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**Algorithm 1** Metropolis-Hastings Sampling of Ising Model with spin configuration  $\sigma$

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1: **function** METROPOLIS-HASTINGS( $\pi, Q, \sigma_0, N$ )

**Input:** target distribution  $\pi$ , proposal distribution  $Q$ , initial state  $\sigma_0$ , number of steps  $N$

**Output:** samples from  $\pi$

2:    $\sigma \leftarrow \sigma_0$

3:   **for**  $t \leftarrow 1$  to  $N$  **do**

4:      $\sigma' \sim Q(\sigma'|\sigma)$

5:      $A \leftarrow \min \left\{ 1, \frac{\pi(\sigma')Q(\sigma|\sigma')}{\pi(\sigma)Q(\sigma'|\sigma)} \right\}$   $\triangleright$  Acceptance probability

6:      $u \sim \text{Uniform}(0, 1)$

7:     **if**  $u < A$  **then**

8:        $\sigma \leftarrow \sigma'$

9:     **end if**

10:   **end for**

11:   **return**  $\sigma$

12: **end function**

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**Algorithm 2** Metropolis-Hastings Sampling of Ising Model with spin configuration  $\sigma$ , plus saving samples every  $L$  steps and warm-up steps  $K$

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1: function METROPOLIS-HASTINGS-SAVE( $\pi, Q, \sigma_0, N, K, L$ )
Input: target distribution  $\pi$ , proposal distribution  $Q$ , initial state  $\sigma_0$ , number
        of steps  $N$ , warm-up steps  $K$ , save steps  $L$ 
Output: samples from  $\pi$ 
2:    $\sigma \leftarrow \sigma_0$ 
3:    $K \leftarrow \min\{K, N - 1\}$  ▷ Ensure feasibility of  $K$ 
4:    $S \leftarrow \emptyset$  ▷ Initialize sample set
5:   for  $t \leftarrow 1$  to  $N$  do
6:      $\sigma' \sim Q(\sigma'|\sigma)$ 
7:      $A \leftarrow \min\left\{1, \frac{\pi(\sigma')Q(\sigma|\sigma')}{\pi(\sigma)Q(\sigma'|\sigma)}\right\}$  ▷ Acceptance probability
8:      $u \sim \text{Uniform}(0, 1)$ 
9:     if  $u < A$  then
10:       $\sigma \leftarrow \sigma'$ 
11:    end if
12:    if  $t > K$  and  $t \bmod L = 0$  then
13:      Save  $\sigma$  as a sample to  $S$ 
14:    end if
15:  end for
16:  return  $S$ 
17: end function

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**Algorithm 3** Gibbs Sampling of Ising Model with spin configuration  $\sigma$

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1: function GIBBS( $\pi, \sigma_0, N$ )
Input: target distribution  $\pi$ , initial state  $\sigma_0$ , number of steps  $N$ 
Output: samples from  $\pi$ 
2:    $\sigma \leftarrow \sigma_0$ 
3:    $i \leftarrow 1$ 
4:   for  $t \leftarrow 1$  to  $N$  do
5:      $\sigma_i \sim P(\sigma_i|\sigma_{-i})$  ▷ Sample  $\sigma_i$  from conditional distribution
6:      $i \leftarrow i + 1$ 
7:     if  $i > |\sigma|$  then ▷  $|\sigma|$  is also the number of vertices
8:        $i \leftarrow 1$ 
9:     end if
10:  end for
11:  return  $\sigma$ 
12: end function

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**Algorithm 4** Gibbs Sampling of Ising Model with spin configuration  $\sigma$ , plus saving samples every  $c$  sweepings and warm-up sweeping count  $k$

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1: **function** GIBBS( $\pi, \sigma_0, N$ )

**Input:** target distribution  $\pi$ , initial state  $\sigma_0$ , number of steps  $N$

**Output:** samples from  $\pi$

2:    $\sigma \leftarrow \sigma_0$

3:    $i \leftarrow 1$

4:    $k \leftarrow \min\{k, \frac{N-1}{|\sigma|}\}$

▷ Ensure feasibility of  $k$

5:    $S \leftarrow \emptyset$

▷ Initialize sample set

6:   **for**  $t \leftarrow 1$  to  $N$  **do**

7:      $\sigma_i \sim P(\sigma_i | \sigma_{-i})$

▷ Sample  $\sigma_i$  from conditional distribution

8:      $i \leftarrow i + 1$

9:     **if**  $i > |\sigma|$  **then**

▷  $|\sigma|$  is also the number of vertices

10:        $i \leftarrow 1$

11:     **end if**

12:     **if**  $t > k|\sigma|$  and  $t \bmod c|\sigma| = 0$  **then**

13:       Save  $\sigma$  as a sample to  $S$

14:     **end if**

15:   **end for**

16:   **return**  $S$

17: **end function**

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