Chapter 09

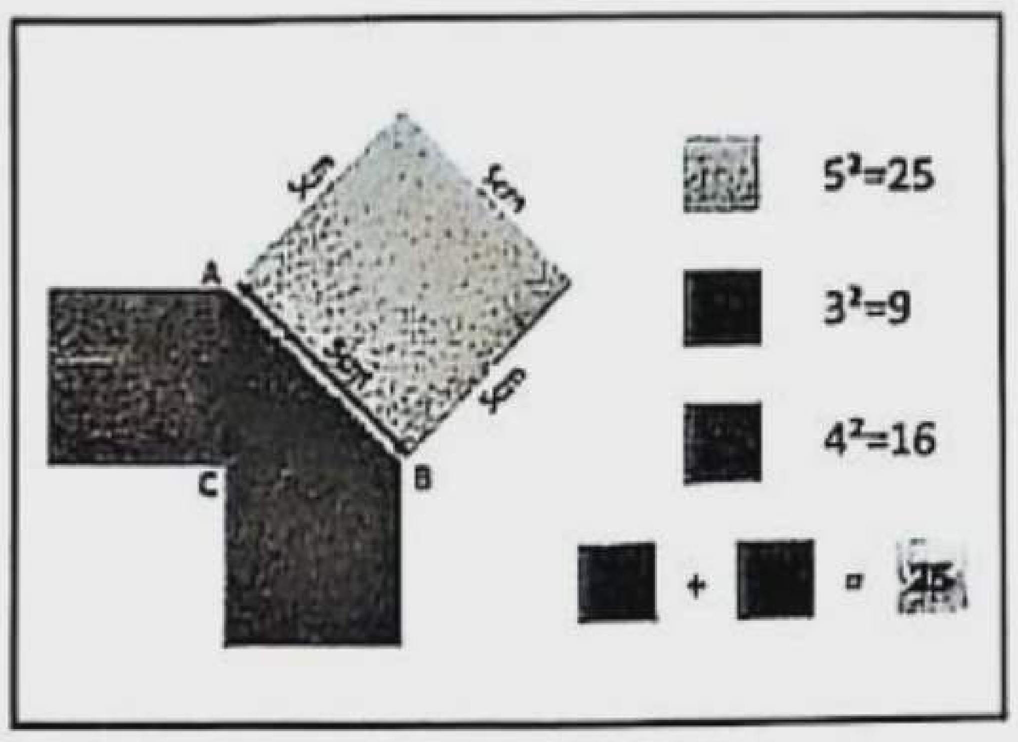
Pythagoras Theorem

Contents for Discussion

• Right angled Triangle • Pythagoras Theorem • Converse of Pythagoras theorem.

Learning Outcomes : After studying this chapter, I will be able to–

- verify and prove Pythagoras theorem
- verify whether the triangle is right-angled when the lengths of three sides of a triangle are given.
- use Pythagoras theorem to solve problems.



Practice

Solutions to Mathematical Problems following 100% accurate format for best prep.

Dear learners, mathematical problems of this chapter have been divided into exercise, multiple choice, short, creative and exercise-based activities in light of the learning outcomes. Practice the solutions well to ensure the best preparation in the exam.

At a Glance — Important Contents of Chapter

- The Greek philosopher Pythagoras, who lived in the 6th century BCE, identified an essential property of right-angled triangles.
- This property of right-angled triangles is known as the Pythagorean property.
- It is said that even before Pythagoras, this property was used in the Egyptian and Babylonian eras.
- The sides of a right-angled triangle are known by specific names:
  i. The side opposite the right angle is called the **hypotenuse**.
  ii. The two sides adjacent to the right angle are called the **base** and the **height**.
- The **Pythagorean theorem** states:
  i. The square constructed on the hypotenuse of a right-angled triangle is **equal in area** to the sum of the squares constructed on the other two sides.
  ii. If, in any triangle, the square on one side is equal to the sum of the squares on the other two sides, then the angle between the latter two sides is a **right angle**.
- Properties of squares and their diagonals:
  i. The square constructed on the **diagonal of any square** is **half** of the original square.
  ii. The area of the square constructed on the diagonal of any square is **twice** the area of the original square.

Solutions to Exercise Problems — Let's solve the textbook problems

MCQs with Answers

1. The ratio of the three sides of a triangle is $1:1:\sqrt{2}$. What is the value of the greatest angle? *(Medium)*

ⓑ ⓐ 80° ⓑ 90° ⓒ 100° ⓓ 120°

Explanation : Let,

Length of triangle a, a, $\sqrt{2}$ a

Here, $a^2 + a^2 = 2a^2 = (\sqrt{2}\,a)^2$

That is squres on one side is equal to sum of the squares on other two sides.

So the triangle is right angled.

∴ The greater angle of the triangle is 90°.

2. For a right angled triangle, the difference between two acute angles is 5°. What is the value of the smallest one? *(Medium)*

ⓑ ⓐ 40° ⓑ 42.5° ⓒ 47.5° ⓓ 50°

Explanation : If, smallest angle of right triangle is x°

∴ Other acute angle is x° + 5°

∴ x° + x° + 5° = 90°

or, 2x° = 90° – 5°

or, $x° = \frac{85°}{2} = 42.5°$

∴ Smallest angle is 42.5°.

3. The hypotenuse of a right angled triangle is x unit and one of the other two sides is y unit. What will be the lenght of the third side? *(Medium)*

(a) $x^2 + y^2$ (b) $\sqrt{x^2 + y^2}$
(c) (c) $\sqrt{x^2 - y^2}$ (d) $x^2 - y^2$

» **Explanation :** Let, length of 3rd side is 'a' unit
We know, for right angled triangle,
Hypotenuse^2 = sum of the squares on other two sides
$\Rightarrow x^2 = y^2 + a^2$
$\Rightarrow a^2 = x^2 - y^2$
$\therefore a = \sqrt{x^2 - y^2}$
$\therefore$ Length of 3rd side $= \sqrt{x^2 - y^2}$ unit.

4. For which of the following measurements, is it possible to draw a right angled triangle? *(Easy)*

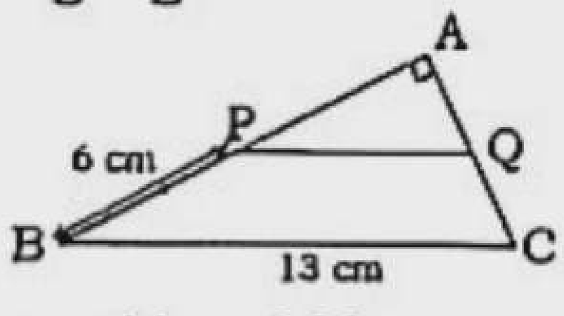
(a) 4, 4, 5 (b) 5, 12, 13
(b) (c) 8, 10, 12 (d) 2, 3, 4

» **Explanation :** Here, $5^2 + 12^2 = 25 + 144 = 169 = 13^2$
So, sum of the squares on two sides of triangle is equal to square on 3rd side. So, right triangle can be drawn by option 'b'.

5. In ΔABC, $\angle A = 1$ right angle — *(Medium)*
i. Hypotenuse is BC
ii. Area $= \frac{1}{2}$ AB.AC
iii. $BC^2 = AB^2 + AC^2$
Which one is correct?
(a) i & ii (b) i & iii
(d) (c) ii & iii (d) i, ii & iii

6. For a right angled triangle —. *(Medium)*
i. the longest side in hupotenuse
ii. sum of the square of the smaller sides is equal to the square of the longest side
iii. Acute angle are complementary to each other.
Which one is correct?
(d) (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

Answer questions 7, 8 and 9 in light with the following figure :



In the figure, $\angle A = 90°$.

7. What will be the length of PQ in cm?
(b) (a) 6 (b) 6.5 (c) 7 (d) 9.5

» **Explanation :** As AP = BP and AQ = CQ
$\therefore$ P and Q are midpoints of AB and AC side respectively
$\therefore PQ = \frac{1}{2}$ BC [The connecting line of two midpoints of two sides of triangle is half of 3rd side.]
$= \frac{1}{2} \times 13 = 6.5$ cm

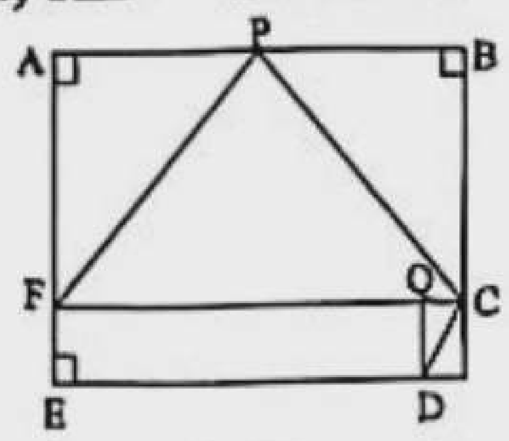
8. What will be the area of ΔABC in sq. cm?
(c) (a) 39 (b) 32.5 (c) 30 (d) 15

» **Explanation :** At ΔABC, $\angle A$ is right angle and BC is hypotenuse
$\therefore AC = \sqrt{BC^2 - AB^2} = \sqrt{13^2 - (6+6)^2}$ [$\because$ P is midpoint of AB]
$= \sqrt{169 - 144} = \sqrt{25} = 5$ cm
$\therefore \Delta ABC = \frac{1}{2} \times AB \times AC = \frac{1}{2} \times 12 \times 5 = 30$ square cm

9. What will be the perimeter of ΔAPQ in cm? *(Hard)*
(a) (a) 15 (b) 12.5 (c) 10 (d) 7.5

» **Explanation :** From '8' we get AC = 5 cm
As P, Q are midpoints of AB and AC respectively.
$\therefore AP = BP = 6$ cm
$AQ = \frac{1}{2} AC = \frac{1}{2} \times 5 = 2.5$ cm
and PQ = 6.5 cm [from '7']
$\therefore$ Perimeter of $\Delta APQ = AP + PQ + AQ$
$= 6 + 2.5 + 6.5$
$= 15$ cm

In the polygon ABCDE, AE || BC, CF $\perp$ AE and DQ $\perp$ CF. ED = 10 mm, EF = 2mm. BC = 8 mm, AB = 12 mm.



On the basis of the above information, answer the questions (10 – 13) :

10. What is the area of the quadrilateral ABCF in square millimetres? *(Medium)*
(b) (a) 64 (b) 96 (c) 100 (d) 144

» **Explanation :** For ABCF rectangle
Length AB = 12 mm
and width BC = 8 mm
$\therefore$ Area of ABCF rectangle = length × width
$= 12 \times 8$
$= 96$ square mm

11. Which one of the following indicates the area of the triangle FPC in sq. mm? *(Easy)*
(b) (a) 32 (b) 48 (c) 72 (d) 80

» **Explanation :** As ABCF is rectangle so AB = F
$\therefore$ Base of FPC triangle = 12 mm
And height BC = 8 mm
$\therefore$ Area of FPC triangle $= \frac{1}{2} \times$ Base × Height
$= \frac{1}{2} \times 12 \times 8 = 48$ mm^2

12. **Which one of the following expresses the length of CD in millimetre?** *(Hard)*

ⓐ ⓐ 2 ⓑ 4 ⓒ $4\sqrt{2}$ ⓓ 8

Explanation : AB = CF = 12 mm and ED = FQ = 10 mm
$\therefore$ CQ = CF – FQ = CF – ED = 12 – 10 = 2 mm
EF = DQ = 2 mm
As DQ $\perp$ CF, So Δ CDQ is right angled CD
$\therefore CD = \sqrt{DQ^2 + CQ^2} = \sqrt{2^2 + 2^2} = \sqrt{4+4} = \sqrt{2.4}$
$= 2\sqrt{2}$ mm

13. **Which one of the following indicates the difference between the areas of ΔFPC and ΔDQC?** *(Hard)*

ⓐ 46 sq. unit ⓑ 48 sq. unit
ⓐ ⓒ 50 sq. unit ⓓ 52 sq. unit

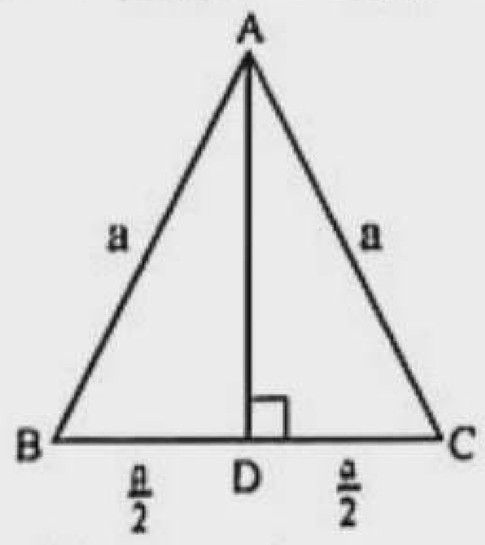
Explanation : From '11'
Area of ΔFPC is 48 mm^2
From '12', CQ = 2 mm and DQ = 2 mm
$\therefore$ Area of $\Delta DQC = \frac{1}{2} \times DC \times CQ$
$= \frac{1}{2} \times 2 \times 2$
$= 2$ mm^2
So, difference between of the area is = 48 – 2 = 46 mm^2

Solutions to Geometrical Problems

14. **ABC is a right angled triangle. AD is the perpendicular to BC. Prove that, $AB^2 + BC^2 + CA^2 = 4AD^2$.**

Solution :

Proposition : Let, ABC is an equilateral triangle. Such that AB = BC = CA = a unit and AD is perpendicular to BC. It is required to prove that $AB^2 + BC^2 + CA^2 = 4AD^2$.



Proof : According to the proposition, AB = BC = CA = a unit and AD $\perp$ BC.

Besides, $BD = CD = \frac{1}{2}a$.

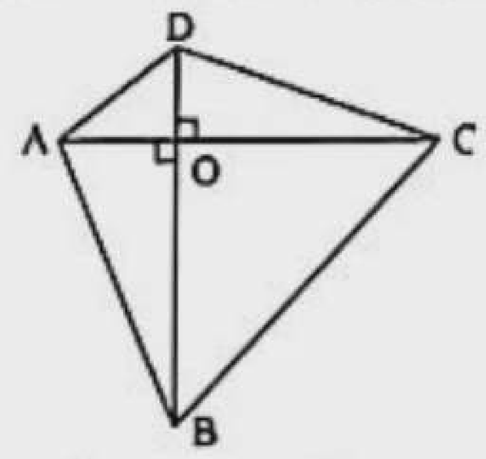
$\angle ADB = 90° = \angle ADC$.
Now from right ΔADB,
$AB^2 = AD^2 + BD^2$
$\Rightarrow a^2 = AD^2 + \frac{1}{4}a^2$ .... (i)
putting the value of AB and BD.
And from right ΔADC,
$AC^2 = AD^2 + CD^2$
$\Rightarrow a^2 = AD^2 + \frac{1}{4}a^2$ ------ (ii)
putting the value of AC and CD.

Adding (i) and (ii),
$$a^2 + a^2 = AD^2 + AD^2 + \frac{1}{4}a^2 + \frac{1}{4}a^2$$
or, $2a^2 - \frac{a^2}{2} = 2AD^2$
or, $\frac{3a^2}{2} = 2AD^2$
or, $3a^2 = 4AD^2$
or, $a^2 + a^2 + a^2 = 4AD^2$
or, $AB^2 + BC^2 + CA^2 = 4AD^2$
$\therefore$ $AB^2 + BC^2 + CA^2 = 4AD^2$. **(Proved)**

15. **Two diagonals of the quadrilateral ABCD intersect each other at right-angled. Prove that, $AB^2 + CD^2 = BC^2 + AD^2$.**

Solution :

Proposition : Let ABCD is any quadrilateral and its diagonals AC and BD mutually intersect at O at right angles. Now it is required to prove that $AB^2 + CD^2 = BC^2 + AD^2$.

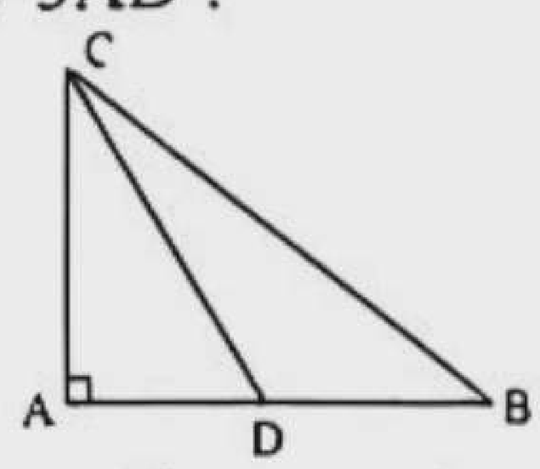


Proof : According to the proposition and the adjoining figure, ΔAOB and ΔCOD are right triangles.
$\therefore AB^2 = OB^2 + OA^2$ .... (1) and
$CD^2 = OC^2 + OD^2$ .... (2)
based on Pythagoras theorem
Now joining (1) and (2), we get,
$AB^2 + CD^2 = OB^2 + OA^2 + OC^2 + OD^2$
$= (OA^2 + OD^2) + (OB^2 + OC^2)$
$= AD^2 + BC^2$
$\therefore AB^2 + CD^2 = AD^2 + BC^2$. **(Proved)**

16. **In ΔABC, $\angle$A is right angle and CD is a median. Prove that, $BC^2 = CD^2 + 3AD^2$.**

Solution :

Proposition : Let, ABC is a right angled triangle where $\angle$A is right angle and CD is a median. Now it is required to be proved that, $BC^2 = CD^2 + 3AD^2$.



Proof : According to the proposition, the adjoining figure refers to a right angled triangle ABC and CD is one of its medians. So, D is the mid point of AB i.e. AD = BD.
Now from right triangle ABC and ACD

$BC^2 = AB^2 + AC^2$

$= (AD + DB)^2 + CD^2 - AD^2$, since $AC^2 = CD^2 - AD^2$ from ΔACD

$= (AD + AD)^2 + CD^2 - AD^2$, since $AD = BD$

$= (2.AD)^2 + CD^2 - AD^2$

$= 4AD^2 + CD^2 - AD^2$

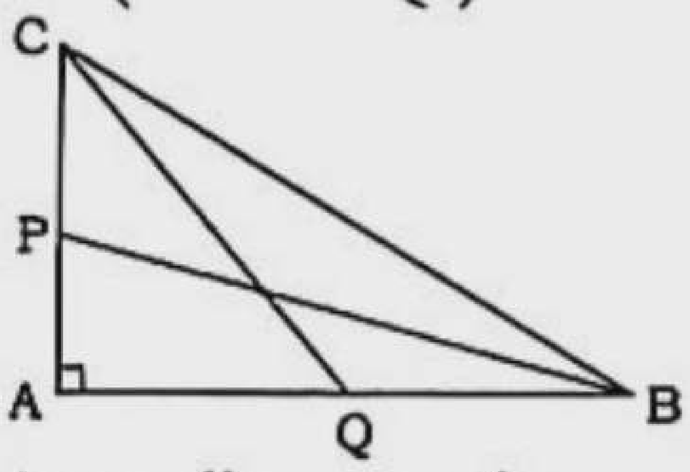
$= CD^2 + 3AD^2$

$\therefore BC^2 = CD^2 + 3AD^2$. **(Proved)**

17. In ΔABC, $\angle A$ is right angle. BP and CQ are two medians. Prove that, $5BC^2 = 4(BP^2 + CQ^2)$.

Solution :

Proposition : Let, ABC is a right angled triangle with $\angle A = 90°$. BP and CQ are two medians of ΔABC. Now, it is required to prove that $5BC^2 = 4(BP^2 + CQ^2)$.



Proof : According to the proposition, the adjoining figure refers to a right triangle ABC where $\angle A = 90°$; BP, CQ are two medians and BC is hypotenuse of ΔABC.

$\therefore$ By using theorem of Pythagoras,

$BC^2 = AB^2 + AC^2$

$= BP^2 - AP^2 + CQ^2 - AQ^2$

$= BP^2 + CQ^2 - AQ^2 - AP^2$

$= BP^2 + CQ^2 - \left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{2}AC\right)^2$

$= BP^2 + CQ^2 - \frac{1}{4}AB^2 - \frac{1}{4}AC^2$

$= BP^2 + CQ^2 - \frac{1}{4}(AB^2 + AC^2)$

$= BP^2 + CQ^2 - \frac{1}{4}(BC^2)$

$= \frac{4.BP^2 + 4.CQ^2 - BC^2}{4}$

$\therefore 4BC^2 = 4.BP^2 + 4CQ^2 - BC^2$, by cross multiplication

or, $4BC^2 + BC^2 = 4.BP^2 + 4CQ^2$

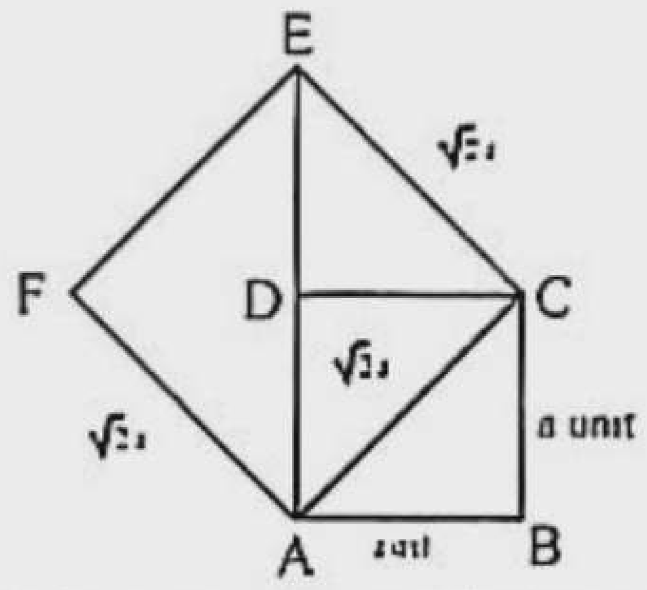
or, $5.BC^2 = 4.BP^2 + 4.CQ^2$. **(Proved)**

18. Prove that, the area of square region on the diagonal of square is the double of the area of the square region.

Solution :

Proposition : Let, ABCD be a square region such that $AB = BC = CD = DA = a$ unit and AC is one of its diagonal.

ACEF be another square region with $AC = CE = EF = FA$. Now it is required to prove that, area of ACEF = 2 × area of ABCD.



Proof : We know, area of a square = (length of one side)^2

$\therefore$ Area of square region,

$ABCD = AB^2$ sq. unit $= a^2$ sq. unit

Again, the diagonal of the square region ABCD,

$AC = \sqrt{AB^2 + BC^2}$ unit

$= \sqrt{a^2 + a^2}$ unit $= \sqrt{2}$ a unit

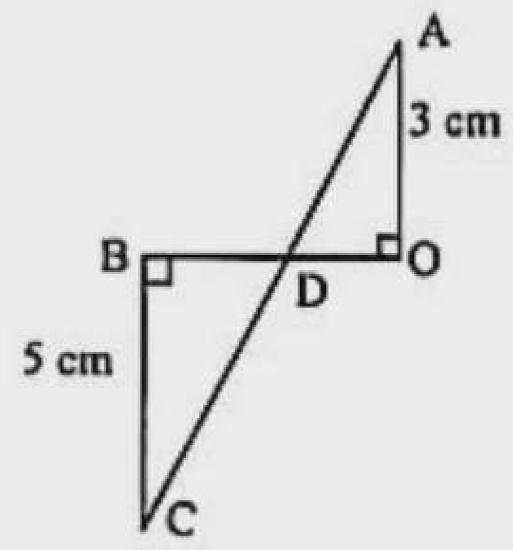
$\therefore$ The area of the square region on ACEF

$= AC^2$ sq. unit $= (\sqrt{2}a)^2$ sq. unit $= 2a^2$ sq. unit

$= 2 \times$ area of the square region ABCD.

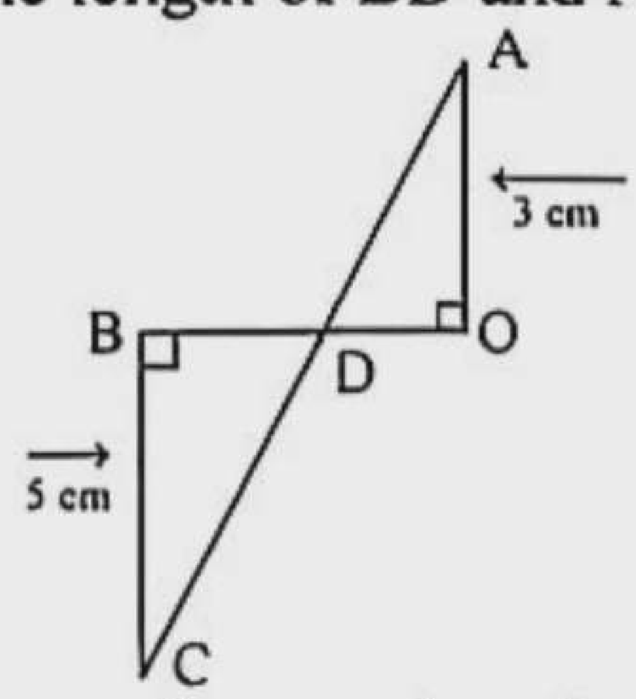
So, the area of square region on the diagonal of square is the double of the area of the square region. **(Proved)**

19. In figure, if OB = 4 cm find the length of BD and AC.



Solution :

Proposition : Let, ΔAOD and ΔBDC are two right angled triangles as shown in the adjoining figure such that OB = 4 cm, OA = 3 cm, BC = 5 cm, $\angle AOD = 90°$ and $\angle CBD = 90°$. It is to be found the length of BD and AC.



Proof : According to the given information, ΔAOD and ΔBCD are two right angled triangle,. $\angle DOA = 90°$, $\angle CBD = 90°$, OA = 3cm, BC = 5 cm and BO = 4cm.

Let, OD = x cm.

So, BD = (4 – x)cm -------------- (i)

Now it is evident that,

$\angle AOD = \angle DBC$,

since they are right angled by proposition.

$\angle ADO = \angle BDC$, since they are vertically opposite angles

$\therefore \angle DAO = \angle BCD$.

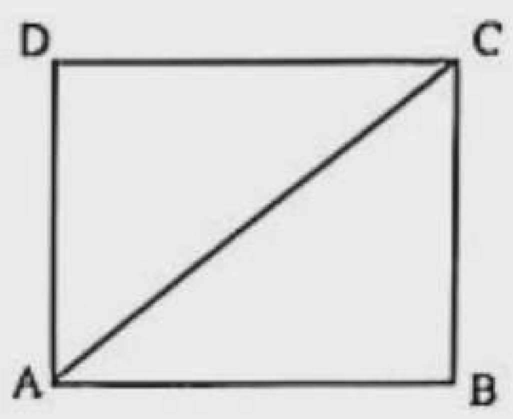
Therefore, ΔAOD and ΔBCD are similar.
Now in the similar ΔAOD and ΔBCD,

$$\frac{OA}{BC} = \frac{OD}{BD}$$

or, $\frac{3cm}{5cm} = \frac{x}{4-x}$, putting the value of OA, OD, BC and BD

or, $3(4 - x) = 5x$, by cross multiplication
or, $12 - 3x = 5x$
or, $5x + 3x = 12$
or, $8x = 12$
or, $x = 1.5$ i.e. $OD = 1.5$ cm
$\therefore$ $BD = (4 - 1.5)$ cm from (i) above
$= 2.5$ cm.
$\therefore$ We have, $BD = 2.5$ and
$OD = 1.5$ cm.
From right ΔAOD,
value of $AD^2 = OD^2 + OA^2$
$= (1.5)^2 + (3)^2$
$= 2.25 + 9 = 11.25$
So, value of $AD = \sqrt{11.25} = 3.35$
$\therefore$ Length of $AD = 3.35$ cm
Again, from right ΔBCD, the value of
$CD^2 = BC^2 + BD^2$
$= (5)^2 + (2.5)^2$
$= 25 + 6.25 = 31.25$
$\therefore$ The value of $CD = 5.59$
So, the length of $CD = 5.59$ cm
$\therefore$ The length of AC
= length of AD + length of DC
$= 3.35$ cm $+ 5.59$ cm $= 8.94$ cm
Therefore, the length of $BD = 2.5$ cm and the length of $AC = 8.94$ cm.

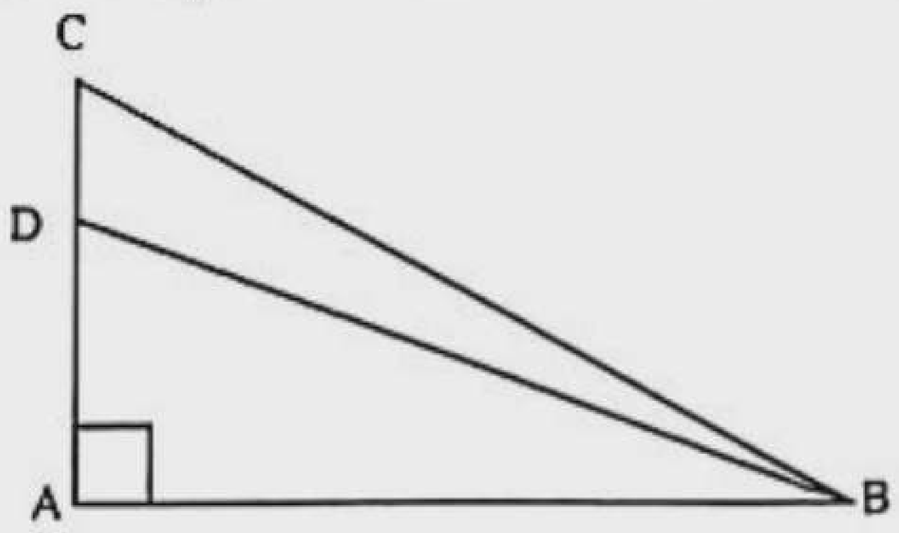
20. Prove that any square region is one half of the square region drawn on its diagonal.
Solution :



Proposition : Let us suppose, ABCD is any square and AC is one of its diagonals. We shall have to prove that $AB^2 = \frac{1}{2} AC^2$.

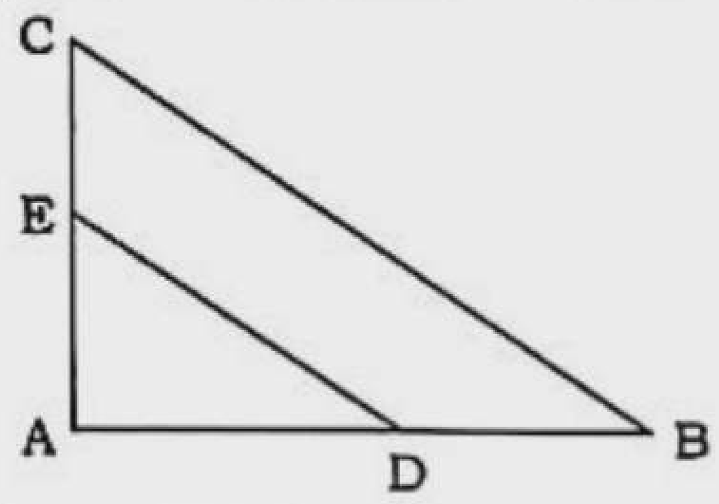
Proof : According to proposition, ABCD is any square with a diagonal AC.
So, the area of $ABCD = AB^2$ .... (i)
Again, the area of the square with side
$AC = AC^2 = AB^2 + BC^2$
$= AB^2 + AB^2$ (since $AB = BC$)
$= 2AB^2$
$= 2 \times$ area of ABCD from (i)
So, $2 \times$ area of ABCD = Area of the square with side AC.
$\therefore$ Area of ABCD $= \frac{1}{2} \times$ area of the square with side AC. **(Proved)**

21. In triangle ABC, $\angle A$= 1 right angle and D is a point on AC. Prove that $BC^2 + AD^2 = BD^2 + AC^2$.
Solution :
Proposition : We have a right angled triangle ABC with $\angle A = 90°$ and D is a point on AC. We shall have to prove that $BC^2 + AD^2 = BD^2 + AC^2$.



Construction : B and D are joined.
Proof : From ΔABC, we have,
$BC^2 = AB^2 + AC^2$ .... (i)
Again, from ΔABD, we have,
$BD^2 = AB^2 + AD^2$
or, $BD^2 - AD^2 = AB^2$ .... (ii)
Now putting the value of AB^2 from (ii) in (i), we have,
$BC^2 = BD^2 - AD^2 + AC^2$
or, $BC^2 + AD^2 = BD^2 + AC^2$ **(Proved)**

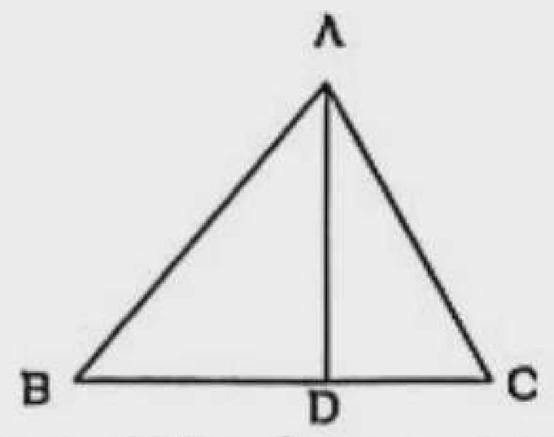
22. In triangle ABC, $\angle A$= 1 right angle. If D and E are respectively the mid-points of AB and AC, prove that $DE^2 = CE^2 + BD^2$.



Proposition : We have ΔABC with $\angle A = 90°$. E and D are the mid points of AC and AB respectively. We shall have to prove that $ED^2 = CE^2 + BD^2$.
Construction : E and D are joined.
Proof : According to proposition, ABC is a right angled trianle where $\angle A = 90°$ and D and E are the mid-points of AB and AC respectively.
$\therefore$ $AD = BD$, $AE = CE$ and ΔDAE is a right angled triangle.
So, $DE^2 = AD^2 + AE^2$ according to theorem of Pythagoras.
$= CE^2 + BD^2$, since $AE = CE$ and $AD = BD$.
Thus, it has been proved that $DE^2 = CE^2 + BD^2$.

23. In ΔABC, AD is the perpendicular to BC and $AB > AC$. Prove that $AB^2 - AC^2 = BD^2 - CD^2$.

Solution :



Proposition : We have any triangle ABC where $AB > AC$ and AD is perpendicular to BC. We shall have to prove that $AB^2 - AC^2 = BD^2 - CD^2$.

Proof : According to proposition, ABD is a right angled triangle with hypotenous AB and $\angle ADB = 90°$. Besides, ACD is also a right angled triangle with hypotenous AC and $\angle ADC = 90°$.

Now according to theorem of Pythagoras, we have from ΔABD :

$AB^2 = AD^2 + BD^2$ .... (i)

Again from ΔACD, we have,

$AC^2 = AD^2 + CD^2$ .... (ii)

Now subtracting (ii) from (i)

We get, $AB^2 - AC^2$

$= \cancel{AD^2} + BD^2 - \cancel{AD^2} - CD^2$

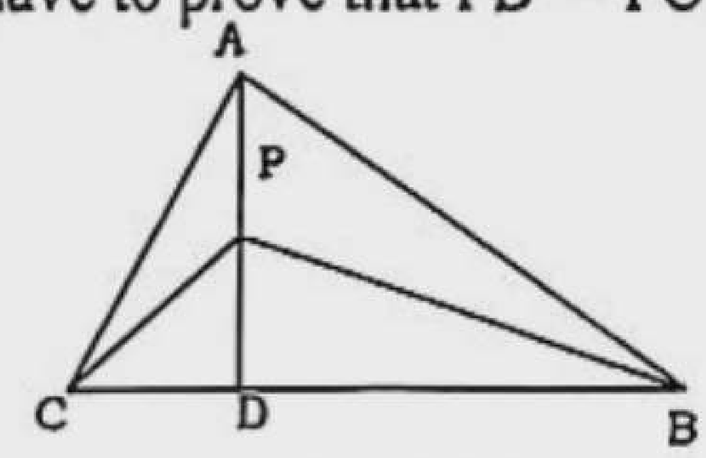
$= BD^2 - CD^2$

Thus proved that $AB^2 - AC^2 = BD^2 - CD^2$.

24. In ΔABC, AD is perpendicular to BC and P is any point on AD and $AB > AC$. Prove that $PB^2 - PC^2 = AB^2 - AC^2$.

Solution :

Proposition : ABC is any triangle where AD is perpendicular to BC and any point P lies on AD. We shall have to prove that $PB^2 - PC^2 = AB^2 - AC^2$.



Construction : C, P and B, P are joined.

Proof : According to proposition,

AD and PD are perpendicular to BC.

So, $\angle PDC$ as well as $\angle PDB$ is right angles.

$\therefore \angle PDC = 90° = \angle PDB$.

Now from ΔABD, we have,

$AB^2 = AD^2 + BD^2$, according to theorem of Pythagoras.

Again from ΔACD, we have, $AC^2 = AD^2 + CD^2$, for the same reason stated above.

$\therefore AB^2 - AC^2 = \cancel{AD^2} + BD^2 - \cancel{AD^2} - CD^2$

$= BD^2 - CD^2$

$= (PB^2 - PD^2) - (PC^2 - PD^2)$

(since from ΔPCD, $CP^2 = PD^2 + CD^2$ and from ΔPBD, $PB^2 = DP^2 + BD^2$)

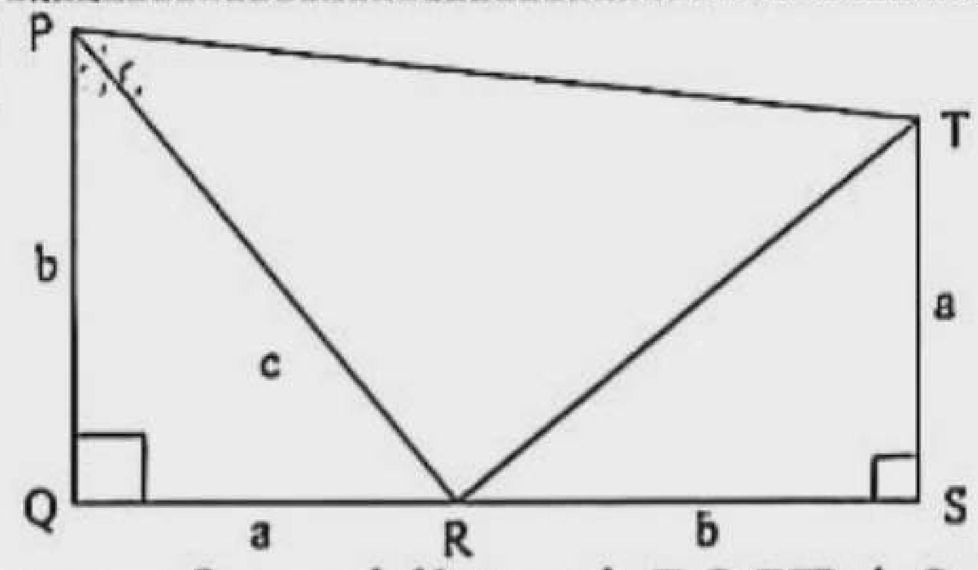
$= PB^2 - \cancel{PD^2} - PC^2 + \cancel{PD^2}$

$= PB^2 - PC^2$.

Thus it is proved that $PB^2 - PC^2 = AB^2 - AC^2$.

Creative Questions with Solutions

Ques. 25



a. What type of quadrilateral PQST is? Justify your answer. *(Easy)* 2

b. Show that ΔPRT is a right angled triangle. *(Medium)* 4

c. Prove that $PR^2 = PQ^2 + QR^2$. *(Hard)* 4

Solution to Question No. 25 :

a The given geometric figure PQST is a quadrilateral of the type of trapezium for the following grounds :

i. Both PQ and TS are perpendicular to QS at Q and S respectively.
So, PQ || TS in PQST.

ii. PQ = b and TS = a in PQST.
So, PT and QS are not parallel.

Now, from the definition of trapezium and based on (i) and (ii) above, the quadrilateral PQST is a trapezium.

b Here we have been given two right triangles ΔPQR and ΔRST where we find that $\angle PQR = 90° = \angle RST$, QR = ST, PQ = RS.

$\therefore \Delta PQR \cong \Delta RST \therefore \angle QRP = \angle RTS$ and $\angle QPR = \angle TRS$ ....(i)

Now we have a point R on the straight line QS. So, at R, $\angle QRS$ = a straight angle

$= 180° = \angle QRP + \angle PRT + \angle SRT$

$= \angle QRP + \angle PRT + \angle QPR$, from (i)

$= \angle QRP + \angle QPR + \angle PRT$

$= 90° + \angle PRT$

$\therefore 90° + \angle PRT = 180°$

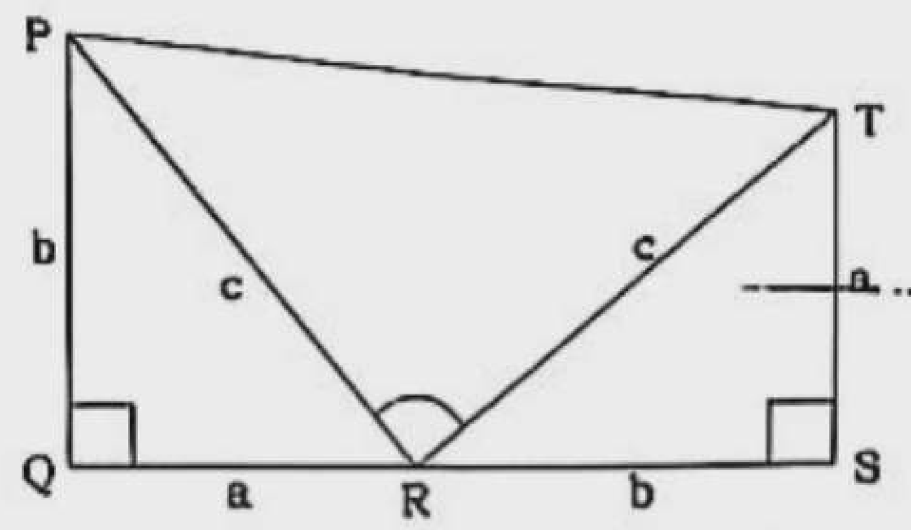
or, $\angle PRT = 180° - 90°$

or, $\angle PRT = 90°$

So, in the given figure, $\angle PRT = 90°$ **(Proved)**

c Let in the triangle PQR, $\angle Q = 90°$, hypotenuse PR = c, PQ = b and QR = a.

It is required to prove that $PR^2 = PQ^2 + QR^2$, i.e. $c^2 = b^2 + a^2$.



According to figure above, both ΔPQR and TSR are right triangles where $\angle Q = \angle S = 90°$. Again, from (b) it is proved that PRT is a right triangle and from (a) we have, quadrilateral PQST is a trapezium whose two parallel lines are a and b, and height (a + b). Now the area of the tranpezium.

PQST $= \Delta PQR + \Delta PRT + \Delta SRT$

$= \frac{1}{2}ab + \frac{1}{2} \times c \times c + \frac{1}{2}ab = ab + \frac{1}{2}c^2$

$\Rightarrow \frac{1}{2}(a+b)(a+b) = ab + \frac{1}{2}c^2$

$\Rightarrow \frac{1}{2}(a+b)^2 = ab + \frac{1}{2}c^2$

$\Rightarrow (a+b)^2 = 2ab + c^2$ (Both sides multiplied by 2)

$\Rightarrow a^2 + b^2 + 2ab = 2ab + c^2$

$\Rightarrow a^2 + b^2 = 2ab + c^2 - 2ab$

$\therefore a^2 + b^2 = c^2$. **(Proved)**

Ques. 26 For, ΔPQR, $\angle P = 90°$, Mid-points of PQ and PR are M and N respectively.

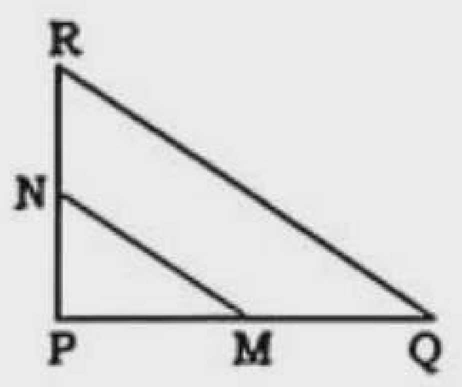
a. Draw the triangle. *(Easy)* 2

b. Prove from the figure A that $PR^2 + PQ^2 = QR^2$. *(Medium)* 4

c. Prove that, $5RQ^2 = 4(RN^2 + QM^2)$. *(Hard)* 4

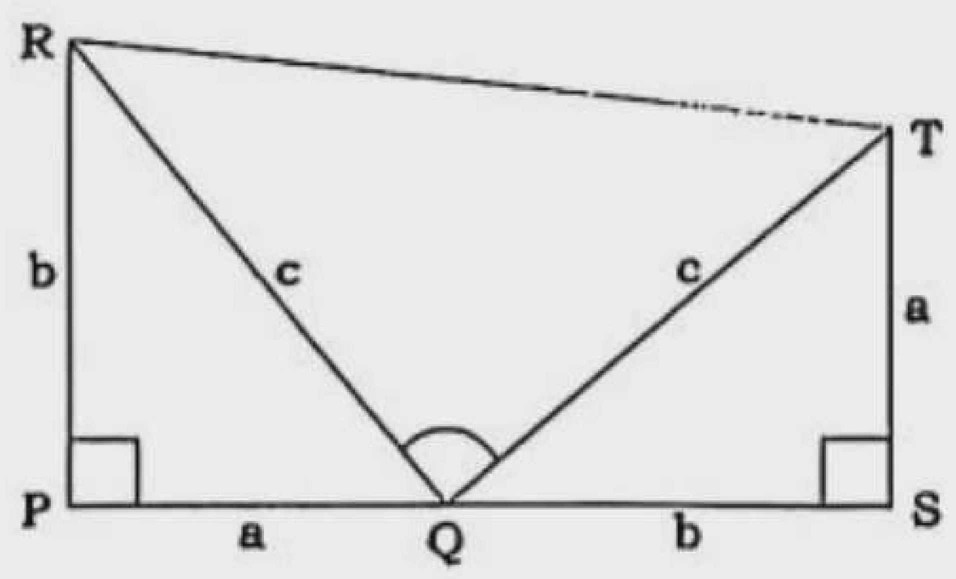
Solution to Question No. 26 :

a A geometric figure based on the given information is constructed below :



b Let in the triangle RPQ, $\angle P = 90°$, hypotenuse $RQ = c$, $RP = b$ and $PQ = a$.

It is required to prove that $RQ^2 = RP^2 + PQ^2$, i.e. $c^2 = b^2 + a^2$.



Construction : Produce PQ up to S such that $QS = RP = b$. Also draw perpendicular ST at S on PQ produced, so that $ST = PQ = a$. Q, T and R, T are joined.

Proof :

Steps	Justification
(1) In ΔRPQ and ΔQST, $RP = QS = b$, $PQ = ST = a$ and included $\angle RPQ$ = included $\angle QST$	[each right angle]
Hence, $\Delta RPQ \cong \Delta QST$.	[SAS theorem]
$\therefore RQ = QT = c$ (say) and $\angle PRQ = \angle TQS$.	
(2) Again, since $RP \perp PS$ and $TS \perp PS$ $\therefore RP \parallel TS$. Therefore, RPST is a trapezium.	$\therefore \angle PRQ = \angle TQS$.
(3) Moreover, $\angle RQP + \angle PRQ = \angle QTS + \angle TQS$ = 1 right angle	
$\therefore \angle RQT$ = 1 right angle	
Now area of the trapezium RPST = area of (Δ region RPQ + Δ region QST + Δ region RQT)	[Area of trapezium = $\frac{1}{2}$ sum of parallel sides × distance between parallel sides]
or, $\frac{1}{2}PS(RP + ST) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$	
or, $\frac{1}{2}(PQ + QS)(RP + ST) = \frac{1}{2}[2ab + c^2]$	
or, $(a+b)(a+b) = 2ab + c^2$ [multiplying by 2]	
or, $a^2 + 2ab + b^2 = 2ab + c^2$	
or, $a^2 + b^2 = c^2$.	
That is, $RQ^2 = RP^2 + PQ^2$ **(Proved)**	

c Here, ΔPQR is a right triangle, where $\angle P = 90°$ and QR = hypotenuse.

$\therefore QR^2 = PQ^2 + PR^2$, according to the theorem of Pythagoras.

$= (2QM)^2 + (2RN)^2$, $= 4QM^2 + 4RN^2$ $\left\{ \begin{array}{l} \text{Since } PQ = 2\,QM \text{ and} \\ PR = 2RN \text{ for M, N are} \\ \text{the mid-points of PQ,PR} \end{array} \right.$

$\therefore RQ^2 = 4(RN^2 + QM^2)$ **(Proved)**

Multiple Choice Q/A Designed as per topic

Lesson-9.1 : Right angled Triangle

▸ Textbook Page 152

1. **A triangle having one right angle is called a — triangle.** *(Easy)*
ⓐ Acute angled ⓑ Obtuse angled triangle
ⓒ ⓒ Right angled ⓓ Isosceles

2. **What is called the opposite side of the right angle of a right angled triangle?** *(Easy)*
ⓐ Hypotenuse ⓑ Base
ⓐ ⓒ Perpendicular ⓓ Diagonal

3. **How many acute angles are there in a right angled triangle?** *(Easy)*
ⓑ ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 4

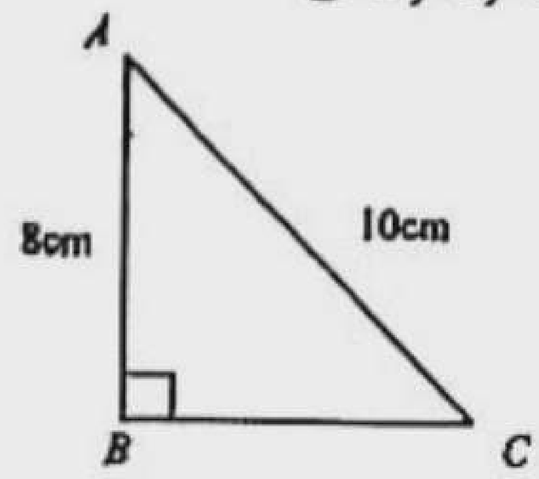
4. **What is the measure of the two angles other than the right angle of a right angled triangle?** *(Easy)*
ⓒ ⓐ 60° ⓑ 180° ⓒ 90° ⓓ 270°

5. **If ΔABC is a right triangle with $\angle B = 90°$, then which one of the following is the hypotenuse of ΔABC?** *(Medium)*
ⓐ AB ⓑ BC
ⓒ ⓒ AC ⓓ None of the above

6. **If 12 cm, 5 cm and 13 cm are the measures of three sides of a triangle, then what type of triangle is it?** *(Medium)*
ⓐ Equilateral triangle ⓑ Isosceles triangle
ⓓ ⓒ Acute angled triangle ⓓ Right triangle

7. **If the ratio of the sides of a triangle is $x : x : x\sqrt{2}$, what is the value of its greatest angle?** *(Easy)* [DB '19]
ⓒ ⓐ 80° ⓑ 85° ⓒ 90° ⓓ 120°

8. **Which of the following measurements of sides is possible to draw a right angle triangle?** *(Easy)* [DB '17]
ⓐ 3, 4, 5 ⓑ 4, 4, 5
ⓐ ⓒ 6, 7, 8 ⓓ 1, 6, 7

9. 

In the figure BC = ? *(Medium)* [SB '17]
ⓐ ⓐ 6 cm ⓑ 12 cm ⓒ 13 cm ⓓ 14 cm

10. **From which sides a triangle can be drawn?** *(Hard)* [SB '17]
ⓐ 3, 4, 6 ⓑ 3, 5, 8
ⓓ ⓒ 3, 5, 9 ⓓ 8, 6, 10

11. **The ratio of sides of a triangle is $1 : 1 : \sqrt{2}$. What kind of triangle is it?** *(Easy)* [JB '16]
ⓐ Right triangle ⓑ Equilateral triangle
ⓐ ⓒ Obtuse angle ⓓ Scalene triangle

12. **$\angle ABE + \angle ACF + \angle CAG$ = what?** *(Hard)*
ⓓ ⓐ 90° ⓑ 120° ⓒ 180° ⓓ 360°

13. **From which sides a right angled triangle can be drawn?** *(Easy)* [DjB '16]
ⓐ 3, 4, 6 ⓑ 4, 5, 9
ⓒ ⓒ 5, 12, 13 ⓓ 12, 13, 17

14. **If the difference of the two acute angle of a right angled triangle is 15°, then what is the value of the greatest between them?** *(Easy)* [Rajuk Uttara Model College, Dhaka]
ⓒ ⓐ 37.5° ⓑ 45° ⓒ 52.5° ⓓ 57.3°

15. **By which of the following right angle triangle can be drawn?** *(Easy)* [Rajuk Uttara Model College, Dhaka]
ⓐ 6 cm, 8 cm, 12 cm ⓑ 5 cm, 12 cm, 13 cm
ⓑ ⓒ 7 cm, 9 cm, 11 cm ⓓ 5 cm, 11 cm, 12 cm

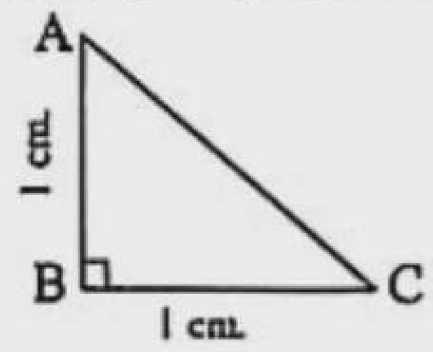
16. **The ratio of the sides of triangle is $1 : 1 : \sqrt{2}$, what is the value of one angle of the triangle?** *(Medium)* [Rajuk Uttara Model College, Dhaka]
ⓑ ⓐ 30° ⓑ 45° ⓒ 60° ⓓ 100°

17. **The ratio of three sides of a triangle is $1 : 1 : \sqrt{2}$. What is the value of the greatest angle?** *(Medium)* [Ideal School & College, Dhaka]
ⓑ ⓐ 80° ⓑ 90° ⓒ 100° ⓓ 180°

18. **If ΔABC is a right angled triangle and $\angle B = 90°$, then —.**
i. $\angle A$ is complementary to $\angle C$
ii. $\angle A + \angle B + \angle C = 180°$
iii. $\angle A + \angle C = 90°$
Which one of the following is correct? *(Hard)*
ⓐ ⓐ i, ii & iii ⓑ i & ii ⓒ i & iii ⓓ ii & iii

19. **A right angled triangle can be constructed if the ratio of the sides are :**
i. $1 : 1 : \sqrt{2}$
ii. $3 : 4 : 5$
iii. $2 : 3 : 4$
Which one of the following is correct? *(Hard)*
ⓐ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

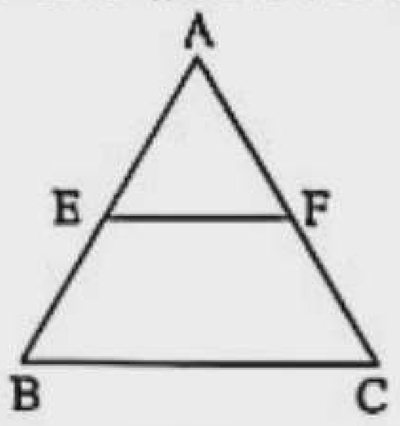
20. **If ΔPQR is a right triangle such that $\angle Q = 90°$, then —.**
i. PR is the largest side of the triangle
ii. $\angle R$ is an obtuse angle
iii. PR + QR > PQ
Which one of the following is correct? *(Medium)*
ⓐ ⓐ i & iii ⓑ ii & iii ⓒ i & ii ⓓ i, ii & iii

21. 

In the above figure —. [RB '16]
i. $\angle A = 45°$
ii. $AC = \sqrt{2}$ cm
iii. Area of ΔABC is 1 sq. cm.
Which one is correct? *(Medium)*
ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

22. **If BC is hypotenuse of triangle ABC.** *[JB '16]*
i. $\angle A$ = right angle
ii. $\angle B$ and $\angle C$ are acute angle
iii. $\angle B + \angle C = 90°$
Which one is correct? *(Medium)*
(d) (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

Answer the questions No. 23 and 24 according to the following information :

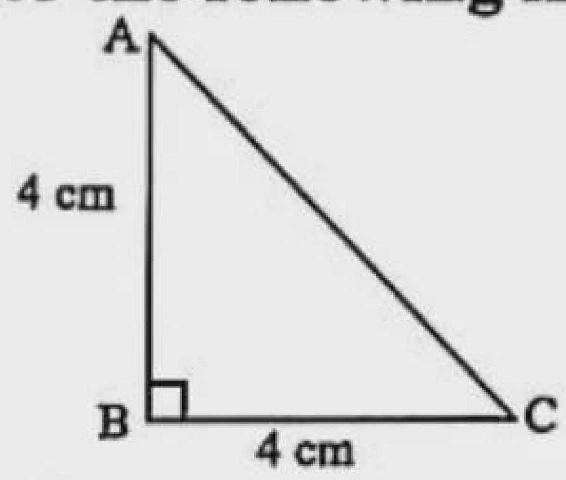


[CB '16]

23. **In the figure, E and F are the middle point of AB and AC. If $\angle AEF = 50°$ then $\angle ABC$ = what?** *(Easy)*
(c) (a) 25° (b) 40° (c) 50° (d) 100°

24. **In figure —.**
i. $EF \parallel BC$
ii. $EF = 2BC$
iii. $\frac{AE}{AB} = \frac{AF}{AC}$
Which one is correct? *(Medium)*
(b) (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

Answer the questions No. 25 and 26 according to the following information :



25. $\angle A$ = **What?** *(Easy)* *[JB' 15]*
(b) (a) 30° (b) 45° (c) 60° (d) 90°

26. **What is the area of triangle ABC in square centimetre?** *(Medium)* *[JB' 15]*
(a) (a) 8 (b) 16 (c) 32 (d) 64

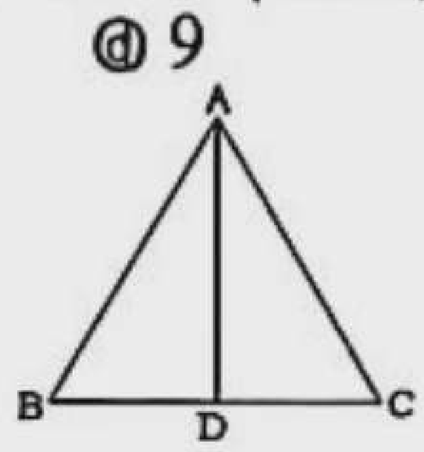
Lesson-9.2 : Pythagoras Theorem ▸ Textbook Page 152

27. **If three sides of a triangle are 5 cm, 12 cm and 13 cm, then what will be its area?** *(Medium)*
(a) 20 sq. cm (b) 30 sq. cm
(b) (c) 60 sq. cm (d) 120 sq. cm

28. **If a = hypotenuse, b = base and c = height of a right triangle, then which of the following is correct according to Pythagoras theorem?** *(Medium)*
(a) $a^2 = b^2 + c^2$ (b) $b^2 = a^2 + c^2$
(a) (c) $c^2 = a^2 + b^2$ (d) None of the above

29. **In right ΔABC, AB = 5, AC = 12, BC = what?** *(Medium)*
(b) (a) 10 (b) 13 (c) 16 (d) 9

30. **In the following figure, ΔABC is an isosceles triangle with AB = AC, D is the mid point of BC, BD = 3 cm, AD = 4 cm. What is the area of ΔABC?** *(Medium)*

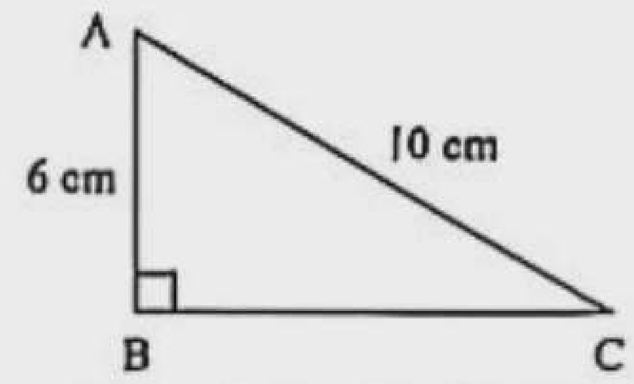


(a) 6 sq. cm (b) 10 sq. cm
(c) (c) 12 sq. cm (d) 24 sq. cm

31. **Who was Pythagoras?** *(Easy)*
(a) A Mathematician (b) A Physicist
(c) (c) A Greek Philosopher (d) A Chemist

32. **When did Pythagoras define Pythagoras theorem?** *(Easy)*
(a) In 6^{th} century B.C. (b) In 5^{th} century B.C.
(a) (c) In 4^{th} century B.C. (d) In 3^{rd} century B.C.

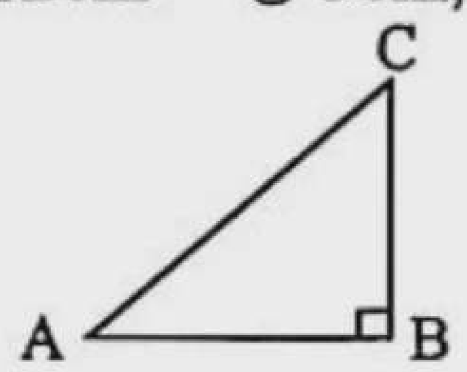
33.



What is the area of the triangle ABC? *(Easy)* *[DB '19]*
(a) (a) 24 sq cm (b) 36 sq cm (c) 48 sq cm (d) 60 sq cm

34. **From which three lengths it is possible to draw a right angled triangle?** *(Medium)* *[RB '19]*
(a) 6cm, 8cm, 9cm (b) 6cm, 7cm, 8cm
(d) (c) 5cm, 11cm, 12cm (d) 5cm, 12cm, 13cm

35.



In the figure, $AB \perp BC$, BC = 3 cm and AC = 5cm. What is the value of AB? *(Hard)* *[DjB '19]*
(b) (a) 3 cm (b) 4 cm (c) 5 cm (d) 6 cm

36. **The hypotenuse of a right angled triangle is 10 metre and one of the other two sides is 6 metre, what will be the length of the another side in metre?** *(Hard)* *[DB '18]*
(d) (a) 136 (b) 64 (c) 60 (d) 8

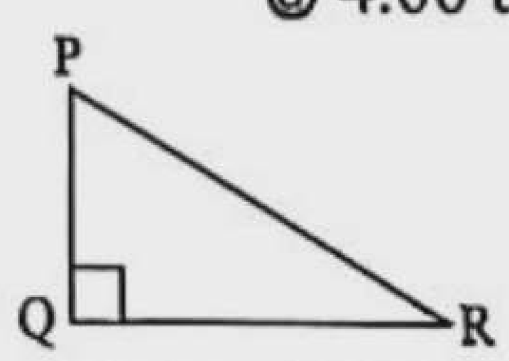
37. **A circle with area 1256 sq metres. What will be the diameter of the circle in centimetre?** *(Hard)* *[DB '18]*
(b) (a) 400 (b) 40 (c) 20 (d) 10

38. **In ΔABC, $\angle B$ = one right angle and AC = 10 cm. What is the sum of the squares of the sides of the triangle in sq cm?** *(Medium)* *[DB '18]*
(c) (a) 24 (b) 100 (c) 200 (d) 480

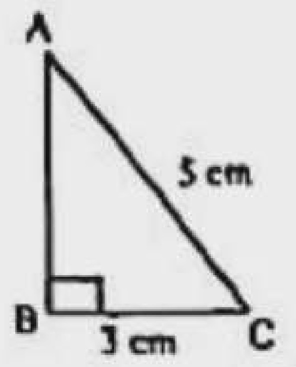
39. **If the length of the perpendicular is 6 cm and the hypotenuse is 9 cm in a right angled triangle, what is the length of the base?** *(Medium)* *[RB '18]*
(a) (a) $3\sqrt{5}$ cm (b) $\sqrt{54}$ cm (c) $4\sqrt{5}$ cm (d) $\sqrt{117}$ cm

40. **What is the length of the diagonal of a square with one side is 1 unit?** *(Hard)* *[BB '18]*
(a) 1.00 unit (b) 1.41 unit
(b) (c) 2.01 unit (d) 4.00 unit

41.

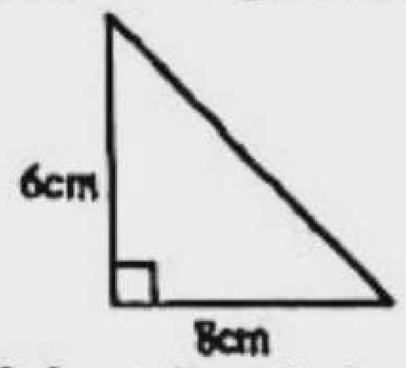


In case of ΔPQR, Which one of the following is correct? *(Hard)* *[DjB '18]*
(a) $PQ^2 = PR^2 + QR^2$ (b) $QR^2 = PR^2 + PQ^2$
(c) (c) $QR^2 = PR^2 - PQ^2$ (d) $PR^2 = PQ^2 - QR^2$

42.


In figure, what is the value of AB? *(Medium)* *[RB '17]*

ⓒ ⓐ 2 cm ⓑ 3 cm ⓒ 4 cm ⓓ 8 cm

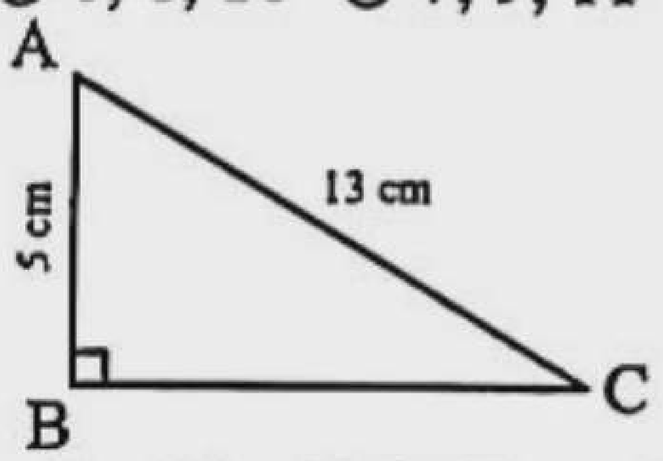
43.


What is the area of the triangle in cm^2? *(Hard)* *[RB '17]*

ⓑ ⓐ 12 ⓑ 24 ⓒ 36 ⓓ 48

44. By which of the following right angled triangle can be drawn? *(Easy)* *[JB '17]*

ⓑ ⓐ 4, 5, 6 ⓑ 6, 8, 10 ⓒ 7, 9, 11 ⓓ 5, 10, 15

45.


What is the length of the side BC in cm? *(Easy)* *[BB '17]*

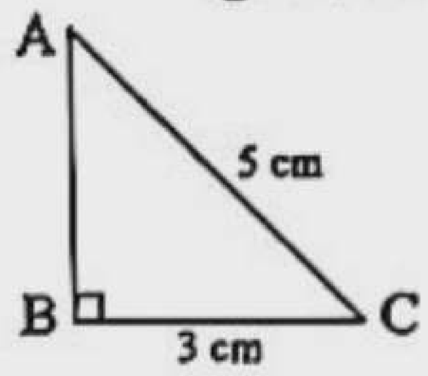
ⓑ ⓐ 8 ⓑ 12 ⓒ 18 ⓓ 144

46. If the difference of the two acute angles of a right angled triangle is 25°, then what is the value of the smallest angle in degree? *(Easy)* *[CtgB '17]*

ⓒ ⓐ 65 ⓑ 57.5 ⓒ 32.5 ⓓ 45

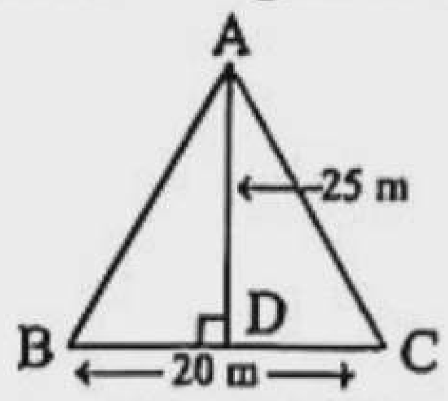
47. If in a ΔABC, $\angle C = 90°$, AB = 13 cm and AC = 12 cm, what is the value of BC in centimeter? *(Hard)* *[CtgB '16]*

ⓑ ⓐ 1 ⓑ 5 ⓒ 17.69 ⓓ 25

48.


What is the value of AB? *(Easy)* *[SB '16]*

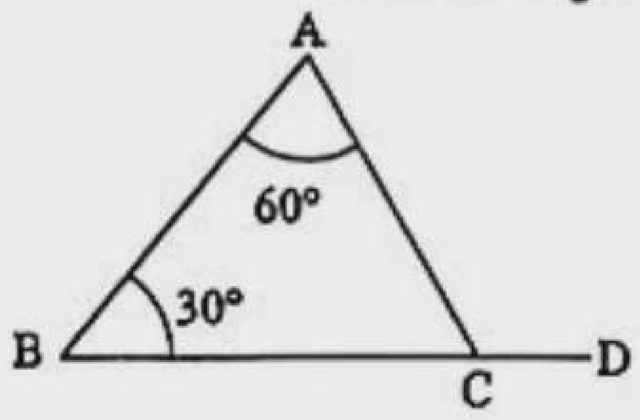
ⓑ ⓐ 2 cm ⓑ 4 cm ⓒ 8 cm ⓓ $\sqrt{34}$ cm

49.


What is the area of ΔABC? *(Medium)* *[SB '16]*

ⓐ 22.5 sq. m ⓑ 45 sq. m

ⓒ ⓒ 250 sq. m ⓓ 500 sq. m

50.


In figure, $\angle ACD$ = ? *(Medium)* *[RB' 15]*

ⓐ ⓐ 90° ⓑ 100° ⓒ 110° ⓓ 120°

51. If the hypotenuse 13 cm and height 12 cm of a right angled triangle, then how many centimetres of its base? *(Hard)* *[RB' 15]*

ⓑ ⓐ 4 ⓑ 5 ⓒ 6 ⓓ 8

52. For a right angled triangle, the difference between two angles is 5°. What is the value of the smallest one? *(Hard)* *[Ideal School & College, Dhaka]*

ⓑ ⓐ 40° ⓑ 42.5° ⓒ 47.5° ⓓ 50°

53. When Pythagoras discovered a special property of right angled triangle? *(Hard)* *[Ideal School & College, Dhaka]*

ⓐ 5th century BC ⓑ 6th century BC

ⓑ ⓒ 7th century BC ⓓ 8th century BC

54. In a right triangle —.

i. one angle is right angle

ii. two angles are acute angles

iii. the hypotenuse is located at the opposite to the right angle

Which one of the following is correct? *(Hard)*

ⓒ ⓐ i & ii ⓑ ii & iii ⓒ i, ii & iii ⓓ i & iii

55. In a right triangle —.

i. $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$

ii. hypotenuse > (base + height)

iii. hypotenuse < (base + height)

Which one of the following is correct? *(Hard)*

ⓑ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

56. In the case of a right triangle —.

i. $\text{Base}^2 = \text{hypotenuse}^2 - \text{height}^2$

ii. Area $= \frac{1}{2} \times$ hypotenuse × base

iii. Hypotenuse is the largest side of the triangle

Which one of the following is correct? *(Medium)*

ⓓ ⓐ i, ii & iii ⓑ i & ii ⓒ ii & iii ⓓ i & iii

57. If in ΔPQR, $\angle R = 90°$ then— *[CB '19]*

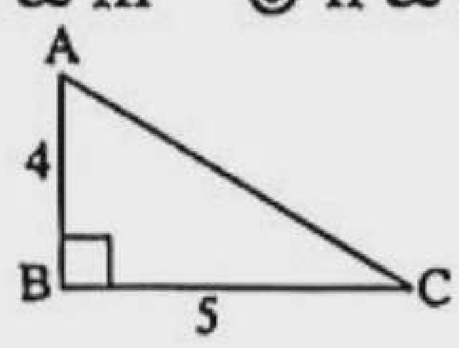
i. the hypotenuse is PQ

ii. the area is $\frac{1}{2} PR \times QR$

iii. $PR^2 = PQ^2 - QR^2$

Which one is correct? *(Medium)*

ⓓ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

58.


In ΔABC —. *[RB '18]*

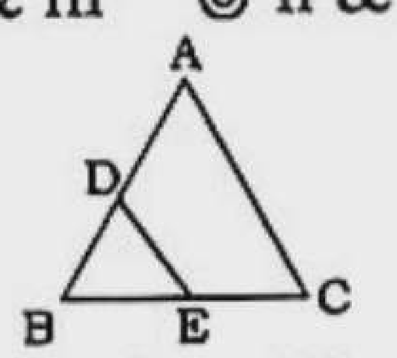
i. Area = 10 sq unit

ii. $AC = \sqrt{41}$ unit

iii. $AB^2 = AC^2 + BC^2$

Which one of the following is correct? *(Medium)*

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

59.


In the fig. D, E are the mid-point of AB and BC respectively then —. *(Hard)* *[JB '18]*

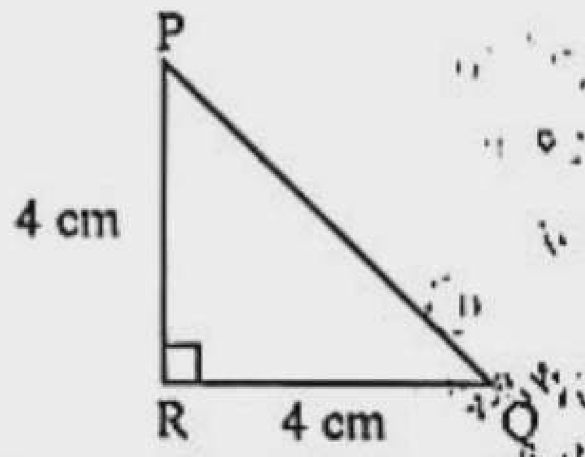
i. DE||AC

ii. $DE = \frac{1}{2} AC$

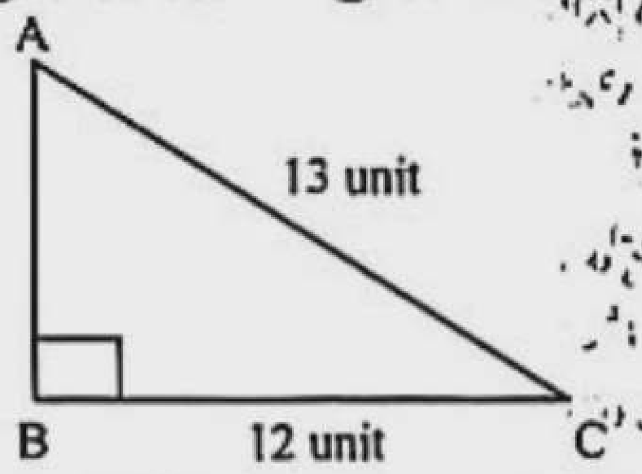
iii. BD = BE

Which one of the following is correct?

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

60. 

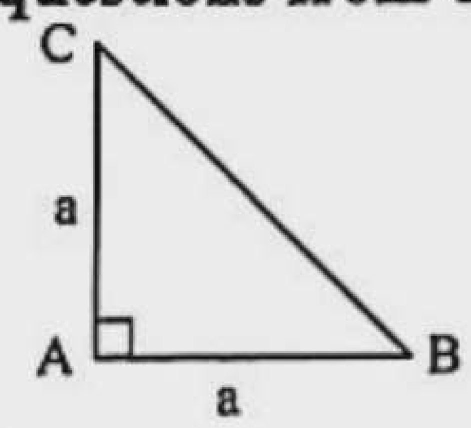
In the above figure —. *[CtgB '18]*
i. $\angle PQR = 45°$
ii. $PQ = 4\sqrt{2}$ cm
iii. The area of ΔPQR is 16 sq cm
Which one of the following is correct? *(Easy)*
a (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

61. 

What is the area of ΔABC in sq unit? *(Medium)* *[SB '18]*
(a) 156 sq unit (b) 78 sq unit
d (c) 60 sq unit (d) 30 sq unit

62. **Observe the following information —** *[DB '16]*
i. Area of parallelogram = base × height
ii. Area of triangle = $\frac{1}{2}$ × base × height
iii. Area of trapezium = base × height
Which one is correct? *(Hard)*
a (a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

■ **Observe the following geometric figure and then answer questions from 63 – 66 based on it.**



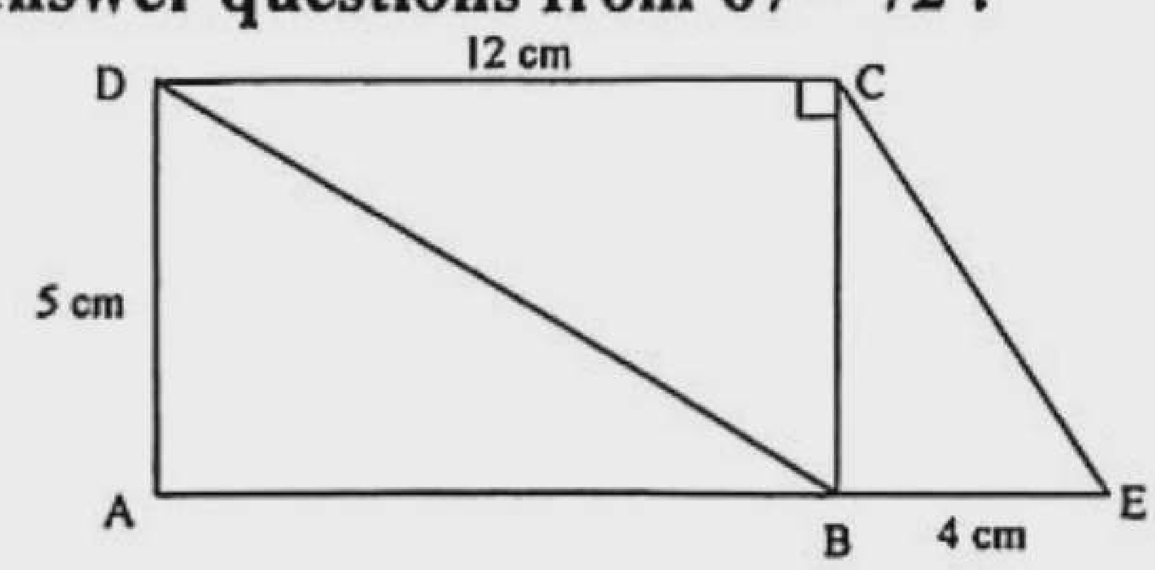
63. **BC = what?** *(Easy)*
a (a) $\sqrt{2}a$ (b) 2a (c) $2\sqrt{a}$ (d) a + a

64. **∠C = what?** *(Medium)*
c (a) 30° (b) 40 (c) 45° (d) 50°

65. **∠A and ∠B can be related as —.** *(Hard)*
(a) ∠A = ∠B (b) $\angle A = \frac{1}{2}\angle B$
c (c) $\frac{1}{2}\angle A = \angle B$ (d) 2∠A = ∠B

66. **If a = 5, what will be the measure of the hypotenuse of ΔABC?** *(Easy)*
c (a) 10 (b) 25 (c) $5\sqrt{2}$ (d) 20

■ **Based on the following geometrical figure, answer questions from 67 – 72 :**



67. **AE = what?** *(Easy)*
a (a) 16 cm (b) 12 cm (c) 20 cm (d) 18 cm

68. **∠ADC + ∠ABC = what?** *(Easy)*
d (a) 90° (b) 145° (c) 160° (d) 180°

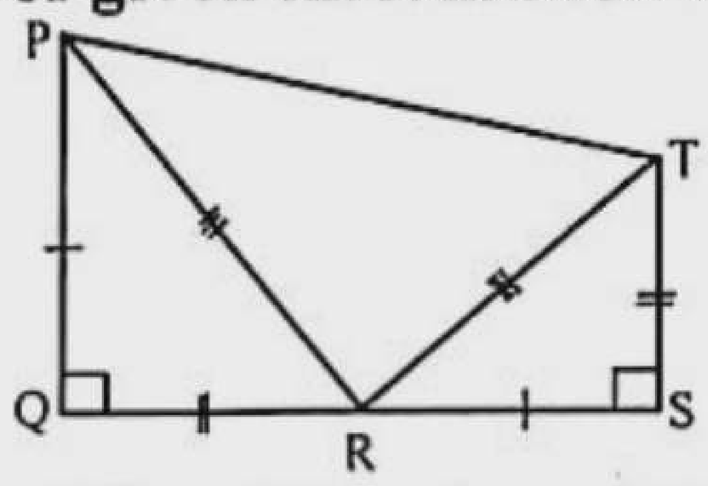
69. **BD = what?** *(Medium)*
c (a) 10 cm (b) 15 cm (c) 13 cm (d) 18 cm

70. **What is the area of the ΔBCE?** *(Medium)*
(a) 10 sq. cm (b) 12 sq. cm
a (c) 8 sq. cm (d) 20 sq. cm

71. **CE = what?** *(Hard)*
c (a) $2\sqrt{5}$ cm (b) $3\sqrt{5}$ cm (c) $\sqrt{41}$ cm (d) 5 cm

72. **What is the area of ΔABD?** *(Hard)*
(a) 25 sq cm (b) 30 sq cm
b (c) 20 sq cm (d) 15 sq cm

■ **Answer to the questions no. 73 and 74 with the help of given information :**

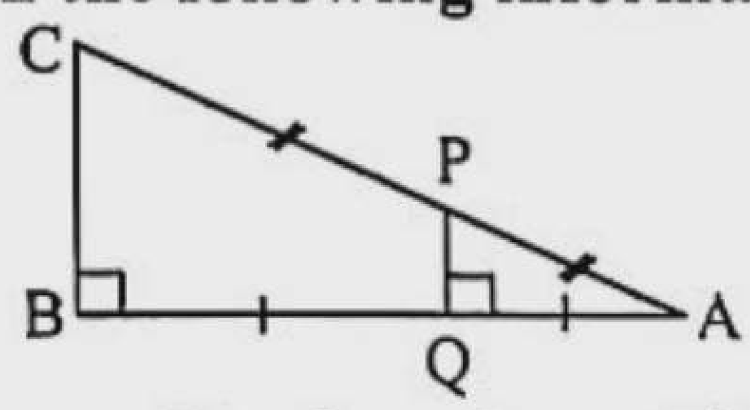


In figure PQ = RS = 4cm, QR = TS = 3cm and PR = RT = 5cm. *[BB '19]*

73. **What is the area of triangle PQR?** *(Easy)*
(a) 5 sq cm (b) 6 sq cm
b (c) 12 sq cm (d) 25 sq cm

74. **What is the area of trapezium PQST?** *(Medium)*
(a) 10.5 sq cm (b) 14 sq cm
c (c) 24.5 sq cm (d) 98 sq cm

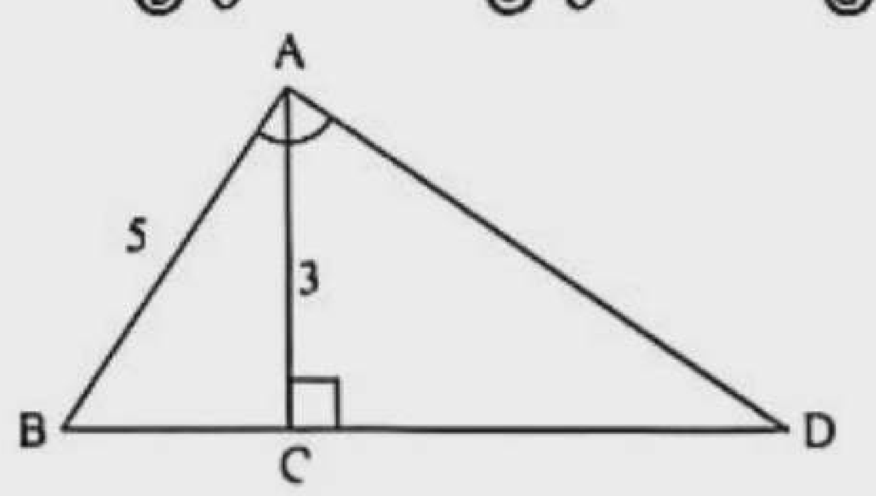
■ **Answer to the questions no. 75 and 76 based on the following information :**



In the figure, AB = 6 cm, Area of ΔAPQ = 6 sq cm. *[DjB '19]*

75. **What is the value of PQ?** *(Hard)*
b (a) 2 (b) 4 (c) 6 (d) 8

76. **What is the value of AC?** *(Hard)*
d (a) 4 (b) 6 (c) 8 (d) 10



Here CD - 2 AB *[MB '19]*

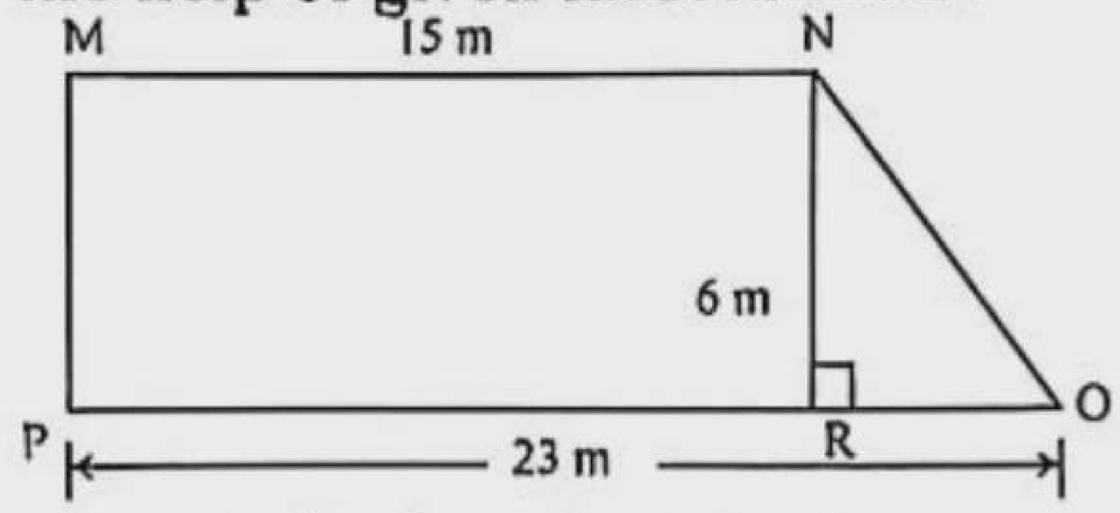
■ **Answer the questions no. 77 and 78 on the basis of the above figure :**

77. **What is the value of BC?** *(Easy)*
d (a) 34 (b) 16 (c) 8 (d) 4

78. **What is the area of ΔACD?** *(Easy)*
b (a) 12 (b) 15 (c) 21 (d) 24

Mathematics

▦ **Answer to the questions no. 79 and 80 with the help of given information : —**



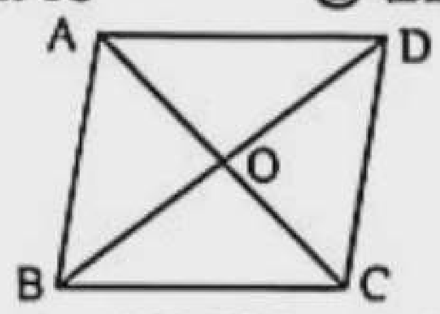
[CtgB '18]

79. What is the length of ON? *(Easy)*

ⓐ 9 metres ⓑ 10 metres
b ⓒ 14 metres ⓓ 17 metres

80. What is the area of MNOP? *(Medium)*

ⓐ 44 sq metres ⓑ 76 sq metres
c ⓒ 114 sq metres ⓓ 228 sq metres



In the rhombus ABCD, AC = 12 metres, BD = 16 metres.

▦ **Answer to the questions No. 81 and 82 on the basis of the above figure :** *[RB '16]*

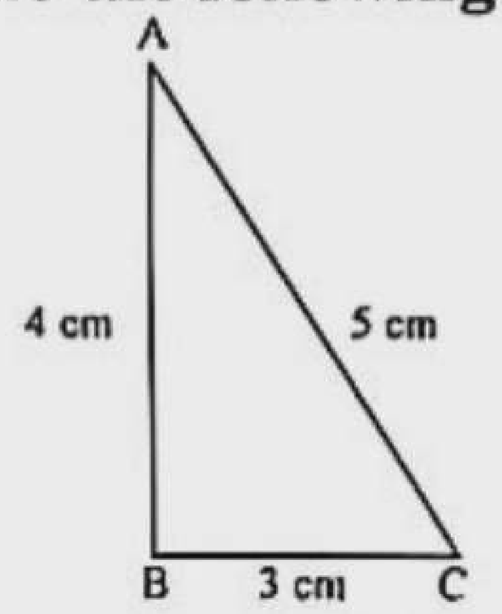
81. What is the area of ΔAOB? *(Hard)*

ⓐ 14 sq. metres ⓑ 24 sq. metres
b ⓒ 48 sq. metres ⓓ 96 sq. metres

82. What is the length of a side of the rhombus? *(Hard)*

ⓐ 6 metres ⓑ $6\sqrt{2}$ metres
c ⓒ 10 metres ⓓ $8\sqrt{2}$ metres

▦ **Answer the questions No. 83 and 84 according to the following information :**



[CB '16]

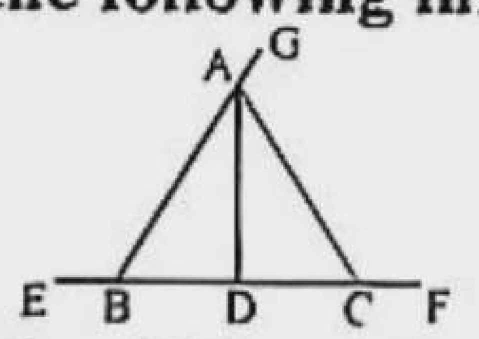
83. What is the value of ∠ABC? *(Easy)*

c ⓐ 45° ⓑ 60° ⓒ 90° ⓓ 120°

84. What is the area of the triangle ABC in sq. cm? *(Medium)*

a ⓐ 6 ⓑ 7.5 ⓒ 12 ⓓ 15

▦ **Answer to the questions No. 85 and 86 in the light of the following information :**



In ΔABC, AB = BC = AC = 6 cm and AD ⊥ BC. *[DjB '16]*

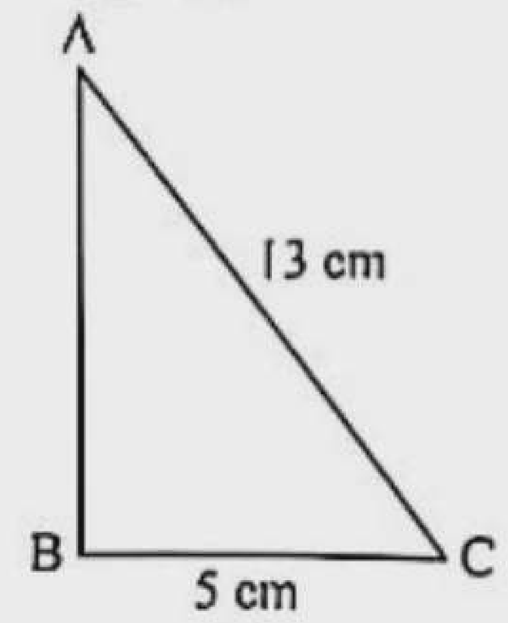
85. What is the length of AD in cm? *(Hard)*

a ⓐ 5.19 ⓑ 6.71 ⓒ 8.49 ⓓ 9.23

86. In which triangle the theorem of Pythagoras is applicable? *(Easy)* *[DB' 15]*

ⓐ Equilateral ⓑ Scalene
c ⓒ Right angled ⓓ Obtuse angled

▦ **Answer question No. 87 and 88 in the light of the following figure —**



87. What is the length of the side AB (in cm)? *(Medium)*

b ⓐ 8 ⓑ 12 ⓒ 18 ⓓ 144

88. What is the area of ΔABC in sq. cm? *(Easy)*

a ⓐ 30 ⓑ 32.5 ⓒ 60 ⓓ 65

Lesson-9.3 : Converse of Pythagoras theorem

▸ Textbook Page 155

89. In ΔABC, ∠A = 90°, AB = 12, AC = 5 then what is the value of BC? *(Hard)*

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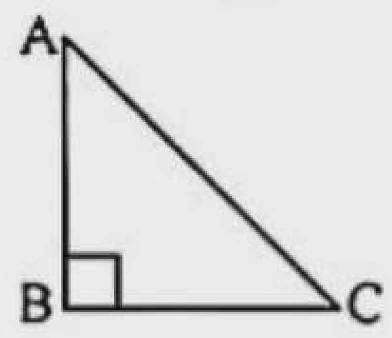
c ⓐ 11 ⓑ 169 ⓒ 13 ⓓ 121

90. In a right angled triangle if the difference between two acute angles is 6° then what is the value of the smallest angle? *(Hard)*

[Viqarunnisa Noon School and College, Dhaka]

d ⓐ 90° ⓑ 60° ⓒ 48° ⓓ 42°

91.



According to the above figure —. *[JB '18]*

i. ∠BAC is complementary to ∠ACB
ii. AC is the greatest side
iii. $BC^2 = AB^2 + AC^2$

Which one of the following is correct? *(Hard)*

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

92. In the right-angled triangle —.

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i. the largest side is called the hypotheses
ii. the largest angle is 90°
iii. the acute angles are complementary to each other

Which one is correct? *(Easy)*

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

93. In ΔPQR, ∠P = 90° then

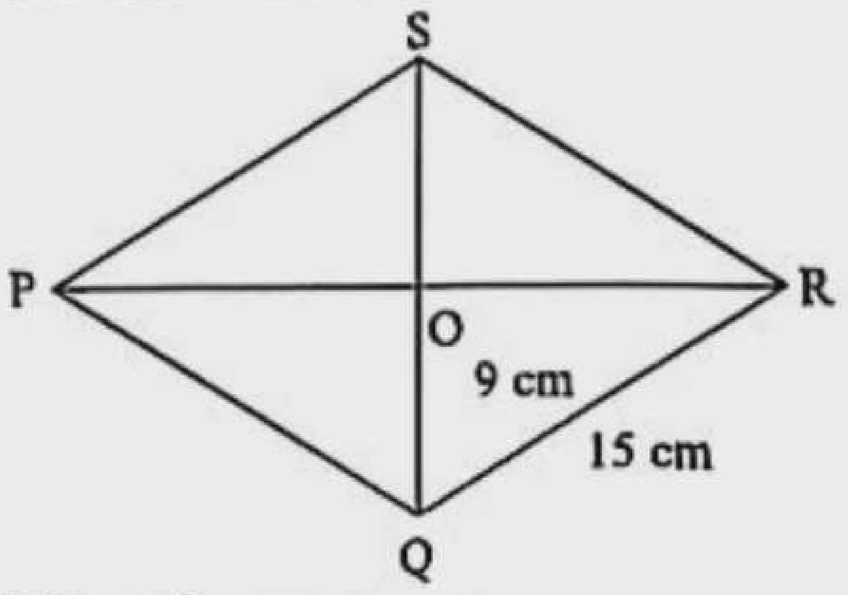
[Viqarunnisa Noon School and College, Dhaka]

i. ∠Q < 90°
ii. $QR^2 = PQ^2 + PR^2$
iii. ∠P = ∠R + ∠Q

Which one is correct? *(Medium)*

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

■ Observe the following rhombus PQRS with QR = 15 cm and OQ = 9 cm where O is the point of intersection of the diagonals PR and QS. Now, **answer questions from 94 – 98 based on the above information :**



94. OR = What? *(Medium)*
ⓐ ⓐ 12 cm ⓑ 10 cm ⓒ 16 cm ⓓ 20 cm

95. What is the area of ΔPQO? *(Hard)*
ⓐ 50 sq. cm ⓑ 52 sq. cm
ⓒ ⓒ 54 sq. cm ⓓ 55 sq. cm

96. What is the area of the rhombus PQRS? *(Hard)*
ⓐ 54 sq. cm ⓑ 108 sq. cm
ⓒ ⓒ 216 sq. cm ⓓ 188 sq. cm

97. PR = what? *(Medium)*
ⓐ 12 cm ⓑ 24 cm
ⓑ ⓒ 16 cm ⓓ 20 cm

98. What is the perimeter of the rhombus PQRS? *(Easy)*
ⓐ 36 cm ⓑ 48 cm
ⓒ ⓒ 60 cm ⓓ 72 cm

Short Q/A Designed as per topic

Lesson-9.1 : Right angled Triangle

▸ Textbook Page 152

Question 1. If the difference between the two acute angles of a right-angled triangle is 5°, what is the value of the greatest angle?

Solution: Let the greatest angle be x.
Then, the other angle is $x - 5°$.
According to the given condition:

$x + (x - 5°) = 90°$
Or, $2x - 5° = 90°$
Or, $2x = 90° + 5°$
Or, $2x = 95°$
$\therefore x = \frac{95°}{2} = 47.5°$

$\therefore$ The greatest angle is 47.5°

Question 2. If the difference between the two acute angles of a right-angled triangle is 15°, what is the value of the smallest angle in degrees?

Solution: Let the greatest angle be x and the smallest angle be y.

$\therefore$ $x - y = 15°$ ....(i)
$x + y = 90°$ ....(ii)
(–) (–) (–)
$-2y = -75°$

$\therefore y = \frac{75°}{-2} = 37.5°$

$\therefore$ The smallest angle is 37.5°.

Question 3. Write down two properties of a right-angled triangle.

Solution: Two properties of a right-angled triangle are as follows:

(i) One angle of a right-angled triangle is a right angle, i.e., 90°.

(ii) The sum of the squares of the two smaller sides is equal to the square of the greatest side, i.e., $(\text{hypotenuse})^2 = (\text{base})^2 + (\text{height})^2$.

Question 4. If the base of a right-angled triangle is 12 cm and the area is 30 square cm, and the perimeter is 30 cm, what are the lengths of the sides in cm?

Solution: Let the base of the right-angled triangle be a cm, the height be b cm, and the hypotenuse be c cm.

$\therefore$ Area $= \frac{1}{2} \times a \times b$

Or, $30 = \frac{1}{2} \times 12 \times b$ [Since base a = 12 cm]

Or, $6b = 30$

Or, $b = \frac{30}{6} = 5$

And Perimeter = a + b + c
Or, $30 = 12 + 5 + c$
$\therefore c = 30 - 17 = 13$
$\therefore$ The lengths of the sides are 5 cm, 12 cm, and 13 cm.

Question 5. If the lengths of the two sides adjacent to the right angle of a right-angled triangle are 6 cm and 8 cm respectively, find the area of the triangle.

Solution: Here, the two sides adjacent to the right angle are 6 cm and 8 cm respectively.

Area of the triangle $= \frac{1}{2} \times 6 \times 8$ square cm

= 24 square cm

$\therefore$ The required area is 24 square cm.

Lesson-9.2 : Pythagoras Theorem

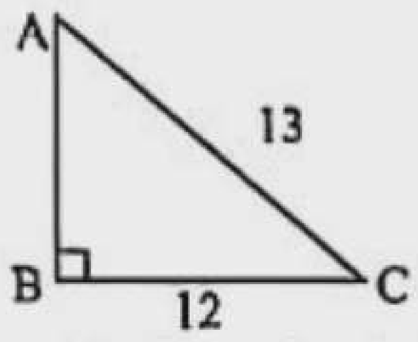
▸ Textbook Page 152

Question 6. State the Pythagoras theorem.

Solution: Pythagoras theorem: In a right-angled triangle, the square on the hypotenuse is equal to the sum of the squares on the two other sides.

Mathematics

Question 7.



What is the area of triangle ABC in square units?

Solution:

In triangle ABC, $\angle B = 90°$.

According to the Pythagoras theorem: $AB^2 = AC^2 - BC^2$

or, $AB^2 = AC^2 - BC^2$

or, $AB^2 = 13^2 - 12^2 = 169 - 144 = 25$

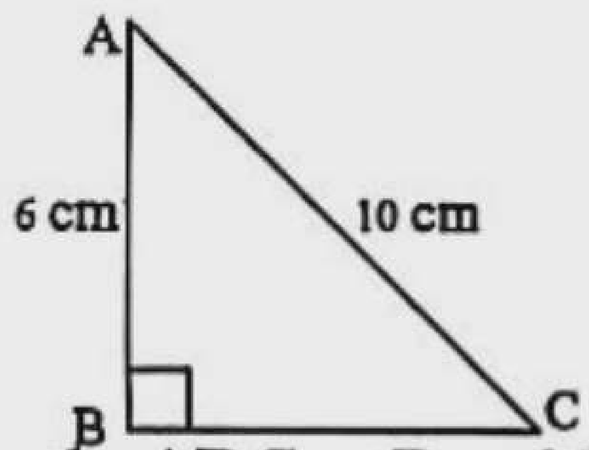
$\therefore AB = \sqrt{25} = 5$ units

Area of triangle ABC = $\frac{1}{2} \times AB \times BC = \frac{1}{2} \times 5 \times 12$

= 30 square units

$\therefore$ The area of triangle ABC is 30 square units.

Question 8. What is the length of side BC in triangle ABC in cm?



Solution: In triangle ABC, $\angle B = 90°$, AB = 6 cm, and AC = 10 cm.

$\therefore AB^2 + BC^2 = AC^2$ [According to the Pythagoras theorem]

Or, $BC^2 = AC^2 - AB^2$

Or, $BC^2 = 10^2 - 6^2$

Or, $BC^2 = 100 - 36$

Or, $BC^2 = 64$

$\therefore BC = \sqrt{64} = 8$

$\therefore$ The length of side BC is 8 cm.

Question 9. If the hypotenuse of a right-angled isosceles triangle is $6\sqrt{2}$ cm, what is the length of its equal sides in cm?

Solution:

Let the length of the equal sides of the right-angled isosceles triangle be x cm.

$(\text{base})^2 + (\text{height})^2 = (\text{hypotenuse})^2$

or, $x^2 + x^2 = (6\sqrt{2})^2$

or, $2x^2 = 72$ or, $x^2 = \frac{72}{2}$

$\therefore x = \sqrt{36} = 6$

$\therefore$ The length of the equal sides is 6 cm.

Question 10. In triangle PQR, $PQ^2 + QR^2 = PR^2$. If PQ = 9 cm and QR = 12 cm, find the value of PR.

Solution: Here, PQ = 9 cm and QR = 12 cm.

In triangle PQR, $PQ^2 + QR^2 = PR^2$

Or, $9^2 + 12^2 = PR^2$

Or, $PR^2 = 81 + 144$

Or, $PR^2 = 225$

$\therefore PR = \sqrt{225} = 15$

$\therefore$ The required value of PR is 15 cm.

Question 11. In triangle ABC, $\angle B = 90°$. If D and E are the midpoints of AB and BC respectively, show that $AC^2 = 4DE^2$.

Solution : Here, in triangle ABC, $\angle B = 90°$ and D and E are the midpoints of AB and BC respectively.

$\therefore AC^2 = AB^2 + BC^2$

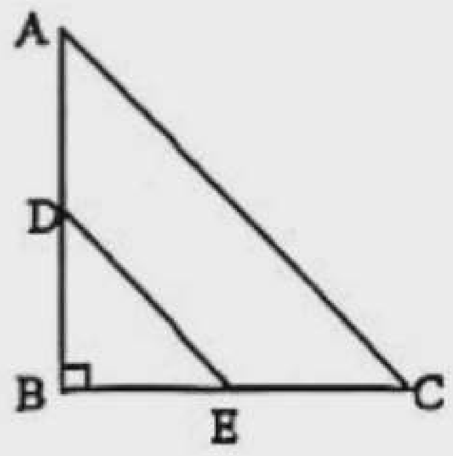
In triangle BDE, $\angle B = 90°$

$\therefore DE^2 = BD^2 + BE^2$

$= \left(\frac{1}{2}AB\right)^2 + \left(\frac{1}{2}BC\right)^2$

$= \frac{1}{4}AB^2 + \frac{1}{4}BC^2$

$= \frac{1}{4}(AB^2 + BC^2) = \frac{1}{4}AC^2$



$\therefore AC^2 = 4DE^2$ **(Showed).**

Question 12. In triangle PQR, $\angle Q = 90°$. If PR = 13 cm and PQ = 12 cm, what is the sum of the squares of the lengths of the sides of the triangle in square cm?

Solution: In triangle PQR, $\angle Q = 90°$.

Here, PR = 13 cm

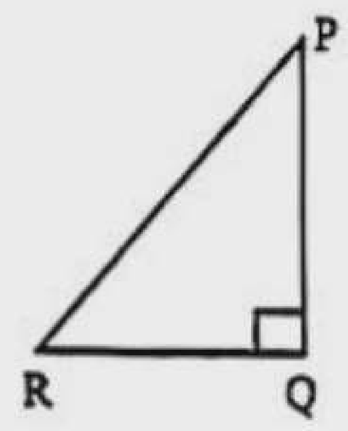
$\therefore \quad PR^2 = PQ^2 + QR^2$ .... (i)

Now, the sum of the squares of the lengths of the sides of triangle PQR is $= PQ^2 + QR^2 + PR^2$

$= PR^2 + PR^2$ [From equation (i)]

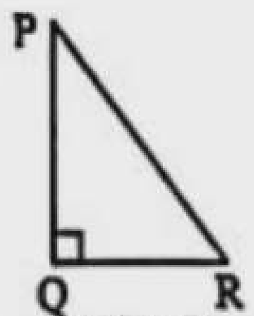
$= 2 \times PR^2$

$= 2 \times 10^2 = 200$ square cm



$\therefore$ The sum of the squares of the lengths of the sides of the triangle is 200 square cm.

Question 13.



In the figure, if PQ = 12 cm and PR = 13 cm, then find the value of OR.

Solution: In the right angled triangle PQR, PQ = 12 cm, PR = 13 cm and $\angle PQR = 90°$

$\therefore PQ^2 + QR^2 = PR^2$

or, $QR^2 = PR^2 - PQ^2$

or, $QR^2 = 13^2 - 12^2$

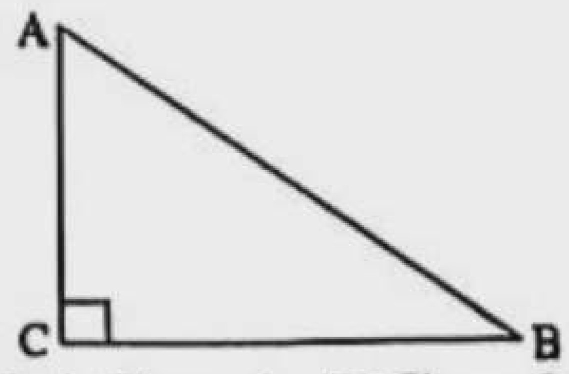
or, $QR^2 = 169 - 144 = 25$

or, $QR = \sqrt{25} = 5$

$\therefore$ QR = 5 cm

$\therefore$ The value of QR is 5 cm.

Question 14.



In the figure, if AC = 1, BC = 2, then find the value of AB.

Solution: Given, AC = 1, BC = 2

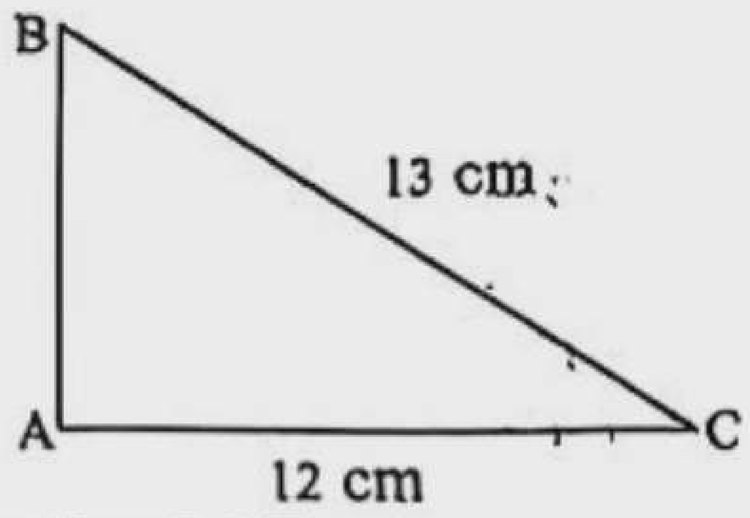
From right angled triangle ACB, we get

$AB^2 = AC^2 + BC^2 = 1^2 + 2^2 = 1 + 4 = 5$

$\therefore AB = \sqrt{5}$ unit

Required value of AB = $\sqrt{5}$ unit.

Question 15.



Find out the length of AB.

Solution : Here, in ΔABC, AC = 12 cm

BC = 13 cm and $\angle BAC$ = Right angle

Here, $BC^2 = AB^2 + AC^2$

or, $AB^2 = BC^2 - AC^2$

or, $AB^2 = 13^2 - 12^2$

or, $AB^2 = 169 - 144$

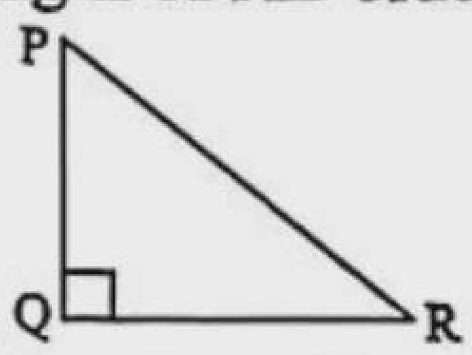
or, $AB^2 = 25$

or, $AB = \sqrt{25} = 5$

$\therefore$ AB = 5 cm

Therefore, the length of AB side is 5 cm.

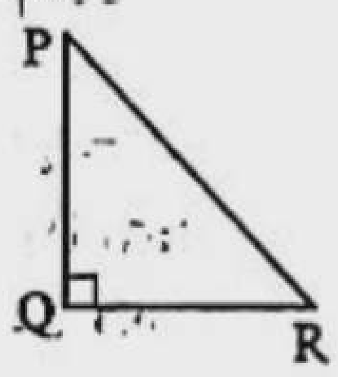
Question 16.



If PR = 5 cm, PQ = 4 cm, then QR = ?

Solution : Given, PR = 5 cm; PQ = 4 cm.

From the right angled triangle PQR, we get–



$PR^2 = PQ^2 + QR^2$

or, $5^2 = 4^2 + QR^2$

or, $25 = 16 + QR^2$

or, $QR^2 = 25 - 16 = 9$

or, $QR = \sqrt{9}$

$\therefore$ QR = 3

Required the value of QR = 3 cm

Lesson-9.3 : Converse of Pythagoras theorem

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Question 17. State the converse of Pythagoras theorem.

Solution:

Converse of Pythagoras theorem: If the square of one side of a triangle is equal to the sum of the squares of the other two sides, then the angle included between the latter two sides will be a right angle.

Question 18. Verify whether a triangle with sides of 8 cm, 15 cm, and 17 cm is a right-angled triangle.

Solution:

Let the sides of triangle ABC be AB = 8 cm, BC = 15 cm, and AC = 17 cm.

Now, $AB^2 + BC^2 = 8^2 + 15^2$

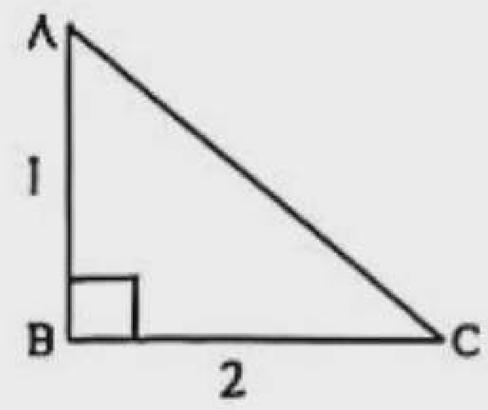
$= 64 + 225$

$= 289 = 17^2 = AC^2$

Since $AB^2 + BC^2 = AC^2$, $\angle B$ is a right angle.

Therefore, a triangle with sides of 8 cm, 15 cm, and 17 cm is a right-angled triangle.

Question 19.



In the figure, if AB = 1 cm and BC = 2 cm, find the value of AC.

Solution: Here, AB = 1 cm, and BC = 2 cm.

Triangle ABC is right-angled, because $\angle B$ = 90 degree,

$\therefore AC^2 = AB^2 + BC^2$

or, $AC^2 = 1^2 + 2^2$

or, $AC^2 = 1 + 4$

or, $AC^2 = 5$

$\therefore AC = \sqrt{5}$ cm = 2.24 cm (approx.)

Question 20. Verify whether it is possible to draw a right-angled triangle with sides of lengths 3 cm, 4 cm, and 5 cm.

Solution: Here, $(3\text{ cm})^2 + (4\text{ cm})^2$

$= 9\text{ cm}^2 + 16\text{ cm}^2$

$= 25\text{ cm}^2 = (5\text{ cm})^2$

Since, $(3\text{ cm})^2 + (4\text{ cm})^2 = (5\text{ cm})^2$

Since $(3\text{ cm})^2 + (4\text{ cm})^2 = (5\text{ cm})^2$, it is possible to draw a right-angled triangle with sides of lengths 3 cm, 4 cm, and 5 cm.

Question 21. In triangle XYZ, XY = 12 cm, YZ = 5 cm, and ZX = 13 cm. Find the value of $\angle Y$.

Solution: Here, $XY^2 = 12^2 = 144$ square cm

$YZ^2 = 5^2 = 25$ square cm

$ZX^2 = 13^2 = 169$ square cm

$\therefore XY^2 + YZ^2 = 144 + 25 = 169 = ZX^2$

Since the square of one side is equal to the sum of the squares of the other two sides, XYZ is a right-angled triangle with hypotenuse ZX.

$\angle Y$, which is opposite to the hypotenuse ZX, is a right angle.

$\therefore \angle Y = 90°$.

Question 22. If in triangle ABC, AB = BC = 2 cm and AC = $2\sqrt{2}$ cm, then find the value of $\angle A$.

Solution:

Here, $AB^2 = BC^2 = 2^2 = 4$ square cm

$AB^2 + BC^2 = 4 + 4 = 8$ square cm

And $AC^2 = (2\sqrt{2})^2 = 8$ square cm

Since $AB^2 + BC^2 = AC^2$, ABC is a right-angled isosceles triangle with hypotenuse AC.

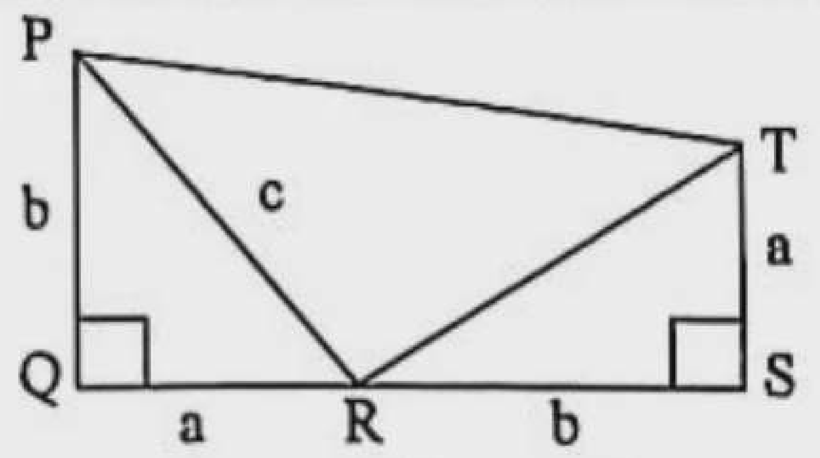
$\angle B$, which is opposite to the hypotenuse AC, is a right angle, i.e., $\angle B = 90°$.

Each of the other two angles of a right-angled isosceles triangle is 45°.

$\therefore \angle A = 45°$.

Creative Q/A — Designed as per learning outcomes

Ques. 01



a. What type of quadrilateral PQST is? Justify your reply. *(Easy)* 2
b. Show that ΔPRT is a right angled triangle. *(Medium)* 4
c. Prove that $PR^2 = PQ^2 + QR^2$. *(Hard)* 4

Solution to Question No. 01 :

a PQST shows a trapezium.
Here $\angle PQS = 90° = \angle TSQ$
So, $PQ \perp QS$ and $ST \perp QS$. That is $PQ \parallel ST$.
Again, PQ = b and opposite side ST = a.
So, QS and PT are non-parallel.
So, PQST is a trapezium.

b From ΔPQR, we have
$\angle PRQ = 90° - \angle QPR$ .... (i)
Again, comparing ΔPQR and ΔRST, we have, PQ = RS, QR = ST and $\angle PQR = 90° = \angle RST$.
$\therefore \Delta PQR \cong \Delta RST$.
So, $\angle QPR = \angle TRS$ .... (ii)
From figure, $\angle QRS$ is a straight angle at R and its measure equal to 180°.
$\therefore \quad \angle QRS = \angle PRQ + \angle PRT + \angle TRS$.
Or, $180° = 90° - \cancel{\angle QPR} + \angle PRT + \cancel{\angle QPR}$ [from (i) and ii.]
Or, $180° - 90° = \angle PRT$
Or, $\angle PRT = 90°$
$\therefore \angle PRT$ is a right angle. (Showed)

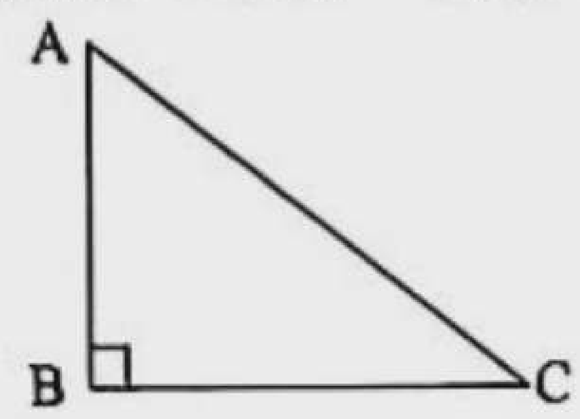
c Let in the triangle PQR, $\angle Q = 90°$, hypotenuse PR = c, PQ = b and QR = c.
t is required to prove that $PR^2 = PQ^2 + QR^2$, i.e. $c^2 = b^2 + a^2$.
Construction : Produce QR upto S such that RS = PQ = b. Also a perpendicular ST at S on QR produced is drawn, so that ST = QR =a. R, T and P, T are joined.
Proof :

Steps	Justification
(1) In ΔPQR and ΔRST, PQ = RS = b, QR = ST = a and included $\angle PQR$ = included $\angle RST$	[each triangle is a right angle]
Hence, $\Delta PQR \cong \Delta RST$. $\therefore$ PR = RT = c and $\angle QPR = \angle TRS$.	[SAS theorem]
(2) Again, since $PQ \perp QS$ and $TS \perp QS$ $\therefore PQ \parallel TS$. Therefore, PQST is a trapezium.	$\therefore \angle QPR = \angle TRS$.
(3) Moreover, $\angle PRQ + \angle QPR = \angle SRT + \angle TRS$ = 1 right angle $\therefore \angle PRT$ = 1 right angle	
Now area of the trapezium PQST = area of (Δ region PQR + Δ region RST + Δ region PRT)	[Area of trapezium = $\frac{1}{2}$ sum of parallel sides × distance between parallel sides]
or, $\frac{1}{2} QS(PQ + ST) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$	
or, $\frac{1}{2}(QR + RS)(PQ + ST) = \frac{1}{2}[2ab + c^2]$	
or, $(a + b)(a + b) = 2ab + c^2$ [multiplying by 2]	
or, $a^2 + 2ab + b^2 = 2ab + c^2$	
or, $a^2 + b^2 = c^2$.	
That is, $PQ^2 + QR^2 = PR^2$ **(Proved)**	

Ques. 02 In triangle ABC, we have, $AB^2 + BC^2 = AC^2$.
a. Draw the triangle with appropriate notations and introduction. *(Easy)* 2
b. Show that $\angle B$ = 1 right angle where $AB^2 + BC^2 = AC^2$ in ΔABC. *(Medium)* 4
c. If D is the mid-point of AB, then prove that $AC^2 + BD^2 = CD^2 + AB^2$. *(Hard)* 4

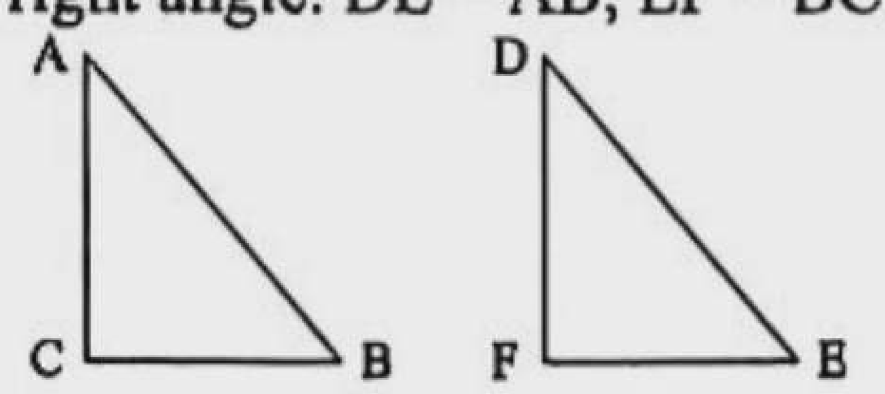
Solution to Question No. 02 :

a According to the stem, let us draw below ΔABC such that $AB^2 + BC^2 = AC^2$:



Here, $\angle B = 90°$, BC = base, AB = height and AC = hypotenuse of the ΔABC.

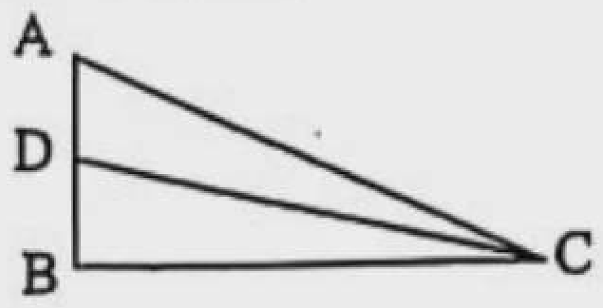
b In ΔABC, we have $AB^2 + BC^2 = AC^2$. We have to show that $\angle B$ = 1 right angle.
Construction : Let us draw a triangle DEF such that $\angle E$ = 1 right angle. DE = AB, EF = BC, DF = AC.



Proof :

Step	Justification
1. In ΔDEF, $DF^2 = DE^2 + EF^2$ $= AB^2 + BC^2$ $\therefore DF = AC$.	[Since ΔDEF is a right triangle and $DE = AB$, $EF = BC$ as per hypothesis]
2. Now comparing ΔDEF and ΔABC, we have, $AC = DF$, $AB = DE$ and $BC = EF$. $\therefore \Delta ABC$ and ΔDEF are congruent i.e. $\Delta ABC \cong \Delta DEF$. $\therefore \angle B = \angle E$. But $\angle E = 90°$ i.e. 1 right angle. So, $\angle B = 90°$ or 1 right angle. **(Shown)**	[Since the corresponding sides of two triangles are mutually equal.]

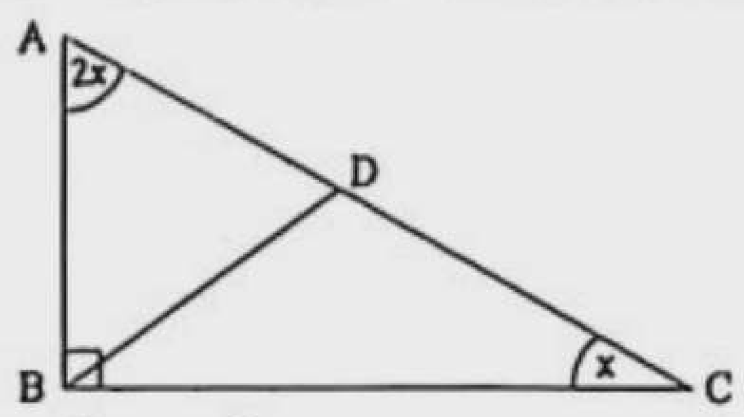
c Here, D is the mid-point of AB and from (b), we have $\angle B = 90°$. Now we have to prove that $AC^2 + BD^2 = CD^2 + AB^2$.



Proof :

Step	Justification
1. According to (b) above, we have $\angle B = 90°$, AC = hypotenuse of ΔABC. $\therefore AC^2 = AB^2 + BC^2$.	[Pythagoras them]
2. Again, ΔBCD is a right triangle with $\angle B = 90°$. $\therefore CD^2 = BD^2 + BC^2$ or, $BD^2 = CD^2 - BC^2$.	[Same reason]
3. From 1 and 2, we have, $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 - BC^2$ $= AB^2 + CD^2$ $\therefore AC^2 + BD^2 = AB^2 + CD^2$. **(Proved)**	

Ques. 03 In the triangle ABC shown below, $\angle B = 90°$, $\angle BCA = x$, $\angle BAC = 2x$ and $AB = AD$.



a. Find the value of x. *(Easy)* 2

b. Show that ΔABD is an equilateral triangle. *(Medium)* 4

c. If $AB = x$ unit and $BC = 2AB$, then show that $CD = (\sqrt{5} - 1)x$ unit. *(Hard)* 4

Solution to Question No. 03 :

a We have ΔABC is a right triangle with $\angle B = 90°$ and $\angle C = x$, $\angle A = 2x$.

We known that the sum of the measures of 3 angles of any triangle is 180°.

$\therefore \angle A + \angle B + \angle C = 180°$

or, $2x + x + 90° = 180°$

or, $3x = 180° - 90°$

or, $3x = 90°$

or, $x = 30°$.

$\therefore$ The value of x is 30°.

b According to the stem, $AB = AD$ in ΔABD.

$\therefore \Delta ABD$ is an isosceles triangle.

Again, from (a), we have,

$\angle A = 2x = 2 \times 30° = 60°$.

$\therefore \angle ADB = 60°$

Since $\angle BAD = \angle ADB$

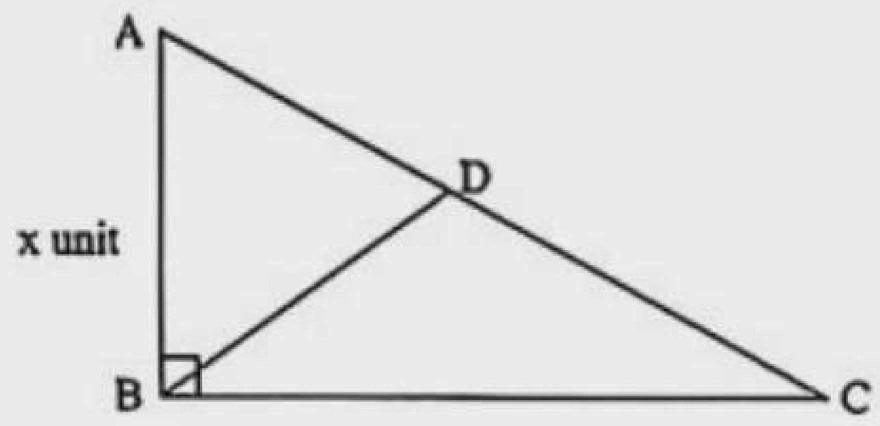
$\therefore \angle ABD = 180° - (60° + 60°) = 60°$

So, $\angle ABD = \angle ADB = \angle BAC = 60°$

Therefore, each of the three angles of ΔABD is 60°.

Thus ΔABD is an equilateral triangle. **(Shown)**

c



Here $AB = x$ unit and

$BC = 2\ AB = 2x$ unit.

Since ΔABC is a right angled triangle with $\angle B = 90°$, hence

$AC^2 = AB^2 + BC^2$.

or, $AC^2 = x^2 + (2x)^2 = x^2 + 4x^2 = 5x^2$.

or, $AC = \sqrt{5x^2} = \sqrt{5}\,x$.

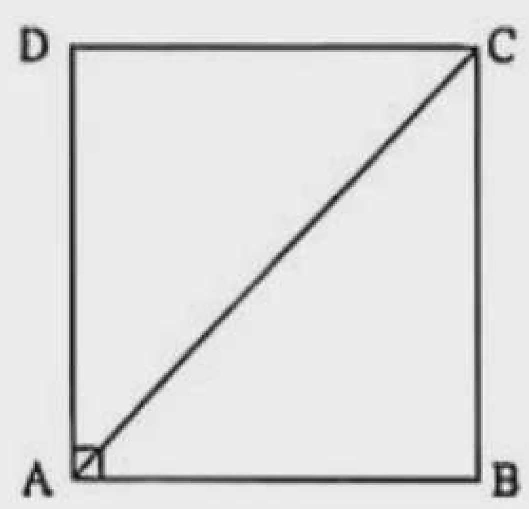
Again, $DC = AC - AD$

$= \sqrt{5}\,x - x$, since $AD = AB = x$

$= (\sqrt{5} - 1)x$

That is, $DC = (\sqrt{5} - 1)x$ unit. **(Proved)**

Ques. 04 The following geometric figure refers to a square :



a. Express AC in terms of AB and AD. *(Easy)* 2

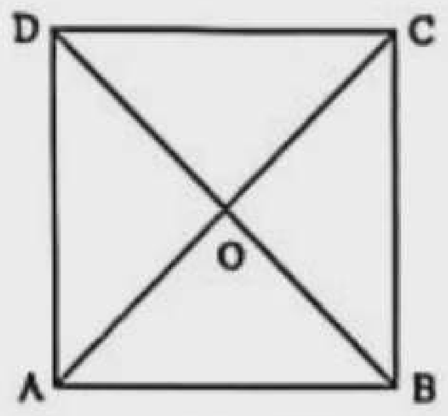
b. Show that AC = BD and the bisectors of AC and BD are mutually equal. *(Medium)* 4

c. Prove that $AB^2 = \frac{1}{2} AC^2$. *(Hard)* 4

Solution to Question No. 04 :

a We have $\angle B = 90°$ in ΔABC.
$\therefore AC^2 = AB^2 + BC^2$
$= AB^2 + AD^2$, since ABCD is a square and $BC = AD$.
So, $AC^2 = AB^2 + AD^2$.

b Let us suppose, the diagonals AC and BD of the square ABCD

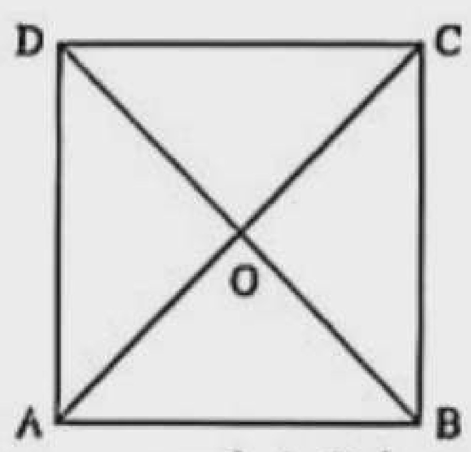


mutually intersect at O. Now, we have to show that $OA = OB = OC = OD$ and $AC = BD$.

Proof :

Step	Justification
1. A square is a parallelogram. $\therefore AB = CD$	[Since opposite sides of parallelogram are equal.]
2. In ΔABD and ΔACD, $\angle DAB = \angle ADC$, $AB = CD$ and AD is common to both the triangles. $\therefore \Delta ABD \cong \Delta ACD$. So, $BD = AC$	[Since they are angles of a square.] [$\because$ They are sides of a square]
3. Now, in ΔOAD and ΔOAB, $AB = AD$ $AO = AO$ and, $\angle OAB = \angle OAD$ $\therefore \Delta OAD \cong \Delta OAD$ $\therefore OD = OB$	[Common side] [Diagonal of a square bisect the angles] [SAS theorem]
4. Since, $BD = AC$ or, $\frac{1}{2}BD = \frac{1}{2}AC$ or, $OB = OA$ So, $OA = OB = OC = OD$. (Showed)	

c

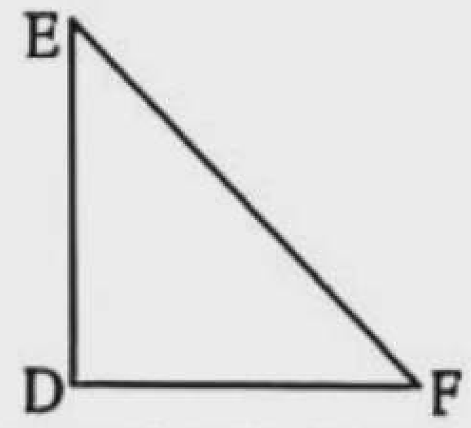


Here ABCD is a square and AC is one of the diagonals of ABCD. We have to prove that $AB^2 = \frac{1}{2}AC^2$.

Proof :

Step	Justification
1. ΔABC is a right triangle with $\angle B = 90°$.	[Since each of the four angles of a square is a right angle.]
2. From right triangle ABC, $AC^2 = AB^2 + BC^2$ $= AB^2 + AB^2$ $= 2AB^2$. or, $2AB^2 = AC^2$ or, $AB^2 = \frac{1}{2}AC^2$. **(Proved)**	[Following theory of Pythagoras.] [Since sides of a square are equal to each other.]

Ques. 05 In the figure $EF^2 = DE + DF^2$ in ΔDEF.

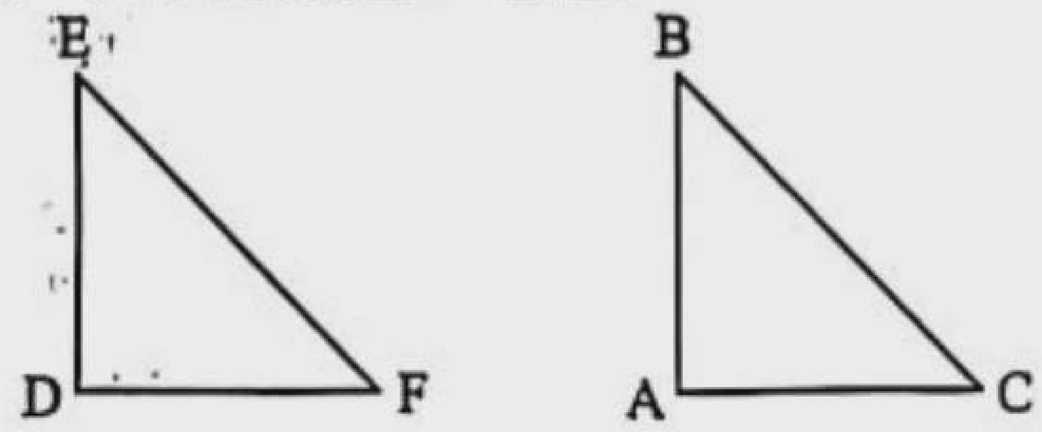


a. If the edge of a cube is 5.5cm, find the area of the entire faces of the cube. *(Easy)* 2
b. Prove that, $\angle D = 90°$. *(Medium)* 4
c. If P and Q are the mid points of DE and EF respectively, prove that, $PQ = \frac{1}{2}DF$. *(Hard)* 4

Solution to Question No. 05 :

a We know that, if the side of a cube is 'a' unit then area of the whole faces of the cube $= 6a^2$ sq. unit.
Here, the edge of a cube is 5.5 cm
$\therefore$ Area of the entire faces of the cube $= 6.(5.5)^2$ sq. cm.
$= 181.5$ sq.cm.

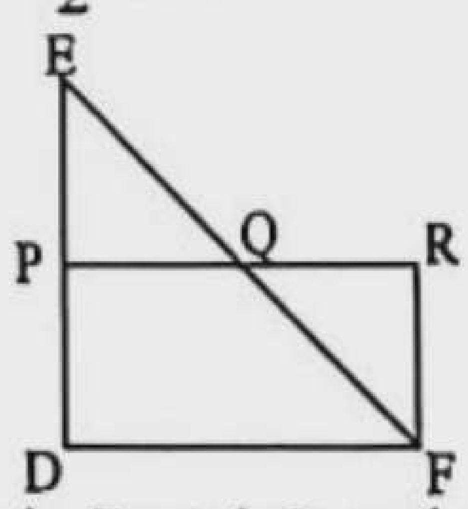
b Let in ΔDEF, $EF^2 = DE^2 + DF^2$. It is required to prove that, $\angle D = 90°$.
Construction : Draw a triangle ABC so that $\angle A = 90°$, $AC = DF$ and $AB = DE$.



Proof :

Steps	Justification
(i) $BC^2 = AB^2 + AC^2$ $= DE^2 + DF^2$ $= EF^2$ $\therefore BC = EF$	[Since in ΔABC, $\angle A = 90°$] [Supposition]
Now, in ΔDEF and ΔABC, $AC = DF$, $AB = DE$ and $BC = EF$. $\therefore \Delta DEF \cong \Delta ABC$; $\therefore \angle D = \angle A$ $\because \angle A = 90°$ $\therefore \angle D = 90°$ **[Proved]**	[SSS Theorem]

c Let, DEF be a triangle. P and Q are the midpoints of DE and EF respectively. It is required to prove that, $PQ = \frac{1}{2}DF$.

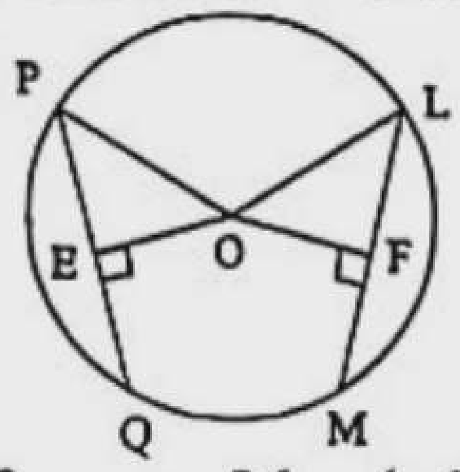


Construction : Join P and Q and extend to R so that $PQ = QR$. Draw a straight line segment from F to R.
Proof : step (i) Between ΔEPQ and ΔFRQ,
$EQ = QF$ [Q is the midpoint of EF]
$PQ = QR$ [By construction]
and $\angle EQP = \angle FQR$ [vertically opposite angle]

Mathematics

$\therefore$ $\Delta EPQ \cong \Delta FRQ$ [SAS theorem]
$\therefore$ $\angle EPQ = \angle QRF$ and $\angle PEQ = \angle QFR$ [Alternate angle]
$\therefore$ $EP \parallel FR$ or, $ED \parallel FR$
Again, $DP = PE = FR$ and $DP \parallel FR$
There fore, DPRF is a parallelogram.
$\therefore$ $PR \parallel DF$ or, $PQ \parallel DF$.
step (ii) Again, $PR = DF$ or, $PQ + QR = DF$
or, $PQ + PQ = DF$ [$\because PQ = QR$]
or, $2PQ = DF$
$\therefore PQ = \frac{1}{2}DF$. **[Proved]**

Ques. 06 PQ and LM are two chords of circle PQML with centre O and OP = 3 cm.



a. Find the circumference of the circle. [$\pi = 3.14$] *(Easy)* 2
b. If QE = OF, then prove that, PQ = LM. *(Medium)* 4
c. From the ΔOLF, prove that, $OL^2 = OF^2 + LF^2$. *(Hard)* 4

Solution to Question No. 06 :

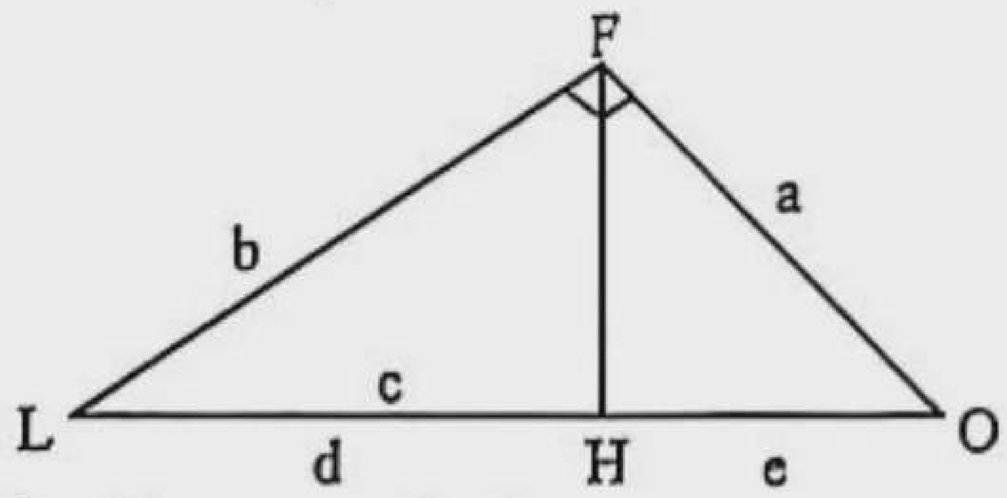
a Here, radius OP = 3 cm
We know that, circumference of a circle having radius $r = 2\pi r$ unit
$\therefore$ Circumference of the circle $= (2 \times \pi \times 3)$ cm
$= (2 \times 3.14 \times 3)$ cm
$= 18.84$ cm.

b Let, PQ and LM be two chords of a circle with centre O. OE and OF are the perpendiculars from O to the chords PQ and LM respectively. Then OE and OF represent the distances from centre to the chords PQ and LM respectively.
If OE = OF, it is to be proved that, PQ = LM.

Construction : Join O, P and O, L.
Proof :

Steps	Justification
1. Since $OE \perp PQ$ and $OF \perp LM$. Therefore, $\angle OEP = \angle OFL$ = 1 right angle	[right angles]
2. Now, between the right-angled ΔOPE and ΔOLF hypotenuse OP = hypotenuse OL and OE = OF	[radius of same circle] [supposition]
$\therefore$ $\Delta OPE \cong \Delta OLF$	[RHS theorem]
$\therefore$ PE = LF.	
3. $PE = \frac{1}{2}PQ$ and $LF = \frac{1}{2}LM$	[Perpendicular from the centre bisects the chord]
4. Therefore, $\frac{1}{2}PQ = \frac{1}{2}LM$ i.e., PQ = LM **(Proved)**	

c Let, in the triangle OLF, $\angle F = 90°$ and hypotenuse OL = c, OF = a and LF = b.



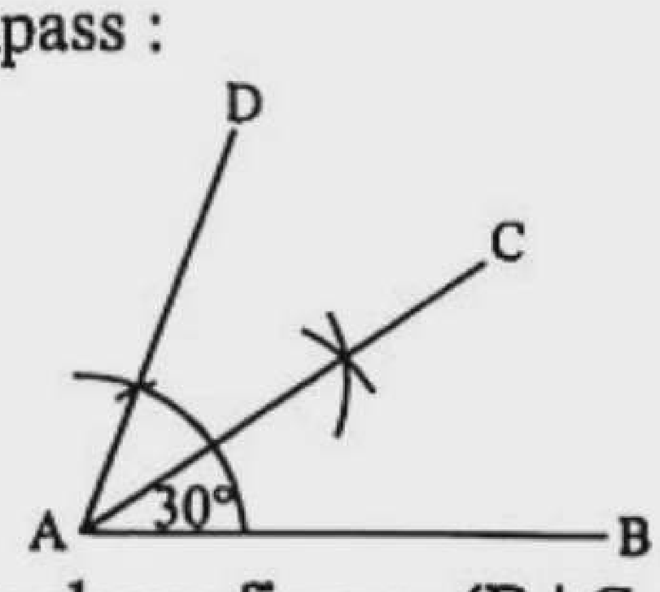
It is required to prove that,
$OL^2 = OF^2 + LF^2$.
Construction : Draw a perpendicular FH from F on hypotenuse OL. The hypotenuse OL is divided at H into the parts of d and e.
Proof :

Steps	Justification
In ΔOFH and ΔOLF, $\angle OHF = \angle OFL$ and $\angle FOH = \angle LOF$ $\therefore$ ΔOFH and ΔOLF are similar. $\therefore \frac{OF}{OL} = \frac{OH}{OF}$	[(i) Both triangles are right angled, (ii) angle $\angle O$ is common]
$\therefore \frac{a}{c} = \frac{e}{a}$ .... (1)	$\because c = e + d$
(ii) similary, ΔLFH and ΔOLF are similar, $\therefore \frac{b}{c} = \frac{d}{b}$ .... (2)	
(iii) From the above ratios, we get, $a^2 = c \times e$, $b^2 = c \times d$ Therefore, $a^2 + b^2 = c \times e + c \times d$ $= c(e + d)$ $= c \times c$ $= c^2$	
$\therefore c^2 = a^2 + b^2$ So, $OL^2 = OF^2 + LF^2$ **[Proved]**	

Ques. 07 In ΔPQR, $PQ > PR$ and $PD \perp QR$.
a. Draw an angle 30° with the help of scale and campus. *(Easy)* 2
b. Prove that, $PQ^2 = PD^2 + QD^2$. *(Medium)* 4
c. If M is any point on PD, prove that, $QM^2 - RM^2 = PQ^2 - PR^2$. *(Hard)* 4

Solution to Question No. 07 :

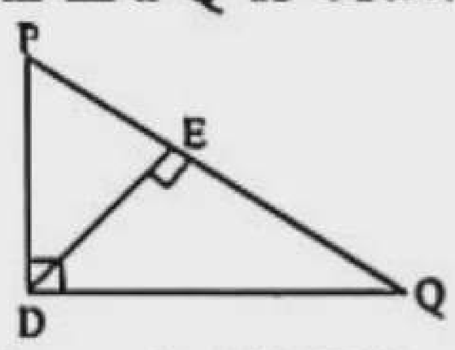
a An angle 30° is drawn below with the help of scale and compass :



In the above figure $\angle BAC = 30°$.

b Let, $\triangle PDQ$ is a right angled triangle where $\angle PDQ = 90°$. Now, it is required to prove that, $PQ^2 = PD^2 + QD^2$.

Construction : $DE \perp PQ$ is drawn.



Proof : In $\triangle PDQ$ and $\triangle DEQ$, $\angle PDQ = \angle DEQ$, $\angle PQD = \angle EQD$ $\therefore \angle DPQ = \angle EDQ$.

So, both triangles are similar.

$\therefore \frac{QD}{PQ} = \frac{EQ}{QD}$

or, $QD^2 = PQ.\ EQ$ .... (i)

Again, in $\triangle PDQ$ and $\triangle DPE$, $\angle PDQ = \angle DEP$, $\angle DPQ = \angle DPE$

$\therefore \angle PQD = \angle PDE$

So, both triangles are similar.

$\therefore \frac{PD}{PQ} = \frac{PE}{PD}$

or, $PD^2 = PQ.\ PE$ .... (ii)

Now, adding equations (i) and (ii), we get,

$QD^2 + PD^2 = PQ.\ EQ + PQ.\ PE$

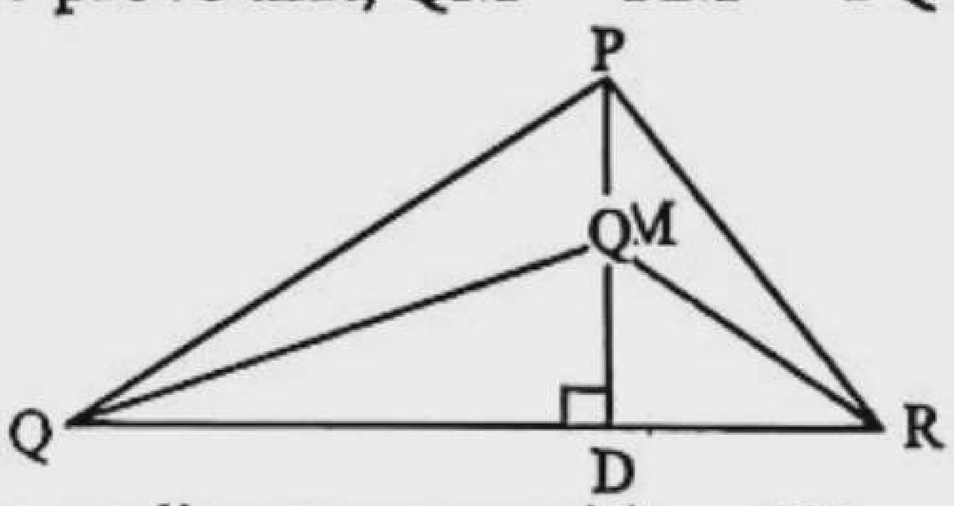
or, $QD^2 + PD^2 = PQ(EQ + PE)$

or, $QD^2 + PD^2 = PQ.\ PQ$ [$\because PQ = PE + EQ$]

or, $QD^2 + PD^2 = PQ^2$

$\therefore PQ^2 = PD^2 + QD^2$ **(Proved)**

c Let, PQR is any triangle where $PQ > PR$, $PD \perp QR$ and M is any point on PD. Q, M and R, M are joined We have to prove that, $QM^2 - RM^2 = PQ^2 - PR^2$.



Proof : According to proposition, PD and MD are perpendicular to QR.

So, $\angle MDQ$ as well as $\angle MDR$ is right angle.

$\therefore \angle MDQ = 90° = \angle MDR$

Now, in $\triangle PQD$ applying Pythagoras's theorem, we get $PQ^2 = PD^2 + QD^2$ .... (i)

Againg, in $\triangle PRD$ applying Pythagoras's theorem, we get,

$PR^2 = PD^2 + RD^2$ .... (ii)

Subtracting equation (ii) from equation (i), we get,

$PQ^2 - PR^2 = PD^2 + QD^2 - PD^2 - RD^2$

$= QD^2 - RD^2$

$= (QM^2 - MD^2) - (RM^2 - MD^2)$

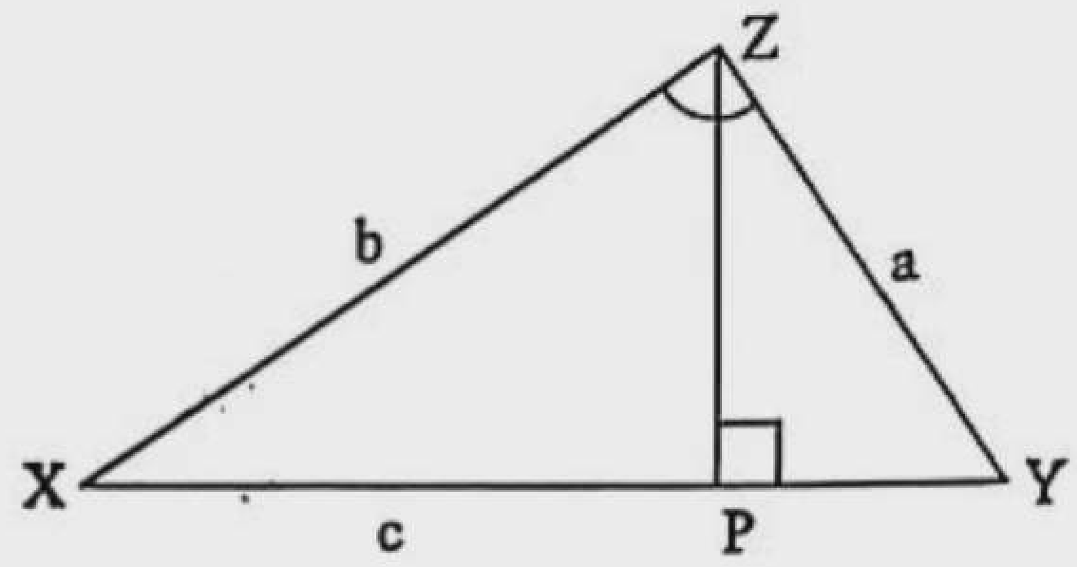
[since from $\triangle MQD$, $QM^2 = MD^2 + QD^2$ and from $\triangle MRD$, $RM^2 = MD^2 + RD^2$]

$= QM^2 - MD^2 - RM^2 + MD^2$

$= QM^2 - RM^2$

$\therefore QM^2 - RM^2 = PQ^2 - PR^2$ **[Proved]**

Ques. 08 In the figure, $XZ = b$, $YZ = a$, $XY = c$ and $XZ > YZ$.



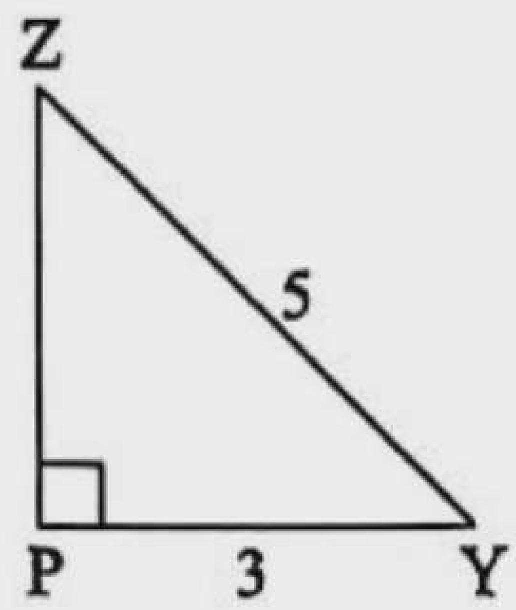
a. In the figure if PY = 3cm, ZY = 5cm, then find the value of ZP. *(Easy)* 2

b. Prove that $a^2 + b^2 = c^2$ from the given figure in the stem. *(Medium)* 4

c. 'O' is any point on ZP. Prove that, $ZX^2 - YZ^2 = XO^2 - YO^2$. *(Hard)* 4

Solution to Question No. 08 :

a From the figure we have, PY = 3 cm and ZY = 5 cm.



Now applying Pythagoras's theorem in the $\triangle PYZ$ we get,

$ZY^2 = PZ^2 + PY^2$

or, $5^2 = PZ^2 + 3^2$

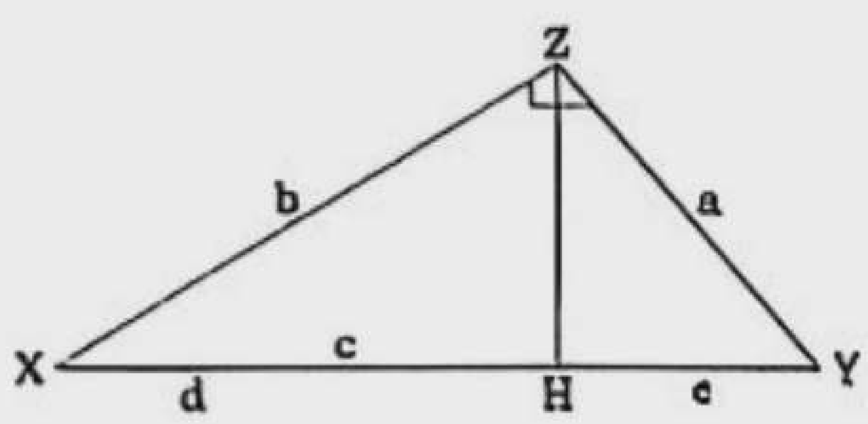
or, $PZ^2 = 25 - 9$

or, $PZ^2 = 16$

$\therefore ZP = 4$

$\therefore$ Determined value of ZP is 4 cm.

b



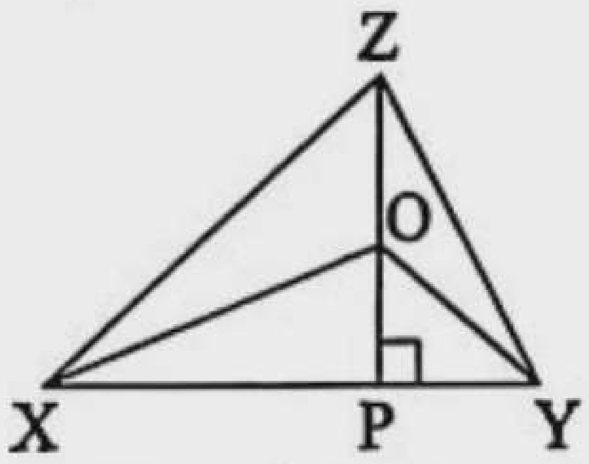
Proposition : Let in the triangle XYZ, $\angle Z = 90°$ and hypotenuse $XY = c$, $YZ = a$ and $XZ = b$. It is required to prove that $XY^2 = XZ^2 + YZ^2$, i.e. $c^2 = a^2 + b^2$.

Construction : Draw perpendicular ZH from Z on hypotenuse XY. The Hypotenuse XY is divided at H into parts of d and e.

Proof :

Steps	Justification
(1) ΔZYH and ΔXYZ are similar. $\therefore \frac{a}{c} = \frac{e}{a}$ .... (1)	[(i) Both triangles are right angled (ii) Angle $\angle X$ is common]
(2) ΔXZH and ΔXYC are similar. $\therefore \frac{b}{c} = \frac{d}{b}$ .... (2)	[(i) Both triangles are right angled (ii) Angle $\angle Y$ is common]
(3) From the two rations, we get $a^2 = c \times e$, $b^2 = c \times d$ Therefore, $a^2 + b^2 = c \times e + c \times d$ $= c(e + d) = c^2$ $\therefore c^2 = a^2 + b^2$. **(Proved)**	

c Let, XYZ is any triangle where ZP is perpendicular to XY and O is any point on ZP. We have to prove that, $ZX^2 - YZ^2 = XO^2 - YO^2$.



Construction : O, X and O, Y are joined.

Proof : According to proposition ZP and OP are perpendicular to XY.

So, $\angle OPX$ as well as $\angle OPY$ is right angle.

$\therefore \angle OPX = \angle OPY = 90°$

Now, from ΔXZP, we have,

$ZX^2 = ZP^2 + XP^2$ [By Pythagoras's theorem]

Again from ΔYZP, we have,

$YZ^2 = ZP^2 + YP^2$ [By Pythagoras's theorem]

$\therefore ZX^2 - YZ^2 = ZP^2 + XP^2 - ZP^2 - YP^2$

$= XP^2 - YP^2$ .... (i)

From ΔOXP, We have,

$XO^2 = XP^2 + OP^2$

or, $XO^2 - OP^2 = XP^2$ .... (ii)

Again, from ΔOYP, We have,

$YO^2 = OP^2 + YP^2$

or, $YO^2 - OP^2 = YP^2$ .... (iii)

Now, Putting the values of XP^2 and YP^2 from the equation (ii) and (iii) into the equation (i), we get,

$ZX^2 - YZ^2 = (XO^2 - OP^2) - (YO^2 - OP^2)$

$= XO^2 - OP^2 - YO^2 + OP^2$

$= XO^2 - YO^2$

$\therefore ZX^2 - YZ^2 = XO^2 - YO^2$. **[Proved]**

Ques. 09 In ΔABC, $\angle A = 90°$, BP and CQ are two medians.

a. Bisect $\angle A$ with the pencil compass. *(Easy)* 2

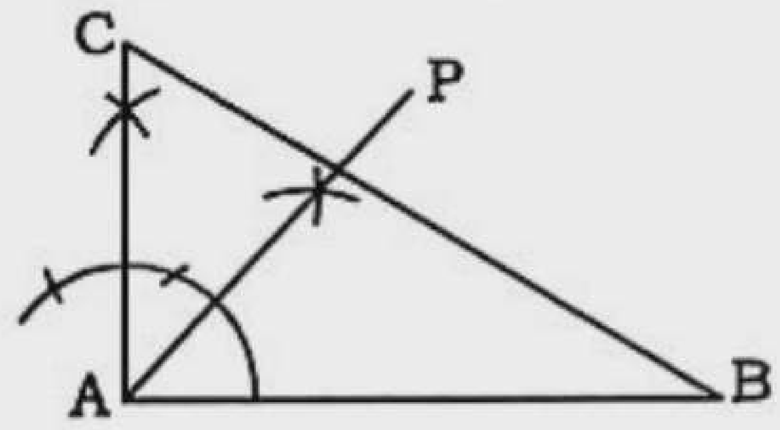
b. Prove that $BC^2 = CQ^2 + 3AQ^2$. *(Medium)* 4

c. Prove that $5BC^2 = 4(BP^2 + CQ^2)$ *(Hard)* 4

Solution to Question No. 09 :

a Given that $\angle A = 90°$.

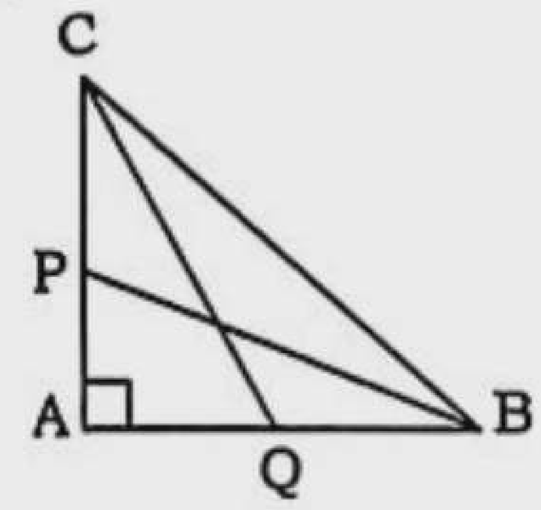
Now, $\angle A$ is bisected as under :



In the above figure, AP is the bisector of $\angle A$.

b Let BP and CQ are two medians of a right triangle ABC where $\angle A = 90°$.

Now, it is to be proved that $BC^2 = CQ^2 + 3AQ^2$



Construction : Two medians BP and CQ are drawn.

Proof : In right ΔABC,

$BC^2 = AC^2 + AB^2$ $= CQ^2 - AQ^2 + (2AQ)^2$, $= CQ^2 - AQ^2 + 4AQ^2$ $= CQ^2 + 3AQ^2$ $\therefore BC^2 = CQ^2 + 3AQ^2$, **(Proved)**	Since in right ΔACQ, $CQ^2 = AC^2 + AQ^2 \Rightarrow AC^2 = CQ^2 - AQ^2$ and Q is the mid-point of AB.

c Here, $BC^2 = AC^2 + AB^2$..............(1)

$CQ^2 = AC^2 + AQ^2$..............................(2)

$BP^2 = AP^2 + AB^2$..............................(3)

Now, adding (2) and (3), we get,

$CQ^2 + BP^2 = AC^2 + AQ^2 + AP^2 + AB^2$

$\therefore 4(BP^2 + CQ^2) = 4(AC^2 + AQ^2 + AP^2 + AB^2)$

$= 4(AB^2 + AC^2) + 4(AQ^2 + AP^2)$

$= 4BC^2 + 4\left\{\left(\frac{1}{2}AB\right)^2 + \left(\frac{1}{2}AC\right)^2\right\}$

$= 4BC^2 + 4\left\{\frac{AB^2}{4} + \frac{AC^2}{4}\right\}$

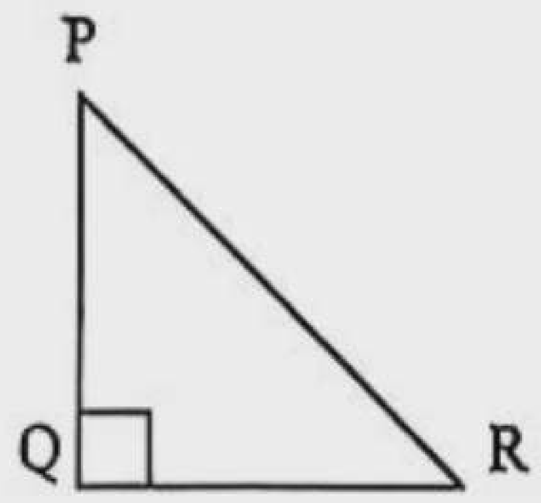
$= 4BC^2 + 4\left(\frac{(AB^2 + AC^2)}{4}\right)$

$= 4BC^2 + (BC^2)$

$= 5BC^2$.

$\therefore 5BC^2 = 4(BP^2 + CQ^2)$. **(Proved)**

Ques. 10 In the figure, PQ = 12 cm, PR = 13 cm.



a. Find out the value of QR. *(Easy)* 2

b. If M is the mid-point of QR, prove that, $PR^2 = PM^2 + 3RM^2$. *(Medium)* 4

c. If QS ⊥ PR, prove that, $PQ^2 - QR^2 = PS^2 - RS^2$. *(Hard)* 4

Solution to Question No. 10 :

a In right triangle PQR,

PQ = 12cm, PR = 13cm, ∠PQR = 90°.

In this case, we have based on theorem of Pythagoras,

$PR^2 = PQ^2 + QR^2$

$\Rightarrow 13^2 = 12^2 + QR^2$

$\Rightarrow QP^2 = 13^2 - 12^2 = 169 - 144 = 25$

$\Rightarrow QR = 5$

∴ Value of QR is 5 cm.

b Here, PQR is a right triangle with ∠PQR = 90°. M is the mid-point of QR.

Now, it is to be proved that $PR^2 = PM^2 + 3RM^2$.

According to the stem,

PQ = 12cm, PR = 13 cm.

And from (a) above,

QR = 5cm,

Since M is the mid-point of QR, QM = MR = $\frac{5}{2}$ cm

= 2.5 cm.

Now, $PR^2 = 13^2$sq cm = 169 sq cm..................(i)

Again, in right ΔPQM,

$PM^2 = PQ^2 + QM^2$

$= 12^2 + 2.5^2 = 144 + 6.25 = 150.25$

∴ $PM^2 + 3RM^2$

$= (150.25) + 3(2.5)^2 = 150.25 + 18.75$

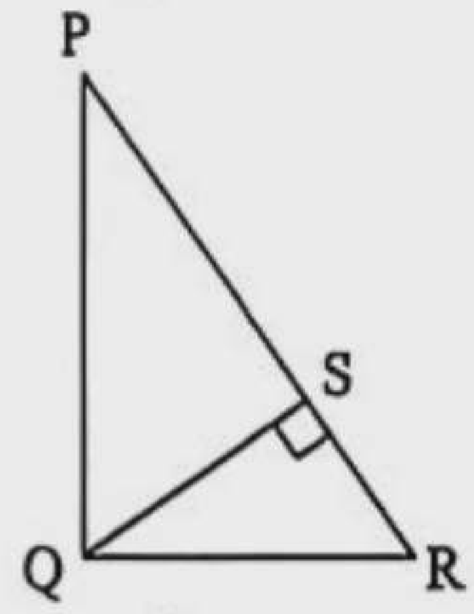
= 169..............................(ii)

∴ From (i) and (ii), we get,

$PR^2 = PM^2 + 3RM^2$. **(Proved)**

c In right ΔPQR, where ∠Q = 90°, QS ⊥ PR.

Now, it is required to prove that,



$PQ^2 - QR^2 = PS^2 - RS^2$.

Proof : From right Δ PQS,

$QS^2 = PQ^2 - PS^2$................(i)

Again, from right ΔQRS,

$QS^2 = QR^2 - RS^2$.................(ii)

Now, from (i) and (ii), we get,

$PQ^2 - PS^2 = QR^2 - RS^2$

or, $PQ^2 - QR^2 = PS^2 - RS^2$.

∴ $PQ^2 - QR^2 = PS^2 - RS^2$. **(Proved)**

Ques. 11 PQR is a right angled triangle where ∠PQR = 90°.

a. Verify the triangle whose sides are 6 cm, 8 cm and 10 cm is right angled triangle or not. *(Easy)* 2

b. According to the stem proof Pythagoras theorem. *(Medium)* 4

c. If PE and RF are two medians of the triangle, prove that $5PR^2 = 4(PE^2 + RF^2)$. *(Hard)* 4

Solution to Question No. 11 :

a Square of the side of 6 cm = 36 cm^2

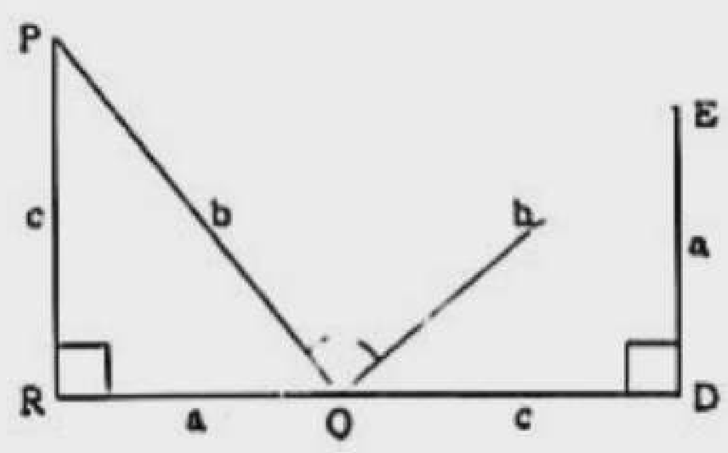
" " " " " 8 cm = 64 cm^2

" " " " " 10 cm = 100 cm^2.

Here, it is obvious that the sum of the square of 1st two sides = (36 + 64) cm^2 or 100 cm^2 which is equal to the square of the third side of the triangle in question.

∴ according to the theorem of Pythagoras, the triangle is a right angled triangle.

b **Proposition:** Let, in the triangle PRQ, ∠R = 90°, hypotenuse PQ = b, PR = c and RQ = a.



It is required to prove that, $PQ^2 = PR^2 + RQ^2$,

Construction : RQ id produced up to D such that QD = PR = c. Also a perpendicular DE at D on RQ produced is drawn, so that DE = RQ = a. Q, E and P, E are joined.

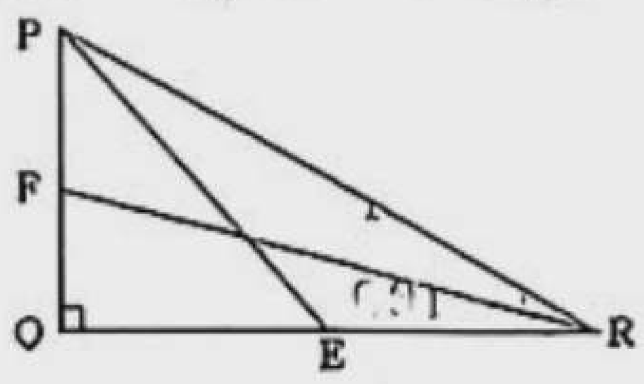
Proof :

Steps	Justification
(1) In ΔPRQ and ΔQDE, $PR = QD = c$, $RQ = DE = a$ and included $\angle PRQ$ = included $\angle QDE$ Hence, $\Delta PRQ \cong \Delta QDE$. $\therefore$ $PQ = QE = b$ and $\angle RPQ = \angle EQD$.	[each right angle] [SPS theorem]
(2) Again, since $PR \perp RD$ and $ED \perp RD$ $\therefore PR \parallel ED$. Therefore, PRDE is a trapezium.	
(3) Moreover, $\angle PQR + \angle RPQ = \angle DEQ + \angle EQD = 1$ right angle $\therefore \angle PQE = 1$ right angle Now area of the trapezium PRDE = area of (Δ region PRQ + Δ region QDE + Δ region PQE) or, $\frac{1}{2}$ RD(PR + DE) = $\frac{1}{2}ac + \frac{1}{2}ac + \frac{1}{2}b^2$ or, $\frac{1}{2}$ (RQ + QD) (PR + DE) = $\frac{1}{2}[2ac + b^2]$ or, $(a + c)(a + c) = 2ac + b^2$ [multiplying by 2] or, $a^2 + 2ac + c^2 = 2ac + b^2$ or, $a^2 + c^2 = b^2$. That is, $PQ^2 = PR^2 + RQ^2$. **(Proved)**	$\therefore \angle RPQ = \angle EQD$. [Area of trapezium $= \frac{1}{2}$ sum of parallel sides $\times$ distance between parallel sides]

c Given a triangle PQR where $\angle PQR = 90°$ and PE, RF are two medians of ΔPQR.

Now, it is to be proved that $5PR^2 = 4(PE^2 + RF^2)$.

Construction : P, E and R, F are joined.



Proof : In right ΔPQE,

$$PE^2 = PQ^2 + QE^2$$

or, $PE^2 = PQ^2 + \left(\frac{1}{2}QR\right)^2$

or, $PE^2 = PQ^2 + \frac{1}{4}QR^2$ ---------------- (1)

Again, in right ΔQRF,

$$RF^2 = QR^2 + FQ^2$$

or, $RF^2 = QR^2 + \left(\frac{1}{2}PQ\right)^2$

or, $RF^2 = QR^2 + \frac{1}{4}PQ^2$ ---------------- (2)

Now, adding (1) and (2), we get,

$PE^2 + RF^2 = PQ^2 + \frac{1}{4} + QR^2 + QR^2 + \frac{1}{4}PQ^2$

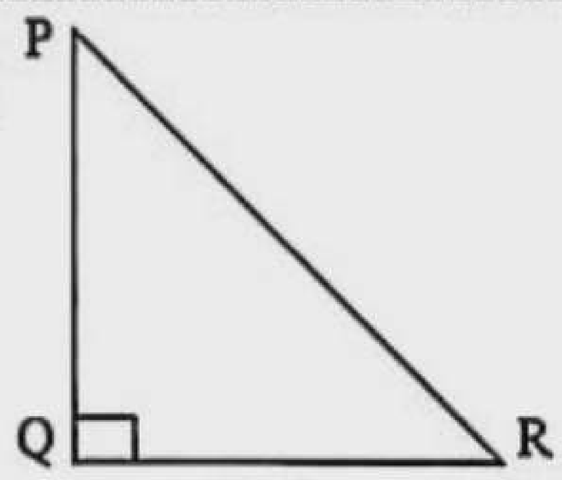
$= \frac{5PQ^2 + 5QR^2}{4}$

or, $4(PE^2 + RF^2) = 5(PQ^2 + QR^2)$

$= 5(PR^2)$, Since in right ΔPQR, $PR^2 = PQ^2 + QR^2$

$\therefore 5PR^2 = 4(PE^2 + RF^2)$. **(Proved)**

Ques. 12



a. The length, width and height of a rectangular solid are 6 cm, 5cm. and 4 cm. respectively. Determine the area of its whole surface. *(Easy)* 2

b. In the light of the stem, prove "Pythagoras Theorem". *(Medium)* 4

c. If S is the middle point of the side QR, Prove that $PR^2 = PS^2 + 3SR^2$. *(Hard)* 4

Solution to Question No. 12 :

a Here, length, breadth and height of a rectangular solid are 6 cm, 5 cm and 4 cm respectively.

We know, area of a rectangular solid with length a, breadth b and height c in unit.

respectively = abc cu. unit.

Here a = 6 cm, b = 5 cm, c = 4 cm

$\therefore$ The desired volume of the rectangular solid in question = $6 \times 5 \times 4$ cu cm or, 120 cu cm.

b Let in the triangle PQR, $\angle Q = 90°$, hypotenuse PR = c, PQ = b and QR = c.

It is required to prove that $PR^2 = PQ^2 + QR^2$, i.e. $c^2 = b^2 + a^2$.

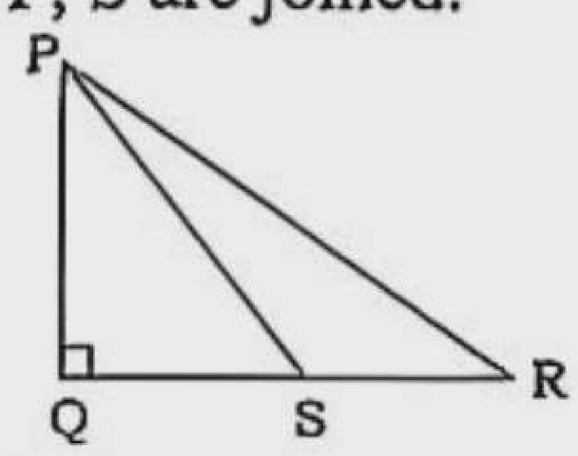
Construction : Produce QR upto S such that RS = PQ = b. Also a perpendicular ST at S on QR produced is drawn, so that ST = QR =a. R, T and P, T are joined.

Proof :

Steps	Justification
(1) In ΔPQR and ΔRST, $PQ = RS = b$, $QR = ST = a$ and included $\angle PQR$ = included $\angle RST$ Hence, $\Delta PQR \cong \Delta RST$. $\therefore$ $PR = RT = c$ and $\angle QPR = \angle TRS$.	[each triangle is a right angle] [SAS theorem]
(2) Again, since $PQ \perp QS$ and $TS \perp QS$ $\therefore$ $PQ \parallel TS$. Therefore, PQST is a trapezium. (3) Moreover, $\angle PRQ + \angle QPR = \angle SRT + \angle TRS$ = 1 right angle $\therefore$ $\angle PRT$ = 1 right angle Now area of the trapezium PQST = area of (Δ region PQR + Δ region RST + Δ region PRT) or, $\frac{1}{2}QS(PQ+ST) = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$ or, $\frac{1}{2}(QR+RS)(PQ+ST) = \frac{1}{2}[2ab+c^2]$ or, $(a+b)(a+b) = 2ab + c^2$ [multiplying by 2] or, $a^2 + 2ab + b^2 = 2ab + c^2$ or, $a^2 + b^2 = c^2$. That is, $PQ^2 + QR^2 = PR^2$ **(Proved)**	$\therefore$ $\angle QPR = \angle TRS$. [Area of trapezium $= \frac{1}{2}$ sum of parallel sides $\times$ distance between parallel sides]

c Let, in right ΔPQR, $\angle Q = 90°$ and S is the mid-point of QR. Now, it is to be proved that $PR^2 = PS^2 + 3SR^2$.

Construction : P, S are joined.



Proof : In ΔPQR,

$PR^2 = PQ^2 + QR^2$ .... (1)

Again, in ΔPQS,

$PS^2 = PQ^2 + QS^2 \Rightarrow PQ^2 = PS^2 - QS^2$ .... (2)

Now, putting for PQ^2 from (2) to (1), we get,

$PR^2 = PS^2 - QS^2 + QR^2$

$\Rightarrow PR^2 = PS^2 - QS^2 + (2QR)^2$

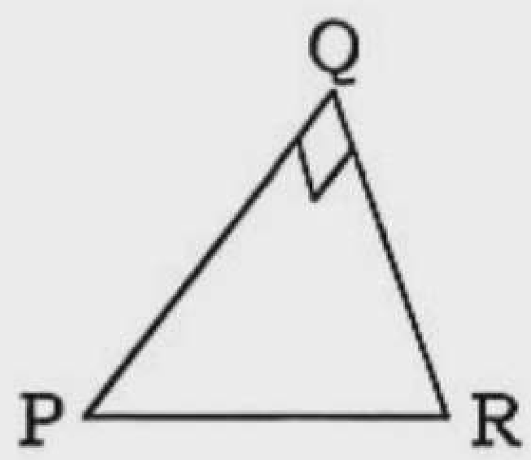
$\Rightarrow PR^2 = PS^2 + 3QS^2$

$\Rightarrow PR^2 = PS^2 + 3\ QS^2$

$\Rightarrow PR^2 = PS^2 + 3\ SR^2$, Since QS = SR

$\therefore$ $PR^2 = PS^2 + 3\ SR^2$ **(Proved)**

Ques. 13



a. Find the area of a circular garden of diameter 12 m. *(Easy)* 2

b. In the light of the stem, prove the Pythagoras Theorem. *(Medium)* 4

c. If in the triangle of the figure, N is a point on QR, prove that, $PR^2 + QN^2 = PN^2 + QR^2$. *(Hard)* 4

Solution to Question No. 13 :

a We know, area of a circular garden $= \pi r^2$ sq. unit

$= \pi \left(\frac{d}{2}\right)^2$ sq. unit, where d = diameter of the circle

Here, d = 12 m.

$\therefore$ Area of the given garden $= \pi \times \left(\frac{12}{2}\right)^2$ sq. m.

$= 113.04$ sq. m.

b Let in the triangle PQR, $\angle Q = 90°$, hypotenuse PR = c, PQ = b and QR = c.

It is required to prove that $PR^2 = PQ^2 + QR^2$, i.e. $c^2 = b^2 + a^2$.

Construction : Produce QR upto S such that RS = PQ = b. Also a perpendicular ST at S on QR produced is drawn, so that ST = QR =a. R, T and P, T are joined.

Proof :

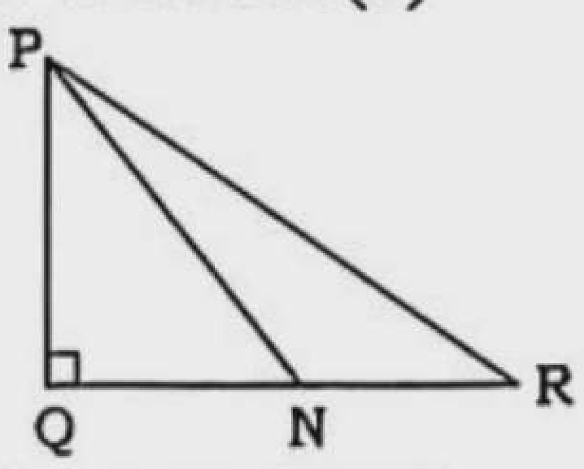
Steps	Justification
(1) In ΔPQR and ΔRST, $PQ = RS = b$, $QR = ST = a$ and included $\angle PQR$ = included $\angle RST$ Hence, $\Delta PQR \cong \Delta RST$. $\therefore$ $PR = RT = c$ and $\angle QPR = \angle TRS$.	[each triangle is a right angle] [SAS theorem]
(2) Again, since $PQ \perp QS$ and $TS \perp QS$ $\therefore$ $PQ \parallel TS$. Therefore, PQST is a trapezium. (3) Moreover, $\angle PRQ + \angle QPR = \angle SRT + \angle TRS$ = 1 right angle $\therefore$ $\angle PRT$ = 1 right angle	$\therefore$ $\angle QPR = \angle TRS$.

Now area of the trapezium PQST = area of (Δ region PQR + Δ region RST + Δ region PRT) or, $\frac{1}{2}$ QS(PQ + ST) $= \frac{1}{2}$ ab $+ \frac{1}{2}$ ab $+ \frac{1}{2}c^2$ or, $\frac{1}{2}$(QR + RS) (PQ + ST) $= \frac{1}{2}[2ab + c^2]$ or, $(a + b)(a + b) = 2ab + c^2$ [multiplying by 2] or, $a^2 + 2ab + b^2 = 2ab + c^2$ or, $a^2 + b^2 = c^2$. That is, $PQ^2 + QR^2 = PR^2$ **(Proved)**	[Area of trapezium $= \frac{1}{2}$ sum of parallel sides $\times$ distance between parallel sides]

c Let, in the right ΔPQR where $\angle Q = 90°$, N is a point on QR. Now, it is required to prove that, $PR^2 + QN^2 = PN^2 + QR^2$.

Proof : From right ΔPQR, We get, by applying Pythagoras theorem,

$PR^2 = PQ^2 + QR^2$ .... (1)



Similarly from the right ΔPQN, we get,

$PN^2 = PQ^2 + QN^2$ .... (2)

Now, adding QN^2 to (1), we get,

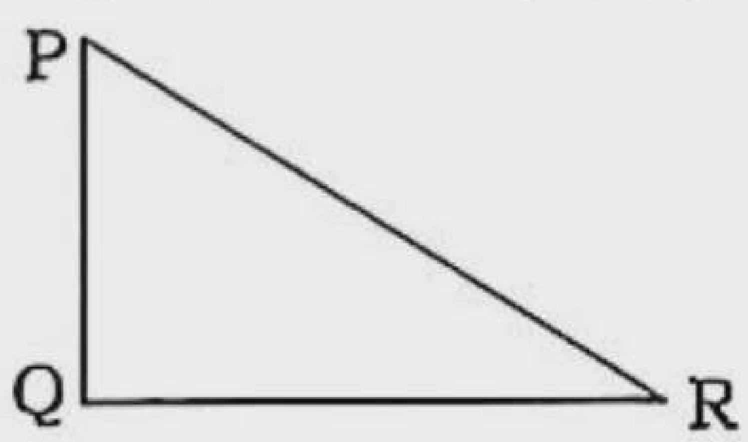
$PR^2 + QN^2 = PQ^2 + QR^2 + QN^2$

$= PQ^2 + QR^2 + PN^2 - PQ^2$, from (2)

$= PN^2 + QR^2$

$\therefore PR^2 + QN^2 = PN^2 + QR^2$. **(Proved)**

Ques. 14 In figure, $PR^2 = PQ^2 + QR^2$.



a. If the two adjacent sides of a right angle triangle are 5 cm and 6 cm respectively. Find the area of the triangle. *(Easy)* 2

b. According to the stem prove that $\angle PQR = 90°$. *(Medium)* 4

c. In ΔPQR, if $\angle Q$ = right angle, D and E are mid-points PQ and QR. Prove that $5PR^2 = 4(PE^2 + RD^2)$. *(Hard)* 4

Solution to Question No. 14 :

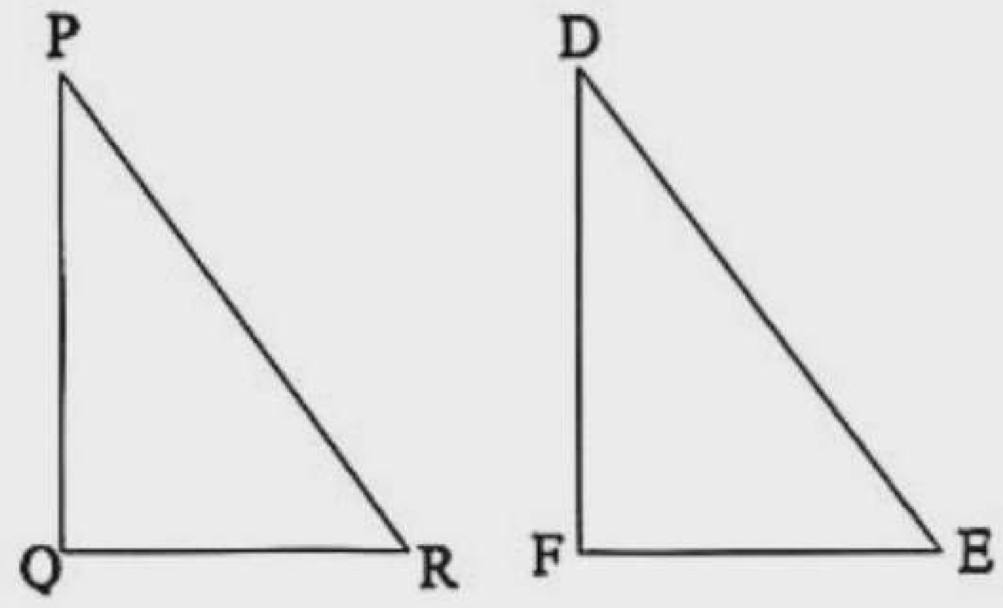
a Let, the two adjacent sides of a right angled triangle are 5cm and 6cm.

$\therefore$ its area $= \frac{1}{2} \times 5 \times 6$sq cm, taking base = 6cm and

Highest = 5cm = 15 sq cm.

b According to the stem, we have ΔPQR such that $PR^2 = PQ^2 + QR^2$. Now, it is to be proved that $\angle PQR = 90°$.

A triangle DEF is drawn so that $\angle F$ = 1 right angle EF = RQ and DF = PQ.

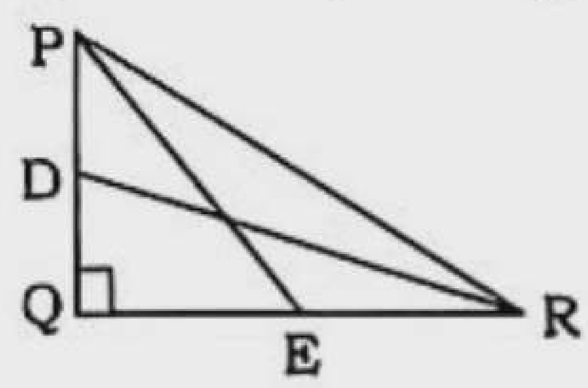


Proof:

Steps	Justification
(1) $DE^2 = EF^2 + DF^2$ $= RQ^2 + PQ^2 = PR^2$ $\therefore$ DE = PR	[Since in ΔDEF, $\angle F$ is a right angle]
(2) Now, in ΔPRQ and ΔDEF, RQ = EF, PQ = DF and PR = DE.	[supposition]
$\therefore \Delta PRQ \cong \Delta DEF$ $\therefore \angle Q = \angle F$ But $\angle F$ = 1 right angle $\therefore$ $\angle Q$ = 1 right angle **[Proved]**	[SSS theorem]

c Let, ΔPQR is a right triangle where $\angle Q = 90°$, D and E are the mid-points of PQ and QR. Now, it is to be proved that $5PR^2 = 4(PE^2 + RD^2)$.

Construction : P, E and R, D are joined.



Proof : In right triangled ΔPEQ,

$PE^- = PQ^2 + QE^2$.....................(1)

Again, in right triangle ΔDQR,

$RD^2 = DQ^2 + QR^2$....................(2)

Now, adding (1) and (2), we get,

$$PE^2 + RD^2 = PQ^2 + QR^2 + QE^2 + DQ^2$$

$$= PR^2 + \left(\frac{1}{2}QR\right)^2 + \left(\frac{1}{2}PQ\right)^2, \text{ since } QE = \frac{1}{2}QR \text{ and } DR = \frac{1}{2}PR$$

$$= PR^2 + \frac{1}{4}QR^2 + \frac{1}{4}PQ^2$$

$$= PR^2 + \frac{1}{4}(PQ^2 + QR^2)$$

$$= PR^2 + \frac{1}{4}PR^2, \text{ because of the reason mentioned above.}$$

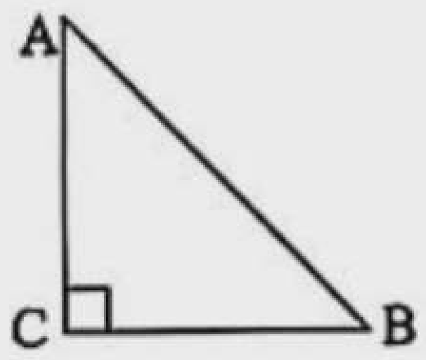
$$= \frac{4PR^2 + PR^2}{4}$$

$\therefore 4(PE^2 + RD^2) = 4PR^2 + PR^2$

or, $4(PE^2 + RD^2) = 5PR^2$.

$\therefore 5PR^2 = 4(PE^2 + RD^2)$. **(Proved)**

Ques. 15 In the figure, in $\triangle ABC$ $\angle C = 90°$



a. Write down two characteristics of a right angled triangle. *(Easy)* 2

b. Prove that, $AB^2 = AC^2 + BC^2$. *(Medium)* 4

c. If P and Q are the middle points of AB and AC respectively prove that PQ||BC and $PQ = \frac{1}{2}BC$. *(Hard)* 4

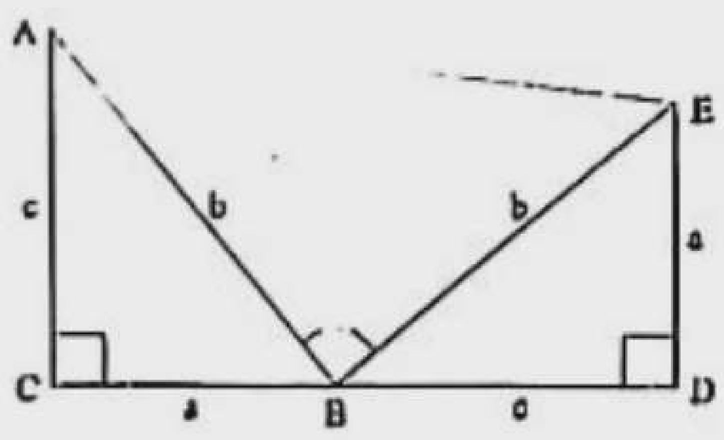
Solution to Question No. 15 :

a Two characteristics of right angled triangle are given below :

i. One angle is equal to 90°.

ii. square of hypotenuse = Sum of square of the other two sides.

b **Proposition:** Let, in the triangle ABC, $\angle C = 90°$, hypotenuse AB = b, AC = c and BC = a. It is required to prove that $AB^2 = AC^2 + BC^2$

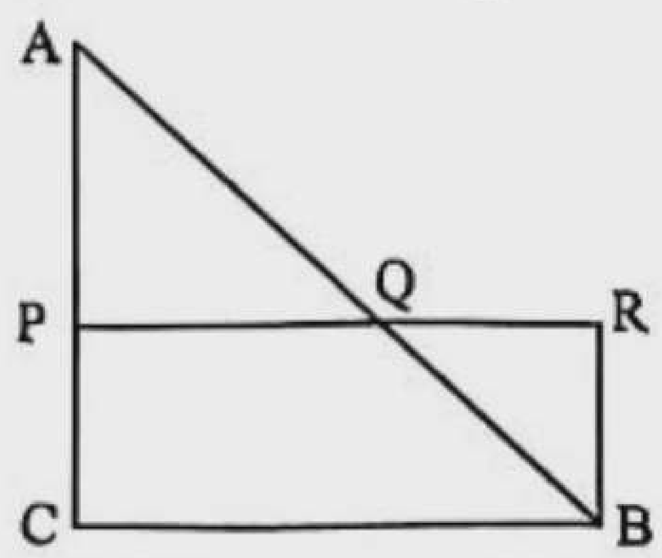


Construction : CB is produced up to D such that BD = AC = c. Also a perpendicular DE at D on CB produced is drawn, so that DE = CB = a. B, E and A, E are joined.

Proof :

Steps	Justification
(1) In $\triangle ABC$ and $\triangle BDE$, AC = BD = c, CB = DE = a and included $\angle ACB$ = included $\angle BDE$ Hence, $\triangle ABC \cong \triangle BDE$. $\therefore$ AB = BE = b and $\angle BAC = \angle EBD$.	[each right angle] [SAS theorem] $\therefore \angle BAC = \angle EBD$.
(2) Again, since $AC \perp CD$ and $ED \perp CD$ $\therefore AC \parallel ED$. Therefore, ACDE is a trapezium.	[Area of trapezium $= \frac{1}{2}$ sum of parallel sides × distance between parallel sides]
(3) Moreover, $\angle ABC + \angle BAC = \angle DBE + \angle EBD$ = 1 right angle $\angle BAC = \angle DBE$ and $\angle ABC = \angle BED$ $\therefore \angle ABE$ = 1 right angle Now, area of the trapezium ACDE = area of (Δ region ABC + Δ region BDE + Δ region ABE) or, $\frac{1}{2}CD(AC + DE) = \frac{1}{2}ac + \frac{1}{2}ac + \frac{1}{2}b^2$ or, $\frac{1}{2}(CB + BD)(AC + DE) = \frac{1}{2}[2ac + b^2]$ or, $(a + c)(a + c) = 2ac + b^2$ [multiplying by 2] or, $a^2 + 2ac + c^2 = 2ac + b^2$ or, $a^2 + c^2 = b^2$. That is, $AB^2 = AC^2 + BC^2$. **(Proved)**	

c According to the given information, $\triangle ACB$ is a right triangle where $\angle C = 90°$. P, Q are the mid points of AB and AC respectively. Now, it is to be proved that PQ||BC and $PQ = \frac{1}{2}BC$.



Construction : PQ is extended to R such that PQ = QR. B, R are joined.

Proof : In ΔAPQ and ΔBRQ, $\angle AQP = \angle BQR$, $PQ = QR$, $AQ = BQ$.
Since they are vertically opposite angles.
According to construction.
Since Q is the mid-point of AB.
$\Delta APQ \cong \Delta BRQ$.
$\therefore AP = BR$
But $AP = PC$ since P is the mid point of AC.
$\therefore PC = BR$.
Again, $\angle PCB = 90°$ and $PC = BR$.
$\therefore$ PCBR is a rectangle.
$\therefore PR \| BC \Rightarrow PQ \| BC$.
And $PR = BC \Rightarrow 2.PQ = BC$, Since $PQ = QR$
$\Rightarrow PQ = \frac{1}{2} BC$.

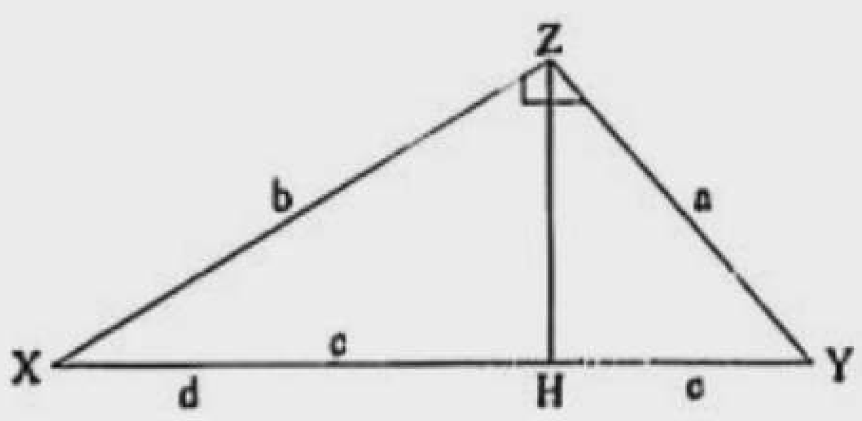
So, $PQ \| BC$ and $PQ = \frac{1}{2} BC$. **(Proved)**

Ques. 16 $\angle D = 1$ right angle of ΔDEF. P, R the mid-points of DE, EF respectively.
a. The base is 4 cm and height is 5 cm of a right angle triangle. Find the area of that triangle. *(Easy)* 2
b. Prove that $EF^2 = DE^2 + DF^2$. *(Medium)* 4
c. Show that $PR \,||\, DF$ and $PR = \frac{1}{2} DF$. *(Hard)* 4

Solution to Question No. 16 :

a We know, area of a right angled triangle $= \frac{1}{2} \times$ base $\times$ height sq unit
Here, base = 4 cm and height = 5 cm.
$\therefore$ The area of the given right triangle $= \frac{1}{2} \times 4 \times 5$ sq cm.
$= 10$ sq cm.

b



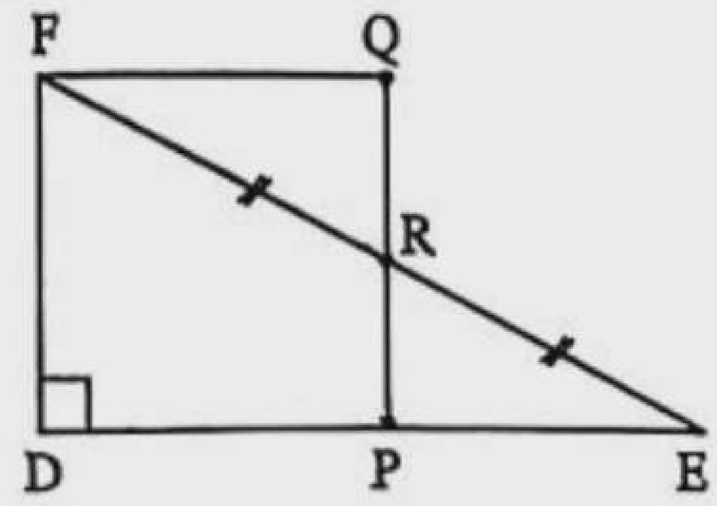
Proposition : Let, in the triangle XYZ, $\angle Z = 90°$ and hypotenuse $XY = c$, $YZ = a$ and $XZ = b$. It is required to prove that $XY^2 = XZ^2 + YZ^2$, i.e. $c^2 = a^2 + b^2$.
Construction : A perpendicular ZH from Z on hypotenuse XY is drawn. The Hypotenuse XY is divided at H into parts of d and e.

Proof :

Steps	Justification
(1) ΔZYH and ΔXYZ are similar. $\therefore \frac{a}{c} = \frac{e}{a}$ .... (1)	[(i) Both triangles are right angled (ii) Angle $\angle X$ is common]
(2) ΔXZH and ΔXYC are similar. $\therefore \frac{b}{c} = \frac{d}{b}$ .... (2)	[(i) Both triangles are right angled (ii) Angle $\angle Y$ is common]
(3) From the two equations, we get $a^2 = c \times e$, $b^2 = c \times d$ Therefore, $a^2 + b^2 = c \times e + c \times d$ $= c(e + d) = c^2$ $\therefore c^2 = a^2 + b^2$. **(Proved)**	

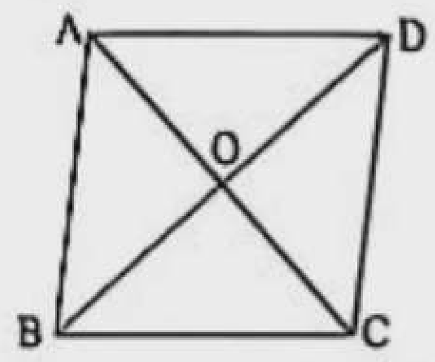
c Let, ΔDEF is a right triangle where $\angle D = 90°$ and P and R are the mid-points of DE and EF respectively. Now it is required to be shown that $PR \| DF$ and $PR = \frac{1}{2} DF$.

Construction : PR is extended to Q such that PR = RQ. F, Q are joined.



Proof : In ΔPER and ΔQFR,
$FR = ER$, since R is the mid-point of EF as per proposition.
$PR = QR$, according to construction.
$\angle PRE = \angle QRF$, since they are vertically opposite angles to each other.
$\therefore \Delta PER$ and ΔQFR are congruents.
$\therefore DP = FQ$.
$\therefore$ DPQF is a rectangle.
$\therefore DF = PQ = 2.PR$, since $PR = RQ$.
$\therefore PR = \frac{1}{2} DF$.
Again, $PR \| DF$, since DPQF is a rectangle.
Thus, it is proved that $PR \| DF$ and $PR = \frac{1}{2} DF$.

Ques. 17 In the figure ABCD is a rhombus.



a. If $\angle B = 75°$, then what is the measurement of $\angle C$ and $\angle D$ in degrees? *(Easy)* 2

b. Prove that AC and BD bisect each other at right angles. *(Medium)* 4

c. Prove that $AB^2 = OA^2 + OB^2$ in the triangle AOB. *(Hard)* 4

Solution to Question No. 17 :

a Here given that, $\angle B = 75°$

$\therefore$ $\angle D = 75°$, since opposite angles of a rhombus are mutually equal.

$\therefore \angle C = \dfrac{360° - 75° - 75°}{2}$,

since $\angle A + \angle B + \angle C + \angle D = 360°$ and $\angle B = \angle D$

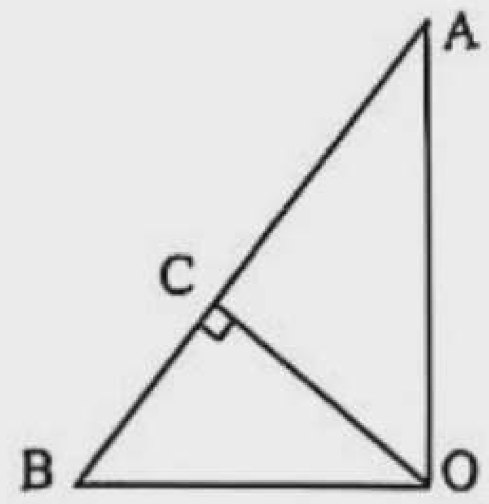
$= \dfrac{360° - 150°}{2} = \dfrac{210°}{2}$

$= 105°$.

b In rhombus ABCD, BC || AC and AB || CD.
Again, diagonal AC is a secant of BC and AD
$\therefore \angle CAD = \angle ACB$, since they are alternate angles.
Besides, AB || CD and diagonal BD is a secant of them.
$\therefore \angle ADB = \angle DBC$, since they are alternate angles
Now in ΔAOD and ΔBOC,
AD = BC, since they are the sides of the same rhombus ABCD.
$\angle ADB = \angle DBC$ and $\angle CAD = \angle ACB$
$\therefore \Delta AOD \cong \Delta BOC \Rightarrow BD = DO$ and $CO = AO$.
So, diagonal BD and AC bisects each other.
Again, in ΔABD, AB = AD, since they are the two sides of the same rhombus ABCD,
OB = OD, that is O is the mid-point of BD, as stated above.
So, ΔABD is an isosceles triangle where side AB = side AD and O is the mid-point of other side BD of ΔABD.
$\therefore$ OD is a median of isosceles ΔABD.
$\therefore OA \perp BD$, which implies AC is a perpendicular bisector of BD.
Therefore, AC and BD mutually bisect each other and AC bisects BD. **(Proved)**

c Let, AOB is a right triangle where $\angle AOB = 90°$. Now it is required to prove that $AB^2 = OA^2 + OB^2$.

Construction : $OC \perp AB$ is drawn.



Proof : In ΔAOB and ΔBOC are similar.

$\therefore \dfrac{AB}{OB} = \dfrac{OB}{BC} \Rightarrow OB^2 = AB \cdot BC$ ---------- (1)

Again, ΔAOB and ΔAOC are similar,

$\therefore \dfrac{AB}{OA} = \dfrac{OA}{AC} \Rightarrow OA^2 = AB \cdot AC$ ---------- (2)

$$\begin{aligned}\therefore OA^2 + OB^2 &= AB \cdot AC + AB \cdot BC \\ &= AB\,(AC + BC) \\ &= AB \cdot AB \\ &= AB^2\end{aligned}$$

$\therefore AB^2 = OA^2 + OB^2$. **(Proved)**

Ques. 18 In ΔPQR, $\angle Q = 90°$ and A and B the middle points of PQ and PR respectively.

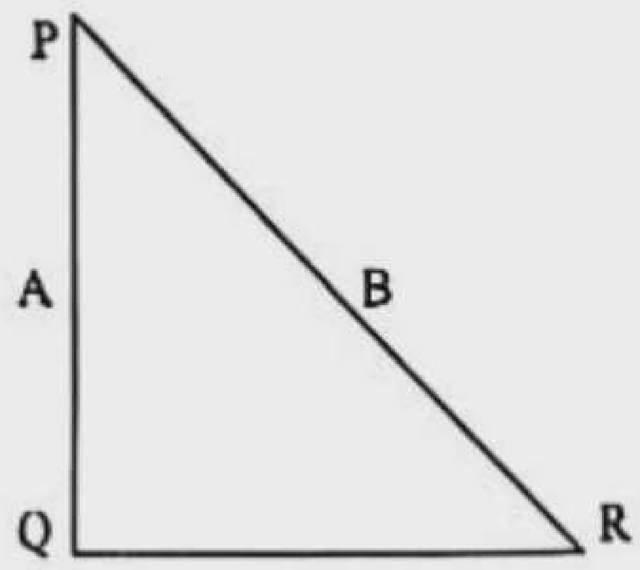
a. Draw the figure with the help of stem. *(Easy)* 2

b. Prove that, $PQ^2 + QR^2 = PR^2$. *(Medium)* 4

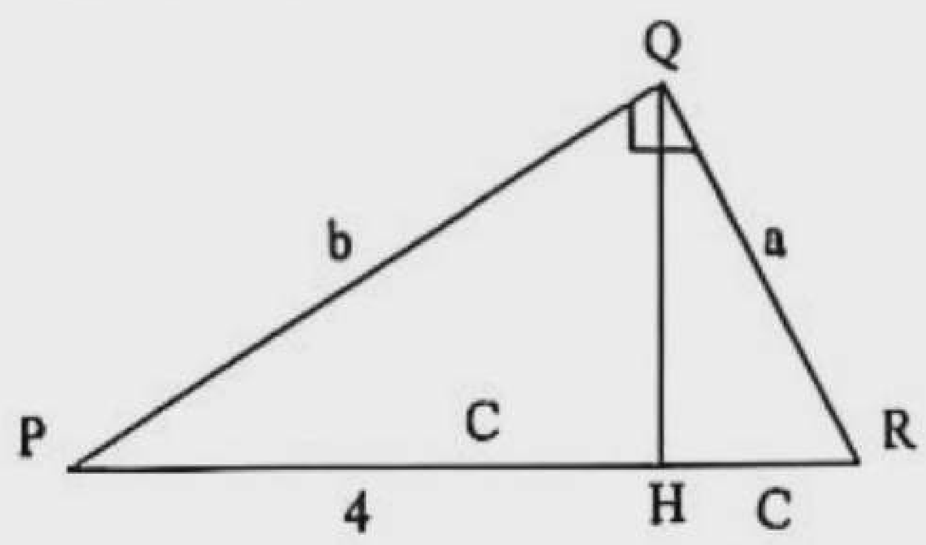
c. Show that, $AB = \frac{1}{2} QR$. *(Hard)* 4

Solution to Question No. 18 :

a



b **Proposition :** In triangle PQR, $\angle Q = 90°$ and hypotenuse PR = C,
PQ = b, QR = a. It is required to prove that $PR^2 = PQ^2 + QR^2$ i.e. $c^2 = a^2 + b^2$.



Construction : Draw perpendicular QH on hypotenuse PR. The hypotenouse PR is divided at H into parts of d and e

Proof :

Steps	Justification
In ΔQRH and ΔPQH $\angle RHQ = \angle PQR$ and $\angle QRH = \angle PRQ$ (1) $\therefore$ ΔRQH and ΔPQR are similar $\frac{QR}{PR} = \frac{RH}{QR}$ $\therefore \frac{a}{c} = \frac{e}{a}$.....................(i)	[(i) Both triangles are right angled (ii) angle $\angle B$ is common. $\therefore c = e + d$.
(2) Similarly, ΔPQH and ΔPQR are similar. $\therefore \frac{b}{c} = \frac{d}{b}$................(2) (3) From above two ratios, we get $a^2 = c \times e$, $b^2 = c \times d$ Therefore, $a^2 + b^2 = c \times e + c \times d$ $= c\,(e + d)$ $= c.\,c$ $= c^2$ $\therefore c^2 = a^2 + b^2$ ie. $PR^2 = QR^2 + PQ^2$	

c A, B are joined. Shown in figure below.

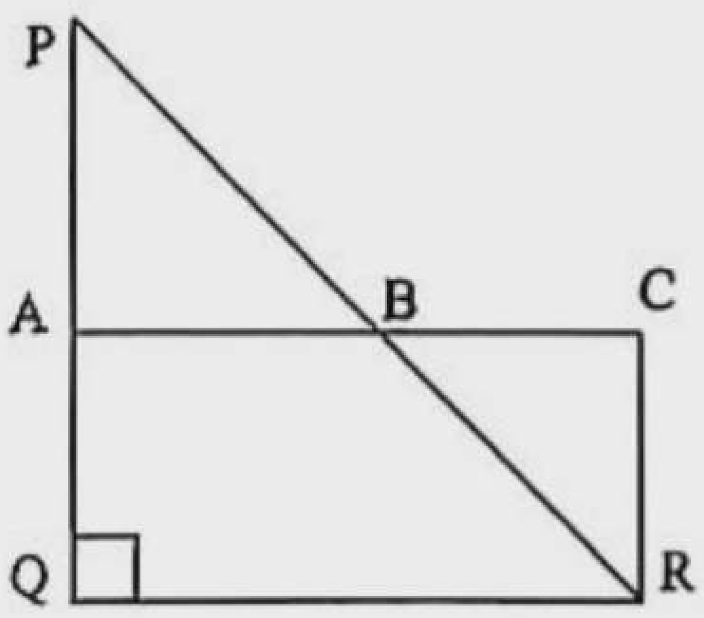
Produce BC so that AB = BC.

ΔAPB and ΔBCR

$\angle ABP = \angle CBR$ [Vertically opposite]

$PB = BR$

and $AB = BC$



$\therefore \Delta APB \cong \Delta\, BCR$

$\therefore AP = CR$

Again $AP = AQ$

So, $AQ = CR$

$\because AC \parallel QR$ and $AQ = CR$

$\therefore AC = = QR$

$\Rightarrow QR = 2\,AB$

$\therefore AB = \frac{1}{2} QR.$

Ques. 19 Suppose ABC is a triangle.

a. If in ΔABC, $AC^2 = AB^2 + BC^2$ then prove that ΔABC is a right angled triangle. *(Easy)* 2

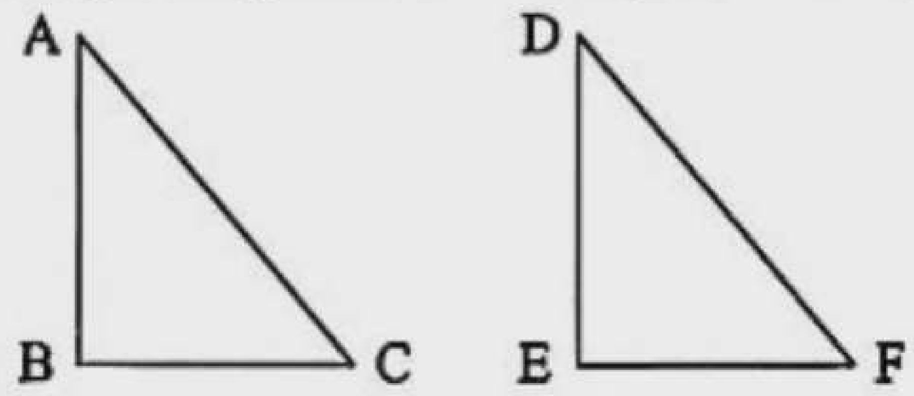
b. If in ΔABC, $\angle A = 90°$ then prove that, $BC^2 = AB^2 + AC^2$. *(Medium)* 4

c. If in ΔABC, $\angle A = 90°$, BP and CQ are two medians. Prove that $5BC^2 = 4(BP^2 + CQ^2)$. *(Hard)* 4

Solution to Question No. 19 :

a In ΔABC, we have $AB^2 + BC^2 = AC^2$. We have to show that $\angle B = 1$ right angle.

Construction : Let us draw a triangle DEF such that $\angle E = 1$ right angle. $DE = AB$, $EF = BC$, $DF = AC$.

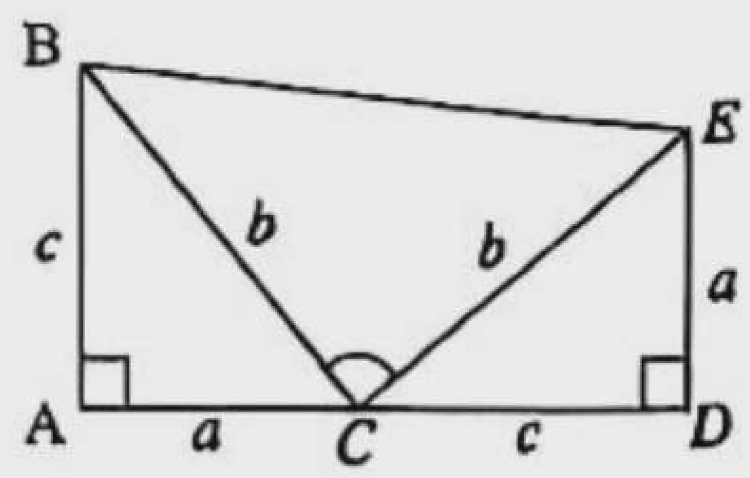


Proof :

Step	Justification
1. In ΔDEF, $DF^2 = DE^2 + EF^2$ $= AB^2 + BC^2$ $\therefore DF = AC$.	[Since ΔDEF is a right triangle and DE = AB, EF = BC as per hypothesis]
2. Now comparing ΔDEF and ΔABC, we have, $AC = DF$, $AB = DE$ and $BC = EF$. $\therefore \Delta ABC$ and ΔDEF are congruent i.e. $\Delta ABC \cong \Delta DEF$. $\therefore \angle B = \angle E$. But $\angle E = 90°$ i.e. 1 right angle. So, $\angle B = 90°$ or 1 right angle. **(Shown)**	[Since the corresponding sides of two triangles are mutually equal.]

So, ΔABC is a right angled triangle.

b **Proposition:** Let in the triangle ABC, $\angle A = 90°$, the hypotenuse $BC = b$, $AB = c$ and $AC = a$. It is required to prove that $BC^2 = AB^2 + AC^2$, i.e. $b^2 = c^2 + a^2$.

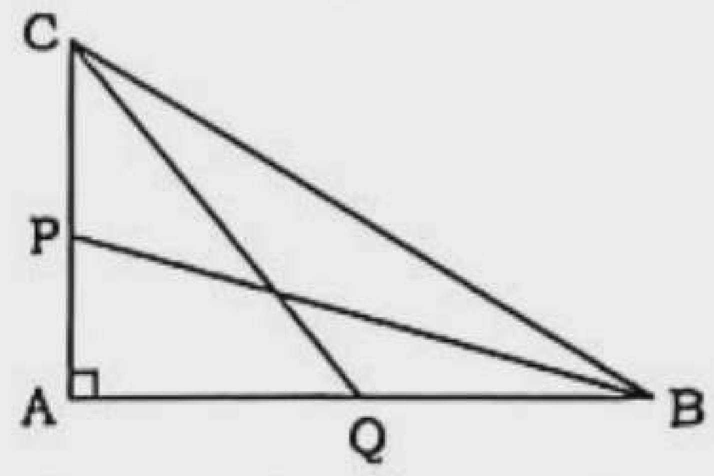


Construction : Produce AC up to D in such a way that $CD = BA = c$. Also, draw perpendicular DE at D on AC produced, so that $DE = AC = a$. Join C, E and B, E.

Proof :

Steps	Justification
(1) In ΔBAC and ΔCDE, $BA = CD = c$, $AC = DE = a$ and included $\angle BAC$ = included $\angle CDE$	[each right angle]
Hence, $\Delta BAC \cong \Delta CDE$. $\therefore BC = CE = b$ and $\angle ABC = \angle ECD$.	[SAS theorem]
(2) Again, since $BA \perp AD$, and $ED \perp AD$, $\therefore BA \| ED$. Therefore, BADE is a trapezium.	
(3) Moreover, $\angle BCA + \angle ABC = \angle BCA + \angle ECD$ = 1 right angle	$\therefore \angle ABC = \angle ECD$
$\therefore$ $\angle BCE$= 1 right angle and ΔBCE is a right- angled triangle	
Now the area of the trapezium BADE = the area of (Δ region BAC + Δ region CDE + Δ region BCE) or, $\frac{1}{2}$ AD(BA + DE) $= \frac{1}{2}ac + \frac{1}{2}ac + \frac{1}{2}b^2$	[Area of trapezium $= \frac{1}{2}$ sum of parallel sides × distance between parallel sides]
or, $\frac{1}{2}$ (AC+CD) (BA + DE) $= \frac{1}{2}[2ac + b^2]$ or, $(a + c)(a + c) = 2ac + b^2$ [multiplying by 2] or, $a^2 + 2ac + c^2 = 2ac = b^2$ $\therefore b^2 = a^2 + c^2$ **(Proved)**	

C Proposition : Let, ABC is a right angled triangle with $\angle A = 90°$. BP and CQ are two medians of ΔABC. Now, it is required to prove that $5BC^2 = 4(BP^2 + CQ^2)$.



Proof : According to the proposition, the adjoining figure refers to a right triangle ABC where $\angle A = 90°$; BP, CQ are two medians and BC is hypotenuse of ΔABC.

$\therefore$ By using theorem of Pythagoras,

$$BC^2 = AB^2 + AC^2$$
$$= BP^2 - AP^2 + CQ^2 - AQ^2$$
$$= BP^2 + CQ^2 - AQ^2 - AP^2$$
$$= BP^2 + CQ^2 - \left(\frac{1}{2}AB\right)^2 - \left(\frac{1}{2}AC\right)^2$$
$$= BP^2 + CQ^2 - \frac{1}{4}AB^2 - \frac{1}{4}AC^2$$
$$= BP^2 + CQ^2 - \frac{1}{4}(AB^2 + AC^2)$$
$$= BP^2 + CQ^2 - \frac{1}{4}(BC^2)$$
$$= \frac{4.BP^2 + 4.CQ^2 - BC^2}{4}$$

$\therefore$ $4BC^2 = 4.BP^2 + 4CQ^2 - BC^2$, by cross multiplication

or, $4BC^2 + BC^2 = 4.BP^2 + 4CQ^2$

or, $5.BC^2 = 4.BP^2 + 4.CQ^2$.

$\therefore 5BC^2 = 4(BP^2 + CQ^2)$ **(Proved)**

Super Suggestions

Super Suggestions with 100% preparatory questions selected by the Master Trainer Panel

Dear learners, important multiple choice, short and creative questions of this chapter selected by Master Trainer Panel for Half-Yearly and Annual Exams are presented below. Learn the answers to the mentioned questions well to ensure 100% preparation.

Question Pattern	7★	5★	3★
○ MCQs with Answers	Learn each MCQs in this chapter thoroughly.		
○ Short Q/A	1, 4, 7, 10, 14, 16	2, 3, 5, 8	6, 9, 11, 12, 17
○ Creative Q/A	1, 3, 7, 12, 14	2, 4, 6, 8	5, 9, 11, 13

Assessment & Evaluation

A question bank presented in the form of a class test to assess the preparation

Class Test Time : 3 hours **Mathematics** Full marks : 100

Class : Eight

Multiple Choice Questions (Each question carries 1 mark) $1 \times 30 = 30$

[N.B. : Answer all the questions. Each question carries one mark. Block fully, with a ball-point pen, the circle of the letter that stands for the correct/best answer in the "Answer Sheet" for Multiple Choice Question Type Examination.]

1. **How many acute angles are there in a right angled triangle?**
(a) 1 (b) 2 (c) 3 (d) 4

2. **From which sides a triangle can be drawn?**
(a) 3, 4, 6 (b) 3, 5, 8 (c) 3, 5, 9 (d) 8, 6, 10

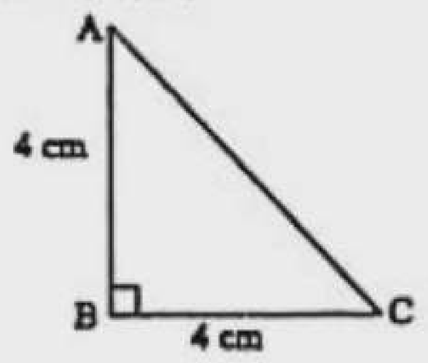
3. **$\angle ABE + \angle ACF + \angle CAG$ = what?**
(a) 90° (b) 120° (c) 180° (d) 360°

4. **From which sides a right angled triangle can be drawn?**
(a) 3, 4, 6 (b) 4, 5, 9
(c) 5, 12, 13 (d) 12, 13, 17

5. **If ΔPQR is a right triangle such that $\angle Q = 90°$, then —.**
i. PR is the largest side of the triangle
ii. $\angle R$ is an obtuse angle
iii. $PR + QR > PQ$
Which one of the following is correct?
(a) i & iii (b) ii & iii (c) i & ii (d) i, ii & iii

6. **Who was Pythagoras?**
(a) A Mathematician (b) A Physicist
(c) A Greek Philosopher (d) A Chemist

■ **Answer the questions No. 7 and 8 according to the following information :**



7. **$\angle A$ = What?**
(a) 30° (b) 45° (c) 60° (d) 90°

8. **What is the area of triangle ABC in square centimetre?**
(a) 8 (b) 16 (c) 32 (d) 64

9. **If three sides of a triangle are 5 cm, 12 cm and 13 cm, then what will be its area?**
(a) 20 sq. cm (b) 30 sq. cm
(c) 60 sq. cm (d) 120 sq. cm

10. **In right ΔABC, AB = 5, AC = 12, BC = what?**
(a) 10 (b) 13 (c) 16 (d) 9

11. **By which of the following right angled triangle can be drawn?**
(a) 4, 5, 6 (b) 6, 8, 10 (c) 7, 9, 11 (d) 5, 10, 15

12. **If in a ΔABC, $\angle C = 90°$, AB = 13 cm and AC = 12 cm, what is the value of BC in centimeter?**
(a) 1 (b) 5 (c) 17.69 (d) 25

13. **AE = what?**
(a) 16 cm (b) 12 cm (c) 20 cm (d) 18 cm

14. **$\angle ADC + \angle ABC$ = what?**
(a) 90° (b) 145° (c) 160° (d) 180°

15. **BD = what?**
(a) 10 cm (b) 15 cm (c) 13 cm (d) 18 cm

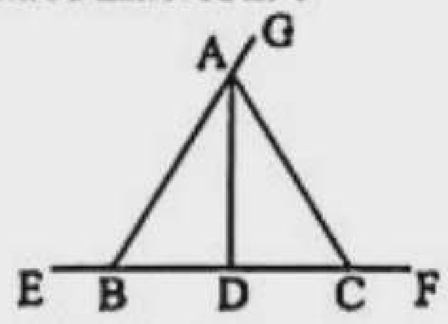
16. **CE = what?**
(a) $2\sqrt{5}$ cm (b) $3\sqrt{5}$ cm (c) $\sqrt{41}$ cm (d) 5 cm

17. **What is the area of ΔABD?**
(a) 25 sq cm (b) 30 sq cm (c) 20 sq cm (d) 15 sq cm

18. **In ΔABC, $\angle A = 90°$, AB = 12, AC = 5 then what is the value of BC?**
(a) 11 (b) 169 (c) 13 (d) 121

19. **In a right angled triangle if the difference between two acute angles is 6° then what is the value of the smallest angle?**
(a) 90° (b) 60° (c) 48° (d) 42°

■ **Answer to the questions No. 20 and 21 in the light of the following information :**



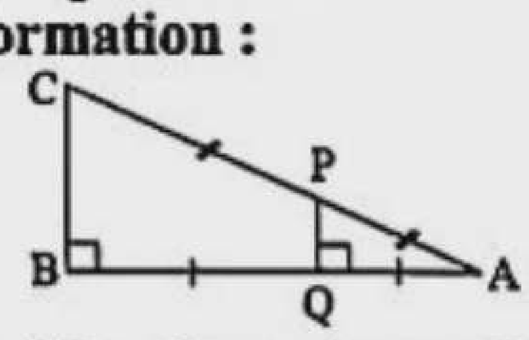
In ΔABC, AB = BC = AC = 6 cm and AD ⊥ BC.

20. **What is the length of AD in cm?**
(a) 5.19 (b) 6.71 (c) 8.49 (d) 9.23

21. **In which triangle the theorem of Pythagoras is applicable?**
(a) Equilateral (b) Scalene
(c) Right angled (d) Obtuse angled

22. **In ΔPQR, $\angle P = 90°$ then**
i. $\angle Q < 90°$
ii. $QR^2 = PQ^2 + PR^2$
iii. $\angle P = \angle R + \angle Q$
Which one is correct?
(a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

■ **Answer to the questions no. 23 and 24 based on the following information :**



In the figure, AB = 6 cm, Area of ΔAPQ = 6 sq cm.

23. **What is the value of PQ?**
(a) 2 (b) 4 (c) 6 (d) 8

24. **What is the value of AC?**
(a) 4 (b) 6 (c) 8 (d) 10

25. **What is the area of the ΔBCE?**
(a) 10 sq. cm (b) 12 sq. cm (c) 8 sq. cm (d) 20 sq. cm

26. **If in ΔPQR, $\angle R = 90°$ then—**
i. the hypotenuse is PQ
ii. the area is $\frac{1}{2}PR \times QR$
iii. $PR^2 = PQ^2 - QR^2$
Which one is correct?
(a) i & ii (b) i & iii (c) ii & iii (d) i, ii & iii

27. **When Pythagoras discovered a special property of right angled triangle?**
(a) 5th century BC (b) 6th century BC
(c) 7th century BC (d) 8th century BC

28. **A circle with area 1256 sq metres. What will be the diameter of the circle in centimetre?**
(a) 400 (b) 40 (c) 20 (d) 10

29. **What is the length of the diagonal of a square with one side is 1 unit?**
(a) 1.00 unit (b) 1.41 unit (c) 2.01 unit (d) 4.00 unit

30. **When did Pythagoras define Pythagoras theorem?**
(a) In 6^{th} century B.C. (b) In 5^{th} century B.C.
(c) In 4^{th} century B.C. (d) In 3^{rd} century B.C.

Mathematics

Short-Answer Question (Each question carries 2 marks)

Answer any 10 of the following questions : $2 \times 10 = 20$

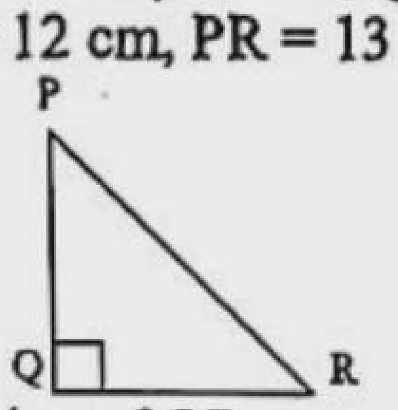
1. Write down two properties of a right-angled triangle.
2. State the Pythagoras theorem.
3. If the hypotenuse of a right-angled isosceles triangle is $6\sqrt{2}$ cm, what is the length of its equal sides in cm?
4. In triangle PQR, $PQ^2 + QR^2 = PR^2$. If PQ = 9 cm and QR = 12 cm, find the value of PR.
5. In triangle ABC, $\angle B = 90°$. If D and E are the midpoints of AB and BC respectively, show that $AC^2 = 4DE^2$.
6. State the converse of Pythagoras theorem.
7. Verify whether a triangle with sides of 8 cm, 15 cm, and 17 cm is a right-angled triangle.
8. Verify whether it is possible to draw a right-angled triangle with sides of lengths 3 cm, 4 cm, and 5 cm.
9. In triangle XYZ, XY = 12 cm, YZ = 5 cm, and ZX = 13 cm. Find the value of $\angle Y$.
10. If in triangle ABC, AB = BC = 2 cm and AC = $2\sqrt{2}$ cm, then find the value of $\angle A$.
11. If the difference between the two acute angles of a right-angled triangle is 5°, what is the value of the greatest angle?
12. If the difference between the two acute angles of a right-angled triangle is 15°, what is the value of the smallest angle in degrees?
13. If the base of a right-angled triangle is 12 cm and the area is 30 square cm, and the perimeter is 30 cm, what are the lengths of the sides in cm?
14. If the lengths of the two sides adjacent to the right angle of a right-angled triangle are 6 cm and 8 cm respectively, find the area of the triangle.
15. In triangle PQR, $\angle Q = 90°$. If PR = 13 cm and PQ = 12 cm, what is the sum of the squares of the lengths of the sides of the triangle in square cm?

Mathematics

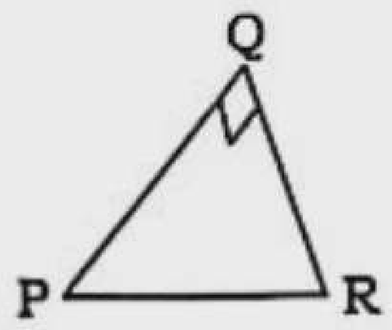
Creative Question (Each question carries 10 marks)

Answer any 5 of the following questions : $10 \times 5 = 50$

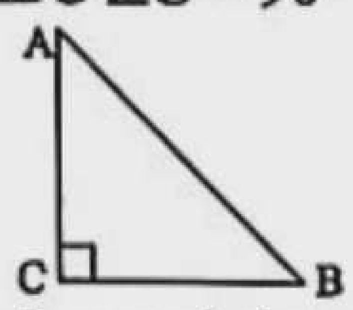
1. In triangle ABC, we have, $AB^2 + BC^2 = AC^2$.
   a. Draw the triangle with appropriate notations and introduction. 2
   b. Show that $\angle B$ = 1 right angle where $AB^2 + BC^2 = AC^2$ in $\triangle ABC$. 4
   c. If D is the mid-point of AB, then prove that $AC^2 + BD^2 = CD^2 + AB^2$. 4
2. In $\triangle PQR$, PQ > PR and PD $\perp$ QR.
   a. Draw an angle 30° with the help of scale and campus. 2
   b. Prove that, $PQ^2 = PD^2 + QD^2$. 4
   c. If M is any point on PD, prove that, $QM^2 - RM^2 = PQ^2 - PR^2$. 4
3. In $\triangle ABC$, $\angle A = 90°$, BP and CQ are two medians.
   a. Bisect $\angle A$ with the pencil compass. 2
   b. Prove that $BC^2 = CQ^2 + 3AQ^2$. 4
   c. Prove that $5BC^2 = 4(BP^2 + CQ^2)$ 4
4. In the figure, PQ = 12 cm, PR = 13 cm.



   a. Find out the value of QR. 2
   b. If M is the mid-point of QR, prove that, $PR^2 = PM^2 + 3RM^2$. 4
   c. If QS $\perp$ PR, prove that, $PQ^2 - QR^2 = PS^2 - RS^2$. 4
5. PQR is a right angled triangle where $\angle PQR = 90°$.
   a. Verify the triangle whose sides are 6 cm, 8 cm and 10 cm is right angled triangle or not. 2
   b. According to the stem proof Pythagoras theorem. 4
   c. If PE and RF are two medians of the triangle, prove that $5PR^2 = 4(PE^2 + RF^2)$. 4
6.



   a. Find the area of a circular garden of diameter 12 m. 2
   b. In the light of the stem, prove the Pythagoras Theorem. 4
   c. If in the triangle of the figure, N is a point on QR, prove that, $PR^2 + QN^2 = PN^2 + QR^2$. 4
7. In the figure, in $\triangle ABC$ $\angle C = 90°$



   a. Write down two characteristics of a right angled triangle. 2
   b. Prove that, $AB^2 = AC^2 + BC^2$. 4
   c. If P and Q are the middle points of AB and AC respectively prove that PQ||BC and $PQ = \frac{1}{2}BC$. 4
8. $\angle D$ = 1 right angle of $\triangle DEF$. P, R the mid-points of DE, EF respectively.
   a. The base is 4 cm and height is 5 cm of a right angle triangle. Find the area of that triangle. 2
   b. Prove that $EF^2 = DE^2 + DF^2$. 4
   o. Show that PR || DF and $PR = \frac{1}{2}DF$. 4

Answer Sheet ▸ Multiple Choice Questions

1	ⓑ	2	ⓓ	3	ⓓ	4	ⓒ	5	ⓐ	6	ⓒ	7	ⓑ	8	ⓐ	9	ⓑ	10	ⓑ	11	ⓑ	12	ⓑ	13	ⓐ	14	ⓓ	15	ⓒ
16	ⓒ	17	ⓑ	18	ⓒ	19	ⓓ	20	ⓐ	21	ⓒ	22	ⓓ	23	ⓑ	24	ⓓ	25	ⓐ	26	ⓓ	27	ⓑ	28	ⓑ	29	ⓑ	30	ⓐ

Solving Reference ▸ Short-Answer Questions

1 ▸ See Page 321; Ques. 03
2 ▸ See Page 321; Ques. 06
3 ▸ See Page 322; Ques. 09
4 ▸ See Page 322; Ques. 10
5 ▸ See Page 322; Ques. 11
6 ▸ See Page 323; Ques. 17
7 ▸ See Page 323; Ques. 18
8 ▸ See Page 323; Ques. 20
9 ▸ See Page 323; Ques. 21
10 ▸ See Page 323; Ques. 22
11 ▸ See Page 321; Ques. 01
12 ▸ See Page 321; Ques. 02
13 ▸ See Page 321; Ques. 04
14 ▸ See Page 321; Ques. 05
15 ▸ See Page 322; Ques. 12

Solving Reference ▸ Creative Questions

1 ▸ See Page 324; Ques. 02
2 ▸ See Page 327; Ques. 07
3 ▸ See Page 329; Ques. 09
4 ▸ See Page 330; Ques. 10
5 ▸ See Page 330; Ques. 11
6 ▸ See Page 332; Ques. 13
7 ▸ See Page 334; Ques. 15
8 ▸ See Page 335; Ques. 16