MATH 262 Linear Algebra Lecture Notes of Özgür Kişisel Week 1

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1 Introduction

2 Free Vector Space Over a Set

Definition 2.1: Let S be a set, F be a field. The set $\mathcal{F}(S) \subseteq Fun(S, F)$ such that

$\mathcal{F}(S) = \{f \in Fun(S, F) | f(s) = 0 \text{ except (possibly) for finitely many } s\}$

Then $\mathcal{F}(S)$ is called the free vector space over S

Proposition 2.1: $\mathcal{F}(S)$ is a subspace of $Fun(S, F)$

2.1 Characteristic Function - Basis for $\mathcal{F}(S)$

Let S be any finite set with n elements and F a field. Let $V = Fun(S, F)$, the set of functions from S to F, viewed as a vector space over F. For every $s \in S$, we can define a characteristic function $\mathcal{X}_s \in Fun(S, F)$ so that

$$\mathcal{X}_s = \begin{cases} 0 & \text{if } s \neq s \\ 1 & \text{otherwise} \end{cases}$$

Suppose that $S = \{s_1, s_2, ..., s_n\}$. We claim that $B = \{\mathcal{X}_{s_1}, \mathcal{X}_{s_2}, ..., \mathcal{X}_{s_n}\}$ is a basis for V over F. Let us first show linear independence. Suppose that $c1\mathcal{X}_{s_1} + ... + cn\mathcal{X}_{s_n} = 0$. This is an equality of functions. Applying both sides to s_i gives $c_i = 0$. Since this is true for every i, we deduce that B is linearly independent. Suppose now that $f \in Fun(S, F)$. We claim that $f = f(s_1)\mathcal{X}_{s_1} + f(s_2)\mathcal{X}_{s_2} + ... + f(s_n)\mathcal{X}_{s_n}$

2.2 Universal Property of $\mathcal{F}(S)$

3 Tensor Product of Two Vector Spaces

3.1 Bilinear Maps

Proposition

3.2 Universal Property of Tensor Product

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3.4 Symmetric Powers of a Vector Space