Monash University

Semester One Examinations 1999

Faculty Of Science

EXAM CODES: MAT1841

TITLE OF PAPER: MATHEMATICS FOR COMPUTER SCIENCE

EXAM DURATION: 180 minutes writing time

READING TIME 10 minutes

THIS PAPER IS FOR STUDENTS STUDYING AT:(office use only - tick where applicable)

Berwick	☐	Clayton	☑	Peninsula	☐	Distance Education	☐	Open Learning	☐
Caulfield	☐	Gippsland	☐	Sunway	☑	Enhancement Studies	☐	Other (specify)	☐

Candidates are reminded that they should have no material on their desks unless their use has been specifically permitted by the following instructions.

Two pages of tables are attached.

AUTHORISED MATERIALS

CALCULATORS	**YES** √	**NO**
OPEN BOOK	**YES**	**NO** √
SPECIFICALLY PERMITTED ITEMS	**YES** √	**NO**

if yes, items permitted are:

Candidates may consult one two-sided A4 sheet of original handwritten material.

EXAMINATION QUESTIONS BEGIN OVER THE PAGE \...

1. (a) Show that the reduced row echelon form of the matrix

$$\begin{bmatrix} 1 & 1 & 2 & -1 & 4 \\ 0 & 3 & -1 & 4 & 2 \\ 1 & 2 & -3 & 5 & 0 \\ 1 & 1 & -5 & 6 & -3 \end{bmatrix}$$

is the matrix

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Specify clearly all row operations you use.

(b) Use the result in part (a) to determine the general solution to the system.

$$\begin{aligned} x + y + 2z - w &= 4 \\ 3y - z + 4w &= 2 \\ x + 2y - 3z + 5w &= 0 \\ x + y - 5z + 6w &= -3 \end{aligned}$$

State clearly which variables, if any, are parameters.

[8 + 6 = 14 marks]

2. (a) Use matrix inversion by row operations to show that

$$\begin{bmatrix} -\frac{3}{2} & -\frac{3}{2} & \frac{11}{2} \\ \frac{1}{2} & \frac{1}{2} & -\frac{3}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \end{bmatrix}^{-1} = \begin{bmatrix} 2 & 7 & 1 \\ 1 & 4 & -1 \\ 1 & 3 & 0 \end{bmatrix}$$

QUESTION 2 CONTINUED OVER THE PAGE \...

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(b) (i) Find the reduced row echelon form B of the matrix

$$A = \begin{bmatrix} 4 & 1 & 2 \\ 3 & 0 & 1 \end{bmatrix}$$

(ii) Determine the matrix U such that $B = UA$

[8 + 6 = 14 marks]

3. Let $A = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 5 & 0 & 0 \\ 0 & 1 & 2 & 3 \end{bmatrix}$

(a) Determine the cofactor C_{11}, and hence find $\det A$.

(b) Given that type 3 row operations (adding a multiple of one row to another) do not alter the value of a determinant, find $\det A$ by reducing A to an upper triangular matrix using type 3 operations.

[6 + 6 = 12 marks]

4. (a) The table below gives values of $c(t)$ the concentration (in milligrams per cubic centimetre) of a drug in the bloodstream at time t (minutes).

t	0	0.1	0.2	0.3
$c(t)$	1.84	1.89	1.94	1.98

(i) Estimate the value of the derivative of c at $t = 0$.

(ii) Obtain an approximation to the equation of the tangent line at $t = 0$ to the graph of the function c.

(iii) Use the result from (ii) to estimate the concentration of the drug at time $t = 0.05$.

QUESTION 4 CONTINUED OVER THE PAGE \...

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(b) Determine the derivatives of the following functions.
Show all working.

(i) $\sqrt{1+x^3}$

(ii) $\sin\left(x^2+1\right)$

(iii) $\dfrac{e^x}{1+e^x}$

[8 + 9 = 17 marks]

5. A soft drink aluminium can is to have a capacity of 375ml and has a cylindrical shape, capped at each end. Determine the dimensions of the can if the amount of aluminium used is to be a minimum.

[12 marks]

6. (a) What **derivative** corresponds to the limit

$$\lim_{h\to 0}\frac{2^h-1}{h} \quad ?$$

i.e. Identify the associated function $f(x)$ and the point at which the derivative is computed.

(b) Use the **limit definition** of the derivative to find $f'(x)$ if $f(x)=1+\dfrac{1}{x}$.

[3 + 9 = 12 marks]

7. (a) If a body moves from rest with an acceleration $a(t)$ then the velocity of the body at time t_0 is given by

$$V(t_0) = \int_0^{t_0} a(t)dt.$$

The table below gives some data on acceleration

t	0	1	2	3	4	5
$a(t)$	10.00	8.0	6.5	4.2	4.0	3.9

Estimate $V(5)$ using the left hand Riemann sum.

(b) Determine the following integrals

(i) $\int xe^{-x^2}\,dx$

(ii) $\int \frac{4x}{1+x^2}\,dx$

(iii) $\int x\cos 2x\,dx$

[8 + 12 = 20 marks]

8. (a) The salaries (in units of \$1,000) of 27 employees in a company are, from highest to lowest,

6200	5917	4000	3375	3000	2312	2300	2150	2100
1500	1012	850	650	635	500	475	220	205
195	195	158	145	109	109	109	109	109

(i) Construct a stem and leaf plot for the data.

(ii) Calculate the median and the interquartile range.

(iii) Which measure of central tendency do you consider best describes this data set? Give reasons. You **do not** have to calculate the mean .

(iv) Draw a box plot for the data.

(b) If test scores are normally distributed with mean 65 and standard deviation 15 determine the test score C such that only 10% of candidates will achieve a score greater than C.

(c) If 65% of a population are in favour of tax reform find the probability that a random sample of 100 people will have fewer than 55 who support tax reform.

[10 + 4 + 4 = 18 marks]

9. (a) In a survey of 200 first year students 132 students reported that they attended all classes. Find a 99% confidence interval for the proportion of first year students that attend all classes.

(b) A test for the presence of antibodies to a certain virus has a probability 0.80 of detecting the antibodies when they are present. Suppose that 20 blood samples with the antibodies present are tested.

(i) If X is the number of these 20 samples that the test detects, what is the probability distribution of X?

(ii) Find $P(X = 20)$.

(iii) Find the probability that the test fails to detect the antibodies in at least one of the samples.

[5 + 10 = 15 marks]

10. (a) A machine manufactures a component for a car engine. The process is supposed to produce components with a mean radius of 4 mm. The results of a sample of 16 components gave the following measurements on the radii

4.120	4.001	4.017	3.982
3.960	4.089	3.987	3.976
4.098	4.057	3.913	3.999
3.989	3.902	3.961	3.980

The manufacturing process is known to vary normally with a standard deviation of $\sigma = 0.060\ mm$. Test the claim $H_0 : \mu = 4.000$ against the alternative $H_1 : \mu \neq 4.000$ at a level of significance of $\alpha = 0.05$.

(b) Suppose now that the standard deviation σ of the process is not known. Carry out a new test of the claim in part (a).

[8 + 6 = 14 marks]

END OF EXAMINATION QUESTIONS

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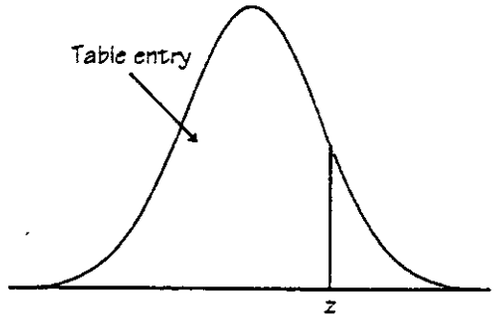

Table entry for z is the area under the standard normal curve to the left of z.

TABLE A Standard normal probabilities (*continued*)

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

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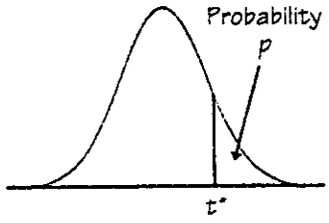

Table entry for p and C is the critical value t^* with probability p lying to its right and probability C lying between $-t^*$ and t^*.

TABLE C t **distribution critical values**

	Upper tail probability p											
df	.25	.20	.15	.10	.05	.025	.02	.01	.005	.0025	.001	.0005
1	1.000	1.376	1.963	3.078	6.314	12.71	15.89	31.82	63.66	127.3	318.3	636.6
2	0.816	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.09	22.33	31.60
3	0.765	0.978	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.21	12.92
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.695	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.611	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z^*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.091	3.291
	50%	60%	70%	80%	90%	95%	96%	98%	99%	99.5%	99.8%	99.9%
	Confidence level C											

END OF EXAMINATION PAPER