CS 600 HW 12
SHUCHI PARAGBHAI MEHTA
CWID: 20009083

- 26.6.7

Convert the following linear program into standard form:

$$\begin{aligned} \text{minimize:} \quad & z = 3y_1 + 2y_2 + y_3 \\ \text{subject to:} \quad & -3y_1 + y_2 + y_3 \geq 1 \\ & 2y_1 + y_2 - y_3 \geq 2 \\ & y_1, y_2, y_3 \geq 0 \end{aligned}$$

y_1	y_2	y_3	C
-3	1	1	1
2	1	-1	2
3	2	1	

Transpose of the above matrix to convert minimize to maximize

x_1	x_2	P
-3	2	3
1	1	2
1	-1	1
1	2	1

$$-3x_1 + 2x_2 \leq 3$$
$$x_1 + x_2 \leq 2$$
$$x_1 - x_2 \leq 1$$
$$x_1 + 2x_2 = P$$

final equation will be after adding slack variable,

Maximize : $-x_1 - 2x_2 + P = 0$

$$-3x_1 + 2x_2 + y_1 = 3$$
$$x_1 + x_2 + y_2 = 2$$
$$x_1 - x_2 + y_3 = 1$$

where, y_1, y_2, y_3 are slack variables

x_1	x_2	y_1	y_2	y_3	P	
-3	$\boxed{2}$	1	0	0	0	3
1	1	0	1	0	0	2
1	-1	0	0	1	0	1
-1	-2	0	0	0	1	0

↑

Smallest value in the row, check for corresponding smallest value when dividing last column with x_2

choose 2 as pivot.

$R_1 \longrightarrow \frac{1}{2} R_1$

x_1	x_2	y_1	y_2	y_3	P	
$-3/2$	$\boxed{1}$	$1/2$	0	0	0	$3/2$
1	1	0	1	0	0	2
1	-1	0	0	1	0	1
-1	-2	0	0	0	1	0

$R_2 \rightarrow R_2 - R_1$, $R_3 \rightarrow R_3 + R_1$, $R_4 \rightarrow R_4 + 2R_1$

x_1	x_2	y_1	y_2	y_3	P	
$-3/2$	1	$1/2$	0	0	0	$3/2$
$\boxed{5/2}$	0	$-1/2$	1	0	0	$1/2$
$-1/2$	0	$1/2$	0	1	0	$5/2$
-4	0	1	0	0	1	3

$R_2 \rightarrow 2/5 R_2$

x_1	x_2	y_1	y_2	y_3	P	
$-3/2$	1	$1/2$	0	0	0	$3/2$
$\boxed{1}$	0	$-1/5$	$2/5$	0	0	$1/5$
$-1/2$	0	$1/2$	0	1	0	$5/2$
-4	0	1	0	0	1	3

$R_1 \to R_1 + \frac{3}{2} R_2$

$R_3 \to R_3 + \frac{1}{2} R_2$

$R_4 \to R_4 + 4R_2$

x_1	x_2	y_1	y_2	y_3	P	
0	1	$1/5$	$3/5$	0	0	$9/5$
1	0	$-1/5$	$2/5$	0	0	$1/5$
0	0	$2/5$	$1/5$	1	0	$13/5$
0	0	$1/5$	$8/5$	0	1	$19/5$

$x_1 = 1/5$

$x_2 = 9/5$

$P = 19/5$ (Maximize)

OR

$y_1 = 1/5$

$y_2 = 9/5$

$y_3 = 0$

$Z = 19/5$

- 26.6.26

Give a linear programming formulation to find the minimum spanning tree of a graph. Recall that a spanning tree T of a graph G is a connected acyclic subgraph of G that contains every vertex of G. The minimum spanning tree of a weighted graph G is a spanning tree T of G such that the sum of the edge weights in T is minimized.

- As given, we can use linear programming to find the minimum spanning tree of a graph. The weighted graph G is created using the vertices V and edges E as-is. The objective is to locate an acyclic subgraph T of G, for example: The remainder of the text is related to the subheading T. The entire edge weights of T are minimized.

- The following are the restrictions to be followed when writing the code:

  - When an edge connects the two vertices I and j, then xij is 1.
  - Otherwise, xij is 0. S is the proprietor of (S being all subsets of E).

- There can only ever be one edge linking two sets of vertices, as a result. If the issue is treated in this manner, a formulation for linear programming may be created. The "weighted, undirected graph G = (V, E)" shouldn't have any cycles, thus locate a subgraph T of G that has each vertex in V and an edge weight of c for each e in E.

- Assume that w(T) is defined as we want to minimize weight w(T).
  the sum in T is equal to w(T)
  A linear program can be used to solve this problem:
  Maximize $x\,e\ +\ y\,e\ -\ z\,e$, where e is a T-expression.
  based on:
  The sum of y u and x v must be less than 1 for each pair (u, v) in $V \times V$, where (u != v).
  $Z\,v\ =\ 2 * z\,v$ for each pair (u, vi).

  For each pair (u, vi), there is at least one edge in G between the pair (u, vi).
  In $E, x\,e\ >\ 0, y\,e\ >\ 0, and\ z\,e\ >\ or\ equals$ 0 or 1 for each e in E.

- To determine a graph's minimum spanning tree, we can perform linear programming. The weighted, undirected graph G = (V, E) shouldn't have any cycles, so locate a subgraph T of G that has each vertex in V and an edge weight of c for each e in E.

- A method for optimizing a mathematical model is linear programming. An objective function can be subjected to Lagrange's inequality and inequality constraints. When the objective function or constraint function is not linear but may be arbitrarily closely approximated by a linear function, convex optimization techniques can be applied.

- 26.6.31

A political candidate has hired you to advise them on how to best spend their advertising budget. The candidate wants a combination of print, radio, and television ads that maximize total impact, subject to budgetary constraints, and available airtime and print space.

type	impact per ad	cost per ad	max ads per week
radio	a	10,000	25
print	b	70,000	7
tv	c	110,000	15

Design and solve a linear program to determine the best combination of ads for the campaign.

Let there be 'x' ads of radio, 'y' ads of print and 'z' ads of tv
The total impact would be impact = (x*a + y*b + z*c) which is to be maximized.

Let total budget be 'B', Then the total cost for all the ads would be,
cost = (10000*x + 70000*y + 110000*z), which should be less than or equal to total budget
(10000*x + 70000*y + 110000*z) <= B
where B is the total budge.

There is a bound to maximum number of each type of ads which gives,

x <= 25
y <= 7
z <= 15

The Linear Program would become,

MAXIMIZE : impact = (x*a + y*b + z*c)
SUBJECT TO : (10000*x + 70000*y + 110000*z) <= B
BOUNDS : x <= 25
y <= 7
z <= 15

This Linear Program would be solvable if numerical values of (a, b, c, B) are given