Monash University

Semester One Examinations 1997

Faculty Of Science

EXAM CODES: **MAT 1841**

TITLE OF PAPER: **Mathematics for Computer Science I**

INSTRUCTIONS TO CANDIDATES

1. Candidates are reminded that they should have no books, notes, paper, calculators, pencil cases or other material in their possession unless their use has been specifically permitted by the following instructions. Materials on or under your desk or chair or on your person are deemed to be in your possession.
2. **EXAM DURATION:** 180 minutes writing time
3. Reading time: 10 minutes
4. Calculators **are** permitted.
5. Each student may take into the examination one A4 sheet (two sides) of original handwritten material.

Questions start on following page (1)

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QUESTIONS

1. (a) Solve the following system by reducing the augmented matrix to row echelon form. Specify clearly the sequence of row operations you employ.

$$\begin{aligned} x_1 + x_2 + 2x_3 - x_4 &= 4 \\ 3x_2 - x_3 + 4x_4 &= 2 \\ x_1 + 2x_2 - 3x_3 + 5x_4 &= 0 \\ x_1 + x_2 - 5x_3 + 6x_4 &= -3 \end{aligned}$$

(b) Let $\mathbf{X} = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$, $\mathbf{Y} = [1 \quad -1]$, $\mathbf{Z} = [-2]$ and $\mathbf{0} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, and let the matrix $\mathbf{A}$ in block form be $\begin{bmatrix} \mathbf{X} & \mathbf{0} \\ \mathbf{Y} & \mathbf{Z} \end{bmatrix}$. Use block multiplication to show that

$$\mathbf{A}^2 = \begin{bmatrix} \mathbf{X}^2 & \mathbf{0} \\ \mathbf{0}^\mathbf{T} & \mathbf{Z}^2 \end{bmatrix}.$$

2. (a) Use matrix inversion by *row operations* to show that

$$\begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 3 & 0 \end{bmatrix}^{-1} = \frac{1}{2}\begin{bmatrix} -3 & -3 & 11 \\ 1 & 1 & -3 \\ 1 & -1 & -1 \end{bmatrix}$$

(b) (i) Define what is meant by an *elementary* matrix.

(ii) What elementary row operation corresponds to the elementary matrix $\mathbf{E}$ below?

$$\mathbf{E} = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(iii) State the inverse row operation to the row operation in part (ii) and hence *write down* the inverse of the matrix $\mathbf{E}$.

3. (a) Show that

$$\det \begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix} = 1 + a^2 + b^2 + c^2$$

(b) Explain why the result in part (a) implies that the system

$$\begin{bmatrix} 1 & a & b \\ -a & 1 & c \\ -b & -c & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} k_1 \\ k_2 \\ k_3 \end{bmatrix}$$

will have a unique solution, for any real number entries a, b, c in the matrix of coefficients.

Question continued on p.2

(c) (i) Let **A** be a square matrix and suppose that **A** is carried to **B** by a single elementary row operation. Describe how the value of det **B** is related to det **A** for each of the three types of elementary row operation.

(ii) Reduce the matrix **A** below to an upper triangular form and hence deduce the value of det **A**.

$$\mathbf{A} = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 5 & 1 & 1 \\ 1 & 1 & 2 & 5 \end{bmatrix}$$

4. (a) The following table gives some values of a function f at specified values of the independent variable

t	1.5	2.0	2.5	3.0
$f(t)$	10.3	8.2	6.5	5.5

(i) Estimate the value of $f'(1.5)$.

(ii) Determine approximately the equation to the tangent line at $t = 1.5$ and use this equation to estimate the value of $f(1.7)$.

(b) Determine all the critical points of

$$f(x) = \frac{x}{x^2+1}$$

and sketch the graph of this function locating any local maxima or minima.

(c) A smokestack deposits soot on the ground with a concentration that is inversely proportional to the distance from the smokestack. Two such smokestacks are situated 20 kilometres apart. The concentration C of soot along the straight line joining the two smokestacks is given by

$$C(x) = \frac{k}{x} + \frac{6k}{20-x}$$

where x is the distance in kilometres to the first stack, and k is a constant. Determine the point on the line joining the stacks when the concentration of soot deposit is a minimum.

5. (a) Evaluate the following limits, giving all reasoning.

(i) $\lim\limits_{t\to 0} \dfrac{\sin 2t}{t}$ (ii) $\lim\limits_{h\to 0} \dfrac{(3+h)^2 - 3^2}{h}$.

(b) Graph the function f defined by

$$f(x) = \begin{cases} \frac{|x|}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$$

Is f continuous at $x = 0$? Give reasons.

Question continued on p.3

(c) Sketch the graph of $g(t)$ where

$$g(t) = \begin{cases} 1 + \cos \frac{\pi t}{2}, & -1 \le t \le 1 \\ 1, & t < -1 \text{ or } t > 1 \end{cases}$$

(i) Is g continuous at $t = 1$? Give reasons.

(ii) Sketch the graph of $g'(t)$ using the graph of $g(t)$. Is g differentiable at $t = 1$? Give reasons.

6. (a) Compute right-hand sums and left-hand sums for the integral

$$\int_0^{2.5} \sin(t^2)dt$$

for the case $n = 10$.

(b) Use substitutions to find

$$\text{(i)} \int x^2 \sin(x^3 + 1)dx \qquad \text{(ii)} \int \frac{e^x}{1 + e^x}dx$$

(c) Evaluate the definite integral

$$\int_0^1 te^{-t}dt$$

7. (a) The number of days to the first service call of eight new photocopiers were 343, 126, 266, 623, 421, 185, 384, 437. Find the mean, median, standard deviation, minimum, maximum, first quartile and third quartile.

(b) A fair coin is tossed three times: find the sample space. Let A be the event that the first toss and the last toss result in heads; let B be the event that the first two tosses result in tails and let C be the event that there are exactly two heads. Find the probabilities $P(A), P(C)$, $P(A \cap C), P(A \cup C)$, $P(B \cap C)$ and $P(A \cap B \cap C)$.

8. An automobile club provides emergency road service to its members. It wishes to estimate the proportion of calls related to starting problems. Upon examining a random sample of 50 calls, 31 calls were found to be related to starting problems. Find a point estimate of the true proportion of service calls that involved starting problems. Find the standard error of such an estimate. Calculate the 95% error margin and the 95% confidence interval for the true proportion. Also, calculate the 90% error margin and the 90% confidence interval for the true proportion and compare its length with that of the 95% confidence interval.

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9. The cure rate for a disease using a standard medication is known from experience to be 35%. A pharmaceutical company claims that the cure rate of a new medication is 50%. Suppose that the new drug is to be tried on a sample of 20 matched patients and that the number cured X in the 20 is to be recorded. State the null hypothesis H_0 and the alternative hypothesis H_a. If the rejection region is specified as $R : X \geq 12$, find the Type I and the Type II error probabilities α and β. Find the power of the test. If X turns out to be 13, find the P-value of such an observed value. Would you reject the null hypothesis at the 1% level of significance if $X = 13$?

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Table C (Continued)

n	k	p = .01	.02	.03	.04	.05	.06	.07	.08	.09
20	0	.8179	.6676	.5438	.4420	.3585	.2901	.2342	.1887	.1516
	1	.1652	.2725	.3364	.3683	.3774	.3703	.3526	.3282	.3000
	2	.0159	.0528	.0988	.1458	.1887	.2246	.2521	.2711	.2818
	3	.0010	.0065	.0183	.0364	.0596	.0860	.1139	.1414	.1672
	4		.0006	.0024	.0065	.0133	.0233	.0364	.0523	.0703
	5			.0002	.0009	.0022	.0048	.0088	.0145	.0222
	6				.0001	.0003	.0008	.0017	.0032	.0055
	7						.0001	.0002	.0005	.0011
	8								.0001	.0002
	9									
	10									
	11									
	12									
	13									
	14									
	15									
	16									
	17									
	18									
	19									
	20									

n	k	p = .10	.15	.20	.25	.30	.35	.40	.45	.50
20	0	.1216	.0388	.0115	.0032	.0008	.0002	.0000	.0000	.0000
	1	.2702	.1368	.0576	.0211	.0068	.0020	.0005	.0001	.0000
	2	.2852	.2293	.1369	.0669	.0278	.0100	.0031	.0008	.0002
	3	.1901	.2428	.2054	.1339	.0716	.0323	.0123	.0040	.0011
	4	.0898	.1821	.2182	.1897	.1304	.0738	.0350	.0139	.0046
	5	.0319	.1028	.1746	.2023	.1789	.1272	.0746	.0365	.0148
	6	.0089	.0454	.1091	.1686	.1916	.1712	.1244	.0746	.0370
	7	.0020	.0160	.0545	.1124	.1643	.1844	.1659	.1221	.0739
	8	.0004	.0046	.0222	.0609	.1144	.1614	.1797	.1623	.1201
	9	.0001	.0011	.0074	.0271	.0654	.1158	.1597	.1771	.1602
	10		.0002	.0020	.0099	.0308	.0686	.1171	.1593	.1762
	11			.0005	.0030	.0120	.0336	.0710	.1185	.1602
	12			.0001	.0008	.0039	.0136	.0355	.0727	.1201
	13				.0002	.0010	.0045	.0146	.0366	.0739
	14					.0002	.0012	.0049	.0150	.0370
	15						.0003	.0013	.0049	.0148
	16							.0003	.0013	.0046
	17								.0002	.0011
	18									.0002
	19									
	20									

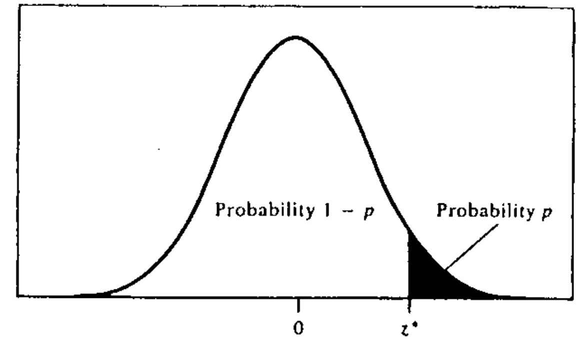

Table entry for p and C is the point z^* with probability p lying above it and probability C lying between $-z^*$ and z^*

Table D Standard normal critical values

C	p	z^*	C	p	z^*
50%	.25	.674	96%	.02	2.054
60%	.20	.841	98%	.01	2.326
70%	.15	1.036	99%	.005	2.576
80%	.10	1.282	99.5%	.0025	2.807
90%	.05	1.645	99.8%	.001	3.091
95%	.025	1.960	99.9%	.0005	3.291

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