Monash University School of Mathematical Sciences

MAT1841 Continuous Mathematics for Computer Science

(Semester 2, 2016)

Assignment 1

This is the first of three assignments worth 10% each. It is to be submitted to your support class instructor in your support class in the week beginning 22 August (week 5 of semester). All assignments must have a signed and dated assignment cover sheet attached. A late penalty of 10% of the total possible mark per day will apply for late work. All late work is to be handed directly to the Unit Coordinator (Daniel McInnes, Room 453, 9 Rainforest Walk).

Show all working. You are being marked on your ability to clearly explain your steps in both English and mathematical statements as appropriate. Writing an answer only will not attract full marks for that question (or part thereof).

Question 1 [10 marks] Consider three vectors in 3-space, $\mathbf{u} = (u_x, u_y, u_z)$, $\mathbf{v} = (v_x, v_y, v_z)$ and $\mathbf{w} = (w_x, w_y, w_z)$.

(a) Using the vector components, prove that $|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2 - (\mathbf{u} \cdot \mathbf{v})^2$ (Lagrange's identity).

(b) If $\mathbf{u} = (1, -1, 0)$, $\mathbf{v} = (2, 0, 3)$ and $\mathbf{w} = (0, 1, -2)$, show that $\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$.

(c) Using $\mathbf{u}$ and $\mathbf{v}$ from (b), find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$, $\mathbf{u}_v$.

Question 2 [10 marks] Answer the following questions about lines and planes:

(a) Find the cartesian form of the equation of a plane passing through the point $(-1, 3, -2)$ with normal vector $\mathbf{n} = (-2, 1, -1)$.

(b) Find the point of intersection of the line

$$x - 9 = -5t, \quad y + 1 = -t, \quad z - 3 = t, \quad (-\infty < t < +\infty),$$

and the plane $2x - 3y + 4z + 7 = 0$.

(c) Find the shortest distance between the point $(-1, 2, 1)$ and the plane $2x+3y-4z = 1$.

Question 3 [10 marks] Answer the following questions about matrices:

(a) Calculate the determinant of the matrix

$$M = \begin{pmatrix} 1 & 3 & -2 \\ 2 & 1 & 3 \\ 1 & -2 & 3 \end{pmatrix}$$

and find its inverse using the Gauss-Jordan algorithm.

(b) Solve the system of linear equations

$$\begin{aligned} x + 3y - 2z &= 1 \\ 2x + y + 3z &= 0 \\ x - 2y + 3z &= 2. \end{aligned}$$