MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #7 and Additional Practice Questions

Tutorial Questions

1. Calculate the following by hand.
   (a) $4!$
   (b) $\frac{10!}{8!}$
   (c) $\binom{10}{8}$
   (d) $\binom{7}{3}$

2. (a) How many ways can a president, treasurer and secretary be chosen from a group of 10 people.
   (b) How many ways can a team of three people be chosen from a group of 10 people.
   (c) What's the essential difference between (a) and (b)? Which answer is larger? Could you have known this without doing any calculation?
   (d) How many ways can three scoops of ice-cream be selected from 10 flavours? (Multiple scoops of the same flavour are allowed.)
   (e) How many ways can five different prizes be divided among Anastasia, Becky and Cadel? (Not everyone has to get a prize.)
   (f) In how many different orders can six horses finish a race? (Assume there are no ties and they all do finish.)

3. Yet another death star is firing missiles at a squadron of 16 rebel A-wings. Each missile will lock on to one of the A-wings but it cannot control which. What is the smallest number of missiles it must fire to ensure that at least one A-wing has 6 missiles locked on to it?

4. What is the coefficient of x^9 in the expansions of the following? (Leave your answer as a mathematical expression rather than a number.)
   (a) $(x+2)^{20}$
   (b) $(3x+2)^{20}$
   (c) $(3x^3+2)^{20}$

5. Let $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.
   (a) How many ways are there to choose subsets A and B of S such that $|A| = 4$ and $|B| = 3$ and $A \cap B = \emptyset$?
   (b) How many ways are there to choose subsets C and D of S such that $|C| = 3$ and $|D| = 4$ and $C \cap D = \emptyset$?
   (c) How many ways are there to choose subsets E and F of S such that $|E| = 7$ and $|F| = 3$ and $F \subseteq E$?
   (d) Calculate your answers to each of the above parts as a number. What do you notice? What's going on?

(See over for practice questions.)

Practice Questions

1. A family of 7 are out to lunch and each of them orders one dish from a menu of 12 dishes.
   (a) From the waiter's perspective, how many orders are possible?
   (b) From the chef's perspective, how many orders are possible?
   (c) What is the effect of changing the perspective in the above?

2. (a) How many PINs consisting of a string of five decimal digits are there?
   (b) How many passwords of length 5 using only decimal digits and upper and lower case letters (0–9, a–z and A–Z) are there?
   (c) What length of PIN is required to be more secure than the password in (b)?

3. A tyrannical tutor demands that a support class of 16 lovely students split themselves into four groups of 4 to work at whiteboards. In how many ways can this be accomplished?

4. A restaurant offers 4 different vegetarian dishes and 6 different meat dishes.
   (a) How many ways are there for to the restaurant to serve a party of 20 different people one dish each so that 8 vegetarian and 12 meat dishes are served?
   (b) How many ways are there for to the restaurant to serve a party of 20 different people one dish each so that 17 vegetarian and 3 meat dishes are served?
   (c) What do your answers above have to do with the expansion of $(4v + 6m)^{20}$ and the binomial theorem?