VICTORIAN CURRICULUM AND ASSESSMENT AUTHORITY

Victorian Certificate of Education 2014

SUPERVISOR TO ATTACH PROCESSING LABEL HERE

Letter

STUDENT NUMBER | | | | | | | | | | |

SAM MURPHY

MATHEMATICAL METHODS (CAS)

Written examination 2

Thursday 6 November 2014

75 mins

Reading time: 3.00 pm to 3.15 pm (15 minutes)
Writing time: 3.15 pm to 5.15 pm (2 hours)

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
1	22	22	21/ 21
2	5	5	56/ 56
			77/ Total 77

- Students are permitted to bring into the examination room: pens, pencils, highlighters, erasers, sharpeners, rulers, a protractor, set-squares, aids for curve sketching, one bound reference, one approved CAS calculator (memory DOES NOT need to be cleared) and, if desired, one scientific calculator. For approved computer-based CAS, their full functionality may be used.
- Students are NOT permitted to bring into the examination room: blank sheets of paper and/or white out liquid/tape.

Materials supplied
- Question and answer book of 22 pages with a detachable sheet of miscellaneous formulas in the centrefold.
- Answer sheet for multiple-choice questions.

Instructions
- Detach the formula sheet from the centre of this book during reading time.
- Write your **student number** in the space provided above on this page.
- Check that your **name** and **student number** as printed on your answer sheet for multiple-choice questions are correct, **and** sign your name in the space provided to verify this.
- All written responses must be in English.

At the end of the examination
- Place the answer sheet for multiple-choice questions inside the front cover of this book.

Students are NOT permitted to bring mobile phones and/or any other unauthorised electronic devices into the examination room.

SECTION 1

Instructions for Section 1

Answer **all** questions in pencil on the answer sheet provided for multiple-choice questions.
Choose the response that is **correct** for the question.
A correct answer scores 1, an incorrect answer scores 0.
Marks will **not** be deducted for incorrect answers.
No marks will be given if more than one answer is completed for any question.

Question 1

The point P (4, –3) lies on the graph of a function f. The graph of f is translated four units vertically up and then reflected in the y-axis.

The coordinates of the final image of P are

A. (–4, 1)
B. (–4, 3)
C. (0, –3)
D. (4, –6)
E. (–4, –1)

Question 2

The linear function $f: D \rightarrow R, f(x) = 4 - x$ has range [–2, 6).

The domain D of the function is

A. [–2, 6)
B. (–2, 2]
C. R
D. (–2, 6]
E. [–6, 2]

Question 3

The area of the region enclosed by the graph of $y = x(x+2)(x-4)$ and the x-axis is

A. $\frac{128}{3}$

B. $\frac{20}{3}$

C. $\frac{236}{3}$

D. $\frac{148}{3}$

E. 36

Question 4

Let f be a function with domain R such that $f'(5)=0$ and $f'(x)<0$ when $x \neq 5$.

At $x=5$, the graph of f has a

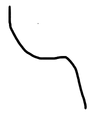
A. local minimum.

B. local maximum.

C. gradient of 5.

D. gradient of –5.

E. stationary point of inflection.



Question 5

The random variable X has a normal distribution with mean 12 and standard deviation 0.5.

If Z has the standard normal distribution, then the probability that X is less than 11.5 is equal to

A. $\Pr(Z > -1)$

B. $\Pr(Z < -0.5)$

C. $\Pr(Z > 1)$

D. $\Pr(Z \geq 0.5)$

E. $\Pr(Z < 1)$

$Pr(X<11.5)$

Question 6

The function $f: D \to R$ with rule $f(x)=2x^3-9x^2-168x$ will have an inverse function for

A. $D = R$

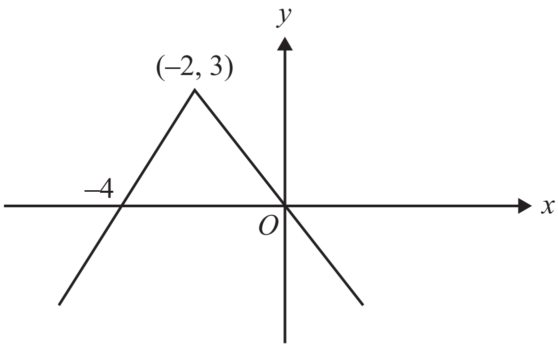
B. $D = (7, \infty)$

C. $D = (-4, 8)$

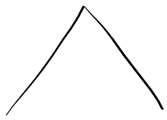
D. $D = (-\infty, 0)$

E. $D = \left[-\frac{1}{2}, \infty\right)$

Question 7



The rule of the function whose graph is shown above is



$-|x|+3$

A. $y = -\frac{3}{2}|x| + 3$

B. $y = \frac{2}{3}|x + 3| + 2$

C. $y = \frac{2}{3}|2 + x| + 3$

D. $y = -\frac{3}{2}|2 - x| + 3$

E. $y = -\frac{3}{2}|x + 2| + 3$

Question 8

If $\int_1^4 f(x)\,dx = 6$, then $\int_1^4 (5 - 2f(x))\,dx$ is equal to

$5x - 12$

A. 3
B. 4
C. 5
D. 6
E. 16

Question 9

The inverse of the function $f: R^+ \to R,\ f(x) = \frac{1}{\sqrt{x}} + 4$ is

$y = \frac{1}{(x-4)^2}$

A. $f^{-1}: (4, \infty) \to R \qquad f^{-1}(x) = \frac{1}{(x-4)^2}$

B. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = \frac{1}{x^2} + 4$

C. $f^{-1}: R^+ \to R \qquad f^{-1}(x) = (x+4)^2$

D. $f^{-1}: (-4, \infty) \to R \qquad f^{-1}(x) = \frac{1}{(x+4)^2}$

E. $f^{-1}: (-\infty, 4) \to R \qquad f^{-1}(x) = \frac{1}{(x-4)^2}$

Question 10

Which one of the following functions satisfies the functional equation $f(f(x)) = x$ for every real number x?

A. $f(x) = 2x$

B. $f(x) = x^2$

C. $f(x) = 2\sqrt{x}$

D. $f(x) = x - 2$

E. $f(x) = 2 - x$

Question 11

A bag contains five red marbles and four blue marbles. Two marbles are drawn from the bag, ~~without replacement~~, and the results are recorded.

The probability that the marbles are different colours is

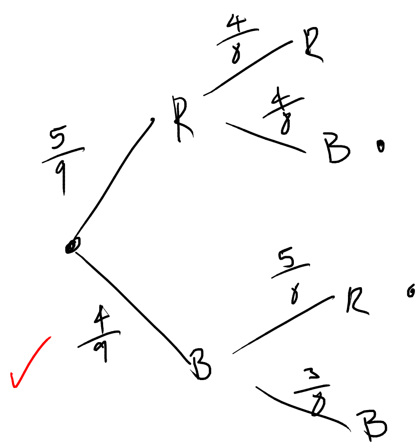
A. $\frac{20}{81}$

B. $\frac{5}{18}$

C. $\frac{4}{9}$

D. $\frac{40}{81}$

E. $\frac{5}{9}$



Question 12

The transformation $T: R^2 \to R^2$ with rule

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

maps the line with equation $x - 2y = 3$ onto the line with equation

A. $x + y = 0$

B. $x + 4y = 0$

C. $-x - y = 4$

D. $x + 4y = -6$

E. $x - 2y = 1$

Question 13

The domain of the function h, where $h(x) = \cos(\log_a(x))$ and a is a real number greater than 1, is chosen so that h is a one-to-one function.

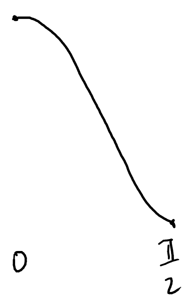
Which one of the following could be the domain?

A. $\left(a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

$0 < \log_a(x) < \frac{\pi}{2}$

$a^0 < x < a^{\frac{\pi}{2}}$

B. $(0, \pi)$



C. $\left[1, a^{\frac{\pi}{2}}\right]$

D. $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right)$

E. $\left[a^{-\frac{\pi}{2}}, a^{\frac{\pi}{2}}\right]$

Question 14

If X is a random variable such that $\Pr(X > 5) = a$ and $\Pr(X > 8) = b$, then $\Pr(X < 5 | X < 8)$ is

A. $\frac{a}{b}$

$$\frac{\Pr(X<5 \cap X<8)}{1-b} = \frac{1-a}{1-b}$$

B. $\frac{a-b}{1-b}$

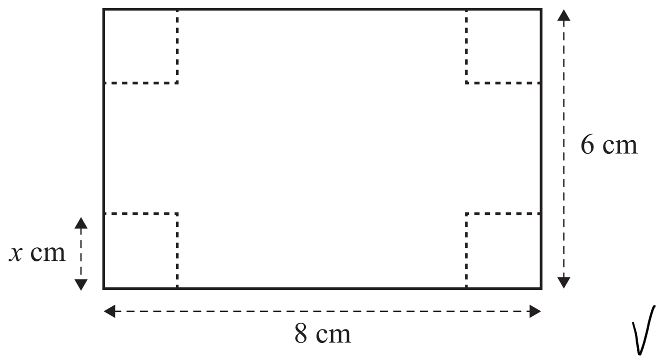
C. $\frac{1-b}{1-a}$

D. $\frac{ab}{1-b}$

E. $\frac{a-1}{b-1}$

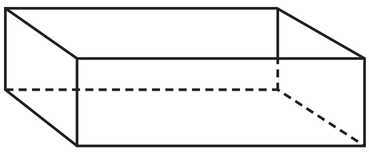
Question 15

Zoe has a rectangular piece of cardboard that is 8 cm long and 6 cm wide. Zoe cuts squares of side length x centimetres from each of the corners of the cardboard, as shown in the diagram below.



$V = (8-2x)(6-2x)x$

Zoe turns up the sides to form an open box.



The value of x for which the volume of the box is a maximum is closest to

A. 0.8
B. 1.1
C. 1.6
D. 2.0
E. 3.6

Question 16

The continuous random variable X, with probability density function $p(x)$, has mean 2 and variance 5.

The value of $\int_{-\infty}^{\infty} x^2 p(x)\,dx$ is

$5 = E(X^2) - 2^2$

9

A. 1
B. 7
C. 9
D. 21
E. 29

Question 17

The simultaneous linear equations $ax - 3y = 5$ and $3x - ay = 8 - a$ have **no solution** for

A. $a = 3$ ✗
B. $a = -3$ ✓
C. both $a = 3$ and $a = -3$
D. $a \in R \backslash \{3\}$
E. $a \in R \backslash [-3, 3]$

Question 18

The graph of $y = kx - 4$ intersects the graph of $y = x^2 + 2x$ at two distinct points for

A. $k = 6$

B. $k > 6$ or $k < -2$ ✓

C. $-2 \le k \le 6$

D. $6 - 2\sqrt{3} \le k \le 6 + 2\sqrt{3}$

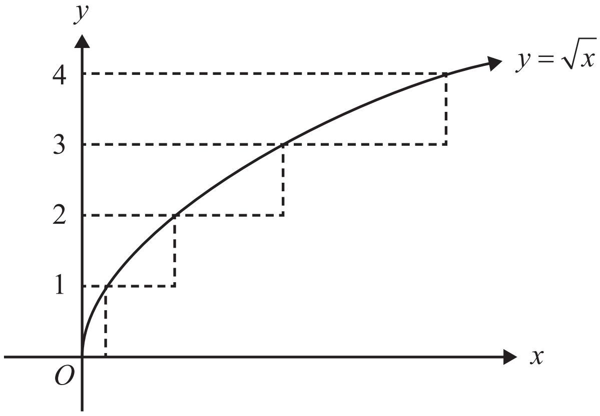
E. $k = -2$

$x^2 + 2x = kx - 4$

$x^2 + (2-k)x + 4 = 0$

Question 19

Jake and Anita are calculating the area between the graph of $y = \sqrt{x}$ and the y-axis between $y = 0$ and $y = 4$. Jake uses a partitioning, shown in the diagram below, while Anita uses a definite integral to find the exact area.



$x = y^2$

J: 30

The difference between the results obtained by Jake and Anita is

A. 0

B. $\frac{22}{3}$ ✓

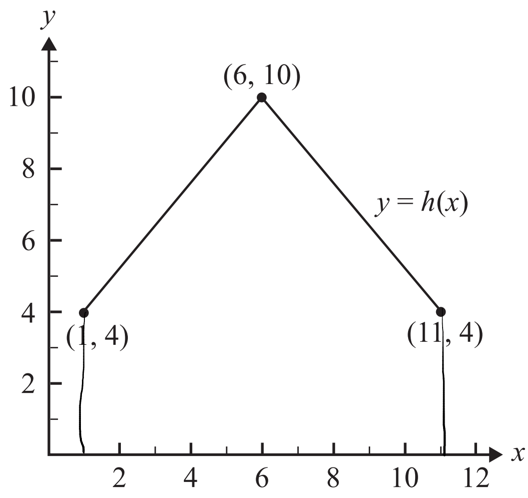
C. $\frac{26}{3}$

D. 14

E. 35

Question 20

The graph of a function, *h*, is shown below.



The average value of *h* is

$\frac{1}{10}$

A. 4

B. 5

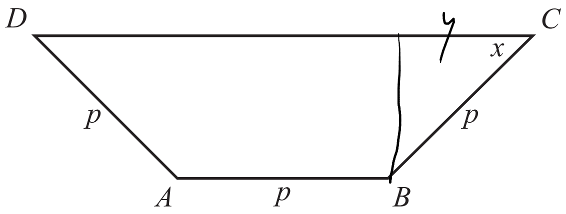
C. 6

D. 7 ✓

E. 10

Question 21

The trapezium *ABCD* is shown below. The sides *AB*, *BC* and *DA* are of equal length, *p*. The size of the acute angle *BCD* is *x* radians.



$y = p\cos(x)$

The area of the trapezium is a maximum when the value of *x* is

$h = p\sin(x)$

$\frac{1}{2}(a+b)h$

$\frac{1}{2}\left(p+p+2p\cos(x)\right)p\sin(x)$

A. $\frac{\pi}{12}$

B. $\frac{\pi}{6}$

C. $\frac{\pi}{4}$

D. $\frac{\pi}{3}$ ✓

E. $\frac{5\pi}{12}$

Question 22

John and Rebecca are playing darts. The result of each of their throws is independent of the result of any other throw. The probability that John hits the bullseye with a single throw is $\frac{1}{4}$. The probability that Rebecca hits the bullseye with a single throw is $\frac{1}{2}$. John has four throws and Rebecca has two throws.

The ratio of the probability of Rebecca hitting the bullseye at least once to the probability of John hitting the bullseye at least once is

A. 1:1

B. 32:27 ✓

C. 64:85

D. 2:1

E. 192:175

$J \sim Bi(4, \frac{1}{4})$

$R \sim Bi(2, \frac{1}{2})$

$$\frac{Pr(R \geq 1)}{Pr(J \geq 1)}$$

END OF SECTION 1

SECTION 2

Instructions for Section 2

Answer **all** questions in the spaces provided.

In all questions where a numerical answer is required, an exact value must be given unless otherwise specified.

In questions where more than one mark is available, appropriate working **must** be shown.

Unless otherwise indicated, the diagrams in this book are **not** drawn to scale.

Question 1 (7 marks)

The population of wombats in a particular location varies according to the rule

$n(t) = 1200 + 400\cos\left(\frac{\pi t}{3}\right)$, where n is the number of wombats and t is the number of months after 1 March 2013.

a. Find the period and amplitude of the function n. 2 marks

period = 6 ✓

amplitude = 400 ✓

b. Find the maximum and minimum populations of wombats in this location. 2 marks

maximum population : 1600 ✓

minimum population : 800 ✓

c. Find $n(10)$. 1 mark

$n(10) = 1000$ ✓

d. Over the 12 months from 1 March 2013, find the fraction of time when the population of wombats in this location was less than $n(10)$. 2 marks

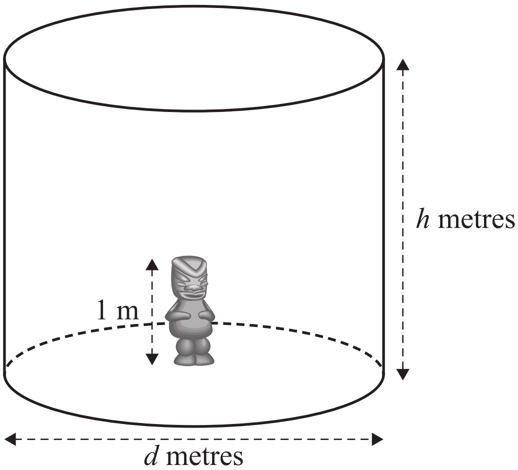
$n(t) = n(10) \Rightarrow t = 2, 4, 8, 10$ ✓

$\therefore \frac{(4-2)+(10-8)}{12} = \frac{1}{3}$ ✓

Question 2 (13 marks)

On 1 January 2010, Tasmania Jones was walking through an ice-covered region of Greenland when he found a large ice cylinder that was made a thousand years ago by the Vikings.

A statue was inside the ice cylinder. The statue was 1 m tall and its base was at the centre of the base of the cylinder.



The cylinder had a height of h metres and a diameter of d metres. Tasmania Jones found that the volume of the cylinder was 216 m^3. At that time, 1 January 2010, the cylinder had not changed in a thousand years. It was exactly as it was when the Vikings made it.

a. Write an expression for h in terms of d. 2 marks

$V = \pi r^2 h$

$216 = \pi \left(\frac{d}{2}\right)^2 h$ ✓

$h = \frac{864}{d^2 \pi}$ ✓

b. Show that the surface area of the cylinder excluding the base, S square metres, is given by the rule $S = \frac{\pi d^2}{4} + \frac{864}{d}$. 1 mark

$S = 2\pi r h + \pi r^2$

$= 2\pi\left(\frac{d}{2}\right)\left(\frac{864}{d^2\pi}\right) + \pi\left(\frac{d}{2}\right)^2$

$= \frac{\pi d^2}{4} + \frac{864}{d}$ as required ✓

Tasmania found that the Vikings made the cylinder so that S is a minimum.

c. Find the value of d for which S is a minimum and find this minimum value of S. 2 marks

$\frac{dS}{dd} = \frac{d\pi}{2} - \frac{864}{d^2}$

Let $\frac{d\pi}{2} - \frac{864}{d^2} = 0$

$\frac{d\pi}{2} = \frac{864}{d^2}$ ✓

$\Rightarrow d = 12\pi^{-\frac{1}{3}}$

$S\big|_{d=12\pi^{-\frac{1}{3}}} = 108\pi^{\frac{1}{3}}$ m^3 ✓

d. Find the value of h when S is a minimum. 1 mark

S is minimum at $d = 12\pi^{-\frac{1}{3}}$

$h = \frac{864}{d^2\pi} = 6\pi^{-\frac{1}{3}}$ m ✓

On 1 January 2010, Tasmania believed that due to recent temperature changes in Greenland, the ice of the cylinder had just started melting. Therefore, he decided to return on 1 January each year to measure the ice cylinder. He observes that the volume of the ice cylinder decreases by a constant rate of 10 m^3 per year. Assume that the cylindrical shape is retained and $d = 2h$ at the beginning and as the cylinder melts.

e. Write down an expression for V in terms of h. 1 mark

$V = \pi r^2 h$

$r = \frac{d}{2} \Rightarrow r = h$

$\therefore V = \pi h^3$

f. Find $\frac{dh}{dt}$ in terms of h. 3 marks

$\frac{dh}{dt} = \frac{dh}{dV} \cdot \frac{dV}{dt}$

$V = \pi h^3$

$\frac{dV}{dh} = 3\pi h^2$

$\frac{dh}{dV} = \frac{1}{3\pi h^2}$

$\frac{dV}{dt} = -10$

$\therefore \frac{dh}{dt} = \frac{-10}{3\pi h^2}$

g. Find the rate at which the height of the cylinder will be decreasing when the top of the statue is just exposed. 1 mark

$h = 1$

$\left.\frac{dh}{dt}\right|_{h=1} = \frac{-10}{3\pi}$ m per year

$\therefore$ decreasing at $\frac{10}{3\pi}$ m per year

h. Find the year in which the top of the statue will just be exposed. (Assume that the melting started on 1 January 2010.)

2 marks

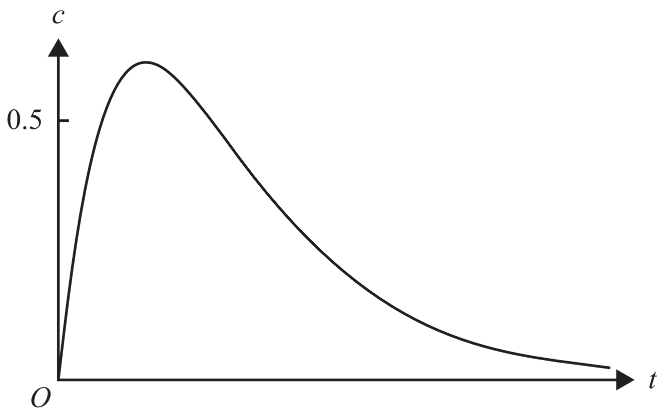
$\frac{dt}{dh} = -\frac{3\pi h^2}{10}$ $\int \frac{dt}{dh} dh = \int \frac{-3\pi h^2}{10} dh$

$t = \int_{6\pi^{-\frac{1}{3}}}^{1} \frac{-3\pi h^2}{10} dh = 21.2858$ ✓

$\therefore$ during 2031 ✓

Question 3 (11 marks)

In a controlled experiment, Juan took some medicine at 8 pm. The concentration of medicine in his blood was then measured at regular intervals. The concentration of medicine in Juan's blood is modelled by the function $c(t) = \frac{5}{2}te^{-\frac{3t}{2}}$, $t \geq 0$, where c is the concentration of medicine in his blood, in milligrams per litre, t hours after 8 pm. Part of the graph of the function c is shown below.



a. What was the maximum value of the concentration of medicine in Juan's blood, in milligrams per litre, correct to two decimal places? 1 mark

$C_{max} = 0.61$

b. **i.** Find the value of t, in hours, correct to two decimal places, when the concentration of medicine in Juan's blood first reached 0.5 milligrams per litre. 1 mark

$t = 0.33$

ii. Find the length of time that the concentration of medicine in Juan's blood was above 0.5 milligrams per litre. Express the answer in hours, correct to two decimal places. 2 marks

$c(t) = 0.5 \Rightarrow t = 0.33, 0.188$

$\therefore$ length of time $= 0.86$ hours

c. **i.** What was the value of the average rate of change of the concentration of medicine in Juan's blood over the interval $\left[\frac{2}{3}, 3\right]$? Express the answer in milligrams per litre per hour, correct to two decimal places.

2 marks

$\frac{c(3) - c\left(\frac{2}{3}\right)}{3 - \frac{2}{3}} = -0.23 \text{ mg } L^{-1} h^{-1}$

ii. At times t_1 and t_2, the instantaneous rate of change of the concentration of medicine in Juan's blood was equal to the average rate of change over the interval $\left[\frac{2}{3}, 3\right]$.

Find the values of t_1 and t_2, in hours, correct to two decimal places.

2 marks

$c'(t) = -0.23$

$t = 0.90, 2.12$

Alicia took part in a similar controlled experiment. However, she used a different medicine. The concentration of this different medicine was modelled by the function $n(t) = Ate^{-kt}$, $t \geq 0$, where A and $k \in R^+$.

d. If the **maximum** concentration of medicine in Alicia's blood was 0.74 milligrams per litre at $t = 0.5$ hours, find the value of A, correct to the nearest integer.

3 marks

$n\left(\frac{1}{2}\right) = \frac{1}{2} A e^{-\frac{k}{2}} = 0.74$ ①

$n'(t) = Ae^{-kt}(1 - kt)$

$n'\left(\frac{1}{2}\right) = 0$

$Ae^{-\frac{1}{2}k}\left(1 - \frac{k}{2}\right) = 0$ ②

$A = 1.48e \approx 4$

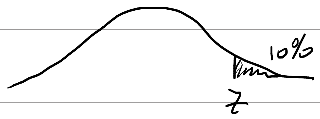
Question 4 (14 marks)

Patricia is a gardener and she owns a garden nursery. She grows and sells basil plants and coriander plants.

The heights, in centimetres, of the basil plants that Patricia is selling are distributed normally with a mean of 14 cm and a standard deviation of 4 cm. There are 2000 basil plants in the nursery.

a. Patricia classifies the tallest 10 per cent of her basil plants as **super**.

What is the minimum height of a super basil plant, correct to the nearest millimetre? 1 mark



$z = 1.28155$

$\frac{h-14}{4} = 1.28155 \Rightarrow h = 19.1$ cm ✓

Patricia decides that some of her basil plants are not growing quickly enough, so she plans to move them to a special greenhouse. She will move the basil plants that are less than 9 cm in height.

b. How many basil plants will Patricia move to the greenhouse, correct to the nearest whole number? 2 marks

$H \sim N(14, 4)$ $n = 2000$

$Pr(H<9) = 0.1056$ ✓

$0.1056 \times 2000 = 211$ plants ✓

The heights of the coriander plants, x centimetres, follow the probability density function $h(x)$, where

$$h(x)=\begin{cases}\frac{\pi}{100}\sin\left(\frac{\pi x}{50}\right) & 0<x<50\\ 0 & \text{otherwise}\end{cases}$$

c. State the mean height of the coriander plants. 1 mark

$\mu = \int_0^{50} \frac{\pi}{100} x \sin\left(\frac{\pi x}{50}\right) dx = 25 \text{ cm}$ ✓

Patricia thinks that the smallest 15 per cent of her coriander plants should be given a new type of plant food.

d. Find the maximum height, correct to the nearest millimetre, of a coriander plant if it is to be given the new type of plant food. 2 marks

Let m be the maximum height, $0<m<50$

$\int_0^{m} h(x)\,dx = 0.15$ ✓ $\Rightarrow m = 12.7 \text{cm}$ ✓

Patricia also grows and sells tomato plants that she classifies as either **tall** or **regular**. She finds that 20 per cent of her tomato plants are tall.

A customer, Jack, selects n tomato plants at random.

e. Let q be the probability that at least one of Jack's n tomato plants is tall.

Find the minimum value of n so that q is greater than 0.95. 2 marks

$T \sim Bi(n, 0.2)$ ✓

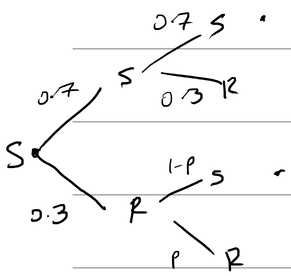
$q = Pr(T \geq 1) > 0.95$

$\Rightarrow n = 14$ ✓

In another section of the nursery, a craftsman makes plant pots. The pots are classified as **smooth** or **rough**.

The craftsman finishes each pot before starting on the next. Over a period of time, it is found that if one plant pot is smooth, the probability that the next one is smooth is 0.7, while if one plant pot is rough, the probability that the next one is rough is p, where $0 < p < 1$. The value of p stays fixed for a week at a time, but can vary from week to week. The first pot made each week is always a smooth pot.

f. **i.** Find, in terms of p, the probability that the **third** pot made in a given week is smooth. 2 marks



$Pr(\text{smooth}) = 0.7 \times 0.7 + 0.3 \times (1-p)$

$= \dfrac{79-30p}{100}$

ii. In one particular week, the probability that the **third** pot made is smooth is 0.61.

Calculate the value of p in this week. 2 marks

$\dfrac{79-30p}{100} = 0.61$

$79-30p = 61$

$p = 0.6$

g. If, in another week, $p = 0.8$, find the probability that the **fifth** pot made that week is smooth. 2 marks

Matrices bullshit

Question 5 (13 marks)

Let $f : R \to R, f(x) = (x-3)(x-1)(x^2+3)$ and $g : R \to R, g(x) = x^4 - 8x$.

a. Express $x^4 - 8x$ in the form $x(x-a)((x+b)^2+c)$. 2 marks

$x^4 - 8x = x(x-2)(x^2+2x+4) = x(x-2)((x+1)^2+3)$

b. Describe the translation that maps the graph of $y = f(x)$ onto the graph of $y = g(x)$. 1 mark

a 1 unit in negative x direction translation

c. Find the values of d such that the graph of $y = f(x+d)$ has

i. one positive x-axis intercept 1 mark

$d = [1, 3)$

ii. two positive x-axis intercepts. 1 mark

$d \in (-\infty, 1)$

d. Find the value of n for which the equation $g(x) = n$ has one solution. 1 mark

$n = -6 \cdot 2^{\frac{1}{3}}$

e. At the point $(u, g(u))$, the gradient of $y = g(x)$ is m and at the point $(v, g(v))$, the gradient is $-m$, where m is a positive real number.

i. Find the value of $u^3 + v^3$. 2 marks

$g'(x) = 4x^3 - 8$

$g'(u) = m$ $\quad$ $g'(v) = -m$

$4u^3 - 8 = m$ $\quad$ $4v^3 - 8 = -m$

$u^3 = \frac{m+8}{4}$ $\quad$ $v^3 = \frac{8-m}{4}$

$u^3 + v^3 = \frac{8}{4} + \frac{8}{4} + \frac{m}{4} - \frac{m}{4}$

$= 4$

ii. Find u and v if $u + v = 1$. 1 mark

$u^3 + v^3 = 4$ ① $\quad$ $u + v = 1$ ②

$u = \frac{1 \pm \sqrt{5}}{2}$ $\quad$ $v = \frac{1 \mp \sqrt{5}}{2}$

$g'\left(\frac{1+\sqrt{5}}{2}\right) = 4\sqrt{5}$ $\quad$ $g'\left(\frac{1-\sqrt{5}}{2}\right) = -4\sqrt{5}$

$\therefore \ u = \frac{1+\sqrt{5}}{2}, \ v = \frac{1-\sqrt{5}}{2}$ for $m > 0$

f. i. Find the equation of the tangent to the graph of $y = g(x)$ at the point $(p, g(p))$. 1 mark

$y - g(p) = g'(p)(x - p)$

$y - (p^4 - 8p) = (4p^3 - 8)(x - p) \Rightarrow y = (4p^3 - 8)x - 3p^4$

ii. Find the equations of the tangents to the graph of $y = g(x)$ that pass through the point with coordinates $\left(\frac{3}{2}, -12\right)$. 3 marks

$y = mx + c$

$-12 = (4p^3 - 8)\frac{3}{2} - 3p^4$

$p = 0, 2$

$p = 0: \ y = -8x$ or $p = 2: \ y = 24x - 48$

END OF QUESTION AND ANSWER BOOK

MATHEMATICAL METHODS (CAS)

Written examinations 1 and 2

FORMULA SHEET

Directions to students

Detach this formula sheet during reading time.

This formula sheet is provided for your reference.

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Mathematical Methods (CAS)
Formulas

Mensuration

area of a trapezium: $\frac{1}{2}(a+b)h$

curved surface area of a cylinder: $2\pi rh$

volume of a cylinder: $\pi r^2 h$

volume of a cone: $\frac{1}{3}\pi r^2 h$

volume of a pyramid: $\frac{1}{3}Ah$

volume of a sphere: $\frac{4}{3}\pi r^3$

area of a triangle: $\frac{1}{2}bc\sin A$

Calculus

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

$$\frac{d}{dx}\left(e^{ax}\right) = ae^{ax}$$

$$\frac{d}{dx}\left(\log_e(x)\right) = \frac{1}{x}$$

$$\frac{d}{dx}\left(\sin(ax)\right) = a\ \cos(ax)$$

$$\frac{d}{dx}\left(\cos(ax)\right) = -a\ \sin(ax)$$

$$\frac{d}{dx}\left(\tan(ax)\right) = \frac{a}{\cos^2(ax)} = a\ \sec^2(ax)$$

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c, n \neq -1$$

$$\int e^{ax} dx = \frac{1}{a}e^{ax} + c$$

$$\int \frac{1}{x} dx = \log_e |x| + c$$

$$\int \sin(ax)dx = -\frac{1}{a}\cos(ax) + c$$

$$\int \cos(ax)dx = \frac{1}{a}\sin(ax) + c$$

product rule: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

quotient rule: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

chain rule: $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

approximation: $f(x+h) \approx f(x) + hf'(x)$

Probability

$\Pr(A) = 1 - \Pr(A')$

$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$

$\Pr(A|B) = \frac{\Pr(A \cap B)}{\Pr(B)}$

transition matrices: $S_n = T^n \times S_0$

mean: $\mu = \mathrm{E}(X)$

variance: $\mathrm{var}(X) = \sigma^2 = \mathrm{E}((X-\mu)^2) = \mathrm{E}(X^2) - \mu^2$

Probability distribution		Mean	Variance
discrete	$\Pr(X = x) = p(x)$	$\mu = \sum x\, p(x)$	$\sigma^2 = \sum (x-\mu)^2\, p(x)$
continuous	$\Pr(a < X < b) = \int_a^b f(x)dx$	$\mu = \int_{-\infty}^{\infty} x\, f(x)dx$	$\sigma^2 = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$

END OF FORMULA SHEET