and we can see that a unique solution of y is obtained if the boundary condition:

$$y(x_0) = y_0$$

is given. What is left is to do the integral — often not possible in closed form — and then solve for y in terms of x, which is also not necessarily possible.

Ex:

$$\frac{dy}{dx} = y^{\frac{1}{2}(1-p)}\sqrt{1-y^{1+p}}$$

with $y(0)=0$. Then:

$$\frac{y^{\frac{p-1}{2}}\,dy}{\sqrt{1-y^{1+p}}} = dx$$

$$x = \int_0^y \frac{s^{\frac{1}{2}(p-1)}\,ds}{\sqrt{1-s^{p+1}}},$$

Let:

$$\sin u = s^{\frac{1}{2}(p+1)}$$

$$\cos u\,du = \tfrac{1}{2}(p+1)\,s^{\frac{1}{2}(p-1)}\,ds$$

$$x = \frac{2}{p+1}\int \frac{\cos u\,du}{\sqrt{1-\sin^2 u}} = \frac{2}{p+1}u$$

$$\tfrac{1}{2}(p+1)x = u \Rightarrow \sin\left[\tfrac{1}{2}(p+1)x\right] = \sin u$$

$$\sin\left[\tfrac{1}{2}(p+1)x\right] = y^{\frac{1}{2}(p+1)} \Rightarrow \boxed{y = \left[\sin\left[\tfrac{1}{2}(p+1)x\right]\right]^{\frac{2}{p+1}}}$$