**MAT1830 - Discrete Mathematics for Computer Science**
**Tutorial Sheet #2 and Additional Practice Questions**

## Tutorial Questions

1. (a) Draw a truth table for the proposition $(\neg b \to \neg p) \wedge (b \vee p) \wedge \neg b$.
   (b) Is it a tautology, a contradiction or neither?
   (c) Give your own examples of a proposition that is a tautology and a proposition that is a contradiction.

2. (a) Dwayne has been looking at fad diets on the internet. He finds one which advises him that if he does not have broccoli with a meal, then he should not eat potatoes either. Another diet tells him he should have at least one out of potatoes or broccoli with every meal. Dwayne decides to follow both these diets and then remembers that he is allergic to broccoli and never eats it.
   Translate the rules Dwayne must follow into propositions using $b$ for "Dwayne eats broccoli with a meal" and $p$ for "Dwayne eats potatoes with a meal".
   (b) What does your answer to 1(b) tell you about Dwayne's decision?

3. (a) Use De Morgan's laws to write "Her car isn't blue or red." in a different way.
   (b) Use De Morgan's laws to write "The integer I am thinking of is not an odd prime number." in a different way.
   (c) Use contrapositives to write "If you use our app, then we can do anything we want with your data." in a different way.

4. Can you write an equivalent proposition to $\neg p \vee (\neg q \to \neg r)$ using only $p, q, r$ and the connectives $\wedge$ and $\neg$?

(See over for practice questions.)

## Practice Questions

1. A digital logic circuit can include AND, OR and NOT gates. These work the way you'd expect (for example, an AND gate accepts two inputs of 1 or 0 and outputs a 1 if both of the inputs were 1 and a 0 otherwise). The diagram in the centre at the bottom of the page is an example of a digital logic circuit corresponding to the proposition $(p \wedge q) \vee r$.

   (a) What proposition does the diagram at the right at the bottom of the page correspond to?

   (b) Draw a logic circuit corresponding to the proposition $(\neg p \wedge q) \vee \neg(p \vee q)$.

   (c) Can you draw a simpler circuit which would do the same job as your answer to 1(b).

2. Another fad diet has the following rules.

   - If you have no broccoli with a meal, then always have tomatoes.
   - When you have both tomatoes and broccoli, never have garlic.
   - If you have garlic or broccoli, then avoid tomatoes.

   (a) Write each of these rules as a proposition.

   (b) Can you simplify the diet's rules? (Drawing a truth table might help, or you can try to just think through it.)

3. You have been hired by Aperture Laboratories to work on their latest weapon. They want the weapon to fire in each of the following situations.

   - Trigger A is pulled and the capacitors are charged.
   - Trigger B is pulled and the capacitors are not charged.

   (a) Let $a$ be the proposition "trigger A is pulled", let $b$ be the proposition "trigger B is pulled", and let $c$ be the proposition "the capacitors are charged". Write a proposition using these symbols which is true exactly when the weapon should fire.

   (b) Aperture want to implement the firing mechanism for their gun using only OR and NOT gates. Use de Morgan's laws to write a proposition that is logically equivalent to your answer to (a) using only the symbols $a$, $b$, $c$, $\vee$, $\neg$.

4. The connective $\uparrow$ (sometimes called NAND) is defined by $p \uparrow q \equiv \neg(p \wedge q)$.

   (a) Show that, for any proposition that can be written using variables the connectives $\wedge$, $\vee$ and $\neg$, an equivalent proposition can be written using only variables and $\uparrow$.

   (b) Using only NAND gates you need to build a circuit equivalent to a circuit that uses 100 gates, each an AND, an OR or a NOT. What's the maximum number of NAND gates you might need?

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