问题 1：

a:证明：

$$\because f在[a,b]连续可导，且x_1,x_2 \in [x_1,x_2]$$

$$\therefore f_{min} \le f(x_1) \le f_{max}$$

$$同理：f_{min} \le f(x_2) \le f_{max}$$

$$\therefore f_{min} \le \frac{f(x_1)+f(x_2)}{2} \le f_{max}$$

$$不妨令k=\frac{f(x_1)+f(x_2)}{2},则k \in [f_{min},f_{max}],由：介值定理可得,\exists \varepsilon \in [x_1,x_2],使得f(\varepsilon)=k$$

$$命题得证$$

b:证明：

$$\because f在[a,b]连续可导，且x_1,x_2 \in [x_1,x_2]$$

$$\therefore f_{min} \le f(x_1) \le f_{max}$$

$$同理：f_{min} \le f(x_2) \le f_{max}$$

$$\because c_1,c_2 > 0$$

$$\therefore \frac{c_1}{c_1+c_2} f_{min} \le \frac{c_1 f(x_1)}{c_1+c_2} \le \frac{c_1}{c_1+c_2} f_{max}$$

$$同理有：\frac{c_1}{c_1+c_2} f_{min} \le \frac{c_2 f(x_2)}{c_1+c_2} \le \frac{c_1}{c_1+c_2} f_{max}$$

$$\therefore f_{min} \le \frac{c_1 f(x_1)+c_2 f(x_2)}{c_1+c_2} \le f_{max}$$

$$不妨令k=\frac{c_1 f(x_1)+c_2 f(x_2)}{c_1+c_2}则k \in [f_{min},f_{max}],根据介值定理,\exists \varepsilon \in [x_1,x_2],使得f(\varepsilon)=k$$

$$命题得证$$

c:证明：

$$不妨令\mathrm{f}(x)=x, c_1=2, c_2=-1, a=-2, b=5, x_1=-2, x_2=5$$

$$\therefore f_{\min}=-1, f_{\max}=5,$$

$$\frac{c_1 f(x_1)+c_2 f(x_2)}{c_1+c_2}=-9$$

$$显然不存在\mathrm{e}满足:\mathrm{f(e)}=\frac{c_1 f(x_1)+c_2 f(x_2)}{c_1+c_2}$$

问题二：

a：

$$\left|f(x_0)-\tilde{f}(x_0)\right| = |f(x_0)-f(x_0+\varepsilon)| = |f^1(c)|\varepsilon$$
$$\because \varepsilon \to 0, c \in [x_0, x_0+\varepsilon]$$
$$\therefore |f^1(c)|\varepsilon \approx |f^1(x_0)|\varepsilon$$
$$\therefore \frac{|f(x_0)-f(x_0+\varepsilon)|}{|f(x_0)|} \approx |\frac{f^1(x_0)}{f(x_0)}|\varepsilon$$

b:

$$当f(x)=e^x, x_0=1\ 时，绝对误差范围$$

$$[|f^1(x_0)|\varepsilon, |f^1(x_0+\varepsilon)|\varepsilon] = [0.1359141\times 10^{-4}, 0.1359148\times 10^{-4}]$$

$$相对误差范围$$

$$\left[\left|\frac{f^1(x_0)}{f(x_0)}\right|\varepsilon, \left|\frac{f^1(x_0+\epsilon)}{f(x_0+\varepsilon)}\right|\varepsilon\right] = [0.5000000\times 10^{-5}, 0.5000000\times 10^{-5}]$$

$$当f(x)=sinx, x_0=1\ 时，绝对误差范围$$

$$[|f^1(x_0)|\varepsilon, |f^1(x_0+\varepsilon)|\varepsilon] = [0.4999238\times 10^{-5}, 0.4999238\times 10^{-5}]$$

$$相对误差$$

$$\left[\left|\frac{f^1(x_0)}{f(x_0)}\right|\varepsilon, \left|\frac{f^1(x_0+\epsilon)}{f(x_0+\varepsilon)}\right|\varepsilon\right] = [0.2864484\times 10^{-3}, 0.2864498\times 10^{-3}]$$

c:

$$当f(x)=e^x, x_0=10\ 时，绝对误差范围$$

$$[|f^1(x_0)|\varepsilon, |f^1(x_0+\varepsilon)|\varepsilon] = [0.1101323, 0.1101328]$$

$$相对误差范围$$

$$\left[\left|\frac{f^1(x_0)}{f(x_0)}\right|\varepsilon, \left|\frac{f^1(x_0+\epsilon)}{f(x_0+\varepsilon)}\right|\varepsilon\right] = [0.500000\times 10^{-5}, 0.500000\times 10^{-5}]$$

$$当f(x)=sinx, x_0=10\ 时，绝对误差范围$$

$$[|f^1(x_0)|\varepsilon, |f^1(x_0+\varepsilon)|\varepsilon] = [0.4924039\times 10^{-5}, 0.4924039\times 10^{-5}]$$

$$相对误差$$

$$\left[\left|\frac{f^1(x_0)}{f(x_0)}\right|\varepsilon, \left|\frac{f^1(x_0+\epsilon)}{f(x_0+\varepsilon)}\right|\varepsilon\right] = [0.2835639\times 10^{-4}, 0.2835641\times 10^{-4}]$$

问题三:

a:

$$\frac{4}{5}+\frac{1}{3}=\frac{17}{15}$$

$$choping=0.113\times10^{1}$$

$$相对误差=-0.294\times10^{-2}$$

$$rounding=0.114\times10^{1}$$

$$相对误差=0.588\times10^{-2}$$

$$\left(\frac{1}{3}+\frac{3}{11}\right)-\frac{3}{20}=\frac{301}{660}$$

$$choping=0.456$$

$$相对误差=-0.132\times10^{-3}$$

$$rounding=0.457$$

$$相对误差=0.205\times10^{-2}$$

问题四:

证明:

a:

$$由已知条件可得:F(x)=c_1L_1+c_2L_2+c_1o(x^{\alpha})+c_2o\left(x^{\beta}\right)$$

$$又因为:\gamma=minmum\{\alpha,\beta\}$$

$$所以:由高阶无穷小量性质可得F(x)=c_1L_1+c_2L_2+o(x^{\gamma})$$

b:

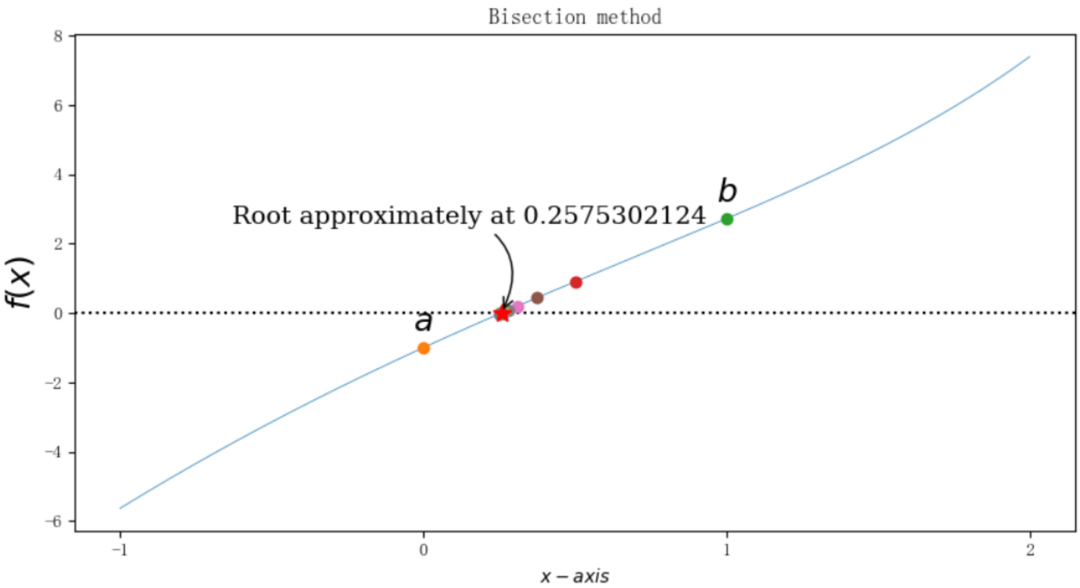
$$由已知条件可得:G(x)=L_1+L_2+o(c_1x^{\alpha})+o\left(c_2x^{\beta}\right)$$

$$又因为:\gamma=minmum\{\alpha,\beta\}$$

$$所以:由高阶无穷小量性质可得G(x)=L_1+L_2+o(x^{\gamma})$$

问题 5:

a: $e^x - x^2 + 3x - 2 = 0 \quad \text{for} \quad 0 \le x \le 1$



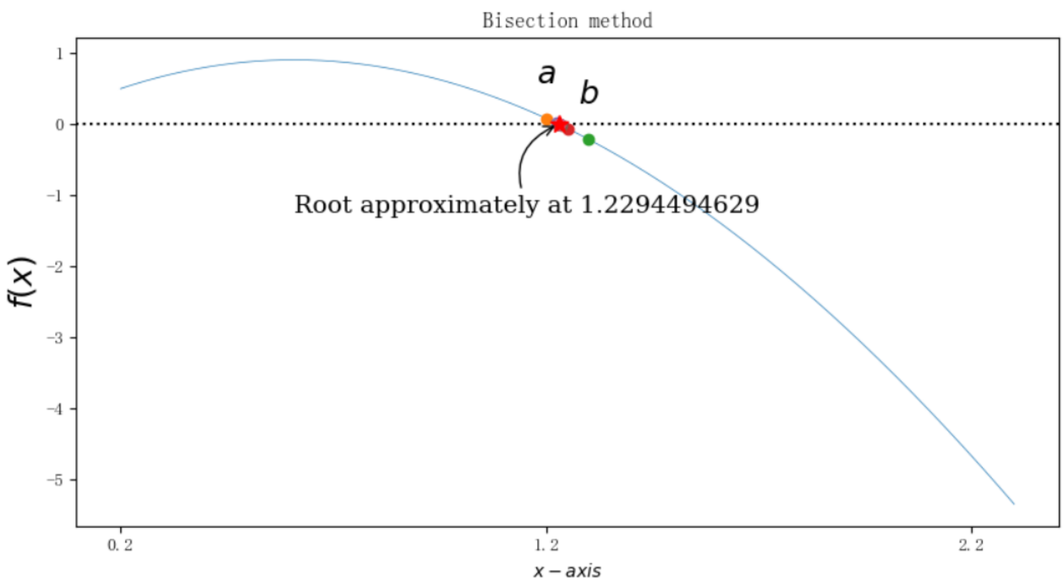
迭代 18 次:

[0.5, 0.25, 0.375, 0.3125, 0.28125, 0.265625, 0.2578125, 0.25390625, 0.255859375, 0.2568359375, 0.25732421875, 0.257568359375, 0.2574462890625, 0.25750732421875, 0.257537841796875, 0.2575225830078125, 0.25753021240234375]

b. $x\cos x - 2x^2 + 3x - 1 = 0$ for $0.2 \le x \le 0.3$ and $1.2 \le x \le 1.3$

在 $0.2 \le x \le 0.3$ 没有根

在 $1.2 \le x \le 1.3$

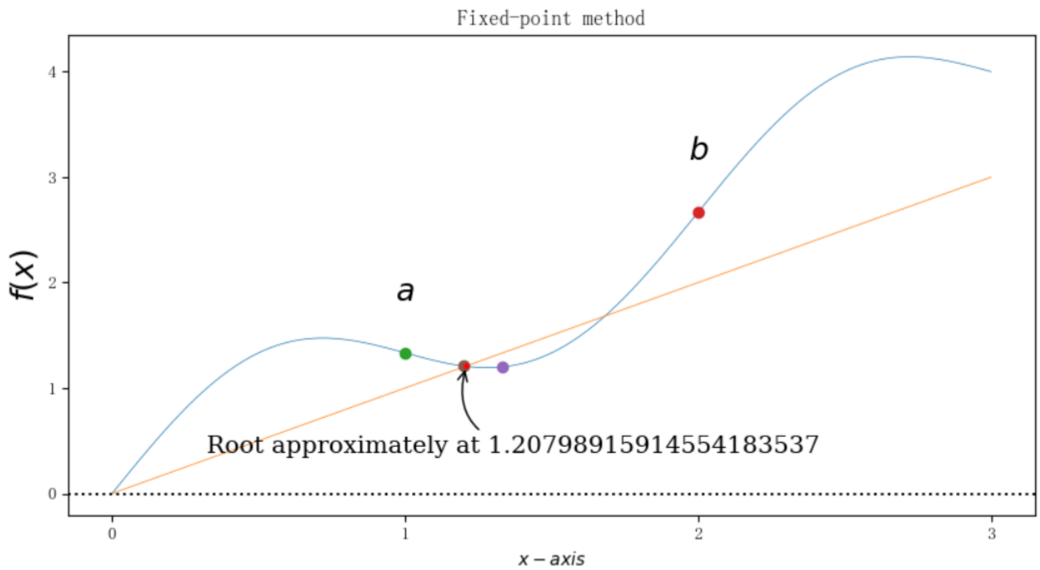


迭代 15 次

[1.25, 1.225, 1.2375, 1.2312500000000002, 1.2281250000000001, 1.2296875000000003, 1.22890625, 1.2292968750000002, 1.2294921875000002, 1.22939453125, 1.2294433593750003, 1.2294677734375004, 1.2294555664062503, 1.2294494628906252]

问题六：

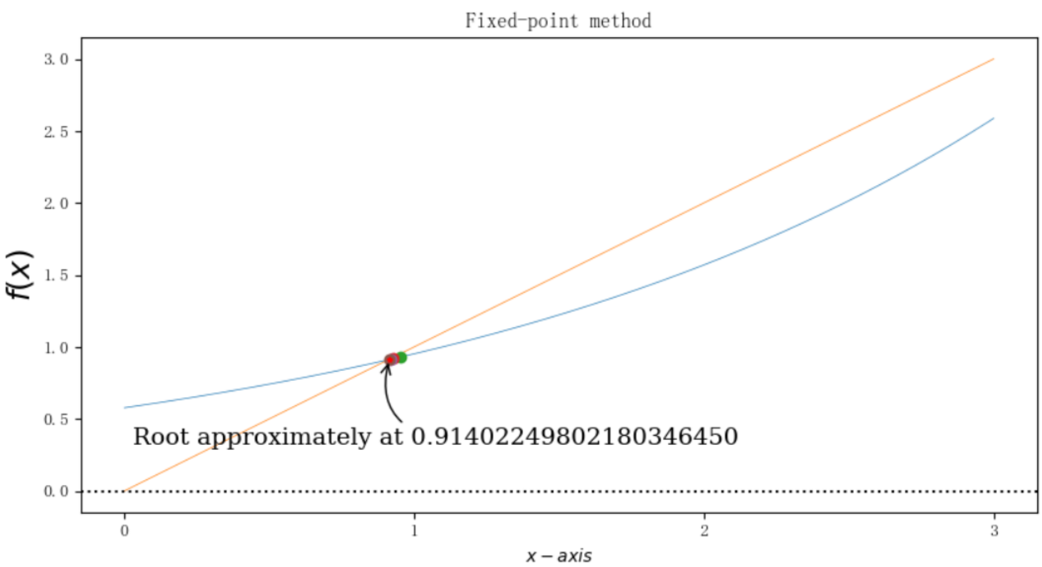
a. $2\sin\pi x + x = 0$ on [1, 2], use $p_0 = 1$



迭代 3 次：

[1.3333333333333335, 1.2004275085881522, 1.2079891591455418]

b. $3x^2 - e^x = 0$ (令$p_0 = 1, g(p) = \sqrt{\frac{e^p}{3}}$)



迭代 4 次

[0.9518896694573808, 0.9292650169866411, 0.918812102813262, 0.9140224980218035]

问题七:

证明:

$$g^1(p) > 1, 0 < |p_0 - p| < \sigma$$

$$如果:\ g^1(p) \geq k > 1, \forall p \in [a, b]$$

$$|p_n - p| = \left|g\ \ (p_{n-1}) - g\ \ (p)\right| = |g^1(\in)| \times |p_{n-1} - p| > k|p_{n-1} - p|$$

$$所以|p_n - p| > k|p_{n-1} - p| > k^2|p_{n-2} - p| > k^n|p_0 - p|$$

$$\lim_{n\to\infty}|p_n - p| > k^n|p_0 - p|, 所以不收敛$$