问题一、

a.

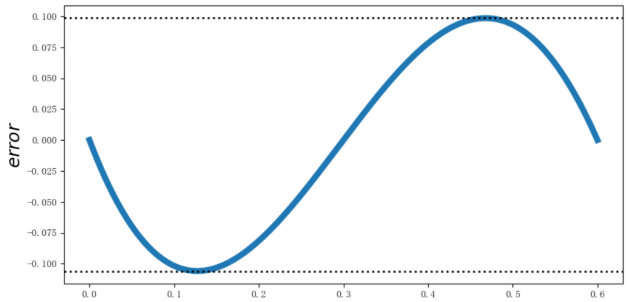
$$f(x) = e^{2x}cos3x$$

$$P(x) = f(0)\frac{(x-0.3)(x-0.6)}{(0-0.3)(0-0.6)} + f(1)\frac{(x-0)(x-0.6)}{(0.3-0)(0.3-0.6)} + f(0)\frac{(x-0.3)(x-0)}{(0.6-0.3)(0.6-0)}$$

$$= -11.22\,x^2 + 3.808\,x + 1$$

$$在[0,2]的误差函数为: g(x) = P(x) - f(x) = e^{2x}cos3x - (-11.22\,x^2 + 3.808\,x + 1)$$

$$所以误差范围: (-0.10594342738134976, 0.09856759303014412)$$



b:

$$f(x) = \sin(\ln(x))$$

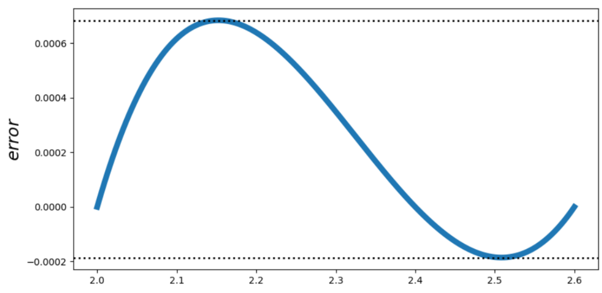
$$P(x) = f(0)\frac{(x-2.4)(x-2.6)}{(2.0-2.4)(2.0-2.6)} + f(1)\frac{(x-2.0)(x-2.6)}{(2.4-2.0)(2.4-2.6)} + f(0)\frac{(x-2.0)(x-2.4)}{(2.6-2.0)(2.6-2.4)}$$

$$= -0.1306\,x^2 + 0.897\,x - 0.6325$$

$$在[0,2]的误差函数为: g(x) = P(x) - f(x)$$

$$= \sin(\ln(x)) - (-0.1306\,x^2 + 0.897\,x - 0.6325)$$

$$所以误差范围: (-0.0001851625083185704, 0.0006835005805589933)$$



问题二、

$$设P_3(x) = a_3x^3 + a_2x^2 + a_1x^1 + a_0$$

$$代入(0,0),(1,3),(2,2)可得$$

$$\begin{cases} a_0 = 0 \\ a_3 = 6 \\ a_1 + a_2 = -3 \\ 2a_1 + 4a_2 = -46 \end{cases}$$

$$解得: \begin{cases} a_0 = 0 \\ a_1 = 17 \\ a_2 = -20 \\ a_3 = 6 \end{cases}$$

$$代入得: P_3(x) = 6x^3 - 20x^2 + 17x^1$$

$$将(0.5, y)代入得: y = 4.25$$

问题三、

(1)

$$P_2 = f(0.5)$$
$$P_3 = f(x_3) = f(0.75) = 8$$
$$P_{2,3} = \frac{(x - x_3)}{(x_2 - x_3)} f(x_2) + \frac{(x - x_2)}{(x_3 - x_2)} f(x_3)$$
$$= \frac{(x - 0.75)}{(0.5 - 0.75)} f(x_2) + \frac{(x - 0.5)}{(0.75 - 0.5)} \times 8$$
$$P_{2,3}(0.4) = 1.4 \times P_2 - 3.2 = 2.4$$
$$\therefore P_2 = 4$$

(2)

$$P_{0,1}(2.5) = 6$$
$$P_{0,2}(2.5) = 3.5$$
$$P_{0,1,2}(2.5) = \frac{P_{0,1}(2.5)(2.5 - 2) - P_{0,2}(2.5)(2.5 - 1)}{1 - 2} = 2.25$$
$$P_{0,1,2,3}(2.5) = \frac{P_{1,2,3}(2.5)(2.5 - 0) - P_{0,1,2}(2.5)(2.5 - 3)}{3 - 0} = 2.875$$

问题四、

$$f[x_0, x_1, x_2] = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0}$$

$$\therefore \frac{50}{7}=\frac{10-f[x_0,x_1]}{0.7}$$
$$f[x_0,x_1]=5$$
$$f[x_1,x_2]=\frac{f[x_2]-f[x_1]}{x_2-x_1}$$
$$10=\frac{6-f[x_1]}{0.3}$$
$$f[x_1]=3$$
$$f[x_0,x_1]=\frac{f[x_1]-f[x_0]}{x_1-x_0}$$
$$5=\frac{3-f[x_0]}{0.4}$$
$$f[x_0]=1$$
$$\therefore f[x_0,x_1]=5, f[x_1]=3, f[x_0]=1$$

问题五、

a．

$$设 S_0=a_0x^3+b_0x^2+c_0x+d_0$$

$$S_1=a_1x^3+b_1x^2+c_1x+d_1$$

$$可列方程:$$

$$\begin{cases} S_0(0)=f(0)=d_0=0 \\ S_0(1)=f(1)=a_0+b_0+c_0=1 \\ S_1(1)=f(1)=a_1+b_1+c_1+d_1=1 \\ S_1(2)=f(2)=8a_1+4b_1+2c_1+d_1=2 \\ S_0{}'(1)=3a_0+c_0=S_1{}'(1)=3a_1+2b_1+c_1 \\ S_0{}''(1)=3a_0+b_0=S_1{}''(1)=3a_1+b_1 \\ S_0{}''(0)=b_0=0 \\ S_1{}''(2)=6a_1+b_1=0 \end{cases}$$

$$解得: a_0=0, b_0=0, c_0=1, d_0=0$$

$$a_0=0, b_0=0, c_0=1, d_0=0$$
$$S_0=S_1=x$$

b.

$$设 S_0=a_0x^3+b_0x^2+c_0x+d_0$$

$$S_1=a_1x^3+b_1x^2+c_1x+d_1$$

$$可列方程:$$

$$
\begin{cases}
S_0(0) = f(0) = d_0 = 0 \\
S_0(1) = f(1) = a_0 + b_0 + c_0 = 1 \\
S_1(1) = f(1) = a_1 + b_1 + c_1 + d_1 = 1 \\
S_1(2) = f(2) = 8a_1 + 4b_1 + 2c_1 + d_1 = 2 \\
S_0{}'(1) = 3a_0 + 2b_0 + c_0 = S_1{}'(1) = 3a_1 + 2b_1 + c_1 \\
S_0{}''(1) = 3a_0 + b_0 = S_1{}''(1) = 3a_1 + b_1 \\
S_0{}'(0) = c_0 = 1 \\
S_1{}'(2) = 12a_1 + 4b_1 + c_1 = 1
\end{cases}
$$

$$\text{解得}: a_0 = 0, b_0 = 0, c_0 = 1, d_0 = 0$$

$$a_0 = 0, b_0 = 0, c_0 = 1, d_0 = 0$$

$$S_0 = S_1 = x$$