
Algorithm 1 Metropolis-Hastings Sampling of Ising Model with spin configuration σ

1: **function** METROPOLIS-HASTINGS(π, Q, σ_0, N)

Input: target distribution π , proposal distribution Q , initial state σ_0 , number of steps N

Output: samples from π

2: $\sigma \leftarrow \sigma_0$

3: **for** $t \leftarrow 1$ to N **do**

4: $\sigma' \sim Q(\sigma'|\sigma)$

5: $A \leftarrow \min \left\{ 1, \frac{\pi(\sigma')Q(\sigma|\sigma')}{\pi(\sigma)Q(\sigma'|\sigma)} \right\}$ \triangleright Acceptance probability

6: $u \sim \text{Uniform}(0, 1)$

7: **if** $u < A$ **then**

8: $\sigma \leftarrow \sigma'$

9: **end if**

10: **end for**

11: **return** σ

12: **end function**

Algorithm 2 Metropolis-Hastings Sampling of Ising Model with spin configuration σ , plus saving samples every L steps and warm-up steps K

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1: function METROPOLIS-HASTINGS-SAVE( $\pi, Q, \sigma_0, N, K, L$ )
Input: target distribution  $\pi$ , proposal distribution  $Q$ , initial state  $\sigma_0$ , number
        of steps  $N$ , warm-up steps  $K$ , save steps  $L$ 
Output: samples from  $\pi$ 
2:    $\sigma \leftarrow \sigma_0$ 
3:    $K \leftarrow \min\{K, N - 1\}$  ▷ Ensure feasibility of  $K$ 
4:    $S \leftarrow \emptyset$  ▷ Initialize sample set
5:   for  $t \leftarrow 1$  to  $N$  do
6:      $\sigma' \sim Q(\sigma'|\sigma)$ 
7:      $A \leftarrow \min\left\{1, \frac{\pi(\sigma')Q(\sigma|\sigma')}{\pi(\sigma)Q(\sigma'|\sigma)}\right\}$  ▷ Acceptance probability
8:      $u \sim \text{Uniform}(0, 1)$ 
9:     if  $u < A$  then
10:       $\sigma \leftarrow \sigma'$ 
11:    end if
12:    if  $t > K$  and  $t \bmod L = 0$  then
13:      Save  $\sigma$  as a sample to  $S$ 
14:    end if
15:  end for
16:  return  $S$ 
17: end function

```

Algorithm 3 Gibbs Sampling of Ising Model with spin configuration σ

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1: function GIBBS( $\pi, \sigma_0, N$ )
Input: target distribution  $\pi$ , initial state  $\sigma_0$ , number of steps  $N$ 
Output: samples from  $\pi$ 
2:    $\sigma \leftarrow \sigma_0$ 
3:    $i \leftarrow 1$ 
4:   for  $t \leftarrow 1$  to  $N$  do
5:      $\sigma_i \sim P(\sigma_i|\sigma_{-i})$  ▷ Sample  $\sigma_i$  from conditional distribution
6:      $i \leftarrow i + 1$ 
7:     if  $i > |\sigma|$  then ▷  $|\sigma|$  is also the number of vertices
8:        $i \leftarrow 1$ 
9:     end if
10:  end for
11:  return  $\sigma$ 
12: end function

```

Algorithm 4 Gibbs Sampling of Ising Model with spin configuration σ , plus saving samples every c sweepings and warm-up sweeping count k

1: **function** GIBBS(π, σ_0, N)

Input: target distribution π , initial state σ_0 , number of steps N

Output: samples from π

2: $\sigma \leftarrow \sigma_0$

3: $i \leftarrow 1$

4: $k \leftarrow \min\{k, \frac{N-1}{|\sigma|}\}$

▷ Ensure feasibility of k

5: $S \leftarrow \emptyset$

▷ Initialize sample set

6: **for** $t \leftarrow 1$ to N **do**

7: $\sigma_i \sim P(\sigma_i | \sigma_{-i})$

▷ Sample σ_i from conditional distribution

8: $i \leftarrow i + 1$

9: **if** $i > |\sigma|$ **then**

▷ $|\sigma|$ is also the number of vertices

10: $i \leftarrow 1$

11: **end if**

12: **if** $t > k|\sigma|$ and $t \bmod c|\sigma| = 0$ **then**

13: Save σ as a sample to S

14: **end if**

15: **end for**

16: **return** S

17: **end function**
