Numerical Analysis – Winter 2019

Assignment #5

Please upload to the 'hw5' directory if you submit your homework in time.

Problem 1:

Use the most accurate three-point formula to determine each missing entry in the following tables.

a.

x	$f(x)$	$f'(x)$
1.1	9.025013	
1.2	11.02318	
1.3	13.46374	
1.4	16.44465	

b.

x	$f(x)$	$f'(x)$
8.1	16.94410	
8.3	17.56492	
8.5	18.19056	
8.7	18.82091	

Problem 2:

Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

$$M = N(h) + K_1h^2 + K_2h^4 + K_3h^6 + \cdots,$$

for some constants $K_1, K_2, K_3, \ldots$. Use the values $N(h)$, $N\left(\frac{h}{3}\right)$, and $N\left(\frac{h}{9}\right)$ to produce an $O(h^6)$ approximation to M.

Problem 3:

a. Derive the Five-Point Midpoint Formula for numerical differentiation.

b Examine the above five-point midpoint formula. Suppose that in evaluating every f(x), we encounter round-off errors bounded by some number ϵ and the fifth derivative of *f*(x) is bounded by a number M>0. Please find the optimal value of h that minimizes the total error (including both truncation error and round-off error) with respect to ε and *M.*

Problem 4:

Approximate the following integrals using the Trapezoidal rule and Simpson's rule, respectively.

a. $\displaystyle\int_{-0.25}^{0.25} (\cos x)^2\, dx$

b. $\displaystyle\int_{-0.5}^{0} x\ln(x+1)\, dx$

c. $\displaystyle\int_{0.75}^{1.3} \left((\sin x)^2 - 2x\sin x + 1\right)\, dx$

d. $\displaystyle\int_{e}^{e+1} \frac{1}{x\ln x}\, dx$

Problem 5:

Use Romberg integration to compute $R_{3,3}$ for the following integrals.

a. $\displaystyle\int_{-1}^{1} (\cos x)^2\, dx$

b. $\displaystyle\int_{-0.75}^{0.75} x\ln(x+1)\, dx$

c. $\displaystyle\int_{1}^{4} \left((\sin x)^2 - 2x\sin x + 1\right)\, dx$

d. $\displaystyle\int_{e}^{2e} \frac{1}{x\ln x}\, dx$

Problem 6:

Use Euler's method to approximate the solutions for each of the following initial-value problems.

a. $y' = y/t - (y/t)^2, \quad 1 \leq t \leq 2, \quad y(1) = 1$, with $h = 0.1$

b. $y' = 1 + y/t + (y/t)^2, \quad 1 \leq t \leq 3, \quad y(1) = 0$, with $h = 0.2$