

Numerical Analysis – Winter 2019

Assignment #2

Issued: Nov. 20, 2019

Due: Dec. 5, 2019

Problem 1:

Let $f(x) = -x^3 - \cos x$ and $p_0 = -1$. Use Newton's method to find p_2 . Could $p_0 = 0$ be used?

Problem 2:

Assume that we wish to use the Newton-Raphson method to approximate the root $\frac{1}{b}$ of the nonlinear equation

$$f(x) = b - \frac{1}{x} = 0,$$

where we assume $b > 0$.

(i) Show that $|\epsilon_{k+1}| = \epsilon_k^2$ where ϵ_k is the relative error in x_k at the k -th iteration, given by

$$\epsilon_k = \frac{\frac{1}{b} - x_k}{\frac{1}{b}}.$$

(ii) Show that the Newton-Raphson iterations will converge to $\frac{1}{b}$ for any starting value x_0 provided that

$$0 < x_0 < \frac{2}{b}.$$

Problem 3:

Use Newton's method with $\mathbf{x}^{(0)} = \mathbf{0}$ to compute $\mathbf{x}^{(2)}$ for each of the following nonlinear systems.

a.
$$\begin{aligned} 3x_1 - \cos(x_2x_3) - \frac{1}{2} &= 0, \\ 4x_1^2 - 625x_2^2 + 2x_2 - 1 &= 0, \\ e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} &= 0. \end{aligned}$$

b.
$$\begin{aligned} x_1^2 + x_2 - 37 &= 0, \\ x_1 - x_2^2 - 5 &= 0, \\ x_1 + x_2 + x_3 - 3 &= 0. \end{aligned}$$

Problem 4:

Use the method of Steepest Descent with $TOL = 0.05$ to approximate the solutions of the following nonlinear systems.

a.
$$\begin{aligned} 15x_1 + x_2^2 - 4x_3 &= 13, \\ x_1^2 + 10x_2 - x_3 &= 11, \\ x_2^3 - 25x_3 &= -22. \end{aligned}$$

b.
$$\begin{aligned} 10x_1 - 2x_2^2 + x_2 - 2x_3 - 5 &= 0, \\ 8x_2^2 + 4x_3^2 - 9 &= 0, \\ 8x_2x_3 + 4 &= 0. \end{aligned}$$