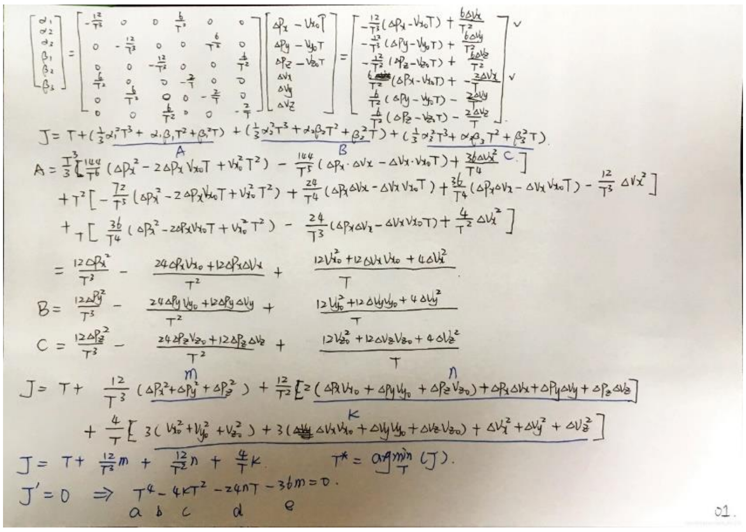
**Optimal analytic expression of T：**



$$\begin{bmatrix}\alpha_1\\ \alpha_2\\ \alpha_3\\ \beta_1\\ \beta_2\\ \beta_3\end{bmatrix} = \begin{bmatrix} -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0 & 0\\ 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2} & 0\\ 0 & 0 & -\frac{12}{T^3} & 0 & 0 & \frac{6}{T^2}\\ \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0 & 0\\ 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T} & 0\\ 0 & 0 & \frac{6}{T^2} & 0 & 0 & -\frac{2}{T}\end{bmatrix}\begin{bmatrix}\Delta P_x - V_{x0}T\\ \Delta P_y - V_{y0}T\\ \Delta P_z - V_{z0}T\\ \Delta V_x\\ \Delta V_y\\ \Delta V_z\end{bmatrix} = \begin{bmatrix} -\frac{12}{T^3}(\Delta P_x - V_{x0}T) + \frac{6\Delta V_x}{T^2}\\ -\frac{12}{T^3}(\Delta P_y - V_{y0}T) + \frac{6\Delta V_y}{T^2}\\ -\frac{12}{T^3}(\Delta P_z - V_{z0}T) + \frac{6\Delta V_z}{T^2}\\ \frac{6}{T^2}(\Delta P_x - V_{x0}T) - \frac{2\Delta V_x}{T}\\ \frac{6}{T^2}(\Delta P_y - V_{y0}T) - \frac{2\Delta V_y}{T}\\ \frac{6}{T^2}(\Delta P_z - V_{z0}T) - \frac{2\Delta V_z}{T}\end{bmatrix}$$

$$J = T + \underbrace{(\tfrac{1}{3}\alpha_1^2T^3 + \alpha_1\beta_1T^2 + \beta_1^2T)}_{A} + \underbrace{(\tfrac{1}{3}\alpha_2^2T^3 + \alpha_2\beta_2T^2 + \beta_2^2T)}_{B} + \underbrace{(\tfrac{1}{3}\alpha_3^2T^3 + \alpha_3\beta_3T^2 + \beta_3^2T)}_{C}$$

$$A = \frac{T^3}{3}\left[\frac{144}{T^6}(\Delta P_x^2 - 2\Delta P_x V_{x0}T + V_{x0}^2T^2) - \frac{144}{T^5}(\Delta P_x\cdot\Delta V_x - \Delta V_x\cdot V_{x0}T) + \frac{36\Delta V_x^2}{T^4}\right]$$
$$+T^2\left[-\frac{72}{T^5}(\Delta P_x^2 - 2\Delta P_x V_{x0}T + V_{x0}^2T^2) + \frac{24}{T^4}(\Delta P_x\Delta V_x - \Delta V_x V_{x0}T) + \frac{36}{T^4}(\Delta P_x\Delta V_x - \Delta V_x V_{x0}T) - \frac{12}{T^3}\Delta V_x^2\right]$$
$$+T\left[\frac{36}{T^4}(\Delta P_x^2 - 2\Delta P_x V_{x0}T + V_{x0}^2T^2) - \frac{24}{T^3}(\Delta P_x\Delta V_x - \Delta V_x V_{x0}T) + \frac{4}{T^2}\Delta V_x^2\right]$$
$$= \frac{12\Delta P_x^2}{T^3} - \frac{24\Delta P_x V_{x0} + 12\Delta P_x\Delta V_x}{T^2} + \frac{12V_{x0}^2 + 12\Delta V_x V_{x0} + 4\Delta V_x^2}{T}$$
$$B = \frac{12\Delta P_y^2}{T^3} - \frac{24\Delta P_y V_{y0} + 12\Delta P_y\Delta V_y}{T^2} + \frac{12V_{y0}^2 + 12\Delta V_y V_{y0} + 4\Delta V_y^2}{T}$$
$$C = \frac{12\Delta P_z^2}{T^3} - \frac{24\Delta P_z V_{z0} + 12\Delta P_z\Delta V_z}{T^2} + \frac{12V_{z0}^2 + 12\Delta V_z V_{z0} + 4\Delta V_z^2}{T}$$
$$J = T + \frac{12}{T^3}\overbrace{(\Delta P_x^2 + \Delta P_y^2 + \Delta P_z^2)}^{m} + \frac{12}{T^2}\Big[\overbrace{2(\Delta P_x V_{x0} + \Delta P_y V_{y0} + \Delta P_z V_{z0}) + \Delta P_x\Delta V_x + \Delta P_y\Delta V_y + \Delta P_z\Delta V_z}^{n}\Big]$$
$$+ \frac{4}{T}\Big[\underbrace{3(V_{x0}^2 + V_{y0}^2 + V_{z0}^2) + 3(\Delta V_x V_{x0} + \Delta V_y V_{y0} + \Delta V_z V_{z0}) + \Delta V_x^2 + \Delta V_y^2 + \Delta V_z^2}_{K}\Big]$$
$$J = T + \frac{12}{T^3}m + \frac{12}{T^2}n + \frac{4}{T}K \qquad T^* = \underset{T}{\arg\min}(J).$$
$$J' = 0 \Rightarrow \underset{a}{T^4} - \underset{b}{} \underset{c}{4KT^2} - \underset{d}{24nT} - \underset{e}{36m} = 0.$$

01.

**Solve using the accompanying matrix of the polynomial:**

Polynomial：

$$p(x) = x^n + c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_1x + c_0$$

Its accompaniment matrix:

$$M_x = \begin{bmatrix} 0 & 0 & \dots & 0 & -c_0\\ 1 & 0 & \dots & 0 & -c_1\\ 0 & 1 & \dots & 0 & -c_2\\ \dots & \dots & \dots & \dots & \dots\\ 0 & 0 & \dots & 1 & -c_{n-1}\end{bmatrix}$$

The feature value is the root of P (x). So use EIGEN for polynomial solution.

```
matrix_44 << 0, 0, 0, -c0,
             1, 0, 0, -c1,
             0, 1, 0, -c2,
             0, 0, 1,  c3;
matrix_eigenvalues = matrix_44.eigenvalues();
```

**Result：**