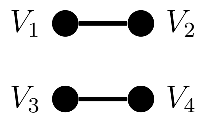
MAT1830 - Discrete Mathematics for Computer Science
Tutorial Sheet #11 Solutions

1. (a)

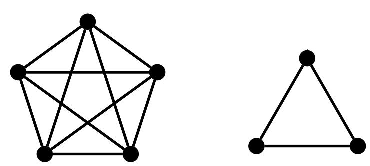


   (b) 1. The walk is V_1, V_2, V_1, V_2.

   (c) For any $n \geq 2$:
   - The 1st entry in the top row of M^n is the number of walks of length n from V_1 to V_1, which is 1 for any even $n \geq 2$ (the walk is $V_1, V_2, V_1, V_2, \ldots, V_1$).
   - The 2nd entry in the top row of M^n is the number of walks of length n from V_1 to V_2, which is 0 for any even $n \geq 2$.
   - The 3rd entry in the top row of M^n is the number of walks of length n from V_1 to V_3, which is 0 for any even $n \geq 2$.
   - The 4th entry in the top row of M^n is the number of walks of length n from V_1 to V_3, which is 0 for any even $n \geq 2$.
   So the top row of M^n for any even $n \geq 2$ is "1 0 0 0".

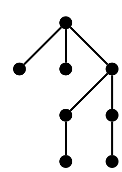
2. (a) No such graph exists. In a simple graph on 7 vertices, each vertex can have degree at most 6 (because it can be joined at most once to each other vertex), but the listed degrees include an 8.

   (b) Such a graph exists. Below is an example of such a graph.



   (c) No such graph exists. The sum of the degrees listed is 12, so such a graph must have $\frac{12}{2} = 6$ edges, but we know any tree on 6 vertices has 5 edges.

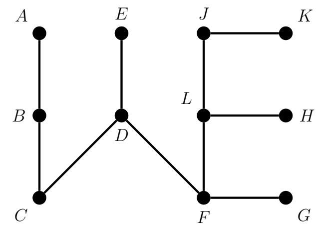
   (d) Such a graph exists. Below is an example of such a graph.



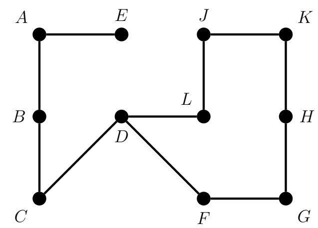
   (e) No such graph exists. The sum of the degrees listed is 17, but we know the sum of the degrees in any graph is even.

3. (a) No such spanning tree exists because the graph we obtain from X by removing the edges JL and HK is disconnected.

(b) Here is one example.



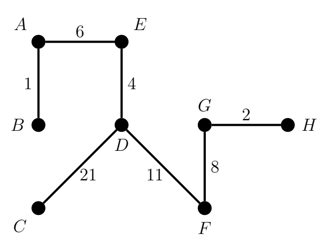
(c) The graph obtained from X by removing the edges DE, FH, FL, and HL is as follows.



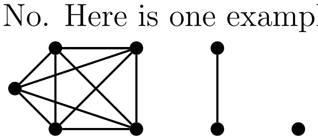
Removing any one of the edges DL, DF, FG, GH, HK, JK, JL from this graph will produce a spanning tree of X with the required properties. Removing any other edge will disconnect the graph. So there are 7 different spanning trees of X are there that contain no edge in $\{DE, FH, FL, HL\}$

4. (a) A,B,C,D,E,A
A,B,C,D,F,G,H,K,J,L,D,E,A (adding the cycle D,F,G,H,K,J,L,D)
A,B,C,D,F,H,L,F,G,H,K,J,L,D,E,A (adding the cycle F,H,L,F)

(b) AB (adding AB)
AB, GH (adding GH)
AB, GH, DE (adding DE)
AB, GH, DE, AE (adding AE)
AB, GH, DE, AE, FG (rejecting BD and adding FG)
AB, GH, DE, AE, FG, DF (adding DF)
$AB, GH, DE, AE, FG, DF, CD$ (rejecting DG, AD and adding CD)
(reject FH, BC)
So our minimum weight spanning tree looks like:



5. (a) No. Here is one example:



(b) 7. A spanning tree has all the vertices of the original graph and any tree with 8 vertices has 7 edges