Monash University

Semester One 2001

Faculty of Science

EXAM CODES: **MAT1841**

TITLE OF PAPER: **MATHEMATICS FOR COMPUTER SCIENCE I**

EXAM DURATION: Three hours writing time

READING TIME: 10 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT:(tick where applicable)

☐ Berwick	✔ Clayton	✔Malaysia	☐ Distance Education	☐ Open Learning
☐ Caulfield	☐ Gippsland	☐ Peninsula	☐ Enhancement Studies	☐ Other (specify)

Candidates are reminded that they should have no material on their desks unless its use has been specifically permitted by the following instructions.

Students should ONLY enter their ID number and desk number on the examination script book, NOT their name. Please take care to ensure that the ID number and desk number are correct and written legibly.

1. **There are nine questions each counting the same number of marks.**
2. **There is an appendix and three pages of statistical tables.**

AUTHORISED MATERIALS

CALCULATORS	☐ **YES**	✔ **NO**
OPEN BOOK	☐ **YES**	✔ **NO**
SPECIFICALLY PERMITTED ITEMS	☐ **YES**	✔ **NO**

if yes, items permitted are:

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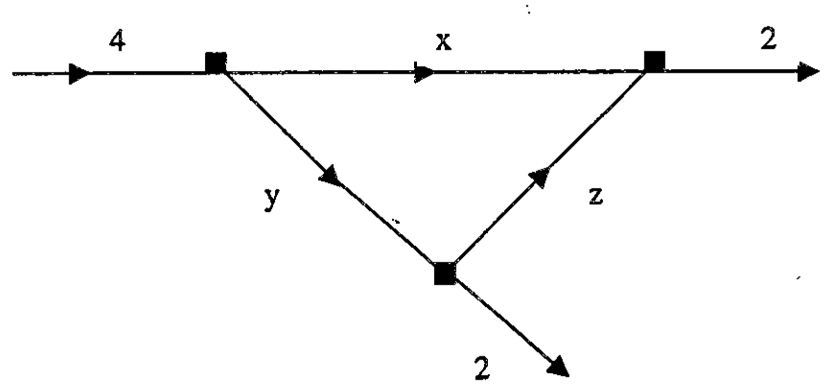
1. Consider the following system of linear equations

$$\begin{aligned} x+4y+2z&=4\\ x+5y+4z&=3\,.\\ 2x+y-z&=6 \end{aligned}$$

   (a) Write down the augmented matrix of the system
   (b) Use Gaussian elimination to bring the augmented matrix to echelon form. State the row operations used at each step.
   (c) Solve the system.

2. Consider the following network



   (a) Set up the flow equations for this network.
   (b) Solve the flow equations. Find all possible solutions consisting of non-negative integers (whole numbers).

3. Consider the matrix $A=\begin{bmatrix}1&2&0\\1&3&0\\2&1&b\end{bmatrix}$.

   (a) Find the determinant of A.
   (b) Find all values of b for which A has an inverse. Give reasons.
   (c) Find the inverse of A if $b = 1$.

4. In each of the following, show your working and give the answer in its simplest form.
   (a) How many 8-bit strings contain exactly six 1s and two 0s ?
   (b) How many 8-bit strings contain at least six 1s ?
   (c) How many subsets are there of a set containing six elements?
   (d) State the inclusion-exclusion principle for three sets A, B and C.
   (e) Three sets A, B and C have 6, 8 and 12 elements respectively. Each pair of sets has two elements in common and there is one element belonging to all three sets. How many elements are there in $A \cup B \cup C$?

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5. A company buys 55% of its computers from the vendor Alpha and finds that 2% are defective. The rest it buys from Beta and finds 3% defective. Let A and B be the events corresponding to a computer being purchased from Alpha and Beta respectively. Let F be the event that a computer is defective. Find
(a) $\Pr(A)$ and $\Pr(B)$;
(b) $\Pr(F \mid A)$ and $\Pr(F \mid B)$;
(c) $\Pr(F \cap A)$ and $\Pr(F \cap B)$;
(d) $\Pr(F)$;
(e) $\Pr(A \mid F)$.

6. (a) A random variable X is normally distributed with mean 5 and standard deviation 2. Use the tables in appendix to find $\Pr(3<X<8)$.
(b) Another random variable Y is also normally distributed with standard deviation 2, but with unknown mean μ. For a random sample of 16 observations of Y, it is found that the sample mean is 3.2. Find a 95% confidence interval for μ.
(c) At the 95% confidence level, it is required to have a margin of error of at most 0.2 for the mean μ in part (b). How large should the sample size be?

7. (a) A random variable X has the t-distribution with 28 degrees of freedom. Find b if
$$\Pr(X < b) = 0.975.$$
(b) What are the three main principles of experimental design?
(c) Find the matrix A for the shearing transformation that takes the triangle with vertices (0,0), (0,1) and (1,0) to the triangle with vertices (0,0), (3,1) and (1,0) respectively. Show working.
(d) A curve C has parametrization $(x, y) = (1 - t, 2 + t^2)$ for $0 \le t \le 1$. Find the parametrization of its image $T(C)$ under the affine transformation with matrix $\begin{bmatrix} 1 & 1 \\ 2 & 0 \end{bmatrix}$ and constant term $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

8. Let $T : \mathbf{R}^2 \to \mathbf{R}^2$ be the orthogonal projector onto the line L through the two points $\mathbf{p} = (1,2)$ and $\mathbf{q} = (2,1)$.
(a) Find a direction vector $\mathbf{d}$ for L.
(b) Find a parametric equation for the line interval from $\mathbf{p}$ to $\mathbf{q}$.
(c) Find a unit direction vector $\mathbf{u}$ for L.
(d) Find the point $\mathbf{b}$ on L nearest to the origin.
(e) Find the matrix and constant term for T.

9. (a) The following plot shows a cubic Bezier curve and a cubic B-spline, each extended by enlarging the parameter domains. They have the same four control points (–2,2), (1,1), (1,–1) and (–2,2) though not necessarily in that order.
(i) State which curve is the Bezier curve and give your reasons.
(ii) State the order of the control points.
(iii) Give the parametrization of the Bezier curve in terms of the defining function $f(x_0, x_1, x_2, x_3, t)$ described in lectures and labs.

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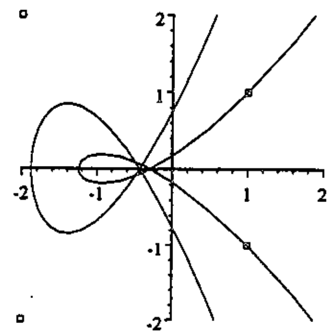

(b) You are required to fit a smooth curve through six data points $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3, \mathbf{p}_4, \mathbf{p}_5, \mathbf{p}_6$ in 2D. The curve is to consist of one or more cubics joined together.
(i) What curves should be used: Bezier curves, interpolating curves, splines or B-splines? Give reasons.
(ii) How many curves will be required? Give reasons.

Appendix

Bayes' theorem: Let $A_1, A_2, \ldots, A_n$ be exhaustive and pairwise mutually exclusive events. For any event F, $\Pr(A_1 \mid F) = \Pr(F \mid A_1) \,.\, \Pr(A_1) \div \sum_{k=1}^{n} \Pr(F \mid A_k) \,.\, \Pr(A_k)$.

Confidence intervals

1. For population mean μ when σ is known and either the population is normal or the sample size is large: $\bar{x} - b\sigma/\sqrt{n} < \mu < \bar{x} + b\sigma/\sqrt{n}$, where b is obtained from the standard normal distribution.
2. For population proportion p when sample size is large and the population is binomial: $\hat{p} - b\hat{\sigma}/\sqrt{n} < p < \hat{p} + b\hat{\sigma}/\sqrt{n}$, where $\hat{\sigma} = \sqrt{\frac{1}{n}\hat{p}(1-\hat{p})}$ and b is obtained from the standard normal distribution.
3. For population mean μ when σ is not known and the population is normal: $\bar{x} - bs/\sqrt{n} < \mu < \bar{x} + b\,s/\sqrt{n}$, where b is obtained from a t-distribution.
4. Confidence intervals for $\mu_1 - \mu_2$ when σ_1 and σ_2 are not known and the populations are normal:
$\bar{x}_1 - \bar{x}_2 - bs\sqrt{2/n} < \mu_1 - \mu_2 < \bar{x}_1 - \bar{x}_2 + bs\sqrt{2/n}$, where b is obtained from a t-distribution.

Projections and reflections

Let L be a line in 2D with unit direction vector $\mathbf{u}$, let $A(\mathrm{i}, \mathrm{j}) = u_i u_j$ and let $\mathbf{b}$ be the point on L closest to the origin.

1. The orthogonal projector onto L has matrix A and constant term $\mathbf{b}$.
2. The reflector through L has matrix $2A$ and constant term $2\mathbf{b}$.

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Table 1 Binomial distribution – probability function

	x						p						
		0.01	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	
n=1	0	.9900	.9500	.9000	.8500	.8000	.7500	.7000	.6500	.6000	.5500	.5000	1
	1	.0100	.0500	.1000	.1500	.2000	.2500	.3000	.3500	.4000	.4500	.5000	0
n=2	0	.9801	.9025	.8100	.7225	.6400	.5625	.4900	.4225	.3600	.3025	.2500	2
	1	.0198	.0950	.1800	.2550	.3200	.3750	.4200	.4550	.4800	.4950	.5000	1
	2	.0001	.0025	.0100	.0225	.0400	.0625	.0900	.1225	.1600	.2025	.2500	0
n=3	0	.9703	.8574	.7290	.6141	.5120	.4219	.3430	.2746	.2160	.1664	.1250	3
	1	.0294	.1354	.2430	.3251	.3840	.4219	.4410	.4436	.4320	.4084	.3750	2
	2	.0003	.0071	.0270	.0574	.0960	.1406	.1890	.2389	.2880	.3341	.3750	1
	3		.0001	.0010	.0034	.0080	.0156	.0270	.0429	.0640	.0911	.1250	0
n=4	0	.9606	.8145	.6561	.5220	.4096	.3164	.2401	.1785	.1296	.0915	.0625	4
	1	.0388	.1715	.2916	.3685	.4096	.4219	.4116	.3845	.3456	.2995	.2500	3
	2	.0006	.0135	.0486	.0975	.1536	.2109	.2646	.3105	.3456	.3675	.3750	2
	3		.0005	.0036	.0115	.0256	.0469	.0756	.1115	.1536	.2005	.2500	1
	4			.0001	.0005	.0016	.0039	.0081	.0150	.0256	.0410	.0625	0
n=5	0	.9510	.7738	.5905	.4437	.3277	.2373	.1681	.1160	.0778	.0503	.0313	5
	1	.0480	.2036	.3281	.3915	.4096	.3955	.3602	.3124	.2592	.2059	.1563	4
	2	.0010	.0214	.0729	.1382	.2048	.2637	.3087	.3364	.3456	.3369	.3125	3
	3		.0011	.0081	.0244	.0512	.0879	.1323	.1811	.2304	.2757	.3125	2
	4			.0005	.0022	.0064	.0146	.0284	.0488	.0768	.1128	.1563	1
	5				.0001	.0003	.0010	.0024	.0053	.0102	.0185	.0313	0
n=6	0	.9415	.7351	.5314	.3771	.2621	.1780	.1176	.0754	.0467	.0277	.0156	6
	1	.0571	.2321	.3543	.3993	.3932	.3560	.3025	.2437	.1866	.1359	.0938	5
	2	.0014	.0305	.0984	.1762	.2458	.2966	.3241	.3280	.3110	.2780	.2344	4
	3		.0021	.0146	.0415	.0819	.1318	.1852	.2355	.2765	.3032	.3125	3
	4		.0001	.0012	.0055	.0154	.0330	.0595	.0951	.1382	.1861	.2344	2
	5			.0001	.0004	.0015	.0044	.0102	.0205	.0369	.0609	.0938	1
	6					.0001	.0002	.0007	.0018	.0041	.0083	.0156	0
n=7	0	.9321	.6983	.4783	.3206	.2097	.1335	.0824	.0490	.0280	.0152	.0078	7
	1	.0659	.2573	.3720	.3960	.3670	.3115	.2471	.1848	.1306	.0872	.0547	6
	2	.0020	.0406	.1240	.2097	.2753	.3115	.3177	.2985	.2613	.2140	.1641	5
	3		.0036	.0230	.0617	.1147	.1730	.2269	.2679	.2903	.2918	.2734	4
	4		.0002	.0026	.0109	.0287	.0577	.0972	.1442	.1935	.2388	.2734	3
	5			.0002	.0012	.0043	.0115	.0250	.0466	.0774	.1172	.1641	2
	6				.0001	.0004	.0013	.0036	.0084	.0172	.0320	.0547	1
	7						.0001	.0002	.0006	.0016	.0037	.0078	0
n=8	0	.9227	.6634	.4305	.2725	.1678	.1001	.0576	.0319	.0168	.0084	.0039	8
	1	.0746	.2793	.3826	.3847	.3355	.2670	.1977	.1373	.0896	.0548	.0313	7
	2	.0026	.0515	.1488	.2376	.2936	.3115	.2965	.2587	.2090	.1569	.1094	6
	3	.0001	.0054	.0331	.0839	.1468	.2076	.2541	.2786	.2787	.2568	.2188	5
	4		.0004	.0046	.0185	.0459	.0865	.1361	.1875	.2322	.2627	.2734	4
	5			.0004	.0026	.0092	.0231	.0467	.0808	.1239	.1719	.2188	3
	6				.0002	.0011	.0038	.0100	.0217	.0413	.0703	.1094	2
	7					.0001	.0004	.0012	.0033	.0079	.0164	.0313	1
	8							.0001	.0002	.0007	.0017	.0039	0
		0.99	0.95	0.90	0.85	0.80	0.75	0.70	0.65	0.60	0.55	0.50	x
							p						

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Table 4 Normal distribution – cumulative distribution function

x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359	4	8	12	16	20	24	28	32	36
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753	4	8	12	16	20	24	28	32	35
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141	4	8	12	15	19	23	27	31	35
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517	4	8	11	15	19	23	26	30	34
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879	4	7	11	14	18	22	25	29	32
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224	3	7	10	14	17	21	24	27	31
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549	3	6	10	13	16	19	23	26	29
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852	3	6	9	12	15	18	21	24	27
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133	3	6	8	11	14	17	19	22	25
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389	3	5	8	10	13	15	18	20	23
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621	2	5	7	9	12	14	16	18	21
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830	2	4	6	8	10	12	14	16	19
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015	2	4	6	7	9	11	13	15	16
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177	2	3	5	6	8	10	11	13	14
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319	1	3	4	6	7	8	10	11	13
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441	1	2	4	5	6	7	8	10	11
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545	1	2	3	4	5	6	7	8	9
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633	1	2	3	3	4	5	6	7	8
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706	1	1	2	3	4	4	5	6	6
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767	1	1	2	2	3	4	4	5	5
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817	0	1	1	2	2	3	3	4	4
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857	0	1	1	2	2	2	3	3	4
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890	0	1	1	1	2	2	2	3	3
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916	0	1	1	1	1	2	2	2	2
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936	0	0	1	1	1	1	1	2	2
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952	0	0	0	1	1	1	1	1	1
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964	0	0	0	0	1	1	1	1	1
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974	0	0	0	0	0	1	1	1	1
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981	0	0	0	0	0	0	0	1	1
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986	0	0	0	0	0	0	0	0	0
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990	0	0	0	0	0	0	0	0	0
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993	0	0	0	0	0	0	0	0	0
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995	0	0	0	0	0	0	0	0	0
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997	0	0	0	0	0	0	0	0	0
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998	0	0	0	0	0	0	0	0	0
3.5	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	.9998	0	0	0	0	0	0	0	0	0
3.6	.9998	.9998	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.7	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.8	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	.9999	0	0	0	0	0	0	0	0	0
3.9	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0	0	0	0	0

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Table 7 t distribution – inverse cdf

ν	0.600	0.750	0.800	0.900	p 0.950	0.975	0.990	0.995	0.9990	0.9995
1	0.325	1.000	1.376	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.289	0.816	1.061	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.277	0.765	0.978	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.271	0.741	0.941	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	0.920	1.476	2.015	2.571	3.365	4.032	5.894	6.869
6	0.265	0.718	0.906	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	0.896	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	0.889	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	0.883	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	0.879	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	0.876	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	0.873	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	0.870	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	0.868	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	0.866	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	0.865	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	0.863	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	0.862	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	0.861	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	0.860	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	0.859	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	0.858	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	0.858	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	0.857	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	0.856	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.256	0.684	0.856	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.256	0.684	0.855	1.314	1.703	2.052	2.473	2.771	3.421	3.689
28	0.256	0.683	0.855	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.256	0.683	0.854	1.311	1.699	2.045	2.462	2.756	3.396	3.660
30	0.256	0.683	0.854	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.255	0.682	0.853	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.255	0.682	0.852	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.255	0.681	0.852	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.255	0.681	0.851	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.255	0.681	0.851	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.255	0.679	0.849	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.254	0.679	0.848	1.296	1.671	2.000	2.390	2.660	3.232	3.460
70	0.254	0.678	0.847	1.294	1.667	1.994	2.381	2.648	3.211	3.435
80	0.254	0.678	0.846	1.292	1.664	1.990	2.374	2.639	3.195	3.416
90	0.254	0.677	0.846	1.291	1.662	1.987	2.368	2.632	3.183	3.402
100	0.254	0.677	0.845	1.290	1.660	1.984	2.364	2.626	3.174	3.390
120	0.254	0.677	0.845	1.289	1.658	1.980	2.358	2.617	3.160	3.373
160	0.254	0.676	0.844	1.287	1.654	1.975	2.350	2.607	3.142	3.352
200	0.254	0.676	0.843	1.286	1.653	1.972	2.345	2.601	3.131	3.340
240	0.254	0.676	0.843	1.285	1.651	1.970	2.342	2.596	3.125	3.332
300	0.254	0.675	0.843	1.284	1.650	1.968	2.339	2.592	3.118	3.323
400	0.254	0.675	0.843	1.284	1.649	1.966	2.336	2.588	3.111	3.315
∞	0.253	0.674	0.842	1.282	1.645	1.960	2.326	2.576	3.090	3.290

Note: Interpolation with respect to ν should be linear in 120/ν.

END OF EXAMINATION PAPER