

Tutorial - 4

Master's Theorem :- $aT(\frac{n}{b}) + (c)(n^k \log n)^p$

$$\textcircled{1} \quad T(n) = 3T(n/2) + n^2$$

$$n \quad 3, n=2, k=2, p=0$$

$$3 < 2^2$$

$$3 < 4$$

$$p > 0$$

$$T(n) = \Theta(n^2 / \log n)$$

$$T(n) = \Theta(n^2)$$

$$\textcircled{2} \quad T(n) = 4T(n/2) + n^2$$

$$a=4, b=2, k=2, p=0$$

$$4 > 2^2$$

$$4 > 4$$

$$p > -1.80$$

$$T(n) = \Theta(n^{\log_2 4} \cdot (\log n)^p)$$

$$T(n) = \Theta(n^2 \log n)$$

$$\textcircled{3} \quad T(n) = T(n/2) + 2^n$$

Master's theorem not applicable.

$$\textcircled{4} \quad T(n) = 2^n T(n/2) + n^n$$

Master's theorem not applicable.

Q5)

$$T(n) = 16T(n/4) + n$$

$$a=16, b=4, k=1, p \geq 0$$

$$16 > 4^1$$

$$T(n) = \Theta(n^{\log_4 16})$$

$$T(n) = \Theta(n^{\log_4 4^2})$$

$$T(n) = \Theta(n^2)$$

$$(6) \quad T(n) = 2T(n/2) + n \log n$$

$$a=2, b=2, k=1, p \geq 1$$

$$2 \geq 2^1$$

$$4 > p+1$$

$$T(n) = \Theta(n^{\log_2 2} \cdot (\log n)^2)$$

$$(7) \quad T(n) = 2T(n/2) + n \log n$$

$$T(n) = 2T(n/2) + n(\log n)^{1/2}$$

$$a=2, b=2, k=1, p=1/2$$

$$2 \geq 2^1$$

$$p \geq -1$$

$$T(n) = \Theta(n^{\log_2 2} \cdot \log \log n)$$

$$T(n) = \Theta(n \log \log n)$$

$$(8) \quad T(n) = 2T(n/4) + n^{0.5}$$

$$a=2, b=4, k=0.51$$

$$2 < 2^{0.51} \quad \& \quad p \geq 0$$

$$T(n) = \Theta(n^{0.51 / \log_4 2})$$

$$T(n) = \Theta(\sqrt{n})$$

(9)

$$T(n) = 0.5 T(n/2) + 1/n$$

$$T(n) = 0.5 T(n/2) + n^{-1}$$

$$a = 0.5, b = 2, k = 1, p = 0$$

$$0.5 = 2^{-1} \quad (0.5 = 1/2)$$

$$p = 0$$

$$T(n) = \Theta(n^{\log_2 0.5} (\log n)^1)$$

$$T(n) = \Theta(n^0 (\log n)^1)$$

(10)

$$T(n) = 4 T(n/2) + \log n$$

$$a = 4, b = 2, k = 0, p = 1$$

$$4 > 2^0$$

$$4 > 1$$

$$T(n) = \Theta(n^{\log_2 4})$$

$$T(n) = \Theta(n^2)$$

(14)

$$T(n) = 3 T(n/3) + \sqrt{n}$$

$$T(n) = 3 T(n/3) + n^{1/2}$$

$$a = 3, b = 3, k = 1/2, p = 0$$

$$3 > 3^{1/2}$$

$$T(n) = \Theta(n^{\log_3 3})$$

$$T(n) = \Theta(n)$$

(15)

$$T(n) = 4 T(n/2) + cn$$

$$a = 4, b = 2, k = 1, p = 0$$

$$4 > 2^1$$

$$T(n) = \Theta(n^{\log_2 4})$$

$$T(n) = \Theta(n^2)$$

(17)

$$T(n) = 2T(n/2) + n/2$$

$$a=2, b=2, k=1, p=0$$

$$2 < 2^1 \text{ and } p > -1$$

$$T(n) = \Theta(n^{\log_2 2} (\log n)^0)$$

$$T(n) = \Theta(n \log n)$$

(18)

$$T(n) = 6T(n/3) + n^2 \log n$$

$$a=6, b=3, k=2, p=1$$

$$6 < 3^2 \text{ and } p > 0$$

$$T(n) = \Theta(n^2 (\log n)^1)$$

$$T(n) = \Theta(n^2 \log n)$$

(20)

$$T(n) = 64T(n/8) - n^2 \log n$$

$$a=64, b=8, k=2, p=1$$

$$64 > 8^2 \text{ and } p > -1$$

$$T(n) = \Theta(n^{\log_8 64} (\log n)^{1+1})$$

$$T(n) = \Theta(n^2 (\log n)^2)$$

(21)

$$T(n) = 7T(n/3) + n^2$$

$$a=7, b=3, k=2, p=0$$

$$7 < 3^2 \text{ and } p > 0$$

$$T(n) = \Theta(n^2 (\log n)^0)$$

$$T(n) = \Theta(n^2)$$