

Under week 10

CHAPTER EIGHTEEN

**Python's Dictionary Implementation:
Being All Things to All People**

Andrew Kuchling

Lecture 32

Binary Trees

FIT 1008
Introduction to Computer Science



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WARNING

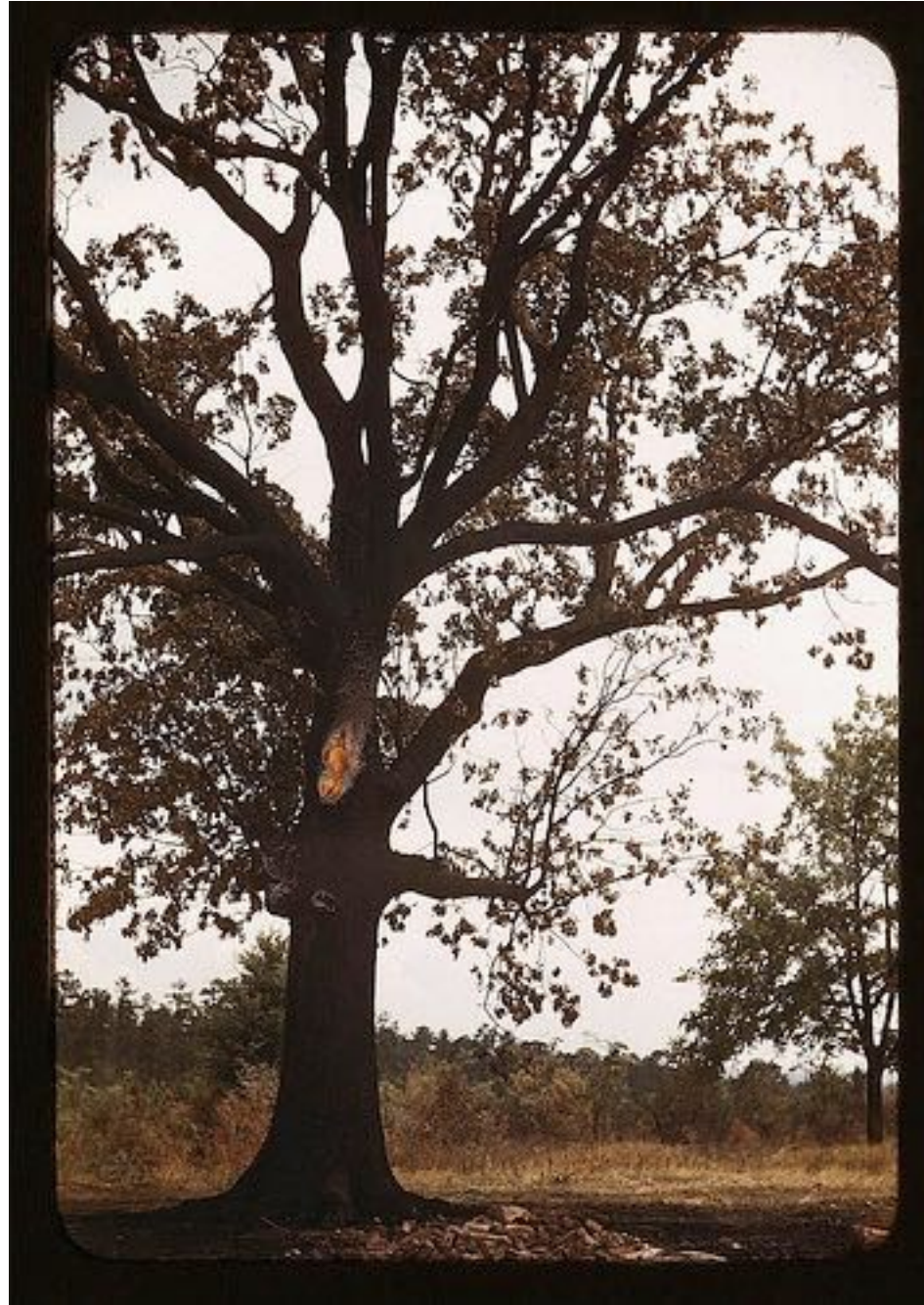
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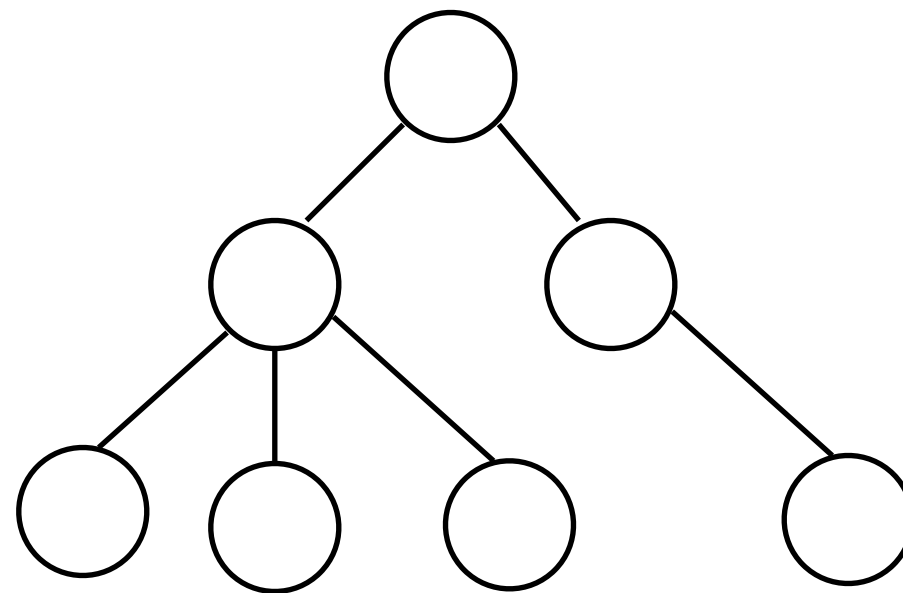
Do not remove this notice.

Objectives

- Revise **Trees**:
 - ➔ Concepts
 - ➔ Operations & Implementation
 - ➔ Complexity Ideas
 - ➔ Traversal

Trees

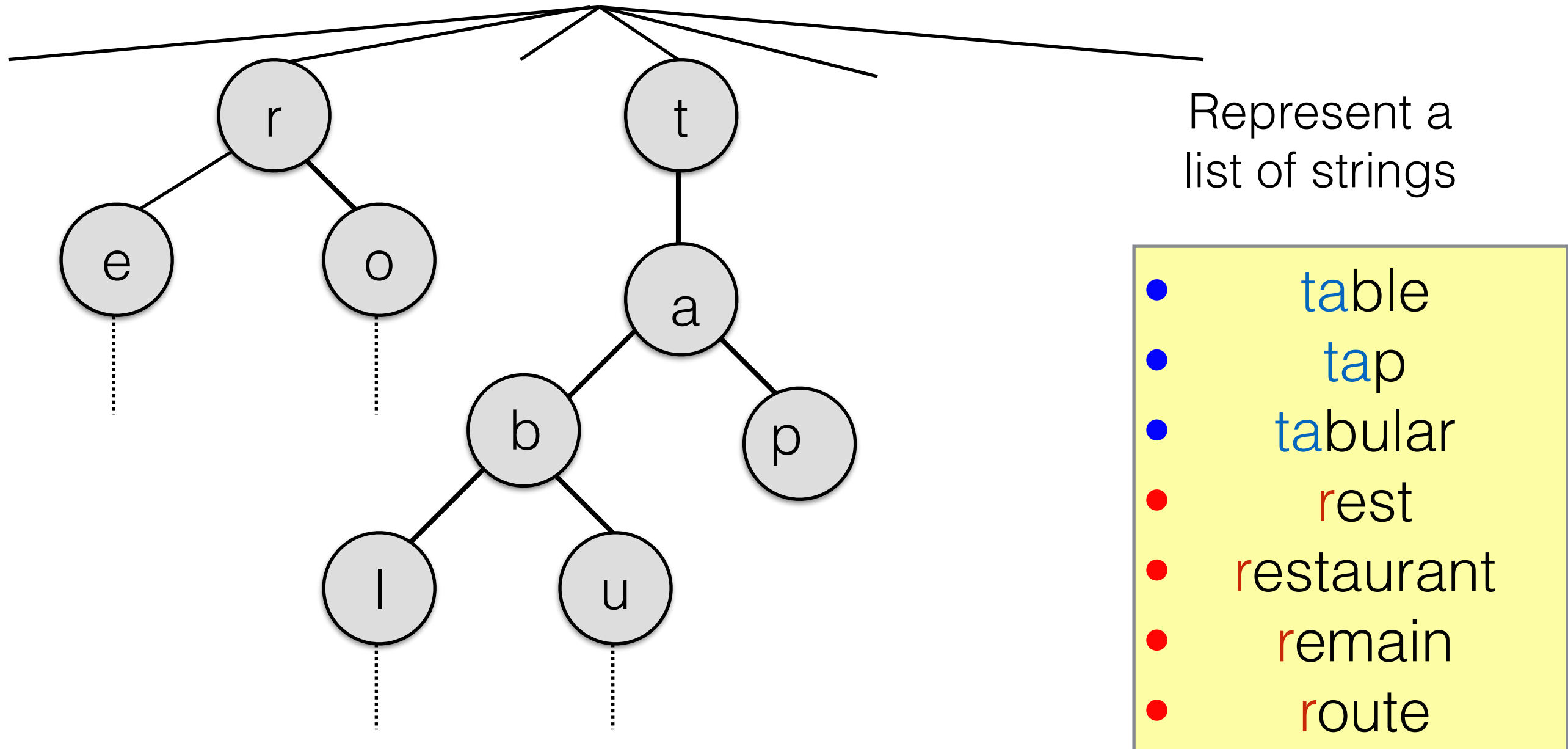




Trees

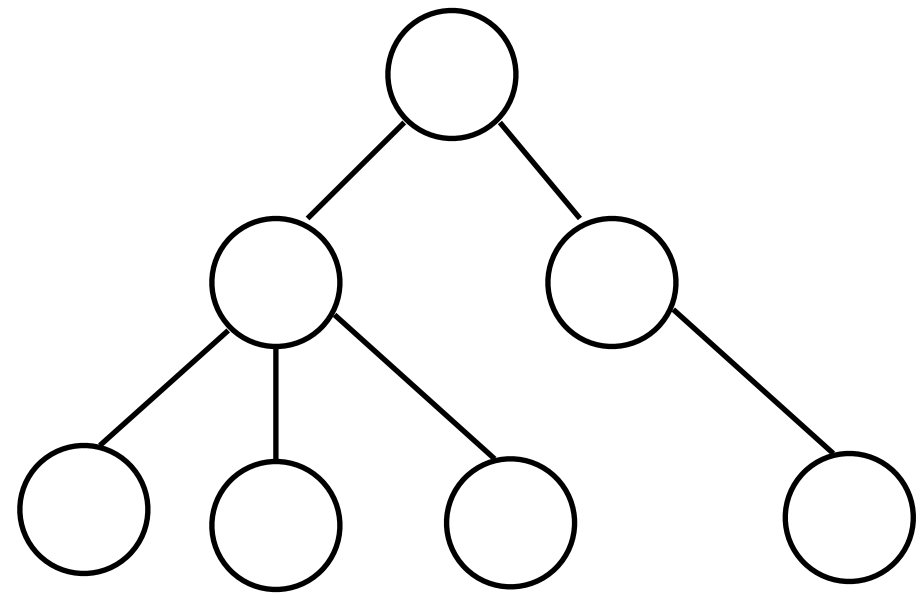
- Extremely useful.
- Natural way of modelling many things:
 - ➔ Family trees
 - ➔ Organisation structure charts
 - ➔ Structure of chapters and sections in a book
 - ➔ Execution/call tree (recall the one for fibonacci)
 - ➔ Object Oriented Class Hierarchies
- Particularly good for some operations (like search)
- Compact representation of data

Compact representation of data



Branches represent different strings.

Trees



Trees

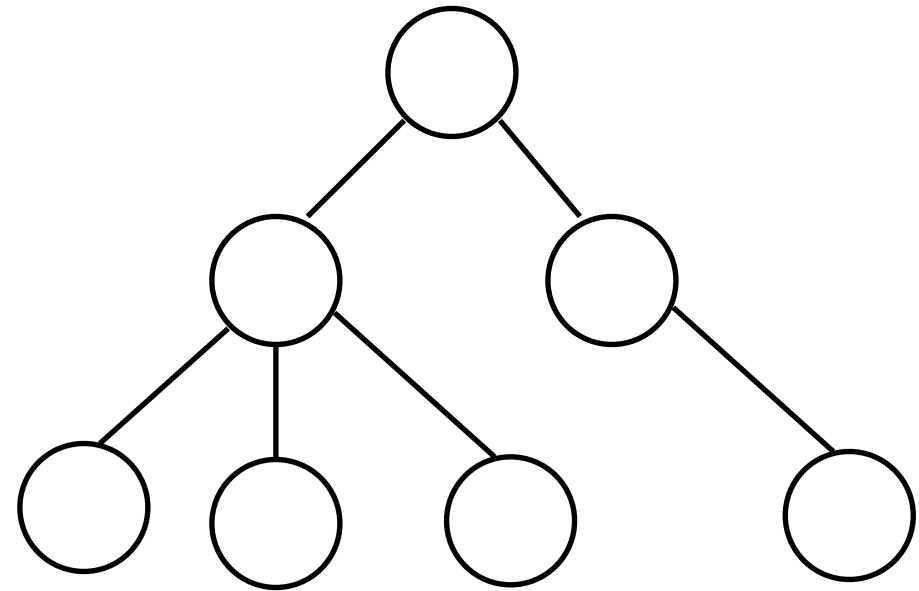
- Graphs which are:

→ Simple

no loops or multiple edges

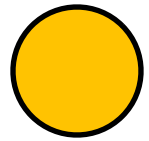
→ Connected

→ No circuits.



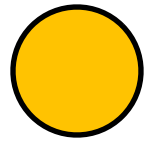
Pre-reading: Binary Trees.

Perfect Binary Trees

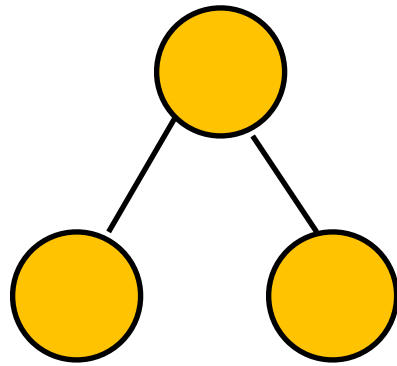


$N = 1$ Height = 0

Perfect Binary Trees

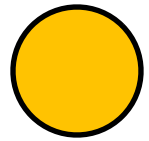


$N = 1$ Height = 0

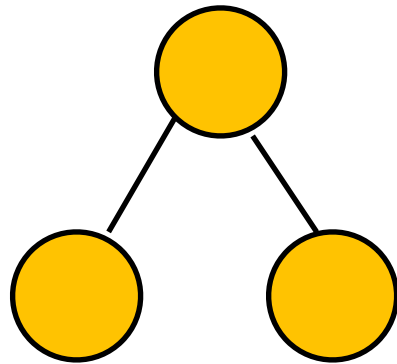


$N = 3$ Height = 1

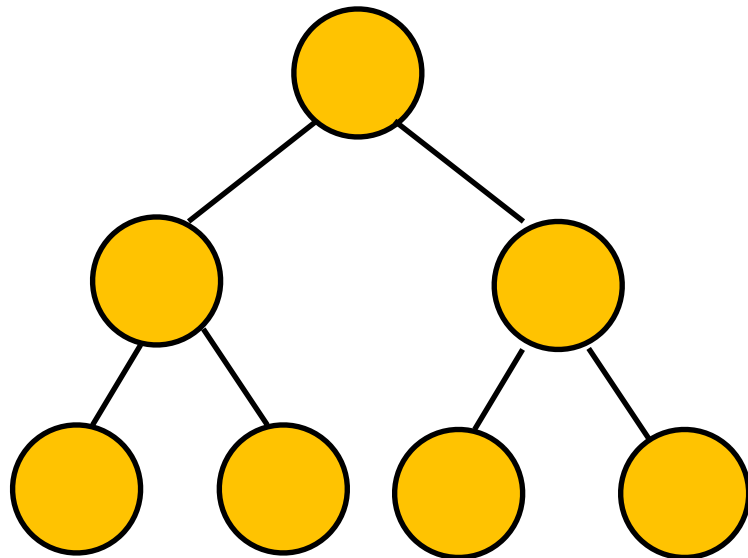
Perfect Binary Trees



$N = 1$ Height = 0

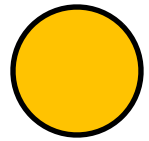


$N = 3$ Height = 1

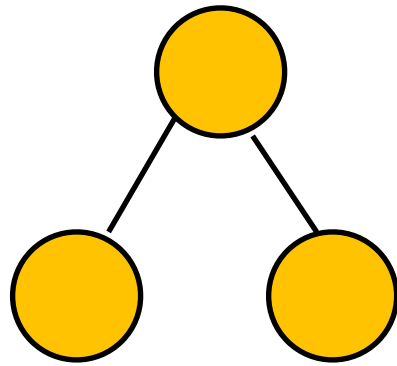


$N = 7$ Height = 2

Perfect Binary Trees

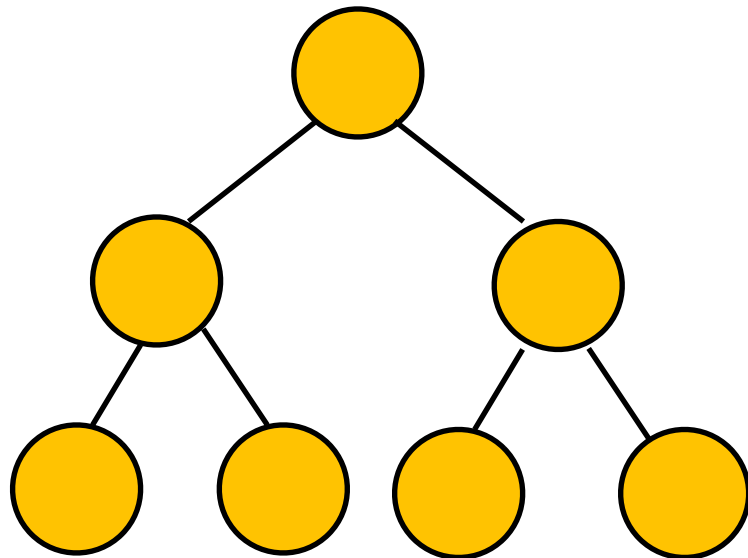


$N = 1$ Height = 0



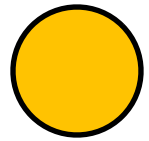
$N = 3$ Height = 1

Each parent has two children

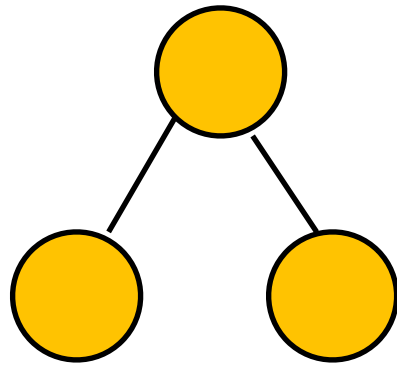


$N = 7$ Height = 2

Perfect Binary Trees



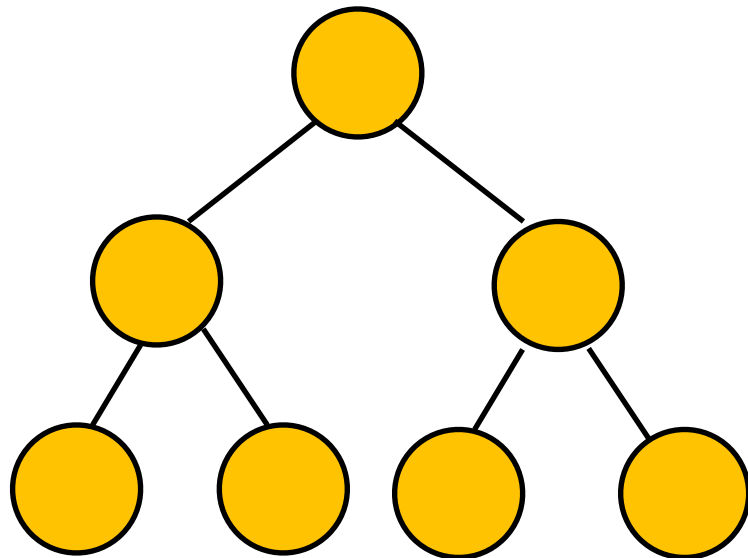
$N = 1$ Height = 0



$N = 3$ Height = 1

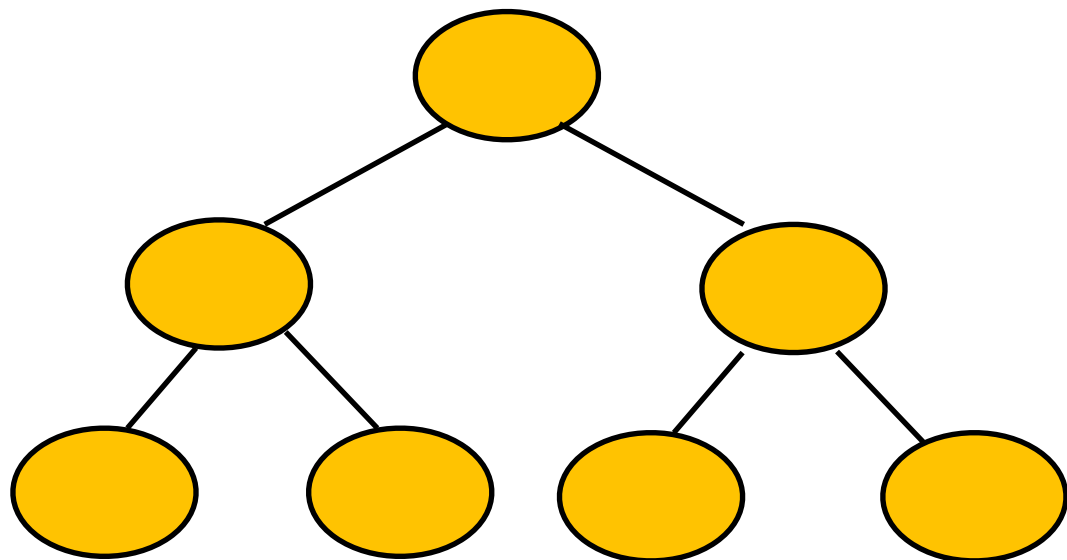
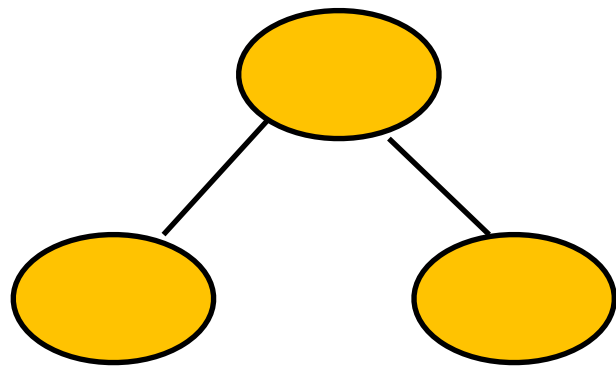
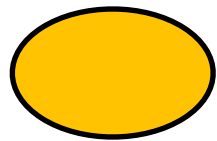
Each parent has two children

All leaves at same level



$N = 7$ Height = 2

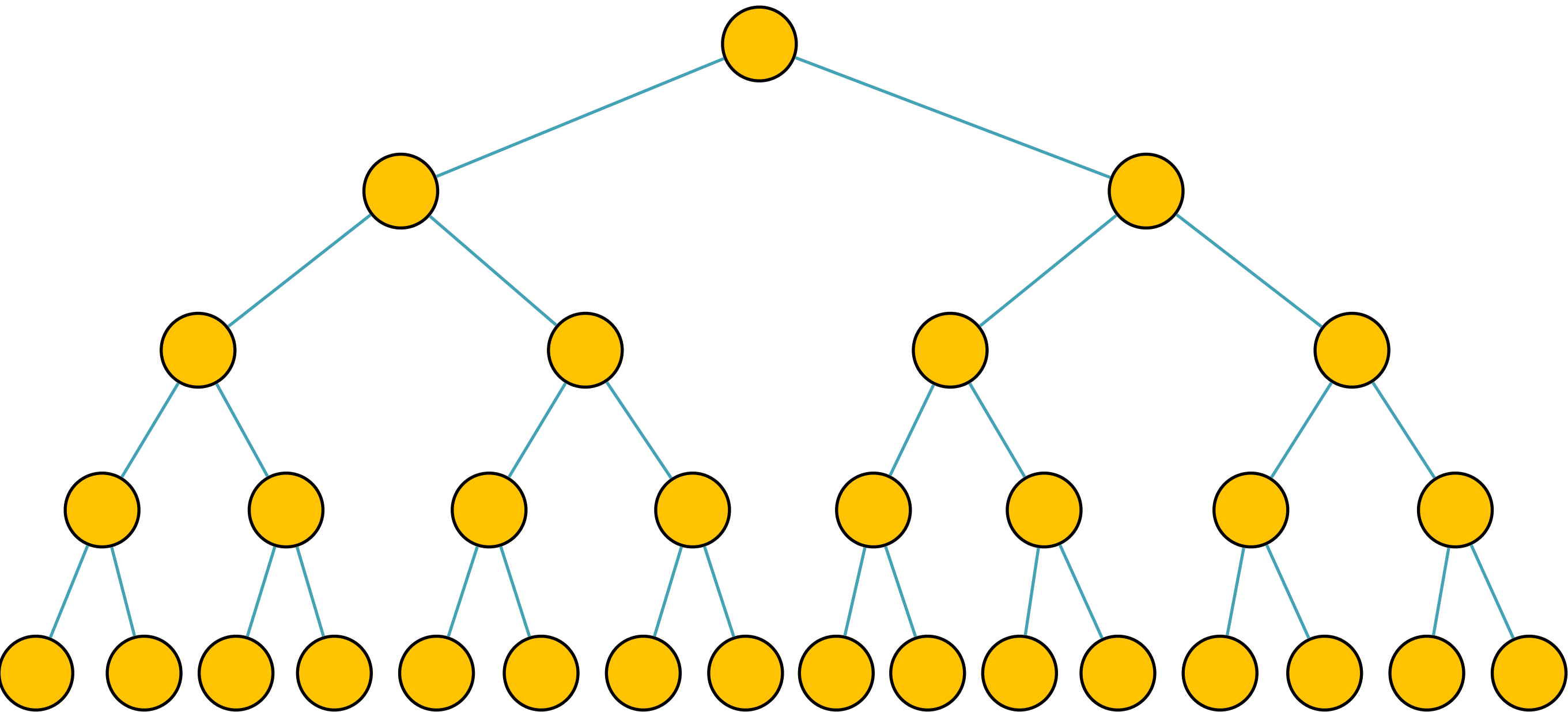
Perfect Binary Trees



height	leaves	nodes
0	1	1
1	2	3
2	4	7
3	8	15
k	2^k	$2^{k+1}-1$

$$N = 2^{k+1} - 1$$

$$\text{Height} = k$$



Perfect Binary Trees

$$N = 2^{k+1} - 1$$

$$N+1 = 2^{k+1}$$

$$\log_2(N+1) = k+1$$

$$\log_2(N+1) - 1 = k$$

Perfect Binary Trees

$$N = 2^{k+1} - 1$$

$$N+1 = 2^{k+1}$$

$$\log_2(N+1) = k+1$$

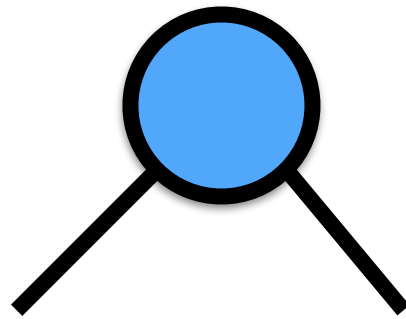
$$\log_2(N+1) - 1 = k$$

In a perfect binary tree with N nodes,
the height is $O(\log N)$

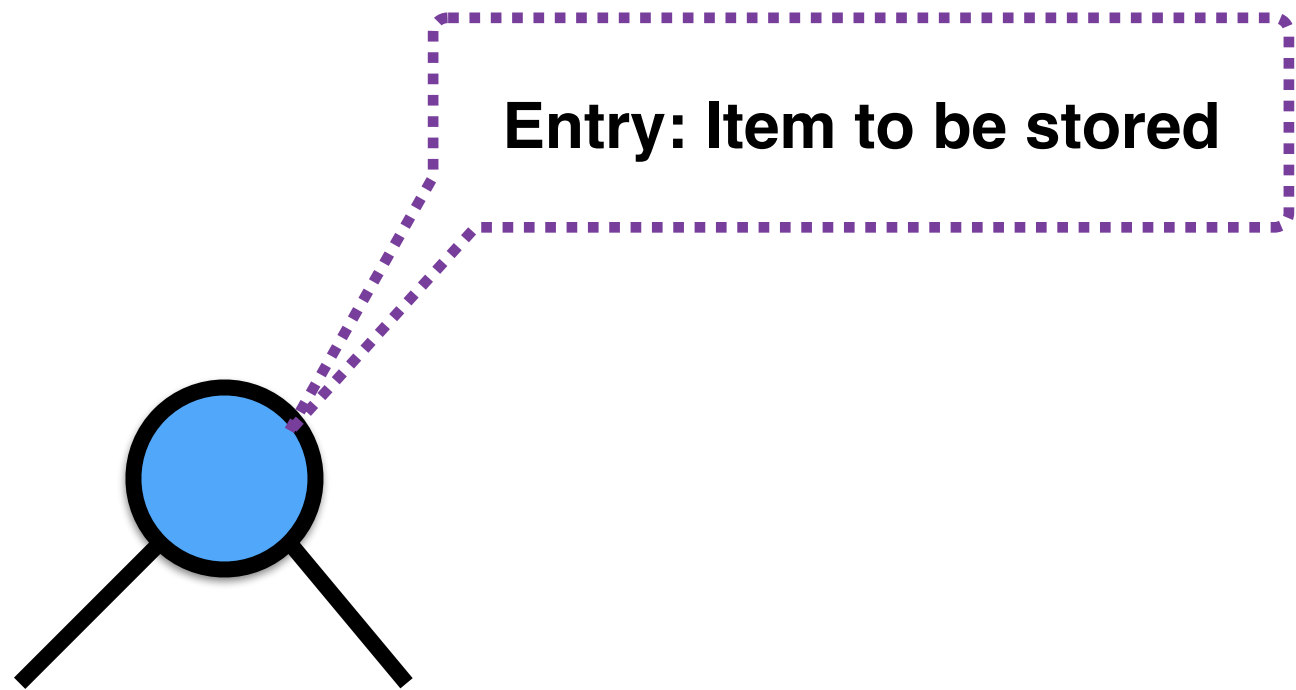
Balanced tree
the height is **$O(\log N)$**

Unbalanced tree
the height is **$O(N)$**

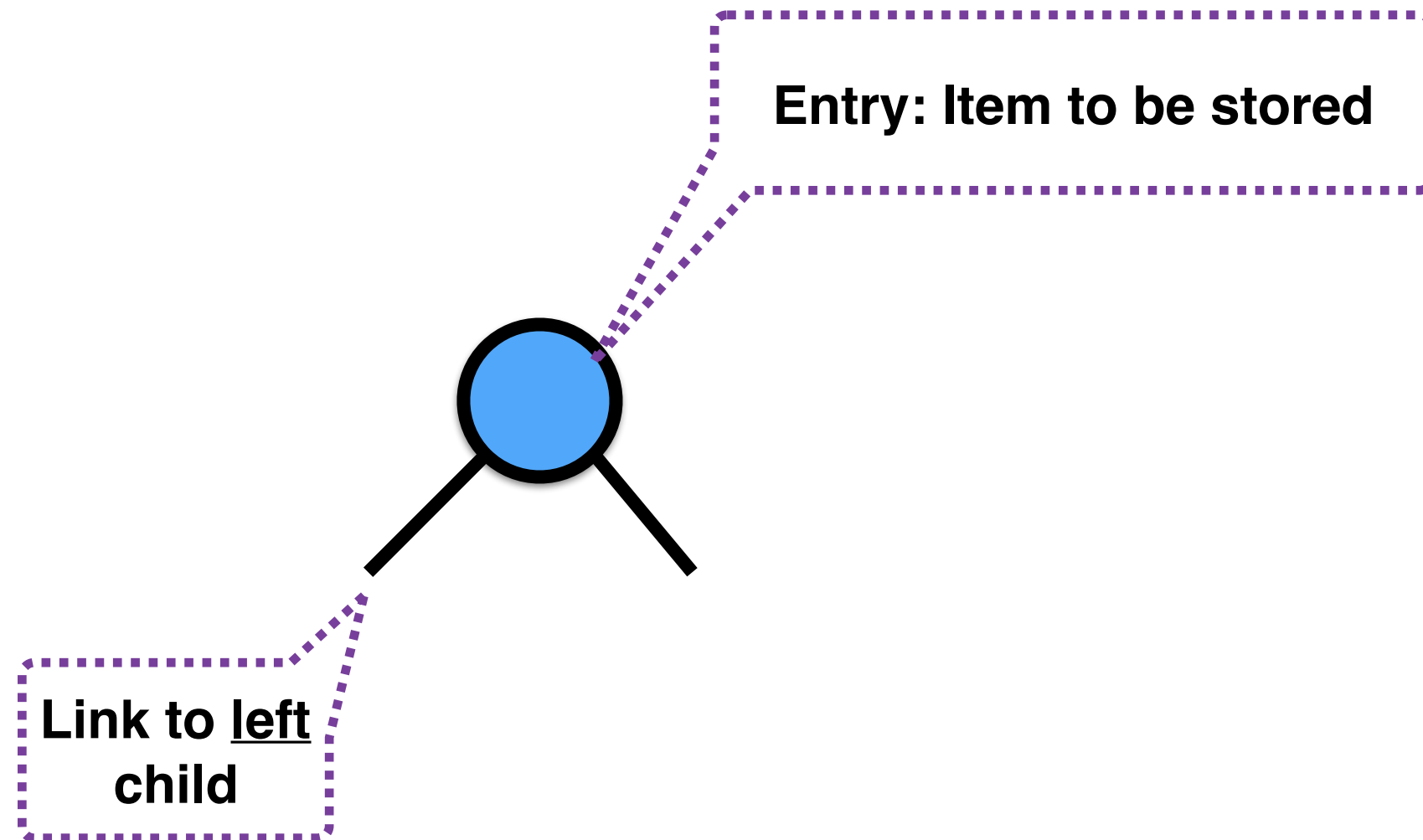
Representing a Binary Tree **Node**



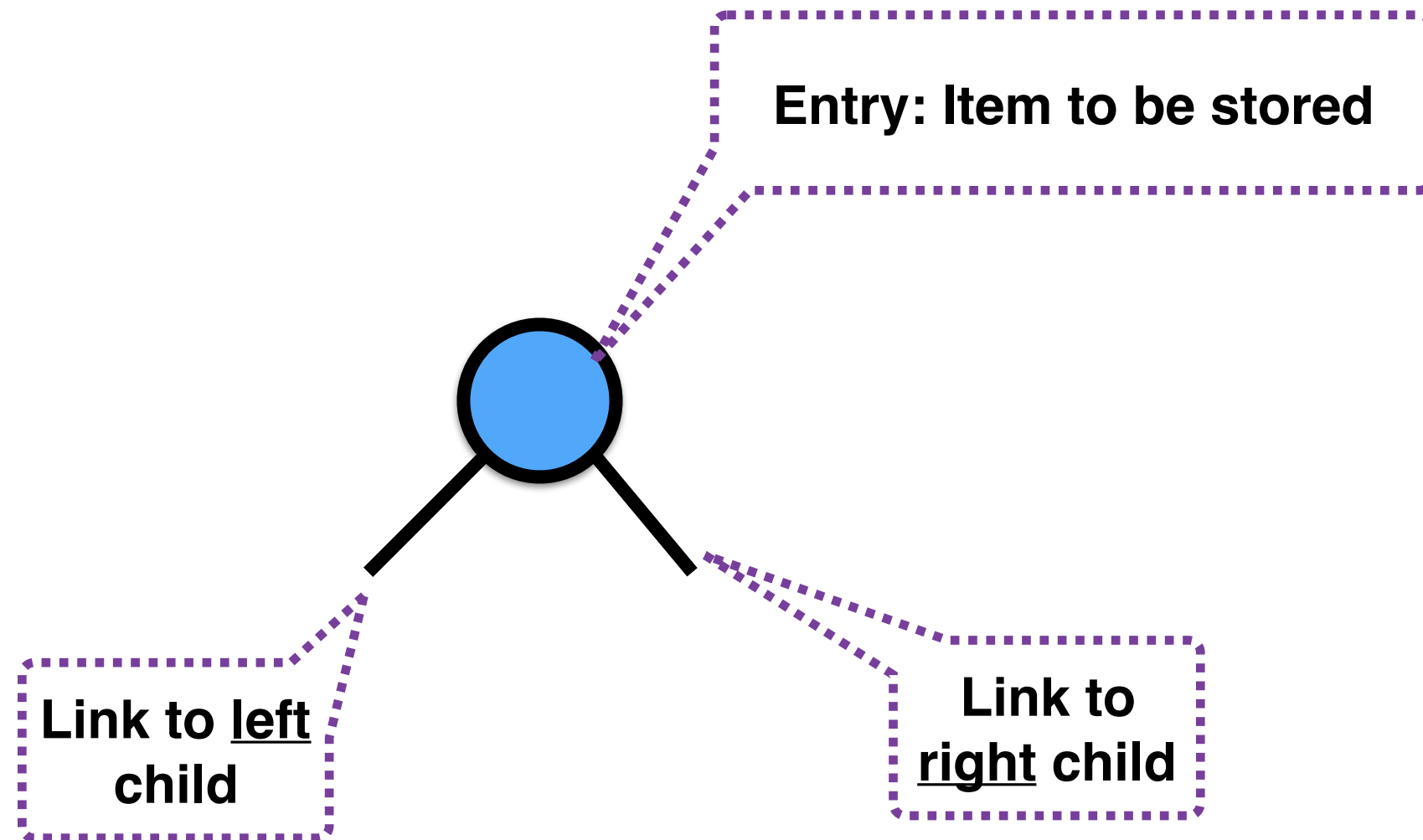
Representing a Binary Tree **Node**



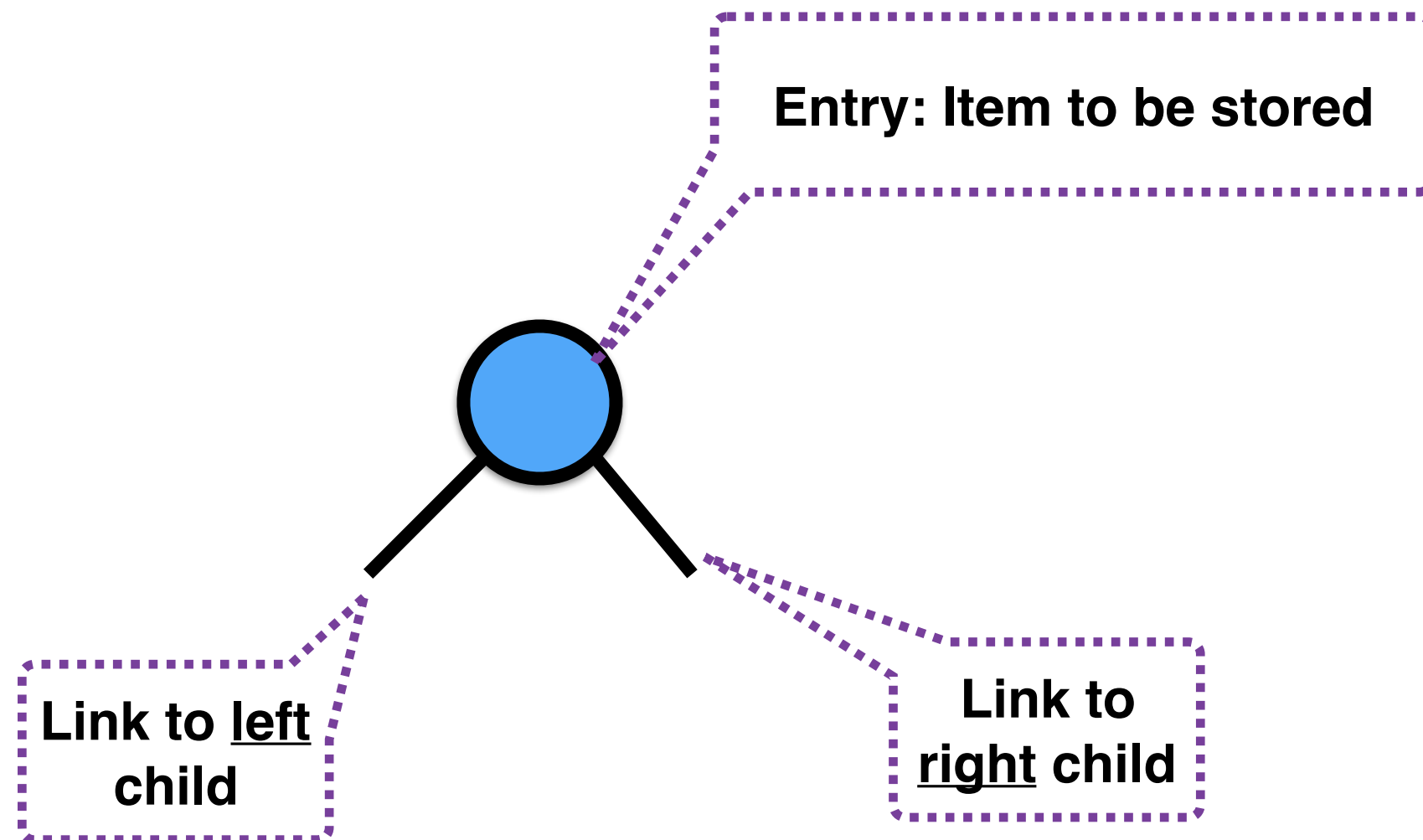
Representing a Binary Tree **Node**



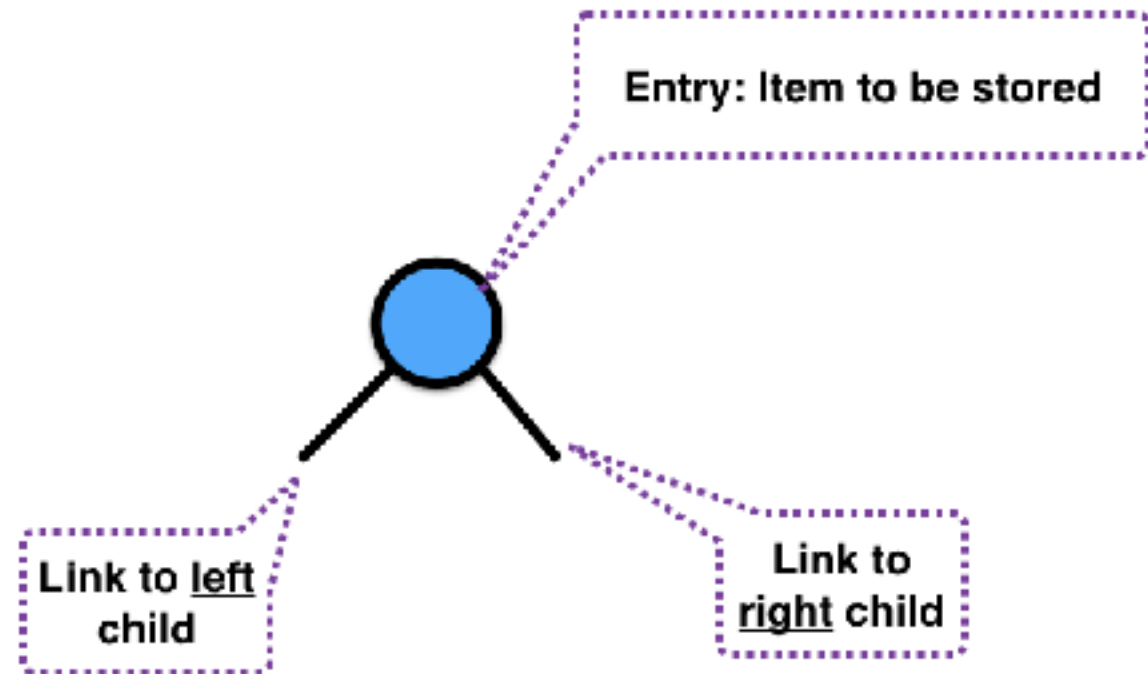
Representing a Binary Tree **Node**



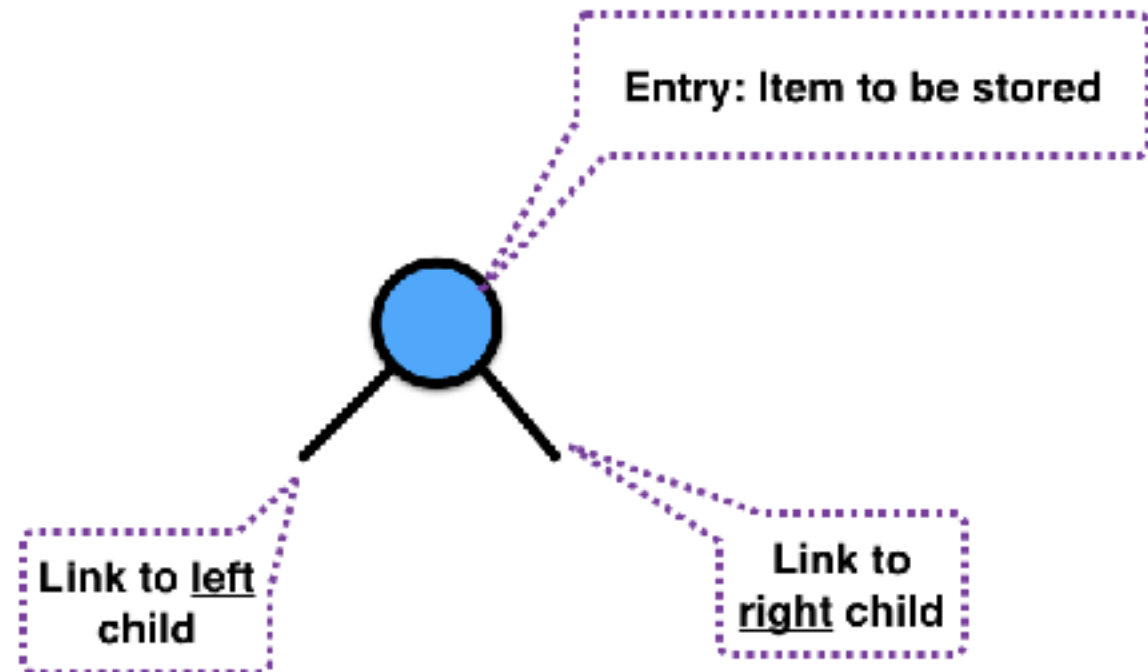
Representing a Binary Tree **Node**



Our implementation: Each link points to a **Node**

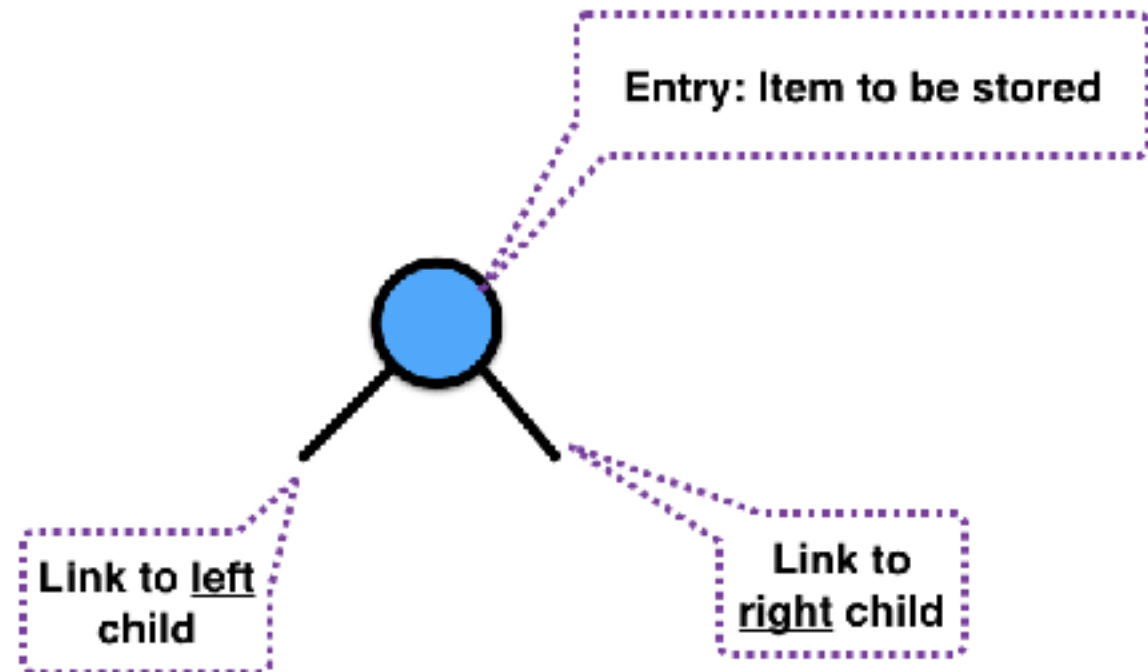


class TreeNode:



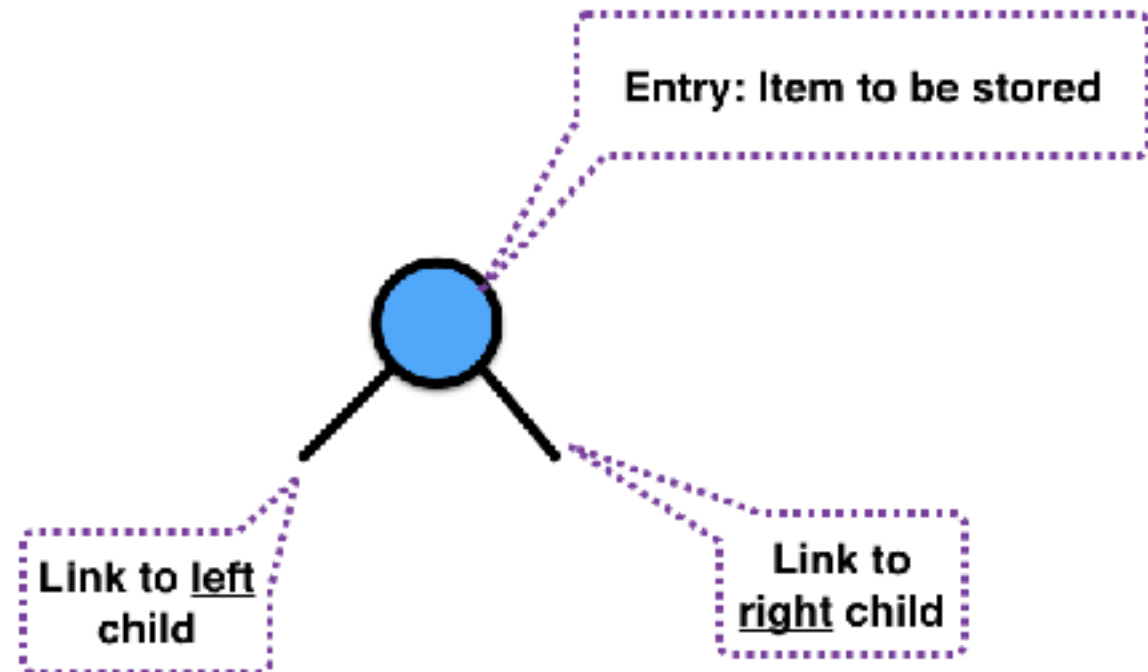
```
class TreeNode:
```

```
    def __init__(self, item=None, left=None, right=None):
```



class TreeNode:

```
def __init__(self, item=None, left=None, right=None):  
    self.item = item  
    self.left = left  
    self.right = right
```



```
class TreeNode:
```

```
    def __init__(self, item=None, left=None, right=None):  
        self.item = item  
        self.left = left  
        self.right = right
```

```
    def __str__(self):  
        return str(self.item)
```

class TreeNode:

```
def __init__(self, item=None, left=None, right=None):  
    self.item = item  
    self.left = left  
    self.right = right  
  
def __str__(self):  
    return str(self.item)
```

class BinaryTree:

class TreeNode:

```
def __init__(self, item=None, left=None, right=None):  
    self.item = item  
    self.left = left  
    self.right = right  
  
def __str__(self):  
    return str(self.item)
```

class BinaryTree:

```
def __init__(self):
```

class TreeNode:

```
def __init__(self, item=None, left=None, right=None):  
    self.item = item  
    self.left = left  
    self.right = right  
  
def __str__(self):  
    return str(self.item)
```

class BinaryTree:

```
def __init__(self):  
    self.root = None
```



```
class TreeNode:
```

```
    def __init__(self, item=None, left=None, right=None):  
        self.item = item  
        self.left = left  
        self.right = right  
  
    def __str__(self):  
        return str(self.item)
```

```
class BinaryTree:
```

```
    def __init__(self):  
        self.root = None  
  
    def is_empty(self):  
        return self.root is None
```

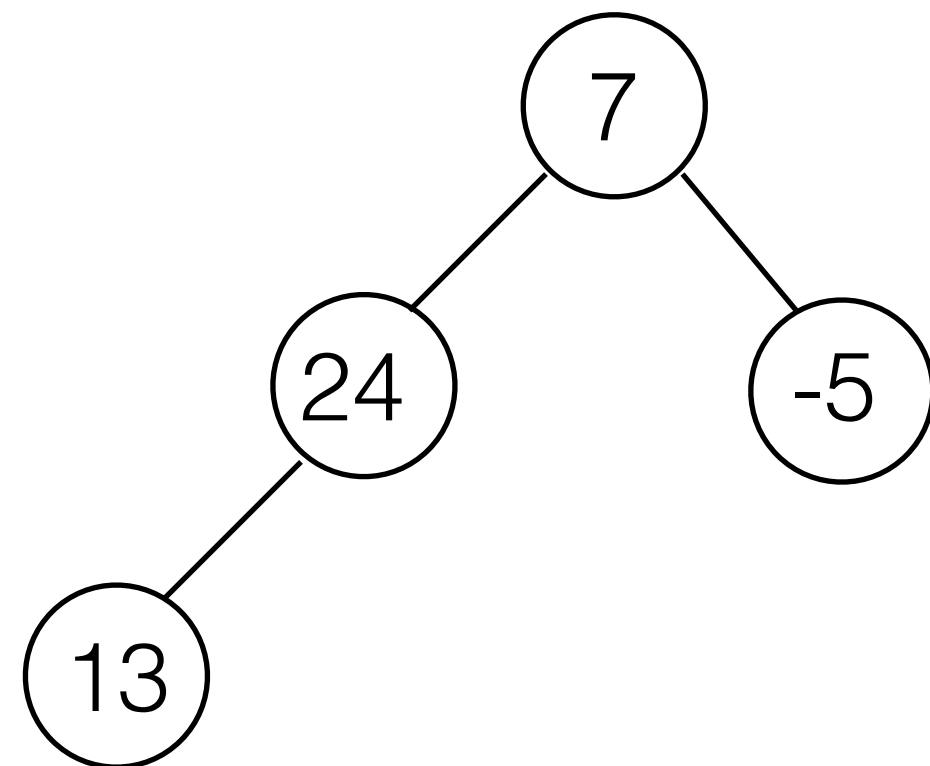
```
class TreeNode:
```

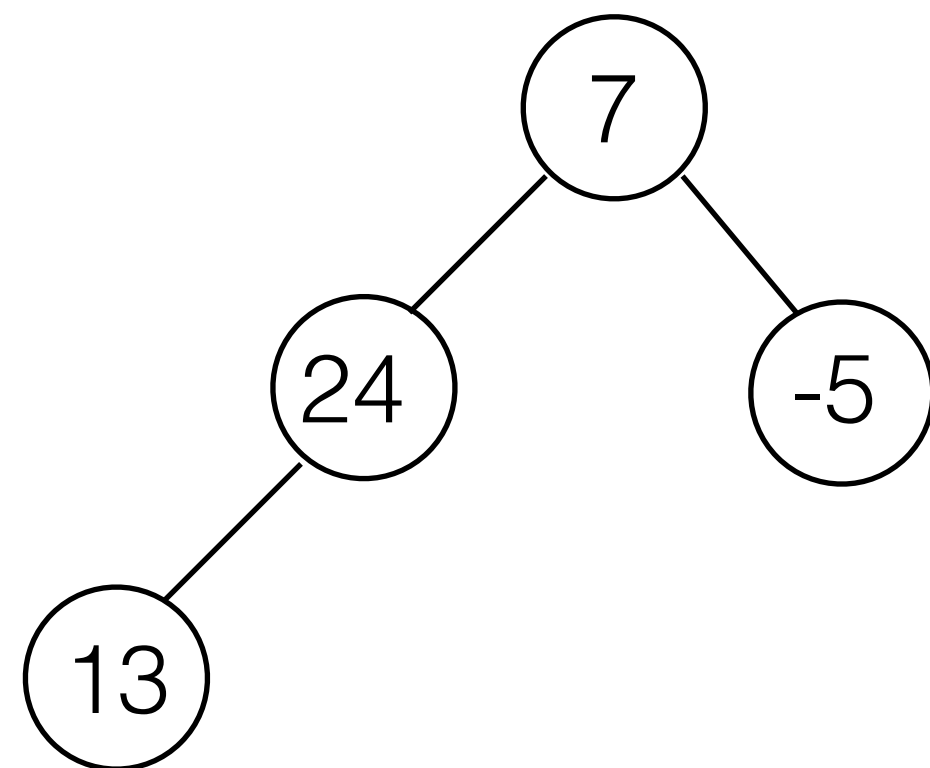
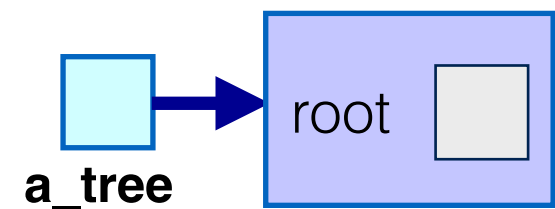
```
    def __init__(self, item=None, left=None, right=None):  
        self.item = item  
        self.left = left  
        self.right = right  
  
    def __str__(self):  
        return str(self.item)
```

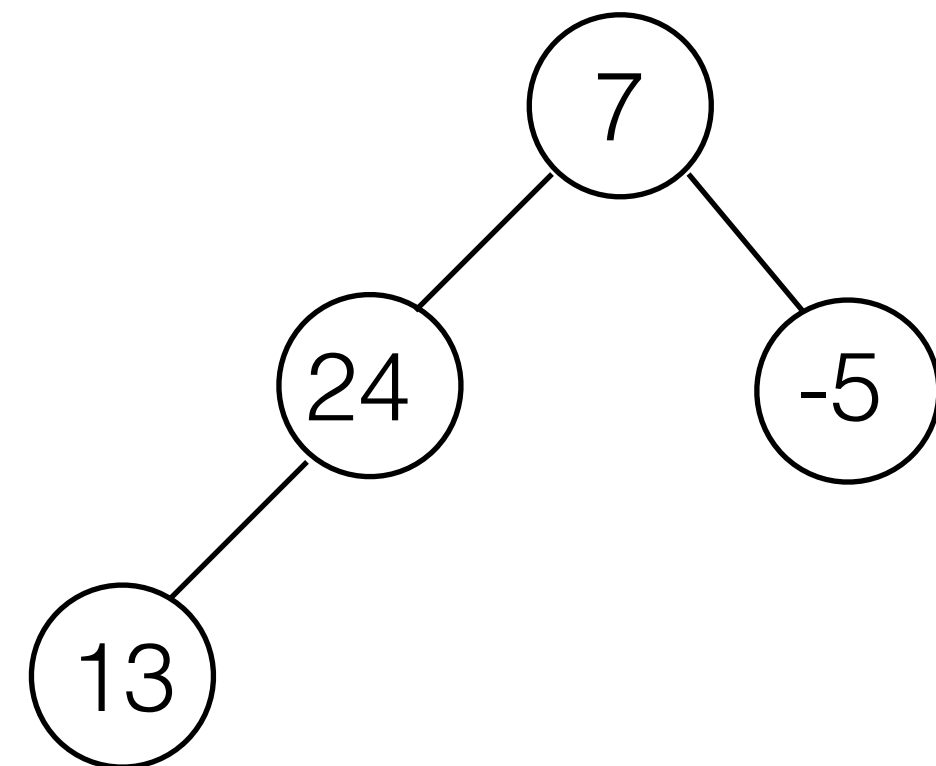
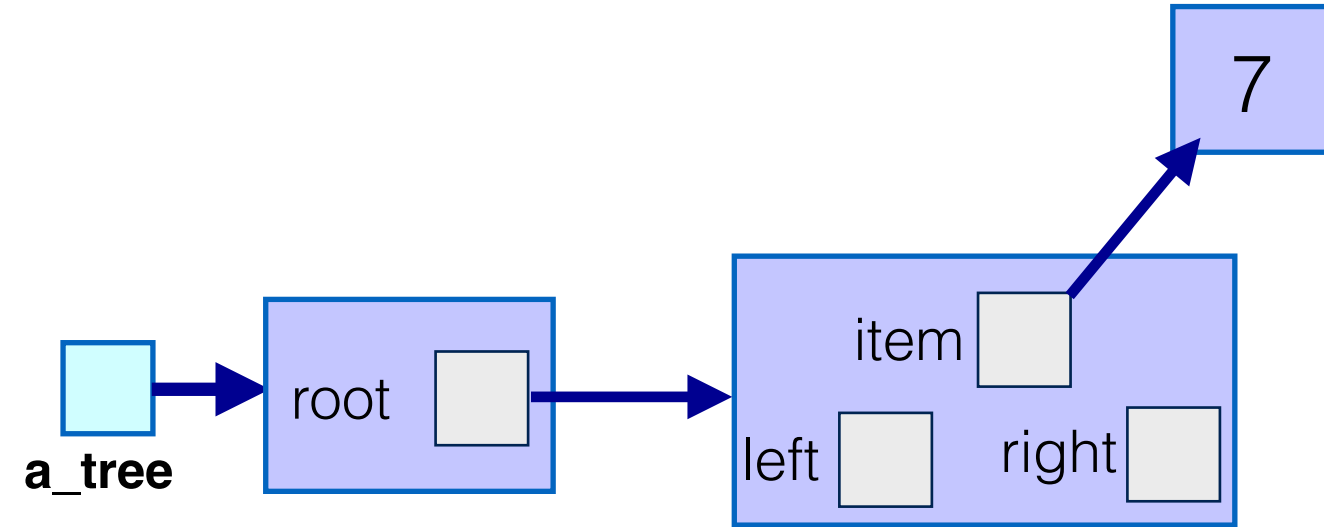
```
class BinaryTree:
```

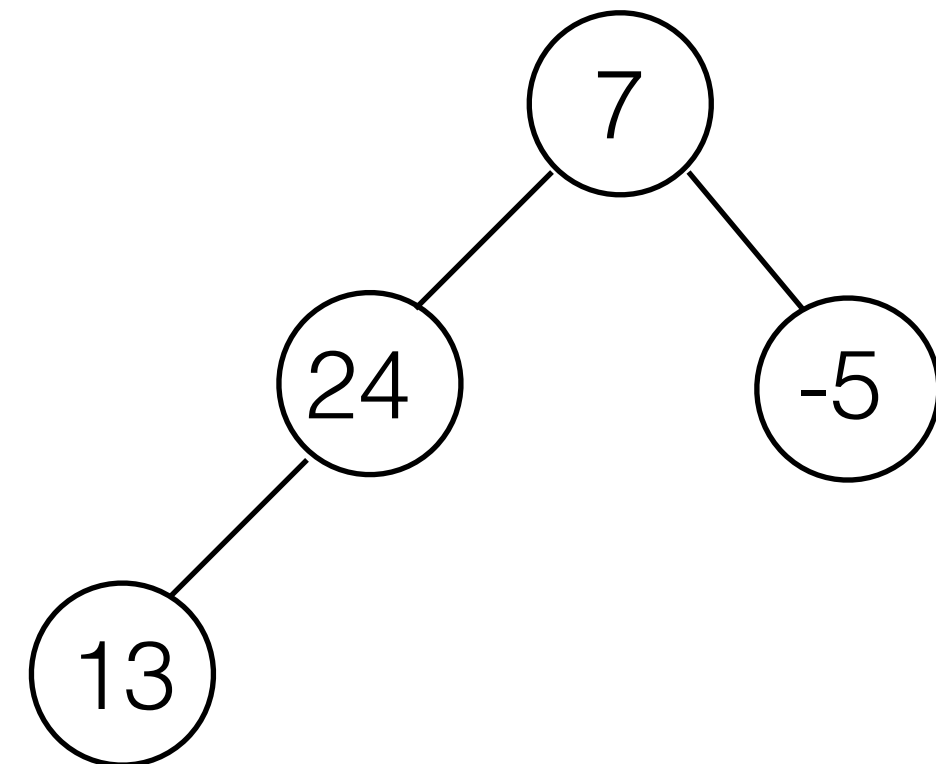
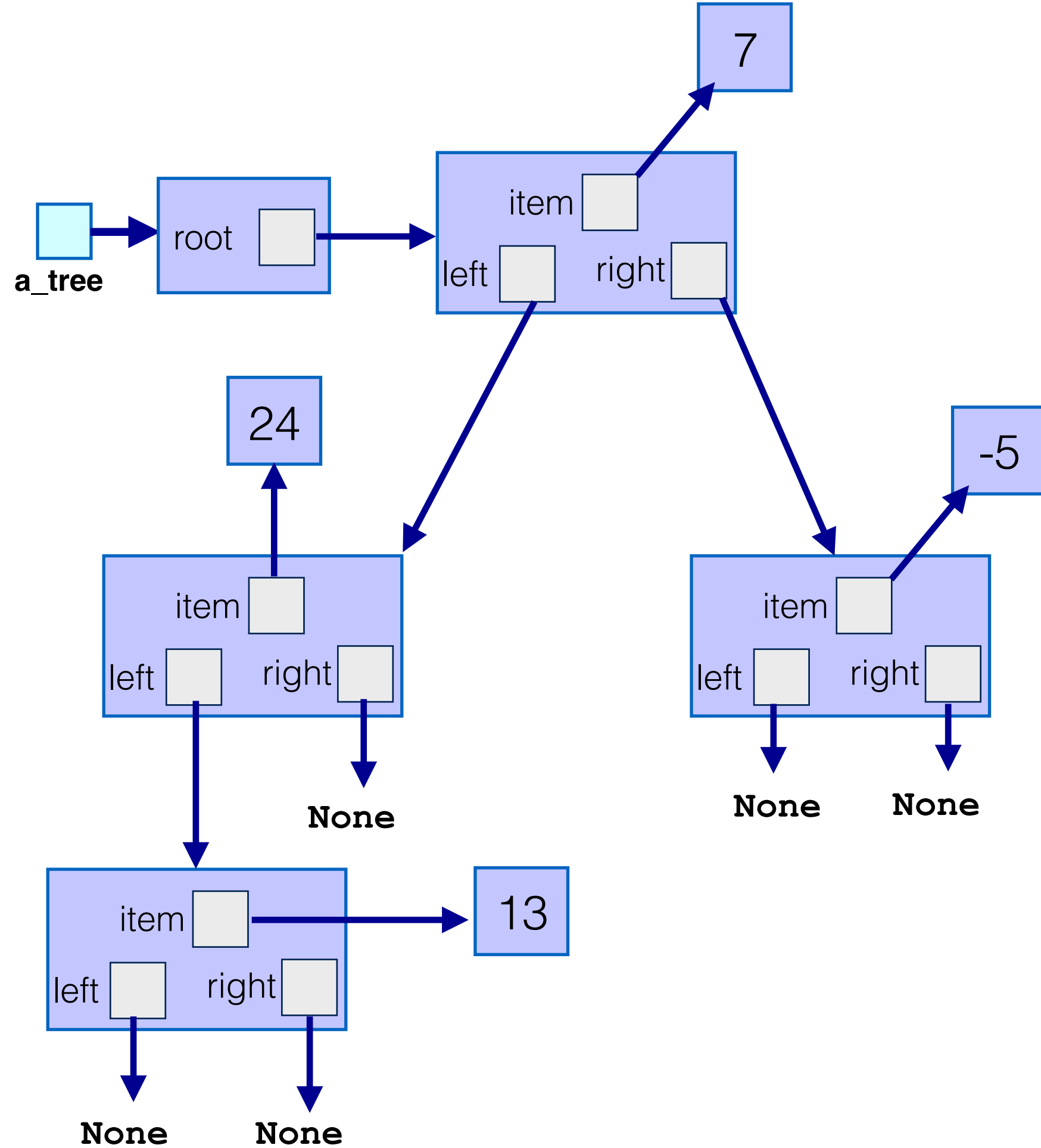
```
    def __init__(self):  
        self.root = None  
  
    def is_empty(self):  
        return self.root is None
```

Only instance variable is a reference to the **root**



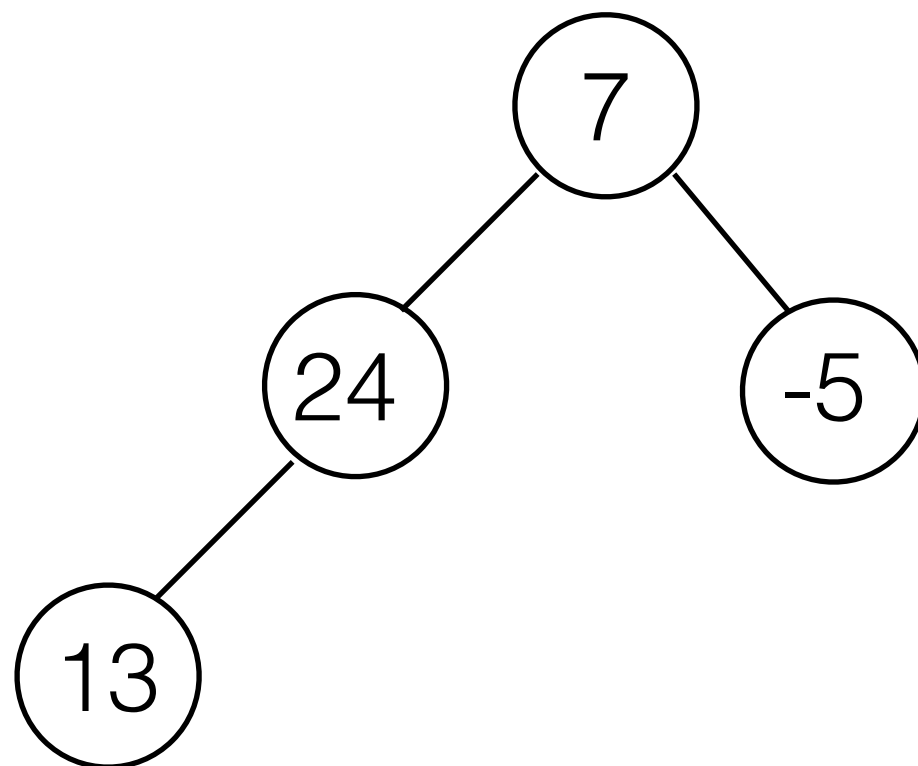




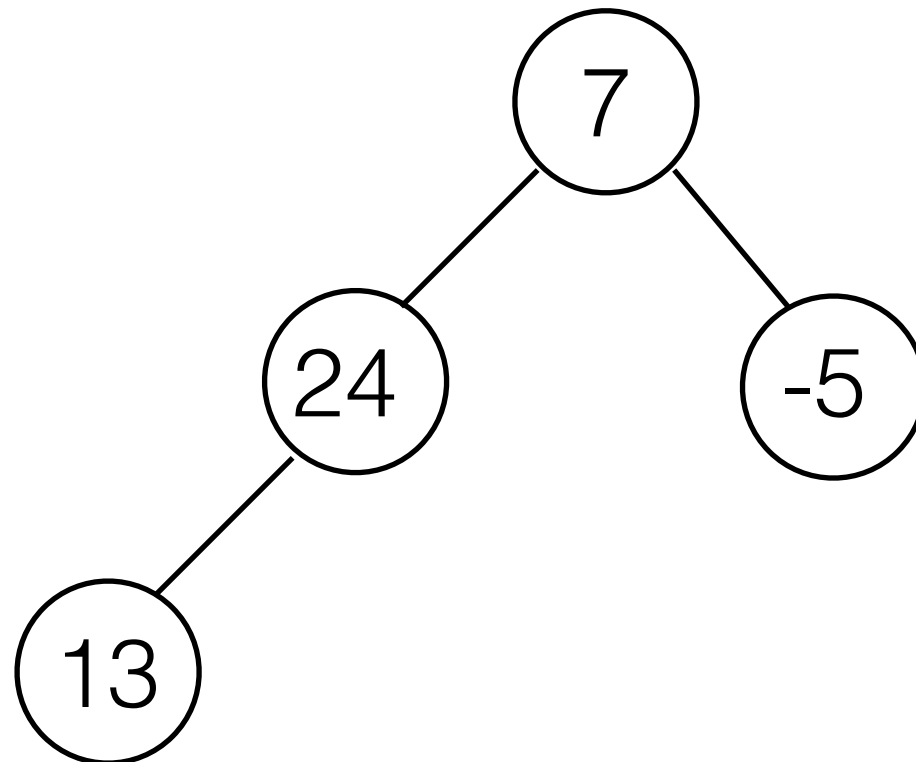


Add an item.

Add 3

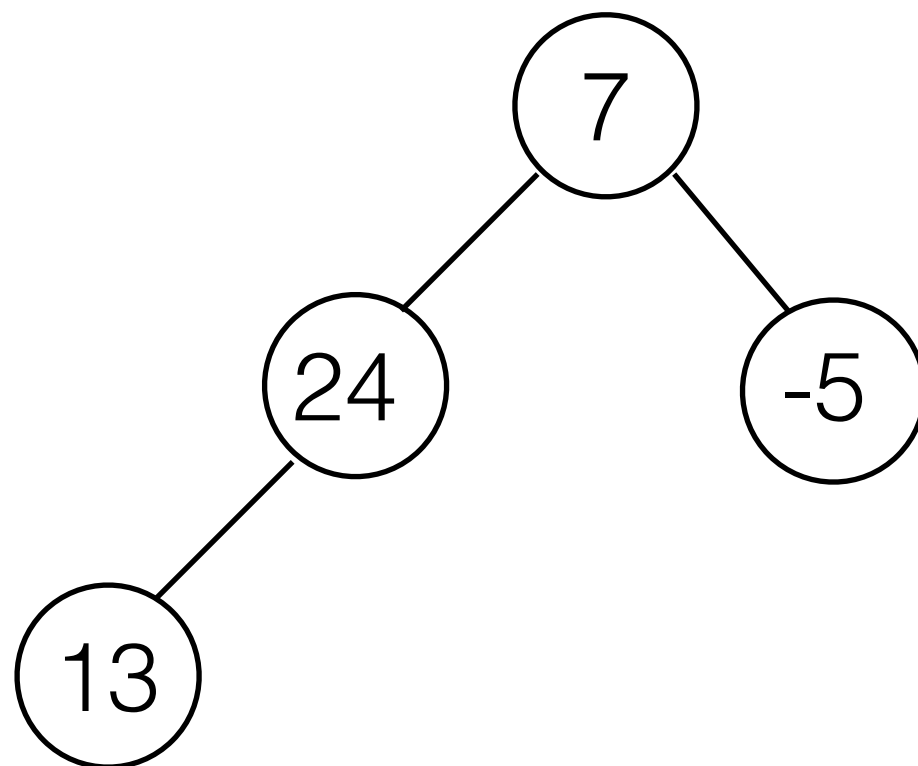


Add 3

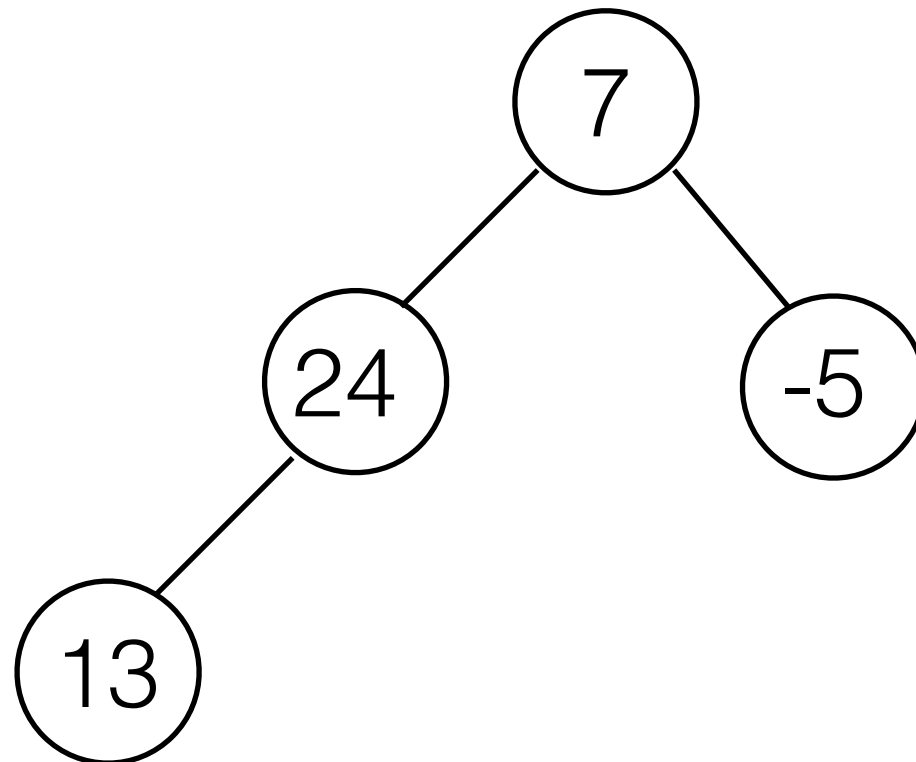


where?

Add 3

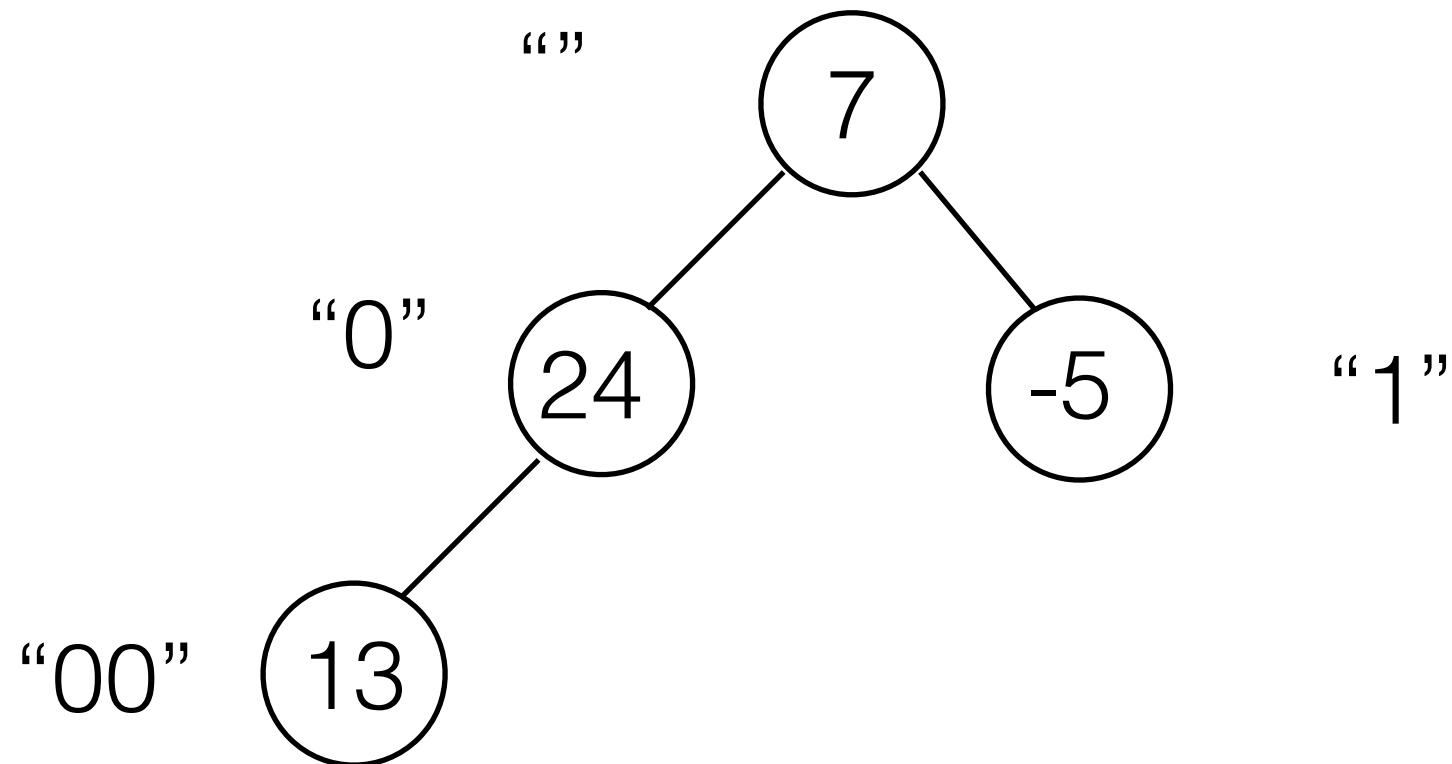


Add 3



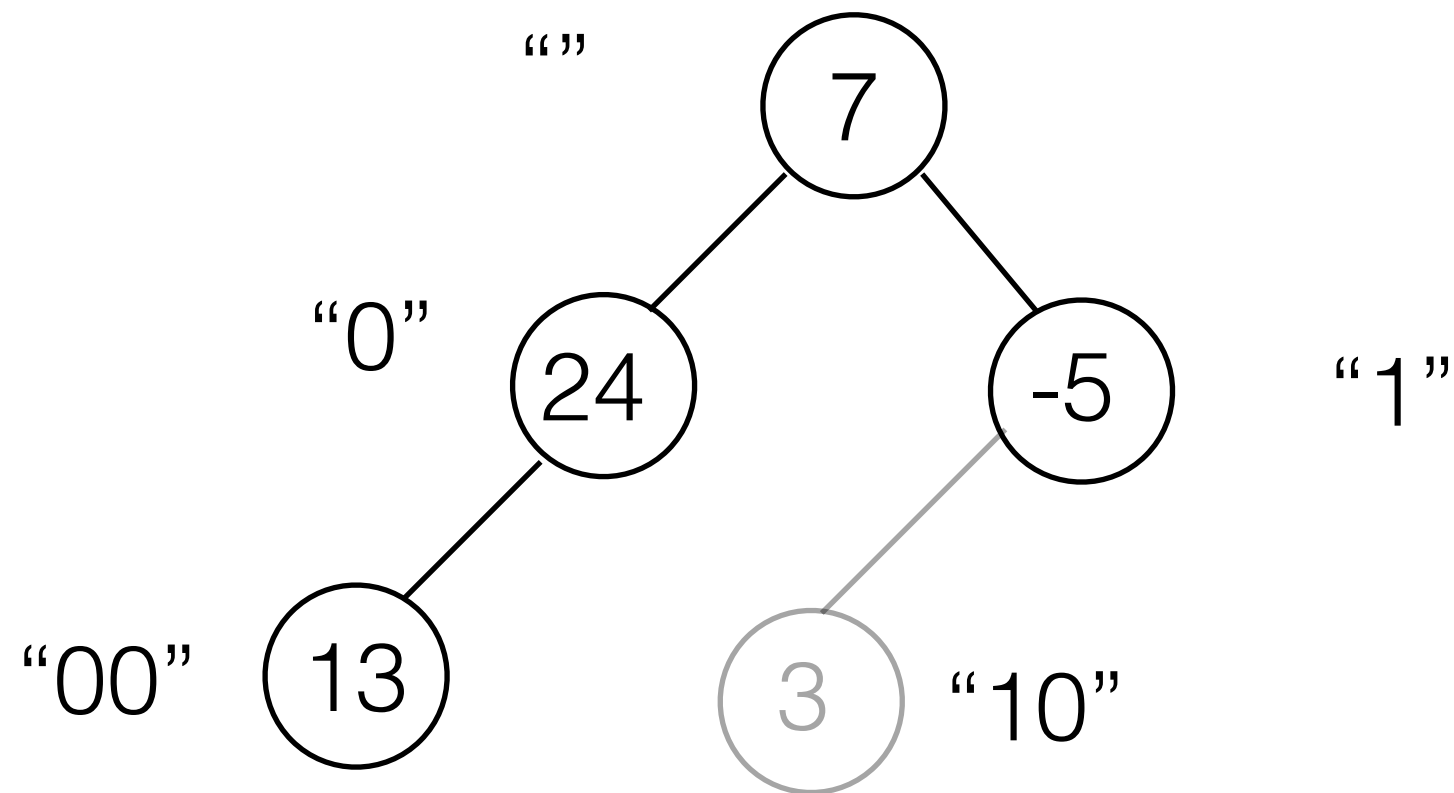
0: Go **left**
1: Go **right**

Add 3



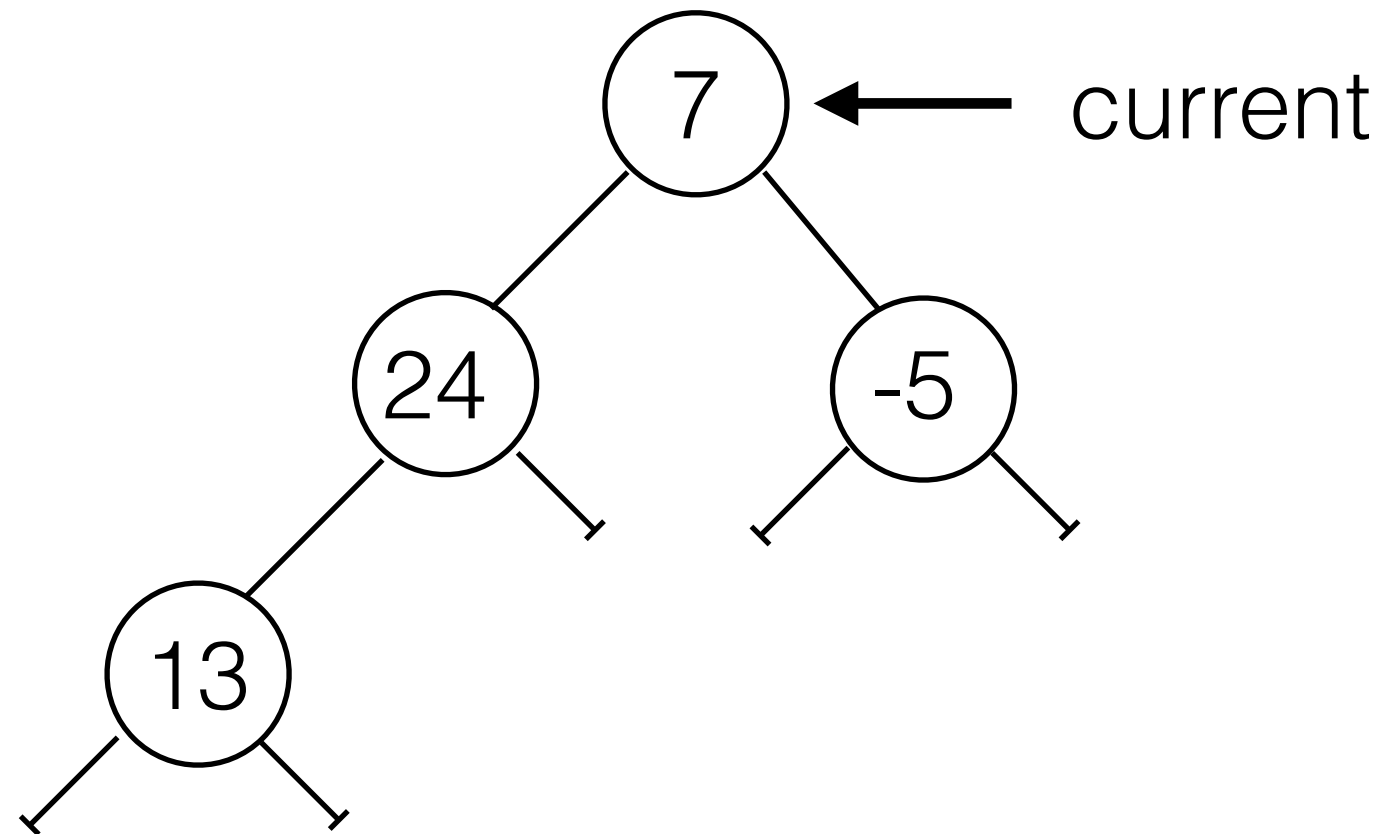
0: Go **left**
1: Go **right**

Add 3



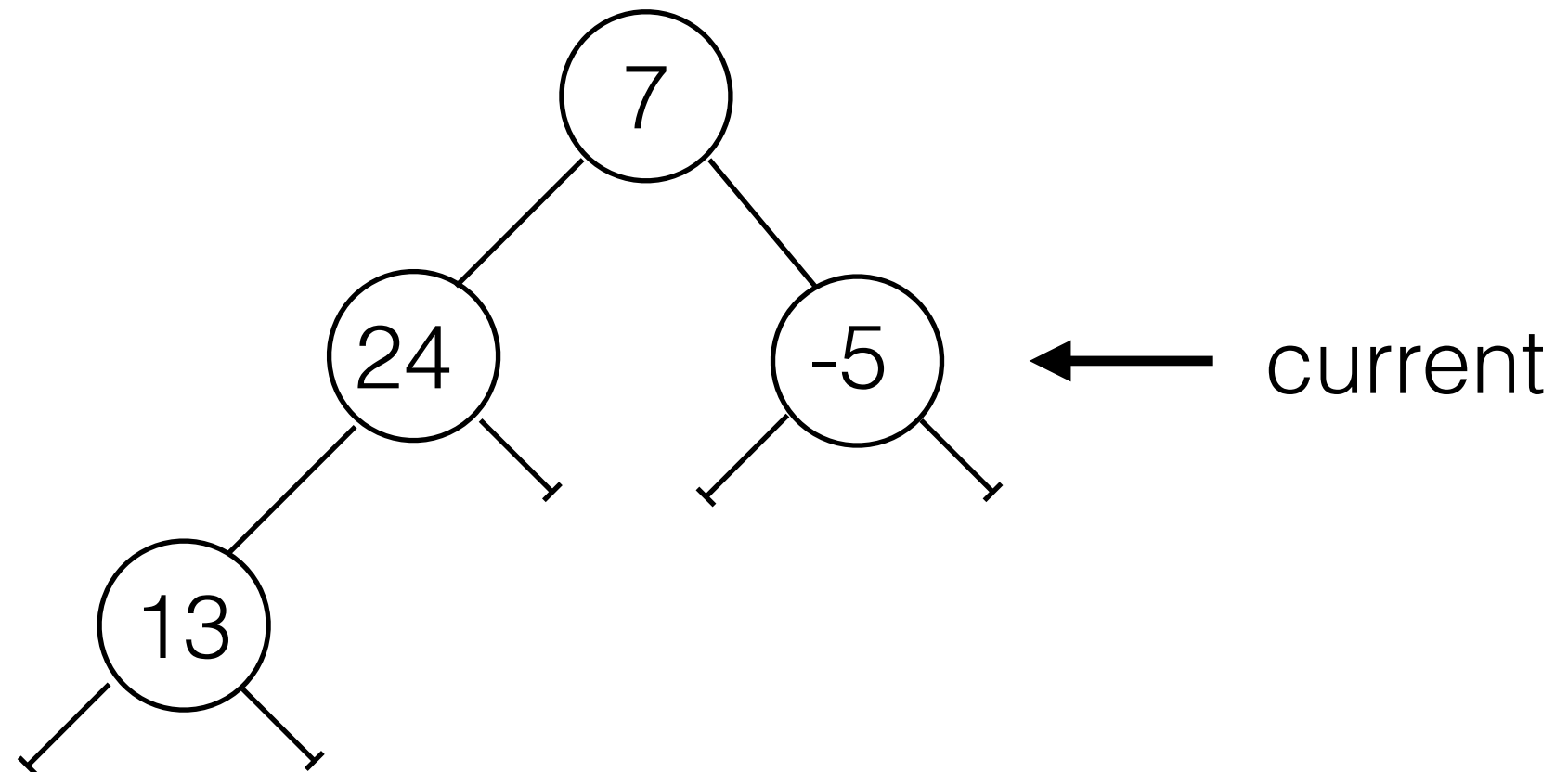
0: Go **left**
1: Go **right**

Add 3



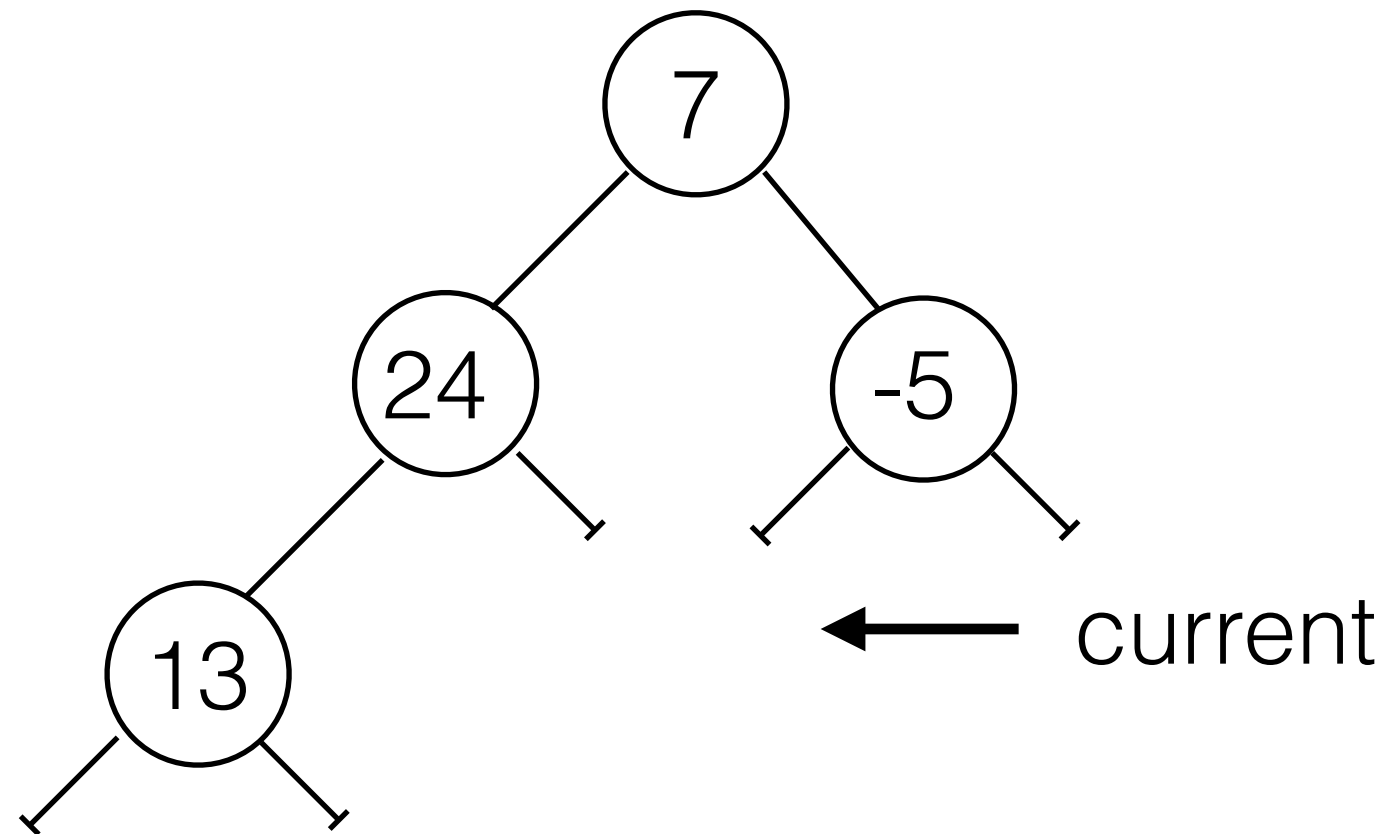
bitstring = "10", **item** = 3

Add 3



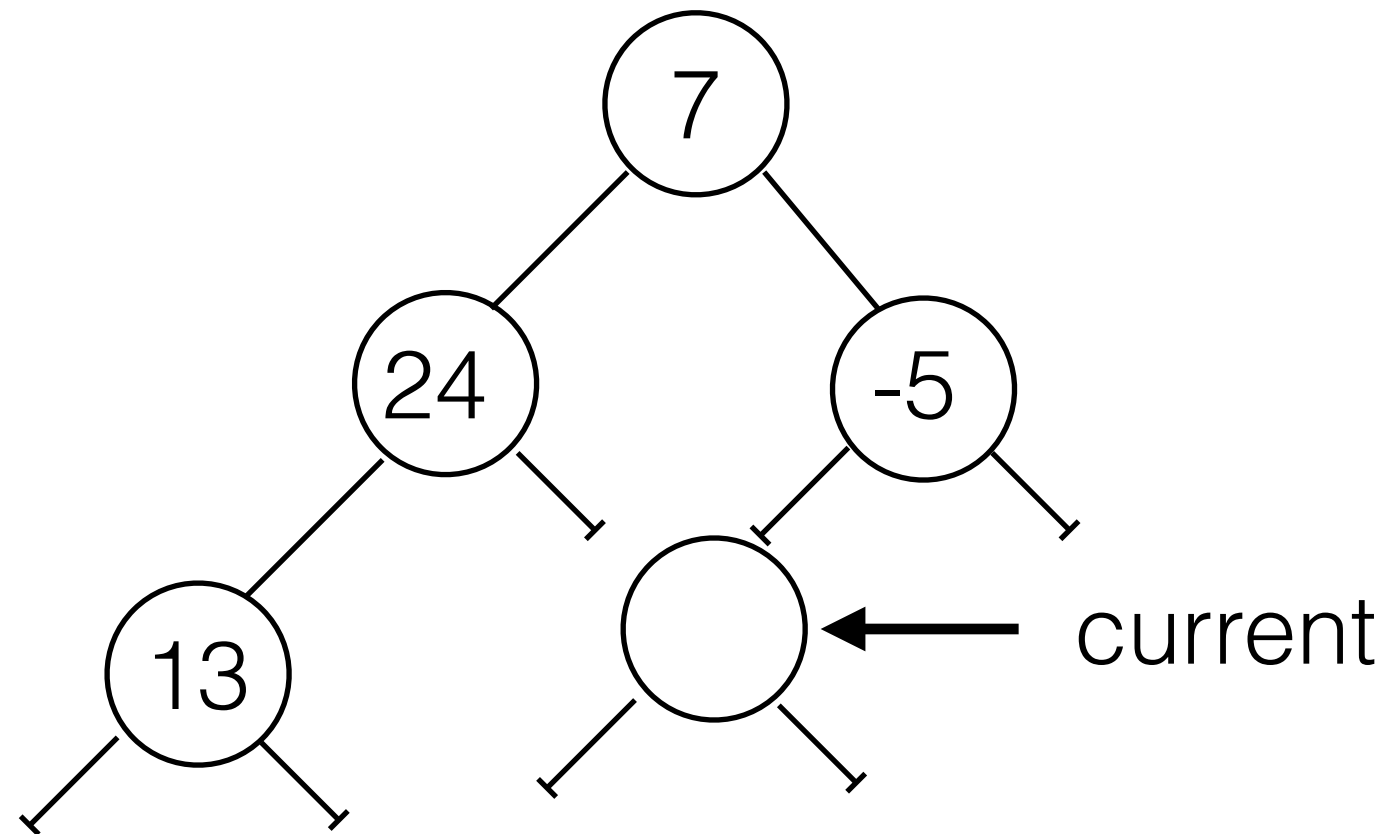
bitstring = “10”, **item**= 3

Add 3



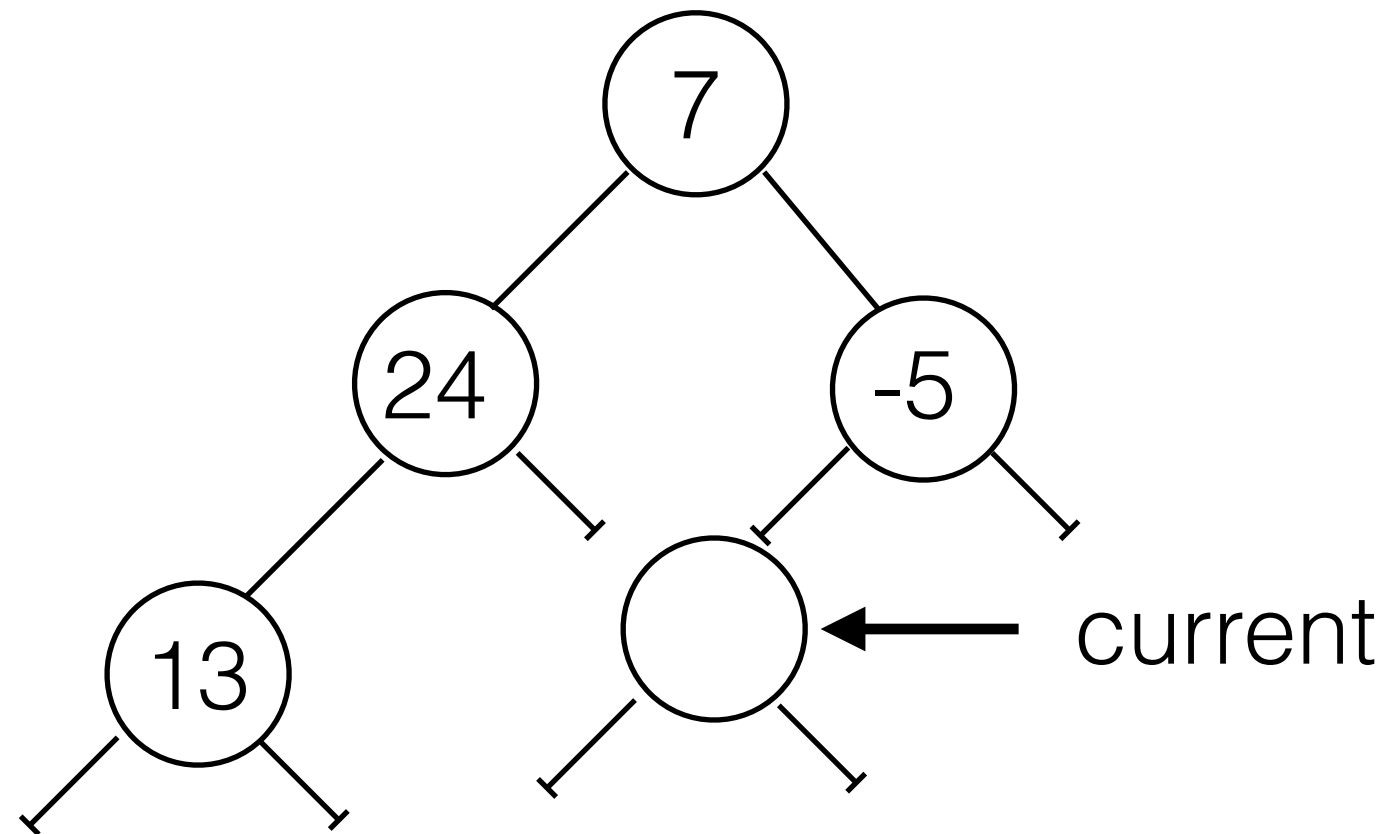
bitstring = "10", **item** = 3

Add 3



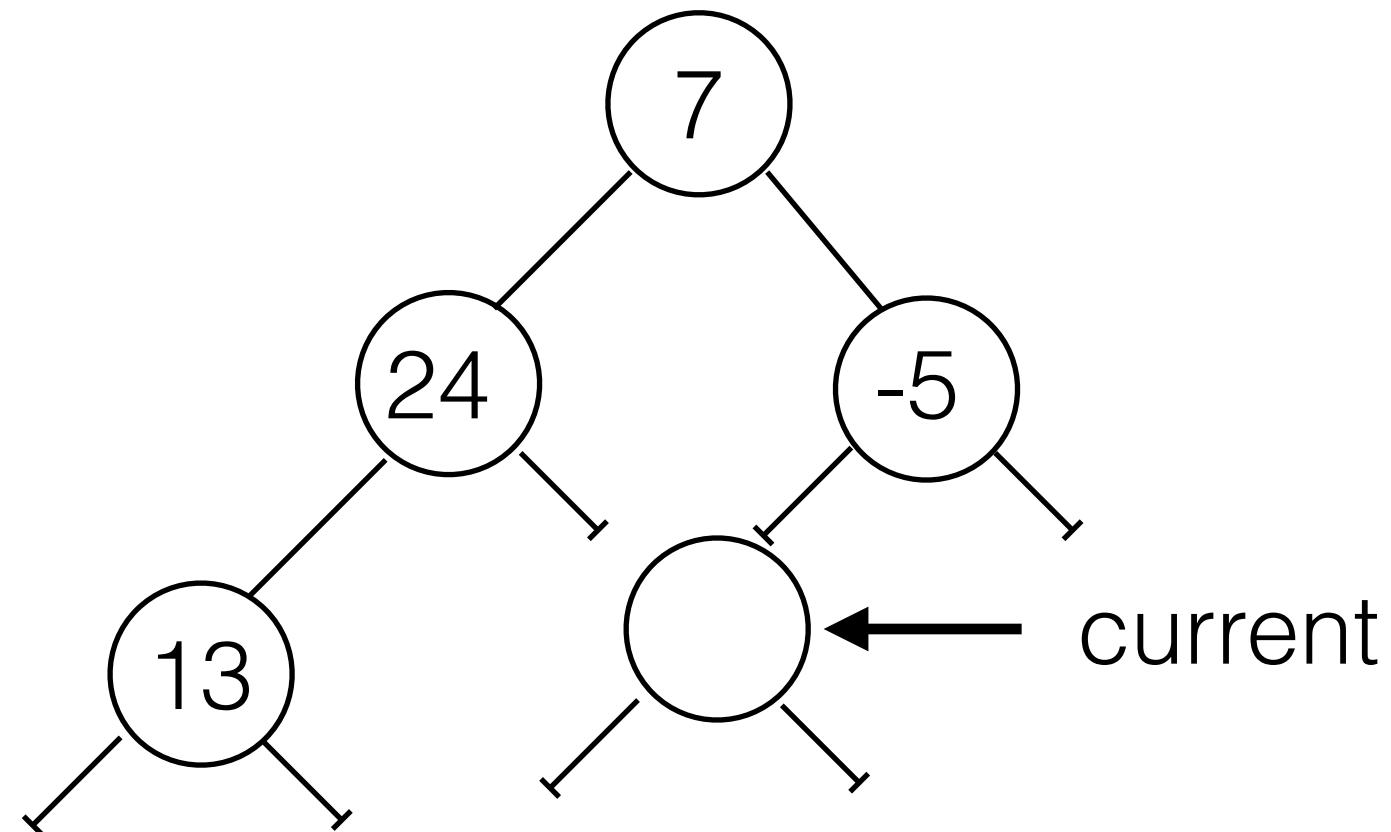
bitstring = "10", **item** = 3

Add 3



bitstring = "10", **item** = 3

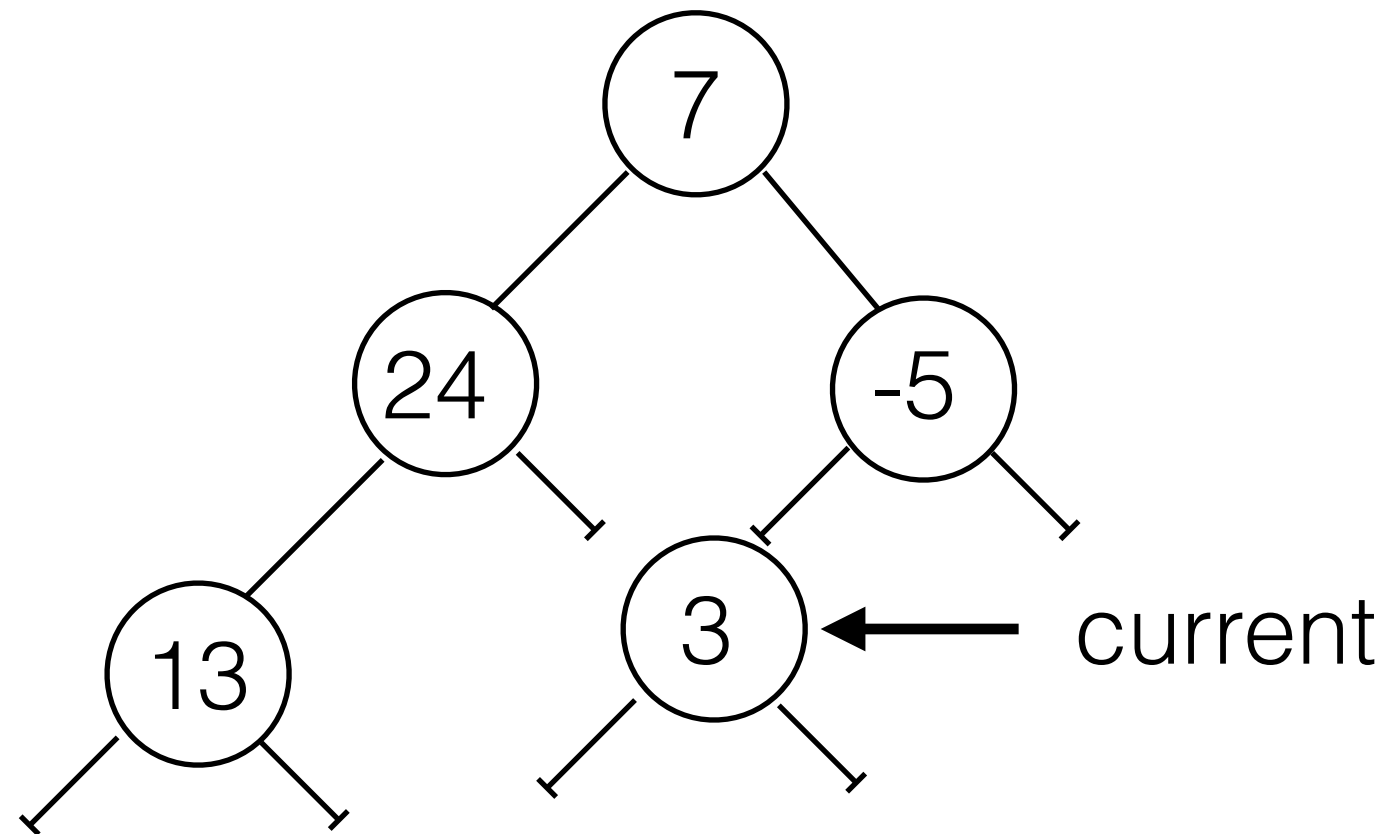
Add 3



bitstring = "10", **item** = 3

Iteration ended, so this must be the place....

Add 3



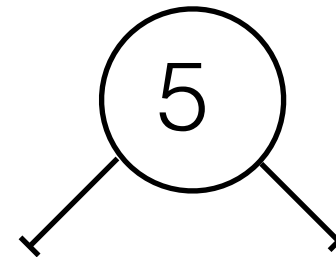
bitstring = "10", **item** = 3

Examples

bitstring = “”, **item**= 5

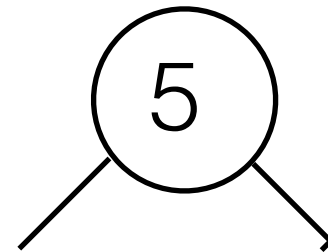
Examples

bitstring = "", **item**= 5



Examples

bitstring = "", **item**= 5

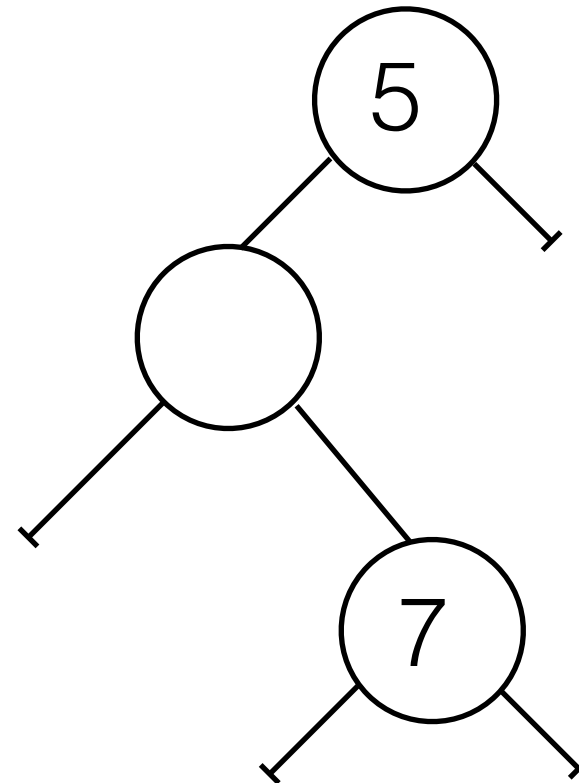


bitstring = "01", **item**= -7

Examples

bitstring = "", **item** = 5

bitstring = "01", **item** = -7

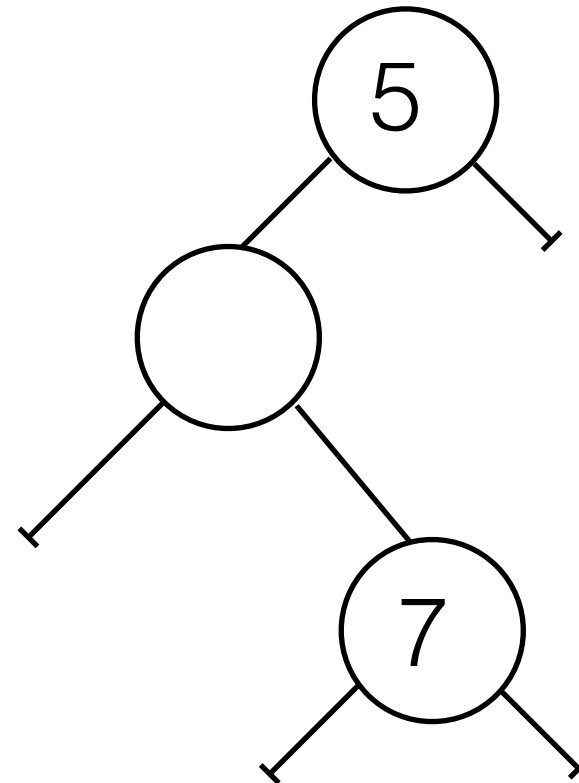


Examples

bitstring = "", **item** = 5

bitstring = "01", **item** = -7

bitstring = " ", **item** = 2

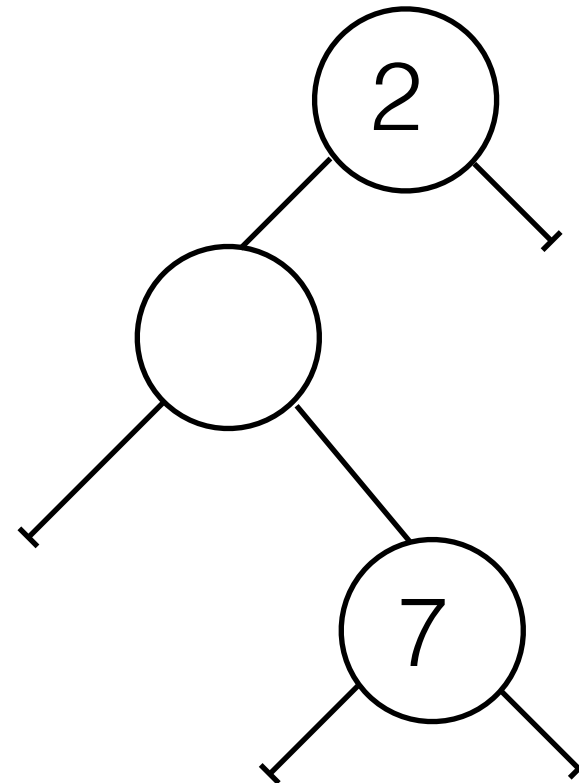


Examples

bitstring = "", **item**= 5

bitstring = "01", **item**= -7

bitstring = " ", **item**= 2

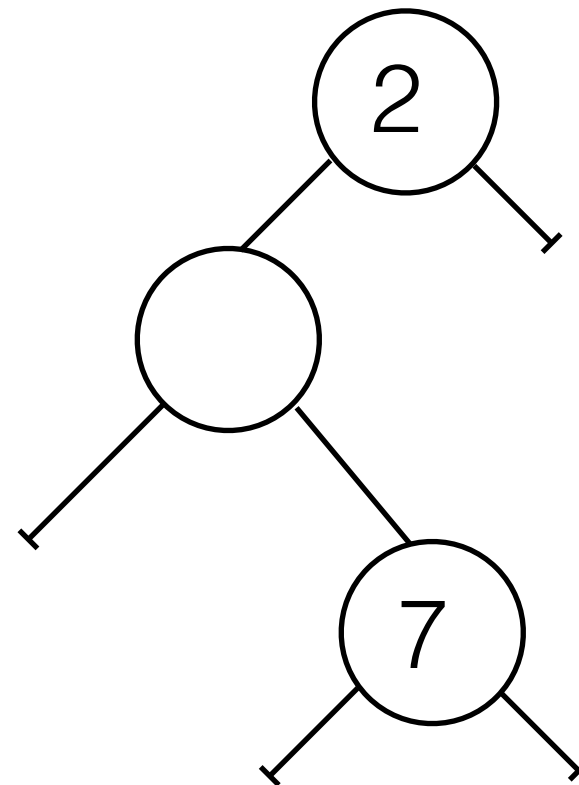


Examples

bitstring = "", **item** = 5

bitstring = "01", **item** = -7

bitstring = " ", **item** = 2



Recursively explore subtree
following "bitstring directions"

```
def add(self, item, position_bitstring):
```

```
def add(self, item, position_bitstring):  
    |    bitstring_iterator = iter(position_bitstring)
```

```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)
```

```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)
```

add sets up the recursion starting at the root
and calls an auxiliary method

```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)  
  
def _add_aux(self, current, item, bitstring_iterator):
```



```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)  
  
def _add_aux(self, current, item, bitstring_iterator):  
    if current is None:
```

```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)  
  
def _add_aux(self, current, item, bitstring_iterator):  
    if current is None:  
        current = TreeNode()
```

```
def add(self, item, position_bitstring):  
    bitstring_iterator = iter(position_bitstring)  
    self.root = self._add_aux(self.root, item, bitstring_iterator)  
  
def _add_aux(self, current, item, bitstring_iterator):  
    if current is None:  
        current = TreeNode()
```

Add empty node if it does not exist

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
```

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
```

Explore left branch

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
        elif bit == "1":
            current.right = self._add_aux(current.right, item, bitstring_iterator)
```

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
        elif bit == "1":
            current.right = self._add_aux(current.right, item, bitstring_iterator)
```

Explore right branch

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
        elif bit == "1":
            current.right = self._add_aux(current.right, item, bitstring_iterator)
    except StopIteration:
        current.item = item
```



```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
        elif bit == "1":
            current.right = self._add_aux(current.right, item, bitstring_iterator)
    except StopIteration:
        current.item = item
```

Bitstring is telling me I have arrived at the correct stop

```
def add(self, item, position_bitstring):
    bitstring_iterator = iter(position_bitstring)
    self.root = self._add_aux(self.root, item, bitstring_iterator)

def _add_aux(self, current, item, bitstring_iterator):
    if current is None:
        current = TreeNode()
    try:
        bit = next(bitstring_iterator)
        if bit == "0":
            current.left = self._add_aux(current.left, item, bitstring_iterator)
        elif bit == "1":
            current.right = self._add_aux(current.right, item, bitstring_iterator)
    except StopIteration:
        current.item = item
    return current
```

Traversal

Traversal

- Systematic way of **visiting**/processing **all the nodes**

Traversal

- Systematic way of **visiting**/processing **all the nodes**
- **Methods**: Preorder, Inorder, and Postorder
- They **all** traverse the left subtree before the right subtree. It's all about the **position of the root**.

Left
subtree

Right
subtree

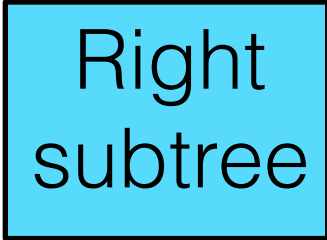
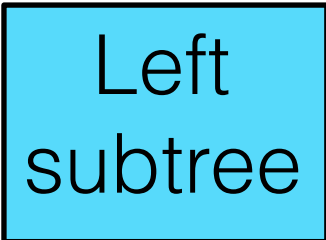
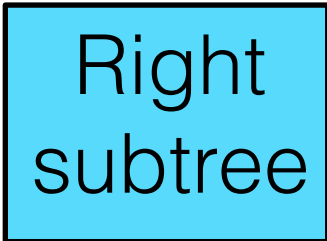
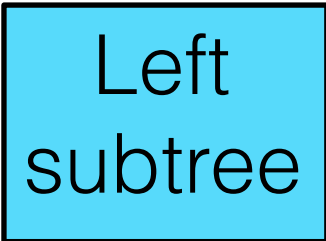
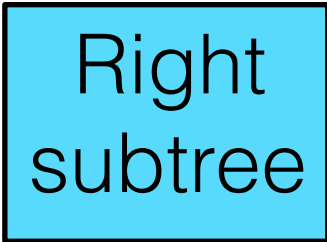
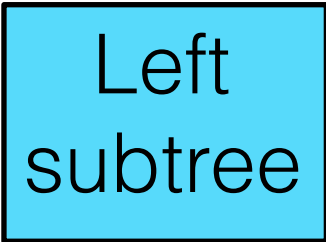
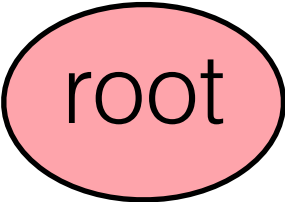
Left
subtree

Right
subtree

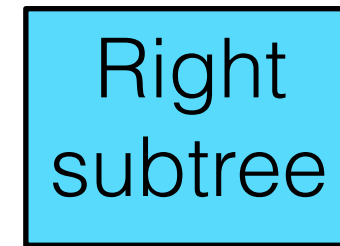
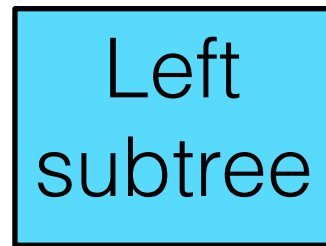
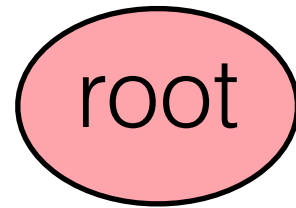
Left
subtree

Right
subtree

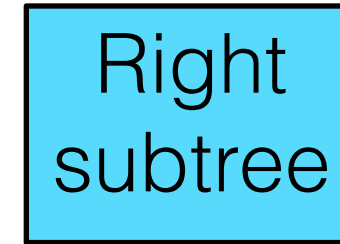
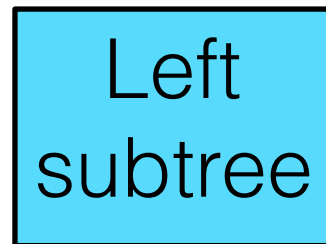
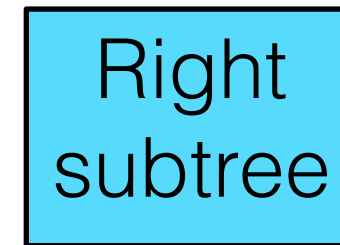
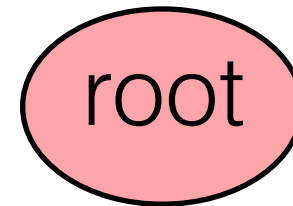
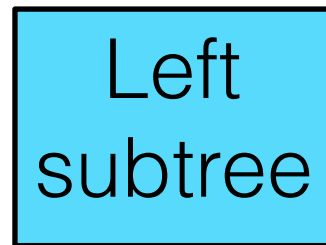
Preorder



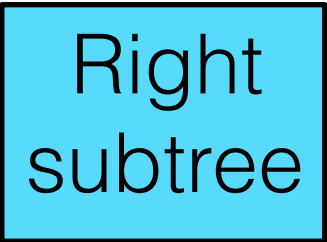
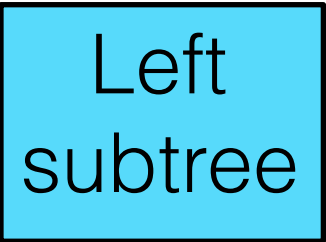
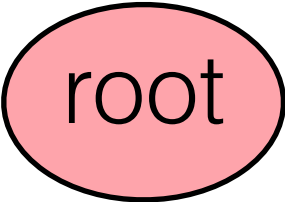
Preorder



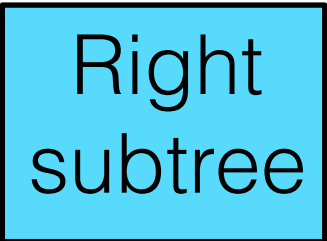
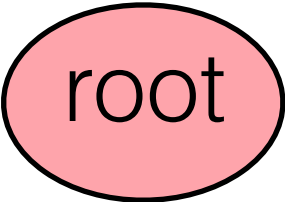
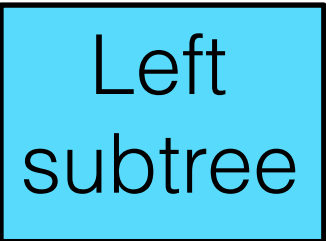
Inorder



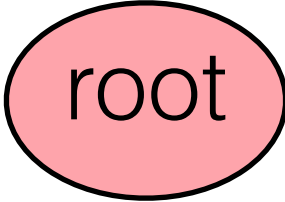
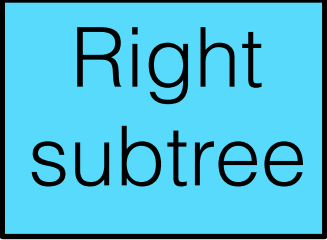
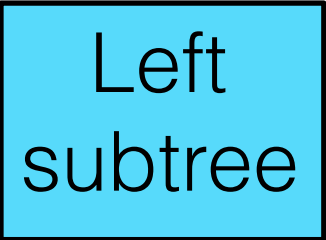
Preorder



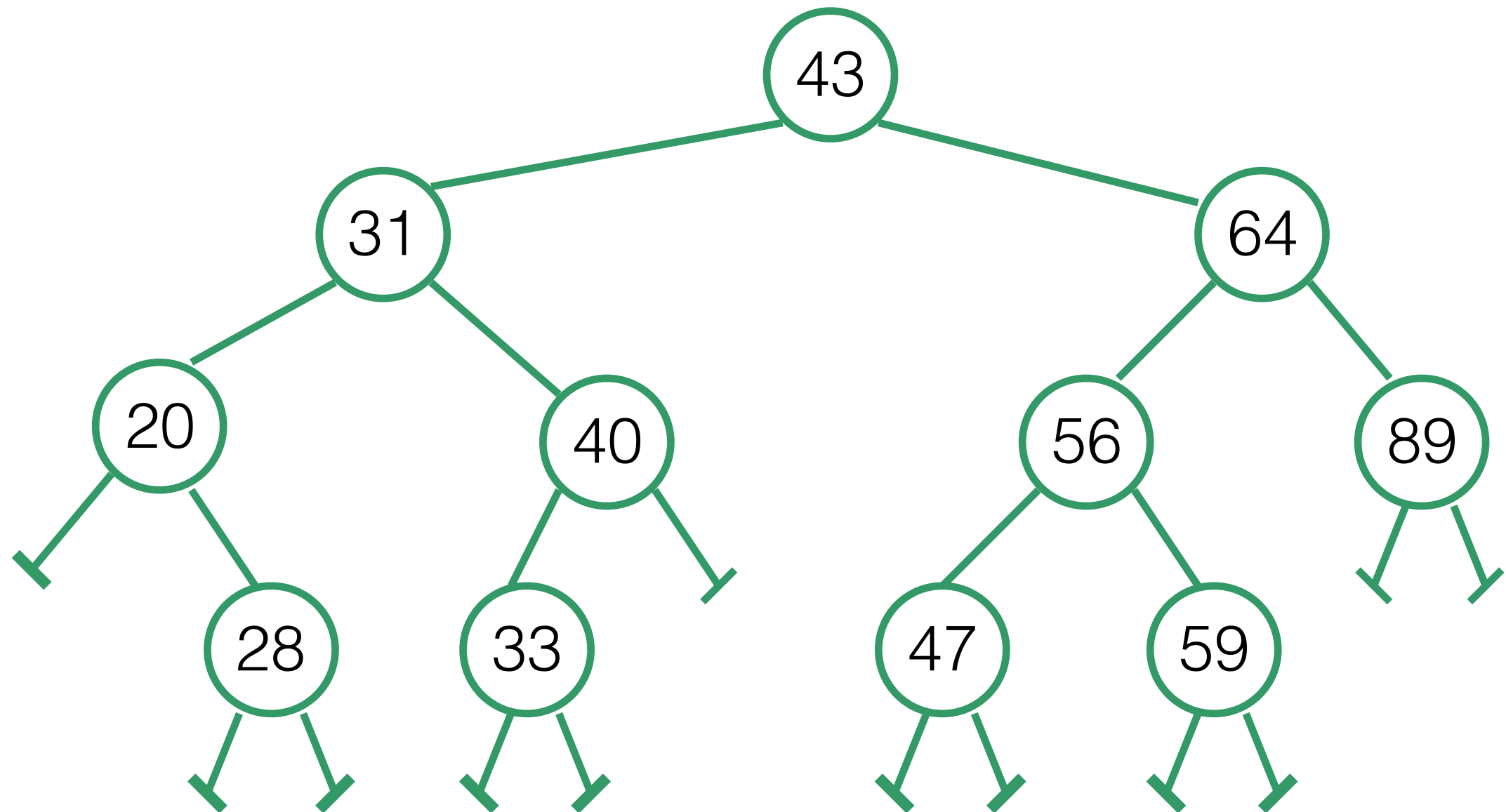
Inorder



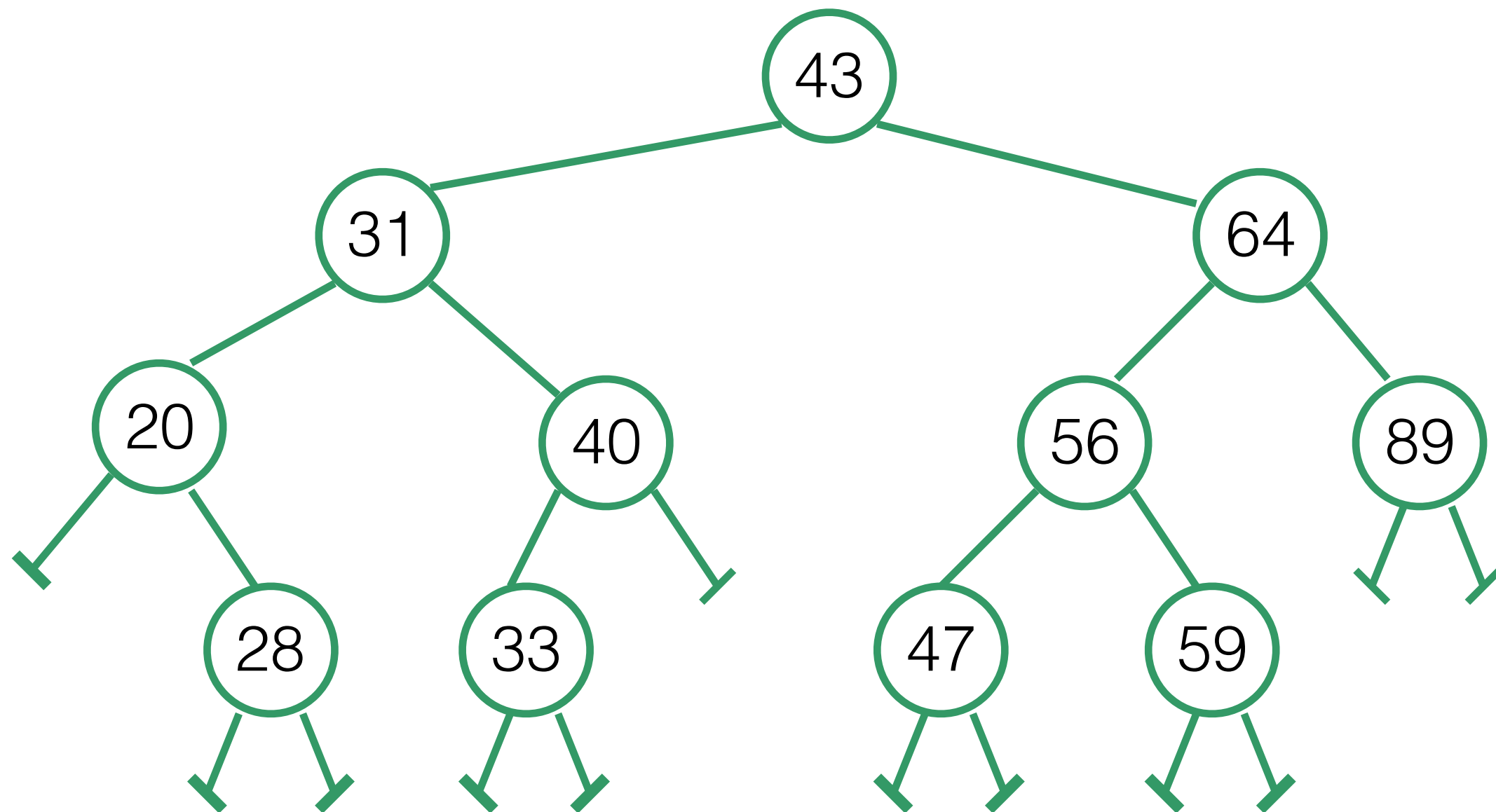
Postorder



Example: Preorder



Example: Preorder



43	31	20	28	40	33	64	56	47	59	89
----	----	----	----	----	----	----	----	----	----	----

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):
```

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)
```

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)
```

Auxiliary method receives a reference to the “next root”

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):
```

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case
```


Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case
```

Work to do...

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case  
        print(current)
```

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
class TreeNode:
    def __init__(self, item=None, left=None, right=None):
        self.item = item
        self.left = left
        self.right = right

    def __str__(self):
        return str(self.item)
```

```
def print_preorder(self):
    self._print_preorder_aux(self.root)
```

```
def _print_preorder_aux(self, current):
    if current is not None: # if not a base case
        print(current)
```

Print Preorder Traversal

- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case  
        print(current)  
        self._print_preorder_aux(current.left)
```

Print Preorder Traversal

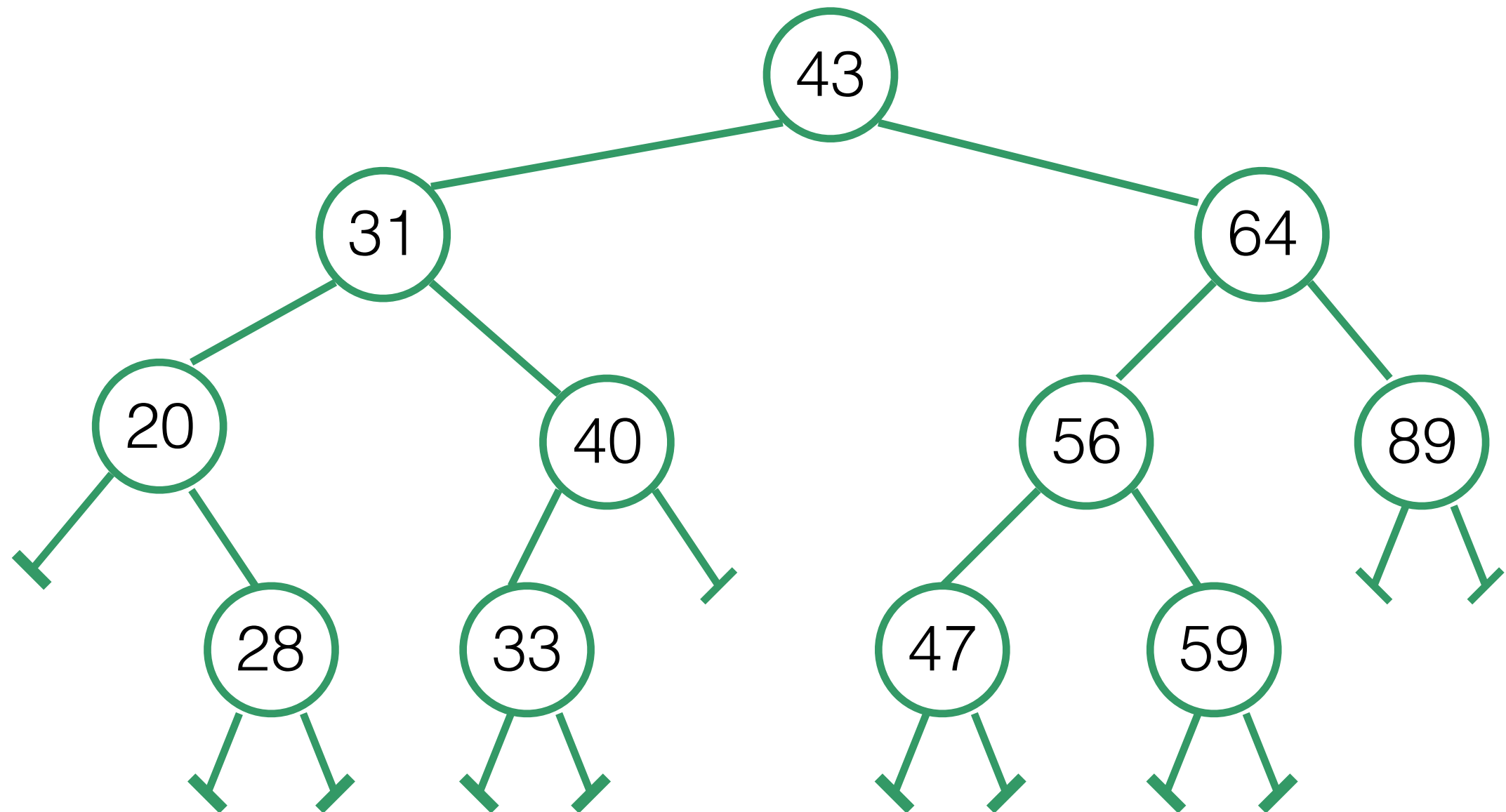
- 1) Print the **root** node
- 2) Traverse the **left** subtree
- 3) Traverse the **right** subtree

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case  
        print(current)  
        self._print_preorder_aux(current.left)  
        self._print_preorder_aux(current.right)
```

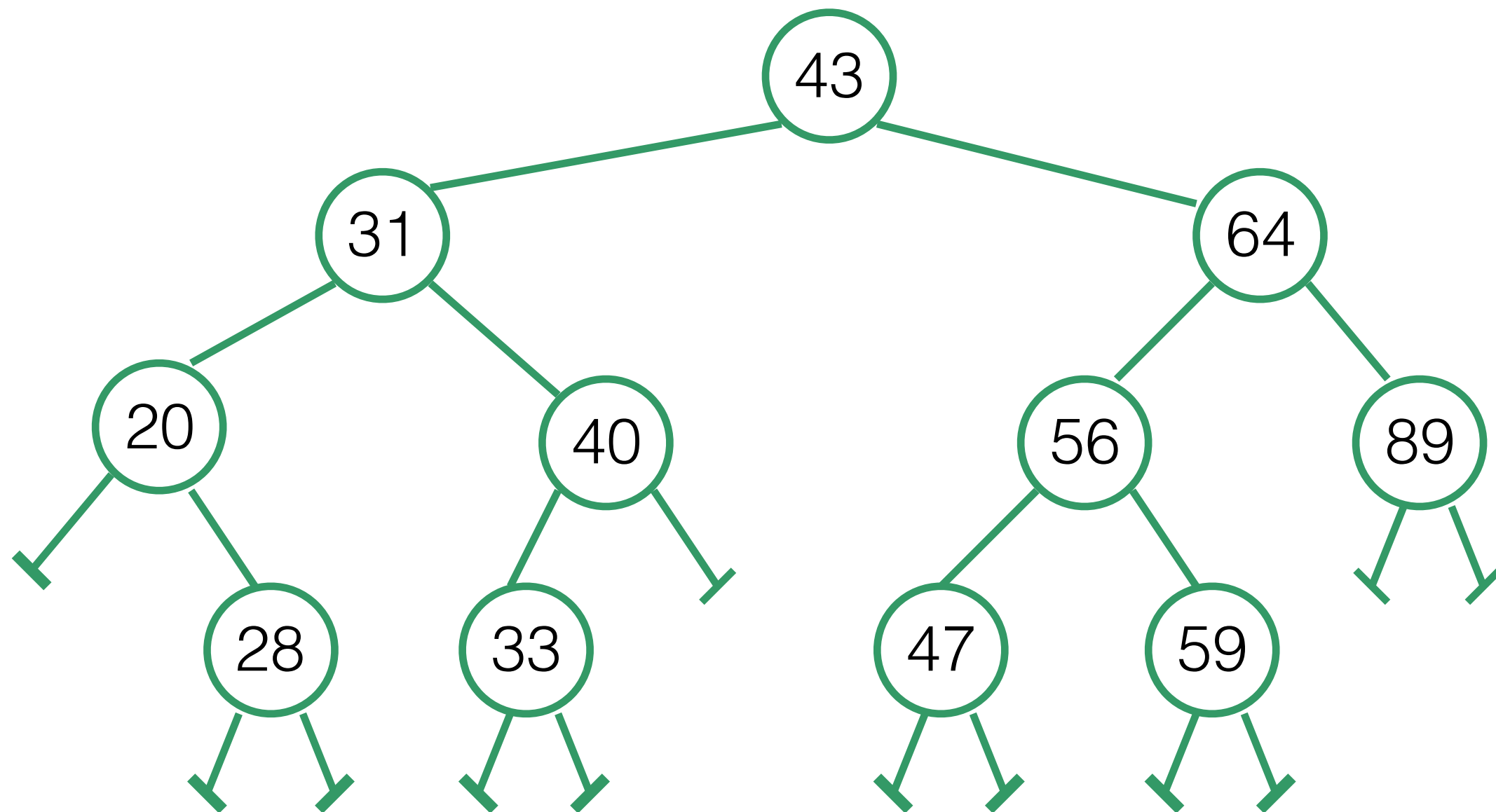
Print Preorder Traversal

```
def print_preorder(self):  
    self._print_preorder_aux(self.root)  
  
def _print_preorder_aux(self, current):  
    if current is not None: # if not a base case  
        print(current)  
        self._print_preorder_aux(current.left)  
        self._print_preorder_aux(current.right)
```

Example: Inorder



Example: Inorder



20	28	31	33	40	43	47	56	59	64	89
----	----	----	----	----	----	----	----	----	----	----

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)
```

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):
```

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case
```

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case
```

Work to do...

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case  
        self._print_inorder_aux(current.left)
```

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case  
        self._print_inorder_aux(current.left)  
        print(current)
```

Print In-order Traversal

- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

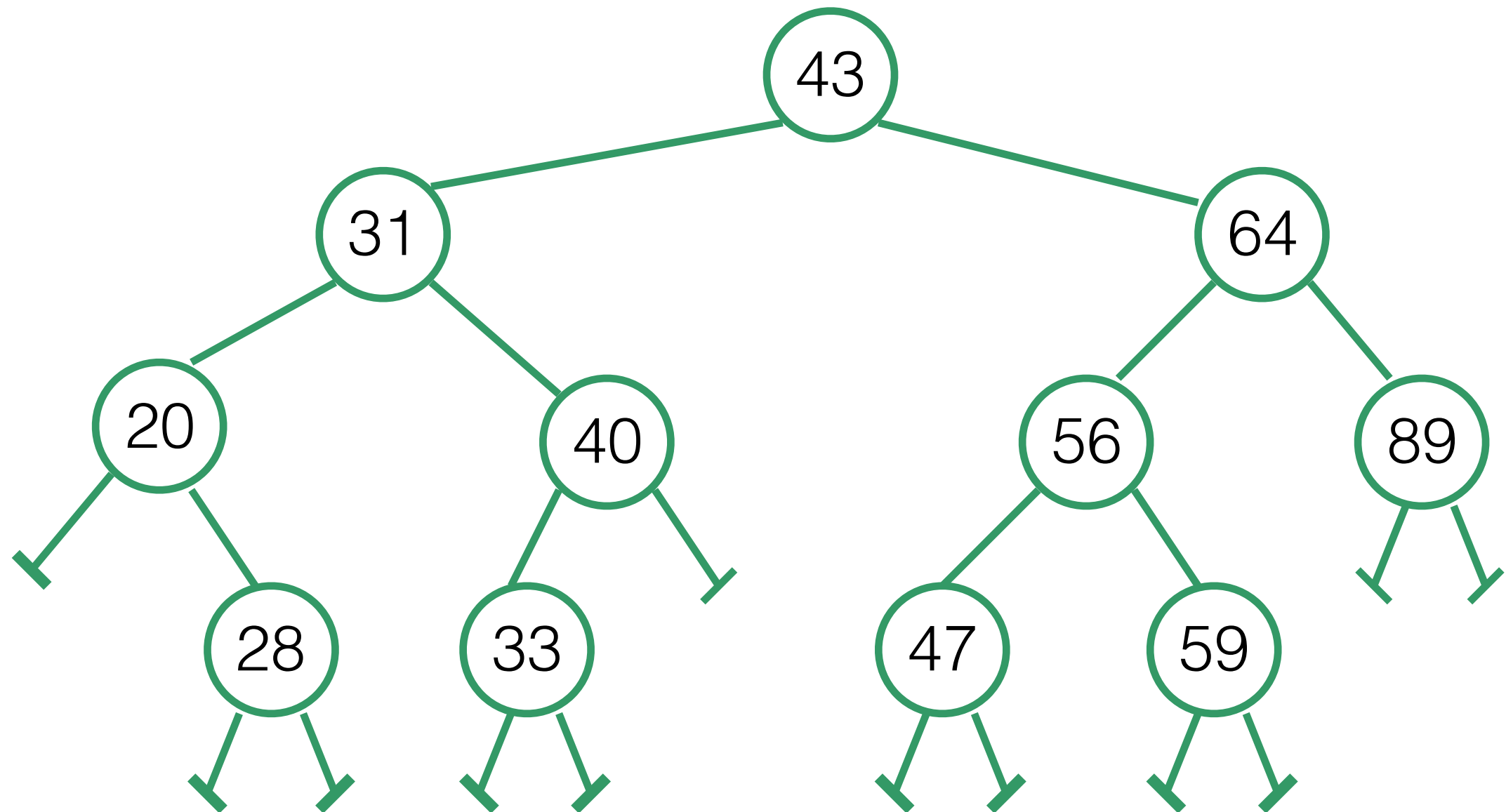
```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case  
        self._print_inorder_aux(current.left)  
        print(current)  
        self._print_inorder_aux(current.right)
```

Print In-order Traversal

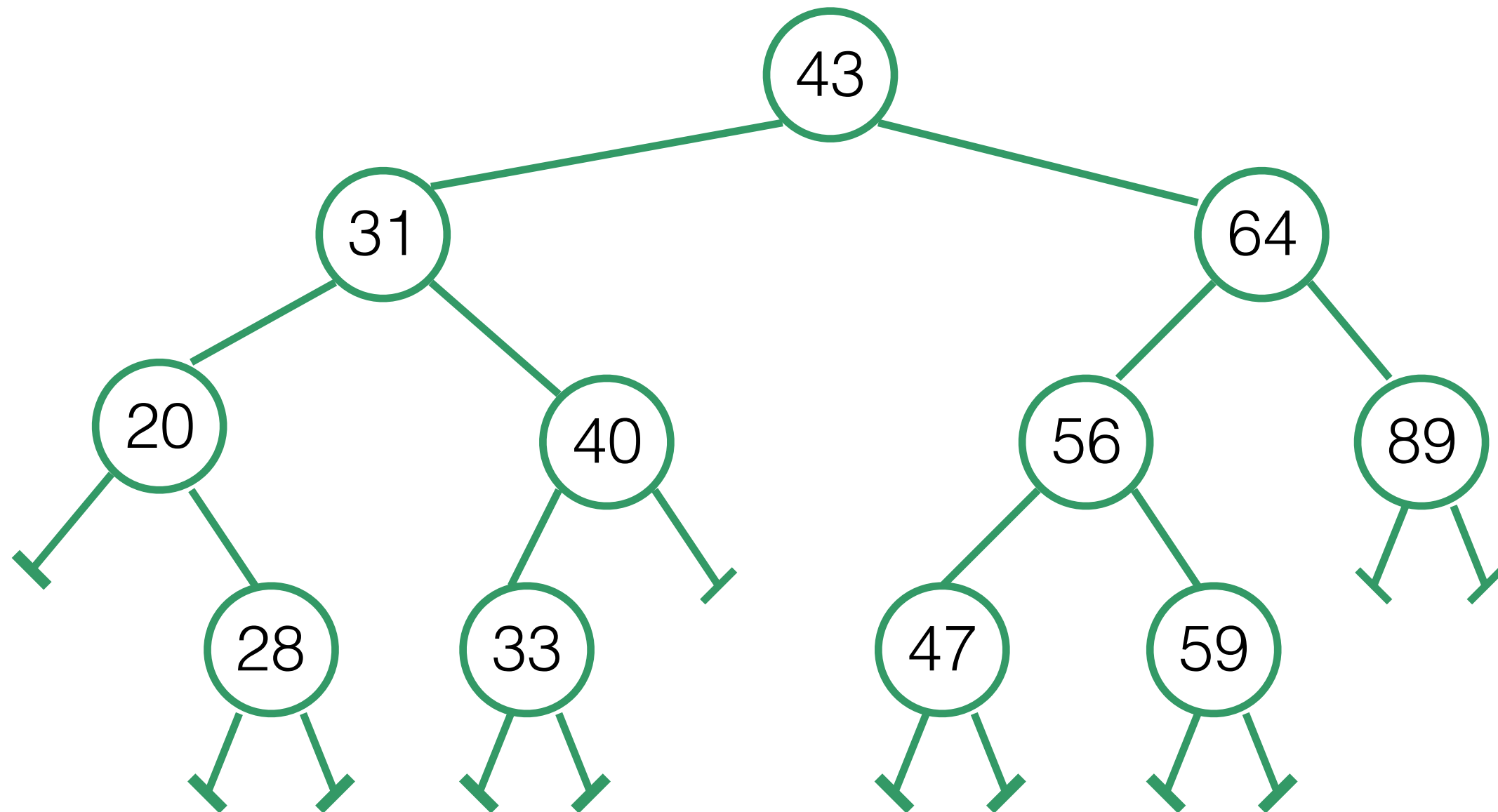
- 1) Traverse the **left** subtree
- 2) Print the **root** node
- 3) Traverse the **right** subtree

```
def print_inorder(self):  
    self._print_inorder_aux(self.root)  
  
def _print_inorder_aux(self, current):  
    if current is not None: # if not a base case  
        self._print_inorder_aux(current.left)  
        print(current)  
        self._print_inorder_aux(current.right)
```


Example: Postorder



Example: Postorder



28	20	33	40	31	47	59	56	89	64	43
----	----	----	----	----	----	----	----	----	----	----

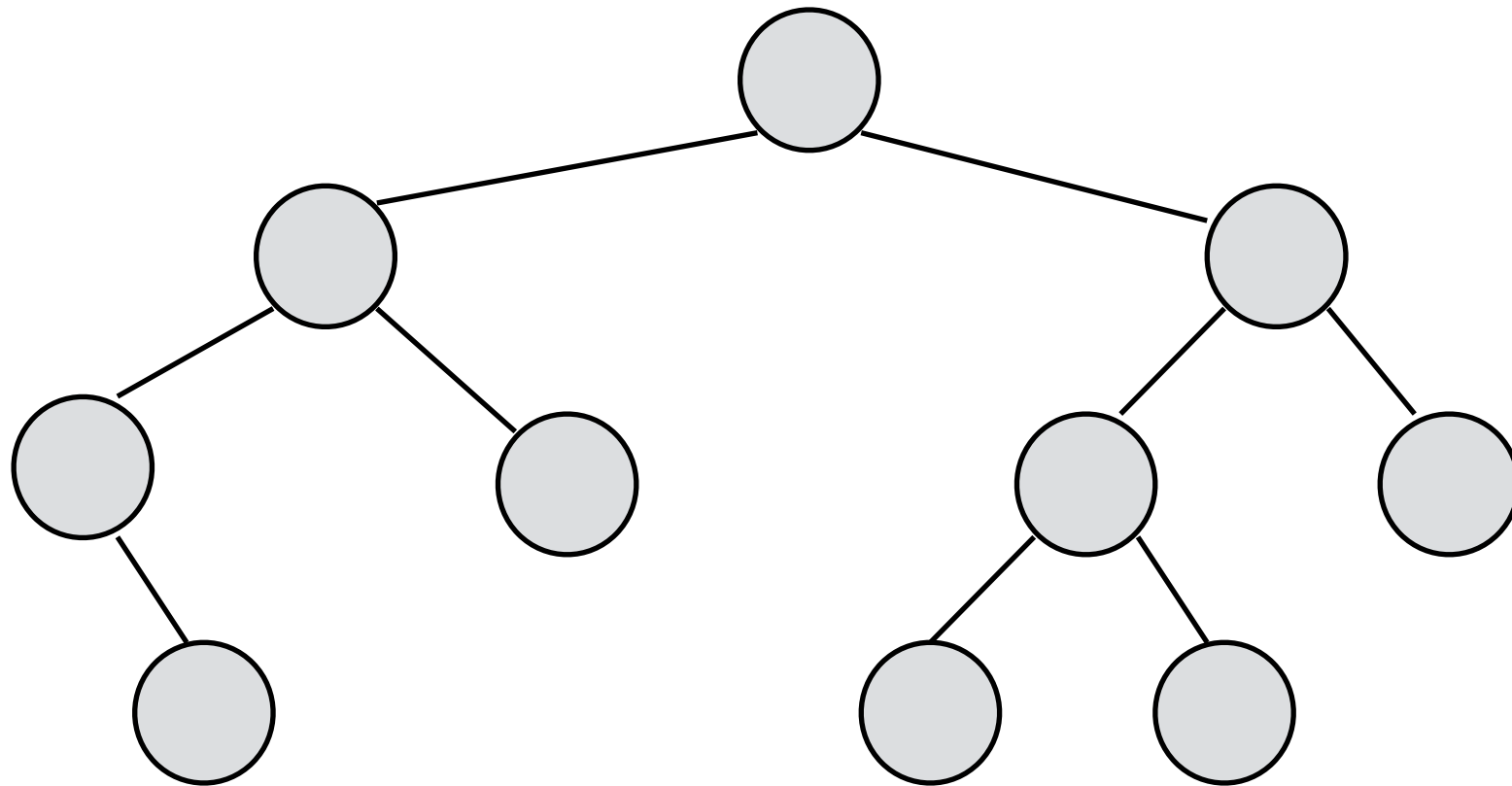
Print Post-order Traversal

- 1) Traverse the **left** subtree
- 2) Traverse the **right** subtree
- 3) Print the **root** node

```
def print_postorder(self):  
    self._print_postorder_aux(self.root)  
  
def _print_postorder_aux(self, current):  
    if current is not None: # if not a base case  
        self._print_postorder_aux(current.left)  
        self._print_postorder_aux(current.right)  
        print(current)
```

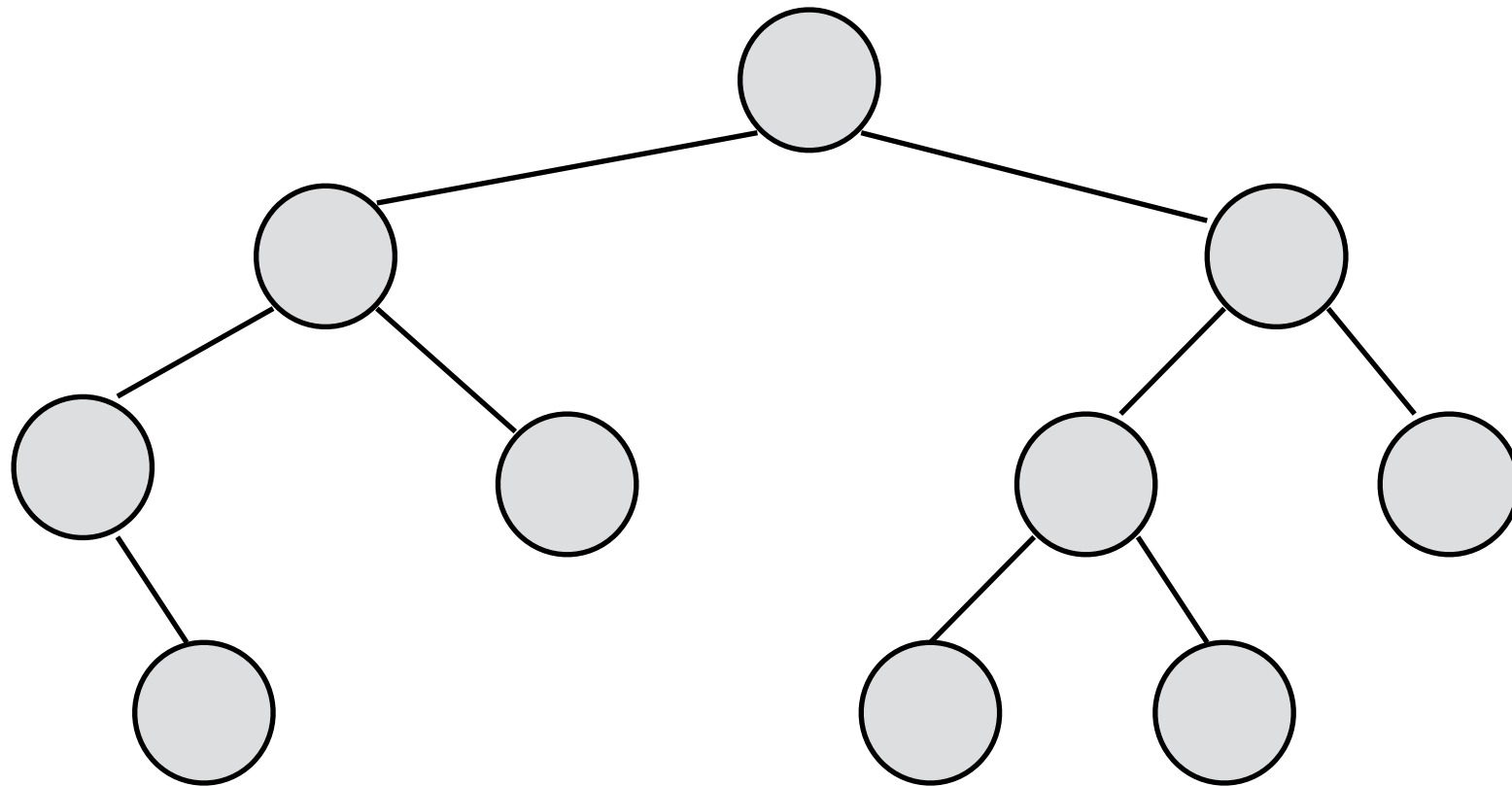
Computing the size of a tree

Returns the **number of nodes in the tree** (without modifying the tree)



Computing the size of a tree

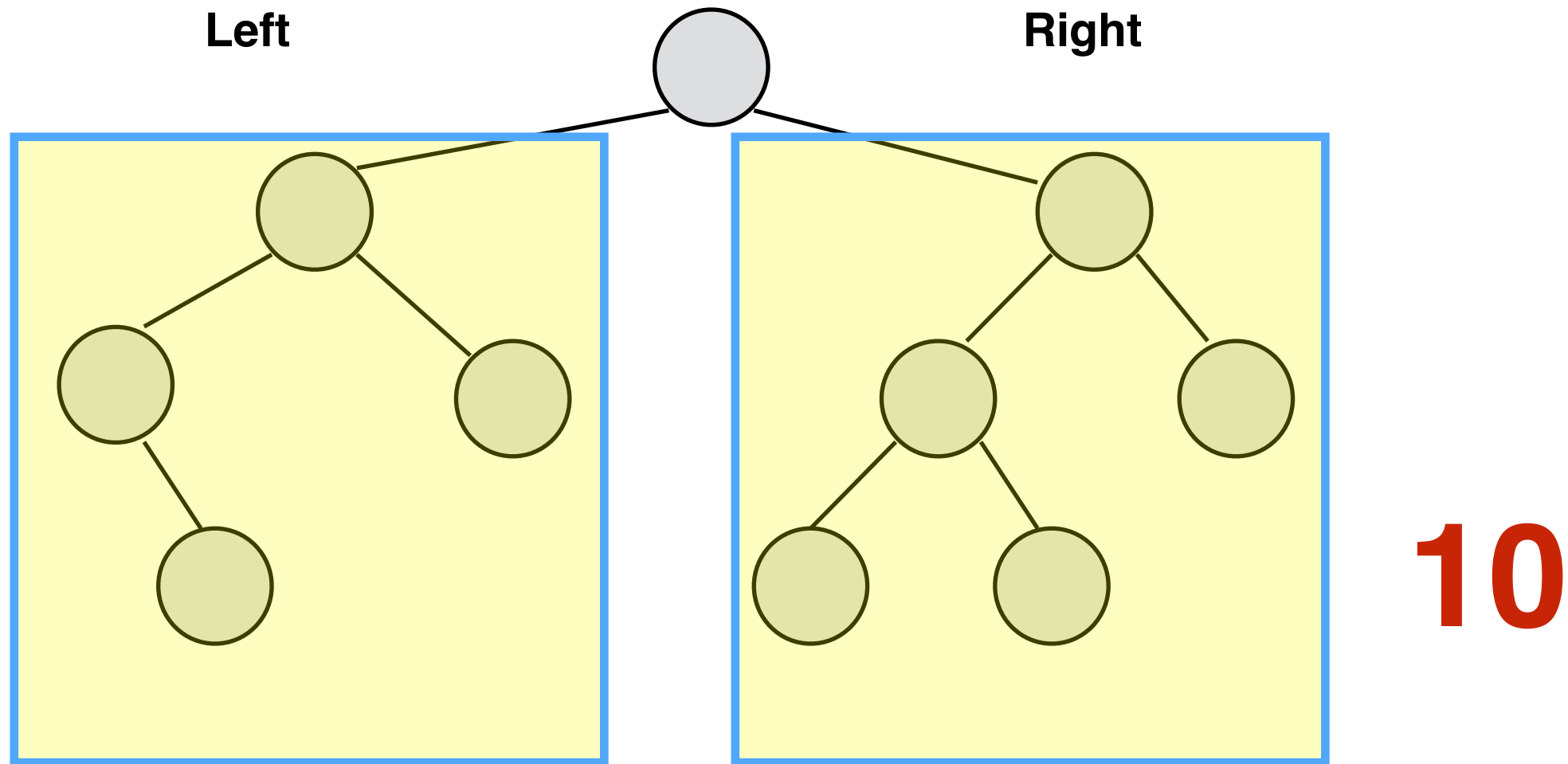
Returns the **number of nodes in the tree** (without modifying the tree)



10

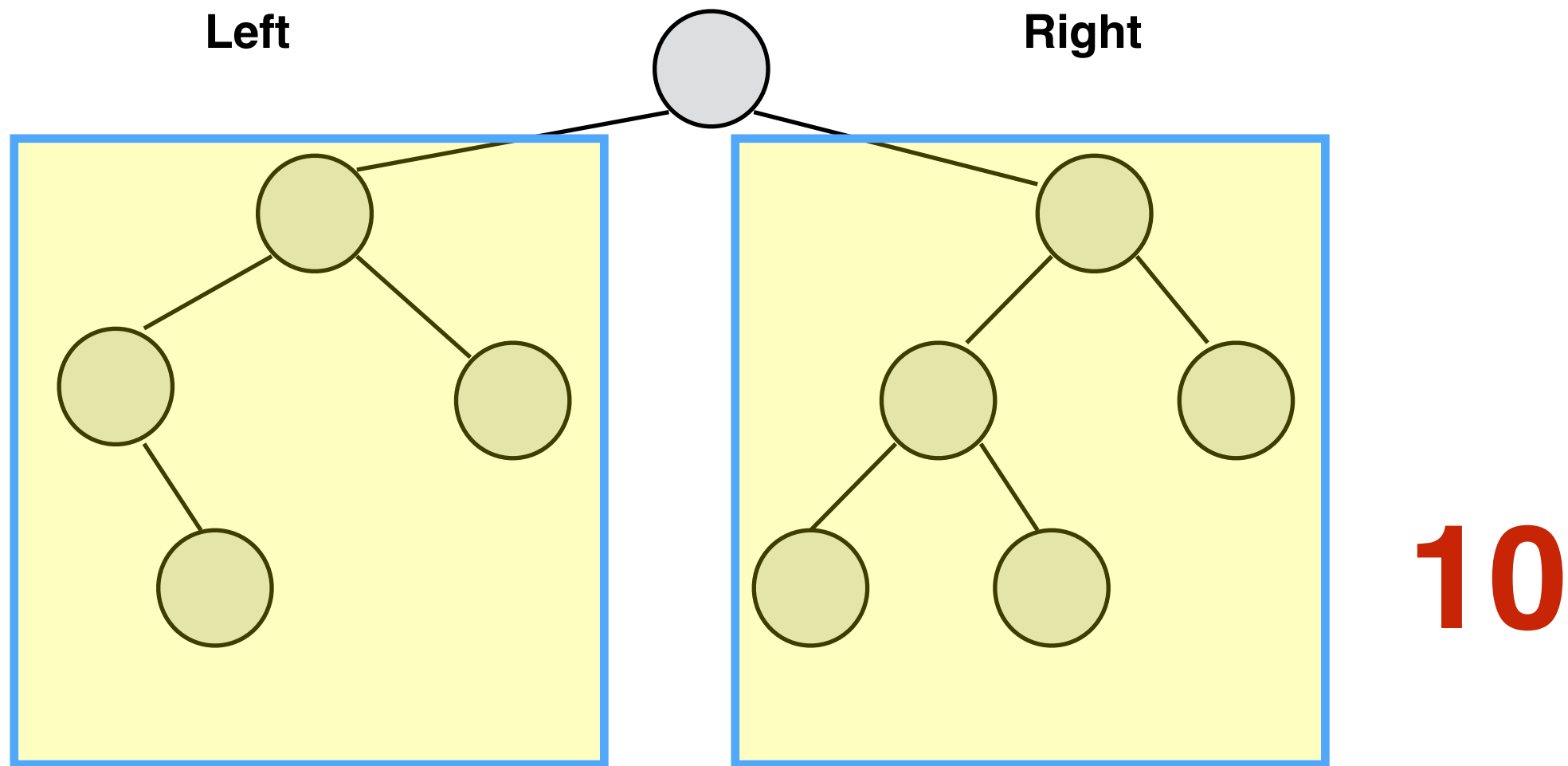
Computing the size of a tree

Returns the **number of nodes in the tree** (without modifying the tree)



Computing the size of a tree

Returns the **number of nodes in the tree** (without modifying the tree)



$$\text{size}(\text{self}) = \text{size}(\text{left}) + 1 + \text{size}(\text{right})$$

Computing the size of a tree

```
def len_aux(self, current):
```


Computing the size of a tree

```
def len_aux(self, current):  
    if current is None:
```

Computing the size of a tree

```
def len_aux(self, current):  
    if current is None:  
        return 0
```

Computing the size of a tree

```
def len_aux(self, current):  
    if current is None:  
        return 0  
    else:  
        return 1 + self.len_aux(current.left) + self.len_aux(current.right)
```

Computing the size of a tree

```
def len_aux(self, current):  
    if current is None:  
        return 0  
    else:  
        return 1 + self.len_aux(current.left) + self.len_aux(current.right)
```

Computing the size of a tree

```
def __len__(self):  
    return self.len_aux(self.root)  
  
def len_aux(self, current):  
    if current is None:  
        return 0  
    else:  
        return 1 + self.len_aux(current.left) + self.len_aux(current.right)
```

Can we implement an Iterator to traverse a Binary Tree?

A) Yes

B) No

C) I have no idea.

Can we implement an Iterator to traverse a Binary Tree?

A)Yes? You may need to use a Stack

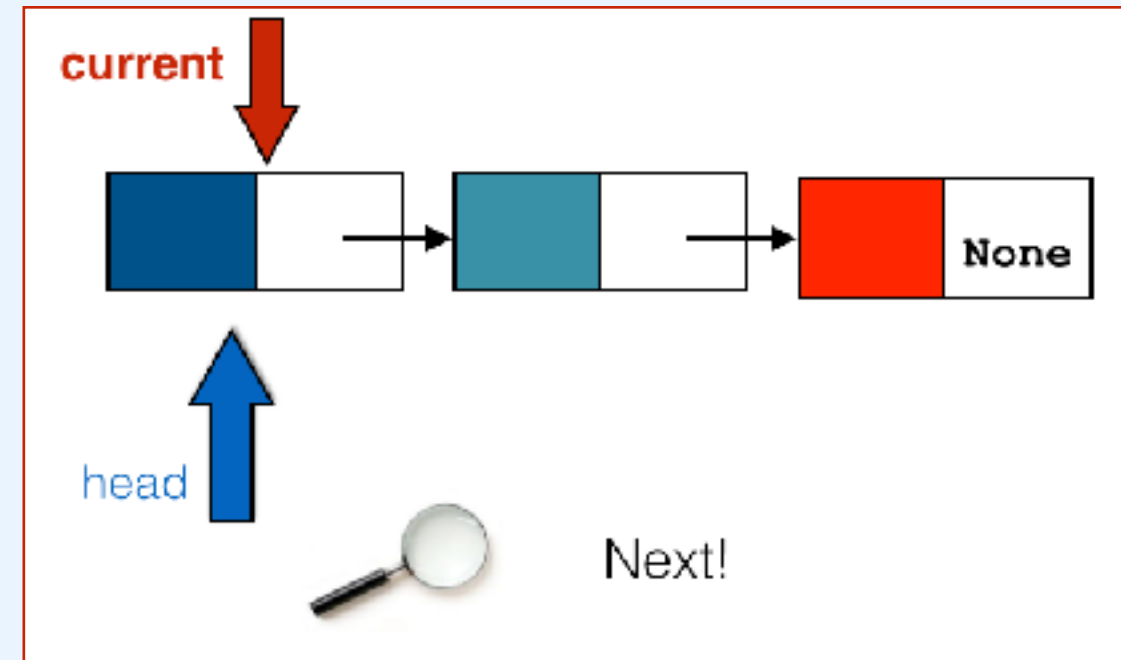
B) No

C) I have no idea.

```
class ListIterator:
    def __init__(self, head):
        self.current = head

    def __iter__(self):
        return self

    def __next__(self):
        if self.current is None:
            raise StopIteration
        else:
            item_required = self.current.item
            self.current = self.current.next
            return item_required
```



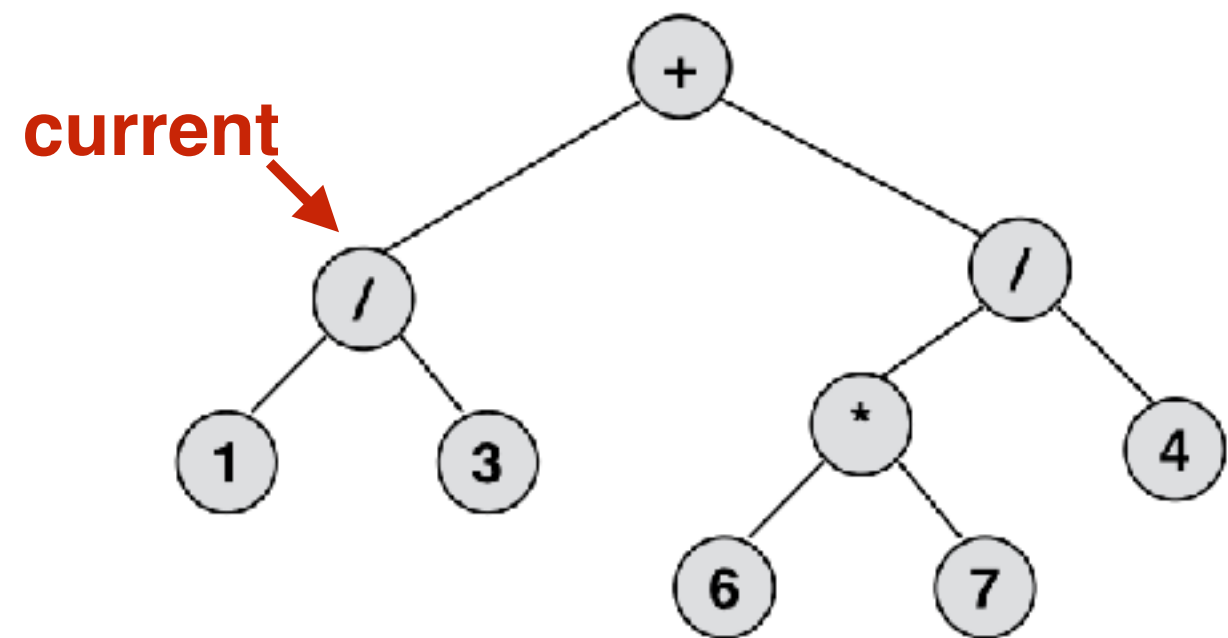
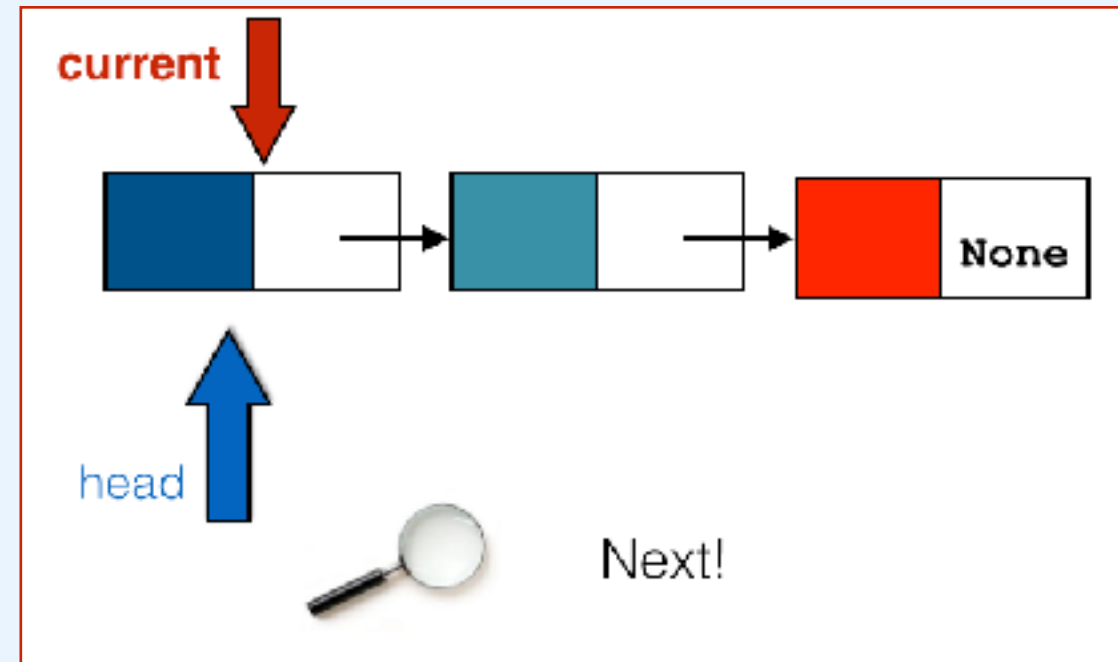

```

class ListIterator:
    def __init__(self, head):
        self.current = head

    def __iter__(self):
        return self

    def __next__(self):
        if self.current is None:
            raise StopIteration
        else:
            item_required = self.current.item
            self.current = self.current.next
            return item_required

```



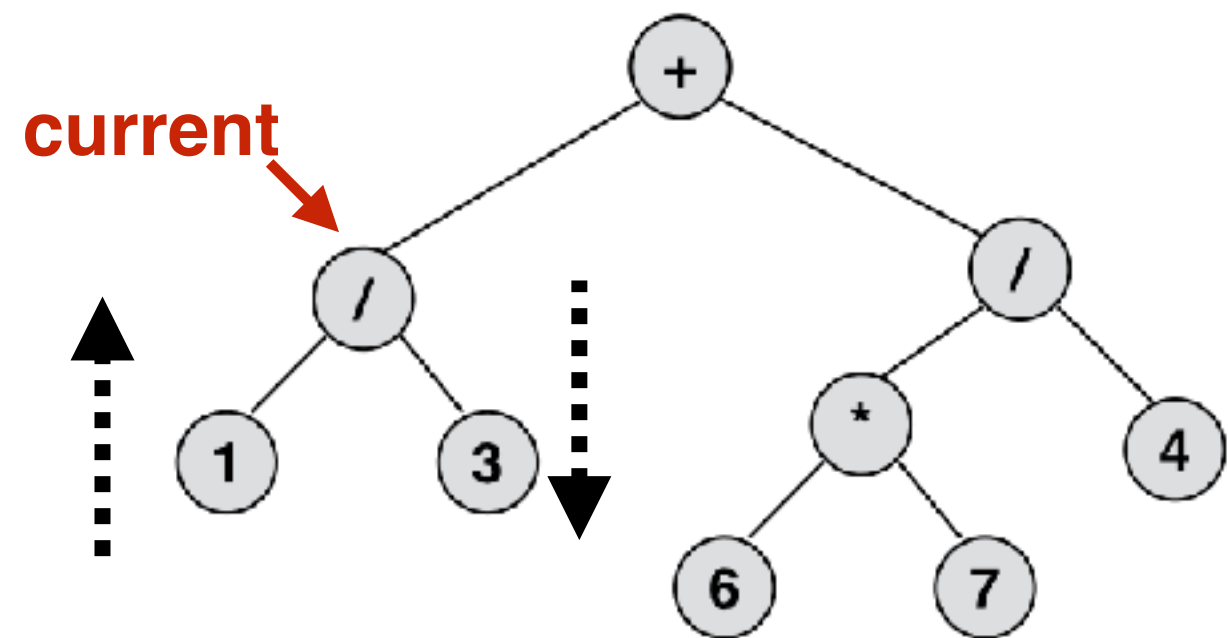
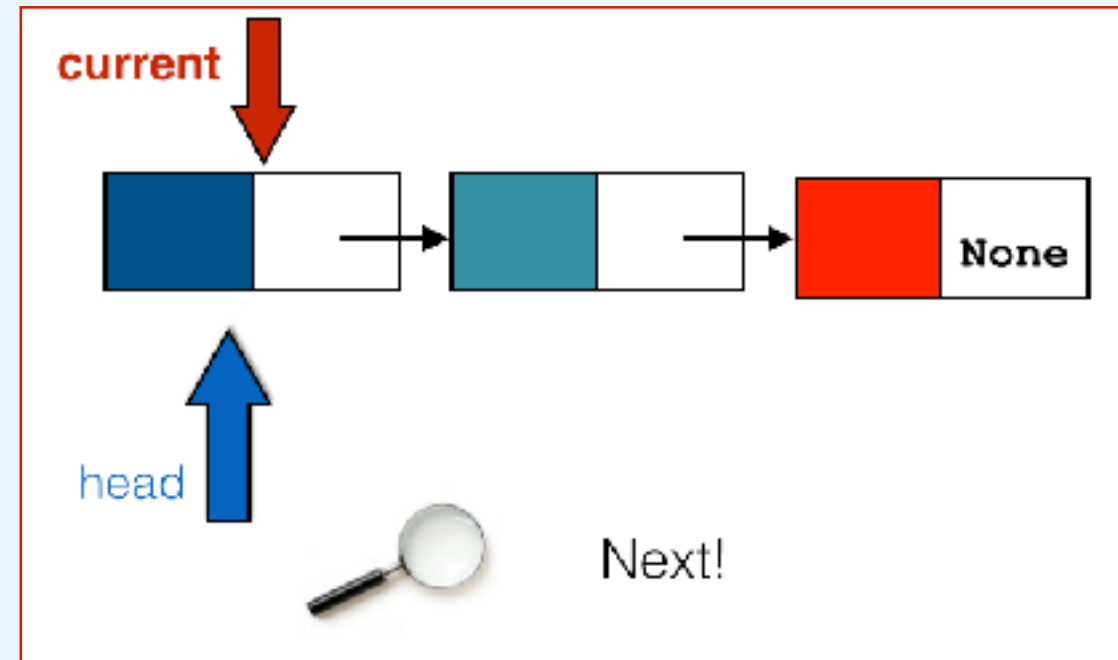
```

class ListIterator:
    def __init__(self, head):
        self.current = head

    def __iter__(self):
        return self

    def __next__(self):
        if self.current is None:
            raise StopIteration
        else:
            item_required = self.current.item
            self.current = self.current.next
            return item_required

```



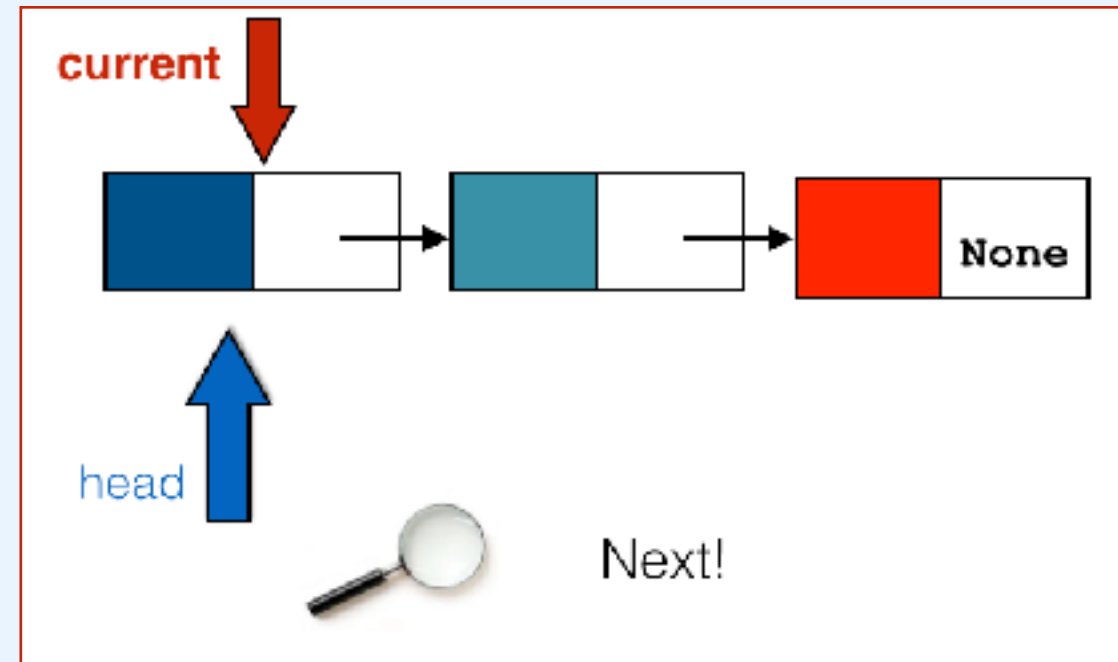
```

class ListIterator:
    def __init__(self, head):
        self.current = head

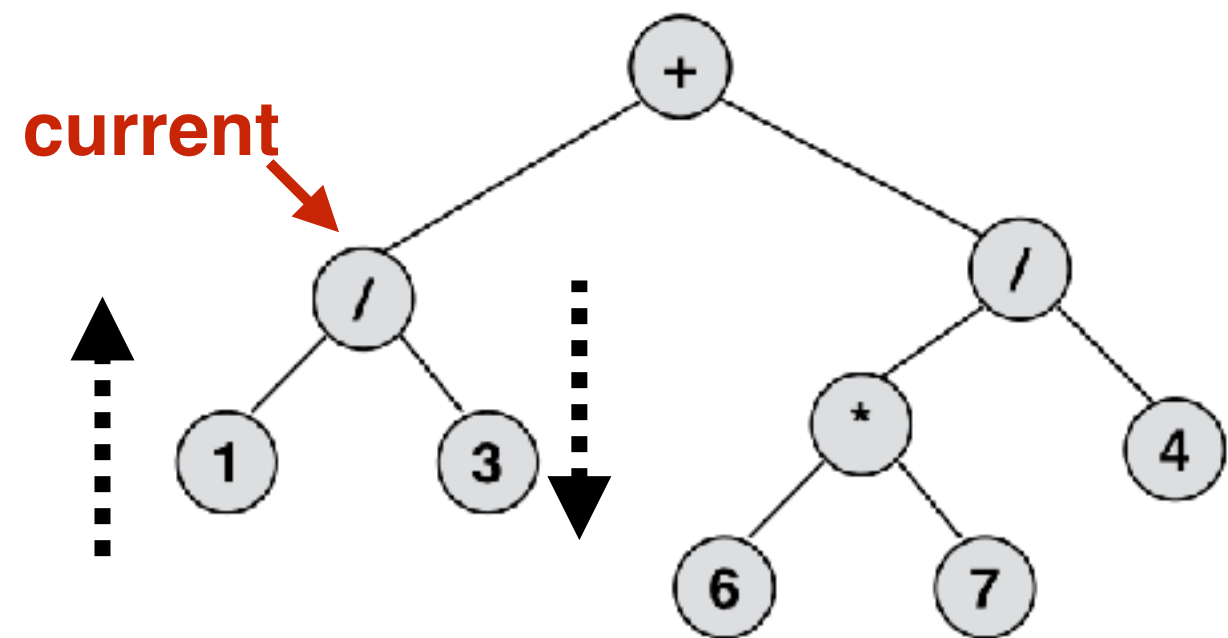
    def __iter__(self):
        return self

    def __next__(self):
        if self.current is None:
            raise StopIteration
        else:
            item_required = self.current.item
            self.current = self.current.next
            return item_required

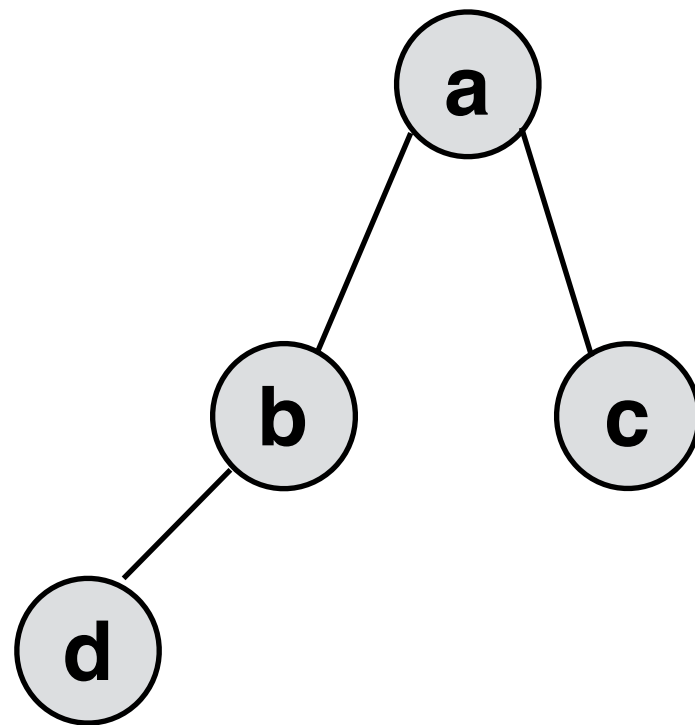
```



?



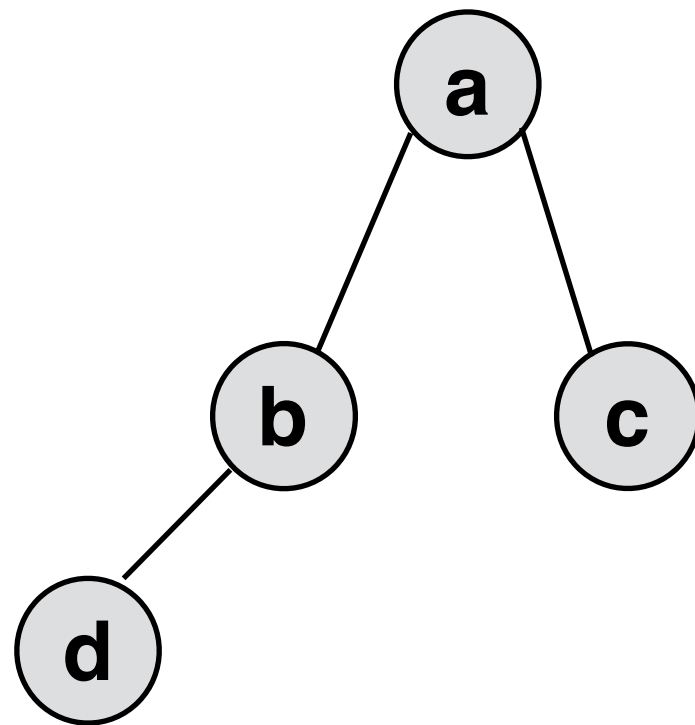
State of the **Iterator** on creation

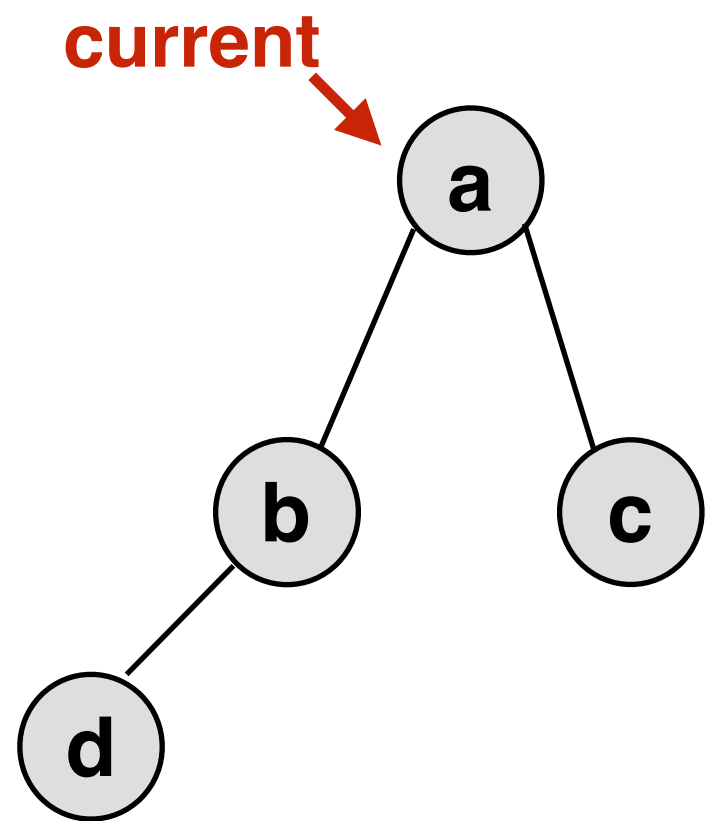


self.stack

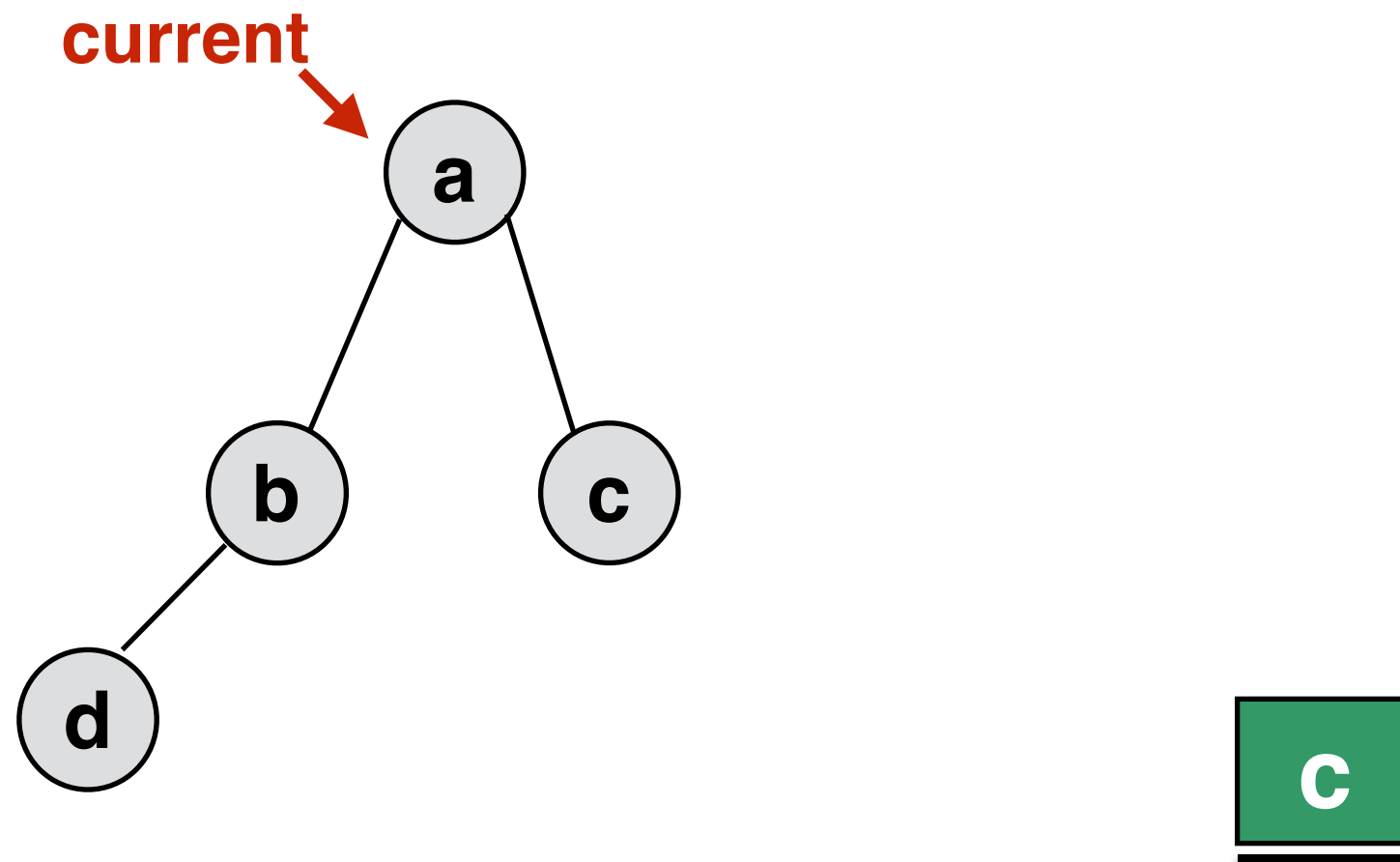


Next!

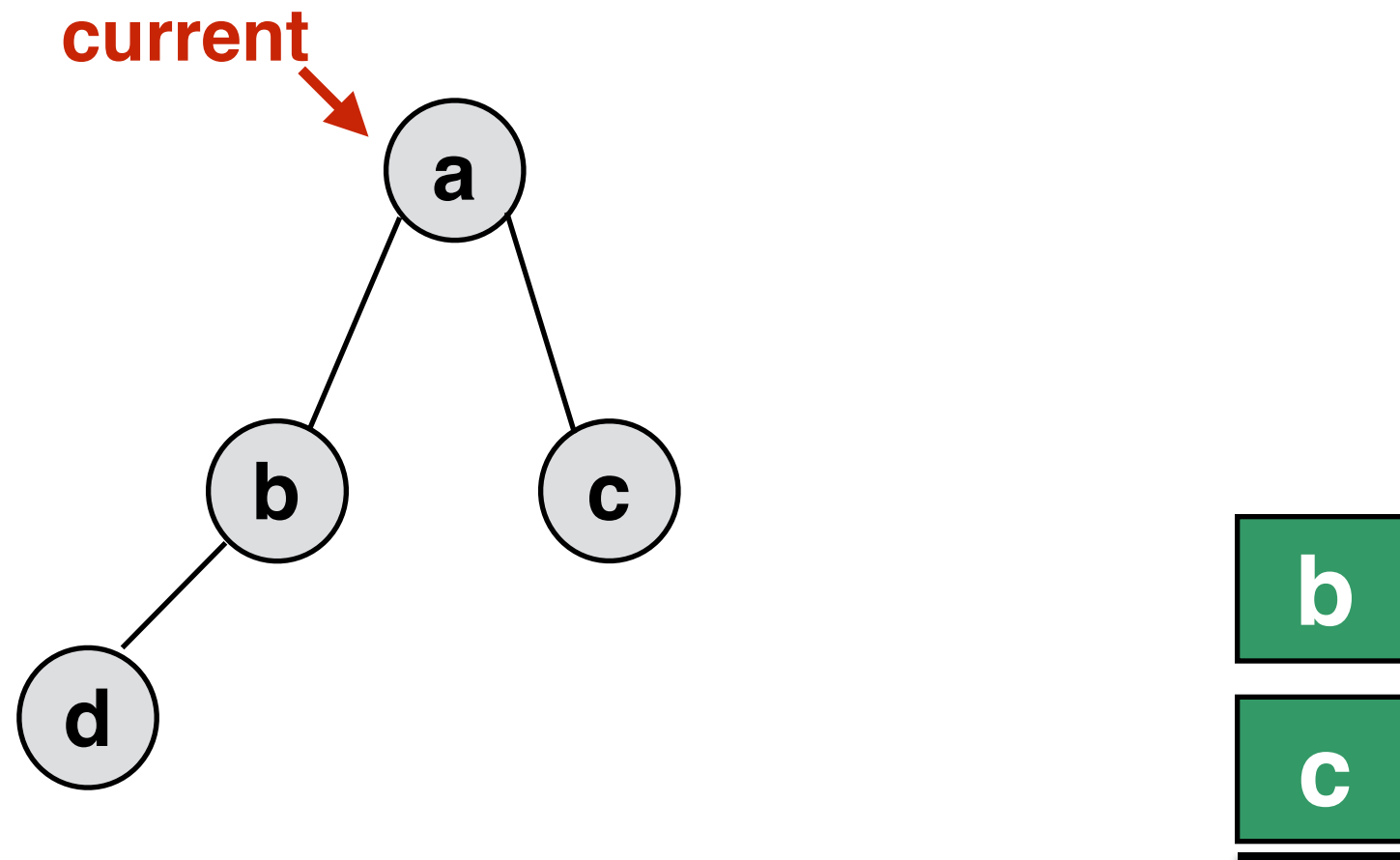




—

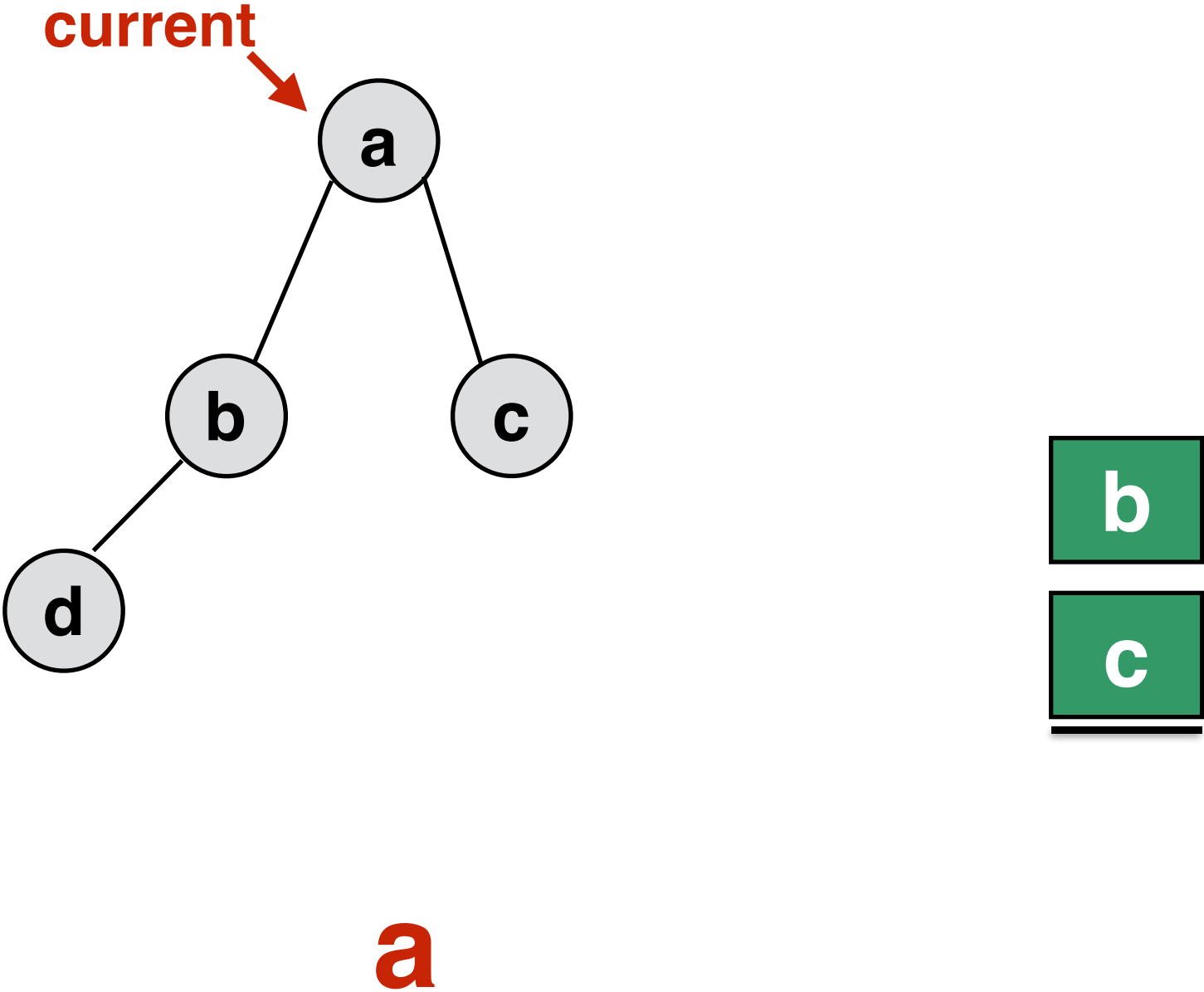


Push what is to the right of current.

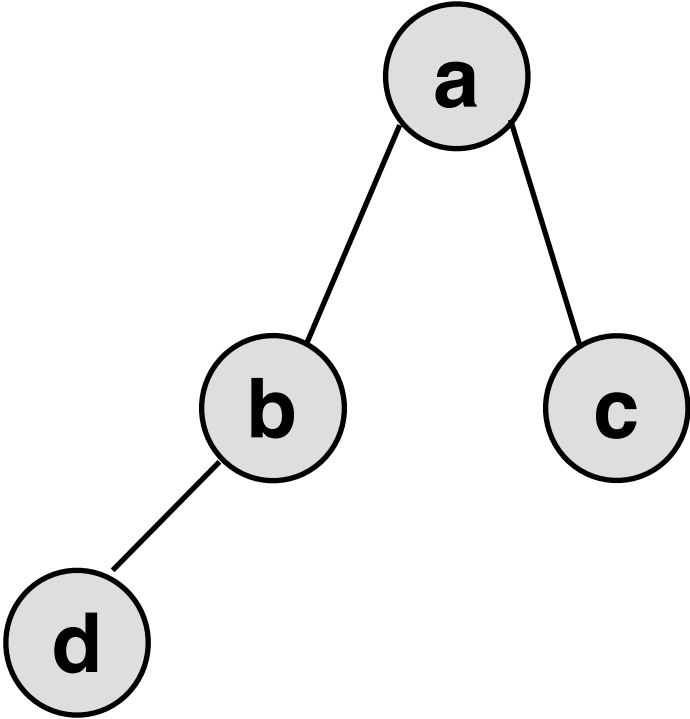


Push what is to the left of current.

return current.item



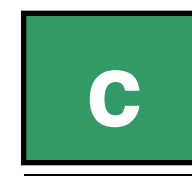
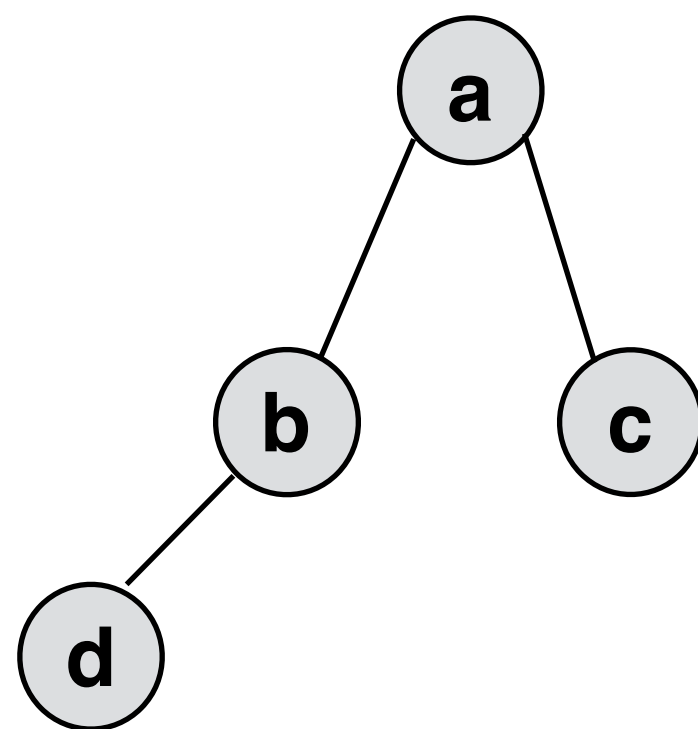
return current.item



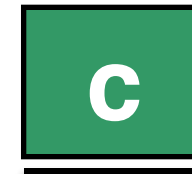
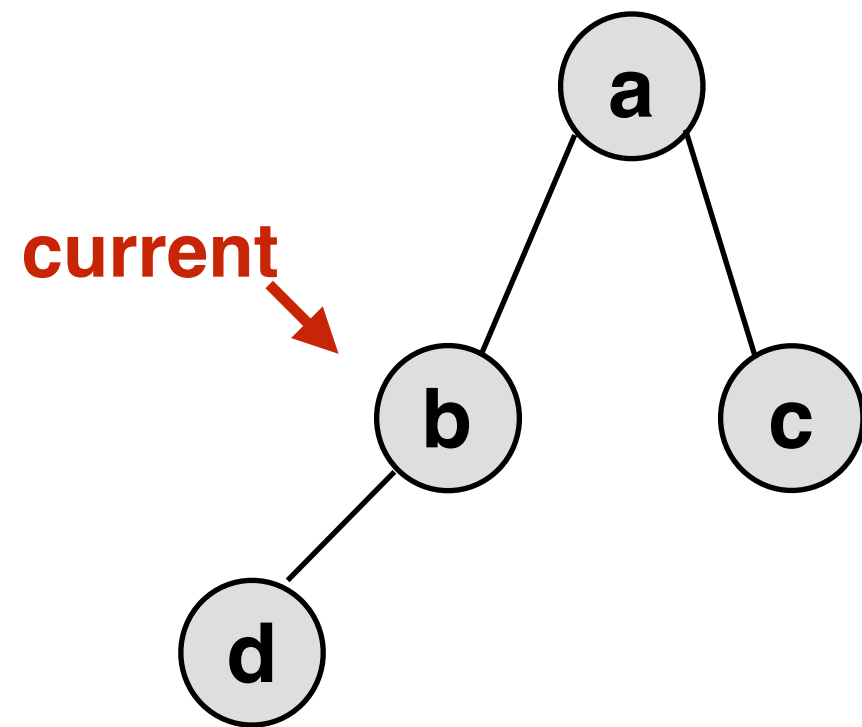
a



Next!

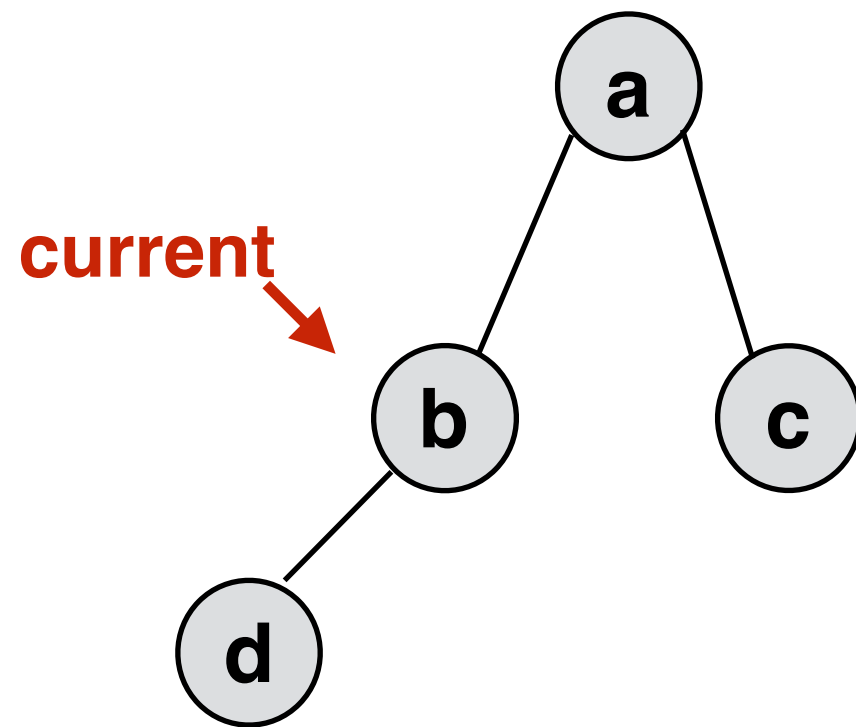


a



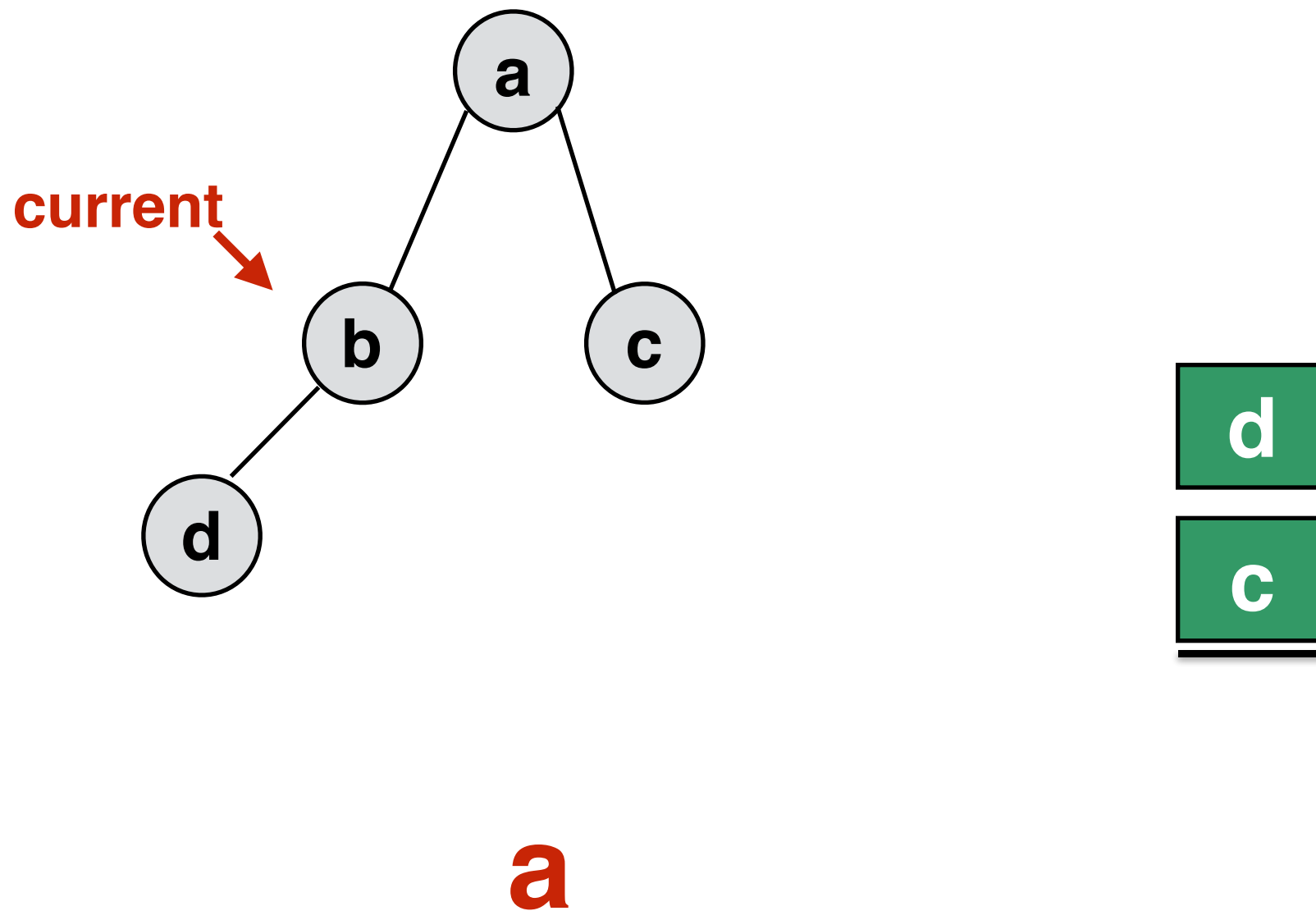
a

Nothing to push on right

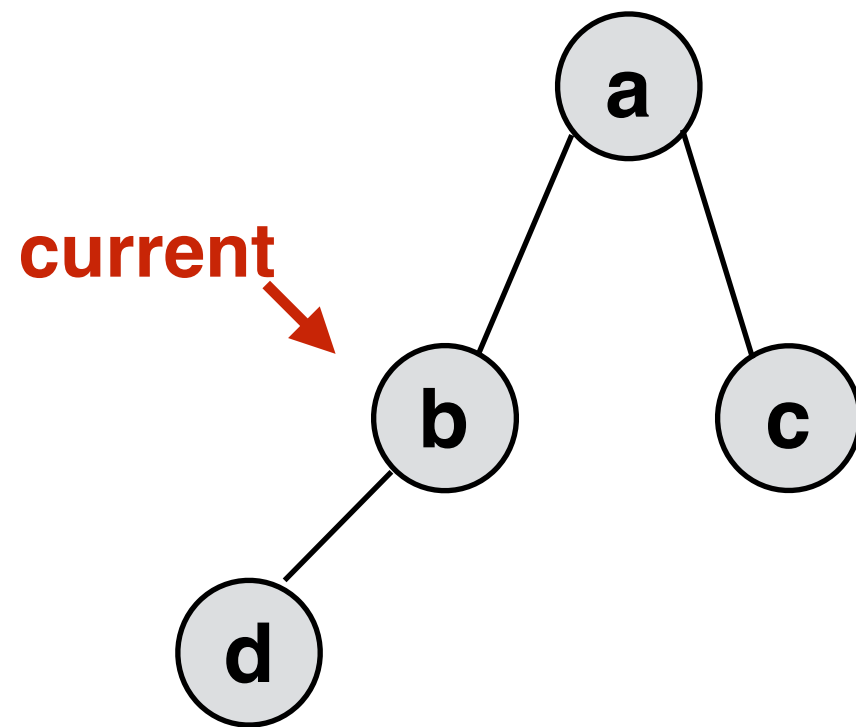


a

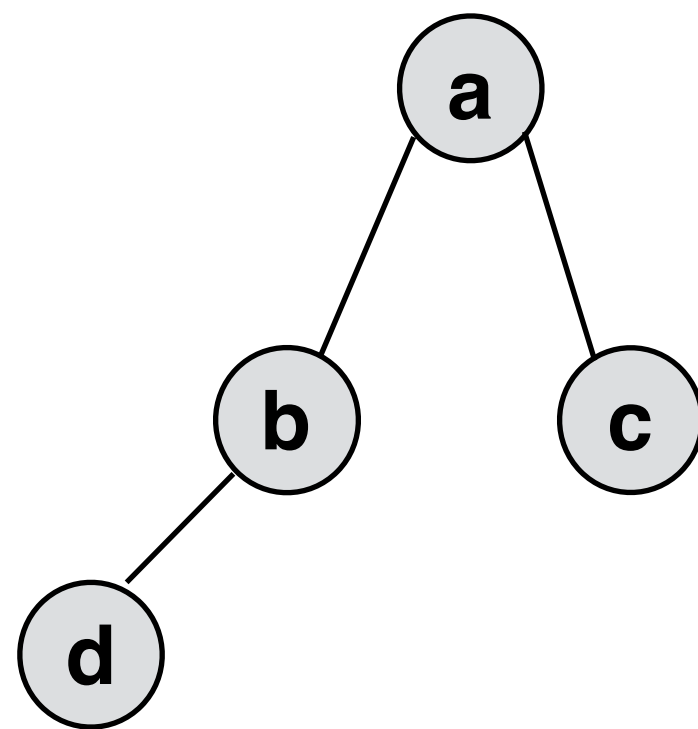
Push what is to the left of current.



return current.item



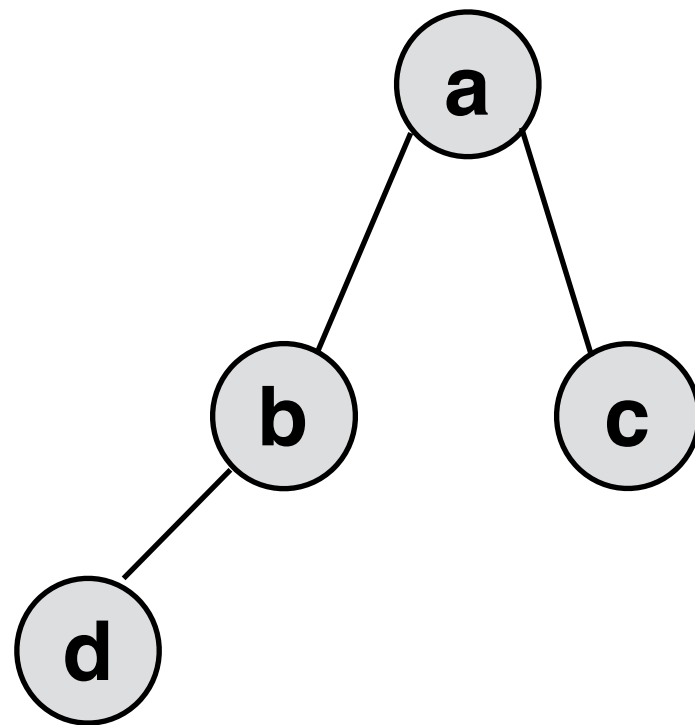
a b



a b



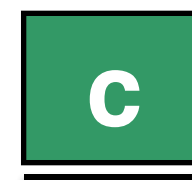
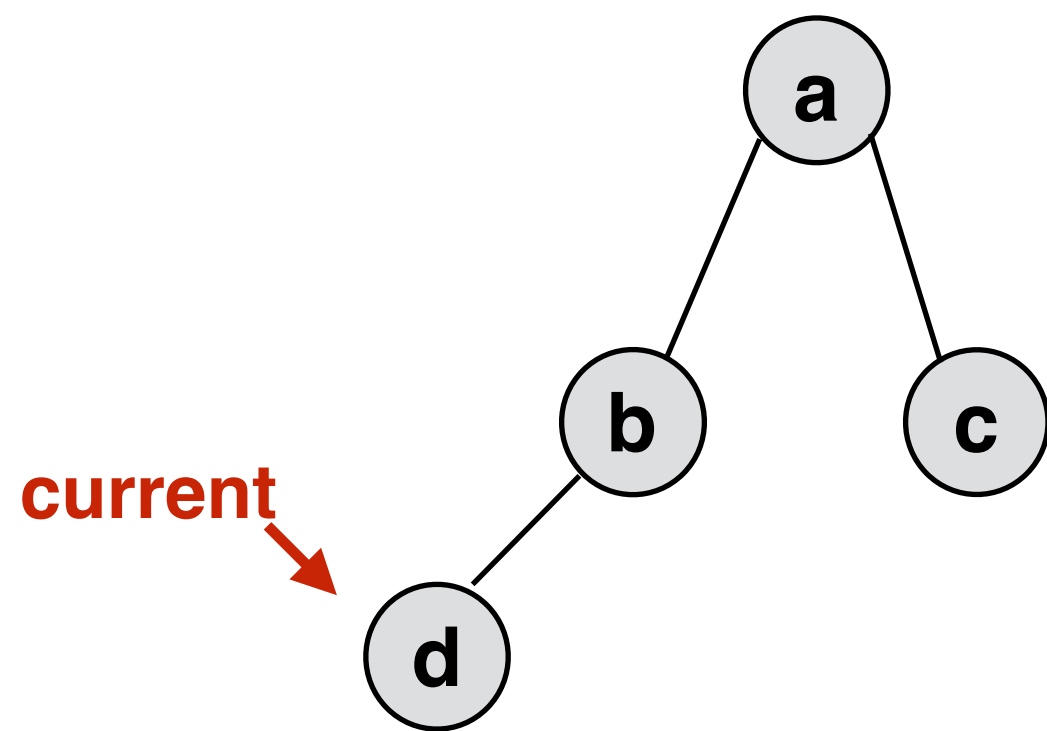
Next!



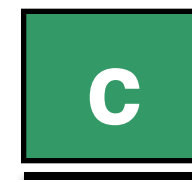
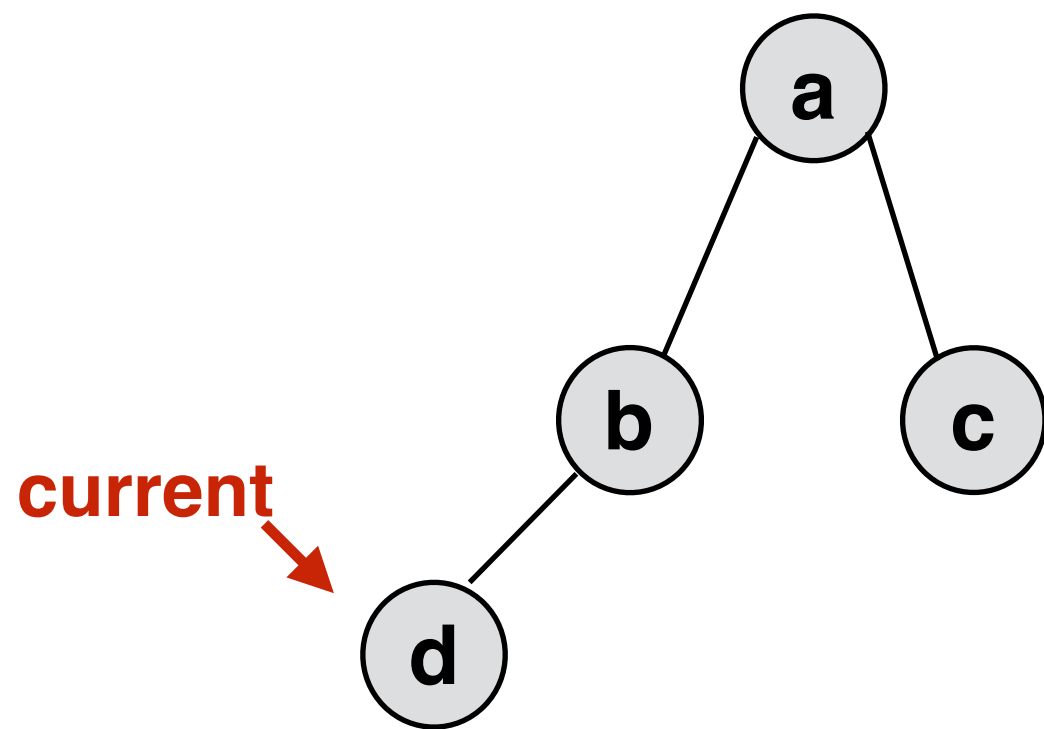
d

c

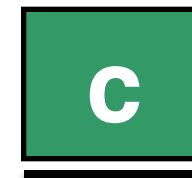
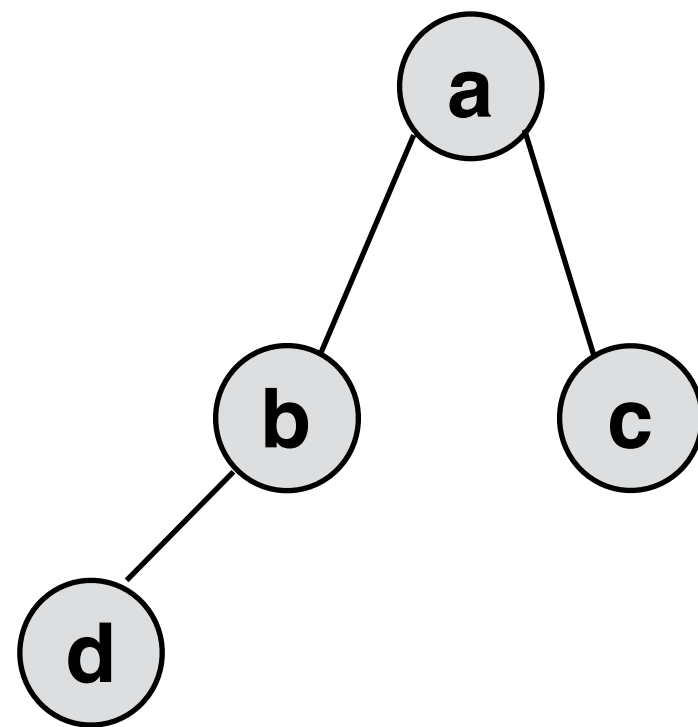
a b



a b



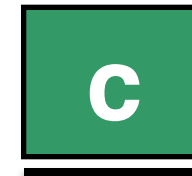
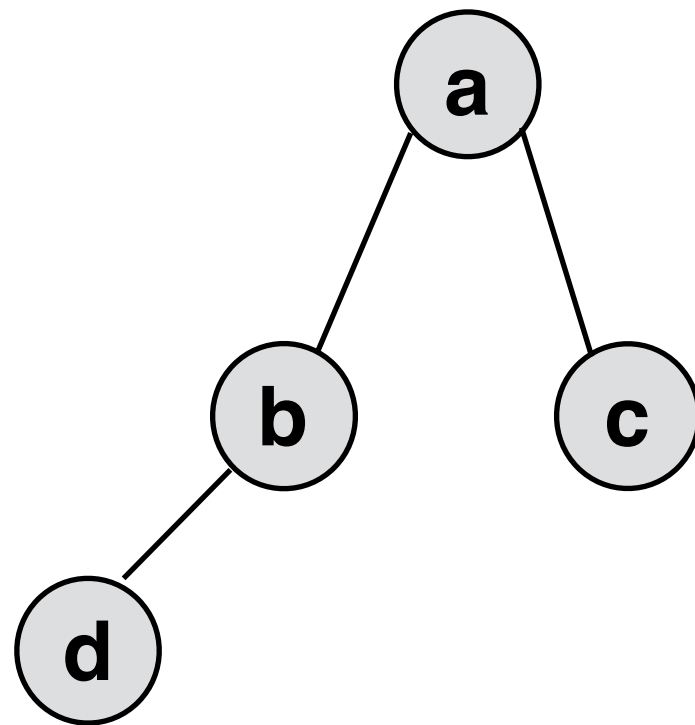
a b d



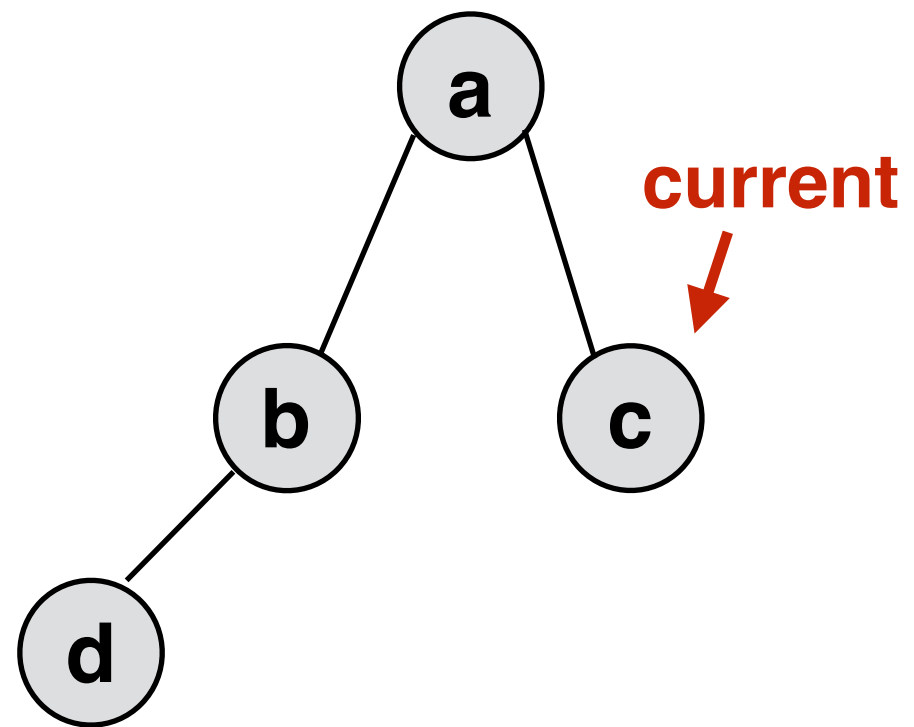
a b d



Next!



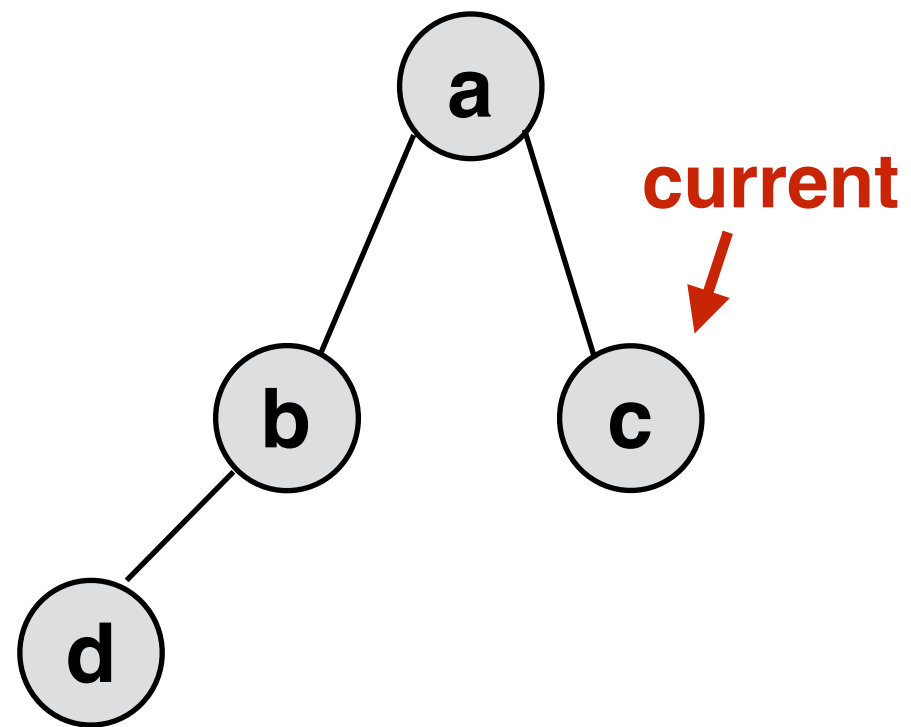
a b d



—

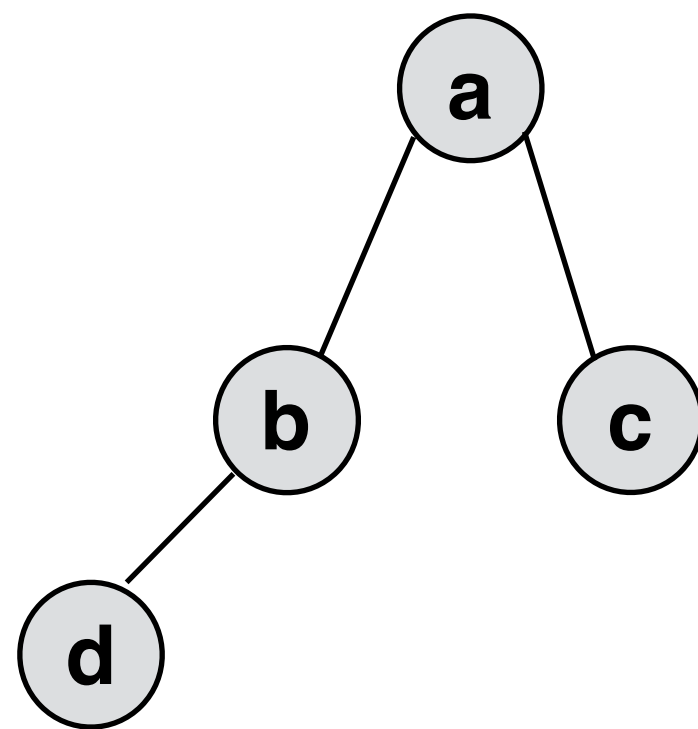
a b d

return current.item



a b d c

—

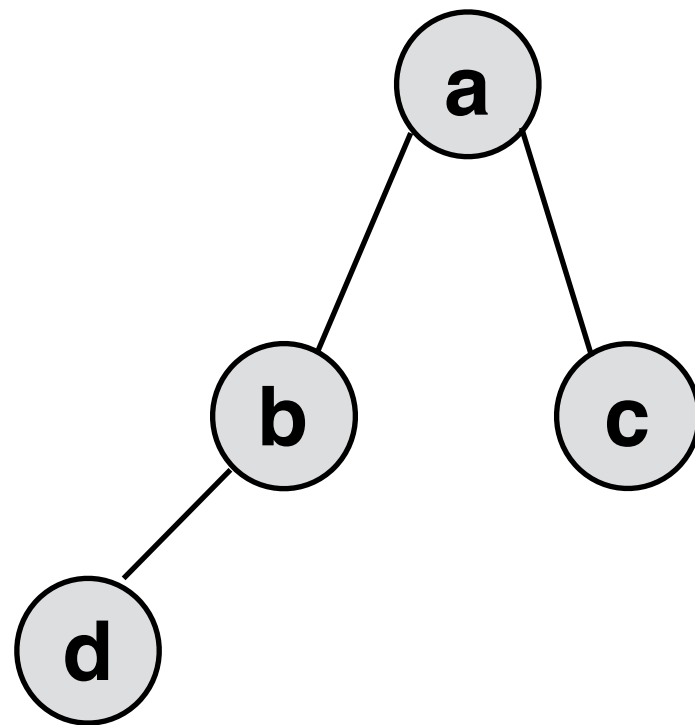


—

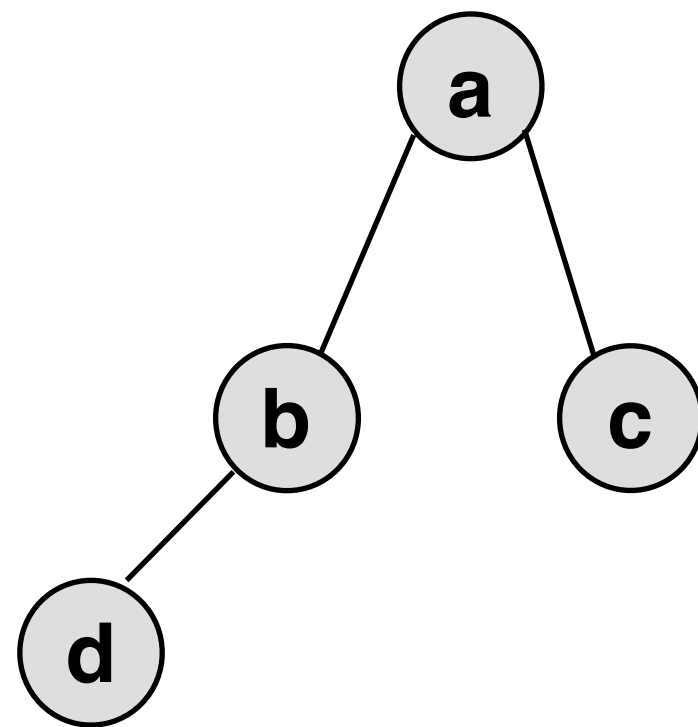
a b d c



Next!



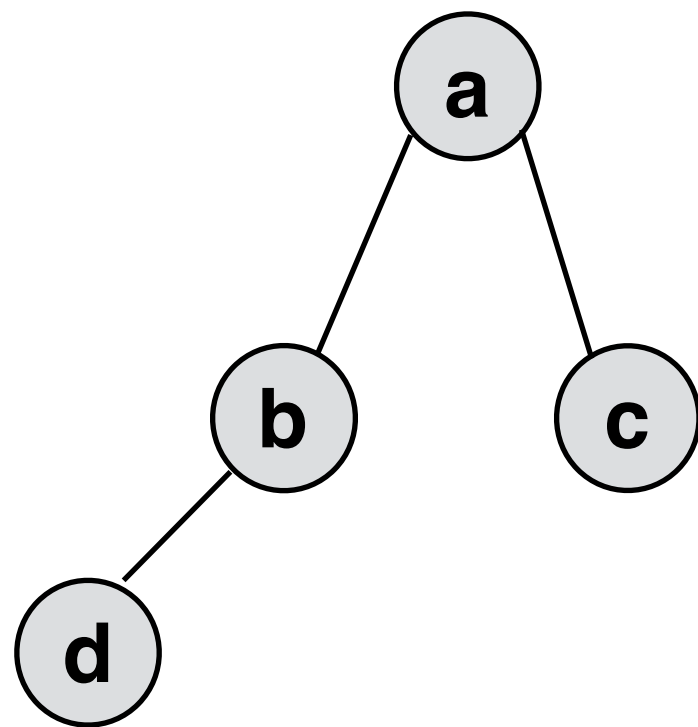
a b d c



StopIteration

—

a b d c



StopIteration

—

a b d c

preorder!

```
self.current = self.stack.pop()  
self.stack.push(self.current.right)  
self.stack.push(self.current.left)  
return current
```

```
class PreOrderIteratorStack:
```

```
    def __init__(self, root):  
        self.current = root  
        self.stack = Stack()  
        self.stack.push(root)
```

```
    def __iter__(self):  
        return self
```

```
    def __next__(self):  
        if self.stack.is_empty():  
            raise StopIteration  
        current = self.stack.pop()  
        if current.right is not None:  
            self.stack.push(current.right)  
        if current.left is not None:  
            self.stack.push(current.left)  
        return current.item
```

```
my_tree.print_preorder()
```

```
2  
5  
3
```

```
for i in my_tree:  
    print(i)
```

```
2  
5  
3
```

In BinaryTree:

```
def __iter__(self):  
    return PreOrderIteratorStack(self.root)
```

What about without a stack?

hint: find out about python generators...
and **yield**

Summary

- **Tree traversal:** inorder, postorder, preorder