Question 6 (10 marks)

A real estate agent's income is comprised of a fixed monthly payment of $5000 plus a commission of $4000 for each house sold. The agent may sell up to three houses in a month with the following probability distribution, where H is the number of houses sold.

h	0	1	2	3
$\Pr(H = h)$	0.2	0.6	0.19	0.01

a. Find the agent's expected total income each month. Give your answer in dollars, correct to the nearest dollar. 2 marks

$I = 5000 + 4000H$

$E(H) = 0 + 0.6 + 0.38 + 0.03 = 1.01$ ✓

$E(I) = 5000 + 4000(1.01) = \$9040$ ✓

b. What extra amount of commission should the agent request for each house sold so that their expected monthly income increases by $2000? Give your answer in dollars and cents, correct to the nearest cent. 2 marks

$2000 = a \times 0.6 + 2a \times 0.19 + 3a \times 0.01$ ✓

$a = \$1980.20$ ✓

To prepare a certain house for sale, the agent recommends that the garden be landscaped and replanted. The time taken for a contractor to landscape the garden is normally distributed with a mean of 18 hours and a standard deviation of four hours. The time taken for replanting is independent of the time taken for landscaping and is normally distributed with a mean of 24 hours and a standard deviation of six hours.

c. What is the probability, correct to four decimal places, that the garden can be landscaped and replanted within a week? Assume that the contractor will work eight hours per day for five days of the week. 2 marks

$L \sim N(18, 4)$ $R \sim N(24, 6)$

$T = L + R$

$E(T) = 42$ $sd(T) = 2\sqrt{13}$

$\therefore T \sim N(42, 2\sqrt{13})$ ✓

$\Pr(T < 40) = 0.3908$ ✓

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d. What period of time should the agent allow the contractor to complete the tasks of landscaping and replanting, in order to be 90% confident that these tasks will be completed within this period of time? Give your answer in hours, correct to two decimal places. 1 mark

$\Pr(T < a) = 0.90$

$\Pr\left(Z < \frac{a - 42}{2\sqrt{13}}\right) = 0.90$

$a = 51.24$ hours ✓

e. Before the annual income review, the agent believes that the company's remuneration level is below the industry average of \$115 000 per annum, with an industry standard deviation of \$12 000. To test this hypothesis, the agent takes a random sample of 36 of the company's employees and finds their mean income to be \$110 800 per annum.

Assuming an industry standard deviation of \$12 000, carry out a one-tailed statistical test at the 5% level of significance to ascertain whether the sample mean of \$110 800 per annum supports the agent's belief. 3 marks

$H_0: \mu = 115\,000$ $H_1: \mu < 115000$ $\alpha = 0.05$ ✓

$p = \Pr(\bar{X} \leq 110\,800 \mid \mu = 115000)$

$= \Pr\left(Z \leq \frac{110\,800 - 115\,000}{\frac{12000}{6}}\right)$

$= 0.0179$ ✓

$p = 0.0179 < 0.05$

$\therefore$ sufficient evidence to reject H_0, agents claim supported ✓

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