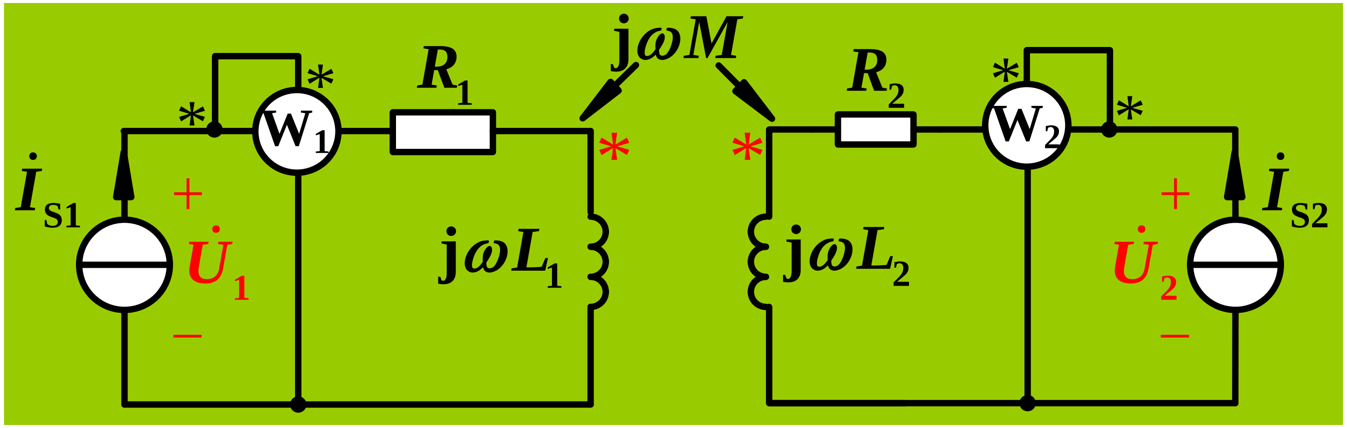
§ 10.1 耦合电感

三、耦合电感的功率

【例】 求图示电路中各功率表的读数。



解：

$$\dot{U}_1 = R_1\dot{I}_{S1} + j\omega L_1\dot{I}_{S1} + j\omega M\dot{I}_{S2}$$

$$\dot{U}_2 = R_2\dot{I}_{S2} + j\omega L_2\dot{I}_{S2} + j\omega M\dot{I}_{S1}$$

电流源发出的复功率：

$$\tilde{S}_1 = \dot{U}_1\dot{I}_{S1}^* = \left(R_1 + j\omega L_1\right)I_{S1}^2 + j\omega M\dot{I}_{S2}\dot{I}_{S1}^*$$

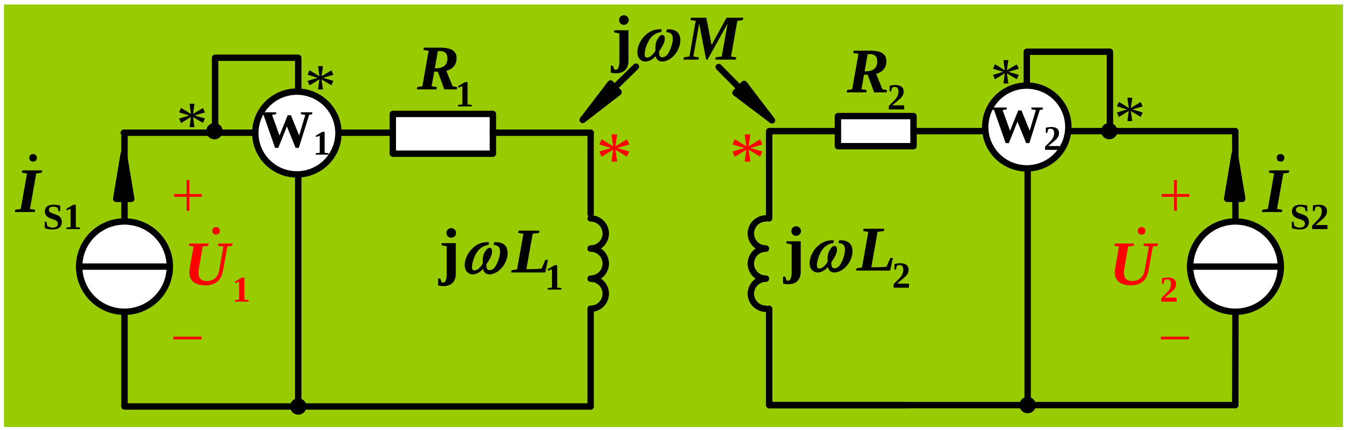
$$\tilde{S}_2 = \dot{U}_2\dot{I}_{S2}^* = \left(R_2 + j\omega L_2\right)I_{S2}^2 + j\omega M\dot{I}_{S1}\dot{I}_{S2}^*$$

线圈1中互感耦合的复功率

线圈2中互感耦合的复功率

三、耦合电感的功率

【例】 求图示电路中各功率表的读数。



解： 假设 $\dot{I}_{S1}=I_{S1}\angle\theta_1 \quad \dot{I}_{S2}=I_{S2}\angle\theta_2$

线圈1中互感耦合的复功率

$$\mathrm{j}\omega M\dot{I}_{S2}\dot{I}_{S1}^{*}=\mathrm{j}\omega MI_{S1}I_{S2}\angle(\theta_2-\theta_1)$$

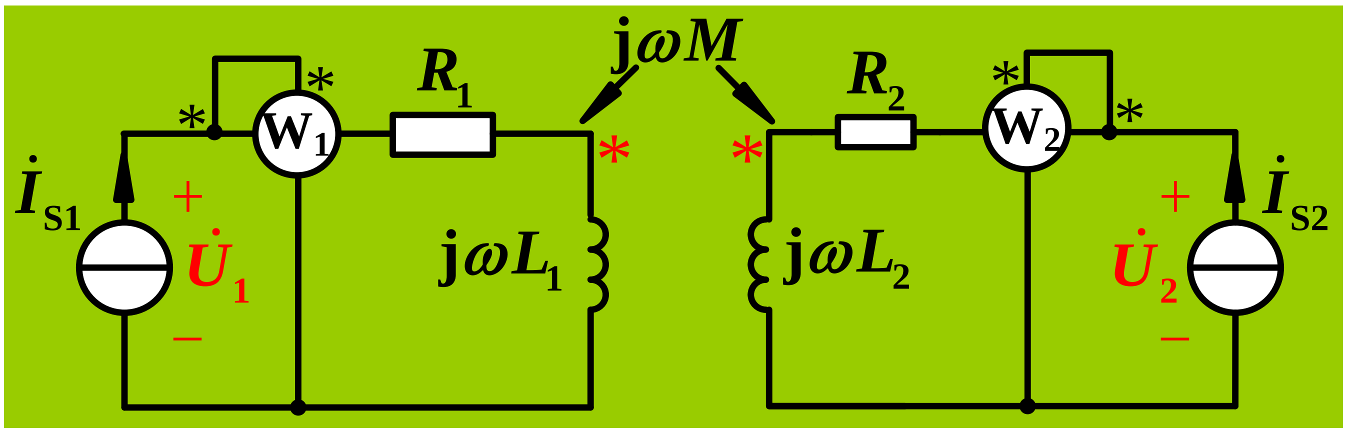
$$=\mathrm{j}\omega MI_{S1}I_{S2}\left[\cos(\theta_2-\theta_1)+\mathrm{j}\sin(\theta_2-\theta_1)\right]$$

$$=\omega MI_{S1}I_{S2}\left[\sin(\theta_1-\theta_2)+\mathrm{j}\cos(\theta_1-\theta_2)\right]$$

§ 10.1 耦合电感

三、耦合电感的功率

【例】 求图示电路中各功率表的读数。



解： 假设 $\dot{I}_{S1}=I_{S1}\angle\theta_1 \quad \dot{I}_{S2}=I_{S2}\angle\theta_2$

线圈2中互感耦合的复功率

$$\mathrm{j}\omega M\dot{I}_{S1}\dot{I}_{S2}^{*}=\mathrm{j}\omega M I_{S1}I_{S2}\angle(\theta_1-\theta_2)$$

$$=\mathrm{j}\omega M I_{S1}I_{S2}\left[\cos(\theta_1-\theta_2)+\mathrm{j}\sin(\theta_1-\theta_2)\right]$$

$$=\omega M I_{S1}I_{S2}\left[-\sin(\theta_1-\theta_2)+\mathrm{j}\cos(\theta_1-\theta_2)\right]$$

三、耦合电感的功率

有功功率　　无功功率

线圈1中互感耦合的复功率　$$\mathrm{j}\omega M\dot{I}_{S2}\dot{I}_{S1}^{*}=\omega M I_{S1}I_{S2}\left[\sin\left(\theta_1-\theta_2\right)+\mathrm{j}\cos\left(\theta_1-\theta_2\right)\right]$$

线圈2中互感耦合的复功率　$$\mathrm{j}\omega M\dot{I}_{S1}\dot{I}_{S2}^{*}=\omega M I_{S1}I_{S2}\left[-\sin\left(\theta_1-\theta_2\right)+\mathrm{j}\cos\left(\theta_1-\theta_2\right)\right]$$

(1) 耦合电感耦合的复功率为虚部同号、实部异号，该特点是耦合电感本身的电磁特性所决定的。

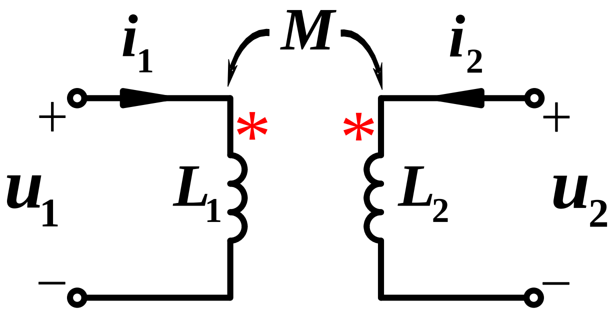
(2) 耦合电感耦合的复功率中有功功率异号，表明有功功率从一个 端口进入，必从另一端口输出，说明耦合电感具有非耗能特性。

(3) 耦合电感耦合的复功率中无功功率同号，表明互感耦合的复功率中无功功率对两个耦合线圈影响的性质相同，即：当互感M起同向耦合作用时，其储能特性与电感元件相同，使耦合电感中的磁能增加；当互感M起反向耦合作用时，其储能特性与电容元件相同，使耦合电感的储能减少。

§ 10.1 耦合电感

四、耦合电感的等效电路

1. 含受控源的等效电路

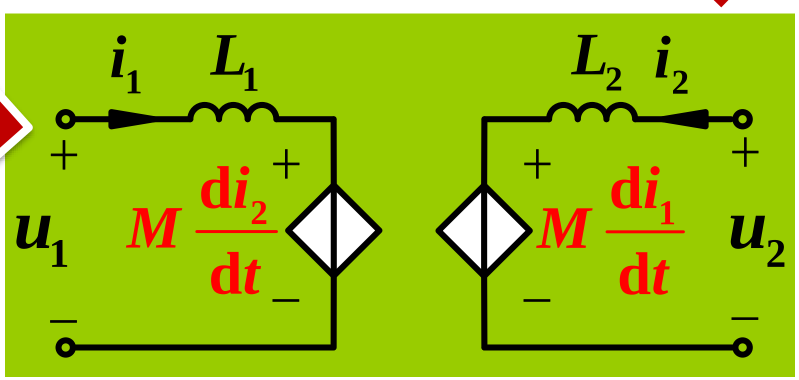


$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

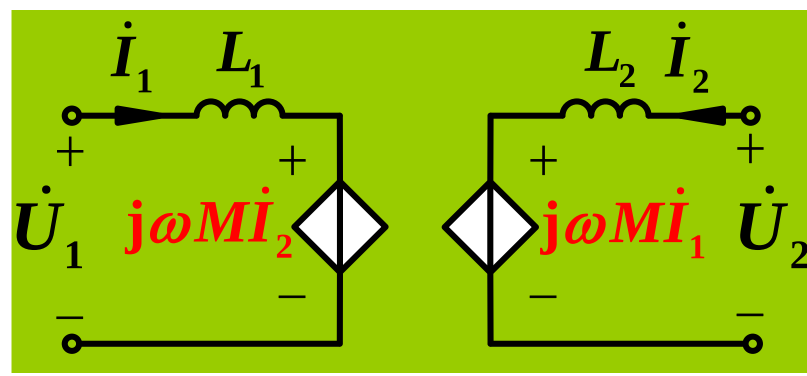
$$u_2 = M\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

等效

VAR方程



含受控源的等效电路（时域形式）

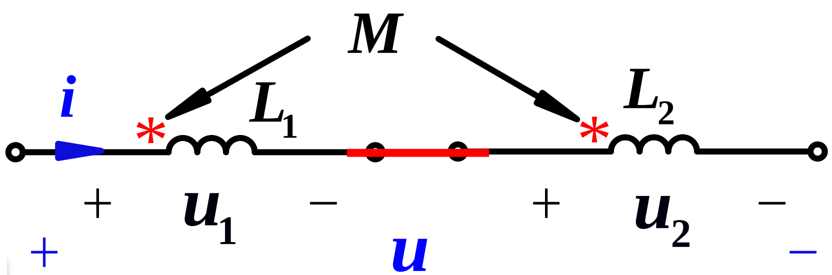


含受控源的等效电路（相量形式）

四、耦合电感的等效电路

2. 去耦等效电路—串联

a. 顺接串联



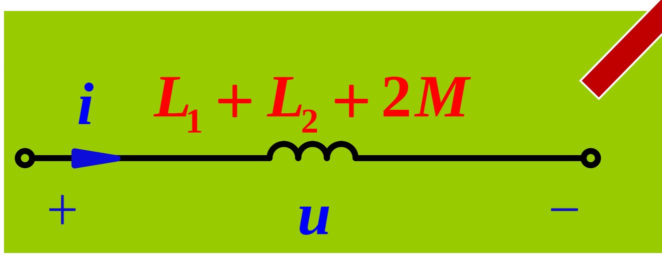
$$u_1 = L_1\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u_2 = M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$u = u_1 + u_2 = L_1\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$= \left(L_1 + L_2 + 2M\right)\frac{\mathrm{d}i}{\mathrm{d}t}$$

等效

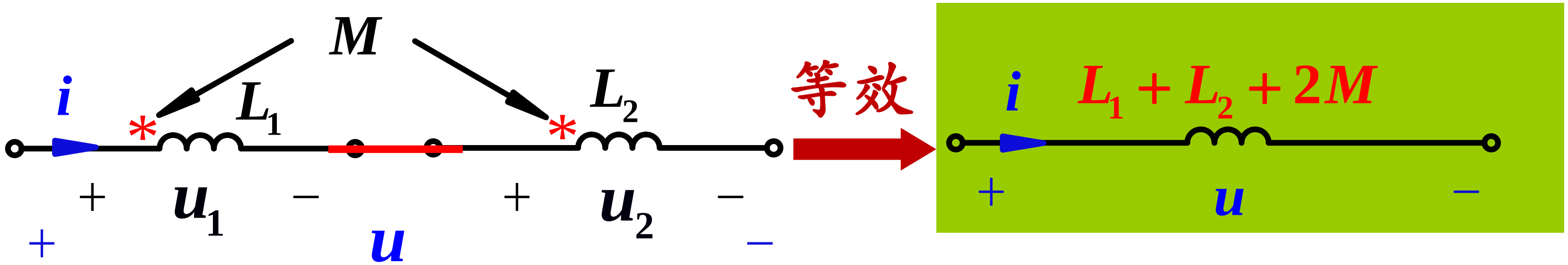


VAR方程

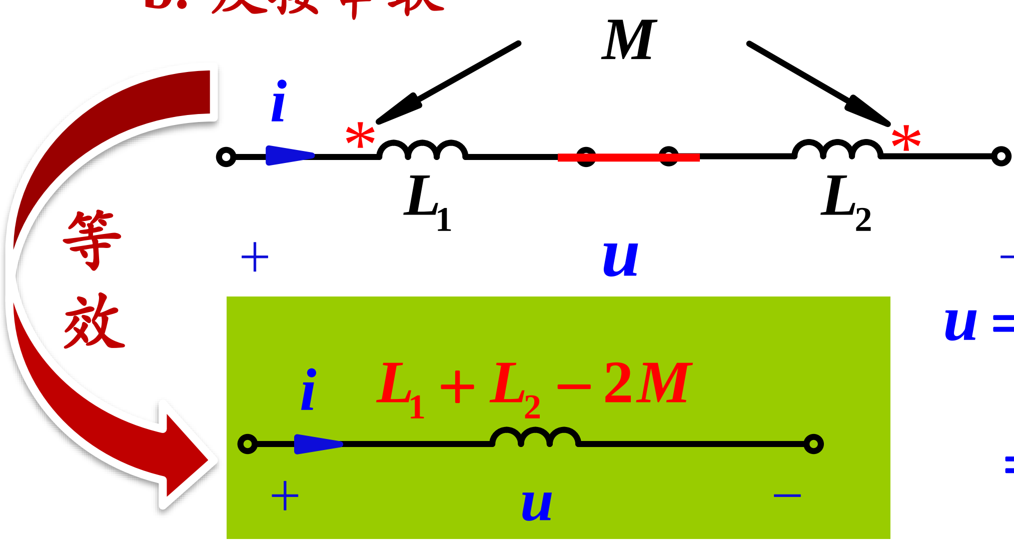
四、耦合电感的等效电路

2. 去耦等效电路—串联

a. 顺接串联



b. 反接串联



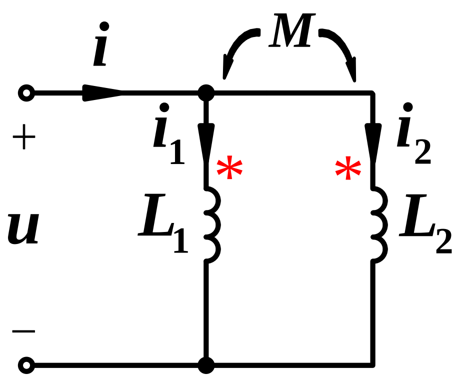
$$u = L_1\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} - M\frac{\mathrm{d}i}{\mathrm{d}t} + L_2\frac{\mathrm{d}i}{\mathrm{d}t}$$

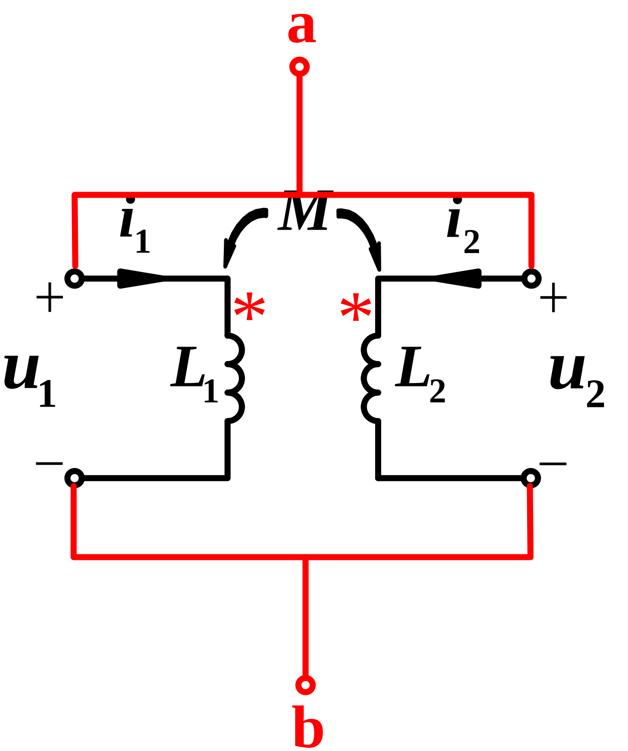
$$= \left(L_1 + L_2 - 2M\right)\frac{\mathrm{d}i}{\mathrm{d}t}$$

四、耦合电感的等效电路

2. 去耦等效电路—并联

a. 同名端并联

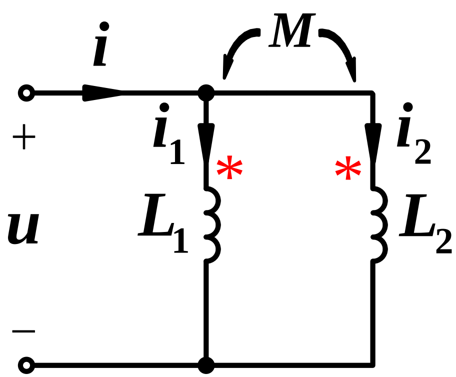




四、耦合电感的等效电路

2. 去耦等效电路—并联

a. 同名端并联



$$\begin{cases} u = L_1\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ \quad = M\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ i = i_1 + i_2 \end{cases}$$

联立求解u，i

$$u = \frac{\left(L_1L_2 + M^2\right)}{L_1 + L_2 - 2M}\frac{\mathrm{d}i}{\mathrm{d}t}$$

$$L_{eq} = \frac{\left(L_1L_2 - M^2\right)}{L_1 + L_2 - 2M}$$

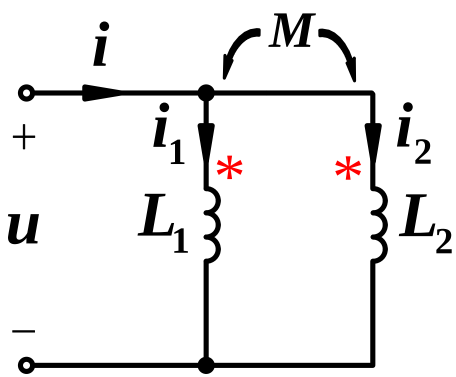
如此复杂的表达式，
如何能记得住啊~ ~ ~

§ 10.1 耦合电感

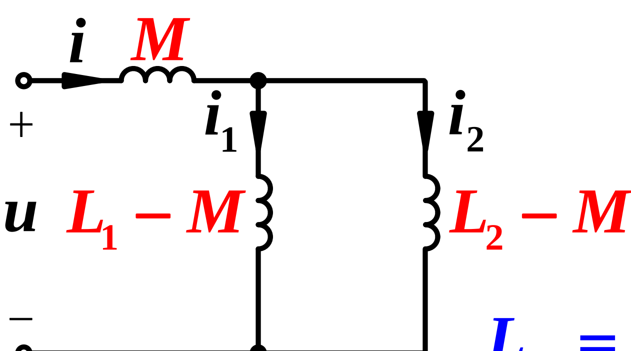
四、耦合电感的等效电路

2. 去耦等效电路—并联

a. 同名端并联



等效



去耦等效电路

$$\begin{cases} u = L_1\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = M\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

代入

$$\begin{cases} i_2 = i - i_1 \\ i_1 = i - i_2 \end{cases}$$

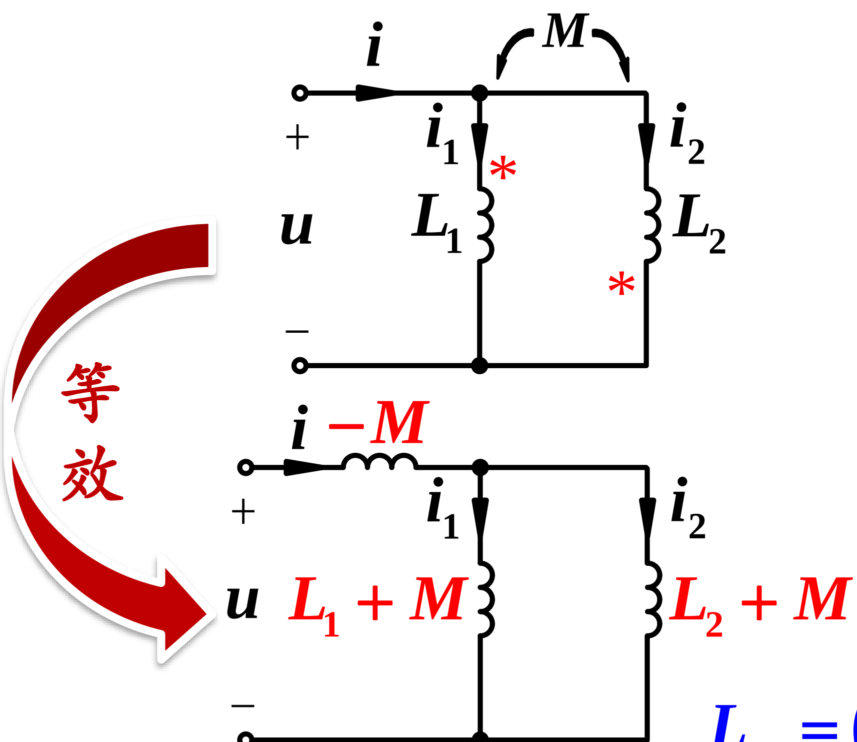
$$\begin{cases} u = (L_1 - M)\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + M\dfrac{\mathrm{d}i}{\mathrm{d}t} \\ u = M\dfrac{\mathrm{d}i}{\mathrm{d}t} + (L_2 - M)\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

$$L_{eq} = (L_1 - M)//(L_2 - M) + M = \frac{(L_1L_2 - M^2)}{L_1 + L_2 - 2M}$$

四、耦合电感的等效电路

2. 去耦等效电路—并联

b. 异名端并联



$$\begin{cases} u = L_1 \dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \\ u = -M\dfrac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

代入

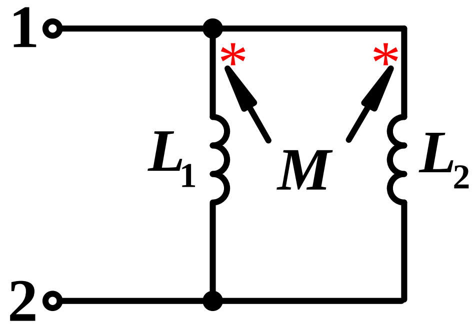
$$\begin{cases} i_2 = i - i_1 \\ i_1 = i - i_2 \end{cases}$$

$$\begin{cases} u = (L_1 + M)\dfrac{\mathrm{d}i_1}{\mathrm{d}t} - M\dfrac{\mathrm{d}i}{\mathrm{d}t} \\ u = -M\dfrac{\mathrm{d}i}{\mathrm{d}t} + (L_2 + M)\dfrac{\mathrm{d}i_2}{\mathrm{d}t} \end{cases}$$

$$L_{\mathrm{eq}} = (L_1 + M)//(L_2 + M) - M = \frac{(L_1L_2 - M^2)}{L_1 + L_2 + 2M}$$

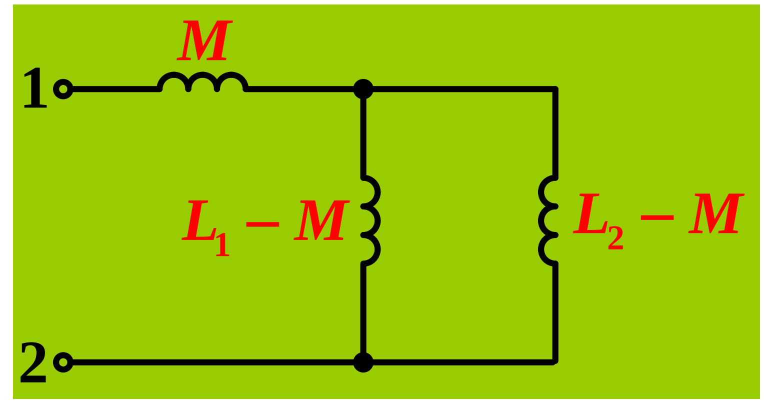
四、耦合电感的等效电路

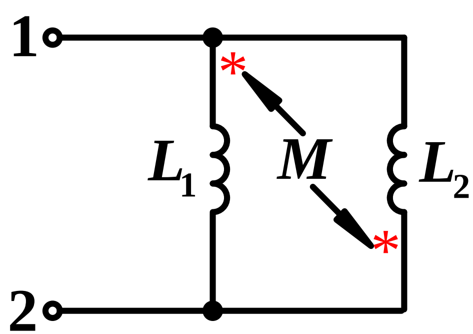
2. 去耦等效电路—并联



去耦

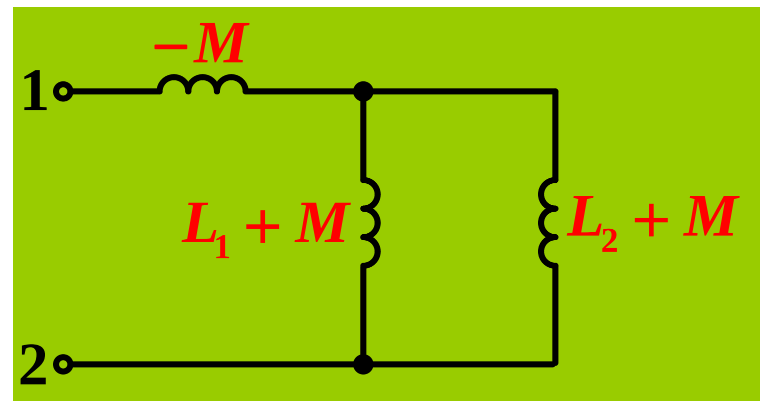
等效





去耦

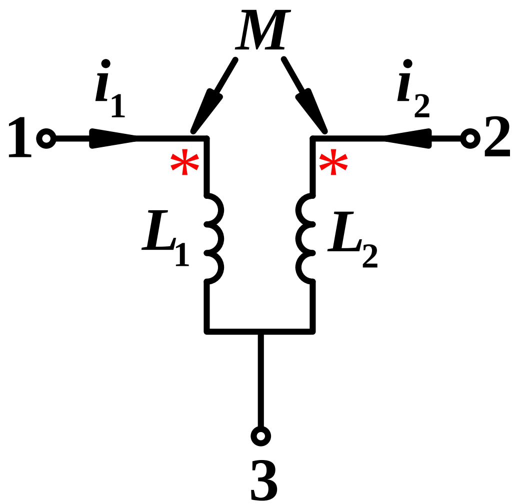
等效

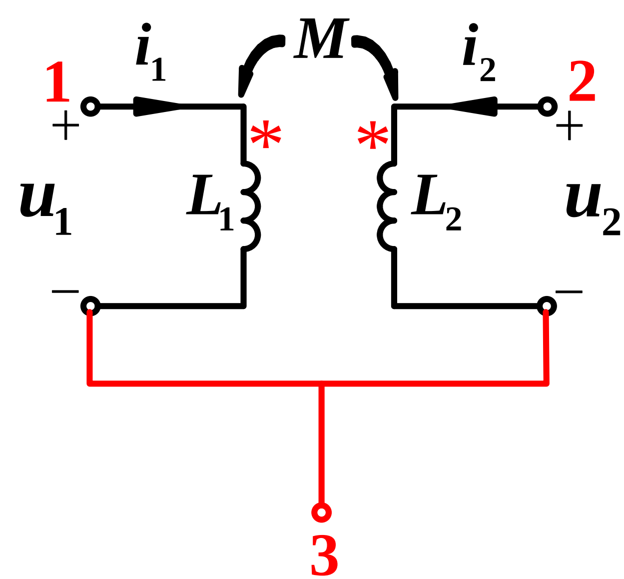


四、耦合电感的等效电路

3. 去耦等效电路—三端耦合电感

a. 同名端相连

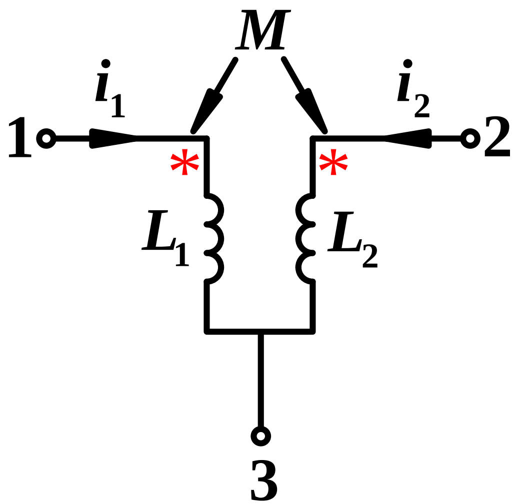




四、耦合电感的等效电路

3. 去耦等效电路—三端耦合电感

a. 同名端相连



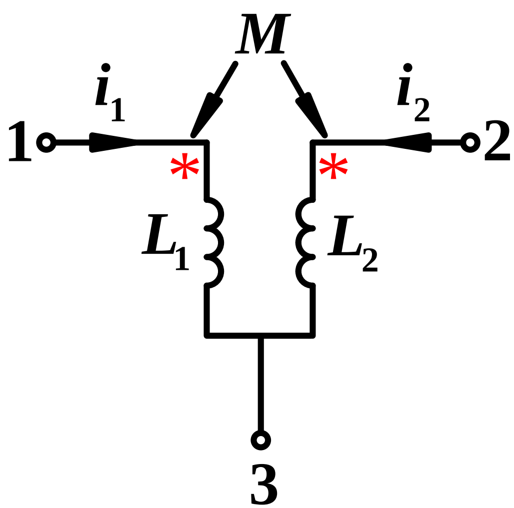
$$u_{13}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}i_2}{\mathrm{d}t}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}i_2}{\mathrm{d}t}+M\frac{\mathrm{d}i_1}{\mathrm{d}t}-M\frac{\mathrm{d}i_1}{\mathrm{d}t}$$

$$=\left(L_1-M\right)\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}\left(i_1+i_2\right)}{\mathrm{d}t}$$

四、耦合电感的等效电路

3. 去耦等效电路—三端耦合电感

a. 同名端相连



$$u_{13}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}i_2}{\mathrm{d}t}=\boxed{\left(L_1-M\right)\frac{\mathrm{d}i_1}{\mathrm{d}t}+M\frac{\mathrm{d}\left(i_1+i_2\right)}{\mathrm{d}t}}$$

$$u_{23}=M\frac{\mathrm{d}i_1}{\mathrm{d}t}+L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}=M\frac{\mathrm{d}i_1}{\mathrm{d}t}+L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}+M\frac{\mathrm{d}i_2}{\mathrm{d}t}-M\frac{\mathrm{d}i_2}{\mathrm{d}t}$$

★注意：

1. 去耦等效后—增加了节点“4”

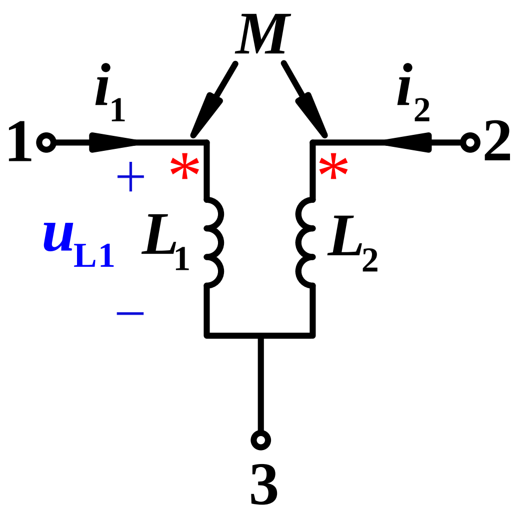
2. 原电路中耦合电感L_1的电压值？

$u_{L1} \neq u_{14}$ $u_{L1} = u_{13}$

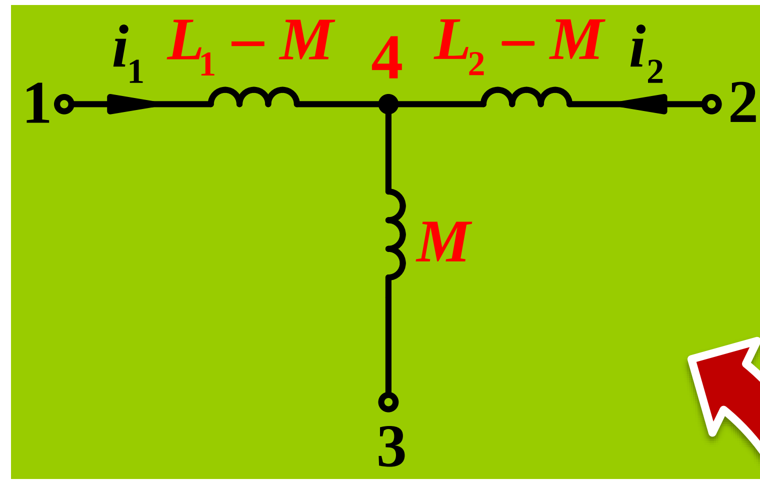
四、耦合电感的等效电路

3. 去耦等效电路—三端耦合电感

a. 同名端相连



等效



构造

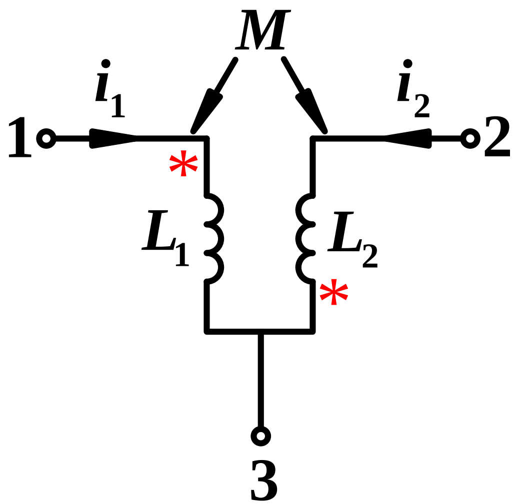
$$u_{13} = L_1 \frac{\mathrm{d}i_1}{\mathrm{d}t} + M \frac{\mathrm{d}i_2}{\mathrm{d}t} = \boxed{(L_1 - M)\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t}}$$

$$u_{23} = M \frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i_2}{\mathrm{d}t} = \boxed{M\frac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t} + (L_2 - M)\frac{\mathrm{d}i_2}{\mathrm{d}t}}$$

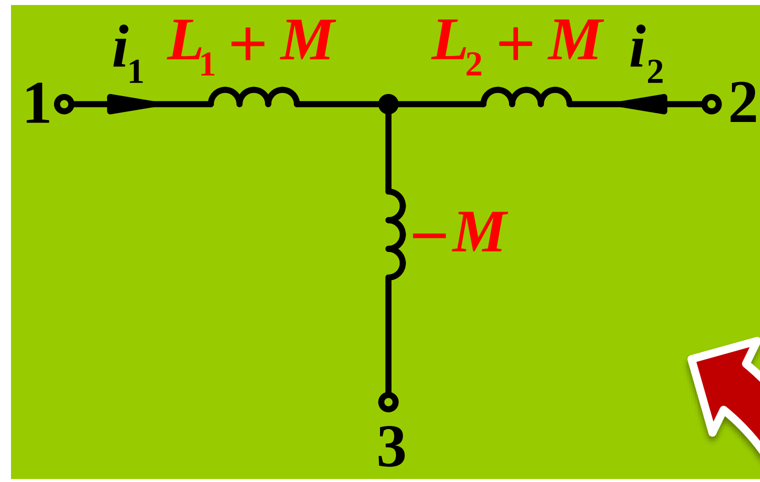
四、耦合电感的等效电路

3. 去耦等效电路—三端耦合电感

b. 异名端相连



等效



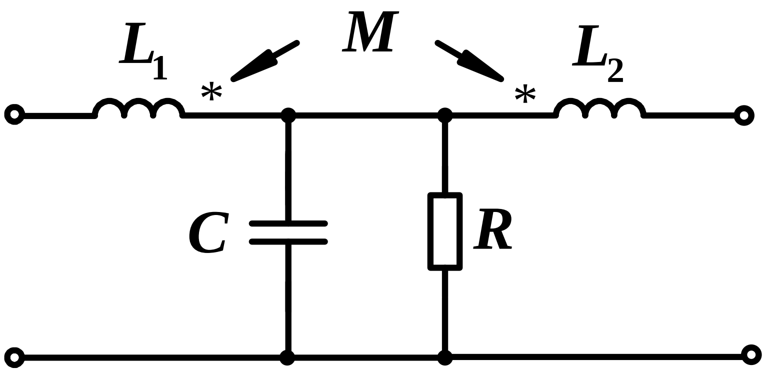
构造

$$u_{13}=L_1\frac{\mathrm{d}i_1}{\mathrm{d}t}-M\frac{\mathrm{d}i_2}{\mathrm{d}t}=\boxed{\left(L_1+M\right)\frac{\mathrm{d}i_1}{\mathrm{d}t}-M\frac{\mathrm{d}\left(i_1+i_2\right)}{\mathrm{d}t}}$$

$$u_{23}=-M\frac{\mathrm{d}i_1}{\mathrm{d}t}+L_2\frac{\mathrm{d}i_2}{\mathrm{d}t}=\boxed{-M\frac{\mathrm{d}\left(i_1+i_2\right)}{\mathrm{d}t}+\left(L_2+M\right)\frac{\mathrm{d}i_2}{\mathrm{d}t}}$$

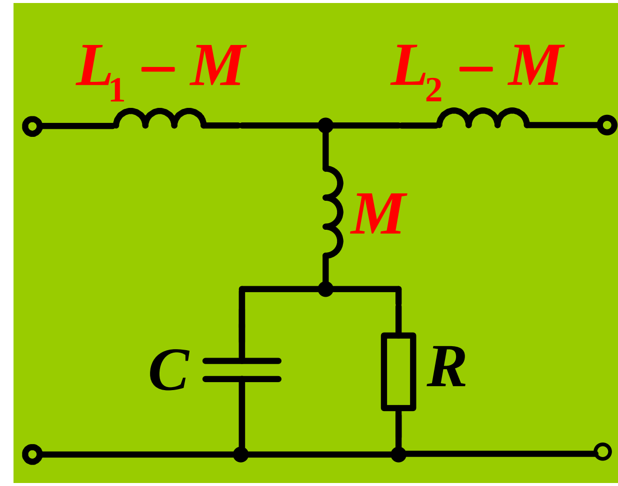
四、耦合电感的等效电路

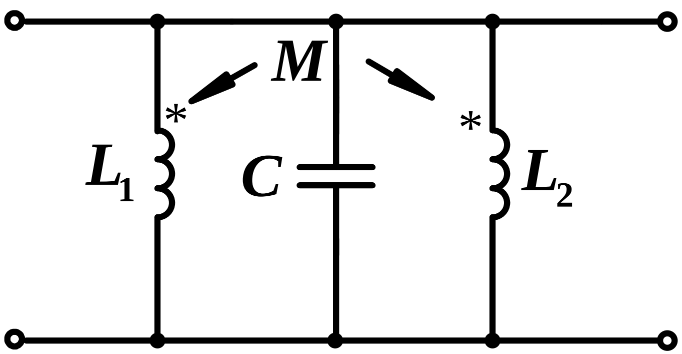
【例1】



去耦

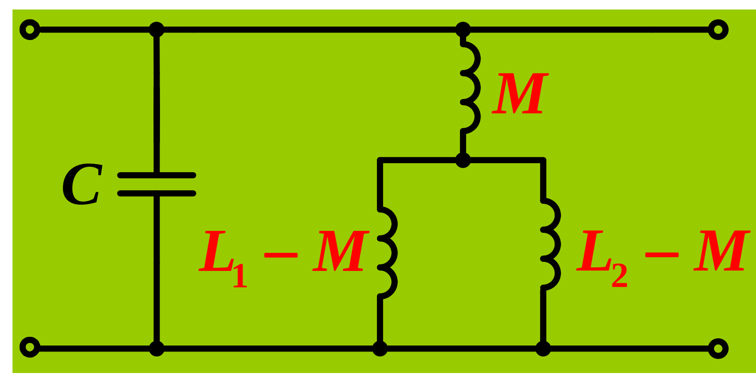
等效





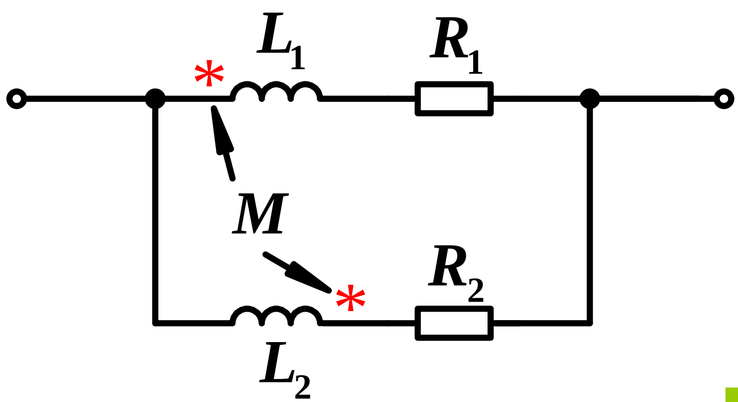
去耦

等效



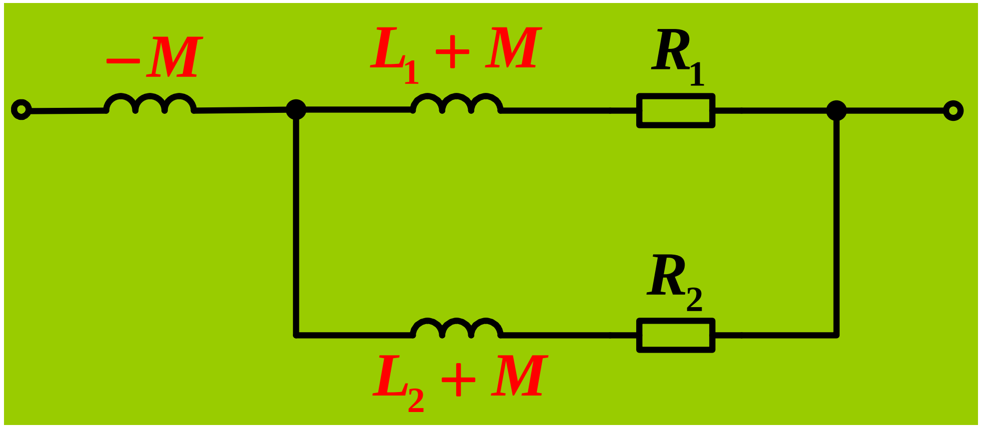
四、耦合电感的等效电路

【例2】



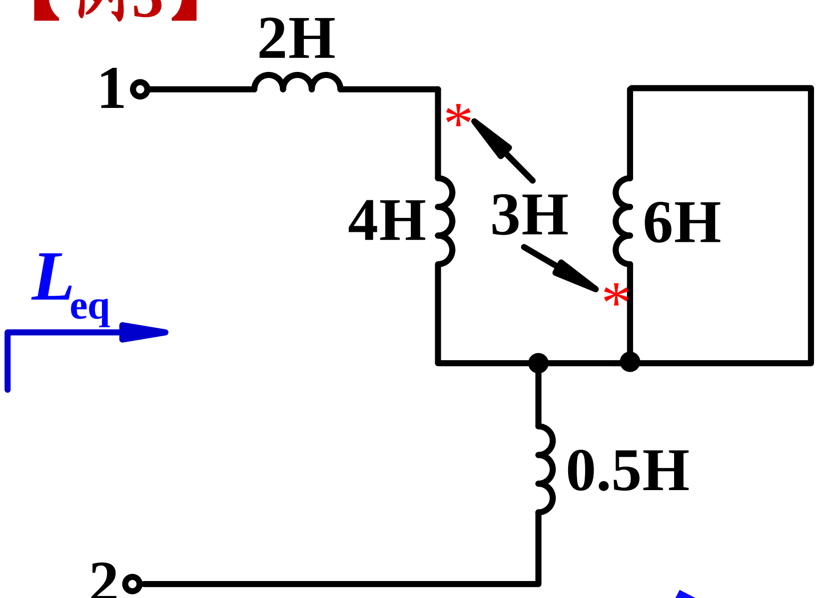
去耦

等效



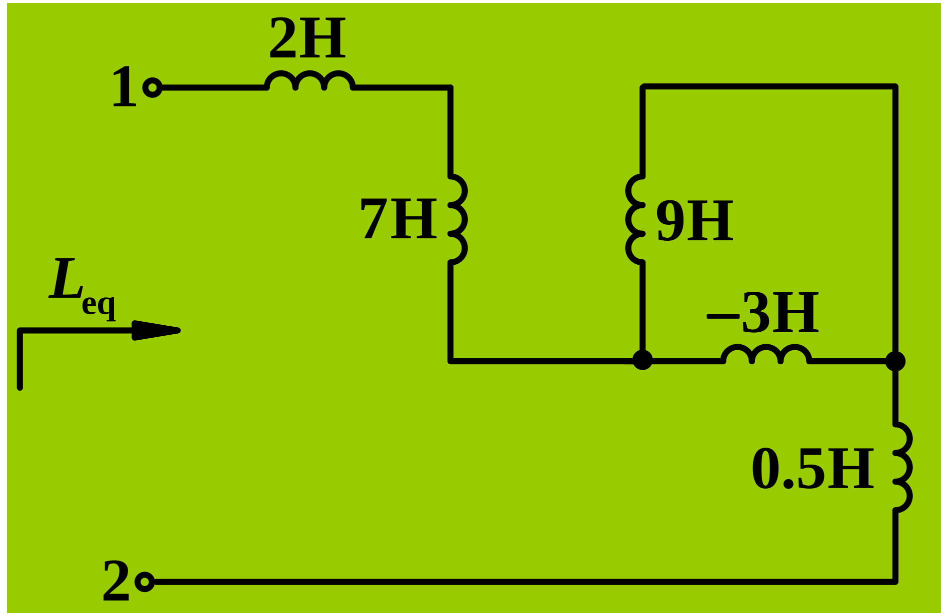
四、耦合电感的等效电路

【例3】



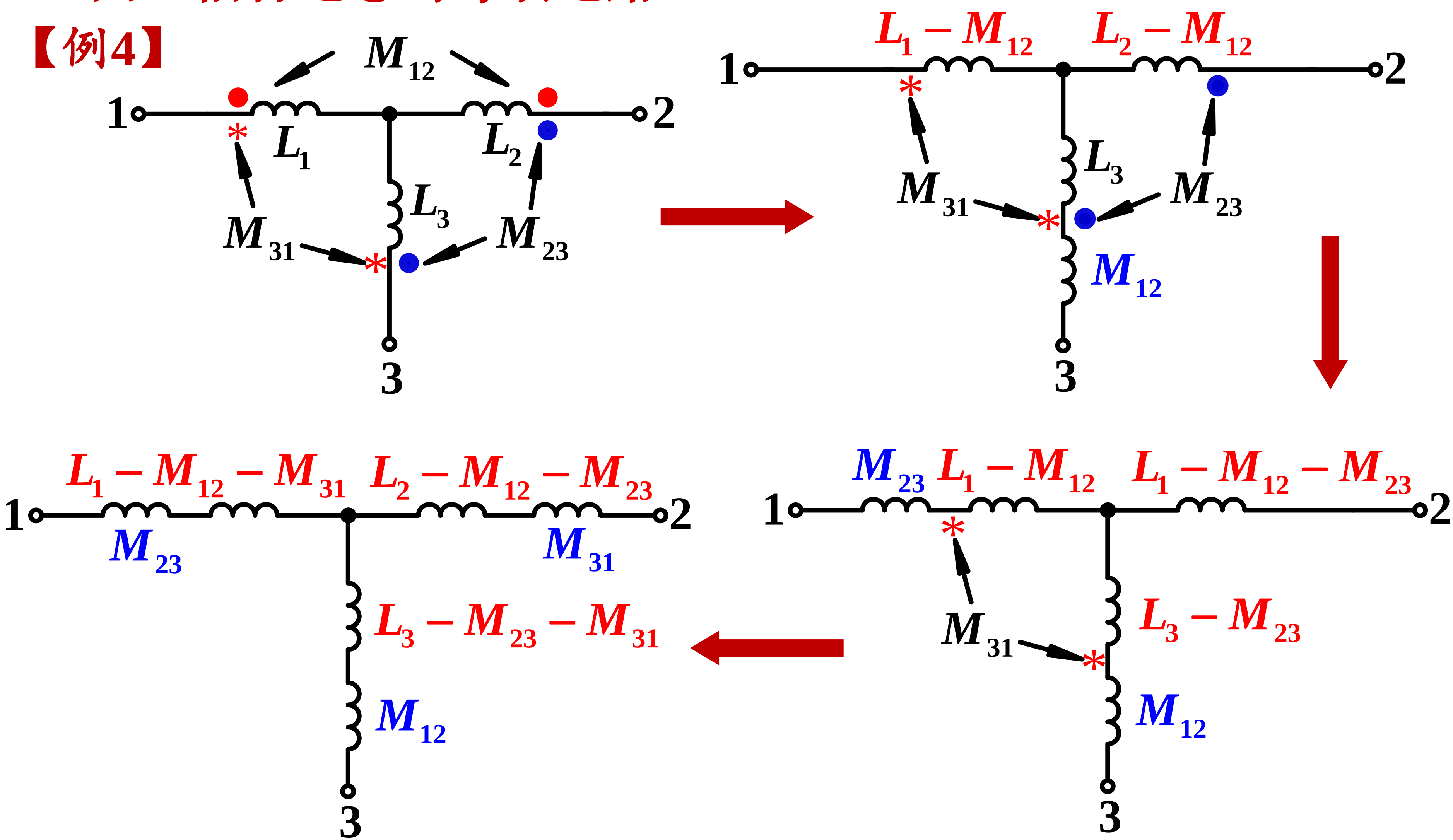
$$L_{eq} = 2H + 7H + 9H // (-3)H + 0.5H$$
$$= 9.5H + \frac{-3\times 9}{9-3}H$$
$$= 5\ H$$

去耦



四、耦合电感的等效电路

【例4】



电 路 理 论
Principles of Electric Circuits

第十章 含耦合电感电路的分析

§ 10.2 含耦合电感电路的分析

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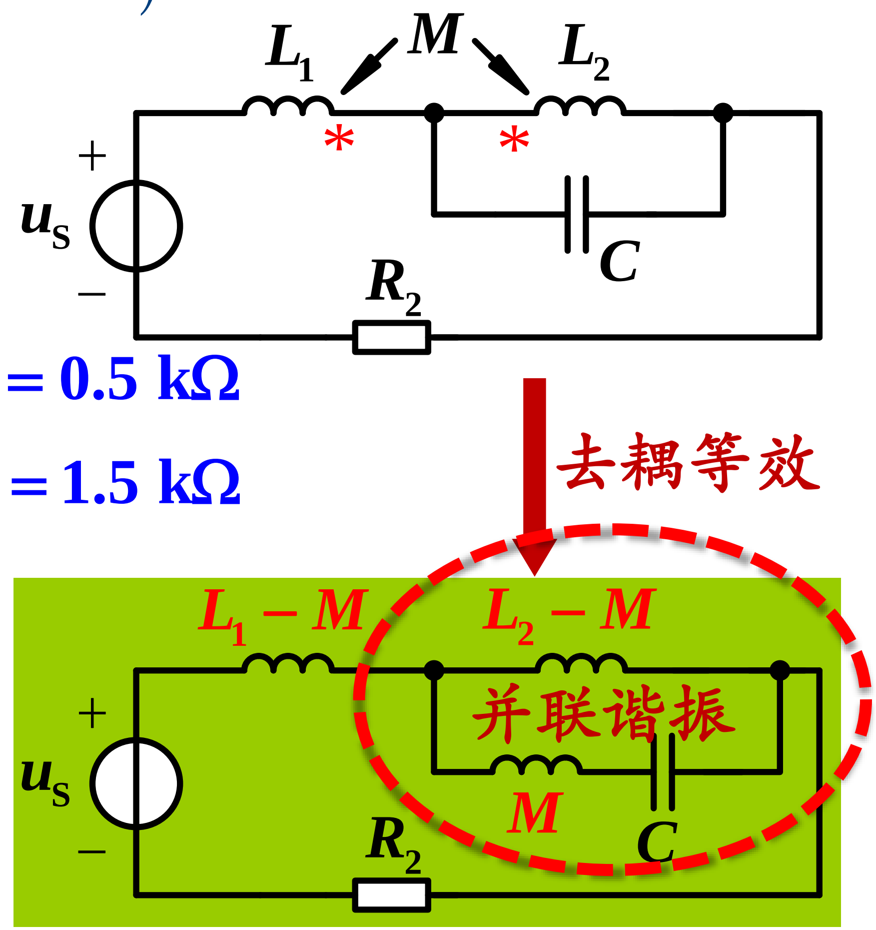
一、互感消去法

【例】图示正弦稳态电路中，$L_1=1\text{H}, L_2=2\text{H}, M=0.5\text{H}, C=0.5\mu\text{F}$
$R=1\text{k}\Omega, u_S(t)=150\sin\left(1000t+30^\circ\right)\text{V}$，求电容支路电流。

解：



$$X_C=-\frac{1}{\omega C}=-\frac{1}{1000\times0.5\times10^{-6}}$$
$$=-2\ \text{k}\Omega$$

$$X_1=\omega\left(L_1-M\right)=1000\times\left(1-0.5\right)=0.5\ \text{k}\Omega$$

$$X_2=\omega\left(L_2-M\right)=1000\times\left(2-0.5\right)=1.5\ \text{k}\Omega$$

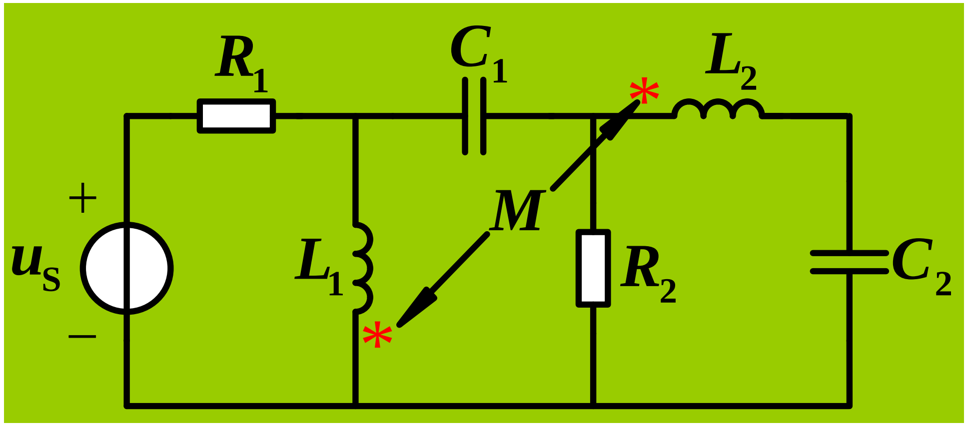
$$X_M=\omega M=1000\times0.5=0.5\ \text{k}\Omega$$

$$\dot{I}_m=\frac{\dot{U}_{sm}}{jX_M+jX_C}=-\frac{150\angle30^\circ}{j0.5-j2}$$
$$=100\angle120^\circ\text{mA}$$

$$i(t)=100\sin\left(1000t+120^\circ\right)\text{mA}$$

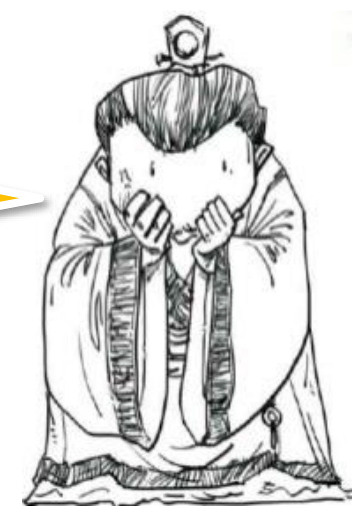
§ 10.2 含耦合电感电路的分析

二、回路分析法

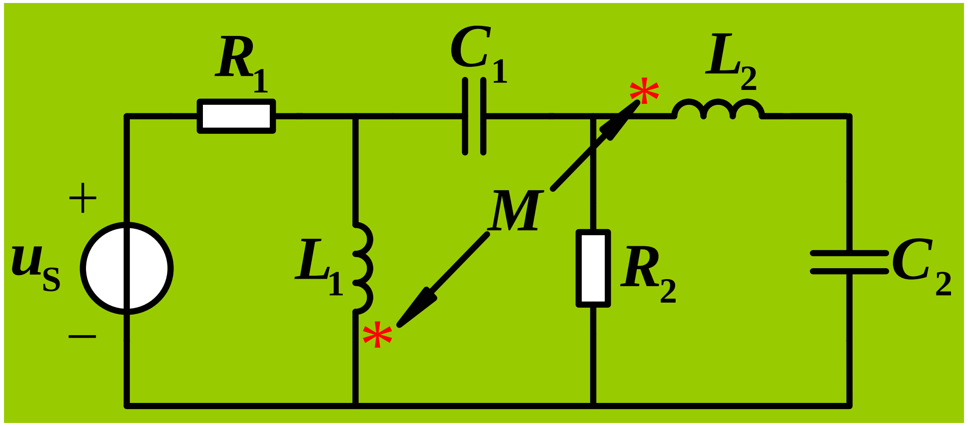


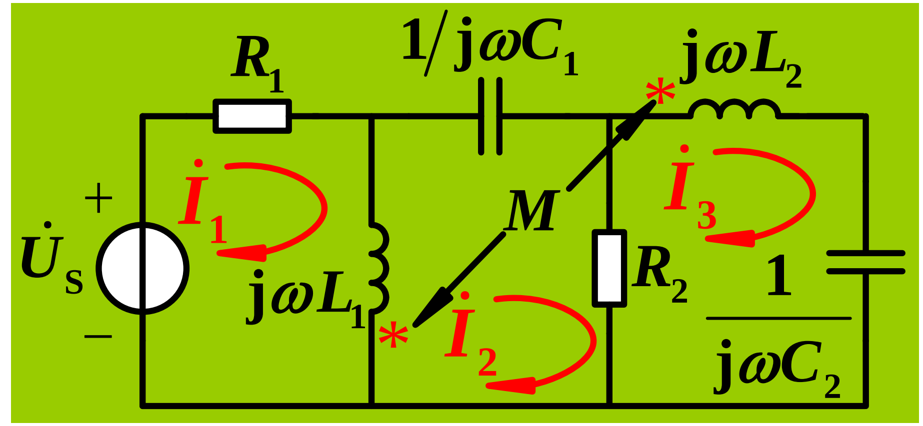
没有公共节点，如何消互感？

去耦等效也不是万能的啊~ ~ ~



二、回路分析法





$$(R_1 + j\omega L_1)\dot{I}_1 - j\omega L_1\dot{I}_2 - j\omega M\dot{I}_3 = \dot{U}_S$$

$$-j\omega L_1\dot{I}_1 + (j\omega L_1 + R_2 + \frac{1}{j\omega C_1})\dot{I}_2 - R_2\dot{I}_3 + j\omega M\dot{I}_3 = 0$$

$$-R_2\dot{I}_2 + (j\omega L_2 + R_2 + \frac{1}{j\omega C_2})\dot{I}_3 + j\omega M(\dot{I}_2 - \dot{I}_1) = 0$$

★**注意：**（1）不要丢掉互感电压项；

（2）正确判断互感电压的正、负。