Algorithm 1 Metropolis-Hastings Sampling of Ising Model with spin configuration $\boldsymbol{\sigma}$

function METROPOLIS-HASTINGS(π, Q, $\boldsymbol{\sigma}_0$, N)

Input: target distribution π, proposal distribution Q, initial state $\boldsymbol{\sigma}_0$, number of steps N

Output: samples from π

$\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}_0$

for $t \leftarrow 1$ to N **do**

$\boldsymbol{\sigma}' \sim Q(\boldsymbol{\sigma}'|\boldsymbol{\sigma})$

$A \leftarrow \min\left\{1, \frac{\pi(\boldsymbol{\sigma}')Q(\boldsymbol{\sigma}|\boldsymbol{\sigma}')}{\pi(\boldsymbol{\sigma})Q(\boldsymbol{\sigma}'|\boldsymbol{\sigma})}\right\}$ ▷ Acceptance probability

$u \sim \text{Uniform}(0, 1)$

if $u < A$ **then**

$\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}'$

end if

end for

return $\boldsymbol{\sigma}$

end function

Algorithm 2 Metropolis-Hastings Sampling of Ising Model with spin configuration $\boldsymbol{\sigma}$, plus saving samples every L steps and warm-up steps K

function METROPOLIS-HASTINGS-SAVE(π, Q, $\boldsymbol{\sigma}_0$, N, K, L)
Input: target distribution π, proposal distribution Q, initial state $\boldsymbol{\sigma}_0$, number of steps N, warm-up steps K, save steps L
Output: samples from π
    $\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}_0$
    $K \leftarrow \min\{K, N-1\}$ ▷ Ensure feasibility of K
    $S \leftarrow \emptyset$ ▷ Initialize sample set
    for $t \leftarrow 1$ to N **do**
        $\boldsymbol{\sigma}' \sim Q(\boldsymbol{\sigma}'|\boldsymbol{\sigma})$
        $A \leftarrow \min\left\{1, \frac{\pi(\boldsymbol{\sigma}')Q(\boldsymbol{\sigma}|\boldsymbol{\sigma}')}{\pi(\boldsymbol{\sigma})Q(\boldsymbol{\sigma}'|\boldsymbol{\sigma})}\right\}$ ▷ Acceptance probability
        $u \sim \text{Uniform}(0,1)$
        if $u < A$ **then**
            $\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}'$
        end if
        if $t > K$ and $t \bmod L = 0$ **then**
            Save $\boldsymbol{\sigma}$ as a sample to S
        end if
    end for
    return S
end function

Algorithm 3 Gibbs Sampling of Ising Model with spin configuration $\boldsymbol{\sigma}$

function GIBBS(π, $\boldsymbol{\sigma}_0$, N)
Input: target distribution π, initial state $\boldsymbol{\sigma}_0$, number of steps N
Output: samples from π
    $\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}_0$
    $i \leftarrow 1$
    for $t \leftarrow 1$ to N **do**
        $\sigma_i \sim P(\sigma_i|\boldsymbol{\sigma}_{-i})$ ▷ Sample σ_i from conditional distribution
        $i \leftarrow i + 1$
        if $i > |\boldsymbol{\sigma}|$ **then** ▷ $|\boldsymbol{\sigma}|$ is also the number of vertices
            $i \leftarrow 1$
        end if
    end for
    return $\boldsymbol{\sigma}$
end function

Algorithm 4 Gibbs Sampling of Ising Model with spin configuration $\boldsymbol{\sigma}$, plus saving samples every c sweepings and warm-up sweeping count k

function GIBBS(π, $\boldsymbol{\sigma}_0$, N)
Input: target distribution π, initial state $\boldsymbol{\sigma}_0$, number of steps N
Output: samples from π
    $\boldsymbol{\sigma} \leftarrow \boldsymbol{\sigma}_0$
    $i \leftarrow 1$
    $k \leftarrow \min\{k, \frac{N-1}{|\boldsymbol{\sigma}|}\}$ ▷ Ensure feasibility of k
    $S \leftarrow \emptyset$ ▷ Initialize sample set
    for $t \leftarrow 1$ to N **do**
        $\sigma_i \sim P(\sigma_i|\boldsymbol{\sigma}_{-i})$ ▷ Sample σ_i from conditional distribution
        $i \leftarrow i + 1$
        if $i > |\boldsymbol{\sigma}|$ **then** ▷ $|\boldsymbol{\sigma}|$ is also the number of vertices
            $i \leftarrow 1$
        end if
        if $t > k|\boldsymbol{\sigma}|$ and $t \bmod c|\boldsymbol{\sigma}| = 0$ **then**
            Save $\boldsymbol{\sigma}$ as a sample to S
        end if
    end for
    return S
end function