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Lecture 19
Sorted Lists
(Array Implementation)

FIT 1008
Introduction to Computer Science

MONASH University
Information Technology

SortedList ADT

- Sequence of items in increasing order
- Possible Operations:
  - **Create** a list
  - **Add item** to the list
  - **Delete** an **item** at a given position from the list
  - Check whether the list **is empty**
  - Check whether the list **is full**
  - Get the **length** of the list.

```python
class SortedList:

    def __init__(self, size):
        if size > 0:
            self.the_array = size*[None]
            self.count = 0

    def length(self):
        return self.count

    def is_empty(self):
        return self.count == 0

    def is_full(self):
        return self.count >= len(self.the_array)
```

Adding an element to a sorted list

- **Sorted list**: Element at position i is <= than that at postion i+1

- **Input:**
  - Sorted list
  - new_item to be added
- **Output:**
  - Sorted list
  - **False** if the list was full; **True**, then the list contains all original elements in the same order together with the new_item (postcondition)
- **Note**:
  - the “Sorted” is also a pre/postcondition

Example: add "Goat" to the sorted list.

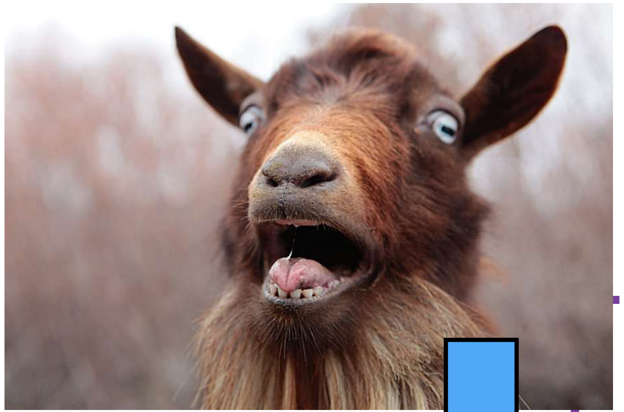
count: 4

the_array

0	1	2	3	4	5
Blobfish	Cheetah	Iguana	Tapir	???	???

Example: add "Goat" to the sorted list.

If there is space, find the correct position



Example: add “Goat” to the sorted list.

Order is maintained!

count: 5

the_array

Blobfish	Cheetah	Goat	Iguana	Tapir	???
0	1	2	3	4	5

If there is space, find the correct position

Make room by moving all to the right.

Put item in position.

Update count then **return True**

If the array has some space left:

find correct ***index*** at which to add item.

make room: move all items from **index** to **count-1** to the right

put item in position **index**.

increment **count**

return **True**.

else:

return **False**.

```python
def add(self, new_item):
    # easy if the list is empty
    if self.is_empty():
        self.the_array[self.count] = new_item
        self.count += 1
        return True
    # if the lis is not empty...
    has_place_left = not self.is_full()
    if has_place_left:
        # find correct position
        index = 0
        while index < self.count and new_item > self.the_array[index]:
            index+=1
        # now index has the correct position
        # we go backwards from count -1 up to index
        for i in range(self.count-1, index-1, -1):
            # "moving" the item in position i to position i+1
            self.the_array[i+1] = self.the_array[i]
        # insert new item
        self.the_array[index] = new_item
        # increment counter
        self.count+=1
    return has_place_left
```

Hence we can use the same delete as for a ListADT

```
def delete(self, index):
    valid_index = index >=0 and index < self.count
    if (valid_index):
        for i in range(index, self.count-1):
            self.the_array[i] = self.the_array[i+1]
        self.count -=1
    return valid_index
```

Print List

```
def print(self):
    for i in range(self.count):
        print(str(self.the_array[i]), end=" ")
```

Convert to string **whatever** is stored

What if the List contains more complex objects?

Overloading operators

- Any class can **redefine** certain special operations:
- By simply defining the **associated method** inside the class

Operation	Class Method
str(obj)	__str__(self)
len(obj)	__len__(self)
item in obj	__contains__(self,item)
y = obj[ndx]	__getitem__(self,ndx)
obj[ndx] = value	__setitem__(self,ndx,value)
obj == rhs	
obj < rhs	
...	
obj + rhs	
...	

Python checks whether the appropriate method is available to the object. If **not defined**, the **built-in operation** (if any) is used.

Creating a list

```
class List:
    def __init__(self, size):
        assert size > 0, "Size should be positive"
        self.the_array = size*[None]
        self.count = 0
```

```
object.__mul__(self, other)
```

Operation * is <u>overloaded</u> in Python's List type

https://docs.python.org/3/reference/datamodel.html#emulating-numeric-types

Operation	Class Method
str(obj)	__str__(self)
len(obj)	__len__(self)
item in obj	__contains__(self,item)
y = obj[ndx]	__getitem__(self,ndx)
obj[ndx] = value	__setitem__(self,ndx,value)
obj == rhs	__eq__(self,rhs)
obj < rhs	__lt__(self,rhs)
...	
obj + rhs	__add__(self,rhs)
...	

```
def __len__(self):
    return self.count

def is_empty(self):
    return len(self) == 0

def is_full(self):
    return len(self) >= len(self.the_array)
```

Operation	Class Method
str(obj)	__str__(self)
len(obj)	__len__(self)
item in obj	__contains__(self,item)
y = obj[ndx]	__getitem__(self,ndx)
obj[ndx] = value	__setitem__(self,ndx,value)
obj == rhs	__eq__(self,rhs)
obj < rhs	__lt__(self,rhs)
…	
obj + rhs	__add__(self,rhs)
…	

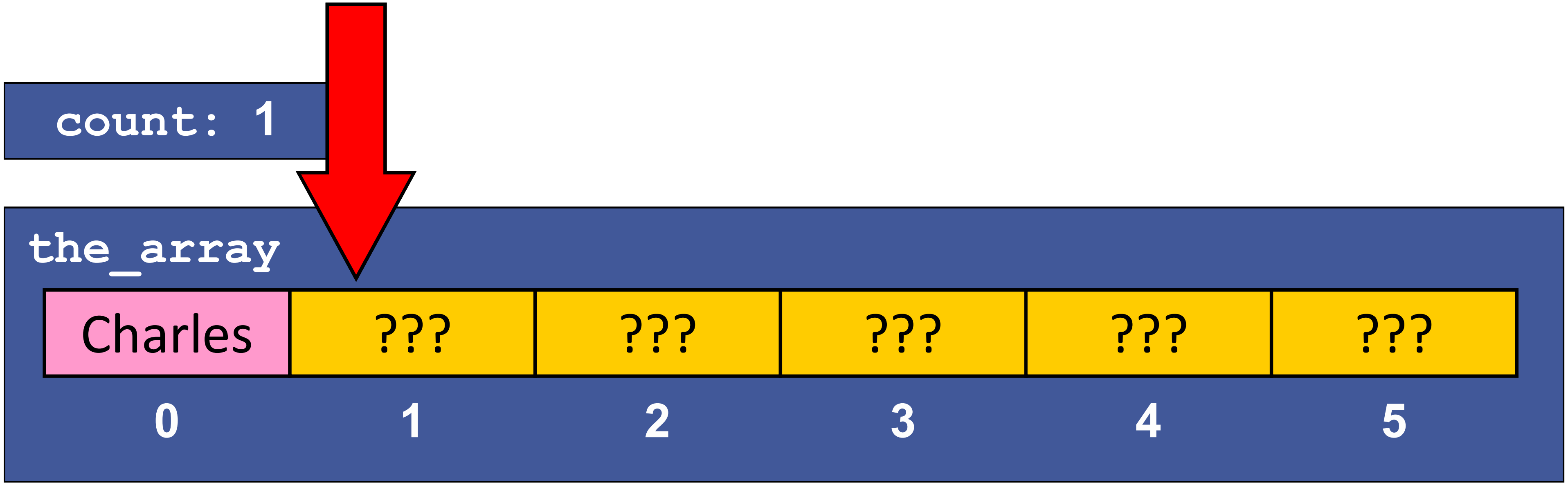
Searching

List ADT

- Sequence of items

- Possible **Operations**:
  - Add item
  - Remove item
  - Find item
  - Retrieve item
  - Next item
  - First item
  - Is last item
  - Is empty
  - Print
  - **Is an item in the list?**
  - **Find the first position of an item in the list.**

- Is an item in the list?
- Find the first position of an item in the list.



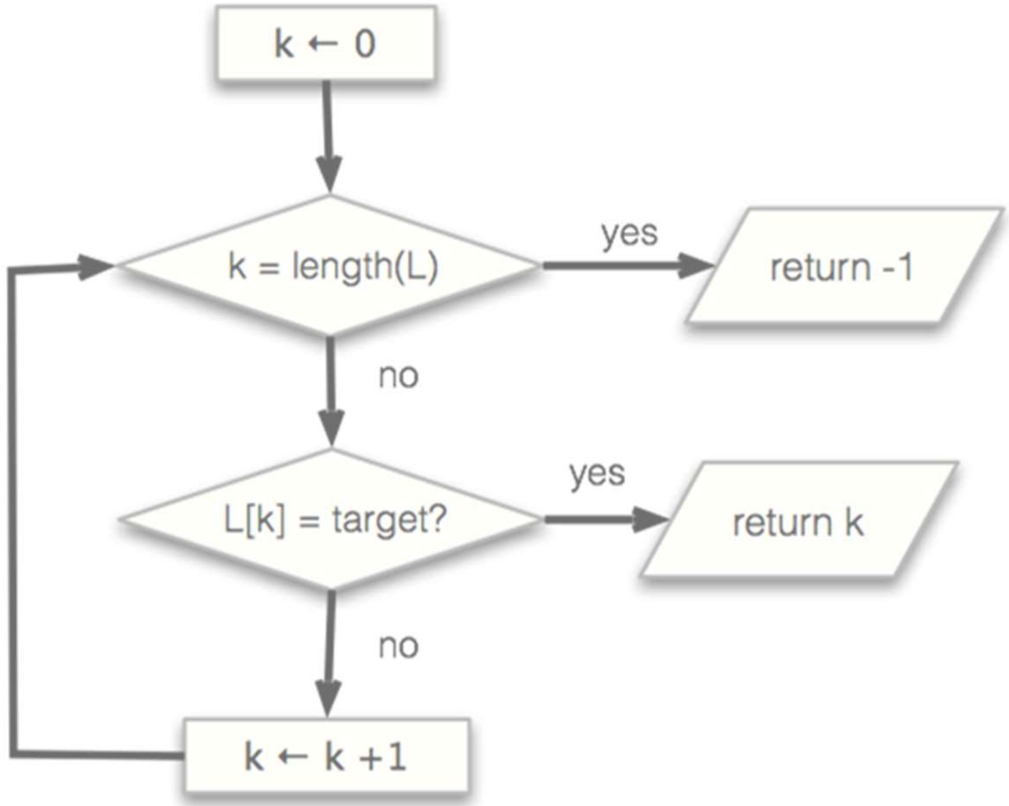
Item in List

```
>>> the_list = [1, 2, 3, 4, 5]
>>> x = 3
>>> x in the_list
True
>>> y = 8
>>> y in the_list
False
```

Item in List

```
def __contains__(self, item):
    for k in range(len(self)):
        if item == self.the_array[k]:
            return True
    return False
```

Linear Search



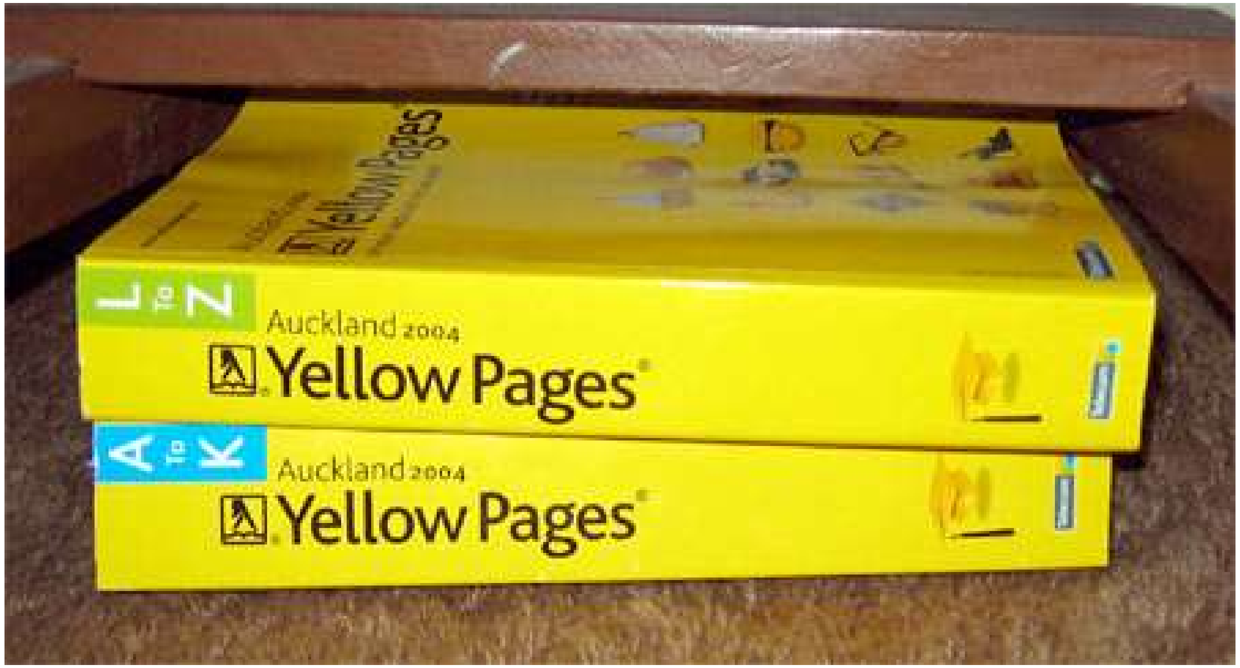
```
class SortedList:

    def __init__(self, size):
        if size > 0:
            self.the_array = size*[None]
            self.count = 0

     def __len__(self):
        return self.count

    def is_empty(self):
        return len(self) == (

    def is_full(self):
        return len(self) >=
```

```
def add(self, new_item):
    # easy if the list is empty
    if self.is_empty():
        self.the_array[self.count] = new_item
        self.count += 1
        return True
    # if the lis is not empty...
    has_place_left = not self.is_full()
    if has_place_left:
        # find correct position
        index = 0
        while index < self.count and new_item > self.the_array[index]:
            index+=1
        # now index has the correct position
        # we go backwards from count -1 up to index
        for i in range(self.count-1, index-1, -1):
            # "moving" the item in position i to position i+1
            self.the_array[i+1] = self.the_array[i]
        # insert new item
        self.the_array[index] = new_item
        # increment counter
        self.count+=1
    return has_place_left
```


```
def add(self, new_item):
    # easy if the list is empty
    if self.is_empty():
        self.the_array[self.count] = new_item
        self.count += 1
        return True
    # if the lis is not empty...
    has_place_left = not self.is_full()
    if has_place_left:
        # find correct position
        index = 0
        while index < self.count and new_item > self.the_array[index]:
            index+=1
        # now index has the correct position
        # we go backwards from count -1 up to index
        for i in range(self.count-1, index-1, -1):
            # "moving" the item in position i to position i+1
            self.the_array[i+1] = self.the_array[i]
        # insert new item
        self.the_array[index] = new_item
        # increment counter
        self.count+=1
    return has_place_left
```


List is always sorted!

Binary Search



Source:http://www.geekets.com/

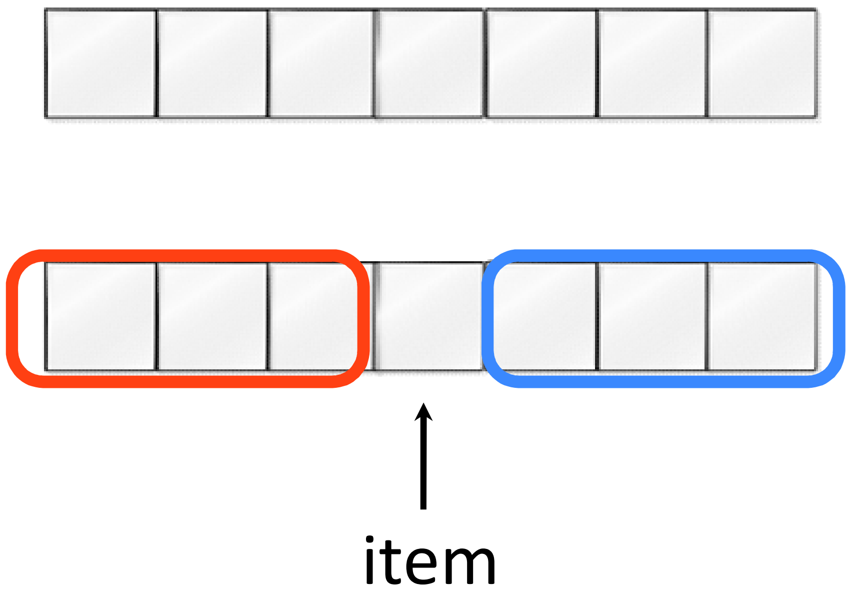
Binary Search Assumptions

- The list is sorted

- We can random access the list
  (you can get the value of any position in the list)

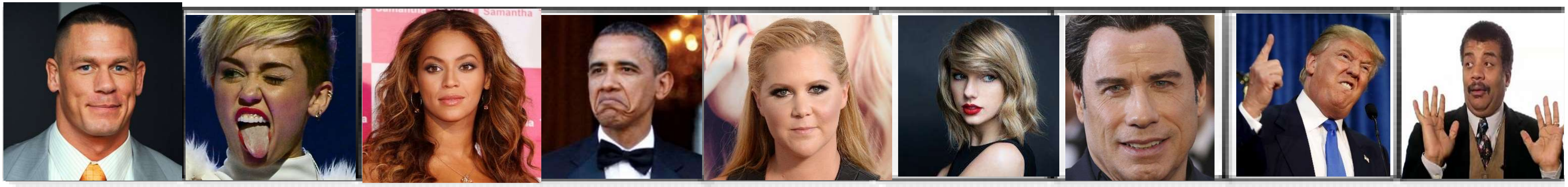
Binary Search

```
item ⟵ the item in the middle of the list

if (item = target)
{
        return True
}

if (target < item)
{
        search the first part of the list
}

if (target > item)
{
        search the second part of the list
}
```



item < target   item = target   item > target

Remember how I found TayTay?



John Cena, Miley Cyrus, Beyonce' Knowles, Barack Obama, Amy Schumer, Taylor Swift, John Travolta, Donald Trump, Neil Degrasse Tyson

Binary Search

Case 2: `target > the_array[mid]`

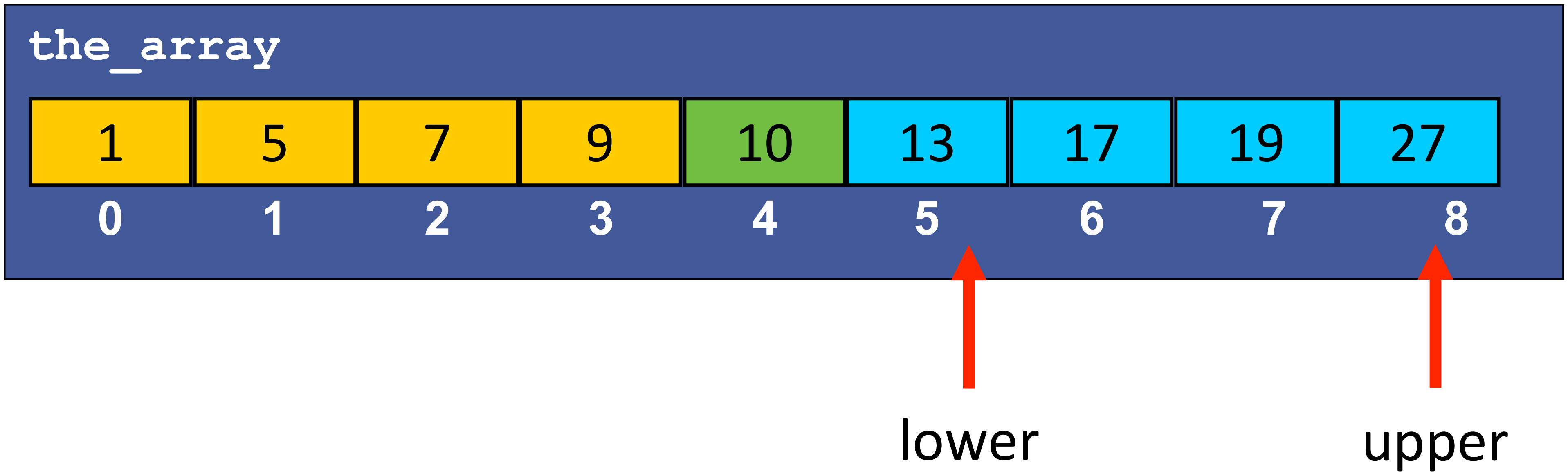
target = 19

lower = 0, upper = 8

mid =(0+8)//2 = 4

update **lower** and keep searching....

lower = mid+1 = 5



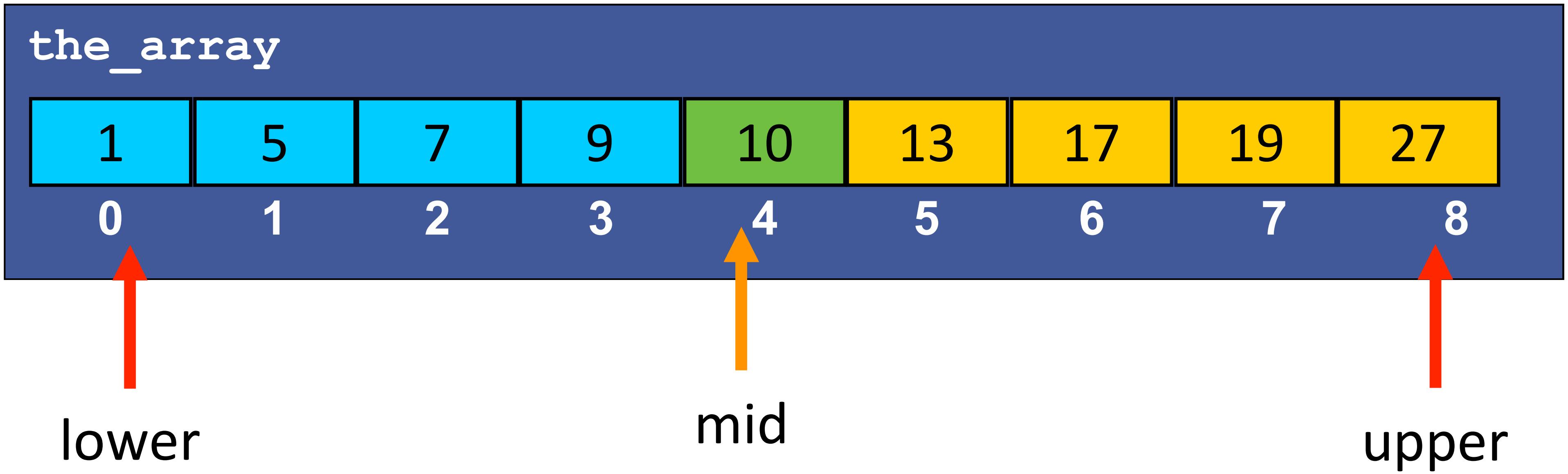
Binary Search

<u>Case 3:</u> `target < the_array[mid]`

target = 7

lower = 0, upper = 8

mid =(0+8)//2 = 4



Binary Search

<u>Case 4:</u> `target` not in the list

target = 11

lower = 0, upper = 8

mid =(0+8)//2 = 4

lower = 5, upper = 8

mid =(5+8)//2 = 13//2 = 6

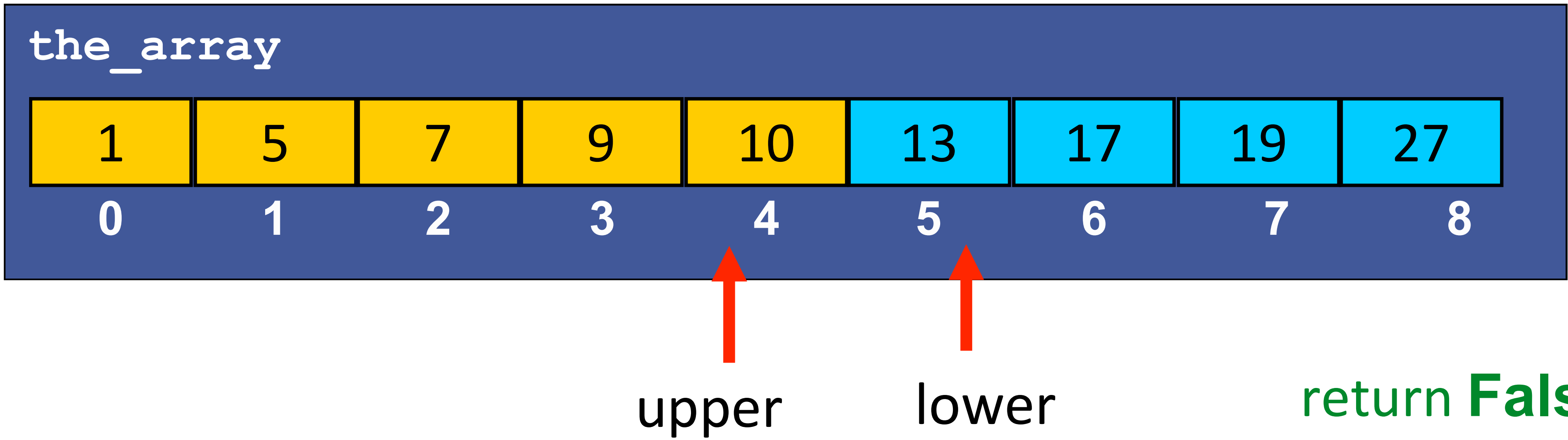
lower = 5, upper = 5

mid =(5+5)//2 = 10//2 = 5

upper = mid -1

upper < lower

if Target is not in the List



return **False**

Algorithm: BinarySearch(target, L[0..n-1])

Search for target in L

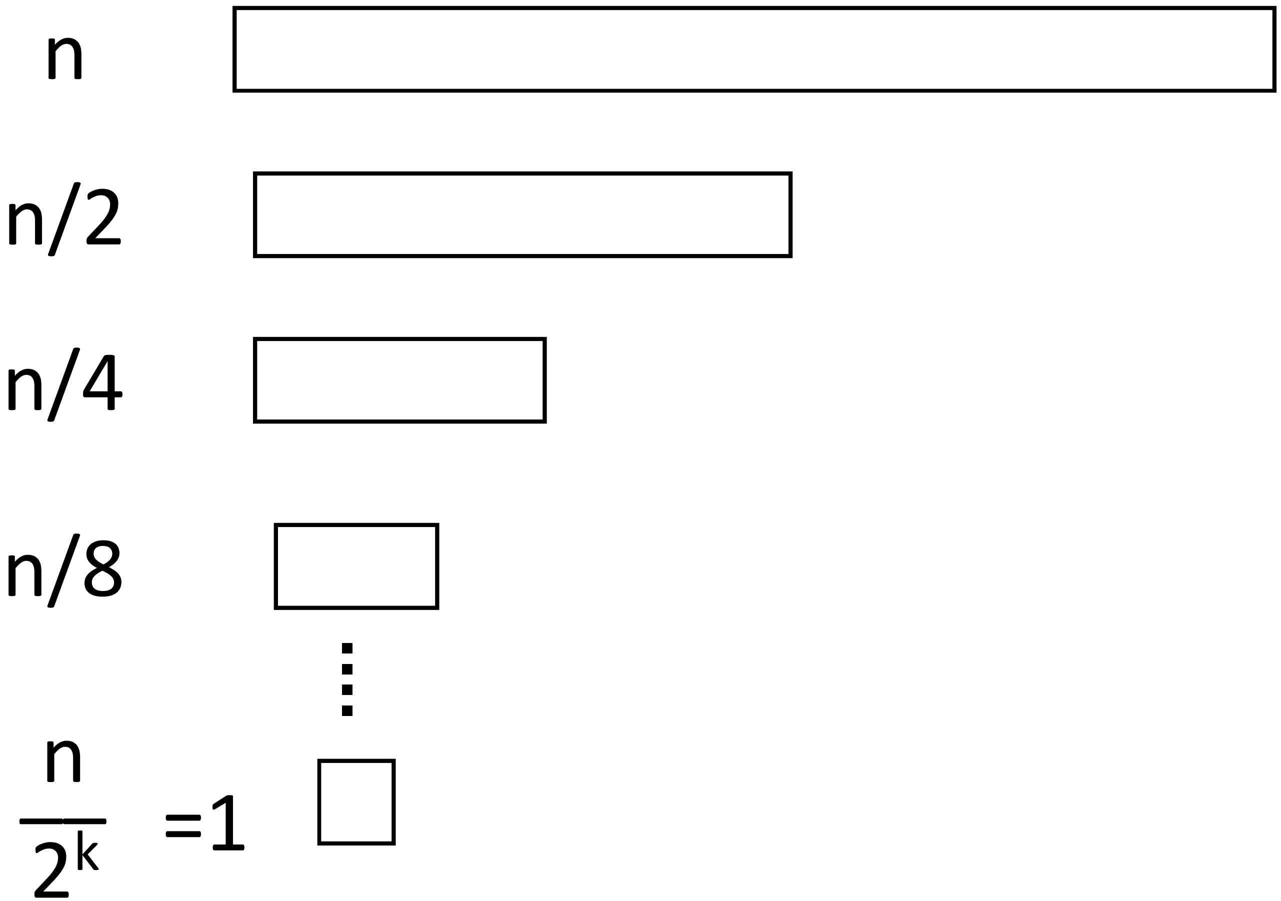
Input: target, and a <u>sorted</u> list L[0, n-1]

Output: If target is in L, returns the index of the first item with that value. Otherwise returns -1.

```
lower ⟵ 0
upper ⟵ n-1
while (lower ≤ upper) do {
    mid ⟵ ⌊ (lower + upper)/2 ⌋
    if (target = L[mid])
        return True
    if (target < L[mid])
        upper ⟵ mid – 1
    if (target > L[mid])
        lower ⟵ mid + 1
}
return False
```

Binary Search

```
def __contains__(self, item):
    lower = 0
    upper = len(self) -1
    while lower <= upper:
        mid = (lower + upper)//2
        if self.the_array[mid] == item:
            return True
        elif self.the_array[mid] > item:
            upper = mid - 1
        else:
            lower = mid + 1
    return False
```



$$\frac{n}{2^k} = 1$$

$$n = 2^k$$

$$\log_2(n) = k$$

Binary Search: **Assumptions**

- To use Binary Search we need:
  - List to be **sorted**
  - Implemented with an **array**

- Why sorted?
  - Otherwise we cannot **guarantee** that the item we are looking for is **not** in the half we discard

- Why implemented using an array?
  - We need access to <u>any element</u> in the list
  - We need to do that efficiently: **constant time** access
  - Arrays ensure that is always the case

list.index(item)

- Finds the **first index** of an item in the list.
- Raises `ValueError` if item is not in list.

```
>>> the_list = [1, 2, 2, 4, 5]
>>> the_list.index(2)
1
>>> the_list.index(8)
Traceback ...
     .....
ValueError: 8 is not in list
```

`list.`**`index`**`(`*x*`)`

Return the index in the list of the first item whose value is *x*. It is an error if there is no such item.

https://docs.python.org/3/tutorial/datastructures.html

Sorted List

`list.`**`index`**`(`*x*`)`

Return the index in the list of the first item whose value is *x*. It is an error if there is no such item.

Sorted List

?

```
def _binary_search(self, item):
    lower = 0
    upper = len(self) -1
    while lower <= upper:
        mid = (lower + upper)//2
        if self.the_array[mid] == item:
            return mid
        elif self.the_array[mid] > item:
            upper = mid - 1
        else:
            lower = mid + 1
    return -1
```

now it <u>returns</u> a valid index **or** -1

Sorted List

```python
def __contains__(self, item):
    if self._binary_search(item) == -1:
        return False
    return True
```

Sorted List

```
def index(self, item):
    position = self._binary_search(item)
    if position == -1:
        raise ValueError("Element " + str(item) + " is not in the list")
    while position >=0 and self.the_array[position] == item:
        position -= 1
    return position + 1
```

Check if there are copies of the target that appear earlier

Sorted List

```
def index(self, item):
    position = self._binary_search(item)
    if position == -1:
        # item is not in the list
        raise ValueError("Element " + str(item) + " is not in the list")
    # there is at least a copy and is in position position
    while position >=0 and self.the_array[position] == item:
        # search back until we find the first appearance
        position -= 1
    return position + 1
```

Best-case time complexity O(1)

you do better than linear?

```
def index(self, item):
    position = self._binary_search(item)
    if position == -1:
        # item is not in the list
        raise ValueError("Element " + str(item) + " is not in the list")
    # there is at least a copy and is in position position
    while position >=0 and self.the_array[position] == item:
        # search back until we find the first appearance
        position -= 1
    return position + 1
```

Summary

- Implementing lists using arrays:
  - Class structure for a list
  - Add an element to an unsorted list
  - Add an element to a sorted list
  - Delete an element