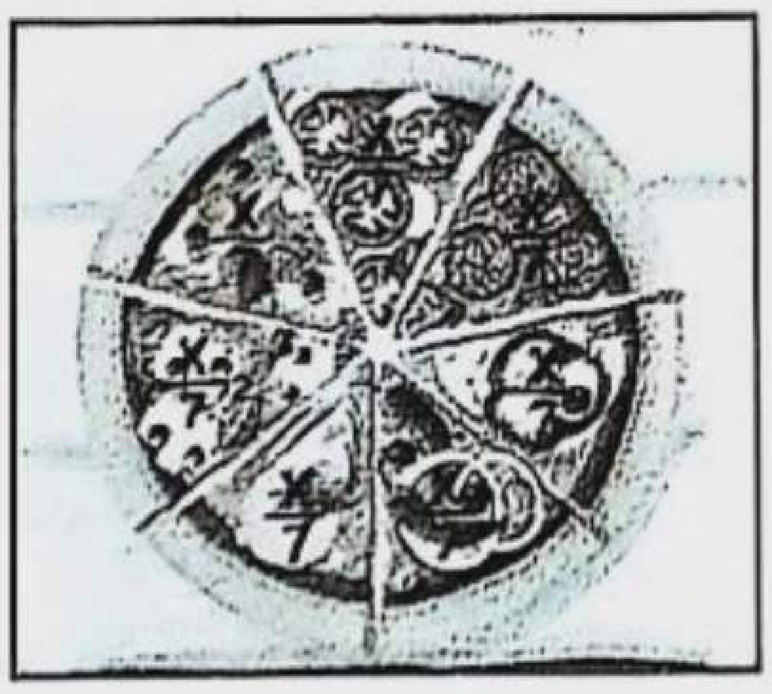
Chapter 05

Algebraic Fractions

Contents for Discussion

• Algebraic Fraction • Lowest From of Fraction • Fractions in the form of common denominator • Addition of Fractions • Subtraction of fractions • Multiplication of fractions • Division of fractions.



Learning Outcomes : After studying this chapter, I will be able to–

- Add, subtract, multiply and divide the algebraic fractions, simplify and solve the problems related to fractions.

Practice

Solutions to Mathematical Problems following 100% accurate format for best prep.

Dear learners, mathematical problems of this chapter have been divided into exercise, multiple choice, short, creative and exercise-based activities in light of the learning outcomes. Practice the solutions well to ensure the best preparation in the exam.

Exercise 5.1 : Algebraic Fraction

At a Glance Important Contents of Exercise

- **Algebraic Fraction :** If m and n are two algebraic expressions, $\frac{m}{n}$ is an algebraic fraction where $n \neq 0$.

  Here, m is called numerator and n is called denominator of the fraction $\frac{m}{n}$.

- **Lowest From of Fraction :** If there are common factors of both numerator and denominator of any algebraic fraction, and if numerator and denominator are divided by H.C.F. of the numerator and the denominator of the fraction, a new fraction is formed and the new fraction is called lowest form of the fraction.

Solutions to Exercise Problems — Let's solve the textbook problems

Solutions to Mathematical Problems

1. Express the following fractions in the lowest form :

(a) $\frac{4x^2y^3z^5}{9x^5y^2z^3}$

Solution : We have, $\frac{4x^2y^3z^5}{9x^5y^2z^3}$

$$= \frac{2 \times 2 \times x^2 \times y^2 \times y \times z^3 \times z^2}{3 \times 3 \times x^2 \times x^3 \times y^2 \times z^3}$$

$$= \frac{4yz^2}{9x^3}$$

(b) $\frac{16(2x)^4 (3y)^5}{(3x)^3 . (2y)^6}$

Solution : We have, $\frac{16(2x)^4 (3y)^5}{(3x)^3 . (2y)^6}$

$$= \frac{16.2^4.x^4.3^5.y^5}{3^3.x^3.2^6.y^6} = \frac{2^4.2^4.3^5.x^4.y^5}{3^3.2^6.x^3.y^6}$$

$$= \frac{2^8.3^5.x^3.x.y^5}{2^6.3^3.x^3.y^5.y} = \frac{2^6.2^2.3^3.3^2.x}{2^6.3^3.y} = \frac{2^2.3^2.x}{y}$$

$$= \frac{4.9x}{y} = \frac{36x}{y}$$

(c) $\frac{x^3y + xy^3}{x^2y^3 + x^3y^2}$

Solution : We have, $\frac{x^3y + xy^3}{x^2y^3 + x^3y^2} = \frac{xy(x^2 + y^2)}{x^2y^2(x + y)}$

$= \frac{xy(x^2 + y^2)}{xy.xy\ (x + y)} = \frac{x^2 + y^2}{xy(x + y)}$

(d) $\frac{(a - b)\ (a + b)}{a^3 - b^3}$

Solution : We have, $\frac{(a - b)\ (a + b)}{a^3 - b^3}$.

$= \frac{(a - b)\ (a + b)}{(a - b)\ (a^2 + ab + b^2)} = \frac{a + b}{a^2 + ab + b^2}$

(e) $\frac{x^2 - 6x + 5}{x^2 - 25}$

Solution : We have, $\frac{x^2 - 6x + 5}{x^2 - 25}$

$= \frac{x^2 - 5x - x + 5}{(x + 5)\ (x - 5)} = \frac{x(x - 5) - 1\ (x - 5)}{(x + 5)\ (x - 5)}$

$= \frac{(x - 5)\ (x - 1)}{(x + 5)\ (x - 5)} = \frac{x - 1}{x + 5}$

(f) $\frac{x^2 - 7x + 12}{x^2 - 9x + 20}$

Solution : We have, $\frac{x^2 - 7x + 12}{x^2 - 9x + 20}$

$= \frac{x^2 - 3x - 4x + 12}{x^2 - 4x - 5x + 20} = \frac{x(x - 3) - 4(x - 3)}{x(x - 4) - 5(x - 4)}$

$= \frac{(x - 3)\ (x - 4)}{(x - 4)\ (x - 5)} = \frac{x - 3}{x - 5}$

(g) $\frac{(x^3 - y^3)\ (x^2 - xy + y^2)}{(x^2 - y^2)\ (x^3 + y^3)}$

Solution : We have, $\frac{(x^3 - y^3)\ (x^2 - xy + y^2)}{(x^2 - y^2)\ (x^3 + y^3)}$

$= \frac{(x - y)\ (x^2 + xy + y^2)\ (x^2 - xy + y^2)}{(x + y)\ (x - y)\ (x + y)\ (x^2 - xy + y^2)}$

$= \frac{x^2 + xy + y^2}{(x + y)^2}$

(h) $\frac{a^2 - b^2 - 2bc - c^2}{a^2 + 2ab + b^2 - c^2}$

Solution : We have, $\frac{a^2 - b^2 - 2bc - c^2}{a^2 + 2ab + b^2 - c^2}$

$= \frac{a^2 - (b^2 + 2bc + c^2)}{(a^2 + 2ab + b^2) - c^2} = \frac{a^2 - (b + c)^2}{(a + b)^2 - c^2}$

$= \frac{(a + b + c)\ (a - b - c)}{(a + b + c)\ (a + b - c)} = \frac{a - b - c}{a + b - c}$

2. **Express the following fractions in the form of a common denominator :**

(a) $\frac{x^2}{xy}, \frac{y^2}{yz}, \frac{z^2}{zx}$

Solution : Here the LCM of the denominators of the given fractions is xyz.

$\therefore\ \frac{x^2}{xy} = \frac{x^2 \times z}{xy \times z} = \frac{x^2z}{xyz}$, multiplying both numerator and denominator by z.

$\therefore\ \frac{y^2}{yz} = \frac{x \times y^2}{x \times yz} = \frac{xy^2}{xyz}$, multiplying both numerator and denominator by x.

$\therefore\ \frac{z^2}{zx} = \frac{y \times z^2}{y \times zx} = \frac{yz^2}{xyz}$, multiplying both numerator and denominator by y.

$\therefore$ The required fractions with common denominators are $\frac{x^2z}{xyz}, \frac{xy^2}{xyz}$ and $\frac{yz^2}{xyz}$

(b) $\frac{x - y}{xy}, \frac{y - z}{yz}, \frac{z - x}{zx}$

Solution : Here the LCM of the denominators of the given fractions is xyz.

$\therefore\ \frac{x - y}{xy} = \frac{(x - y) \times z}{xy \times z} = \frac{zx - yz}{xyz}$, multiplying numerator and denominator by z.

$\therefore\ \frac{y - z}{yz} = \frac{x \times (y - z)}{x \times yz} = \frac{xy - zx}{xyz}$, multiplying numerator and denominator by x.

$\therefore\ \frac{z - x}{zx} = \frac{(z - x) \times y}{zx \times y} = \frac{yz - xy}{xyz}$, multiplying numerator and denominator by y.

$\therefore$ The required fractions with common denominators are $\frac{zx - yz}{xyz}, \frac{xy - zx}{xyz}$ and $\frac{yz - xy}{xyz}$

(c) $\frac{x}{x - y}, \frac{y}{x + y}, \frac{z}{x(x + y)}$

Solution : Here, the LCM of the denominators of the given fractions is $x(x + y)\ (x - y)$.

$\therefore\ \frac{x}{x - y} = \frac{x \times x(x + y)}{(x - y) \times x(x + y)} = \frac{x^2(x + y)}{x(x + y)\ (x - y)}$, multiplying both numerator and denominator by $x(x + y)$.

$\therefore\ \frac{y}{x + y} = \frac{y \times x\ (x - y)}{(x + y) \times x(x - y)} = \frac{xy(x - y)}{x(x + y)\ (x - y)}$, multiplying both numerator and denominator by $x(x - y)$.

$\therefore\ \frac{z}{x(x + y)} = \frac{z \times (x - y)}{x(x + y) \times (x - y)} = \frac{z(x - y)}{x(x + y)\ (x - y)}$, multiplying both numerator and denominator by $x - y$.

$\therefore$ The required fractions with common denominators are $\frac{x^2(x + y)}{x(x + y)\ (x - y)}, \frac{xy(x - y)}{x(x + y)\ (x - y)}$ and $\frac{z(x - y)}{x(x + y)\ (x - y)}$

Mathematics

(d) $\dfrac{x+y}{(x-y)^2}, \dfrac{x-y}{x^3+y^3}, \dfrac{y-z}{x^2-y^2}$

Solution : Here the LCM of the denominators of the given fractions is $(x-y)^2(x+y)(x^2-xy+y^2)$.

$\therefore \dfrac{x+y}{(x-y)^2} = \dfrac{(x+y)\times(x+y)(x^2-xy+y^2)}{(x-y)^2\times(x+y)(x^2-xy+y^2)}$

$= \dfrac{(x+y)(x^3+y^3)}{(x-y)^2(x^3+y^3)}$, multiplying both numerator and denominator by $(x+y)(x^2-xy+y^2)$

$\therefore \dfrac{x-y}{x^3+y^3} = \dfrac{(x-y)\times(x-y)^2}{(x^3+y^3)(x-y)^2} = \dfrac{(x-y)^3}{(x-y)^2(x^3+y^3)}$, multiplying both numerator and denominator by $(x-y)^2$.

$\therefore \dfrac{y-z}{x^2-y^2} = \dfrac{(y-z)(x-y)(x^2-xy+y^2)}{(x^2-y^2)(x-y)(x^2-xy+y^2)}$

$= \dfrac{(y-z)(x-y)(x^2-xy+y^2)}{(x-y)^2(x^3+y^3)}$, multiplying both numerator and denominator by $(x-y)(x^2-xy+y^2)$

$\therefore$ The required fractions with common denominators are $\dfrac{(x+y)(x^3+y^3)}{(x-y)^2(x^3+y^3)}, \dfrac{(x-y)^3}{(x-y)^2(x^3+y^3)}$ and $\dfrac{(y-z)(x-y)(x^2-xy+y^2)}{(x-y)^2(x^3+y^3)}$

(e) $\dfrac{a}{a^3+b^3}, \dfrac{b}{(a^2+ab+b^2)}, \dfrac{c}{a^3-b^3}$

Solution : Here, the LCM of the denominators of the given fractions a^3+b^3, (a^2+ab+b^2) and a^3-b^3 is $(a+b)(a-b)(a^2+ab+b^2)(a^2-ab+b^2)$

$\therefore \dfrac{a}{a^3+b^3}$

$= \dfrac{a}{(a+b)(a^2-ab+b^2)}$

$= \dfrac{a(a-b)(a^2+ab+b^2)}{(a+b)(a-ab+b^2)(a-b)(a^2+ab+b^2)}$

$= \dfrac{a(a^3-b^3)}{(a^3+b^3)(a^3-b^3)}$

and $\dfrac{b}{a^2+ab+b^2}$

$= \dfrac{b\times(a+b)(a-b)(a^2-ab+b^2)}{(a^2+ab+b^2)(a+b)(a-b)(a^2-ab+b^2)}$

$= \dfrac{b(a-b)(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}$

and $\dfrac{c}{a^3-b^3}$

$= \dfrac{c}{(a-b)(a^2+ab+b^2)}$

$= \dfrac{c(a+b)(a^2-ab+b^2)}{(a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)}$

$= \dfrac{c(a^3+b^3)}{(a^3+b^3)(a^3-b^3)} = \dfrac{c(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}$

$\therefore$ The required fractions with common denominators are $\dfrac{a(a^3-b^3)}{(a^3+b^3)(a^3-b^3)}, \dfrac{b(a-b)(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}$ and $\dfrac{c(a^3+b^3)}{(a^3+b^3)(a^3-b^3)}$

(f) $\dfrac{1}{x^2-5x+6}, \dfrac{1}{x^2-7x+12}, \dfrac{1}{x^2-9x+20}$

Solution : Here, the denominator of the 1st fraction,

$x^2-5x+6 = x^2-3x-2x+6$
$= x(x-3)-2(x-3)$
$= (x-3)(x-2)$

the denominator of the 2nd fraction,

$x^2-7x+12 = x^2-4x-3x+12$
$= x(x-4)-3(x-4)$
$= (x-4)(x-3)$

the denominator of the 3rd fraction,

$x^2-9x+20 = x^2-5x-4x+20$
$= x(x-5)-4(x-5)$
$= (x-5)(x-4)$

$\therefore$ The LCM of the denominators of the fractions is $(x-2)(x-3)(x-4)(x-5)$.

So, $\dfrac{1}{x^2-5x+6} = \dfrac{1}{(x-2)(x-3)}$

$= \dfrac{1\times(x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)}$

$= \dfrac{(x-4)(x-5)}{(x^2-5x+6)(x^2-9x+20)}$

and $\dfrac{1}{x^2-7x+12} = \dfrac{1}{(x-3)(x-4)}$

$= \dfrac{1\times(x-2)(x-5)}{(x-3)(x-4)(x-2)(x-5)}$

$= \dfrac{(x-2)(x-5)}{(x^2-5x+6)(x^2-9x+20)}$

and $\dfrac{1}{x^2-9x+20} = \dfrac{1}{(x-4)(x-5)}$

$= \dfrac{1\times(x-2)(x-3)}{(x-4)(x-5)(x-2)(x-3)}$

$= \dfrac{(x-2)(x-3)}{(x^2-5x+6)(x^2-9x+20)}$

$\therefore$ The required fractions with common denominators are $\dfrac{(x-4)(x-5)}{(x^2-5x+6)(x^2-9x+20)}, \dfrac{(x-2)(x-5)}{(x^2-5x+6)(x^2-9x+20)}$ and $\dfrac{(x-2)(x-3)}{(x^2-5x+6)(x^2-9x+20)}$

(g) $\frac{a-b}{a^2b^2}, \frac{b-c}{b^2c^2}, \frac{c-a}{c^2a^2}$

Solution : The LCM of the denominators of the given fractions is $a^2b^2c^2$.

$\therefore \frac{a-b}{a^2b^2} = \frac{(a-b) \times c^2}{a^2b^2 \times c^2} = \frac{c^2(a-b)}{a^2b^2c^2}$

and $\frac{b-c}{b^2c^2} = \frac{(b-c) \times a^2}{b^2c^2 \times a^2} = \frac{a^2(b-c)}{a^2b^2c^2}$

and $\frac{c-a}{c^2a^2} = \frac{(c-a) \times b^2}{c^2a^2 \times b^2} = \frac{b^2(c-a)}{a^2b^2c^2}$

$\therefore$ The required fractions with common denominators are $\frac{c^2(a-b)}{a^2b^2c^2}, \frac{a^2(b-c)}{a^2b^2c^2}$ and $\frac{b^2(c-a)}{a^2b^2c^2}$.

(h) $\frac{x-y}{x+y}, \frac{y-z}{y+z}, \frac{z-x}{z+x}$

Solution : The LCM of the denominators of the fractions is $(x+y)(y+z)(z+x)$.

$\therefore \frac{x-y}{x+y} = \frac{(x-y) \times (y+z)(z+x)}{(x+y) \times (y+z)(z+x)}$

$= \frac{(x-y)(y+z)(z+x)}{(x+y)(y+z)(z+x)}$,

and $\frac{y-z}{y+z} = \frac{(y-z) \times (x+y)(z+x)}{(y+z) \times (x+y)(z+x)}$

$= \frac{(x+y)(y-z)(z+x)}{(x+y)(y+z)(z+x)}$

and $\frac{z-x}{z+x} = \frac{(z-x) \times (x+y)(y+z)}{(z+x) \times (x+y)(y+z)}$

$= \frac{(z-x)(x+y)(y+z)}{(x+y)(y+z)(z+x)}$,

$\therefore$ The required fractions with common denominators are $\frac{(x-y)(y+z)(z+x)}{(x+y)(y+z)(z+x)}, \frac{(y-z)(x+y)(z+x)}{(x+y)(y+z)(z+x)}$ and $\frac{(z-x)(x+y)(y+z)}{(x+y)(y+z)(z+x)}$.

3. Find the sum :

(a) $\frac{a-b}{a} + \frac{a+b}{b}$

Solution : $\frac{a-b}{a} + \frac{a+b}{b} = \frac{b(a-b) + a(a+b)}{ab}$

$= \frac{ab - b^2 + a^2 + ab}{ab}$

$= \frac{a^2 + 2ab - b^2}{ab}$

(b) $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab}$

Solution : $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} = \frac{a \times a + b \times b + c \times c}{abc}$

$= \frac{a^2 + b^2 + c^2}{abc}$

(c) $\frac{x-y}{x} + \frac{y-z}{y} + \frac{z-x}{z}$

Solution : $\frac{x-y}{x} + \frac{y-z}{y} + \frac{z-x}{z}$

$= \frac{yz(x-y) + xz(y-z) + xy(z-x)}{xyz}$

$= \frac{xyz - y^2z + xyz - xz^2 + xyz - x^2y}{xyz}$

$= \frac{3xyz - x^2y - y^2z - z^2x}{xyz}$

(d) $\frac{x+y}{x-y} + \frac{x-y}{x+y}$

Solution : $\frac{x+y}{x-y} + \frac{x-y}{x+y} = \frac{(x+y)(x+y) + (x-y)(x-y)}{(x-y)(x+y)}$

$= \frac{x^2 + 2xy + y^2 + x^2 - 2xy + y^2}{x^2 - y^2}$

$= \frac{2x^2 + 2y^2}{x^2 - y^2} = \frac{2(x^2 + y^2)}{x^2 - y^2}$

(e) $\frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 4}$

Solution : $\frac{1}{x^2 - 3x + 2} + \frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 4}$

$= \frac{1}{(x-1)(x-2)} + \frac{1}{(x-1)(x-3)} + \frac{1}{(x-1)(x-4)}$

$= \frac{(x-3)(x-4) + (x-2)(x-4) + (x-2)(x-3)}{(x-1)(x-2)(x-3)(x-4)}$

$= \frac{x^2 - 7x + 12 + x^2 - 6x + 8 + x^2 - 5x + 6}{(x-1)(x-2)(x-3)(x-4)}$

$= \frac{3x^2 - 18x + 26}{(x-1)(x-2)(x-3)(x-4)}$

(f) $\frac{1}{a^2 - b^2} + \frac{1}{a^2 + ab + b^2} + \frac{1}{a^2 - ab + b^2}$

Solution : $\frac{1}{a^2 - b^2} + \frac{1}{a^2 + ab + b^2} + \frac{1}{a^2 - ab + b^2}$

$= \frac{(a^2 + ab + b^2)(a^2 - ab + b^2) + (a^2 - b^2)(a^2 - ab + b^2) + (a^2 - b^2)(a^2 + ab + b^2)}{(a^2 - b^2)(a^2 + ab + b^2)(a^2 - ab + b^2)}$

$= \frac{a^4 + 2a^2b^2 + b^4 - a^2b^2 + a^4 - a^3b + a^2b^2 - a^2b^2 + ab^3 - b^4 + a^4 + a^3b + a^2b^2 - a^2b^2 - ab^3 - b^4}{(a^3 + b^3)(a^3 - b^3)}$

$= \frac{a^4 + 2a^2b^2 + b^4 - a^2b^2 + a^4 - a^3b + a^2b^2 + ab^3 - b^4 + a^4 + a^3b + a^2b^2 - a^2b^2 - ab^3 - b^4}{(a^3 + b^3)(a^3 - b^3)}$

$= \frac{3a^4 + a^2b^2 - b^4}{(a^3 + b^3)(a^3 - b^3)}$

(g) $\frac{1}{x-2}-\frac{1}{x+2}+\frac{4}{x^2-4}$

Solution : $\frac{1}{x-2}-\frac{1}{x+2}+\frac{4}{x^2-4}$

$=\frac{x+2-x+2}{(x-2)(x+2)}+\frac{4}{x^2-4}$

$=\frac{4}{x^2-4}+\frac{4}{x^2-4}$

$=\frac{4+4}{x^2-4}=\frac{8}{x^2-4}$

(h) $\frac{1}{x^2-1}+\frac{1}{x^4-1}+\frac{4}{x^8-1}$

Solution : $\frac{1}{x^2-1}+\frac{1}{x^4-1}+\frac{4}{x^8-1}$

$=\frac{1}{x^2-1}+\frac{1}{(x^2+1)(x^2-1)}+\frac{4}{x^8-1}$

$=\frac{x^2+1+1}{(x^2-1)(x^2+1)}+\frac{4}{x^8-1}$

$=\frac{x^2+2}{x^4-1}+\frac{4}{(x^4-1)(x^4+1)}$

$=\frac{(x^2+2)(x^4+1)+4}{(x^4-1)(x^4+1)}$

$=\frac{x^6+x^2+2x^4+2+4}{x^8-1}$

$=\frac{x^6+2x^4+x^2+6}{x^8-1}$

4. Find the difference :

(a) $\frac{a}{x-3}-\frac{a^2}{x^2-9}$

Solution : $\frac{a}{x-3}-\frac{a^2}{x^2-9}=\frac{a}{x-3}-\frac{a^2}{(x+3)(x-3)}$

$=\frac{a(x+3)-a^2}{(x-3)(x+3)}$

$=\frac{ax+3a-a^2}{(x-3)(x+3)}$

$=\frac{ax+3a-a^2}{x^2-9}$

(b) $\frac{1}{y(x-y)}-\frac{1}{x(x+y)}$

Solution : $\frac{1}{y(x-y)}-\frac{1}{x(x+y)}=\frac{x(x+y)-y(x-y)}{xy(x+y)(x-y)}$

$=\frac{x^2+xy-xy+y^2}{xy(x+y)(x-y)}$

$=\frac{x^2+y^2}{xy(x^2-y^2)}$

(c) $\frac{x+1}{1+x+x^2}-\frac{x-1}{1-x+x^2}$

Solution : $\frac{x+1}{1+x+x^2}-\frac{x-1}{1-x+x^2}$

$=\frac{(x+1)(1-x+x^2)-(x-1)(1+x+x^2)}{(1+x+x^2)(1-x+x^2)}$

$=\frac{x^3+1-(x^3-1)}{1+x^2+x^4}$

$=\frac{x^3+1-x^3+1}{1+x^2+x^4}=\frac{2}{1+x^2+x^4}$

(d) $\frac{a^2+16b^2}{a^2-16b^2}-\frac{a-4b}{a+4b}$

Solution : $\frac{a^2+16b^2}{a^2-16b^2}-\frac{a-4b}{a+4b}$

$=\frac{a^2+16b^2}{(a+4b)(a-4b)}-\frac{a-4b}{a+4b}$

$=\frac{a^2+16b^2-(a-4b)(a-4b)}{(a+4b)(a-4b)}$

$=\frac{a^2+16b^2-(a^2-8ab+16b^2)}{a^2-16b^2}$

$=\frac{a^2+16b^2-a^2+8ab-16b^2}{a^2-16b^2}=\frac{8ab}{a^2-16b^2}$

(e) $\frac{1}{x-y}-\frac{x^2-xy+y^2}{x^3+y^3}$

Solution : $\frac{1}{x-y}-\frac{x^2-xy+y^2}{x^3+y^3}$

$=\frac{1}{x-y}-\frac{x^2-xy+y^2}{(x+y)(x^2-xy+y^2)}$

$=\frac{(x+y)(x^2-xy+y^2)-(x-y)(x^2-xy+y^2)}{(x-y)(x+y)(x^2-xy+y^2)}$

$=\frac{x^3+y^3-(x^3-x^2y+xy^2-x^2y+xy^2-y^3)}{(x-y)(x^3+y^3)}$

$=\frac{x^3+y^3-x^3+x^2y-xy^2+x^2y-xy^2+y^3}{(x-y)(x^3+y^3)}$

$=\frac{2x^2y-2xy^2+2y^3}{(x-y)(x^3+y^3)}$

$=\frac{2y(x^2-xy+y^2)}{(x-y)(x+y)(x^2-xy+y^2)}=\frac{2y}{x^2-y^2}$

5. Simplify :

(a) $\frac{x-y}{xy}+\frac{y-z}{yz}+\frac{z-x}{zx}$

Solution : $\frac{x-y}{xy}+\frac{y-z}{yz}+\frac{z-x}{zx}$

$=\frac{z(x-y)+x(y-z)+y(z-x)}{xyz}$

$=\frac{zx-yz+xy-zx+yz-xy}{xyz}$

$=\frac{0}{xyz}=0$

(b) $\frac{x-y}{(x+y)(y+z)}+\frac{y-z}{(y+z)(z+x)}+\frac{z-x}{(z+x)(x+y)}$

Solution : $\frac{x-y}{(x+y)(y+z)}+\frac{y-z}{(y+z)(z+x)}+\frac{z-x}{(z+x)(x+y)}$

$=\frac{(x-y)(z+x)+(y-z)(x+y)+(z-x)(y+z)}{(x+y)(y+z)(z+x)}$

$=\frac{zx+x^2-yz-xy+xy+y^2-zx-yz+yz+z^2-xy-zx}{(x+y)(y+z)(z+x)}$

$=\frac{x^2+y^2+z^2-xy-yz-zx}{(x+y)(y+z)(z+x)}$

(c) $\frac{y}{(x-y)(y-z)}+\frac{x}{(z-x)(x-y)}+\frac{z}{(y-z)(z-x)}$

Solution : $\frac{y}{(x-y)(y-z)}+\frac{x}{(z-x)(x-y)}+\frac{z}{(y-z)(z-x)}$

$=\frac{y(z-x)+x(y-z)+z(x-y)}{(x-y)(y-z)(z-x)}$

$=\frac{yz-xy+xy-zx+zx-yz}{(x-y)(y-z)(z-x)}$

$=\frac{0}{(x-y)(y-z)(z-x)}=0.$

(d) $\frac{1}{x+3y}+\frac{1}{x-3y}-\frac{2x}{x^2-9y^2}$

Solution : $\frac{1}{x+3y}+\frac{1}{x-3y}-\frac{2x}{x^2-9y^2}$

$=\frac{1}{x+3y}+\frac{1}{x-3y}-\frac{2x}{(x+3y)(x-3y)}$

$=\frac{x-3y+x+3y-2x}{(x+3y)(x-3y)}$

$=\frac{2x-2x}{(x+3y)(x-3y)}=0$

(e) $\frac{1}{x-y}-\frac{2}{2x+y}+\frac{1}{x+y}-\frac{2}{2x-y}$

Solution : $\frac{1}{x-y}-\frac{2}{2x+y}+\frac{1}{x+y}-\frac{2}{2x-y}$

$=\frac{1}{x-y}+\frac{1}{x+y}-\frac{2}{2x+y}-\frac{2}{2x-y}$

$=\frac{x+y+x-y}{(x-y)(x+y)}-\left(\frac{2(2x-y)+2(2x+y)}{(2x+y)(2x-y)}\right)$

$=\frac{2x}{(x+y)(x-y)}-\frac{4x-2y+4x+2y}{(2x+y)(2x-y)}$

$=\frac{2x}{(x+y)(x-y)}-\frac{8x}{(2x+y)(2x-y)}$

$=\frac{2x(2x+y)(2x-y)-8x(x+y)(x-y)}{(x+y)(x-y)(2x+y)(2x-y)}$

$=\frac{8x^3-2xy^2-8x^3+8xy^2}{(x^2-y^2)(4x^2-y^2)}=\frac{6xy^2}{(x^2-y^2)(4x^2-y^2)}$

(f) $\frac{1}{x-2}-\frac{x-2}{x^2+2x+4}+\frac{6x}{x^3+8}$

Solution : $\frac{1}{x-2}-\frac{x-2}{x^2+2x+4}+\frac{6x}{x^3+8}$

$=\frac{x^2+2x+4-(x-2)(x-2)}{(x-2)(x^2+2x+4)}+\frac{6x}{x^3+8}$

$=\frac{x^2+2x+4-x^2+4x-4}{x^3-8}+\frac{6x}{x^3+8}$

$=\frac{6x}{x^3-8}+\frac{6x}{x^3+8}=\frac{6x(x^3+8)+6x(x^3-8)}{(x^3-8)(x^3+8)}$

$=\frac{6x^4+48x+6x^4-48x}{x^6-64}=\frac{12x^4}{x^6-64}$

(g) $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}+\frac{4}{x^4+1}$

Solution : $\frac{1}{x-1}-\frac{1}{x+1}-\frac{2}{x^2+1}+\frac{4}{x^4+1}$

$=\frac{x+1-x+1}{(x-1)(x+1)}-\frac{2}{x^2+1}+\frac{4}{x^4+1}$

$=\frac{2}{x^2-1}-\frac{2}{x^2+1}+\frac{4}{x^4+1}$

$=\frac{2(x^2+1)-2(x^2-1)}{(x^2-1)(x^2+1)}+\frac{4}{x^4+1}$

$=\frac{2x^2+2-2x^2+2}{x^4-1}+\frac{4}{x^4+1}$

$=\frac{4}{x^4-1}+\frac{4}{x^4+1}=\frac{4(x^4+1)+4(x^4-1)}{(x^4-1)(x^4+1)}$

$=\frac{4x^4+4+4x^4-4}{x^8-1}=\frac{8x^4}{x^8-1}$

(h) $\frac{x-y}{(y-z)(z-x)}+\frac{y-z}{(z-x)(x-y)}+\frac{z-x}{(x-y)(y-z)}$

Solution : $\frac{x-y}{(y-z)(z-x)}+\frac{y-z}{(z-x)(x-y)}+\frac{z-x}{(x-y)(y-z)}$

$=\frac{(x-y)(x-y)+(y-z)(y-z)+(z-x)(z-x)}{(x-y)(y-z)(z-x)}$

$=\frac{x^2-2xy+y^2+y^2-2yz+z^2+z^2-2zx+x^2}{(x-y)(y-z)(z-x)}$

$=\frac{2x^2+2y^2+2z^2-2xy-2yz-2zx}{(x-y)(y-z)(z-x)}$

$=\frac{2(x^2+y^2+z^2-xy-yz-zx)}{(x-y)(y-z)(z-x)}$

(i) $\frac{1}{a-b-c}+\frac{1}{a-b+c}+\frac{a}{a^2+b^2-c^2-2ab}$

Solution : $\frac{1}{a-b-c}+\frac{1}{a-b+c}+\frac{a}{a^2+b^2-c^2-2ab}$

$=\frac{a-b+c+a-b-c}{(a-b-c)(a-b+c)}+\frac{a}{a^2+b^2-c^2-2ab}$

Mathematics

$$= \frac{2a-2b}{\{(a-b)-c\}\ \{(a-b)+c\}} + \frac{a}{a^2+b^2-c^2-2ab}$$

$$= \frac{2a-2b}{(a-b)^2-c^2} + \frac{a}{a^2+b^2-c^2-2ab}$$

$$= \frac{2a-2b}{a^2-2ab+b^2-c^2} + \frac{a}{a^2+b^2-c^2-2ab}$$

$$= \frac{2a-2b+a}{a^2-2ab+b^2-c^2} = \frac{3a-2b}{a^2-2ab+b^2-c^2}$$

(j) $\frac{1}{a^2+b^2-c^2+2ab} + \frac{1}{b^2+c^2-a^2+2bc} + \frac{1}{c^2+a^2-b^2+2ca}$

Solution : $\frac{1}{a^2+b^2-c^2+2ab} + \frac{1}{b^2+c^2-a^2+2bc} + \frac{1}{c^2+a^2-b^2+2ca}$

$$= \frac{1}{a^2+2ab+b^2-c^2} + \frac{1}{b^2+2bc+c^2-a^2} + \frac{1}{c^2+2ca+a^2-b^2}$$

$$= \frac{1}{(a+b)^2-c^2} + \frac{1}{(b+c)^2-a^2} + \frac{1}{(c+a)^2-b^2}$$

$$= \frac{1}{(a+b-c)(a+b+c)} + \frac{1}{(b+c-a)(a+b+c)} + \frac{1}{(c+a-b)(a+b+c)}$$

$$= \frac{(b+c-a)(c+a-b)+(a+b-c)(c+a-b)+(a+b-c)(b+c-a)}{(a+b-c)(b+c-a)(c+a-b)(a+b+c)}$$

From numerator we have,

$$= (b+c-a)(c+a-b)+(a+b-c)(c+a-b) + (a+b-c)(b+c-a)$$

$$= bc+ab-b^2+c^2+ca-bc-ca-a^2+ab+ca + a^2-ab+bc+ab-b^2-c^2-ca+bc+ab + ca-a^2+b^2+bc-ab-bc-c^2+ca$$

$$= 2ab+2bc+2ca-a^2-b^2-c^2$$

$\therefore$ The given expression =

$$\frac{2ab+2bc+2ca-b^2-c^2-a^2}{(a+b-c)(b+c-a)(c+a-b)(a+b+c)}$$

Multiple Choice Q/A — Designed as per topic

5.1 & 5.2 Algebraic Fraction and Lowest From of Fraction ▸ Textbook Page 77

1. **What is called a broken part of something whole?** *(Easy)*
   ⓐ Factor ⓑ Fraction ⓒ Friction ⓓ Fracture
   Ans: ⓑ

2. **What is called the fraction if the numerator or the denominator or both them of any fraction is expressed by algebraic letter symbol?** *(Easy)*
   ⓐ Arithmetic fraction ⓑ Digit ⓒ Algebraic fraction ⓓ Algebraic equation
   Ans: ⓒ

3. **Which one of the following is an equivalent fraction of $\frac{3}{4}$?** *(Medium)*
   ⓐ $\frac{6}{12}$ ⓑ $\frac{6}{9}$ ⓒ $\frac{12}{16}$ ⓓ $\frac{12}{15}$
   Ans: ⓒ

4. **What type of fraction is $\frac{a}{2}$?** *(Medium)*
   ⓐ Arithmetic fraction ⓑ Decimal fraction ⓒ Algebraic Fraction ⓓ Mixed fraction
   Ans: ⓒ

5. **Which is the lowest term of $\frac{a^5b^3c^4}{a^3b^5c^2}$?** *(Medium)*
   ⓐ $\frac{a^2b^2c^2}{b}$ ⓑ $\frac{a^3b^2c^2}{a^2b^2}$ ⓒ $\frac{a^2c^2}{b^2}$ ⓓ $\frac{a^3c^4}{b^2c^2}$
   Ans: ⓒ

6. **What is the lowest term of $\frac{x^2+x-6}{x^2-9}$?** *(Hard)*
   ⓐ $\frac{x-2}{x+3}$ ⓑ $\frac{x+3}{x-3}$ ⓒ $\frac{x-2}{x-3}$ ⓓ $\frac{x+2}{x+3}$
   Ans: ⓒ

7. **What is the lowest form of the fraction $\frac{a^2+6a+5}{a^2+10a+25}$?** *(Medium)* [DjB '19]
   ⓐ $\frac{a+1}{a+5}$ ⓑ $\frac{a+3}{a+5}$ ⓒ $\frac{a-1}{a-5}$ ⓓ $\frac{a+3}{a-5}$
   Ans: ⓐ

8. **$\frac{x^2+5x-14}{x^2+3x-28}$ which one of the following is the lowest value of the fraction?** *(Hard)* [MB '19]
   ⓐ $\frac{x+7}{x-4}$ ⓑ $\frac{x-2}{x+7}$ ⓒ $\frac{x+2}{x-4}$ ⓓ $\frac{x-2}{x-4}$
   Ans: ⓓ

9. **Which is the lowest form of $\frac{x^3+3x^4}{x+3x^2}$?** *(Hard)* [DB '18]
   ⓐ x^2 ⓑ $x+1$ ⓒ x ⓓ x^3
   Ans: ⓐ

10. **Which one is the lowest form of $\frac{x^2-x-6}{x^2-5x+6}$** *(Hard)* [RB '18]
    ⓐ $\frac{x-3}{x-2}$ ⓑ $\frac{x-2}{x-3}$ ⓒ $\frac{x+2}{x-2}$ ⓓ $\frac{x-2}{x+2}$
    Ans: ⓒ

11. **Which one of the following is the lowest value of $\frac{x^2-7x+12}{x^2-6x+9}$?** *(Hard)* [CB '18]
    ⓐ $\frac{x-4}{x-3}$ ⓑ $\frac{x+4}{x-3}$ ⓒ $\frac{x-4}{x+3}$ ⓓ $\frac{x+4}{x+3}$
    Ans: ⓐ

12. **Which one of the following is the lowest value of $\frac{x^3+3x^2}{x^2-9}$?** *(Easy)* [BB '18]
    ⓐ $\frac{x^2}{x-3}$ ⓑ $\frac{x^2}{x+3}$ ⓒ $\frac{x}{x-3}$ ⓓ $\frac{x+3}{x-3}$
    Ans: ⓐ

Mathematics

13. Which is the lowest form of $\frac{x^2-6x+9}{x^2-9}$. *(Medium) [DjB '18]*

ⓐ $\frac{x-3}{x+3}$ ⓑ $\frac{(x-3)^2}{x^2-9}$

a ⓒ $\frac{x+3}{x-3}$ ⓓ $\frac{(x+3)^2}{x^2-9}$

14. Which one of the following is the lowest form of $\frac{x^2-1}{x+1}$? *(Easy) [DB '17]*

c ⓐ x ⓑ $x+1$ ⓒ $x-1$ ⓓ x^2+1

15. Which one of the following is the lowest value of $\frac{x^2-7x+12}{x^2-9x+20}$? *(Medium) [RB '17]*

ⓐ $\frac{x-3}{x-5}$ ⓑ $\frac{x-1}{x-5}$

a ⓒ $\frac{x-2}{x-3}$ ⓓ $\frac{x-2}{x-5}$

16. Which is the lowest form of $\frac{x^2-7x+12}{x^2-9x+20}$ *(Easy) [JB '17]*

ⓐ $\frac{x-3}{x-5}$ ⓑ $\frac{x-4}{x-5}$

a ⓒ $\frac{x-4}{x-3}$ ⓓ $\frac{x-5}{x-3}$

17. Which is the lowest form of $\frac{a^2-6a+5}{a^2-25}$? *(Medium) [SB '17]*

a ⓐ $\frac{a-1}{a+5}$ ⓑ $\frac{a+5}{a-1}$ ⓒ $\frac{a-5}{a+5}$ ⓓ $\frac{a-3}{a+5}$

18. Which one of the following is the lowest form of $\frac{x^3y-xy^3}{x^2y^3+x^3y^2}$? *(Hard) [CB '16]*

ⓐ $\frac{1}{xy}$ ⓑ $\frac{x-y}{xy}$

b ⓒ $\frac{x+y}{xy}$ ⓓ $\frac{x^2-y^2}{xy}$

19. Which is the lowest form of $\frac{x^2-6x+5}{x^2+4x-45}$? *(Easy) [DjB '16]*

ⓐ $\frac{x+1}{x+9}$ ⓑ $\frac{x-1}{x+9}$

b ⓒ $\frac{x+9}{x-1}$ ⓓ $\frac{x-1}{x-9}$

20. Which one is the lowest form of $\frac{a^2+3a}{a^2-9}$? *(Medium) [DB' 15]*

ⓐ $\frac{a}{a+3}$ ⓑ $\frac{1}{a-3}$

c ⓒ $\frac{a}{a-3}$ ⓓ $\frac{a-3}{a}$

21. Which is the lowest form of $\frac{x^2-7x+12}{x^2-9x+20}$? *(Easy) [CB' 15]*

b ⓐ $\frac{x-4}{x-5}$ ⓑ $\frac{x-3}{x-5}$ ⓒ $\frac{x-3}{x-4}$ ⓓ $\frac{x-5}{x-3}$

22. i. Algebraic fractions are expressed by algebraic letter symbols

ii. $\frac{3}{5}$ is an algebraic fraction

iii. $\frac{b}{a}$ is the lowest term of $\frac{ab^3}{a^2b^2}$

Which one of the following is correct? *(Hard)*

b ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

23. i. In $\frac{a^2}{b^2}$, numerator is b^2 and denominator is a^2.

ii. $\frac{x}{a}, \frac{p}{3}$ and $\frac{x+a}{y+b}$ are algebraic expressions.

iii. The lowest term of $\frac{15x^5a^3bc^4}{20x^9ab^6c^3}$ is $\frac{3a^2c}{4x^4b^5}$

Which one of the following is correct? *(Hard)*

b ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

5.3 Fractions in the form of common denominator

▸ Textbook Page 77

24. Which is the equivalent fraction of $\frac{a}{b}$? *(Medium)*

a ⓐ $\frac{a^2}{ab}$ ⓑ $\frac{a^3}{b^3}$ ⓒ $\frac{a^2}{b^2}$ ⓓ $\frac{ab}{ab^2}$

25. Which of the following is the fraction of common denominators of $\frac{a}{b}, \frac{b}{c}$ and $\frac{c}{d}$? *(Hard)*

b ⓐ $\frac{abd}{bcd}$ ⓑ $\frac{acd}{bcd}, \frac{b^2d}{bcd}, \frac{bc^2}{bcd}$ ⓒ $\frac{ac^2d}{bcd}$ ⓓ $\frac{bcd^2}{bcd}$

26. Which are the fractions of common denominator of $\frac{a}{a-b}, \frac{b}{a+b}, \frac{c}{a(a+b)}$? *(Hard)*

ⓐ $\frac{a^2(a+b)}{a(a-b)(a+b)}, \frac{ab(a-b)}{a(a-b)(a+b)}, \frac{c(a-b)}{a(a-b)(a+b)}$

ⓑ $\frac{a(a+b)}{a(a-b)(a+b)}, \frac{ab^2(a-b)}{a(a-b)(a+b)}, \frac{bc(a-b)}{a(a-b)(a+b)}$

ⓒ $\frac{a^2(a^2-b^2)}{a(a-b)(a+b)}, \frac{ab(a-b)^2}{a(a-b)(a+b)}, \frac{ac(a+b)^2}{a(a-b)(a+b)}$

a ⓓ $\frac{ab(a+b)}{a(a-b)(a+b)}, \frac{bc(a^2-b^2)}{a(a-b)(a+b)}, \frac{ca(a-b)}{a(a-b)(a+b)}$

27. Which one is the lowest form of $\frac{x^3-49x}{x^2+7x}$? *(Hard) [DB '19]*

ⓐ $(x-7)$ ⓑ $(x+7)$

a ⓒ $x(x-7)$ ⓓ $x(x+7)$

28. Which one is the H.C.F. of x^2-2 and x^2-4? *(Hard) [CtgB '19]*

a ⓐ 1 ⓑ $x+2$ ⓒ $x-2$ ⓓ x^2-2

Mathematics

29. **$x + 1$, $x^2 - 1$ and $x^3 - 1$ are three algebraic expressions. What is the H.C.F. of the three expressions?** *(Hard) [BB '19]*
ⓐ $x - 1$ ⓑ $x + 1$
ⓓ ⓒ $(x + 1)(x^3 - 1)$ ⓓ 1

30. **Which one of the following is the H.C.F. of $2x^3y^2z^2$, $12x^2yz$, $20xy^3z^3$?** *(Hard) [CB '18]*
ⓒ ⓐ $60x^3y^3z^3$ ⓑ $2x^3y^3z^3$ ⓒ $2xyz$ ⓓ xyz

31. **What is the H.C.F. of $3(a + b)$, $9(a^2 - b^2)$ and $18(a^3 + b^3)$?** *(Hard) [SB '18]*
ⓐ $a + b$ ⓑ $3(a + b)$
ⓑ ⓒ $(a - b)(a^3 + b^3)$ ⓓ $18(a - b)(a^3 + b^3)$

32. **Which one is the factorized form of expression $x^2 + 2x - 143$?** *(Easy) [SB '18]*
ⓐ $(x + 11)(x - 13)$ ⓑ $(x - 11)(x - 13)$
ⓓ ⓒ $(x + 11)(x + 13)$ ⓓ $(x - 11)(x + 13)$

33. **What is the H.C.F. of $a^3 - b^3$ and $a^3 + b^3$?** *(Easy) [BB '18]*
ⓑ ⓐ 0 ⓑ 1 ⓒ $a - b$ ⓓ $a + b$

34. **Which one of the following is the H.C.F of $9a^3b^2c^2$, $12a^2bc$, $15ab^3c^3$?** *(Medium) [DB '17]*
ⓐ abc ⓑ 3abc
ⓑ ⓒ $135a^2b^2c^2$ ⓓ $180a^3b^3c^3$

35. **What is the H.C.F of $x^2 - 4$, $x^2(x - 2)$ and $x^2y - 2xy$?** *(Easy) [CB '16]*
ⓐ $x - 2$ ⓑ $x + 2$
ⓐ ⓒ $x(x - 2)$ ⓓ $(x + 2)(x - 2)$

36. i. Equivalent fraction of $\frac{2a^2bc}{3xyz^2}$ is $\frac{6a^2b^2cx}{9bx^2yz^2}$
ii. $\frac{x + y}{x - y}$ is an algebraic fraction with numerator, $(x + y)$ and denominator, $(x - y)$.
iii. $\frac{a}{b}$, $\frac{b}{c}$ and $\frac{c}{a}$ correspond to $\frac{a^2c}{abc}$, $\frac{ab^2}{abc}$ and $\frac{bc^2}{abc}$ respectively as the fractions with common denominator.
Which one of the following is correct? *(Hard)*
ⓐ ⓐ i, ii & iii ⓑ i & iii ⓒ ii & iii ⓓ i & ii

37. **$5x^3yz^2$, $15xy^3z$ and $20x^4y^2z^3$ are three algebraic expression, then—** *[JB '19]*
i. HCF = $10xyz$
ii. LCM = $60x^4y^3z^3$
iii. Product = $1,500\ x^8y^6z^6$
Which one is correct? *(Hard)*
ⓒ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

38. **The equivalent fraction of $\frac{a^2 - 5a + 6}{a^2 - 7a + 12}$ is —.**
(Medium) [Viqarunnisa Noon School and College, Dhaka]
i. $\frac{a - 2}{a - 4}$
ii. $\frac{a^2 - 4a + 4}{a^2 - 2a - 8}$
iii. $\frac{a^2 - a - 2}{a^2 - 3a - 4}$
Which one is correct?
ⓑ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer to the questions No. 39 and 40 based on the following information :
$\frac{x^2 + x - 12}{x^2 + 2x - 15}$ is a algebraic fraction. *[JB '18]*

39. **What is the factor of numerator of the given fraction?** *(Hard)*
ⓐ $(x - 3)(x - 4)$ ⓑ $(x - 3)(x + 4)$
ⓑ ⓒ $(x + 3)(x - 4)$ ⓓ $(x + 3)(x + 4)$

40. **The lowest form of the fraction is subtracted from which number to get the result $\frac{1}{x + 5}$?** *(Hard)*
ⓑ ⓐ -1 ⓑ 1 ⓒ $x + 4$ ⓓ $x + 5$

5.4 Addition of Fractions ▸ Textbook Page 80

41. **$\frac{a}{b} + \frac{b}{a}$ = What?** *(Medium)*
ⓑ ⓐ $\frac{a + b}{ab}$ ⓑ $\frac{a^2 + b^2}{ab}$ ⓒ $\frac{ab + b^2}{ab}$ ⓓ $\frac{a^2 + ab}{ab}$

42. **What will be the difference if $\frac{q + r}{qr}$ is subtracted from $\frac{p + q}{pq}$?** *(Hard)*
ⓐ ⓐ $\frac{(r - p)}{pr}$ ⓑ $\frac{p(r - q)}{pqr}$ ⓒ $\frac{r(p - q)}{pqr}$ ⓓ $\frac{q(p - r)}{pqr}$

43. **Which is the simplified value of $\frac{x - y}{xy} + \frac{y - z}{yz} + \frac{z - x}{zx}$?** *(Hard)*
ⓓ ⓐ $\frac{1}{xyz}$ ⓑ $\frac{2}{xyz}$ ⓒ $\frac{3}{xyz}$ ⓓ 0

44. **$\frac{a}{a + b} + \frac{b}{a - b}$ = What?** *(Hard)*
ⓑ ⓐ 1 ⓑ $\frac{a^2 + b^2}{a^2 - b^2}$ ⓒ $\frac{2a^2}{a^2 - b^2}$ ⓓ $\frac{2b^2}{a^2 - b^2}$

45. **Which one is the value of $\left(\frac{2x}{y} + \frac{1}{y}\right)$?** *(Medium) [DjB '19]*
ⓐ $\frac{2x + y}{y}$ ⓑ $\frac{2x + 1}{y}$
ⓑ ⓒ $\frac{2x + 1}{2y}$ ⓓ $\frac{2xy + 1}{y}$

46. **$\frac{1}{x - 3} - \frac{1}{x + 3} - \frac{6}{x^2 - 9}$ = ?** *(Hard) [RB '18]*
ⓐ ⓐ 0 ⓑ 6 ⓒ $\frac{12}{x^2 - 9}$ ⓓ $\frac{2x}{x^2 - 9}$

47. **$\frac{x - y}{xy} + \frac{y - z}{yz} + \frac{z + x}{zx}$ = what?** *(Easy) [CtgB '18]*
ⓓ ⓐ $\frac{2x}{yz}$ ⓑ $\frac{2}{xyz}$ ⓒ $\frac{x}{2}$ ⓓ $\frac{2}{z}$

48. **What is the value of $\frac{1}{x + y} + \frac{1}{x - y}$** *(Easy) [JB '17]*
ⓐ ⓐ $\frac{2x}{x^2 - y^2}$ ⓑ $\frac{y}{x^2 - y^2}$ ⓒ $\frac{x}{x^2 - y^2}$ ⓓ $\frac{2y}{x^2 - y^2}$

49. What is the sum of $\frac{1}{x-y}$ and $\frac{-xy}{x^3-y^3}$? *(Medium) [CtgB '17]*

ⓐ $\frac{-(x^2+y^2)}{(x-y)(x^2+xy+y^2)}$ ⓑ $\frac{x^2+y^2}{x^2-xy+y^2}$

ⓒ $\frac{x^2+y^2}{(x-y)(x^2+xy+y^2)}$ ⓓ $\frac{x^2-y^2}{(x-y)(x^2+xy+y^2)}$

Correct Answer is : $\frac{x^2+y^2}{x-y}$

50. $\frac{a}{bc}+\frac{c}{ab}+\frac{b}{ac}$ = what? *(Easy) [DB '16]*

ⓐ $\frac{b^2+2ac}{abc}$ ⓑ $\frac{a^2+2bc}{abc}$

ⓓ ⓒ $\frac{c^2+2ab}{abc}$ ⓓ $\frac{a^2+b^2+c^2}{abc}$

51. Which one of the following is the simplified value of $\frac{a}{a+b}+\frac{ab}{a^2-b^2}$? *(Medium) [CtgB '16]*

ⓐ $\frac{ab}{a^2-b^2}$ ⓑ $\frac{a^2b}{a^2-b^2}$

ⓓ ⓒ $\frac{a+ab}{a^2-b^2}$ ⓓ $\frac{a^2}{a^2-b^2}$

52. $\frac{x^3+1}{(x+1)^2-3x}$ = what? *(Easy) [SB' 15]*

ⓓ ⓐ $\frac{x-4}{x+4}$ ⓑ $\frac{x+4}{x-4}$ ⓒ $\frac{x-1}{x+4}$ ⓓ $x+1$

53. Which one is the readuced form $\frac{p^2+4p-21}{p^2+5p-14}$? *(Medium)[Ideal School & College, Dhaka]*

ⓐ $\frac{p-7}{p+7}$ ⓑ $\frac{p-3}{p+2}$

ⓓ ⓒ $\frac{p+7}{p-2}$ ⓓ $\frac{p-3}{p-2}$

54. $\left(1+\frac{1}{x}\right)+\left(1-\frac{1}{x^2}\right)$ = what? *(Medium) [Ideal School & College, Dhaka]*

ⓐ $\frac{x}{x+1}$ ⓑ $\frac{x}{x-1}$

ⓑ ⓒ $\frac{x+1}{x}$ ⓓ $\frac{x-1}{x}$

55. i. The value of $\frac{x}{a}$ will be 1 if $x = a$.

ii. The simplified value of $\frac{x-y}{xy}+\frac{y-z}{yz}$ and $\frac{z-x}{zx}$ is 0.

iii. The sum of $\frac{a^2}{b}$ and $\frac{b^2}{a}$ is $\frac{a^3+b^3}{ab}$

Which one of the following is correct? *(Hard)*

ⓓ ⓐ i & iii ⓑ ii & iii ⓒ i & ii ⓓ i, ii & iii

56. If $x=\frac{p}{q}-1$ and $y=1-\frac{p}{q}$, then— *[CtgB '19]*

i. $x+y=0$ ii. $\frac{x}{y}=-1$

iii. $xy=\frac{(p-q)^2}{q^2}$

Which one is correct? *(Hard)*

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

57. The following will be the equivalent fraction of $\frac{m^2+6m+5}{m^2+10m+25}$— *[SB '19]*

i. $\frac{m+1}{m+5}$ ii. $\frac{m^2-2m-3}{m^2+2m-15}$

iii. $\frac{m^2+2m+1}{m^2-3m-10}$

Which one is correct? *(Hard)*

ⓐ ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

58. If p^2-4p+3 and p^2-9 are two algebraic expressions —. *[CtgB '18]*

i. HCF of the two expressions is $p-3$

ii. LCM of the two expressions is $p(p+3)(p-3)(p-1)$

iii. The coefficient of p^2 in the first expression is 1.

Which one of the following is correct? *(Medium)*

ⓒ ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

■ **Answer questions from 59 to 60 on the basis of the following information :**

$\frac{2x}{x^2-4y^2}, \frac{x}{xy+2y^2}$ are two algebraic expressions.

59. Which is the numerator of the 2nd expression? *(Easy)*

ⓓ ⓐ $xy+2y^2$ ⓑ xy ⓒ $2y^2$ ⓓ x

60. $\frac{2x}{x^2-4y^2}-\frac{x}{xy+2y^2}$ = what? *(Hard)*

ⓐ $\frac{1}{x+2y}$ ⓑ $\frac{1}{x(x+2y)(x-2y)}$

ⓓ ⓒ $\frac{1}{x(x+2y)}$ ⓓ $\frac{x(4y-x)}{y(x^2-4y^2)}$

61. What is the LCM of the denominators of the given expressions? *(Hard)*

ⓐ $x(x+2y)$ ⓑ $x(x-2y)$

ⓒ ⓒ $y(x+2y)(x-2y)$ ⓓ $(x+2y)(x-2y)$

5.5 Subtraction of fractions ▸ Textbook Page 83

62. $\frac{a}{p}-\frac{b}{q}$ = What? *(Medium)*

ⓓ ⓐ $\frac{ap-aq}{p-q}$ ⓑ $\frac{ab-pq}{pq}$ ⓒ $\frac{aq-bp}{p-q}$ ⓓ $\frac{aq-bp}{pq}$

63. Which one of the following is the factorized form of x^2-x-42? *(Medium) [DB '19]*

ⓐ $(x-6)(x-7)$ ⓑ $(x-6)(x+7)$

ⓓ ⓒ $(x+6)(x+7)$ ⓓ $(x-7)(x+6)$

64. **Which one is the lowest form of $\frac{x^2-x-6}{x^2-4}$?** *(Hard) [RB '19]*

a ⓐ $\frac{x-3}{x-2}$ ⓑ $\frac{x+3}{x+2}$ ⓒ $\frac{x-3}{x+2}$ ⓓ $\frac{x+3}{x-2}$

65. **Which one is the lowest form of $\frac{x^2-x-12}{x^2-16}$?** *(Hard) [JB '19]*

d ⓐ $\frac{x+3}{x-4}$ ⓑ $\frac{x-3}{x-4}$ ⓒ $\frac{x-3}{x+4}$ ⓓ $\frac{x+3}{x+4}$

66. $\frac{a}{a-5}-\frac{a^2}{a^2-25}$ **= What?** *(Hard) [CB '19]*

d ⓐ $\frac{5a}{a-5}$ ⓑ $\frac{2a^2-5}{2a^2-25}$ ⓒ $\frac{2a^2-5}{a^2+25}$ ⓓ $\frac{5a}{a^2-25}$

67. **If $a-1$ is a factor of a^3-1, then which one is the another factor?** *(Hard) [CtgB '19]*

ⓐ $a+1$ ⓑ a^2-a-1

c ⓒ $1+a+a^2$ ⓓ $1-a-a^2$

68. **What is the simplified value of $\frac{2}{a}+\frac{3}{a}-\frac{4}{a}$?** *(Hard) [CtgB '19]*

d ⓐ $\frac{4}{a}$ ⓑ $\frac{3}{a}$ ⓒ $\frac{2}{a}$ ⓓ $\frac{1}{a}$

69. **What is the simplified value of $\frac{1}{a-2}-\frac{a}{a^2-4}$?** *(Hard) [CtgB '19]*

d ⓐ $\frac{2}{a-2}$ ⓑ $\frac{-2}{a+2}$ ⓒ $\frac{-2}{a^2+4}$ ⓓ $\frac{2}{a^2-4}$

70. **What is the simplified form of $\frac{x-y}{x}-\frac{x+y}{y}$?** *(Hard) [JB '18]*

ⓐ $\frac{-(x^2+y^2)}{xy}$ ⓑ $\frac{-(x^2-y^2)}{xy}$

a ⓒ $\frac{(x-y)^2}{xy}$ ⓓ $\frac{(x+y)^2}{xy}$

71. $\frac{x^2}{x^2-16}-\frac{x}{x+4}$ **= what?** *(Hard) [BB '17]*

ⓐ $\frac{2x^2}{x^2-16}$ ⓑ $\frac{4x}{x^2-16}$

b ⓒ $\frac{2x(x-2)}{x^2-16}$ ⓓ $\frac{-4x}{x^2-16}$

72. **What is the value of $\frac{1}{a+2}-\frac{1}{a-2}+\frac{4}{a^2+4}$?** *(Medium) [SB '16]*

ⓐ $-\frac{32}{2a^3-8}$ ⓑ $-\frac{32}{a^4-16}$

b ⓒ $\frac{32}{a^3-16}$ ⓓ $\frac{32}{a^4+16}$

73. **If 7 is added to the numerator of a proper fraction then the fraction will be 2 and if 2 is subtracted from the denominator, the fraction will be 1. Which is the fraction?** *(Hard) [Dih.B' 15]*

c ⓐ $\frac{1}{4}$ ⓑ $\frac{5}{7}$ ⓒ $\frac{3}{5}$ ⓓ $\frac{5}{6}$

74. **What is the simplified value of $\frac{1}{x-2}-\frac{1}{x+1}-\frac{4}{x^2-4}$?** *(Easy) [Ideal School & College, Dhaka]*

ⓐ $\frac{8}{x^2-4}$ ⓑ $\frac{2x}{x^2-4}$

c ⓒ $\frac{-1}{(x^2+4)(x+1)}$ ⓓ 0

75. i. $\frac{a}{a+b}+\frac{b}{a-b}=\frac{a^2+b^2}{a^2-b^2}$

ii. $\frac{x}{2}+\frac{x}{5}+\frac{3x}{10}=\frac{x}{10}$

iii. $\frac{x}{yz}-\frac{y}{zx}+\frac{z}{xy}=\frac{x^2-y^2+z^2}{xyz}$

Which one of the following is correct? *(Hard)*

c ⓐ i, ii & iii ⓑ i & ii ⓒ i & iii ⓓ ii & iii

76. i. Simplification of fraction is nothing but transformation of two or more fractions associated with operational signs into one fraction.

ii. $\frac{1}{x+2}-\frac{1}{x^2-4}=\frac{x-3}{x^2-4}$

iii. $\frac{x+y}{2a}-\frac{x-y}{2a}=\frac{y}{a}$

Which one of the following is correct? *(Hard)*

a ⓐ i, ii & iii ⓑ i & ii ⓒ ii & iii ⓓ i & iii

77. **The equivalent fraction of $\frac{x^3y^2-x^2y^3}{x^3y-xy^3}$ will be—.** *(Medium) [JB '18]*

i. $\frac{xy}{x+y}$ ii. $\frac{x^2y}{x^2+xy}$

iii. $\frac{xy^2}{xy+y^2}$

Which one of the following is correct?

d ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Answer questions from 78 to 80 on the basis of the following information :

$\frac{5}{a^2-6a+5}+\frac{1}{a-1}$ is an algebraic expression.

78. **How many terms are there in the given expression?** *(Easy)*

d ⓐ 5 ⓑ 4 ⓒ 3 ⓓ 2

79. **What is the common factor of the denominators of the expression?** *(Medium)*

ⓐ 5 ⓑ a^2-6a+5

c ⓒ $a-1$ ⓓ $a-5$

80. **What is the simplified value of the expression?** *(Hard)*

ⓐ $\frac{10}{(a-5)(a-1)}$ ⓑ $\frac{a}{(a-5)(a-1)}$

b ⓒ $\frac{a^2}{(a-5)(a-1)}$ ⓓ $\frac{10a}{(a-5)(a-1)}$

Short Q/A — Designed as per topic

5.1 and 5.2 Algebraic fraction and Lowest From of Fraction
▸ Textbook Page 77

Question 1. Express in the lowest form $\frac{m^3n^2 - m^2n^3}{m^3n - mn^3}$.

Solution : Given fraction $= \frac{m^3n^2 - m^2n^3}{m^3n - mn^3}$

$= \frac{m^2n^2(m-n)}{mn(m^2-n^2)} = \frac{m^2n^2(m-n)}{mn(m+n)(m-n)}$

$\therefore$ H.C.F. of numberator and denominator $= mn(m-n)$
Dividing the numerator and the denominator of the fraction by $mm(m-n)$ we get, $\frac{mn}{m+n}$

$\therefore$ The lowest form of the fraction is $\frac{mn}{m+n}$.

Question 2. Express in the lowest form $\frac{x^2+3x+2}{x^2-1}$.

Solution : Given fraction $= \frac{x^2+3x+2}{x^2-1} = \frac{x^2+2x+x+2}{x^2-1^2}$

$= \frac{x(x+2)+1(x+2)}{(x+1)(x-1)}$

$= \frac{(x+2)(x+1)}{(x+1)(x-1)}$

$\therefore$ H.C.F. of numberator and denominator $= x+1$
Dividing the numerator and the denominator of the fraction by $(x+1)$ we get, $\frac{x+2}{x-1}$

$\therefore$ The lowest form of the fraction is $\frac{x+2}{x-1}$.

Question 3. Express in the lowest form $\frac{34(x-a)^2}{17(x-a)(x^2-a^2)}$.

Solution : Given fraction $= \frac{34(x-a)^2}{17(x-a)(x^2-a^2)}$

$= \frac{2 \times 17 \times (x-a)(x-a)}{17(x-a)(x+a)(x-a)}$

$= \frac{2 \times 17(x-a)^2}{17(x-a)^2(x+a)}$

$\therefore$ H.C.F. of numberator and denominator $= 17(x-a)^2$
Dividing the numerator and the denominator of the fraction by $17(x-a)^2$ We get, $\frac{2}{x+a}$

$\therefore$ The lowest form of the fraction is $\frac{2}{x+a}$.

Question 4. Express in the lowest form $\frac{x^3-49x}{x^2+7x}$.

Solution : Given fraction $= \frac{x^3-49x}{x^2+7x}$

$= \frac{x(x^2-49)}{x(x+7)}$

$= \frac{x(x^2-7^2)}{x(x+7)} = \frac{x(x+7)(x-7)}{x(x+7)}$

$\therefore$ H.C.F. of numberator and denominator $= x(x+7)$
Dividing the numerator and the denominator of the fraction by $x(x+7)$ We get, $x-7$
The lowest form of the fraction is $x-7$.

Question 5. Express in the lowest form $\frac{m^2+6m+5}{m^2+10m+25}$.

Solution : Given fraction $= \frac{m^2+6m+5}{m^2+10m+25}$

$= \frac{m^2+m+5m+5}{m^2+5m+5m+25}$

$= \frac{m(m+1)+5(m+1)}{m(m+5)+5(m+5)}$

$= \frac{(m+1)(m+5)}{(m+5)(m+5)}$

$\therefore$ H.C.F. of numberator and denominator $= m+5$
Dividing the numerator and the denominator of the fraction by $(m+5)$ We get, $\frac{m+1}{m+5}$

The lowest form of the fraction is $\frac{m+1}{m+5}$.

5.3 Fractions in the form of common denominator
▸ Textbook Page 77

Question 6. Express the fractions with a common denominator $\frac{p^2+pq}{p^2q}$ and $\frac{p^2-pq}{pq^2}$.

Solution : Given fraction are : $\frac{p^2+pq}{p^2q}, \frac{p^2-pq}{pq^2}$
L.C.M. of the denominators p^2q, pq^2 of the given fraction $= p^2q^2$

Now, $\frac{p^2+pq}{p^2q} = \frac{(p^2+pq) \times q}{p^2q \times q}$ [$\because p^2q^2 \div p^2q = q$]

$= \frac{p^2q+pq^2}{p^2q^2}$

$\frac{p^2-pq}{pq^2} = \frac{(p^2-pq) \times p}{pq^2 \times p}$ [$\because p^2q^2 \div pq^2 = p$]

$= \frac{p^3-p^2q}{p^2q^2}$

$\therefore$ Fractions with common denominator : $\frac{p^2q+pq^2}{p^2q^2}$ and $\frac{p^3-p^2q}{p^2q^2}$.

Mathematics

Question 7. Express the fractions with a common denominator $\frac{x}{x^3-y^3}$ and $\frac{y}{x-y}$.

Solution : Given fraction are : $\frac{x}{x^3-y^3}, \frac{y}{x-y}$

Denominator of 1st fraction $= x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

Denominator of 2nd fraction $= x - y$

$\therefore$ L.C.M. of the denominators $= (x-y)(x^2+xy+y^2)$

Now, $\frac{x}{x^3-y^3} = \frac{x}{(x-y)(x^2+xy+y^2)} = \frac{x}{x^3-y^3}$

$\frac{y}{x-y} = \frac{y \times (x^2+xy+y^2)}{(x-y)\times(x^2+xy+y^2)} = \frac{y(x^2+xy+y^2)}{x^3-y^3}$

$\therefore$ Fractions with common denominator $\frac{x}{x^3-y^3}, \frac{y(x^2+xy+y^2)}{x^3-y^3}$.

Question 8. Express the fractions with a common denominator $\frac{m^2}{mn}, \frac{n^2}{nl}, \frac{l^2}{lm}$.

Solution : Given fraction are : $\frac{m^2}{mn}, \frac{n^2}{nl}, \frac{l^2}{lm}$

L.C.M. of the denominators mn, nl and lm of the given fraction = mn$l$

$\therefore \frac{m^2}{mn} = \frac{m^2 \times l}{mn \times l} = \frac{m^2 l}{mnl}$

$\frac{n^2}{nl} = \frac{n^2 \times m}{nl \times m} = \frac{mn^2}{mnl}$

and $\frac{l^2}{lm} = \frac{l^2 \times m}{nl \times m} = \frac{nl^2}{mnl}$

$\therefore$ Fractions with common denominator : $\frac{m^2 l}{mnl}, \frac{mn^2}{mnl}, \frac{nl^2}{mnl}$.

Question 9. Express the fractions with a common denominator $\frac{a-b}{ab}, \frac{b-c}{bc}, \frac{c-a}{ca}$.

Solution : Given fraction are : $\frac{a-b}{ab}, \frac{b-c}{bc}, \frac{c-a}{ca}$

L.C.M. of the denominators ab, bc and ca of the given fractions = abc

$\therefore \frac{a-b}{ab} = \frac{(a-b)\times c}{ab \times c} = \frac{c(a-b)}{abc}$

$\frac{b-c}{bc} = \frac{(b-c)\times a}{bc \times a} = \frac{a(b-c)}{abc}$

and $\frac{c-a}{ca} = \frac{(c-a)\times b}{ca \times b} = \frac{b(c-a)}{abc}$

$\therefore$ Fractions with common denominator : $\frac{c(a-b)}{abc}, \frac{a(b-c)}{abc}, \frac{b(c-a)}{abc}$.

Question 10. Express the fractions with a common denominator $\frac{p-q}{p^2q^2}, \frac{q-r}{q^2r^2}, \frac{r-p}{r^2p^2}$.

Solution : Given fractions are : $\frac{p-q}{p^2q^2}, \frac{q-r}{q^2r^2}, \frac{r-p}{r^2p^2}$

L.C.M. of the denominators p^2q^2, q^2r^2 and r^2p^2 of the given fractions $= p^2q^2r^2$

$\therefore \frac{p-q}{p^2q^2} = \left\{\frac{(p-q)\times r^2}{p^2q^2 \times r^2}\right\} = \frac{r^2(p-q)}{p^2q^2r^2}$

$\frac{q-r}{q^2r^2} = \frac{(q-r)\times p^2}{q^2r^2 \times p^2} = \frac{p^2(q-r)}{p^2q^2r^2}$

and $\frac{r-p}{r^2p^2} = \frac{(r-p)\times q^2}{r^2p^2 \times q^2} = \frac{q^2(r-p)}{p^2q^2r^2}$

$\therefore$ Fractions with common denominator :

$\frac{r^2(p-q)}{p^2q^2r^2}, \frac{p^2(q-r)}{p^2q^2r^2}, \frac{q^2(r-p)}{p^2q^2r^2}$.

5.4 Addition of Fractions

▸ Textbook Page 80

Question 11. $\frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx} = ?$

Solution : Given expression $= \frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$

$= \frac{zx - yz + xy - zx + yz - xy}{xyz} = \frac{0}{xyz} = 0$

Required summation 0.

Question 12. Find the sum : $\frac{a+b}{a-b} + \frac{a-b}{a+b} + 1$.

Solution : Given expression $= \frac{a+b}{a-b} + \frac{a-b}{a+b} + 1$

$= \frac{(a+b)^2 + (a-b)^2 + (a-b)(a+b)}{(a-b)(a+b)}$

$= \frac{a^2 + 2ab + b^2 + a^2 - 2ab + b^2 + a^2 - b^2}{a^2 - b^2} = \frac{3a^2+b^2}{a^2-b^2}$

Required summation : $\frac{3a^2+b^2}{a^2-b^2}$.

Question 13. Find the sum of $\frac{x}{x^2-xy+y^2}$ and $\frac{xy}{x^3+y^3}$.

Solution : The sum of $\frac{x}{x^2-xy+y^2}$ and $\frac{xy}{x^3+y^3}$

$= \frac{x}{x^2-xy+y^2} + \frac{xy}{x^3+y^3}$

$= \frac{x}{x^2-xy+y^2} + \frac{xy}{(x+y)(x^2-xy+y^2)}$

$= \frac{x(x+y)+xy}{(x+y)(x^2-xy+y^2)} = \frac{x^2+xy+xy}{x^3+y^3} = \frac{x^2+2xy}{x^3+y^3}$

Required summation : $\frac{x^2+2xy}{x^3+y^3}$.

Question 14. $\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c+a}{ca}=?$

Solution : Given expression $=\frac{a-b}{ab}+\frac{b-c}{bc}+\frac{c+a}{ca}$

$=\frac{c(a-b)+a(b-c)+b(c+a)}{abc}$

$=\frac{ac-bc+ab-ac+bc+ab}{abc}=\frac{2ab}{abc}=\frac{2}{c}$

Required summation : $\frac{2}{c}$.

Question 15. $\frac{1}{x+2}+\frac{4}{x^2-4}=?$

Solution : Given expression

$=\frac{1}{x+2}+\frac{4}{x^2-4}=\frac{1}{x+2}+\frac{4}{x^2-2^2}$

$=\frac{1}{x+2}+\frac{4}{(x+2)(x-2)}$

$=\frac{x-2+4}{(x+2)(x-2)}=\frac{x+2}{(x+2)(x-2)}=\frac{1}{x-2}$

Required summation : $\frac{1}{x-2}$.

Mathematics

5.5 Subtraction of fractions ▸ Textbook Page 83

Question 16. Find the difference : $\frac{a}{(a-b)^2}-\frac{a+b}{a^2-b^2}$.

Solution : Given expression $=\frac{a}{(a-b)^2}-\frac{a+b}{a^2-b^2}$

$=\frac{a}{(a-b)^2}-\frac{a+b}{(a+b)(a-b)}=\frac{a}{(a-b)^2}-\frac{1}{a-b}$

Here, the L.C.M. of denominators, $(a-b)^2$ and $(a-b)=(a-b)^2$

$\therefore$ Given expression $=\frac{a-(a-b)}{(a-b)^2}=\frac{a-a+b}{(a-b)^2}=\frac{b}{(a-b)^2}$

Required difference $\frac{b}{(a-b)^2}$.

Question 17. $\frac{1}{1-a+a^2}-\frac{1}{1+a+a^2}=?$

Solution : Here, the L.C.M. of denominators, $(1-a+a^2)$ and $(1+a+a^2)=(1-a+a^2)(1+a+a^2)$

Given expression $=\frac{1}{1-a+a^2}-\frac{1}{1+a+a^2}$

$=\frac{1.(1+a+a^2)-1.(1-a+a^2)}{(1-a+a^2)(1+a+a^2)}=\frac{1+a+a^2-1+a-a^2}{\{(1+a^2)-a\}\{(1+a^2)+a\}}$

$=\frac{2a}{(1+a^2)^2-a^2}=\frac{2a}{1+2a^2+a^4-a^2}=\frac{2a}{1+a^2+a^4}$

Required value: $\frac{2a}{1+a^2+a^4}$.

Question 18. Subtract $\frac{1}{b(a-b)}$ **from** $\frac{1}{a(a+b)}$.

Solution : The difference between $\frac{1}{b(a-b)}$ and

$\frac{1}{a(a+b)}=\frac{1}{b(a-b)}-\frac{1}{a(a+b)}$

Here, the L.C.M. of denominators $b(a-b)$ and $a(a+b)=ab(a-b)(a+b)$

Now, $\frac{1}{b(a-b)}-\frac{1}{a(a+b)}$

$=\frac{a(a+b)-b(a-b)}{ab(a-b)(a+b)}=\frac{a^2+ab-ab+b^2}{ab(a^2-b^2)}=\frac{a^2+b^2}{ab(a^2-b^2)}$

Required difference : $\frac{a^2+b^2}{ab(a^2-b^2)}$.

Question 19. Subtract $\frac{a^3-b^3}{a^2+b^2+ab}$ **from** $\frac{a^3+b^3}{a^2+b^2-ab}$.

Solution : The difference between $\frac{a^3-b^3}{a^2+b^2+ab}$ from $\frac{a^3+b^3}{a^2+b^2-ab}$

$=\frac{a^3-b^3}{a^2+b^2+ab}-\frac{a^3+b^3}{a^2+b^2-ab}$

$=\frac{(a-b)(a^2+ab+b^2)}{(a^2+ab+b^2)}-\frac{(a+b)(a^2-ab+b^2)}{(a^2-ab+b^2)}$

$=(a-b)-(a+b)=a-b-a-b=-2b$

Required difference : $-2b$.

Question 20. $\frac{x^2}{x^2-16}-\frac{x}{x+4}=?$

Solution : Here, $x^2-16=x^2-4^2=(x+4)(x-4)$

$\therefore$ Here, the L.C.M. of denominators x^2-16 and $x+4=(x+4)(x-4)$

Given expression $=\frac{x^2}{x^2-16}-\frac{x}{x+4}=\frac{x^2}{(x+4)(x-4)}-\frac{x}{x+4}$

$=\frac{x^2-x(x-4)}{(x+4)(x-4)}=\frac{x^2-x^2+4x}{x^2-16}=\frac{4x}{x^2-16}$

Required difference $\frac{4x}{x^2-16}$.

Creative Q/A — Designed as per learning outcomes

Ques. 01 $\frac{a}{x+1}$, $\frac{a}{2x+2}$ and $\frac{3a}{x^2-1}$ are three algebraic expressions :

a. Using relation symbol, relate the 1st two expressions. 2

b. Find the L.C.M. of the denominators of the expressions. 4

c. Transform the expressions into fractions with common denominator. 4

Solution to Question No. 01 :

a 1st fraction $= \frac{a}{x+1}$

2nd fraction $= \frac{a}{2x+2} = \frac{a}{2(x+1)}$

$\therefore$ The relation between the 1st and the 2nd expressions is $\frac{a}{x+1} > \frac{a}{2(x+1)}$ **(Ans.)**

b Denominator of 1st expression $= x+1$

" " 2nd " $= 2x+2 = 2(x+1)$

" " 3rd " $= x^2-1 = (x+1)(x-1)$

$\therefore$ L. C. M. of the denominators is

$(x+1)\times 2\times(x-1) = 2(x-1)(x+1) = 2(x^2-1)$

Ans. $2(x^2-1)$.

c From (b), we have,

L. C. M. of denominators of the given expressions is $2(x^2-1)$ or $2(x+1)(x-1)$.

$$\therefore \text{1st expression} = \frac{a}{x+1} = \frac{a\times 2(x-1)}{2(x-1)(x+1)} = \frac{2a(x-1)}{2(x-1)(x+1)}$$

$$\therefore \text{2nd expression} = \frac{a}{2x+2} = \frac{a}{2(x+1)} = \frac{a(x-1)}{2(x+1)(x-1)} = \frac{a(x-1)}{2(x-1)(x+1)}$$

$$\therefore \text{3rd expression} = \frac{3a}{x^2-1} = \frac{3a\times 2}{(x-1)(x+1)\times 2} = \frac{6a}{2(x-1)(x+1)}$$

$\therefore$ The required fractions are respectively $\frac{2a(x-1)}{2(x-1)(x+1)}$, $\frac{a(x-1)}{2(x-1)(x+1)}$ and $\frac{6a}{2(x-1)(x+1)}$. **(Ans.)**

Ques. 02 $\frac{x-y}{(y+z)(z+x)}$, $\frac{y-z}{(x+y)(z+x)}$, $\frac{z-x}{(x+y)(y+z)}$ are three algebraic expressions.

a. Determine the L. C. M. of the denominators of the three fractions. 2

b. Add the 2nd expression to the 1st expression. 4

c. Find the H. C. F. of the three expressions. 4

Solution to Question No. 02 :

a Denominator of the 1st expression $= (y+z)(z+x)$

" " " 2nd " $= (x+y)(z+x)$

" " " 3rd " $= (x+y)(y+z)$

$\therefore$ The required L. C. M. $= (x+y)(y+z)(z+x)$ **(Ans.)**

b Here, 1st expression $= \frac{x-y}{(y+z)(z+x)}$

and 2nd expression $= \frac{y-z}{(x+y)(z+x)}$

$\therefore$ 1st expression + 2nd expression

$$= \frac{x-y}{(y+z)(z+x)} + \frac{y-z}{(x+y)(z+x)}$$
$$= \frac{(x-y)(x+y)+(y-z)(y+z)}{(x+y)(y+z)(z+x)}$$
$$= \frac{x^2-y^2+y^2-z^2}{(x+y)(y+z)(z+x)}$$
$$= \frac{x^2-z^2}{(x+y)(y+z)(z+x)}$$
$$= \frac{(x+z)(x-z)}{(x+y)(y+z)(z+x)}$$
$$= \frac{x-z}{(x+y)(y+z)} \textbf{ (Ans.)}$$

c From (a), L.C.M. of the denominators of the expression $5 = (x+y)(y+z)(z+x)$

Numerator of the 1st expression $= x-y$

Numerator of the 2nd expression $= y-z$

Numerator of the 3rd expression $= z-x$

$\therefore$ There is no common factor except 1 of the numerators of the fractions.

$\therefore$ H.C.F. of the numerators is 1.

$\therefore$ The required H. C. F. of the fractions is

$$\frac{1}{(x+y)(y+z)(z+x)} \textbf{ (Ans.)}$$

Ques. 03 $A = x - 3$, $B = x^2 + 3x + 9$ and $C = x^3 - 27$ are three algebraic expressions.

a. Express $\frac{x^3y^2 - x^2y^3}{x^3y - xy^3}$ in the lowest form. 2

b. Prove that, $\frac{1}{A} \times \frac{x+3}{B} \div \frac{x+3}{C} = 1$. 4

c. Express $\frac{1}{A}, \frac{1}{B}, \frac{1}{C}$ into the fractions with a common denominator. 4

● Dhaka Board 2019

Solution to Question No. 03 :

a Given fraction $= \frac{x^3y^2 - x^2y^3}{x^3y - xy^3}$

Here, the numerator $= x^3y^2 - x^2y^3$
$= x^2y^2(x - y)$

and the denominator $= x^3y - xy^3$
$= xy(x^2 - y^2)$
$= xy(x + y)(x - y)$

$\therefore$ H.C.F of the numerator and the denominator $= xy(x-y)$

Dividing the numerator and the denominator of the given fraction by $xy(x - y)$,

we get, $\frac{xy}{x+y}$

$\therefore$ The lowest form of the fraction is $\frac{xy}{x+y}$.

b Given,

$A = x - 3$
$B = x^2 + 3x + 9$
and $C = x^3 - 27$

Now, L.H.S $= \frac{1}{A} \times \frac{x+3}{B} \div \frac{x+3}{C}$

$= \frac{1}{x-3} \times \frac{x+3}{x^2+3x+9} \div \frac{x+3}{x^3-27}$

$= \frac{1}{x-3} \times \frac{x+3}{x^2+3x+9} \times \frac{x^3-27}{x+3}$

$= \frac{1}{x-3} \times \frac{x+3}{x^2+3x+9} \times \frac{x^3-3^3}{x+3}$

$= \frac{1}{(x-3)} \times \frac{(x+3)}{(x^2+3x+9)} \times \frac{(x-3)(x^2+3x+9)}{(x+3)}$

$= 1$

$=$ R.H.S

$\therefore \frac{1}{A} \times \frac{x+3}{B} \div \frac{x+3}{C} = 1$ **(Proved)**

c Here, $\frac{1}{A} = \frac{1}{x-3}$

$\frac{1}{B} = \frac{1}{x^2+3x+9}$

and $\frac{1}{C} = \frac{1}{x^3-27} = \frac{1}{(x-3)(x^2+3x+9)}$

Now, L.C.M of the denominators of $\frac{1}{A}, \frac{1}{B}$ and $\frac{1}{C}$ is $(x - 3)(x^2 + 3x + 9)$.

Now again, $\frac{1}{A} = \frac{1}{x-3} = \frac{x^2+3x+9}{(x-3)(x^2+3x+9)}$ .... (i)

$\frac{1}{B} = \frac{1}{x^2+3x+9} = \frac{x-3}{(x-3)(x^2+3x+9)}$ ...... (ii)

and $\frac{1}{C} = \frac{1}{(x-3)(x^2+3x+9)}$ .... (iii)

Here, (i), (ii) and (iii) are the required fractions with a common denominator.

Ques. 04 $A = 4x^2 - 9$, $B = 2x^2 - 7x + 6$, $C = x^3 - 1$, $D = x^3 + 1$ and $E = 1 + x^2 + x^4$.

a. Find the sum : $\frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$. 2

b. Simplify : $\left(\frac{1}{A} + \frac{1}{B}\right) \div \frac{6x+2}{(4x^2-9)(x-2)}$ 4

c. Express $\frac{1}{C}, \frac{1}{D}, \frac{1}{E}$ in the form of common denominator. 4

● Rajshahi Board 2019

Solution to Question No. 04 :

a $\frac{x-y}{xy} + \frac{y-z}{yz} + \frac{z-x}{zx}$

$= \frac{z(x-y) + x(y-z) + y(z-x)}{xyz}$

$= \frac{zx - yz + xy - zx + yz - xy}{xyz}$

$= \frac{0}{xyz}$

$= 0$

b Given, $A = 4x^2 - 9$
and $B = 2x^2 - 7x + 6$

Now,

$\left(\frac{1}{A} + \frac{1}{B}\right) \div \frac{6x+2}{(4x^2-9)(x-2)}$

$= \left(\frac{1}{4x^2-9} + \frac{1}{2x^2-7x+6}\right) \div \frac{6x+2}{\{(2x)^2 - 3^2\}(x-2)}$

$= \left\{\frac{1}{(2x)^2 - 3^2} + \frac{1}{2x^2 - 4x - 3x + 6}\right\} \div \frac{6x+2}{(2x+3)(2x-3)(x-2)}$

$= \left\{\frac{1}{(2x+3)(2x-3)} + \frac{1}{2x(x-2) - 3(x-2)}\right\} \div \frac{6x+2}{(2x+3)(2x-3)(x-2)}$

$= \left\{\frac{1}{(2x+3)(2x-3)} + \frac{1}{(2x-3)(x-2)}\right\} \div \frac{6x+2}{(2x+3)(2x-3)(x-2)}$

$= \left\{\frac{x - 2 + 2x + 3}{(2x+3)(2x-3)(x-2)}\right\} \div \frac{2(3x+1)}{(2x+3)(2x-3)(x-2)}$

$= \frac{(3x+1)}{(2x+3)(2x-3)(x-2)} \times \frac{(2x+3)(2x-3)(x-2)}{2(3x+1)} = \frac{1}{2}$

$\therefore$ Simplified value of $\left(\frac{1}{A} + \frac{1}{B}\right) \div \frac{6x+2}{(4x^2-9)(x-2)}$ is $\frac{1}{2}$.

c Here,

$$C = x^3 - 1$$
$$= (x-1)(x^2+x+1)$$
$$D = x^3 + 1$$
$$= (x+1)(x^2-x+1)$$
and
$$E = 1 + x^2 + x^4$$
$$= 1 + 2x^2 + (x^2)^2 - x^2$$
$$= (1+x^2)^2 - x^2$$
$$= (1+x+x^2)(1-x+x^2)$$
$$= (x^2+x+1)(x^2-x+1)$$

$\therefore$ LCM of C, D and E $= (x-1)(x^2+x+1)(x+1)(x^2-x+1)$

Now,

$$\frac{1}{C} = \frac{1}{(x-1)(x^2+x+1)} = \frac{(x+1)(x^2-x+1)}{(x-1)(x^2+x+1)(x+1)(x^2-x+1)} \text{.... (i)}$$

$$\frac{1}{D} = \frac{1}{(x+1)(x^2-x+1)} = \frac{(x-1)(x^2+x+1)}{(x-1)(x^2+x+1)(x+1)(x^2-x+1)} \text{.... (ii)}$$

and
$$\frac{1}{E} = \frac{1}{(x^2+x+1)(x^2-x+1)} = \frac{(x+1)(x-1)}{(x-1)(x^2+x+1)(x+1)(x^2-x+1)} \text{... (iii)}$$

$\therefore$ (i), (ii) and (iii) are the required form of the three fractions with common denominator.

Ques. 05 $A = \frac{xy}{x^4+x^2y^2+y^4}$, $B = \frac{1}{x^2+xy+y^2}$, $C = 3x^2+2x-8$, $D = x^2-4$ and $E = x^2+5x-14$ are five algebraic expressions.

a. Divide $\frac{x^3+y^3}{x^4-y^4}$ by $\frac{(x+y)^3-3xy(x+y)}{x^2+y^2}$. 2

b. Simplify : $(A+B) \times \frac{x^2-xy+y^2}{-x^2-y^2}$ 4

c. Express $\frac{1}{C}, \frac{1}{D}, \frac{1}{E}$ in the form of the common denominator. 4

• *Jashore Board 2019*

Solution to Question No. 05 :

a
$$\frac{x^3+y^3}{x^4-y^4} \div \frac{(x+y)^3-3xy(x+y)}{x^2+y^2}$$
$$= \frac{x^3+y^3}{(x^2)^2-(y^2)^2} \times \frac{x^2+y^2}{x^3+y^3}$$
$$= \frac{1}{(x^2+y^2)(x^2-y^2)} \times (x^2+y^2)$$
$$= \frac{1}{x^2-y^2}$$
$$= \frac{1}{(x+y)(x-y)} \textbf{ (Ans.)}$$

b Given,

$$A = \frac{xy}{x^4+x^2y^2+y^4}$$
$$= \frac{xy}{(x^2)^2+2x^2y^2+(y^2)-x^2y^2}$$
$$= \frac{xy}{(x^2+y^2)^2-(xy)^2}$$
$$= \frac{xy}{(x^2+y^2+xy)(x^2+y^2-xy)}$$
$$B = \frac{1}{x^2+xy+y^2}$$

Given expression : $(A+B) \times \frac{x^2-xy+y^2}{x^2+y^2}$

$$= \frac{xy+x^2-xy+y^2}{(x^2+xy+y^2)(x^2-xy+y^2)} \times \frac{x^2-xy+y^2}{x^2+y^2}$$
$$= \frac{x^2+y^2}{x^2+xy+y^2} \times \frac{1}{x^2+y^2}$$
$$= \frac{1}{x^2+xy+y^2} \textbf{ (Ans.)}$$

c Given,

$$C = 3x^2+2x-8$$
$$= 3x^2+6x-4x-8$$
$$= 3x(x+2)-4(x+2)$$
$$= (x+2)(3x-4)$$
and
$$D = x^2-4$$
$$= (x+2)(x-2)$$
and
$$E = x^2+5x-14$$
$$= x^2+7x-2x-14$$
$$= x(x+7)-2(x+7)$$
$$= (x+7)(x-2)$$

Now, L.C.M of C, D and E are : $(x+2)(x-2)(3x-4)(x+7)$

The common denominator form of fractions are now,

$$\frac{1}{C} = \frac{1}{3x^2+2x-8} = \frac{1}{(x+2)(3x-4)}$$
$$= \frac{(x-2)(x+7)}{(x+2)(3x-4)(x-2)(x+7)}$$

$$\frac{1}{D} = \frac{1}{x^2-4} = \frac{1}{(x-2)(x+2)} = \frac{(3x-4)(x+7)}{(x+2)(x-2)(3x-4)(x+7)}$$

and
$$\frac{1}{E} = \frac{1}{x^2+5x-14} = \frac{1}{(x+7)(x-2)}$$
$$= \frac{(x+2)(3x-4)}{(x+7)(x+2)(x-2)(3x-4)}$$

Therefore, required fractions are :

$$\frac{(x-2)(x+7)}{(x+2)(x-2)(3x-4)(x+7)}, \frac{(3x-4)(x+7)}{(x+2)(x-2)(3x-4)(x+7)},$$
$$\frac{(x+2)(3x-4)}{(x+2)(x-2)(3x-4)(x+7)} \textbf{ (Ans.)}$$

Ques. 06 $P = 1-x+x^2$, $Q = 1+x+x^2$ and $R = 1+x^2+x^4$ are three algebraic expressions.

a. Resolve into factors of R. 2

b. Determine the value of $\frac{1}{P} - \frac{1}{Q} - \frac{2x}{R}$. 4

c. Simplify : $\left(\frac{1}{P} + \frac{1}{Q} - \frac{2x}{R}\right) \times \frac{Q}{2x}$. 4

• *Cumilla Board 2019*

Solution to Question No. 06 :

a Given, $R = 1 + x^2 + x^4$
$= 1 + 2x^2 + x^4 - x^2$
$= 1^2 + 2.1.x^2 + (x^2)^2 - x^2$
$= (1 + x^2)^2 - x^2$
$= (1 + x + x^2)(1 - x + x^2)$ **(Ans.)**

b From 'a' we get,
$R = (1 + x + x^2)(1 - x + x^2)$
Given, $P = 1 - x + x^2$
$Q = 1 + x + x^2$
Given, expression : $\frac{1}{P} - \frac{1}{Q} - \frac{2x}{R}$

$$= \frac{1}{1-x+x^2} - \frac{1}{1+x+x^2} - \frac{2x}{(1+x+x^2)(1-x+x^2)}$$
$$= \frac{1+x+x^2-1+x-x^2-2x}{(1+x+x^2)(1-x+x^2)}$$
$$= \frac{2x-2x}{(1+x+x^2)(1-x+x^2)}$$
$$= \frac{0}{(1+x+x^2)(1-x+x^2)} = 0 \textbf{ (Ans.)}$$

c Here, $P = (1 - x + x^2)$
$Q = (1 + x + x^2)$
and $R = (1 - x + x^2)(1 + x + x^2)$
Given expression : $\left(\frac{1}{P} + \frac{1}{Q} - \frac{2x}{R}\right) \times \frac{Q}{2x}$

$$= \left\{\frac{1}{1-x+x^2} + \frac{1}{1+x+x^2} - \frac{2x}{(1+x+x^2)(1-x+x^2)}\right\} \times \frac{1+x+x^2}{2x}$$
$$= \frac{1+x+x^2+1-x+x^2-2x}{(1+x+x^2).(1-x+x^2)} \times \frac{1+x+x^2}{2x}$$
$$= \frac{2-2x+2x^2}{1-x+x^2} \times \frac{1}{2x}$$
$$= \frac{2(1-x+x^2)}{(1-x+x^2)} \times \frac{1}{2x} = \frac{1}{x} \textbf{ (Ans.)}$$

Ques. 07 $S = x + 3$, $T = x - 3$ and $V = x^2 - 9$.

a. Express the lowest form of $\frac{y^4-1}{y^3+y}$. 2

b. Simplify $\frac{x}{S} + \frac{x}{T} + \frac{6x}{V}$. 4

c. Express $\frac{S}{x^2-6x+5}$, $\frac{T}{x^2+2x-3}$ and $\frac{V}{x^2-2x-15}$ in the form of a common denominator. 4

● *Sylhet Board 2019*

Solution to Question No. 07 :

a Given fraction $= \frac{y^4-1}{y^3+y}$
Now, numerator of the fraction $= y^4 - 1$
$= (y^2)^2 - 1^2$
$= (y^2 + 1)(y^2 - 1)$
$= (y^2 + 1)(y + 1)(y - 1)$
and denominator of the fraction $= y^3 + y$
$= y(y^2 + 1)$
$\therefore$ H.C.F of the numerator and the denominator $= (y^2 + 1)$
Dividing the numerator and the denominator of the given fraction by $(y^2 + 1)$, we get $\frac{(y+1)(y-1)}{y}$
$\therefore$ The lowest form of the fraction is $\frac{(y+1)(y-1)}{y}$

b Given, $S = x + 3$,
$T = x - 3$
and $V = x^2 - 9$.

Now, $\frac{x}{S} + \frac{x}{T} + \frac{6x}{V} = \frac{x}{x+3} + \frac{x}{x-3} + \frac{6x}{x^2-9}$
$$= \frac{x}{x+3} + \frac{x}{x-3} + \frac{6x}{(x+3)(x-3)}$$
$$= \frac{x(x-3)+x(x+3)+6x}{(x+3)(x-3)}$$
$$= \frac{x^2-3x+x^2+3x+6x}{(x+3)(x-3)}$$
$$= \frac{2x^2+6x}{(x+3)(x-3)}$$
$$= \frac{2x(x+3)}{(x+3)(x-3)} = \frac{2x}{x-3}$$
$\therefore$ Simplified value of $\frac{x}{S} + \frac{x}{T} + \frac{6x}{V}$ is $\frac{2x}{x-3}$.

c Here, denominator of 1st fraction $= x^2 - 6x + 5$
$= x^2 - 5x - x + 5$
$= x(x - 5) - 1(x - 5)$
$= (x - 1)(x - 5)$
denominator of 2nd fraction. $= x^2 + 2x - 3$
$= x^2 + 3x - x - 3$
$= x(x + 3) - 1(x + 3)$
$= (x - 1)(x + 3)$
and denominator of 3rd fraction $= x^2 - 2x - 15$
$= x^2 - 5x + 3x - 15$
$= x(x - 5) + 3(x - 5)$
$= (x + 3)(x - 5)$
LCM of denominators of the three fractions
$= (x - 1)(x + 3)(x - 5)$
Now,
$$\frac{S}{x^2-6x+5} = \frac{x+3}{(x-1)(x-5)} = \frac{(x+3)(x+3)}{(x-1)(x+3)(x-5)} \text{ ..... (i)}$$
$$\frac{T}{x^2+2x-3} = \frac{x-3}{(x-1)(x+3)} = \frac{(x-3)(x-5)}{(x-1)(x+3)(x-5)} \text{ ..... (ii)}$$
and $$\frac{V}{x^2-2x-15} = \frac{x^2-9}{(x+3)(x-5)} = \frac{(x+3)(x-3)(x-1)}{(x-1)(x+3)(x-5)} \text{ ...... (iii)}$$
$\therefore$ (i), (ii) and (iii) are the required form of the three fractions with common denominator.

Mathematics

Ques. 08 $A = 3x^2 - 2x - 1$, $B = 2x^2 - 3x + 1$, $C = 6x^2 - x - 1$ and $D = 3x^2 - 2x - 1$ are four algebraic expressions.

a. Express in the lowest form of $\frac{3x^2 + x}{9x^2 - 1}$. 2

b. Show that, $\left(\frac{2}{A} - \frac{1}{B}\right) \div \frac{1}{C} = \frac{x-3}{x-1}$. 4

c. Turn $\frac{1}{B}$, $\frac{1}{C}$ and $\frac{1}{D}$ into the fraction with a common denominator. 4

● *Barishal Board 2019*

Solution to Question No. 08 :

a Given expression $= \frac{3x^2 + x}{9x^2 - 1}$

$$= \frac{x(3x+1)}{(3x)^2 - (1)^2}$$

$$= \frac{x(3x+1)}{(3x+1)(3x-1)}$$

$$= \frac{x}{3x-1} \textbf{ (Ans.)}$$

b Given, $A = 3x^2 - 2x - 1$

$= 3x^2 - 3x + x - 1$

$= 3x(x-1) + 1(x-1)$

$= (x-1)(3x+1)$

$B = 2x^2 - 3x + 1$

$= 2x^2 - 2x - x + 1$

$= 2x(x-1) - 1(x-1)$

$= (x-1)(2x-1)$

and $C = 6x^2 - x - 1$

$= 6x^2 - 3x + 2x - 1$

$= 3x(2x-1) + 1(2x-1)$

$= (3x+1)(2x-1)$

Now, L.H.S. $= \left(\frac{2}{A} - \frac{1}{B}\right) \div \frac{1}{C}$

$$= \left\{\frac{2}{(x-1)(3x+1)} - \frac{1}{(x-1)(2x-1)}\right\} \div \frac{1}{(3x+1)(2x-1)}$$

$$= \frac{2(2x-1) - 1(3x+1)}{(x-1)(3x+1)(2x-1)} \times (3x+1)(2x-1)$$

$$= \frac{4x - 2 - 3x - 1}{x-1}$$

$$= \frac{x-3}{x-1} = \text{R.H.S.}$$

$$\therefore \left(\frac{2}{A} - \frac{1}{B}\right) \div \frac{1}{C} = \frac{x-3}{x-1} \textbf{ (Shown)}$$

c From 'b' we get,

$B = (x-1)(2x-1)$

and $C = (3x+1)(2x-1)$

Given, $D = 3x^2 - 2x - 1$

$= 3x^2 - 3x + x - 1$

$= 3x(x-1) + 1(x-1)$

$= (x-1)(3x+1)$

Here, L.C.M of B, C and D $= (x-1)(2x-1)(3x+1)$

Now, the common denominator form of the fractions are

$$\frac{1}{B} = \frac{1}{(x-1)(2x-1)} = \frac{(3x+1)}{(x-1)(2x-1)(3x+1)}$$

$$\frac{1}{C} = \frac{1}{(3x+1)(2x-1)} = \frac{(x-1)}{(x-1)(2x-1)(3x+1)}$$

and $\frac{1}{D} = \frac{1}{(x-1)(3x+1)} = \frac{(2x-1)}{(x-1)(2x-1)(3x+1)}$

$\therefore$ Required fractions are :

$$\frac{3x+1}{(x-1)(2x-1)(3x+1)}, \frac{x-1}{(x-1)(2x-1)(3x+1)}, \frac{2x-1}{(x-1)(2x-1)(3x+1)} \text{ (Ans.)}$$

Ques. 09 $P = \frac{a^4 - b^4}{a^2 + b^2 - 2ab}$, $Q = \frac{(a+b)^2 - 4ab}{a^3 - b^3}$, $R = \frac{a+b}{a^2 + ab + b^2}$ are three algebraic fractions.

a. Resolve into factors : $x2 - x - (m-1)(m-2)$ 2

b. Simplify : $\left\{\frac{(a-b)}{a^2+b^2} \times P + (a^2 + ab + b^2) \times Q\right\} \frac{1}{2a}$. 4

c. Show that, $P \div [(a^2 + b^2) \times R] \times Q = 1$. 4

● *Dinajpur Board 2019*

Solution to Question No. 09 :

a $x^2 - x - (m-1)(m-2)$

$= x^2 - x - (m-1)(m-1-1)$

$= x^2 - x - p(p-1)$ [Let, $m - 1 = p$]

$= x^2 - x - p^2 + p$

$= x^2 - p^2 - x + p$

$= (x+p)(x-p) - 1(x-p)$

$= (x-p)(x+p-1)$

$= (x-m+1)(x+m-1-1)$ [Putting $p = m - 1$]

$= (x-m+1)(x+m-2)$ **(Ans.)**

b Given, $P = \frac{a^4 - b^4}{a^2 + b^2 - 2ab}$

and $Q = \frac{(a+b)^2 - 4ab}{a^3 - b^3}$

Now, $\left\{\frac{(a-b)}{a^2+b^2} \times P + (a^2 + ab + b^2) \times Q\right\} \frac{1}{2a}$

$$= \left\{\frac{(a-b)}{a^2+b^2} \times \frac{a^4 - b^4}{a^2 + b^2 - 2ab} + (a^2 + ab + b^2) \times \frac{(a+b)^2 - 4ab}{a^3 - b^3}\right\} \frac{1}{2a}$$

[putting the values of P and Q]

$= \left\{\frac{(a-b)}{a^2+b^2} \times \frac{(a^2+b^2)(a^2-b^2)}{(a-b)^2} + (a^2+ab+b^2) \times \frac{(a-b)^2}{(a-b)(a^2+ab+b^2)}\right\} \frac{1}{2a}$

$= \left\{\frac{(a-b)}{(a^2+b^2)} \times \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} + (a^2+ab+b^2) \times \frac{(a-b)(a-b)}{(a-b)(a^2+ab+b^2)}\right\} \frac{1}{2a}$

$= \{a+b+a-b\} \frac{1}{2a}$

$= \{2a\} \frac{1}{2a}$

$= \frac{2a}{2a}$

$= 1$

$\therefore$ Simplified value of $\left\{\frac{(a-b)}{a^2+b^2} \times P + (a^2+ab+b^2) \times Q\right\} \frac{1}{2a}$ is 1

c Given, $P = \frac{a^4-b^4}{a^2+b^2-2ab}$

$Q = \frac{(a+b)^2-4ab}{a^3-b^3}$

and $R = \frac{a+b}{a^2+ab+b^2}$

Now, L.H.S $= P \div \{(a^2+b^2) \times R\} \times Q$

$= \frac{a^4-b^4}{a^2+b^2-2ab} \div \left\{(a^2+b^2) \times \frac{a+b}{a^2+ab+b^2}\right\} \times \frac{(a+b)^2-4ab}{a^3-b^3}$

$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a^2+ab+b^2)}{(a^2+b^2)(a+b)} \times \frac{(a-b)(a-b)}{(a-b)(a^2+ab+b^2)}$

$= 1 =$ R.H.S

$\therefore P \div \{(a^2+b^2) \times R\} \times Q = 1$ **[Showed]**

Solutions to Textual Activities — Along with textual reference

Mathematics

Activity 01 Express the fractions with a common denominator : ▸ Textbook Page 80

1. $\frac{x^2+xy}{x^2y}$ **and** $\frac{x^2-xy}{xy^2}$

Solution :

$\frac{x^2+xy}{x^2y} = \frac{y(x^2+xy)}{x^2y^2}$ (since LCM of x^2y and xy^2 is x^2y^2)

$\frac{x^2-xy}{xy^2} = \frac{x(x^2-xy)}{x^2y^2}$

$\therefore$ The required fractions with common denominator are $\frac{y(x^2+xy)}{x^2y^2}$ and $\frac{x(x^2-xy)}{x^2y^2}$.

2. $\frac{a-b}{a+2b}$ **and** $\frac{2a+b}{a^2-4b}$

Solution : Here, the denominator of 1st fraction $= a+2b$

The denominator of 2nd fraction $= a^2-4b$

$\therefore$ L.C.M. of the denominators $= (a+2b)(a^2-4b)$

$\therefore \frac{a-b}{a+2b} = \frac{(a-b)(a^2-4b)}{(a+2b)(a^2-4b)}$

and $\frac{2a+b}{a^2-4b} = \frac{(2a+b)(a+2b)}{(a^2-4b)(a+2b)}$

$\therefore$ Required fractions are $\frac{(a-b)(a^2-4b)}{(a+2b)(a^2-4b)}$ and $\frac{(2a+b)(a+2b)}{(a^2-4b)(a+2b)}$.

Activity 02 Add the expressions : ▸ Textbook Page 83

1. $\frac{2a}{3x^2y}, \frac{3b}{2xy^2}$ **and** $\frac{a+b}{xy}$

Solution : $\frac{2a}{3x^2y} + \frac{3b}{2xy^2} + \frac{a+b}{xy}$

$= \frac{4ay+9bx+6axy+6bxy}{6x^2y^2}$

[Here, L.C.M. of the denominators $= 6x^2y^2$]

$\therefore$ Required summation $= \frac{4ay+9bx+6axy+6bxy}{6x^2y^2}$.

2. $\frac{2}{x^2y-xy^2}, \frac{3}{xy(x^2-y^2)}, \frac{1}{x^2y^2}$

Solution :

LCM of $x^2y-xy^2 = xy(x-y)$, $xy(x^2-y^2) = xy(x+y)(x-y)$ and $\frac{1}{x^2y^2}$ is $x^2y^2(x+y)(x-y)$.

$\therefore \frac{2}{x^2y-xy^2} + \frac{3}{xy(x+y)(x-y)} + \frac{1}{x^2y^2}$

$= \frac{2(xy)(x+y)+3(xy)(1)+(x+y)(x-y)}{x^2y^2(x+y)(x-y)}$

$= \frac{2x^2y+2xy^2+3xy+x^2-y^2}{x^2y^2(x+y)(x-y)}$. **(Ans.)**

Activity 03 Simplify : ▸ Textbook Page 85

1. $\frac{x}{x^2+xy+y^2} - \frac{xy}{x^3-y^3}$

Solution : $\frac{x}{x^2+xy+y^2} - \frac{xy}{x^3-y^3}$

$= \frac{x}{x^2+xy+y^2} - \frac{xy}{(x-y)(x^2+xy+y^2)}$

$= \frac{x(x-y)-xy(1)}{(x-y)(x^2+xy+y^2)}$

$= \frac{x^2-xy-xy}{(x-y)(x^2+xy+y^2)}$

$= \frac{x^2-2xy}{(x-y)(x^2+xy+y^2)}$

$= \frac{x(x-2y)}{x^3-y^3}$ **(Ans.)**

2. $\frac{1}{1+a+a^2}-\frac{2a}{1+a^2+a^4}$

Solution : $\frac{1}{1+a+a^2}-\frac{2a}{1+a^2+a^4}$

$=\frac{1}{a^2+a+1}-\frac{2a}{a^4+a^2+1}$

$=\frac{1}{a^2+a+1}-\frac{2a}{(a^2)^2+2a^2.1+1^2-a^2}$

$=\frac{1}{a^2+a+1}-\frac{2a}{(a^2+1)^2-a^2}=\frac{1}{a^2+a+1}-\frac{2a}{(a^2+1+a)(a^2+1-a)}$

$=\frac{1}{(a^2+a+1)}-\frac{2a}{(a^2+a+1)(a^2-a+1)}=\frac{a^2-a+1-2a}{(a^2+a+1)(a^2-a+1)}$

$=\frac{a^2-3a+1}{(a^2+a+1)(a^2-a+1)}=\frac{a^2-3a+1}{a^4+a^2+1}$ (Ans.)

Exercise 5.2 : Multiplication and Division of Fractions

At a Glance Important Contents of Exercise

- **Multiplication of fractions :** By multiplying two or more fractions, we can also get a fraction. Its numerator is equal to the product of the numerators of two or more fractions and the denominator is equal to the product of their denominators.
- **Division of fractions :** Division of one fraction by another fraction means multiplication of the first fraction by the inverse of the second fraction.
- **Simplification of algebraic fractions :** The process of combining two or more algebraic fractions connected by operational signs into a single fraction or expression is called the simplification of fractions. In this process, the resulting fraction is expressed in its simplest form.

Mathematics

Solutions to Exercise Problems — Let's solve the textbook problems

MCQs with Answers

1. **Which one is correct if $\frac{a}{x}, \frac{b}{y}, \frac{c}{z}, \frac{p}{q}$ are reduced to the common denominator?**

ⓐ $\frac{ayzq}{xyzq}, \frac{bxzq}{xyzq}, \frac{cxyq}{xyzq}, \frac{pxyz}{xyzq}$

ⓑ $\frac{axy}{xyzq}, \frac{byz}{xyzq}, \frac{czx}{xyzq}, \frac{pxy}{xyzq}$

ⓒ $\frac{a}{xyzq}, \frac{b}{xyzq}, \frac{c}{xyzq}, \frac{p}{xyzq}$

ⓓ $\frac{axyzq}{zyzq}, \frac{bxyzq}{xyzq}, \frac{cxyzq}{xyzq}, \frac{pxyzq}{xyzq}$

Answer: ⓐ

2. **Which one of the following is the product of $\frac{x^2y^2}{ab}$ and $\frac{c^3d^2}{x^5y^3}$?**

ⓐ $\frac{x^2y^2c^3d^2}{abx^3y^2}$ ⓑ $\frac{c^3d^2}{abx^3y}$ ⓒ $\frac{x^2y^2c^3}{x^3y}$ ⓓ $\frac{xyd^2}{ab}$

Answer: ⓑ

Explanation : Product of $\frac{x^2y^2}{ab}$ and $\frac{c^3d^2}{x^5y^3}$ = $\frac{x^2y^2}{ab}\times\frac{c^3d^2}{x^5y^3}=\frac{c^3d^2}{abx^3y}$.

3. **What is the quotient if $\frac{x^2-2x+1}{a^2-2a+1}$ is divided by $\frac{x-1}{a-1}$?**

ⓐ $\frac{x+1}{a-1}$ ⓑ $\frac{x-1}{a-1}$ ⓒ $\frac{x-1}{a+1}$ ⓓ $\frac{a-1}{x-1}$

Answer: ⓑ

Explanation : $\frac{x^2-2x+1}{a^2-2a+1}\div\frac{x-1}{a-1}$

$=\frac{(x-1)^2}{(a-1)^2}\times\frac{a-1}{x-1}=\frac{x-1}{a-1}$.

4. **Which one of the following is the simple value of $\frac{a-b}{a}-\frac{a+b}{b}$?**

ⓐ $\frac{a^2-2ab-b^2}{ab}$ ⓑ $\frac{a^2-2ab+b^2}{ab}$

ⓒ $\frac{-a^2-b^2}{ab}$ ⓓ $\frac{a^2-b^2}{ab}$

Answer: ⓒ

Explanation :

$\frac{a-b}{a}-\frac{a+b}{b}=\frac{b(a-b)-a(a+b)}{ab}$

$=\frac{ab-b^2-a^2-ab}{ab}=\frac{-a^2-b^2}{ab}$.

5. **Which one of the following is the value of $\frac{p+x}{p-x}+\frac{(p+x)^2}{p^2-x^2}$?**

ⓐ 1 ⓑ $p-x$ ⓒ $p+x$ ⓓ $\frac{p-x}{p+x}$

Answer: ⓐ

Explanation :

$\frac{p+x}{p-x}\div\frac{(p+x)^2}{p^2-x^2}=\frac{p+x}{p-x}\div\frac{(p+x)^2}{(p+x)(p-x)}$

$=\frac{p+x}{p-x}\div\frac{(p+x)}{(p-x)}=\frac{p+x}{p-x}\times\frac{(p-x)}{(p+x)}=1.$

6. Which one of the following expressions will be if $\frac{x+y}{x-y}$ and $\frac{x-y}{x+y}$ are turned into fractions with common denominator?

ⓐ $\frac{(x+y)^2}{x^2-y^2}, \frac{(x-y)^2}{x^2-y^2}$ ⓑ $\frac{(x+y)^2}{x-y}, \frac{(x-y)^2}{x+y}$

ⓒ $\frac{(x+y)^2}{x^2+y^2}, \frac{(x-y)^2}{x^2+y^2}$ ⓓ $\frac{x-y}{(x+y)^2}, \frac{x+y}{(x-y)^2}$

Ans: (a)

⮞ **Explanation :** Here, the L.C.M of $(n-y)$ and $(n+y)$ is First fraction, $\frac{x+y}{x-y} = \frac{(x+y)\times(x+y)}{(x-y)\times(x+y)}$

$= \frac{(x+y)^2}{x^2-y^2} (x+y)(x-y)$

Second fraction, $\frac{x-y}{x+y} = \frac{(x-y)\times(x-y)}{(x+y)\times(x-y)}$

$= \frac{(x-y)^2}{x^2-y^2}$

Answer questions 7 to 9 in the light of the following information of the stimulus :

$\frac{x^2+4x-21}{x^2+5x-14}$ is an algebraic fraction.

7. Which one is the factorized form of the numerator?

ⓐ $(x+7)(x-3)$ ⓑ $(x-1)(x+21)$

ⓒ $(x-3)(x-7)$ ⓓ $(x+3)(x-7)$

Ans: (a)

⮞ **Explanation :** The numerator of the fraction $= x^2+4x-21$

$= x^2+7x-3x-21$

$= x(x+7)-3(x+7)$

$= (x+7)(x-3)$

8. Which one of the following is the lowest value of the fraction?

ⓐ $\frac{x-7}{x+7}$ ⓑ $\frac{x-3}{x+2}$ ⓒ $\frac{x+7}{x-2}$ ⓓ $\frac{x-3}{x-2}$

Ans: (d)

⮞ **Explanation :**

$\frac{x^2+4x-21}{x^2+5x-14} = \frac{x^2+7x-3x-21}{x^2+7x-2x-14}$

$= \frac{x(x+7)-3(x+7)}{x(x+7)-2(x+7)} = \frac{(x+7)(x-3)}{(x+7)(x-2)} = \frac{x-3}{x-2}.$

9. What would be added to the lowest value to get the sum $\frac{1}{2-x}$?

ⓐ -1 ⓑ 1 ⓒ $x-2$ ⓓ $x-3$

Ans: (a)

10. The following will be the equivalent fraction of $\frac{x^2+6x+5}{x^2+10x+25}$.

i. $\frac{x+1}{x+5}$ ii. $\frac{x^2-2x-3}{x^2+2x-15}$

iii. $\frac{x^2+2x+1}{x^2-3x-10}$

Which one of the following is correct?

ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Ans: (a)

⮞ **Explanation :**

(i) Given expression $= \frac{x^2+6x+5}{x^2+10x+25}$

$= \frac{x^2+5x+x+5}{x^2+5x+5x+25}$

$= \frac{x(x+5)+1(x+5)}{x(x+5)+5(x+5)} = \frac{(x+5)(x+1)}{(x+5)(x+5)} = \frac{x+1}{x+5}$

(ii) Given expression $= \frac{x^2-2x-3}{x^2+2x-15} = \frac{x^2-3x+x-3}{x^2+5x-3x-15}$

$= \frac{x(x-3)+1(x-3)}{x(x+5)-3(x+5)} = \frac{(x-3)(x+1)}{(x+5)(x-3)} = \frac{x+1}{x+5}.$

(iii) Given expression $= \frac{x^2+2x+1}{x^2-3x-10}$

$= \frac{x^2+2.x.1+1^2}{x^2-5x+2x-10} = \frac{(x+1)^2}{x(x-5)+2(x-5)} = \frac{(x+1)(x+1)}{(x-5)(x+2)}$

So, (i) and (ii) correct.

11. Which one of the following is the quotient of $\frac{x^2+2x-3}{x^2+x-2}$ and $\frac{x^2+x-6}{x^2-4}$.

ⓐ $\frac{x+3}{x+2}$ ⓑ $\frac{x-1}{x+3}$ ⓒ 1 ⓓ 0

Ans: (c)

⮞ **Explanation :** $\frac{x^2+2x-3}{x^2+x-2} \div \frac{x^2+x-6}{x^2-4}$

$= \frac{x^2+3x-x-3}{x^2+2x-x-2} \div \frac{x^2+3x-2x-6}{x^2-2^2}$

$= \frac{x(x+3)-1(x+3)}{x(x+2)-1(x+2)} \div \frac{x(x+3)-2(x+3)}{(x+2)(x-2)}$

$= \frac{(x+3)(x-1)}{(x+2)(x-1)} \div \frac{(x+3)(x-2)}{(x+2)(x-2)}$

$= \frac{x+3}{x+2} \div \frac{x+3}{x+2} = 1.$

12. Which is the simplified value of $\frac{1}{x-2} - \frac{1}{x+2} - \frac{4}{x^2-4}$?

ⓐ $\frac{8}{x^2-4}$ ⓑ $\frac{2x}{x^2-4}$ ⓒ 1 ⓓ 0

Ans: (d)

⮞ **Explanation :** $\frac{1}{x-2} - \frac{1}{x+2} - \frac{4}{x^2-4}$

$= \frac{x+2-(x-2)}{(x-2)(x+2)} - \frac{4}{x^2-4}$

$= \frac{x+2-x+2}{x^2-2^2} - \frac{4}{x^2-4}$

$= \frac{4}{x^2-4} - \frac{4}{x^2-4}$

$= \frac{4-4}{x^2-4} = \frac{0}{x^2-4} = 0.$

Solutions to Mathematical Problems

13. Multiply :

(a) $\frac{9x^2y^2}{7y^2z^2}, \frac{5b^2c^2}{3z^2x^2}$ and $\frac{7c^2a^2}{x^2y^2}$

Solution : $\frac{9x^2y^2}{7y^2z^2} \times \frac{5b^2c^2}{3z^2x^2} \times \frac{7c^2a^2}{x^2y^2}$

$= \frac{9 \times 5 \times 7 \times x^2 \times y^2 \times b^2 \times c^2 \times a^2 \times c^2}{7 \times 3 \times y^2 \times z^2 \times x^2 \times z^2 \times x^2 \times y^2} = \frac{15a^2b^2c^4}{x^2y^2z^4}$

(b) $\frac{16a^2b^2}{21z^2}, \frac{28z^4}{9x^3y^4}$ and $\frac{3y^7z}{10x}$

Solution : $\frac{16a^2b^2}{21z^2} \times \frac{28z^4}{9x^3y^4} \times \frac{3y^7z}{10x}$

$= \frac{16 \times 28 \times 3 \times a^2 \times b^2 \times z^4 \times y^7 \times z}{21 \times 9 \times 10 \times z^2 \times x^3 \times y^4 \times x}$

$= \frac{32a^2b^2y^7z^5}{45x^4y^4z^2} = \frac{32a^2b^2y^3z^3}{45x^4}$

(c) $\frac{yz}{x^2}, \frac{zx}{y^2}$ and $\frac{xy}{z^2}$

Solution : $\frac{yz}{x^2} \times \frac{zx}{y^2} \times \frac{xy}{z^2} = \frac{x^2y^2z^2}{x^2y^2z^2} = 1$

(d) $\frac{x-1}{x+1}, \frac{(x-1)^2}{x^2+x}$ and $\frac{x^2}{x^2-4x+5}$

Solution : $\frac{x-1}{x+1} \times \frac{(x-1)^2}{x^2+x} \times \frac{x^2}{x^2-4x+5}$

$= \frac{(x-1)(x-1)^2(x^2)}{(x+1)\,x(x+1)(x^2-4x+5)}$

$= \frac{x(x-1)^3}{(x+1)^2(x^2-4x+5)}$

(e) $\frac{x^4-y^4}{x^2-2xy+y^2}, \frac{x-y}{x^3+y^3}$ and $\frac{x+y}{x^3+y^3}$

Solution : $\frac{x^4-y^4}{x^2-2xy+y^2} \times \frac{x-y}{x^3+y^3} \times \frac{x+y}{x^3+y^3}$

$= \frac{(x^2+y^2)(x+y)(x-y)\ (x-y)(x+y)}{(x-y)^2\ (x+y)(x^2-xy+y^2)(x^3+y^3)}$

$= \frac{(x^2+y^2)(x+y)^2(x-y)^2}{(x-y)^2(x+y)(x^2-xy+y^2)(x+y)(x^2-xy+y^2)}$

$= \frac{x^2+y^2}{(x^2-xy+y^2)^2}$

(f) $\frac{1-b^2}{1+x}, \frac{1-x^2}{b+b^2}$ and $\left(1+\frac{1-x}{x}\right)$

Solution : $\frac{1-b^2}{1+x} \times \frac{1-x^2}{b+b^2} \times \left(1+\frac{1-x}{x}\right)$

$= \frac{(1+b)(1-b)}{1+x} \times \frac{(1+x)(1-x)}{b(1+b)} \times \frac{x+1-x}{x}$

$= \frac{(1-b)(1-x)}{bx}$

(g) $\frac{x^2-3x+2}{x^2-4x+3}, \frac{x^2-5x+6}{x^2-7x+12}$ and $\frac{x^2-16}{x^2-9}$

Solution : $\frac{x^2-3x+2}{x^2-4x+3} \times \frac{x^2-5x+6}{x^2-7x+12} \times \frac{x^2-16}{x^2-9}$

$= \frac{(x-2)(x-1)}{(x-3)(x-1)} \times \frac{(x-2)(x-3)}{(x-3)(x-4)} \times \frac{(x+4)(x-4)}{(x+3)(x-3)}$

$= \frac{(x-2)^2(x+4)}{(x-3)^2(x+3)}$

(h) $\frac{x^3+y^3}{a^2b+ab^2+b^3}, \frac{a^3-b^3}{x^2-xy+y^2}$ and $\frac{ab}{x+y}$

Solution : $\frac{x^3+y^3}{a^2b+ab^2+b^3} \times \frac{a^3-b^3}{x^2-xy+y^2} \times \frac{ab}{x+y}$

$= \frac{(x+y)(x^2-xy+y^2)(a-b)(a^2+ab+b^2)\,ab}{b(a^2+ab+b^2)(x^2-xy+y^2)(x+y)}$

$= a(a-b).$

(i) $\frac{x^3+y^3+3xy(x+y)}{(a+b)^3}, \frac{a^3+b^3+3ab(a+b)}{x^2-y^2}$ and $\frac{(x-y)^2}{(x+y)^2}$

Solution :

$\frac{x^3+y^3+3xy(x+y)}{(a+b)^3} \times \frac{a^3+b^3+3ab(a+b)}{x^2-y^2} \times \frac{(x-y)^2}{(x+y)^2}$

$= \frac{(x+y)^3}{(a+b)^3} \times \frac{(a+b)^3}{(x+y)(x-y)} \times \frac{(x-y)^2}{(x+y)^2}$

$= \frac{(x+y)^3(x-y)^2}{(x+y)^3(x-y)} = (x-y)$

14. Divide (1st expression by 2nd expression) :

(a) $\frac{3x^2}{2a}, \frac{4y^2}{15zx}$

Solution : $\frac{3x^2}{2a} \div \frac{4y^2}{15zx} = \frac{3x^2}{2a} \times \frac{15xz}{4y^2}$

$= \frac{3 \times 15 \times x^2 \times xz}{2 \times 4 \times a \times y^2} = \frac{45x^3z}{8ay^2}$

(b) $\frac{9a^2b^2}{4c^2}, \frac{16a^3b}{3c^3}$

Solution : $\frac{9a^2b^2}{4c^2} \div \frac{16a^3b}{3c^3} = \frac{9a^2b^2}{4c^2} \times \frac{3c^3}{16a^3b}$

$= \frac{9 \times 3 \times a^2b^2c^3}{4 \times 16 \times a^3bc^2} = \frac{27bc}{64a}$

(c) $\frac{21a^4b^4c^4}{4x^3y^3z^3}, \frac{7a^2b^2c^2}{12xyz}$

Solution : $\frac{21a^4b^4c^4}{4x^3y^3z^3} \div \frac{7a^2b^2c^2}{12xyz}$

$= \frac{21a^4b^4c^4}{4x^3y^3z^3} \times \frac{12xyz}{7a^2b^2c^2}$

$= \frac{21 \times 12 \times a^4b^4c^4 \times xyz}{4 \times 7 \times a^2b^2c^2 \times x^3y^3z^3}$

$= \frac{9a^2b^2c^2}{x^2y^2z^2}$

(d) $\frac{x}{y}, \frac{x+y}{y}$

Solution : $\frac{x}{y} \div \frac{x+y}{y} = \frac{x}{y} \times \frac{y}{x+y} = \frac{x}{x+y}$

(e) $\frac{(a+b)^2}{(a-b)^2}, \frac{a^2-b^2}{a+b}$

Solution : $\frac{(a+b)^2}{(a-b)^2} \div \frac{(a^2-b^2)}{(a+b)}$

$= \frac{(a+b)(a+b)}{(a-b)(a-b)} \div \frac{(a+b)(a-b)}{a+b}$

$= \frac{(a+b)(a+b)}{(a-b)(a-b)} \times \frac{a+b}{(a+b)(a-b)} = \frac{(a+b)^2}{(a-b)^3}$

(f) $\frac{x^3-y^3}{x+y}, \frac{x^2+xy+y^2}{x^2-y^2}$

Solution : $\frac{x^3-y^3}{x+y} \div \frac{x^2+xy+y^2}{x^2-y^2}$

$= \frac{(x-y)(x^2+xy+y^2)}{x+y} \div \frac{x^2+xy+y^2}{(x+y)(x-y)}$

$= \frac{(x-y)(x^2+xy+y^2)}{x+y} \times \frac{(x+y)(x-y)}{x^2+xy+y^2}$

$= (x-y)(x-y) = (x-y)^2.$

(g) $\frac{a^3+b^3}{a-b}, \frac{a^2-ab+b^2}{a^2-b^2}$

Solution : $\frac{a^3+b^3}{a-b} \div \frac{a^2-ab+b^2}{a^2-b^2}$

$= \frac{(a+b)(a^2-ab+b^2)}{a-b} \div \frac{a^2-ab+b^2}{(a+b)(a-b)}$

$= \frac{(a+b)(a^2-ab+b^2)}{a-b} \times \frac{(a+b)(a-b)}{a^2-ab+b^2}$

$= (a+b)^2$

(h) $\frac{x^2-7x+12}{x^2-4}, \frac{x^2-16}{x^2-3x+2}$

Solution : $\frac{x^2-7x+12}{x^2-4} \div \frac{x^2-16}{x^2-3x+2}$

$= \frac{x^2-3x-4x+12}{x^2-2^2} \div \frac{x^2-4^2}{x^2-2x-x+2}$

$= \frac{(x-4)(x-3)}{(x+2)(x-2)} \div \frac{(x+4)(x-4)}{(x-2)(x-1)}$

$= \frac{(x-4)(x-3)}{(x+2)(x-2)} \times \frac{(x-2)(x-1)}{(x+4)(x-4)}$

$= \frac{(x-1)(x-3)}{(x+2)(x+4)}$

(i) $\frac{x^2-x-30}{x^2-36}, \frac{x^2+13x+40}{x^2+x-56}$

Solution : $\frac{x^2-x-30}{x^2-36} \div \frac{x^2+13x+40}{x^2+x-56}$

$= \frac{x^2-6x+5x-30}{x^2-6^2} \div \frac{x^2+5x+8x+40}{x^2+8x-7x-56}$

$= \frac{x(x-6)+5(x-6)}{(x+6)(x-6)} \div \frac{x(x+5)+8(x+5)}{x(x+8)-7(x+8)}$

$= \frac{(x-6)(x+5)}{(x+6)(x-6)} \times \frac{(x+8)(x-7)}{(x+8)(x+5)} = \frac{x-7}{x+6}$

15. Simplify :

(a) $\left(\frac{1}{x}+\frac{1}{y}\right) \times \left(\frac{1}{y}-\frac{1}{x}\right)$

Solution : $\left(\frac{1}{x}+\frac{1}{y}\right) \times \left(\frac{1}{y}-\frac{1}{x}\right)$

$= \frac{y+x}{xy} \times \frac{x-y}{yx} = \frac{x^2-y^2}{x^2y^2}$

(b) $\left(\frac{1}{1+x}+\frac{2x}{1-x^2}\right)\left(\frac{1}{x}-\frac{1}{x^2}\right)$

Solution : $\left(\frac{1}{1+x}+\frac{2x}{1-x^2}\right)\left(\frac{1}{x}-\frac{1}{x^2}\right)$

$= \left(\frac{1-x+2x}{1-x^2}\right)\left(\frac{x-1}{x^2}\right)$

$= \frac{1+x}{(1+x)(1-x)} \times \frac{x-1}{x^2}$

$= \frac{x-1}{x^2(1-x)} = \frac{-1(1-x)}{x^2(1-x)} = -\frac{1}{x^2}$

(c) $\left(1-\frac{c}{a+b}\right)\left(\frac{a}{a+b+c}-\frac{a}{a+b-c}\right)$

Solution : $\left(1-\frac{c}{a+b}\right)\left(\frac{a}{a+b+c}-\frac{a}{a+b-c}\right)$

$= \frac{a+b-c}{a+b} \times \frac{a(a+b-c)-a(a+b+c)}{(a+b+c)(a+b-c)}$

$= \frac{a+b-c}{a+b} \times \frac{a^2+ab-ac-a^2-ab-ac}{(a+b+c)(a+b-c)}$

$= \frac{a+b-c}{a+b} \times \frac{-2ac}{(a+b+c)(a+b-c)}$

$= -\frac{2ac}{(a+b)(a+b+c)}$

(d) $\left(\frac{1}{1+a}+\frac{a}{1-a}\right)\left(\frac{1}{1+a^2}-\frac{1}{1+a+a^2}\right)$

Solution : $\left(\frac{1}{1+a}+\frac{a}{1-a}\right)\left(\frac{1}{1+a^2}-\frac{1}{1+a+a^2}\right)$

$= \frac{1-a+a(1+a)}{(1+a)(1-a)} \times \frac{1+a+a^2-(1+a^2)}{(1+a^2)(1+a+a^2)}$

$= \frac{1-a+a+a^2}{(1+a)(1-a)} \times \frac{1+a+a^2-1-a^2}{(1+a^2)(1+a+a^2)}$

$= \frac{1+a^2}{(1+a)(1-a)} \times \frac{a}{(1+a^2)(1+a+a^2)}$

$= \frac{a}{(1-a^2)(1+a+a^2)}$

(e) $\left(\frac{x}{2x-y}+\frac{x}{2x+y}\right)\left(4+\frac{3y^2}{x^2-y^2}\right)$

Solution : $\left(\frac{x}{2x-y}+\frac{x}{2x+y}\right)\left(4+\frac{3y^2}{x^2-y^2}\right)$

$$=\frac{x(2x+y)+x(2x-y)}{(2x-y)\,(2x+y)}\times\frac{4(x^2-y^2)+3y^2}{x^2-y^2}$$

$$=\frac{2x^2+xy+2x^2-xy}{(2x-y)\,(2x+y)}\times\frac{4x^2-4y^2+3y^2}{(x+y)\,(x-y)}$$

$$\frac{4x^2}{(2x-y)\,(2x+y)}\times\frac{4x^2-y^2}{(x+y)\,(x-y)}$$

$$=\frac{4x^2}{(4x^2-y^2)}\times\frac{(4x^2-y^2)}{x^2-y^2}=\frac{4x^2}{x^2-y^2}$$

(f) $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{y}{x+y}\right)$

Solution : $\left(\frac{2x+y}{x+y}-1\right)\div\left(1-\frac{y}{x+y}\right)$

$$=\frac{2x+y-x-y}{x+y}\div\frac{x+y-y}{x+y}=\frac{x}{x+y}\times\frac{x+y}{x}=1$$

(g) $\left(\frac{a}{a+b}+\frac{b}{a-b}\right)\div\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$

Solution : $\left(\frac{a}{a+b}+\frac{b}{a-b}\right)\div\left(\frac{a}{a-b}-\frac{b}{a+b}\right)$

$$=\frac{a(a-b)+b(a+b)}{(a+b)\,(a-b)}\div\frac{a(a+b)-b(a-b)}{(a-b)\,(a+b)}$$

$$=\frac{a^2-ab+ab+b^2}{(a+b)\,(a-b)}\div\frac{a^2+ab-ab+b^2}{(a-b)\,(a+b)}$$

$$=\frac{a^2+b^2}{(a+b)\,(a-b)}\div\frac{a^2+b^2}{(a+b)\,(a-b)}$$

$$=\frac{a^2+b^2}{(a+b)\,(a-b)}\times\frac{(a+b)\,(a-b)}{a^2+b^2}=1$$

(h) $\left(\frac{a^2+b^2}{2ab}-1\right)\div\left(\frac{a^3-b^3}{a-b}-3ab\right)$

Solution : $\left(\frac{a^2+b^2}{2ab}-1\right)\div\left(\frac{a^3-b^3}{a-b}-3ab\right)$

$$=\frac{a^2+b^2-2ab}{2ab}\div\frac{a^3-b^3-3ab(a-b)}{a-b}$$

$$=\frac{(a-b)^2}{2ab}\div\frac{(a-b)^3}{a-b}$$

$$=\frac{(a-b)^2}{2ab}\times\frac{a-b}{(a-b)^3}=\frac{(a-b)^3}{2ab\,(a-b)^3}=\frac{1}{2ab}$$

(i) $\frac{(x+y)^2-4xy}{(a+b)^2-4ab}\div\frac{x^3-y^3-3xy(x-y)}{a^3-b^3-3ab(a-b)}$

Solution : $\frac{(x+y)^2-4xy}{(a+b)^2-4ab}\div\frac{x^3-y^3-3xy(x-y)}{a^3-b^3-3ab(a-b)}$

$$=\frac{(x-y)^2}{(a-b)^2}\div\frac{(x-y)^3}{(a-b)^3}=\frac{(x-y)^2}{(a-b)^2}\times\frac{(a-b)^3}{(x-y)^3}$$

$$=\frac{a-b}{x-y}$$

(j) $\left(\frac{a}{b}+\frac{b}{a}+1\right)\div\left(\frac{a^2}{b^2}+\frac{a}{b}+1\right)$

Solution : $\left(\frac{a}{b}+\frac{b}{a}+1\right)\div\left(\frac{a^2}{b^2}+\frac{a}{b}+1\right)$

$$=\frac{a^2+b^2+ab}{ab}\div\frac{a^2+ab+b^2}{b^2}$$

$$=\frac{a^2+ab+b^2}{ab}\times\frac{b^2}{a^2+ab+b^2}=\frac{b}{a}$$

16. Simplify :

(a) $\frac{x^2+2x-15}{x^2+x-12}\div\frac{x^2-25}{x^2-x-20}\times\frac{x-2}{x^2-5x+6}$

Solution : $\frac{x^2+2x-15}{x^2+x-12}\div\frac{x^2-25}{x^2-x-20}\times\frac{x-2}{x^2-5x+6}$

$$=\frac{x^2+5x-3x-15}{x^2+4x-3x-12}\div\frac{(x+5)\,(x-5)}{x^2-5x+4x-20}\times\frac{x-2}{x^2-3x-2x+6}$$

$$=\frac{(x+5)\,(x-3)}{(x+4)\,(x-3)}\div\frac{(x+5)\,(x-5)}{(x-5)\,(x+4)}\times\frac{x-2}{(x-3)\,(x-2)}$$

$$=\frac{(x+5)\,(x-3)}{(x+4)\,(x-3)}\times\frac{(x-5)\,(x+4)}{(x+5)\,(x-5)}\times\frac{(x-2)}{(x-3)\,(x-2)}$$

$$=\frac{1}{x-3}$$

(b) $\left(\frac{x}{x-y}-\frac{x}{x+y}\right)\div\left(\frac{y}{x-y}-\frac{y}{x+y}\right)+\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\div\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right)$

Solution : $\left(\frac{x}{x-y}-\frac{x}{x+y}\right)\div\left(\frac{y}{x-y}-\frac{y}{x+y}\right)+\left(\frac{x+y}{x-y}+\frac{x-y}{x+y}\right)\div\left(\frac{x+y}{x-y}-\frac{x-y}{x+y}\right)$

$$=\frac{x(x+y)-x(x-y)}{(x-y)\,(x+y)}\div\frac{y(x+y)-y(x-y)}{(x-y)\,(x+y)}+\frac{(x+y)^2+(x-y)^2}{(x-y)\,(x+y)}\div\frac{(x+y)^2-(x-y)^2}{(x-y)\,(x+y)}$$

$$=\frac{x^2+xy-x^2+xy}{x^2-y^2}\div\frac{xy+y^2-xy+y^2}{x^2-y^2}+\frac{2(x^2+y^2)}{x^2-y^2}\div\frac{4xy}{x^2-y^2}$$

$$=\frac{2xy}{(x^2-y^2)}\times\frac{(x^2-y^2)}{2y^2}+\frac{2(x^2+y^2)}{(x^2-y^2)}\times\frac{(x^2-y^2)}{4xy}$$

$$=\frac{2xy}{2y^2}+\frac{2(x^2+y^2)}{4xy}=\frac{x}{y}+\frac{(x^2+y^2)}{2xy}$$

$$=\frac{2x^2+x^2+y^2}{2xy}=\frac{3x^2+y^2}{2xy}$$

(c) $\frac{x^2+2x-3}{x^2+x-2} \div \frac{x^2+x-6}{x^2-4}$

Solution : $\frac{x^2+2x-3}{x^2+x-2} \div \frac{x^2+x-6}{x^2-4}$

$= \frac{x^2+3x-x-3}{x^2+2x-x-2} \div \frac{x^2+3x-2x-6}{(x+2)(x-2)}$

$= \frac{(x+3)(x-1)}{(x+2)(x-1)} \div \frac{(x+3)(x-2)}{(x+2)(x-2)}$

$= \frac{(x+3)(x-1)}{(x+2)(x-1)} \times \frac{(x+2)(x-2)}{(x+3)(x-2)} = 1$

(d) $\frac{a^4-b^4}{a^2+b^2-2ab} \times \frac{(a+b)^2-4ab}{a^3-b^3} \div \frac{a+b}{a^2+ab+b^2}$

Solution : $\frac{a^4-b^4}{a^2+b^2-2ab} \times \frac{(a+b)^2-4ab}{a^3-b^3} \div \frac{a+b}{a^2+ab+b^2}$

$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)^2} \times \frac{(a-b)^2}{(a-b)(a^2+ab+b^2)} \div \frac{a+b}{a^2+ab+b^2}$

$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a-b)(a-b)}{(a-b)(a^2+ab+b^2)} \times \frac{(a^2+ab+b^2)}{(a+b)}$

$= a^2+b^2$

Creative Questions with Solutions

Ques. 17 $\frac{a^4-b^4}{a^2-2ab+b^2}, \frac{a-b}{a^3+b^3}, \frac{a+b}{a^3+b^3}$ are three algebraic expressions.

a. Express the first expression into the lowest form. 2

b. Show that, the product of the three expressions is $\frac{a^2+b^2}{(a^2-ab+b^2)^2}$. 4

c. Divide the first expression by $\frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}$. Add $\frac{a^2}{a+b}$ to the quotient you get. 4

Solution to Question No. 17 :

a 1^{st} expression $= \frac{a^4-b^4}{a^2-2ab+b^2} = \frac{(a^2+b^2)(a^2-b^2)}{(a-b)^2}$

$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)^2} = \frac{(a^2+b^2)(a+b)}{a-b}$,

which is the required lowest form.

b The product of the given 3 expressions

$= \frac{a^4-b^4}{a^2-2ab+b^2} \times \frac{a-b}{a^3+b^3} \times \frac{a+b}{a^3+b^3}$

$= \frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a-b)}{(a+b)(a^2-ab+b^2)} \times \frac{(a+b)}{(a+b)(a^2-ab+b^2)}$

$= \frac{a^2+b^2}{(a^2-ab+b^2)^2}$

$\therefore$ The product of the 3 expressions $= \frac{a^2+b^2}{(a^2-ab+b^2)^2}$.

c $\left(1\text{st expression} \div \frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}\right) + \frac{a^2}{a+b}$

$= \left(\frac{a^4-b^4}{a^2-2ab+b^2} \div \frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}\right) + \frac{a^2}{a+b}$

$= \left(\frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a-b)(a-b)}{(a+b)(a^2+b^2)}\right) + \frac{a^2}{a+b}$

$= (a-b) + \frac{a^2}{a+b} = \frac{(a+b)(a-b)+a^2}{a+b}$

$= \frac{a^2-b^2+a^2}{a+b} = \frac{2a^2-b^2}{a+b}$

Ques. 18 $A = x^2-5x+6$, $B = x^2-7x+12$, $C = x^2-9x+20$ are three algebraic expressions.

a. Find out the difference between $\frac{x}{y}$ and $\frac{x+y}{y}$. 2

b. Express $\frac{1}{B}+\frac{1}{C}$ into the lowest form. 4

c. Turn $\frac{1}{A}, \frac{1}{B}, \frac{1}{C}$ into the fractions with a common denominator. 4

Solution to Question No. 18 :

a $\frac{x}{y} - \frac{x+y}{y} = \frac{x-(x+y)}{y} = \frac{x-x-y}{y} = \frac{-y}{y} = -1$

$\therefore \frac{x}{y} - \frac{x+y}{y} = -1$.

b Here, $\frac{1}{B}+\frac{1}{C} = \frac{1}{x^2-7x+12} + \frac{1}{x^2-9x+20}$

$= \frac{1}{(x-4)(x-3)} + \frac{1}{(x-5)(x-4)} = \frac{x-5+x-3}{(x-3)(x-4)(x-5)}$

$= \frac{2x-8}{(x-3)(x-4)(x-5)} = \frac{2(x-4)}{(x-3)(x-4)(x-5)}$

$= \frac{2}{(x-3)(x-5)}$, which is the required lowest form.

c $\frac{1}{A} = \frac{1}{x^2-5x+6} = \frac{1}{(x-3)(x-2)}$

Now LCM of the denominators of the three fractions is—

$(x-2)(x-3)(x-4)(x-5)$

$\therefore \frac{1}{A} = \frac{1}{(x-3)(x-2)}$

$= \frac{(x-4)(x-5)}{(x-2)(x-3)(x-4)(x-5)}$ ....(i)

And $\frac{1}{B} = \frac{1}{x^2-7x+12} = \frac{1}{(x-4)(x-3)}$

$= \frac{(x-2)(x-5)}{(x-2)(x-3)(x-4)(x-5)}$ ....(ii)

And $\frac{1}{C} = \frac{1}{x^2-9x+20} = \frac{1}{(x-5)(x-4)}$

$= \frac{(x-2)(x-3)}{(x-2)(x-3)(x-4)(x-5)}$ ....(iii)

$\therefore$ (i), (ii), (iii) are the required fractions with common denominators against $\frac{1}{A}, \frac{1}{B}, \frac{1}{C}$ respectively.

Mathematics

Ques. 19 $A = x - 2$, $B = x^2 + 2x + 4$, $C = x^3 - 8$ are three algebraic expressions.

a. Find out the sum of : $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} + \frac{a-b}{ac}$. 2

b. Simplify : $\frac{1}{A} \times \frac{x-2}{B} + \frac{6x}{C}$. 4

c. Prove that, $\frac{1}{A} \times \frac{x+2}{B} \div \frac{x+2}{C} = 1$. 4

Solution to Question No. 19 :

a Here $\frac{a}{bc} + \frac{b}{ca} + \frac{c}{ab} + \frac{a-b}{ac}$

$= \frac{a^2 + b^2 + c^2 + b(a-b)}{abc}$

$= \frac{a^2 + b^2 + c^2 + ab - b^2}{abc} = \frac{a^2 + c^2 + ab}{abc}$.

b $\frac{1}{A} \times \frac{x-2}{B} + \frac{6x}{C}$

$= \frac{1}{x-2} \times \frac{x-2}{x^2+2x+4} + \frac{6x}{x^3-8}$

$= \frac{1}{(x-2)} \times \frac{(x-2)}{x^2+2x+4} + \frac{6x}{(x-2)(x^2+2x+4)}$

$= \frac{1}{x^2+2x+4} + \frac{6x}{(x-2)(x^2+2x+4)}$

$= \frac{x-2+6x}{(x-2)(x^2+2x+4)} = \frac{7x-2}{(x-2)(x^2+2x+4)}$

c $\frac{1}{A} \times \frac{x+2}{B} \div \frac{x+2}{C} = 1$ .... (i)

LHS of (i) $= \frac{1}{A} \times \frac{x+2}{B} \div \frac{x+2}{C}$

$= \frac{1}{x-2} \times \frac{x+2}{x^2+2x+4} \div \frac{x+2}{x^3-8}$

$= \frac{1}{x-2} \times \frac{x+2}{x^2+2x+4} \times \frac{x^3-8}{x+2}$

$= \frac{1}{(x-2)} \times \frac{(x+2)}{x^2+2x+4} \times \frac{(x-2)(x^2+2x+4)}{x+2}$

$= 1$

$=$ RHS of (i)

$\therefore \frac{1}{A} \times \frac{x+2}{B} \div \frac{x+2}{C} = 1$ **(Proved)**

Ques. 20 $A = \frac{x^2+3x-4}{x^2+7x+12}$, $B = \frac{x^2+2x-3}{x^2+6x-7}$, $C = \frac{x^2+12x+35}{x^2+4x-5}$ are three algebraic expressions.

a. Turn the expression A into the lowest form. 2

b. Simplify : A + B. 4

c. Show that, $B \times C \div \frac{x^2-9}{x-1} = \frac{1}{x-3}$. 4

Solution to Question No. 20 :

a Here, $A = \frac{x^2+3x-4}{x^2+7x+12} = \frac{(x+4)(x-1)}{(x+3)(x+4)} = \frac{x-1}{x+3}$, which is the lowest form of A.

b $A + B = \frac{x^2+3x-4}{x^2+7x+12} + \frac{x^2+2x-3}{x^2+6x-7}$

$= \frac{(x+4)(x-1)}{(x+4)(x+3)} + \frac{x^2+2x-3}{(x+7)(x-1)}$

$= \frac{x-1}{x+3} + \frac{(x+3)(x-1)}{(x+7)(x-1)} = \frac{x-1}{x+3} + \frac{x+3}{x+7}$

$= \frac{(x-1)(x+7)+(x+3)(x+3)}{(x+3)(x+7)}$

$= \frac{x^2+6x-7+x^2+6x+9}{(x+3)(x+7)}$

$= \frac{2x^2+12x+2}{(x+3)(x+7)} = \frac{2(x^2+6x+1)}{(x+3)(x+7)}$

c L.H.S $= B \times C \div \frac{x^2-9}{x-1}$

$= \frac{x^2+2x-3}{x^2+6x-7} \times \frac{x^2+12x+35}{x^2+4x-5} \div \frac{x^2-9}{x-1}$

$= \frac{x^2+2x-3}{(x+7)(x-1)} \times \frac{(x+7)(x+5)}{(x+5)(x-1)} \times \frac{x-1}{(x+3)(x-3)}$

$= \frac{x^2+2x-3}{(x-1)(x+3)(x-3)}$

$= \frac{(x^2+2x-3)}{(x^2+2x-3)(x-3)} = \frac{1}{x-3} =$ R.H.S

$\therefore B \times C \div \frac{x^2-9}{x-1} = \frac{1}{x-3}$ **(Showed)**

Multiple Choice Q/A Designed as per topic

5.6 Multiplication of fractions ▸ Textbook Page 89

1. **What is called transformation of two or more fractions associated with operational signs into simple expression?** *(Easy)*
   ⓐ Simplification ⓑ Lowest term
   ⓒ Multiple ⓓ Done of the above

   ⓐ

2. $\frac{1-x^2}{b+b^2} \times \frac{1-b^2}{1+x} = ?$ *(Hard) [MB '19]*
   ⓐ $\frac{(1+x)(1-b)}{b}$ ⓑ $\frac{(1-x)(1+b)}{b}$
   ⓒ $\frac{(1-x)(1-b)}{b}$ ⓓ $\frac{(1+x)(1+b)}{b}$

   ⓒ

3. **What will be the product if the fraction $\frac{x^2-5x+6}{x^2-9x+20}$ is multiplied by $(x-4)(x-5)$?** *(Medium) [CtgB '17]*

ⓐ $x^2-9x+20$ ⓑ x^2-6x+5
ⓒ x^2-5x+6 ⓓ $x^2-8x+180$

Ⓒ

4. $\frac{x^2-y^2}{x^3+y^3}\times\frac{x^2-xy+y^2}{x^3-y^3}$ **= what?** *(Medium)[DB' 15]*

ⓐ x^2+xy+y^2 ⓑ $\frac{1}{x^2+xy+y^2}$
ⓒ x^2-xy+y^2 ⓓ $\frac{x+y}{x^2+xy+y^2}$

Ⓑ

5. $\frac{a^2}{ab}\times\frac{b^2}{bc}\times\frac{c^2}{ca}$ **= what?** *(Easy) [Ctg.B' 15]*

ⓐ $a^2b^2c^2$ ⓑ $\frac{1}{a^2b^2c^2}$ ⓒ 1 ⓓ $a^4b^4c^4$

Ⓒ

6. $\left(\frac{1}{x}+\frac{1}{y}\right)\times\left(\frac{1}{y}-\frac{1}{x}\right)$ **= What?** *(Hard) [Din.B' 15]*

ⓐ $\frac{x^2-y^2}{x^2y^2}$ ⓑ $\frac{x^2+y^2}{x^2y^2}$ ⓒ $\frac{x^2-y^2}{xy}$ ⓓ 1

Ⓐ

7. **Which one of the simple value of $\frac{a-b}{a}\times\frac{a+b}{a}$?** *(Easy) [Ideal School & College, Dhaka]*

ⓐ $\frac{a^2-2ab-b^2}{ab}$ ⓑ $\frac{a^2-2ab+b^2}{ab}$
ⓒ $\frac{-a^2-b^2}{ab}$ ⓓ $\frac{a^2-b^2}{ab}$

Ⓓ

8. **What is the product of $\frac{a^2-b^2}{a^3+b^3}\times\frac{a^2-ab+b^2}{a^3-b^3}$?** *(Easy) [Viqarunnisa Noon School and College, Dhaka]*

ⓐ $\frac{a+b}{a^2+ab+b^2}$ ⓑ a^2-ab+b^2
ⓒ $\frac{1}{a^2-ab+b^2}$ ⓓ $\frac{1}{a^2+ab+b^2}$

Ⓓ

9. **For the two fractions $\frac{1}{x-3},\frac{1}{x+3}$.** *(Medium) [DjB '16]*

i. the product of denominator is x^2-9
ii. the required quotient is $\frac{x+3}{x-3}$
iii. the required product is $\frac{1}{x^2-9}$

Which one is correct?

ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Ⓓ

10. **The value of the two fractions is $\frac{x}{x+y},\frac{x}{x-y}$.** *(Easy) [RB' 15]*

i. the product of denominators is x^2-y^2
ii. the required product is $\frac{x^2}{x^2-y^2}$
iii. the required quotient is $\frac{x+y}{x-y}$

Which one of the following is true?

ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

Ⓐ

5.7 Division of fractions ▸ Textbook Page 92

11. **Which one of the following is the value of $\frac{a^2+2a+1}{a^2-2a+1}\div\frac{a+1}{a-1}$?** *(Hard) [BB '19]*

ⓐ $\frac{a+1}{a^2-1}$ ⓑ $\frac{a^2-1}{a+1}$ ⓒ $\frac{a-1}{a+1}$ ⓓ $\frac{a+1}{a-1}$

Ⓓ

12. **Which one of the following is the simplified value of $\left(\frac{1}{2x}+\frac{1}{y}\right)\div\left(\frac{1}{y}-\frac{1}{2x}\right)$?** *(Hard) [MB '19]*

ⓐ $\frac{(2x+y)^2}{4xy}$ ⓑ $\frac{4x^2-y^2}{4x^2y^2}$
ⓒ $\frac{2x+y}{2x-y}$ ⓓ $\frac{(2x+y)^2}{4x^2y^2}$

Ⓒ

13. $\frac{a-p}{a+p}\div\frac{(a-p)^2}{a^2-p^2}$ **= ?** *(Hard) [DB '18]*

ⓐ 1 ⓑ $\left(\frac{a-p}{a+p}\right)^2$ ⓒ $(a+p)$ ⓓ $\left(\frac{a+p}{a-p}\right)^2$

Ⓐ

14. **Which one of the following is the value of $\frac{a+b}{a-b}\div\frac{(a+b)^2}{a^2-b^2}$?** *(Medium) [CtgB '18]*

ⓐ 0 ⓑ 1 ⓒ a + b ⓓ a − b

Ⓑ

15. **The present age of Summo and Alok are 35 years and 25 years respectively. What was the ratio of the their age before 10 years?** *(Hard) [CtgB '18]*

ⓐ 9 : 7 ⓑ 7 : 5 ⓒ 5 : 3 ⓓ 3 : 5

Ⓒ

16. **The present age of sister and brother is 40 and 30 years respectively. What was the ratio between their ages 10 years ago?** *(Medium) [BB '18]*

ⓐ 2 : 3 ⓑ 3 : 2 ⓒ 4 : 2 ⓓ 5 : 4

Ⓑ

17. **What is the simplified value of $\frac{m^2-n^2}{m^2+n^2-2mn}\div\frac{(m-n)^2}{(m+n)^2-4mn}$?** *(Medium) [DB '17]*

ⓐ m − n ⓑ m + n
ⓒ $\frac{m-n}{m+n}$ ⓓ $\frac{m+n}{m-n}$

Ⓓ

18. $\left(\frac{2a}{a+b}-2\right)\div\left(4-\frac{4a}{a+b}\right)$ **= ?** *(Medium) [RB '17]*

ⓐ $\frac{3}{2}$ ⓑ $\frac{2}{3}$ ⓒ $\frac{1}{2}$ ⓓ $-\frac{1}{2}$

Ⓓ

19. $\frac{4a}{1+a^2}\div\frac{8a^3}{1-a^4}$ **= ?** *(Hard) [DjB '17]*

ⓐ $\frac{1-a^2}{2a^2}$ ⓑ $\frac{2a^2}{1+a^2}$ ⓒ $\frac{1+a^2}{2a^2}$ ⓓ $\frac{2a^2}{1-a^2}$

Ⓐ

20. **Which one is the equivalent fraction of $\frac{x}{y}$?** *(Easy) [DB '16]*

a ⓐ $\frac{x^2}{xy}$ ⓑ $\frac{x^3}{y^3}$ ⓒ $\frac{x^2}{y^2}$ ⓓ $\frac{xy}{xy^2}$

21. **What is the value of $\frac{a^2-ab+b^2}{a^2+ab+b^2} \div \frac{(a-b)^2}{a^2-b^2}$?** *(Hard) [RB '16]*

ⓐ $\frac{a^3-b^3}{a^3+b^3}$ ⓑ $\frac{a^3+b^3}{a^3-b^3}$

b ⓒ $\frac{(a+b)(a^2+ab+b^2)}{(a-b)(a^2-ab+b^2)}$ ⓓ $\frac{a^2-ab+b^2}{a^2+ab+b^2}$

22. $\frac{x+y}{x^2-y^2} \div \frac{1}{x-y} = ?$ *(Hard) [RB' 15]*

d ⓐ $\frac{x+y}{x-y}$ ⓑ $\frac{x+y}{x^2-y^2}$ ⓒ $\frac{1}{xy}$ ⓓ 1

23. **What is the simplified value of $\frac{x^2-y^2}{(x+y)^2} \div \frac{(x+y)^2-4xy}{x^3+y^3} \times \frac{x+y}{x^2-xy+y^2}$?** *(Hard) [Ctg.B' 15]*

ⓐ $\frac{(x-y)^3}{(x^3+y^3)(x^2-xy+y^2)}$ ⓑ $\frac{x+y}{x-y}$

b ⓒ $\frac{(x^2-xy+y^2)^2}{x^2-y^2}$ ⓓ $\frac{x-y}{x+y}$

24. $\frac{by^2+y^2}{b^2-z^2} \div \frac{bx^2+x^2}{b^2-z^2} = ?$ *(Hard) [SB' 15]*

a ⓐ $\frac{y^2}{x^2}$ ⓑ $\frac{x^2}{y^2}$ ⓒ $\frac{y^2}{bx^2}$ ⓓ $\frac{y^2b}{x^2}$

25. $\left(\frac{2a}{a+b}-2\right) \div \left(4-\frac{4a}{a+b}\right) = ?$ *(Medium) [SB' 15]*

c ⓐ $\frac{1}{2}$ ⓑ 1 ⓒ $-\frac{1}{2}$ ⓓ 2

26. $\left(\frac{x^2-2x+1}{a^2-2a+1}\right) \div \left(\frac{x-1}{a-1}\right)$ = what? *(Hard) [DjB '15]*

c ⓐ $\frac{x+1}{a-1}$ ⓑ $\frac{x-1}{a+1}$ ⓒ $\frac{x-1}{a-1}$ ⓓ $\frac{a-1}{x-1}$

27. $\left(\frac{a}{b}+\frac{b}{a}+1\right) \div \left(\frac{a^2}{b^2}+\frac{a}{b}+1\right) = ?$ *(Medium)*
[Rajuk Uttara Model College, Dhaka]

ⓐ 1 ⓑ (a^2+ab+b^2)

d ⓒ $\frac{a}{b}$ ⓓ $\frac{b}{a}$

28. **Which one of the following is the quotient of $\frac{a^2+2a-3}{a^2+a-2}$ and $\frac{a^2+a-6}{a^2-4}$?** *(Medium)*
[Ideal School & College, Dhaka]

c ⓐ $\frac{a+3}{a+2}$ ⓑ $\frac{a-1}{a+3}$ ⓒ 1 ⓓ 0

29. $\frac{x^4-y^4}{x^2-2xy+y^2} \div \frac{x^3+y^3}{x-y}$ = what? *(Hard)*
[Viqarunnisa Noon School and College, Dhaka]

ⓐ $\frac{x^2+y^2}{x^2-xy+y^2}$ ⓑ $\frac{x^2+y^2}{x^2+xy+y^2}$

a ⓒ $(x+y)^2$ ⓓ x^2-y^2

30. **Expression** $\left(\frac{a}{b}-1\right)$ **and** $\left(1-\frac{a}{b}\right)$— *(Hard) [JB '17]*

i. sumation is zero
ii. quotient is – 1
iii. product is $\frac{(a-b)^2}{b^2}$

Which one of the following is correct?

a ⓐ i & ii ⓑ i & iii ⓒ ii & iii ⓓ i, ii & iii

Short Q/A Designed as per topic

5.6 Multiplication of fractions ▸ Textbook Page 89

Question 1. Multiply $\frac{a^2-b^2}{a^3+b^3}$ by $\frac{a^2-ab+b^2}{a^3-b^3}$.

Solution : The product of $\frac{a^2-b^2}{a^3+b^3}$ and $\frac{a^2-ab+b^2}{a^3-b^3}$

$$= \frac{a^2-b^2}{a^3+b^3} \times \frac{a^2-ab+b^2}{a^3-b^3}$$

$$= \frac{(a+b)(a-b)}{(a+b)(a^2-ab+b^2)} \times \frac{(a^2-ab+b^2)}{(a-b)(a^2+ab+b^2)} = \frac{1}{a^2+ab+b^2}$$

Required product : $\frac{1}{a^2+ab+b^2}$.

Question 2. Multiply $\frac{m^2-5m+6}{m^2-9m+20}$ by $\frac{m-5}{m-3}$.

Solution : The product of $\frac{m^2-5m+6}{m^2-9m+20}$ and $\frac{m-5}{m-3}$

$$= \frac{m^2-5m+6}{m^2-9m+20} \times \frac{m-5}{m-3} = \frac{m^2-3m-2m+6}{m^2-4m-5m+20} \times \frac{m-5}{m-3}$$

$$= \frac{m(m-3)-2(m-3)}{m(m-4)-5(m-4)} \times \frac{m-5}{m-3} = \frac{(m-3)(m-2)}{(m-4)(m-5)} \times \frac{(m-5)}{(m-3)} = \frac{m-2}{m-4}$$

Required product : $\frac{m-2}{m-4}$.

Question 3. $\frac{1-4b^2}{2+x} \times \frac{4-x^2}{b+2b^2} = ?$

Solution : Given expression $= \frac{1-4b^2}{2+x} \times \frac{4-x^2}{b+2b^2}$

$$= \frac{1^2-(2b)^2}{2+x} \times \frac{2^2-x^2}{b(1+2b)}$$

$$= \frac{(1+2b)(1-2b)}{(2+x)} \times \frac{(2+x)(2-x)}{b(1+2b)}$$

$$= \frac{(1+2b)(2-x)}{b}$$

Required product : $\frac{(1+2b)(2-x)}{b}$.

Mathematics

Question 4. $\left(\frac{b}{a}+\frac{a}{b}\right) \times \frac{ab}{a^2+b^2} = ?$

Solution : Given expression $= \left(\frac{b}{a}+\frac{a}{b}\right) \times \frac{ab}{a^2+b^2} =$ $\frac{b^2+a^2}{ab} \times \frac{ab}{a^2+b^2} = 1$

Required product : 1.

Question 5. $\frac{1-y^2}{a+a^2} \times \frac{1-a^2}{1+y}$.

Solution : Given expression $= \frac{1-y^2}{a+a^2} \times \frac{1-a^2}{1+y}$

$$= \frac{1^2-y^2}{a(1+a)} \times \frac{1^2-a^2}{1+y}$$

$$= \frac{(1+y)(1-y)}{a(1+a)} \times \frac{(1+a)(1-a)}{(1+y)} = \frac{(1-y)(1-a)}{a}$$

Required product : $\frac{(1-y)(1-a)}{a}$.

5.7 Division of fractions

▸ Textbook Page 92

Question 6. **Divide** $\frac{x^2-y^2}{x^2+xy+y^2}$ **by** $\frac{x+y}{x^3-y^3}$.

Solution : $\frac{x^2-y^2}{x^2+xy+y^2} \div \frac{x+y}{x^3-y^3} = \frac{x^2-y^2}{x^2+xy+y^2} \times \frac{x^3-y^3}{x+y}$

$$= \frac{(x+y)(x-y)}{(x^2+xy+y^2)} \times \frac{(x-y)(x^2+xy+y^2)}{(x+y)} = (x-y)(x-y) = (x-y)^2$$

Required quotient $(x-y)^2$.

Question 7. $\frac{9a^2}{3+12a^2} \div \frac{18a^3}{1-16a^4} = ?$

Solution : Given expression $= \frac{9a^2}{3+12a^2} \div \frac{18a^3}{1-16a^4}$

$$= \frac{9a^2}{3(1+4a^2)} \div \frac{18a^3}{(1)^2-(4a^2)^2}$$

$$= \frac{9a^2}{3(1+4a^2)} \div \frac{18a^3}{(1+4a^2)(1-4a^2)}$$

$$= \frac{9a^2}{3(1+4a^2)} \times \frac{(1+4a^2)(1-4a^2)}{18a^3}$$

$$= \frac{1-4a^2}{6a}$$

Required quotient : $\frac{1-4a^2}{6a}$.

Question 8. What is the simplified value of $\frac{m^2-n^2}{m^2+n^2-2mn} \div \frac{(m-n)^2}{(m+n)^2-4mn}$**?**

Solution : Given expression

$$= \frac{m^2-n^2}{m^2+n^2-2mn} \div \frac{(m-n)^2}{(m+n)^2-4mn}$$

$$= \frac{m^2-n^2}{m^2-2mn+n^2} \div \frac{(m-n)^2}{(m-n)^2}$$

$$= \frac{(m+n)(m-n)}{(m-n)^2} \times \frac{(m-n)^2}{(m-n)^2} = \frac{m+n}{m-n}$$

Required value : $\frac{m+n}{m-n}$.

Question 9. Divide $\frac{x^2-2x+1}{a^2-2a+1}$ **by** $\frac{x-1}{a-1}$.

Solution : $\frac{x^2-2x+1}{a^2-2a+1} \div \frac{x-1}{a-1} = \frac{(x-1)^2}{(a-1)^2} \times \frac{(a-1)}{(x-1)}$

$$= \frac{a-1}{x-1}$$

Required quotient : $\frac{a-1}{x-1}$.

Question 10. $\frac{p^3+q^3+3pq(p+q)}{(p+q)^2-4pq} \div \frac{(p+q)^2}{(p-q)^3} = ?$

Solution : $\frac{p^3+q^3+3pq(p+q)}{(p+q)^2-4pq} \div \frac{(p+q)^2}{(p-q)^3}$

$$= \frac{(p+q)^3}{(p-q)^2} \times \frac{(p-q)^3}{(p+q)^2} = (p+q)(p-q) = p^2-q^2$$

Required quotient p^2-q^2.

Creative Q/A — Designed as per learning outcomes

Ques. 01 $P = a^2 - 2a - 8$, $Q = a^2 - 3a - 10$ and $R = a^2 - 8a + 15$ are three algebraic expressions.

a. Factorize 'R'. 2

b. Turn $\frac{1}{P}, \frac{1}{Q}, \frac{1}{R}$ into the fraction with a common denominator. 4

c. Show that, $P \times \frac{a-5}{Q} \div \frac{R}{a-3} = \frac{a-4}{a-5}$. 4

• *Dhaka Board 2018*

Solution to Question No. 01 :

a Here, $R = a^2 - 8a + 15$

$$= a^2 - 5a - 3a + 15$$
$$= a(a-5) - 3(a-5)$$
$$= (a-5)(a-3).$$

b We have, $P = a^2 - 2a - 8 = (a-4)(a+2)$

$Q = a^2 - 3a - 10 = (a-5)(a+2)$

$R = a^2 - 8a + 15 = (a-5)(a-3)$

$\therefore$ LCM of P, Q, R is $(a-5)(a-4)(a-3)(a+2)$

$$\therefore \frac{1}{P} = \frac{1}{(a-4)(a+2)}$$
$$= \frac{(a-5)(a-3)}{(a-5)(a-4)(a-3)(a+2)} \text{.............(i)}$$
$$\therefore \frac{1}{Q} = \frac{1}{(a-5)(a+2)}$$
$$= \frac{(a-4)(a-3)}{(a-5)(a-4)(a-3)(a+2)} \text{........(ii)}$$
$$\therefore \frac{1}{R} = \frac{1}{(a-5)(a-3)}$$
$$= \frac{(a-4)(a+2)}{(a-5)(a-4)(a-3)(a+2)} \text{.......(iii)}$$

So, expressions (i), (ii) and (iii) are the required fractions with common dinominators of $\frac{1}{P}, \frac{1}{Q}$ and $\frac{1}{R}$.

c From (b) above, we have,

$P = (a-4)(a+2)$, $Q = (a-5)(a+2)$,

$R = (a-5)(a-3)$

$$\therefore P \times \frac{a-5}{Q} = (a-4)(a+2) \times \frac{a-5}{(a-5)(a+2)}$$
$$= (a-4) \text{.....................(i)}$$

$$\text{Again, } \frac{R}{a-3} = \frac{(a-5)(a-3)}{a-3}$$
$$= (a-5) \text{..................(ii)}$$

$$\therefore P \times \frac{a-5}{Q} \div \frac{R}{a-3} = (a-4) \div (a-5)$$
$$= \frac{a-4}{a-5} \textbf{ (Showed)}$$

Ques. 02 $A = \frac{(a-b)^2 + 2ab}{(a-b)(a^2+2ab+b^2)}$, $B = \frac{a^3+b^3}{(a+b)^3(a^2-b^2)}$ and $C = \frac{a^3-b^3}{a^4-b^4}$.

a. Determine the value of $\left(\frac{1}{x} - \frac{1}{y}\right) \div \left(\frac{1}{y} - \frac{1}{x}\right)$. 2

b. Express A, B and C in the form of common denominator. 4

c. Prove that, $A \div B \times C = \frac{a^2+ab+b^2}{a^2-ab+b^2}$. 4

• *Rajshahi Board 2018*

Solution to Question No. 02 :

a Given that, $\left(\frac{1}{x} - \frac{1}{y}\right) \div \left(\frac{1}{y} - \frac{1}{x}\right)$

$$= \frac{y-x}{xy} \div \frac{x-y}{xy} = \frac{y-x}{xy} \times \frac{xy}{x-y}$$
$$= \frac{-(x-y)}{xy} \times \frac{xy}{x-y}$$
$$= -1.$$

b Here, we have,

$$A = \frac{(a-b)^2 + 2ab}{(a-b)(a^2+2ab+b^2)}$$
$$= \frac{a^2+b^2}{(a-b)(a+b)(a+b)}$$
$$= \frac{a^2+b^2}{(a^2-b^2)(a+b)}$$
$$= \frac{(a^2+b^2)(a^2+b^2)(a+b)^2}{(a^2-b^2)(a^2+b^2)(a+b)^3} \text{.................(i)}$$

$$B = \frac{a^3+b^3}{(a+b)^3(a^2-b^2)}$$
$$= \frac{(a+b)(a^2-ab+b^2)}{(a+b)^3(a+b)(a-b)}$$
$$= \frac{(a+b)(a^2-ab+b^2)}{(a+b)^3(a^2-b^2)}$$
$$= \frac{(a+b)(a^2-ab+b^2)(a^2+b^2)}{(a+b)^3(a^2-b^2)(a^2+b^2)} \text{...............(ii)}$$

$$C = \frac{a^3-b^3}{(a^4-b^4)}$$
$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a+b)(a-b)}$$
$$= \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}$$
$$= \frac{(a^3-b^3)(a+b)^3}{(a^2+b^2)(a^2-b^2)(a+b)^3} \text{.................(iii)}$$

Thus, A, B, C are expressed in (i), (ii) and (iii) in the form of common denominator.

c L.H.S = A ÷ B × C

$$= \frac{(a-b)^2+2ab}{(a-b)(a^2+2ab+b^2)} \div \frac{a^3+b^3}{(a+b)^3(a^2-b^2)} \times \frac{a^3-b^3}{a^4-b^4}$$

$$= \frac{a^2+b^2}{(a-b)(a+b)^2} \div \frac{(a+b)(a^2-ab+b^2)}{(a+b)^3(a^2-b^2)} \times \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}$$

$$= \frac{a^2+b^2}{(a-b)(a+b)^2} \times \frac{(a+b)^3(a^2-b^2)}{(a+b)(a^2-ab+b^2)} \times \frac{(a-b)(a^2+ab+b^2)}{(a^2+b^2)(a^2-b^2)}$$

$$= \frac{a^2+ab+b^2}{a^2-ab+b^2} = \text{R.H.S}$$ **(Proved)**

Ques. 03 $P = \dfrac{a^3-b^3-3ab(a-b)}{(a+b)^2-4ab}$,

$Q = \dfrac{(a-b)^2+4ab}{a^3+b^3+3ab(a+b)}$ are two algebraic fractions.

a. Resolve into factors of m^4+m^2+1. 2

b. Simplify : $\dfrac{a}{a-b} \times (P \div Q)$. 4

c. Prove that, $\left(\dfrac{1}{P}-Q\right)(a^2-b^2) = 2b$. 4

● *Jashore Board 2018*

Mathematics

Solution to Question No. 03 :

a $m^4+m^2+1 = (m^2)^2+1^2+m^2$

$$= (m^2+1)^2-2m^2+m^2$$
$$= (m^2+1)^2-m^2$$
$$= (m^2+1+m)(m^2+1-m)$$
$$= (m^2+m+1)(m^2-m+1).$$

b Given that,

$$P = \frac{a^3-b^3-3ab(a-b)}{(a+b)^2-4ab} = \frac{(a-b)^3}{(a-b)^2} = a-b$$

$$\text{And } Q = \frac{(a-b)^2+4ab}{a^3+b^3+3ab(a+b)} = \frac{(a+b)^2}{(a+b)^3} = \frac{1}{a+b}$$

$$\text{Now, } P \div Q = \frac{P}{Q} = \frac{a-b}{\frac{1}{a+b}} = (a+b)(a-b) = a^2-b^2.$$

$$\therefore \frac{a}{a-b} \times (P \div Q)$$

$$= \frac{a}{a-b} \times a^2-b^2 = \frac{a(a+b)(a-b)}{(a-b)} = a(a+b).$$

c From (b) above,

$P = a-b \Rightarrow \dfrac{1}{p} = \dfrac{1}{a-b}$ and

$$Q = \frac{1}{a+b}$$

$$\therefore \frac{1}{P} - Q = \frac{1}{a-b} - \frac{1}{a+b} = \frac{a+b-a+b}{a^2-b^2} = \frac{2b}{a^2-b^2}$$

$$\text{Now, } \left(\frac{1}{P}-Q\right)(a^2-b^2) = \frac{2b}{a^2-b^2} \times (a^2-b^2) = 2b.$$

$$\therefore \left(\frac{1}{P}-Q\right)(a^2-b^2) = 2b.$$ **(Proved)**

Ques. 04 $A = \dfrac{(p-q)^2+4pq}{p^3-q^3-3pq(p-q)}$,

$B = \dfrac{p^3+q^3+3pq(p+q)}{(p+q)^2-4pq}$,

$C = x^3+y^3$ and $D = x^3-y^3$.

a. Express in the lowest form : $\dfrac{a^2+2a-15}{a^2-9}$. 2

b. Simplify : A ÷ B. 4

c. Express the fractions $\dfrac{x}{C}$ and $\dfrac{y}{D}$ with common denominator. 4

● *Chattogram Board 2018*

Solution to Question No. 04 :

a
$$\frac{a^2+2a-15}{a^2-9} = \frac{a^2+5a-3a-15}{(a+3)(a-3)} = \frac{a(a+5)-3(a+5)}{(a+3)(a-3)} = \frac{(a+5)(a-3)}{(a+3)(a-3)} = \frac{a+5}{a+3},$$ which is the desired lowest form.

b Here, we have,

$$A = \frac{(p-q)^2+4pq}{p^3-q^3-3pq(p-q)} = \frac{p^2-2pq+q^2+4pq}{p^3-q^3-3p^2q+3pq^2} = \frac{p^2+2pq+q^2}{p^3-3p^2q+3pq^2-q^3} = \frac{(p+q)^2}{(p-q)^3}$$

$$B = \frac{p^3+q^3+3pq(p+q)}{(p+q)^2-4pq} = \frac{p^3+3p^2q+3pq^2+q^3}{p^2-2pq+q^2} = \frac{(p+q)^3}{(p-q)^2}$$

$$\therefore A \div B = \frac{(p+q)^2}{(p-q)^3} \div \frac{(p+q)^3}{(p-q)^2}$$
$$= \frac{(p+q)^2}{(p-q)^3} \times \frac{(p-q)^2}{(p+q)^3}$$
$$= \frac{1}{p-q} \times \frac{1}{p+q} = \frac{1}{p^2-q^2}$$
$$\therefore A \div B = \frac{1}{p^2-q^2}$$

c Here, $C = x^3 + y^3 = (x+y)(x^2 - xy + y^2)$

$$\therefore \frac{1}{C} = \frac{1}{(x+y)(x^2-xy+y^2)}$$
$$\text{or, } \frac{x}{C} = \frac{x}{(x+y)(x^2-xy+y^2)} \text{ .... (1)}$$

Again, $D = x^3 - y^3 = (x-y)(x^2+xy+y^2)$

$$\therefore \frac{1}{D} = \frac{1}{(x-y)(x^2+xy+y^2)}$$
$$\text{or, } \frac{y}{D} = \frac{y}{(x-y)(x^2+xy+y^2)} \text{ .... (2)}$$

Now, LCM of denominators of (1) and (2) are
$(x+y)(x-y)(x^2+xy+y^2)(x^2-xy+y^2)$
$= (x^2-y^2)(x^4+x^2y^2+y^4)$

From (1) and (2), we get,

$$\frac{x}{C} = \frac{x(x-y)(x^2+xy+y^2)}{(x^2-y^2)(x^4+x^2y^2+y^4)} \text{ and}$$
$$\frac{y}{D} = \frac{y(x+y)(x^2-xy+y^2)}{(x^2-y^2)(x^4+x^2y^2+y^4)}$$ which are the desired fractions with common denomination.

Ques. 05 $A = 6p^2 - p - 1$, $B = 4p^2 - 1$, $C = p^3 - q^3$ and $D = p^4 + p^2q^2 + q^4$.

a. Find the sum : $\frac{x-2}{x} + \frac{x-2}{2}$. 2

b. Simplify : $\left(\frac{1}{A} + \frac{1}{B}\right) \div \frac{5p+2}{(4p^2-1)(3p+1)}$. 4

c. Express $\frac{1}{C}$, $\frac{1}{D}$ in the form of common denominators. 4

• *Sylhet Board 2018*

Solution to Question No. 05 :

a Here, $\frac{x-2}{x} + \frac{x-2}{2}$

$$= \frac{2x - 4 + x^2 - 2x}{2x}$$
$$= \frac{x^2-4}{2x}.$$

$\therefore$ The sum is $\frac{x^2-4}{2x}$.

b We have,

$A = 6P^2 - P - 1$
$= 6P^2 - 3p + 2p - 1$
$= 3p(2p-1) + 1(2p-1)$
$= (2p-1)(3p+1)$.

$B = 4p^2 - 1 = (2p-1)(2p+1)$

$$\therefore \frac{1}{A} + \frac{1}{B} = \frac{1}{(2P-1)(3P+1)} + \frac{1}{(2p-1)(2p+1)}$$
$$= \frac{2P+1+3P+1}{(2P-1)(2P+1)(3P+1)}$$
$$= \frac{5P+2}{(2P-1)(2P+1)(3P+1)}$$
$$= \frac{5P+2}{(4P^2-1)(3P+1)}.$$

$$\text{Now, } \left(\frac{1}{A} + \frac{1}{B}\right) \div \frac{5P+2}{(4P^2-1)(3P+1)}$$
$$= \frac{5P+2}{(4P^2-1)(3P+1)} \div \frac{5P+2}{(4P^2-1)(3P+1)}$$
$$= \frac{(5P+2)}{(4P^2-1)(3P+1)} \times \frac{(4P^2-1)(3P+1)}{(5P+2)}$$
$$= 1$$

c Here, we have,

$C = p^3 - q^3 = (p-q)(p^2+pq+q^2)$
and $D = p^4 + p^2q^2 + q^4$
$= (p^2+q^2)^2 - 2p^2q^2 + p^2q^2$
$= (p^2+q^2)^2 - (pq)^2$
$= (p^2+pq+q^2)(p^2-pq+q^2)$

Now, LCM of C and D
$= (p-q)(p^2-pq+q^2)(p^2+pq+q^2)$

$$\therefore \frac{1}{C} = \frac{1}{(p-q)(p^2+pq+q^2)}$$
$$= \frac{1(p^2-pq+q^2)}{(p-q)(p^2+pq+q^2)(p^2-pq+q^2)},$$

multiplying both numerator and denominator by $p^2 - pq + q^2$.

$$= \frac{p^2-pq+q^2}{(p-q)(p^4+p^2q^2+q^4)} \text{.................. (1)}$$

$$\text{And } \frac{1}{D} = \frac{1}{(p^2+q^2+pq)(p^2+pq+q^2)}$$
$$= \frac{p-q}{(p-q)(p^2+pq+q^2)(p^2-pq+q^2)}$$
$$= \frac{p-q}{(p-q)(p^4+p^2q^2+q^4)} \text{.....................(2)}$$

Thus, (1) and (2) are the desired fractions with common denominators against fractions $\frac{1}{C}$ and $\frac{1}{D}$ respectively.

Ques. 06 $A = \frac{x^2-5x-14}{x^2-4x-21}$, $B = \frac{x+2}{x^2+7x+12}$, $C = \frac{4x}{x^2-9}$ are three algebraic fractions.

a. Determine the difference form $\frac{1}{x-1}$ to $\frac{2x}{x^2-1}$. 2

b. $A \div B \times C = ?$ 4

c. Express A, B and C in the form of the common denominator. 4

• *Barishal Board 2018*

Mathematics

Solution to Question No. 06 :

a The difference from $\frac{1}{x-1}$ to $\frac{2x}{x^2-1}$

$$= \frac{2x}{x^2-1} - \frac{1}{x-1}$$
$$= \frac{2x-x-1}{(x+1)(x-1)}$$
$$= \frac{x-1}{(x+1)(x-1)} = \frac{1}{x+1}$$

b Here, $A \div B = \frac{x^2-5x-14}{x^2-4x-21} \div \frac{x+2}{x^2+7x+12}$

$$= \frac{(x-7)(x+2)}{(x-7)(x+3)} \div \frac{x-2}{(x+3)(x+4)}$$
$$= \frac{(x-7)(x+2)}{(x-7)(x+3)} \times \frac{(x+3)(x+4)}{x-2}$$
$$= \frac{(x+2)(x+4)}{x-2}$$

Now, $A \div B \times C = \frac{(x+2)(x+4)}{x-2} \times \frac{4x}{x^2-9}$

$$= \frac{4x(x^2+6x+8)}{x^3-2x^2-9x+18}$$
$$= \frac{4x^3+24x^2+32x}{x^3-2x^2-9x+18}$$

c Here, denominator of A $= x^2-4x-21$
$= (x-7)(x+3)$,
denominator of B $= x^2+7x+12 = (x+3)(x+4)$
denominator of C $= x^2-9 = (x-3)(x+3)$
$\therefore$ L.C.M. of denominators of A, B, C $= (x-3)(x+3)(x+4)$, after simplifying A as under :

$$A = \frac{x^2-5x-14}{x^2-4x-21} = \frac{(x-7)(x+2)}{(x-7)(x+3)}$$
$$= \frac{x+2}{x+3}$$
$$= \frac{(x+2)(x-3)(x+4)}{(x-3)(x+3)(x+4)} \text{ .... (1)}$$

$$B = \frac{x+2}{x^2+7x+12} = \frac{x+2}{(x+4)(x+3)}$$
$$= \frac{(x+2)(x-3)}{(x-3)(x+3)(x+4)} \text{ .... (2)}$$

$$C = \frac{4x}{x^2-9} = \frac{4x}{(x+3)(x-3)}$$
$$= \frac{4x(x+4)}{(x-3)(x+3)(x+4)} \text{..................(3)}$$

$\therefore$ (1), (2) and (3) are the required form of A, B, C with common denominator.

Ques. 07 $A = x^2-5x+6$, $B = x^2-9$, $C = x^2+4x+3$.

a. Express $\frac{C}{x^2+x}$ in the lowest form. 2

b. Simplify : $\frac{1}{A}+\frac{1}{B}$. 4

c. Express $\frac{1}{A}, \frac{1}{B}, \frac{1}{C}$ in the form of common denominator. 4

• *Dhaka Board 2017*

Solution to Question No. 07 :

a Given that, $C = x^2+4x+3$

$$\therefore \frac{c}{x^2+x} = \frac{x^2+4x+3}{x(x+1)}$$
$$= \frac{(x+3)(x+1)}{x(x+1)}$$
$$= \frac{x+3}{x}, \text{ which is the lowest form.}$$

b $\frac{1}{A}+\frac{1}{B} = \frac{1}{x^2-5x+6}+\frac{1}{x^2-9}$, putting
$A = x^2-5x+6$ and
$B = x^2-9$

$$= \frac{1}{(x-3)(x-2)} + \frac{1}{(x+3)(x-3)}$$
$$= \frac{x+3+x-2}{(x-3)(x-2)(x+3)}$$
$$= \frac{2x+1}{(x-3)(x-2)(x+3)}$$

c $\frac{1}{A}+\frac{1}{B}+\frac{1}{C}$

$$= \frac{1}{x^2-5x+6} + \frac{1}{x^2-9} + \frac{1}{x^2+4x+3}$$
$$= \frac{1}{(x-3)(x-2)} + \frac{1}{(x-3)(x+3)} + \frac{1}{(x+3)(x+1)}$$
$$= \frac{1(x+1)(x+3)}{(x-3)(x-2)(x+1)(x+3)} + \frac{1(x-2)(x+1)}{(x-3)(x-2)(x+1)(x+3)} + \frac{1(x-3)(x-2)}{(x-3)(x-2)(x+1)(x+3)}$$
$$= \frac{(x+1)(x+3)}{(x-3)(x-2)(x+1)(x+3)} + \frac{(x-2)(x+1)}{(x-3)(x-2)(x+1)(x+3)} + \frac{(x-3)(x-2)}{(x-3)(x-2)(x+1)(x+3)}$$

which is the required form of common denominator.

Ques. 08 $M = x^2 - 3x + 2$, $N = x^2 - 5x + 6$ and $K = x^2 - 4x + 3$ are three algebraic expressions.

a. Express, $\frac{M}{x-2}$ in the lowest form. 2

b. Simplify : $\frac{1}{M} + \frac{1}{N} + \frac{1}{K}$. 4

c. Express $\frac{1}{M}, \frac{1}{N}, \frac{1}{K}$ in the form of the common denominator. 4

Rajshahi Board- 2017

Solution to Question No. 08 :

a Here, $\frac{M}{x-2} = \frac{x^2-3x+2}{x-2}$

$$= \frac{x^2-2x-x+2}{x-2}$$
$$= \frac{x(x-2)-1(x-2)}{x-2}$$
$$= \frac{(x-2)(x-1)}{x-2} = (x-1)$$

b Here, $\frac{1}{M} + \frac{1}{N} + \frac{1}{K}$

$$= \frac{1}{x^2-3x+2} + \frac{1}{x^2-5x+6} + \frac{1}{x^2-4x+3}$$
$$= \frac{1}{(x-2)(x-1)} + \frac{1}{(x-3)(x-2)} + \frac{1}{(x-3)(x-1)}$$
$$= \frac{x-3+x-1+x-2}{(x-1)(x-2)(x-3)} = \frac{3x-6}{(x-1)(x-2)(x-3)}$$
$$= \frac{3(x-2)}{(x-1)(x-2)(x-3)} = \frac{3}{(x-1)(x-3)}$$

c $\frac{1}{M} = \frac{1}{x^2-3x+2}$

$$= \frac{1}{(x-2)(x-1)}$$
$$= \frac{x-3}{(x-1)(x-2)(x-3)},$$

multiplying both numerator and denominator by $(x-3)$.

$\frac{1}{N} = \frac{1}{x^2-5x+6}$

$$= \frac{1}{(x-3)(x-2)}$$
$$= \frac{x-1}{(x-3)(x-2)(x-1)},$$

multiply both numerator and denominator by $(x-1)$.

$\frac{1}{K} = \frac{1}{x^2-4x+3}$

$$= \frac{1}{(x-3)(x-1)}$$
$$= \frac{x-2}{(x-3)(x-2)(x-1)},$$

multiply both numerator and denominator by $(x-2)$.

Ques. 09 $\frac{1}{1-x+x^2}, \frac{1}{1+x+x^2}, \frac{2x}{1+x^2+x^4}$ and $\frac{(x+1)^2-(x^2+x)}{x^3+1}$ are four algebraic fractions.

a. Express the 1st and 2nd fractions in the form of common denominator. 2

b. Show that, 3rd fraction + 2nd fraction − 1st fraction = 0. 4

c. Find the simplified value of 2nd fraction ÷ 3rd fraction ÷ 4th fraction. 4

Jashore Board 2017

Solution to Question No. 09 :

a $\frac{1}{1-x+x^2} = \frac{1(1+x+x^2)}{(1-x+x^2)(1+x+x^2)}$

$$= \frac{1+x+x^2}{(1+x^2)^2-x^2} \text{..................(1)}$$

$\frac{1}{1+x+x^2} = \frac{1(1-x+x^2)}{(1+x+x^2)(1-x+x^2)}$

$$= \frac{1-x+x^2}{(1+x^2)^2-x^2} \text{......................(2)}$$

Here, (1) and (2) refers to two given fractions in the from of fractions of common denominator.

b Here, 3^{rd} fraction + 2^{nd} fraction

$$= \frac{2x}{1+x^2+x^4} + \frac{1}{1+x+x^2}.$$
$$= \frac{2x}{(1+x+x^2)(1-x+x^2)} + \frac{1}{1+x+x^2}.$$
$$= \frac{2x+1-x+x^2}{(1+x+x^2)(1-x+x^2)}$$
$$= \frac{1+x+x^2}{(1+x+x^2)(1-x+x^2)}$$
$$= \frac{1}{1-x+x^2} \text{......................(1)}$$

$\therefore$ 3^{rd} fraction + 2^{nd} fraction − 1^{st} fraction

$$= \frac{1}{1-x+x^2} - \frac{1}{1-x+x^2}, \text{ from (1) above,}$$

$= 0$. **(Showed)**

c 2^{nd} fraction ÷ 3^{rd} fraction ÷ 4^{th} fraction

$$= \frac{1}{1+x+x^2} \div \frac{2x}{1+x^2+x^4} \div \frac{(x+1)^2-(x^2+x)}{x^3+1}$$
$$= \frac{1}{1+x+x^2} \times \frac{(1+x+x^2)(1-x+x^2)}{2x} \div \frac{x+1}{(x+1)(x^2-x+1)}$$
$$= \frac{1-x+x^2}{2x} \times \frac{(x+1)(x^2-x+1)}{x+1}$$
$$= \frac{(1-x+x^2)^2}{2x}.$$

Ques. 10 $P = \dfrac{x^2+3x-4}{x^2+7x+12}$, $Q = \dfrac{x^2+2x-3}{x^2+6x-7}$ $R = \dfrac{x^2+12x+35}{x^2+4x-5}$

a. Express P into the lowest form. 2

b. Simplify; P + Q 4

c. Show that, $Q \times R \div \dfrac{x^2-9}{x-1} = \dfrac{1}{x-3}$. 4

● Comilla Board 2017

Solution to Question No. 10 :

a Given that,

$$P = \frac{x^2+3x-4}{x^2+7x+12} + \frac{x^2+4x-x-4}{x^2+4x+3x+12}$$

$$= \frac{(x+4)(x-1)}{(x+4)(x+3)} = \frac{x-1}{x+3},$$

which is the lowest form of P.

b Here we have,

$$P+Q = \frac{x^2+2x-3}{x^2+6x-7} + \frac{x^2+12x+35}{x^2+4x-5}$$

$$= \frac{(x+3)(x-1)}{(x+7)(x-1)} + \frac{(x+5)(x+7)}{(x+5)(x-1)}$$

$$= \frac{x+3}{x+7} + \frac{x+7}{x-1}$$

$$= \frac{(x+3)(x-1)+(x+7)(x+7)}{(x+7)(x-1)}$$

$$= \frac{x^2+2x-3+x^2+14x+49}{(x+7)(x-1)}$$

$$= \frac{2x^2+16x+46}{(x+7)(x-1)} = \frac{2(x^2+8x+23)}{(x+7)(x-1)}.$$

c Here we have,

$$Q \times R \div \frac{x^2-9}{x-1}$$

$$= \frac{x^2+2x-3}{x^2+6x-7} \times \frac{x^2+12x+35}{x^2+4x-5} \div \frac{(x+3)(x-3)}{x-1}$$

$$= \frac{(x+3)(x-1)}{(x+7)(x-1)} \times \frac{(x+5)(x+7)}{(x+5)(x-1)} \times \frac{x-1}{(x+3)(x-3)}$$

$$= \frac{1}{x-3}$$

$\therefore Q \times R \div \dfrac{x^2-9}{x-1} = \dfrac{1}{x-3}$. **(Showed)**

Ques. 11 $A = \dfrac{x^3-y^3}{x^4+x^2y^2+y^4}$, $B = \dfrac{1}{1-x+x^2}$ and $D = \dfrac{1}{1+x^2+x^4}$ are the four algebraic expressions.

a. Express A into the lowest form. 2

b. Prove that $B - C - 2x \times D = 0$. 4

c. Simplify : $\dfrac{1+x^2}{D} \div (B+C)$. 4

● Chittagong Board 2017

Solution to Question No. 11 :

a Here, $A = \dfrac{x^3-y^3}{x^4+x^2y^2+y^4}$

$$= \frac{(x-y)(x^2+xy+y^2)}{(x^2)^2+(y^2)^2+x^2y^2}$$

$$= \frac{(x-y)(x^2+xy+y^2)}{(x^2+y^2)^2-2x^2y^2+x^2y^2}$$

$$= \frac{(x-y)(x^2+xy+y^2)}{(x^2+y^2)^2-x^2y^2}$$

$$= \frac{(x-y)(x^2+xy+y^2)}{(x^2+y^2+xy)(x^2+y^2-xy)}$$

$= \dfrac{x-y}{x^2-xy+y^2}$, which is the desired lowest form

b Here, $B - C = \dfrac{1}{1-x+x^2} - \dfrac{1}{1+x+x^2}$

$$= \frac{1+x+x^2-1+x-x^2}{(1-x+x^2)(1+x+x^2)}$$

$$= \frac{2x}{(1-x+x^2)(1+x+x^2)} \text{ .... (1)}$$

Again, $2x \times D = 2x \times \dfrac{1}{1+x^2+x^4}$

$$= \frac{2x}{1+x^2+x^4} = \frac{2x}{1^2+(x^2)^2+x^2}$$

$$= \frac{2x}{(1+x^2)^2-x^2}$$

$$= \frac{2x}{(1-x+x^2)(1+x+x^2)} \text{ .... (2)}$$

Now from (1) and (2), it is obvious that,

$B - C - 2x \times D$

$$= \frac{2x}{(1+x+x^2)(1-x+x^2)} - \frac{2x}{(1+x+x^2)(1-x+x^2)}$$

$= 0$

$\therefore$ $B - C - 2x \times D = 0$. **(Proved)**

c $\dfrac{1+x^2}{D} \div (B+C)$

$$= \frac{1+x^2}{\frac{1}{1+x^2+x^4}} \div \left(\frac{1}{1-x+x^2} + \frac{1}{1+x+x^2}\right)$$

$$= (1+x^2)(1+x^2+x^4) \div \left(\frac{1+x+x^2+1-x+x^2}{(1-x+x^2)(1+x+x^2)}\right)$$

$$= (1+x^2)(1+x^2+x^4) \div \frac{2(1+x^2)}{(1+x^2)^2-x^2}$$

$$= (1+x^2)(1+x^2+x^4) \times \frac{(1+x+x^2)(1-x+x^2)}{2(1+x^2)}$$

$$= \frac{(1+x^2+x^4)\{(1+x^2)^2-x^2\}}{2}$$

$$= \frac{(1+x^2+x^4)(1+x^2+x^4)}{2} = \frac{(1+x^2+x^4)^2}{2}$$

Ques. 12 $x = \dfrac{2p}{1+p^2+p^4}$, $y = \dfrac{1}{1-p+p^2}$ and

$z = \dfrac{1}{1+p+p^2}$ are three algebraic expressions.

a. Express y and z with common denominator of fraction. 2

b. Simplify : $x - y + z$. 4

c. Simplify : $(y - z) \div x$. 4

• *Barisal Board 2017*

Solution to Question No. 12 :

a Here, $y = \dfrac{1}{1-p+p^2}$

$$= \frac{1\{(1+p^2)+p\}}{\{(1+p^2)-p\}\{(1+p^2)+p\}}$$

$$= \frac{1+p+p^2}{(1+p^2)^2 - p^2} = \frac{1+p+p^2}{1+p^2+p^4}$$

And $z = \dfrac{1}{1+p+p^2}$

$$= \frac{1\{p^2+1\}-p}{\{(p^2+1)+p\}\{(p^2+1)^2-p\}}$$

$$= \frac{p^2-p+1}{(p^2+1)^2-p^2} = \frac{p^2-p+1}{1+p^2+p^4}.$$

b Here, $x - y + z$

$$= \frac{2p}{1+p^2+p^4} - \frac{1}{1-p+p^2} + \frac{1}{1+p+p^2}$$

$$= \frac{2p}{(1-p+p^2)(1+p+p^2)} - \frac{1}{1-p+p^2} + \frac{1}{1+p+p^2}$$

$$= \frac{2p(1) - 1(1+p+p^2) + 1(1-p+p^2)}{(1-p+p^2)(1+p+p^2)}$$

$$= \frac{2p - 1 - p - p^2 + 1 - p + p^2}{(1+p^2+p^4)}$$

$$= \frac{0}{1+p^2+p^4} = 0.$$

c Here, $(y - z) \div x$

$$= \left(\frac{1}{1-p+p^2} - \frac{1}{1+p+p^2}\right) \div \frac{2p}{1+p^2+p^4}$$

$$= \frac{1+p+p^2-1+p-p^2}{(1-p+p^2)(1+p+p^2)} \div \frac{2p}{1+p^2+p^4}$$

$$= \frac{2p}{1+p^2+p^4} \div \frac{2p}{1+p^2+p^4}$$

$$= \frac{2p}{1+p^2+p^4} \times \frac{1+p^2+p^4}{2p}$$

Ques. 13 $\dfrac{1}{2x+3y}$, $\dfrac{1}{2x-3y}$, $\dfrac{2x}{4x^2-9xy^2}$ are three algebraic fractions.

a. Subtract the 2nd fraction from the 1st one. 2

b. Multiply the 1st and 2nd fractions, then divide the required product by the 3rd fraction. 4

c. Express the three fractions in the form of common denominator. 4

• *Dinajpur Board 2017*

Solution to Question No. 13 :

a Here, 1st fraction – 2nd fraction

$$= \frac{1}{2x+3y} - \frac{1}{2x-3y}$$

$$= \frac{(2x-3y)-(2x+3y)}{(2x+3y)(2x-3y)}$$

$$= \frac{2x-3y-2x-3y}{4x^2-9y^2} = \frac{-6y}{4x^2-9y^2}$$

$\therefore$ The difference of 1st and 2nd fraction is $\dfrac{-6y}{4x^2-9y^2}$.

b Here, 1st fraction × 2nd fraction.

$$= \frac{1}{2x+3y} \times \frac{1}{2x-3y}$$

$= \dfrac{1}{4x^2-9y^2}$, which is the product of 1st and 2nd fraction

Again, product of 1st and 2nd fraction ÷ 3rd fraction.

$$= \frac{1}{4x^2-9y^2} \div \frac{2x}{4x^2-9y^2}$$

$$= \frac{1}{4x^2-9y^2} \times \frac{4x^2-9y^2}{2x}$$

$$= \frac{1}{2x}$$

$\therefore$ The required answer is $\dfrac{1}{2x}$.

c L.C.M. of the denomenotors of the given fraction is $4x^2 - 9y^2 = (2x+3y)(2x-3y)$

Now, $\dfrac{1}{2x+3y} = \dfrac{2x-3y}{(2x+3y)(2x-3y)}$

$= \dfrac{2x-3y}{4x^2-9y^2}$ .... (1)

$\dfrac{1}{2x-3y} = \dfrac{2x+3y}{(2x-3y)(2x+3y)}$

$= \dfrac{2x+3y}{4x^2-9y^2}$ .... (2)

$\dfrac{2x}{4x^2-9y^2}$ .... (3)

Now the three fractions (1), (2) and (3) are the desirous expressions expressed in the form of common denominator.

Solutions to Textual Activities — Along with textual reference

Activity 01 Multiply : ▸ Textbook Page 92

1. $\frac{7a^2b}{36a^3b^2}$ by $\frac{24ab^2}{35a^4b^5}$

Solution : $\frac{7a^2b}{36a^3b^2} \times \frac{24ab^2}{35a^4b^5}$

$= \frac{7a^2b \times 24ab^2}{36a^3b^2 \times 35a^4b^5}$

$= \frac{2a^3b^3}{15a^7b^7} = \frac{2}{15a^4b^4}.$

2. $\frac{x^2+3x-4}{x^2-7x+12}$ by $\frac{x^2-9}{x^2-16}.$

Solution : $\frac{x^2+3x-4}{x^2-7x+12} \times \frac{x^2-9}{x^2-16}$

$= \frac{x^2+4x-x-4}{x^2-3x-4x+12} \times \frac{(x+3)(x-3)}{(x+4)(x-4)}$

$= \frac{(x+4)(x-1)}{(x-3)(x-4)} \times \frac{(x+3)(x-3)}{(x+4)(x-4)}$

$= \frac{(x-1)(x+3)}{(x-4)(x-4)}.$ **(Ans.)**

Activity 02 Divide : ▸ Textbook Page 94

1. $\frac{16a^2b^2}{21z^2}$ by $\frac{28ab^4}{35xy}$

Solution : $\frac{16a^2b^2}{21z^2} \div \frac{28ab^4}{35xy}$

$= \frac{16a^2b^2}{21z^2} \times \frac{35xy}{28ab^4}$

$= \frac{20a^2b^2xy}{21ab^4z^2} = \frac{20axy}{21b^2z^2}$

2. $\frac{x^4-y^4}{x^2-2xy+y^2}$ by $\frac{x^3+y^3}{x-y}.$

Solution : $\frac{x^4-y^4}{x^2-2xy+y^2} \div \frac{x^3+y^3}{x-y}$

$= \frac{(x^2-y^2)(x^2-y^2)}{(x-y)^2} \div \frac{(x+y)(x^2-xy+y^2)}{x-y}$

$= \frac{(x^2+y^2)(x+y)(x-y)}{(x-y)(x-y)} \times \frac{x-y}{(x+y)(x^2-xy+y^2)}$

$= \frac{x^2+y^2}{x^2-xy+y^2}.$ **(Ans.)**

Appendix

Multiple Choice Q/A — Designed as per topic

1. What is the reduce form of $\frac{4a^2bc}{6ab^2c}$?

a ⓐ $\frac{2a}{3b}$ ⓑ $\frac{3b}{2a}$ ⓒ $\frac{2b}{3a}$ ⓓ $\frac{3a}{2b}$

Explanation : $\frac{4a^2bc}{6ab^2c} = \frac{2\times2\times a\times a\times b\times c}{2\times3\times a\times b\times b\times c} = \frac{2a}{3b}.$

2. What is the reduce form of $\frac{2a^2+3ab}{4a^2-9b^2}$?

b ⓐ $\frac{a}{2a+3b}$ ⓑ $\frac{a}{2a-3b}$ ⓒ $\frac{b}{2a+3b}$ ⓓ $\frac{b}{2a-3b}$

Explanation : $\frac{2a^2+3ab}{4a^2-9b^2} = \frac{a(2a+3b)}{(2a)^2-(3b)^2}$

$= \frac{a(2a+3b)}{(2a+3b)(2a-3b)}$

$= \frac{a}{2a-3b}.$

3. What is the reduce form of $\frac{x^2+7x+12}{x^2+9x+20}$?

c ⓐ $\frac{x+3}{x-3}$ ⓑ $\frac{x+3}{x+4}$ ⓒ $\frac{x+3}{x+5}$ ⓓ $\frac{x+3}{x-4}$

Explanation : $\frac{x^2+7x+12}{x^2+9x+20} = \frac{x^2+3x+4x+12}{x^2+4x+5x+20}$

$= \frac{x(x+3)+4(x+3)}{x(x+4)+5(x+4)}$

$= \frac{(x+3)(x+4)}{(x+4)(x+5)}$

$= \frac{x+3}{x+5}.$

4. $\frac{a}{a+b} + \frac{b}{a-b} =$ what?

a ⓐ $\frac{a^2+b^2}{a^2-b^2}$ ⓑ $\frac{a^2-b^2}{a^2+b^2}$ ⓒ $\frac{b^2-a^2}{a^2+b^2}$ ⓓ 1

Explanation : $\frac{a}{a+b} + \frac{b}{a-b}$

$= \frac{a(a-b)+b(a+b)}{(a+b)(a-b)}$

$= \frac{a^2-ab+ab+b^2}{a^2-b^2} = \frac{a^2+b^2}{a^2-b^2}.$

Mathematics

5. $\frac{x+y}{xy} - \frac{y+z}{yz}$ = what?

ⓐ $\frac{z-x}{zx}$ ⓑ $\frac{z-x}{xy}$

a ⓒ $\frac{z+x}{xz}$ ⓓ $\frac{z+x}{xy}$

Explanation : $\frac{x+y}{xy} - \frac{y+z}{yz}$

$= \frac{(x+y)\times z - (y+z)\times x}{xyz}$

$= \frac{zx + yz - xy - zx}{xyz}$

$= \frac{yz - xy}{xyz} = \frac{y(z-x)}{xyz} = \frac{z-x}{zx}$.

Answer questions from 6 to 8 on the basis of the following information :

Two algebraic fractions are $\frac{a}{4x}, \frac{b}{2x^2}$

6. **What is the L.C.M. of the denominators of the fractions?**

ⓐ $4x$ ⓑ $4x^2$

b ⓒ $8x$ ⓓ $8x^3$

7. **Which fraction will be used to convert into equivalent fractions with a common denominator?**

ⓐ $\frac{ax}{4x^2}, \frac{2b}{4x^2}$ ⓑ $\frac{a^2x}{4x^2}, \frac{2b}{4x^2}$

a ⓒ $\frac{a^2x}{4x^2}, \frac{2b^2}{4x^2}$ ⓓ $\frac{a^3x}{4x^2}, \frac{2b^2}{3x^2}$

Explanation : $\frac{a}{4x} = \frac{a\times x}{4x\times x} = \frac{ax}{4x^2}$ and $\frac{b}{2x^2}$

$= \frac{b\times 2}{2x^2\times 2} = \frac{2b}{4x^2}$.

8. **What is the result when the second fraction is subtracted from the first fraction?**

ⓐ $\frac{a-2b}{4x^2}$ ⓑ $\frac{ax-b}{4x^2}$

d ⓒ $\frac{ax-2}{4x^2}$ ⓓ $\frac{ax-2b}{4x^2}$

Explanation : $\frac{a}{4x} - \frac{b}{2x^2}$

$= \frac{a\times x - b\times 2}{4x^2} = \frac{ax-2b}{4x^2}$.

Mathematics

Short Q/A — Designed as per topic

Question 1. Express in the lowest form $\frac{2p^2+3pq}{4p^2-9q^2}$.

Solution : $\frac{2p^2+3qp}{4p^2-9q^2}$

$= \frac{p(2p+3q)}{(2p)^2-(3q)^2} = \frac{p(2p+3q)}{(2p+3q)(2p-3q)}$

$\therefore$ L.C.M. of numerator and denominator $= 2p+3q$

Dividing the numerator and the denominator of the fraction by $(2p+3q)$ we get, $\frac{p}{2p-3q}$

The lowest form of the fraction is : $\frac{p}{2p-3q}$

Question 2. Express in the lowest form $\frac{a^2+7a+12}{a^2-9}$.

Solution : $\frac{a^2+7a+12}{a^2-9}$

$= \frac{a^2+3a+4a+12}{a^2-3^2} = \frac{a(a+3)+4(a+3)}{(a+3)(a-3)} = \frac{(a+3)(a+4)}{(a+3)(a-3)}$

$\therefore$ L.C.M. of numerator and denominator $= (a+3)$

Dividing the numerator and the denominator of the fraction by $(a+3)$ we get, $\frac{a+4}{a-3}$

The lowest form of the fraction is : $\frac{a+4}{a-3}$

Question 3. Express in the lowest form $\frac{m^2+2m-15}{m^2+9m+20}$.

Solution : $= \frac{m^2+5m-3m-15}{m^2+4m+5m+20}$

$= \frac{m(m+5)-3(m+5)}{m(m+4)+5(m+4)} = \frac{(m+5)(m-3)}{(m+4)(m+5)}$

$\therefore$ L.C.M. of numerator and denominator $= m+5$

Dividing the numerator and the denominator of the fraction by $(m+5)$ We get, $\frac{m-3}{m+4}$

The lowest form of the fraction is : $\frac{m-3}{m+4}$

Question 4. Express the fractions with a common denominator $\frac{2a}{3c}$ and $\frac{3b}{2d}$.

Solution : Here, Given fraction are : $\frac{2a}{3c}, \frac{3b}{2d}$

L.C.M. of denominators $3c$ and $2d = 6cd$

$\therefore \frac{2a}{3c} = \frac{2a\times 2d}{3c\times 2d}$ [$\because 6cd \div 3c = 2d$]

$= \frac{4ad}{6cd}$

and $\frac{3b}{2d} = \frac{3b \times 3c}{2d \times 3c}$ [$\because$ 6cd ÷ 2d = 3c]

$= \frac{9bc}{6cd}$

$\therefore$ Two fractions with common denominator : $\frac{4ad}{6cd}, \frac{9bc}{6cd}$.

Question 5. Express the fractions with a common denominator $\frac{2}{3a}$ and $\frac{3}{5ab}$.

Solution : Given fraction are : $\frac{2}{3a}, \frac{3}{5ab}$

L.C.M. of denominators 3a and 5ab = 15ab

$\therefore \frac{2}{3a} = \frac{2 \times 5b}{3a \times 5b}$ [$\because$ 15ab ÷ 3a = 5b]

$= \frac{10b}{15ab}$

and $\frac{3}{5ab} = \frac{3 \times 3}{5ab \times 3}$ [$\because$ 15ab ÷ 5ab = 3]

$= \frac{9}{15ab}$

$\therefore$ Two fractions with common denominator : $\frac{10b}{15ab}, \frac{9}{15ab}$.

Question 6. Add the expressions : $\frac{2x}{8}$ and $\frac{4y}{8}$.

Solution : The sum of $\frac{2x}{8}$ and $\frac{4y}{8} = \frac{2x}{8} + \frac{4y}{8}$

$= \frac{2x + 4y}{8} = \frac{2(x + 2y)}{8} = \frac{x + 2y}{4}$

Required summation : $\frac{x + 2y}{4}$.

Question 7. Find the sum : $\frac{3a}{2m} + \frac{b}{4n}$.

Solution : Given expression $= \frac{3a}{2m} + \frac{b}{4n}$

$= \frac{3a \times 2n + b \times m}{4mn}$ [$\because$ L.C.M. 2m and 4n = 4mn]

$= \frac{6an + bm}{4mn}$

Required summation $\frac{6an + bm}{4mn}$.

Question 8. Subtract $\frac{p + q}{pq}$ form $\frac{q + r}{qr}$.

Solution : $\frac{p + q}{pq} - \frac{q + r}{qr}$

$= \frac{r(p + q) - p(q + r)}{pqr}$

$= \frac{pr + qr - pq - pr}{pqr} = \frac{qr - pq}{pqr}$

$= \frac{q(r - p)}{pqr} = \frac{r - p}{pr}$

Required difference : $\frac{r - p}{pr}$

Question 9. Simplify : $\frac{x}{y} - \frac{3x}{2y} + \frac{2x}{3y}$.

Solution : Given expression $= \frac{x}{y} - \frac{3x}{2y} + \frac{2x}{3y}$

$= \frac{x \times 6 - 3x \times 3 + 2x \times 2}{6y}$

[$\because$ L.C.M. of denominators y, 2y and 3y = 6y]

$= \frac{6x - 9x + 4x}{6y} = \frac{10x - 9x}{6y} = \frac{x}{6y}$.

Question 10. Simplify : $\frac{a + b}{ab} + \frac{b + c}{bc}$.

Solution : Given expression $= \frac{a + b}{ab} + \frac{b + c}{bc}$

$= \frac{c(a + b) + a(b + c)}{abc}$ [$\because$ L.C.M. of denominators ab and bc = abc]

$= \frac{ca + bc + ab + ca}{abc}$

$= \frac{ab + bc + 2ca}{abc}$.

Creative Q/A — Designed as per learning outcomes

Ques. 01 $P = \frac{x^2 - 9x + 20}{x^2 - 7x + 12}$, $Q = \frac{x^2 - 5x + 6}{x^2 - 8x + 15}$, $R = \frac{x^2 + 3x + 2}{x^2 + 4x + 4}$, $S = \frac{m^2 - mn + n^2}{m^2 + mn + n^2}$ and $T = \frac{m^4 + m^2n^2 + n^4}{m^6 - n^6}$

a. Express $\frac{a^2(a^2 - 2ab + b^2)(a^6 - b^6)}{(a^3 + b^3)(a^5b - b^5a)(a^2 + ab + b^2)}$ in the lowest form. 2

b. Express $\frac{1}{P(x + 2)}$, Q and $\frac{R}{x^2 - 25}$ into common dinominator. 4

c. Prove that $(S \div T) \times \frac{1}{m^2 - mn + n^2} - \frac{2mn - 2n^2}{m^2 + mn + n^2} = \frac{(m - n)^2}{m^2 + mn + n^2}$. 4

• *Rajuk Uttara Model College, Dhaka*

Mathematics

Solution to Question No. 01 :

a Given expression

$$= \frac{a^2(a^2-2ab+b^2)(a^6-b^6)}{(a^3+b^3)(a^5b-b^5a)(a^2+ab+b^2)}$$

$$= \frac{a^2(a-b)^2\{(a^3)^2-(b^3)^2\}}{(a^3+b^3)\,ab(a^4-b^4)(a^2+ab+b^2)}$$

$$= \frac{a^2(a-b)(a-b)(a^3+b^3)(a^3-b^3)}{ab(a^3+b^3)\{(a^2)^2-(b^2)^2\}(a^2+ab+b^2)}$$

$$= \frac{a^2(a-b)(a-b)(a^3+b^3)(a-b)(a^2+ab+b^2)}{ab(a^3+b^3)(a^2+b^2)(a^2-b^2)(a^2+ab+b^2)}$$

$$= \frac{a^2(a-b)(a-b)(a^3+b^3)(a-b)(a^2+ab+b^2)}{ab(a^3+b^3)(a^2+b^2)(a+b)(a-b)(a^2+ab+b^2)}$$

$$= \frac{a(a-b)(a-b)}{b(a^2+b^2)(a+b)}.$$ **(Ans.)**

b Given,

$$P = \frac{x^2-9x+20}{x^2-7x+12}$$

$$Q = \frac{x^2-5x+6}{x^2-8x+15}$$

$$R = \frac{x^2+3x+2}{x^2+4x+4}$$

Now,

$$\frac{1}{p(x+2)} = \frac{1}{\dfrac{x^2-9x+20}{x^2-7x+12}\times(x+2)}$$

$$= \frac{(x^2-7x+12)}{(x+2)(x^2-9x+20)}$$

$$= \frac{(x^2-4x-3x+12)}{(x+2)(x^2-5x-4x+20)}$$

$$= \frac{x(x-4)-3(x-4)}{(x+2)\{x(x-5)-4(x-5)\}}$$

$$= \frac{(x-4)(x-3)}{(x+2)(x-5)(x-4)}$$

Again, $$Q = \frac{x^2-5x+6}{x^2-8x+15}$$

$$= \frac{x^2-3x-2x+6}{x^2-5x-3x+15}$$

$$= \frac{x(x-3)-2(x-3)}{x(x-5)-3(x-5)}$$

$$= \frac{(x-3)(x-2)}{(x-5)(x-3)} = \frac{(x-2)}{(x-5)}$$

And, $$\frac{R}{x^2-25} = \frac{\dfrac{x^2+3x+2}{x^2+4x+4}}{x^2-25}$$

$$= \frac{(x^2+2x+x+2)(x^2-5^2)}{(x^2+2x+2x+4)}$$

$$= \frac{x(x+2)+1(x+2)(x+5)(x-5)}{x(x+2)+2(x+2)}$$

$$= \frac{(x+2)(x+1)(x+5)(x-5)}{(x+2)(x+2)}$$

$$= \frac{(x+1)(x+5)(x-5)}{(x+2)}$$

$\therefore$ L.C.M of the denominators $= (x+2)(x-5)(x-4)$.

Now,

$$\frac{1}{p(x+2)} = \frac{(x-4)(x-3)}{(x+2)(x-5)(x-4)}$$

$$Q = \frac{(x-2)}{(x-5)} = \frac{(x-2)(x+2)(x-4)}{(x+2)(x-5)(x-4)}$$

And $$\frac{R}{x^2-25} = \frac{(x+1)(x+5)(x-5)}{(x+2)}$$

$$= \frac{(x+1)(x+5)(x-5)(x-5)(x-4)}{(x+2)(x-5)(x-4)}$$

$\therefore$ Required fractions are, $\dfrac{(x-4)(x-3)}{(x+2)(x-5)(x-4)}$, $\dfrac{(x-2)(x+2)(x-4)}{(x+2)(x-5)(x-4)}$, $\dfrac{(x+1)(x+5)(x-5)(x-5)(x-4)}{(x+2)(x-5)(x-4)}$.

c Given that

$$S = \frac{m^2-mn+n^2}{m^2+mn+n^2} \text{ and } T = \frac{m^4+m^2n^2+n^4}{m^6-n^6}$$

$$\text{L.H.S.} = (S \div T)\times\frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \left(\frac{m^2-mn+n^2}{m^2+mn+n^2} \div \frac{m^4+m^2n^2+n^4}{m^6-n^6}\right) \times \frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \left\{\frac{m^2-mn+n^2}{m^2+mn+n^2} \div \frac{(m^2)^2+2m^2n^2+(n^2)^2-m^2n^2}{(m^3)^2-(n^3)^2}\right\} \times \frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \left\{\frac{m^2-mn+n^2}{m^2+mn+n^2} \div \frac{(m^2+n^2)-(mn)^2}{(m^3+n^3)(m^3-n^3)}\right\} \times \frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn-n^2}$$

$$= \left\{\frac{m^2-mn+n^2}{m^2+mn+n^2} \times \frac{(m^3+n^3)(m^3-n^3)}{(m^2+mn+n^2)(m^2-mn+n^2)}\right\} \times \frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \left\{\frac{(m^2-mn+n^2)}{m^2+mn+n^2} \times \frac{(m+n)(m^2-mn+n^2)(m-n)(m^2+mn+n^2)}{(m^2+mn+n^2)}\right\} \times \frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \frac{m^2-n^2}{m^2+mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2}$$

$$= \frac{m^2-n^2-2mn+2n^2}{m^2+mn+n^2}$$

$$= \frac{m^2-2mn+n^2}{m^2+mn+n^2}$$

$$= \frac{(m-n)^2}{m^2+mn+n^2}$$

$= \text{R.H.S.}$

$$\therefore (S \div T)\times\frac{1}{m^2-mn+n^2} - \frac{2mn-2n^2}{m^2+mn+n^2} = \frac{(m-n)^2}{m^2+mn+n^2}.$$

(Proved)

Ques. 02 $\frac{a^4-b^4}{a^2-2ab+b^2}, \frac{a-b}{a^3+b^3}, \frac{a+b}{a^3+b^3}$ are the algebraic expressions.

a. Express $\frac{a(a^2+2ab+b^2)(a^3-b^3)}{(a^3+b^3)(a^4b-b^5)}$ into lowest form. 2

b. Divide the 1st expression by $\frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}$ and then add $\frac{a^2}{a+b}$ to the quotient. 4

c. Prove that, the product of three given expressions is $\frac{a^2+b^2}{(a^2-ab+b^2)}$. 4

● *Ideal School & College, Dhaka*

Mathematics

Solution to Question No. 02 :

a Given expression

$$=\frac{a(a^2+2ab+b^2)(a^3-b^3)}{(a^3+b^3)(a^4b-b^5)}$$

$$=\frac{a(a+b)^2(a-b)(a^2+ab+b^2)}{(a+b)(a^2-ab+b^2)b(a^4-b^4)}$$

$$=\frac{a(a+b)^2(a-b)(a^2+ab+b^2)}{b(a+b)(a^2-ab+b^2)\{(a^2)^2-(b^2)^2\}}$$

$$=\frac{a(a+b)^2(a-b)(a^2+ab+b^2)}{b(a+b)(a^2-ab+b^2)(a^2+b^2)(a^2-b^2)}$$

$$=\frac{a(a+b)(a+b)(a-b)(a^2+ab+b^2)}{b(a+b)(a^2-ab+b^2)(a^2+b^2)(a+b)(a-b)}$$

$$=\frac{a(a^2+ab+b^2)}{b(a^2-ab+b^2)(a^2+b^2)} \textbf{ (Ans.)}$$

b Given, 1[st] expression $=\frac{a^4-b^4}{a^2-2ab+b^2}$

Now,

$$\left(\text{1st expression} \div \frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}\right)+\frac{a^2}{a+b}$$

$$=\left\{\frac{a^4-b^4}{a^2-2ab+b^2} \div \frac{a^3+a^2b+ab^2+b^3}{(a+b)^2-4ab}\right\}+\frac{a^2}{a+b}$$

$$=\left\{\frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a-b)(a-b)}{(a+b)(a^2+b^2)}\right\}+\frac{a^2}{a+b}$$

$$=(a-b)+\frac{a^2}{a+b}=\frac{(a+b)(a-b)+a^2}{a+b}$$

$$=\frac{a^2-b^2+a^2}{a+b}=\frac{2a^2-b^2}{a+b}$$

c Given,

1[st] expression $=\frac{a^4-b^4}{a^2-2ab+b^2}$

2[nd] expression $=\frac{a-b}{a^3+b^3}$

3[rd] expression $=\frac{a+b}{a^3+b^3}$

The product of the given 3 expressions

$$=\frac{a^4-b^4}{a^2-2ab+b^2} \times \frac{a-b}{a^3+b^3} \times \frac{a+b}{a^3+b^3}$$

$$=\frac{(a^2+b^2)(a+b)(a-b)}{(a-b)(a-b)} \times \frac{(a-b)}{(a+b)(a^2-ab+b^2)} \times \frac{(a+b)}{(a+b)(a^2-ab+b^2)}$$

$$=\frac{a^2+b^2}{(a^2-ab+b^2)^2}$$

$\therefore$ The product of the 3 expressions $=\frac{a^2+b^2}{(a^2-ab+b^2)^2}$.

Super Suggestions

Super Suggestions with 100% preparatory questions selected by the Master Trainer Panel

Dear learners, important multiple choice, short and creative questions of this chapter selected by Master Trainer Panel for Half-Yearly and Annual Exams are presented below. Learn the answers to the mentioned questions well to ensure 100% preparation.

Question Pattern		7★	5★	3★
MCQs with Answers	Learn each MCQs in this chapter thoroughly.			
Short Q/A	Exercise 5.1	2, 5, 9, 14, 18	1, 7, 10, 15, 19	4, 8, 12, 16, 20
	Exercise 5.2	1, 6, 8	2, 5, 10	4, 7
Creative Q/A	Exercise 5.1	1, 5, 8	3, 6	4, 7
	Exercise 5.2	2, 4, 9, 13	1, 6, 11	3, 7, 12
	Appendix	2, 5, 8	1, 4, 7	3

Assessment & Evaluation

A question bank presented in the form of a class test to assess the preparation

Class Test Time : 3 hours **Mathematics** Full marks : 100

Class : Eight

Multiple Choice Questions (Each question carries 1 mark) $1 \times 30 = 30$

[N.B. : Answer all the questions. Each question carries one mark. Block fully, with a ball-point pen, the circle of the letter that stands for the correct/best answer in the "Answer Sheet" for Multiple Choice Question Type Examination.]

1. **What is called a broken part of something whole?**
ⓐ Factor ⓑ Fraction ⓒ Friction ⓓ Fracture

2. **What type of fraction is $\frac{a}{2}$?**
ⓐ Arithmetic fraction ⓑ Decimal fraction
ⓒ Algebraic Fraction ⓓ Mixed fraction

3. **Which is the lowest form of $\frac{x^3+3x^4}{x+3x^2}$?**
ⓐ x^2 ⓑ $x+1$ ⓒ x ⓓ x^3

4. i. In $\frac{a^2}{b^2}$, numerator is b^2 and denominator is a^2.
ii. $\frac{x}{a}, \frac{p}{3}$ and $\frac{x+a}{y+b}$ are algebraic expressions.
iii. The lowest term of $\frac{15x^5a^3bc^4}{20x^9ab^6c^3}$ is $\frac{3a^2c}{4x^4b^5}$
Which one of the following is correct?
ⓐ i & ii ⓑ ii & iii ⓒ i & iii ⓓ i, ii & iii

5. **Which is the equivalent fraction of $\frac{a}{b}$?**
ⓐ $\frac{a^2}{ab}$ ⓑ $\frac{a^3}{b^3}$ ⓒ $\frac{a^2}{b^2}$ ⓓ $\frac{ab}{ab^2}$

6. Which one of the following is the H.C.F. of $2x^3y^2z^2$, $12x^2yz$, $20xy^3z^2$?
ⓐ $60x^3y^3z^3$ ⓑ $2x^3y^3z^3$ ⓒ $2xyz$ ⓓ xyz

Answer to the questions No. 7 and 8 based on the following information :
$\frac{x^2+x-12}{x^2+2x-15}$ is a algebraic fraction.

7. **What is the factor of numerator of the given fraction?**
ⓐ $(x-3)(x-4)$ ⓑ $(x-3)(x+4)$
ⓒ $(x+3)(x-4)$ ⓓ $(x+3)(x+4)$

8. **The lowest form of the fraction is subtracted from which number to get the result $\frac{1}{x+5}$?**
ⓐ -1 ⓑ 1 ⓒ $x+4$ ⓓ $x+5$

9. $\frac{a}{b}+\frac{b}{a}=$ **What?**
ⓐ $\frac{a+b}{ab}$ ⓑ $\frac{a^2+b^2}{ab}$ ⓒ $\frac{ab+b^2}{ab}$ ⓓ $\frac{a^2+ab}{ab}$

10. $\frac{1}{x-3}-\frac{1}{x+3}-\frac{6}{x^2-9}=?$
ⓐ 0 ⓑ 6 ⓒ $\frac{12}{x^2-9}$ ⓓ $\frac{2x}{x^2-9}$

11. Which one is the lowest form of $\frac{x^2-x-6}{x^2-4}$?
ⓐ $\frac{x-3}{x-2}$ ⓑ $\frac{x+3}{x+2}$ ⓒ $\frac{x-3}{x+2}$ ⓓ $\frac{x+3}{x-2}$

12. If $a-1$ is a factor of a^3-1, then which one is the another factor?
ⓐ $a+1$ ⓑ a^2-a-1 ⓒ $1+a+a^2$ ⓓ $1-a-a^2$

13. What is the simplified value of $\frac{2}{a}+\frac{3}{a}-\frac{4}{a}$?
ⓐ $\frac{4}{a}$ ⓑ $\frac{3}{a}$ ⓒ $\frac{2}{a}$ ⓓ $\frac{1}{a}$

14. **What is the quotient if $\frac{x^2-2x+1}{a^2-2a+1}$ is divided by $\frac{x-1}{a-1}$?**
ⓐ $\frac{x+1}{a-1}$ ⓑ $\frac{x-1}{a-1}$ ⓒ $\frac{x-1}{a+1}$ ⓓ $\frac{a-1}{x-1}$

15. $\frac{a^2}{ab}\times\frac{b^2}{bc}\times\frac{c^2}{ca}=$ **what?**
ⓐ $a^2b^2c^2$ ⓑ $\frac{1}{a^2b^2c^2}$ ⓒ 1 ⓓ $a^4b^4c^4$

16. **Which one of the simple value of $\frac{a-b}{a}\times\frac{a+b}{a}$?**
ⓐ $\frac{a^2-2ab-b^2}{ab}$ ⓑ $\frac{a^2-2ab+b^2}{ab}$ ⓒ $\frac{-a^2-b^2}{ab}$ ⓓ $\frac{a^2-b^2}{ab}$

17. $\frac{a-p}{a+p}+\frac{(a-p)^2}{a^2-p^2}=?$
ⓐ 1 ⓑ $\left(\frac{a-p}{a+p}\right)^2$ ⓒ $(a+p)$ ⓓ $\left(\frac{a+p}{a-p}\right)^2$

18. $\left(\frac{2a}{a+b}-2\right)+\left(4-\frac{4a}{a+b}\right)=?$
ⓐ $\frac{3}{2}$ ⓑ $\frac{2}{3}$ ⓒ $\frac{1}{2}$ ⓓ $-\frac{1}{2}$

19. $\frac{4a}{1+a^2}+\frac{8a^3}{1-a^4}=?$
ⓐ $\frac{1-a^2}{2a^2}$ ⓑ $\frac{2a^2}{1+a^2}$ ⓒ $\frac{1+a^2}{2a^2}$ ⓓ $\frac{2a^2}{1-a^2}$

20. $\frac{x+y}{x^2-y^2}+\frac{1}{x-y}=?$
ⓐ $\frac{x+y}{x-y}$ ⓑ $\frac{x+y}{x^2-y^2}$ ⓒ $\frac{1}{xy}$ ⓓ 1

21. $\frac{by^2+y^2}{b^2-z^2}+\frac{bx^2+x^2}{b^2-z^2}=?$
ⓐ $\frac{y^2}{x^2}$ ⓑ $\frac{x^2}{y^2}$ ⓒ $\frac{y^2}{bx^2}$ ⓓ $\frac{y^2b}{x^2}$

22. $\left(\frac{2a}{a+b}-2\right)+\left(4-\frac{4a}{a+b}\right)=?$
ⓐ $\frac{1}{2}$ ⓑ 1 ⓒ $-\frac{1}{2}$ ⓓ 2

23. $\left(\frac{x^2-2x+1}{a^2-2a+1}\right)+\left(\frac{x-1}{a-1}\right)=$ what?
ⓐ $\frac{x+1}{a-1}$ ⓑ $\frac{x-1}{a+1}$ ⓒ $\frac{x-1}{a-1}$ ⓓ $\frac{a-1}{x-1}$

24. $\left(\frac{a}{b}+\frac{b}{a}+1\right)+\left(\frac{a^2}{b^2}+\frac{a}{b}+1\right)=?$
ⓐ 1 ⓑ (a^2+ab+b^2) ⓒ $\frac{a}{b}$ ⓓ $\frac{b}{a}$

25. Which one of the following is the quotient of $\frac{a^2+2a-3}{a^2+a-2}$ and $\frac{a^2+a-6}{a^2-4}$?
ⓐ $\frac{a+3}{a+2}$ ⓑ $\frac{a-1}{a+3}$ ⓒ 1 ⓓ 0

26. $\frac{x^4-y^4}{x^2-2xy+y^2}+\frac{x^3+y^3}{x-y}=$ what?
ⓐ $\frac{x^2+y^2}{x^2-xy+y^2}$ ⓑ $\frac{x^2+y^2}{x^2+xy+y^2}$ ⓒ $(x+y)^2$ ⓓ x^2-y^2

27. What is the reduce form of $\frac{4a^2bc}{6ab^2c}$?
ⓐ $\frac{2a}{3b}$ ⓑ $\frac{3b}{2a}$ ⓒ $\frac{2b}{3a}$ ⓓ $\frac{3a}{2b}$

28. What is the reduce form of $\frac{x^2+7x+12}{x^2+9x+20}$?
ⓐ $\frac{x+3}{x-3}$ ⓑ $\frac{x+3}{x+4}$ ⓒ $\frac{x+3}{x+5}$ ⓓ $\frac{x+3}{x-4}$

29. $\frac{x+y}{xy}-\frac{y+z}{yz}=$ what?
ⓐ $\frac{z-x}{zx}$ ⓑ $\frac{z-x}{xy}$ ⓒ $\frac{z+x}{xz}$ ⓓ $\frac{z+x}{xy}$

30. $\frac{a}{a+b}+\frac{b}{a-b}=$ what?
ⓐ $\frac{a^2+b^2}{a^2-b^2}$ ⓑ $\frac{a^2-b^2}{a^2+b^2}$ ⓒ $\frac{b^2-a^2}{a^2+b^2}$ ⓓ 1

Mathematics

Short-Answer Question (Each question carries 2 marks)

Answer any 10 of the following questions : $2 \times 10 = 20$

1. Express in the lowest form $\frac{m^3n^2 - m^2n^3}{m^3n - mn^3}$.
2. Express in the lowest form $\frac{x^3 - 49x}{x^2 + 7x}$.
3. Express the fractions with a common denominator $\frac{p^2 + pq}{p^2q}$ and $\frac{p^2 - pq}{pq^2}$.
4. $\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c+a}{ca} = ?$
5. Find the difference : $\frac{a}{(a-b)^2} - \frac{a+b}{a^2 - b^2}$.
6. $\frac{x^2}{x^2 - 16} - \frac{x}{x+4} = ?$
7. Divide $\frac{x^2 - y^2}{x^2 + xy + y^2}$ by $\frac{x+y}{x^3 - y^3}$.
8. What is the simplified value of $\frac{m^2 - n^2}{m^2 + n^2 - 2mn} \div \frac{(m-n)^2}{(m+n)^2 - 4mn}$?
9. $\frac{p^3 + q^3 + 3pq(p+q)}{(p+q)^2 - 4pq} \div \frac{(p+q)^2}{(p-q)^3} = ?$
10. Express in the lowest form $\frac{2p^2 + 3pq}{4p^2 - 9q^2}$.
11. Express in the lowest form $\frac{m^2 + 2m - 15}{m^2 + 9m + 20}$.
12. Express the fractions with a common denominator $\frac{2}{3a}$ and $\frac{3}{5ab}$.
13. Find the sum : $\frac{3a}{2m} + \frac{b}{4n}$.
14. Simplify : $\frac{a+b}{ab} + \frac{b+c}{bc}$.
15. Subtract $\frac{p+q}{pq}$ form $\frac{q+r}{qr}$.

Creative Question (Each question carries 10 marks)

Mathematics

Answer any 5 of the following questions : $10 \times 5 = 50$

1. $\frac{x-y}{(y+z)(z+x)}, \frac{y-z}{(x+y)(z+x)}, \frac{z-x}{(x+y)(y+z)}$ are three algebraic expressions.
   a. Determine the L. C. M. of the denominators of the three fractions. 2
   b. Add the 2nd expression to the 1st expression. 4
   c. Find the H. C. F. of the three expressions. 4
2. $P = 1 - x + x^2$, $Q = 1 + x + x^2$ and $R = 1 + x^2 + x^4$ are three algebraic expressions.
   a. Resolve into factors of R. 2
   b. Determine the value of $\frac{1}{P} - \frac{1}{Q} - \frac{2x}{R}$. 4
   c. Simplify : $\left(\frac{1}{P} + \frac{1}{Q} - \frac{2x}{R}\right) \times \frac{Q}{2x}$. 4
3. $S = x + 3$, $T = x - 3$ and $V = x^2 - 9$.
   a. Express the lowest form of $\frac{y^4 - 1}{y^3 + y}$. 2
   b. Simplify $\frac{x}{S} + \frac{x}{T} + \frac{6x}{V}$. 4
   c. Express $\frac{S}{x^2 - 6x + 5}, \frac{T}{x^2 + 2x - 3}$ and $\frac{V}{x^2 - 2x - 15}$ in the form of a common denominator. 4
4. $P = \frac{a^4 - b^4}{a^2 + b^2 - 2ab}$, $Q = \frac{(a+b)^2 - 4ab}{a^3 - b^3}$, $R = \frac{a+b}{a^2 + ab + b^2}$ are three algebraic fractions.
   a. Resolve into factors : x2 − x − (m − 1) (m − 2)2
   b. Simplify : $\left\{\frac{(a-b)}{a^2 + b^2} \times P + (a^2 + ab + b^2) \times Q\right\} \frac{1}{2a}$. 4
   c. Show that, $P \div [(a^2 + b^2) \times R] \times Q = 1$. 4
5. $P = a^2 - 2a - 8$, $Q = a^2 - 3a - 10$ and $R = a^2 - 8a + 15$ are three algebraic expressions.
   a. Factorize 'R'. 2
   b. Turn $\frac{1}{P}, \frac{1}{Q}, \frac{1}{R}$ into the fraction with a common denominator. 4
   c. Show that, $P \times \frac{a-5}{Q} \div \frac{R}{a-3} = \frac{a-4}{a-5}$. 4
6. $P = \frac{a^3 - b^3 - 3ab(a-b)}{(a+b)^2 - 4ab}$, $Q = \frac{(a-b)^2 + 4ab}{a^3 + b^3 + 3ab(a+b)}$ are two algebraic fractions.
   a. Resolve into factors of $m^4 + m^2 + 1$. 2
   b. Simplify : $\frac{a}{a-b} \times (P \div Q)$. 4
   c. Prove that, $\left(\frac{1}{P} - Q\right)(a^2 - b^2) = 2b$. 4
7. $A = \frac{x^2 - 5x - 14}{x^2 - 4x - 21}$, $B = \frac{x+2}{x^2 + 7x + 12}$, $C = \frac{4x}{x^2 - 9}$ are three algebraic fractions.
   a. Determine the difference form $\frac{1}{x-1}$ to $\frac{2x}{x^2 - 1}$. 2
   b. $A \div B \times C = ?$ 4
   c. Express A, B and C in the form of the common denominator. 4
8. $\frac{1}{2x + 3y}, \frac{1}{2x - 3y}, \frac{2x}{4x^2 - 9xy^2}$ are three algebraic fractions.
   a. Subtract the 2nd fraction from the 1st one. 2
   b. Multiply the 1st and 2nd fractions, then divide the required product by the 3rd fraction. 4
   c. Express the three fractions in the form of common denominator. 4

Answer Sheet ▸ Multiple Choice Questions

1	ⓑ	2	ⓒ	3	ⓐ	4	ⓑ	5	ⓐ	6	ⓒ	7	ⓑ	8	ⓑ	9	ⓑ	10	ⓐ	11	ⓐ	12	ⓒ	13	ⓓ	14	ⓑ	15	ⓒ
16	ⓓ	17	ⓐ	18	ⓓ	19	ⓐ	20	ⓓ	21	ⓐ	22	ⓒ	23	ⓒ	24	ⓓ	25	ⓒ	26	ⓐ	27	ⓐ	28	ⓒ	29	ⓐ	30	ⓐ

Solving Reference ▸ Short-Answer Questions

1 ▸ See Page 150; Ques. 01
2 ▸ See Page 150; Ques. 04
3 ▸ See Page 150; Ques. 06
4 ▸ See Page 152; Ques. 14
5 ▸ See Page 152; Ques. 16
6 ▸ See Page 152; Ques. 20
7 ▸ See Page 168; Ques. 06
8 ▸ See Page 168; Ques. 08
9 ▸ See Page 168; Ques. 10
10 ▸ See Page 177; Ques. 01
11 ▸ See Page 177; Ques. 03
12 ▸ See Page 178; Ques. 05
13 ▸ See Page 178; Ques. 07
14 ▸ See Page 178; Ques. 10
15 ▸ See Page 178; Ques. 08

Solving Reference ▸ Creative Questions

1 ▸ See Page 153; Ques. 02
2 ▸ See Page 155; Ques. 06
3 ▸ See Page 156; Ques. 07
4 ▸ See Page 157; Ques. 09
5 ▸ See Page 169; Ques. 01
6 ▸ See Page 170; Ques. 03
7 ▸ See Page 171; Ques. 06
8 ▸ See Page 175; Ques. 13