Monash University
Faculty of Information Technology
2^{nd} Semester 2019

FIT2014

Tutorial $2\frac{3}{4}$

Games, puzzles, paradoxes, ...

This "tutorial" is intended to be interesting, fun, and entirely optional. Much of it goes beyond the FIT2014 syllabus. There will be a special session in Week 6 for interested students to discuss any of the problems that interest them.

1. Finiteness of Chess

(a) Give an upper bound on the number of legal positions in a game of chess.

(b) Prove by contradiction that every game of chess is finite.

2. Russell's Paradox

A *set* is defined by giving a precise condition that its elements must satisfy. Normally, we don't think of a set as being able to contain *itself*, but let's ignore that outdated prejudice. For example, let Q be the set of all infinite sets. Since the number of infinite sets is itself infinite, Q is an infinite set, so it must be a member of itself: $Q \in Q$.

Now let R be the set of all sets that do not contain themselves. For example, the set $A = \{a, b, c\}$ does not contain itself, since its elements (namely a, b, c) do not include the set $\{a, b, c\}$. So $A \in R$. On the other hand, our set Q does contain itself, so $Q \notin R$. Since most sets we meet do not contain themselves, it's natural to think of R as a very big "inclusive" set that contains most sets we might be interested in.

Question: Is R a member of R?

How do you resolve this paradox?

3. The Hypergame paradox

In this question, a *game* has two players who take turns to move and must result in a win for one of the players after a finite number of moves. (It's ok for a player to have an infinite number of available moves at a given turn, but the sequence of moves chosen by the players as they play the game must be finite.) There is no randomness involved, so that the initial position together with the sequence of the players' moves is enough to determine who wins. So, Chess is not allowed because it can end in a draw; Backgammon is not allowed because of its use of dice; and Paper-Scissors-Rock is not allowed because the players move simultaneously rather than alternately. Allowable games include Hex, Nim and Dots-and-Boxes (with an odd number of cells).

Let *Hypergame* be the game in which the first player's first move is to choose a game, after which the players play that game in the usual way, but with the second player starting first in that game because it's now the second player's turn (because the first player has just had a turn, by choosing the game).

Is Hypergame a game?

Surely it is! The first player must choose a valid game, and each of those games must always finish in a finite amount of time, so Hypergame itself must always finish in finite time (since it takes only one move longer than the time taken to play the game chosen by the first player).

However, if Hypergame is a game, then the first player can choose it on the first move. Then the second player is now the *first* player in a new game of Hypergame, and must choose a game to play. That choice could also be Hypergame. Now we're back to the (original) First Player, who must choose a game, and could choose Hypergame. And so on! The players could keep choosing Hypergame forever, and the game never ends, violating one of the requirements for a game.

So, it looks like Hypergame is not a valid game because it can go on forever.

But, if Hypergame is not a valid game, then it cannot be chosen as its own first move. Furthermore, any game chosen as that first move of Hypergame is valid, and so finishes in finite time. So Hypergame is a valid game after all!

How do you resolve this paradox?

- W. S. Zwicker, Playing games with games: the hypergame paradox, *American Mathematical Monthly* **94** (1987) 507–514.

4. How to play 1D-Go

You have met 1D-Go in the first two tutorials. Those exercises only concerned the legality of 1D-Go positions. But 1D-Go is also a game, and can be played. Check out the rules of Go, as they apply to 1D-Go, and see if you can determine a good strategy for playing the game.

5. Regular expression for English

Find the shortest regular expression you can that matches precisely the words of the English language (e.g., in `/usr/share/dict/words`).

Find regular expressions E_1 and E_2 such that the English words are precisely those that match E_1 but do not match E_2. See if you can make the total length of E_1 and E_2 as short as possible.

6. Mazes

You are in a maze represented by a rectangular grid. One cell is the start cell, another is the cell you want to get to. Some pairs of adjacent cells have a wall between them, preventing you from moving directly from one to the other. There is at least one path from the start to the end. Suppose that the characters U, D, L, R represent moving up, down, left and right by one grid cell. A string of these characters represents a sequence of movements through the maze, but is only valid if it does not make you bump into a wall.

Describe an algorithm for converting a given maze into a regular expression which matches exactly those strings which correspond to sequences of moves which solve the maze.

Thanks to FIT2014 tutor Nathan Companez for this question.

7. History

Check out the new banner image at the top of the FIT2014 Moodle page.

Find the names of as many of the people shown as you can. What did they each do?