Formulae:

Scalar and vector projections:
The **scalar projection**, v_w, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$v_w = \frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|}$$

The **vector projection**, $\mathbf{v}_w$, of $\mathbf{v}$ in the direction of $\mathbf{w}$ is given by

$$\mathbf{v}_w = \left(\frac{\mathbf{v} \cdot \mathbf{w}}{|\mathbf{w}|^2} \right) \mathbf{w}$$

Vector cross product:

The vector cross product of vectors $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$ is

$$\mathbf{v} \times \mathbf{w} = (v_y w_z - v_z w_y, v_z w_x - v_x w_z, v_x w_y - v_y w_x)$$

Vector equation of a plane:

$$\mathbf{n} \cdot (\mathbf{r} - \mathbf{d}) = 0$$

Matrix inverse (2×2):

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

for $ad - bc \neq 0$.

Schematic of Gauss-Jordan algorithm:

$$[A|I] \underset{G.A}{\longrightarrow} [U|*] \underset{J.A}{\longrightarrow} [I|B] \quad \text{where } B = A^{-1}.$$

Derivative definition:
The **derivative** of $f(x)$ at the point x is defined as

$$\frac{\mathrm{d}f}{\mathrm{d}x} = f'(x) = \lim_{\Delta x \to 0} \left(\frac{\Delta f}{\Delta x} \right) = \lim_{\Delta x \to 0} \left(\frac{f(x + \Delta x) - f(x)}{\Delta x} \right).$$

Some rules for finding derivatives:

Description	**Function**	**Derivative**
Sum (or difference) of functions	$f(x) \pm g(x)$	$f'(x) \pm g'(x)$
Product of functions	$f(x)g(x)$	$f(x)g'(x) + g(x)f'(x)$
Quotient of functions	$\dfrac{f(x)}{g(x)}$	$\dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

Chain rule for composite functions

If $u = g(x)$ and $y = f(u)$ so that $y = f(g(x))$ then

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u}\frac{\mathrm{d}u}{\mathrm{d}x} = f'(u)g'(x)$$

Derivative rule for inverse functions

$$\text{If } y = f^{-1}(x) \Leftrightarrow x = f(y), \text{ then } \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\mathrm{d}x/\mathrm{d}y} = \frac{1}{f'(y)}$$

Parametric differentiation:

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}t}\Big/\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{g'(t)}{f'(t)} \quad \text{where } f'(t) = \frac{\mathrm{d}f}{\mathrm{d}t} \text{ and } g'(t) = \frac{\mathrm{d}g}{\mathrm{d}t}.$$

Taylor series at $x = 0$:

$$T_n(x) = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x^2 + \frac{f^{(3)}(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n.$$

Integration by substitution:

$$I = \int f(x)dx = \int f(x(u))\frac{dx}{du}du$$

Integration by parts:

$$\int f\frac{dg}{dx}dx = fg - \int g\frac{df}{dx}dx$$

Fundamental Theorem of Calculus:

If $f(x)$ is a continuous function on the interval $[a, b]$ and there is a function $F(x)$ such that $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Area between two curves. Given two continuous functions $f(x)$ and $g(x)$ where $f(x) \geq g(x)$ for all x in the interval $[a, b]$, the area of the region bounded by the curves $y = f(x)$ and $y = g(x)$, and the lines $x = a$ and $x = b$ is given by the definite integral

$$\int_a^b \big[f(x) - g(x)\big]dx$$

Trapezoidal rule:

$$\int_a^b f(x)dx \approx \frac{b-a}{2n}\left(f(a) + f(b) + 2\sum_{i=1}^{n-1} f(x_i)\right)$$

Tangent plane to surface:

$$z = f(a, b) + f_x(a, b) \cdot (x - a) + f_y(a, b) \cdot (y - b)$$

Multivariate chain-rule:

$$\frac{df}{ds} = \frac{\partial f}{\partial x}\frac{dx}{ds} + \frac{\partial f}{\partial y}\frac{dy}{ds}$$

Directional derivative:

The *directional derivative* df/ds of a function f in the direction $\underset{\sim}{t}$ is given by

$$\frac{df}{ds} = \underset{\sim}{t} \cdot \nabla f = \nabla_{\underset{\sim}{t}} f$$

where the *gradient* ∇f is defined by

$$\nabla f = \frac{\partial f}{\partial x}\underset{\sim}{i} + \frac{\partial f}{\partial y}\underset{\sim}{j}$$

and $\underset{\sim}{t}$ is a unit vector, $\underset{\sim}{t} \cdot \underset{\sim}{t} = 1$.

Quadratic approximation to surface:

$$\begin{aligned} T_2(x, y) = f(a, b) &+ f_x(a, b) \cdot (x - a) + f_y(a, b) \cdot (y - b) \\ &+ \frac{1}{2!}\big[f_{xx}(a, b)(x - a)^2 + 2f_{xy}(a, b)(x - a)(y - b) + f_{yy}(a, b)(y - b)^2\big] \end{aligned}$$

Table of the derivatives of the basic functions of calculus	
Original function f	**Derivative function f'**
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x \equiv 1 + \tan^2 x$
$\operatorname{cosec} x \equiv 1/\sin x$	$-\operatorname{cosec} x \cdot \cot x$
$\sec x \equiv 1/\cos x$	$\sec x \cdot \tan x$
$\cot x \equiv 1/\tan x$	$-\operatorname{cosec}^2 x$
$\sin^{-1} x \quad$ domain: $-1 \leq x \leq 1$ (ie $\lvert x\rvert \leq 1$)	$\dfrac{1}{\sqrt{1-x^2}}$
$\cos^{-1} x \quad$ domain: $-1 \leq x \leq 1$ (ie $\lvert x\rvert \leq 1$)	$-\dfrac{1}{\sqrt{1-x^2}}$
$\tan^{-1} x \quad$ domain: $-\infty < x < \infty$	$\dfrac{1}{1+x^2}$
e^x	e^x
$\ln x \quad$ domain: $x > 0$	$\dfrac{1}{x}$

Table of Useful Power Series

Series	**Domain**
$\dfrac{1}{1-x} = 1 + x + x^2 + x^3 + \ldots + x^n + \ldots$	$-1 < x < 1$
$\dfrac{1}{1+x} = 1 - x + x^2 - x^3 + \ldots + (-1)^n x^n + \ldots$	$-1 < x < 1$
$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \ldots + \frac{1}{n!}x^n + \ldots$	$-\infty < x < \infty$
$\ln(1+x) \equiv \log_e(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$ $+(-1)^n \dfrac{x^{n+1}}{n+1} + \ldots$	$-1 < x \leq 1$
$\sin x = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \ldots + (-1)^n \dfrac{x^{2n+1}}{(2n+1)!} + \ldots$	$-\infty < x < \infty$
$\cos x = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \ldots + (-1)^n \dfrac{x^{2n}}{(2n)!} + \ldots$	$-\infty < x < \infty$