

# DESIGN OF PRESTRESSED SLAB

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# Introduction

- Post-tensioned concrete floors form a large proportion of all prestressed concrete construction.
- It is economically competitive with RC slabs in most practical medium- to long-span situations.

# Advantages

- 1) Deflection – better controlled in post-tensioned slab.
- 2) More slender slab system - increased head room /reduced floor-to-floor heights.
- 3) Pre-stressed inhibits cracking – will produced crack-free and watertight floors.
- 4) Simple uncluttered steel layouts – speeding steel fixing and concrete placing work.
- 5) Improve punching shear.
- 6) Reduce formworks and stripping times.

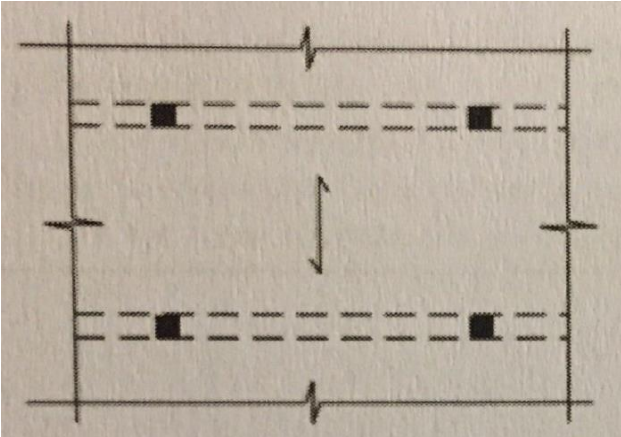
# Info!

- Typical products for pre-tensioned concrete are roof slabs, piles, poles, bridge girders, wall panels, and railroad ties.
- Whereas, post-tensioned concrete is used for cast-in-place concrete and for bridges, large girders, floor slabs, shells, roofs, and pavements.

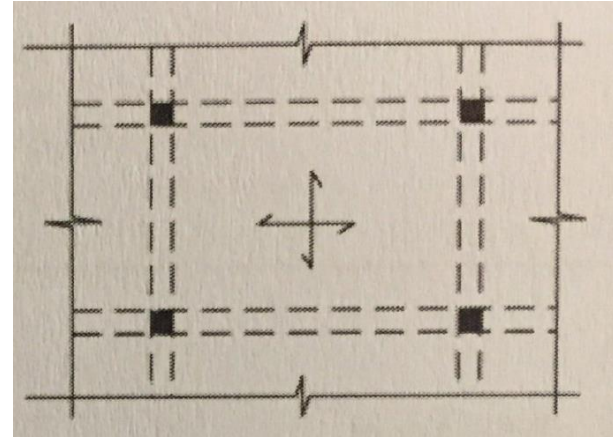
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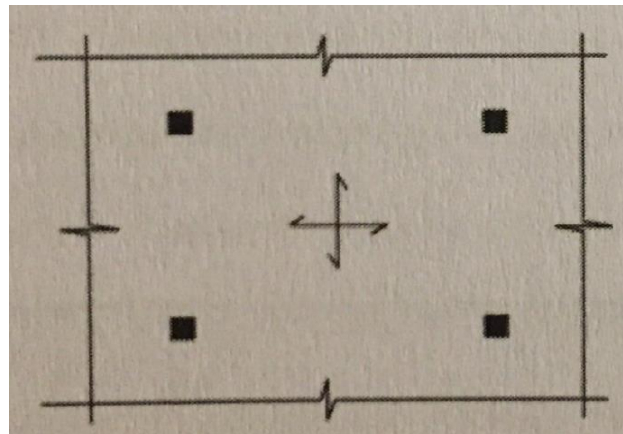
# Slab Systems



(1) One-way slab

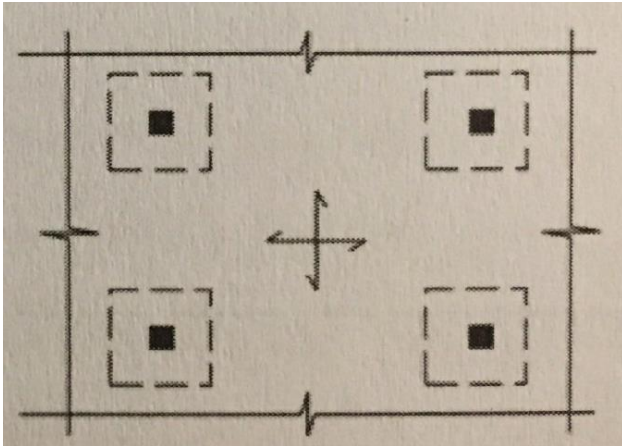


(2) Edge-supported two-way slab

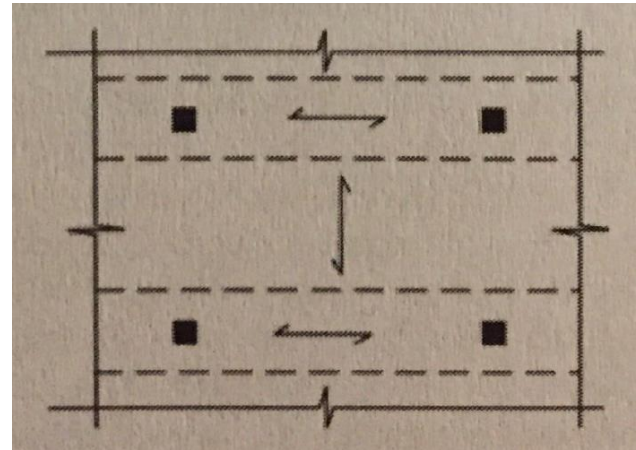


(3) Flat plate

# Slab Systems



(4) Flat slab with drop panels



(5) Band beam and slab

# Design Approach : General

- After making an initial selection of the slab thickness, the second step in slab design is to determine the amount and distribution of prestress.
- Load balancing is used, a portion of the load on slab is balanced by the transverse forces imposed by the draped tendons in each direction.
- Under the balanced load, the slab remains plane (without curvature) and is subjected only to the resultant longitudinal compressive  $P/A$  stresses.



# One-way Slabs

- Normally design as a beam with cables running in the direction of the span at uniform centres.
- A slab strip of unit width is analysed using simple beam theory.
- In any span, the max cable sag,  $z_d$  depends on the concrete cover requirements and the tendon dimensions.
- When  $z_d$  is determined, the prestressing force required to balance an external load,  $w_{bal}$  is calculated from eqn. 1

$$P = \frac{w_{bal} l^2}{8z_d} \quad \text{eqn. 1}$$

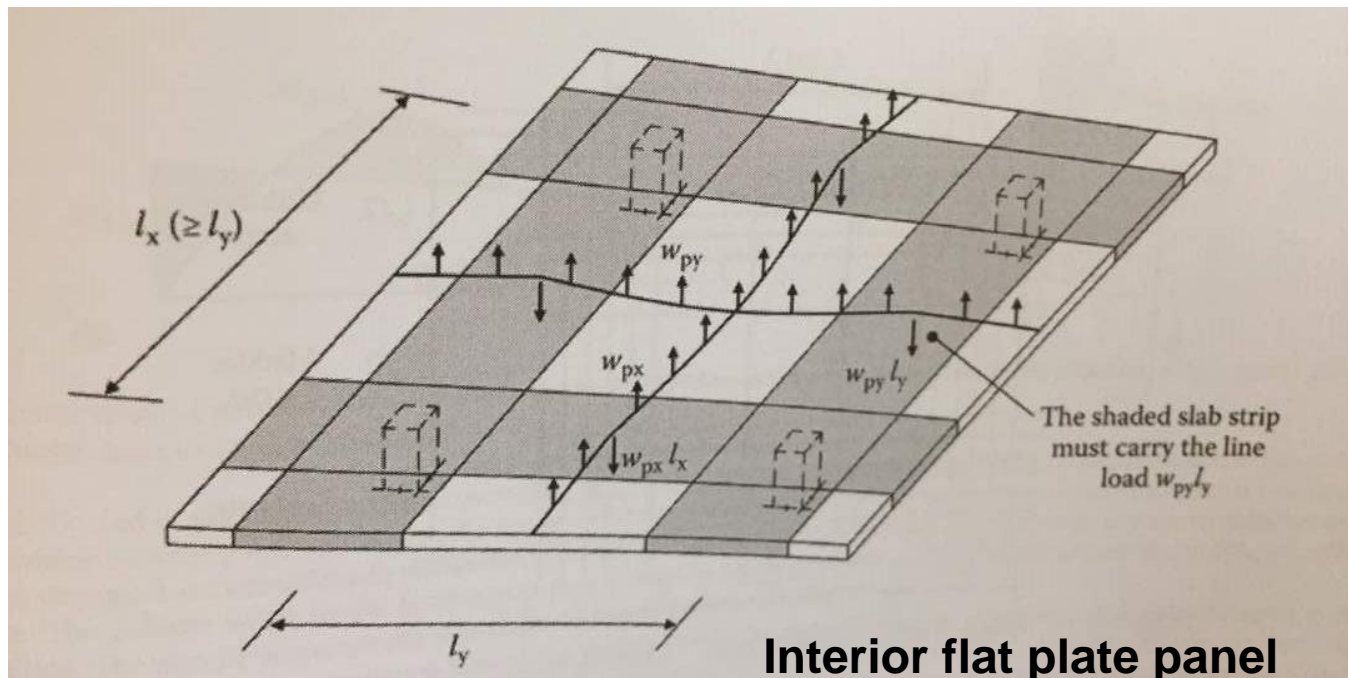
- In the transverse direction, conventional reinforcement may be used (to control shrinkage, temperature cracking & to distribute local load concentrations).



# Flat Plate Slabs

## [1] LOAD BALANCING

-Flat plates behave in a similar manner to edge-supported slabs (except that the edge beams are strips of slab located on the column lines).



# Flat Plate Slabs

- The edge beams have the same depth as the remainder of the slab panel, and therefore the system tends to be less stiff and more prone to serviceability problems.
- The load paths for both the flat plate and the edge-supported slabs are, however, essentially the same.
- In the flat plate panel, the total load to be balanced is  $w_{ballxly}$ .
- The upward forces per unit area exerted by the slab tendons in each direction are;

$$w_{px} = \frac{8P_x z dx}{l_x^2} \quad \text{eqn. 2}$$

$$w_{py} = \frac{8P_y z dy}{l_y^2} \quad \text{eqn. 3}$$

# Flat Plate Slabs

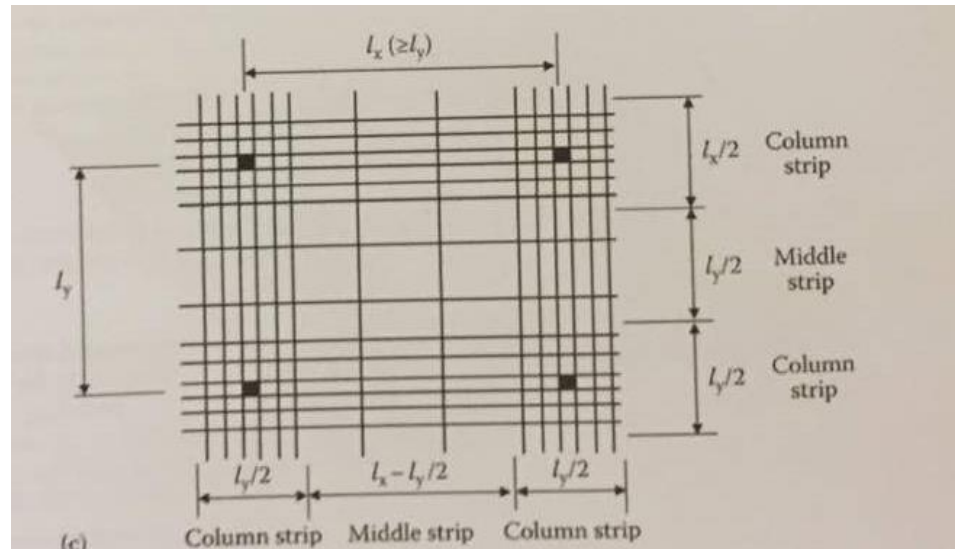
- And, the slab tendons impose a total upward force of:

$$w_{px}l_xl_y + w_{py}l_yl_x = w_{bal}l_xl_y \quad \text{eqn. 4}$$

- For perfect load balancing, the column line tendons would have to be placed within the width of slab in which the slab tendons exert downward load due to reverse curvature.
- However, this is not a strict requirement and considerable variation in tendon spacing can occur without noticeably affecting slab behaviour.

# Flat Plate Slabs

- Column line tendons are frequently spread out over a width of slab as large as one half the shorter span;

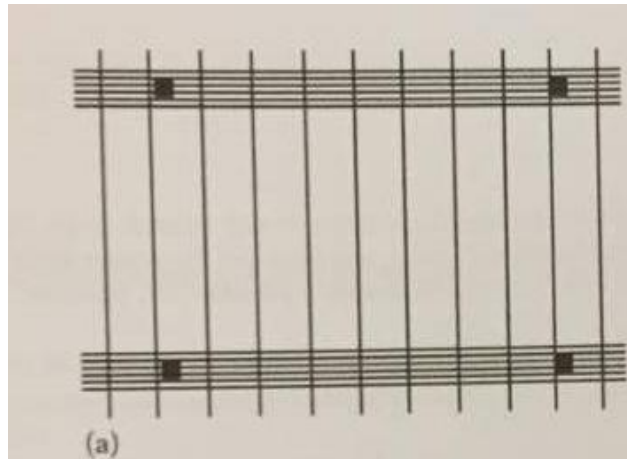


**Two-way slab arrangement with column line tendons distributed over column strips**

- The total upward force that must be provided in the slab along the column lines is also as given in Eqn. 4.
- Therefore, prestressing tendons (slab tendons + column line tendons) must be provided in each panel to give a total upward force of  $2w_{bal}l_xl_y$ .

# Flat Plate Slabs

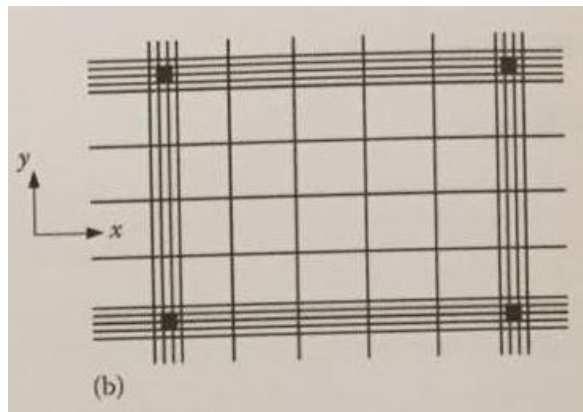
- As shown in the figure, the entire load to be balanced is carried by slab tendons in the y-direction, i.e.  $w_{py} = w_{bal}$ ,  $w_{px} = 0$ .
- The entire load is deposited as a line load on the column lines in the x-direction and must be balanced by column line tendons in this vicinity.
- This slab has in effect been designed as a one-way slab spanning in the y-direction and supported by shallow heavily stressed slab strips on the x-direction column lines.



Alternative tendon layout : One-way slab arrangement

# Flat Plate Slabs

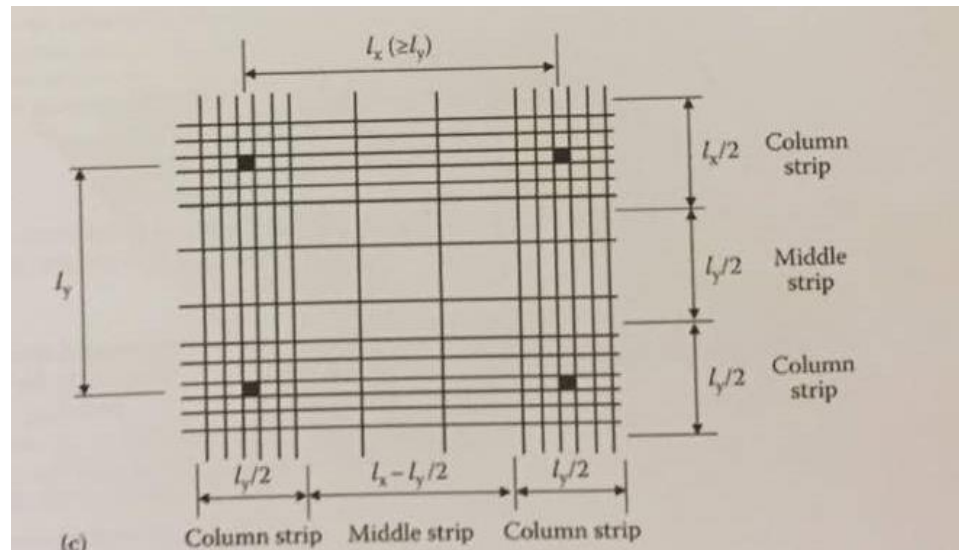
- The two way system as shown in the following figure is more likely to perform better under unbalanced loads, particularly when the orthogonal spans  $l_x$  and  $l_y$  are similar and the panel is roughly square.
- In practice, however, steel congestion over the supporting columns and minimum spacing requirements (often determined by the size of the anchorages) make the concentration of tendons on column lines impossible.



Alternative tendon layout : Two-way slab arrangement with column line tendons in narrow band

# Flat Plate Slabs

- Therefore, the following figure is a more practical and generally acceptable layout.
- Approximately, 75% of the tendons in each direction are located in the column strips, as shown, the remainder being uniformly spread across the middle strip regions.



**Alternative tendon layout : Two-way slab arrangement with column line tendons distributed over column strips**



# Flat Plate Slabs

- Under unbalanced loads, moments and shears are induced in the slab. To calculate the moments and stresses due to unbalanced service loads and to calculate the factored design moments and shears in the slab (in order to check for strength), one of the methods described in the following sections may be adopted.

(1) Frame analysis

(2) Direct design method

# Frame Analysis

- A commonly used technique for the analysis of flat plates.
- The structure is idealised into a set of parallel two-dimensional frames running in orthogonal directions through building.
- Each frame consists of a series of vertical columns spanned by horizontal beams.
- These idealised beams consist of the strip of slab of width on each side of the column line equal to half the distance to the adjacent parallel row of columns and include any floor beams forming part of the floor system.

# BS EN 1992-1-1

- 1) The frames are analysed using a linear-elastic frame system.
- 2) Permits the stiffness of the members to be based on gross-sections and, for vertical gravity loads, the stiffness may be based on the full width of the panels.
- 3) For flat slab structures subjected to lateral (horizontal) loads, the stiffness of the horizontal (floor) members should be based on 40% of the full width of the panels. This reduction of stiffness reflects the increased flexibility of the slab-column connections compared to column-beam connections in beam and slab floor systems.
- 4) For a flat plate building in which shear walls, or some other bracing system, are provided to resist all lateral loads, it is usually permissible to analyse each floor of the building separately, with the columns above and below the slab assumed to be fixed at the remote ends.

# Assumptions

- 1) A two-way plate is idealised by orthogonal one-way strips
- 2) The stiffness of a cracked slab may be significantly less than that based on gross sections
- 3) A linear-elastic analysis is applied to a structure that is non-linear and inelastic both at service loads and at overloads.

Loading patterns to be considered to assess deflection and cracking should include at least the following:

- (1) Where the loading pattern is known, the frame should be analysed under that known loading. This includes the factored permanent dead load
- (2) With regard to live loads, where the pattern of loaded and unloaded spans is variable, the factored live load should be applied;
  - (a) On alternate spans (this will permit the determination of the max factored positive moment near the middle of the loaded spans)
  - (b) On two adjacent spans (this will permit the determination of the max factored negative moment at the interior support between the loaded spans)
  - (c) On all spans

# BS EN 1992-1-1

- The frame moments calculated at the critical sections of the idealised horizontal members are distributed across the floor slab into the column and middle strips. Therefore, in BS EN 1992-1-1 apportions that the total frame moments to column and middle strips:

## Fraction of frame moments distributed to the column and middle strips

	-ve moments (%)	+ve moments (%)
Column strip	60 - 80	50 - 70
Middle strip	40 - 20	50 - 30

\*Note : The sum of moments resisted by the column and middle strips at any location must always equal the frame moment at that location.

- When the design resistances of the column and middle strips are being checked, it is advisable to ensure that the depth to the neutral axis at any section does not exceed about  $0.25d$ .
- This will ensure sufficient ductility for the slab to redistribute the bending moments towards the bending moment diagram predicted by the idealised frame analysis and will allow the designer to safely ignore the secondary moments.

# Shear Resistance

- Punching shear strength requirements often control the thickness of a flat slab at the supporting columns (therefore must always be checked).
- If frame analyses are performed to check the flexural resistance of a slab, the design moment,  $M_{ed}$  transferred from the slab to a column and the design shear,  $V_{ed}$  are obtained from the relevant analysis.
- $M_{ed}$  is that part of the unbalanced slab bending moments that is transferred into the column at the support.

$$M_{Ed} = 0.06 \left[ (1.35w_G + 0.75w_Q)l_t(l_{eff})^2 - 1.35w_Gl_t(l'_{eff'})^2 \right]$$

# Shear Resistance

$$M_{Ed} = 0.06 \left[ (1.35w_G + 0.75w_Q)l_t(l_{eff})^2 - 1.35w_Gl_t(l'_{eff'})^2 \right]$$

Where;

$w_G$  = uniformly distributed dead load (per unit area)

$w_Q$  = uniformly distributed live load

$l_t$  = transverse width of the slab

$l_{eff}$  = longer and shorter effective spans on either side of the column



# Shear Resistance

- For an edge column,  $M_{ed}$  is equal to the design moment at the exterior edge of the slab and may be taken as  $0.25M_o$  (where  $M_o$  is the static moment for the end span of the slab calculated using eqn.

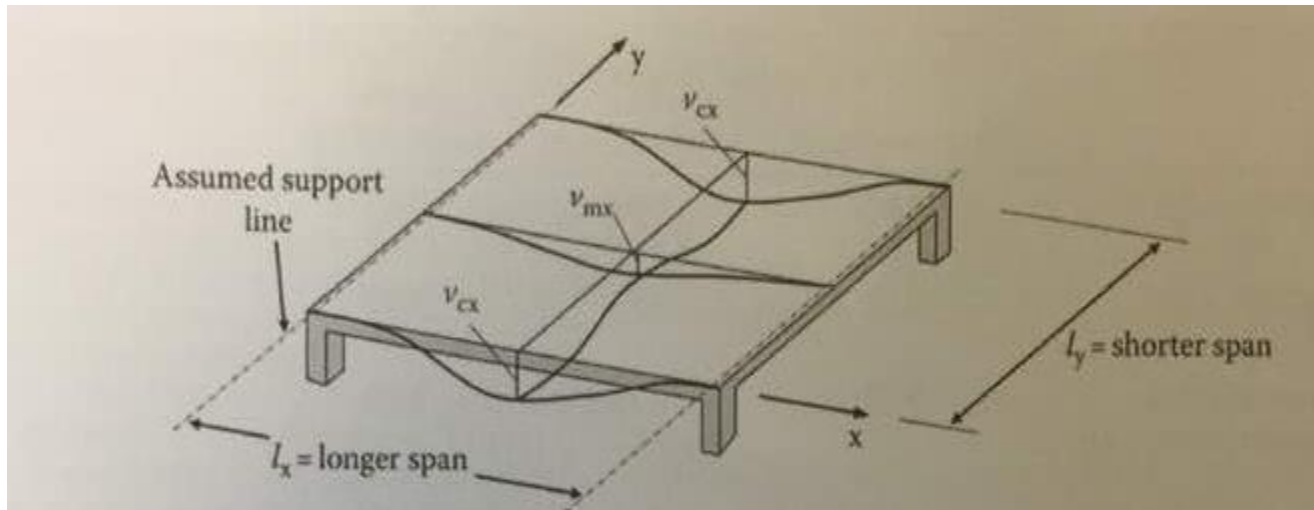
$$M_o = \frac{w_{Ed} l_t l_{eff}^2}{8}$$

- When detailing the slab-column connection, it is advisable to have at least two prestressing tendons crossing the critical shear perimeter in each direction.
- Additional well anchored non-prestressed reinforcement crossing the critical perimeter will also prove beneficial (both in terms of crack control and ductility) in the event of unexpected overloads.

# Deflection Calculations

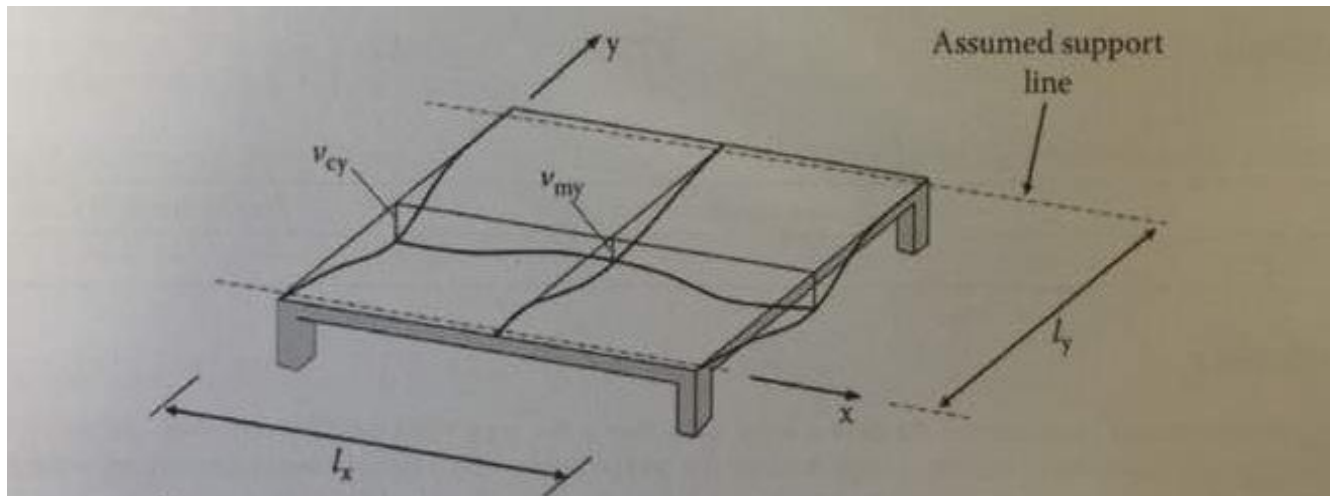
- The deflection of a uniformly loaded flat slab may be estimated using the wide beam method which was formalised by Nilson and Walters.
- Originally developed for RC slabs, the method is particularly appropriate for prestressed flat slabs which are usually uncracked at service loads.
- The basis of the method is illustrated in the following figure;

- In the following figure, the slab is considered to act as a wide shallow beam of width equal to the smaller panel dimension  $l_y$  and span equal to the longer panel effective span  $l_x$ .
- This wide beam is assumed to rest on unyielding supports.
- Because of variations in the moments caused by the unbalanced loads and the flexural rigidity across the width of the slab, all unit strips in the x-direction will not deform identically.
- Unbalanced moments and hence curvatures in the regions near the column lines (the column strips) are greater than in the middle strips. This is particularly so for uncracked prestressed concrete slabs or cracked only in the column strips.



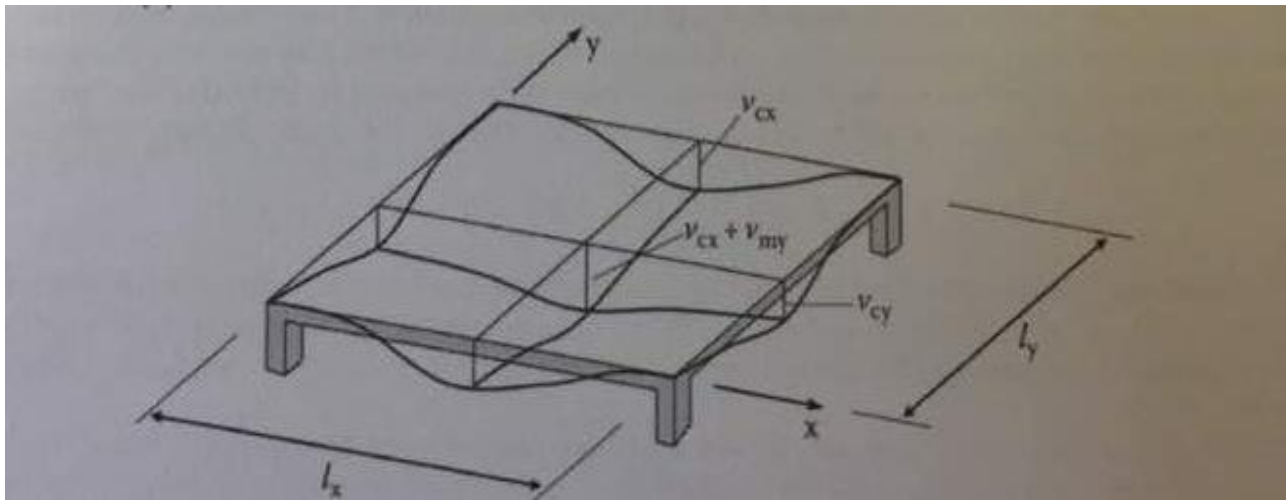
**The basis of the wide beam method : bending in x-direction**

The slab is next considered to act as a wide shallow beam spanning in the y-direction.



**The basis of the wide beam method : bending in y-direction**

- The mid-panel deflection is the sum of the mid-span deflection of the column strip in the long direction and that of the middle strip in the short direction (as shown in the following figure).



**The basis of the wide beam method : combined bending**

$$v_{mid} = v_{cx} + v_{my}$$

- The actual deflection calculations are more easily performed for strips of floor in either direction bounded by the panel centre lines, as is used for the moment analysis.
- In each direction, an average deflection  $v_{ave}$  at mid-span of the wide beam is calculated from the previously determined moment diagram and the moment of inertia of the entire wide beam  $I_{beam}$  using the deflection calculation procedure.
- For the wide beam in the x-direction, the column and middle strip deflections are;

$$v_{cx} = v_{avge.x} \frac{M_{col} E_{cm} I_{beam}}{M_{beam} E_{cm} I_{col}}$$

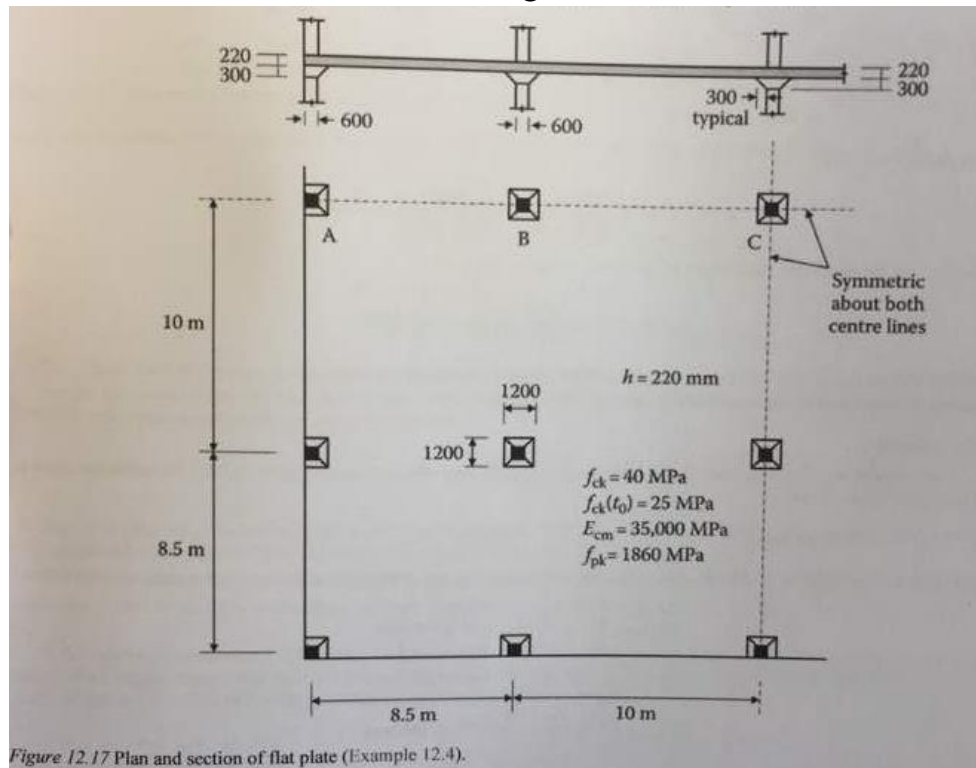
$$v_{mx} = v_{avge.x} \frac{M_{mid} E_{cm} I_{beam}}{M_{beam} E_{cm} I_{mid}}$$

Where  $M_{col}/M_{beam} = 0.7$ ,  $M_{mid}/M_{beam} = 0.3$ .

- A reasonable estimate of the weighted average effective moment of inertia of an interior span of the wide beam is obtained by taking 0.7 times the value at mid-span plus 0.3 times the average of values at each span.
- For an exterior span, a reasonable weighted average is 0.85 times the mid-span value plus 0.15 times the value at the continuous end.
- The moment of inertia of wide beam is the sum of  $I_{col}$  and  $I_{mid}$ .

# Example

- Determine the tendons required in the 220 mm thick flat slab shown in the following figure. The live load on the slab is 3.0 kPa and the dead load is 1.0 kPa plus the slab self-weight. All columns are 600 mm by 600 mm and are 4 m long above and below the slab. At the top of each column, a 300 mm column capital is used to increase the supported area, as shown. In this example, the dead load  $w_G$  is to be effectively balanced by prestress and is given by  $w_G = 1 \text{ kPa} + \text{self-weight} = 1 + (24 \times 0.22) = 6.3 \text{ kPa}$ .





# Solution

## 1. *Checking punching shear:*

Before proceeding too far into the design, it is prudent to make a preliminary check of punching shear at typical interior and exterior columns. Consider the interior column B in Figure 12.17. The area of slab supported by the column is  $10 \times (8.5 + 10)/2 = 92.5 \text{ m}^2$ . Using the strength load factors specified in EN 1992-1-1 [1] (Equation 2.2), the factored design load is:

$$w_{Ed} = 1.35w_G + 1.5w_Q = (1.35 \times 6.3) + (1.5 \times 3.0) = 13.0 \text{ kN/m}^2$$

and therefore the shear force crossing the critical section may be approximated by:

$$V_{Ed} \approx 13.0 \times 92.5 = 1203 \text{ kN}$$

$$M_{Ed} = 0.06[(1.35 \times 6.3 + 0.75 \times 3.0) \times 10 \times 9.02^2 - 1.35 \times 6.3 \times 10 \times 7.52^2]$$

The design value of the maximum punching shear resistance on any control section is obtained from Equation 7.45:

$$v_{Rd,max} = 0.3 \times \left[ 1 - \frac{40}{250} \right] \times 26.67 = 6.72 \text{ MPa}$$

Referring to Figure 7.18,  $l_{t1} = 300 \text{ mm}$ ,  $c = 600 \text{ mm}$  and  $l_1 = l_2 = 1200 \text{ mm}$ . The average effective depth around the shear perimeter is taken to be  $d_{eff} = 220 - 50 = 170 \text{ mm}$ , and from Equation 7.38,  $r_{cont} = 2 \times 170 + 0.56 \times 1200 = 1012 \text{ mm}$ . The critical shear perimeter is located at a distance  $r_{cont}$  from the centroid of the column (constructed so that its length is minimised). In each span direction, the critical shear perimeter is located at 412 mm from the edge of the loaded area (i.e. the edge of the column capital). The critical shear perimeter is therefore:

$$u_1 = 4 \times 600 + 2\pi \times 412 = 4989 \text{ mm}$$

The design punching shear resistance for a slab without shear reinforcement is obtained from Equation 7.42. With  $k = 2.0$ ,  $v_{min} = 0.035 \times k^{1.5} \times f_{ck}^{0.5} = 0.626$ . We will assume that  $p_1 = 0.006$  and the average prestress in the concrete is assumed to be  $\sigma_{cp} = 2.4 \text{ MPa}$ . These assumptions will need to be checked subsequently. Equation 7.42 gives:

$$v_{Rd,c} = 0 \times 2.0 \times (100 \times 0.006 \times 40)^{1/3} + 0 \times 2.4 = 0.932 \text{ MPa} \\ (> v_{min} + k_1 \sigma_{cp} |)$$

From Equation 7.49:

$$W_1 = 0.5 \times 600^2 + 600 \times 600 + 4 \times 600 \times 170 + 16 \times 170^2 + 2\pi \times 170 \times 600 \\ = 2051 \times 10^3 \text{ mm}^2$$

and from Equation 7.47 and Table 7.2:

$$\beta = 1 + 0.6 \times \frac{236 \times 10^6}{1203 \times 10^3} \times \frac{4989}{2051 \times 10^3} = 1.286$$

From Equation 7.46, the maximum shear stress on the basic control perimeter is:

$$v_{Ed} = 1.286 \times \frac{1203 \times 10^3}{4989 \times 170} = 1.82 \text{ MPa}$$

which is much less than  $v_{Rd,max}$ , but greater than  $v_{Rd,c}$ , and therefore shear reinforcement is required.

Punching shear at edge and corner columns should similarly be checked.

### 2. Establish cable profiles:

Using four 12.5 mm strands in a flat duct, with 25 mm concrete cover to the duct (the same as in Figure 12.12b), the maximum depth to the centre of gravity of the strand is:

$$d_p = 220 - (25 + 19 - 7) = 183 \text{ mm}$$

and the corresponding eccentricity is  $e = 73 \text{ mm}$ . The maximum cable drapes in an exterior span and in an interior span are, respectively:

$$\left(z_{d,max}\right)_{ext} = \frac{73}{2} + 73 = 109.5 \text{ mm}$$

and

$$\left(z_{d,max}\right)_{int} = \frac{73 + 73}{2} + 73 = 146 \text{ mm}$$

Consider the trial cable profile shown in Figure 12.18. For the purposes of this example, it is assumed that jacking occurs simultaneously from both ends of a tendon so that the prestressing force in a tendon is symmetrical with respect to the centre line of the structure (shown in Figure 12.17). The friction losses have been calculated using Equation 5.148 with  $\mu = 0.19$  and  $k = 0.016$  for flat ducts, and the losses due to a 6 mm draw-in at the anchorage are calculated as outlined in Section 5.10.2.4 using Equation 5.151. The immediate losses (friction + draw-in) are also shown in Figure 12.18.

### 3. Determine tendon layout:

It is assumed here that the average time-dependent loss of prestress in each low-relaxation tendon is 15%. Of course, this assumption should be checked.

The effective prestressing forces per metre width required to balance 6.3 kPa using the fully available drapes in the exterior span (AB) and in the interior span (BC) are found using Equation 12.10:

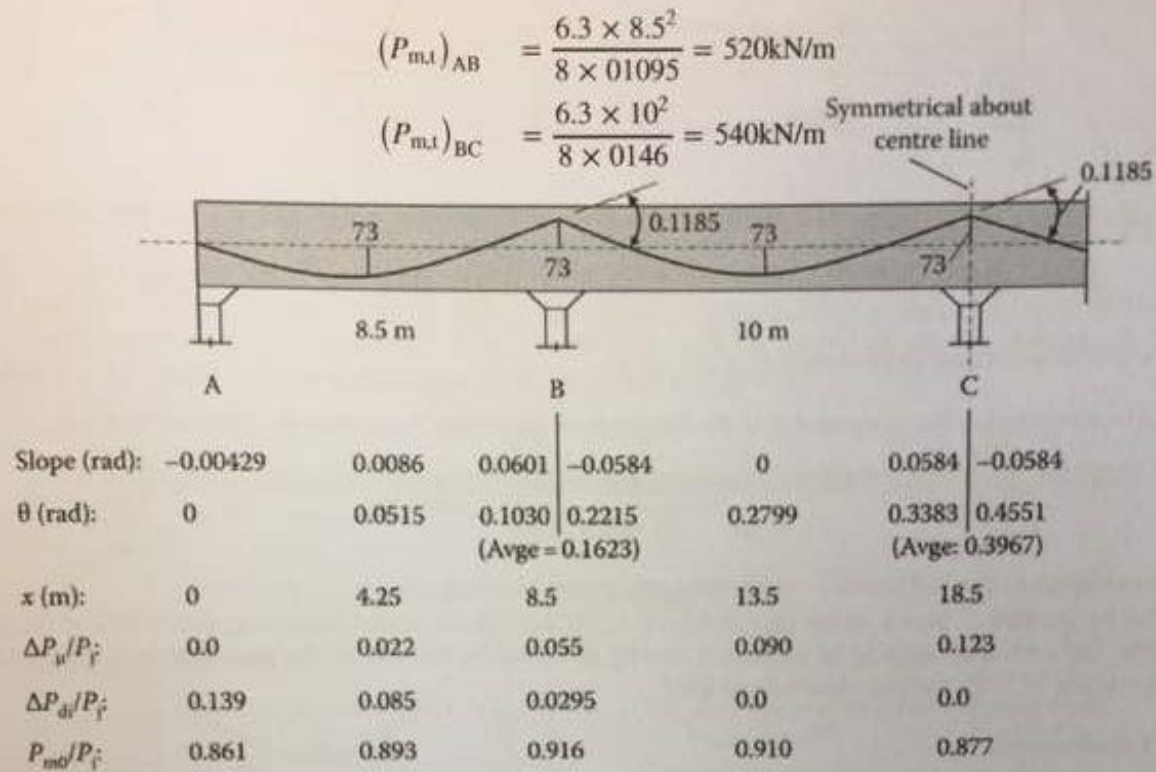


Figure 12.18 Cable profile and immediate loss details (Example 12.4).

and the corresponding forces required at the jack prior to the instantaneous and time-dependent losses are:

$$P_j = \frac{520}{0.890 \times 0.85} = 687 \text{ kN/m}$$

(required to balance 6.3 kPa in the exterior span)

$$P_j = \frac{540}{0.901 \times 0.85} = 705 \text{ kN/m}$$

(required to balance 6.3 kPa in the interior span)

The jacking force is therefore governed by the requirements for the interior span.

For the 10 m wide panel, the total jacking force required is  $705 \times 10 = 7050$  kN. If the maximum stress in the tendon is  $0.9f_{pk} = 1440$  MPa, the total area of prestressing steel is therefore:

$$A_p = \frac{7050 \times 10^3}{1440} = 4896 \text{ mm}^2$$

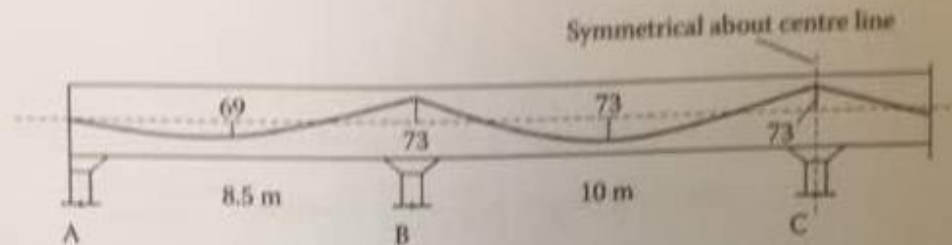
At least 14 flat-ducted cables are required in the 10 m wide panel (with the area of prestressing steel in each cable  $A_p = 372 \text{ mm}^2/\text{cable}$ ) with an initial jacking force of  $7050/14 = 504$  kN per cable (i.e.  $\sigma_{pj} = 0.728f_{pk}$ ).

The required jacking force in the 8.5 m wide panel is  $705 \times 8.5 = 5993$  kN and therefore  $A_p = 4162 \text{ mm}^2$ . At least 12 flat-ducted cables are needed in the 8.5 m wide panels ( $A_p = 4462 \text{ mm}^2$ ) with an initial jacking force of  $5993/12 = 500$  kN per cable (i.e.  $\sigma_{pj} = 0.722f_{pk}$ ).

In the interests of uniformity, all tendons will be initially stressed with a jacking force of 504 kN/cable (i.e.  $\sigma_{pj} = 0.728f_{pk}$ ). This means that a slightly higher load than 6.3 kPa will be balanced in the 10 m wide panels. The average prestress at the jack in each metre width of slab is  $(26 \times 504)/(8.5 + 10) = 708$  kN/m, and the revised drape in the exterior span is:

$$h_{AB} = \frac{6.3 \times 8.5^2}{8 \times 708 \times 0.890 \times 0.85} = 0.106 \text{ m}$$

The final cable profile and effective prestress per panel after all losses are shown in Figure 12.19.



For the 10 m wide panel:

$P_{\text{eff}}$ (kN):	5160	5351	5489	5453	5255
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Figure 12.19 Cable profile and effective prestress (Example 12.4).

The maximum average stress in the concrete due to the longitudinal anchorage force after the deferred losses is:

$$\frac{P}{A} = \frac{5,489 \times 10^3}{10,000 \times 220} = 2.50 \text{ MPa}$$

which is within the range mentioned in Section 12.3 as being typical for flat slabs.

The cable layout for the slab is shown on the plan in Figure 12.20. For effective load balancing, about 75% of the cables are located in the column strips. The minimum spacing of tendons is usually governed by the size of the anchorage and is taken here as 300 mm, while a maximum spacing of 1600 mm has also been adopted.



