Lecture 3a:

Specific-factor Model

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C181 – International Trade

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- **CHAPTER 2: Ricardian model:**
  - Only one factor of production: labor
  - Labor is mobile across sectors
  - ➔ Everyone gains from trade

The next model relaxes these assumptions:

- **CHAPTER 3: But what if:**
  - We have more than one factor of production?
  - What if these factors are NOT mobile across sectors?
  - ➔ Then there may be losers and winners!
    (unequal effects of globalization)

- **CHAPTER 3: Road map:**
  - Setting up the specific factor model
  - Change in production and employment
  - Aggregate gains from trade > 0
  - Effect on labor wages
  - Effect on returns to K and Land

inequality

1 Setup of Factor-Specific Model

Setup

- Two countries: Home and Foreign.
- Two sectors: Manufacturing and Agriculture

- Manufacturing uses labor and **capital**
- Agriculture uses labor and **land**.

three factors: L, K, T

"terrain"

1 Setup of Factor-Specific Model

Setup

- Two countries: Home and Foreign.
- Two sectors: Manufacturing and Agriculture

- Manufacturing uses labor and **capital**
- Agriculture uses labor and **land**.

- *Diminishing returns* for labor in each industry:

  The marginal product of labor declines if the amount of labor used in the industry increases.

1 Setup of Factor-Specific Model

Alternative interpretation

NOTE:

We can also use the same model and interpret "capital" and "land" as fixed labor:

Capital: equivalent to Labor that is stuck in manufacturing

Land: equivalent to Labor that is stuck in Agriculture

Labor: Labor that is mobile across industries

→ Three types of labor depending on its mobility

(for the lecture, we'll keep talking about capital and land as it's easier to follow)

1 Setup of Factor-Specific Model

Econ 100A

Production function with Constant Returns to Scale:

- Manufacturing output: $Y = F(K, L)$

such that: $F(\lambda K, \lambda L) = \lambda F(K, L)$

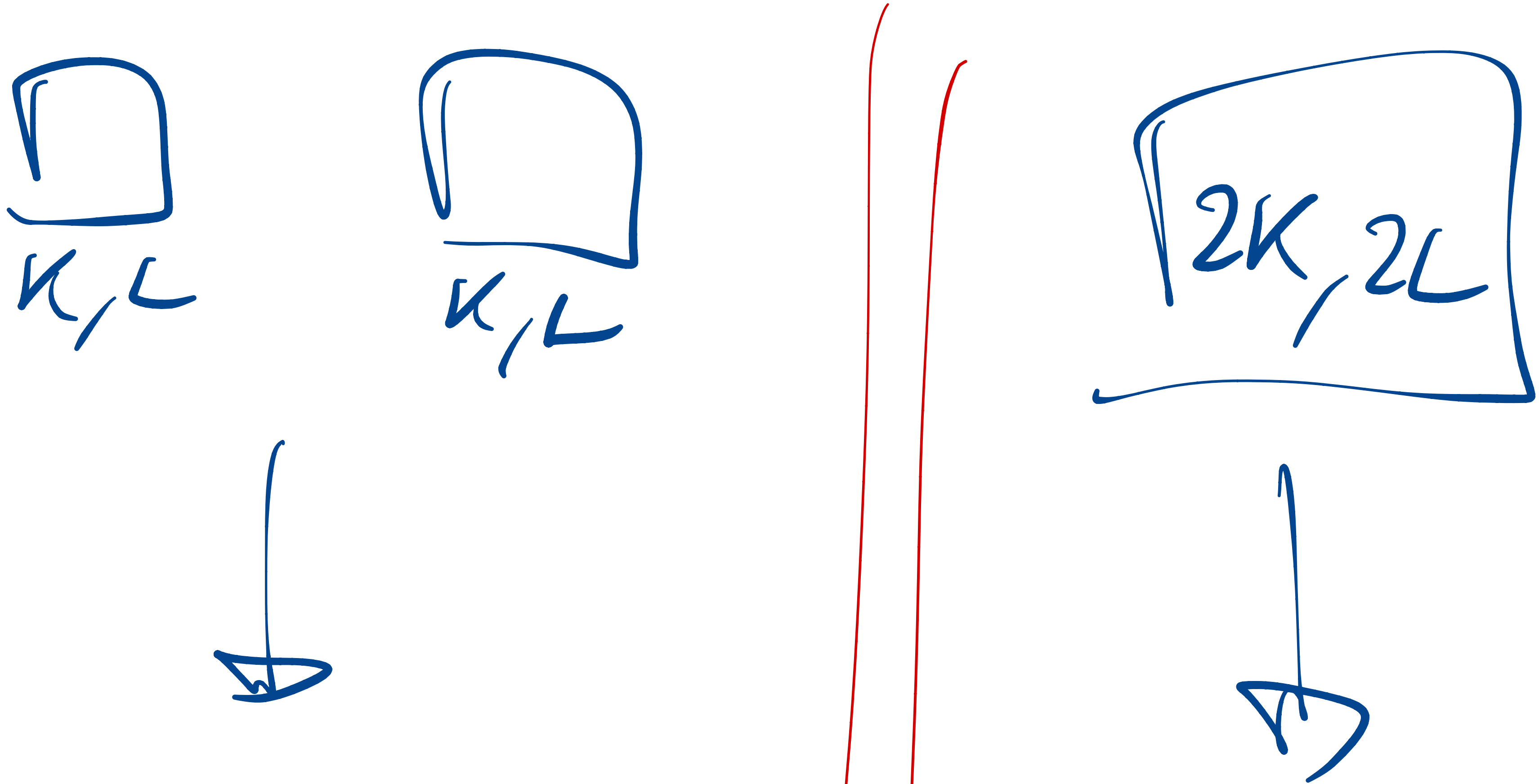
$$\underbrace{F(2K, 2L)}_{Y'} = 2\underbrace{F(K, L)}_{}$$

$$= 2 \times Y$$

CRS $\boxed{F(2K, 2L) = 2F(K, L)}$

$$F(K, 2L) < F(2K, 2L)$$

$$\Rightarrow F(K, 2L) < 2F(K, L)$$



$$2F(K,L) \neq F(2K,2L)$$

1 Setup of Factor-Specific Model

Production function with Constant Returns to Scale:

- Manufacturing output: $Y = F(K, L)$

  such that: $F(\lambda K, \lambda L) = \lambda F(K, L)$

- This implies *decreasing* returns to scale if we focus on one input:

  $$F(K, \lambda L) < F(\lambda K, \lambda L)$$

  $$\rightarrow \quad F(K, \lambda L) < \lambda F(K, L)$$

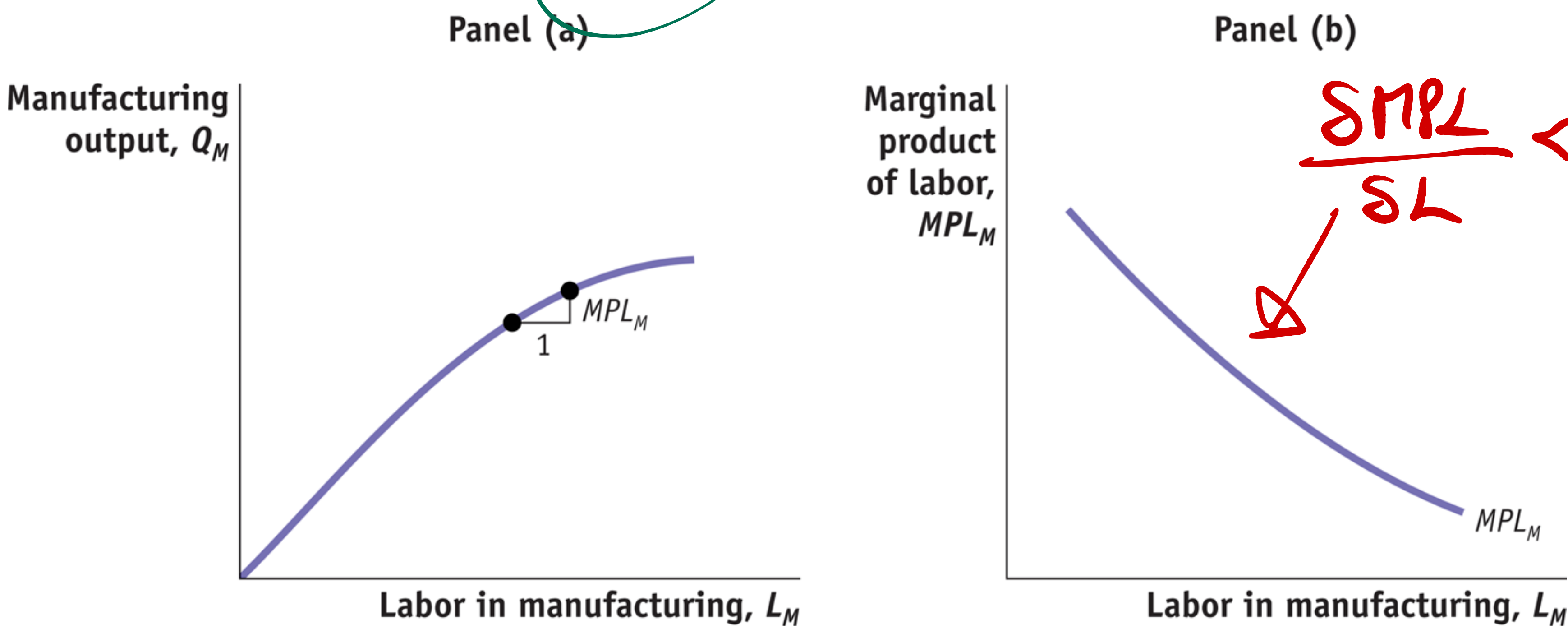
- In each industry: $\dfrac{\partial MPL}{\partial L} < 0$

1 Setup of Factor-Specific Model

Keeping
K fixed
T fixed

Diminishing returns for labor in each industry:

$\frac{\delta MPL}{\delta L} < 0$



(same for Agriculture: MPL decreases with production)

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{2/3}$

Cobb Douglas production functions

parameters for productivity

add-up $\Longrightarrow$ constant returns to scale

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{2/3}$

$MPL_M = \frac{\delta Y_M}{\delta L_M} = \frac{2}{3} a_M K^{1/3} L_M^{\frac{2}{3}-1}$

→ Marginal product of Labor:

- MPL in Manufactures: $MPL_M = \frac{2}{3} a_M (K/L_M)^{1/3}$

$\Uparrow K/L_M \Rightarrow \Uparrow MPL_M$

- MPL in Agriculture: $MPL_A = \frac{2}{3} a_A (T/L_A)^{1/3}$

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{\ 2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{\ 2/3}$

$MPL_M = \frac{\delta Y_M}{\delta L_M}$

→ Marginal product of Labor:

- MPL in Manufactures: $MPL_M = \frac{2}{3} a_M \left(K/L_M\right)^{1/3}$

  Increases with K/L_M

- MPL in Agriculture: $MPL_A = \frac{2}{3} a_A \left(T/L_A\right)^{1/3}$

  Increases with T/L_A

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{\;2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{\;2/3}$

→ Marginal product of Capital and Land:

- MPK in Manufactures: $MPK = \frac{1}{3} a_M \left(L_M / K\right)^{2/3}$
- MPT in Agriculture: $MPT = \frac{1}{3} a_A \left(L_A / T\right)^{2/3}$

1 Setup of Factor-Specific Model

Example of production function:

- Manufactures: $Y_M = a_M K^{1/3} L_M^{\ 2/3}$
- Agriculture: $Y_A = a_A T^{1/3} L_A^{\ 2/3}$

→ Marginal product of Capital and Land:

- MPK in Manufactures: $MPK = \frac{1}{3} a_M \left(L_M / K\right)^{2/3}$

  Decreases with K/L_M

- MPT in Agriculture: $MPT = \frac{1}{3} a_A \left(L_A / T\right)^{2/3}$

  Decreases with T/L_A

opposite of effect on $MPL_{M,A}$

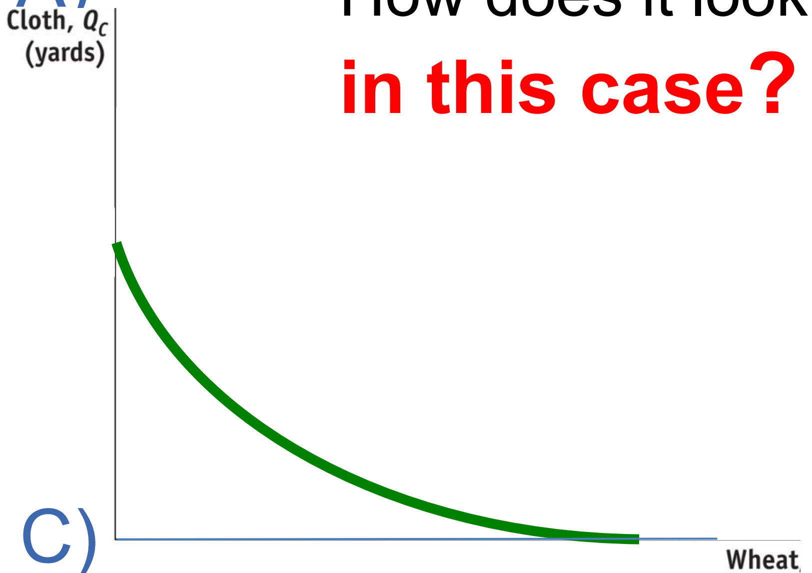
Production Possibility Frontier:

How does it look like

in this case?

Hint:
$$\text{slope} = \frac{MPL}{MPL}$$

A)



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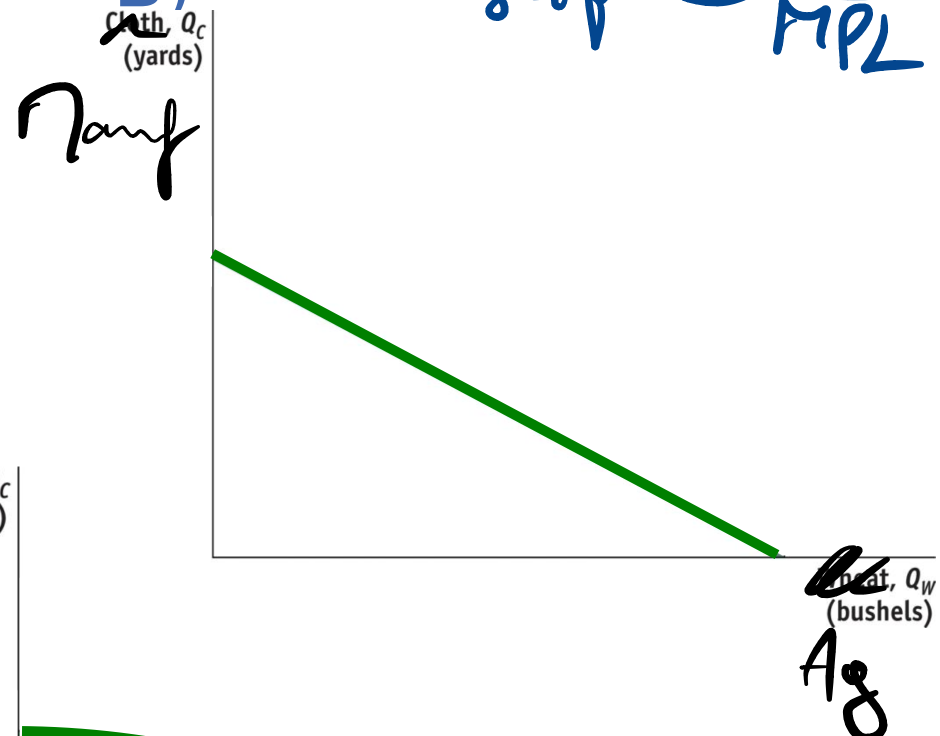
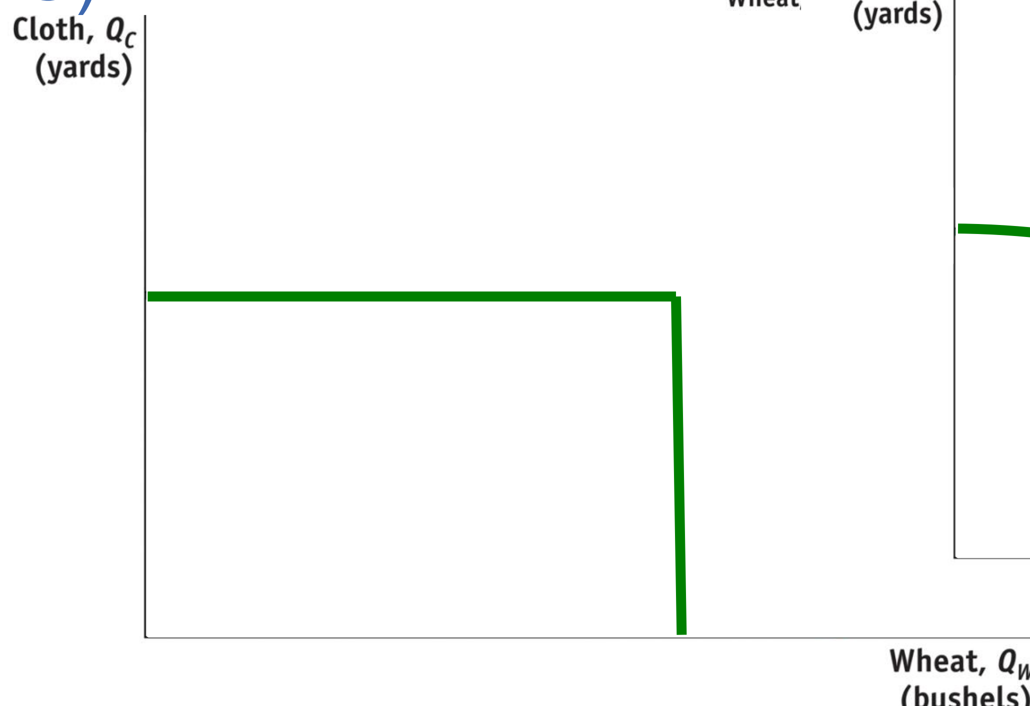


C)



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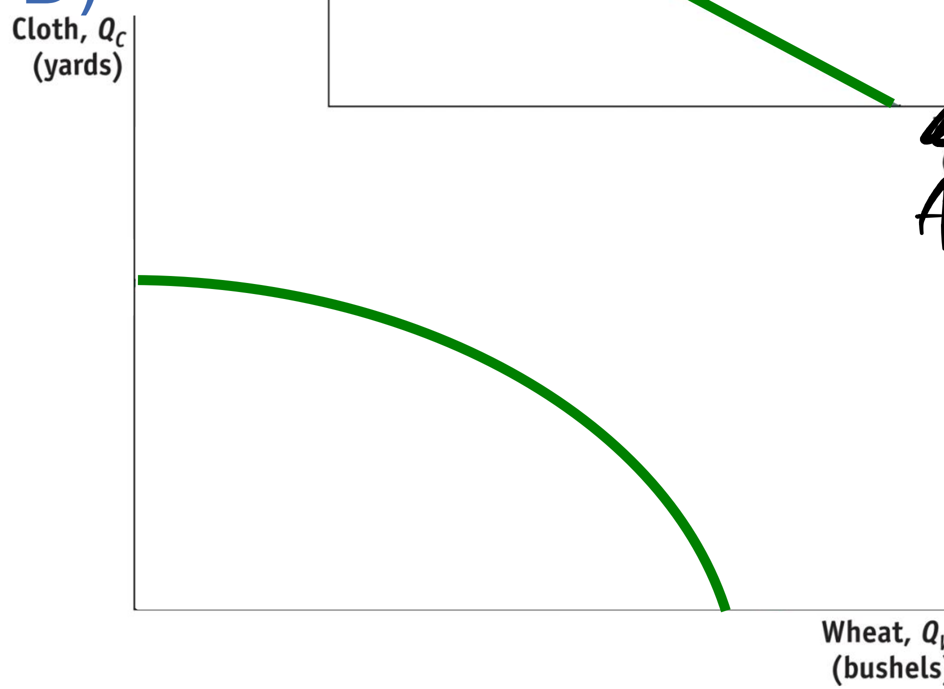
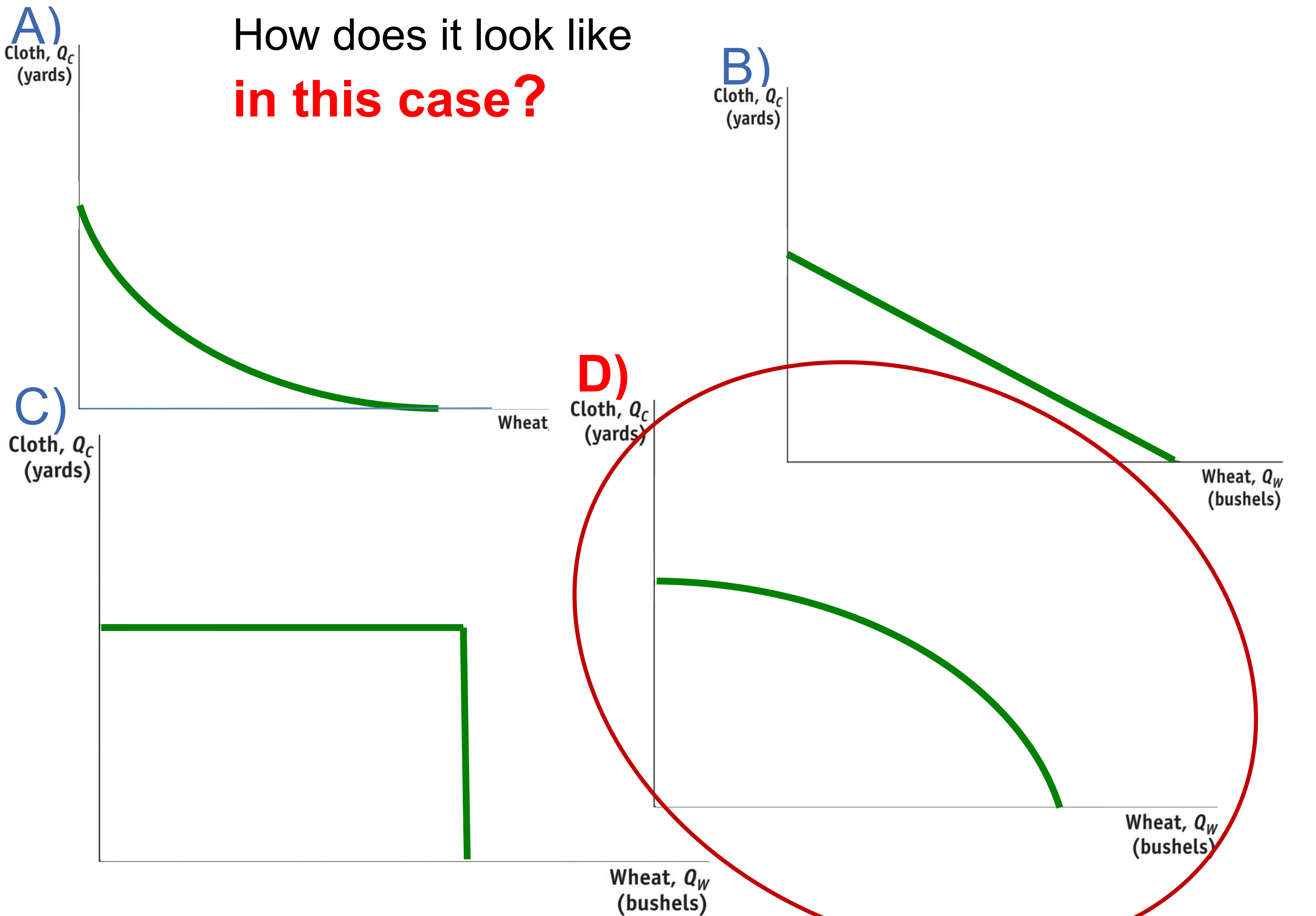


Production Possibility Frontier:

How does it look like **in this case?**



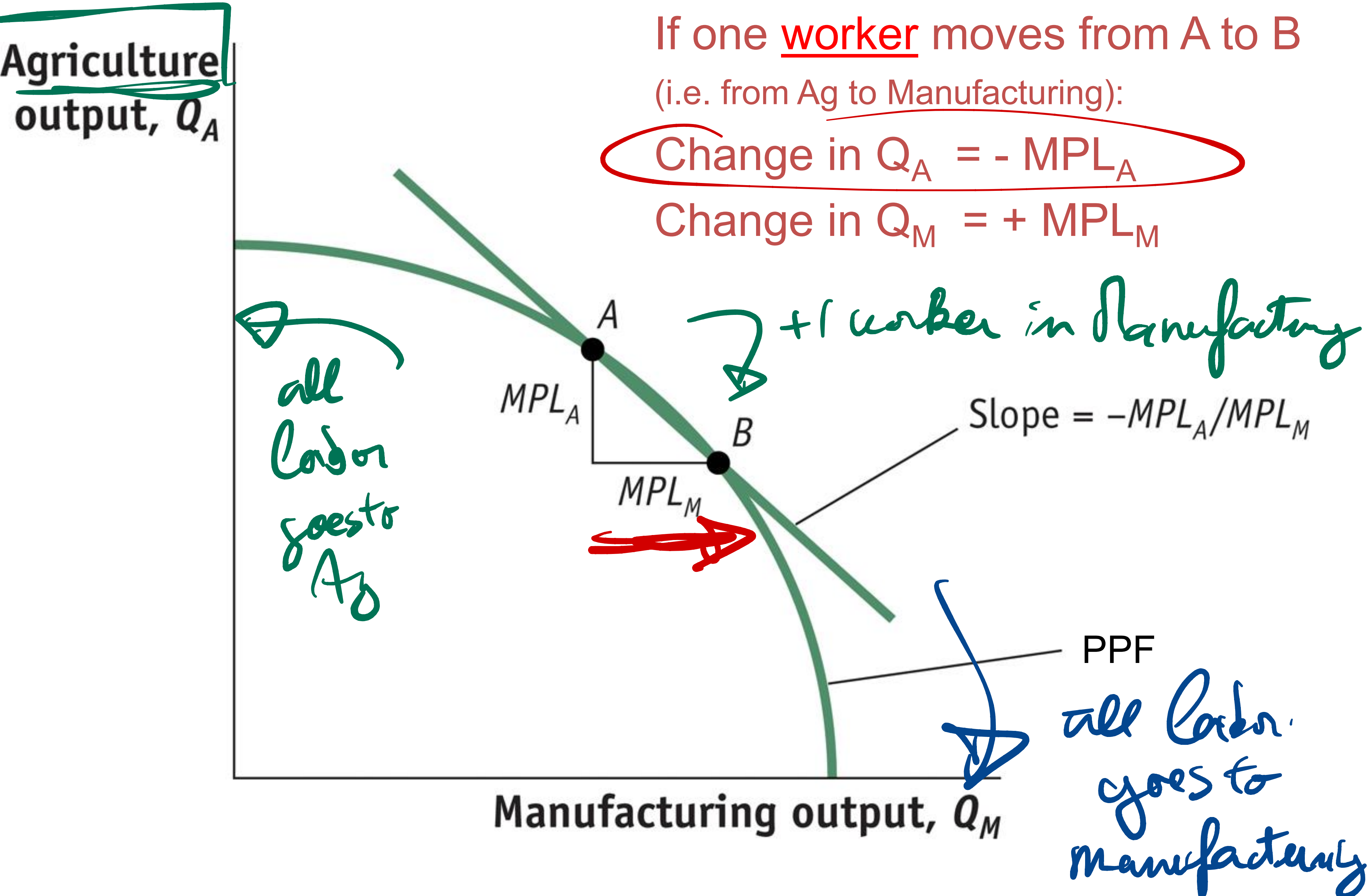
Slope of PPF reflects the opportunity cost of manuf. output:

If one worker moves from A to B

(i.e. from Ag to Manufacturing):

Change in Q_A = - MPL_A

Change in Q_M = + MPL_M



Slope of PPF reflects the opportunity cost of manuf. output:

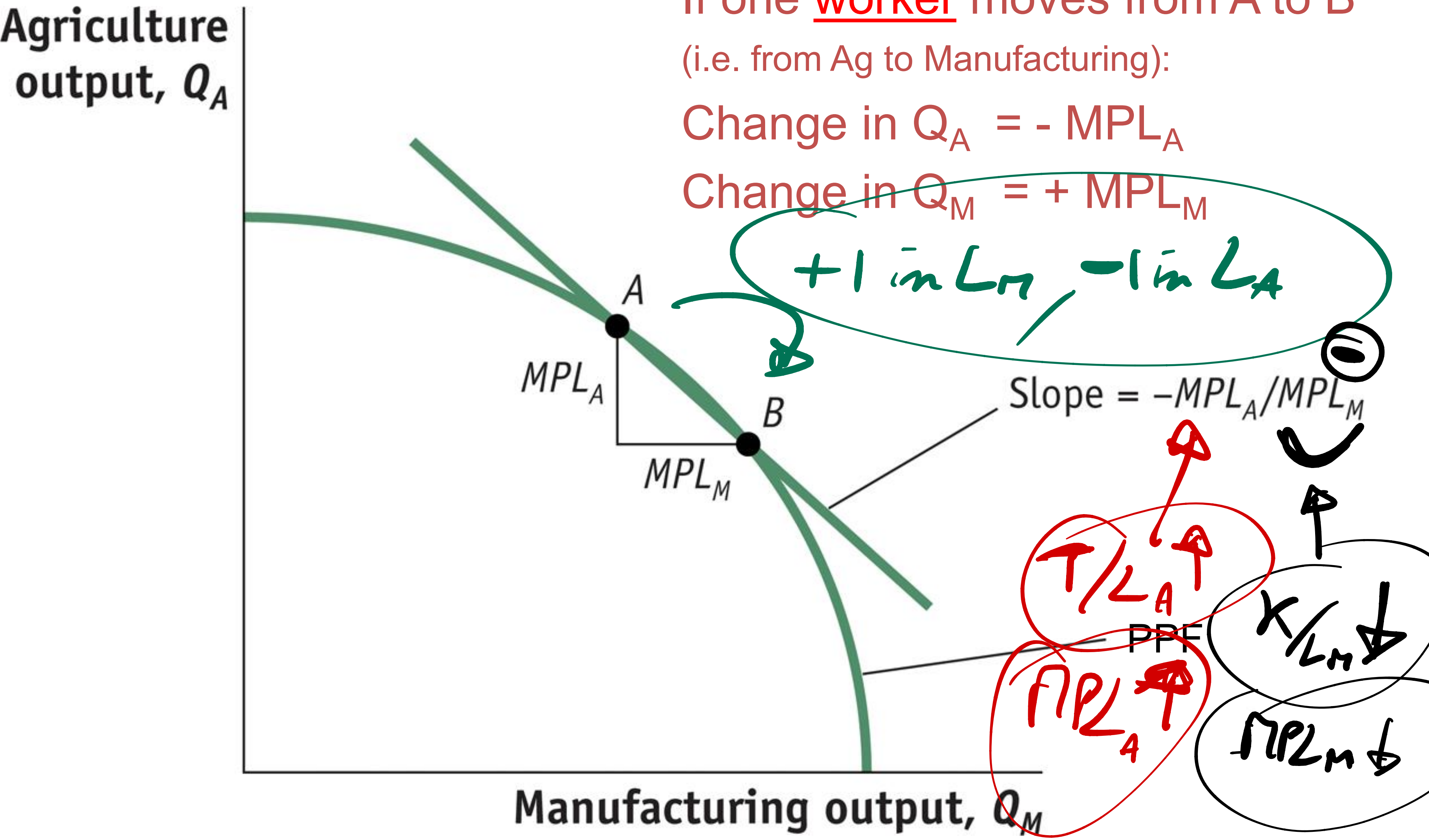
If one worker moves from A to B

(i.e. from Ag to Manufacturing):

Change in Q_A = - MPL_A

Change in Q_M = + MPL_M

+1 in L_M, -1 in L_A

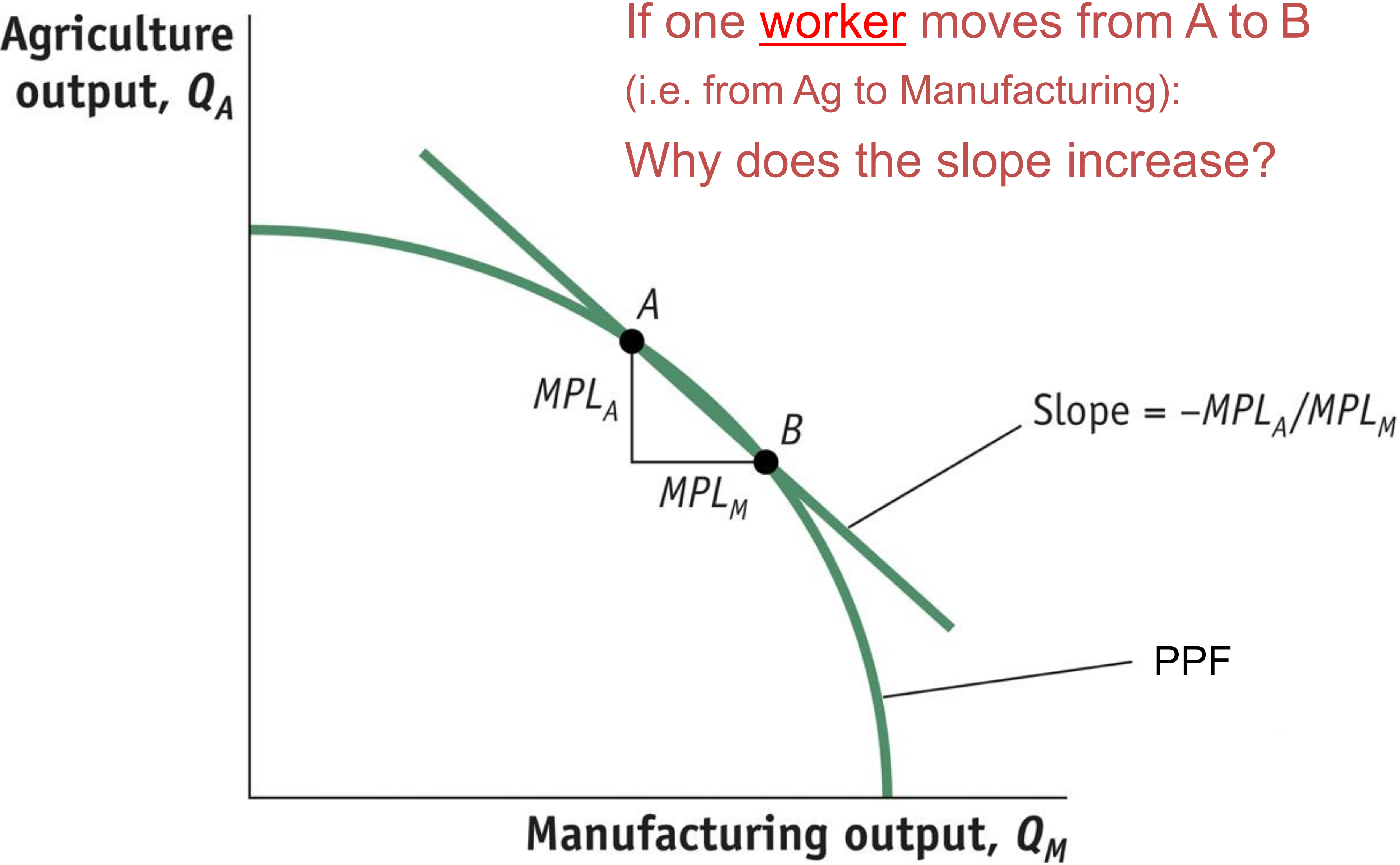


Slope of PPF reflects the opportunity cost of manuf. output:

If one <u>worker</u> moves from A to B

(i.e. from Ag to Manufacturing):

Why does the slope increase?

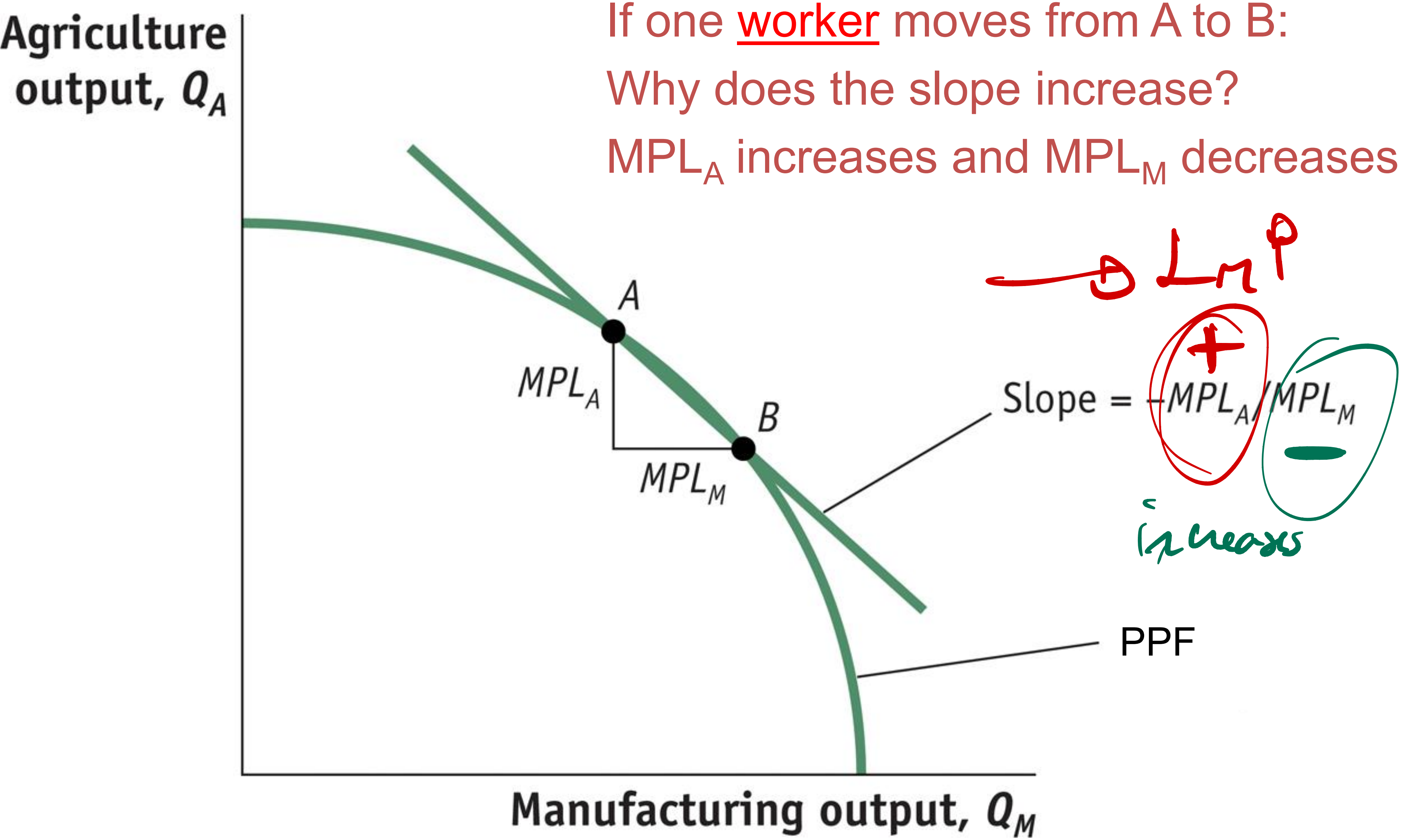


Slope of PPF reflects the opportunity cost of manuf. output:

If one worker moves from A to B:

Why does the slope increase?

MPL_A increases and MPL_M decreases



1 Setup of Factor-Specific Model

Slope of PPF

Why does the slope increase from point A to B?

- Slope equals MPL_A/MPL_M
- As L_A decreases, MPL_A increases
- As L_M increases, MPL_M decreases

➔ Hence the ratio increases!

1 Setup of Factor-Specific Model

Labor market and relative prices

- Labor is mobile across sectors
- Hence **wages** are equalized:

$$W = P_M \cdot MPL_M$$

$$W = P_A \cdot MPL_A$$

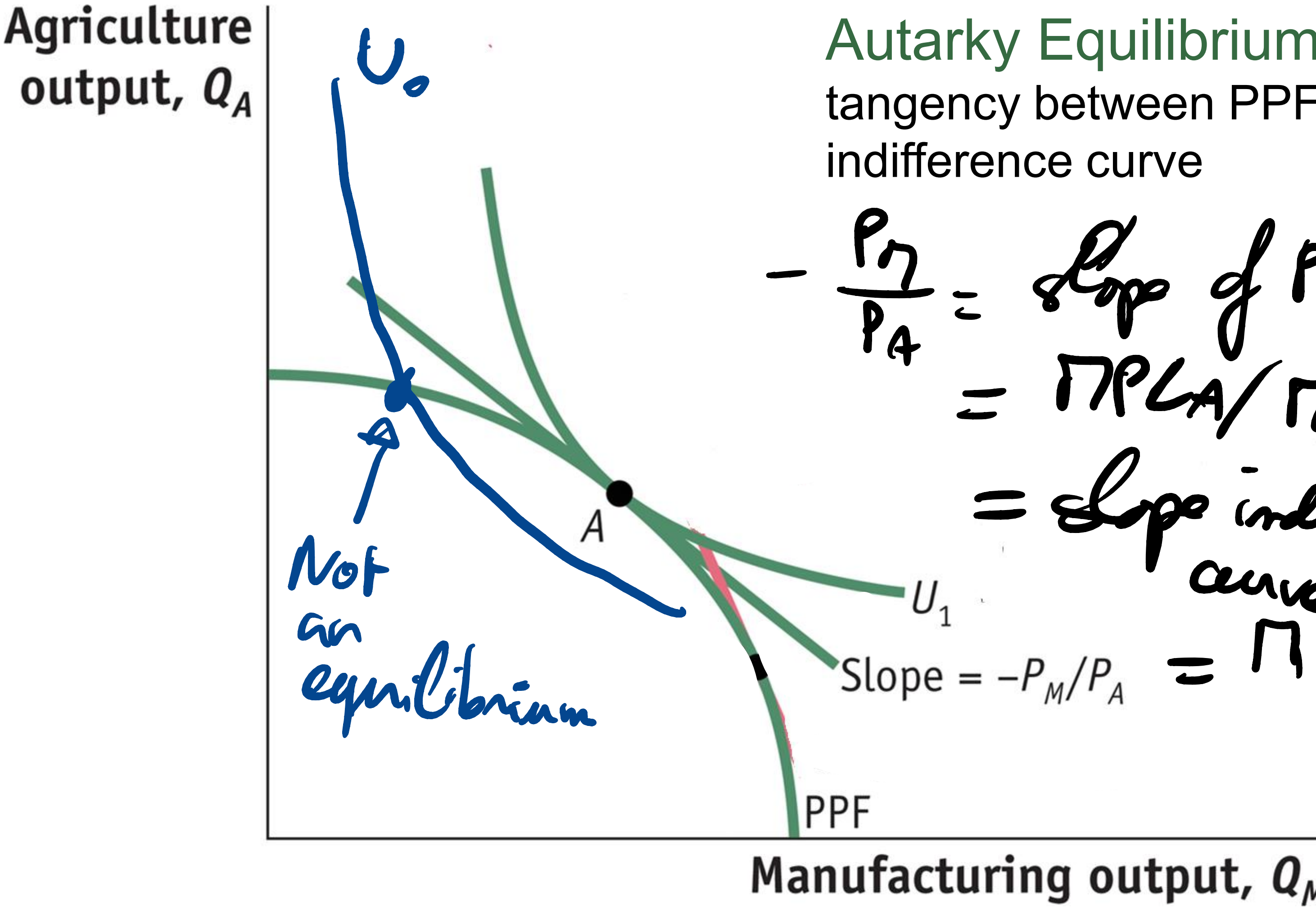
- And should be the same across sectors. Hence:

$$P_M / P_A = MPL_A / MPL_M$$

= Slope of the PPF

1 Setup of Factor-Specific Model

Equilibrium in Autarky:



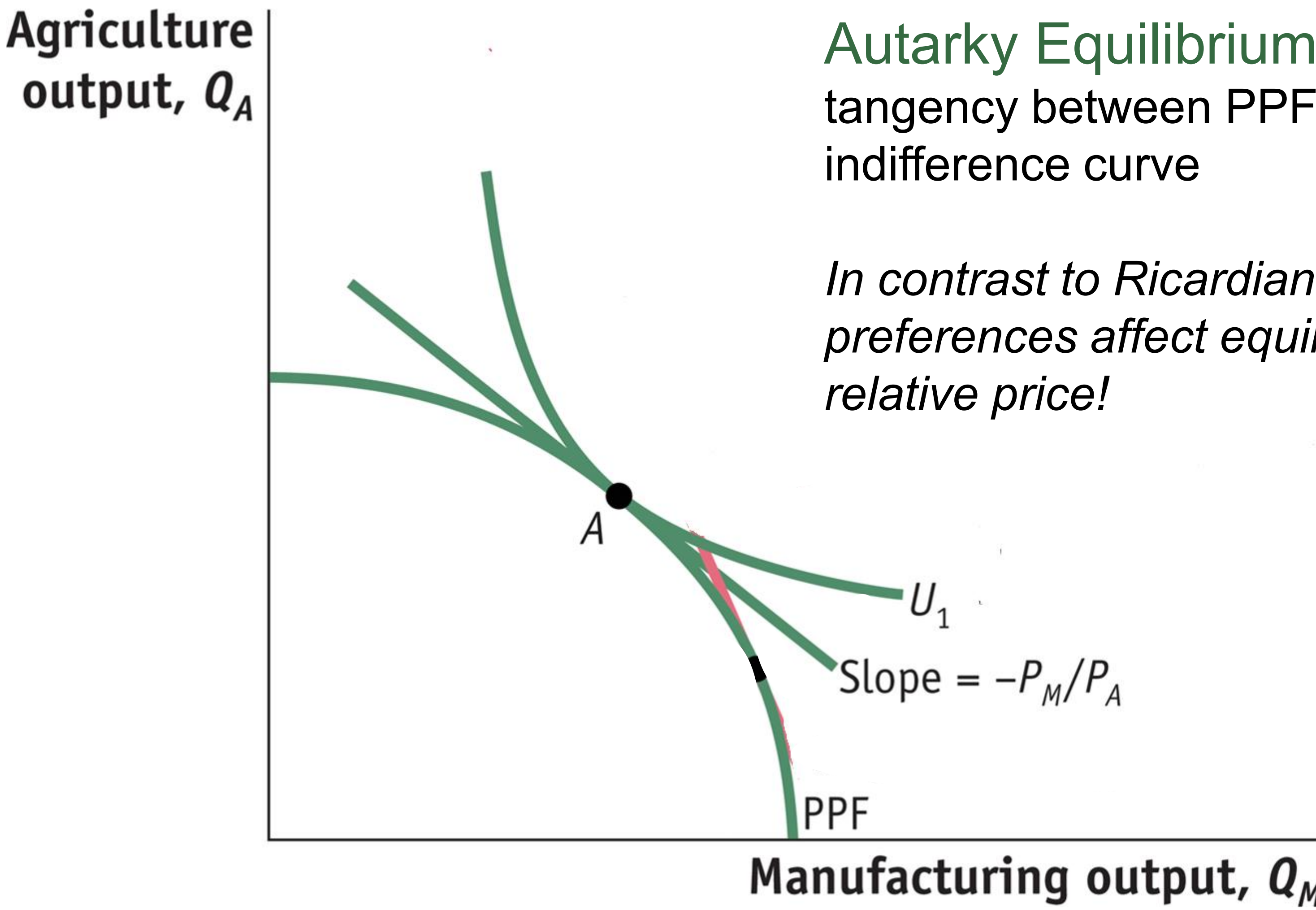
Autarky Equilibrium:

tangency between PPF and indifference curve

$$-\frac{P_M}{P_A} = \text{slope of PPF} = MPL_A/MPL_M = \text{slope indifference curve} = MU_M/MU_A$$

1 Setup of Factor-Specific Model

Equilibrium in Autarky:



Autarky Equilibrium:

tangency between PPF and indifference curve

In contrast to Ricardian model, preferences affect equilibrium relative price!

- **CHAPTER 3: Road map:**

  - Setting up the specific factor model

  → Change in production and employment

  - Aggregate gains from trade

  - Effect on labor wages

  - Effect on returns to K and Land

2 Effect of Trade on production

The Foreign Country

- Let us assume that Home has a comparative advantage in manufacturing

⬄ *Equivalent to assuming that the Home no-trade relative price of manufacturing is lower than Foreign rel. price:*

$$(P_M / P_A) < (P^*_M / P^*_A).$$

New world price?

2 Effect of Trade on production

The Foreign Country

- Let us assume that Home has a comparative advantage in manufacturing

⬄ *Equivalent to assuming that the Home no-trade relative price of manufacturing is lower than Foreign rel. price:*

$$(P_M / P_A) < (P^*_M / P^*_A).$$

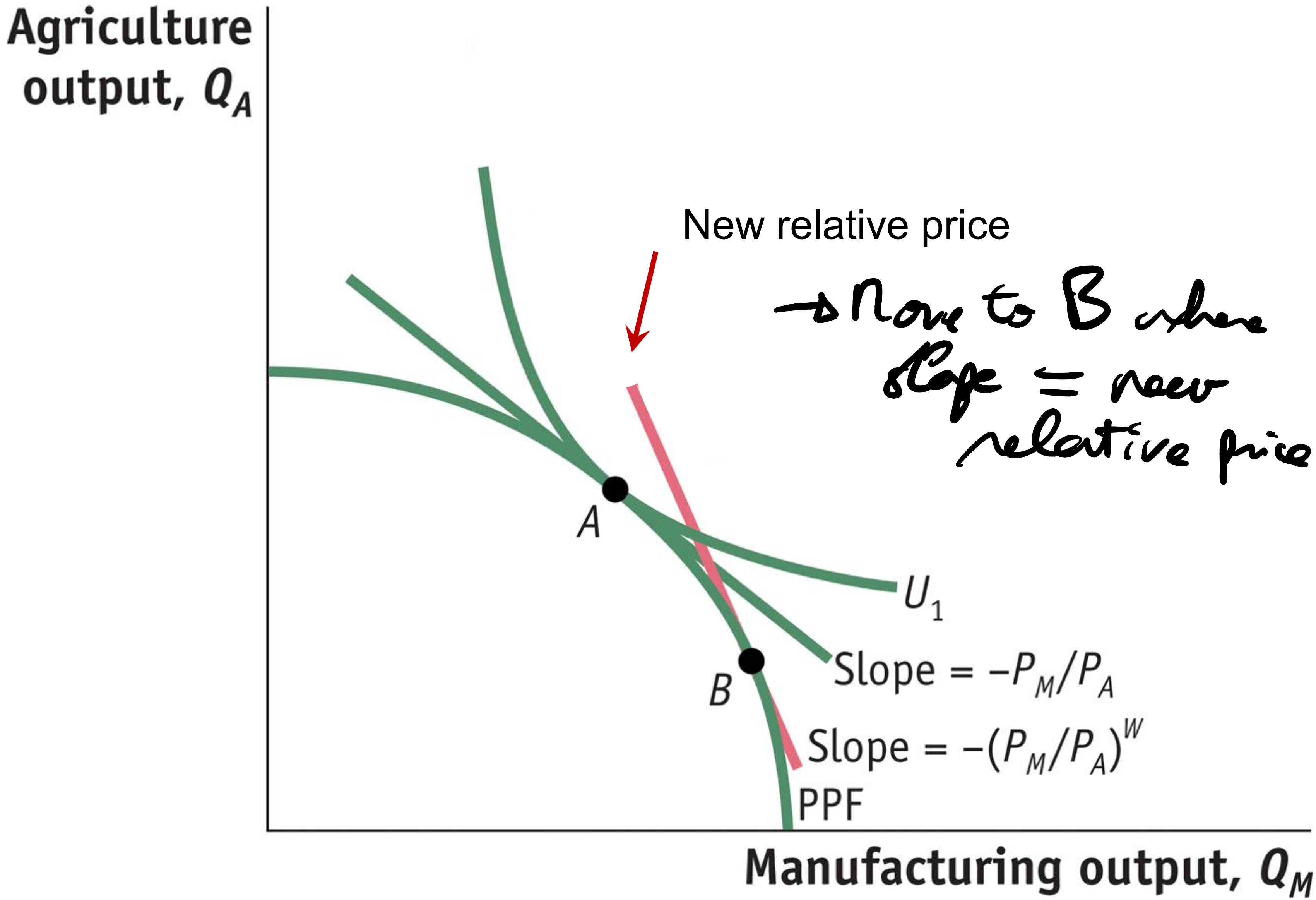
Now, Trade: $P_M / P_A \uparrow$

New world price:

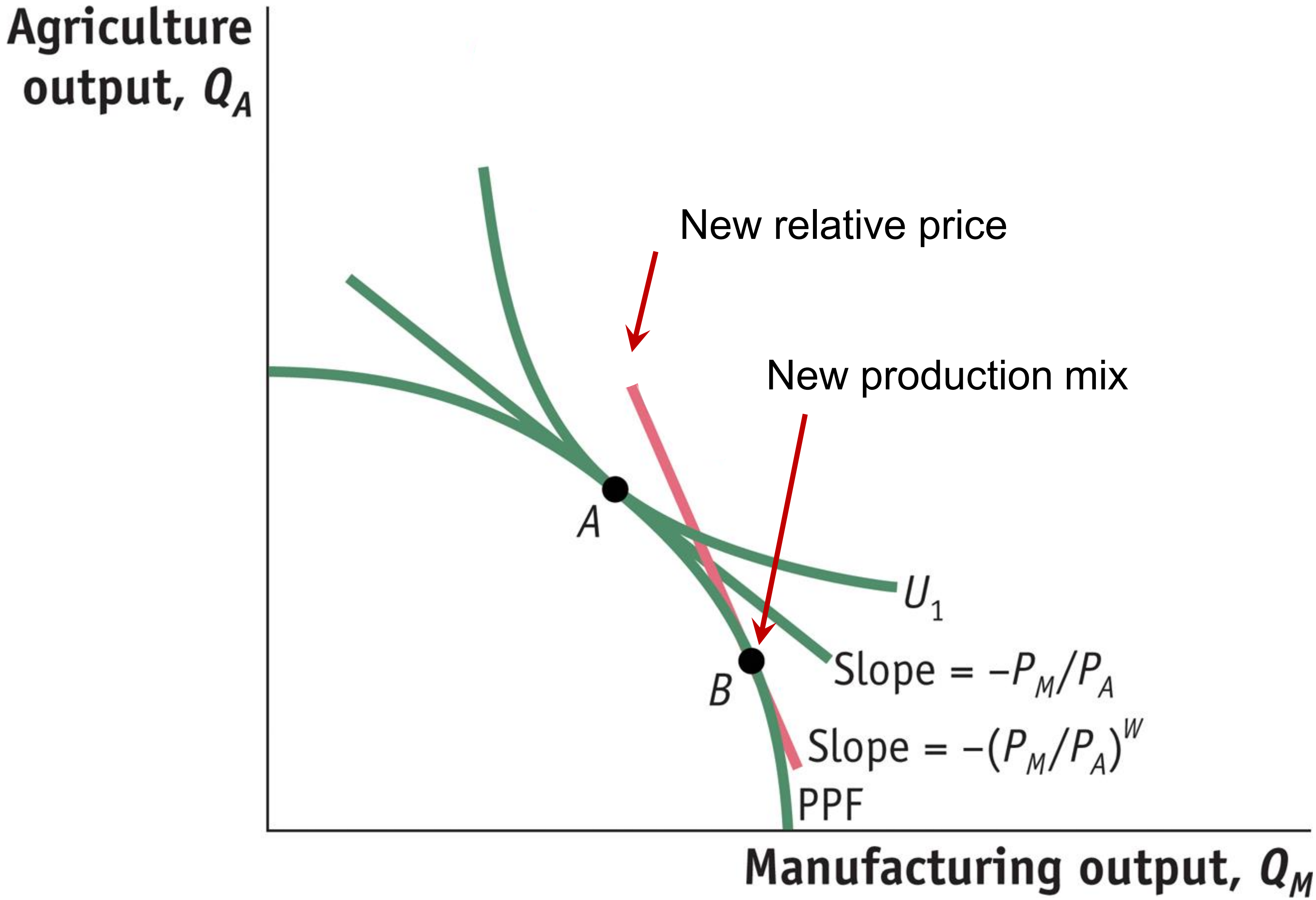
$$(P_M / P_A) < \mathbf{(P_M / P_A)^W} < (P^*_M / P^*_A).$$

Effect on production?

2 Effect of Trade on production



2 Effect of Trade on production



Notes

- Position on PPF depends on $\frac{L_M}{L_A}$
- How does $\frac{L_M}{L_A}$ respond to change in $\frac{P_M}{P_A}$?

1) Look at wages $\Longrightarrow$ $\frac{MPL_A}{MPL_M} = \frac{P_M}{P_A}$

2) $\left.\begin{array}{l} MPL_M(L_M) \searrow L_M \\ MPL_A(L_A) \searrow L_A \end{array}\right\} \Longrightarrow \frac{MPL_A}{MPL_M}\left(L_A/L_M\right)$

$\longrightarrow$ equation: $\frac{P_M}{P_A} = \frac{MPL_A}{MPL_M}\overset{\text{function of}}{\left(\frac{L_A}{L_M}\right)}$

Typically: assume Cobb Douglas

$$\frac{P_M}{P_A} = \frac{MPL_A}{MPL_M} = \frac{\frac{2}{3} a_A \left(T/L_A\right)^{1/3}}{\frac{2}{3} a_M \left(K/L_M\right)^{1/3}}$$

$$= \frac{a_A}{a_M} \times \frac{T^{1/3}}{K^{1/3}} \times \left(\frac{L_M}{L_A}\right)^{1/3}$$

This determines how a change in (P_M/P_A) leads to a change in L_M/L_A

e.g.: a 1% increase in P_M/P_A $\longrightarrow$ leads to a 3% increase in L_M/L_A

2 Effect of Trade on production

Effect on production:

- This leads to an increase in Manufacturing output

- Equilibrium production is where the slope of the PPF is equal to the relative price of Manufacturing.

    - Since the PPF is concave, a higher relative price leads to a larger production of Manufacturing

- **CHAPTER 3: Road map:**

  - Setting up the specific factor model

  - Change in production and employment

  → Aggregate gains from trade

  - Effect on labor wages

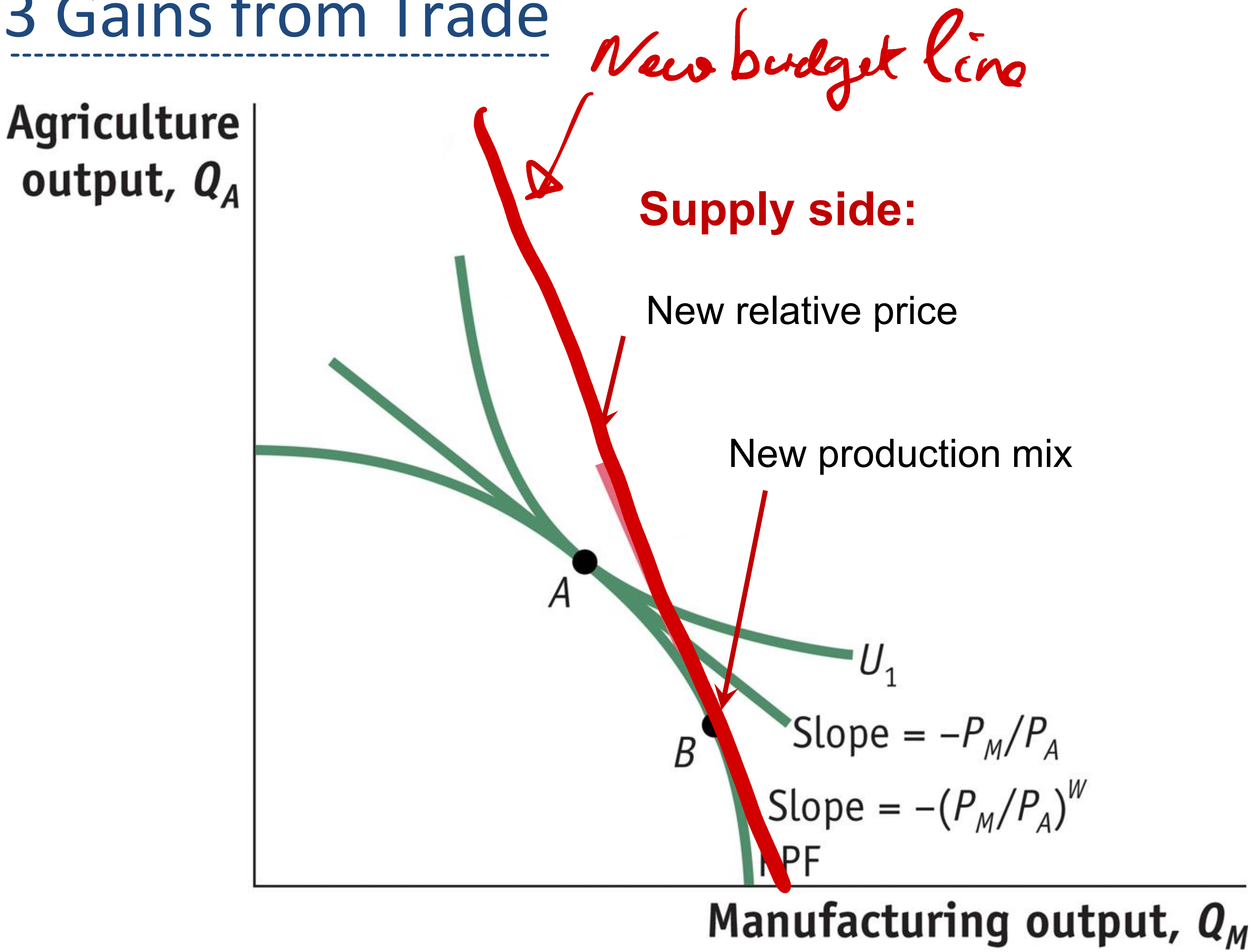
  - Effect on returns to K and Land

3 Gains from Trade

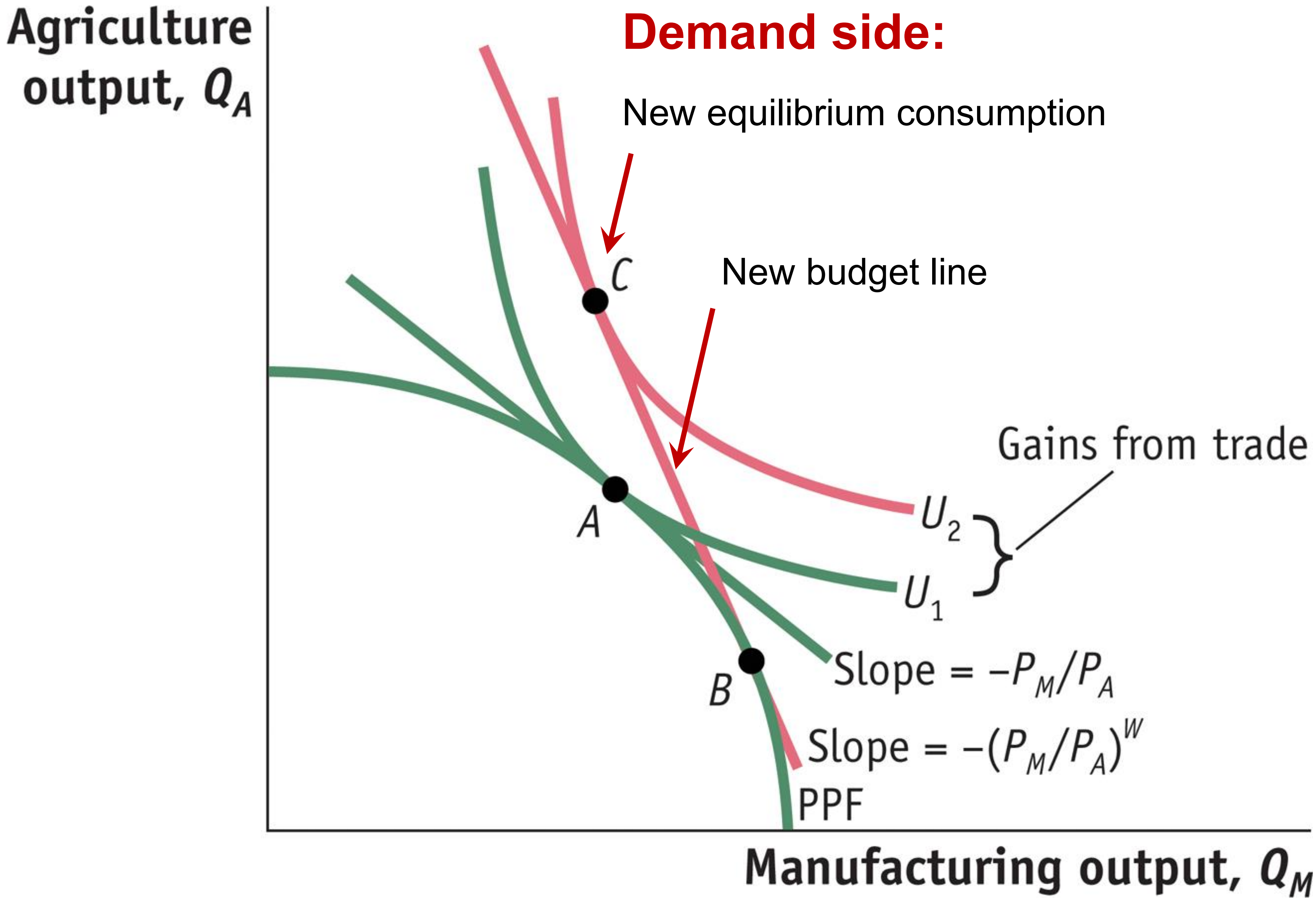
Overall Gains from Trade?

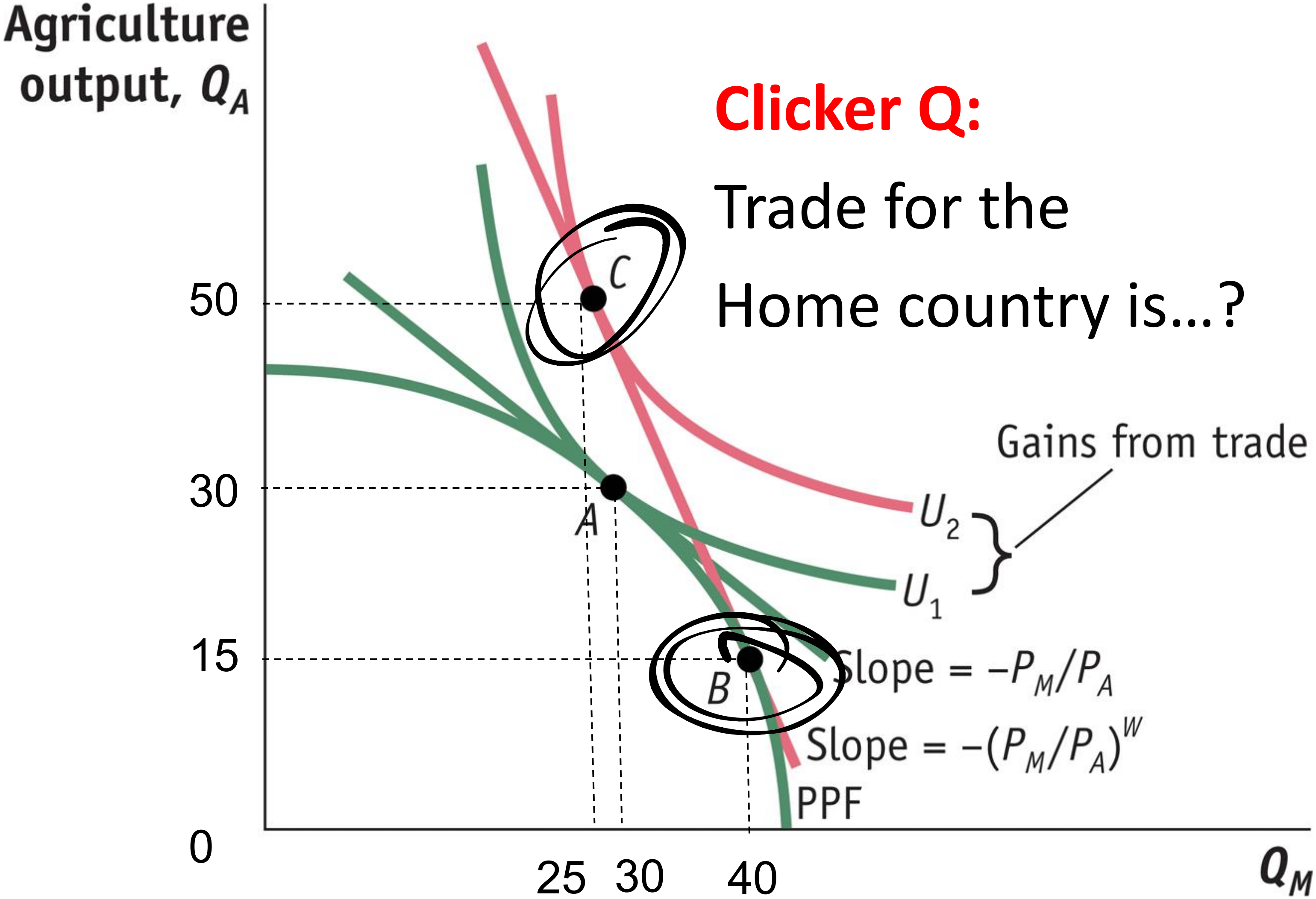
- We start by looking at the average consumer

3 Gains from Trade



3 Gains from Trade





Clicker Q:

Trade for the Home country is…?

a) $X_M = 5$, $M_A = 20$; b) $X_M = 20$, $M_A = 20$;

c) $X_M = 15$, $M_A = 35$; d) $X_M = 20$, $M_A = 15$;

3 Gains from Trade

Overall Gains from Trade

So far, things are not very different from Ricardo:

New world price:

$$(P_M / P_A) < \mathbf{(P_M / P_A)^W} < (P^*_M / P^*_A).$$

- Manufacturing goods are exported,
- Agricultural goods are imported
- For an average consumer, Home is better off with trade.

3 Gains from Trade

Gains for everyone?

- When there are gains from trade *on average*, it does not imply that everyone gains from trade

- The interesting part of the model is to examine what happens to the return to each factor:
  1) Labor wage
  2) Rental rate of Capital and Land

Do workers gain? Do land and capital owner gain?

- **CHAPTER 3: Road map:**
  - Setting up the specific factor model
  - Change in production
  - Aggregate gains from trade

→ Effect on labor wages?

→ Effect on returns to Capital and Land?

3 Gains from Trade

Gains for everyone?

1) What about laborers?

   Does income from labor increase?

➔ Ambiguous! Workers do not necessarily gain

3 Gains from Trade

Do workers gain?

Assume for now that P_A doesn't change and P_M increases

(Note: we would obtain similar results if P_A decreases and P_M doesn't change; it's all about relative prices)

- Can they buy more Agricultural goods?
  … i.e. does W/P_A increase?

- Can they buy more Manufacturing goods?
  … i.e. does W/P_M increase?

3 Gains from Trade

Do workers gain?

- Can they buy more Agricultural goods?

Starting from: $W = P_A \, MPL_A$ (wage in Ag), we get:

$\boxed{W / P_A = MPL_A}$

$L_A \downarrow$

Does MPL_A increase?

Yes! As labor moves away from Ag, MPL increases in Ag.

➔ Workers can buy more Ag goods

$\Leftrightarrow$ wage w increases faster than P_A

$\Leftrightarrow \quad \frac{\Delta w}{w} > \frac{\Delta P_A}{P_A}$

3 Gains from Trade

Do workers gain?

- Can they buy more Manufacturing goods?

Now using: $W = P_M\ MPL_M$ (wage in Manuf.), we get:

$W / P_M = MPL_M$

Does MPL_M increase? $MPL_M \downarrow$ $\Leftarrow$ $L_M \uparrow$

No! Labor moves to Manuf., so MPL decreases in Manuf.

➔ Workers **cannot** buy as many Manuf goods as before

$\Leftrightarrow$ wage W does not increase as fast as P_M

$\Leftrightarrow \frac{\Delta W}{W} < \frac{\Delta P_M}{P_M} = 30\%$

3 Gains from Trade

$\frac{P_M}{P_A} \uparrow$

$\Delta P_M = 0$

Polling question:

Suppose that the price of manufacturing does not change but that the price of agricultural goods decreases by 1%.

$\frac{\Delta P_A}{P_A} = -1\%$

We get:

a) Wages decrease but not as fast as the price of Agricultural goods, i.e. decline by less than 1%

b) Wages decrease faster than the price of Agricultural goods, i.e. decline by more than 1%

c) Wages increase by more than 1%

d) Wages increase but increase by less than 1%

3 Gains from Trade

Gains for everyone?

1) So what about laborers?

- Their income grow faster than the price of Agricultural products, but slower than the price of manuf. goods

$$\frac{\Delta P_A}{P_A} = 0 < \frac{\Delta W}{W} < \frac{\Delta P_M}{P_M}$$

- Overall effect is ambiguous and depends on preferences:
  - Workers may loose if they care a lot about manufacturing goods

3 Gains from Trade

Gains for Land and Capital owners?

2) What about income from Capital and Land?

(Capital is used in Manuf, Land in Agriculture)

- Rental rate of capital (machines)?

- Rental rate of land?

3 Gains from Trade

Gains for Land and Capital owners?

2) What about income from Capital and Land?

(Capital is used in Manuf, Land in Agriculture)

- Rental rate of capital (machines):

$$R_K = P_M \cdot MPK_M$$

$ = income for capital owners

- Rental rate of land:

$$R_T = P_A \cdot MPT_A$$

$ = income for land owners

3 Gains from Trade

Gains for Land and Capital owners?

Checking whether their budget line shifts:

- Can K owners buy more agricultural goods?
  Does R_K / P_A increase?
- Can K owners buy more manufacturing goods?
  Does R_K / P_M increase?
- Can Land owners buy more agricultural goods?
  Does R_T / P_A increase?
- Can Land owners buy more manufacturing goods?
  Does R_T / P_M increase?

3 Gains from Trade

Gains for Land and Capital owners?

Recall that: $R_K = P_M . MPK_M$

If workers move towards the manufacturing sector, the marginal product of capital increases

(because there are more workers to operate each machine)

- Since MPK_M increases, $R_K / P_M = MPK_M$ also increases: Capital owners can *buy more manuf. goods*

$$\Rightarrow \frac{\Delta R_K}{R_K} > \frac{\Delta P_M}{P_M}$$

- Since P_M / P_A increases, we can also conclude that: $R_K / P_A = MPK_M . P_M / P_A$ increases and that Capital owners can also *buy more agricultural goods*

$$R_K = P_M \times MPK_M$$

$$\frac{R_K}{P_A} = \frac{P_M}{P_A} \times MPK_M$$

$$\frac{P_M}{P_A}\uparrow \qquad \frac{L_M}{K}\uparrow \qquad MPK\uparrow$$

$$\Rightarrow \frac{R_K}{P_A}\uparrow \iff \frac{\Delta R_K}{R_K} > \frac{\Delta P_A}{P_A}$$

Here

$K \rightarrow$ only in Manuf

T (land) $\rightarrow$ only in Ag

$L \rightarrow L_M \uparrow \Rightarrow K/L_M \downarrow \Rightarrow MPK \uparrow$

$L \rightarrow L_A \downarrow \Rightarrow (T/L_A) \uparrow \Rightarrow MPT \downarrow$

3 Gains from Trade ($P_M/P_A \uparrow$)

Gains for Land and Capital owners?

$\frac{R_T}{P_A}$?

Recall that: $R_T = P_A . MPT_A$

$\frac{R_T}{P_M}$?

If workers move away from the agricultural sector, the marginal product of land decreases

(because there are *fewer* ~~workers~~ farmers to operate each ~~machine~~ plot of land)

$\frac{T}{L_A} \uparrow$

- Since MPT_A decreases, $R_T / P_A = MPT_A$ also decreases: Land owners can *buy less agricul. goods*

$$\frac{\Delta R_T}{R_T} < \frac{\Delta P_A}{P_A} < \frac{\Delta P_M}{P_M}$$

- Since P_A / P_M decreases, we can also conclude that: $R_T / P_M = MPT_A . P_A / P_M$ decreases and that Land owners can also *buy less manufacturing goods*

3 Gains from Trade

CONCLUSION: Who gains from trade?

- Gains on average for the economy
- Ambiguous for workers that are mobile between sectors
- Gains for factors trapped in the sector with a comparative advantage
- Loss for factors trapped in the sector with a comparative disadvantage

3 Gains from Trade

Clicker question:

If the price of manufacturing goods increases, whose income increases faster than the price of manufacturing goods?

a) Both Land owners and K owners' income increases faster than the price of manufacturing goods

b) Only Land owners' income increases faster than the price of manufacturing goods

c) Only K owners' income increases faster than the price of manufacturing goods

d) Neither Land or K owners' income increases faster than the price of manufacturing goods

3 Gains from Trade

Clicker question:

In the specific-factor model, an increase in the price of manufacture $\Delta P_M > 0$ (keeping P_A constant) yields:

$$a) \quad \frac{\Delta R_T}{R_T} < 0 < \frac{\Delta W}{W} < \frac{\Delta P_M}{P_M} < \frac{\Delta R_K}{R_K}$$

$$b) \quad \frac{\Delta R_T}{R_T} = 0 < \frac{\Delta P_M}{P_M} < \frac{\Delta W}{W} < \frac{\Delta R_K}{R_K}$$

$$c) \quad \frac{\Delta R_T}{R_T} < 0 < \frac{\Delta P_M}{P_M} < \frac{\Delta R_K}{R_K} < \frac{\Delta W}{W}$$

3 Gains from Trade

Clicker question:

In the specific-factor model, suppose that P_M increases by 15% while P_A increases by 10%, we get:

$$a) \quad \frac{\Delta R_K}{R_K} < 10\% < \frac{\Delta W}{W} < 15\% < \frac{\Delta R_T}{R_T}$$

$$b) \quad \frac{\Delta R_T}{R_T} < 10\% < \frac{\Delta W}{W} < 15\% < \frac{\Delta R_K}{R_K}$$

$$c) \quad \frac{\Delta R_K}{R_K} < 10\% < 15\% < \frac{\Delta W}{W} < \frac{\Delta R_T}{R_T}$$

3 Gains from Trade

Clicker question:

In the specific-factor model, suppose that P_M decreases by 10% while P_A increases by 5%, we get:

$$a)\quad \frac{\Delta R_T}{R_T} < -10\% < \frac{\Delta W}{W} < 5\% < \frac{\Delta R_K}{R_K}$$

$$b)\quad \frac{\Delta R_K}{R_K} < -10\% < \frac{\Delta W}{W} < 5\% < \frac{\Delta R_T}{R_T}$$

$$c)\quad \frac{\Delta R_T}{R_T} < \frac{\Delta W}{W} < -10\% < 5\% < \frac{\Delta R_K}{R_K}$$

$$d)\quad \frac{\Delta R_K}{R_K} < \frac{\Delta W}{W} < -10\% < 5\% < \frac{\Delta R_T}{R_T}$$

3 Gains from Trade

Attention: Flipping price changes

If the price of Agriculture increases more than the price of Manufacturing goods:

Results are flipped!

- It implies that Home has a comparative advantage in Agriculture
- Gains for Land owners
- Loss for Capital owners
- Ambiguous effects for (mobile) workers