

Numerical Analysis – Winter 2019

Assignment #4

Issued: Dec. 4, 2019

Due: Dec.25, 2019

Please upload to the 'hw4' directory if you submit your homework in time.

Problem 1:

Construct the Lagrange interpolating polynomials for the following functions, and find a bound for the absolute error on the interval $[x_0, x_n]$.

a. $f(x) = e^{2x} \cos 3x$, $x_0 = 0, x_1 = 0.3, x_2 = 0.6, n = 2$

b. $f(x) = \sin(\ln x)$, $x_0 = 2.0, x_1 = 2.4, x_2 = 2.6, n = 2$

Problem 2:

Let $P_3(x)$ be the interpolating polynomial for the data $(0, 0)$, $(0.5, y)$, $(1, 3)$, and $(2, 2)$. The coefficient of x^3 in $P_3(x)$ is 6. Find y .

Problem 3:

Neville's method is used to approximate $f(0.4)$, giving the following table.

$x_0 = 0$	$P_0 = 1$				
$x_1 = 0.25$	$P_1 = 2$	$P_{01} = 2.6$			
$x_2 = 0.5$	P_2	$P_{1,2}$	$P_{0,1,2}$		
$x_3 = 0.75$	$P_3 = 8$	$P_{2,3} = 2.4$	$P_{1,2,3} = 2.96$	$P_{0,1,2,3} = 3.016$	

Determine $P_2 = f(0.5)$.

Suppose $x_j = j$, for $j = 0, 1, 2, 3$ and it is known that

$$P_{0,1}(x) = 2x + 1, \quad P_{0,2}(x) = x + 1, \quad \text{and} \quad P_{1,2,3}(2.5) = 3.$$

Find $P_{0,1,2,3}(2.5)$.

Problem 4:

For a function f , the forward-divided differences are given by

$x_0 = 0.0$	$f[x_0]$			
		$f[x_0, x_1]$		
$x_1 = 0.4$	$f[x_1]$		$f[x_0, x_1, x_2] = \frac{50}{7}$	
		$f[x_1, x_2] = 10$		
$x_2 = 0.7$	$f[x_2] = 6$			

Determine the missing entries in the table.

Problem 5:

Determine the natural cubic spline S that interpolates the data $f(0) = 0$, $f(1) = 1$, and $f(2) = 2$.

Determine the clamped cubic spline s that interpolates the data $f(0) = 0$, $f(1) = 1$, $f(2) = 2$ and satisfies $s'(0) = s'(2) = 1$.