MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #8 and Additional Practice Questions

Tutorial Questions

1. A fair coin is flipped three times. Let X be the number of times heads is flipped. Find the probability distribution of X.

2. About one in every three pairs of twins worldwide is a pair of identical twins. An ultrasound shows that a woman is pregnant with twins.

   (a) The ultrasound does not give any further information. The woman asks her doctor what the chance is that her twins are identical. What should the doctor answer?

   (b) The ultrasound also reveals that both of the twins are male. The woman asks her doctor what the chance is that her twins are identical. What should the doctor answer?

   (Assume that 50% of pairs of identical twins are two females and 50% are two males. Assume that 25% of pairs of non-identical twins are two females, 50% are one female and one male, and 25% are two males.)

   (This question approximates real life, but it's a simplification. Real life is more complicated and interesting: using IVF changes things, twinning rates vary in different regions, a significant number of people are intersex, etc.)

3. A standard die is rolled twice. (Remember that rolling a standard die is equivalent to selecting an integer from $\{1, 2, 3, 4, 5, 6\}$ uniformly at random.) Let X, Y and Z be random variables such that X gives the value of the first roll, Y gives the value of the second roll, and Z gives the sum of the two rolls.

   (a) What is $\Pr(Z = 7 \mid Y = 2)$?

   (b) What is $\Pr(Y = 2 \mid Z = 7)$?

   (c) Are "$Y = 2$" and "$Z = 7$" independent events?

   (d) What is $\Pr(Z = 6 \mid Y = 2)$?

   (e) What is $\Pr(Y = 2 \mid Z = 6)$?

   (f) Are "$Y = 2$" and "$Z = 6$" independent events?

   (g) Are Y and Z independent random variables?

4. A binary string of length 8 is chosen uniformly at random. Let X be the number of 1s in the string and Y be the number of 0s in the string.

   (a) Are X and Y independent random variables?

   (b) Let $Z = X + Y$. Find the probability distribution for Z.

(See over for practice questions.)

Practice Questions

1. On a game show there are three identical looking boxes. One contains a prize and the other two are empty. The contestant doesn't know which box contains the prize, but the host does. The game goes like this.

   - The contestant is asked to choose one of the boxes – this box is not opened yet, though.
   - The host then opens one of the two boxes that the contestant did not choose. She never opens the box with the prize.
   - The contestant is then given the option to "stay" with the box they originally chose or to "switch" to the other unopened box. They win if their final choice contains the prize.

   What is the contestant's probability of winning if they stay? What if they switch?

   (To find more takes on this question than you could ever want to read, google "Monty Hall problem".)

2. A Bayes spam filter is being trained on a library of spam emails and a library of legitimate emails. It finds that 10% of the spam emails contain the word "winner" but only 1% of the legitimate emails do. The filter assumes that 53% of emails are spam. Use Bayes' theorem to calculate probability that an email is spam given that it contains "winner".

   (Bayes spam filters are a thing – google them. That 53% of emails are spam is in line with recent statistics, but the "winner" percentages are just made up.)

3. Let $S = \{1, 2, \ldots, 10\}$. Out of all the equivalence relations on S that have exactly 2 equivalence classes, an equivalence relation is chosen uniformly at random. What is the probability that it has two equivalence classes of size 5?