

① Asymptation notation One the mathematical notation used to describe the running time of an algorithm corresponding to the input ~~not~~ trends the particular value as a limiting value.

~~ex~~ → Types of notation

- 1) Θ Notation : - It is the way to express the both the lower bound and the upper bound of an algorithm's running time
- 2) O Notation - It is the formal way to express the upper bound of an algorithm's running time.
- 3) Omega Notation Ω - It is the formal way to express the lower bound of an algorithm's running time.

② $\sum_{i=1}^n 1 + 1 + 1 \dots k \text{ times}$

$$i = 1 \left(i \times 2 \right)$$

$$2^k \geq n$$

$$2^k = n$$

taking log both side

$$1 \cdot \log 2 = \log n$$

$$k \log 2 = \log n$$

$$k = \frac{\log n}{\log 2}$$

$$k = \log_2 n$$

Ans $O(\log n)$

$$(3) \quad T(n) = \begin{cases} 3T(n-1) & n > 0 \\ 1 & n = 0 \end{cases}$$

$$T(n) = 3T(n-1) - 0$$

$$\text{let } n = n-1$$

putting n in eq (1)

$$T(n-1) = 3T(n-2) - 0$$

$$\text{let } n = n-2$$

putting n in eq (1)

$$T(n) = 3T(n-2) - 0 \quad (4)$$

putting (4) in (3)

$$T(n) = 3^2 T(n-3)$$

$$T(n) = 3^k T(n-k)$$

$$\text{let } n-k = 0$$

$$n = k$$

$$T(n) = 3^n T(0)$$

$$= O(3^n)$$

$$(5) \quad i = 1, 2, 3, 4, 5, 6 \dots$$

$$\text{sum of } = 1 + 3 + 6 + 10 + 15 + 21 \dots \quad (5)$$

$$\text{also } = 1 + 3 + 6 + 10 \dots T_{n-1} + T_2 - 0$$

$$0 = 1 + 2 + 3 + 4 + \dots - n = T_n$$

$$T_k = 1 + 2 + 3 + 4 + \dots + k$$

$$T_k = \frac{1}{2} k(k+1)$$

for k iteration

$$1 + 2 + 3 \dots + k \leq n$$

$$\frac{k(k+1)}{2} \leq n$$

$$O(k^2) \leq n$$

$$k \leq O(\sqrt{n})$$

$$T(n) = O(\sqrt{n})$$

$$1^2 = n$$

$$1 = \sqrt{n}$$

$$i = 1 \ 2 \ 3 \ 4 \dots \sqrt{n}$$

$$T(n) = \frac{\sqrt{n}(\sqrt{n}+1)}{2} = \frac{n + \sqrt{n}}{2}$$

$$T(n) = O(n)$$

for $k = k \cdot 2$

$$k = 1, 2, 4, 8 \dots n$$

$$Qp = 1, 2, 4, 8 \dots n$$

$$\text{Sum of } n \text{ terms} = \frac{q(r^n - 1)}{r - 1}$$

$$n = \frac{1(2^k - 1)}{2 - 1} = \frac{2^k - 1}{2}$$

$$n = 2^k - 1$$

taking log on both side

$$\log_2 n = k \log_2 2 - \log_2 1$$

$$\log_2 n = k$$

*

1

1

2

$\log n$

$\log n \times \log n$

1

$\log n$

1

n

$\log n$

$$\Rightarrow O(\log n \times \log n) \Rightarrow O(n \log^2 n)$$

~~funct(n)~~
2. log

$$T(n) = T(n/3) + n^2$$

$$a = 1, b = 3$$

$$f(n) = n^2$$

$$c = \log_3 1 = 0$$

$$2) nc = 1 > f(n)$$

$$\Rightarrow T(n) = O(n^2)$$

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for $i = 1 \Rightarrow i = 1, 2, 3, \dots, n$
 for $i = 2 \Rightarrow j = 1, 3, 5, \dots, n$
 for $i = 3 \Rightarrow 1, 4, 7, \dots, n$

for $i = n \Rightarrow f = 1$

$$\sum_{j=n} n + \frac{n}{2} + \frac{n}{3} + \dots + 1$$

$$\sum_{j=n} n \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n} \right)$$

$$\sum_{j=n} n \log n$$

$$\Rightarrow T(n) = n \log n$$

$$T(n) = O(n \log n)$$

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as given n^k & c^n
 relation b/w n^k & c^n is

$$n^k = O(c^n)$$

$$\text{as } n^k \leq a c^n$$

$\forall n \geq n_0$ as $a > 0$
 for $n_0 = 1$

$$c = 2$$

$$\Rightarrow k \leq a$$

$$\Rightarrow n_0 \geq 1 \text{ \& } c \geq 2$$