MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #10 Solutions

1. (a) Note that $a_0 = 2^0 = 1$.
   Also, $a_n = 2^n = 2(2^{n-1}) = 2a_{n-1}$ for $n \geq 1$.
   So "$a_0 = 1$ and $a_n = 2a_{n-1}$ for all integers $n \geq 1$" is a recursive definition for the sequence.

   (b) Note that $b_0 = 0^2 = 0$.
   Also, $b_n = n^2 = (n-1)^2 + 2n - 1 = b_{n-1} + 2n - 1$ for $n \geq 1$.
   So "$b_0 = 0$ and $b_n = b_{n-1} + 2n - 1$ for all integers $n \geq 1$" is a recursive definition the sequence.

2. (a) Vertices: P, Q, R, S
   Edges: PQ, PR, PS

   (b) Vertices: P, Q, R, S, T
   Edges: PQ, PR, PS, QR, QS, RS

   (c) Vertices: P, Q, R, S, T
   Edges: PS, QS, QT, RS, ST

3. (a) $\frac{1}{13} + \frac{1}{15} + \frac{1}{17} + \frac{1}{19} + \frac{1}{21}$

   (b) $(\frac{x^4}{8} + 4)(\frac{x^5}{10} + 5)(\frac{x^6}{12} + 6)$

4. There is only one way of writing 1 in this form: "1".

   There are two ways of writing 2: "$1 + 1$" and "2".

   So $s_1 = 1$ and $s_2 = 2$.

   If the first term in such a sum is a 1, then there are s_{n-1} ways to finish it so that it adds to n.
   If the first term in such a sum is a 2, then there are s_{n-2} ways to finish it so that it adds to n.
   These two cases account for every possible sum.

   So $s_1 = 1$, $s_2 = 2$ and $s_n = s_{n-1} + s_{n-2}$ for all integers $n \geq 3$.

5. One line creates two regions, so $r_1 = 2$.
   The nth line added intersects each of the $n-1$ previous lines exactly once because it is not parallel to any of them. The $n-1$ intersection points created are all different because no three lines meet at a point. These $n-1$ intersection points divide the nth line into n segments and each of these segments splits a region in two. So the new number of regions is n more than the previous number.

   Thus $r_1 = 2$ and $r_n = r_{n-1} + n$ for all integers $n \geq 2$.