MATH 251 Advanced Calculus Lecture Notes of Y. Week 1

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Contents

1 Topological Terminology

1.1 Distance between p and q

Definition 1.1: Let $p = (x_1, \ldots, x_n), p = (y_1, \ldots, y_n) \in \mathbb{R}^n$

Then the distance between p and q, denoted $||p - q||$, is given by

$$||p - q|| = \sqrt{\sum_{i=1}^{n} (x_i - y_i)^2}$$

1.2 Open Ball

Definition 1.2: Let $p_0 \in \mathbb{R}^n$ and $r > 0$. The open ball of radius r centered at p_0 is the set
$B(p_0, r) = \{p \in \mathbb{R}^n : ||p - p_0|| < r\}$

1.3 Interior of S

Definition 1.3: Let $S \subseteq \mathbb{R}^n$. The interior of S, denoted $int(S)$, is the set $int(S) = \{p \in S : \exists r > 0, B(p, r) \subseteq S\}$

1.3.1 Open Set

1.4 Exterior of S

1.4.1 Closed Set

1.5 Boundary of S

1.6 Closure of S

1.7 Bounded and Unbounded sets

Theorem

Proposition