# Connected Graphs and Spanning Trees

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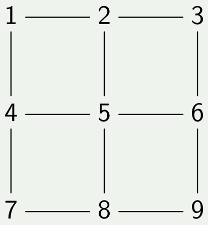
# Describing the problem I

$G = (V, E)$ is a graph, where

1. $V$ is a set of vertices
2. $E$ is a (multi)set of edges

## Example 1




- $V = \{1, \ldots, 9\}$
- $E = \{[1,2]; [1,4]; [2,3]; [2,5]; [3,6]; [4,5]; [4,7]; [5,6]; [5,8]; [6,9]; [7,8]; [8,9]\}$

# Describing the problem II

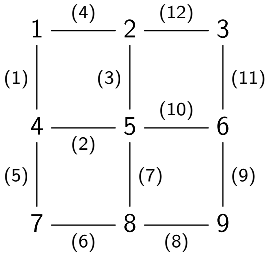
Let $G = (V, E)$ be a connected graph.

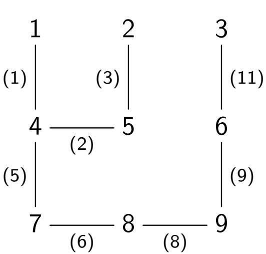
$T = (V, E')$ is spanning tree of $G$ when

1. $T$ is tree, and
2. $E' \subseteq E$.

**Theorem 2**

*Every connected graph has a spanning tree.*



# Towards formalization

## Specification

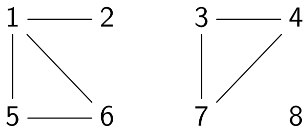
1. Define data types to represent graphs
2. Define the following functions:
   - `connected`
   - `nocycle`
   - `mktree`

## Verification

1. $\texttt{connected(G)} \Rightarrow \exists \texttt{G'} \subseteq \texttt{G}.\texttt{tree(G')}$
2. $\texttt{connected(G)} \Rightarrow \texttt{connected(mktree(G))} \wedge \texttt{nocycle(mktree(G))}$

# Functions on graphs

$$G = \left( \begin{array}{l} \{1,2,3,4,5,6,7,8\}, \\ \texttt{[1,2];[1,6];[1,5];[3,4];[3,7];[4,7];[5,6]} \end{array} \right)$$




- `mcc(A,G)` = max. connected comp. of `A` in `G`.
  `mcc(6,G)` = $\{1,2,5,6\}$, `mcc(8,G)` = $\{8\}$
- \#`cc(G)` = no. of max. connected components
  \#`cc(G) = 3`
- `nocycle(G) = false`

# Spanning forests

In the attempt of proving the desired properties we realized is much easier to prove a more general result:

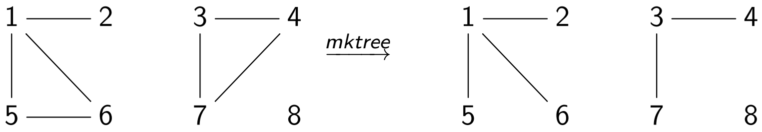
**Every graph has a spanning forest!**

## Definition 3

A **spanning forest** of a graph is a subgraph that consists of a set of spanning trees, one for each maximal connected component of the initial graph.

We define the function `mktree` which returns the spanning forest of a graph.

$$G = \left( \begin{array}{l} \{1,2,3,4,5,6,7,8\}, \\ \texttt{[1,2];[1,6];[1,5];[3,4];[3,7];[4,7];[5,6]} \end{array} \right)$$




$$\texttt{mktree(G)} = \left( \begin{array}{l} \{1,2,3,4,5,6,7,8\} \\ \texttt{[1,2];[1,6];[1,5];[3,4];[3,7]} \end{array} \right)$$

**Remark 4**

*The value is relative to the order chosen for the edges.*

## Properties to prove

1. $\texttt{mcc(A,G)=mcc(A,mktree(G))}$
2. $\texttt{\#cc(G)=\#cc(mktree(G))}$
3. $\texttt{nocycle(mktree(G))}$

Then we define $\texttt{connected(G):= (\#cc(G)=1)}$ which implies

$$\texttt{connected(G)} \Rightarrow \texttt{connected(mktree(G))} \wedge \texttt{nocycle(mktree(G))}$$