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Lecture 34
Priority Queues

FIT 1008
Introduction to Computer Science

MONASH University
Information Technology

Objectives

- To understand the Priority Queue ADT.
- Consider different implementations (advantages/disadvantages)
- To understand the Heaps, and the Heap-based implementation of Priority Queues.

The doctor will see you now…

http://www.nma.gov.au/__data/assets/image/0018/218250/News_of_day_17_480w.jpg

Probably should have seen the doctor earlier

The doctor will see you now…

http://www.nma.gov.au/__data/assets/image/0018/218250/News_of_day_17_480w.jpg

Priority Queue

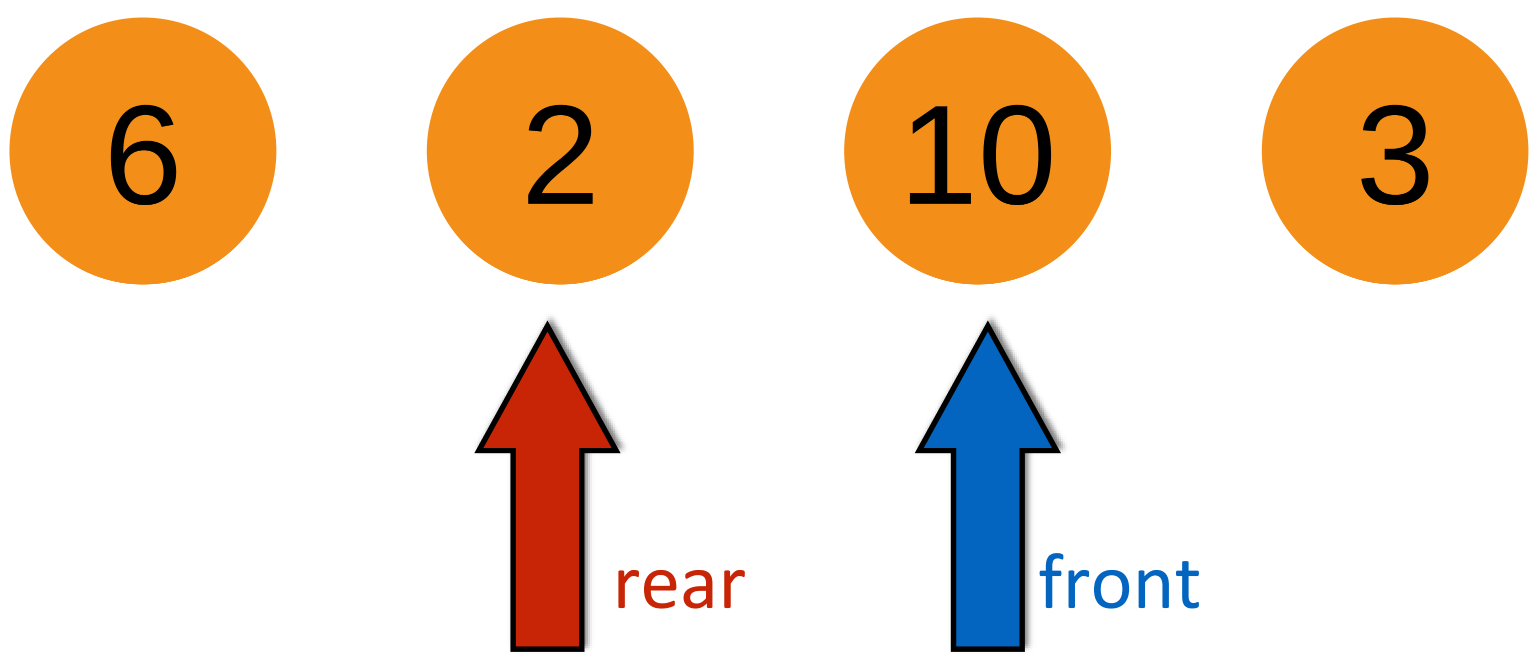
lowest in a dual implementation

- Each element has a numeric priority.
- Element with highest priority is processed first.

Operations:

`add(element)`

`get_max()`



The following data structures can be used to support the implementation of a Priority Queue ADT...

- Array-based List (sorted and unsorted)
- Linked List (sorted and unsorted)
- Binary Search Trees
- Heaps (Binary Tree-based)
- Heaps (Array-based)

Implementing Priority Queues

- Standard operations:
  `is_empty, __len__, __init__`

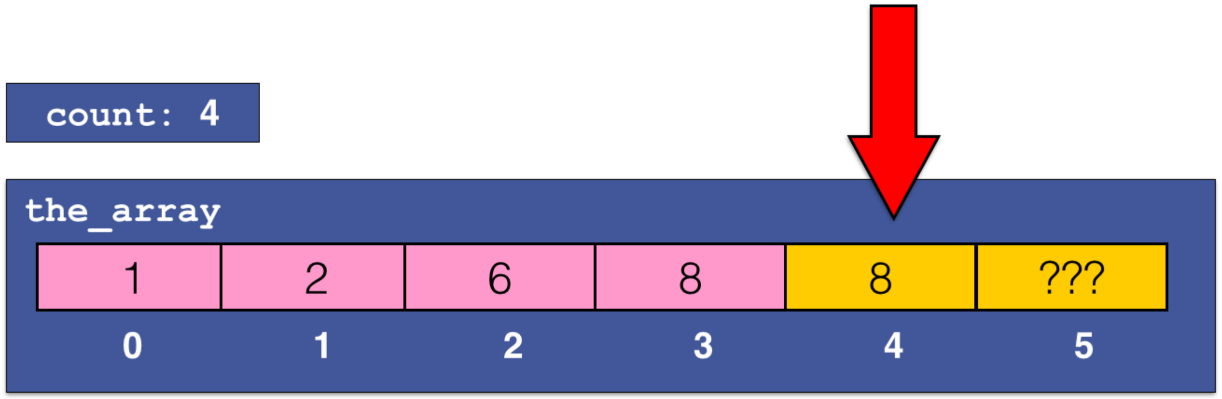
- Core operations:

  - **`get_max()`**:
    returns the max element (and removes it from the queue)

  - **`add(element)`**: adds element to the priority queue

Complexity

(Priority Queue using Unsorted Lists)



get_max()

- Find max item.
- Remove and reshuffle.

O(n)

---

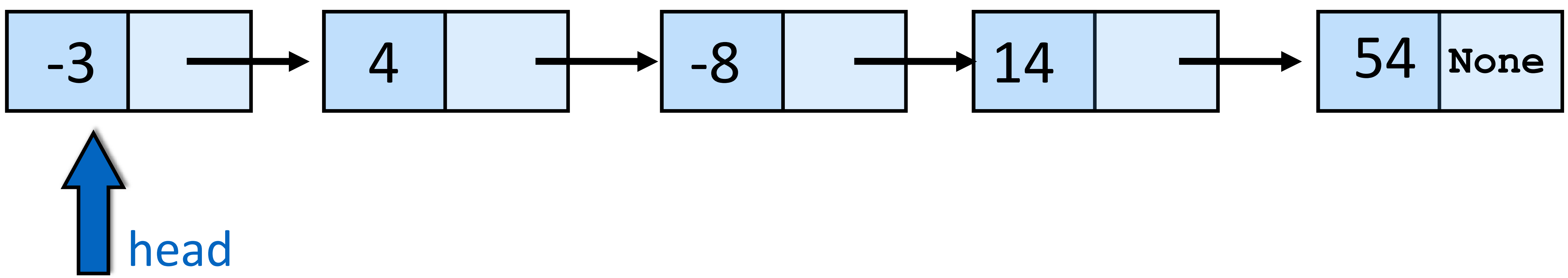
add()

O(1)

- Add item at the back.

Complexity

(Priority Queue using Unsorted Linked Lists)



`get_max()`

- Find max item.
- Remove.

O(n)

`add()`

O(1)

- Add item at the head.

Adding to an array-based sorted list

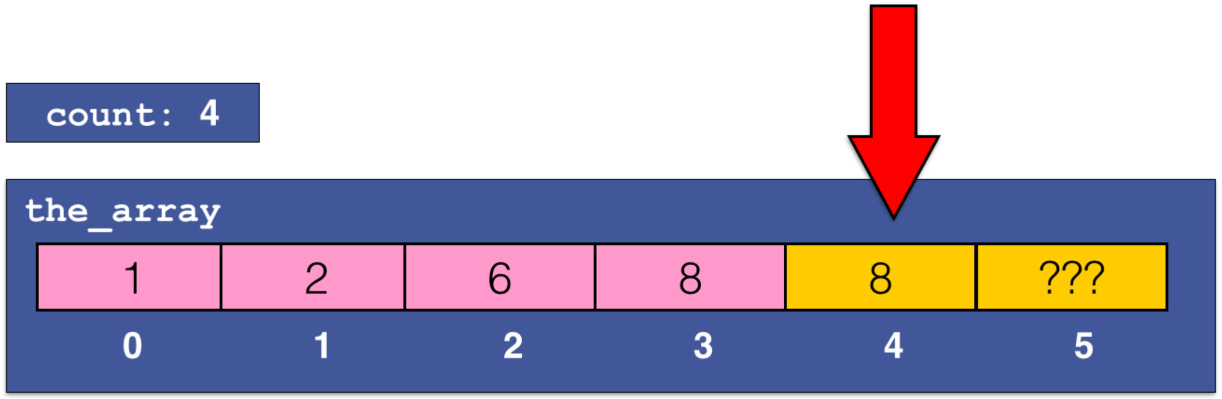
```
def add(self, new_item):
    # easy if the list is empty
    if self.is_empty():
        self.the_array[self.count] = new_item
        self.count += 1
        return True
    # if the lis is not empty...
    has_place_left = not self.is_full()
    if has_place_left:
        # find correct position
        index = 0
        while index < self.count and new_item > self.the_array[index]:
            index+=1
        # now index has the correct position
        # we go backwards from count -1 up to index
        for i in range(self.count-1, index-1, -1):
            # "moving" the item in position i to position i+1
            self.the_array[i+1] = self.the_array[i]
        # insert new item
        self.the_array[index] = new_item
        # increment counter
        self.count+=1
    return has_place_left
```

Find correct position

Move things to make space

Complexity

(Priority Queue using Sorted Lists)



`get_max()`

- Find max item.
- Remove.

Position
`count-1`

O(1)

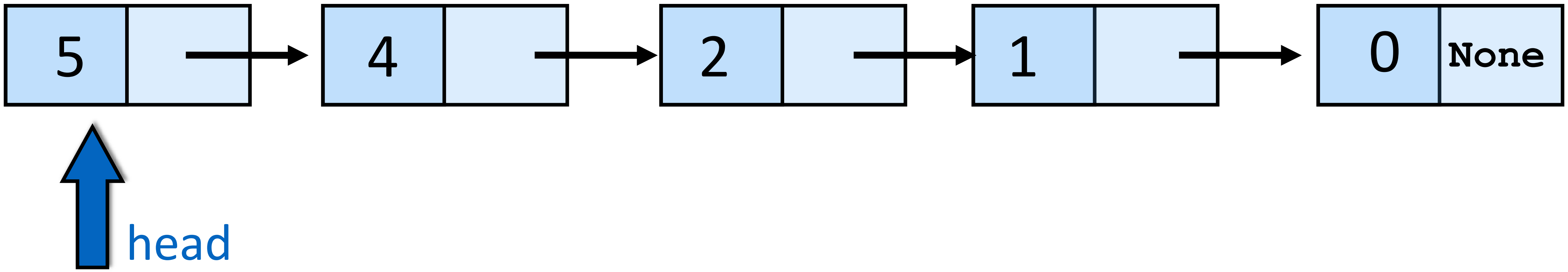
`add()`

- Add item.

O(n)

Complexity

(Priority Queue using Sorted Linked Lists)



`get_max()`

at head

- Find max item.
- Remove.

O(1)

`add()`

- Find correct position.

O(n)

Priority Queues using linear structures…

Implementation	`get_max()`	`add`
Unsorted array	O(n)	O(1)
Unsorted linked list	O(n)	O(1)
Sorted array	O(1)	O(n)
Sorted linked list	O(1)	O(n)

Consider comparisons complexity

Linear structure gives linear complexity

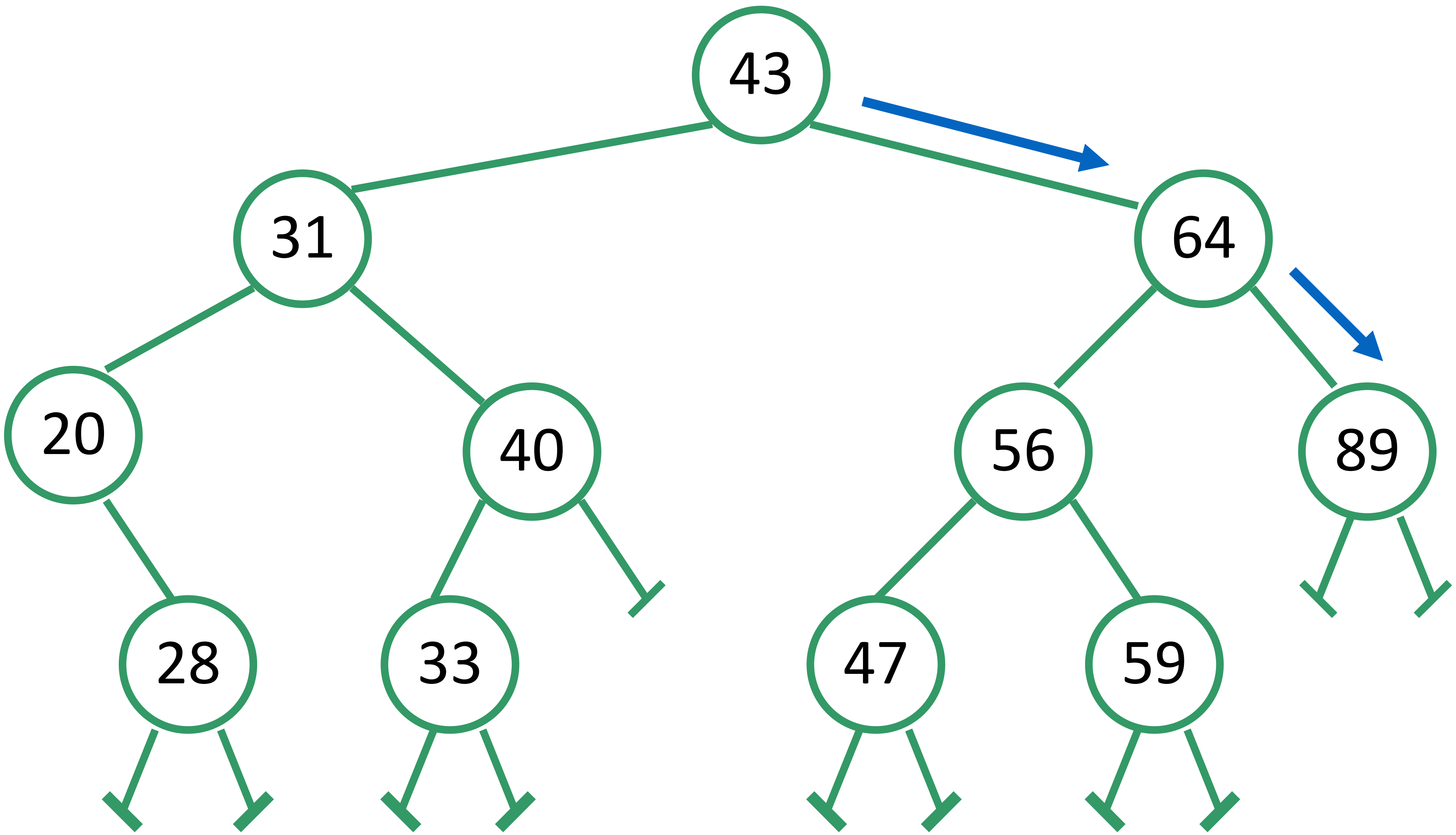
Let’s try a non-linear structure!

```python
class BinarySearchTreeNode:
    def __init__(self, key, item=None, left=None, right=None):
        self.key = key
        self.item = item
        self.left = left
        self.right = right


class BinarySearchTree:
    def __init__(self):
        self.root = None

    def is_empty(self):
        return self.root is None
```

`add()` trivial.

`get_max()` ?

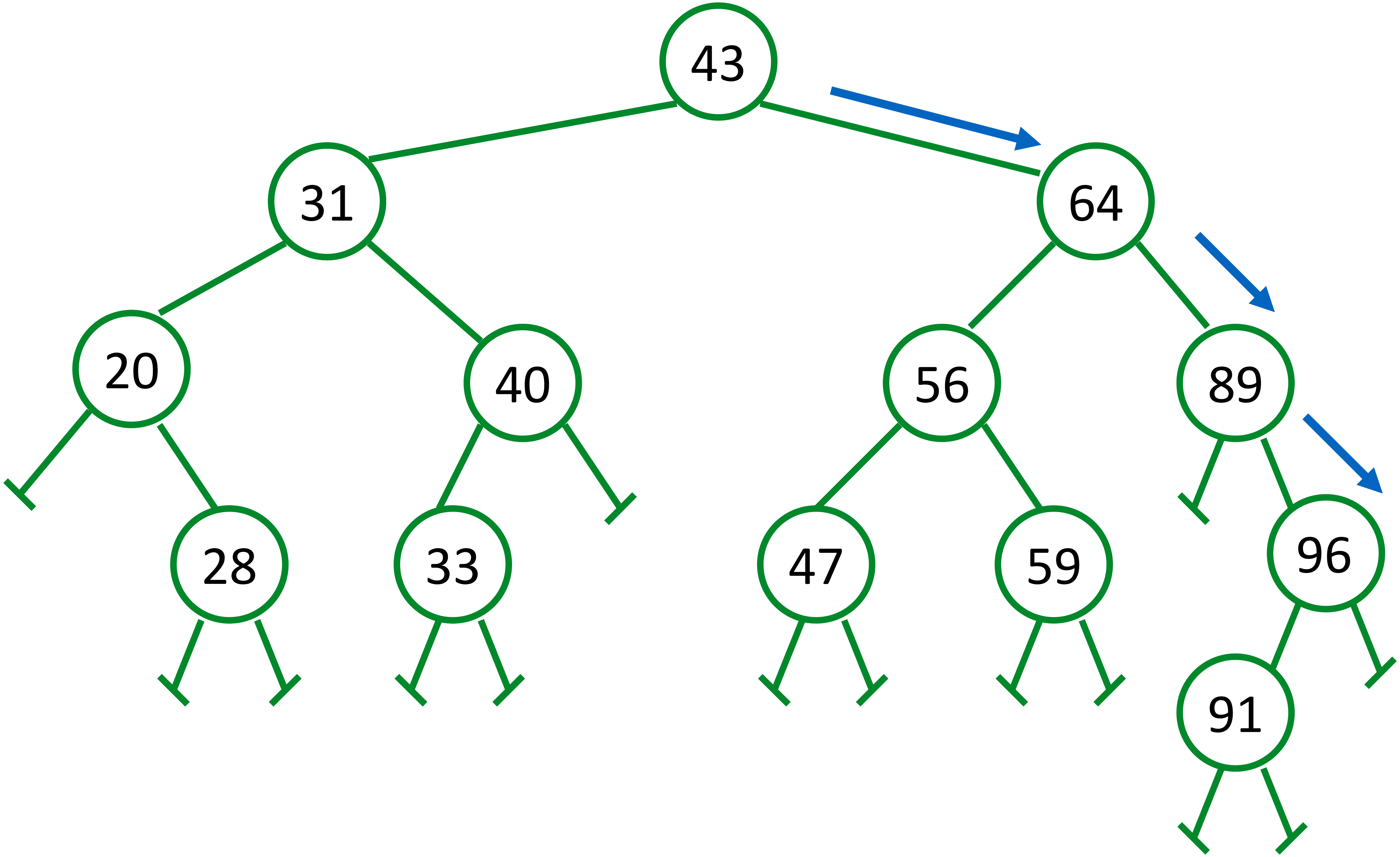
BST: **`get_max()`** ?



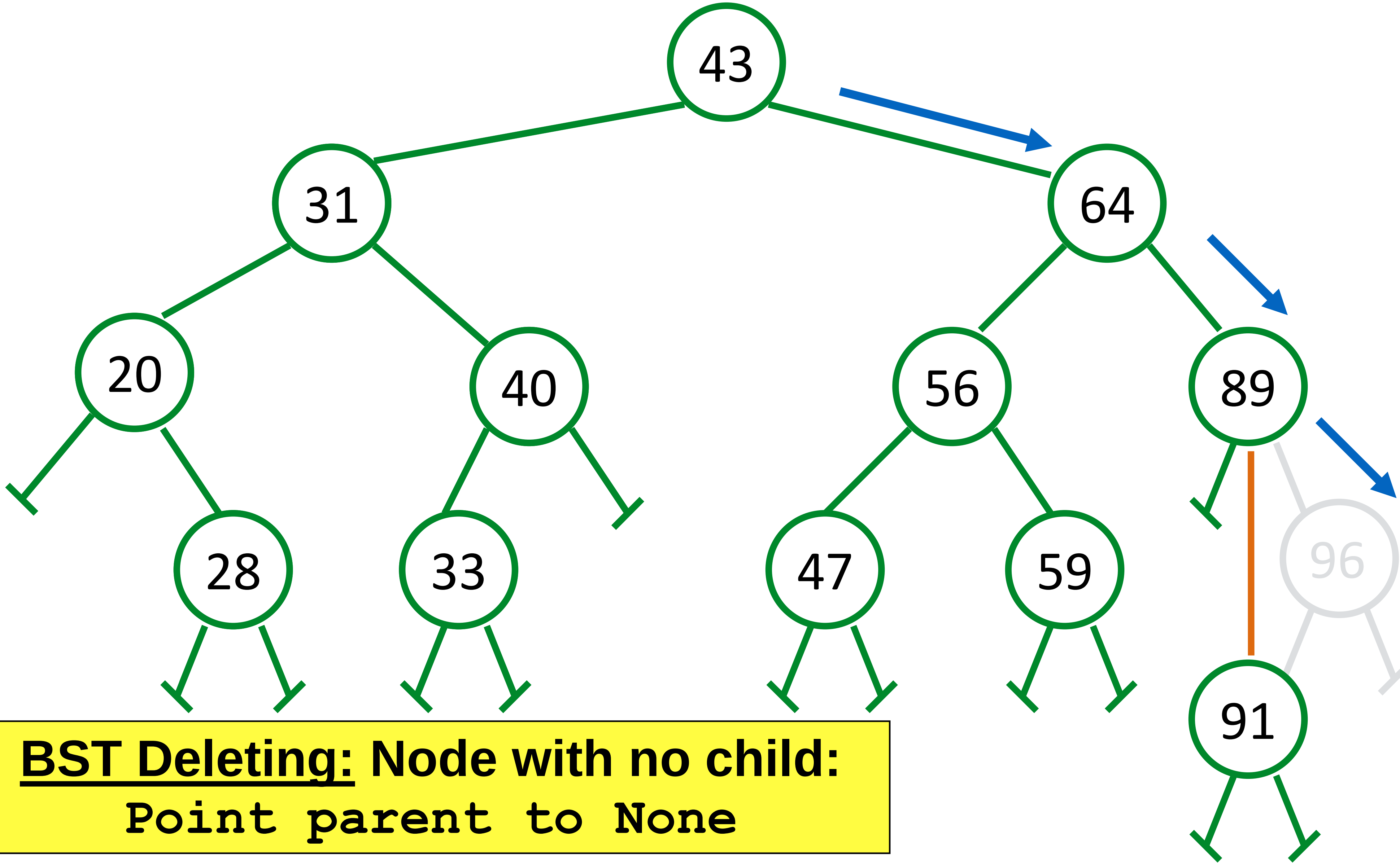
Right-most node: Go right until you can

Complexity depends on the height!

`get_max()`



`get_max()`



<u>BST Deleting:</u> Node with no child:
`Point parent to None`

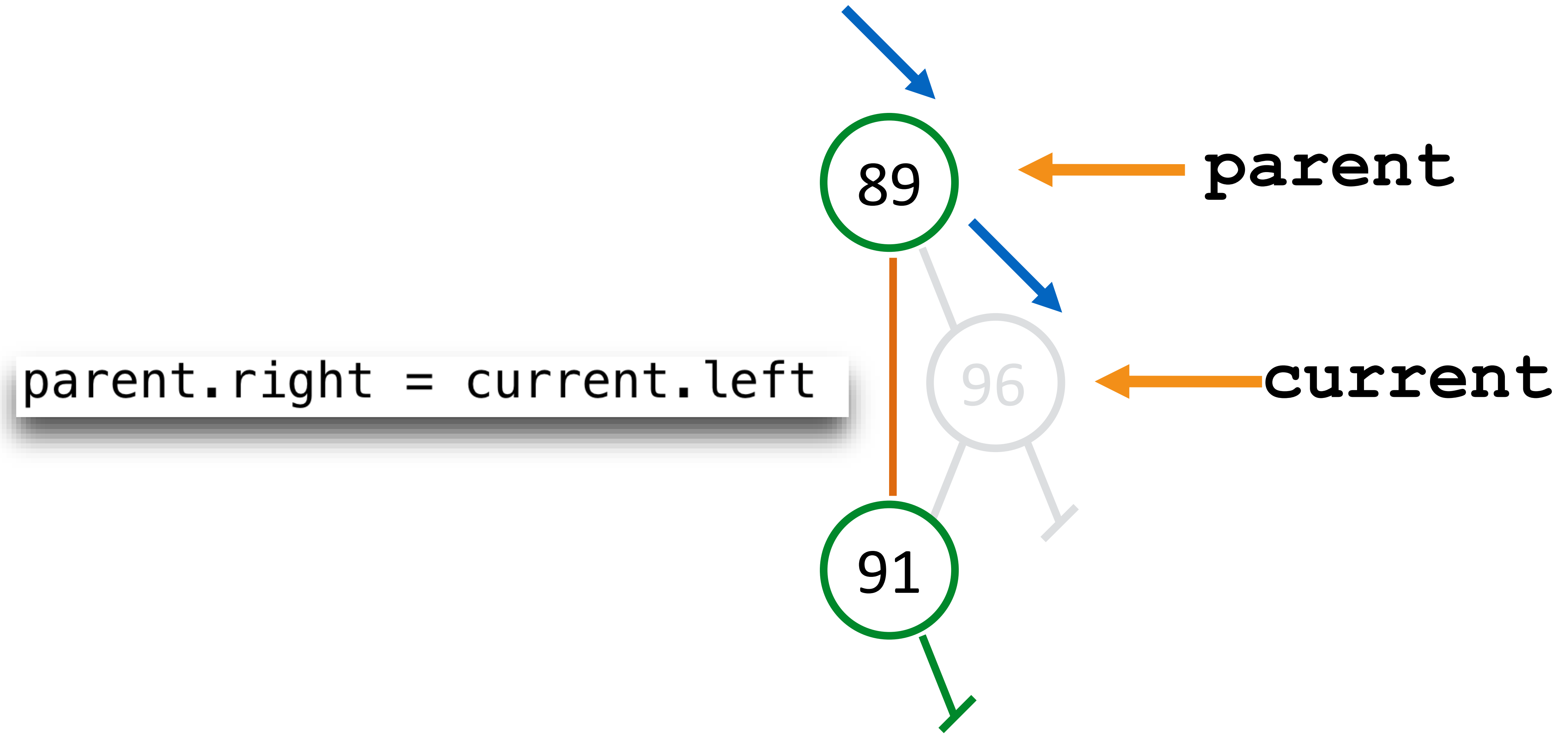
<u>BST Deleting:</u> Node with one child:
`Point parent to child of deleted node`

Simple version (without deleting)

```python
def get_max(self):
    if self.root is None:
        raise ValueError("Empty Priority Queue")
    else:
        return self.get_max_aux(self.root)

def get_max_aux(self, current):
    if current.right is None:  # base case: at max
        return current.item
    else:
        return self.get_max_aux(current.right)
```

With delete?

[Remember the parent]



Node with no child:
Point parent to None

Node with one child:
Point parent to child of deleted node

```python
def get_max(self):
    if self.root is None:
        raise ValueError("Priority Queue is empty")
    elif self.root.right is None: # root has the max
```

We can only call the aux method if we have a parent!

```python
def get_max_aux(self, current, parent):
    if current.right is None: # base case: at max
        parent.right = current.left
        return current.item
    else:
        return self.get_max_aux(current.right,current)
```

```
def get_max(self):
    if self.root is None:
        raise ValueError("Priority Queue is empty")
    elif self.root.right is None: # root has the max
        temp = self.root.item
        self.root = self.root.left # delete root
        return temp
    else:
        return self.get_max_aux(self.root.right, self.root)
```

```
def get_max_aux(self, current, parent):
    if current.right is None: # base case: at max
        parent.right = current.left
        return current.item
    else:
        return self.get_max_aux(current.right,current)
```

Alternative implementation

```python
def get_max(self):
    if self.root is None:
        raise ValueError("Priority Queue is empty")
    elif self.root.right is None: # root has the max
        temp = self.root.item
        self.root = self.root.left # delete root
        return temp
    else:
        return self.get_max_aux(self.root)


def get_max_aux(self, parent):
    if parent.right.right is None: # base case: at max
        temp = parent.right.item
        parent.right = parent.right.left
        return temp
    else:
        return self.get_max_aux(parent.right)
```

only passing the parent, but “looking down” two levels

A better implementation

- Each of the previous choices has one O(N) operation.
- O(Depth) for the binary tree with depth being N-1 if unbalanced.
- Can we do better? Use a (max) heap.
  - →One could also use a min-heap
  - →In FIT1008 we use max-heaps but the ideas are the same for a min-heap.

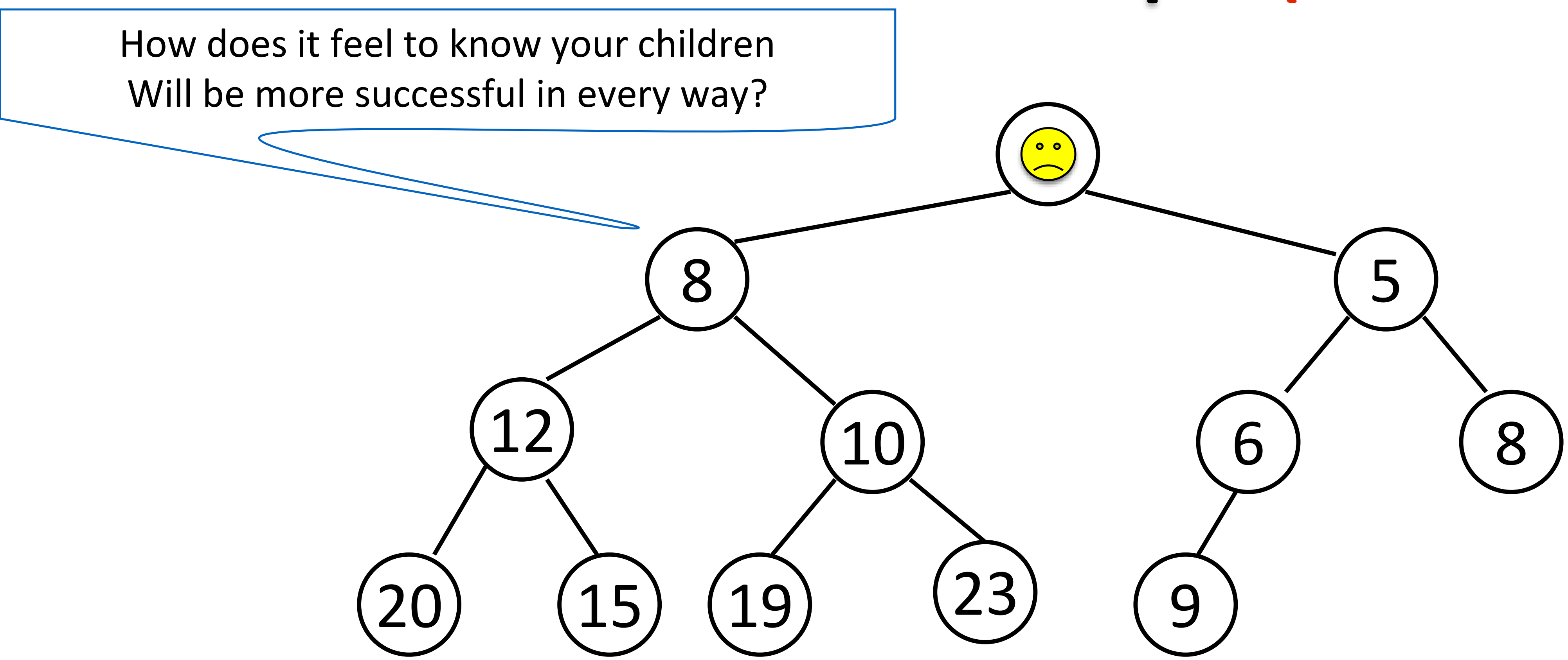
big bricks boss it up

Soil and silt sink through

Max Heaps!

http://static3.depositphotos.com/1003450/176/i/950/depositphotos_1760575-Heap-of-red-brick.jpg

Heap (Min-Heap)

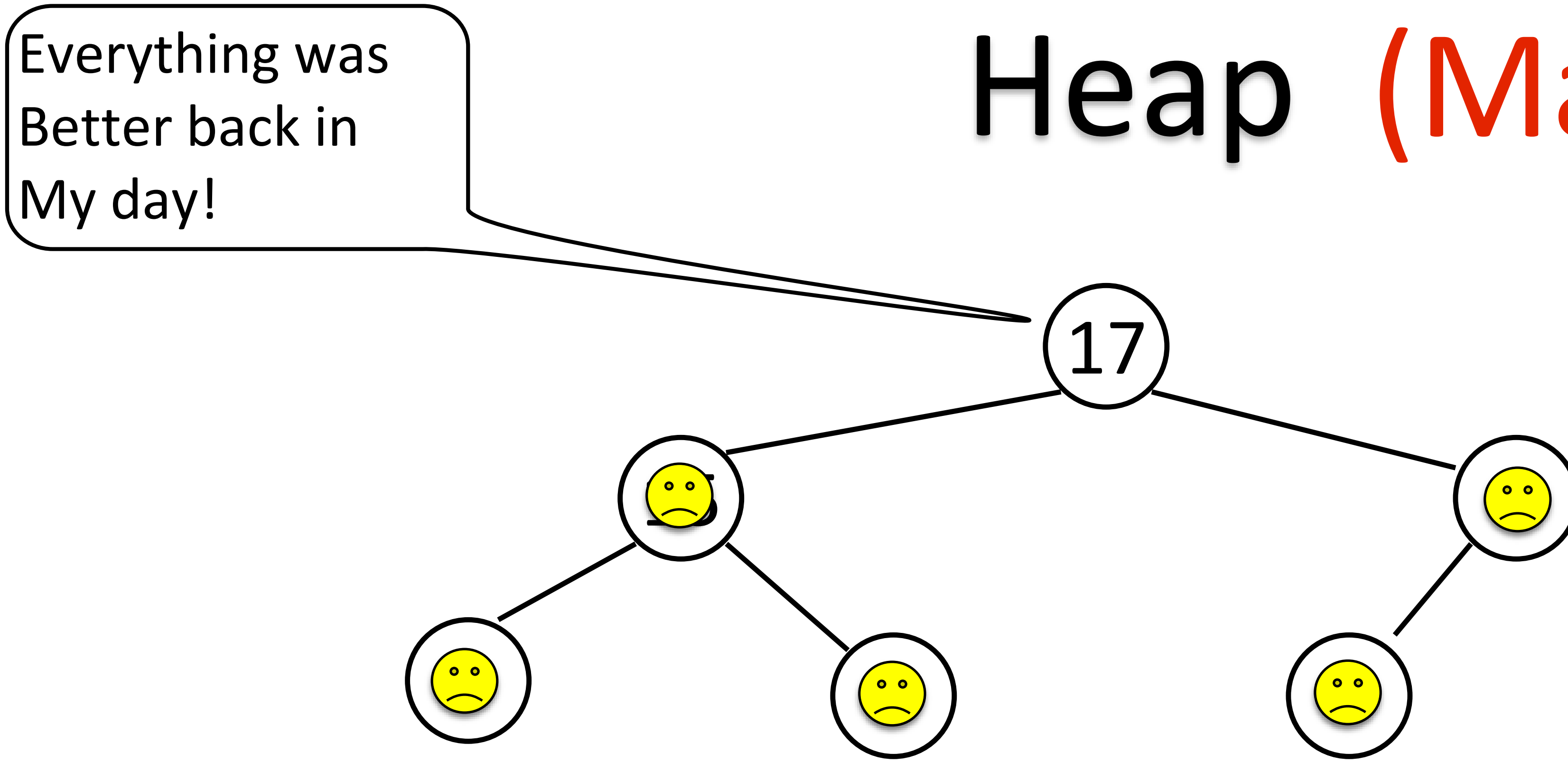


For **every** node:

- The values of the children are **greater or equal** to its value.
- **All the levels are filled**, except possibly the last one, which is filled left to right.

Note: The minimum is always at the root of the tree.

Heap (Max-Heap)

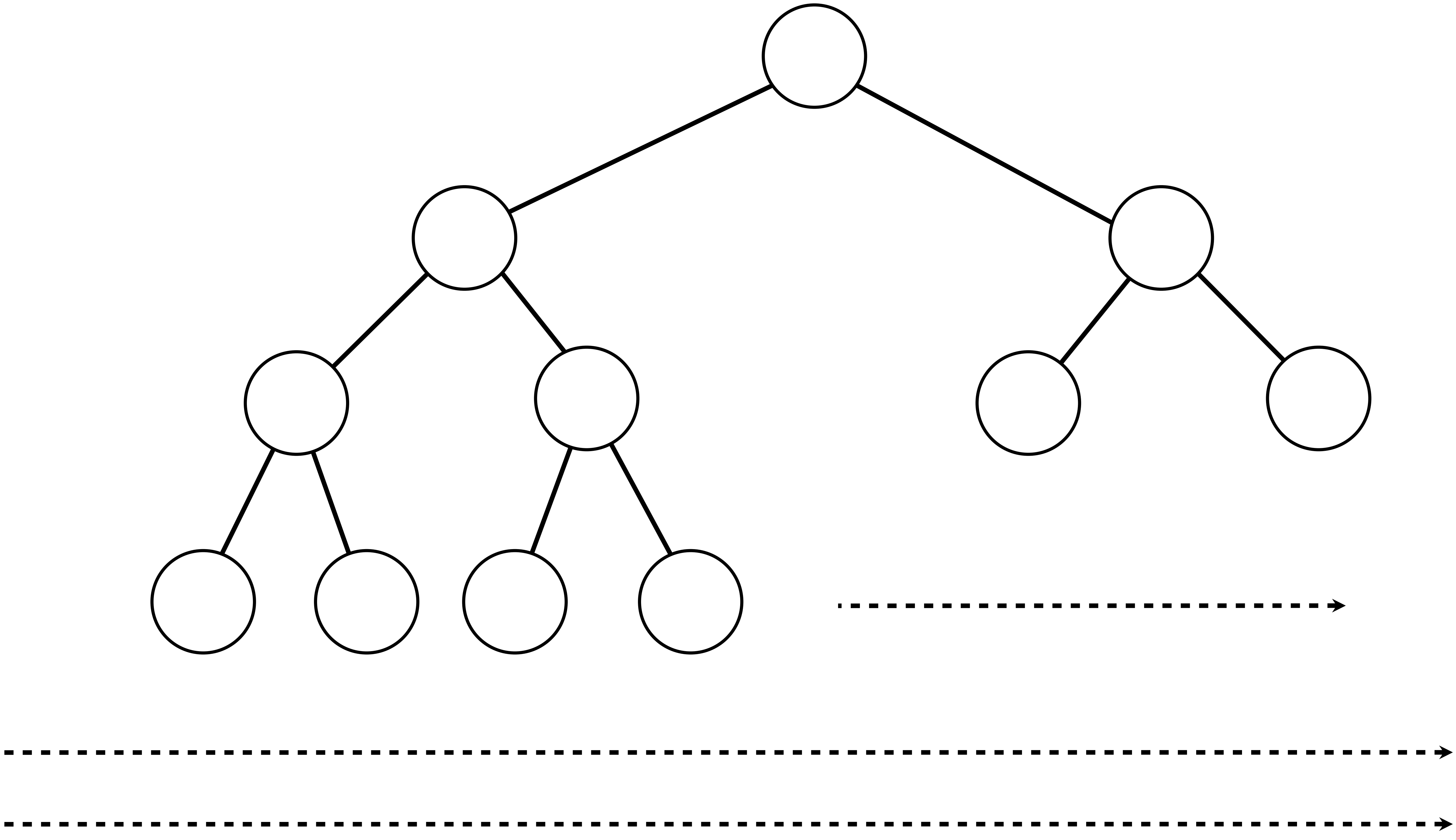


For **every** node:

- The values of the children are **smaller or equal** to its value.
- **All the levels are filled**, except possibly the last one, which is filled left to right.

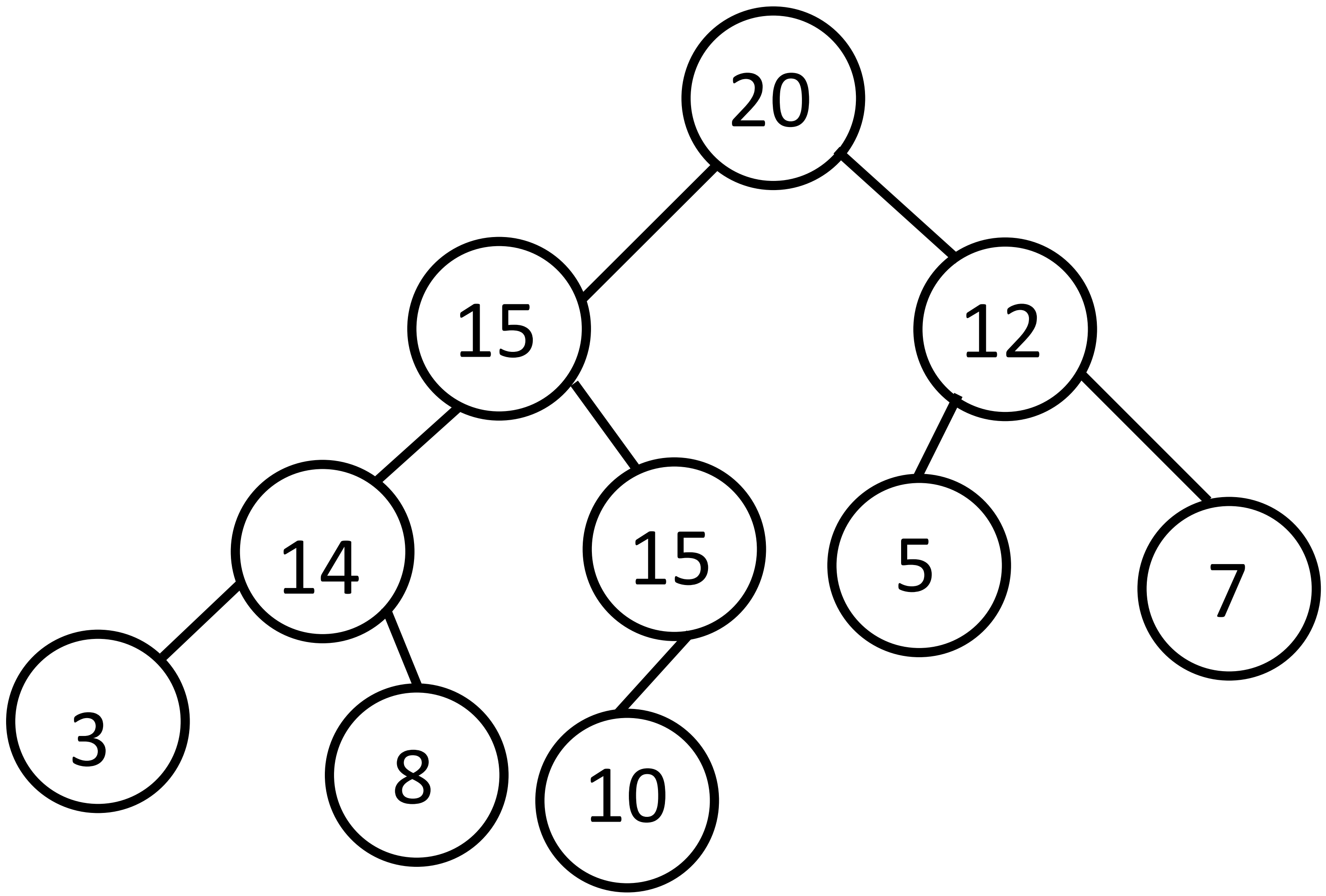
Note: The **maximum** is always at the root of the tree.

Building a *binary* heap



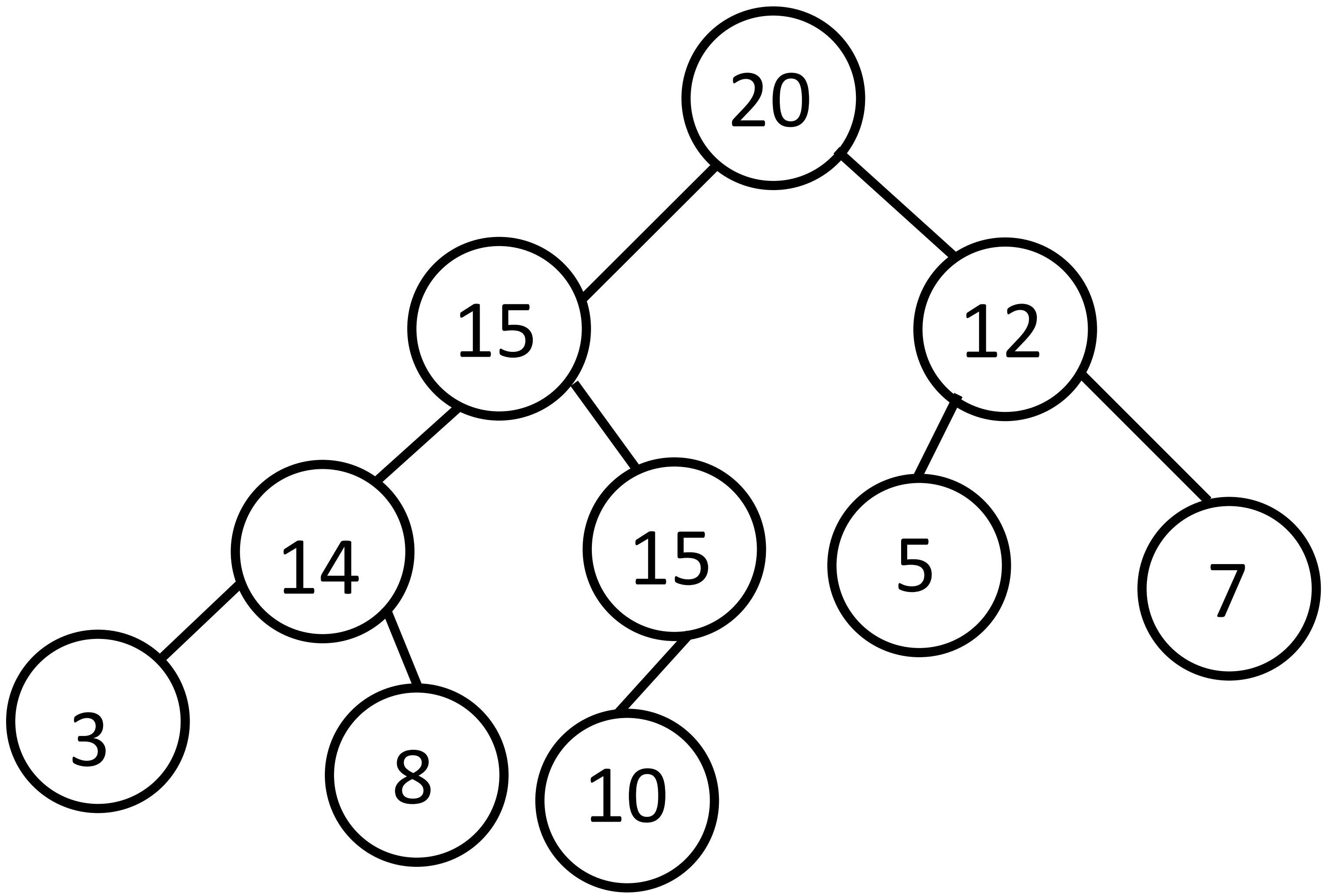
Force the tree to be balanced…

Example

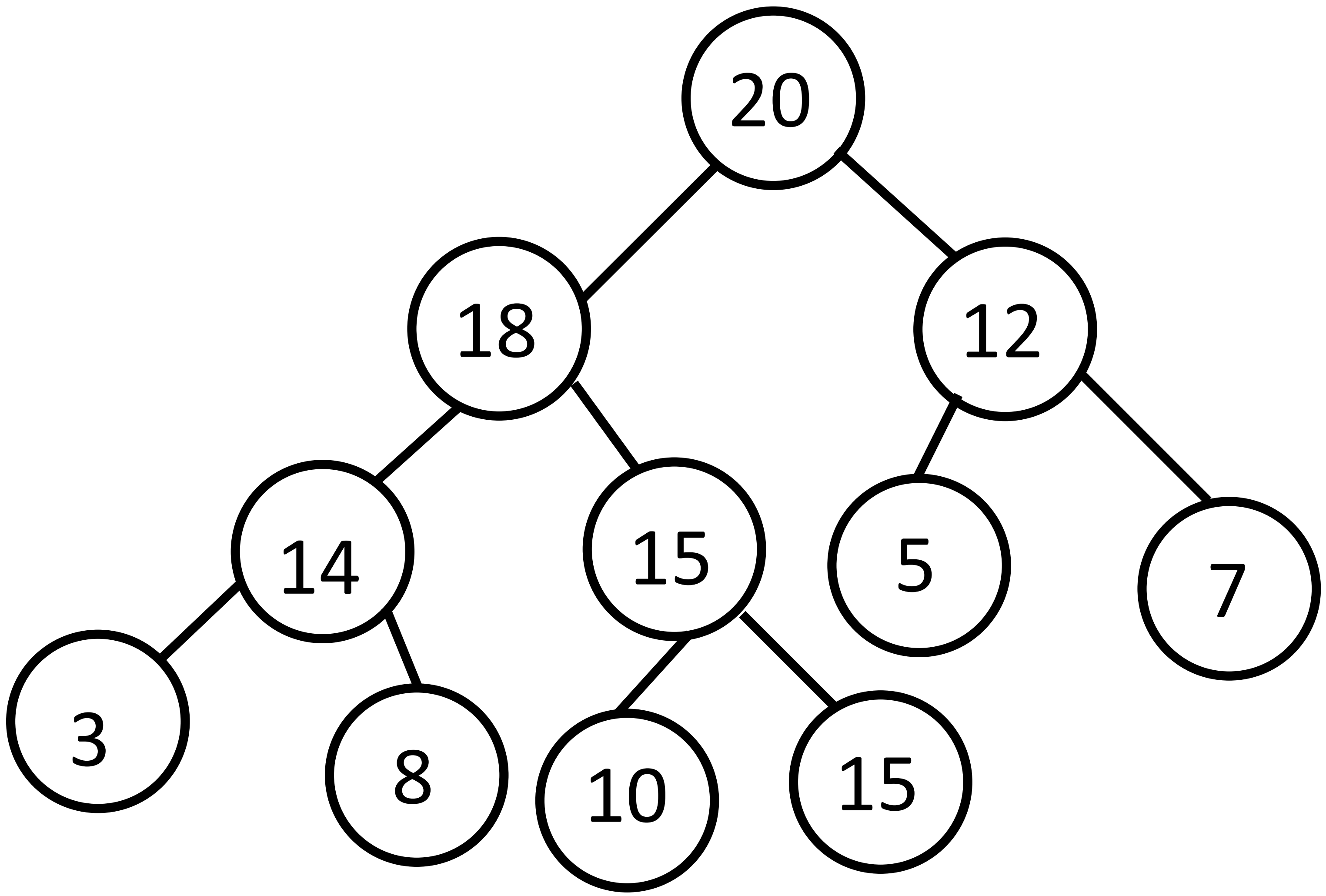


“Not a binary search tree”

Add 18



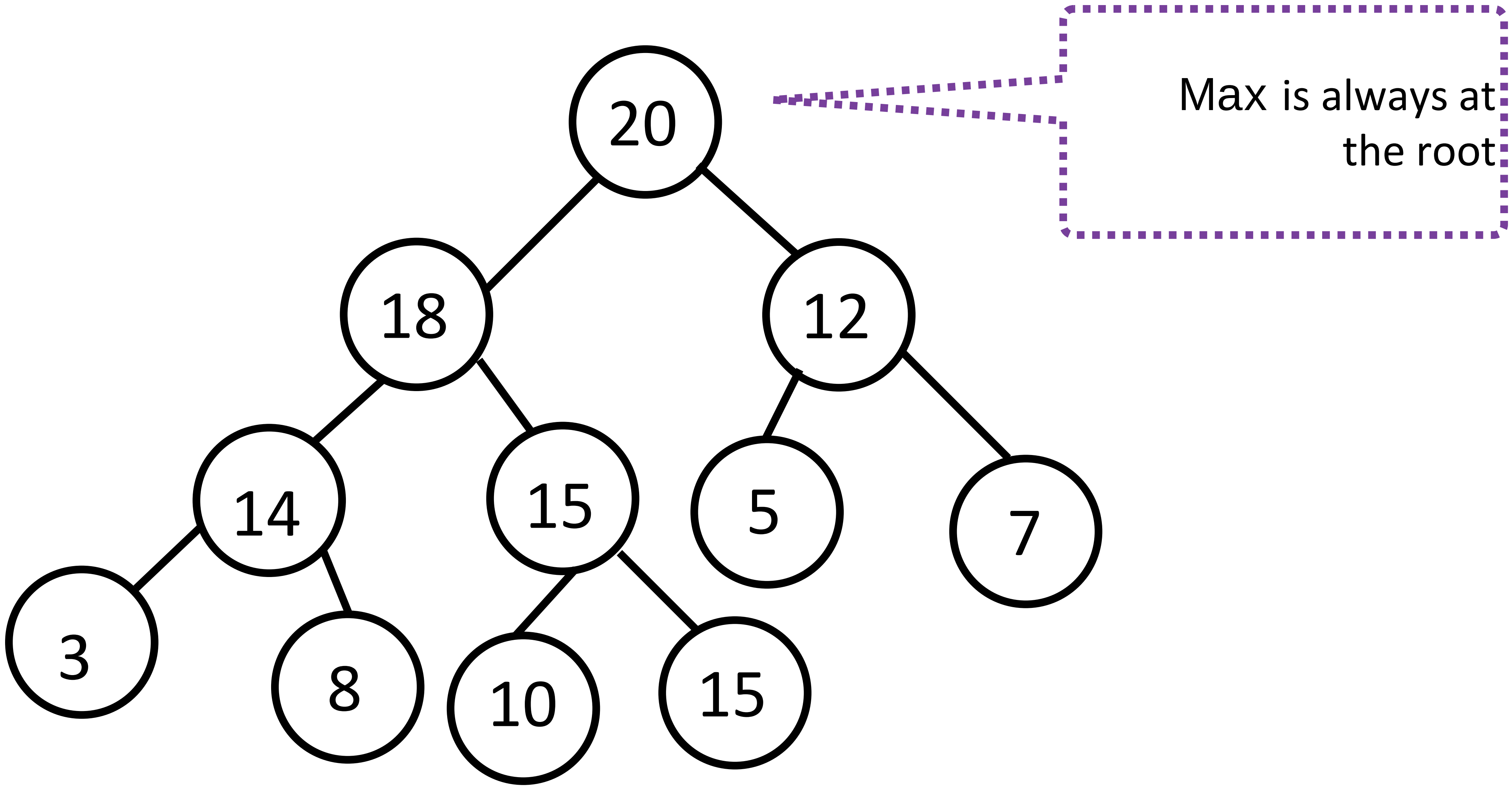
Add 18



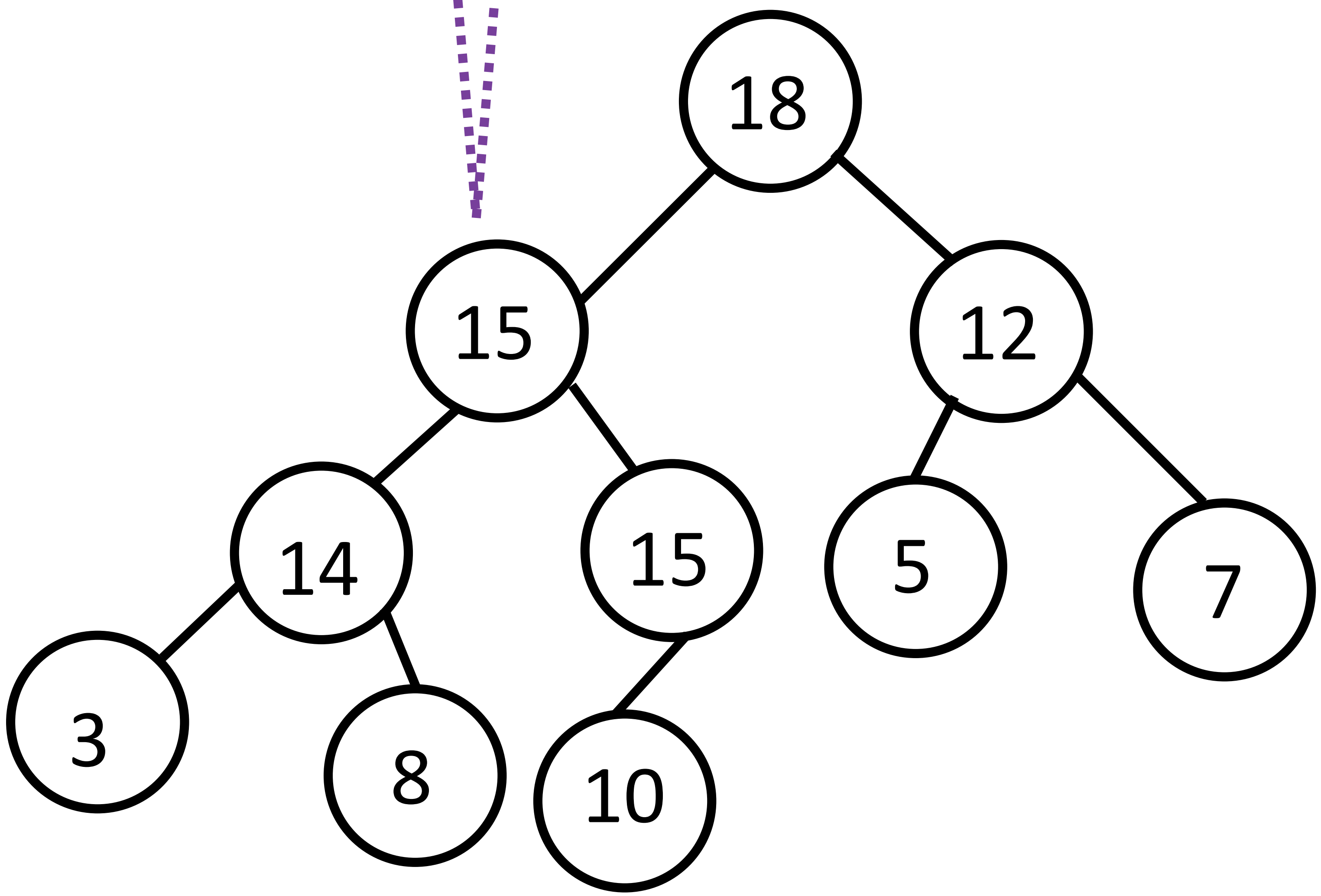
all good now!

issue: swaps need a reference to parent

```
get_max()
```


20
18
12
14
15
5
7
3
8
10
15
Max is always at the root

“Sink” until the element is larger than or equal to children



Complexity?

Implementation?

http://www.brainlazy.com/wp-content/uploads/Disgaea-3-Absence-of-Detention-14.jpg

I don't know what this is… but we'll cover complexity and implementation next time

Summary

- Priority Queues:
  - **`add`**
  - **`get_max`**

- Possible implementations: Lists, BST.

- Heaps: binary tress that are
  - Complete
  - Heap ordered

- Heap operations for Priority Queues: Complexity and correctness