



深蓝学院
shenlanxueyuan.com

第三章作业思路提示



主讲人 陈铭楠



- 第一部分：MATLAB实现RRT
- 第二部分：C++实现RRT*
- 第三部分：C++实现Informed RRT*

MATLAB实现RRT

●这一部分具体流程可参考代码注解

```
for iter = 1:3000
    x_rand=[0 + xL * rand(1,1), 0 + yL * rand(1,1)];
    %Step 1: 在地图中随机采样一个点x_rand
    %提示: 用 (x_rand(1),x_rand(2)) 表示环境中采样点的坐标

    %Step 2: 遍历树, 从树中找到最近邻近点x_near
    %提示: x_near已经在树T里
    %put the distance into and array
    distArray = arrayfun(@(x) getdis(x_rand(1),x_rand(2),x.x,x.y),T.v);
    %The ~ represents an output that is discarded. It is essentially equivalent to
    %reference: https://de.mathworks.com/matlabcentral/answers/73735-what-does-it-mean-by-writing-idx-in-code
    [~,idx]=min(distArray);
    x_near=[T.v(idx).x,T.v(idx).y];

    %x_new=[];
    %Step 3: 扩展得到x_new节点
    %提示: 注意使用扩展步长Delta
    %The direction of extension theta
    theta = atan2(x_rand(2) - x_near(2),x_rand(1) - x_near(1));
    x_new = [x_near(1)+Delta*cos(theta),x_near(2)+Delta*sin(theta)];

    %检查节点是否是collision-free
    if ~collisionChecking(x_near,x_new,lmp)
        continue;
    end
    count=count+1;

    if count>2000
        bFind = false;
        end
```

```
%Step 4: 将x_new插入树T
%提示: 新节点x_new的父节点是x_near
T.v(count).x = x_new(1);
T.v(count).y = x_new(2);
T.v(1).xPrev = x_near(1); % 父节点改为x_near
T.v(1).yPrev = x_near(2);
T.v(1).dist=Delta; % 从父节点到该节点的距离, 这里可取欧氏距离
T.v(1).indPrev = idx; % 编号
```

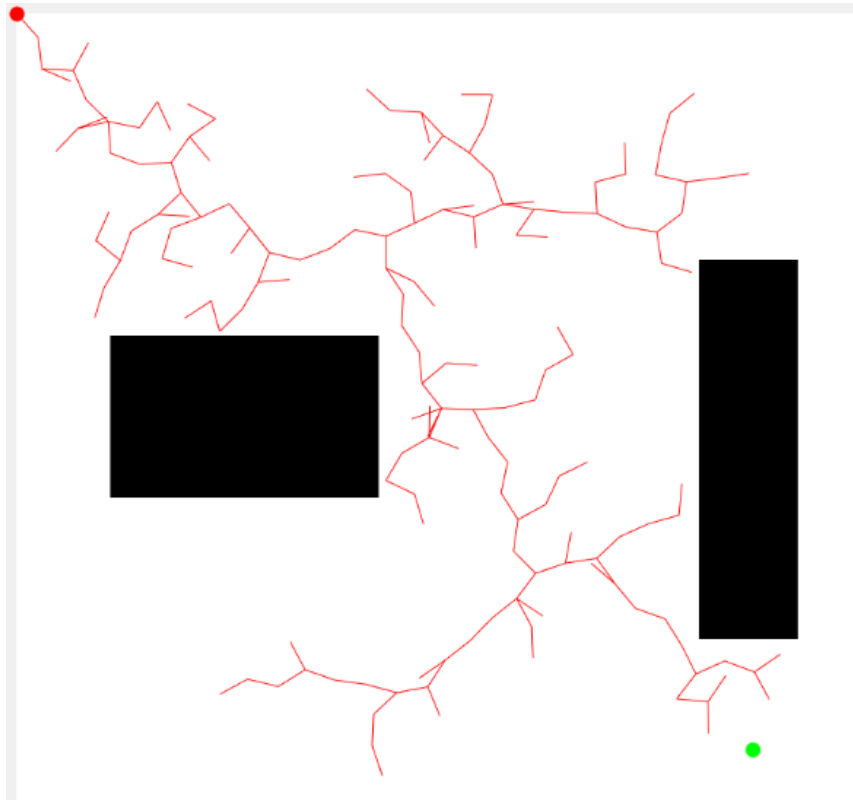
```
%Step 5:检查是否到达目标点附近
%提示: 注意使用目标点阈值Thr, 若当前节点和终点的欧式距离小于Thr, 则跳出当前for循环
if getdis(x_new(1),x_new(2),x_G,y_G)<Thr
    plot([x_near(1),x_new(1)],[x_near(2),x_new(2)],'r');
    hold on;
    break;
end
```

```
%Step 6:将x_near和x_new之间的路径画出来
%提示 1: 使用plot绘制, 因为要多次在同一张图上绘制线段, 所以每次使用plot后需要接上hold on命令
%提示 2: 在判断终点条件弹出for循环前, 记得把x_near和x_new之间的路径画出来
plot([x_near(1),x_new(1)],[x_near(2),x_new(2)],'r');
hold on;
```

```
pause(0.05); %暂停一会, 使得RRT扩展过程容易观察
end
```

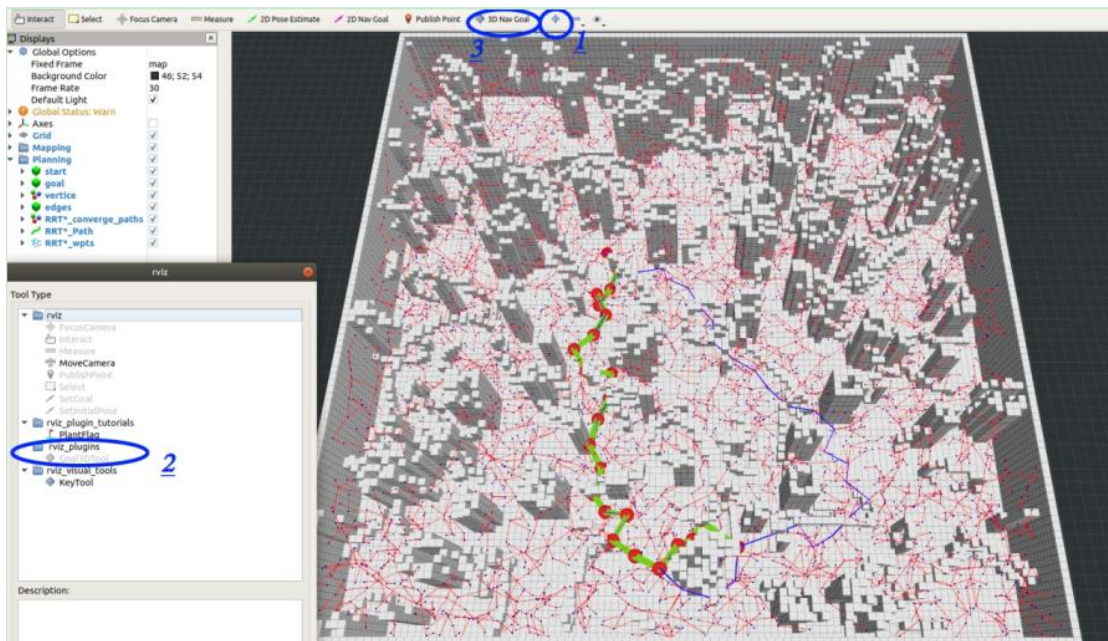
MATLAB实现RRT

- 效果如下图



C++实现RRT*

- 参考《hw3作业说明》创建工作空间与编译（这一部分比较简单，这里不做赘述）。按照文档要求操作，将能成功实现RRT效果。



能执行到这一步，说明环境配置没问题，接下来进行RRT*代码编写。

C++实现RRT*

- 首先我们梳理下RRT*的代码流程，其伪代码如下图：

Algorithm 2: RRT*Algorithm

Input: $\mathcal{M}, x_{init}, x_{goal}$

Result: A path Γ from x_{init} to x_{goal}

$\mathcal{T}.init();$

for $i = 1$ **to** n **do**

$x_{rand} \leftarrow Sample(\mathcal{M});$

$x_{near} \leftarrow Near(x_{rand}, \mathcal{T});$

$x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize);$

if $CollisionFree(x_{new})$ **then**

$X_{near} \leftarrow NearC(\mathcal{T}, x_{new});$

$x_{min} \leftarrow ChooseParent(X_{near}, x_{near}, x_{new});$

$\mathcal{T}.addNodeEdge(x_{min}, x_{new});$

$\mathcal{T}.rewire();$

作业需要补全rrt_star.h/rrt_star()函数中两处空白部分，分别为ChooseParent部分和rewire部分。

●ChooseParent部分：注解提示：

// ! Hints:

// ! 1. Use `map_ptr->isSegmentValid(p1, p2)` to check line edge validity;

// ! 2. Default parent is `[nearest_node]`;

// ! 3. Store your chosen parent-node-pointer, the according cost-from-parent and cost-from-start

// ! in `[min_node]`, `[cost_from_p]`, and `[min_dist_from_start]`, respectively;

// ! 4. [Optional] You can sort the potential parents first in increasing order by cost-from-start value;

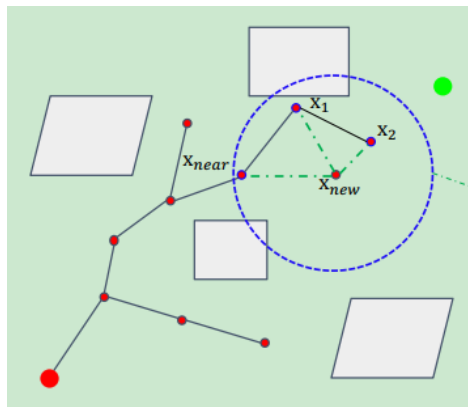
// ! 5. [Optional] You can store the collision-checking results for later usage in the Rewire procedure

● ChooseParent部分代码讲解:

```
double next_node_cost_start;  
for (auto &curr_node : neighbour_nodes)  
{  
    if (map_ptr->isSegmentValid(p0: x_new, p1: curr_node->x) && (curr_node->cost_from_start +  
        calDist(p1: curr_node->x, p2: x_new)) < min_dist_from_start){  
        min_node = curr_node;  
        cost_from_p = calDist(p1: curr_node->x, p2: x_new);  
        next_node_cost_start = curr_node->cost_from_start + cost_from_p;  
        min_dist_from_start = next_node_cost_start;  
    }  
}
```

首先 for (auto &curr_node : neighbour_nodes) 表示的是遍历 x_{new} 周围的所有的neighbour_node。该循环的目的在于：在所有curr_node中找到最小的dist_from_start，然后记录下min_dist_from_start，cost_from_p和min_node

以右图为例，neighbour_nodes有 x_{near} , x_1 , x_2



接下来for循环每执行一次，将检查两个条件：

1. x_{new} 与当前计算的neighbour_node的连线(图中某条绿色连线)是否穿过障碍物
2. 判断该neighbour_node的cost是否会比初始的最小cost（以 x_{near} 为父节点， x_{new} 为子节点的cost）小。

●rewire部分代码讲解：

```
/* 3.rewire */  
    // TODO Rewire according to potential cost-from-start values  
    // ! Hints:  
    // ! 1. Use map_ptr->isSegmentValid(p1, p2) to check line edge validity;  
    // ! 2. Use changeNodeParent(node, parent, cost_from_parent) to change  
    a node's parent;  
    // ! 3. the variable [new_node] is the pointer of X_new;  
    // ! 4. [Optional] You can test whether the node is promising before  
    checking edge collision.  
    // ! Implement your own code between the dash lines [-----] in the  
    following loop
```

●rewire部分代码讲解：

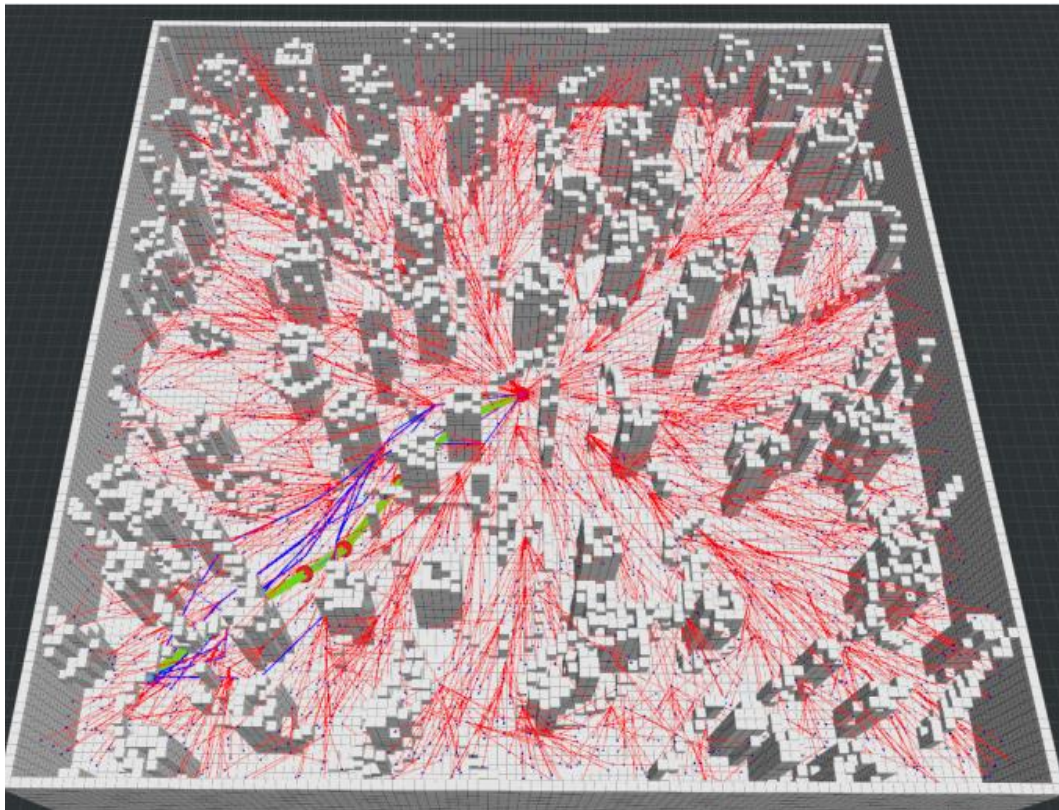
```
for (auto &curr_node:TreeNode* & : neighbour_nodes)
{
    double best_cost_before_rewire = goal_node->cost_from_start;
    // ! -----start write-----
    double dist_start_xnew_currentnode = new_node->cost_from_start + calDist(p1: curr_node->x, p2: x_new);
    if(dist_start_xnew_currentnode < curr_node->cost_from_start && map_ptr->isSegmentValid(p0: curr_node->x, p1: x_new)){
        curr_node->parent = new_node;
        curr_node->cost_from_start = dist_start_xnew_currentnode;
    }
    // ! -----end write-----
}
```

主要过程如下：

1. 计算curr_node与new_node的距离
2. 计算curr_node通过new_node连接与起点的距离dist_start_xnew_currentnode
3. 对比curr_node按照原路径到起点的距离curr_node->cost_from_start和dist_start_xnew_currentnode，若dist_start_xnew_currentnode更小，则更换curr_node的父节点为new_node

C++实现RRT*

●效果如下：



C++实现Informed RRT*

●伪代码如右图：

可以发现，Informed RRT*的实现思路和RRT*思路基本一样，不同的是采样函数的不同，Informed RRT*利用椭圆的特殊性质，在椭圆/球区域内采样能更快地获得最优路径。

Algorithm 1: Informed RRT*($\mathbf{x}_{\text{start}}, \mathbf{x}_{\text{goal}}$)

```
1  $V \leftarrow \{\mathbf{x}_{\text{start}}\};$ 
2  $E \leftarrow \emptyset;$ 
3  $X_{\text{soln}} \leftarrow \emptyset;$ 
4  $\mathcal{T} = (V, E);$ 
5 for iteration = 1 ...  $N$  do
6    $c_{\text{best}} \leftarrow \min_{\mathbf{x}_{\text{soln}} \in X_{\text{soln}}} \{\text{Cost}(\mathbf{x}_{\text{soln}})\};$ 
7    $\mathbf{x}_{\text{rand}} \leftarrow \text{Sample}(\mathbf{x}_{\text{start}}, \mathbf{x}_{\text{goal}}, c_{\text{best}});$ 
8    $\mathbf{x}_{\text{nearest}} \leftarrow \text{Nearest}(\mathcal{T}, \mathbf{x}_{\text{rand}});$ 
9    $\mathbf{x}_{\text{new}} \leftarrow \text{Steer}(\mathbf{x}_{\text{nearest}}, \mathbf{x}_{\text{rand}});$ 
10  if CollisionFree( $\mathbf{x}_{\text{nearest}}, \mathbf{x}_{\text{new}}$ ) then
11     $V \leftarrow V \cup \{\mathbf{x}_{\text{new}}\};$ 
12     $X_{\text{near}} \leftarrow \text{Near}(\mathcal{T}, \mathbf{x}_{\text{new}}, r_{\text{RRT}^*});$ 
13     $\mathbf{x}_{\text{min}} \leftarrow \mathbf{x}_{\text{nearest}};$ 
14     $c_{\text{min}} \leftarrow \text{Cost}(\mathbf{x}_{\text{min}}) + c \cdot \text{Line}(\mathbf{x}_{\text{nearest}}, \mathbf{x}_{\text{new}});$ 
15    for  $\forall \mathbf{x}_{\text{near}} \in X_{\text{near}}$  do
16       $c_{\text{new}} \leftarrow \text{Cost}(\mathbf{x}_{\text{near}}) + c \cdot \text{Line}(\mathbf{x}_{\text{near}}, \mathbf{x}_{\text{new}});$ 
17      if  $c_{\text{new}} < c_{\text{min}}$  then
18        if CollisionFree( $\mathbf{x}_{\text{near}}, \mathbf{x}_{\text{new}}$ ) then
19           $\mathbf{x}_{\text{min}} \leftarrow \mathbf{x}_{\text{near}};$ 
20           $c_{\text{min}} \leftarrow c_{\text{new}};$ 
21     $E \leftarrow E \cup \{(\mathbf{x}_{\text{min}}, \mathbf{x}_{\text{new}})\};$ 
22    for  $\forall \mathbf{x}_{\text{near}} \in X_{\text{near}}$  do
23       $c_{\text{near}} \leftarrow \text{Cost}(\mathbf{x}_{\text{near}});$ 
24       $c_{\text{new}} \leftarrow \text{Cost}(\mathbf{x}_{\text{new}}) + c \cdot \text{Line}(\mathbf{x}_{\text{new}}, \mathbf{x}_{\text{near}});$ 
25      if  $c_{\text{new}} < c_{\text{near}}$  then
26        if CollisionFree( $\mathbf{x}_{\text{new}}, \mathbf{x}_{\text{near}}$ ) then
27           $\mathbf{x}_{\text{parent}} \leftarrow \text{Parent}(\mathbf{x}_{\text{near}});$ 
28           $E \leftarrow E \setminus \{(\mathbf{x}_{\text{parent}}, \mathbf{x}_{\text{near}})\};$ 
29           $E \leftarrow E \cup \{(\mathbf{x}_{\text{new}}, \mathbf{x}_{\text{near}})\};$ 
30    if InGoalRegion( $\mathbf{x}_{\text{new}}$ ) then
31       $X_{\text{soln}} \leftarrow X_{\text{soln}} \cup \{\mathbf{x}_{\text{new}}\};$ 
32 return  $\mathcal{T};$ 
```

C++实现Informed RRT*

●伪代码

在椭球区域采样过程如伪代码所示：

Algorithm 2: Sample ($\mathbf{x}_{\text{start}}, \mathbf{x}_{\text{goal}}, c_{\text{max}}$)

```
1 if  $c_{\text{max}} < \infty$  then
2    $c_{\text{min}} \leftarrow \|\mathbf{x}_{\text{goal}} - \mathbf{x}_{\text{start}}\|_2$ ;
3    $\mathbf{x}_{\text{centre}} \leftarrow (\mathbf{x}_{\text{start}} + \mathbf{x}_{\text{goal}}) / 2$ ;
4    $\mathbf{C} \leftarrow \text{RotationToWorldFrame}(\mathbf{x}_{\text{start}}, \mathbf{x}_{\text{goal}})$ ;
5    $r_1 \leftarrow c_{\text{max}} / 2$ ;
6    $\{r_i\}_{i=2,\dots,n} \leftarrow (\sqrt{c_{\text{max}}^2 - c_{\text{min}}^2}) / 2$ ;
7    $\mathbf{L} \leftarrow \text{diag}\{r_1, r_2, \dots, r_n\}$ ;
8    $\mathbf{x}_{\text{ball}} \leftarrow \text{SampleUnitNBall}$ ;
9    $\mathbf{x}_{\text{rand}} \leftarrow (\mathbf{C}\mathbf{L}\mathbf{x}_{\text{ball}} + \mathbf{x}_{\text{centre}}) \cap X$ ;
10 else
11    $\mathbf{x}_{\text{rand}} \sim \mathcal{U}(X)$ ;
12 return  $\mathbf{x}_{\text{rand}}$ ;
```

构造椭球的过程如下：

1. **unit ball**(单位球): 首先在单位圆内均匀采样（单位圆内均匀采样是有方法的，可以去搜一下）。
2. **Scale**: 对单位球进行放缩，也就是长轴和短轴拉伸一下，呈椭球形状。
3. **Rotation**: 做一个旋转。为求得旋转矩阵 \mathbf{C} ，我们可以求出旋转后的椭球相对于原来椭球的欧拉角，再通过欧拉角转旋转矩阵的函数求得旋转矩阵 \mathbf{C} 。通过求起点与终点的连线的向量求出yaw和pitch角（椭球长轴方向），由于椭球的长轴为对称轴，因此roll可取任意值。
4. **Translation**: 做一个平移。平移量为起点终点连线的中点。

C++实现Informed RRT*

●代码（个人作业代码仅供参考，欢迎交流~）

复制rrt_star.h文件并重命名为informed_rrt_star.h

旋转矩阵函数如下图：

```
//void Sample_infrt_star()
static Eigen::Matrix3d euler2RotationMatrix(const double roll, const double pitch, const double yaw)
{
    Eigen::AngleAxisd rollAngle( angle: roll, axis: Eigen::Vector3d::UnitZ());
    Eigen::AngleAxisd yawAngle( angle: yaw, axis: Eigen::Vector3d::UnitY());
    Eigen::AngleAxisd pitchAngle( angle: pitch, axis: Eigen::Vector3d::UnitX());

    Eigen::Quaterniond q = rollAngle * yawAngle * pitchAngle;
    Eigen::Matrix3d R = q.matrix();
    cout << "Euler2RotationMatrix result is:" <<endl;
    cout << "R = " << endl << R << endl<<endl;
    return R;
}
```

C++实现Informed RRT*

●代码（个人作业代码仅供参考，欢迎交流~）

由于我重写单位球采样的函数，因此我将此句注解：
#include “sampler.h”,实现代码添加如下：

```
bool rrt_star(const Eigen::Vector3d &s, const Eigen::Vector3d &g)
{
    ros::Time rrt_start_time = ros::Time::now();
    bool goal_found = false;
    const double pai = 3.141596;

    std::random_device rd;
    std::mt19937_64 gen_(sd::rd());
    std::uniform_real_distribution<double> uniform_rand_;
    uniform_rand_ = std::uniform_real_distribution<double>(a:0, b:2.0);

    /* kd tree init */
    kdtree *kd_tree = kd_create(k:3);
    //Add start and goal nodes to kd tree
    kd_insert3(tree:kd_tree, x:start_node->x[0], y:start_node->x[1], z:start_node->x[2], data:start_node_);
    //informed RRT*:
    //Sample(x_start,x_goal,Cmax)
    //ellipse: (x_start,0) and (x_goal,0),
    //STEP1: c_min
    double c_min = calDist(p1:start_node->x, p2:goal_node->x);
    goal_node->cost_from_start = 1.4 * c_min;
    double c_best = goal_node->cost_from_start;
    //double c_best = 1000;
    cout << "goal_cost:"<<goal_node->cost_from_start<<endl;
    //STEP2: the centre point between start_node and goal_node
    Eigen::Vector3d x_centre = (start_node->x + goal_node->x)/2.0;
    //STEP4:rotation matrix
```

C++实现Informed RRT*

- 代码（个人作业代码仅供参考，欢迎交流~）

```
double e_yaw = atan2(y: goal_node_ -> x[1] - start_node_ -> x[1], x: goal_node_ -> x[0] - start_node_ -> x[0]); // y/x
if(goal_node_ -> x[0] - start_node_ -> x[0] < 0 && goal_node_ -> x[1] - start_node_ -> x[1] > 0){
    e_yaw += pai;
}
if(goal_node_ -> x[0] - start_node_ -> x[0] < 0 && goal_node_ -> x[1] - start_node_ -> x[1] < 0){
    e_yaw -= pai;
}
double e_pitch = atan2(y: goal_node_ -> x[1] - start_node_ -> x[1], x: std::sqrt(x: std::pow(x: (goal_node_ -> x[0] - start_node_ -> x[0]), y: 2)
    + std::pow(x: (goal_node_ -> x[1] - start_node_ -> x[1]), y: 2)));
double e_roll = 0.01;
Eigen::Matrix3d Rotation_matrix = euler2RotationMatrix(roll: e_roll, pitch: e_pitch, yaw: e_yaw);
```


C++实现Informed RRT*

● 代码

```
/* main loop */
int idx = 0;
for (idx = 0; (ros::Time::now() - rrt_start_time).toSec() < search_time_ && valid_tree_node_nums_ < max_tree_node_nums_; ++idx)
{
    /* biased random sampling */
    //STEP3
    double r1 = c_best / 2.0; // Cmax/2
    //cout << "c_best"<<c_best<<endl;
    //cout << "idx:"<<idx<<endl;
    double r2 = std::sqrt(x: std::pow(x: goal_node_ -> cost_from_start, y: 2) - std::pow(x: c_min, y: 2))/2;
    double r3 = r2;

    //STEP 5: L_ellipse <- diag
    Eigen::Matrix<double,3,3> L_ellipse;
    L_ellipse << r1,0,0,
                0,r2,0,
                0,0,r3;

    //STEP 6: x_ball <- SampleUnitNball    x_ball=x_rand
    Eigen::Vector3d x_rand;
    /.../
    //sampler_.samplingOnce(x_rand);
    x_rand[0] = uniform_rand_(&c: gen_);
    x_rand[1] = uniform_rand_(&c: gen_);
    x_rand[2] = uniform_rand_(&c: gen_);
    x_rand[0] = x_rand[0] -1.0;
    x_rand[1] = x_rand[1] -1.0;
    x_rand[2] = x_rand[2] -1.0;
```

C++实现Informed RRT*

●代码

```
312 x_rand[2] = x_rand[2] -1.0;
313 //cout <<"x_rand"<<x_rand<<endl;
314 if(std::sqrt(x_rand[0]*x_rand[0] + x_rand[1]*x_rand[1] + x_rand[2]*x_rand[2]) > 1.0 ){
315     continue;
316 }
317 //cout <<"x_rand"<<x_rand<<endl;
318 //STEP 7:x_rand <- (Rotation_matrix * L * x_ball) x_ball=x_rand 3*1 = 3*3 3*3 3*1
319 //x_rand = Rotation_matrix * L_ellipse * x_rand + x_centre;
320 x_rand[0] = x_rand[0]*r1 ;
321 x_rand[1] = x_rand[1]*r2 ;
322 x_rand[2] = x_rand[2]*r3 ;
323 x_rand = Rotation_matrix * x_rand + x_centre;
324 //cout <<"x_rand"<<x_rand<<endl;
325 // samplingOnce(x_rand);
326 if (!map_ptr->isStateValid(x_rand))
327 {
328     continue;
329 }
330
331 struct kdres *p_nearest = kd_nearest3(tree, x_rand[0], x_rand[1], x_rand[2]);
332 if (p_nearest == nullptr)
333 {
334     ROS_ERROR("nearest query error");
335     continue;
336 }
337 RRTNode3DPtr nearest_node = (RRTNode3DPtr)kd_res_item_data(set, p_nearest);
338 kd_res_free(set, p_nearest);
339
```

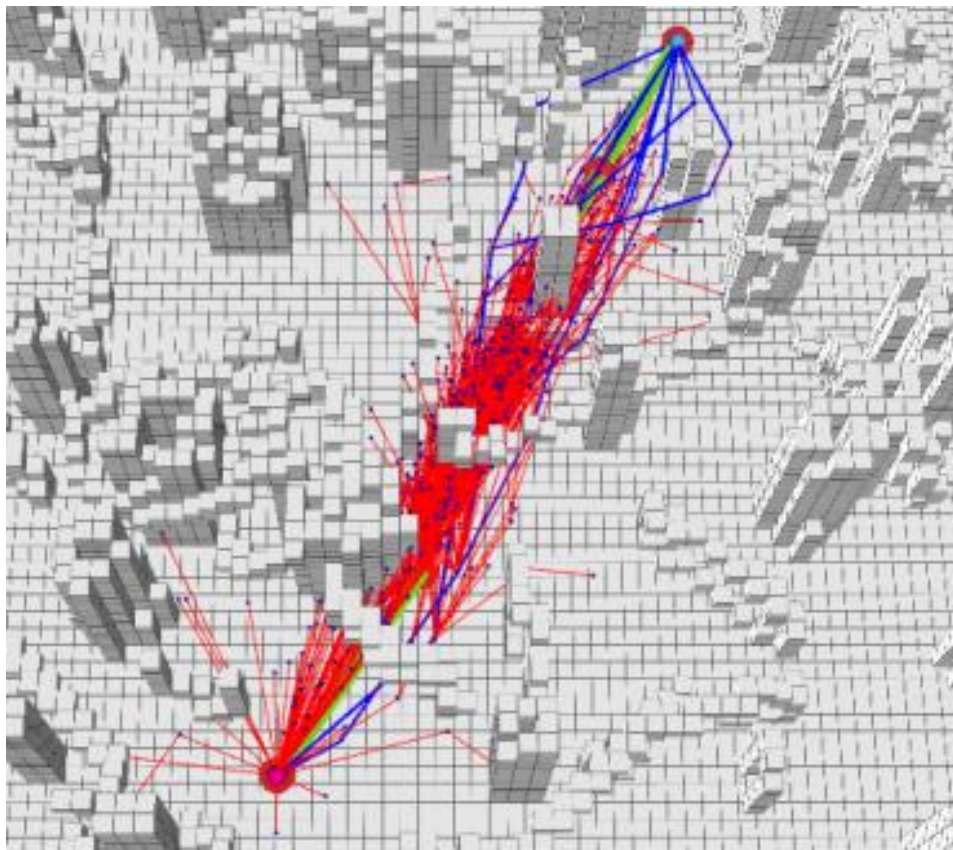
C++实现Informed RRT*

- 代码（个人作业代码仅供参考，欢迎交流~）

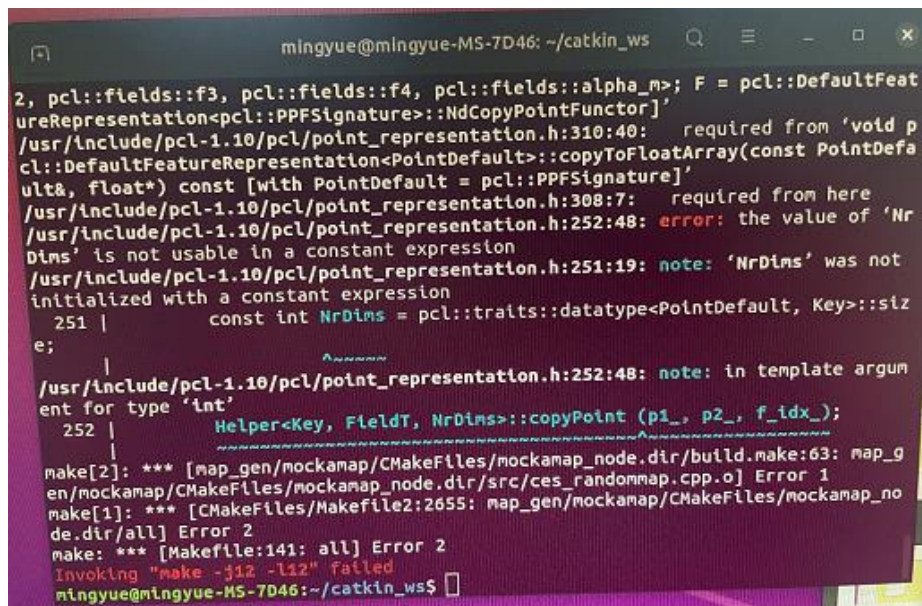
```
340 Eigen::Vector3d x_new = steer(nearest_node_p: nearest_node->x, rand_node_p: x_rand, len: steer_length_);
341 // Eigen::Vector3d x_new_ball = L_ellipse.inverse() * Rotation_matrix.inverse() * (x_rand - x_centre);
342 Eigen::Vector3d x_new_ball = L_ellipse.inverse() * (x_rand - x_centre);
343 x_new_ball = Rotation_matrix.inverse() * x_new_ball;
344 //cout <<"x_new_Ball:"<<x_new_ball<<endl;
345 if(std::sqrt(x: std::pow(x: x_new_ball[0], y: 2) + std::pow(x: x_new_ball[1], y: 2) + std::pow(x: x_new_ball[2], y: 2)) > 1.0){
346     continue;
347 }
348 if (!map_ptr_->isSegmentValid(nearest_node->x, x_new))
349 {
350     continue;
351 }
```

C++实现Informed RRT*

- 效果如图



●第三章作业群里反馈的问题



```
mingyue@mingyue-MS-7D46: ~/catkin_ws
2, pcl::fields::f3, pcl::fields::f4, pcl::fields::alpha_m>; F = pcl::DefaultFeatureRepresentation<pcl::PPFSignature>::NdCopyPointFuncor]
/usr/include/pcl-1.10/pcl/point_representation.h:310:40: required from 'void pcl::DefaultFeatureRepresentation<PointDefault>::copyToFloatArray(const PointDefault&, float*) const [with PointDefault = pcl::PPFSignature]'
/usr/include/pcl-1.10/pcl/point_representation.h:308:7: required from here
/usr/include/pcl-1.10/pcl/point_representation.h:252:48: error: the value of 'NrDims' is not usable in a constant expression
/usr/include/pcl-1.10/pcl/point_representation.h:251:19: note: 'NrDims' was not initialized with a constant expression
251 |         const int NrDims = pcl::traits::datatype<PointDefault, Key>::size;
    |
    |
/usr/include/pcl-1.10/pcl/point_representation.h:252:48: note: in template argument for type 'int'
252 |         Helper<Key, FieldT, NrDims>::copyPoint(p1_, p2_, f_idx_);
    |
make[2]: *** [map_gen/mockamap/CMakeFiles/mockamap_node.dir/build.make:63: map_gen/mockamap/CMakeFiles/mockamap_node.dir/src/ces_randommap.cpp.o] Error 1
make[1]: *** [CMakeFiles/Makefile2:2655: map_gen/mockamap/CMakeFiles/mockamap_node.dir/all] Error 2
make: *** [Makefile:141: all] Error 2
Invoking "make -j12 -l12" failed
mingyue@mingyue-MS-7D46:~/catkin_ws$
```

第一章与第二章作业能编译成功，第三章报错如图错误（系统版本：ubuntu 20.04）

解决方法：

- 1.在cmakelist中将find_package(pcl 1.7 required)的1.7删除；
- 2.在cmakelist中加入set(CMAKE_CXX_STANDARD 14);
- 3.在cmakelist中将所有的能看到c++11全改成c++14





深蓝学院
shenlanxueyuan.com

感谢各位聆听 !
Thanks for Listening

