Analysis of Campus Fountain Algae Mitigation using Hydrodynamic Modelling

Harshvardhan Agarwal and Sunny Kumar
Mentored by Prof. Akash Chaudhary

August 4, 2025

Abstract

This report aims to develop and analyze a mathematical model for algal growth within a campus fountain to understand the key factors driving its proliferation. The model is based on established frameworks in surface water-quality modeling, primarily referencing the work of Steven C. Chapra. We formulate a differential equation for algal biomass concentration that incorporates environmental limitations, including light intensity, temperature, and pH levels. Experimental work was conducted to identify the dominant algal species through microscopic observation and to measure nutrient concentrations (nitrates and phosphates) in the fountain water. The model parameters were tuned using species-specific data and local environmental data for light and temperature over a one-month observation period. The resulting first-order differential equation was solved using MATLAB to simulate algal concentration over time. The simulation shows an initial rapid growth in biomass, which aligns with real-world observations, followed by a decline. However, the model's prediction of a decline contrasts with the expected stagnation of the algal population, suggesting that inaccuracies in pH data interpolation or the omission of nutrient dynamics may be limiting factors. This work provides a foundational model for predicting algal blooms in the fountain and identifies key areas for future refinement.

1 Introduction

The proliferation of algae in managed water bodies, such as campus fountains, presents both an aesthetic and maintenance challenge. To effectively mitigate unwanted algal blooms, it is crucial to understand the underlying dynamics of their growth. This report details the development of a mathematical model designed to simulate the growth of algae in our campus fountain. The primary objective is to create a predictive tool that accounts for the most significant environmental drivers of algal growth.

Our approach is grounded in established principles of aquatic biology and water quality modeling. We began by reviewing foundational texts and research papers, notably "Surface Water-Quality Modelling" by Steven C. Chapra and related studies on algal-growth hydrodynamics. This provided a robust theoretical framework for our model.

The report is structured as follows: First, we present the core mathematical formulation, detailing the differential equation that governs algal biomass and the functions that represent limitations imposed by nutrients, light, temperature, and pH. Next, we describe the experimental data acquisition process, which included identifying the specific algal species present in the fountain and analyzing water samples for key nutrients. Subsequently, we detail the parameterization of the model using species-specific constants and local environmental data. Finally, we present the results of our MATLAB simulation, compare them to qualitative observations, and discuss the model's limitations and potential avenues for future improvement.

2 Theoretical Framework and Model Formulation

The model is built upon a differential equation that describes the change in algal biomass concentration over time. The foundational equation, adapted from established hydrodynamic models, is as follows:

$$\frac{\partial B}{\partial t} = (P - B_M - P_R)B + \frac{\partial}{\partial z}(w_s B) + \frac{B_L}{V} \tag{1}$$

Where:

- **B** is the biomass concentration (g/m^3)
- **P** is the production (growth) rate (day^{-1})
- B_M is the basal metabolic rate (day^{-1})
- P_R is the predation rate (day^{-1})
- w_s is the settling velocity (m/day)
- B_L represents external loadings (g/day)
- **V** is the model cell volume (m^3)

Given the shallow depth of the fountain and the absence of significant external loadings, the final two terms of Equation (1) were considered negligible. The metabolic and predation rates were combined into a single respiration rate, simplifying the equation.

The production rate, **P**, is the most critical component, as it is dependent on multiple environmental factors. It is represented by:

$$P = P_M f(v) g(I) h(T) i(pH) \tag{2}$$

Where:

- P_M is the maximum production rate under optimal conditions.
- **f(v), g(I), h(T), and i(pH)** are limitation functions (ranging from 0 to 1) for nutrients, light, temperature, and pH, respectively.

3 Experimental Data Acquisition and Parameterization

3.1 Nutrient Level Analysis

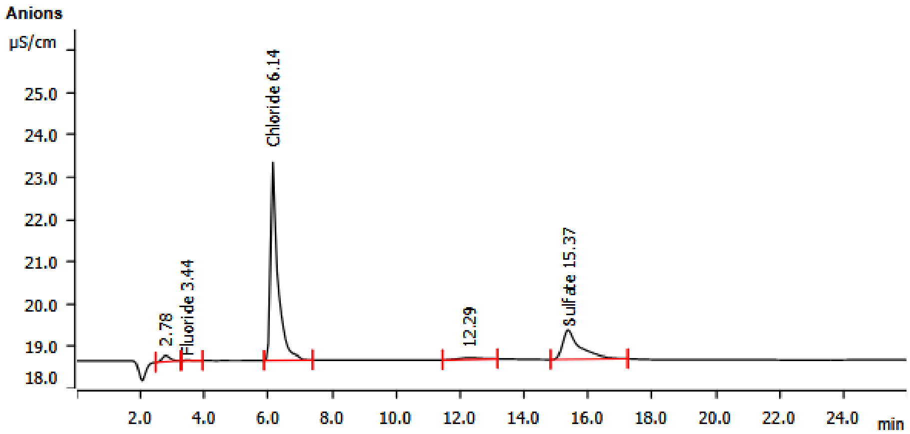
To determine the nutrient limitation factor, f(v), water samples from the fountain were analyzed for anion concentrations, specifically nitrates and phosphates. The tests, conducted on three separate occasions, consistently revealed negligible or only trace amounts of these nutrients.

Due to these findings, the nutrient limitation factor was assumed to be non-critical for this modeling period, and the focus shifted to the other environmental parameters.

3.2 Algal Specie Identification

Identifying the dominant algal species was essential for finding accurate, species-specific parameters for the model. Algae samples were collected from the fountain and observed under a microscope.

The microscopic analysis revealed structures closely resembling *Spirulina*, which allowed us to source relevant growth parameters from existing literature for the effects of light, temperature, and respiration.



Peak number	Retention time	Area	Height	Concentration	Component name
	min	(µS/cm) x min	µS/cm	ppm	
1	2.783	0.0414	0.143	invalid	
2	3.437	0.0043	0.017	0.379	Fluoride
3	6.135	1.2136	4.690	157.595	Chloride
4	12.293	0.0319	0.039	invalid	
5	15.368	0.4352	0.688	76.903	Sulfate

Figure 1: Ion chromatography results from three separate water samples. In each test, nitrate and phosphate concentrations were found to be negligible, leading to the exclusion of the nutrient limitation factor from the final model.

3.3 Model Parameterization

The model was configured to run over a one-month period (June 23rd to July 23rd), starting from a time when the fountain had been recently cleaned. The time step was set to 12 hours to approximate the effects of day and night.

- **Effect of Light, g(I):** Light intensity data for Kanpur was obtained from the NASA Power Data Access Viewer. The limitation was modeled using parameters specific to *Spirulina platensis*, with a maximum growth rate (P_M) of approximately 0.89 per half-day.
- **Effect of Temperature, h(T):** The temperature limitation was modeled using a quadratic formula centered around an optimal temperature (T_{opt}) of 33°C. Half-day average temperature data was used.
- **Effect of pH, i(pH):** The pH limitation was calculated using a standard formulation dependent on hydrogen ion concentration. As daily pH data was unavailable, values were interpolated between measurements, introducing a layer of approximation.
- **Respiration Rate:** The combined respiration and predation rate was modeled as a function of temperature, with a dependency on the ratio of dark volume to total volume of the pool (assumed to be 0.25 during the day and 1 at night).

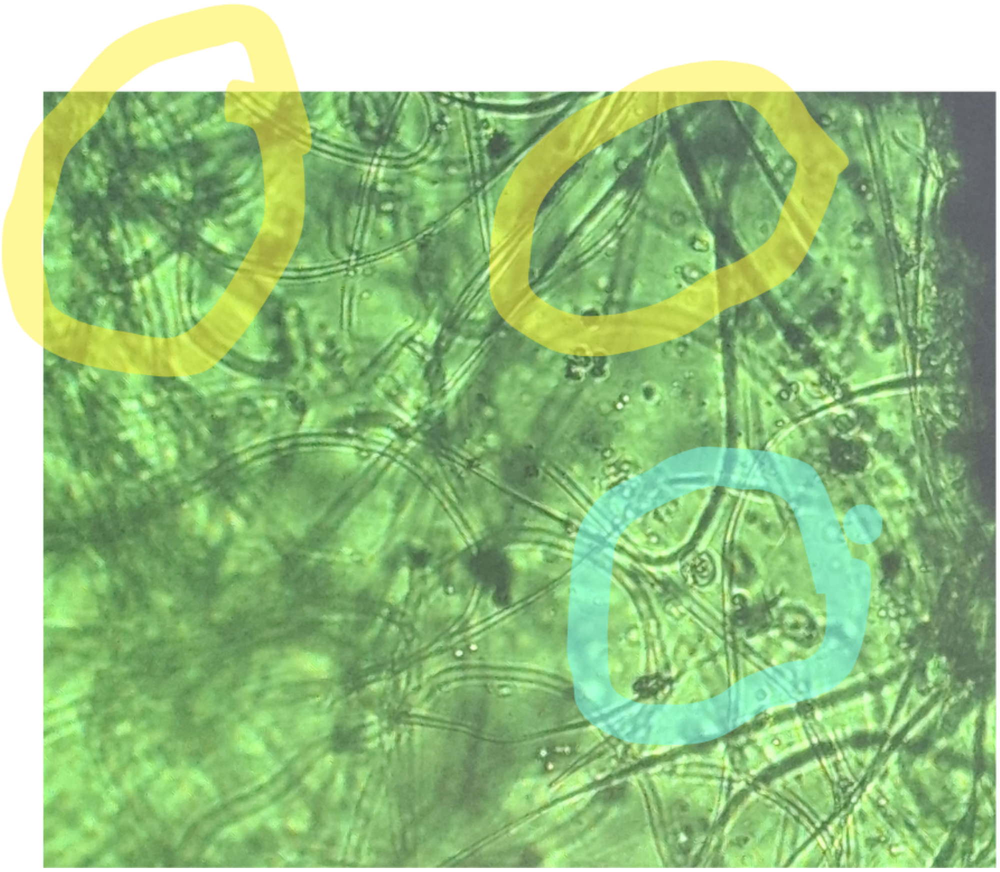


Figure 2: Microscopic view of the fountain algae (left) compared with a chart of common algal species (right). The observed structures showed strong similarities to *Spirulina Maxima* (yellow highlights) and *Phormidium Ambiguum* (blue highlights).

4 Governing Equations and Parameter Definitions

$$g(I) = \frac{I^m}{I^m + K_I^m}$$

$$h(T) = 1 - \frac{(\tau - T_{\text{opt}})^2}{\Delta T^2}$$

$$i([H^+]) = \frac{[H^+]}{[H^+] + k_{\text{OH}}(T) + [H^+]^2/k_H(T)}$$

The parameters and temperature-dependent constants used in the equations above are defined as:

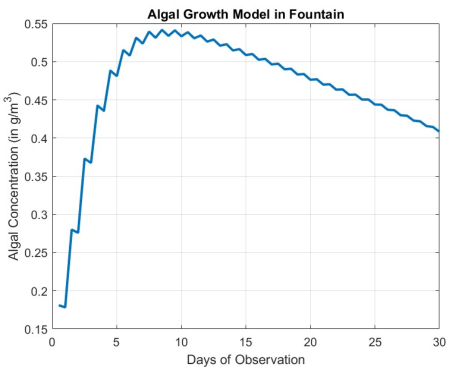


Figure 3: The simulated concentration of algae (g/m3)

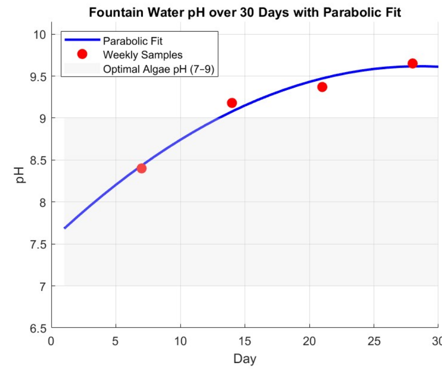


Figure 4: Best Fit Curve of the pH values

$$\mu_{\max} = 2.06e - 5$$
$$K_I = 160$$
$$m = 1.49$$

$$T_{\text{opt}} = 33$$
$$\Delta T = 25$$

$$[H^+] = 10^{(-\text{pH})}$$

$$k_{\text{OH}}(T) = 10^{-13}(8T^2 - 500T + 8000)$$

$$k_H(T) = 10^{-7}(-5T^3 + 300T^2 - 5000T + 30000)$$

5 Simulation and Results

With all parameters defined, the simplified differential equation was solved using MATLAB's 'ode45' solver. The simulation produced a time-series graph of the algal concentration in the fountain over the 30-day observation period.

The initial phase of the simulation, showing a rapid increase in algal concentration, aligns well with the visual observation of the fountain during that period. The model successfully captures the initial bloom. However, the model predicts a peak concentration around day 10, followed by a steady decline. In reality, the algal population appeared to stagnate at a high level rather than decrease.

6 Discussion and Conclusion

This study successfully developed a foundational mathematical model for simulating algal growth in the campus fountain. The model correctly predicted the initial exponential growth phase observed after the fountain was cleaned. This confirms that the core framework, which incorporates limitations from light, temperature, and pH, is valid for describing the system's dynamics.

However, a key discrepancy exists between the model's prediction of a population decline and the observed stagnation. We attribute this primarily to two factors:

1. **pH Data Inaccuracy:** The use of interpolated pH values is a significant source of potential error. Since algal growth is sensitive to pH, more frequent and accurate measurements are needed to improve the model's predictive power.

2. **Nutrient Dynamics:** While initial tests showed low nutrient levels, it is possible that nutrients are recycled within the system or that samples from deeper water would reveal different concentrations. A more complex model incorporating nutrient cycles could yield more accurate long-term predictions.

In conclusion, the model serves as a valuable first step. Future work should focus on gathering higher-resolution data, particularly for pH and nutrients, to refine the model and improve its accuracy in predicting the long-term, steady-state behavior of the algal population.