FIT2093: Sample Tutorial 6 Solutions

Introduction to Number Theory

Problems

1. Write the following composites numbers as a multiplication of their prime factors.
   a. 12 = 3*2*2
   b. 78 = 3*2*13
   c. 99 = 3*3*11
   d. 128 = 2*2*2*2*2*2*2
2. Check whether the following pairs of numbers are relative primes.
   a. 12 and 48

      12 = 3*2*2, 48 = 12*4, a common divisor is 12, therefore they are NOT relatively prime.

   b. 5 and 125

      5 , 125 = 5*25, a common divisor is 5, therefore they are NOT relatively prime.

   c. 6 and 44

      6 = 3*2, 44 = 2*2*11, a common divisor is 2, therefore they are NOT relatively prime.

   d. 8 and 51

      8 = 2*2*2*, 51 = 3*17, a common divisor is 1, therefore they ARE relatively prime.

   e. 7 and 64

      7, 64= 2*2*2*2*2*2, a common divisor is 1, therefore they ARE relatively prime.
3. What is the greatest common divisor of the following set of numbers?
   a. 12, 24 and 18

      6*2, 6*4, 6*3

      6 is the GCD

   b. 5, 125 and 60

5, 5*5*5 and 5*3*4

5 is GCD

c. 49 and 175

7*7, 7*25

7 is the GCD

4. Find the congruent class of each number in modulo 8
   a. 28 = 8*3 + 4 or 4 is a congruent class of 28 in module 8
   b. 33 = 8*4 + 1 or 1 is a congruent class of 33 in module 8
   c. 5 = 8*0 + 5 or 5 is a congruent class of 5 in module 8
   d. 12 = 8*1 + 4 or 4 is a congruent class of 12 in module 8
5. Complete the following modular arithmetic operations and determine the result:
   a. (12+8) mod 6 = 2
   b. (2x12) mod 6 = 0
   c. (20+125) mod 5 = 0
   d. (20-35) mod 5 = 0
   e. 10^4 mod 3 = 1 (9999 +1) mod 3 = 0+1= 1
6. What is the value of Euler Totient φ(n) of the following:
   a. 3

      The positive coprimes less than or equal to 3 are: {1, 2}

      Therefore Euler Totient function of 3 is the number of elements in the above t set which is 2.

   b. 7x5

      The positive coprimes less than or equal to 7*5 (35) are: {1, 2, 3, 4, 6, 8, 9, 11, 12, 13, 16, 17, 18, 19, 22, 23, 24, 26, 27, 29, 31, 32, 33, 34}, Euler totient function of 35 is 24 = (p-1) * (q-1) = 6 * 4 = 24

   c. 3X11

      ϕ(n) = (p-1)(q-1) if p & q are primes.

      = (3-1)(11-1) = 2* 10 = 20.
7. Let X to be the set of all the possible relative primes of 15 that is less then 15. Note, you can write 15 as a multiplication of 3 * 5. List the members of X.

The relative primes (coprimes) of 15 are {1, 2, 4, 7, 8, 11, 13, 14}.
Euler Totient function of 3 * 5 = 8 = 2 *4 = 8

8. Check whether the following pair of numbers in a given modulo is a multiplicative inverse.
    a. Numbers 3 and 7 in modulo 10.

       3 * 7 = 21 mod 10 = 1 , therefore 3 and 7 are multiplicative inverse in modular arithmetic of 10

    b. Numbers 7 and 11 in modulo 13.

       7 * 11 = 77 mode 13 = 12, hence 7 and 11 are NOT multiplicative inverse in modular arithmetic of 13

    c. Numbers 3 and 4 in modulo 11.

       3 * 4 = 12 mod 11 = 1, therefore 3 and 4 are multiplicative inverse in modular arithmetic of 11.