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Lecture 33
Binary Search Trees

FIT 1008
Introduction to Computer Science

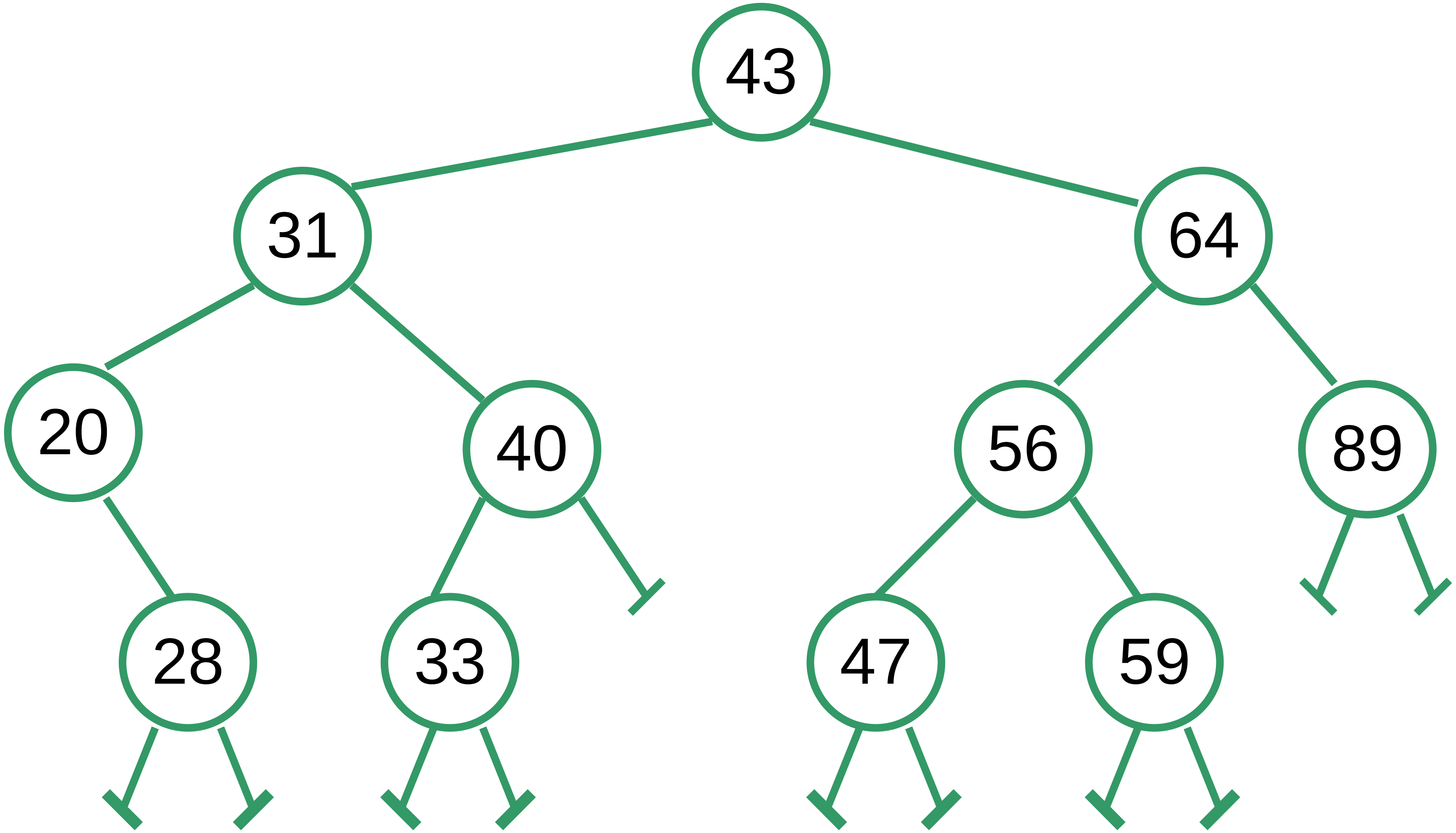
Objectives

- To understand Binary Search Trees
- Implement Binary Search Trees:
  - → search
  - → insert
- Advantages and disadvantages of Binary Search Trees over sorted lists.

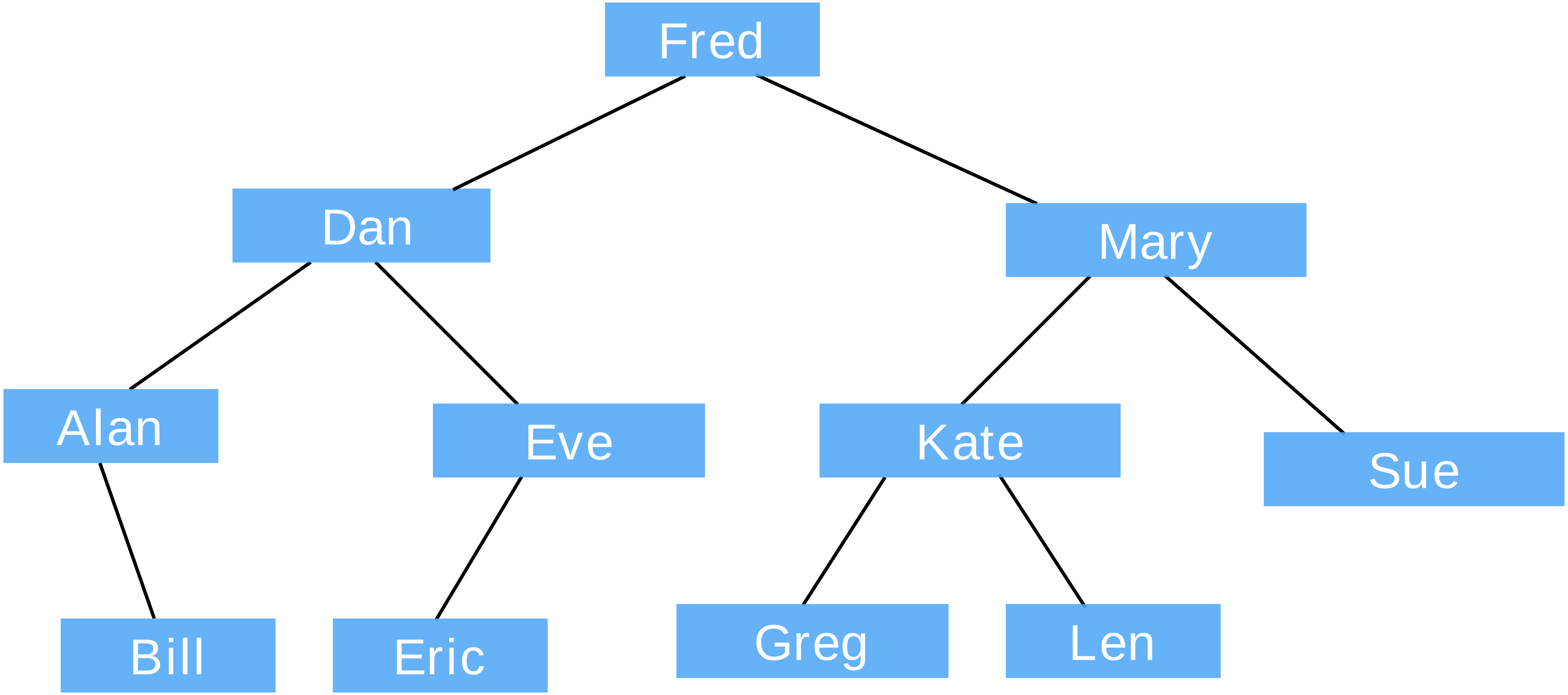
Binary Search Tree

A Binary Tree such that:

- Every node entry has a key
- All keys in the left subtree of a node are less than the key of the node
- All keys in the right subtree of a node are greater than the key of the node

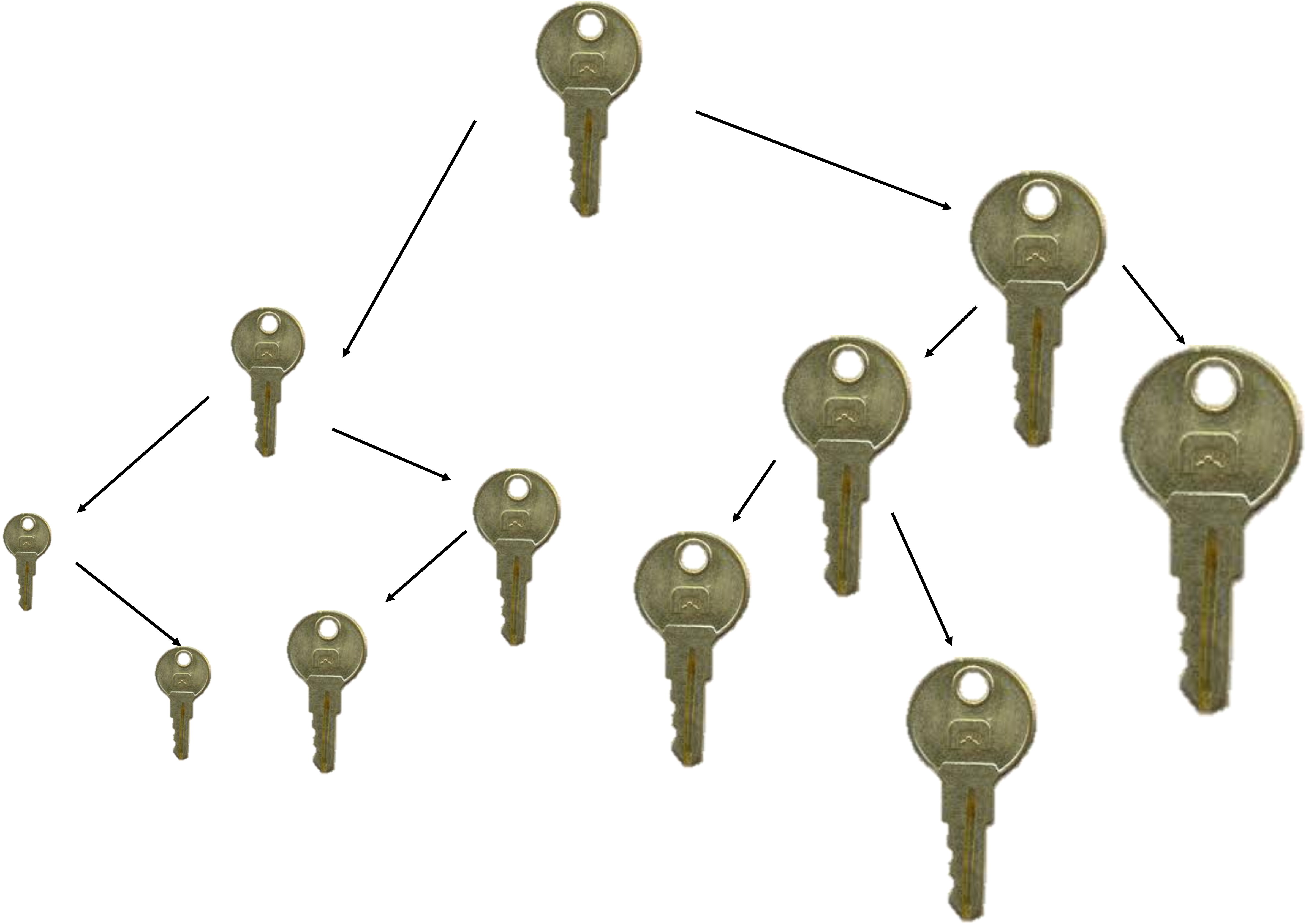


key is an integer.

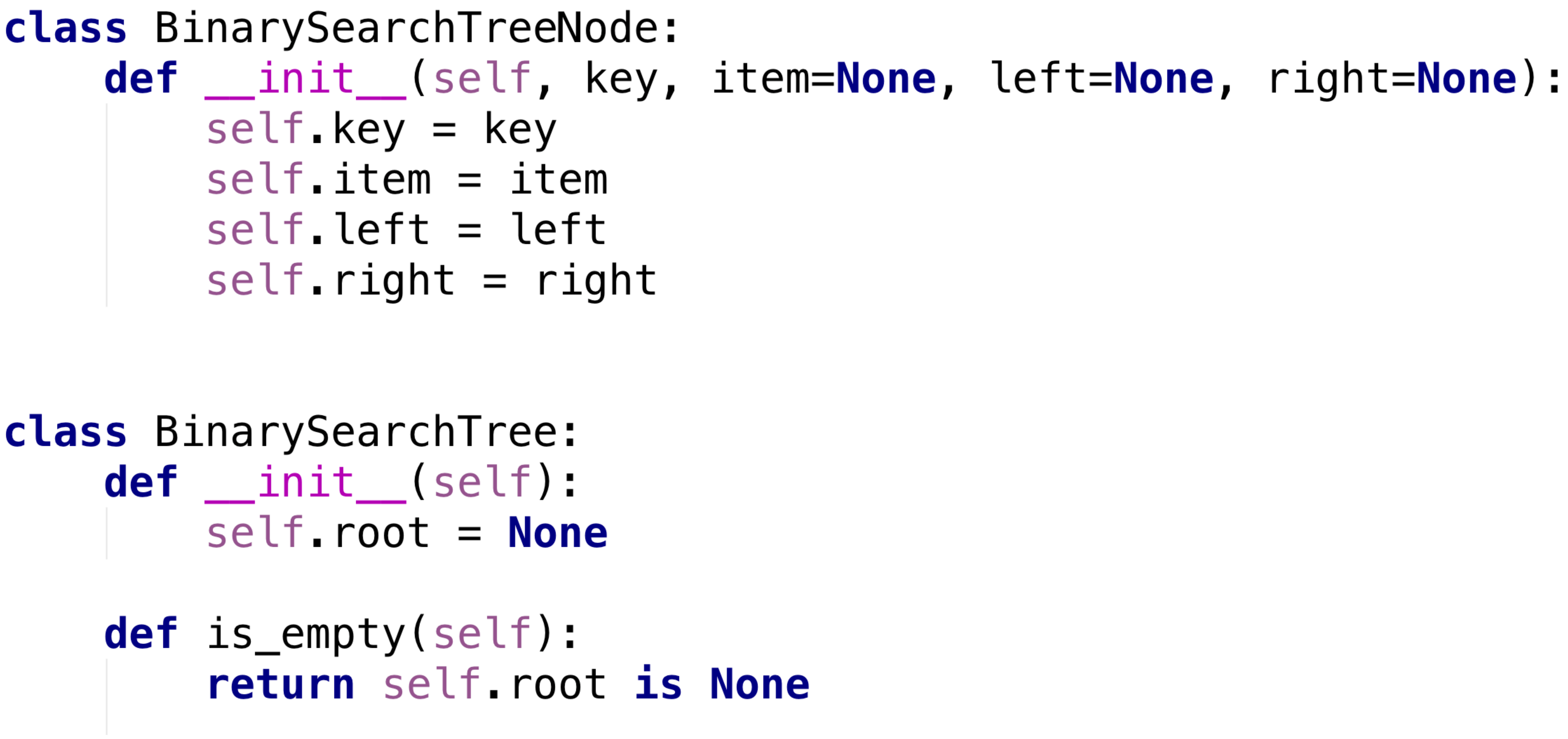


key is a string

(here not showing the associated items)



key is an key.



```python
class BinarySearchTreeNode:
    def __init__(self, key, item=None, left=None, right=None):
        self.key = key
        self.item = item
        self.left = left
        self.right = right


class BinarySearchTree:
    def __init__(self):
        self.root = None

    def is_empty(self):
        return self.root is None
```

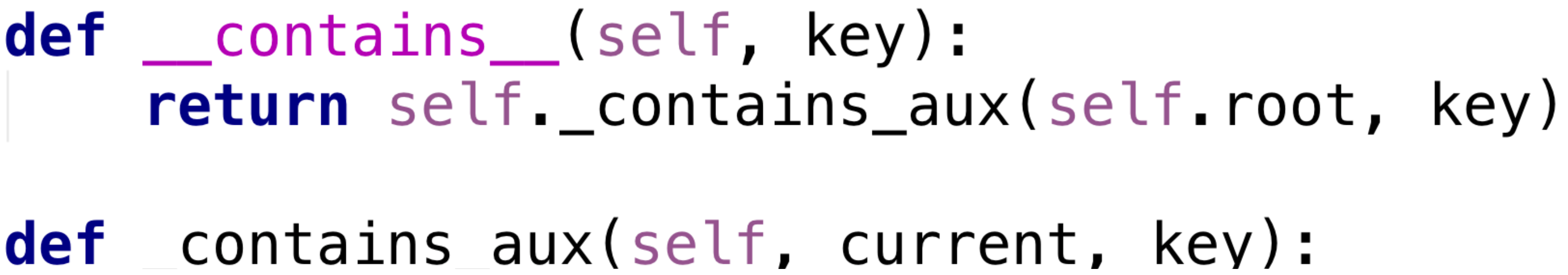
Search

Search algorithm

- If we reach an empty node, item is not there… return **`False`**.
- Else, if target key is equal to the current node’s key, return **`True`**
- Else, if target key is less than current node’s key, search the left sub-tree
- Else, if target key is greater than current node’s key, search the right sub-tree

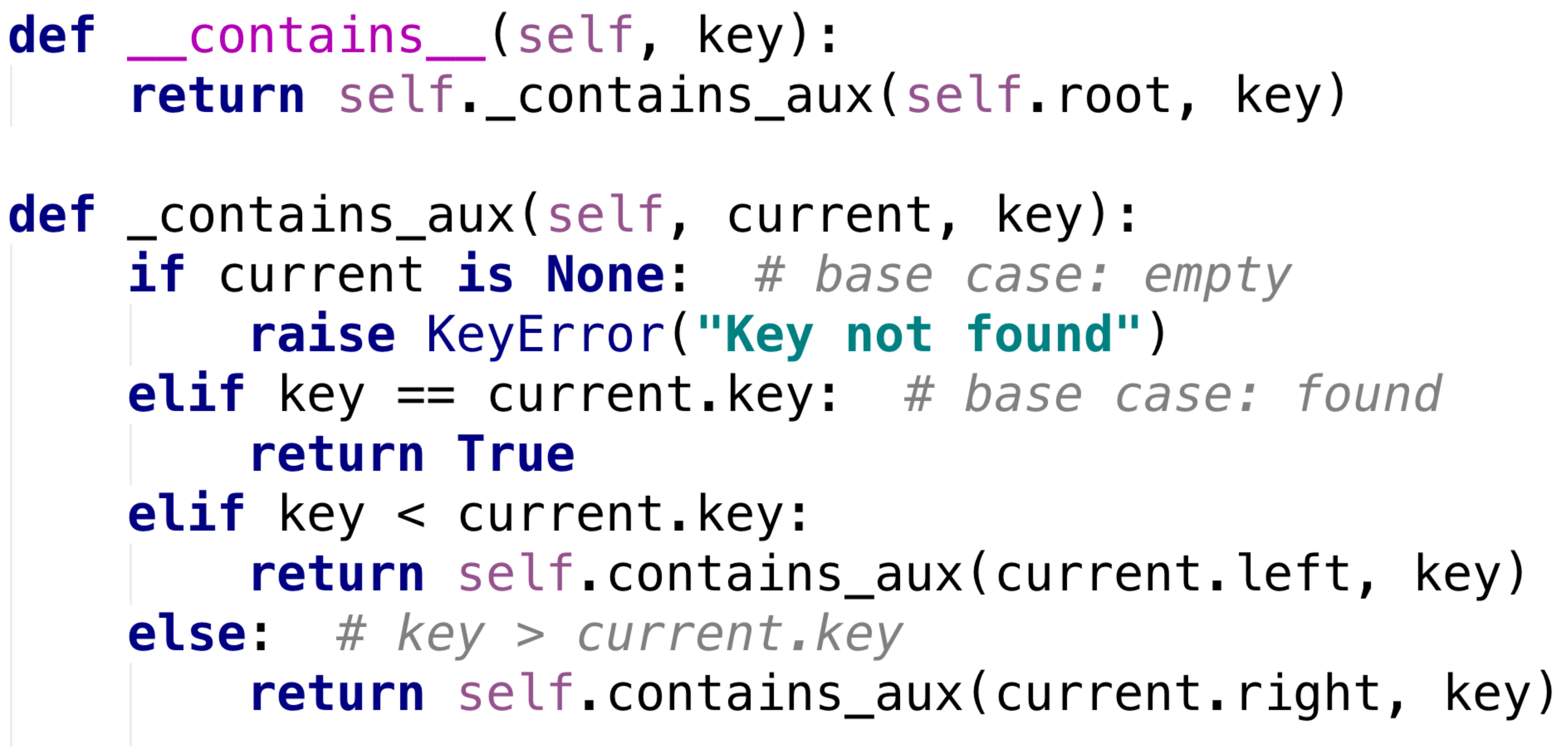
search can be implemented by **`__contains__`**

__contains__



```
def __contains__(self, key):
    return self._contains_aux(self.root, key)

def _contains_aux(self, current, key):
```

__contains__



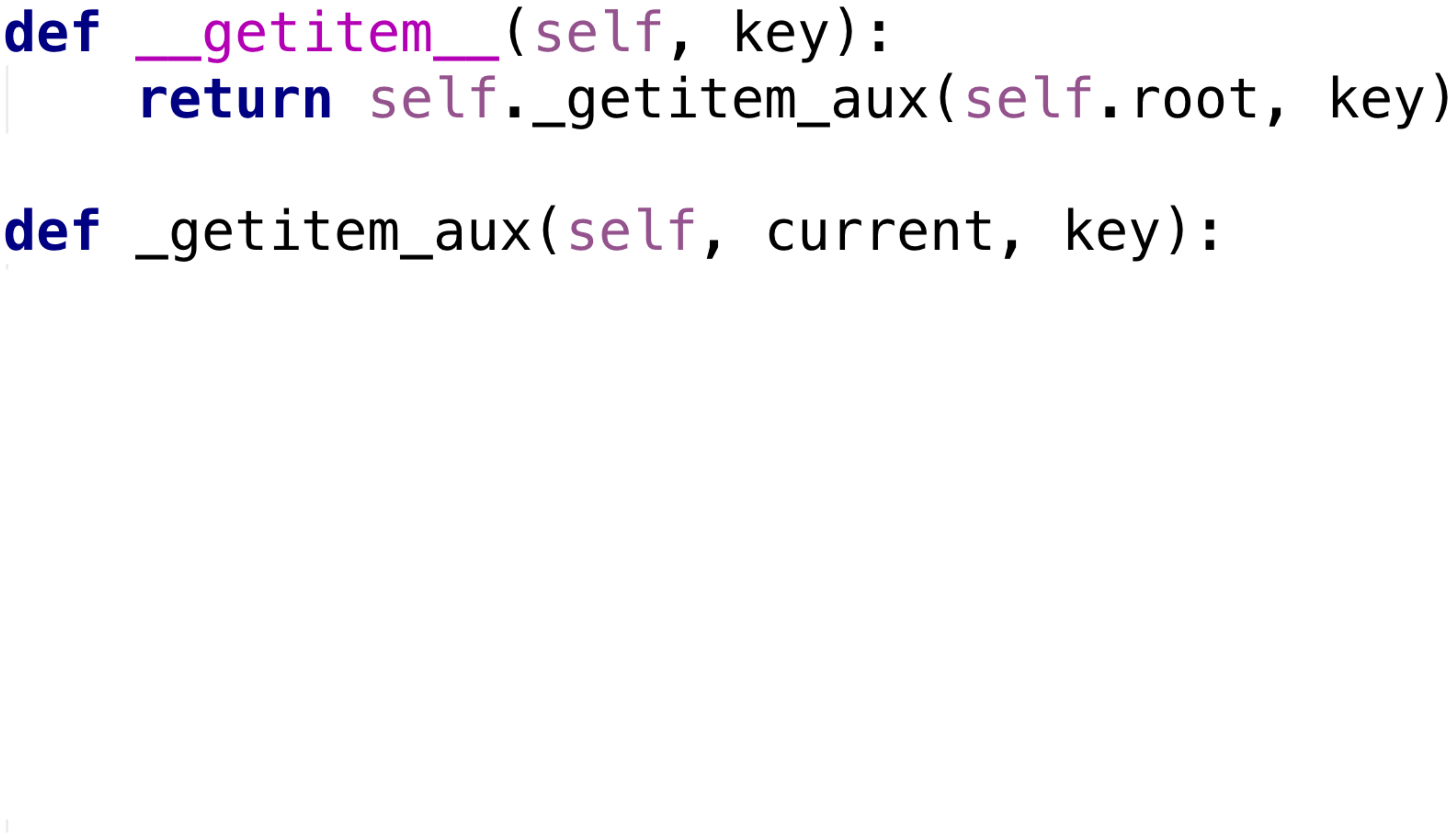
```python
def __contains__(self, key):
    return self._contains_aux(self.root, key)

def _contains_aux(self, current, key):
    if current is None:  # base case: empty
        raise KeyError("Key not found")
    elif key == current.key:  # base case: found
        return True
    elif key < current.key:
        return self.contains_aux(current.left, key)
    else:  # key > current.key
        return self.contains_aux(current.right, key)
```

we want to get the item associated to a key…

`__getitem__`

Search for key and retrieve item associated with that key

__getitem__



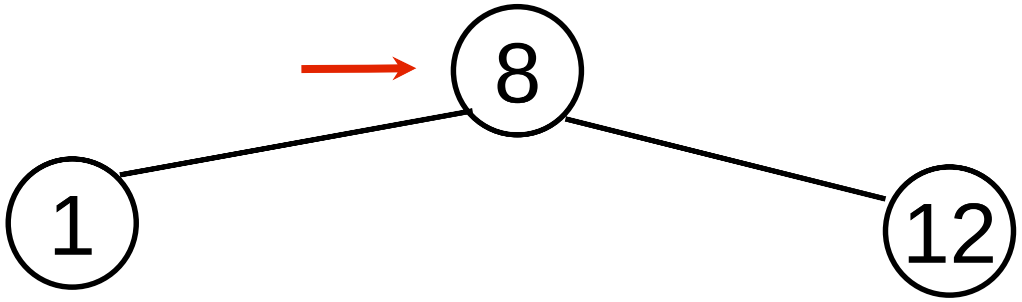
```python
def __getitem__(self, key):
    return self._getitem_aux(self.root, key)

def _getitem_aux(self, current, key):
```

__getitem__

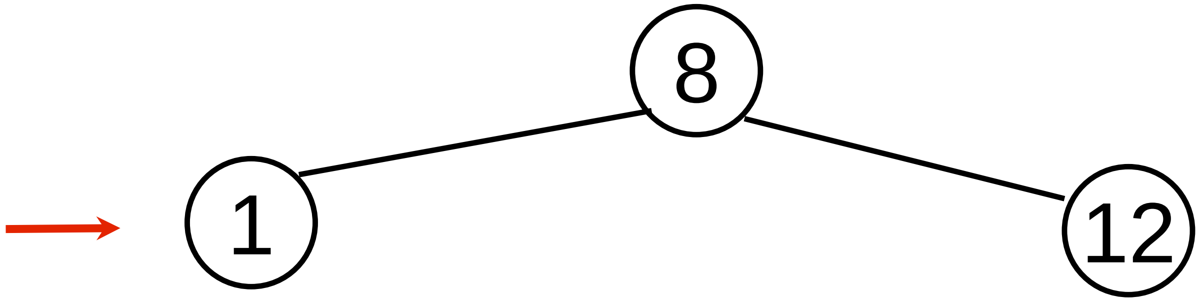
```
def __getitem__(self, key):
    return self._getitem_aux(self.root, key)

def _getitem_aux(self, current, key):
    if current is None:  # base case: empty
        raise KeyError("Key not found")
    elif key == current.key: # base case: found
        return current.item
    elif key < current.key:
        return self.getitem_aux(current.left, key)
    else:  # key > current.key
     return self.getitem_aux(current.right, key)
```

Insert

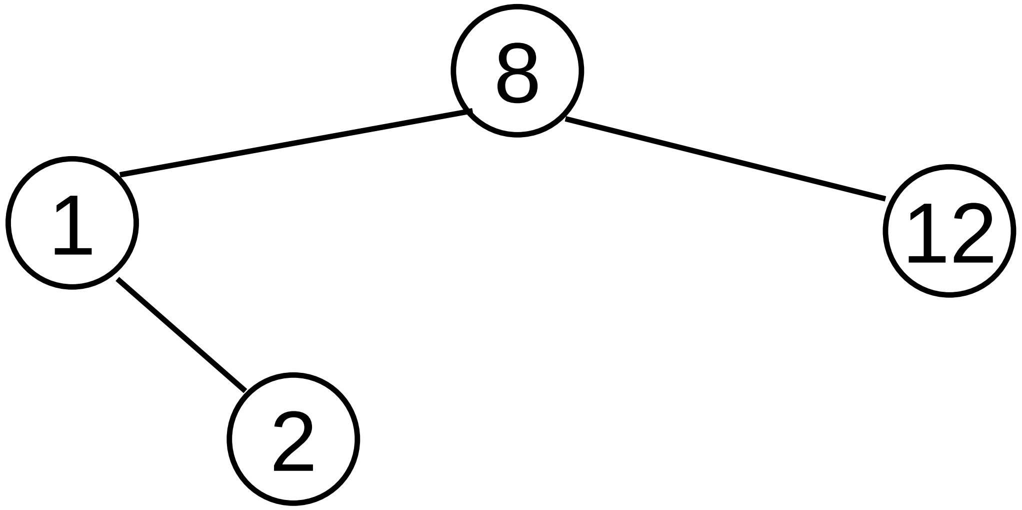
Insert 2



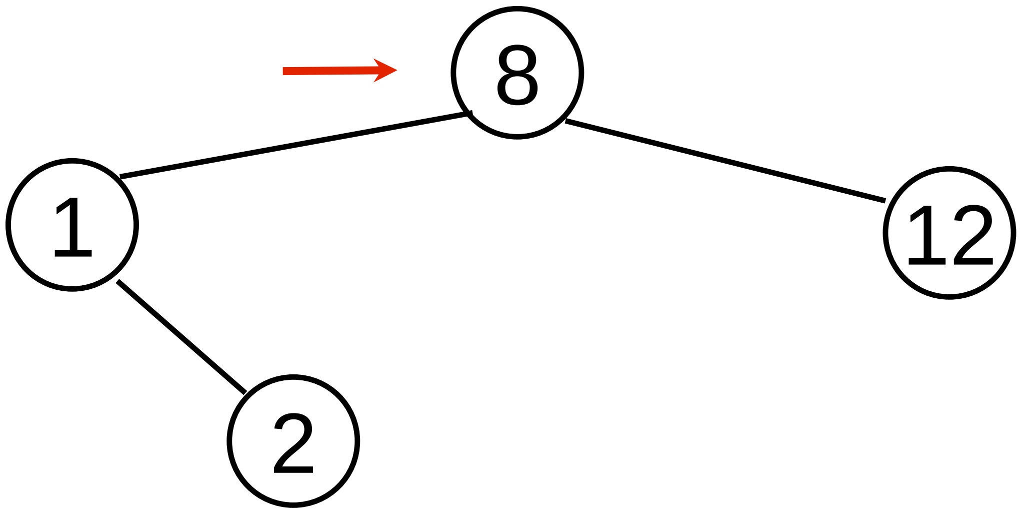
Insert 2



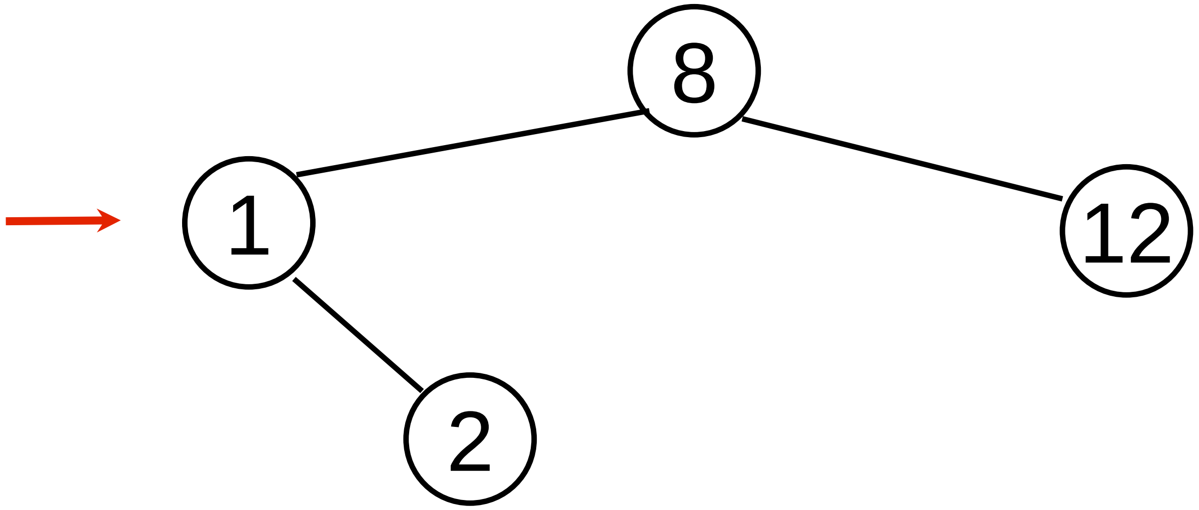
Insert 2



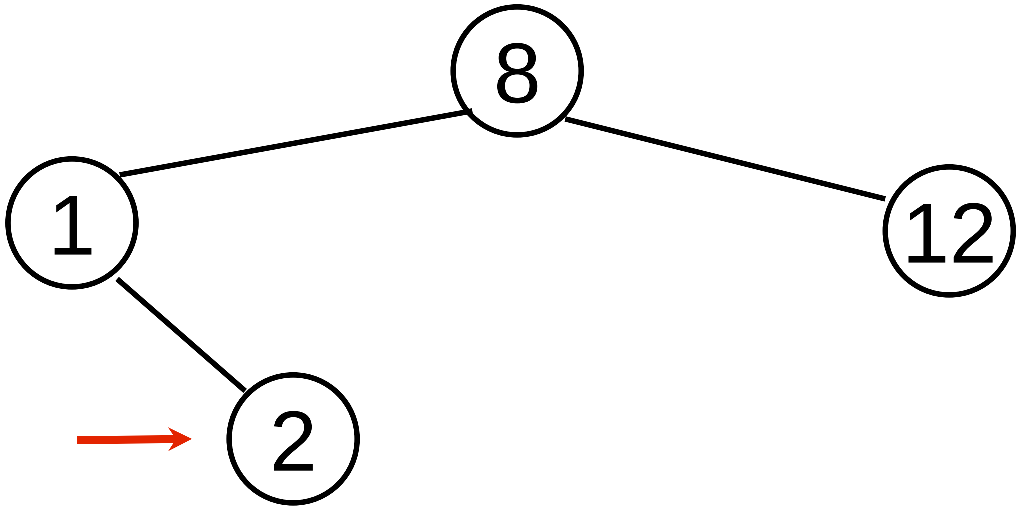
Insert 7



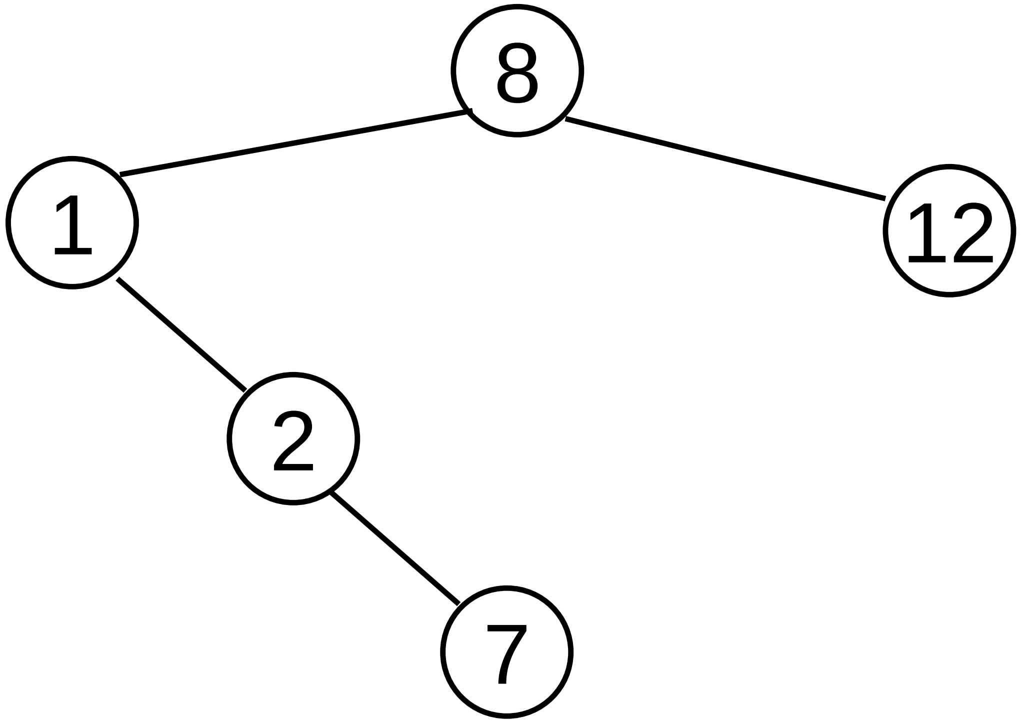
Insert 7



Insert 7



Insert 7



Our BST does not allow for duplicates, so we need to do something if we find the key in the tree…

Insert algorithm

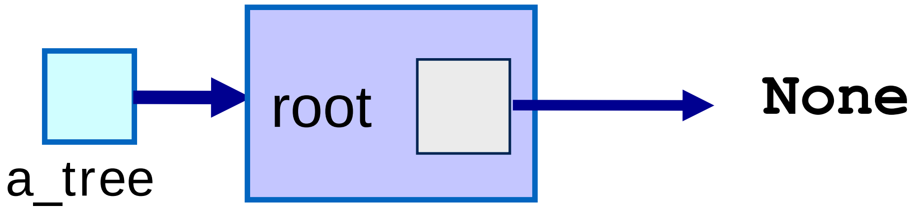
Input: key and associated item to insert.

Idea: Find the right spot (search) then create new node.

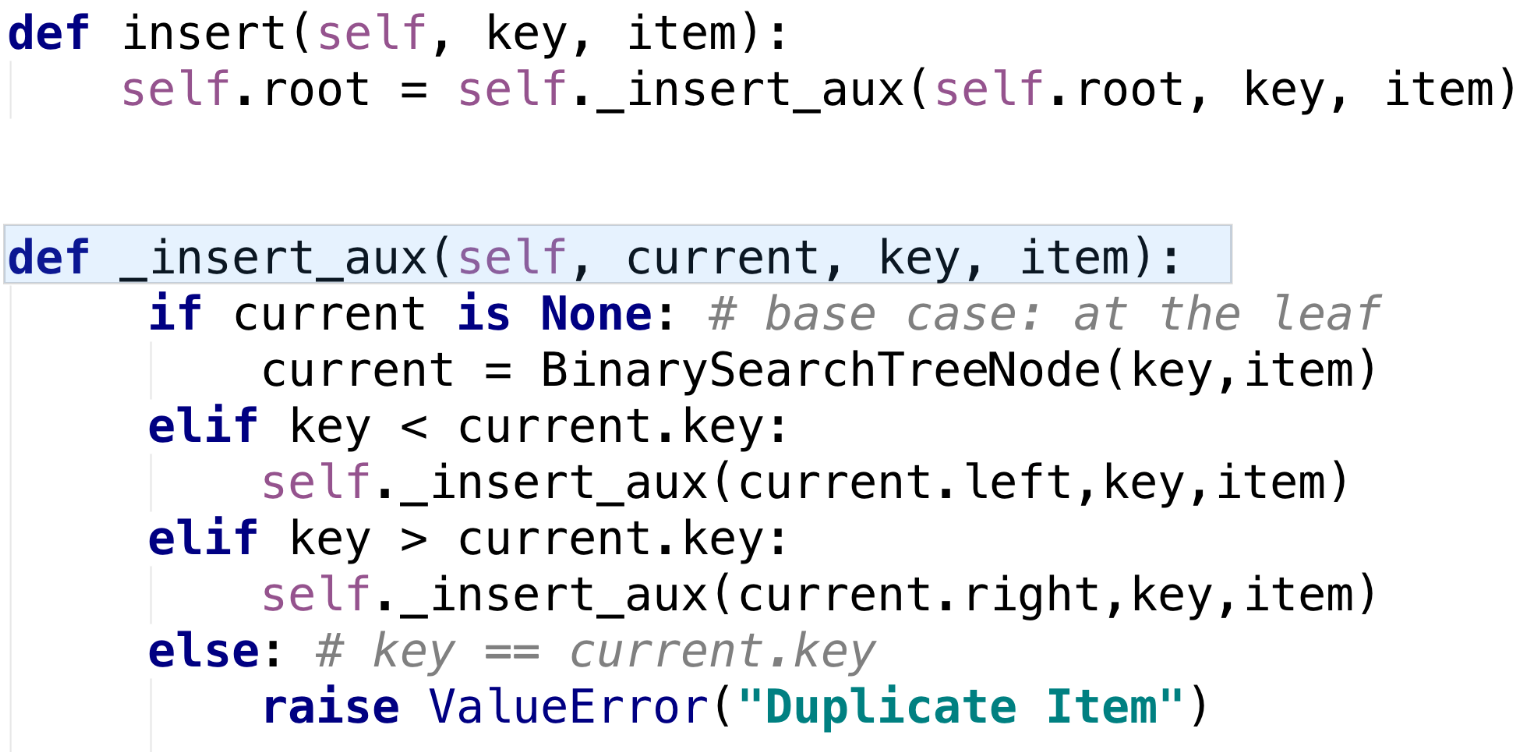
- Try to find the key…
    - ➞ Found? Raise an exception (insert expects no duplicates)
    - ➞ Not found? parent of **`None`** should be the parent of new node, which needs to be created.

```python
def insert(self, key, item):
    self.root = self._insert_aux(self.root, key, item)


def _insert_aux(self, current, key, item):
     if current is None: # base case: at the leaf
         current = BinarySearchTreeNode(key,item)
     elif key < current.key:
         self._insert_aux(current.left,key,item)
     elif key > current.key:
         self._insert_aux(current.right,key,item)
     else: # key == current.key
         raise ValueError("Duplicate Item")
```



```
a_tree.insert(57, “Coco”)
```



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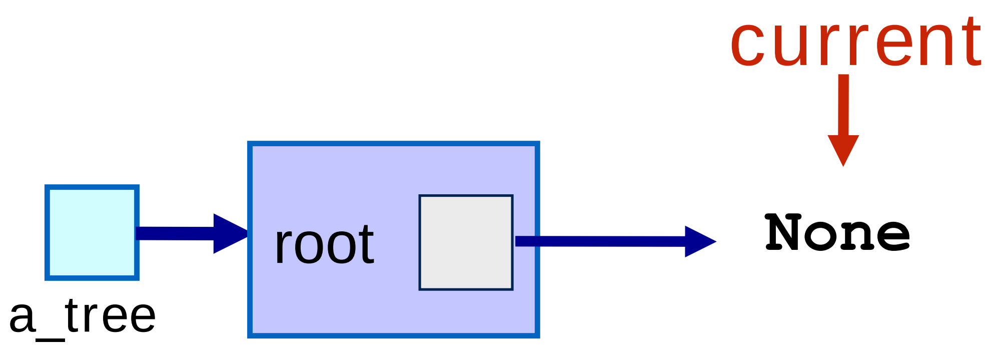
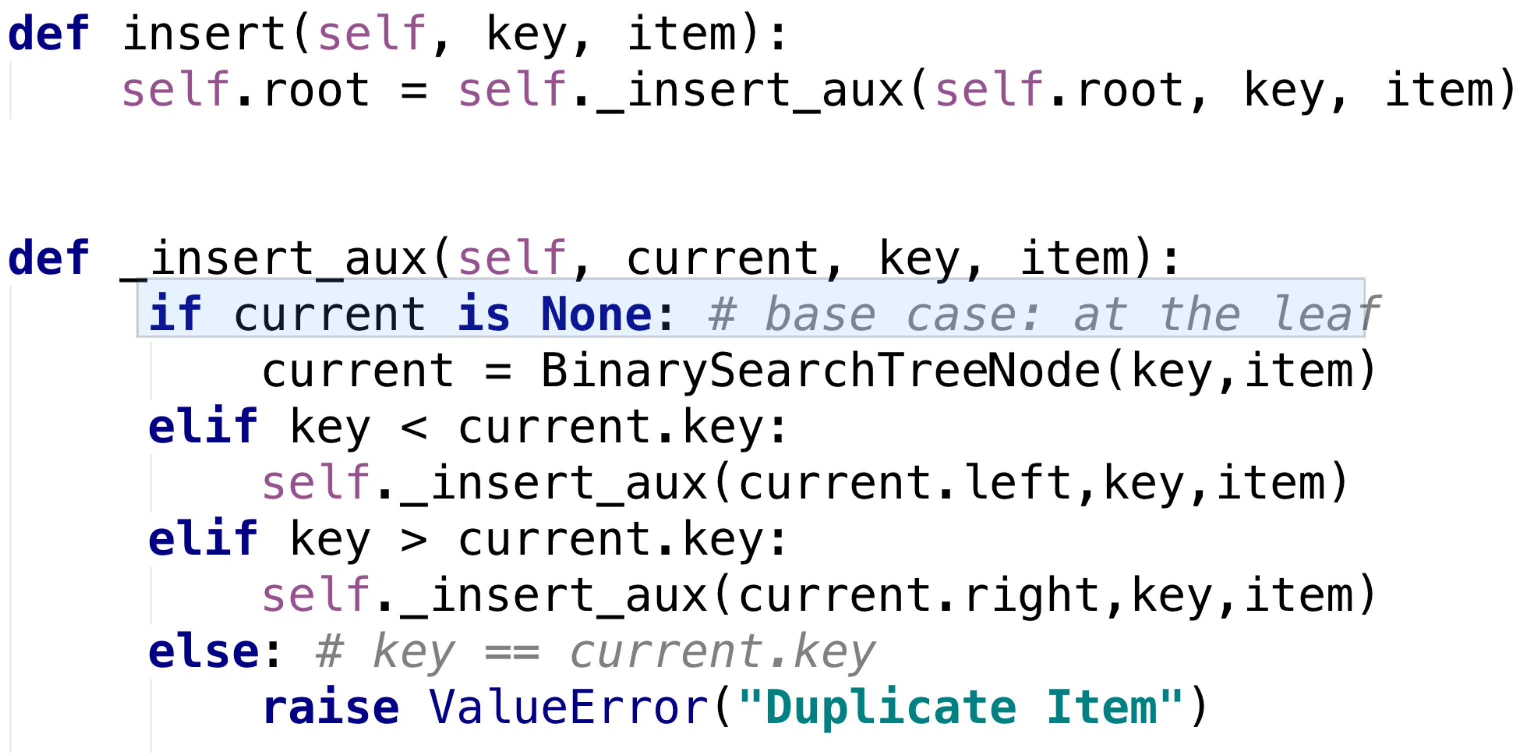


key → 57

item → “Coco”

a_tree.insert(57, “Coco”)



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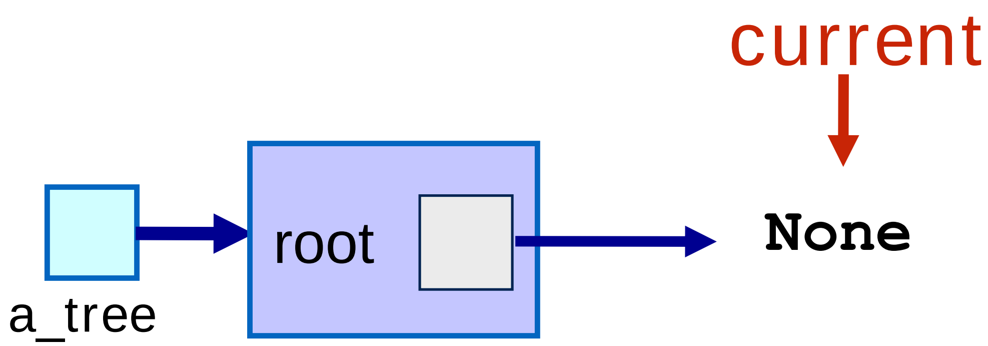
        self._insert_aux(current.right,key,item)
    else: # key == current.key
        raise ValueError("Duplicate Item")
```



key → 57

item → **“Coco”**

a_tree.insert(57, “Coco”)

```python
def insert(self, key, item):
    self.root = self._insert_aux(self.root, key, item)


def _insert_aux(self, current, key, item):
     if current is None: # base case: at the leaf
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     elif key < current.key:
         self._insert_aux(current.left,key,item)
     elif key > current.key:
         self._insert_aux(current.right,key,item)
     else: # key == current.key
         raise ValueError("Duplicate Item")
```

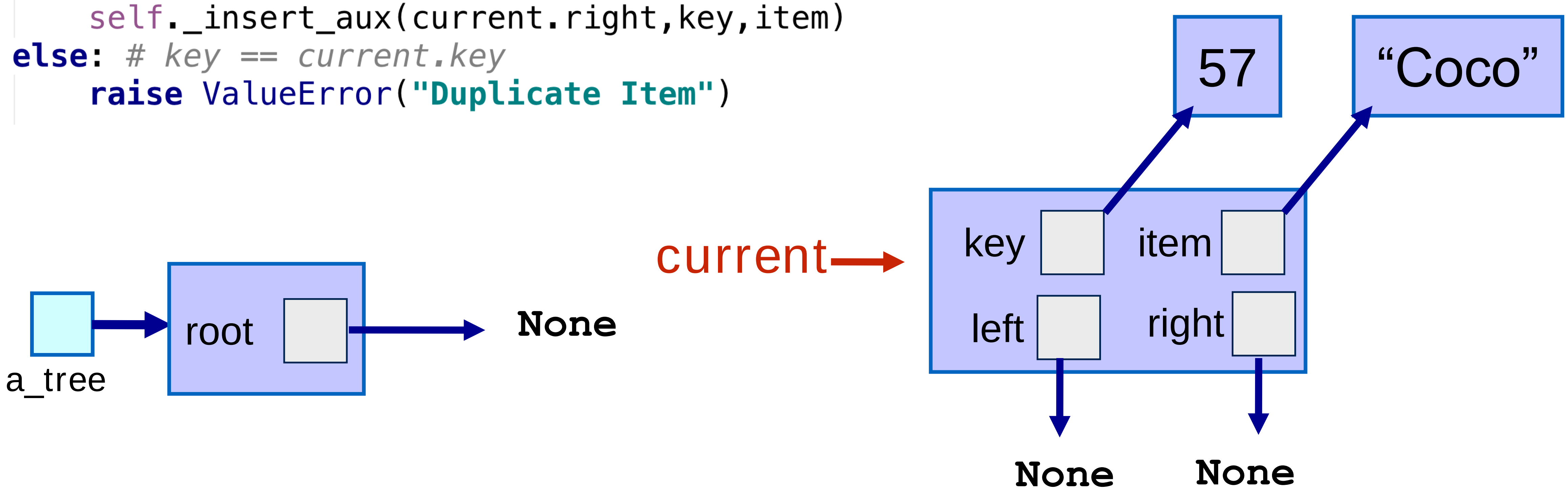

a_tree.insert(57, “Coco”)

```python
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def _insert_aux(self, current, key, item):
    if current is None: # base case: at the leaf
        current = BinarySearchTreeNode(key,item)
    elif key < current.key:
        self._insert_aux(current.left,key,item)
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        self._insert_aux(current.right,key,item)
    else: # key == current.key
        raise ValueError("Duplicate Item")
```
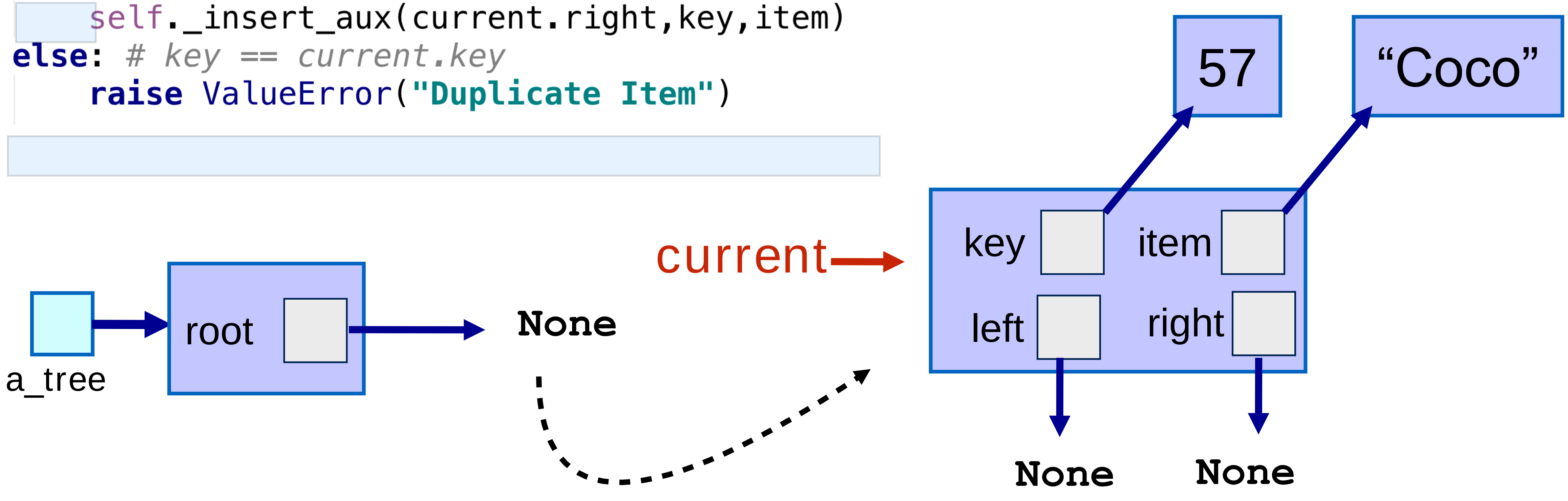



missing link!

current needs to be returned!

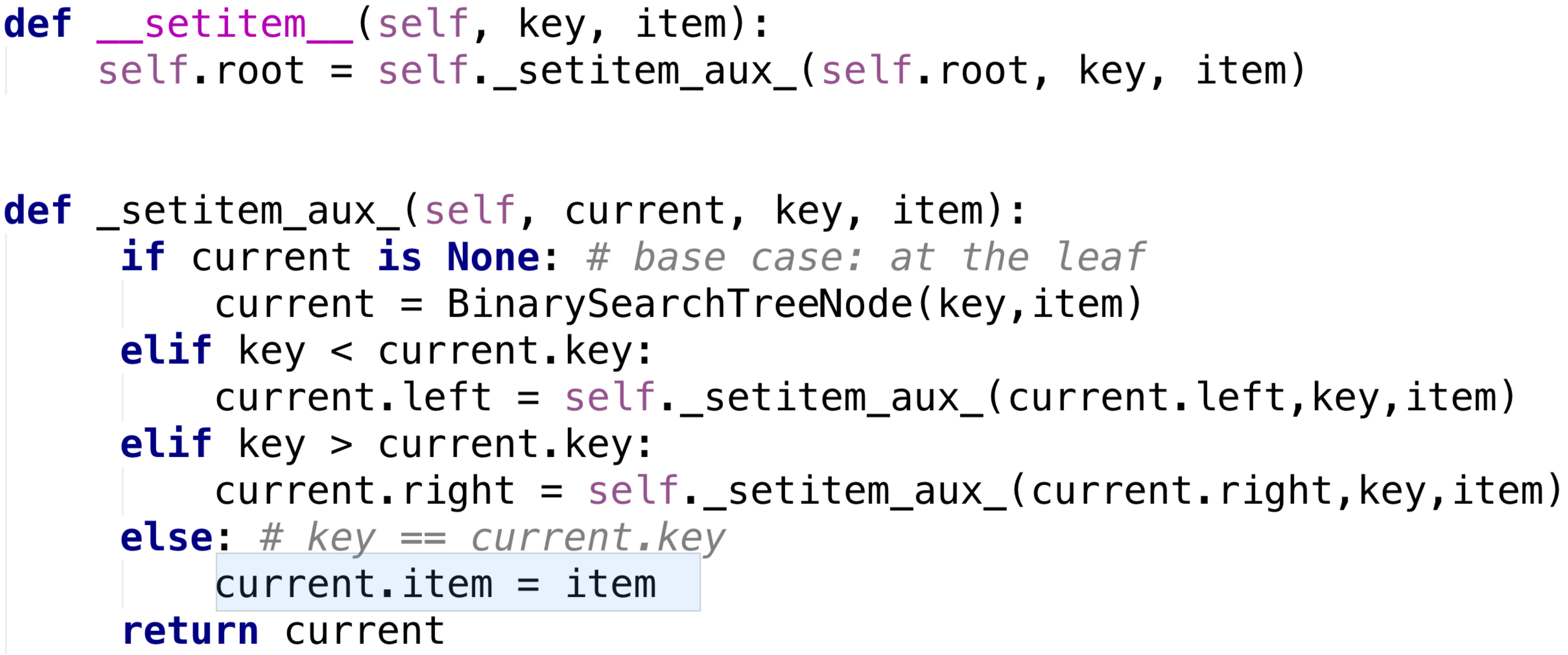
```
def insert(self, key, item):
    self.root = self._insert_aux(self.root, key, item)


def _insert_aux(self, current, key, item):
```

)

```python
def insert(self, key, item):
    self.root = self._insert_aux(self.root, key, item)


def _insert_aux(self, current, key, item):
     if current is None: # base case: at the leaf
         current = BinarySearchTreeNode(key,item)
     elif key < current.key:
         current.left = self._insert_aux(current.left,key,item)
     elif key > current.key:
         current.right = self._insert_aux(current.right,key,item)
     else: # key == current.key
         raise ValueError("Duplicate Item")
     return current
```

```
def insert(self, key, item):
    self.root = self._insert_aux(self.root, key, item)


def _insert_aux(self, current, key, item):
     if current is None: # base case: at the leaf
         current = BinarySearchTreeNode(key,item)
     elif key < current.key:
         current.left = self._insert_aux(current.left,key,item)
     elif key > current.key:
         current.right = self._insert_aux(current.right,key,item)
     else: # key == current.key
         raise ValueError("Duplicate Item")
     return current
```

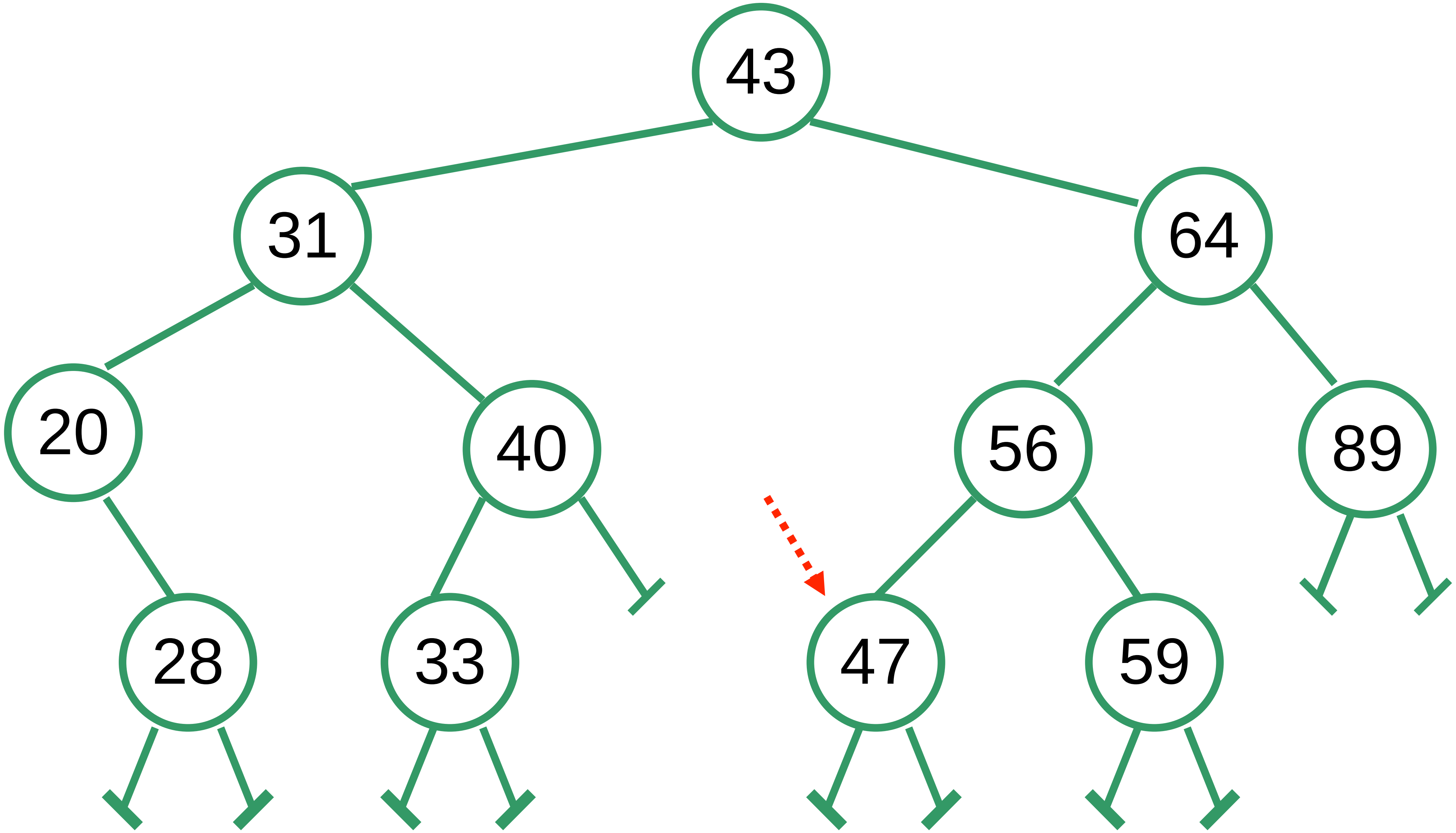
__setitem__



```
def __setitem__(self, key, item):
    self.root = self._setitem_aux_(self.root, key, item)


def _setitem_aux_(self, current, key, item):
     if current is None: # base case: at the leaf
         current = BinarySearchTreeNode(key,item)
     elif key < current.key:
         current.left = self._setitem_aux_(current.left,key,item)
     elif key > current.key:
         current.right = self._setitem_aux_(current.right,key,item)
     else: # key == current.key
         current.item = item
     return current
```

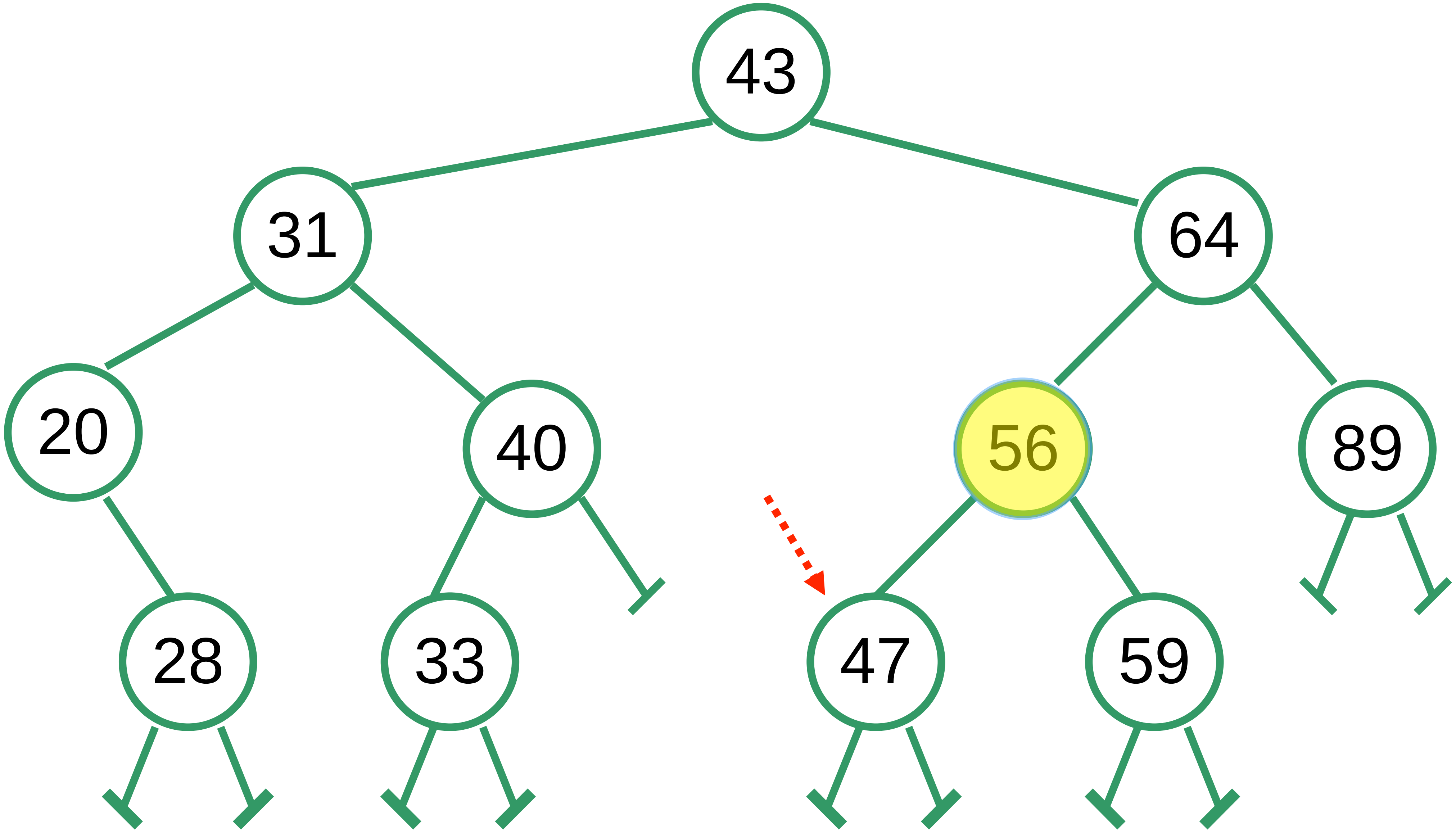
Delete

Delete 47



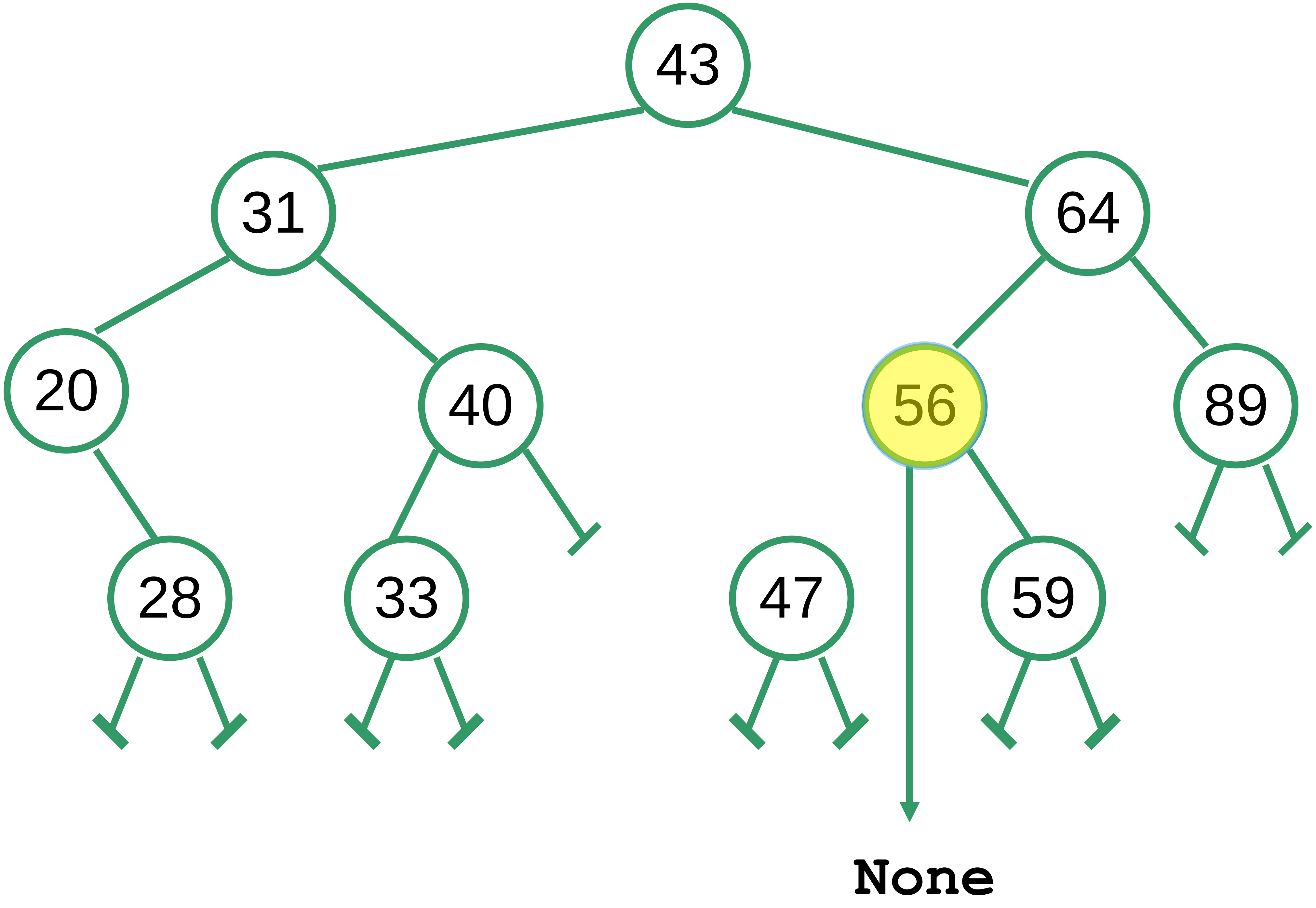
only showing key

Delete 47



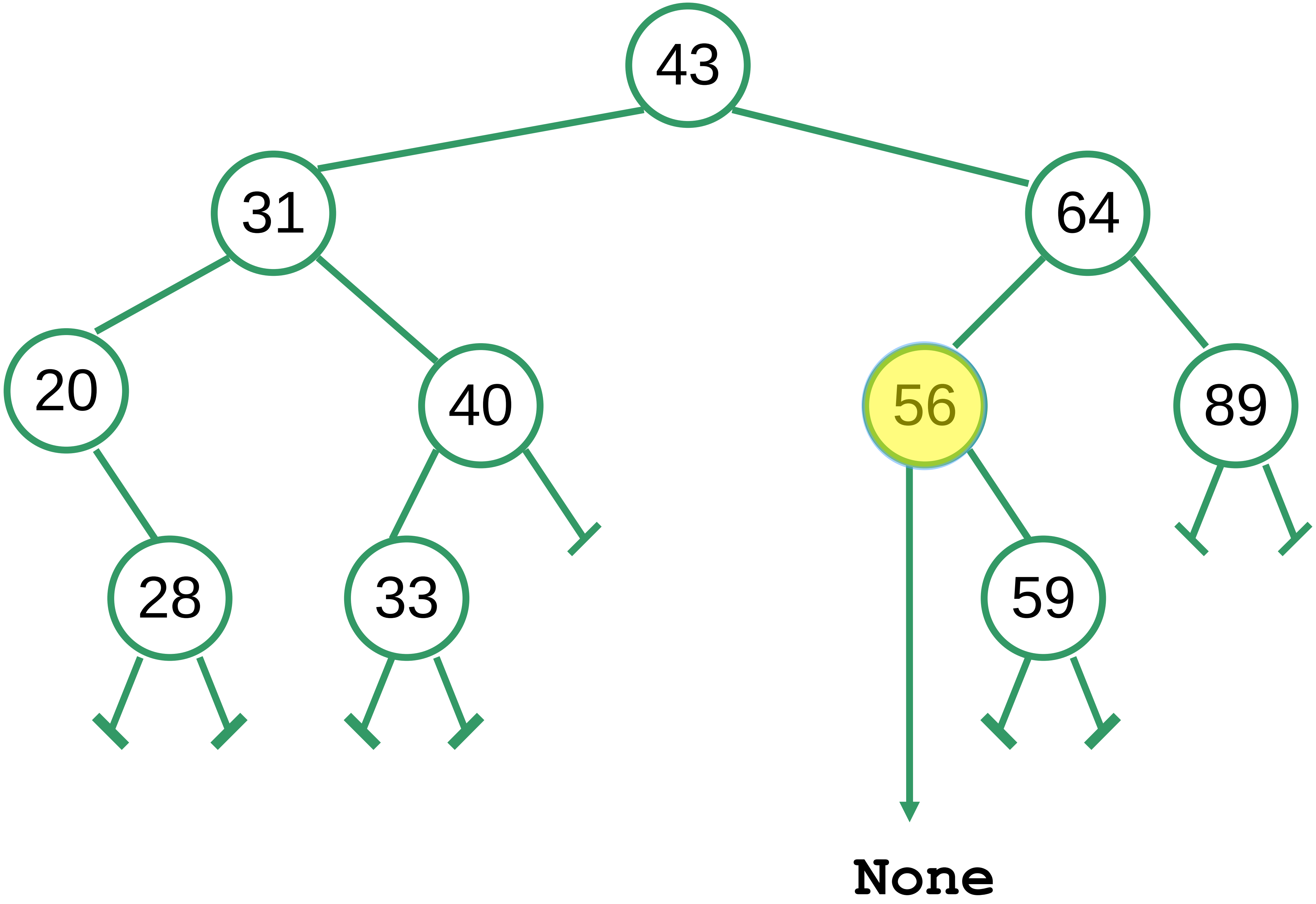
only showing key

Delete 47



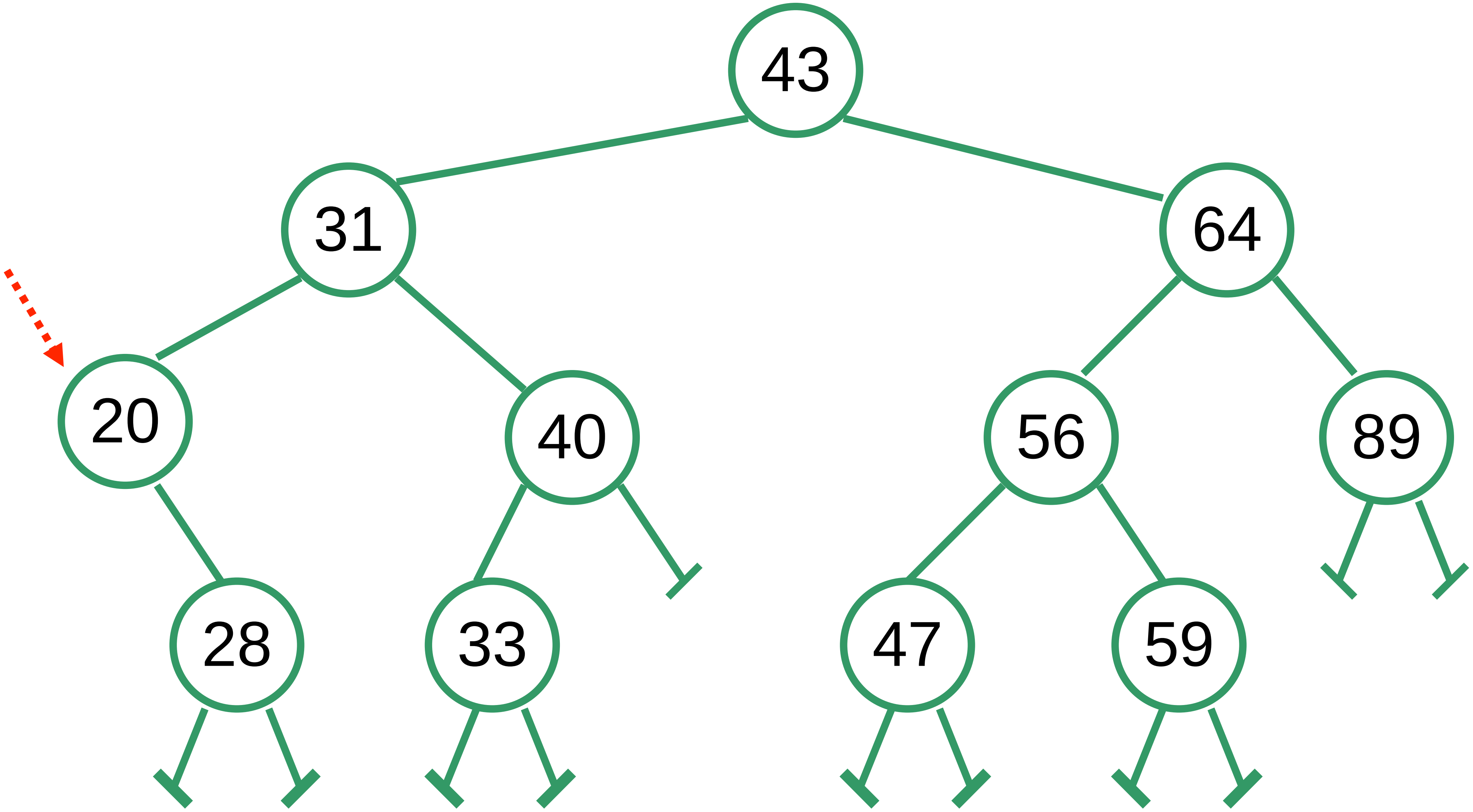
only showing key

Delete 47



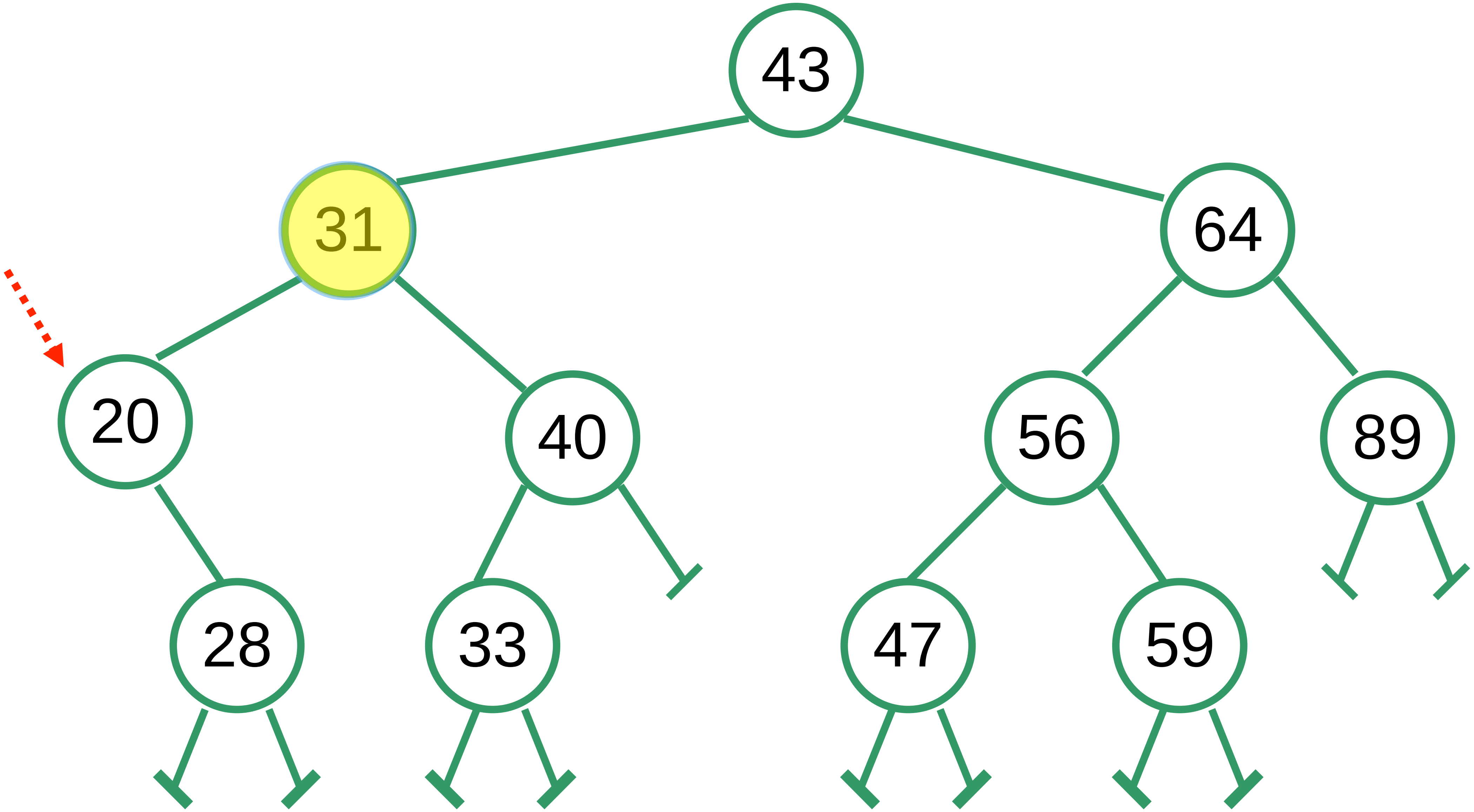
<u>Node with no child:</u>
Find parent - point to None

Delete 20



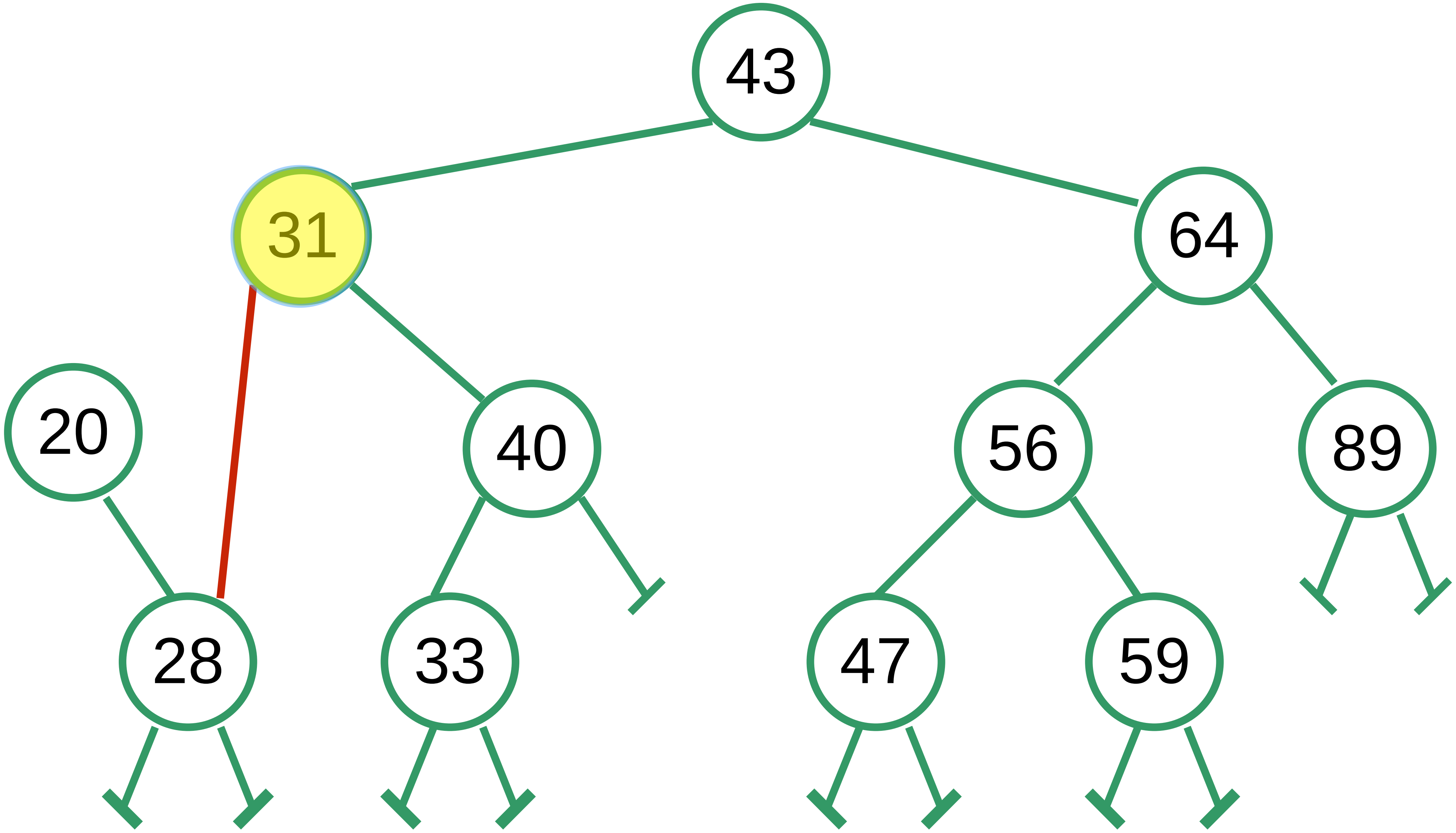
only showing key

Delete 20



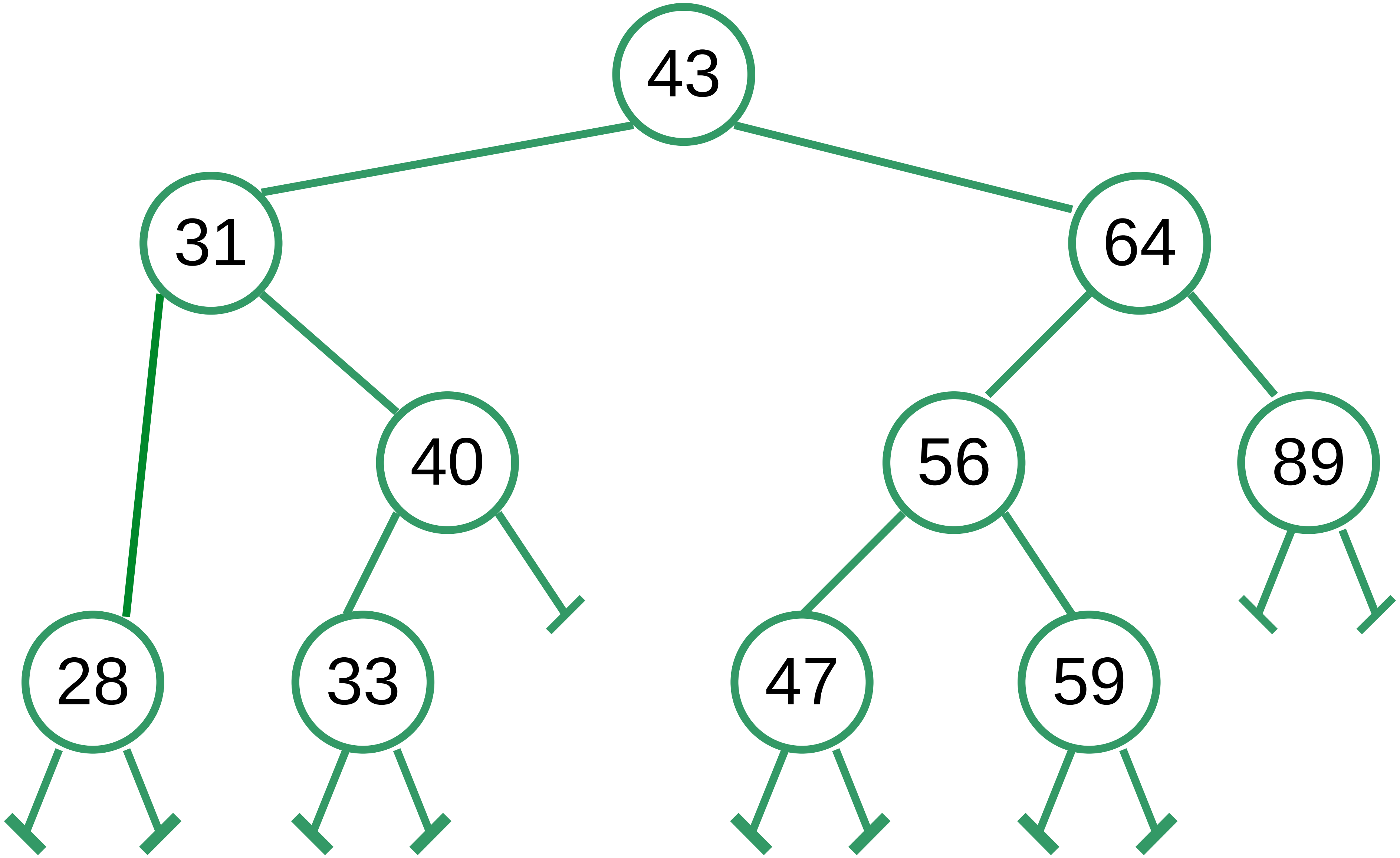
only showing key

Delete 20



only showing key

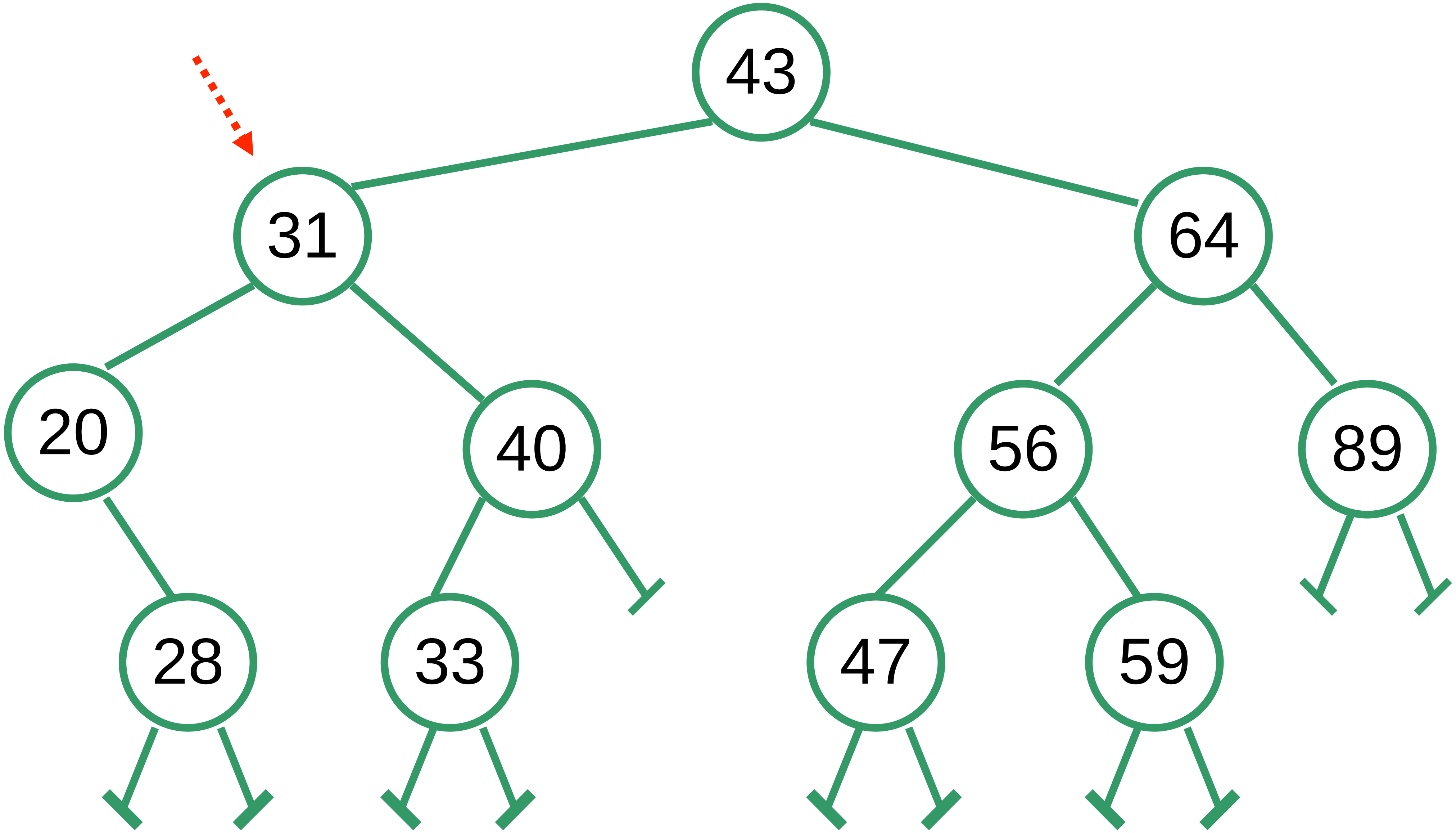
Delete 20



Node with one child:

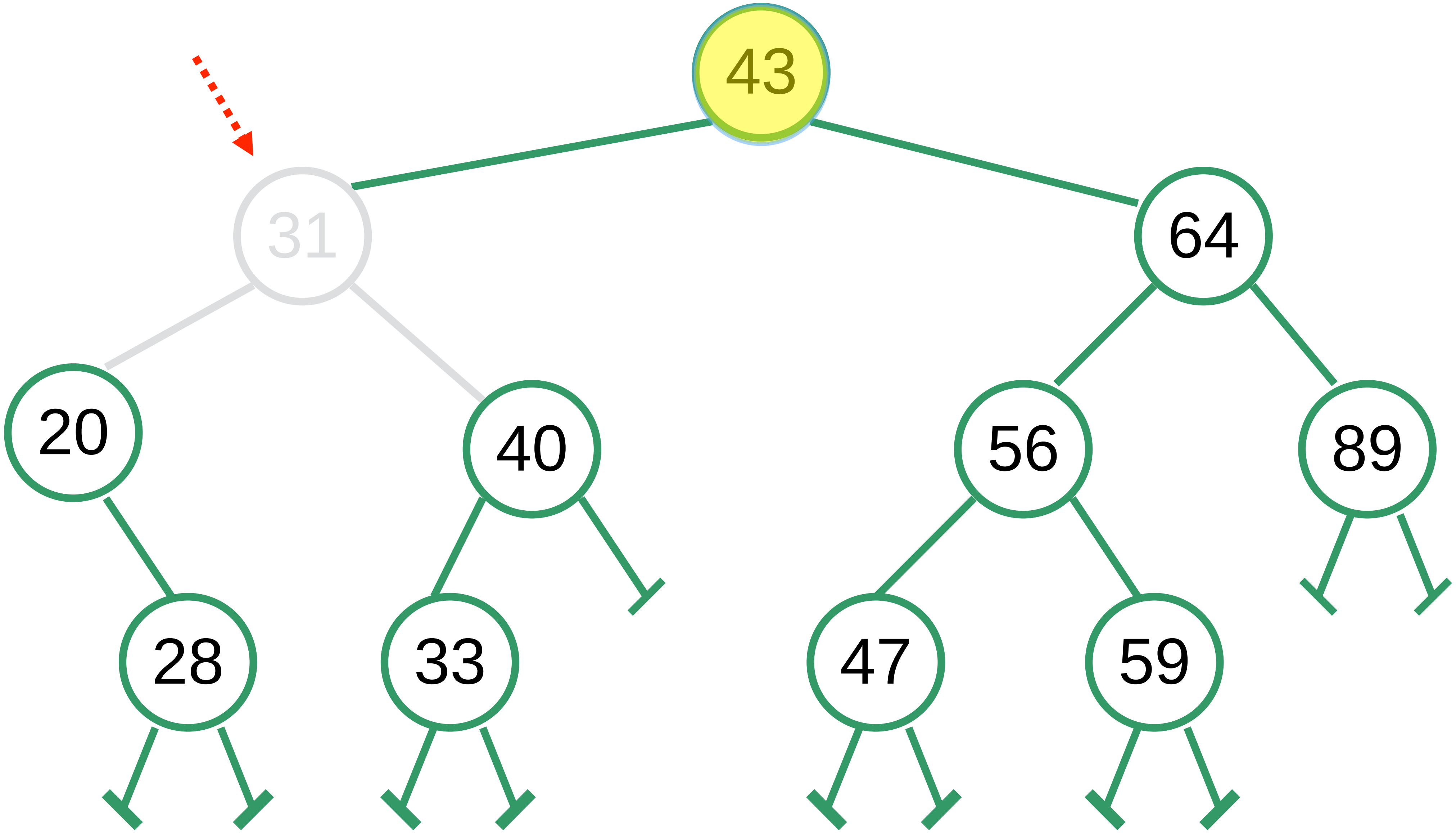
Find parent – point to child of deleted node

Delete 31



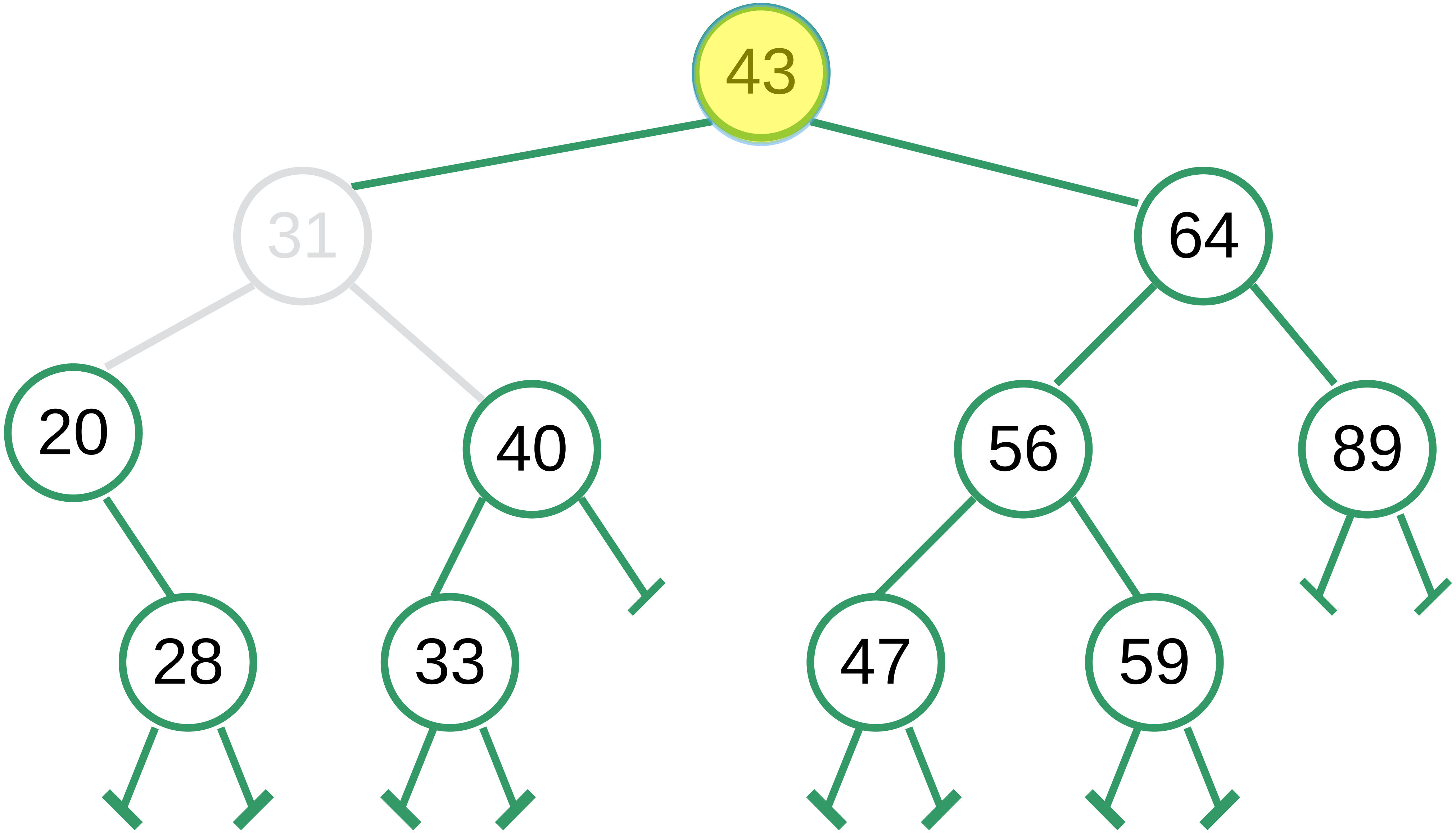
only showing key

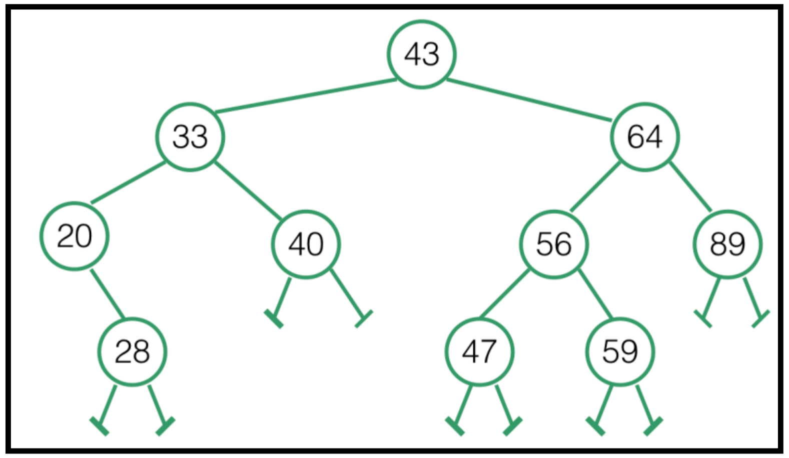
Delete 31



only showing key

Delete 31





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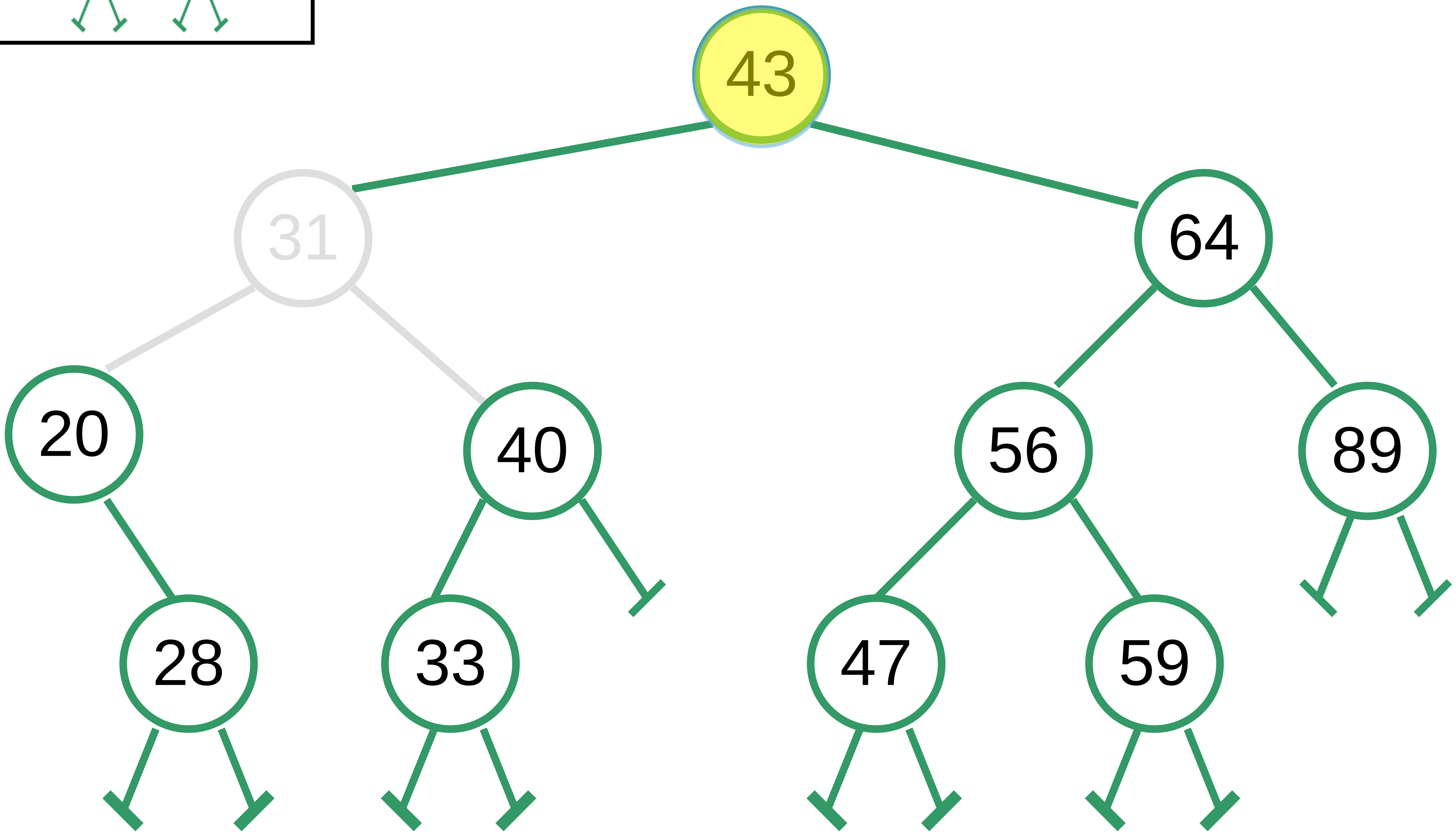

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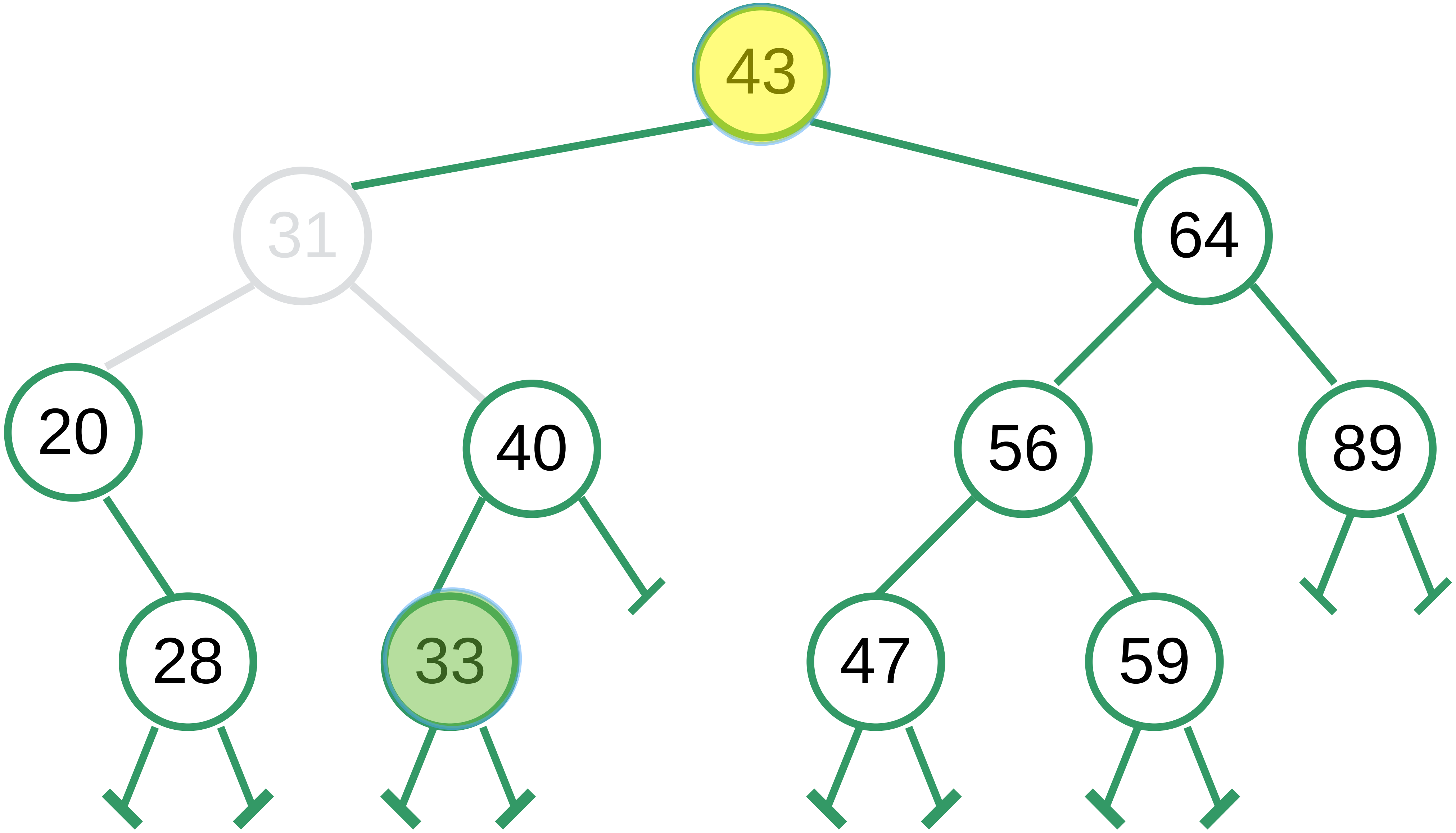
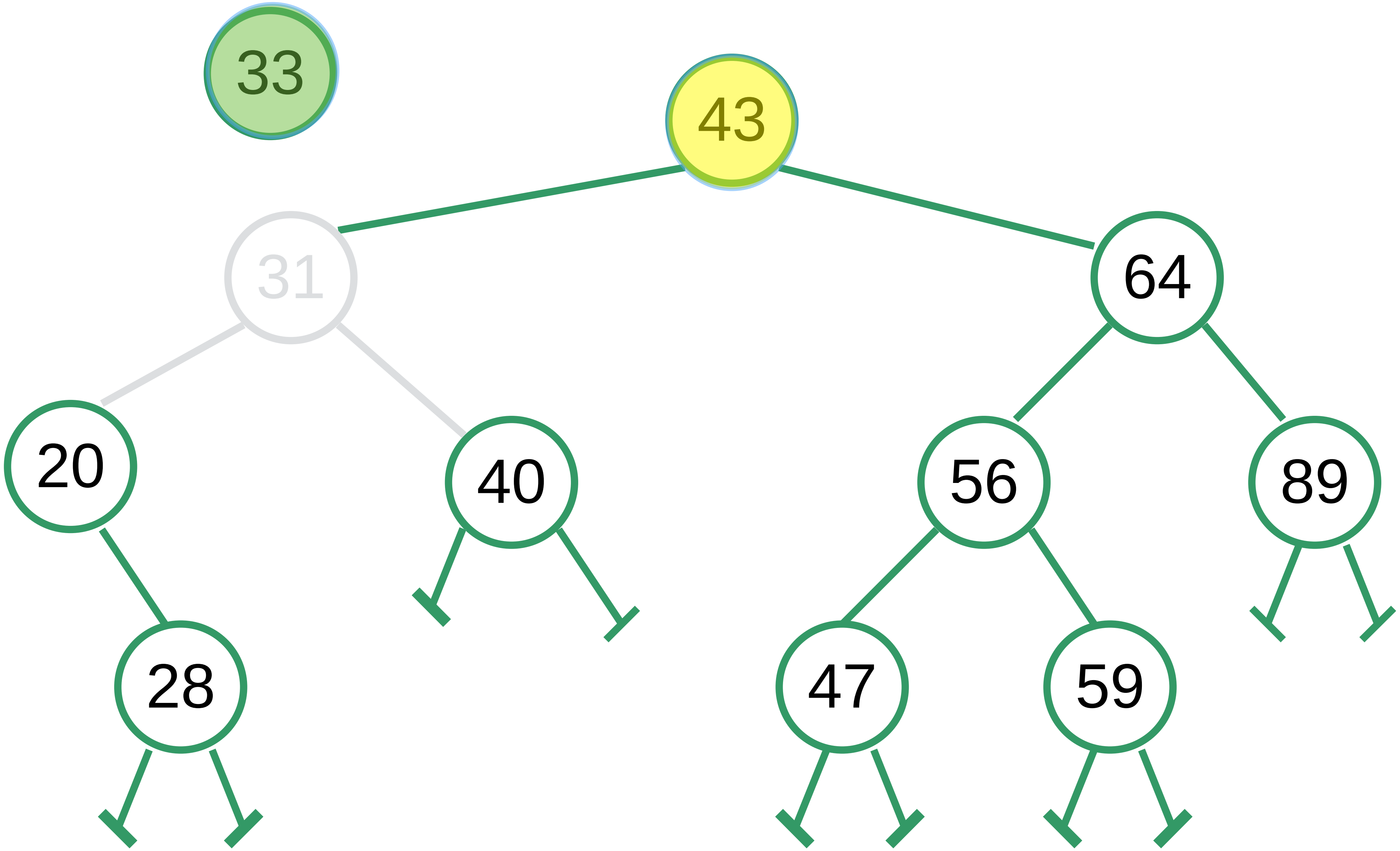


Successor of a node: node with next larger key.

Delete 31



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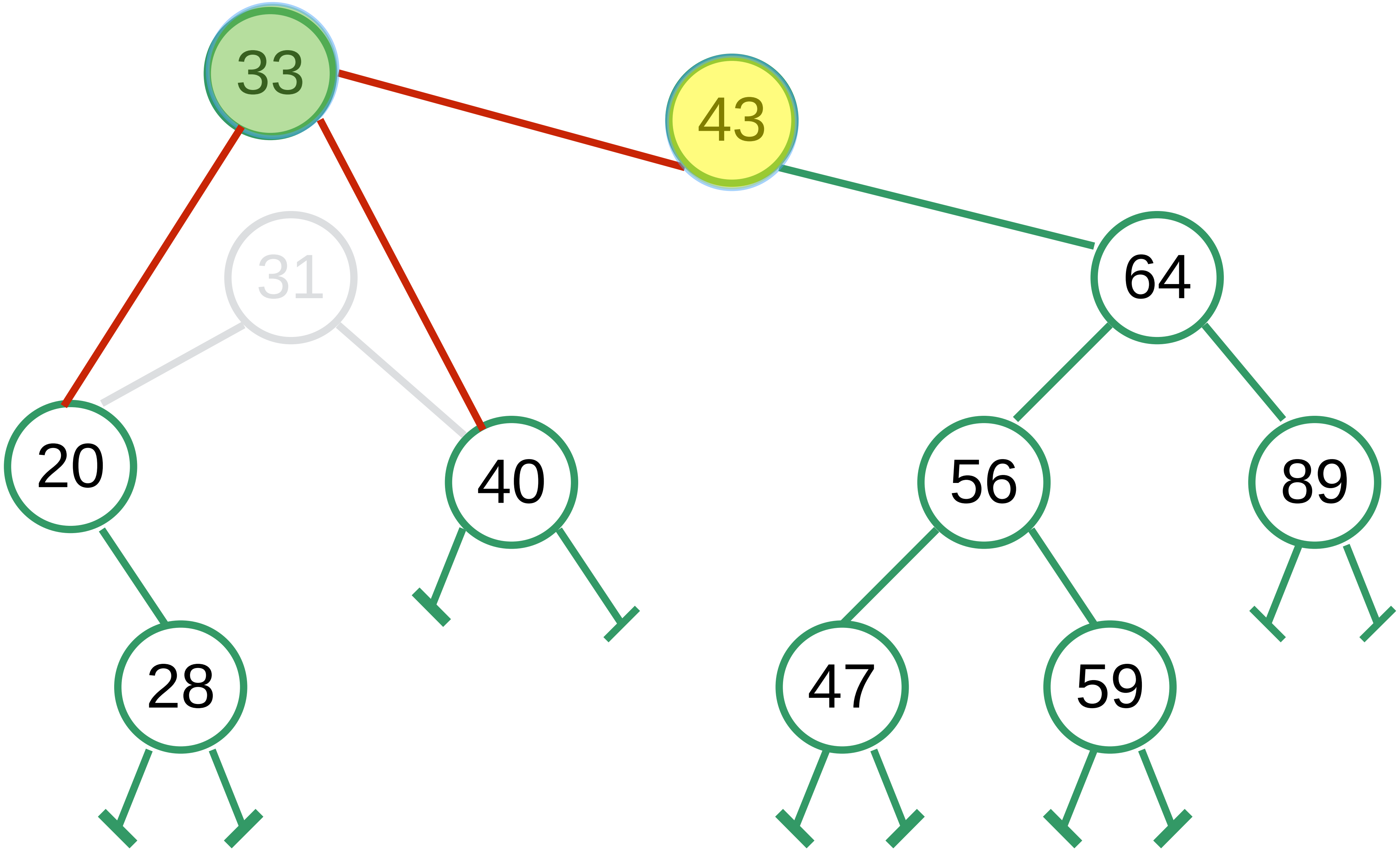

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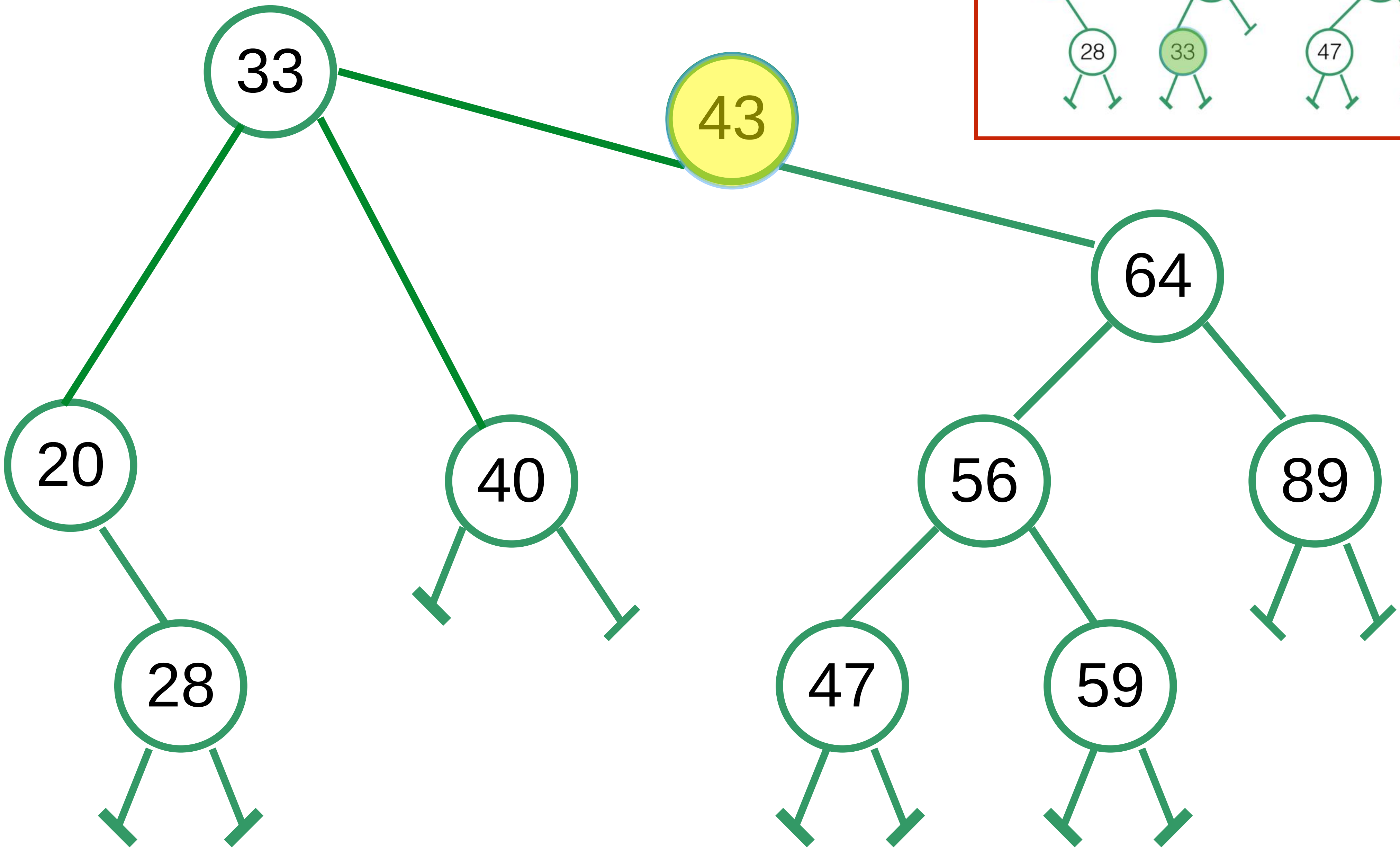
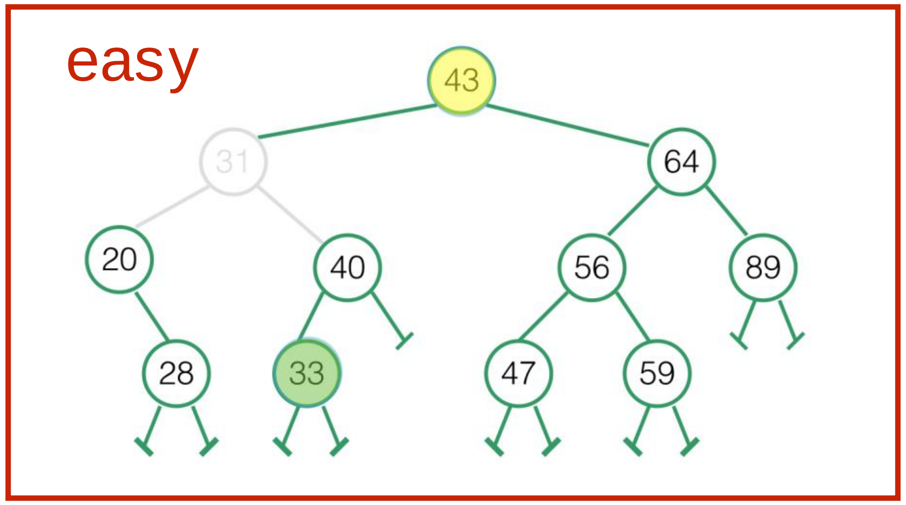
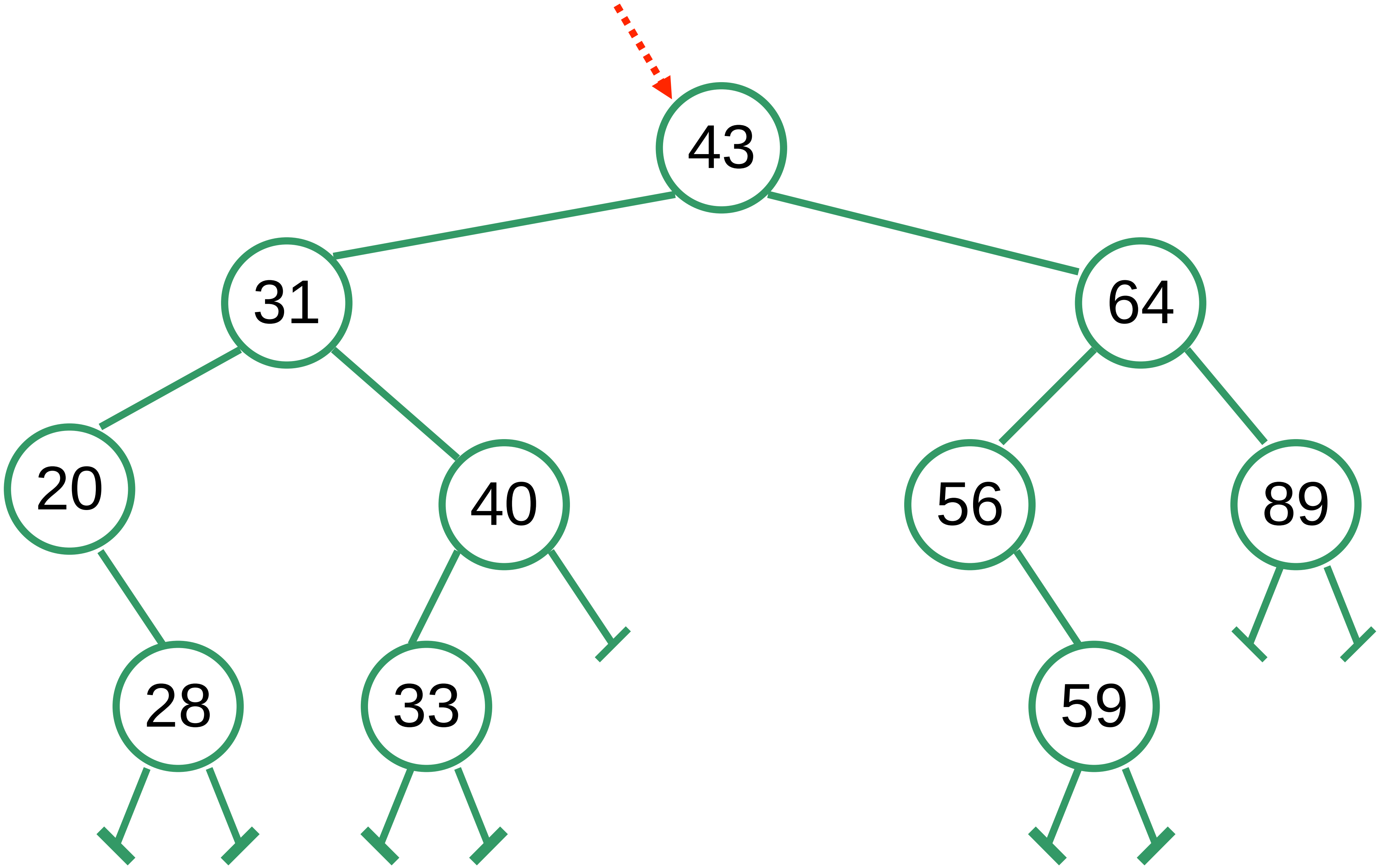




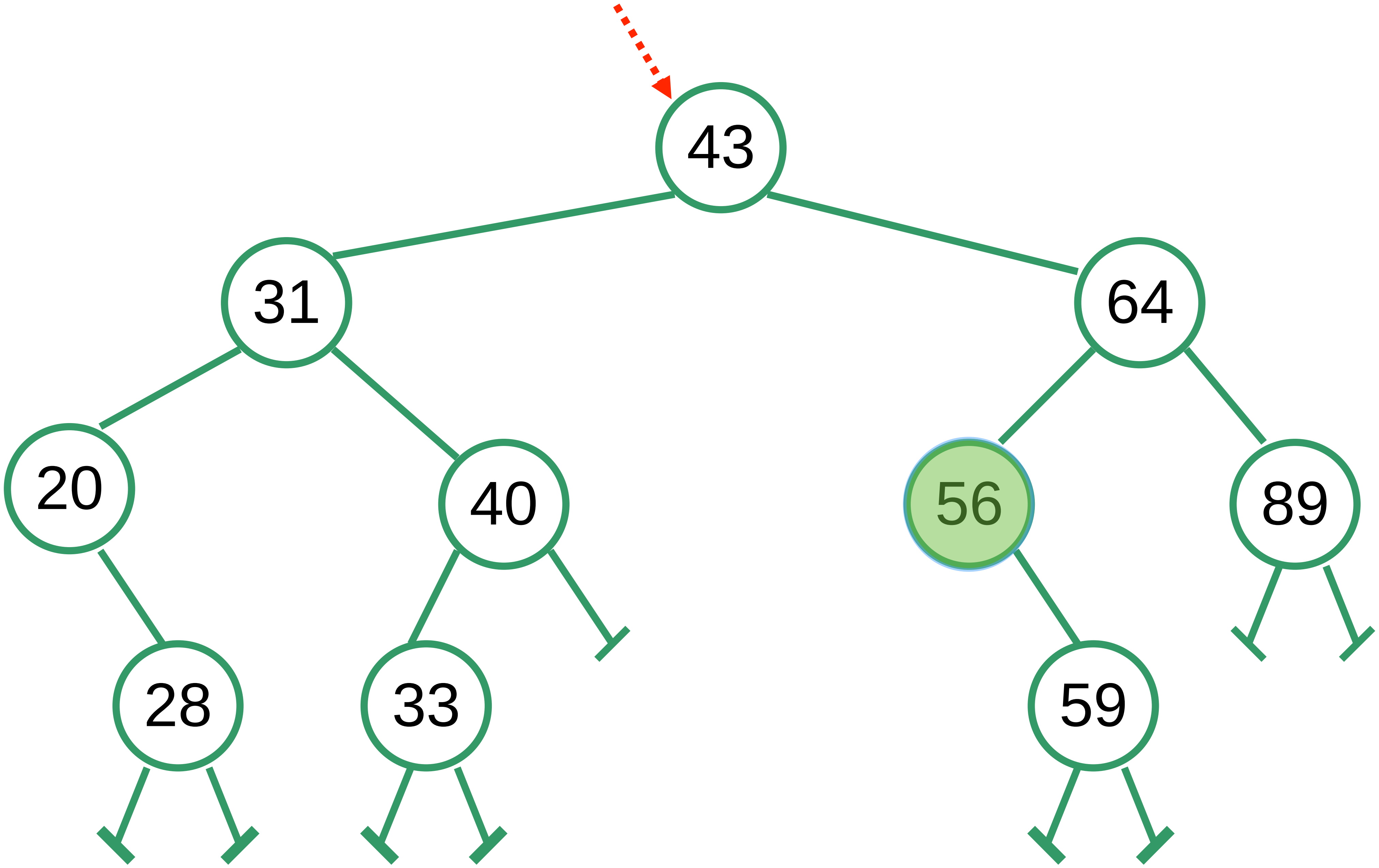
Node with two children:

Find parent and successor - successor is the new parent of the (orphan) children

Delete 43



Delete 43



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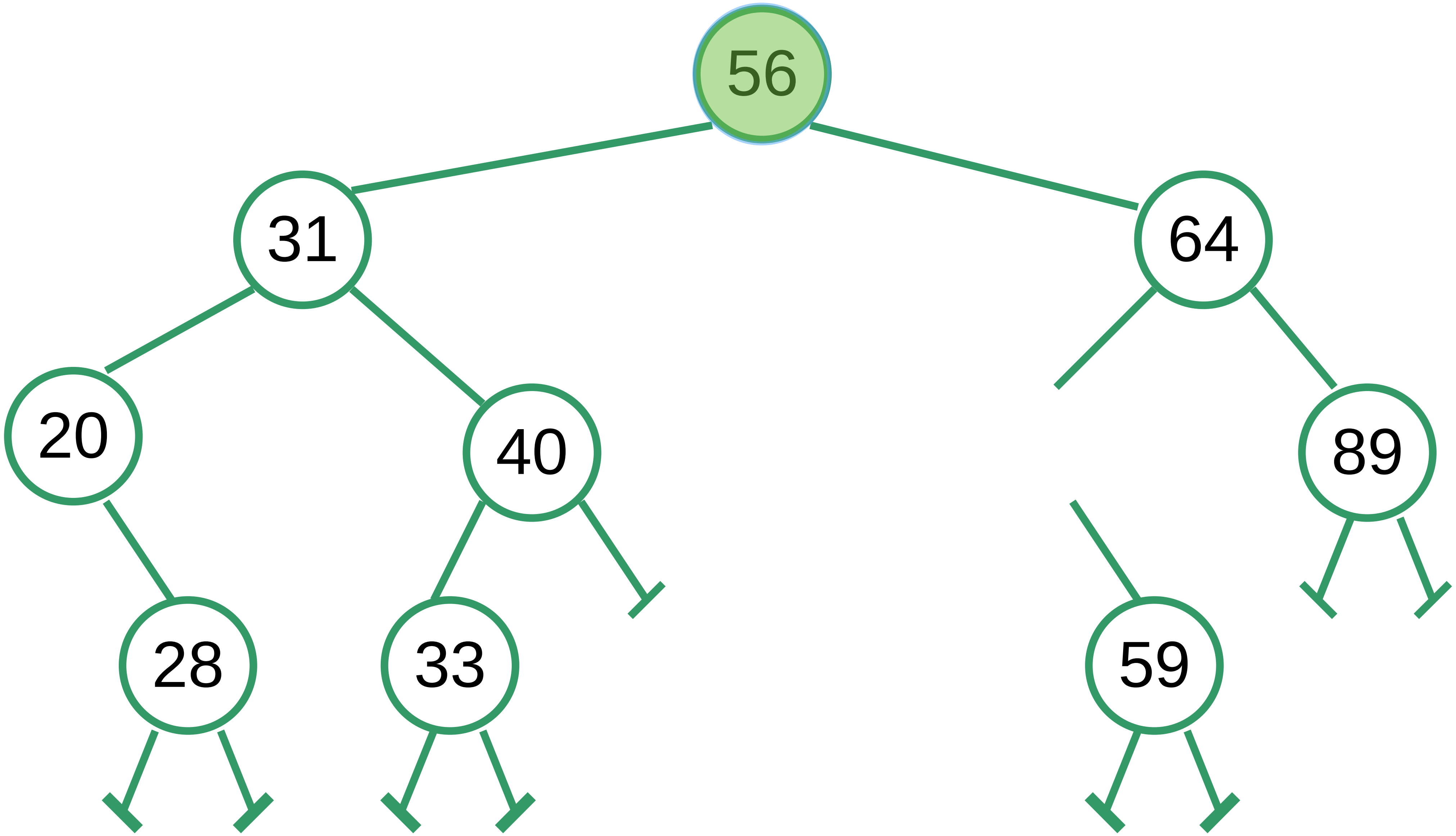
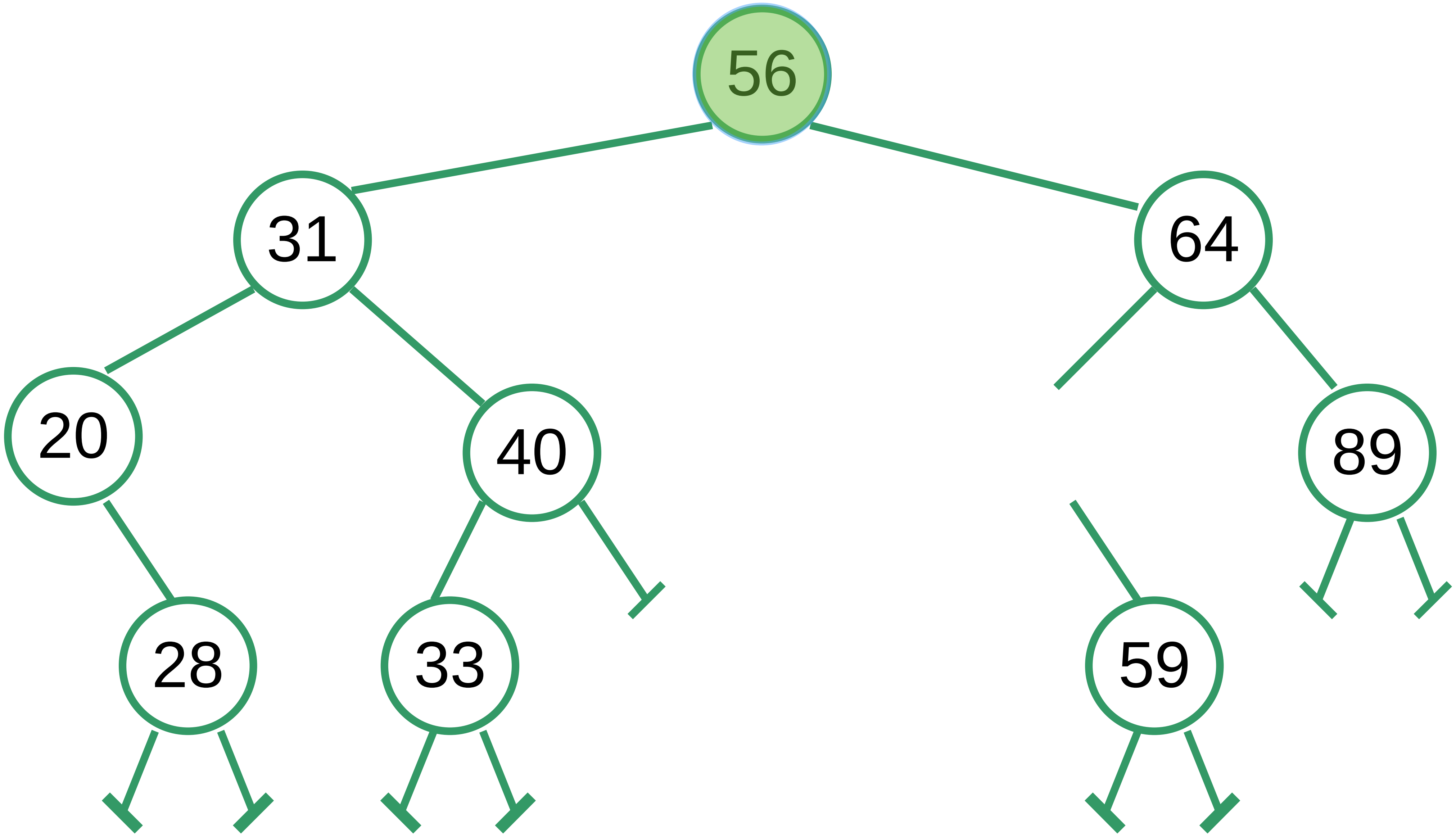


Delete 43



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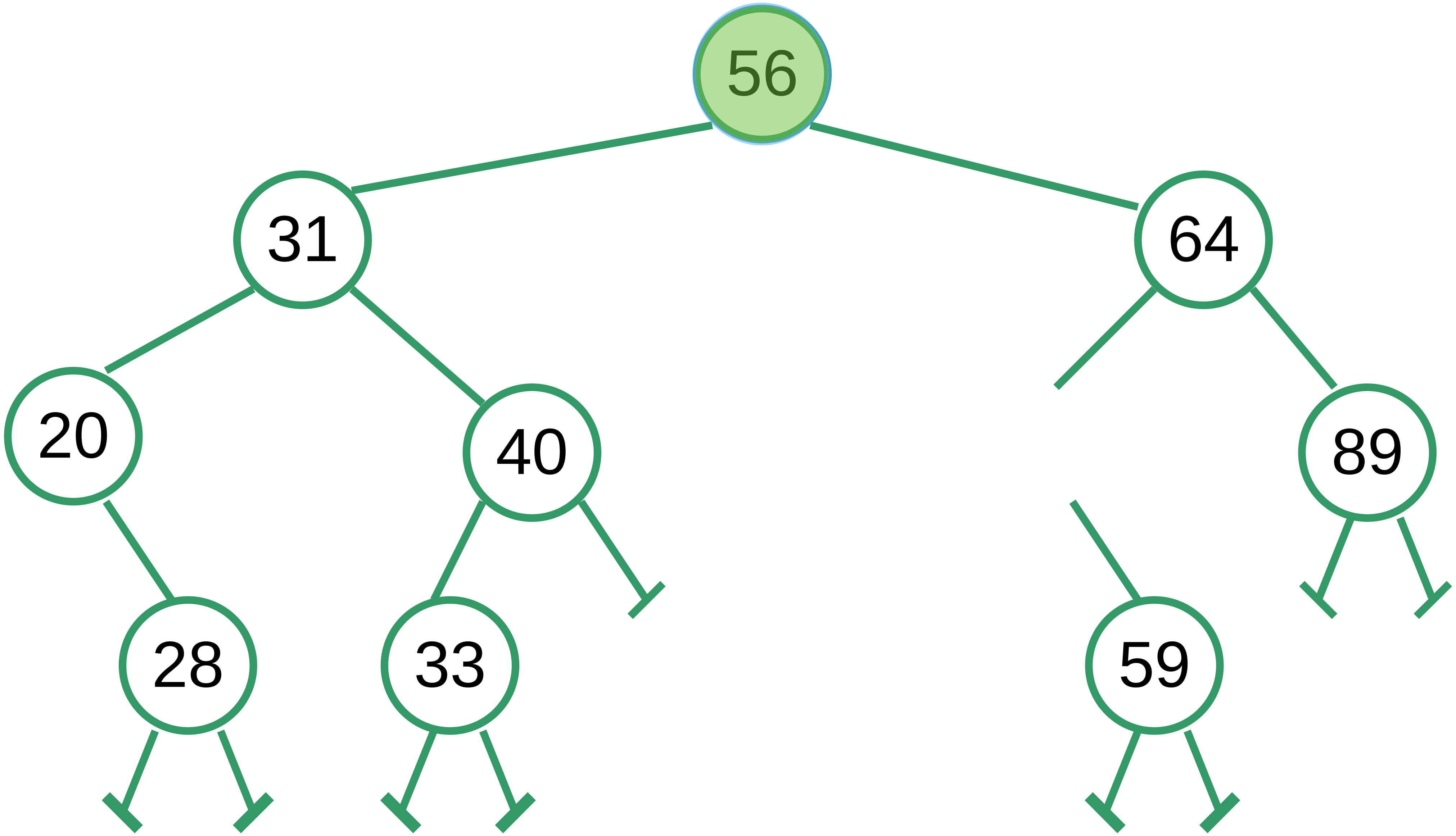
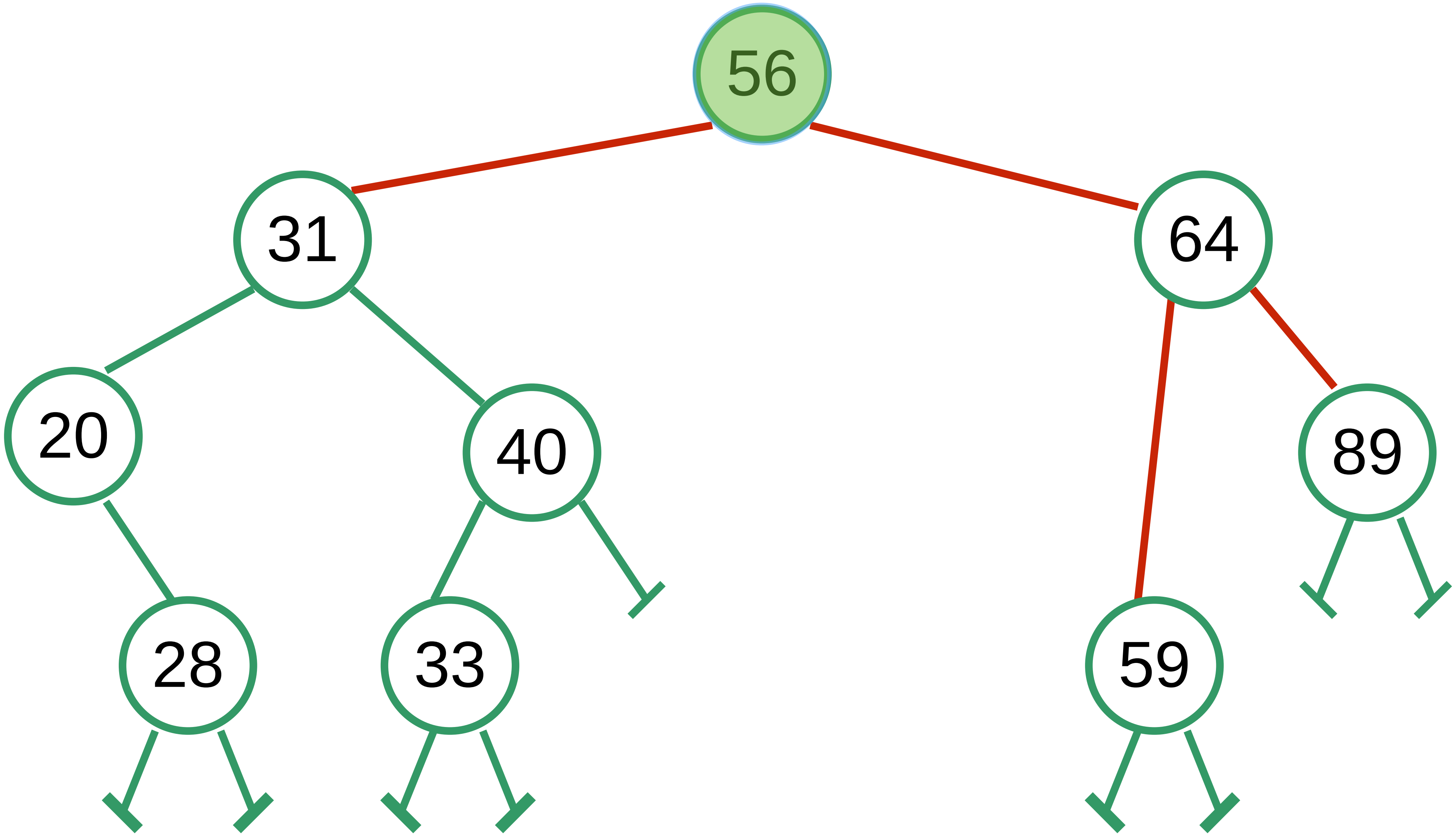


Delete 43



Delete

Input: key of element to delete.

Idea: Find key and successor…

- Try to find the key…
  - ➞ If it is a leaf? Set parent’s reference to None
  - ➞ It has one child? Parent’s reference set to child (“bypass”).
  - ➞ It has two children? Find successor. Successor takes position of deleted node. If successor leaves an orphan child, it should be linked to the successor’s parent.

__delitem__

left as an exercise.

Summary

- Binary search trees: search, insertion and deletion