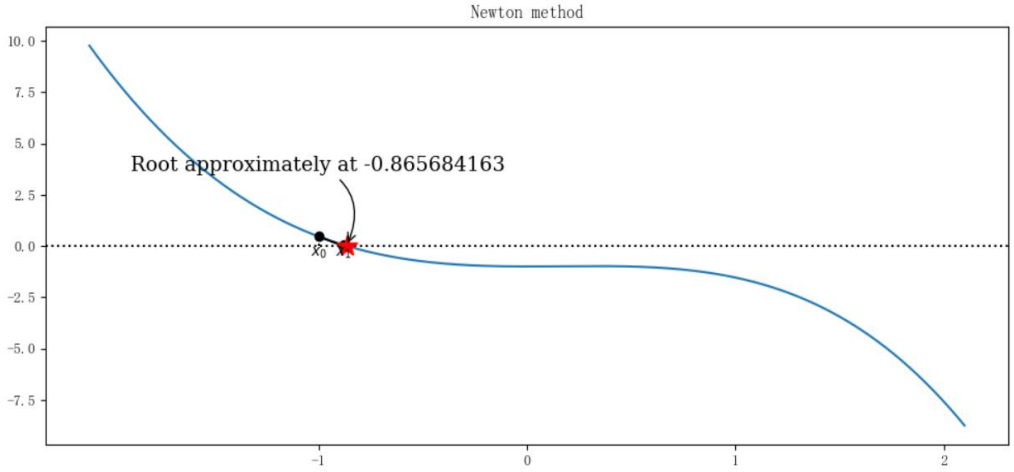
问题一、

当 p_0=-1 时:



可以看出当 p_0=-1 时，牛顿迭代收敛，

当 tolerance=$1*10^{-8}$时，迭代 4 次

[-1, -0.880332899571582, -0.8656841631760818, -0.865474075952977]

当 p_0=0 时:

$$p_1 = p_0 - \frac{-{p_0}^3 - cosp_0}{-3{p_0}^2 + sinp_0}|_{p_0=0}$$

所以

$$p_1 = \infty(不收敛)$$

程序会发生报错提醒。

问题二、

(i)证明:

$$因为 f(x) = b - \frac{1}{x}, 所以 f'(x) = \frac{1}{x^2}$$

$$所以可以得出迭代式：x_{k+1} = x_k + \frac{b - \frac{1}{x}}{\frac{1}{x^2}} = 2x_k - b{x_k}^2$$

$$进行变换可得：\frac{x_{k+1} - \frac{1}{b}}{\frac{1}{b}} = \frac{2x_k - b{x_k}^2 - \frac{1}{b}}{\frac{1}{b}} = \frac{2\frac{1}{b}x_k - {x_k}^2 - \frac{1}{b}^2}{(\frac{1}{b})^2}$$

$$所以\varepsilon_{k+1} = \frac{x_{k+1} - \frac{1}{b}}{\frac{1}{b}} = \frac{2\frac{1}{b}x_k - {x_k}^2 - \frac{1}{b}^2}{(\frac{1}{b})^2} = -\frac{(x_k - \frac{1}{b})^2}{(\frac{1}{b})^2} = -{\varepsilon_k}^2$$

$$所以 |\varepsilon_{k+1}| = {\varepsilon_k}^2$$

(ii)证明:

$$因为有: |\varepsilon_{k+1}| = {\varepsilon_k}^2 = {\varepsilon_{k-1}}^4 = {\varepsilon_{k-2}}^8 = \cdots = {\varepsilon_1}^{2^k}$$

$$因为 0 < x < \frac{2}{k},$$

$$所\ 以\quad \varepsilon_1 = \left|\frac{x_1 - \frac{1}{b}}{\frac{1}{b}}\right| < 1$$

$$所以: \lim_{k\to\infty}\left|\frac{x_{k+1} - \frac{1}{b}}{\frac{1}{b}}\right| = \lim_{k\to\infty}|\varepsilon_{k+1}| = \lim_{k\to\infty}{\varepsilon_1}^{2^k} = 0,\ 证毕$$

问题三、

a:

$X^{(2)}$= [0.50016668 0.2508036 -0.51738736]

b:

$X^{(2)}$= [5.36385742 9.25513344 -11.61900885]

问题四、

a:

X= [1.03640047 1.08570656 0.93119144]

b:

X= [0.49999973 1.00000105 -0.50000148]