Todo list

Contents

1 Introduction

The non-linearity exhibited by superconducting nanowires is of key importance to many of its applications, from superconducting nanowire single-photon detectors (SNSPDs) to neuromorphic computing. This non-linearity however is also the reason simulating nanowires is an increasingly hard problem, especially when we start to care about the microwave properties of these devices. A common way of simulating these nanowire electronics builds on top of existing circuit simulation environments that were optimized for classical electronics – where the microwave and superconducting properties of the models did not matter.

Simulating the predominant effects in our superconducting electronics is an important step in designing superconducting devices.

While the requirement for better nanowire simulations and the complexity of our models are steadily increasing, the tools used to simulate

Sending pulses is cool

not done

1.1 Non-linearity in superconducting nanowires

Superconducting nanowires are highly non-linear and present three main forms of non-linearity: kinetic inductance, state transitions between the normal and superconducting state and coupling to other non-linear dynamics.

Kinetic inductance is a continuous form of non-linearity introduced by the inertia of cooper pairs in a nanowire. In thin films, kinetic inductance is highly dependent on the film thickness, temperature, and magnetic field [1]. In nanowires, the effect of a bias current on the kinetic inductance is a well-studied phenomenon. Of particular interest to more complicated electronics and SNSPDs is the ability to simulate the effects of pulses and non-DC current behavior on the kinetic inductance. This is an important effect since the non-linearity of some nanowire geometry can change the shapes of pulses traveling in the nanowire. This causes even simple designs such as a superconducting transmission line operating only in the superconducting regime to behave in a difficult to anticipate non-linear fashion.

The second form of non-linearity pertains to the superconducting state. By assuming the device is experiencing a constant magnetic field and temperature, we can find a threshold critical current, i_c, where if the current exceeds that value locally along a nanowire, it switches out of the superconducting state and into the resistive state. This switching behavior is a non-linearity over a boolean state that is dependent on the current flowing through each portion of the nanowire. Non-linearities over a boolean state are particularly hard to simulate as they involve sudden large magnitude changes. Typical non-linear solvers are optimized for continuous non-linear systems where the solver enters a loop making the timestep smaller until the magnitude of change is small. In boolean states, there is no sense of continuity, and in the limit of smaller timesteps, the change in response magnitude will be just as large.

1.2 Nanowire Elements

From an electronics standpoint, a nanowire's lumped model is a non-linear inductor when superconducting. When resistive, an additional resistor is in series with that inductor. These two building blocks (a continuously non-linear inductor and a discrete non-linear resistance) are the basis for modeling the behavior of superconducting nanowires in the electronics picture and cover the two main types of non-linearities exhibited.

A more complicated - but necessary - picture includes coupling to a thermal equation. A nanowires critical temperature i_c and critical temperature T_c are in reality functions of the current state of the superconductor, they are related by the critical surface, $T_c(i = 0) \neq T_c(i = 0.75i_c)$. The superconducting-to-normal state transition begins a coupled chain reaction between a thermal system and an electrical system, making modeling nanowires harder. When a portion of the nanowire switches into the resistive state, a normal region starts to form in the wire that dissipates thermal energy. This energy heats up the surrounding portions of the nanowire, decreasing their critical current. At the same time, the normal region has a higher impedance than the nanowire diverting current around it, allowing portions of the nanowire to see a higher density of the current, making it more likely to switch in the plane of the hotspot. The hotspot also dissipates heat to the stack and fridge. These are well-studied phenomena for nanowires and tend to be modeled through experimentally fitted parameters for the electrical-thermal coupling.

add details: maybe talk about simulating noise transition??

add ref: ref karl nanowire. ref others??

Another picture that tends to be neglected is the distributed picture of the nanowire. In reality, the nanowire has a spatial dimension to it and is a microwave device. This picture tends to enforce simulation constraints as the discretization and network size increase. This picture accounts for time delays introduced for a signal entering and leaving a nanowire, resonances that might occur in the nanowire, as well as distributed thermal and electrical effects that don't make sense in the lumped picture. For meanders longer than the wavelength of frequencies carried, modeling them as distributed devices is essential. Not doing so can result in the device not switching when it has to. Another big effect that can't be captured with non-distributed pictures is pulse reshaping, more on that in Section 1.2.3.

idea:
tlines

add fig:
tline fig

idea:
CPW geometry

add ref:
ref https://arxiv.org/pdf/1805.05601.pdf

Missing figure

nanowire model simple circuit

1.2.1 SNSPDs

One geometry a nanowire can be designed for are superconducting nanowire single-photon detectors (SNSPDs). By having a nanowire meander biased near its critical current, a small energy perturbation (such as photon incidence) cause a transition into the normal state. As a result a single photon injecting a small amount of energy into the nanowire has an amplified output from the bias current flwoing across no resistor to a large resistor (usually on the order of $1k\Omega$).

add ref:
refernece for snspds

For simple SNSPD designs, it is enough to model the device using a lumped element model nanowire in parallel with a shunt resistor. Assuming an SNSPD with 50Ω impedance shunted with a 50Ω resistor, in this topology, the current is split and equally diverted into the shunt and nanowire. If biased at the right threshold, a photon count would correspond to a tiny spike in the current flowing through the nanowire leading to a switching event. The nanowire produces a voltage pulse as a $\sim 7k\Omega$ resistor forms (this number is dependent on multiple design parameters).

Due to the impedance mismatch, the current prefers the shunt resistor branch, which allows the hotspot to cool off and resets the SNSPD.

add fig: typical snspd readout setup figure

1.2.2 SNSPIs

Superconducting nanowire single-photon imagers (SNSPIs) are used in a similar fashion to SNSPDs but take advantage of the distributed picture for larger lengths of nanowire. SNSPIs tend to be be designed to have a slower propagation velocity and longer meanders. These effects cause a switching effect in the wire to take time to propagate to the two ends of the nanowire. By differentially reading out the meander, we can spatially resolve the photon incidence location as a function of the delay between the 2 ports of the SNSPI.

In this image, the SNSPI can be thought of as a non-linear transmission line that has both the non-linearities described above. By discretizing the transmission line into multiple lumped elements, the local non-linear contributions can be modeled by a non-linear inductor and resistor, while keeping the capacitor linear. In this image, this can be thought of as a long chain of discrete lumped nanowire elements in parallel with linear capacitor elements. This topology captures the distributed picture of pulses propagating in nanowire meanders and allows us to simulate SNSPIs.

1.2.3 Impedance Matching Tapers

Usually the nanowire's impedance is not similar enough to that of the input and output circuitry. This mismatch causes the signal to reflect back into the wire instead of propagating into the next stage, causing interference and distortions. Impedance matching is done by designing tapers: extensions of the same line that increase in width slowly as shown in figure 1(b). This slow increase ensures that there is minimal step change in impedance allowing for less reflections on the impedance boundaries. A Klopfenstein taper is the optimal taper geomtery for its length in decreasing the total amount of reflections [**klopfenstein˙transmission˙1956**]. The slow change in width still causes internal reflections along the length of the wire, however, their overall magnitude is smaller than one step change as illustrated in figure 2.

add ref:
cite klopf

idea:
mayhaps exponential taper, etc.

Impedance matching tapers are often used on nanowire devices when we care about preseving the signal shape and magnitude allowing us to use the SNSPDs readout pulses in more interesting ways and preserve logic pulses. Impedance matching tapers allow us to perform photon number resolution - infer the number of incident photons on a nanowire from the readout pulse as shown in more details in section 2.6.1. In devices like SNSPIs, the reflections caused by impedance mismatch at the edges can prohibit us from timing the pulses correctly and as a result spatially resolving the photon. This can also be remedied by the introduction of a taper.

add ref:
marco's snspd paper

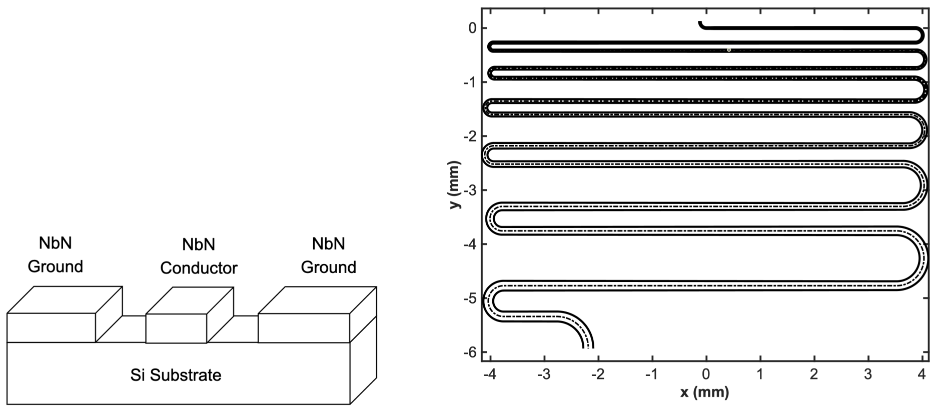
add ref:
DiZhu MW paper

add ref:
taper on SNSPI?

One side effect of using a Klopfenstein taper is the change in propagation velocity caused by the change of electrical characteristics in the wire. Using $L(x)$ and $C(x)$ as the inductance and capacitance along the length of the wire, the continuous version of equation 1b that $v_p(x)$ is dependant on $L \cdot C$. The Klopfenstein does not scale L and C inversely, which means that $v_p(x)$ is not constant along the wire. Coupling this with the fact that the thinner wire will have a higher current density, this becomes a huge source of distortion.

place tline v prop eq in nanowire section...

Long taper meander are folded into strips of straight wires connected by curved edges with large radii. This curvature is chosen in a manner that minimizes the



(a) (b)

Figure 1: (a) Vertical slice a Co-Planar Waveguide (CPW) geometry for a Niobium Nitride (NbN) nanowire. (b) Top view of a Klopfenstein taper with a folded meander.

add fig:
Forgot a layer lol

amount of reflections and current crowding: an unwanted effect caused by non-homogenous distributions of current density through a conductor that change the frequency response and impedance of the wire.[**hazra˙superconducting˙2016**] Impedance matching and the folding of the wire both introduce distortions to the input signal and require the simulator to account for them.

1.2.4 hTron

Since the nanowire is also a thermal system that can generate heat and has its accessible state space affected by its current thermal state, modeling its thermal behavior is important for accurate characterization. A CPW geometry nanowire, as shown in figure 1(a), consists of multiple layers. Having multiple layers of devices stacked on-top of each other can cause thermal coupling between the devices.

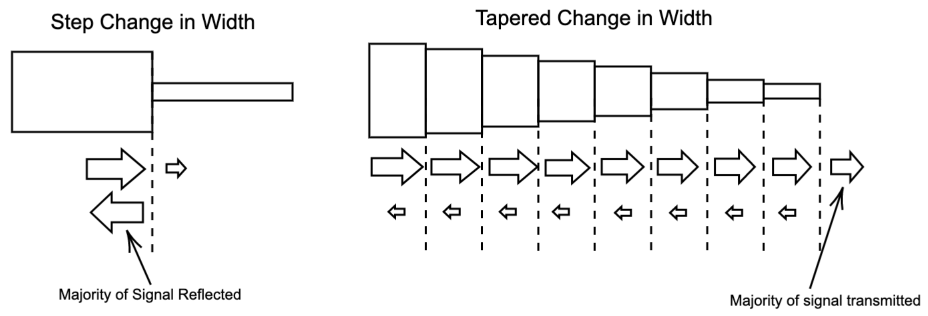


Figure 2: A tapered change in width causes smaller reflections at each boundary which add up to a lower total amount of reflection than a singular step change in width. Change in width is roughly proportional to change in impedance. The inclusion of multiple step changes in width increases the number of individual reflections occurring step and the number of interferences a simulator would need to keep track of grows exponentially increasing the complexity of the simulation.

While this coupling can be unwanted, devices that take advantage of that exist to create a new set of superconducting circuits. One such device is the heater-tron (hTron). A simple device topology that uses the hTron is having two superconducting stacks separated by an insulating layer. One of the stacks contains a resistor that generates heat while the other stack contains a nanowire above the resistor. When current flows through the resistor it generates heat that gets transmitted through the stack to the nanowire. Through this thermal coupling, the nanowire can be thermally biased.

add ref: htron!

Simulating heat transport in stacks

1.3 Problems Simulating Nanowires

A full-model that simulates all the dynamics of superconducting nanowires is hard to achieve due to the complexity, long simulation time, convergence issues and experimental fitting. Previous parts of this section demonstrated multiple regimes nanowires can be used in, from thermal coupling to a distributed microwave picture, each of which involves solving stiff non-linear differential equations in the time domain. Getting a model to simualte the electro-thermal coupling of a nanowire with respect to the stack, noise and photons as well as the distributed effect accounting for non-linearities results in stiff non-linear equations. As a result of this complexity, work is usually done on the individual parts with experimental fits for each picture but no conception of the greater picture of how these systems interact. This, in part, is also due to having multiple hard to define models for each device topology that look different even though the fabricated device is identical.

Berggren et al. implemented a nanowire model in LTspice based on the phenomenological hotspot velocity model by Kermal et al. This model is a lumped element nanowire model that accounts for the hotspot dynamics with experimentally fitted parameters. It is implemented in LTspice and contains both non-linearities exhibited by a nanowire using integrators and state saving, discussed in more depth in section 3. This model only accounts for the small signal solution and cannot be used in noise, AC or DC analysis. The model also suffers from instability around the non-linearities. The instability and simulation modes are further discussed in section 3.4.1.

add ref: karl spice

Simulating tapers made out of the superconducting material cause the pulse to reshape, an effect that isn't modeled in nanowire tapers. Pulses traveling in a taper experience reshaping due to the linear taper aspect, the change in impedance reflects certain components from the pulse while leaving other frequencies untouched. This reshaping can be completely captured by the scattering parameters and linear trans-

add ref: idk?

mission lines. However, linear parameters do not account for non-linear reshaping that is caused by kinetic inductance. Modeling tapers in LTspice tends to use a sequence of transmission lines (namely the lossy transmission line model with $R = 0$, discussed in 2.6.1) in series that have decreasing impedance. This model is sufficient in simulating the linear part of the reshaping under the assumption that the current flowing in the taper is much lower than the critical current, or that in the DC picture, this effect reaches equilibrium and causes a final shift in the perceived critical current of the device. In reality, this is not accurate, as pulses being carried down a biased line also exhibit non-linear reshaping and this method cannot be used to simulate pulse shapes accurately.

Non-linear simulation in superconductors has been studied a lot in the frequency domain using techniques like Harmonic Balance. These methods can account for the continuous non-linearity presented by kinetic inductance, but not the state transition non-linearity. JosephsonCircuits.jl for instance is designed to simulate JTWPA topologies in the frequency domain using Harmonic Balance. WRspice and Xyce both have Harmonic Balance backends that are very efficient. However, for topologies that utilize the binary state non-linearity, time-domain simulation is needed to characterize the device behavior. Given the nature of WRspice and Xyce, this implies that the entire circuit must be simulated in the time-domain. This is addressed in section 4.2.

The non-linearity of the electrical model gets even harder to simulate when coupled to a thermal equation. As a result, the thermal coupling is usually linearized around the regime we care about. For example, for nanowires that have no need to thermally interact with other elements, the hotspot growth is simulated via the phenomenological hotspot velocity model **??**. For geometries that rely on thermal

add ref: phen model

coupling such as the hTron, the critical current of the device is presumed to be a function of the electron temperature **??**. These models are sufficient but don't encompass a full thermal image of the device, neglecting the thermal coupling that might occur between two nanowires.

add ref: matteos thesis

idea: think about this more idk...

While most of these effects are hard to simulate on their own, the simulation of multiple nanowires is essential for scaling devices. As a result, it is important to develop a more stable nanowire model, a dedicated efficient simulator, and a more standardized simulation environment for various topologies.

This thesis proposes having an integrated simulator environment designed with the goal of simulating superconducting nanowires. Existing simulators like WRSpice and JosephsonCircuits.jl exist for simulating superconducting electronics, but tend to favor the frequency domain and weren't designed to account for things beyond the electrical non-linearities exhibited by nanowires. The work presented in this thesis will be divided into 3 sections tackling:

add ref: WRspice, KO-brien sim

1. an integrated environment for LTspice to design specific models and tools for superconducting devices and accompanying experiments.

2. a simple procedure to measure the stability of nanowire models and presenting improved models for the nanowire.

3. and finally, present preliminary work done to construct an efficient Julia-based simulator optimized for superconducting nanowire devices.

idea: is it one or multiple that I will include?

add ref: WRSpice, JosephsonCircuits.jl

add ref: Latching: https://studylib.net/ feedback-in-superconducting-nanowire-singl...

add ref: Check this out: https://ieeexplore.ie

2 spice-daemon — a Python wrapper for SPICE solvers

2.1 SPICE

One popular way of simulating electronics is using SPICE (Simulation Program with Integrated Circuit Emphasis). SPICE solvers are an industry standard method of simulation that combines DC analysis (also known as operating point analysis), AC analysis (linear small-signal frequency domain analysis), and transient analysis (time-domain large-signal solution of nonlinear differential algebraic equations) among other analysis methods.

Since then, Berkley SPICE inspired multiple other SPICE solvers including LT-spice. LTspice is a popular free circuit simulator that is widely used . SPICE models for superconducting electronics exist .

add details: more on spice and ref

idea: maybe FDM?

add ref: ref

nanowire, hTron, nTron...

2.1.1 Interacting with LTspice

Interfacing with SPICE software involves generating a netlist — a code snippet that defines how the different circuit elements are connected to each other. Netlists have a '.net' (and sometimes a '.cir') extension and can be used across different SPICE implementations. Netlists are encoded as ASCII files and as such editing them is straightforward.

Some commercial versions of SPICE software, including LTspice, add Schematic Capture capability. Schematic Capture allows for a native GUI encoding of a circuit to be converted into a netlist (in LTspice, that is a schematic file with the extension '.asc').

LTspice generates multiple types of files after each successful run. The most common type is a compressed binary '.raw' file that is generated after AC and transient simulations. An optional '.op.raw' file is also generated that saves the DC solution and can be imported to skip DC simulation. LTspice also generated a '.plt' file that encodes the layout of and variables plotted in the LTspice plotting window. A '.log' file is always generated, regardless of the type of simulation and/or its success (after passing any topology and expression checks). For a smoother experience using LTspice on Mac with spice-daemon, it is recommended you uncheck "automatically delete .raw and .log files" under the operation sub-menu in the LTspice preferences. This setting is by default checked on Mac (but not on Windows) and deletes files that spice-daemon uses to track LTspice simulations (discussed in section 2.4).

2.1.2 Models

Regular SPICE models are composed of two main types of files, symbol ('.asy') and library ('.lib') files. A symbol file defines the visual metaphor used by the schematic capture part of LTspice to visualize the element and its ports. The library file contains the subcircuit definitions for models. A subcircuit defines how ports connect to each other using other components or subcircuits. A library file can contain the subcircuits and they can each be referenced individually by a separate symbol file.

LTspice allows these files to exist in two locations. The Model Library folder and in the directory of the circuit being currently used. In other words, whenever the

LTspice schematic needs to reference a symbol or library file, unless a path explicitly references a full path, LTspice checks the Model Library folders and the parent folder for the schematic. This makes it hard to continuously develop models in a repository while still being able to use the most up to date version. This issue is addressed in section 2.3.

2.2 Installating spice-daemon and qnn-spice

2.3 QNN SPICE

In a collaborative setup where SPICE models might be edited (either continuously or with infrequent small fixes,) having the ability to track the version of the models is important. One solution is to include a version string that the editor updates between revisions. Doing so however, does not handle merge conflicts natively and does not track file differences. From these requirements, the widely used version tracking software git can be used to track the file differences and users need to always re-download the latest version of the model.

One way to manage models in LTspice is to download them individually and place them in the library folder that contains all the base models. However, this can be tedious as it requires repeating the process for each model and your models can't be version tracked easily. An alternative approach is to store all the models in the same directory as the circuits and manually download each model as needed. This has the advantage of forcing the models to be version tracked in your repository, but it can result in a cluttered directory and multiple copies of each model on the system. Both methods don't guarantee that you are using the latest version of a model.

This is where qnn-spice comes in, MIT's Quantum Nanostructures and Nanofabrication group (QNN) has multiple repositories, each with multiple spice models. By having a single repository track every repository containing SPICE models, a single repository could track all the changes across every model produced by the QNN group. This single repository method takes advantage of git submodules, which track the head of each sub-repository. A helper update script pulls every submodule and creates symbolic links in LTspice's library folder to each model. The model library and symbol files to be included are specified in a YAML file – a human-readable data-serialization language.

The use of symbolic links means if a user edits the model in the cloned repository, LTspice sees the updated file. When the update script pulls the main and sub repositories, the previous symbolic links are deleted and new ones are made. The sub-repository structure is copied into two `qnn-spice` folders are created in the `sub/` and `sym/` subfolders of LTspice's library folder.

The YAML file maps the path of each file to include in the repositories to a destination path in the two `qnn-spice` subfolders based on their extension.

idea:
how to make a custom module grouping?

add details:
Note for updates: lib files automatically updated. Schematic Capture related files (asy) aren't updated.

add details:
Include library location? vs. repo loc.

2.4 spice-daemon assisted LTspice simulations

The main input for spice-daemon is a YAML file that defines simulation parameters, spice-daemon models, and toolkits. A YAML file can also be version tracked, allowing all parameterizations to be known by the host python script.

idea:
cool because u can save runs behind hidden menus and monte carlo things

LTspice generates multiple files during every run, including: a log file, a netlist file, an optional operating point analysis raw binary file, and a raw binary file that includes the code of the simulation that is run. spice-daemon tracks the edit history of the log file, a YAML specification file, and the circuit schematic using the WatchDog object.

Every Simulation object defines a couple of important File objects that are always present regardless of the user's setup for LTspice. Files are an extension of python's Path object that can additionally:

1. track edit timelines,
2. detect LTspice-native file encodings,
3. generate dictionaries from YAML files, and
4. read/write to files.

The WatchDog module periodically checks for edits on a Simulation's watch_files, a set of files that indicate a need to regenerate some (or all) spice-daemon produced files. For instance, if someone edits an attribute for a component in the YAML specification file, the component library file needs to be regenerated to reflect the change in the attribute.

spice-daemon's WatchDog can be called from the terminal or from a Python script. The terminal bash script suffices for basic usage of spice-daemon intended for non-experimental environments. When you call spiced from the terminal, spice-daemon launches the WatchDog that checks for periodic changes in files and runs the module and toolkit initialization and post-processing logic accordingly. spice-daemon needs to access simulation parameters - such as simulation time, steps, etc. - before LTspice starts solving the circuit. This is through spice-daemon's parameter acquisition and injection features.

Larger sweeps, use python!

idea:
Setting up a simulation, how is tran command and stepping handled

idea:
setting up a spice-daemon simulation

idea:
design a model together!!!

add fig:
Diagram of the listen and write files

add fig:
design a model together!!!

2.5 Handling Simulation Parameters

spice-daemon creates a `trancmd.txt` file in the `.spice-daemon-data` directory that contains the simulation time and steps parameters as well as other user (or daemon) specified simulation parameters. This file contains all the

finish this

2.6 Dynamic Models

LTspice components are parametrizable using a constant global parameter space that can be used when math expressions are being evaluated (such as the output voltage of a behavioral source or the inductance of an inductor). spice-daemon adds the ability to parameterize components beyond expressions by granting the ability to edit a PWL file and the netlist of the model between runs.

note on all libs being merged into one!

2.6.1 Lumped Element Transmission Lines and Tapers

One type of dynamic model that is incorporated into spice-daemon is lumped element transmission lines. Instead of using LTspice's built-in transmission line models (either the Lossless Transmission Lines (T elements) or the Lossy Transmission Lines

(O elements)), spice-daemon allows you to specify a variable discretization length lumped-element version.

The Lossless Transmission Line model has a bunch of limitations: it models only one propagation mode, does not support non-linear response functions, and does not model the DC behavior correctly. The Lossy Transmission Line also suffers from multiple caveats, it does not support frequency dependence for loss and it also does not support non-linear response functions.

For well-defined behavior with non-convolution based models, it is helpful to be able to run a lumped element model from within LTspice. However, this would involve laying down thousands of repeating chunks of elements manually. One use of dynamic models is generating a model that encodes variable length logic. In this method of programming a lumped element transmission line, the circuit topology can be affected by a single parameter in the configuration file (in this case number of nodes). This type of automation is not possible using LTspice's built-in parameterization.

This method of simulating a transmission line not only solves the issues introduced by the T and O models, but also give us the ability to simulate more complicated transmission lines. For instance, inductors on a transmission line can be non-linear, making simulating a superconducting transmission line more accurate. Other possibilities that aren't possible in the LTspcie environment are adding custom elements instead of a repeating sequence of inductors and capacitors allowing us to model JTWPAs of variable length easily.

Another extension to this one-to-many mapping for the transmission line can be extended to model lumped-element tapers. The transmission line models in LTspice work for lines with constant parameters (impedance, propagation velocity, loss, etc.).

With a lumped element model that is fully controlled by spice-daemon, changing the impedance of one port can map the inductance and capacitance of each finite element to a pair of values based on the taper geometry chosen. This adds another layer of abstraction where we can define an impedance-matched transmission line with a Klopfenstein geometry between two impedances Z_{in}, Z_{out}. If we change Z_{in}, the spice-daemon instance calculates new tapering parameters smoothly perturbing the impedance of the line from Z_{in} to Z_{out} and updates the library file for the taper element. When LTspice runs a new simulation, it pulls the latest lib file with the new impedance-matched taper. This non-uniform version of the transmission line is included as a separate taper model in spice-daemon that has additional logic pertaining to impedance-matching geometries.

idea:
PNR!

add fig:
pnr

2.6.2 Generating Noise

The ability to model different types of noise when designing superconducting nanowire based devices is important to yield realistic operation margins. Since we care about the non-linear transition between the superconducting and normal state, the addition of noise severly limits the operational margin and maximum performance of our device. In the example of using an SNSPD, optimal operation of the device requires it to be biased at $i_c - \epsilon$, where ϵ is a factor that correlates the magnitude of noise and the rate of dark counts (false switching events). For instance, if we assume a gaussian noise centered around $\mu = 0$ and a standard deviation (related to the magnitude of noise) of σ, an $\epsilon = 2\sigma$ means 2.28% of all detected counts are dark counts caused by this noise distribution. Alongside the noise constraint, the signal being measured (in the case of an SNSPD, the energy added by a photon) must always be greater than ϵ to cause a detection event.

The generation of uncorrelated noise is important in analysing logic devices. There are various methods for generating uncorrelated random numbers that have been developed over time, however, LTspice uses one global random number generator causing distributions to be inherently linked.

Weird noise distributions (shot noise, 1/f) why? further studying nanowires

why this is important for simulating nanowires

Noise analysis in LTspice is limited, especially on non-linear systems. Since superconducting electronics are highly non-linear, it is essential that we run all our analysis in the transient analysis mode (time-basis small-signal AC simulation). In this mode, we can use a voltage source with an LTspice native math command to generate noise such as `noise`, `random`, `gaussian`, `white`. However, these noise commands generate noise that is correlated amongst instances and is not gaussian in nature. One workaround that was discovered by the LTspice community was using 4 voltage sources each producing a shifted seed value for gaussian noise that guarantees that the seed does not overlap with the simulation time. By concatenating the outputs of the behavioral voltage sources, the central limit theorem makes the noise distribution behave more gaussian. Note that the number 4 was picked due to a trade-off between the complexity of generating and simulating that noise and how gaussian it is – the more sources there are, the more gaussian the noise distribution is.

One other workaround is to use PWL files. PWL files allow you to input piecewise linear functions into LTspice sources that are not necessarily behavioral. The PWL operation mode maps (time, value) pairs to a continuous output value based on the simulation time. However, if a simulation has N points it is solved at and you provide N noise points, then there is no extrapolation that occurs, provided these points are chosen at random from a gaussian distribution, then the voltage source will generate noise that has a gaussian probability density function. The process of re-creating this noise file, ensuring there are enough data points as timesteps in the simulation and that the noise data follows a certain distribution is tedious.

To solve this issue, spice-daemon can handle the creation of noise sources and their accompanying PWL files, abstracting them behind one symbol file. The user defines a noise source type (voltage or current), the noise distribution it should follow (Poisson, Gaussian, $1/f$, etc.) and distribution parameters (mean, standard deviation, etc.). The spice-daemon instance then generates a symbol file for a noise source that references a separate library sub-circuit for each noise instance. Each sub-circuit references a separate PWL file that encodes a list of (time, value) pairs generated in Python using NumPy. As a result, we know that the noise inputs to LTspice actually follow a specific distribution, and we can verify that the correct noise distribution is being simulated inside LTspice.

This method also allows us to easily have multiple non-correlated noise sources, as well as noise distributions that are not pseudo-gaussian or pseudo-uniform. The scaling and math required to generate LTspice native gaussian noise for multiple sources was unfeasible in terms of simulation time as well as calculating seed values off of the simulation time to guarantee there was no time correlation between the sources.

add fig: noise example

section on how to design model

2.6.3 Creating a new dynamic model – a coupled SNSPI

2.7 Arbitrary S$_{xy}$ models

Diagram of 2port and n-port Sxy networks
Inductor, Cap model for T.D.

2.7.1 2-port model

E.g. Tapers!

2.7.2 n-port model

E.g. Bias Tee

2.8 Post-processing using spice-daemon Toolkits

While LTspice transient simulations are sufficient for characterizing the time behavior of a superconducting circuit, it is also essential for a good simulation environment to have the ability to post-process the data in a meaningful way, producing plots that are familiar to those seen in a lab setting. The data generated by a SPICE simulation is a raw file that contains the node voltages as a function of time, however, we might care about things that are functions of bias currents, functions of temperature, noise statistics, and other factors where time is not the independent variable. One example of a widely used metric for SNSPDs is a PCR curve (Photon Count Rate), where the y-axis is the count rate (how many hotspots form on the SNSPD) and the x-axis is the bias current.

Creating a PCR curve requires creating a plot where the x-axis is the bias current used for multiple periods of simulation time. The y-axis would contain the sum of spikes that the SNSPD has over each period of time. This sort of rate measurement cannot be performed in LTspice without a complicated secondary circuit that is not feasible. Using a separate circuit within the simulation is also detrimental to the simulation performance as it can introduce a non-linear coupling between the counting circuit and the nanowire circuit. This coupling can cause the SPICE solver to take more steps near counts and cause the nanowire model to misbehave, this will be further discussed in Section 3.

By hooking a post-processing function to the WatchDog class, spice-daemon can produce a plot using that function at the end of each simulation. Using PyLTSpice's LTSpiceRawRead function, the contents of LTspice's RAW output file can be dumped into a Python object. This object separates out each individual trace (voltage at a node, current through a component, time, etc.) from each simulation run (produced by the .step SPICE directive) as separate waves. The post-processor can then perform any kind of analysis needed from one or multiple simulations just as you would if you exported the simulation data from LTspice and post-processed it in NumPy. spice-daemon can then update the plot automatically after each simulation run is completed.

Packages like PyLTSpice can be used to manually run a script, however, spice-daemon provides the ability to generate the plots after each simulation invocation automatically. It also gives a standard way of building post-processing blocks that are directly related and parametrized by the circuit schematic. Since spice-daemon has access to the global variables used in the SPICE simulation, it can also use them in its computation. Some toolkits can also be made to be component specific. By instantiating a PCR object for instance, spice-daemon automatically infers that the

topic sentence doesn't match paragraph

nanowire circuit is of interest for this object and allows for more compact toolkit specifications.

2.8.1 IV curves

IV (current-voltage) curves are an important tool for characterizing the electrical characteristics of superconducting devices. By plotting the current passing through the device as a function of the applied voltage, IV curves provide valuable information about the device's behavior: the critical current and normal resistance. A common way to confirm that the fabrication process was successful is to measure the material's electrical properties, using an IV curve, and compare them to the expected values. This form of process verification and quality analysis requires an accurate comparison to provide a useful metric. Being able to simulate the curve for a sample given the desired geometry and comparing it to the actual fabricated sample provides useful insight into what could have gone wrong during fabrication.

Given the standardization of IV curves, a good simulation environment for superconducting nanowire based devices should be able to generate IV plots readily. While a pseudo-DC simulation for the IV curve can be performed using a slow ramp of bias current, it is bandwidth limited, not an accurate representation of the inductor physics, and takes much longer to perform than doing parallel measurements for the IV curve of a device. Since spice-daemon has the ability to control models and post-analyze signals, we are able to perform an IV curve measurement through the spice-daemon toolkits framework. By running multiple simulations of a PULSE voltage bias source across the nanowire we can accurately get the DC operating point solution for the nanowire and reconstruct the measured current and voltage across the nanowire from each simulation into an IV curve. The PULSE form is important for two reasons: (1) simulating hysteresis and (2) the nanowire model is not DC-compatible (discussed in section 3).

example of IV curve obtained from sd

2.8.2 Power Spectral Density

Power Spectral Density (PSD) is a useful tool for designing superconducting single photon detectors. It is a measure of the power present in a signal as a function of frequency and can provide valuable information about the frequency content of the signal. This can be particularly important in the design of superconducting single photon detectors, as the PSD can reveal the level of noise present in the system and help identify any potential sources of noise that may impact the performance of the detector. The spice-daemon toolkit can be used to generate PSD plots, making it a valuable tool for analyzing the frequency content of signals and optimizing the performance of superconducting circuits.

LTspice cannot generate a PSD plot natively and there is probably no way of doing it in a SPICE environment without any kind of post-processing or additional features not inside LTspice. Note that LTspice allows users to plot the Fast Fourier transform (FFT) of a signal but not the PSD. spice-daemon handles that by taking the FFT in Python and plotting it either in Python's plotting package matplotlib or by injecting a new node into the RAW output.

example of a PSD to replicate noise input

2.8.3 Output Methods

Post-processing modules need to somehow display the signal back to the user. spice-daemon handles this in one of two ways. The preferred method of displaying a plot uses matplotlib to spawn a separate window that contains all the post-processing plots. The other optional way of doing it is by relying on PyLTSpice's LTSpiceRawWrite to write new waveforms as specified by the post-processing module. This method is less preferred as it allows for less customization for plots, it only supports a square plotting window, shares the x-axis, and doesn't allow for additional markers. As such, unless explicitly specified, the matplotlib backend is used by default. The matplotlib plotting backend also has multiple choices of backends and allows for integration with jupyter notebooks.

Post-processing modules also allow for saving data in a csv and/or Python objects. This is useful when using the python spice-daemon package, as this allows for simulation

add details: finish thought...

2.9 spice-tikz?

3 Model Stability

3.1 Integration in SPICE

ltspice-diff-post: https://www.analog.com/en/technical-articles/spice-differentiation.html

spice-book: https://designers-guide.org/analysis/dg-spice/index.html

SPICE implementations in general rely on second-order integration, namely trapezoidal and Gear integration, each with their own benefits. LTspice allows the use to pick between 4 options: trapezoidal, Gear, (1st Order) Backward Euler and a proprietary modified trapezoidal method. In general, Backwards is the most stable and least accurate, followed by Gear integration.

Trapezoidal integration is more accurate and faster than the other options but introduces a ringing numerical artifact that occurs on adjacent timesteps on stiff systems. Ringing is dampened by the gear method which means this numerical artifact is mostly eliminated, however, the gear method dampens most ringing. We typically care about oscillatory behavior in nanowires that can be filtered by the gear method and as a result, we choose to use the trapezoidal method. LTspice's default integration method is a proprietary modified version of trapezoidal integration that cancels out the trapezoidal ringing introduced by the regular implementation without numerical dampening **??**.

add ref: ltspice-diff-post by Mike E.

LC Tank in trapz vs gear showing decay. Maybe include dampened trapz? and trapz oscillations?

section on DC being wrong. Start simulation from PULSE

3.1.1 Dependent Sources

idea:
There are multiple versions of dependent sources in LTspice. b-sources etc.

Dependent sources are a powerful model in SPICE software that allows for the dependence of current and voltage outputs to other node voltages and currents. Along with the multiple types of dependent sources that are supported by SPICE (e-, f-, g- and h-sources), LTspice supports an additional type of dependent source called the behavioral source (b-source). Behavioral sources are powerful (they can mimic the behavior of any other source) and can have multiple inputs as opposed to the other sources' dependence on one value only. B-sources also allow for the use of

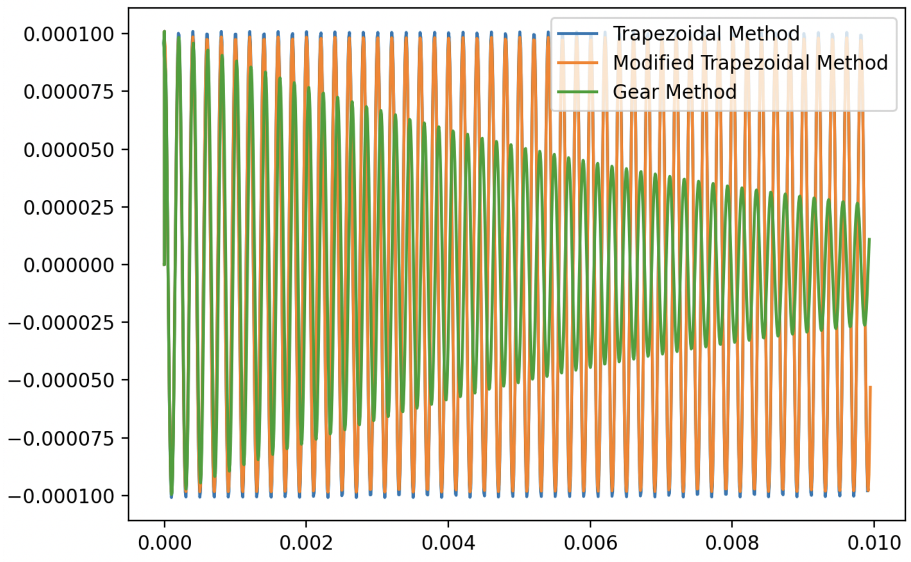


Figure 3:

add fig:
change caption and better plot! include circuit diagram. swap it out with nanowire.

arbitrary maths functions that allow them to compute non-trivial expressions, such as time integrals, derivatives and modulos. B-sources can use the outputs of functions defined using the .func directive as a current, voltage, resistance and power outputs (resistance and power aren't well documented).

For example, one can construct a gaussian pulse source using dependent sources by defining these two functions:

```
.func mod(x, y) x-floor(x/y)*y
.func gaussian_time(a, b, c, M) a*exp(-square((mod(time, M)-b)/c)/2)
```

The b-source expression can be V=gaussian_time(0.1, 100f, 10f, 200f) and that would output a gaussian pulse with magnitude $0.1V$ with peaks spaced out by 100fs with a standard deviation of 10fs.

add fig:

figure of bsource, filename gaussian_pulses_bsource.asc

3.2 Stability in Transient Simulations

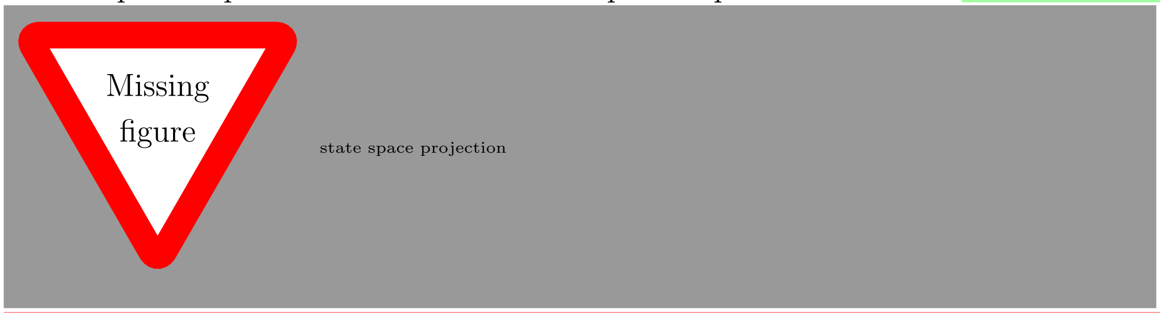
add ref:
end of page 201 in the book.

Stability of a finite element method is intimately related to the consistency and convergence of the method through the Dahlquist Equivalence Theorem. One result of this for non-linear systems, such as the nanowire model, is their solution should be smooth as you decrease the timestep. As in, there must exist a timestep Δt for which all timesteps $< \Delta t$ the method gives a result bounded around that of using the timestep Δt. Transient simulations, which are a type of continuation method parametrized with time, often use straightforward convergence correction. In this method, the timestep for continuous signals is decreased until convergence is achieved, which is guaranteed to occur for continuous signals.

add ref:
spice-book

One way of visualizing solving a continuous system using a finite element method is via corrections as projections. For instance, solving for the final state $u(T)$ for a circuit C at a time T transforms $u(0) \to u(T)$ smoothly when continuous. However, when a finite method is used with coarse discretizations, the method steps around this continuous evolution. Overshoots due to the coarseness could exist outside the state-space and solution trajectory but are corrected for. These corrections (decreasing the timestep and adding gmin capacitances) can be thought of as projections back into the subspace of possible solutions. The subspace of possible solutions

idea:
epsilon ball around state



add fig:
nw sim state space

3.2.1 Stability of the Nanowire Model

Boolean state non-linearities are integral to optimizing and training Neural Networks, and as a result are a well studied concept. A common way of solving this issue is smoothening out the change by modeling the boolean state as a continuous state transition with a smooth interpolating function, such as a sigmoid function. In nanowires, we particularly care about smoothness of state transition over time while the transition dependence is on current (which is also a function of time). Macroscopically, this state transition involves a chain reaction and can be modelled as a smooth ramp up (the hotspot thermal growth is a smooth change). This is how the hotspot integrator in the existing nanowire model handles the non-linear transition into the resistive state.

The continuous nonlinearity exhibited in nanowires is important for all sorts of superconducting topologies including Josephson traveling-wave parametric amplifier. As the network size increases

3.2.2 Relative Tolerance for the Nanowire Model

idea:

Nanowire operating region is 1e-6 reltol. Reltol vs nanowire-C tank!

3.3 Malicious Circuits

Why? How? (tline timestepping with half res?)

3.3.1 Proof of Equivalence to Stability

3.4 Improving the Nanowire Model

3.4.1 Current nanowire model

Layout of in-depth model

3.4.2 Stability of the original nanowire model

3.4.3 Different Integrator

3.4.4 1 Element Models

3.4.5 0 Resistance Models

4 Efficient Simulation

Julia simulator

4.1 Tline Model

4.1.1 Equivalent Circuit

4.1.2 Kernel

4.1.3 GPU

4.2 Harmonic Balance

4.2.1 TD Assist

4.3 Device Symmetries

4.4 Coupling Diff. Eq. (or Thermal Model?)

Maybe sections should be separate. Phonon-Electron transport.

Complex electric coupling

4.4.1 TDC

4.4.2 SNSPI coupling

4.5 Precomputation

4.6 ML Optimization

4.6.1 Symbolic Solver

4.6.2 Tapers

Note on resistive groups? -¿ Sonnet?

4.6.3 Differentiable Simulator

4.6.4 Inverse design

4.6.5 Monte Carlo Simulation

References

[1] Di Zhu. “Microwave engineering in superconducting nanowires for single-photon detection”. eng. Accepted: 2020-03-09T18:59:00Z. Thesis. Massachusetts Institute of Technology, 2019. URL: https://dspace.mit.edu/handle/1721.1/124123 (visited on 12/14/2022).