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MONASH University

**Semester Two 2016**
**Examination Period**

**Faculty of Information Technology**

**EXAM CODES:** **MAT1841**

**TITLE OF PAPER:** **Continuous Mathematics for Computer Science – Practice Exam**

**EXAM DURATION:** 3 hours writing time

**READING TIME:** 10 minutes

***THIS PAPER IS FOR STUDENTS STUDYING AT: (tick where applicable)***

☐ Berwick ☑ Clayton ☑ Malaysia ☐ Off Campus Learning ☐ Open Learning
☐ Caulfield ☐ Gippsland ☐ Peninsula ☐ Monash Extension ☐ Sth Africa
☐ Parkville ☐ Other (specify)

During an exam, you must not have in your possession any item/material that has not been authorised for your exam. This includes books, notes, paper, electronic device/s, mobile phone, smart watch/device, calculator, pencil case, or writing on any part of your body. Any authorised items are listed below. Items/materials on your desk, chair, in your clothing or otherwise on your person will be deemed to be in your possession.

**No examination materials are to be removed from the room.** This includes retaining, copying, memorising or noting down content of exam material for personal use or to share with any other person by any means following your exam.
Failure to comply with the above instructions, or attempting to cheat or cheating in an exam is a discipline offence under Part 7 of the Monash University (Council) Regulations.

**Instructions:**
1. The exam has 7 questions with a total of 85 marks.
2. Attempt all questions in each section (Pages 2—12).
3. Answers are to be written in the spaces provided in this paper.
4. Some relevant formulae are provided on Moodle page. The formula sheet will be attached to the exam.

**AUTHORISED MATERIALS**

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| **OPEN BOOK** | ☐ **YES** | ☑ **NO** |
| **CALCULATORS** | ☐ **YES** | ☑ **NO** |
| **SPECIFICALLY PERMITTED ITEMS** | ☐ **YES** | ☑ **NO** |

***Candidates must complete this section if required to write answers within this paper***

STUDENT ID: __ __ __ __ __ __ __ __ DESK NUMBER: __ __ __ __ __

| Q1 | Q2 | Q3 | Q4 | Q5 | Q6 | Q7 | | | |
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**1.** Find an equation of the form $ax + by + cz = d$ of the plane $P$ that contains the three points $(1, 0, 0), (2, 0, 1)$ and $(1, 1, 1)$. (ii) What is the distance of the origin $(0, 0, 0)$ from this plane? (iii) Find an equation of a plane $Q$ through the origin such that the angle between $P$ and $Q$ is a right angle.

10 marks

2. **[15 Marks]**

(a) Consider the matrix $M$ given below:

$$M = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$$

(i) Find the determinant of $M$ and show that $M$ has an inverse. [2 marks]

(ii) Form the matrix $[M|I]$ and using the Gauss-Jordan algorithm find the inverse of $M$. Carefully record all row operations. [5 Marks]

(iii) Check that the matrix you have found is indeed the inverse of $M$. [2 Marks]

(b) Solve

$$\begin{aligned} x + y + 2z &= 9 \\ 2x + 4y - 3z &= 1 \\ 3x + 6y - 5z &= 0 \end{aligned}$$

using Gaussian elimination and back-substitution. Carefully record all row operations.

[6 Marks]

**3.** What are the determinants of the following matrices:

$$\text{(i)} \begin{pmatrix} 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -1 & 0 \end{pmatrix}, \text{(ii)} \begin{pmatrix} 0 & 5 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ -1 & 0 & 0 & 0 \end{pmatrix}, \text{(iii)} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 3 & 1 & 5 \\ 1 & 2 & 3 & 4 \\ 5 & 8 & 11 & 14 \end{pmatrix},$$

$$\text{(iv)} \begin{pmatrix} 1 & -1 & 0 & 2 \\ 2 & 3 & 0 & 7 \\ 0 & 5 & 3 & 3 \\ 8 & -4 & 0 & 0 \end{pmatrix}$$

10 marks

4. **[15 Marks]**

(a) Find the first derivative of the following functions with respect to $x$:

(i) $y = x^4 + 3x^3 + 2$ [1 Mark]

(ii) $y = \cos^2(2x^2 + 3)$ [2 Marks]

(iii) $y = \dfrac{x^3}{2x + 1}$ [2 Marks]

(b) Find the second derivative with respect to $x$ of the following functions:

(i) $e^{3x^2+1}$ [5 Marks]

(ii) $y = \cos(3x)\sin(x)$ [5 Marks]

5.

Evaluate the following integrals:

$$\text{(i)} \int \sin^2(x)dx, \quad \text{(ii)} \int e^{2x}\cos(e^x)dx$$

10 marks

6. **[15 marks]**

(a) Recall that a geometric power series is written as

$$\sum_{n=0}^{+\infty} ax^n = \frac{a}{1-x},$$

for $-1 < x < 1$.

(i) Fill in the coefficients of the first five terms in the power series expansion of $\dfrac{1}{1-x}$. Show any required working in the space provided. [1 Mark]

$$\frac{1}{1-x} = \quad x^0 + \quad x^1 + \quad x^2 + \quad x^3 + \quad x^4 + \ldots$$

(ii) Fill in the coefficients of the first five terms in the power series expansion of $\dfrac{1}{1+x}$. Show any required working in the space provided. [1 Mark]

$$\frac{1}{1+x} = \quad x^0 + \quad x^1 + \quad x^2 + \quad x^3 + \quad x^4 + \ldots$$

(iii) Fill in the coefficients of the first five terms in the power series expansion of $\dfrac{1}{1-x} - \dfrac{1}{1+x}$. Show any required working in the space provided. [1 Mark]

$$\frac{1}{1-x} - \frac{1}{1+x} = \quad x^0 + \quad x^1 + \quad x^2 + \quad x^3 + \quad x^4 + ...$$

(iv) Find the values of $a, b, c$ and $d$, as shown below, that correspond to the sum found in (iii). Show any working required in the space provided. [2 Marks]

$$\frac{1}{1-x} - \frac{1}{1+x} = \sum_{n=a}^{b} cx^d$$

(b) Recall that the Taylor polynomial of degree $n$ for approximating a function $f(x)$ is given by

$$T_n(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + ... + \frac{f^{(n)}(0)}{n!}x^n.$$

Find the first four Taylor polynomials, that is $T_0(x), T_1(x), T_2(x)$ and $T_3(x)$, of the function $f(x) = \ln(3x+2)$, at $x = 0$. [10 Marks]

7. **[10 Marks]**

Let $f(x, y) = ye^{3x+y}$ be a function of two variables, $x$ and $y$.

(i) Find the first partial derivatives, $f_x(x, y)$ and $f_y(x, y)$. [6 Marks]

(ii) Evaluate the derivatives found in (i) at $(x, y) = (-1/3, 1)$. [2 Marks]

(iii) Recall that the tangent plane to the surface $z = f(x, y)$ at the point $(x, y) = (a, b)$ is given by

$$z = f(a, b) + f_x(a, b)(x - a) + f_y(a, b)(y - b).$$

Using your answers from (i), write the equation of the tangent plane to the surface $z = f(x, y)$ defined in (i) at the point $(x, y) = (-1/3, 1)$, in simplified form. [2 Marks]

**End of examination questions**