Monash University

Semester One - 2000

Examinations

Faculty of Science

EXAM CODES: **MAT1841**

TITLE OF PAPER: **Mathematics For Computer Science 1**

EXAM DURATION: **180** minutes writing time

READING TIME **10** minutes

THIS PAPER IS FOR STUDENTS STUDYING AT: (tick where applicable)

Berwick	☐	Clayton	√	Peninsula	☐	Distance Education	☐	Open Learning	☐
Caulfield	☐	Gippsland	☐	Malaysia	√	Enhancement Studies	☐	Other (specify)	☐

Instructions to Candidates

Candidates are reminded that they should have no material on their desks unless their use has been specifically permitted by the following instructions.

Students should ONLY enter their ID number and desk number on the examination script book, NOT their name. Please take care to ensure that the ID number and desk number are correct and are written legibly.

1. Two pages of tables are attached

AUTHORISED MATERIALS

CALCULATORS	√ **YES**	**NO**
OPEN BOOK	**YES**	√ **NO**
SPECIFICALLY PERMITTED ITEMS	√ **YES**	**NO**

if yes, items permitted are:

One A4 sheet (2 sides) of handwritten material

EXAMINATION QUESTIONS BEGIN OVER THE PAGE\...

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1. (a) Use row operations to show that the reduced echelon form for the matrix

$$\begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 500 \\ 1 & 0 & 0 & 1 & 0 & 1 & 400 \\ 0 & 0 & 1 & 0 & 1 & -1 & 100 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 \end{pmatrix}$$

is the matrix

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 400 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & -1 & 100 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Clearly state the row operation you use at each step, no marks will be given for answers without working shown.

(b) Use the result in part (a) to solve the following system of linear equations and state which variables, if any, are parameters in your solution.

$$\begin{aligned} u+v+w+0x+0y+0z &= 500 \\ u+0v+0w+x+0y+z &= 400 \\ 0u+0v+w+0x+y-z &= 100 \\ 0u+v+0w-x-y+0z &= 0 \end{aligned}$$

[7+3=10 marks]

2. (a) Let $X=\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$, $Y=\begin{pmatrix} 1 & -1 \end{pmatrix}$, $Z=\begin{pmatrix} -2 \end{pmatrix}$, and $O=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. The matrix A in block form is $A=\begin{pmatrix} X & O \\ Y & Z \end{pmatrix}$. Determine the matrix A^2 by using the method of block multiplication, show all calculation steps.

QUESTION 2 CONTINUED OVER THE PAGE\...

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QUESTION 2 CONTINUED\...

(b) Let $C = \begin{pmatrix} 4 & 1 & 2 & 1 \\ 3 & 0 & 1 & 6 \end{pmatrix}$, use row operations to reduce C to reduced row echelon form, B.

Obtain an expression for the matrix D such that $B = DC$. (You do not have to evaluate D, you can express D as a product of elementary matrices.)

[6+4=10 marks]

3. (a) Use row operations to invert the matrix $A = \begin{pmatrix} -3 & -3 & 11 \\ 1 & 1 & -3 \\ 1 & -1 & -1 \end{pmatrix}$.

Show all working

(b) Use your answer to part (a) to solve for X the system.

$$AX = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

[8+2=10 marks]

4. (a) The following table shows the concentration $C(t)$ of a pollutant at a certain point in a river t hours after the pollutant was released upstream.

$C(t)$	4.6	9.5	7.5	5.4	3.4
t	1	2	2.5	3.0	4.0

(i) Estimate the value of $\frac{dc}{dt}$ at $t = 3.0$ using the average of the two estimates obtained by forward differencing and backward differencing.

(ii) Write down (an approximation to) the equation to the tangent line at $t = 3.0$.

QUESTION 4 CONTINUED OVER THE PAGE\...

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QUESTION 4 CONTINUED\...

(b) Differentiate the following functions showing all working.

(i) $\frac{e^{-x}}{1+x^2}$ (ii) $\sin\left(1+\sqrt{x}\right)$ (iii) $\sqrt{1-x}\,\ln\left(x+x^3\right)$.

[4+6=10 marks]

5. The diagram below shows the cross section of a proposed tunnel consisting of a rectangle of height h surmounted by a semi-circle of radius r. If the cross sectional area is fixed at A determine the dimensions that will minimize the perimeter.

[10 marks]

6. (a) If a car travels in a straight line from O with velocity $v(t)$ then its displacement from O at time t_0 is given by

$$s(t_0) = \int_0^{t_0} v(t)dt\,.$$

The table below gives data on the velocity.

t	0	1	2	3	4
v	60	70	75	70	50

Estimate the value of $s(4)$ using a right hand Riemann sum.

QUESTION 6 CONTINUED OVER THE PAGE\...

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QUESTION 6 CONTINUED \...

(b) Determine the following integrals

(i) $\int x\cos\left(x^2\right)dx$

(ii) $\int xe^{2x}dx$

[5+5 = 10 marks]

7. (a) The table below gives some data on the yield of a certain crop.

Yield	3.22	3.04	3.06	2.64	3.19	2.49	3.31	2.99	3.17	3.40	2.37	3.26	3.27
Yield	2.93	2.59	3.11	2.38	3.25	3.20	3.09	2.62	3.23	2.37			

Construct a stem and leaf plot with a depth column for this data set; use a leaf unit of 0.01.

(b) Use your plot in (a) to determine the median of the data set; show all working.

[7+3=10 marks]

8. (a) The diagram below shows the probability density function for the depth below ground $\left(X \text{ meters}\right)$ at which a certain mineral is found.

(i) Determine the probability of finding the mineral at a depth between 20 meters and 60 meters.

(ii) Find the median value of X.

QUESTION 8 CONTINUED OVER THE PAGE \...

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QUESTION 8 CONTINUED \....

(b) Bags of cement have weights distributed with mean $50kg$. and standard deviation $5kg$. A truck is permitted to carry a maximum weight of $1000kg$ and the truck is loaded with 21 bags. Use the central limit theorem to find the probability that the total weight of the bags exceeds $1000kg$.

(c) If 20% of a population have a certain type of genetic defect find the probability that fewer than 4 people will be found to have the defect in a randomly drawn sample of size 30.

[4 +3+3 = 10 marks]

9. (a) A politician wishes to estimate the proportion of the population who support gun law reform. A random sample of size 80 revealed that 35 were in favour of gun law reform. Determine a 95% confidence interval for the proportion of the population in favour of gun law reform.

(b) The results of 10 measurements on pollution levels of a car exhaust system are given below. Find a 90% confidence interval for the mean. Assume that the measurements are normally distributed.

22.1 23.5 27.4 29.0 28.5 30.5 28.4 30.6 29.4 30.0

[4+6=10 marks]

10. (a) A group of researchers into educational standards claim that standards have dropped. Previous records show that the mean mark on a standard achievement test was 65 and the standard deviation of marks was 15. This year the test was given to a group of 100 students and the sample average was 62. Carry out a test at the significance level of 5% to see if the claim is substantiated by the results.

(b) Another research claims that students in a particular part of the country are better than average. A sample of 10 from the region gave the results below.

72 67 82 64 65 76 88 64 77 80

Test the claim using a t-test at the level of significance 1%.

[4+6 = 10 marks]

END OF EXAMINATION QUESTIONS

TABLES FOLLOW OVER THE PAGE\...

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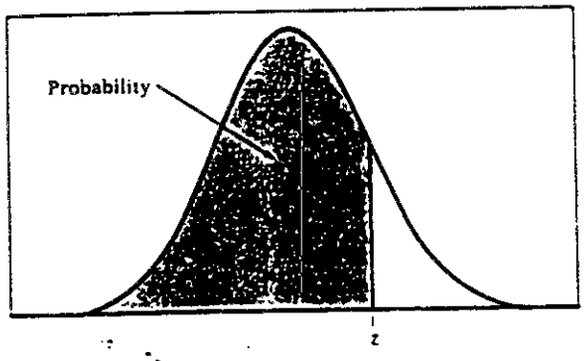

Table entry is probability at or below z.

Table A *(Continued)*

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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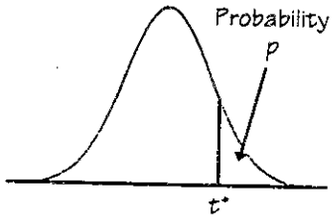

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

TABLE C t **distribution critical values**

	Upper tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

END OF EXAMINATION PAPER