

问题 1

a:

x	f(x)	f'(x)
1.1	9.025013	17.769705
1.2	11.02318	22.191635
1.3	13.46374	27.10375
1.4	16.44465	32.51085

计算过程:

$$\begin{aligned}f'(1.1) &= \frac{1}{0.2} \{-3f(1.1) + 4f(1.2) - f(1.3)\} = 17.769705 \\f'(1.2) &= \frac{1}{0.2} \{f(1.3) - f(1.1)\} = 22.191635 \\f'(1.3) &= \frac{1}{0.2} \{f(1.4) - f(1.2)\} = 27.10375 \\f'(1.4) &= \frac{1}{0.2} \{3f(1.4) - 4f(1.3) + f(1.2)\} = 32.51085\end{aligned}$$

b:

x	f(x)	f'(x)
8.1	16.94410	3.09205
8.3	17.56492	3.11615
8.5	18.19056	3.139975
8.7	18.82091	3.163525

计算过程:

$$\begin{aligned}f'(8.1) &= \frac{1}{0.4} \{-3f(8.1) + 4f(8.3) - f(8.5)\} = 3.09205 \\f'(8.3) &= \frac{1}{0.4} \{f(8.5) - f(8.1)\} = 3.11615 \\f'(8.5) &= \frac{1}{0.4} \{f(8.7) - f(8.3)\} = 3.139975 \\f'(8.7) &= \frac{1}{0.4} \{3f(8.7) - 4f(8.5) + f(8.3)\} = 3.163525\end{aligned}$$

问题 2

$$\begin{cases} M = N(h) + k_1 h^2 + k_2 h^4 + k_3 h^6 + \cdots & \textcircled{1} \\ M = N\left(\frac{h}{3}\right) + k_1 \left(\frac{h}{3}\right)^2 + k_2 \left(\frac{h}{3}\right)^4 + k_3 \left(\frac{h}{3}\right)^6 + \cdots & \textcircled{2} \\ M = N\left(\frac{h}{9}\right) + k_1 \left(\frac{h}{9}\right)^2 + k_2 \left(\frac{h}{9}\right)^4 + k_3 \left(\frac{h}{9}\right)^6 + \cdots & \textcircled{3} \end{cases}$$

由①②消去 h^2 可得:

$$8M = 9N\left(\frac{h}{3}\right) - N(h) - \frac{8}{9}k_2h^4 - \frac{80}{81}k_3h^6 \dots \quad (4)$$

由②③消去 h^2 可得:

$$8M = 9N\left(\frac{h}{9}\right) - N(h) - \frac{8}{729}k_2h^4 - \frac{80}{59049}k_3h^6 \dots \quad (5)$$

由④⑤消去 h^4 可得:

$$M = \frac{729}{640}N\left(\frac{h}{9}\right) - \frac{9}{64}N\left(\frac{h}{3}\right) + \frac{1}{640}N(h) - \frac{1}{729}k_3h^6 \dots$$

问题 3

a:

$$\begin{aligned} f(x) = & f(x_0) \frac{(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)(x_0-x_4)} \\ & + f(x_1) \frac{(x-x_1)(x-x_0)(x-x_3)(x-x_4)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)(x_1-x_4)} \\ & + f(x_2) \frac{(x-x_0)(x-x_1)(x-x_3)(x-x_4)}{(x_2-x_0)(x_2-x_2)(x_2-x_3)(x_1-x_4)} \\ & + f(x_3) \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_4)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)(x_3-x_4)} \\ & + f(x_4) \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)}{(x_4-x_0)(x_4-x_1)(x_4-x_2)(x_4-x_3)} \\ & + \frac{(x-x_0)(x-x_1)(x-x_2)(x-x_3)(x-x_4)}{5!} f^{(5)}(\varepsilon(x)) \end{aligned}$$

对 x 求导: 将 $x_0=x_0-2h, x_1=x_0-h, x_2=x_0, x_3=x_0+h, x_4=x_0+2$ 代入可得:

$$\begin{aligned} f'(x_0) = & f(x_0-2h) \frac{h * -h * -2h}{-h * -2h * -3h * -4h} + f(x_0-h) \frac{2h * -h * -2h}{-h * h * -2h * -4h} + 0 \\ & + f(x_0+h) \frac{2h * h * -2h}{3h * 2h * h * -h} + f(x_0+2h) \frac{2h * h * -2h}{3h * 2h * h * 4h} \\ & + \frac{f^{(5)}(\varepsilon(x))}{5!} * (2h * h * -h * -2h) \end{aligned}$$

化简可得:

$$f'(x_0) = \frac{1}{12h} \{f(x_0-2h) - 8f(x_0-h) + 8f(x_0+h) - f(x_0+2h)\} + \frac{h^4}{30} f^{(5)}(\varepsilon)$$

b:

$$\begin{aligned} & \left| f'(x_0) - \frac{1}{12h} \{ \tilde{f}(x_0-2h) - 8\tilde{f}(x_0-h) + 8\tilde{f}(x_0+h) - \tilde{f}(x_0+2h) \} \right| \\ & \leq \frac{\varepsilon + 8\varepsilon + 8\varepsilon + \varepsilon}{12h} + \frac{h^4}{30} M = \frac{3\varepsilon}{2h} + \frac{h^4}{30} M \geq \sqrt[4]{\frac{3^4 \varepsilon^4 M}{8^4 * 30}} \end{aligned}$$

当且仅当:

$$\frac{3\varepsilon}{8h} = \frac{3\varepsilon}{8h} = \frac{3\varepsilon}{8h} = \frac{3\varepsilon}{8h} = \frac{h^4}{30} M$$

所以:

$$h = \sqrt[5]{\frac{90\varepsilon}{8M}}$$

问题 4

a: Trapezoidal rule:

$$\int_{-0.25}^{0.25} (\cos x)^2 dx = \frac{(0.25 - -0.25)}{2} [f(-0.25) + f(0.25)] = 0.46939564$$

Simpson's rule:

$$\int_{-0.25}^{0.25} (\cos x)^2 dx = \frac{0.25}{3} [f(-0.25) + 4f(0) + f(0.25)] = 0.489798547$$

b: Trapezoidal rule:

$$\int_{-0.5}^0 x \ln(x+1) dx = \frac{(0 - -0.5)}{2} [f(0) + f(-0.5)] = 0.08664339757$$

Simpson's rule:

$$\int_{-0.5}^0 x \ln(x+1) dx = \frac{0.25}{3} [f(-0.5) + 4f(-0.25) + f(0)] = 0.052854639$$

c: Trapezoidal rule:

$$\int_{0.75}^{1.3} ((\sin x)^2 - 2x \sin x + 1) dx = \frac{(1.3 - 0.75)}{2} [f(1.3) + f(0.75)] = -0.037024253$$

Simpson's rule:

$$\int_{0.75}^{1.3} ((\sin x)^2 - 2x \sin x + 1) dx = \frac{0.275}{3} [f(0.75) + 4f(1.025) + f(1.3)] = -0.02027159$$

d: Trapezoidal rule:

$$\int_e^{e+1} \frac{1}{x \ln x} dx = \frac{1}{2} [f(e+1) + f(e)] = 0.28633417248$$

Simpson's rule:

$$\int_e^{e+1} \frac{1}{x \ln x} dx = \frac{0.5}{3} [f(e) + 4f(e+0.5) + f(e+1)] = 0.27267045244$$

问题 5

a;

$$R_{1,1} = \frac{2}{2} (f(-1) + f(1)) = 0.583853163$$

$$R_{2,1} = \frac{2}{4} (f(-1) + 2f(0) + f(1)) = 1.291926582$$

$$R_{3,1} = \frac{2}{8} (f(-1) + 2[f(-0.5) + f(0) + f(0.5)] + f(1)) = 1.416114444$$

$$R_{2,2} = R_{2,1} + \frac{1}{3} (R_{2,1} - R_{1,1}) = 1.527951054$$

$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 1.457510398$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 1.45281435$$

b:

$$R_{1,1} = \frac{1.5}{2}(f(0.75) + f(-0.75)) = 1.09457445884$$

$$R_{2,1} = \frac{1.5}{4}(f(-0.75) + 2f(0) + f(0.75)) = 0.54728722942$$

$$R_{3,1} = \frac{1.5}{8}(f(-0.75) + 2[f(-0.375) + f(0) + f(0.375)] + f(0.75)) = 0.38452043101$$

$$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.364858153$$

$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 0.330264832$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 0.32795861$$

c:

$$R_{1,1} = \frac{3}{2}(f(1) + f(4)) = 11.47845214$$

$$R_{2,1} = \frac{3}{4}(f(1) + 2f(2.5) + f(4)) = 3.28793835$$

$$R_{3,1} = \frac{3}{8}(f(1) + 2[f(1.75) + f(2.5) + f(3.25)] + f(4)) = 1.82340824$$

$$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.557767088$$

$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 1.335231542$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 1.38706251$$

d:

$$R_{1,1} = \frac{e}{2}(f(e) + f(2e)) = 0.647654027$$

$$R_{2,1} = \frac{e}{4}(f(e) + 2f(1.5e) + f(2e)) = 0.560996426$$

$$R_{3,1} = \frac{e}{8}(f(e) + 2[f(1.25e) + f(1.5e) + f(1.75e)] + f(2e)) = 0.535608969$$

$$R_{2,2} = R_{2,1} + \frac{1}{3}(R_{2,1} - R_{1,1}) = 0.532110558$$

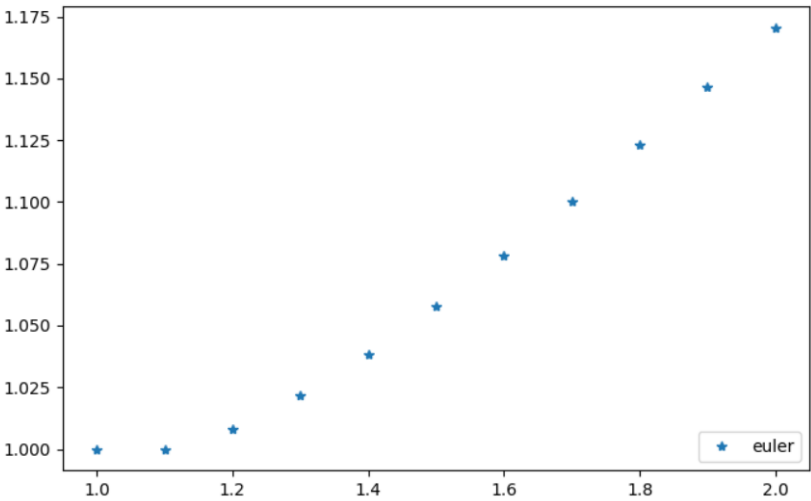
$$R_{3,2} = R_{3,1} + \frac{1}{3}(R_{3,1} - R_{2,1}) = 0.527146483$$

$$R_{3,3} = R_{3,2} + \frac{1}{15}(R_{3,2} - R_{2,2}) = 0.526815545$$

问题 6

a:

t_i	w_i
1.0	1.0
1.1	1.0
1.2	1.0082644628099173
1.3	1.0216894717270375
1.4	1.038514734248178
1.5	1.0576681921408762
1.6	1.0784610936317547
1.7	1.100432164699466
1.8	1.1232620515812632
1.9	1.1467235965295264
2.0	1.1706515695646647



b:

t_i	w_i
1.0	0
1.2	0.2
1.4	0.4388888888888889
1.6	0.721242756361804
1.8	1.0520380316573712
2.0	1.4372511475238394
2.2	1.884260805329153
2.4	2.4022695885615417
2.6	3.002837164557213
2.8	3.700600704932798
3.0	4.5142774281767

