Monash University
Faculty of Information Technology

Lecture 6
Finite Automata

Slides by David Albrecht (2011), modified by Graham Farr (2013).

FIT2014 Theory of Computation

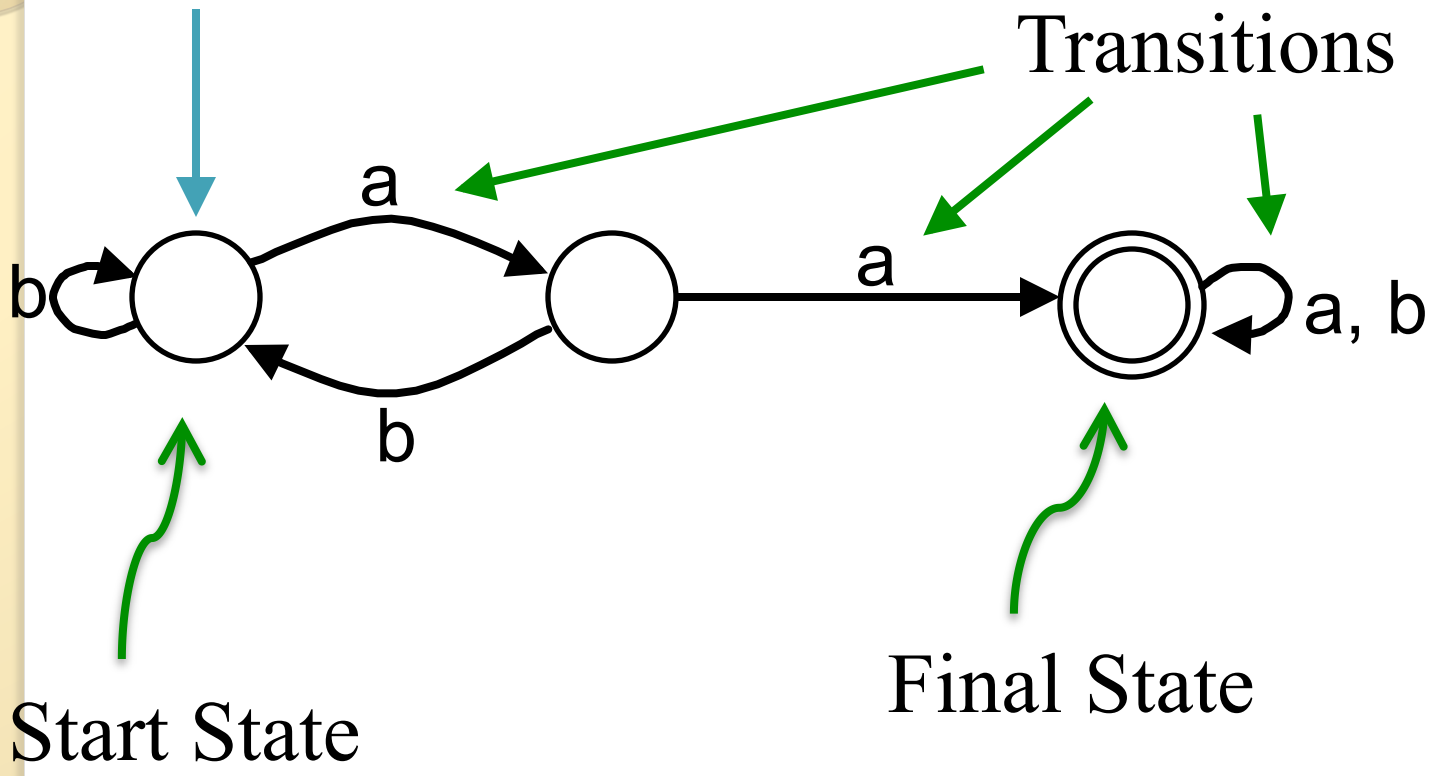
Overview

- Definition
- How they are used to define languages
- Representations
- Complement Languages
- Comparison with Regular Expressions

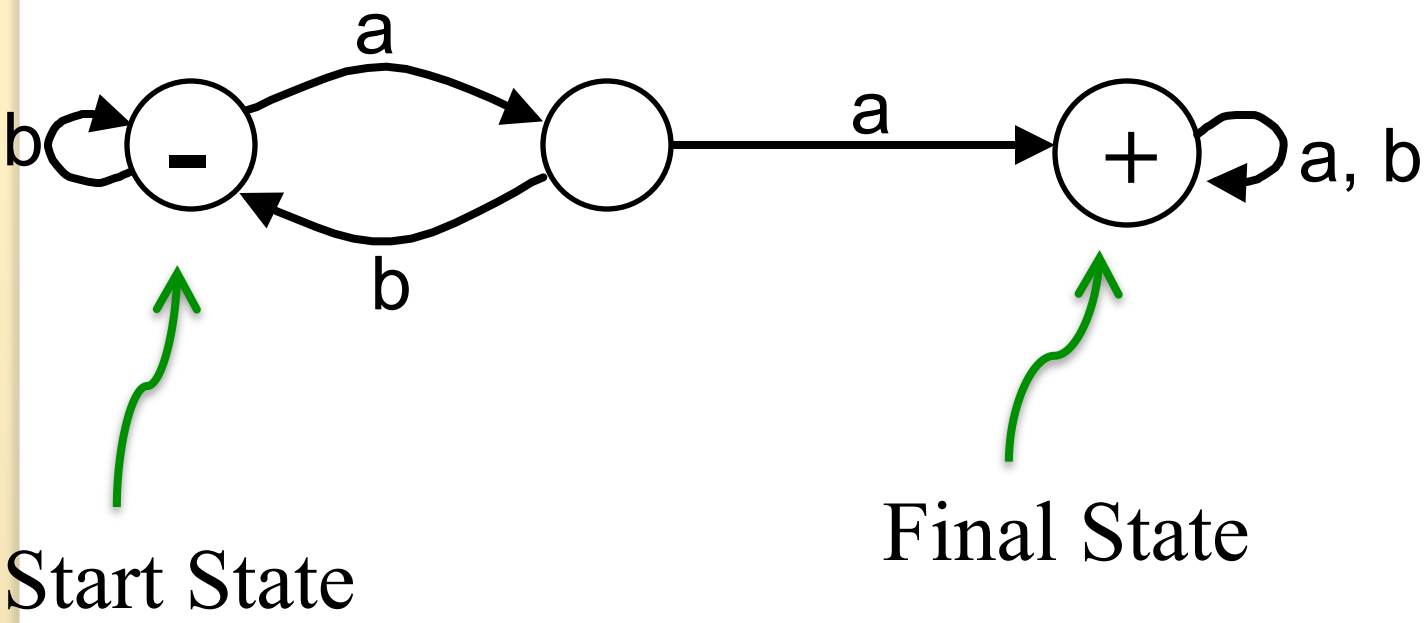
Finite Automaton

- Sometimes known as a **Deterministic Finite Automaton.**
- Used for determining whether a word does or does not belong to a **Regular Language**.
- Used for defining a **Regular Language**.
- Used in **Lexical Analysers**.

Notation



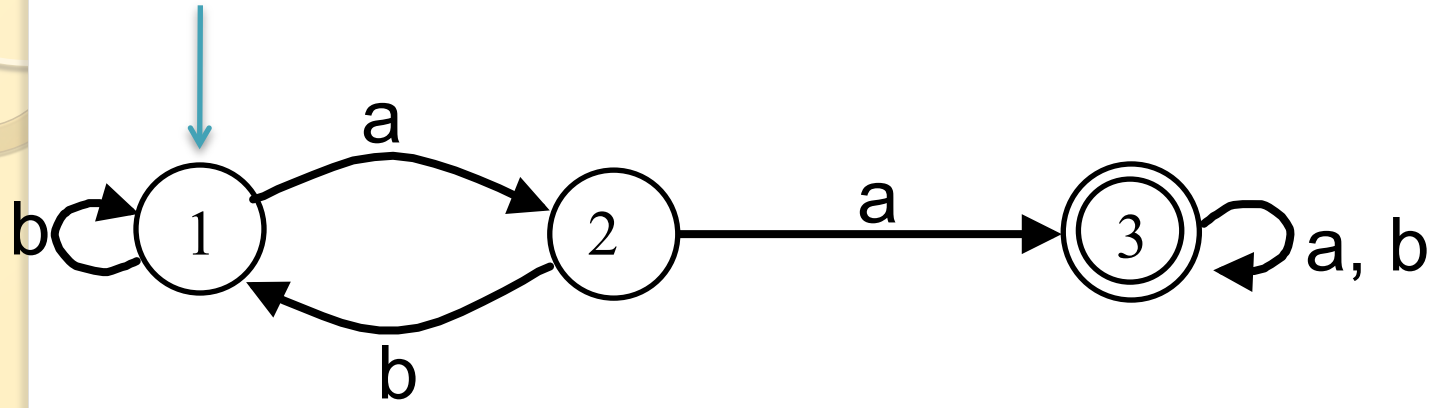
Alternative Notation



Finite automaton: definition

- A finite set of *states*
  - One called the Start State
  - Some (maybe none) called Final States
- An alphabet of possible *input letters*
- A finite set of *transitions*
  - that tell, for each state and each letter in the alphabet, which state to go to next.
  - There is an unique transition from any state for each letter in the alphabet.

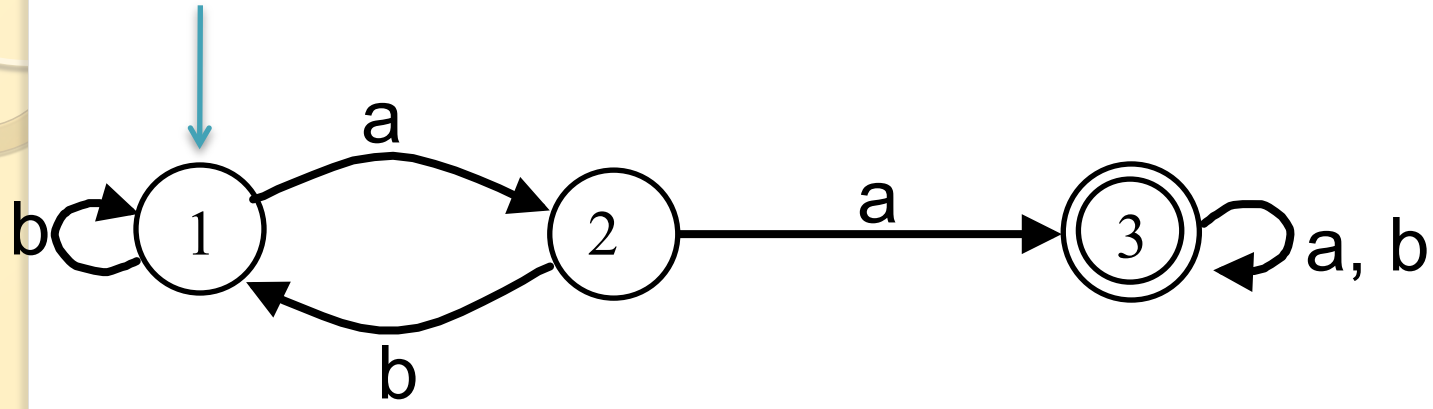
Finite automata: representations



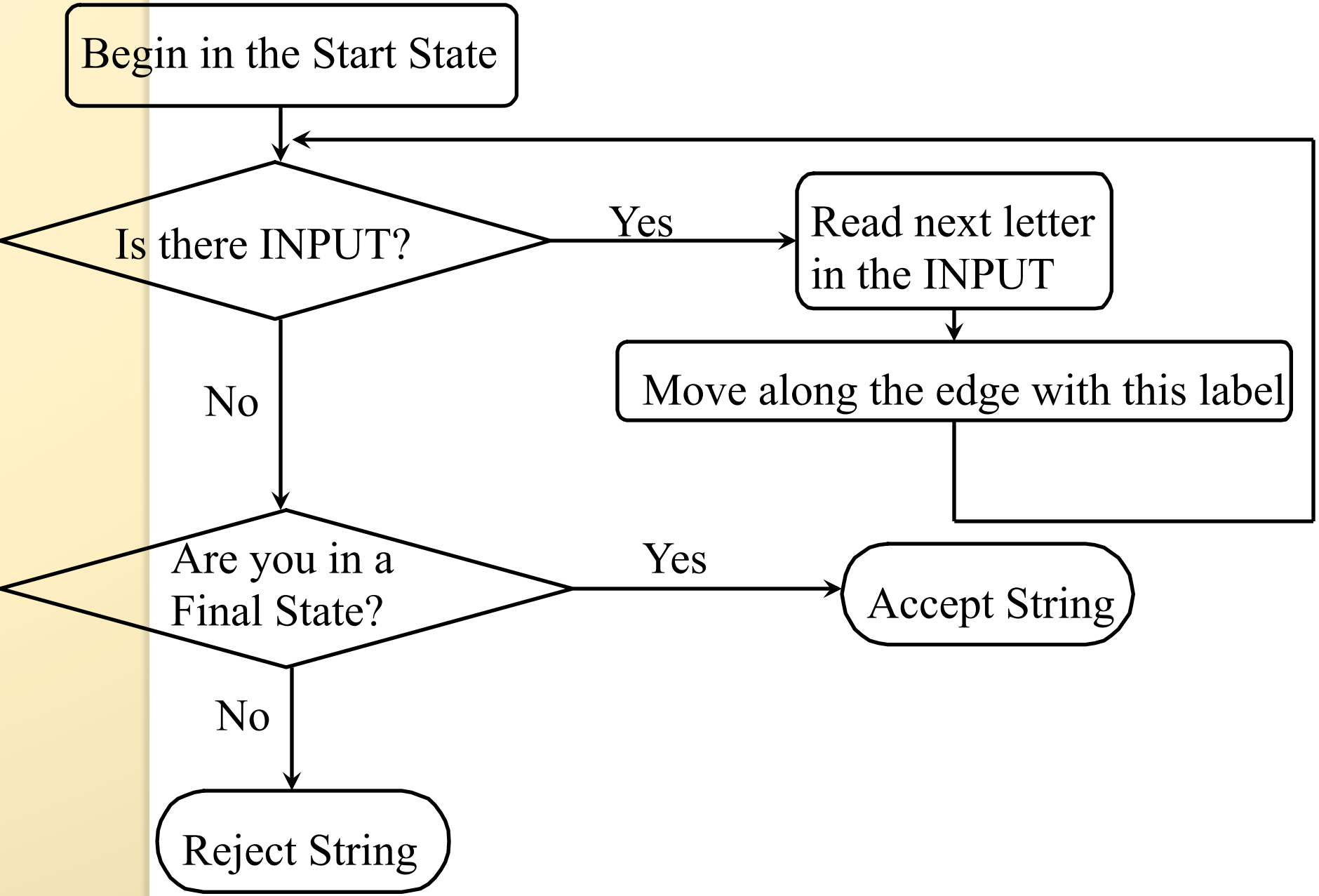
	a	b
Start 1	2	1
2	3	1
Final 3	3	3

Finite automata

Every string traces a *unique* path in the automaton, starting from the Start State and following the transitions, letter by letter.



	a	b
Start 1	2	1
2	3	1
Final 3	3	3


Begin in the Start State
Is there INPUT?
Yes
Read next letter in the INPUT
Move along the edge with this label
No
Are you in a Final State?
Yes
Accept String
No
Reject String

Finite automata

Every string traces a *unique* path in the automaton, starting from the Start State and following the transitions, letter by letter.

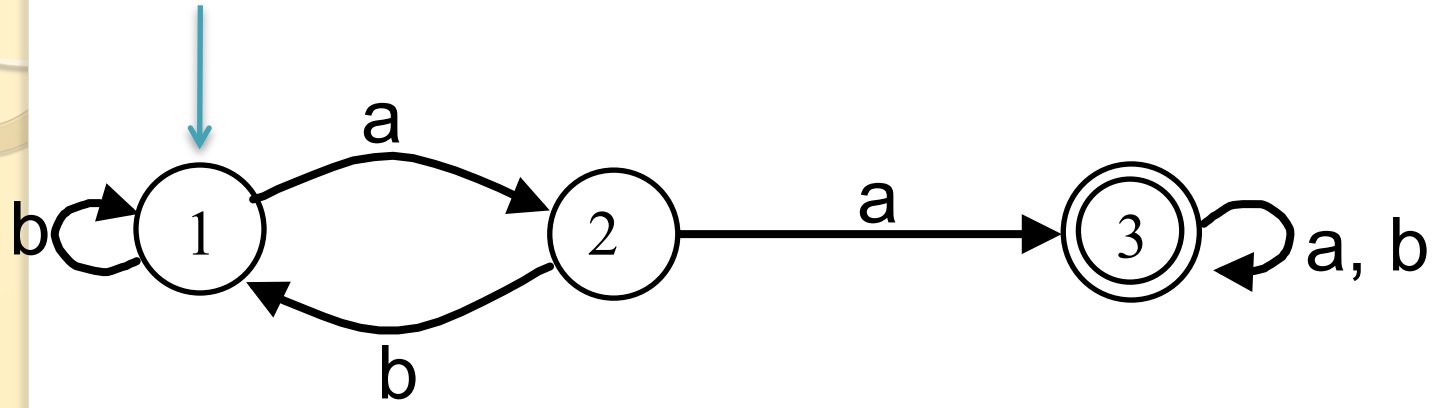
Definitions

A string is ***accepted*** by a FA if its path ends on a Final State. Otherwise the string is ***rejected***.

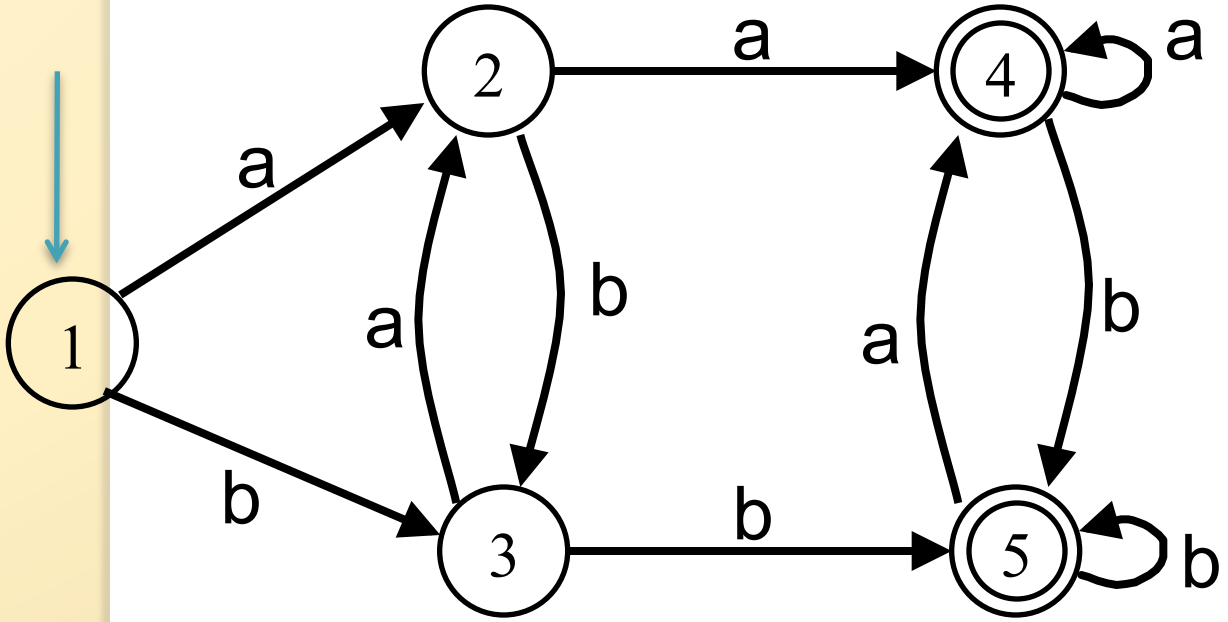
The ***language recognised*** by a FA is the set of all strings it accepts.

We say the FA ***recognises*** the language, or ***accepts*** the language.

Representations



	a	b
Start 1	2	1
2	3	1
Final 3	3	3



	a	b
Start 1	**2**	**3**
2	**4**	**3**
3	**2**	**5**
Final 4	**4**	**5**
Final 5	**4**	**5**

Special Cases

- All words accepted.
- Only the empty word accepted.
- A single word accepted.

Complements

If L is a language over an alphabet, then its ***complement*** $\overline{L}$ is set of strings of letters from the alphabet that are not words in L.

The complement of L is sometimes denoted by L' or L^c.

EVEN-EVEN

EVEN-EVEN is the set of strings that contain an **even** number of **a**'s **and** an **even** number of **b**'s.

- E.g.

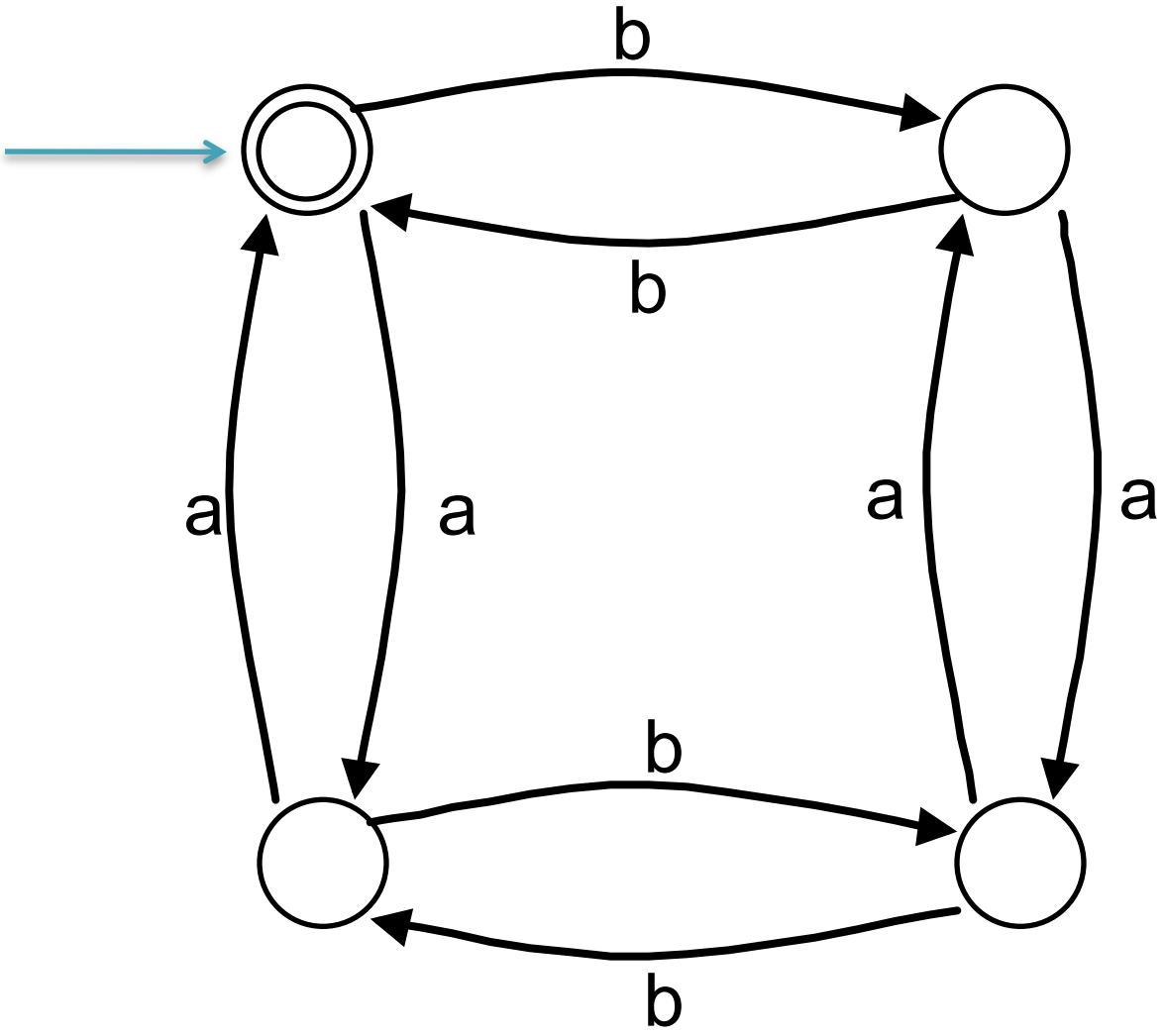
  ε aa bb aaaa aabb abab abba baab

EVEN-EVEN is the set of strings which contain an **odd** number of **a**'s **or** an **odd** number of **b**'s.

- E.g.

  a b ab ba aaa aab aba abb baa …

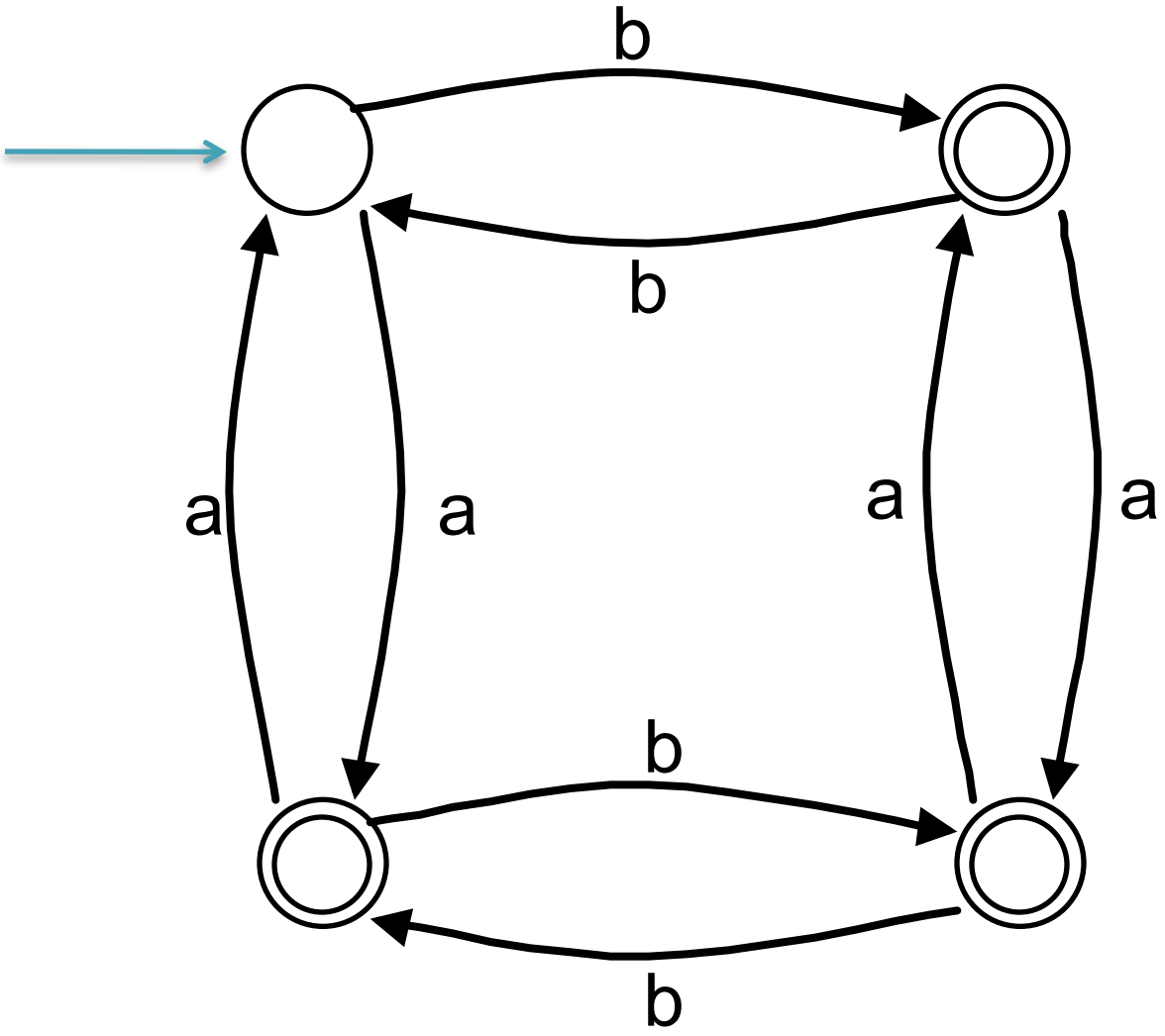
EVEN-EVEN



Complement Finite Automaton (FA)

- Suppose some FA accepts the language L
- Change all the final states in this FA to non-final states, and all the non-final states to final states.
- This new FA now accepts all the strings not accepted by the original FA (ie. all the words in $\overline{L}$), and rejects all the words that the original accepted (i.e., the words in L).
- So the new FA accepts $\overline{L}$.

EVEN-EVEN



Comparison with Regular Expressions

- It is easier to write down a regular expression that defines a language than to design a FA to accept this language.
- It is easier to check whether a given string is accepted by a FA than it is to see whether it matches a regular expression.
- It is easier to find complements using a FA than by using a regular expression.

Some Generalizations of Finite Automata

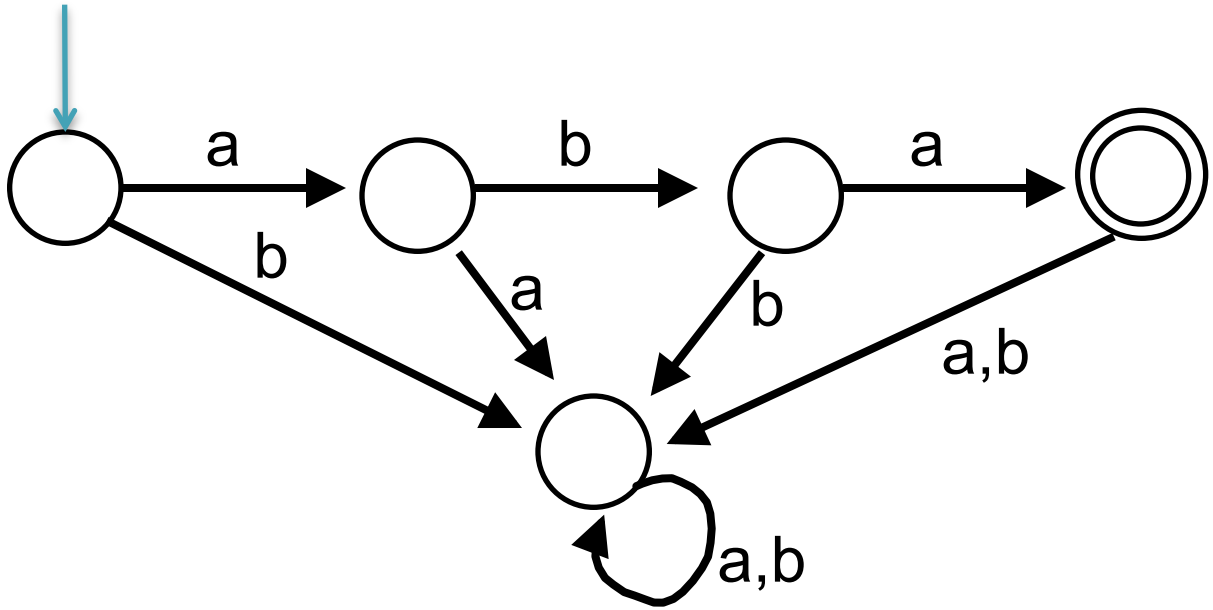
- For every state and letter, there is **not** a **unique** transition.
- **Change state without** reading any **letter**.
- Read **more than one** letter at a time.
- Read **strings** which **match regular expressions**.

Nondeterministic Finite Automaton (NFA) Definition

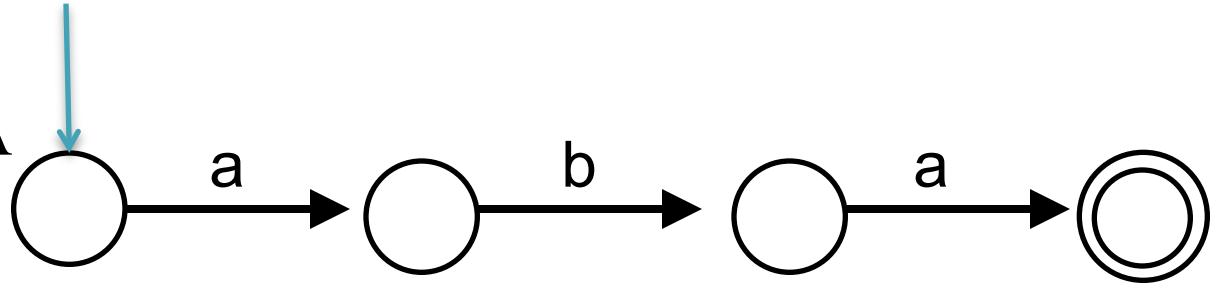
- Like a Finite Automaton (FA) except for transitions
- Transitions
  - For ***some*** states and letters there is a transition.
  - **The labels may include the empty word** ε .
- So for a given letter and state there may be:
  - **No transition**
  - **More than one transition**
- For a given string, the path it takes …
  - might **not exist**
  - might **not be unique**

aba

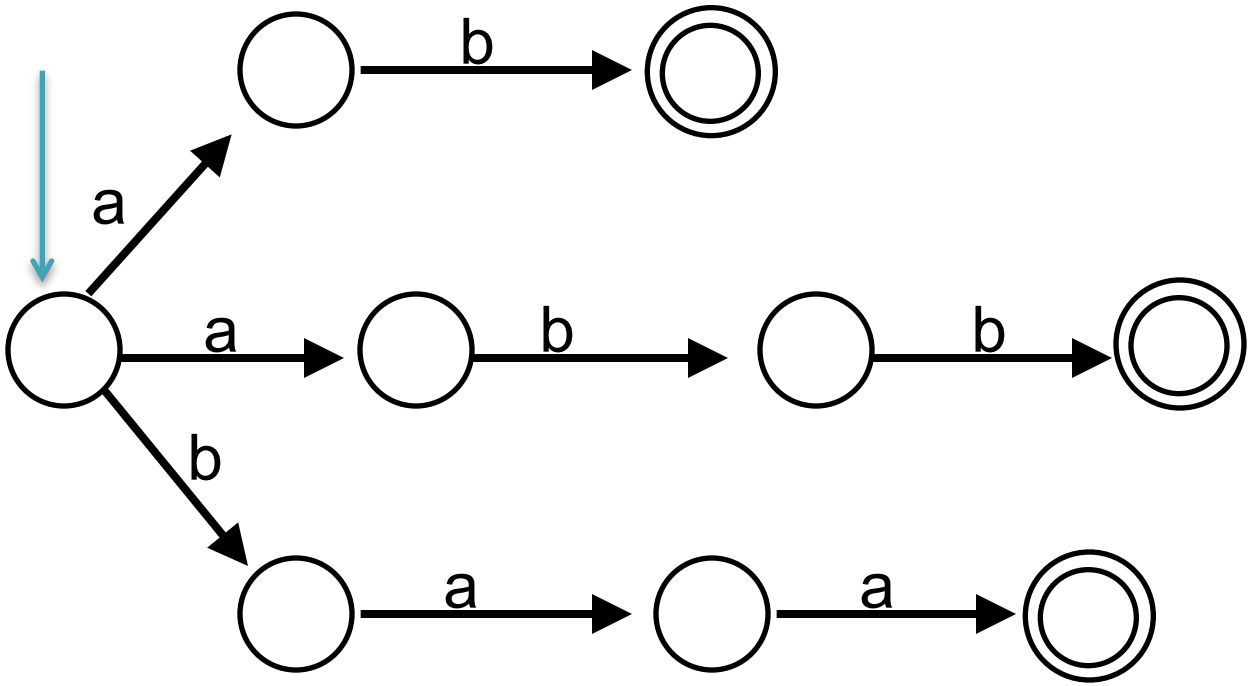
FA



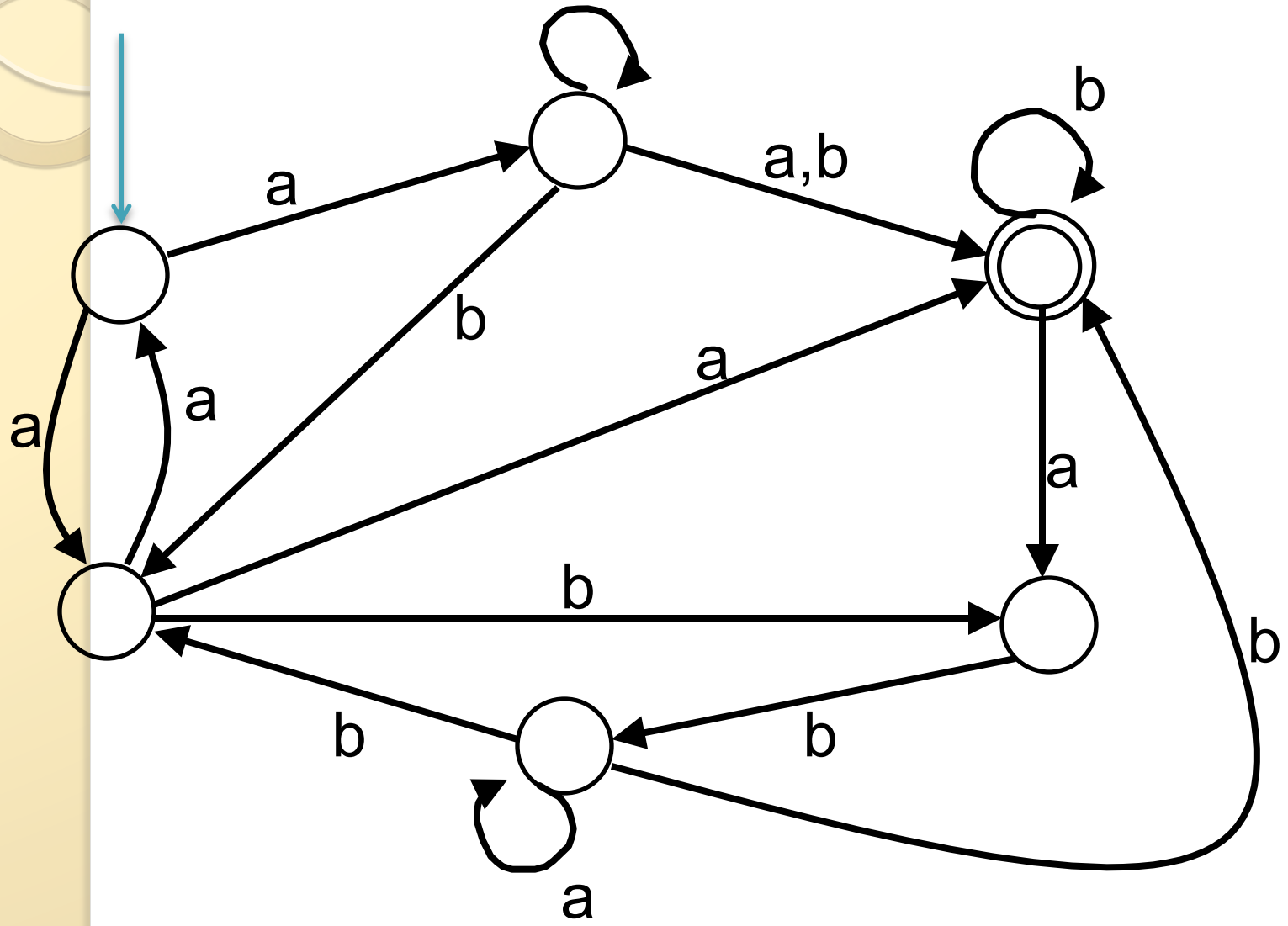
NFA



ab ∪ abb ∪ baa



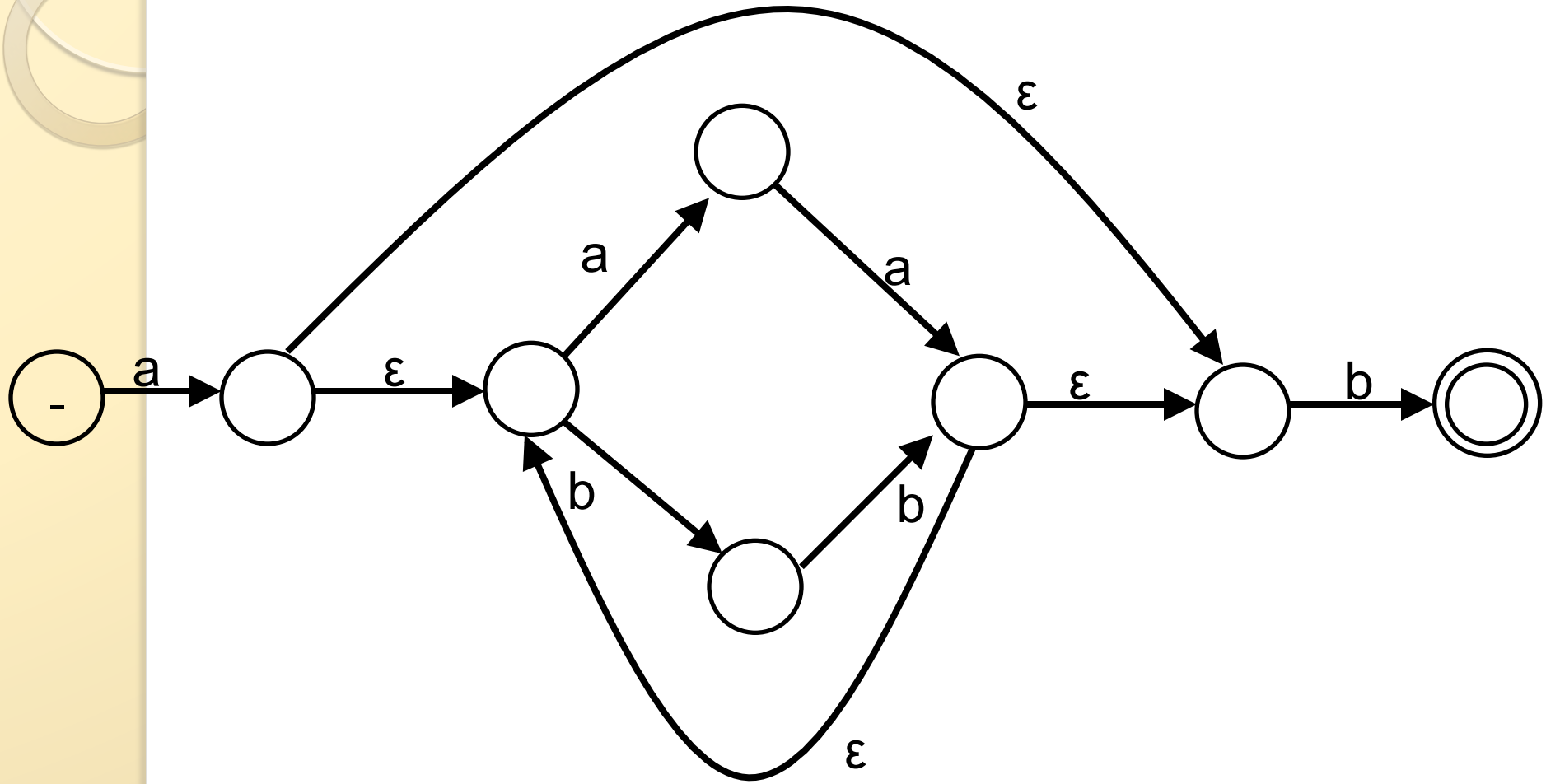
Is abbbabbbbabba accepted?



Properties

- If there is **no transition** for the **current letter and state** the machine **crashes**.
- Paths from the **Start State** to a **Final State** for a given input:
  - **One**
  - **None**
  - **Several** (Nondeterministic)
- Accept a string if there is at ***least one*** path from the **Start State** to a **Final State**.
- Reject a string if there are ***no paths*** from the **Start State** to a **Final State**.

a(aa ∪ bb)*b



Revision

- Finite Automata (FA)
  - Definition. How to use them.
  - How to construct a Finite Automaton to accept a language.
- Complement Languages
  - What they are. Designing FA to accept them.
- Nondeterministic Finite Automata (NFA)
  - Definition. How to use them
  - How to construct a Nondeterministic Finite Automata to accept a language.

- Reading: Sipser Ch 1.