

Question 5 (10 marks)

A machine is calibrated to produce ball bearings such that their diameters are normally distributed with a mean of 0.5 cm and a standard deviation of 0.005 cm. To determine whether the machine is working correctly, random samples of 10 ball bearings are selected at various times and the mean diameter of the 10 ball bearings in each sample is calculated.

For a particular sample of 10 ball bearings, the mean diameter is 0.5003 cm. It is proposed to carry out a statistical test at the 5% level of significance to determine whether the machine is now producing ball bearings with a mean diameter greater than 0.5 cm and possibly needs resetting. Assume that the standard deviation is still 0.005 cm.

- a. Write down the mean and the standard deviation of the sampling distribution for samples of 10 ball bearings. Give the value of the standard deviation correct to four decimal places. (2 marks)

Let \bar{D} be the sampling distribution of diameters of 10 balls

$$E(\bar{D}) = 0.5 \quad \text{sd}(\bar{D}) = \frac{0.005}{\sqrt{10}} = 0.0016$$

- b. Write down suitable null and alternative hypotheses for the statistical test. (1 mark)

$H_0: \mu = 0.5 \quad H_1: \mu > 0.5$

- c. Find, correct to three decimal places, the p value for the statistical test. (1 mark)

$p = 0.425$

- d. Draw an appropriate conclusion about the hypotheses in **part b.** from the p value found in **part c.** Give a reason involving p for your conclusion. (1 mark)

$p = 0.425 > 0.05$

\therefore insufficient evidence to reject H_0

- e. Find the largest sample mean of 10 ball bearings that could be observed for the null hypothesis not to be rejected at the 5% level of significance. Assume that the machine is still producing ball bearings with a standard deviation of 0.005 cm. Give your answer correct to four decimal places. (1 mark)

$\Pr(\bar{D} < \bar{d} | \mu = 0.5) > 0.05$

$\bar{d} = 0.5026 \text{ cm}$

The settings of the machine will be checked if a sample mean \bar{x} of 10 ball bearings falls outside the interval (a, b) , where $\Pr(\bar{x} < a) = 0.01$ and $\Pr(\bar{x} > b) = 0.01$

- f. Find the values of a and b , correct to three decimal places, assuming that the machine is producing ball bearings with a mean diameter of 0.5 cm and a standard deviation of 0.005 cm. (2 marks)

$$a = 0.496$$

✓

$$b = 0.504$$

✓

- g. On a certain day, five random samples of 10 ball bearings are selected.

Find the probability that only one of the five sample means falls outside the interval found in **part f**. Give your answer correct to three decimal places. (2 marks)

$$X \sim B_1(5, 0.02)$$

✓

$$\Pr(X=1) = 0.092$$

✓