - 4$
$= \left(x - \frac{1}{x}\right) + 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right) - 4$
$= 1^3 + 3(1) - 4$, putting for $x - \frac{1}{x} = 1$
$= 4 - 4 = 0$
$\therefore x^3 - \frac{1}{x^3} - 4 = 0$
or, $x^6 - 4x^3 - 1 = 0$. **(Proved)**

C Here, $A = a^2 + 4a + 4 = (a + 2)(a + 2)$
$B = a^2 - 4 = a^2 - 2^2 = (a + 2)(a - 2)$
$C = a^3 - 8 = a^3 - 2^3 = (a - 2)(a^2 + 2a + 4)$
Since there exist no common factor in A, B and C except 1.
$\therefore$ H.C.F. of A, B and C = 1.

Mathematics

Ques. 07 $P = x^2 - ax + 1$, $Q = p^2 + q^2 - r^2$, $R = x^6 - 1$ are three algebraic expressions.

a. Resolve into factors of R. 2

b. If P = 0, show that $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$. 4

c. If Q = 0, prove that, $p^6 + q^6 + 3p^2q^2r^2 = r^6$. 4

Barishal Board 2018

Solution to Question No. 07 :

a Given that, $R = x^6 - 1$

$= (x^3)^2 - 1^2$

$= (x^3 - 1)(x^3 + 1)$

$= (x - 1)(x^2 + x + 1)(x + 1)(x^2 - x + 1)$

b Here, $P = 0$

$\therefore x^2 - ax + 1 = 0$

or, $x + \frac{1}{x} = a$, dividing both sides by x and then transposing a.

Now, $x^4 + \frac{1}{x^4} = \left(x^2 + \frac{1}{x^2}\right)^2 - 2$

$= \left\{\left(x + \frac{1}{x}\right)^2 - 2\right\}^2 - 2$, putting $x + \frac{1}{x} = a$

$= (a^2 - 2)^2 - 2$

$= a^4 - 4a^2 + 4 - 2$

$= a^4 - 4a^2 + 2$

$\therefore x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$. **(Shown)**

c Given, $Q = 0$

$\therefore p^2 + q^2 - r^2 = 0, \Rightarrow p^2 + q^2 = r^2$ ..... (i)

Now, $p^6 + q^6 + 3p^2q^2r^2$

$= (p^2 + q^2)^3 - 3p^2q^2(p^2 + q^2) + 3p^2q^2r^2$

$= (r^2)^3 - 3p^2q^2(r^2) + 3p^2q^2r^2$, putting $p^2 + q^2 = r^2$ from (i)

$= r^6$

$\therefore p^6 + q^6 + 3p^2q^2r^2 = r^6$ **(Proved)**

Ques. 08 $x^2 - \sqrt{5}x + 1 = 0$.

a. Find the value of $x + \frac{1}{x}$. 2

b. Prove that, $x^3 - \frac{1}{x^3} = 4$. 4

c. Find the value of $\left(x^6 - \frac{1}{x^6}\right)$. 4

Dhaka Board 2017

Solution to Question No. 08 :

a Here we have,

$x^2 - \sqrt{5}x + 1 = 0$

or, $\frac{x^2 - \sqrt{5}x + 1}{x} = \frac{0}{x}$

or, $x - \sqrt{5} + \frac{1}{x} = 0$

or, $x = \frac{1}{x} = \sqrt{5}$

$\therefore x + \frac{1}{x} = \sqrt{5}$

b We know,

$\left(x - \frac{1}{x}\right) = \sqrt{\left(x + \frac{1}{x}\right)^2 - 4.x.\frac{1}{x}} = \sqrt{(\sqrt{5})^{2-4}}$,

putting $x + \frac{1}{x} = \sqrt{5}$ from (a)

$= \sqrt{5 - 4} = \sqrt{1} = 1$

$\therefore x^3 - \frac{1}{a^3} = \left(x - \frac{1}{x}\right)^3 + 3.x - \frac{1}{x}\left(x - \frac{1}{x}\right)$

$= 1^3 + 3.1$, putting $x - \frac{1}{x} = 1$

$= 4$

$\therefore x^3 - \frac{1}{x^3} = 4$ **(Proved)**

c From (a) and (b) above, we get,

$x + \frac{1}{x} = \sqrt{5}$ and $x - \frac{1}{x} = 1$

Now $x^6 - \frac{1}{x^6} = (x^3)^2 - \left(\frac{1}{x^3}\right)^2 = \left(x^3 + \frac{1}{x^3}\right)\left(x^3 - \frac{1}{x^3}\right)$

$= \left\{\left(x + \frac{1}{x}\right)^3 - 3.x.\frac{1}{x}\left(x + \frac{1}{x}\right)\right\}\left\{\left(x - \frac{1}{x}\right)^3 + 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right)\right\}$

$= \left\{(\sqrt{5})^3 - 3\sqrt{5}\right\}\{1^3 + 3.1\}$

$= \{5\sqrt{5} - 3\sqrt{5}\}\{4\} = 2\sqrt{5} \times 4 = 8\sqrt{5}$.

$\therefore x^6 - \frac{1}{x^6} = 8\sqrt{5}$.

Ques. 09 If $x^2 = 3x - 1$ then —.

a. $\left(x + \frac{1}{x}\right)^2$ = what? 2

b. Show that $x^4 = 47 - \frac{1}{x^4}$. 4

c. Find the value of $\frac{x^6 - 1}{x^3}$. 4

Jashore Board 2017

Solution to Question No. 09 :

a Here we have,

$x^2 = 3x - 1$

or, $x = 3 - \frac{1}{x}$, dividing both sides by x.

or, $x + \frac{1}{x} = 3$

or, $\left(x + \frac{1}{x}\right)^2 = 3^2$ or 9.

$\therefore \left(x + \frac{1}{x}\right)^2 = 9$.

b From (a), we have, $\left(x+\frac{1}{x}\right)^2=9$.

$\therefore x^4=47-\frac{1}{x^4}$

or, $x^4+\frac{1}{x^4}=47$

Now, $x^4+\frac{1}{x^4}$.

$=\left(x^2+\frac{1}{x^2}\right)^2-2$

$=\left\{\left(x+\frac{1}{x}\right)^2-2\right\}^2-2$

$=(9-2)^2-2=7^2-2$

$=49-2=47$

$\therefore x^4+\frac{1}{x^4}=47$

or, $x^4=47-\frac{1}{x^4}$ **(Showed)**

c Here given that,

$\frac{x^6-1}{x^3}=x^3-\frac{1}{x^3}$.

Now, $x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3.x.\frac{1}{x}\left(x-\frac{1}{x}\right)$ .....(1)

But from (a), we have,

$x+\frac{1}{x}=3$

$\therefore \left(x-\frac{1}{x}\right)=\sqrt{\left(x+\frac{1}{x}\right)^2-4}=\sqrt{5}$

$\therefore$ From (1), we get,

$x^3-\frac{1}{x^3}=\left(x-\frac{1}{x}\right)^3+3\left(x-\frac{1}{x}\right)$

$=(\sqrt{5})^3-3\sqrt{5}$, putting $x-\frac{1}{x}=\sqrt{5}$.

$=5\sqrt{5}-3\sqrt{5}=2\sqrt{5}$.

$\therefore x^3-\frac{1}{x^3}=2\sqrt{5}$.

Ques. 10 $P^2-2p-1=0$.

a. Find our the value of $\left(p-\frac{1}{p}\right)$. 2

b. Evaluate the value of $\left(p^2+\frac{1}{p^2}\right)\left(p^3+\frac{1}{p^3}\right)$. 4

c. In light of the stem prove that $p^8-34p^4+1=0$. 4

● *Cumilla Board 2017*

Solution to Question No. 10 :

a Here we have,

$p^2-2p-1=0$

or, $p-2-\frac{1}{p}=0$, dividing both sides by p

or, $p-\frac{1}{p}=2$

$\therefore p-\frac{1}{p}=2$.

b From (a) above, we get,

$p-\frac{1}{p}=2$

$\therefore p+\frac{1}{p}=\sqrt{\left(p-\frac{1}{p}\right)^2+4}=\sqrt{4+4}=\sqrt{8}$

Now, $\left(p^2+\frac{1}{p^2}\right)\left(p^3+\frac{1}{p^3}\right)$

$=\left\{\left(p+\frac{1}{p}\right)^2-2\right\}\left\{\left(p+\frac{1}{p}\right)^3-3.\left(p+\frac{1}{p}\right)\right\}$

$=\{(\sqrt{8})^2-2\}\ \{(\sqrt{8})^3-3(\sqrt{8})\}$

$=\{8-2\}\ \{8\sqrt{8}-3\sqrt{8}\}$

$=6\times5\sqrt{8}=30\sqrt{8}$

$=30\times2\sqrt{2}=60\sqrt{2}$.

c We have,

$p^2-2p-1=0$

or, $p-2-\frac{1}{p}=0$, dividing both sides by p

or, $p-\frac{1}{p}=2$

$\therefore p+\frac{1}{p}=\sqrt{\left(p-\frac{1}{p}\right)^2+4}=\sqrt{2^2+4}=\sqrt{8}=2\sqrt{2}$

Now it is required to prove that,

$p^8-34p^4+1=0$

or, $p^4-34+\frac{1}{p^4}=0$, dividing both sides by p^4

or, $p^4+\frac{1}{p^4}=34$ ---------- (1)

From L.H.S. of (1), we have,

$p^4+\frac{1}{p^4}=\left(p^2+\frac{1}{p^2}\right)^2-2=\left\{\left(p+\frac{1}{p}\right)^2-2\right\}^2-2$

$=\{(2\sqrt{2}^2-2\}^2-2$, putting $p+\frac{1}{p}=2\sqrt{2}$ from the above

$=(8-2)^2-2=36-2$

$=34$

$\therefore p^4+\frac{1}{p^4}=34$ is proved

$\Rightarrow p^8-34p^4+1=0$. **(Proved)**

Mathematics

Ques. 11 $x^2 + y^2 = z^2$.

a. Factorize $x^6 - y^6$. 2

b. Prove that, $x^6 + y^6 + 3x^2y^2z^2 = z^6$. 4

c. If $x + y = 5$ and $x - y = 3$, then determine the value of z^2. 4

● *Chattogram Board 2017*

Solution to Question No. 11 :

a $x^6 - y^6 = (x^3)^2 - (y^3)^2$

$= (x^3 + y^3)(x^3 - y^3)$

$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$

$= (x - y)(x + y)(x^2 - xy + y^2)(x^2 + xy + y^2)$

b Here given that,

$x^2 + y^2 = z^2$

Now, $x^6 + y^6 + 3x^2y^2z^2$

$= (x^2)^3 + (y^2)^3 + 3x^2y^2z^2$

$= (x^2 + y^2)^3 - 3x^2.y^2(x^2 + y^2) + 3x^2y^2z^2$

$= (z^2)^3 - 3x^2y^2z^2 + 3x^2y^2z^2$, putting $x^2 + y^2 = z^2$

$= z^6$

$\therefore$ $x^6 + y^6 + 3x^2y^2z^2 = z^6$. **(Proved)**

c We know, $2(x^2 + y^2) = (x + y)^2 + (x - y)^2$

$= 5^2 + 3^2 = 34$

$\therefore x^2 + y^2 = \frac{34}{2}$ or, 17.

But we have, $x^2 + y^2 = z^2$

$\therefore z^2 = 17$.

Ques. 12 $P^3 - 3p^2 - 10p$, $p^3 + 6p^2 + 8p$ and $p^4 - 5p^3 - 14p^2$ are three algebraic expressions.

a. Find the factorial analysis of 2nd expression. 2

b. Find the H.C.F of the expressions. 4

c. Find the value of $\frac{p^6 + 1}{p^3}$ if the value of H.C.F is -1 of the expressions. 4

● *Sylhet Board 2017*

Solution to Question No. 12 :

a 2nd expression $= p^3 + 6p^2 + 8p$

$= p(p^2 + 6p + 8)$

$= p(p^2 + 4p + 2p + 8)$

$= p\{p(p + 4) + 2(p + 4)\}$

$= p(p + 2)(p + 4)$

b 1st expression,

$p^3 - 3p^2 - 10p$

$= p(p^2 - 3p - 10)$

$= p(p^2 - 5p + 2p - 10)$

$= p\{p(p - 5) + 2(p - 5)\}$

$= p(p - 5)(p + 2)$

2nd expression,

$p^3 + 6p^2 + 8p$

$= p(p + 2)(+4)$ [From questions answer 4(a)]

3rd expression $= p^4 - 5p^3 - 14p^2$

$= p^2(p^2 - 5p - 14)$

$= p^2(p^2 - 7p + 2p - 14)$

$= p^2\{p(p - 7) + 2(p - 7)\}$

$= p^2(p - 7)(p + 2)$

$\therefore$ The H. C. F $= p(p + 2)$

c $p(p + 2) = -1$

$\Rightarrow p^2 + 2p + 1 = 0$

$\Rightarrow p + \frac{1}{p} = 0$ [Dividing by p]

$\Rightarrow p + 2 + \frac{1}{p} = -2$

$\therefore p + \frac{1}{p} = -2$.....(1)

$\therefore \frac{p^6 + 1}{p^3}$

$= p^3 + \frac{1}{p^3}$

$= \left(p + \frac{1}{p}\right)^3 - 3p\frac{1}{p}\left(p + \frac{1}{p}\right)$

$= (-2)^3 - 3(-2)$ [Using equation (1)]

$= -8 + 6 = -2$

Ques. 13 If $a^2 - 1 = 5a$, where $a > 0$.

a. Find the value of $a^2 + \frac{1}{a^2}$. 2

b. Find the value of $\left(a^2 - \frac{1}{a^2}\right)^2$. 4

c. Prove that $\frac{a^6 - 1}{a^3} = 140$. 4

● *Dinajpur Board 2017*

Solution to Question No. 13 :

a Here we have,

$a^2 - 1 = 5a$

or, $a - \frac{1}{a} = 5$, dividing both sides by a

$\therefore a^2 + \frac{1}{a^2} = \left(a - \frac{1}{a}\right)^2 + 2.a.\frac{1}{a} = 5^2 + 2 = 27$.

b From (a) above,

$a - \frac{1}{a} = 5$

$\therefore a + \frac{1}{a} = \sqrt{\left(a - \frac{1}{a}\right)^2 + 4} = \sqrt{29}$

Now $\left(a^2-\frac{1}{a^2}\right)^2=\left\{\left(a+\frac{1}{a}\right)\left(a-\frac{1}{a}\right)\right\}^2$

$=\left(a+\frac{1}{a}\right)^2\left(a-\frac{1}{a}\right)^2$

$=(\sqrt{29})^2\,(5)^2$, putting $a+\frac{1}{a}=\sqrt{29}$ and $a-\frac{1}{a}=5$

$=29\times25=725.$

C From (a) above, we get,

$a-\frac{1}{a}=5$

Now $\frac{a^6-1}{a^3}=\frac{a^6}{a^3}-\frac{1}{a^3}$

$=a^3-\frac{1}{a^3}$

$=\left(a-\frac{1}{a}\right)^3+3a.\frac{1}{a}\left(a-\frac{1}{a}\right)$

$=5^3+3\times5$, putting $a-\frac{1}{a}=5$

$=125+15=140$

$\therefore \frac{a^6-1}{a^3}=140.$ **(Proved)**

Solutions to Textual Activities Along with textual reference

Mathematics

Task 01 **Find the cube with the help of an appropriate formulae :** *[Textbook Page 58]*

1. $ab+bc$
2. $2x-5y$
3. $2x-3y-z$

Solution :

1. The cube of $(ab+bc)$ is $(ab+bc)^3$.
   Now, $(ab+bc)^3$
   $=(ab)^3+(bc)^3+3(ab)^2(bc)+3(ab)(bc)^2$
   $=a^3b^3+b^3c^3+3a^2b^3c+3ab^3c^2.$

2. $2x-5y$
   The cube of $2x-5y=(2x-5y)^3$
   $=(2x)^3-3\times(2x)^2\times5y+3\times2x\times(5y)^2-(5y)^3$
   $=8x^3-3\times4\times5\times x^2y+3\times2\times25\times xy^2-125y^3$
   $=8x^3-60x^2y+150xy^2-125y^3$

3. The cube of $(2x-3y-z)$ is $(2x-3y-z)^3$.
   Now, $(2x-3y-z)^3$
   $=(2x)^3+(-3y)^3+(-z)^3+3.(2x)^2(-3y)$
   $+3.(2x)(-3y)^2+3(-3y)^2(-z)$
   $+3(-3y)(-z)^2+3(-z)^2(2x)$
   $+3(-z)(2x)^2+6.2x.(-3y)(-z)$
   $=8x^3-27y^3-z^3-36x^2y+54xy^2-27y^2z$
   $-9yz^2+6z^2x-12zx^2+36xyz.$

Task 02 *[Textbook Page 60]*

1. **Simplify : $(7x-6)^3-(5x-6)^3-6x(7x-6)(5x-6)$.**
2. **If $a+b=10$ and $ab=21$, find the value of a^3+b^3.**
3. **If $a+\frac{1}{a}=3$, show that, $a^3+\frac{1}{a^3}=18$.**

Solution :

1. $(7x-6)^3-(5x-6)^3-6x(7x-6)(5x-6)$
   $=a^3-b^3-3ab(a+b)$, considering $a=7x-6$ and $b=5x-6$ which implies $a-b=2x$.
   $=(a-b)^3=(7x-6-5x+6)^3$, putting for a and b.
   $=(2x)^3=8x^3.$

2. $a^3+b^3=(a+b)^3-3ab(a+b)$
   Given, $a+b=10$
   and $ab=21$
   Now, $a^3+b^3=(a+b)^3-3ab(a+b)$
   $=(10)^3-3\times21\times10$
   $=1000-630$
   $=370$
   Required value is 370.

3. $a^3+\frac{1}{a^3}=\left(a+\frac{1}{a}\right)^3-3a.\frac{1}{a}\left(a+\frac{1}{a}\right).$
   Given, $a+\frac{1}{a}=3$
   or, $\left(a+\frac{1}{a}\right)^3=3^3$ [Cubing]
   or, $a^3+\left(\frac{1}{a}\right)^3+3.a.\frac{1}{a}\left(a+\frac{1}{a}\right)=27$
   or, $a^3+\frac{1}{a^3}+3\times3=27\left[\because a+\frac{1}{a}=3\right]$
   or, $a^3+\frac{1}{a^3}=27-9$
   $\therefore a^3+\frac{1}{a^3}=18.$ **(Showed)**

Task 03 **Find the product of $(2a+3b)$ and $(4a^2-6ab+9b^2)$ by an appropriate formula.**

[Textbook Page 61]

Solution :

$(2a+3b)(4a^2-6ab+9b^2)$
$=(2a+3b)\{(2a)^2-2a.3b+(3b)^2\}$
$=(2a)^3+(3b)^3$
$=8a^3+27b^3$

Exercise 4.3 : Resolving into Factors

At a Glance Important Contents of Exercise

- **Factor :** If an expression is the product of two or more expressions, each of these two or more latter expression is termed as factor of the first expression. For example, $a^2 - b^2 = (a + b)(a - b)$, here $(a + b)$ and $(a - b)$ are two factors of the expression $(a^2 - b^2)$.
- **Resolving into factors :** When any exprfession is expressed as the product of two or more of expressions, it is said to have been resolved into factors and each of such expressions is called the factor of the first expression.
  For example, $x^2 + 2x = x(x + 2)$ [here, x and $(x + 2)$ are the factors].
- **Rules of resolving expressions into factors are stated below :**
  (a) Arranging conveniently : $px - qy + qx - py$ is arranged as, $px + qx - py - qy$.
  Now, $px + qx - py - qy = x(p + q) - y(p + q) = (p + q)(x - y)$.
  Again, $px + qy + qx - py$ is arranged as, $px - py + qx - qy$.
  Now, $px - py + qx - qy = p(x - y) + q(x - y) = (x - y)(p + q)$.
  (b) Expressing an expression in the form of square : $x^2 + 4xy + 4y^2 = (x)^2 + 2 \times x \times 2y + (2y)^2$
  $= (x + 2y)^2 = (x + 2y)(x + 2y)$
  (c) Expressing an expression as the difference of two squares and applying the formula $a^2 - b^2$.

Solutions to Exercise Problems Let's solve the textbook problems

Solutions to Mathematical Problems

Resolve into factors :

1. $a^3 + 8 = (a)^3 + (2)^3$
   $= (a + 2)\{(a)^2 - a \cdot 2 + (2)^2\}$
   $= (a + 2)(a^2 - 2a + 4)$
2. $8x^3 + 343 = (2x)^3 + (7)^3$
   $= (2x + 7)\{(2x)^2 - 2x \times 7 + (7)^2\}$
   $= (2x + 7)(4x^2 - 14x + 49)$
3. $8a^4 + 27ab^3 = a(8a^3 + 27b^3)$
   $= a\{(2a)^3 + (3b)^3\}$
   $= a(2a + 3b)\{(2a)^2 - 2a \cdot 3b + (3b)^2\}$
   $= a(2a + 3b)(4a^2 - 6ab + 9b^2)$
4. $8x^3 + 1 = (2x)^3 + (1)^3$
   $= (2x + 1)\{(2x)^2 - 2x \cdot 1 + (1)^2\}$
   $= (2x + 1)(4x^2 - 2x + 1)$
5. $64a^3 - 125b^3$
   $= (4a)^3 - (5b)^3$
   $= (4a - 5b)\{(4a)^2 + 4a \times 5b + (5b)^2\}$
   $= (4a - 5b)(16a^2 + 20ab + 25b^2)$
6. $729a^3 - 64b^3c^6$
   $= (9a)^3 - (4bc^2)^3$
   $= (9a - 4bc^2)\{(9a)^2 + 9a \times 4bc^2 + (4bc^2)^2\}$
   $= (9a - 4bc^2)(81a^2 + 36abc^2 + 16b^2c^4)$
7. $27a^3b^3 + 64b^3c^3$
   $= b^3(27a^3 + 64c^3)$
   $= b^3\{(3a)^3 + (4c)^3\}$
   $= b^3(3a + 4c)\{(3a)^2 - 3a \times 4c + (4c)^2\}$
   $= b^3(3a + 4c)(9a^2 - 12ac + 16c^2)$
8. $56x^3 - 189y^3 = 7(8x^3 - 27y^3)$
   $= 7\{(2x)^3 - (3y)^3\}$
   $= 7(2x - 3y)(4x^2 + 6xy + 9y^2)$
9. $3x - 75x^3 = 3x(1 - 25x^2)$
   $= 3x\{(1)^2 - (5x)^2\}$
   $= 3x(1 + 5x)(1 - 5x)$
10. $4x^2 - y^2 = (2x)^2 - (y)^2$
    $= (2x + y)(2x - y)$
11. $3ay^2 - 48a = 3a(y^2 - 16)$
    $= 3a\{(y)^2 - (4)^2\}$
    $= 3a(y + 4)(y - 4)$
12. $a^2 - 2ab + b^2 - p^2 = (a - b)^2 - p^2$
    $= (a - b + p)(a - b - p)$
13. $16y^2 - a^2 - 6a - 9 = 16y^2 - (a^2 + 6a + 9)$
    $= (4y)^2 - (a + 3)^2$
    $= (4y + a + 3)(4y - a - 3)$
14. $8a + ap^3 = a(8 + p^3)$
    $= a\{(2)^3 + (p)^3\}$
    $= a(2 + p)(2^2 - 2p + p^2)$
    $= a(2 + p)(4 - 2p + p^2)$
15. $2a^3 + 16b^3 = 2(a^3 + 8b^3)$
    $= 2\{(a)^3 + (2b)^3\}$
    $= 2(a + 2b)(a^2 - 2ab + 4b^2)$
16. $x^2 + y^2 - 2xy - 1 = x^2 - 2xy + y^2 - 1$
    $= (x - y)^2 - (1)^2$
    $= (x - y + 1)(x - y - 1)$

17. $a^2 - 2ab + 2b - 1 = (a^2 - 1) - 2ab + 2b$
$= (a + 1)(a - 1) - 2b(a - 1)$
$= (a - 1)(a + 1 - 2b)$
$= (a - 1)(a - 2b + 1)$

18. $x^4 - 2x^2 + 1 = (x^2)^2 - 2.x^2.1 + 1^2$
$= (x^2 - 1)^2$
$= \{(x + 1)(x - 1)\}^2 = (x + 1)^2 (x - 1)^2$

19. $36 - 12x + x^2 = (6)^2 - 2 . 6 . x + (x)^2$
$= (6 - x)^2 = (x - 6)^2$

20. $x^6 - y^6$
$= (x^3)^2 - (y^3)^2 = (x^3 + y^3)(x^3 - y^3)$
$= (x + y)(x^2 - xy + y^2)(x - y)(x^2 + xy + y^2)$
$= (x + y)(x - y)(x^2 + xy + y^2)(x^2 - xy + y^2)$

21. $(x - y)^3 + z^3$
$= (x - y + z)\{(x - y)^2 - (x - y)z + z^2\}$
$= (x - y + z)(x^2 - 2xy + y^2 - xz + yz + z^2)$
$= (x - y + z)(x^2 + y^2 + z^2 - 2xy + yz - zx)$

22. $64x^3 - 8y^3 = (4x)^3 - (2y)^3$
$= (4x - 2y)\{(4x)^2 + 4x . 2y + (2y)^2\}$
$= (4x - 2y)(16x^2 + 8xy + 4y^2)$

23. $x^2 + 14x + 40 = x^2 + 10x + 4x + 40$
$= x(x + 10) + 4(x + 10)$
$= (x + 10)(x + 4)$

24. $x^2 + 7x - 120 = x^2 + 15x - 8x - 120$
$= x(x + 15) - 8(x + 15)$
$= (x + 15)(x - 8)$

25. $x^2 - 51x + 650 = x^2 - 26x - 25x + 650$
$= x(x - 26) - 25(x - 26)$
$= (x - 26)(x - 25)$

26. $a^2 + 7ab + 12b^2 = a^2 + 3ab + 4ab + 12b^2$
$= a(a + 3b) + 4b(a + 3b)$
$= (a + 3b)(a + 4b)$

27. $p^2 + 2pq - 80q^2 = p^2 + 10pq - 8pq - 80q^2$
$= p(p + 10q) - 8q(p + 10q)$
$= (p + 10q)(p - 8q)$

28. $x^2 - 3xy - 40y^2 = x^2 - 8xy + 5xy - 40y^2$
$= x(x - 8y) + 5y(x - 8y)$
$= (x - 8y)(x + 5y)$

29. $(x^2 - x)^2 + 3(x^2 - x) - 40$
Let, $x^2 - x = p$
$\therefore$ The given expression turns into
$p^2 + 3p - 40$
$= p^2 + 8p - 5p - 40 = p(p + 8) - 5(p + 8)$
$= (p + 8)(p - 5)$
$= (x^2 - x + 8)(x^2 - x - 5)$ [Putting $p = x^2 - x$]

30. $(a^2 + b^2)^2 - 18(a^2 + b^2) - 88$
Let, $a^2 + b^2 = x$
$\therefore$ The given expression turns into,
$x^2 - 18x - 88$
$= x^2 - 22x + 4x - 88 = x(x - 22) + 4(x - 22)$
$= (x - 22)(x + 4)$
$= (a^2 + b^2 - 22)(a^2 + b^2 + 4)$ [Putting the value of x]

31. $(a^2 + 7a)^2 - 8(a^2 + 7a) - 180.$
Let, $a^2 + 7a = x$
$\therefore$ The given expression turns into,
$x^2 - 8x - 180$
$= x^2 - 18x + 10x - 180$
$= x(x - 18) + 10(x - 18)$
$= (x - 18)(x + 10)$
$= (a^2 + 7a - 18)(a^2 + 7a + 10)$ [Putting the value of x]
$= (a^2 + 9a - 2a - 18)(a^2 + 5a + 2a + 10)$
$= \{a(a + 9) - 2(a + 9)\}\{a(a + 5) + 2(a + 5)\}$
$= (a + 9)(a - 2)(a + 5)(a + 2)$
$= (a + 9)(a + 5)(a + 2)(a - 2)$

32. $x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)$
$= x^2 + (3a + 4b)x + (2a^2 + 3ab + 2ab + 3b^2)$
$= x^2 + (3a + 4b)x + \{a(2a + 3b) + b(2a + 3b)\}$
$= x^2 + (3a + 4b)x + (2a + 3b)(a + b)$
$= x^2 + (2a + 3b)x + (a + b)x + (2a + 3b)(a + b)$
$= x(x + 2a + 3b) + (a + b)(x + 2a + 3b)$
$= (x + 2a + 3b)(x + a + b)$

33. $6x^2 - x - 15 = 6x^2 - 10x + 9x - 15$
$= 2x(3x - 5) + 3(3x - 5) = (3x - 5)(2x + 3)$

34. $x^2 - x - (a + 1)(a + 2)$
$= x^2 - (a + 2)x + (a + 1)x - (a + 1)(a + 2)$
$= x(x - a - 2) + (a + 1)(x - a - 2)$
$= (x - a - 2)(x + a + 1)$

35. $3x^2 + 11x - 4 = 3x^2 + 12x - x - 4$
$= 3x(x + 4) - 1(x + 4)$
$= (x + 4)(3x - 1)$

36. $3x^2 - 16x - 12 = 3x^2 - 18x + 2x - 12$
$= 3x(x - 6) + 2(x - 6)$
$= (3x + 2)(x - 6)$

37. $2x^2 - 9x - 35 = 2x^2 - 14x + 5x - 35$
$= 2x(x - 7) + 5(x - 7)$
$= (x - 7)(2x + 5)$

38. $2x^2 - 5xy + 2y^2 = 2x^2 - 4xy - xy + 2y^2$
$= 2x(x - 2y) - y(x - 2y)$
$= (x - 2y)(2x - y)$

39. $x^3 - 8(x - y)^3$
$= x^3 - \{2(x - y)\}^3 = x^3 - (2x - 2y)^3$
$= (x - 2x + 2y)\{x^2 + x(2x - 2y) + (2x - 2y)^2\}$
$= (2y - x)(7x^2 - 10xy + 4y^2)$

40. $10p^2 + 11pq - 6q^2$
$= 10p^2 + 15pq - 4pq - 6q^2$
$= 5p(2p + 3q) - 2q(2p + 3q)$
$= (2p + 3q)(5p - 2q)$

41. $2(x + y)^2 - 3(x + y) - 2$
Let, $x + y = p$
$\therefore$ The given expression turns into,
$2p^2 - 3p - 2$
$= 2p^2 - 4p + p - 2$
$= 2p(p - 2) + 1(p - 2)$
$= (p - 2)(2p + 1)$
$= (x + y - 2)(2x + 2y + 1)$ [Putting the value of p]

42. $ax^2 + (a^2 + 1)x + a$
$= ax^2 + a^2x + x + a$
$= ax(x + a) + 1(x + a)$
$= (x + a)(ax + 1)$

43. $15x^2 - 11xy - 12y^2$
$= 15x^2 - 20xy + 9xy - 12y^2$
$= 5x(3x - 4y) + 3y(3x - 4y)$
$= (3x - 4y)(5x + 3y)$

44. $a^3 - 3a^2b + 3ab^2 - 2b^3$
$= (a^3 - 3a^2b + 3ab^2 - b^3) - b^3$
$= (a - b)^3 - b^3$
$= (a - b - b)\{(a - b)^2 + (a - b)b + b^2\}$
$= (a - 2b)(a^2 - 2ab + b^2 + ab - b^2 + b^2)$
$= (a - 2b)(a^2 - ab + b^2)$

Multiple Choice Q/A Designed as per topic

4.4 Resolving into Factors ▸ Textbook Page 63

1. Which one is the factor of $a^2 - 2ab + 2b - 1$? *[CB '19]*
ⓐ $(a + 1)$ ⓑ $(a - 1)$
ⓑ ⓒ $(a - 2b)$ ⓓ $(a + 2b)$

2. Which one of the following is the product of $(x + 11)$ and $(x - 8)$? *(Medium)* *[CB '16]*
ⓐ $x^2 - 19x + 88$ ⓑ $x^2 - 3x + 88$
ⓒ ⓒ $x^2 + 3x - 88$ ⓓ $x^2 + 19x - 88$

3. Which one of the following is the lowest form of $\frac{x^2 - 6x + 5}{x^2 - 1}$? *(Medium)* *[CtgB '16]*
ⓐ ⓐ $\frac{x - 5}{x + 1}$ ⓑ $\frac{x + 1}{x - 5}$ ⓒ $\frac{x - 1}{x + 1}$ ⓓ $\frac{x - 5}{x - 1}$

4. Which one is the lowest form of $\frac{x^2 - x - 30}{x^2 - 36}$? *(Medium)* *[SB '16]*
ⓑ ⓐ $\frac{x + 5}{x - 6}$ ⓑ $\frac{x + 5}{x + 6}$ ⓒ $\frac{x - 5}{x + 6}$ ⓓ $\frac{x - 5}{x - 6}$

5. Which one of the following is the product of $(a + 13)$ and $(a - 11)$? *(Hard)* *[SB '16]*
ⓐ $a^2 - 2a + 143$ ⓑ $a^2 + 2a - 143$
ⓑ ⓒ $a^2 - 24a + 143$ ⓓ $a^2 + 24a - 143$

6. The factors of $27a^3 - 8$ is? *(Hard)* *[DjB '17]*
ⓐ $(3a + 2)(9a^2 - 6a + 4)$
ⓑ $(3a + 2)(9a^2 + 6a + 4)$
ⓒ $(3a - 2)(9a^2 - 6a + 4)$
ⓓ ⓓ $(3a - 2)(9a^2 + 6a + 4)$

4.5 Factors of the expression of the form $x^2 + px + q$ ▸ Textbook Page 65

7. Which one of the following expression is the factors of $x^2 - x - 6$? *(Easy)* *[BB '19]*
ⓐ $(x + 3)(x - 2)$ ⓑ $(x - 6)(x + 1)$
ⓓ ⓒ $(x + 6)(x - 1)$ ⓓ $(x - 3)(x + 2)$

8. Which one of the following is the factorization of $(x^2 + 5x - 6)$? *(Medium)* *[RB '16]*
ⓐ $(x - 6)(x + 1)$ ⓑ $(x + 6)(x + 1)$
ⓓ ⓒ $(x - 2)(x - 3)$ ⓓ $(x - 1)(x + 6)$

9. Which one of the following is the factor of $x^2 - 5x + 6$? *(Medium)* *[CB '16]*
ⓐ ⓐ $x - 3$ ⓑ $x + 2$ ⓒ $x + 3$ ⓓ $x - 6$

10. Which one is the factorization of $x^2 + 5x - 6$? *(Easy)* *[SB '16]*
ⓐ $(x + 6)(x - 1)$ ⓑ $(x - 6)(x + 1)$
ⓐ ⓒ $(x - 6)(x - 1)$ ⓓ $(x + 6)(x + 1)$

11. Which one of the following is the factorization of $x^2 - 2x - 35$? *(Easy)* *[DjB '16]*
ⓐ $(x - 7)(x + 5)$ ⓑ $(x + 7)(x - 5)$
ⓐ ⓒ $(x - 7)(x - 5)$ ⓓ $(x + 7)(x + 5)$

12. A factor of $x^4 - 2x^2 + 1$ is —. *(Medium)* *[CtgB '17]*
ⓐ $2x + 1$ ⓑ $x^2 + 1$
ⓒ ⓒ $(x + 1)$ ⓓ $2x - 1$

13. For the expression $x^2 + 7x + 12$ *(Easy)* *[CB '16]*
i. coefficient of x is 7
ii. constant term is 12
iii. one factor is $x + 3$
Which one is correct?
ⓓ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

4.6 Factors of the expression in the form of $ax^2 + bx + c$ ▸ Textbook Page 67

14. Which one of the following expressions is the factors of $2x^2 - 7xy + 6y^2$? *(Medium)* *[JB '19]*
ⓐ $(x - 2y)(2x - 3y)$ ⓑ $(x + 2y)(2x - 3y)$
ⓐ ⓒ $(x - 2y)(2x + 3y)$ ⓓ $(x + 2y)(2x + 3y)$

15. Which one of the following expression is the factors of $2y^2 + y - 10$? *(Easy)* *[SB '19]*
ⓐ $(2y - 5)(y + 2)$ ⓑ $(2y + 5)(y - 1)$
ⓒ ⓒ $(2y + 5)(y - 2)$ ⓓ $(y + 5)(2y - 1)$

Mathematics

16. **Which one is the factorized form of $2x^2 + 7x - 4$?** *(Easy)* *[MB '19]*
ⓐ $(2x - 1)(x - 4)$ ⓑ $(2x + 1)(x - 4)$
ⓒ ⓒ $(2x - 1)(x + 4)$ ⓓ $(2x + 1)(x + 4)$

17. **Which one of the following expression is the factors of $2a^2 - 5ab + 2b^2$?** *(Hard)* *[CtgB '18]*
ⓐ $(a - 2b)(2a - b)$ ⓑ $(a + 2b)(2a - b)$
ⓐ ⓒ $(a + 2b)(2a + b)$ ⓓ $(a - 2b)(2a + b)$

18. **Which one of the following is a factor of $3x^2 + 11x - 4$?** *(Easy)* *[DB '17]*
ⓒ ⓐ $x - 4$ ⓑ $3x + 1$ ⓒ $3x - 1$ ⓓ $x - 1$

19. **Which is the factor of $3x^2 + x - 10$?** *(Medium)* *[SB '17]*
ⓑ ⓐ $x - 2$ ⓑ $x + 2$ ⓒ $3x + 5$ ⓓ $2x + 5$

20. **Which one of the following is a factor of $3x^2 - 16x - 12$?** *(Easy)* *[CtgB '16]*
ⓑ ⓐ $3x - 2$ ⓑ $3x + 2$ ⓒ $x + 6$ ⓓ $3x - 5$

Short Q/A Designed as per topic

4.4 Resolving into factors ▸ Textbook Page 63

Mathematics

Question 1. Resolve into factors $m^2 + 2mn - 2n - 1$.
Solution : $m^2 + 2mn - 2n - 1$
$= m^2 + 2mn + n^2 - n^2 - 2n - 1$
$= (m^2 + 2mn + n^2) - (n^2 + 2n + 1)$
$= (m + n)^2 - (n + 1)^2$
$= (m + n + n + 1)(m + n - n - 1)$
$= (m + 2n + 1)(m - 1)$

Question 2. Resolve into factors $x^2 - 2xy + 4y - 4$.
Solution : $x^2 - 2xy + 4y - 4$
$= x^2 - 2^2 - 2y(x - 2)$
$= (x + 2)(x - 2) - 2y(x - 2)$
$= (x - 2)(x + 2 - 2y) = (x - 2)(x - 2y + 2)$

Question 3. Resolve into factors $27a^4 + 8ab^3$.
Solution : $27a^4 + 8ab^3$
$= a(27a^3 + 8b^3)$
$= a\{(3a)^3 + (2b)^3\}$
$= a(3a + 2b)\{(3a)^2 - 3a.2b + (2b)^2\}$
$= a(3a + b)(9a^2 - 6ab + 4b^2)$

Question 4. Resolve into factors : $p^4 + p^2 + 1$.
Solution : $p^4 + p^2 + 1$
$= (p^2)^2 + 2.p^2.1 + (1)^2 - p^2$
$= (p^2 + 1)^2 - p^2$
$= (p^2 + 1 + p)(p^2 + 1 - p)$
$= (p^2 + p + 1)(p^2 - p + 1)$

Question 5. Resolve into factors : $a^4 + a^2b^2 + b^4$.
Solution : $a^4 + a^2b^2 + b^4$
$= (a^2)^2 + 2.a^2.b^2 + (b^2)^2 - a^2b^2$
$= (a^2 + b^2) - (ab)^2$
$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$
$= (a^2 + ab + b^2)(a^2 - ab + b^2)$

Question 6. Resolve into factors : $x^2 + x - (a + 1)(a + 2)$
Solution : Let, $a + 1 = m$
$\therefore \quad a + 2 = (a + 1) + 1 = m + 1$
Given expression $= x^2 + x - m(m + 1)$
$= x^2 + x - m^2 - m$
$= x^2 - m^2 + x - m$
$= (x + m)(x - m) + 1(x - m)$
$= (x - m)(x + m + 1)$
$= \{x - (a + 1)\}\{x + (a + 1) + 1\}$
$= (x - a - 1)(x + a + 2)$

Question 7. Resolve into factors : $a^4 - 23a^2 + 1$
Solution : $a^4 - 23a^2 + 1$
$= (a^2)^2 + 2.a^2.1 + (1)^2 - 25a^2$
$= (a^2 + 1)^2 - (5a)^2$
$= (a^2 + 1 + 5a)(a^2 + 1 - 5a)$
$= (a^2 + 5a + 1)(a^2 - 5a + 1)$

Question 8. Resolve into factors $mx^2 - (m^2 + 1)x + m$.
Solution : $mx^2 - (m^2 + 1)x + m$
$= mx^2 - m^2x - x + m$
$= mx(x - m) - 1(x - n)$
$= (x - m)(mx - 1)$

Question 9. Resolve into factors : $x^3 - 3x^2y + 3xy^2 - 9y^3$
Solution : $x^3 - 3x^2y + 3xy^2 - 9y^3$
$= x^3 - 3x^2y + 3xy^2 - y^3 - 8y^3$
$= (x - y)^3 - (2y)^3$
$= (x - y - 2y)\{(x - y)^2 + (x - y)2y + (2y)^2\}$
$= (x - 3y)(x^2 - 2xy + y^2 + 2xy - 2y^2 + 4y^2)$
$= (x - 3y)(x^2 - 3y^2)$.

Question 10. Resolve into factors $x^2 + \left(a + \frac{1}{a}\right)x + 1$

Solution : $x^2 + \left(a + \frac{1}{a}\right)x + 1$
$= x^2 + ax + \frac{x}{a} + 1$
$= x(x + a) + \frac{1}{a}(x + a) = (x + a)\left(x + \frac{1}{a}\right)$

Question 11. If a factor of $a^3 - 1$ is $a - 1$, what is the another?
Solution : $a^3 - 1$
$= a^3 - 1^3$
$= (a - 1)(a^2 + a.1 + 1^2)$
$= (a - 1)(a^2 + a + 1)$
$\therefore$ Another factor $(a^2 + a + 1)$

4.5 Factors of the expression of the form $x^2 + px + q$

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Question 12. Resolve into factors $x^2 - 18x + 72$.

Solution : $x^2 - 18x + 72$

$= x^2 - 6x - 12x + 72$

$= x(x - 6) - 12(x - 6)$

$= (x - 6)(x - 12)$

Question 13. Resolve into factors $a^2 + 7a - 120$.

Solution : $a^2 + 7a - 120$

$= a^2 + 15a - 8a - 120$

$= a(a + 15) - 8(a + 15)$

$= (a + 15)(a - 8)$

Question 14. Resolve into factors $p^2 - 9p - 36$.

Solution : $p^2 - 9p - 36$

$= p^2 - 12p + 3p - 36$

$= p(p - 12) + 3(p - 12)$

$= (p - 12)(p + 3)$

4.6 Factors of the expression in the form of $ax^2 + bx + c$

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Question 15. Resolve into factors $4x^2 + 3x - 10$.

Solution : Here, $4 \times (-10) = -40$

Now, $8 \times (-5) = -40$ and $8 + (-5) = 3$

$\therefore\ 4x^2 + 3x - 10 = 4x^2 + 8x - 5x - 10$

$= 4x(x + 2) - 5(x + 2)$

$= (x + 2)(4x - 5)$

Question 16. Resolve into factors $3y^2 - 15y + 18$.

Solution : Here, $3 \times 18 = 54$

Now, $(-9) \times (-6) = 54$ and $(-9) + (-6) = -15$

$\therefore\ 3y^2 - 15y + 18 = 3y^2 - 9y - 6y + 18$

$= 3y(y - 3) - 6(y - 3)$

$= (y - 3)(3y - 6)$

Question 17. Resolve into factors $2x^2 - 9x - 35$.

Solution : Here, $2 \times (-35) = -70$

Now, $(-14) \times 5 = -70$ and $(-14) + 5 = -9$

$\therefore\ 2x^2 - 9x - 35 = 2x^2 - 14x + 5x - 35$

$= 2x(x - 7) + 5(x - 7)$

$= (x - 7)(2x + 5)$

Creative Q/A Designed as per learning outcomes

Ques. 01 $Q = 3a^2 - 7a - 6$ and $R = p + \frac{1}{p}$ are algebraic expressions.

a. Resolve into factors : $x^3 - 3x^2y + 3xy^2 - 2y^3$. 2

b. If $R = a + 2$, find out the value of $p^4 + \frac{1}{p^4}$. 4

c. Express the given expression 'Q' as the difference of two squares. 4

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Solution to Question No. 01 :

a $x^3 - 3x^2y + 3xy^2 - 2y^3$

$= x^3 - 3x^2y + 3xy^2 - y^3 - y^3$

$= (x - y)^3 - y^3$

$= (x - y - y)\{(x - y)^2 + (x - y)y + y^2\}$

$= (x - 2y)(x^2 - 2xy + y^2 + xy - y^2 + y^2)$

$= (x - 2y)(x^2 - xy + y^2)$ **(Ans.)**

b Given, $R = p + \frac{1}{p}$

and $R = a + 2$

$\therefore\ p + \frac{1}{p} = a + 2$

or, $p^2 + \frac{1}{p^2} + 2.p.\frac{1}{p} = a^2 + 4a + 4$ [Squaring both sides]

or, $p^2 + \frac{1}{p^2} = a^2 + 4a + 4 - 2$

or, $p^2 + \frac{1}{p^2} = a^2 + 4a + 2$

or, $p^4 + \frac{1}{p^4} + 2.p^2.\frac{1}{p^2} = a^4 + 16a^2 + 4 + 8a^3 + 16a + 4a^2$ [Squaring again]

or, $p^4 + \frac{1}{p^4} = a^4 + 8a^3 + 20a^2 + 16a + 4 - 2$

or, $p^4 + \frac{1}{p^4} = a^4 + 8a^3 + 20a^2 + 16a + 2$

$\therefore$ Determined value of $p^4 + \frac{1}{p^4}$ is $a^4 + 8a^3 + 20a^2 + 16a + 2$ **(Ans.)**

c Here, $Q = 3a^2 - 7a - 6$

$= 3a^2 - 9a + 2a - 6$

$= 3a(a - 3) + 2(a - 3)$

$= (3a + 2)(a - 3)$

$= \left(\frac{3a + 2 + a - 3}{2}\right)^2 - \left(\frac{3a + 2 - a + 3}{2}\right)^2$

$= \left(\frac{4a - 1}{2}\right)^2 - \left(\frac{2a + 5}{2}\right)^2$

$\therefore\ Q = \left(\frac{4a - 1}{2}\right)^2 - \left(\frac{2a + 5}{2}\right)^2$; Which is expressed as the difference of two squares.

Ques. 02 $x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)$ is an algebraic expression.

a. Factorize the constant part. (Easy) 2

b. Factorize the algebrac expression. (Medium) 4

c. Show that, if $a = -b$, then $(x + b)$ is a factor of the expression. (Hard) 4

Solution to Question No. 02 :

a Given constant $= 2a^2 + 5ab + 3b^2$
$= 2a^2 + 2ab + 3ab + 3b^2$
$= 2a(a + b) + 3b(a + b)$
$= (a + b)(2a + 3b)$
Required factor $(a + b)(2a + 3b)$.

b Given expression $= x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)$
$= x^2 + (3a + 4b)x + \{(a + b)(2a + 3b)\}$ [From 'a']

Let, $3a + 4b = p$ ..... (i)
$2a + 3b = q$ ..... (ii)
(i) – (ii) $\Rightarrow$,
$3a + 4b = p$
$2a + 3b = q$
(–) (–) (–)
$a + b = p - q$
Now, $x^2 + (3a + 4b)x + \{(a + b)(2a + 3b)\}$
$= x^2 + px + (p - q)q$ [Putting the value]
$= x^2 + px + pq - q^2$
$= x^2 - q^2 + px + pq$
$= (x + q)(x - q) + p(x + q)$
$= (x + q)(x - q + p)$
$= (x + q)(x + p - q)$
$= (x + 2a + 3b)(x + 3a + 4b - 2a - 3b)$ [Putting the value]
$= (x + 2a + 3b)(x + a + b)$
Required factorr is $(x + 2a + 3b)(x + a + b)$.

c Given expression $= x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)$
Given, If $a = -b$,
$x^2 + \{3(-b) + 4b\}x + \{2(-b)^2 + 5(-b)b + 3b^2\}$
$= x^2 + (4b - 3b)x + (2b^2 - 5b^2 + 3b^2)$
$= x^2 + bx + (5b^2 - 5b^2)$
$= x^2 + bx$
$= x(x + b)$
$\therefore$ Required factor is $(x + b)$. **(Showed)**

Mathematics

Solutions to Textual Activities Along with textual reference

Task 01 **Resolve into factors :** *[Textbook Page 64]*

1. $4x^2 - y^2$
2. $6ab^2 - 24a$
3. $x^2 + 2px + p^2 - 4$
4. $x^3 + 27y^3$
5. $27a^3 - 8$

Solution :

1. $4x^2 - y^2 = (2x)^2 - (y)^2 = (2x + y)(2x - y)^2$.
2. $6ab^2 - 24a = 6a(b^2 - 4) = 6a(b + 2)(b - 2)$.
3. $x^2 + 2px + p^2 - 4$
$= (x)^2 + 2.p.x + (p)^2 - (2)^2$
$= (x + p)^2 - 2^2$
$= (x + p + 2)(x + p - 2)$, since $a^2 - b^2 = (a + b)(a - b)$
4. $x^3 + 27y^3 = (x)^3 + (3y)^3 = (x + 3y)\{x^2 - x \times 3y + (3y)^2\}$
$= (x + 3y)(x^2 - 3xy + 9y^2)$.
5. $27a^3 - 8 = (3a)^3 - (2)^3$
$= (3a - 2)\{(3a)^2 + 3a \times 2 + (2)^2\}$.
$= (3a - 2)(9a^2 + 6a + 4)$.

Task 02 **Resolve into factors :** *[Textbook Page 67]*

1. $x^2 - 18x + 72$
2. $x^2 - 9x - 36$
3. $x^2 - 23x + 132$

Solution :

1. $x^2 - 18x + 72 = x^2 - 6x - 12x + 72 = x(x - 6) - 12(x - 6) = (x - 6)(x - 12)$.
2. $x^2 - 9x - 36 = x^2 - 12x + 3x - 36 = x(x - 12) + 3(x - 12) = (x - 12)(x + 3)$.
3. $x^2 - 23x + 132 = x^2 - 12x - 11x + 132 = x(x - 12) - 11(x - 12) = (x - 12)(x - 11)$.

Task 03 **Resolve into factors :** *[Textbook Page 68]*

1. $8x^2 + 18x + 9$
2. $27x^2 + 15x + 2$
3. $2a^2 - 6a - 20$

Solution :

1. $8x^2 + 18x + 9 = 8x^2 + 12x + 6x + 9$
Now,
$8x^2 + 18x + 9$
$= 8x^2 + 12x + 6x + 9$ | Here, $8 \times 9 = 72$
$= 4x(2x + 3) + 3(2x + 3)$ | Here, $(+12) \times (+6) = 72$
$= (2x + 3)(4x + 3)$ | and $12 + 6 = 18$
2. $27x^2 + 15x + 2 = 27x^2 + 6x + 9x + 2$.
Now,
$27x^2 + 15x + 2$
$= 27x^2 + 9x + 6x + 2$ | Here, $27 \times 2 = 54$
$= 9x(3x + 1) + 2(3x + 1)$ | Here, $9 \times 6 = 54$
$= (3x + 1)(x + 2)$ | and $9 + 6 = 15$
3. $2a^2 - 6a - 20 = 2a^2 - 10a + 4a - 20$.
Now,
$2a^2 - 6a - 20$
$= 2(a^2 - 3a - 10)$ | Here, $1 \times (-10) = -10$
$= 2\{a^2 - 5a + 2a - 10\}$ | Here, $2 \times (-5) = -10$
$= 2\{a(a - 5) + 2(a - 5)\}$ | and $2 - 5 = -3$
$= 2(a - 5)(a + 2)$

Exercise 4.4 : H.C.F. and L.C.M. of Algebraic Expressions

At a Glance Important Contents of Exercise

- **Common factor :** The expression which is a factor of each of two or more expressions, is called common factor. For example x is the common factor of the expressions x^2y, xy, xy^2, $5x$.
- **Highest Common Factor (H.C.F) :** The product of common factors of two or more expressions is called the Highest Common Factor or in brief H.C.F. of those two or more expressions.
For example, the H.C.F. of three expressions $a^3b^3c^3$, $a^5b^3c^4$ and $a^4b^3c^3$ is $a^3b^2c^2$.
Again, the H.C.F. of three expressions $(x+y)^2$, $(x+y)^3$ and (x^2-y^2) is $(x+y)$.
- **Rules of finding H.C.F :** The H.C.F. of the numerical coefficients of expressions of those algebraic expressions should be determined first by applying the rules of Arithmetic. Then the prime factors of those algebraic expressions have to be found. After that, the successive product of the H.C.F. of numerical coefficients and the highest number of algebraic common factors of given expressions is the required H.C.F.
- **Common Multiple :** If any expression is completely divisible by two or more expressions, the dividend is called the common multiple of those two or more divisors. For example, the expression a^2b^2c is divisible by each expression of a, b, c, ab, bc, ac, a^2b, ab^2, a^2c, b^2c, a^2b^2. Hence, the expression a^2b^2c is the common multiple of expressions a, b, c ab, bc, ac, a^2b, ab^2, a^2c, b^2c, a^2c, a^2b^2. Again, the expression $(a+b)^2(a-b)$ is the common multiple of three expressions $(a+b)$, $(a+b)^2$ and (a^2-b^2).
- **Least Common Multiple (L.C.M) :** Among different multiples of two or more expressions the common multiple which consists of lowest number of factors is called Least Common Multiple or L.C.M. in short.
For example, the expression x^2y^2z is the L.C.M. of three expressions, x^2yz, xy^2 and xyz. Again, the expression $(x+y)^2(x-y)$ is the L.C.M. of three expressions $(x+y)$, $(x+y)^2$ and (x^2-y^2).
- **Rules of finding L.C.M :** At first, the L.C.M. of the given expressions of the numerical coefficients have to be determined. Then, we have to find the highest power of common factors. After that, the product of both is the L.C.M. of the given expressions.

Solutions to Exercise Problems Let's solve the textbook problems

MCQs with Answers

1. Which one of the following is the square of $(-5-y)$?

ⓐ $y^2+10y+25$ ⓑ $y^2-10y+25$
ⓐ ⓒ $25-10y+y^2$ ⓓ $y^2-10y-25$

Explanation : Square of $(-5-y)$
$= (-5-y)^2 = \{-(5+y)\}^2$
$= (5+y)^2$
$= 5^2 + 2 \cdot 5 \cdot y + y^2$
$= 25 + 10y + y^2$
$= y^2 + 10y + 25.$

2. Which one of the following is the product of $(x-2)$ and $(4x+3)$?

ⓐ $4x^2-5x+6$ ⓑ $4x^2-11x-6$
ⓓ ⓒ $4x^2+5x-6$ ⓓ $4x^2-5x-6$

Explanation : $(x-2)(4x+3)$
$= 4x^2+3x-8x-6$
$= 4x^2-5x-6.$

3. What is the H.C.F of x^2-2x-3 and x^2+2x-3?

ⓒ ⓐ $x+1$ ⓑ $x-1$ ⓒ 1 ⓓ 0

Explanation : 1st quantity $= x^2-2x-3$
$= x^2-3x+x-3$
$= x(x-3)+1(x-3)$
$= (x+1)(x-3)$
2nd quantity $= x^2+2x-3 = x^2+3x-x-3$
$= x(x+3)-1(x+3)$
$= (x-1)(x+3)$
$\therefore$ H.C.F $= 1$.

4. Which one of the following will be right if we express $(3x-5)(5+3x)$ in the form of the difference between two squares?

ⓐ $3x^2-25$ ⓑ $9x^2-5$
ⓒ ⓒ $(3x)^2-5^2$ ⓓ $9x^2-25$

Explanation : $(5+3x)(3x-5)$
$= (3x+5)(3x-5)$
$= (3x)^2-(5)^2.$

Answer to the questions No. 5 – 7 in accordance with the information given below :
If $x^2 - \sqrt{3}x + 1 = 0$

5. **Which one of the following is the value of $x + \frac{1}{x}$?**

ⓓ ⓐ $-\sqrt{3}x$ ⓑ $\sqrt{3}x$ ⓒ $-\sqrt{3}$ ⓓ $\sqrt{3}$

Explanation : $x^2 - \sqrt{3}x + 1 = 0$
or, $x^2 + 1 = \sqrt{3}x$
or, $\frac{x^2+1}{x} = \sqrt{3}$
$\therefore x + \frac{1}{x} = \sqrt{3}$.

6. **Which one of the following is the value of $x^2 + \frac{1}{x^2}$?**

ⓐ ⓐ 1 ⓑ 5 ⓒ 7 ⓓ 11

Explanation : $x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2 . x . \frac{1}{x}$
$= (\sqrt{3})^2 - 2 = 3 - 2 = 1.$

7. **Which one of the following is the value of $x^3 + \frac{1}{x^3}$?**

ⓓ ⓐ 12 ⓑ $6\sqrt{3}$ ⓒ $3\sqrt{3} + 3$ ⓓ 0

Explanation : $x^3 + \frac{1}{x^3}$
$= \left(x + \frac{1}{x}\right)^3 - 3 . x . \frac{1}{x}\left(x + \frac{1}{x}\right)$
$= (\sqrt{3})^3 - 3 . \sqrt{3}$
$= 3\sqrt{3} - 3\sqrt{3} = 0.$

8. **Which one of the following expressions is the factors of $x^2 - x - 30$?**
ⓐ $(x - 5)(x + 6)$ ⓑ $(x + 5)(x - 6)$
ⓑ ⓒ $(x - 5)(x - 6)$ ⓓ $(x + 5)(x + 6)$

Explanation : $x^2 - x - 30$
$= x^2 - 6x + 5x - 30$
$= x(x - 6) + 5(x - 6)$
$= (x + 5)(x - 6).$

9. **If $x^2 - 10x + 21$ and $x^2 - 6x - 7$ are two algebraic expressions —.**
i. The H.C.F. of the two expressions is $x - 7$
ii. The L.C.M. of the two expressions is $(x + 1)(x - 3)(x - 7)$
iii. The product of the two expressions is $x^4 - 60x^2 - 147$
Which one of the following is correct?
ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii
Note : Correct answer is absent.

Explanation : 1st Quantity $= x^2 - 10x + 21$
$= x^2 - 7x - 3x + 21$
$= x(x - 7) - 3(x - 7)$
$= (x - 7)(x - 3)$
and 2nd Quantity $= x^2 - 6x - 7 = x^2 - 7x + x - 7$
$= x(x - 7) + 1(x - 7)$
$= (x - 7)(x + 1)$
$\therefore$ H.C.F $= (x - 7)$
and L.C.M. $= (x - 7)(x - 3)(x + 1) = (x + 1)(x - 3)(x - 7)$
Multiplication of two quantity
$= (x^2 - 10x + 21)(x^2 - 6x - 7)$
$= x^4 - 6x^3 - 7x^2 - 10x^3 + 60x^2 + 70x + 21x^2 - 126x - 147$
$= x^4 - 16x^3 + 74x^2 - 56x - 147$
$\therefore$ (i) and (ii) are correct

10. **In algebraic formulae —.**
i. $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$
ii. $ab = \left(\frac{a+b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2$
iii. $x^3 + y^3 = (x + y)^3 + 3xy(x + y)$
Which one of the following is correct according to the above information?
ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

11. **If $x + y = 5$ and $x - y = 3$, then**
(1) What is the value of $x^2 + y^2$?
ⓒ ⓐ 15 ⓑ 16 ⓒ 17 ⓓ 18

Explanation : $2(x^2 + y^2)$
$= (x + y)^2 + (x - y)^2$
$= (5)^2 + (3)^2$ [Putting the value]
$= 25 + 9 = 34$
$\therefore x^2 + y^2 = \frac{34}{2} = 17$

(2) What is the value of xy?
ⓓ ⓐ 10 ⓑ 8 ⓒ 6 ⓓ 4

Explanation : $xy = \left(\frac{x+y}{2}\right)^2 - \left(\frac{x-y}{2}\right)^2$
$= \left(\frac{5}{2}\right)^2 - \left(\frac{3}{2}\right)^2$ [Putting the value]
$= \frac{25}{4} - \frac{9}{4} = \frac{25-9}{4} = \frac{16}{4} = 4$

(3) What is the value of $x^2 - y^2$?
ⓒ ⓐ 13 ⓑ 14 ⓒ 15 ⓓ 16

Explanation : $x^2 - y^2 = (x + y)(x - y)$
$= 5 \times 3 = 15$

12. **If $x + \frac{1}{x} = 2$, then**
(1) What is the value of $\left(x - \frac{1}{x}\right)^2$?
ⓐ ⓐ 0 ⓑ 1 ⓒ 2 ⓓ 4

Explanation : $\left(x - \frac{1}{x}\right)^2 = \left(x + \frac{1}{x}\right)^2 - 4 . x . \frac{1}{x}$
$= (2)^2 - 4 = 4 - 4 = 0$

(2) What is the value of $x^3 + \frac{1}{x^3}$?

ⓑ ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

Explanation : $(x)^3 + \left(\frac{1}{x}\right)^3 = \left(x + \frac{1}{x}\right)^3 - 3 . x . \frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (2)^3 - 3 \times 2 = 8 - 6 = 2$

(3) What is the value of $x^4 + \frac{1}{x^4}$?

ⓓ ⓐ 8 ⓑ 6 ⓒ 4 ⓓ 2

Explanation : $x^4 + \frac{1}{x^4} = (x^2)^2 + \frac{1}{(x^2)^2}$

$= \left(x^2 + \frac{1}{x^2}\right)^2 - 2 . x^2 . \frac{1}{x^2}$

$= \left\{\left(x + \frac{1}{x}\right)^2 - 2 . x . \frac{1}{x}\right\}^2 - 2$

$= (2^2 - 2)^2 - 2$

$= 2^2 - 2 = 4 - 2 = 2$

Solutions to Mathematical Problems

Find the H.C.F. of the following (13 – 20) :

13. $36a^2b^2c^4d^5$, $54a^5c^2d^4$ and $90\,a^4b^3c^2$

Solution : Here,

1^{st} quantity $= 36a^2b^2c^4d^5$

$= 3 \times 3 \times 2 \times 2 \times a^2 \times b^2 \times c^2 \times c^2 \times d \times d^4$

2^{nd} quantity $= 54a^5c^2d^4$

$= 3 \times 3 \times 3 \times 2 \times a^2 \times a^3 \times c^2 \times d^4.$

3^{rd} quantity $= 90a^4b^3c^2$

$= 5 \times 3 \times 3 \times 2 \times a^2 \times a^2 \times b^2 \times b \times c^2$

∴ The required H.C.F. is $3 \times 3 \times 2 \times a^2 \times c^2 = 18a^2c^2$

14. $20x^3y^2a^3b^4$, $15x^4y^3a^4b^3$ and $35x^2y^4a^3b^2$

Solution : Here, 1^{st} quantity $= 20x^3y^2a^3b^4$

$= 2 \times 2 \times 5 \times x^2 \times x \times y^2 \times a^3 \times b^2 \times b^2.$

2^{nd} quantity $= 15x^4y^3a^4b^3$

$= 3 \times 5 \times x^2 \times x^2 \times y^2 \times y \times a^3 \times a \times b^2 \times b$

3^{rd} quantity $= 35x^2y^4a^3b^2$

$= 5 \times 7 \times x^2 \times y^2 \times y^2 \times a^3 \times b^2$

∴ The required H.C.F. is $5x^2y^2a^3b^2$

15. $15x^2y^3z^4a^3$, $12x^3y^2z^3a^4$ and $27\,x^3y^4z^5a^7$;

Solution : Here, 1^{st} quantity $= 15x^2y^3z^4a^3$

$= 3 \times 5 \times x^2 \times y^2 \times y \times z^3 \times z \times a^3$

2^{nd} quantity $= 12x^3y^2z^3a^4$

$= 3 \times 2 \times 2 \times x^2 \times x \times y^2 \times z^3 \times a^3 \times a.$

3^{rd} quantity $= 27\,x^3y^4z^5a^7$

$= 3 \times 3 \times 3 \times x^2 \times x \times y^2 \times y^2 \times z^3 \times z^2 \times a^3 \times a^4.$

∴ The required H.C.F. is $3x^2y^2z^3a^3$.

16. $18a^3b^4c^5$, $42a^4c^3d^4$, $60b^3c^4d^5$ and $78a^2b^4d^3$.

Solution : Here, 1^{st} quantity $= 18a^3b^4c^5$

$= 3 \times 3 \times 2 \times a^2 \times a \times b^3 \times b \times c^3 \times c^2.$

2^{nd} quantity $= 42a^4c^3d^4$

$= 7 \times 3 \times 2 \times a^2 \times a^2 \times c^3 \times d^3 \times d.$

3^{rd} quantity $= 60b^3c^4d^5$

$= 3 \times 5 \times 2 \times 2 \times b^3 \times c^3 \times c \times d^3 \times d^2$

4^{th} quantity $= 78a^2b^4d^3$

$= 3 \times 2 \times 13 \times a^2 \times b^3 \times b \times d^3$

∴ The required H.C.F. is $3 \times 2 = 6$

17. $x^2 - 3x$, $x^2 - 9$ and $x^2 - 4x + 3$.

Solution : Here,

1^{st} quantity $= x^2 - 3x = x(x - 3)$

2^{nd} quantity $= x^2 - 9 = (x + 3)(x - 3)$

3^{rd} quantity $= x^2 - 4x + 3 = (x - 3)(x - 1)$

∴ The required H.C.F. is $(x - 3)$

18. $18(x + y)^3$, $24(x + y)^2$ and $32(x^2 - y^2)$

Solution : Here, 1^{st} quantity $= 18(x + y)^3$

$= 3 \times 3 \times 2 (x + y)(x + y)(x + y)$

2^{nd} quantity $= 24 (x + y)^2$

$= 3 \times 2 \times 2 \times 2 (x + y (x + y)$

3^{rd} quantity $= 32(x^2 - y^2)$

$= 2 \times 2 \times 2 \times 2 \times 2 (x + y)(x - y)$

∴ The required H.C.F. is $2(x + y)$

19. $a^2b\,(a^3 - b^3)$, $a^2b^2\,(a^4 + a^2b^2 + b^4)$ and $a^3b^2 + a^2b^3 + ab^4$

Solution :

Here,

1^{st} quantity $= a^2b\,(a^3 - b^3)$

$= a \times a \times b\,(a - b)(a^2 + ab + b^2)$

2^{nd} quantity $= a^2b^2\,(a^4 + a^2b^2 + b^4)$

$= a \times a \times b \times b\,(a^2 + ab + b^2)(a^2 - ab + b^2)$

3^{rd} quantity $= a^3b^2 + a^2b^3 + ab^4$

$= ab^2\,(a^2 + ab + b^2)$

∴ The required H.C.F. is $ab\,(a^2 + ab + b^2)$

20. $a^3 - 3a^2 - 10a$, $a^3 + 6a^2 + 8a$ and $a^4 - 5a^3 - 14a^2$

Solution : Here,

1^{st} quantity $= a^3 - 3a^2 - 10a = a(a^2 - 3a - 10)$

$= a(a - 5)(a + 2)$

2^{nd} quantity $= a^3 + 6a^2 + 8a$

$= a(a^2 + 6a + 8) = a(a + 4)(a + 2)$

3^{rd} quantity $= a^4 - 5a^3 - 14a^2$

$= a^2(a^2 - 5a - 14) = a^2(a - 7)(a + 2)$

∴ The required H.C.F. is $a(a + 2)$

Find the L.C.M. of the following (21 – 28) :

21. a^5b^2c, ab^3c^2 and $a^7b^4c^3$.

Solution : Here, 1^{st} quantity $= a^5b^2c$

2^{nd} quantity $= ab^3c^2$

3^{rd} quantity $= a^7b^4c^3$

Now, highest power of a is a^7, of b is b^4 and of c is c^3.

∴ The L.C.M. of the given quantities is $a^7b^4c^3$.

22. $5a^2b^3c^2$, $10ab^2c^3$ and $15ab^3c$.

Solution : Here, 1st quantity = $5a^2b^3c^2$

2nd quantity = $10ab^2c^3$

3rd quantity = $15ab^3c$.

Now we find that the L.C.M. of 5, 10 and 15 is 30 and the highest power of a is a^2, of b is b^3 and of c is c^3.

∴ The required L.C.M. is $30\,a^2b^3c^3$.

23. $3x^3y^2$, $4xy^3z$, $5x^4y^2z^2$ and $12\,xy^4z^2$.

Solution : Here, 1st quantity = $3x^3y^2$

2nd quantity = $4xy^3z$

3rd quantity = $5\,x^4y^2z^2$

4th quantity = $12xy^4z^2$

Now we find that L.C.M. of 3, 4, 5 and 12 is 60 and the highest power of x is x^4, of y is y^4 and of z is z^2.

∴ The required L.C.M. is $60\,x^4y^4z^2$.

24. $3a^2d^3$, $9d^2b^2$, $12c^3d^2$, $24a^3b^2$ and $36c^3d^2$.

Solution : Here, 1st quantity = $3a^2d^3$

2nd quantity = $9d^2b^2$

3rd quantity = $12c^3d^2$

4th quantity = $24\,a^3b^2$

5th quantity = $36\,c^3d^2$

Now we find that L.C.M. of 3, 9, 12, 24 and 36 is 72 and the highest powerf of a is a^3, of b is b^2, of c is c^3 and of d is d^3.

∴ The required L.C.M. is $72\,a^3b^2c^3d^3$.

25. $x^2 + 3x + 2$, $x^2 - 1$ and $x^2 + x - 2$.

Solution : Here,

1st quantity = $x^2 + 3x + 2 = (x + 1)(x + 2)$

2nd quantity = $x^2 - 1 = (x + 1)(x - 1)$

3rd quantity = $x^2 + x - 2 = (x + 2)(x - 1)$

Now we find that highest power of $(x + 1)$ is 1, of $(x - 1)$ is 1, of $(x + 2)$ is 1.

∴ The required L.C.M. is $(x + 1)(x - 1)(x + 2)$ or, $(x^2 - 1)(x + 2)$.

26. $x^2 - 4$, $x^2 + 4x + 4$ and $x^3 - 8$.

Solution : Here,

1st quantity = $x^2 - 4 = (x + 2)(x - 2)$

2nd quantity = $x^2 + 4x + 4 = (x + 2)(x + 2)$

3rd quantity = $x^3 - 8 = (x - 2)(x^2 + 2x + 4)$

∴ The required L.C.M. is $(x - 2)(x + 2)(x + 2)(x^2 + 2x + 4)$

$= (x + 2)^2 (x - 2)(x^2 + 2x + 4)$

$= (x + 2)^2 (x^3 - 8)$

27. $6x^2 - x - 1$, $3x^2 + 7x + 2$ and $2x^2 + 3x - 2$.

Solution :

Here, 1st quantity = $6x^2 - x - 1 = (2x - 1)(3x + 1)$

2nd quantity = $3x^2 + 7x + 2 = (x + 2)(3x + 1)$

3rd quantity = $2x^2 + 3x - 2 = (x + 2)(2x - 1)$

∴ The required L.C.M. is $(x + 2)(2x - 1)(3x + 1)$

28. $a^3 + b^3$, $(a + b)^3$, $(a^2 - b^2)^2$ and $(a^2 - ab + b^2)^2$

Solution :

Here, 1st quantity = $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

2nd quantity = $(a + b)^3 = (a + b)(a + b)(a + b)$

3rd quantity = $(a^2 - b^2)^2 = (a + b)(a + b)(a - b)(a - b)$

4th quantity = $(a^2 - ab + b^2)^2 = (a^2 - ab + b^2)(a^2 - ab + b^2)$

∴ The required L.C.M. is $(a + b)(a + b)(a + b)(a - b)(a - b)(a^2 - ab + b^2)(a^2 - ab + b^2)$

$= (a + b)^3 (a - b)^2 (a^2 - ab + b^2)^2$.

Creative Questions with Solutions

Ques. 29 If $x^2 + \frac{1}{x^2} = 3$,

a. Determine the value of $\left(x + \frac{1}{x}\right)^2$. 2

b. What is the value of $\frac{x^6 + 1}{x^3}$? 4

c. Determine the value of $\left(x^2 - \frac{1}{x^2}\right)^3$. 4

Solution to Question No. 29 :

a $\left(x + \frac{1}{x}\right)^2 = x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2} = x^2 + \frac{1}{x^2} + 2 = 3 + 2 = 5$

b We have, $x^2 + \frac{1}{x^2} = 3$

or, $\left(x + \frac{1}{x}\right)^2 - 2.x.\frac{1}{x} = 3$ [$\because a^2 + b^2 = (a + b)^2 - 2ab$]

or, $\left(x + \frac{1}{x}\right)^2 = 3 + 2$

or, $\left(x + \frac{1}{x}\right)^2 = 5$ or, $x + \frac{1}{x} = \sqrt{5}$

Now, $\frac{x^6 + 1}{x^3}$

$= \frac{x^6}{x^3} + \frac{1}{x^3} = x^3 + \frac{1}{x^3} = \left(x + \frac{1}{x}\right)^3 - 3x.\frac{1}{x}\left(x + \frac{1}{x}\right)$

$= (\sqrt{5})^3 - 3(\sqrt{5}) = 5\sqrt{5} - 3\sqrt{5} = 2\sqrt{5}$.

c Here given that,

$x^2 + \frac{1}{x^2} = 3$

∴ $\left(x + \frac{1}{x}\right)^2 - 2 \cdot x \cdot \frac{1}{x} = 3$

or, $\left(x + \frac{1}{x}\right)^2 = 3 + 2$

or, $\left(x + \frac{1}{x}\right)^2 = 5$ .....(i)

∴ $\left(x - \frac{1}{x}\right) = \sqrt{\left(x + \frac{1}{x}\right)^2 - 4 \cdot x \cdot \frac{1}{x}}$

$= \sqrt{5 - 4}$ {from (i)} = 1

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That is, $x-\frac{1}{x}=1$ .....(ii)

Now cube of $x^2-\frac{1}{x^2}=\left(x^2-\frac{1}{x^2}\right)^3$

$$=(x^2)^3-3\cdot(x^2)^2\cdot\frac{1}{x^2}+3\cdot x^2\cdot\left(\frac{1}{x^2}\right)^2-\left(\frac{1}{x^2}\right)^3$$

$$=x^6-3x^2+\frac{3}{x^2}-\frac{1}{x^6}=x^6-\frac{1}{x^6}-3\left(x^2-\frac{1}{x^2}\right)$$

$$=\left(x^2-\frac{1}{x^2}\right)^3+3\cdot x^2\cdot\frac{1}{x^2}\left(x^2-\frac{1}{x^2}\right)-3\left(x^2-\frac{1}{x^2}\right)$$

$$=\left\{\left(x+\frac{1}{x}\right)\left(x-\frac{1}{x}\right)\right\}^3+3\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$$-3\left(x-\frac{1}{x}\right)\left(x+\frac{1}{x}\right)$$

$=(\sqrt{5}\cdot1)^3+3\cdot\sqrt{5}\cdot1-3.\sqrt{5}.1$ {from (i) & (ii)}

$=5\sqrt{5}$

Ques. 30 $3x-5y+3z$ and $3x+5y-z$ are two algebraic expressions.

a. Find out the square of the first expression. 2

b. Express the product of the two expressions in the form of difference of two squares. 4

c. If the second expression is '0' (zero), prove that $27x^3+125y^3+45xyz=z^3$. 4

Solution to Question No. 30 :

a Square of the 1st quantity

$=(3x-5y+3z)^2$

$=\{(3x-5y)+3z\}^2$

$=(3x-5y)^2+2(3x-5y)\,3z+(3z)^2$

$=9x^2-30xy+25y^2+6z(3x-5y)+9z^2$

$=9x^2-30xy+25y^2+18zx-30yz+9z^2$

$=9x^2+25y^2+9z^2-30xy-30yz+18zx$

b The product of the two expressions

$=(3x-5y+3z)(3x+5y-z)$

$=\left(\frac{3x-5y+3z+3x+5y-z}{2}\right)^2-\left(\frac{3x-5y+3z-3x-5y+z}{2}\right)^2$,

since $ab=\left(\frac{a+b}{2}\right)^2-\left(\frac{a-b}{2}\right)^2$

$=\left(\frac{6x+2z}{2}\right)^2-\left(\frac{-10y+4z}{2}\right)^2$

$=\left\{\frac{2(3x+z)}{2}\right\}^2-\left\{\frac{-2(5y-2z)}{2}\right\}^2$

$=(3x+z)^2-(5y-2z)^2$

c If the 2nd expression is zero, then we get,

$3x+5y-z=0$

or, $3x+5y=z$

or, $(3x+5y)^3=z^3$ [Cubing both sides]

or, $(3x)^3+(5y)^3+3\cdot3x\cdot5y(3x+5y)=z^3$

or, $27x^3+125y^3+45xy(z)=z^3$, putting $3x+5y=z$

or, $27x^3+125y^3+45xyz=z^3$

$\therefore$ $27x^3+125y^3+45xyz=z^3$ **(Proved)**

Ques. 31 $P=3x^2-16x-12$, $Q=3x^2+5x+2$, $R=3x^2-x-2$ are three algebraic expressions.

a. What do you mean by factorization? 2

b. If $Q=0$ and $x\neq0$ find out the value of $9x^2+\frac{4}{x^2}$. 4

c. Find out the L.C.M. of P, Q and R. 4

Solution to Question No. 31 :

a When an algebraic expression is expressed as the product of two or more simple expressions, then it is said to have been resolved the expression into its factors.

In such cases, each of the simple expression is termed as the factor of the original expression. If we have an expression x^2-5x or $x(x-5)$, then both x and $x-5$ are the factors of x^2-5x.

b Here we have,

$Q=3x^2+5x+2$,

Now if $Q=0$, then we get,

$3x^2+5x+2=0$

or, $\frac{3x^2+5x+2}{x}=\frac{0}{x}$ [Dividing both sides by x]

or, $3x+5+\frac{2}{x}=0$

or, $3x+\frac{2}{x}=-5$

or, $\left(3x+\frac{2}{x}\right)^2=(-5)^2$ [Squaring both sides]

or, $9x^2+\frac{4}{x^2}+2\cdot3x\cdot\frac{2}{x}=25$

or, $9x^2+\frac{4}{x^2}+12=25$

or, $9x^2+\frac{4}{x^2}=25-12$

or, $9x^2+\frac{4}{x^2}=13$

$\therefore$ $9x^2+\frac{4}{x^2}=13$

c Given that,

$P=3x^2-16x-12=3x^2-18x+2x-12$

$=3x(x-6)+2(x-6)=(x-6)(3x+2)$

$Q=3x^2+5x+2=3x^2+3x+2x+2$

$=3x(x+1)+2(x+1)=(x+1)(3x+2)$

$R=3x^2-x-2=3x^2-3x+2x-2$

$=3x(x-1)+2(x-1)=(x-1)(3x+2)$

$\therefore$ L.C.M. of P, Q and R is $(x-6)(x-1)(x+1)(3x+2)$.

Multiple Choice Q/A Designed as per topic

4.7 H.C.F. and L.C.M. of Algebraic Expressions
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1. **What is the full name of the acronym of H.C.F.?** *(Easy)*
   ⓐ Highest Common Fraction
   ⓑ Highest Common Factor
   ⓒ Highest Common Faction
   ⓓ Highest Common Factorial — **ⓑ**
2. **What is the full name of the acronym of L.C.M.?** *(Easy)*
   ⓐ Lowest Common Measure
   ⓑ Lowest Common Multiple
   ⓒ Lowest Common Multitude
   ⓓ Lowest Common Multiplication — **ⓑ**
3. **What is called an algebraic expression which is the product of two or more expressions?** *(Easy)*
   ⓐ Measure ⓑ Multiple
   ⓒ Multitude — **ⓑ** ⓓ Measurement

4.7.1 Highest Common Factor (H.C.F.)
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4. **What is the H.C.F. of ab, bc, ca?** *(Medium)*
   ⓐ abc ⓑ a ⓒ b ⓓ 1 — **ⓓ**
5. **What is the H.C.F. of (a + 1), $a^2 - 1$ and $a^4 - 1$?** *(Medium)*
   ⓐ (a + 1) ⓑ (a − 1)
   ⓒ $a^2 - 1$ ⓓ $a^4 - 1$ — **ⓐ**
6. **What is the H.C.F. of $3ab$, $6a^2b$, $9ab^2$?** *(Hard)*
   ⓐ 9ab ⓑ 6ab ⓒ 3ab ⓓ $18a^2b^2$ — **ⓒ**
7. **If $2a^2 + 6a - 80$ is resolved into factors, what will be the result?** *(Hard)*
   ⓐ (a + 8) (a − 5) ⓑ 2(a + 8) (a − 5)
   ⓒ 2(a − 8) (a + 5) — **ⓑ** ⓓ (a − 8) (a + 5)
8. **Which is the H.C.F. of the given three expressions?** *(Easy)*
   ⓐ (x − 3) (x − 5) (x + 5) ⓑ (x + 5)
   ⓒ x − 5 — **ⓓ** ⓓ 1
9. **What is the H.C.F. of $3x^3y^2z$, $6xy^3z^2$ and $12x^2yz^3$?** *(Hard)* *[RB '19]*
   ⓐ xyz — **ⓑ** ⓑ 3xyz
   ⓒ $3x^3y^3z^3$ ⓓ $12x^3y^3z^3$
10. **What is the H.C.F. of $x^3 - 2x^2$, $x^2 - 4$ and $xy - 2y$?** *(Easy)* *[SB '19]*
    ⓐ 1 ⓑ x − 2
    ⓒ $x^2y(x - 2)$ — **ⓑ** ⓓ $x^2y(x^2 - 4)$
11. **Which one will be the H.C.F. of $18x^3y^2$, $27x^2y$, $45xy^3$?** *(Easy)* *[MB '19]*
    ⓐ $9x^2y^2$ ⓑ $9xy^2$ ⓒ $9x^2y$ ⓓ 9xy — **ⓓ**
12. **Which one is the H.C.F. of the expressions, $x^2 - 5x$, $x^2 - 25$ and $x^2 - 7x + 10$?** *(Easy)* *[RB '18]*
    ⓐ x ⓑ x − 5 ⓒ x − 2 ⓓ x + 5 — **ⓑ**
13. **What is the H.C.F. of $x^2 - 3x$ and $x^2 - 9$?** *(Easy)* *[JB '18]*
    ⓐ $x(x^2 - 9)$ ⓑ $x^2 - 9$
    ⓒ x + 3 — **ⓓ** ⓓ x − 3
14. **What is the H.C.F. of a + 1, $a^2 - 1$ and $a^4 - 1$?** *(Hard)* *[RB '17]*
    ⓐ a + 1 ⓑ a − 1 ⓒ $a^2 - 1$ ⓓ $a^4 - 1$ — **ⓐ**
15. **Which one of the following is the H.C.F. of $(a^3 + b^3)$ and $(a^3 - b^3)$?** *(Hard)* *[RB '16]*
    ⓐ 1 ⓑ a − b
    ⓒ a + b — **ⓐ** ⓓ $a^2 + ab + b^2$
16. **Which one of the following is the H.C.F. of $x^2y + xy^2$ and $x^3 + y^3$?** *(Easy)* *[JB '16]*
    ⓐ x + y — **ⓐ** ⓑ x(x + y)
    ⓒ $x^2 + y^2$ ⓓ $x^3 + y^3$
17. **Which one of the following is H.C.F of $a^3b^4c^4$, $12a^2b^3c^2$, $3a^4b^2c^3$?** *(Hard)* *[CtgB '16]*
    ⓐ a^2bc^2 — **ⓒ** ⓑ 3abc
    ⓒ $a^2b^2c^2$ ⓓ $12a^4b^4c^4$
18. **What is the H.C.F. of the numerator and the denominator of fraction $\frac{x(x-5)}{x^2-25}$?** *(Medium)*
    [Rajuk Uttara Model College, Dhaka]
    ⓐ 1 ⓑ (x + 5)
    ⓒ (x − 5) — **ⓐ** ⓓ $(x^2 - 25)$
19. **What is the H.C.F. of $2x(x^3 - 1)$, $4x^2(x^2 - 1)$** *(Medium)* *[JB '17]*
    ⓐ 2x ⓑ 2x(x + 1)
    ⓒ 2x(x − 1) — **ⓒ** ⓓ $4x^2(x^2 - 1)$
20. **Which one is the H.C.F. of $x^3 + x^2y$, $x^2y + xy^2$ and $x^3 + y^3$?** *(Medium)* *[DjB '16]*
    ⓐ x + y ⓑ x(x + y)
    ⓒ $x^2(x + y)$ — **ⓐ** ⓓ xy(x + y)

Using the following information answer the question No. 21 and 22 :
$16x^4 - 81y^4$ and $2a^2 + 6a - 80$ are two algebraic expressions.

21. **What is the H.C.F. of 2, 6 and 80?** *(Medium)*
    ⓐ 1 — **ⓑ** ⓑ 2 ⓒ 6 ⓓ 80
22. **What will be obtained if the 1st expression is resolved into factors?** *(Hard)*
    ⓐ $(4x^2 + 9y^2)(2x + 3y)(2x - 3y)$
    ⓑ $(4x^2 + 9y^2)(3x + 2y)(3x - 2y)$
    ⓒ $(4x^2 - 9y^2)(3x + 2y)(3x - 2y)$
    ⓓ $(4x^2 - 9y^2)(2x + 3y)(2x - 3y)$ — **ⓐ**

Mathematics

Using the following information answer the question No. 23 and 24 :

$x^2 - 8x + 15$, $x^2 + 2x - 15$ and $x^2 - 25$ are three algebraic expressions.

23. $x^2 - 8x + 15$ = what when it is resolved into factors? *(Medium)*

ⓐ $(x - 5)(x + 3)$ ⓑ $(x + 5)(x - 3)$

c ⓒ $(x - 5)(x - 3)$ ⓓ $(x - 15)(x + 1)$

24. What are the factors when $x^2 + 2x - 15$ is resolved into factors? *(Medium)*

ⓐ $(x + 5), (x - 3)$ ⓑ $(x - 5), (x + 3)$

a ⓒ $(x - 5), (x - 3)$ ⓓ $(x - 15)(x + 1)$

4.7.2 Least Common Multiple (L.C.M.)

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25. What is the L.C.M. of $4ax$, $12a^2x$, $20ax^2$? *(Hard)*

c ⓐ $120ax$ ⓑ $60ax$

ⓒ $60a^2x^2$ ⓓ $120a^2x^2$

26. Which is the L. C. M. of xy, $x + y$ and $x^2 - y^2$? *(Easy)* *[SB '17]*

ⓐ $x^2y^2(x^2 - y^2)$ ⓑ $xy(x^2 - y^2)$

b ⓒ $xy(x - y)$ ⓓ $xy(x^2 + y^2)$

27. Which one will be the L.C.M. of x^2yz, xy^2 and xyz? *(Medium)* *[DjB '17]*

d ⓐ xyz ⓑ xy^2z

ⓒ x^2yz ⓓ x^2y^2z

28. What is the L.C.M. of $4ab^2x^3$, $9a^3c$ and $12a^3bc^4x$? *(Medium)* *[DB '16]*

ⓐ $36a^3b^2c^4x^3$ ⓑ $36a^3b^3c^4$

a ⓒ $36ab^3c^4x$ ⓓ $a^3b^3c^4x$

29. Which one of the following is the L.C.M. of $x^2 - 9$ and $x^2 - 3x$? *(Medium)* *[JB '16]*

ⓐ $x^2 - 3x$ ⓑ $x^2 - 9$

d ⓒ $x + 3$ ⓓ $x(x^2 - 9)$

30. i. $(x + a)(x + b) = x^2 + (a + b)x + ab$ *(Medium)*

ii. $(a - b)^2$ means the product of $(a - b)$ and $(a - b)$

iii. L.C.M. refers to Lowest Common Measure.

Which one of the following is correct?

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

31. i. $(x - a)(x + b) = x^2 - (a + b)x - ab$ *(Easy)*

ii. H.C.F. refers to Highest Common Factor

iii. L.C.M. of $x^2 - 3x$ and $x^2 + 6x + 9$ is $x(x - 3)(x + 3)(x + 3)$

Which one of the following is correct?

c ⓐ i ⓑ i & ii ⓒ ii & iii ⓓ i, ii & iii

32. i. L.C.M. of $3x^2yz$, $6xy^2z$ and $15xyz^2$ is $30x^2y^2z^2$ *(Medium)*

ii. H.C.F. of $3x^2yz$, $6xy^2z$ and $15xyz^2$ is $3xyz$.

iii. H.C.F. of abc and xyz is 1.

Which one of the following is correct?

d ⓐ i ⓑ i & ii ⓒ ii & iii ⓓ i, ii & iii

33. i. The first 3 multiples of 5 are 5, 10 and 15. *(Medium)*

ii. The prime factors of 15 are 3 and 5.

iii. The Highest Common Factor of 14, 21 and 35 is 7.

Which one of the following is correct?

d ⓐ i ⓑ i & ii ⓒ ii & iii ⓓ i, ii & iii

34. For the two expressions of $x^3 - 8$ and $x^2 + 2x + 4$— *(Hard)* *[CB '19]*

i. the H.C.F. is $x^2 + 2x + 4$

ii. the L.C.M. is $x^3 - 8$

iii. the product is $(x - 2)(x^2 + 2x + 4)^2$

Which one is correct?

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

35. $x + 1$, $x^2 - 1$, $x^3 + 1$ are the three algebraic expression, whose —. *(Hard)* *[BB '17]*

i. L.C.M. $= (x^2 - 1)(x^2 - x + 1)$

ii. Sum $= x^3 + x^2 + x + 1$

iii. H.C.F. $= x + 1$

Which one is correct?

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

36. If $A = x^2 - 9$, $B = x - 3$ then —. *(Hard)* *[RB '16]*

i. L.C.M of A and B is $(x^2 - 9)$

ii. H.C.F. of A and B is $(x - 3)$

iii. $A \div B = (x - 3)$

Which one is correct?

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

37. If $a - 1$, $a^2 - 1$, $a^3 - 1$ are three algebraic expression, then —. *(Medium)* *[SB '16]*

i. H.C.F $= a - 1$

ii. L.C.M $= (a - 1)(a^2 + a + 1)$

iii. Sum $= a^3 + a^2 + a - 3$

Which one is correct?

b ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer the questions No. 38 and 39 in the light of the following information :

$2(a + b)$, $4(a^2 - b^2)$ and $12(a^2b + ab^2)$ are three algebraic expression? *[DjB '18]*

38. Which one of the following is the H.C.F. of the given expression? *(Hard)*

ⓐ $a + b$ ⓑ $2(a + b)$

b ⓒ $4(a + b)$ ⓓ $12(a + b)$

39. Which one of the following is the L.C.M. of the given expression? *(Easy)*

ⓐ $12(a + b)$ ⓑ $12(a^2 - b^2)$

c ⓒ $12ab(a^2 - b^2)$ ⓓ $2ab(a^2 - b^2)$

Short Q/A Designed as per topic

4.7.1 Highest common Factor (H.C.F.)

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Question 1. Find the H.C.F. of $2x^3 y^2z^2$, $12x^2yz$ and $20xy^3z^3$.

Solution : Here, H.C.F. of 2, 12, 20 = 2

H.C.F. of x^3, x^2, $x = x$

H.C.F. of y^2, y, $y^3 = y$

H.C.F. of z^2, z, $z^3 = z$

Required H.C.F. = 2xyz

Question 2. Find the H.C.F. of $(a + 2)^2$, $(a^2 + 2a)$ and $(a^2 + 5a + 6)$

Solution : Here,

1st Quantity = $(a + 2)^2 = (a + 2)(a + 2)$

2nd Quantity = $a^2 + 2a = a(a + 2)$

3rd Quantity = $a^2 + 5a + 6$

$= a^2 + 2a + 3a + 6$

$= a(a + 2) + 3(a + 2)$

$= (a + 2)(a + 3)$

Required H.C.F. = a + 2

Question 3. Find the H.C.F. of $6x^2 + 3xy$, $2x^3 + 5x^2 - 12x$ and $x^4 - 8x$.

Solution : Here,

1st Quantity = $6x^2 + 3xy$

$= 3x(2x + y)$

2nd Quantity = $2x^3 + 5x^2 - 12x$

$= x(2x^2 + 5x - 12)$

$= x(2x^2 + 8x - 3x - 12)$

$= x\{2x(x + 4) - 3(x + 4)\}$

$= x(x + 4)(2x - 3)$

3rd Quantity = $x^4 - 8x$

$= x(x^3 - 8)$

$= x(x^3 - 2^3)$

$= x(x - 2)(x^2 + x.2 + 2^2)$

$= x(x - 2)(x^2 + 2x + 4)$

Required H.C.F. = x

Question 4. Find the H.C.F. of $a^3b^4c^4$, $12a^2b^3c^2$ and $3a^4b^2c^3$.

Solution : Here, H.C.F. of 1, 12, 3 = 1

H.C.F. of a^3, a^2, $a^4 = a^2$

H.C.F. of b^4, b^3, $b^2 = b^2$

H.C.F. of c^4, c^2, $c^3 = c^2$

Required H.C.F. = $a^2b^2c^2$

Question 5. Find the H.C.F. of $x + 1$, $x^2 - 1$ and $x^3 - 1$.

Solution : Here, 1st Quantity = x + 1

2nd Quantity = $x^2 - 1 = x^2 - 1^2 = (x + 1)(x - 1)$

3rd Quantity = $x^3 - 1 = x^3 - 1^3 = (x - 1)(x^2 + x + 1)$

Required H.C.F. = 1

Question 6. Find the H.C.F. $3(m + n)$, $9(m^2 - n^2)$ and $18(m^3 + n^3)$?

Solution : Here,

1st Quantity = 3(m + n)

2nd Quantity = $9(m^2 - n^2) = 3.3(m + n)(m - n)$

3rd Quantity = $18(m^3 + n^3) = 2.3.3(m + n)(m^2 - mn + n^2)$

Required H.C.F. = 3(m + n)

4.7.2 Least Common Multiple (L.C.M.)

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Question 7. Find the L.C.M. of $4ab^2x^3$, $9a^3c$ and $12a^3bc^4x^4$.

Solution : Here, L.C.M. of 4, 9 and 12 = 36

The common factors with highest common power among the given expressions are respectively a^3, b^2, c^4, x^4

∴ Required L.C.M. = $36a^3b^2c^4x^4$

Question 8. Find the L.C.M. $5x^2y^3z^2$, $10xy^2z^3$ and $15xy^3z$.

Solution : Here, L.C.M. of 5, 10 and 15 = 30

The common factors with highest common power among the given expressions are respectively x^2, y^3, z^3

Required L.C.M. = $30x^2y^3z^3$

Question 9. Find the L.C.M. of $a^2 - 4$, $a^2 + 4a + 4$ and $a^3 - 8$?

Solution : Here,

1st Quantity = $a^2 - 4 = a^2 - 2^2 = (a + 2)(a - 2)$

2nd Quantity = $a^2 + 4a + 4$

$= a^2 + 2a + 2a + 4$

$= a(a + 2) + 2(a + 2)$

$= (a + 2)(a + 2)$

3rd Quantity = $a^3 - 8$

$= a^3 - 2^3$

$= (a - 2)(a^2 + 2a + 4)$

Required L.C.M. = $(a + 2)(a - 2)(a + 2)(a^2 + 2a + 4)$

$= (a + 2)^2 (a^3 - 8)$

Question 10. Find the L.C.M. of $a^2 - b^2$, $a^2 - ab + b^2$ and $a^3 + b^3$.

Solution : Here,

1st Quantity = $a^2 - b^2 = (a + b)(a - b)$

2nd Quantity = $a^2 - ab + b^2$

3rd Quantity = $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Required L.C.M. = $(a + b)(a - b)(a^2 - ab + b^2)$

$= (a - b)(a^3 + b^3)$

Question 11. Find the L.C.M. of $x + 1$, $x^2 - 1$ and $x^3 + 1$.

Solution : Here, 1st Quantity $= x + 1$

2nd Quantity $= x^2 - 1$
$= x^2 - 1^2$
$= (x + 1)(x - 1)$

3rd Quantity $= x^3 + 1$
$= x^3 + 1^3$
$= (x + 1)(x^2 - x + 1)$

Required L.C.M. $= (x + 1)(x - 1)(x^2 - x + 1)$
$= (x - 1)(x^3 + 1)$

Question 12. Find the L.C.M. of $a^2 - b^2$ and $a^4 + a^2b^2 + b^4$?

Solution : Here, 1st Quantity $= a^2 - b^2$
$= (a + b)(a - b)$

2nd Quantity $= a^4 + a^2b^2 + b^4$
$= (a)^2 + 2a^2b^2 + (b^2)^2 - a^2b^2$
$= (a^2 + b^2)^2 - (ab)^2$
$= (a^2 + b^2 + ab)(a^2 + b^2 - ab)$

Required L.C.M. $= (a + b)(a - b)(a^2 + ab + b^2)(a^2 - ab + b^2)$
$= (a^3 + b^3)(a^3 - b^3)$
$= a^6 - b^6$

Creative Q/A — Designed as per learning outcomes

Ques. 01 $A = x^2 - 9x - 10$ and $B = x^2 - 10x + 25$.

a. What is the L.C.M. of the two or more expressions? 2
b. Find the H.C.F. of the two expressions. 4
c. If $x = -2$, find the value of AB. 4

Solution to Question No. 01 :

a Among different multiples of two or more expressions the common multiple which consists of lowest number of factors is called Least Common Multiple or L.C.M. in short.

b $A = x^2 - 9x - 10 = x^2 - 10x + x - 10$
$= x(x - 10) + 1(x - 10) = (x - 10)(x + 1)$
$B = x^2 - 10x + 25$
$= x^2 - 5x - 5x + 25$
$= x(x - 5) - 5(x - 5)$
$= (x - 5)(x - 5)$.
It is obvious that there is no common factor between A and B other than 1.
So, 1 is the required H.C.F. of A and B. **(Ans.)**

c $A = x^2 - 9x - 10$
$= (x - 10)(x + 1)$, from (b)
$= (-2 - 10)(-2 + 1)$, putting the value of x.
$= (-12) \times (-1) = 12$
$B = x^2 - 10x + 25$
$= (x - 5)(x - 5)$, from (b)
$= (-2 - 5)(-2 - 5)$, putting the value of x
$= (-7) \times (-7) = 49$.
$\therefore AB = 12 \times 49 = 588$
Ans. 588.

Ques. 02 $x + \frac{1}{x} = 5$, where $x \neq 0$ and $P = 4a^2 - 1$, $Q = 2a^2 + 3a - 2$, $R = 6a^2 - a - 1$.

a. Find the square of $(-x^3 + 2)$. 2
b. Prove that, $x^3 - \frac{1}{x^3} = 24\sqrt{21}$. 4
c. Find the H.C.F. of P, Q and R. 4

● *Rajshahi Board 2019*

Solution to Question No. 02 :

a Square of $(-x^3 + 2)$
$= (-x^3 + 2)^2$
$= (-x^3)^2 + 2.(-x^3).2 + 2^2$ $[\because (a + b)^2 = a^2 + 2ab + b^2]$
$= x^6 - 4x^3 + 4$ **(Ans.)**

b Given,
$x + \frac{1}{x} = 5$
or, $\left(x + \frac{1}{x}\right)^2 = 5^2$ [Squaring both sides]
or, $\left(x - \frac{1}{x}\right)^2 + 4.x.\frac{1}{x} = 25$ $[\because (a + b)^2 = (a - b)^2 + 4ab]$
or, $\left(x - \frac{1}{x}\right)^2 = 25 - 4$
or, $x - \frac{1}{x} = \sqrt{21}$
or, $x^3 - \frac{1}{x^3} - 3.x.\frac{1}{x}\left(x - \frac{1}{x}\right) = 21\sqrt{21}$ [Cubing both sides]
or, $x^3 - \frac{1}{x^3} - 3.\sqrt{21} = 21\sqrt{21}$ $[\because x - \frac{1}{x} = \sqrt{21}]$
or, $x^3 - \frac{1}{x^3} = 21\sqrt{21} + 3\sqrt{21}$
$\therefore x^3 - \frac{1}{x^3} = 24\sqrt{21}$ **[Proved]**

c Here,
$P = 4a^2 - 1$
$= (2a)^2 - 1^2$
$= (2a + 1)(2a - 1)$
$Q = 2a^2 + 3a - 2$
$= 2a^2 + 4a - a - 2$
$= 2a(a + 2) - 1(a + 2)$
$= (a + 2)(2a - 1)$
and $R = 6a^2 - a - 1$
$= 6a^2 - 3a + 2a - 1$
$= 3a(2a - 1) + 1(2a - 1)$
$= (3a + 1)(2a - 1)$
$\therefore$ H.C.F. of P, Q and R $= (2a - 1)$.

Mathematics

Ques. 03 $y^2 - 7y - 1$, $15a^2 + 4a - 3$, $9a^3 - a$ and $27a^3 - 1$ are four algebraic expressions.

a. Find the square of $(b^2 + c^3)$ 2

b. If the first expression = 0, then find out the value of $\frac{y^8 + 1}{y^4}$. 4

c. Find out toe L.C.M. of 2nd, 3rd and 4th expressions. 4

● *Sylhet Board 2019*

Solution to Question No. 03 :

a Square of $(b^2 + c^3) = (b^2 + c^3)^2$
$= (b^2)^2 + 2.b^2.c^3 + (c^3)^2$
$[\because (a + b)^2 = a^2 + 2ab + b^2]$
$= b^4 + 2b^2c^3 + c^6$.

Mathematics

b Given,
$y^2 - 7y - 1 = 0$
or, $y^2 - 1 = 7y$
or, $\frac{y^2 - 1}{y} = 7$
or, $y - \frac{1}{y} = 7$
or, $y^2 - 2.y.\frac{1}{y} + \frac{1}{y^2} = 49$ [Squaring both sides]
or, $y^2 + \frac{1}{y^2} = 49 + 2$
or, $y^2 + \frac{1}{y^2} = 51$
or, $y^4 + 2.y^2.\frac{1}{y^2} + \frac{1}{y^4} = 2601$ [Squaring again]
or, $y^4 + \frac{1}{y^4} = 2601 - 2$
or, $\frac{y^8 + 1}{y^4} = 2599$
∴ Determined value of $\frac{y^8 + 1}{y^4}$ is 2599

c Here,
2nd expression $= 15a^2 + 4a - 3$
$= 15a^2 + 9a - 5a - 3$
$= 3a(5a + 3) - 1(5a + 3)$
$= (3a - 1)(5a + 3)$
3rd expression $= 9a^3 - a$
$= a(9a^2 - 1)$
$= a\{(3a)^2 - 1^2\}$
$= a(3a + 1)(3a - 1)$
and 4th expression $= 27a^3 - 1$
$= (3a)^3 - 1^3$
$= (3a - 1)(9a^2 + 3a + 1)$
∴ L.C.M. of 2nd, 3rd and 4th expressions
$= a(3a - 1)(3a + 1)(5a + 3)(9a^2 + 3a + 1)$
∴ Determined L.C.M. is $a(3a - 1)(3a + 1)(5a + 3)(9a^2 + 3a + 1)$.

Ques. 04 $A = x + \frac{1}{x}$, $B = 18(x^2 - 3x)$, $C = 24(x^2 - 9)$, $D = 32(x^2 - 4x + 3)$ are four algebraic expressions.

a. Find the cube of $(xy + yz)$. 2

b. If $A = \sqrt{6}$, then prove that, $\frac{x^6 - 1}{x^3} = 5\sqrt{2}$ 4

c. Find the H.C.F. of B, C and D. 4

● *Chattogram Board 2018*

Solution to Question No. 04 :

a Cube of $(xy + yz) = (xy + yz)^3$
$= (xy)^3 + (yz)^3 + 3xy.yz(xy + yz)$
$= x^3y^3 + y^3z^3 + 3xy^2z(xy + yz)$
$= x^3y^3 + y^3z^3 + 3x^2y^3z + 3xy^3z^2$

b Here, $A = \sqrt{6} \Rightarrow x + \frac{1}{x} = \sqrt{6}$
$\therefore x - \frac{1}{x} = \sqrt{\left(x + \frac{1}{x}\right)^2 - 4} = \sqrt{6 - 4} = \sqrt{2}$ ..... (i)
Now, $\frac{x^6 - 1}{x^3} = x^3 - \frac{1}{x^3}$.
$= \left(x - \frac{1}{x}\right)^3 + 3x \cdot \frac{1}{x}\left(x - \frac{1}{x}\right)$
$= (\sqrt{2})^3 + 3\sqrt{2}$, putting $x - \frac{1}{x} = \sqrt{2}$ from (i).
$= 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$

c Here, $B = 18(x^2 - 3x) = 2 \times 3 \times 3x(x - 3)$ ... (1)
$C = 24(x^2 - 9) = 2 \times 2 \times 2 \times 3(x + 3)(x - 3)$ ..... (2) and
$D = 32(x^2 - 4x + 3) = 2 \times 2 \times 2 \times 2 \times 2(x - 3)(x - 1)$ .... (3)
Now, H.C.F. of 18, 24, 32, = 2 and H.C.F. of $x(x - 3)$, $(x - 3)(x + 3)$ and $(x - 3)(x - 1)$ is $(x - 3)$ from (1), (2) and (3).
∴ H.C.F. of B, C and D is $2(x - 3)$.

Ques. 05 $P = a^2 - 7a + 6$, $Q = a^2 - 2a + 1$, $R = a^2 - 5a + 4$ and $S = ax^2 + (a^2 - 1)x - a$ are four algebraic expressions.

a. Resolve into factors of S. 2

b. Determine $a^3 - \frac{1}{a^3}$, if $Q = 2$. 4

c. Find the H.C.F. of P, Q, R. 4

● *Rajshahi Board 2017*

Solution to Question No. 05 :

a Here, $S = ax^2(a^2 - 1)x - a = ax^2 + a^2x - x - a$
$= ax^2 - x + a^2x - a = x(ax - 1) + a(ax - 1)$
$= (ax - 1)(x + a)$.

b Here, $Q = a^2 - 2a + 1 = 2 \Rightarrow (a-1)^2 = 2 \Rightarrow a - 1 = \pm\sqrt{2} \Rightarrow a = 1 + \sqrt{2}$,

$\therefore \frac{1}{a} = \frac{1}{\sqrt{2}+1} = \frac{\sqrt{2}-1}{2-1} = \sqrt{2} - 1$ when $a = \sqrt{2} + 1$.

And $\frac{1}{a} = \frac{1}{-\sqrt{2}+1} = \frac{1+\sqrt{2}}{(1-\sqrt{2})(1+\sqrt{2})} = \frac{1+\sqrt{2}}{1-2} = -(1+\sqrt{2})$ when $a = -\sqrt{2} + 1$

$\therefore a - \frac{1}{a} = \sqrt{2} + 1 - (\sqrt{2} - 1) = \sqrt{2} + 1 - \sqrt{2} + 2 = 2$ when $a = \sqrt{2} + 1$.

And $a - \frac{1}{a} = -\sqrt{2} + 1 - \{-(1\sqrt{2})\} = -\sqrt{2} + 1 + 1\sqrt{2} = 2$ when $a = -\sqrt{2} + 1$

That is, $a - \frac{1}{a} = 2$ when $a = 1 \pm \sqrt{2}$.

Now, $a^3 - \frac{1}{a^3} = \left(a - \frac{1}{a}\right)^3 + 3.a - \frac{1}{a}\left(a - \frac{1}{a}\right)$

$= 2^3 + 3.2$

$= 8 + 6$

$= 14.$

So, $a^3 - \frac{1}{a^3} = 14$.

c Given that,

$P = a^2 - 7a + 6 = a^2 - 6a - a + 6$

$= a(a-6) - 1(a-6) = (a-6)(a-1)$.

$Q = a^2 - 2a + 1 = a^2 - a - a + 1$

$= a(a-1) - 1(a-1) = (a-1)(a-1)$.

$R = a^2 - 5a + 4 = a^2 - 4a - a + 4$

$= a(a-4) - 1(a-4) = (a-4)(a-1)$.

From the above, it is evident that $(a - 1)$ is common to P, Q and R.

$\therefore$ H. C. F. of P, Q and R is $(a - 1)$.

Solutions to Textual Activities Along with textual reference

Task 01 Find the value H.C.F. : ▸ Textbook Page 70

1. $15a^3b^2c^4$, $25a^2b^4c^3$ and $20a^4b^3c^2$

Solution : H.C.F. of 15, 25, 20 = 5

H.C.F. of $a^3, a^2, a^4 = a^2$

H.C.F. of $b^2, b^4, b^3 = b^2$

H.C.F. of $c^4, c^3, c^2 = c^2$

Required H.C.F. $= 5 \times a^2 \times b^2 \times c^2 = 5a^2b^2c^2$.

2. $(x+2)^2$, (x^2+2x) and (x^2+5x+6)

Solution : 1st Quantity $= (x+2)^2$

$= (x+2)(x+2)$

2nd Quantity $= (x^2 + 2x) = x(x+2)$

3rd Quantity $= x^2 + 5x + 6$

$= x^2 + 3x + 2x + 6$

$= x(x+3) + 2(x+3)$

$= (x+3)(x+2)$

Here, $(x + 2)$ is the common factor in 1st 2nd and 3rd quantity

Required H.C.F. $= (x + 2)$.

3. $6a^2 + 3ab$, $2a^3 + 5a^2 - 12a$ and $a^4 - 8a$

Solution : 1st Quantity $= 6a^2 + 3ab$

$= 3a(2a + b)$

2nd Quantity $= 2a^3 + 5a^2 - 12a$

$= a(2a^2 + 5a - 12)$

$= a(2a^2 + 8a - 3a - 12)$

$= a\{2a(a+4) - 3(a+4)\}$

$= a(a+4)(2a-3)$

3rd Quantity $= a^4 - 8a$

$= a(a^3 - 8)$

$= a(a^3 - 2^3)$

$= a(a-2)(a^2 + 2a + 4)$

Here, $(x + 2)$ is the common factor in 1st 2nd and 3rd quantity

Required H.C.F. $= a$.

Task 02 Find the value L.C.M. : ▸ Textbook Page 71

1. $5x^3y$, $10x^2y$ and $20x^4y^2$

Solution : Here, L.C.M. of 5, 10 and 20 = 20

L.C.M. of $x^3, x^2, x^4 = x^4$

L.C.M. of $y, y, y^2 = y^2$

Required L.C.M. $= 20 \times x^4 \times y^2$

$= 20x^4y^2$

2. $x^2 - y^2$, $2(x+y)$, $2x^2y + 2xy^2$

Solution : 1st Quantity $= x^2 - y^2$

$= (x+y)(x-y)$

2nd Quantity $= 2(x + y)$

3rd Quantity $= 2x^2y + 2xy^2$

$= 2xy(x+y)$

Required L.C.M. $= 2xy(x+y)(x-y)$

$= 2xy(x^2 - y^2)$

3. $a^3 - 1$, $a^3 + 1$, $a^4 + a^2 + 1$

Solution : 1st Quantity $= a^3 - 1$

$= (a-1)(a^2 + a + 1)$

2nd Quantity $= a^3 + 1$

$= (a+1)(a^2 - a + 1)$

3rd Quantity $= a^4 + a^2 + 1$

$= (a^2)^2 + 2.a^2.1 + 1^2 - a^2$

$= (a^2 + 1)^2 - a^2$

$= (a^2 + a + 1)(a^2 - a + 1)$

Required L.C.M. $= (a+1)(a-1)(a^2 + a + 1)(a^2 - a + 1)$

$= (a+1)(a^2 - a + 1)(a-1)(a^2 + a + 1)$

$= (a^3 + 1)(a^3 - 1)$

$= (a^3)^2 - 1^2$

$= a^6 - 1$.

Mathematics

Appendix

Multiple Choice Q/A Designed as per topic

1. $(a+b)^2$? *(Medium)*

ⓐ a^2-b^2 ⓑ a^2+b^2

ⓓ ⓒ $(a-b)(a+b)$ ⓓ $a^2+2ab+b^2$

2. $(a-b)^2+2ab$ = what? *(Hard)*

ⓐ a^2+b^2 ⓑ a^2-b^2

ⓐ ⓒ $(a-b)^2$ ⓓ $(a+b)^2$

▸▸ **Explanation :** $(a-b)^2+2ab$

$= a^2-2ab+b^2+2ab$

$= a^2+b^2$.

3. What is the square of $(3x-2y)$? *(Medium)*

ⓐ $9x^2+12xy+4y^2$ ⓑ $9x^2-12xy+4y^2$

ⓑ ⓒ $9x^2+12xy-4y^2$ ⓓ $9x^2-12xy-4y^2$

▸▸ **Explanation :** Square of $(3x-2y)$

$= (3x-2y)^2$

$= (3x)^2-2\times3x\times2y+(2y)^2$

$= 9x^2-12xy+4y^2$

4. If $p+q=7$ and $pq=9$, p^2+q^2 = what? *(Medium)*

ⓒ ⓐ 40 ⓑ 67 ⓒ 31 ⓓ 58

▸▸ **Explanation :** $p^2+q^2=(p+q)^2-2pq$

$=(7)^2-2\times9$

$=49-18=31$

5. What is the square of 98? *(Medium)*

ⓓ ⓐ 10404 ⓑ 9600 ⓒ 10396 ⓓ 9604

▸▸ **Explanation :** Square of $98=(98)^2$

$=(100-2)^2$

$=(100)^2-2\times100\times2+(2)^2$

$=10000-400+4=9604$

Using the following information answer the question No. 6 and 7 :

$x-\frac{1}{x}=5$

6. $x^2+\frac{1}{x^2}$ = what? *(Hard)*

ⓑ ⓐ 25 ⓑ 27 ⓒ 29 ⓓ 23

▸▸ **Explanation :** $x^2+\frac{1}{x^2}=\left(x-\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}$

$=(5)^2+2=25+2=27$

7. $\left(x^2-\frac{1}{x^2}\right)^2$ = what? *(Medium)*

ⓐ ⓐ 725 ⓑ 729 ⓒ 733 ⓓ 727

▸▸ **Explanation :** $\left(x^2-\frac{1}{x^2}\right)^2=\left(x^2+\frac{1}{x^2}\right)^2-4.x^2.\frac{1}{x^2}$

$=\left\{\left(x-\frac{1}{x}\right)^2+2\times x\times\frac{1}{x}\right\}^2-4$

$=(5^2+2)^2-4$

$=(25+2)^2-4$

$=(27)^2-4=729-4=725$

Using the following information answer the question No. 8 and 9 :

$x+y=10$ and $xy=1$

8. What is the value of $(x-y)^2$? *(Medium)*

ⓒ ⓐ 6 ⓑ 90 ⓒ 96 ⓓ 98

▸▸ **Explanation :** $(x-y)^2=(x+y)^2-4xy$

$=(10)^2-4.1$

$=100-4=96$

9. x^2+y^2 = what? *(Hard)*

ⓐ ⓐ 98 ⓑ 102 ⓒ 96 ⓓ 104

▸▸ **Explanation :** $x^2+y^2=(x+y)^2-2xy$

$=(10)^2-2\times1$

$=100-2=98$

Short Q/A Designed as per topic

Question 1. If $x-y=3$ and $xy=10$, Find the value of x^2+y^2

Solution : Given, $x-y=3$ and $xy=10$

Given expression $=x^2+y^2=(x-y)^2+2xy=(3)^2+2\times10=29$

Required value 29.

Question 2. Find the squire of $5p^3-3q^2$

Solution : The squire of $5p^3-3q^2$

$=(5p^3-3q^2)^2$

$=(5p^3)^2-2\times5p^3\times3q^2+(3q^2)^2$

$=25p^6-30p^3q^2+9q^4$

Mathematics

Question 3. Find the squire of 202
Solution : The squire of 202
$= (202)^2 = (200 + 2)^2$
$= (200)^2 + 2 \times 200 \times 2 + (2)^2$
$= 40000 + 800 + 4 = 40804$

Question 4. Find the squire of 399.
Solution : The squire of 399
$= (399)^2 = (400 - 1)^2$
$= (400)^2 - 2 \times 400 \times 1 + (1)^2$
$= 160000 - 800 + 1 = 159201$

Question 5. If $m - \frac{1}{m} = 9$, Prove that, $m^2 + \frac{1}{m^2} = 83$.

Solution : Given, $m - \frac{1}{m} = 9$

L.H.S. $= m^2 + \frac{1}{m^2}$

$= \left(m - \frac{1}{m}\right)^2 + 2.m.\frac{1}{m}$ [$\because a^2 + b^2 = (a - b)^2 + 2ab$]

$= (9)^2 + 2 = 81 + 2 = 83 =$ R.H.S.

$\therefore m^2 + \frac{1}{m^2} = 83$ **(Proven)**

Question 6. If $x + \frac{1}{x} = 3$, Find the value of $x^4 + \frac{1}{x^4}$

Solution : Given, $x + \frac{1}{x} = 3$

or, $\left(x + \frac{1}{x}\right)^2 = (3)^2$ [Squaring]

or, $x^2 + 2.x.\frac{1}{x} + \frac{1}{x^2} = 9$

or, $x^2 + \frac{1}{x^2} = 9 - 2$

or, $x^2 + \frac{1}{x^2} = 7$

or, $\left(x^2 + \frac{1}{x^2}\right)^2 = (7)^2$ [Squaring again]

or, $x^4 + 2.x^2.\frac{1}{x^2}. + \frac{1}{x^4} = 49$

$\therefore x^4 + \frac{1}{x^4} = 49 - 2 = 47$

Required value 47.

Question 7. Simplify : $(3a + 5b)^2 - 2(3a + 5b)(3a - 5b) + (3a - 5b)^2$.
Solution : Let, $3a + 5b = x$
and $3a - 5b = y$
Given expression $= x^2 - 2xy + y^2$
$= (x - y)^2$
$= \{(3a + 5b) - (3a - 5b)\}^2$ [Putting the value of x and y]
$= (3a + 5b - 3a + 5b)^2$
$= (10b)^2$
$= 100\ b^2$

Question 8. Multiply 7x + 6y by 7x − 6y withe help of formula.
Solution : $(7x + 6y)(7x - 6y)$
$= (7x)^2 - (6y)^2$ [$\because a^2 - b^2 = (a + b)(a - b)$]
$= 49x^2 - 36y^2$

Question 9. a + 3 by a + 5 withe help of formula.
Solution : We know, $(x + a)(x + b) = x^2 + (a + b)x + ab$
$\therefore (a + 3)(a + 5) = a^2 + (3 + 5) \times a + 3 \times 5$
$= a^2 + 8 \times a + 3 \times 5$
$= a^2 + 8a + 15$

Question 10. (2x + 5) by (2x − 7) withe help of formula.
Solution : We know, $(x + a)(x + b) = x^2 + (a + b)x + ab$
$\therefore (2x + 5)(2x - 7) = (2x + 5)\{2x + (-7)\}$
$= (2x)^2 + \{(5) + (-7)\}.\ 2x + 5 \times (-7)$
$= (2x)^2 + (5 - 7)2x - 5 \times 7$
$= 4x^2 - 4x - 35$

Creative Q/A Designed as per learning outcomes

Ques. 01 If $p - \frac{1}{p} = 8$

a. Show that $p^2 - 8p = 1$. (Easy) 2

b. Proved that $p^2 + \frac{1}{p^2} = 66$. (Medium) 4

c. Determine the value of $p^4 + \left(\frac{1}{p}\right)^4$. (Hard) 4

Solution to Question No. 01 :

a Given, $p - \frac{1}{p} = 8$

or, $\frac{p^2 - 1}{p} = 8$

or, $p^2 - 1 = 8p$

$\therefore p^2 - 8p = 1$ **(Showed)**

b Given, $p - \frac{1}{p} = 8$

L.H.S. $= p^2 + \frac{1}{p^2}$

$= \left(p - \frac{1}{p}\right)^2 + 2 \times p \times \frac{1}{p}$ [As $a^2 + b^2 = (a - b)^2 + 2ab$]

$= (8)^2 + 2 = 64 + 2 = 66 =$ R.H.S.

$\therefore p^2 + \frac{1}{p^2} = 66$ **(Proved)**

c Given, $p - \frac{1}{p} = 8$

Given expression $= p^4 + \left(\frac{1}{p}\right)^4$

$= (p^2)^2 + \left(\frac{1}{p^2}\right)^2$

$= \left(p^2 + \frac{1}{p^2}\right)^2 - 2 \times p^2 \times \frac{1}{p^2}$ [$\because a^2 + b^2 = (a+b)^2 - 2ab$]

$= \left\{(p)^2 + \left(\frac{1}{p}\right)^2\right\}^2 - 2$

$= \left\{\left(p - \frac{1}{p}\right)^2 + 2 \times p \times \frac{1}{p}\right\}^2 - 2$ [$\because a^2 + b^2 = (a-b)^2 + 2ab$]

$= \{(8)^2 + 2\}^2 - 2$

$= (64 + 2)^2 - 2$

$= (66)^2 - 2 = 4356 - 2 = 4354$

Required value 4354.

Ques. 02 $m + \frac{1}{m} = 5$

a. Determine 99^2 by formula. *(Easy)* 2

b. Prove that, $\left(m^2 - \frac{1}{m^2}\right)^2 = 525$. *(Medium)* 4

c. Shown that, $m^4 + \frac{1}{m^4} = 527$. *(Hard)* 4

Solution to Question No. 02 :

a Square of $99 = (99)^2$

$= (100 - 1)^2$

$= (100)^2 - 2 \times 100 \times 1 + (1)^2$ [$\because (a-b)^2 = a^2 - 2ab + b^2$]

$= 10000 - 200 + 1 = 9800 + 1 = 9801.$

b Given, $m + \frac{1}{m} = 5$

L.H.S. $= \left(m^2 - \frac{1}{m^2}\right)^2$

$= \left(m^2 + \frac{1}{m^2}\right)^2 - 4 \times m^2 \times \frac{1}{m^2}$ [$\because (a-b)^2 = (a+b)^2 - 4ab$]

$= \left\{(m)^2 + \left(\frac{1}{m}\right)^2\right\}^2 - 4$

$= \left\{\left(m + \frac{1}{m}\right)^2 - 2 \times m \times \frac{1}{m}\right\}^2 - 4$

[$\because a^2 + b^2 = (a+b)^2 - 2ab$]

$= \{5^2 - 2\}^2 - 4$

$= (25 - 2)^2 - 4$

$= (23)^2 - 4$

$= 529 - 4$

$= 525 = $ R.H.S.

$\therefore \left(m^2 - \frac{1}{m^2}\right)^2 = 525$. **(Proved)**

c Given, $m + \frac{1}{m} = 5$

L.H.S. $= m^4 + \frac{1}{m^4}$

$= (m^2)^2 + \left(\frac{1}{m^2}\right)^2$

$= \left(m^2 + \frac{1}{m^2}\right)^2 - 2 \times m^2 \times \frac{1}{m^2}$

$= \left\{(m)^2 + \left(\frac{1}{m}\right)^2\right\}^2 - 2$

$= \left\{\left(m + \frac{1}{m}\right)^2 - 2 \times m \times \frac{1}{m}\right\}^2 - 2$

$= \{(5)^2 - 2\}^2 - 2$

$= (25 - 2)^2 - 2$

$= (23)^2 - 2 = 529 - 2 = 527 = $ R.H.S.

$\therefore \; m^4 + \frac{1}{m^4} = 527$. **(Showed)**

Mathematics

Super Suggestions

Super Suggestions with 100% preparatory questions selected by the Master Trainer Panel

Dear learners, important multiple choice, short and creative questions of this chapter selected by Master Trainer Panel for Half-Yearly and Annual Exams are presented below. Learn the answers to the mentioned questions well to ensure 100% preparation.

Question Pattern		7★	5★	3★
○ MCQs with Answers	Learn each MCQs in this chapter thoroughly.			
○ Short Q/A	Exercise 4.1	1, 4, 6, 8, 12, 15	2, 3, 7, 9, 11	5, 10, 13, 14
	Exercise 4.2	1, 3, 7, 10, 15	2, 4, 5, 8, 9	6, 11, 12
	Exercise 4.3	2, 5, 8, 9, 13	1, 3, 7, 10, 11	4, 6, 12, 15
	Exercise 4.4	2, 5, 6	1, 3, 7	4, 8, 9
	Appendix	1, 5, 9	2, 4, 8	3, 6, 10
○ Creative Q/A	Exercise 4.1	1, 3	2	4
	Exercise 4.2	1, 4, 8, 12	2, 5, 7, 9	3, 6, 10
	Exercise 4.3	1	2	
	Exercise 4.4	1, 4	2, 5	3
	Appendix	1	2	

Assessment & Evaluation

A question bank presented in the form of a class test to assess the preparation

Class Test Time : 3 hours **Mathematics** Full marks : 100

Class : Eight

Multiple Choice Questions (Each question carries 1 mark) $1 \times 30 = 30$

[N.B. : Answer all the questions. Each question carries one mark. Block fully, with a ball-point pen, the circle of the letter that stands for the correct/best answer in the "Answer Sheet" for Multiple Choice Question Type Examination.]

1. What is the square of 95?
   ⓐ 9025 ⓑ 9015 ⓒ 9035 ⓓ 9005
2. Which one is the product of $(x - 11)$ and $(x + 5)$?
   ⓐ $x^2 - 6x + 55$ ⓑ $x^2 + 6x - 55$
   ⓒ $x^2 + 6x + 55$ ⓓ $x^2 - 6x - 55$
3. If $p + q = 7$ and $pq = 9$, what is the value of $p^2 + q^2$?
   ⓐ 67 ⓑ 40 ⓒ 31 ⓓ 13
4. What is the value of $36x^2 - 48xy + 16y^2$, when $x = 3$ and $y = 2$?
   ⓐ 100 ⓑ 110 ⓒ 120 ⓓ 140
5. If $x = p + \frac{1}{p}$ and $y = p - \frac{1}{p}$ then $(x + y)^2$ = what?
   ⓐ $2p$ ⓑ $4p$ ⓒ $2p^2$ ⓓ $4p^2$
6. Which one of the following is the square of $(x - 3y)$?
   ⓐ $x^2 + 6xy + 9y^2$ ⓑ $x^2 - 6xy + 9y^2$
   ⓒ $x^2 + 6xy - 9y^2$ ⓓ $x^2 - 6xy - 9y^2$
7. If $\left(3a + \frac{3}{a}\right)^2 = 9$, then—
   i. $\left(a + \frac{1}{a}\right)^2 = 3$ ii. $a + \frac{1}{a} = 1$
   iii. $a^2 + \frac{1}{a^2} = -1$
   Which one is correct?
   ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

■ According to the following information answer to the questions No. 8 and 9 :
$x^2 - 3x + 1 = 0$.

8. What is the value of $\left(x - \frac{1}{x}\right)$?
   ⓐ $\sqrt{5}$ ⓑ $\sqrt{7}$ ⓒ 5 ⓓ 7
9. Which one of the following is the value of $x^2 + \frac{1}{x^2}$?
   ⓐ 3 ⓑ 5 ⓒ 7 ⓓ 9
10. If $a + b = 4$ and $a - b = 2$, then $a^3 - b^3 = ?$
    ⓐ 27 ⓑ 26 ⓒ 8 ⓓ 1
11. Which one of the cubic value of $x^3 + 2$?
    ⓐ $x^6 + 8$ ⓑ $x^9 + 8$
    ⓒ $x^6 4x^3 + 4$ ⓓ $x^9 + 6x^6 + 12x^3 + 8$

■ Answer the questions No. 12 and 13 in the light of the following information :
$x + y = 5$ and $x - y = 1$.

12. What is the value of $2(x^2 + y^2)$?
    ⓐ 12 ⓑ 13 ⓒ 24 ⓓ 26
13. What is the value of $x^3 - y^3$?
    ⓐ -35 ⓑ -17 ⓒ 19 ⓓ 35
14. $\frac{x^3 + y^3 + 3xy(x + y)}{(x + y)^2 - 4xy} + \frac{(x + y)^2}{(x - y)^3}$ = What?
    ⓐ $x^2 - y^2$ ⓑ $x^2 + y^2$ ⓒ $(x + y)^3$ ⓓ $x^3 + y^3$
15. If $p + q = 3$, $pq = 2$, what is the value of $p^3 + q^3 = ?$
    ⓐ 9 ⓑ 18 ⓒ 27 ⓓ 45
16. The factor of $x^6 - y^6$ is —.
    i. $x^2 - xy + y^2$
    ii. $x^2 + xy + y^2$
    iii. $x^2 + y^2$
    Which one of the following is correct?
    ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii
17. Which one is the factor of $a^2 - 2ab + 2b - 1$?
    ⓐ $(a + 1)$ ⓑ $(a - 1)$
    ⓒ $(a - 2b)$ ⓓ $(a + 2b)$
18. A factor of $x^4 - 2x^2 + 1$ is —.
    ⓐ $2x + 1$ ⓑ $x^2 + 1$
    ⓒ $(x + 1)$ ⓓ $2x - 1$
19. Which one is the factorized form of $2x^2 + 7x - 4$?
    ⓐ $(2x - 1)(x - 4)$ ⓑ $(2x + 1)(x - 4)$
    ⓒ $(2x - 1)(x + 4)$ ⓓ $(2x + 1)(x + 4)$
20. Which is the factor of $3x^2 + x - 10$?
    ⓐ $x - 2$ ⓑ $x + 2$ ⓒ $3x + 5$ ⓓ $2x + 5$
21. Which one of the following is a factor of $3x^2 - 16x - 12$?
    ⓐ $3x - 2$ ⓑ $3x + 2$ ⓒ $x + 6$ ⓓ $3x - 5$
22. Which one of the following is the square of $(-5 - y)$?
    ⓐ $y^2 + 10y + 25$ ⓑ $y^2 - 10y + 25$
    ⓒ $25 - 10y + y^2$ ⓓ $y^2 - 10y - 25$
23. What is the H.C.F of $x^2 - 2x - 3$ and $x^2 + 2x - 3$?
    ⓐ $x + 1$ ⓑ $x - 1$ ⓒ 1 ⓓ 0
24. What is the H.C.F. of $x^2 - 3x$ and $x^2 - 9$?
    ⓐ $x(x^2 - 9)$ ⓑ $x^2 - 9$
    ⓒ $x + 3$ ⓓ $x - 3$
25. What is the H.C.F. of $a + 1$, $a^2 - 1$ and $a^4 - 1$?
    ⓐ $a + 1$ ⓑ $a - 1$ ⓒ $a^2 - 1$ ⓓ $a^4 - 1$
26. What is the H.C.F. of $3ab$, $6a^2b$, $9ab^2$?
    ⓐ $9ab$ ⓑ $6ab$ ⓒ $3ab$ ⓓ $18a^2b^2$
27. What is the H.C.F. of $2x(x^3 - 1)$, $4x^2(x^2 - 1)$
    ⓐ $2x$ ⓑ $2x(x + 1)$
    ⓒ $2x(x - 1)$ ⓓ $4x^2(x^2 - 1)$
28. Which one is the H.C.F. of $x^3 + x^2y$, $x^2y + xy^2$ and $x^3 + y^3$?
    ⓐ $x + y$ ⓑ $x(x + y)$
    ⓒ $x^2(x + y)$ ⓓ $xy(x + y)$
29. What is the H.C.F. of $3x^3y^2z$, $6xy^3z^2$ and $12x^2yz^3$?
    ⓐ xyz ⓑ $3xyz$
    ⓒ $3x^3y^3z^3$ ⓓ $12x^3y^3z^3$
30. Which one will be the H.C.F. of $18x^3y^2$, $27x^2y$, $45xy^3$?
    ⓐ $9x^2y^2$ ⓑ $9xy^2$ ⓒ $9x^2y$ ⓓ $9xy$

Short-Answer Question (Each question carries 2 marks)

Answer any 10 of the following questions : 2 × 10 = 20

1. If $p + q = 7$ and $pq = 9$, Find the value of $p^2 + q^2$.
2. Simplify : $(3a^2 + 7b^2)^2 + 2(3a^2 + 7b^2)(3a^2 - 7b^2) + (3a^2 - 7b^2)^2$
3. If $m - \frac{1}{m} = 4$, Find the value of $m^2 + \frac{1}{m^2}$
4. If $a + b = 7$ and $a - b = 3$, Find the value of $a^2 + b^2$?
5. If $a + b = 9$ and $a - b = 5$, Find the value of ab?
6. If $x + \frac{1}{x} = 2$, Show that, $x^4 + \frac{1}{x^4} = 2$.
7. If $4a - 3 = 5$, Find the value of $64a^3 - 27 - 180a$
8. If $a + b + c = 0$, Prove that, $a^3 + b^3 + c^3 = 3abc$.
9. Find the product of $(2p - 3q)$ and $(4p^2 + 6pq + 9q^2)$ with the help of formula.
10. Resolve into factors $m^2 + 2mn - 2n - 1$.
11. Resolve into factors : $x^2 + x - (a + 1)(a + 2)$
12. Resolve into factors : $x^3 - 3x^2y + 3xy^2 - 9y^3$
13. Find the H.C.F. of $2x^3y^2z^2$, $12x^2yz$ and $20xy^3z^3$.
14. Find the L.C.M. of $4ab^2x^3$, $9a^3c$ and $12a^3bc^4x^4$.
15. Find the L.C.M. of $a^2 - b^2$, $a^2 - ab + b^2$ and $a^3 + b^3$.

Creative Question (Each question carries 10 marks)

Answer any 5 of the following questions : 10 × 5 = 50

Mathematics

1. $(2x - 3y + 5z)^2 - (x - 2y + 3z)^2$ is an algebraic expression.
   a. Simplify the given expression using formula. 2
   b. Find the value of $(2x - 3y + 5z)^2$. 4
   c. Determine the value of the expression when $x = 3$, $y = -1$ and $z = 1$. 4
2. $x^2 + \frac{1}{x^2} = 4$ and $p^4 + \frac{1}{p^4} = 322$ are two algebraic equations.
   a. Find the value of $x + \frac{1}{x}$ 2
   b. Prove that $\frac{x^6 - 1}{x^3} = 5\sqrt{2}$ 4
   c. Show that, $P - \frac{1}{P} = 4$ 4
3. $x - y = 3$, $xy = 4$ and $p^2 - 2\sqrt{3}p + 1 = 0$.
   a. Resolve into factors : $a^4 + a^2b^2 + b^4$. 2
   b. Find the value of $x^3 - y^3 - 2(x + y)^2$. 4
   c. Prove that, $p^3 + \frac{1}{p^3} = 18\sqrt{3}$. 4
4. $P = 5x - 3$, $Q = 2x - 1$ and $R = 3x^2 - 10x + 3$ are three algebraic expressions.
   a. Express PQ in the form of difference of two squares. 2
   b. If $P = \frac{5}{x}$, determine the value of $\left(x^2 - \frac{1}{x^2}\right)^2$. 4
   c. If $R = 0$, show that, $x^3 + \frac{1}{x^3} = \frac{730}{27}$. 4
5. $P = x^2 - ax + 1$, $Q = p^2 + q^2 - r^2$, $R = x^6 - 1$ are three algebraic expressions.
   a. Resolve into factors of R. 2
   b. If $P = 0$, show that $x^4 + \frac{1}{x^4} = a^4 - 4a^2 + 2$. 4
   c. If $Q = 0$, prove that, $p^6 + q^6 + 3p^2q^2r^2 = r^6$. 4
6. $x^2 + (3a + 4b)x + (2a^2 + 5ab + 3b^2)$ is an algebraic expression.
   a. Factorize the constant part. (Easy) 2
   b. Factorize the algebrac expression. (Medium) 4
   c. Show that, if $a = -b$, then $(x + b)$ is a factor of the expression. (Hard) 4
7. $y^2 - 7y - 1$, $15a^2 + 4a - 3$, $9a^3 - a$ and $27a^3 - 1$ are four algebraic expressions.
   a. Find the square of $(b^2 + c^3)$ 2
   b. If the first expression = 0, then find out the value of $\frac{y^8 + 1}{y^4}$. 4
   c. Find out toe L.C.M. of 2nd, 3rd and 4th expressions. 4
8. If $p - \frac{1}{p} = 8$
   a. Show that $p^2 - 8p = 1$. (Easy) 2
   b. Proved that $p^2 + \frac{1}{p^2} = 66$. (Medium) 4
   c. Determine the value of $p^4 + \left(\frac{1}{p}\right)^4$. (Hard) 4

☑ Answer Sheet ▸ Multiple Choice Questions

1	ⓐ	2	ⓓ	3	ⓒ	4	ⓐ	5	ⓓ	6	ⓑ	7	ⓒ	8	ⓐ	9	ⓒ	10	ⓑ	11	ⓓ	12	ⓓ	13	ⓒ	14	ⓐ	15	ⓐ
16	ⓐ	17	ⓑ	18	ⓒ	19	ⓒ	20	ⓑ	21	ⓑ	22	ⓐ	23	ⓒ	24	ⓓ	25	ⓐ	26	ⓒ	27	ⓒ	28	ⓐ	29	ⓑ	30	ⓓ

☑ Solving Reference ▸ Short-Answer Questions

1 ▸ See Page 97; Ques. 02
2 ▸ See Page 97; Ques. 05
3 ▸ See Page 98; Ques. 10
4 ▸ See Page 98; Ques. 13
5 ▸ See Page 99; Ques. 19
6 ▸ See Page 99; Ques. 23
7 ▸ See Page 109; Ques. 07
8 ▸ See Page 109; Ques. 13
9 ▸ See Page 110; Ques. 17
10 ▸ See Page 120; Ques. 01
11 ▸ See Page 120; Ques. 06
12 ▸ See Page 120; Ques. 09
13 ▸ See Page 130; Ques. 01
14 ▸ See Page 130; Ques. 07
15 ▸ See Page 130; Ques. 10

☑ Solving Reference ▸ Creative Questions

1 ▸ See Page 100; Ques. 01
2 ▸ See Page 101; Ques. 04
3 ▸ See Page 110; Ques. 01
4 ▸ See Page 111; Ques. 04
5 ▸ See Page 113; Ques. 07
6 ▸ See Page 121; Ques. 02
7 ▸ See Page 132; Ques. 03
8 ▸ See Page 135; Ques. 01