

ON COMPLETE DERANGEMENT NUMBER; PROPERTIES, RELATIONSHIP, AND APPLICATION

Alaguia, Riden C., Alferez, Shaira H., Luzano, Jomar G.

Department: College of Arts and Science

Course: BS Mathematics

INTRODUCTION

Mathematics tends to be hated by most students for they found out that this is difficult to understand. Basically, it promotes and presents the consciousness of students in analytical and critical thinking as compared to other subjects.

On the other hand, mathematics is the language of the universe. It is the civilizations greatest achievement. Mathematics became the foundation of everything in the world. To figure out what is something within, scientific methods we used which is another form of mathematics. Mathematics is everywhere where most of the number of people think that mathematics is just about computation. But they don't know that this encompasses analysis, observation, inferring, and such.

Mathematics deals with everything. In nature, there are patterns. Most of these patterns are according to a pattern called Fibonacci Sequence. Aside from that, to attain symmetry in a lot of things, Mathematics is needed.

It only shows that Mathematics is everything.

The value of Mathematics plays an important role in the fast growing world. It used as effective and essential tool in many field including natural sciences. It is divided in different branches and one of its braches is Combinatorial Mathematics. Combinatorial Mathematics is branch of mathematics which is about where everyone could discover many exciting example of things they can count. First combinatorial problems have been studied by ancient Indian, Arabian, and Greek mathematics. Combinatorial Mathematics has many applications in other area of mathematics, including graph theory, coding and cryptography, and probability.

The derangement problem was formulated by Pierre Raymond de Montmort in 1708 and solved by him in 1713. Nicholas Bernoulli also solved the problem using inclusion-exclusion principle.

Pierre Raymond de Montmort was known for his book on probability and game of chance,

Essay d'analyse sur les jeux de hasard, which was also the the first to introduce the combinatorial study of derangements. He was also known for naming Pascal's Triangle after Blaise Pascal, calling it "Table de M. Pascal pour les combinations."

The researchers described or exposed this study to broaden their knowledge regarding Combinatorial Mathematics. They had chosen this topic because there's a lot of interesting applications that is worth studying.

GENERAL OBJECTIVE

SPECIFIC OBJECTIVES

STATEMENT OF THE PROBLEM

The purpose of this study is to discuss the definition of Complete Derangement number. Specially, it sought answer the following questions:

- 1. How Compete Derangement generated?
- 2. What are the properties of Compete Derangement Number?
- 3. Relation to other number, An,Bn and Mousetrap Number.
- 4. How Cayley Game of "mousetrap" be applied in Complete Derangement?

METHODOLOGY

The type of research used in this study is mathematical research. The appropriate methods are descriptive expository methods. The definitions were provided with illustrations for better understanding of the readers. The theorems were presented with a clear and understanding proof. Through this approaches the readers can comprehend the generalization and familiarized themselves highly with general steps, like proving the theorems.

Evaluation Method

The project was evaluated on the following criteria, namely: Functionality, Usability, Reliability, Efficiency, and Maintainability.

A. Statistical Treatment

The mean was used as the tool for evaluating the project.

The Formula is:

$$x = \Sigma X / N$$

Where:

Σ, represents the summation

X represents scores

N represents number of scores

The Likert scale was used for descriptive ratings.

Table 1: Likert Scale for descriptive ratings.

Numerical Scale	Average Response	Adjective Rating	Verbal Interpretation
5	4.50 – 5.00	Excellent	E
4	3.50 – 4.49	Very Satisfactory	VS
3	2.50 – 3.49	Satisfactory	S
2	1.50 – 2.49	Fair	F
1	1.00 – 1.49	Poor	P

Analysis of variance (ANOVA)

Analysis of variance (ANOVA) is a set of statistical models and estimate processes for analyzing variations between means. Ronald Fisher, a statistician, invented ANOVA.

$$F = \frac{MST}{MSE}$$

Where:

SUMMARY OF FINDINGS

The following were defined accordingly:

1. In combinatorial mathematics, a Complete Derangement is permutation of the elements of a set, such that no element appears in its original position. In other words, Complete Derangement of a set of size n , usually written D_n , is generated by using the formula

$$D_n = n! \left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^{n-1} \frac{1}{n!} \right)$$

The first 10 elements are (0, 1, 2, 9.44.265, 1854, 14833, 133496, 1334961)

2. Using the recurrence formula and the exponential generating function, the n th term of complete derangement can be derived.
3. The theorems were established in relation to the generating function of Complete Derangement Number. $D_0, D_1, D_2, D_3, \dots, D$ is sequence of Complete Derangement then its generating function $D(x)$ given by $D(x) = e^{-x}/(1-x)$
4. The Recurrence of Complete Derangement are given if the following:
 - 4.1 Let $A = \{1, 2, \dots, n\}$ be infinite n -element set of positive integers. Then the number of derangements D_n of A can be

obtained by $D_n = (n-1)(D_{n-1} + D_{n-2})$ for all $n > 3$ where $D_1 = 0, D_2 = 1$

4.2 Recursive Formula. Derangement Number defined recursively by the equation: $D_n = nD_{n-1} - 1(-1)^{n-2}$ where $D_1 = 0, D_2 = 1$ and $D_6 = 265$

5. The properties of Complete Derangement number are given for the following:
 - 5.1 The only prime Complete Derangement Number is 2.
 - 5.2 The number of zero fixed point to the permutation of n are the Complete Derangement.
 - 5.3 D_n is the number of permutation of $\{1, 2, 3, \dots\}$ having exactly small one ascent.
6. The number of A_n, B_n , and Mousetrap number are related to Complete Derangement Number because using the sequence the Complete Derangement the said numbers will be formulated. These are the formula of A_n and b_n .
 - 6.1 $A_n = D_n + D_{n-1}$, where $A_n = nA_{n-1} + (n-1)A_{n-2}$.
 - 6.2 $D_n = b_n - n!$, where $B_n = nb_{n-1} - (-1)^n$.
 - 6.3 $M(n, 2) = D(n-1) - D(n-2) - 2D(n-3)$.
7. The Complete Derangement can applied to Game of Mousetrap with n Card, Mousetrap is the name of a game introduced by the English mathematician Arthur Cayley. In the game, cards numbered 1 through n are shuffled to place them in some random permutation and are arranged in a circle with their faces up. Then, starting with the first card, the player begins counting 1, 2, 3... and moving to the card as the count is incremented. If at any point the player's

current, count matches the number on the card currently being pointed to, that card is removed from the circle and the player starts all over at 1 on the next card. If the player ever removes all of the cards from the permutation, in this manner, then the player wins. If the player reaches the count $n+1$ and cards still remain, then the game is lost.

CONCLUSIONS

Based from the findings of the study, the following conclusions are formulated.

1. Constructing the Complete Derangement is given by the formula.

$$D_n = n!$$

$$(1 - 1/1! + 1/2 - 1/3 + \dots + (-1)^{n+1}/n!)$$

2. The sequence can be formulated using the Recurrences;

$$D_n = (n-1)(D_{n-1} + D_{n-2}) \quad \text{and} \quad D_n = nD_{n-1} + (-1)^{n-2}.$$

3. Complete Derangement Number can be generated by generating function.
4. The relationship of Complete Derangement has relation to A_n, B_n and Mousetrap Number.
5. The properties of Complete Derangement are used to solve its term.
6. The Complete Derangement can be applied to Cayley's Game of Mousetrap.

RECOMMENDATION

The summary of findings as well as the conclusion led to the formulation of the following recommendations;

1. Further study on Complete Derangement Number.
2. A relationship on the A_n number B_n number maybe investigated
3. Discover other properties of Complete Derangement Number.
4. Further study about the Cayley's Games of Mousetrap,