

## Data Structures and Algorithms using Python

Efficiently using data structures to collect, organize, and retrieve information is one of the core abilities modern computer engineers are expected to have. Python currently is one of the most popular programming languages, and as such, it has become vital for students to understand this concept in this language.

This student-friendly textbook provides a complete view of data structures and algorithms using the Python programming language, striking a balance between theory and practical application. All major algorithms have been discussed and analysed in detail, and the corresponding codes in Python have been provided. Diagrams and examples have been extensively used for better understanding. Running time complexities are also discussed for each algorithm, allowing the student to better understand how to select the appropriate one.

The book has been written with both undergraduate and graduate students in mind. Each chapter ends with a large number of problems, including multiple choice questions, to help consolidate the knowledge gained. This will also be helpful with competitive examinations for engineering in India such as GATE and NET. As such, the book will be a vital resource for students as well as professionals who are looking for a handbook on data structures in Python.

**Subrata Saha** is Head of the Department of Computer Applications at Techno India Hooghly, West Bengal. He has more than 20 years of teaching experience in various subjects in computer science and engineering, including data structures and algorithms, programming in C, C++, Java, and Python, basic computation, operating systems, and so on. He has previously published *Basic Computations and Programming with C* with the Cambridge University Press in 2016.

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**Subrata Saha**



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*To my beloved wife,  
Mrs Sriparna Saha, and my princess, Miss Shreya  
Saha, whose constant support motivated me to  
write this book*

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