

问题一、

a:

$$\text{有 } X - \tilde{X} = (0, -7, 5)^t - (-0.2, -7.5, 5.4)^t = (0.2, 0.5, -0.4)^t$$

$$\text{所以 } \|X - \tilde{X}\|_{\infty} = 0.5$$

$$\text{因为 } A\tilde{X} - b = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \times (-0.2, -7.5, 5.4)^t - (1, -1, 2)^t = (0, -0.3, -0.2)^t$$

$$\text{所以 } \|A\tilde{X} - b\|_{\infty} = 0.3$$

b:

$$\text{因为 } X - \tilde{X} = (0, -7, 5)^t - (-0.33, -7.9, 5.8)^t = (0.33, 0.9, -0.8)^t$$

$$\text{所以 } \|X - \tilde{X}\|_{\infty} = 0.8$$

$$\begin{aligned} \text{因为 } A\tilde{X} - b &= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 6 \end{bmatrix} \times (-0.33, -7.9, 5.8)^t - (1, -1, 2)^t \\ &= (0.27, -0.16, 0.21)^t \end{aligned}$$

$$\text{所以 } \|A\tilde{X} - b\|_{\infty} = 0.27$$

问题二、

证明:

如果 A 为对称矩阵, 那么 A 一定为 $n \times n$ 矩阵

$$\text{因为 } A^T A = A^2$$

$$\text{所以 } A^2 X = \gamma X$$

$$\begin{aligned} |A^2 - \gamma E| &= |A - \sqrt{\gamma} E| \times |A + \sqrt{\gamma} E| = 0 \\ |A - \sqrt{\gamma} E| &= 0 \end{aligned}$$

$$\text{而 } |A - \lambda E| = 0$$

$$\text{所以有 } \sqrt{\gamma} = \lambda, \quad \gamma = \lambda^2$$

$$\text{所以 } \|A\|_2 = [\rho(A^T A)]^{1/2} = [\rho(A^2)]^{1/2} = [\max\{\gamma\}]^{1/2} = \max\{\lambda\} = \rho(A)$$

问题三、

a;

运算结果: solution: [10. 1.]

b:

运算结果:

solution: [0. 10. 0.14285714]

问题四、

a:

前三次运算结果:

[1.25, -1.33333333, 0.2]
[1.63333333, -0.98333333, 0.23333333]
[1.55416667, -0.86666667, -0.06]

b:

前三次运算结果:

[-2, 2, 0]
[-1, 1, -1]
[-1.75, 1.75, -0.5] (此时误差很大)

问题五、

a:

Jacobi method: [0.03510079, -0.23663751, 0.65812732]

Gauss-Seidel: [0.03535107, -0.23678863, 0.65775895]

b:

Jacobi method: [0.995725, 0.957775, 0.79145]

Gauss-Seidel: [0.9957475, 0.95787375, 0.79157475]