Question 3

The Bouncy Ball Company (BBC) makes tennis balls whose diameters are normally distributed with mean 67 mm and standard deviation 1 mm. The tennis balls are packed and sold in cylindrical tins that each hold four balls. A tennis ball fits into such a tin if the diameter of the ball is less than 68.5 mm.

a. What is the probability, correct to ~~four~~ decimal places, that a randomly selected tennis ball produced by BBC fits into a tin?

$D \sim N(67, 1)$

$\Pr(D < 68.5) = 0.9332$ *normcdf(-∞, 68.5, 67, 1)

✓

2 marks

BBC management would like each ball produced to have diameter between 65.6 and 68.4 mm.

b. What is the probability, correct to four decimal places, that the diameter of a randomly selected tennis ball made by BBC is in this range?

$D \sim N(67, 1)$

$\Pr(65.6 < D < 68.4) = 0.8385$ *normcdf(65.6, 68.4, 67, 1)

✓

2 marks

c. **i.** What is the probability, correct to four decimal places, that the diameter of a tennis ball which fits into a tin is between 65.6 and 68.4 mm?

$D \sim N(67, 1)$

$\Pr(65.6 < D < 68.4 \mid D < 68.5) = \dfrac{\Pr(65.6 < D < 68.4)}{\Pr(D < 68.5)}$ *part b *part a

$= 0.8985$ ✓

ii. A tin of four balls is selected at random. What is the probability, correct to four decimal places, that at least one of these balls has diameter outside the desired range of 65.6 to 68.4 mm?

$X \sim Bi(4, 1-0.8985)$ *do not use rounded value from i

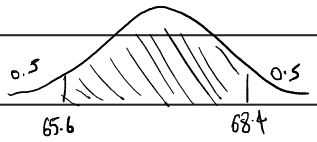
$\Pr(X \geq 1) = 0.3482$ ✓ *binomcdf(4, 1-0.8985, 1, 4)

1 + 2 = 3 marks

BBC management wants engineers to change the manufacturing process so that 99% of all balls produced have diameter between 65.6 and 68.4 mm. The mean is to stay at 67 mm but the standard deviation is to be changed.

d. What should the new standard deviation be (correct to two decimal places)?

$D \sim N(67, \sigma)$ $Z \sim N(0,1)$



$\Pr(D < 68.4) = 0.995$

$\Pr\left(Z < \dfrac{68.4-67}{\sigma}\right) = 0.995$ *typical standardisation

$\dfrac{68.4-67}{\sigma} = 2.5758$

$\sigma = 0.54$ mm ✓

invNorm(0.995,0,1) 2.575829303

3 marks

BBC sells tennis balls directly to tennis clubs once a year. If a tennis club buys its balls from BBC one year, there is an 80% chance it will buy its balls from BBC the next year. If a tennis club does not buy its balls from BBC one year, there is a 15% chance it will buy its balls from BBC the next year.

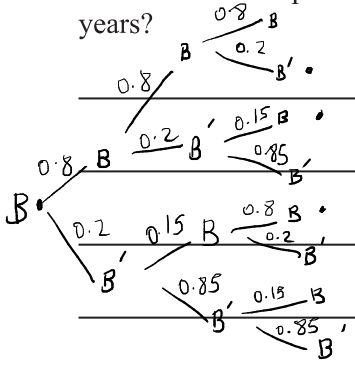
Suppose the Melbourne Tennis Club buys its tennis balls from BBC this year.

e. What is the exact probability that it will buy its tennis balls from BBC for the next three years?

$Pr(\text{buys for next three years}) = (0.8)^3 = \frac{64}{125}$ ✓

2 marks

f. What is the exact probability that it will buy its tennis balls from BBC for exactly two of the next three years?



$Pr = 0.8 \times 0.8 \times 0.2 + 0.8 \times 0.2 \times 0.15 + 0.2 \times 0.15 \times 0.8$

$= \frac{22}{125}$

3 marks

Let p be the probability that the Melbourne Tennis Club will buy its tennis balls from BBC n years after 2009 given that it buys them from BBC in 2009.

g. Find the smallest value of n such that $p \leq 0.45$.

OLD SD

2 marks

Total 17 marks