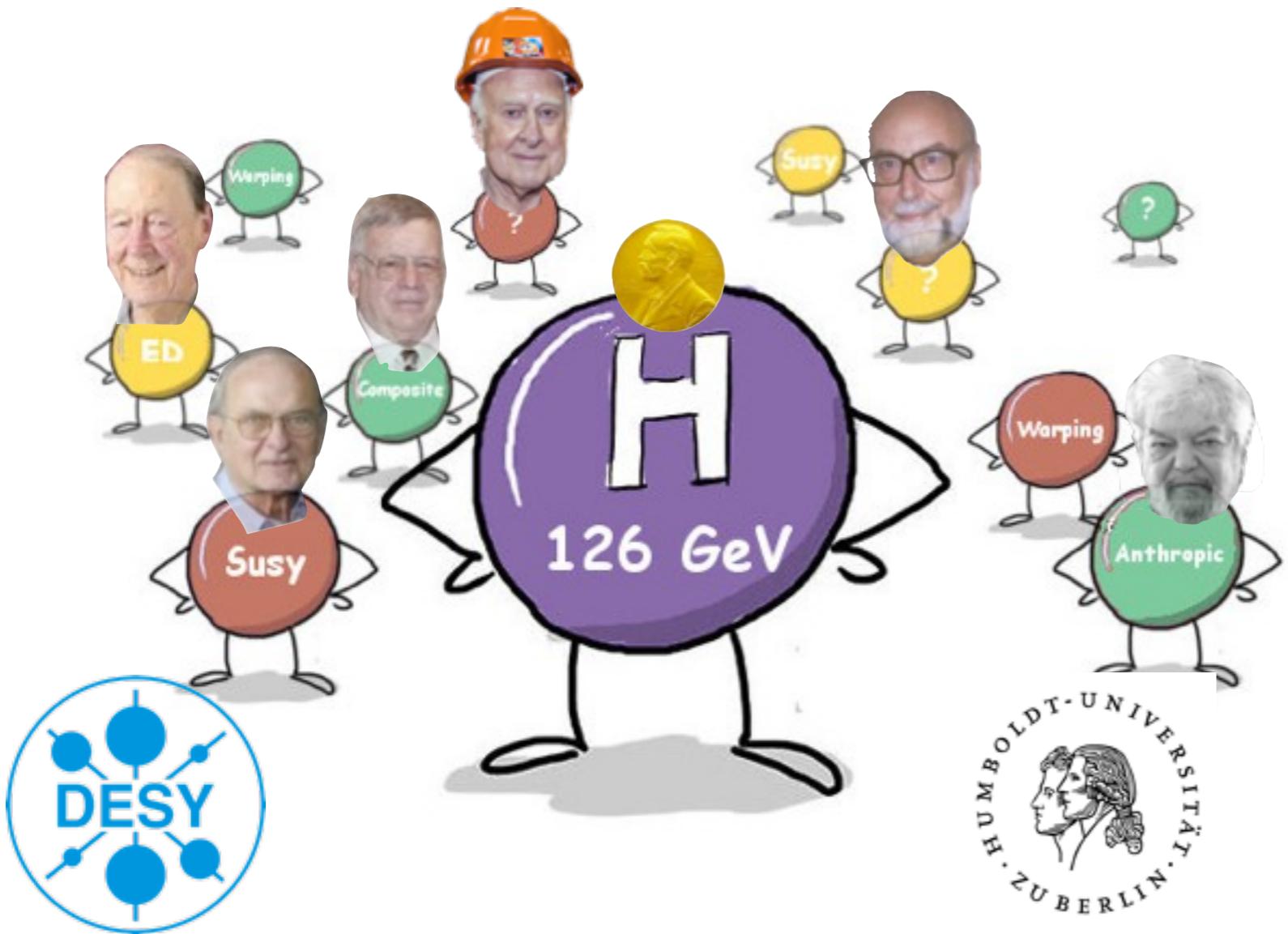


# Introduction

# HEP Theory

DESY summer student lectures 2017



Lectures 1+2/6

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Humboldt University (Berlin)

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# Outline

## 1. Friday

Quantum field theory, dimensional analysis, symmetries (space-time and internal gauge symmetries, continuous, global), group theory

## 2. Friday

Local symmetries and gauge invariance principle, QED, Standard Model

## 3. Monday

Spontaneous symmetry breaking, Goldstone theorem, Higgs mechanism

## 4. Monday

Electroweak precision test, stability of the EW vacuum, the hierarchy problem

## 5. Tuesday

Field quantization, S-matrix, Feynman rules, scattering, cross sections, decay rates, calculation tricks, sample calculation

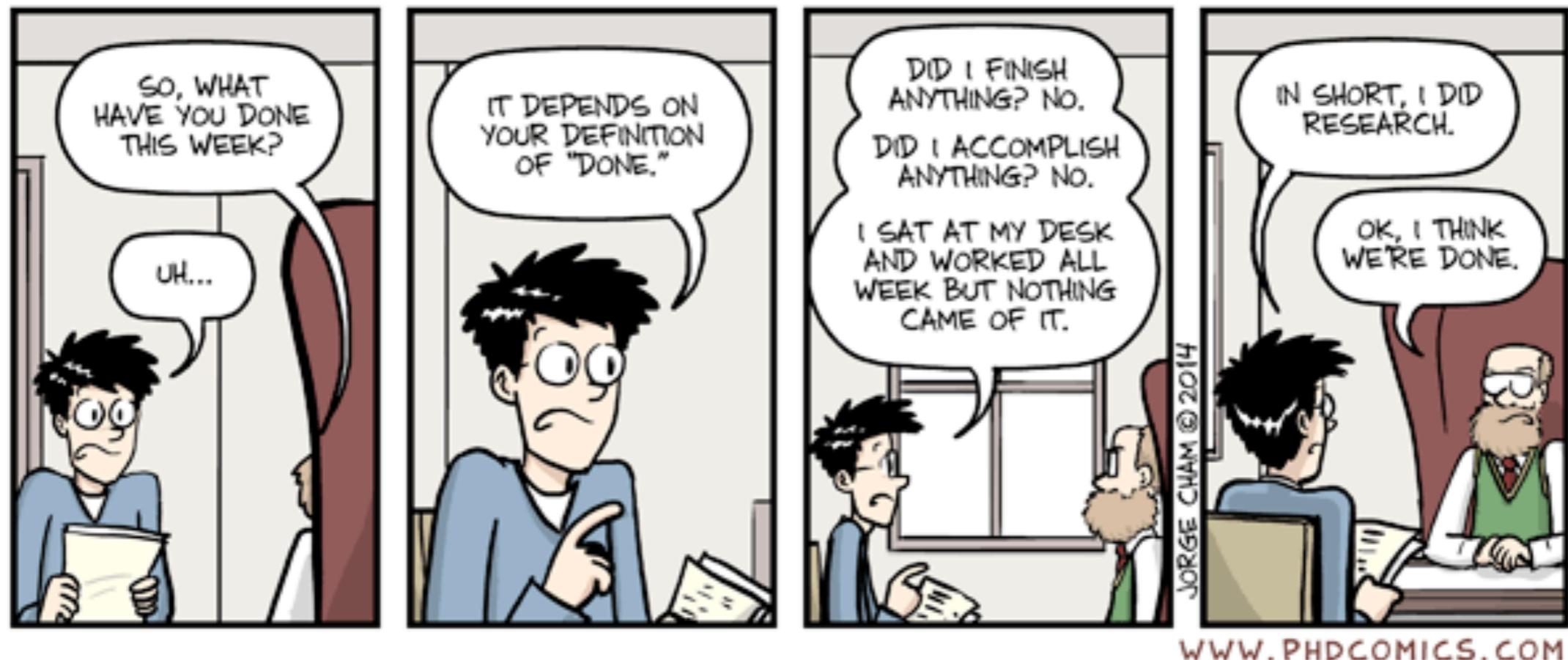
## 6. Tuesday

Non-abelian gauge theories, Standard Model Lagrangian and its phenomenological properties, observables

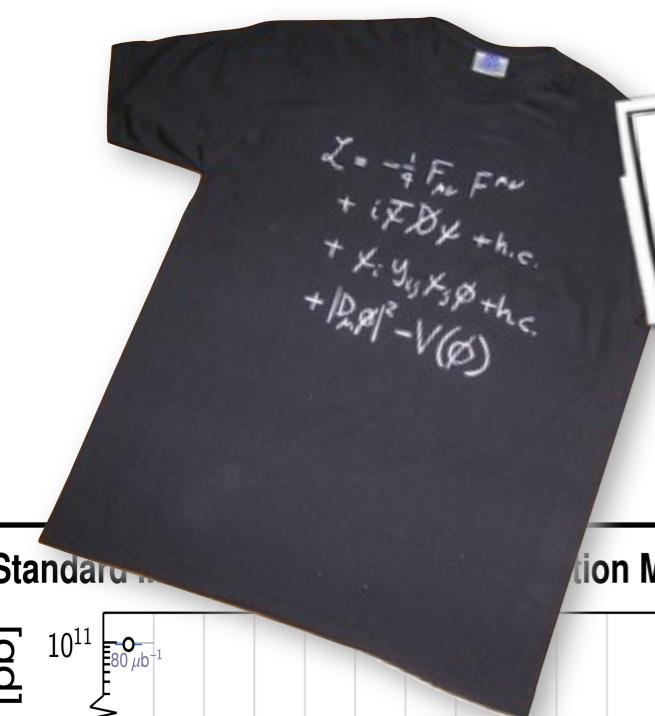
the order will probably change

# Ask questions

Your work, as students, is to question all what you are listening during the lectures...

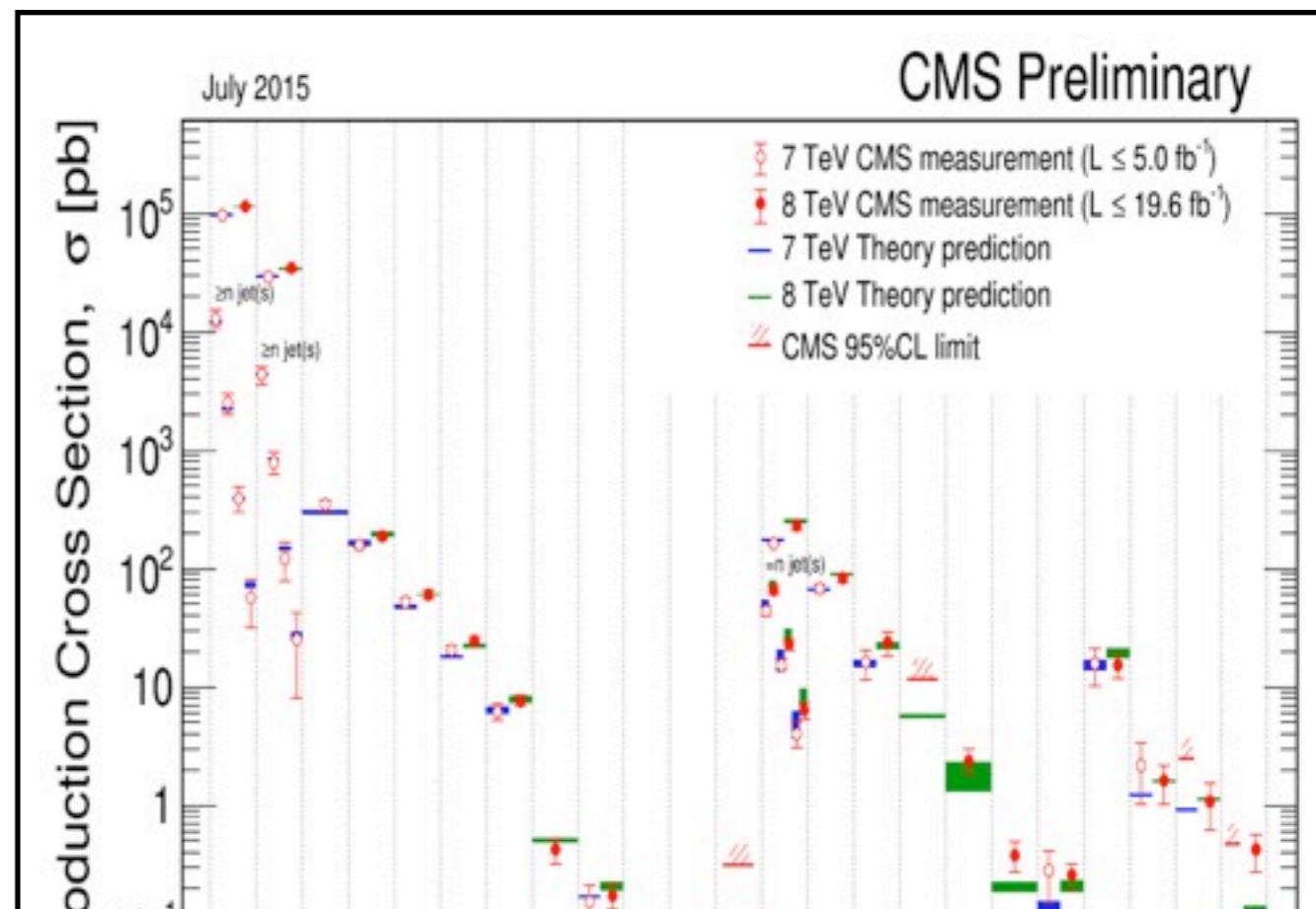
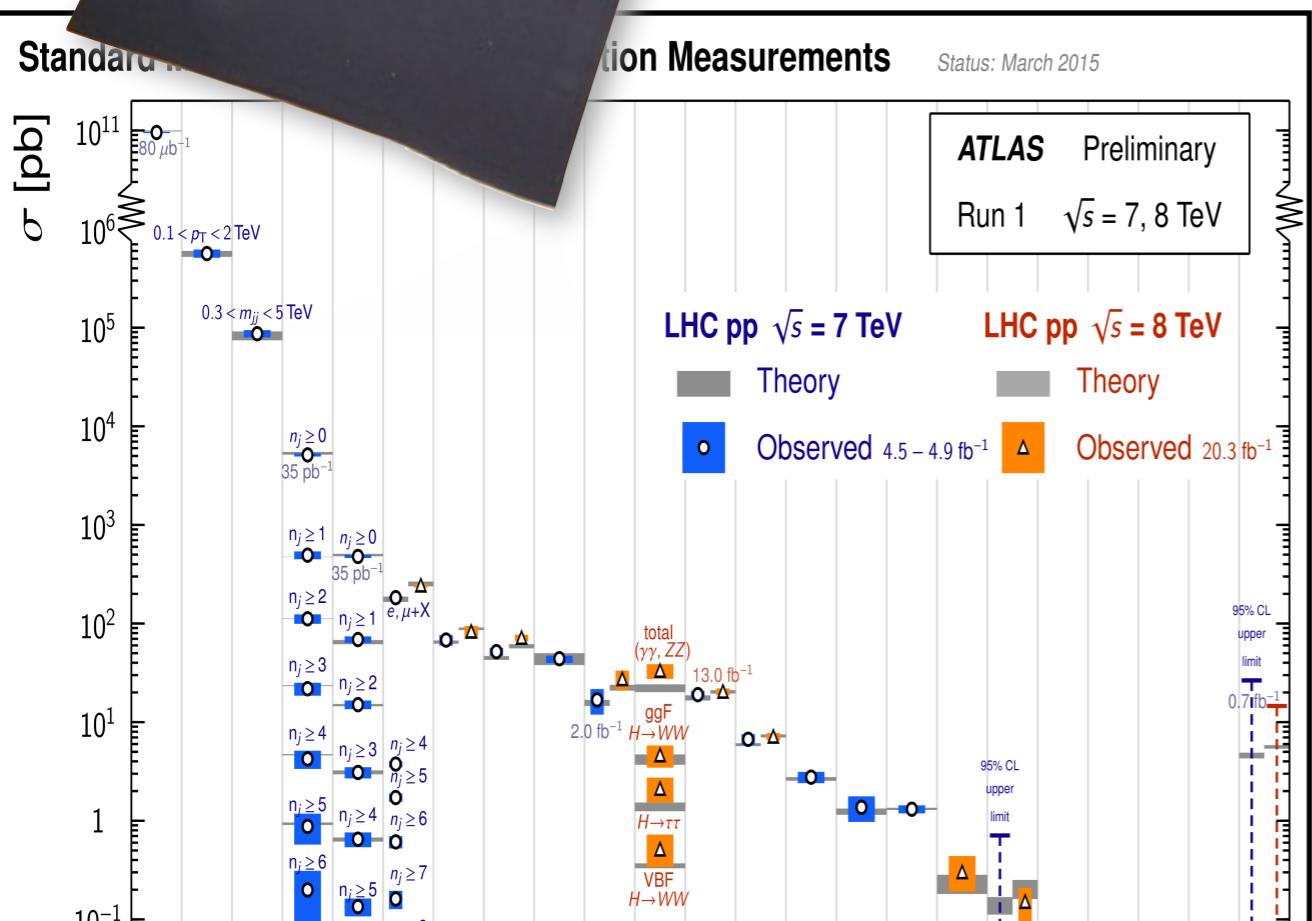


# The SM and... the LHC data so far

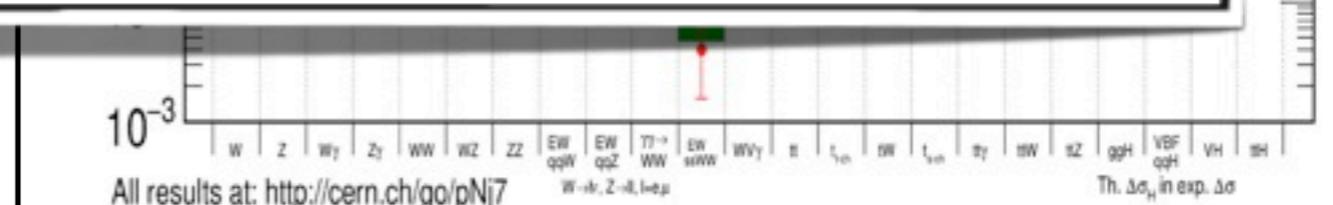
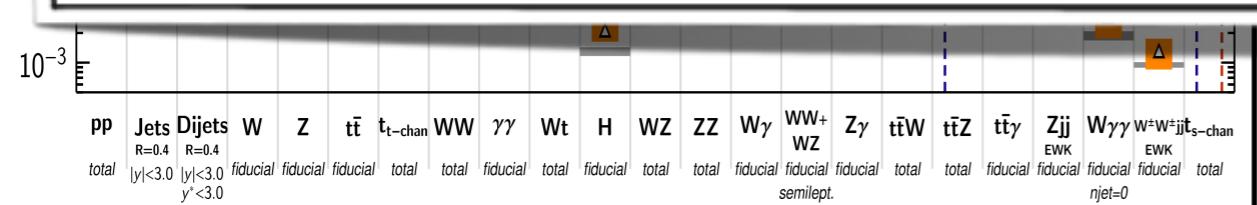


rules the world!

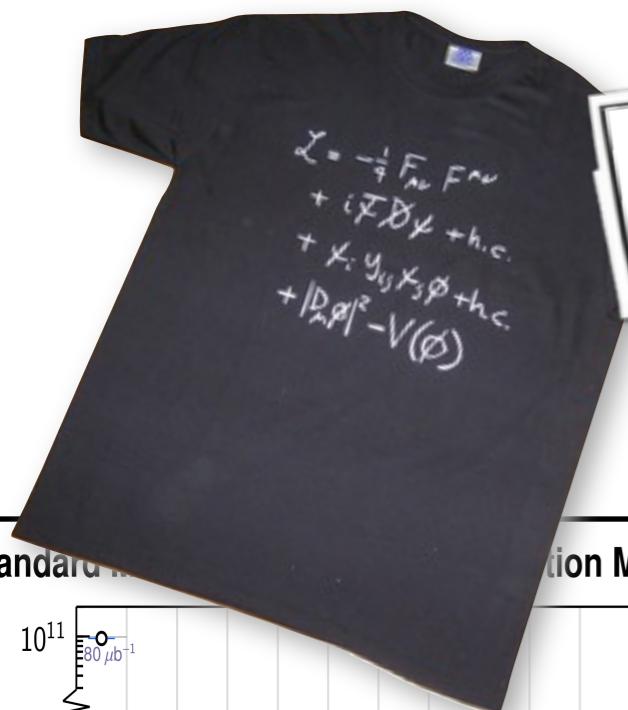
[and we, HEP practitioners, are all entitled for some royalties!]



<sup>1</sup> the same set of eqs. describe phenomena over 15 orders of magnitude

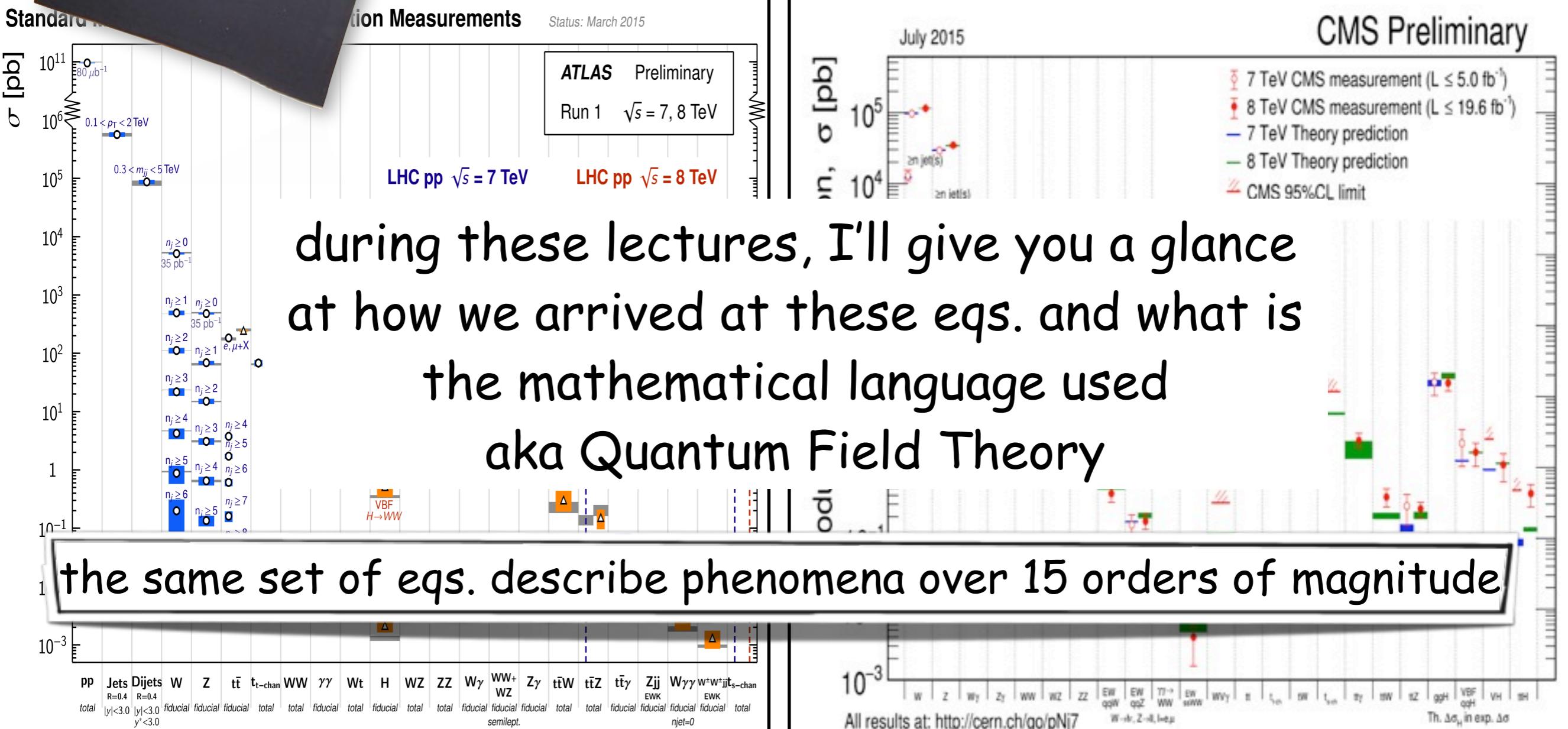


# The SM and... the LHC data so far



rules the world!

[and we, HEP practitioners, are all entitled for some royalties!]



# *The elementary particles*

# What is a particle?

*A small, quantic and fast-moving object*

## Quantum Mechanics

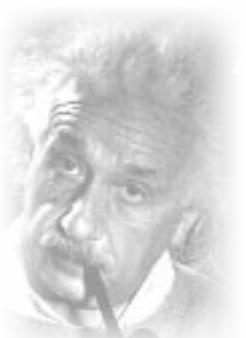
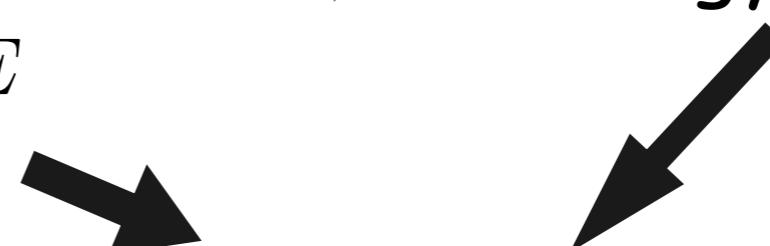
- duality
- wave-particle
- Heisenberg inequalities
- energy non-conservation  
on time intervals  $\Delta t$  energy fluctuations  $\Delta E$

$$\Delta t \times \Delta E \sim \hbar$$



## Special Relativity

- space-time constant speed of light  
ex: how long does it take for a photon to travel from your feet to your brain?
- energy = mass



number of particles  
is not constant

# What is a particle?

*A small, quantic and fast-moving object*

## Quantum Mechanics

- duality
- wave-particle
- Heisenberg inequalities
- energy non-conservation

on time intervals  $\Delta t$  energy fluctuations  $\Delta E$

try to localize a particle  $\Delta x \ll 1/m \Rightarrow \Delta k \gg m$  ie  $\Delta E \gg m$

ultrarelativistic particles  
particle creations



## Special Relativity

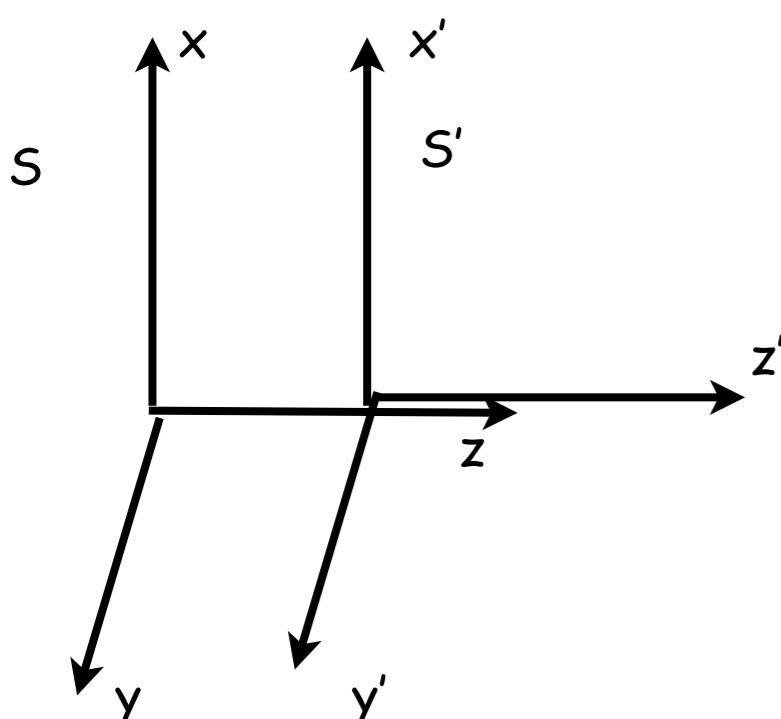
- space-time constant speed of light  
ex: how long does it take for a photon to travel from your feet to your brain?
- energy = mass



# Special relativity

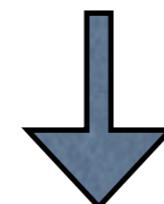
The two postulates of  
Special relativity

- Speed of light is the same in all reference frames
- Causality



Unique choice:  
Lorentz transformations

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix}$$



Look for coordinate  
transformations that satisfy  
these requirements

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Lorentz transformations keep the lightcone fixed

# Special relativity

consider two systems of coordinates

$$\begin{array}{c} S \qquad \qquad S' \\ \left( \begin{array}{c} ct \\ x \\ y \\ z \end{array} \right) \rightarrow \left( \begin{array}{c} ct' = \gamma(ct - \beta x) \\ x' = \gamma(-\beta ct + x) \\ y' = y \\ z' = z \end{array} \right) \\ \beta = \frac{v}{c} \text{ and } \gamma = \frac{1}{\sqrt{1 - \beta^2}} \end{array}$$

consider two events  $E_1$  and  $E_2$  characterized by their space-time coordinates

$E_1$	
$t_1 = 0$	$t'_1 = 0$
$x_1 = 0$	$x'_1 = 0$

$E_2$	
$t_2 > 0$	$ct'_2 = \gamma(ct_2 - \beta x_2)$
$x_2 > 0$	$x'_2 = \gamma(-\beta ct_2 + x_2)$

$t'_2$  can be positive or negative  
causality  $\neq$  time ordering

$$\Delta'^2 = (ct'_2)^2 - (x'_2)^2 = (ct_2)^2 - x_2^2 = \Delta^2$$

# Lorentz transformations

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix} \quad \beta = \frac{v}{c} \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

## Time dilation

consider time interval  $\tau = t'_2 - t'_1$  in  $S'$ , the rest frame of a particle located at  $z'_1 = z'_2 = 0$ .

then in frame  $S$  where the particle is moving:  $t_2 - t_1 = \gamma\tau$

-->The observed lifetime of a particle is  $\gamma \times \tau$   
so it can travel over a distance  $\beta c \gamma \tau$

-->muons which have a lifetime  $\tau \sim 2 \times 10^{-6}$  s produced by reaction of cosmic rays with atmosphere at 15-20 km altitude can reach the surface  
ex: estimate the energy of the cosmic rays

## length contraction

an object at rest in  $S'$  has length  $L_0 = z'_2 - z'_1$

It measures in  $S$   $z_2 - z_1 = L_0/\gamma$

--> densities increase  $\rho_0 = \Delta n / (\Delta x' \Delta y' \Delta z')$   $\rho = \Delta n / (\Delta x \Delta y \Delta z) = \gamma \rho_0$

$\Delta x \Delta y \Delta z \Delta t$  is invariant

# Four vectors

Time and space get mixed-up under Lorentz transformations. They are considered as different components of a single object, a four-component spacetime vector:

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$$

By construction:  $x_\mu x^\mu = \mathbf{x}^2 = x^{0^2} - \vec{x}^2$  is invariant

$$d\tau = \sqrt{dt^2 - d\vec{x}^2} \quad \text{is invariant}$$

Lorentz invariant action built with the proper time  $d\tau$

$$S = -m \int d\tau = \int \mathcal{L} dt$$

$$\left. \begin{aligned} \mathcal{L} &= -m\sqrt{1 - \dot{x}^2} \\ \vec{p} &= \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\gamma \vec{\beta} \\ E &= \vec{p} \cdot \dot{x} - \mathcal{L} = m\gamma \end{aligned} \right\} \vec{p} = E \vec{\beta}$$

# Energy momentum

so we find:

$$m^2 = E^2 - \vec{p}^2$$

This suggests to define the four-vector

$$p^\mu = \left( \frac{E}{c}, p_x, p_y, p_z \right)$$

$$m = 0 \rightarrow \beta = 1 \rightarrow \gamma = \infty \rightarrow \tau = \infty$$

a massless particle cannot decay

# Conservation energy-momentum

Consider collision between A and B

$$\vec{p}_A + \vec{p}_B = 0$$

Define center of mass (CM) frame as where

Energy available in center of mass frame  
is an invariant:  $\sqrt{s} = E_* = E_A + E_B$

$$p_{tot}^2 = E_*^2$$

## 1) Collision on fixed target

B is at rest in lab frame,  $E_B = m_B$  and  $E_A$  is energy of incident particle

$$E_*^2 = m_A^2 + m_B^2 + 2m_B E_A$$

## 2) Colliding beams

A and B travel in opposite directions

$$E_*^2 = m_A^2 + m_B^2 + 2(E_A E_B + |p_A||p_B|) \approx 4E_A E_B$$

if  $m_A, m_B \ll E_A, E_B$

So for fixed target machine

$$E_* \sim \sqrt{2m_B E_A}$$

While for colliding beam accelerators

$$E_* \sim 2E$$

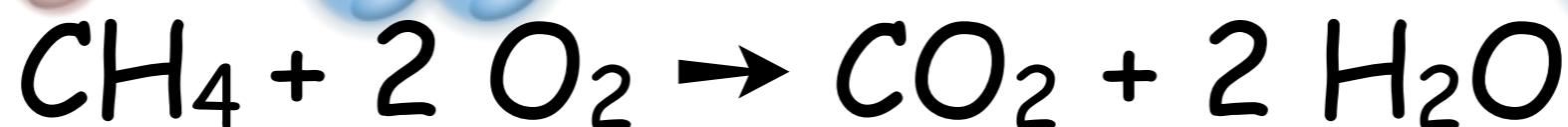
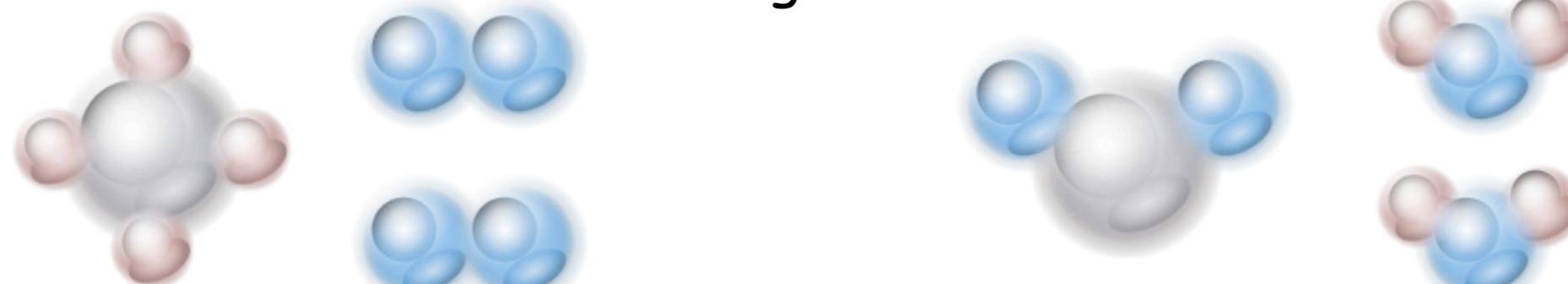
To obtain 2 TeV in the CM with a fixed proton target accelerator the energy of a proton beam would need to be 2000 TeV!

# *The elementary particles*

# Particle physics is special

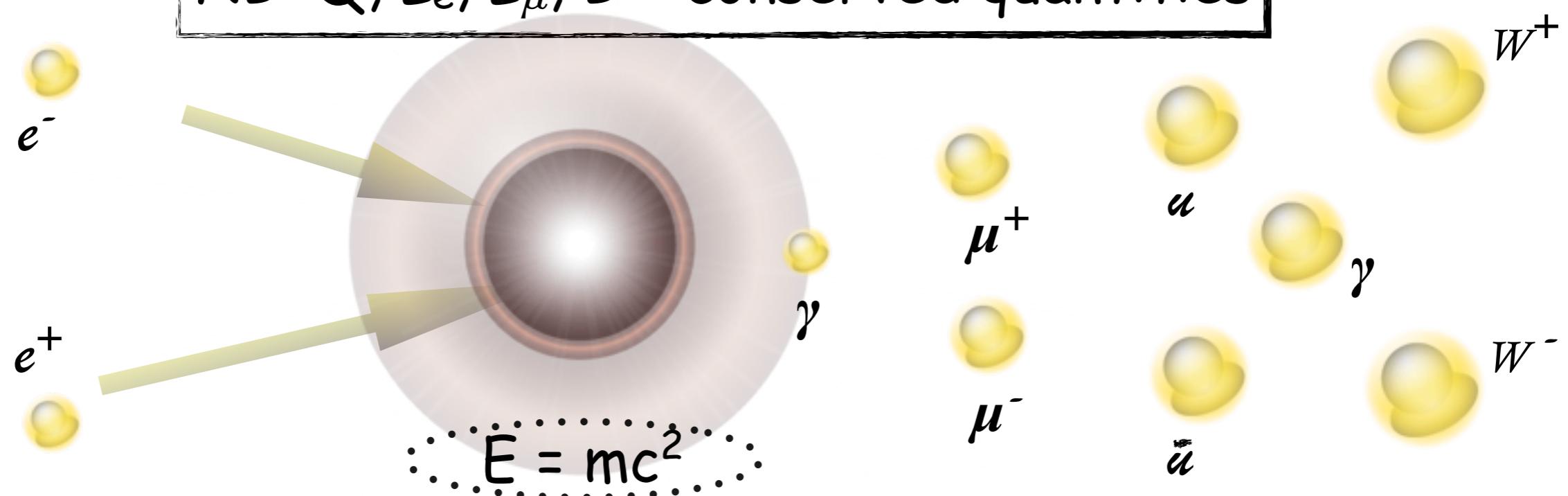
## • Chemistry: reorganization of matter

the various constituents of matter reorganize themselves in different structures



## • Particle physics: transformation matter $\leftrightarrow$ energy

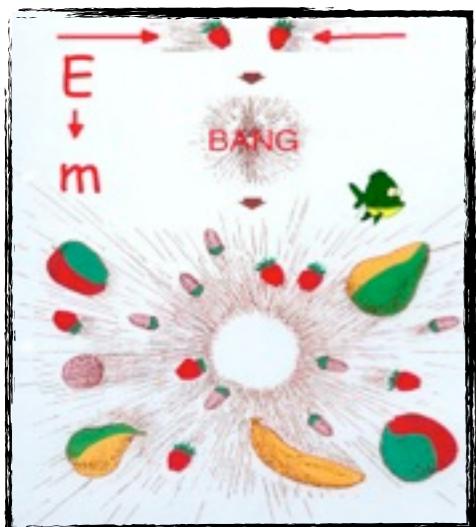
NB:  $Q, L_e, L_\mu, B$  = conserved quantities



# Classical vs. Quantum Collisions

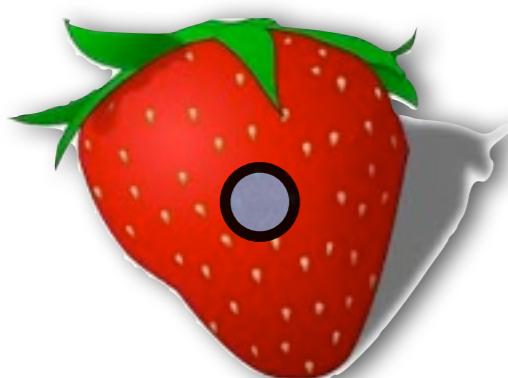


# Classical vs. Quantum Collisions



Compton  
wavelength

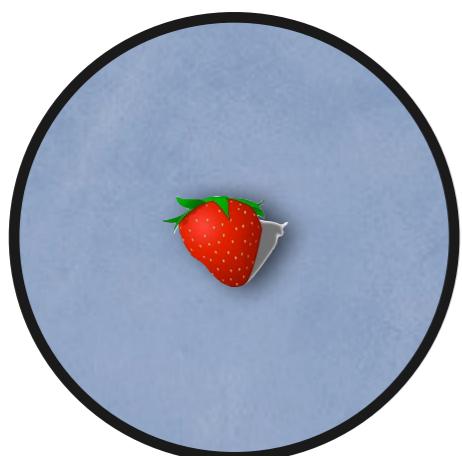
$$\lambda = \frac{\hbar}{mc}$$



strawberry :  $m \sim 30 \text{ g} \sim 10^{25} \text{ GeV}/c^2 \rightarrow \lambda \sim 10^{-40} \text{ m}$

classical :  $\lambda \ll R$

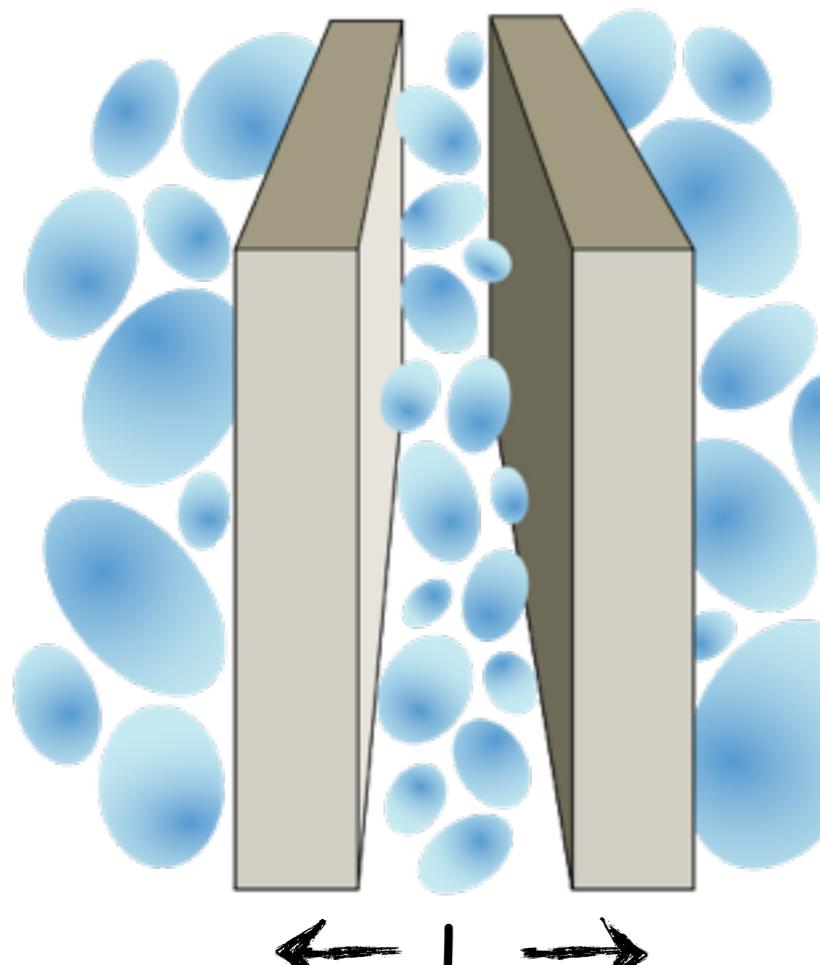
quantum :  $\lambda \gg R$



$e^-$  :  $m \sim 9.1 \times 10^{-31} \text{ kg} \sim 0.5 \text{ MeV}/c^2 \rightarrow \lambda \sim 10^{-13} \text{ m}$

$p$  :  $m \sim 1.6 \times 10^{-27} \text{ kg} \sim 1 \text{ GeV}/c^2 \rightarrow \lambda \sim 10^{-16} \text{ m}$

# Vacuum fluctuations



( $S \gg L^2$ : no boundary effects)

Casimir (1948)

attractive force between two neutral plates

ideal conductors (electric conductivity=∞) and uncharged

$E$ =energy of virtual photons between the plates

$E \downarrow$  when  $L \downarrow \rightarrow$  attractive force

Force per unit area

$$\frac{dF}{dS} = -\frac{\pi^2}{240} \frac{\hbar c}{L^4}$$

QM   Special Relativity

non-trivial coefficient

dim. analysis

numerically: pressure of ~ 1 atm for a 10nm separation

The quantum vacuum is not empty

# Energy Scales of Particle Physics



$e^-$   
 $v=0$        $e^- \rightarrow$   
 $v \sim 700 \text{ km/s}$

$$1 \text{ TeV} = 10^{12} \text{ eV}$$

$1 \text{ eV}$  = energy of an electron accelerated  
by a potential difference of 1 volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joule}$$



$1 \text{ kg sugar} = 4000 \text{ kCalories} = 17 \text{ millions of Joule}$   
but  $1 \text{ kg sugar} \approx 10^{27} \text{ protons}$   
 $0.1 \text{ eV / protons}$

If one wanted to accelerate each protons contained in 1kg of sugar to 14 TeV, (s)he would need the caloric energy contained in  $10^{14} \text{ kg}$  of sugar\* or 1% of the total energy produced yearly

\*yearly worldwide production of sugar = 150 millions of tons  $\approx 10^{11} \text{ kg}$

# The LHC: some numbers

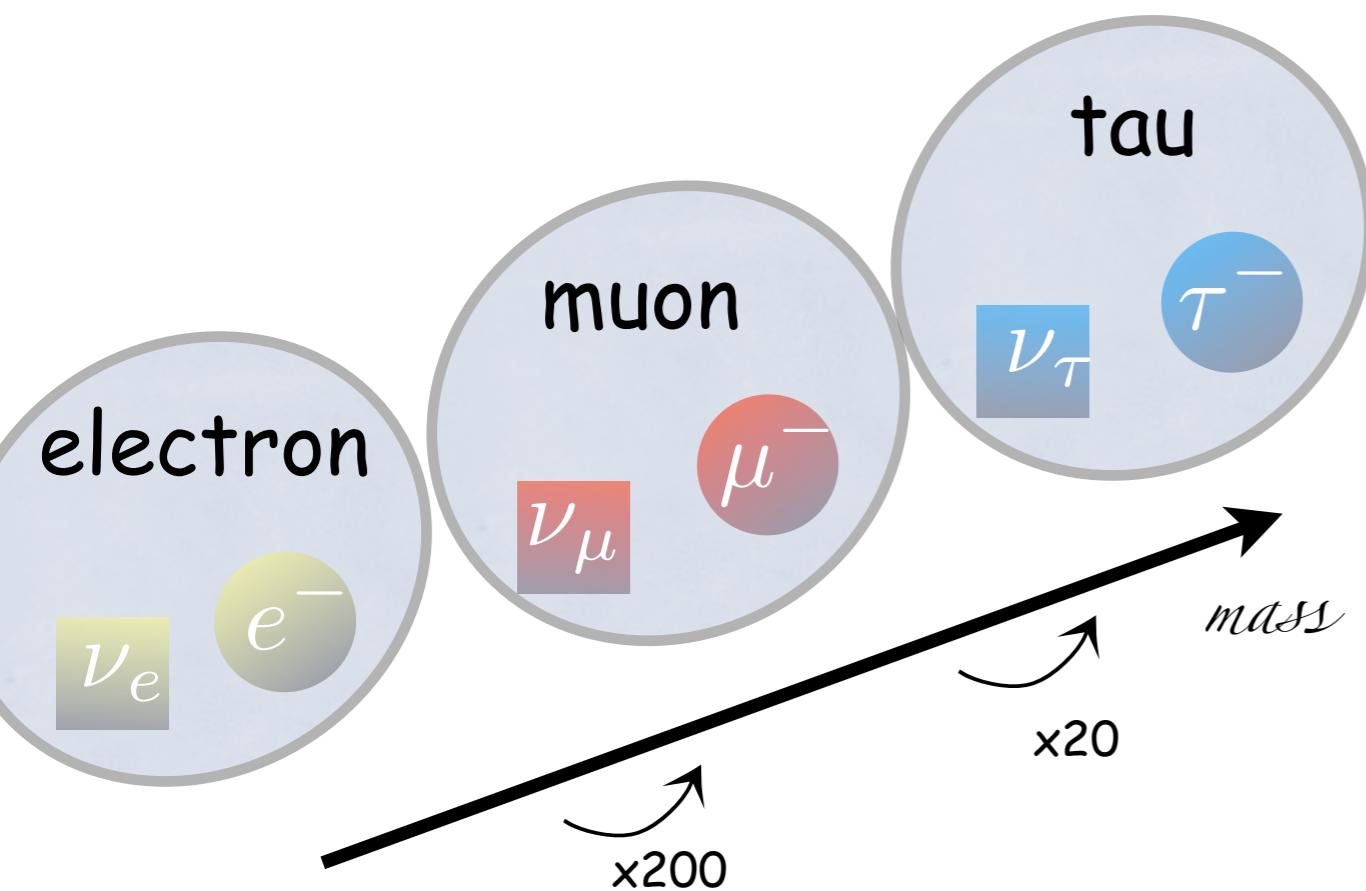
- total amount of matter accelerated by the LHC during 10 years:  
~ 5  $\mu\text{g}$  (hydrogen nuclei)
- yearly energy consumption:  
~800,000 MWh (~ $2 \times 10^9$  x the energy stored in the battery of my laptop)
- beam energy (~ protons) :  
350 MJ  $\approx E_{\text{kin}}$  (TGV) can melt 500 kg of copper
- energy stored in the magnets  
10 GJ
- amount of information produced :  
40 M collisions/s, 1 collision=1MB  
trigger: only 100 collisions/s are recorded  
15 PB data/year  
(1 TB=yearly book production, 1000 PB=total information produced on earth in one year)
- CERN budget :  
~ 1 Bn CHF/year (worldwide military expenses: 1200 Bn \$)

# The Standard Model: matter

*the genetic code of matter*

the elementary particles all the other particles are made of

## LEPTONS

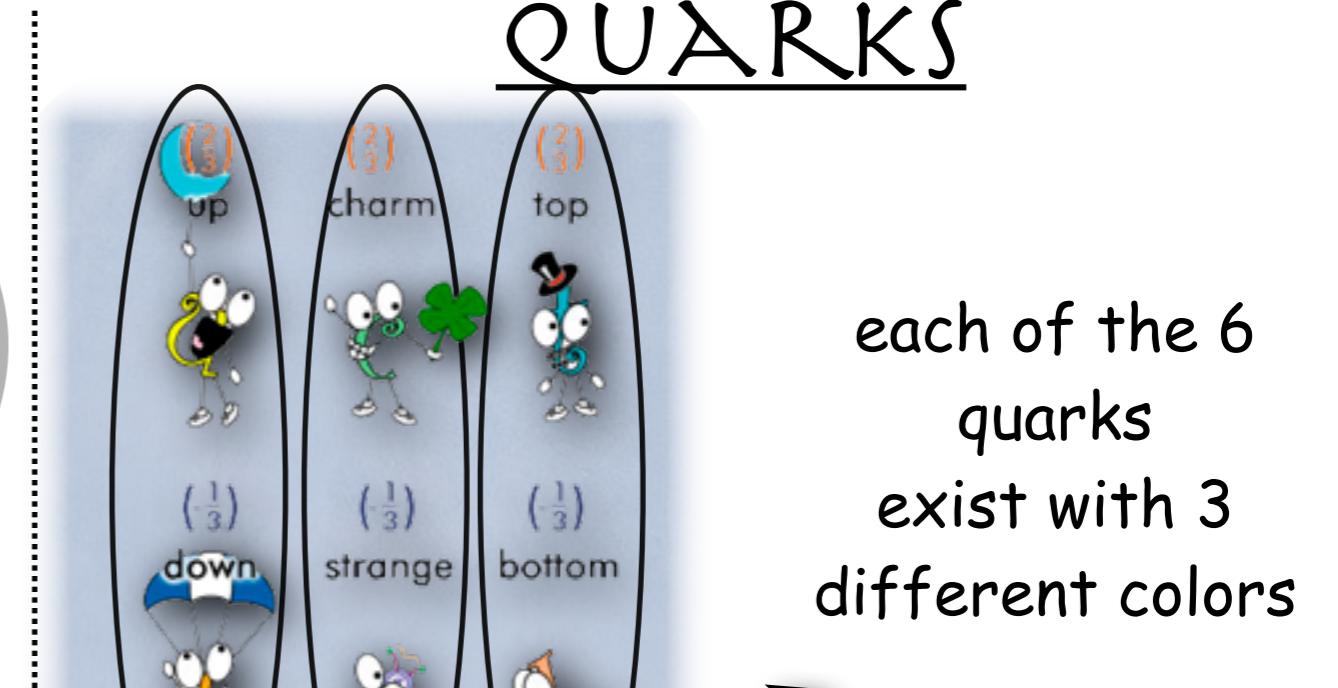


no composite states

made of leptons

+ antiparticles

## QUARKS



each of the 6  
quarks  
exist with 3  
different colors

composite states (white object)

0 baryons

proton       $p=uud$

neutron     $n=udd$

0 mésons

# Classical/Quantum EM & Antimatter

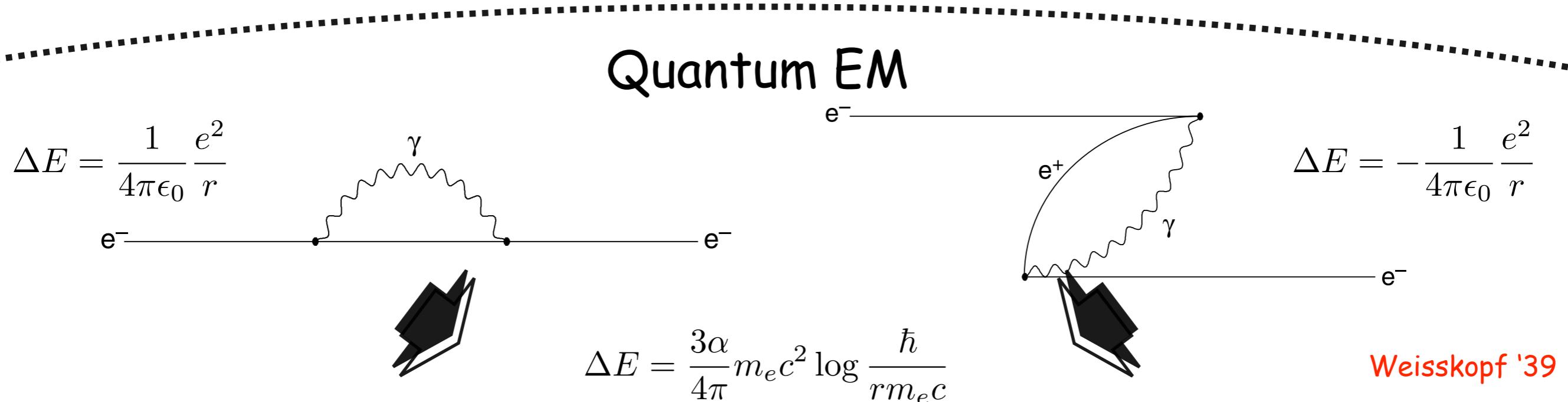
an electron makes an electric field which carries an energy

$$\Delta E_{\text{Coulomb}}(r) = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r}$$

and interacts back to the electron and contributes to its mass  $\delta mc^2 = \Delta E$

$$\delta m < m_e \quad \rightarrow \quad r > r_e \equiv \frac{e^2}{4\pi\epsilon_0 m_e c^2} \sim 10^{-13} \text{ m} \quad \text{i.e.} \quad E < \frac{\hbar c}{r_e} \sim 5 \text{ MeV}$$

At shortest distances or larger energies, classical EM breaks down



new states  $\approx$  softer high-energy (UV) behavior:  $\delta m < 0.1 m_e$   $E < 10^{21} \text{ GeV}$

# Antimatter and Dirac equation

Schrödinger's equation (1926) is non-relativistic  
(cannot account for creation/annihilation of particles)

Schrödinger Equation (1926): 
$$\left( i\hbar \frac{\partial}{\partial t} + \frac{\hbar^2}{2m} \Delta - V \right) \Phi = 0$$

$E = \frac{p^2}{2m} + V$       classical  $\leftrightarrow$  quantum correspondance

$E \rightarrow i\hbar \frac{\partial}{\partial t}$  &  $p \rightarrow i\hbar \frac{\partial}{\partial x}$

Klein-Gordon Equation (1927): 
$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2 c^2}{\hbar^2} \right) \Phi = 0$$

$\frac{E^2}{c^2} = p^2 + m^2 c^2$

Dirac Equation (1928): 
$$\left( i\gamma^\mu \partial_\mu - \frac{mc}{\hbar} \right) \Psi = 0$$

$E = \begin{cases} +\sqrt{p^2 c^2 + m^2 c^4} & \text{matter} \\ -\sqrt{p^2 c^2 + m^2 c^4} & \text{antimatter} \end{cases}$

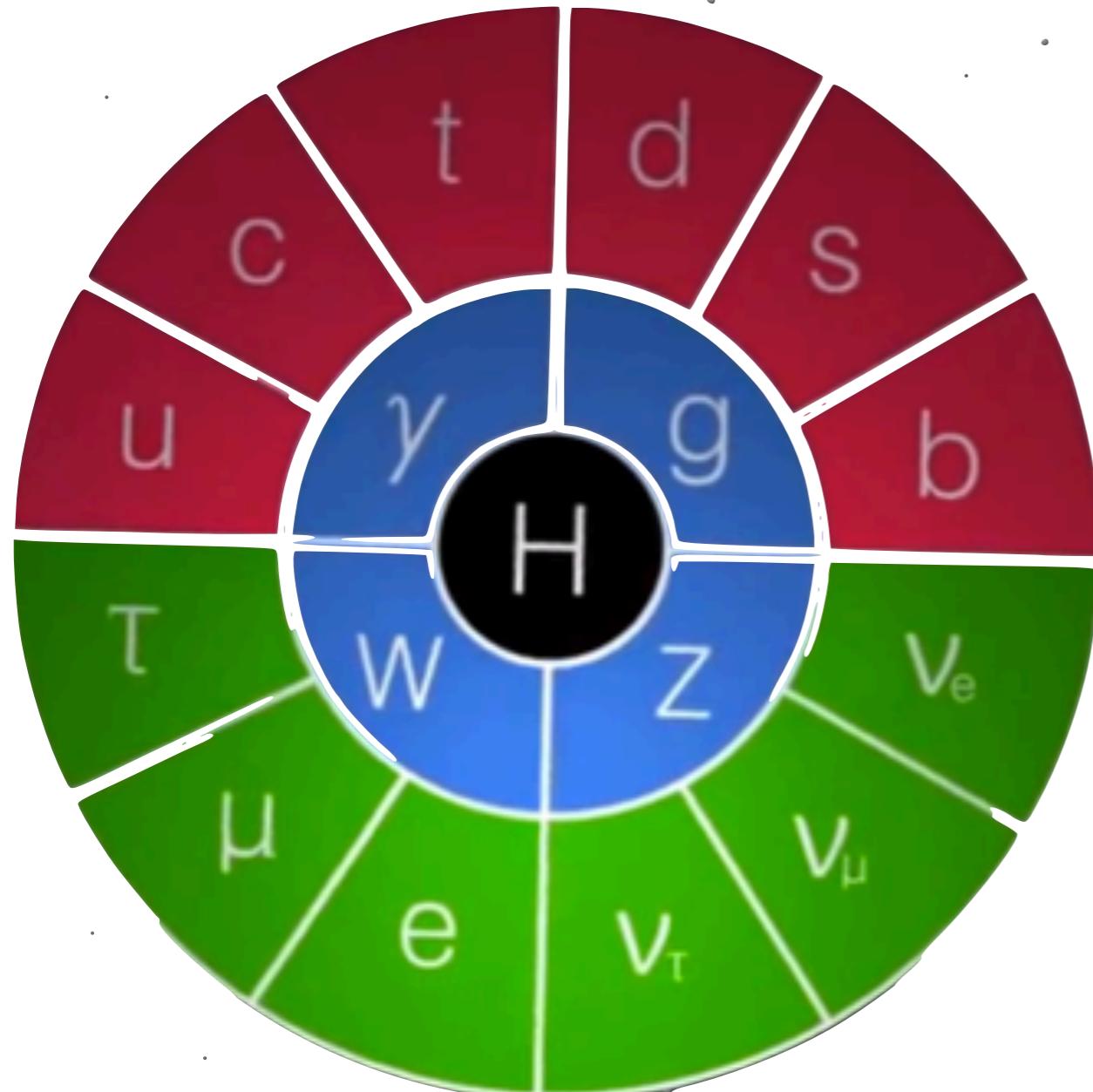
$E = \vec{\alpha} \vec{p} c + \beta mc^2$

$\gamma^0 = \beta, \gamma^i = \beta \alpha^i, \{\gamma^\mu, \gamma^\nu\} = 2\eta^{\mu\nu}$

positron ( $e^+$ ) discovered by C. Anderson in 1932

# The Standard Model: Matter

How many quarks and leptons?



Three Generations of Matter (Fermions) spin $\frac{1}{2}$			
	I	II	
mass →	2.4 MeV	1.27 GeV	173.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
name →	u up	c charm	t top
Quarks	Left	Left	Left
	d down	s strange	b bottom
Leptons	Left	Left	Left
	${}^0 \bar{\nu}_e$ electron neutrino	${}^0 \bar{\nu}_\mu$ muon neutrino	${}^0 \bar{\nu}_\tau$ tau neutrino
	e electron	$\mu$ muon	$\tau$ tau

# The Standard Model: Matter

How many quarks and leptons?

$$6+6=12?$$

$$6 \times 3 = 18?$$

shouldn't we count different color states?

$$6 \times 3 \times 2 + 3 \times 2 + 3 = 45?$$

it is an accident that  $e_L \sim e_R$  for QED  
 SM is a chiral theory:  $e_L \neq e_R$

$$6 \times 3 \times 2 + 6 \times 2 = 48?$$

are there  $\nu_R$ ?  
 are they part of the SM?

Three Generations of Matter (Fermions) spin $\frac{1}{2}$			
	I	II	III
mass →	2.4 MeV	1.27 GeV	173.2 GeV
charge →	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$
name →	u Left up	c Left charm	t Left top
Quarks	d Left down	s Left strange	b Left bottom
Leptons	$\nu_e$ Left electron neutrino	$\nu_\mu$ Left muon neutrino	$\nu_\tau$ Left tau neutrino
	e Left electron	$\mu$ Left muon	$\tau$ Left tau

# The Standard Model: Matter

How many quarks and leptons?

Is the SM theoretically consistent?

SM = theory based on (chiral) gauge symmetries  
a symmetry is consistent with QM

iff the "sum" of the charges of the different fermions vanishes

$$Q = T_L^3 - Y$$

**Exercise 1:** within the SM, check that

$$(1) \text{Tr}_L Y - \text{Tr}_R Y = 0$$

$$(2) \text{Tr}_L Y^3 - \text{Tr}_R Y^3 = 0$$

note that there was a priori no guarantee to find a solution to this system of non-linear equations.

It works because EM is a vector-like theory

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$L_L^i = \begin{cases} N^i = (\nu^i, \tilde{\nu}^i) \\ E^i = (l_L^i, \tilde{l}_L^i) \end{cases}$	1	2	1/2
$E^i = (CP(l_R^i), CP(\tilde{l}_R^i))$	1	1	-1
$Q_L^i = \begin{cases} U_L^i = (u_L^i, \tilde{u}_L^i) \\ D_L^i = (d_L^i, \tilde{d}_L^i) \end{cases}$	3	2	-1/6
$U_R^i = (CP(u_R^i), CP(\tilde{u}_R^i))$	$\bar{3}$	1	2/3
$D_R^i = (CP(d_R^i), CP(\tilde{d}_R^i))$	$\bar{3}$	1	-1/3

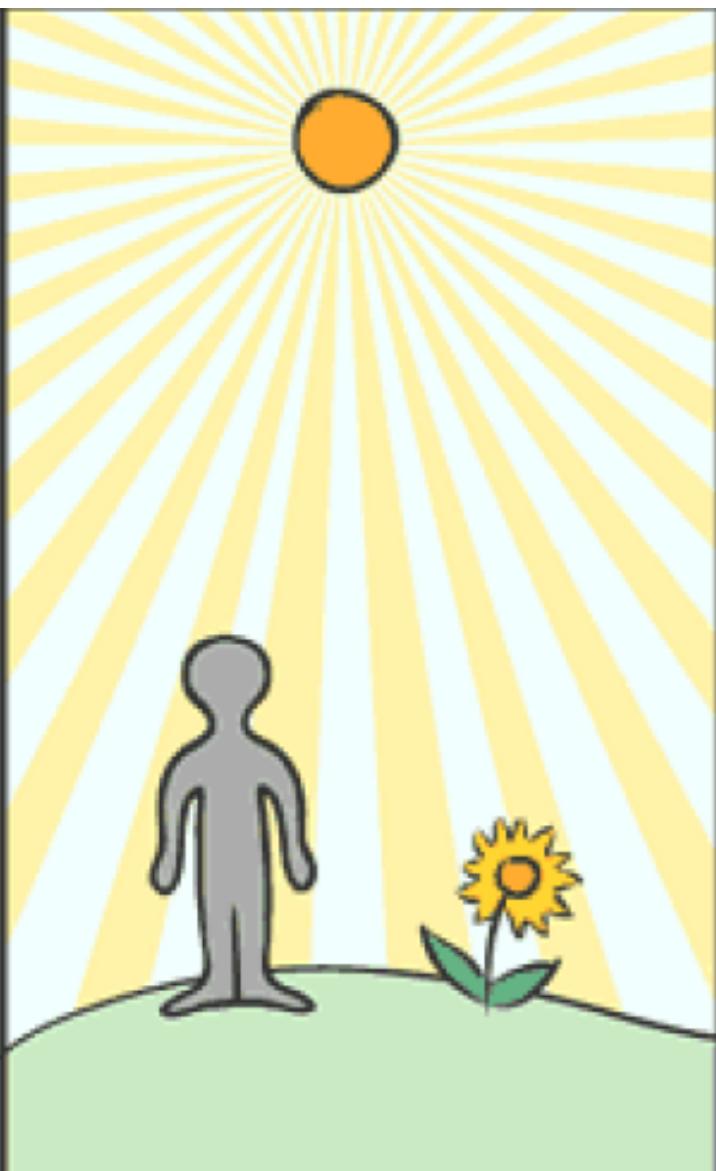
**Exercise 2:** Within the SM, the anomaly cancelation fixes the relative electric charges of the leptons and quarks. Show that with the addition of a right-handed neutrino, this ratio of electric charges is free. Still the cancelation of the anomaly imposes that the proton is electrically neutral

# *The fundamental interactions*

# Interactions between Particles

The very observation that the sun is shining for several millennia tells us that there are various mechanisms of energy production.

Sun = gigantic source of energy

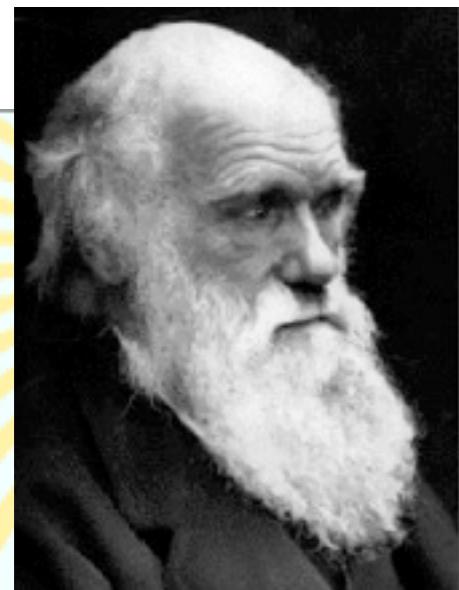


- 1 cm<sup>3</sup> of ice under the sun (in the summer) melts in ~ 40mn
- an ice cap 1 cm thick and 300 million km of diameter centered around the sun will melt in 40mn

energy produced by burning  $10^{19}$  liters of oil  
(~ volume Sun-Mercury/1000)

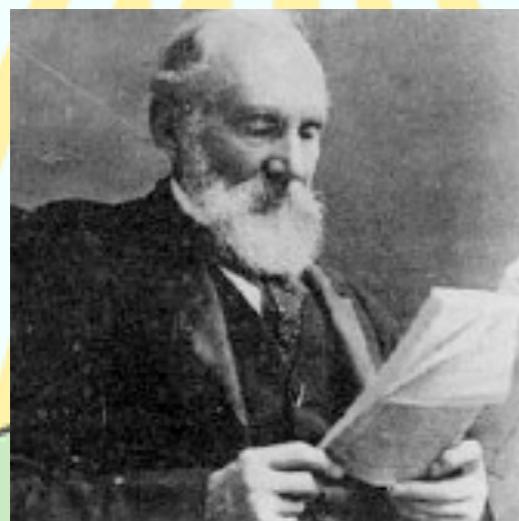
# Interactions between Particles

*The very observation that the sun is shining for several millennia tells us that there are various mechanisms of energy production.*



Sun = gigantic source of energy

Darwin ("On the origin of species by means of natural selection", 1<sup>st</sup> edition, 1859 ) estimates that the age of the Earth, and thus the age of the Sun also, has to be larger than 300 millions of years to account for the erosion of hills in South England.



Thompson, Lord Kelvin, computes the gravitational energy of the Sun and with the assumption that it is entirely converted in heat, concludes that the Sun cannot be older than 20 million years (chemical energy would allow the Sun to shine for at most 3000 years)

We know today that the Sun is more than 4.5 billion years

# Different Interactions

## 0 Atomic Physics

mass of an atom = mass of nucleus + masses of electrons

example : hydrogen atom, mass  $\sim 1 \text{ GeV}$ , binding energy  $\sim 13 \text{ eV}$

$\Rightarrow 10^{-8}$

## 0 Nuclear Physics

mass of a nucleus <  $\Sigma$  masses of protons and neutrons

example : Helium nucleus, mass  $\sim 4 \text{ GeV}$ , binding energy  $\sim 28 \text{ MeV}$

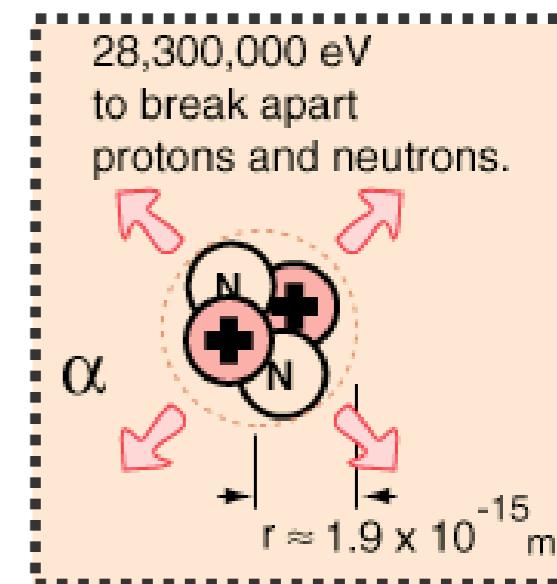
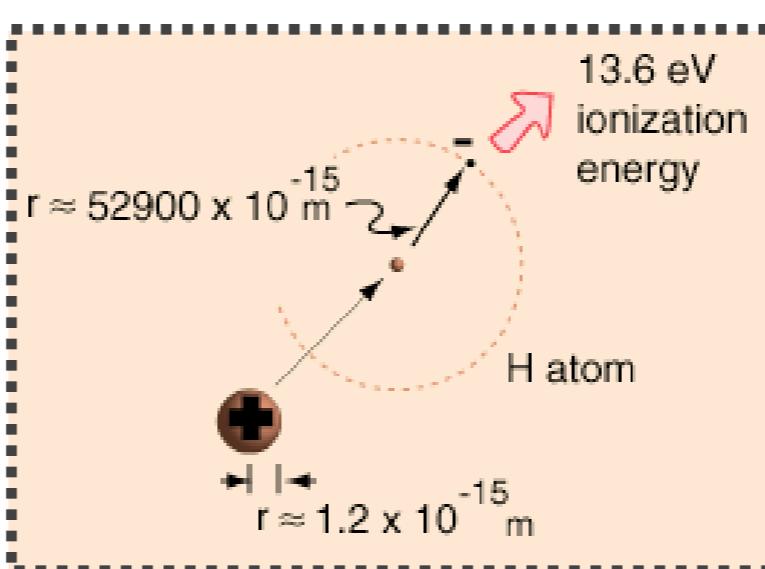
$\Rightarrow 10^{-2}$

## 0 Particle Physics

mass of a proton or a neutron  $\gg \Sigma$  masses of quarks

proton mass  $\sim 1 \text{ GeV}$ , constituent quarks masses  $\sim 12 \text{ MeV}$

$\Rightarrow 10^2$



# The Standard Model: Interactions

Even though EM is way stronger than gravity, it was unnoticed until  $\sim 300$  years because  $1-1=0$

## electromagnetic interactions

(1873, Maxwell)

tested with an accuracy of  $10^{-8}$

## weak interactions

(1933, Fermi)

tested with an accuracy of  $10^{-3}$

## strong interactions

tested with an accuracy of  $10^{-1}$

(1911, Rutherford; 1921, Chadwick and Biesler)

• gravity

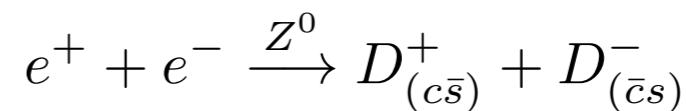
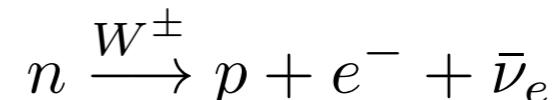
$10^{-40}$

light

atoms

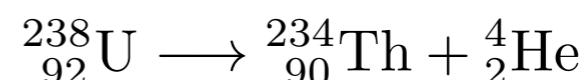
molecules

$\beta$  decay



atomic nuclei

$\alpha$  decay



strength

1

# Gauge Theories

## Fermi Theory

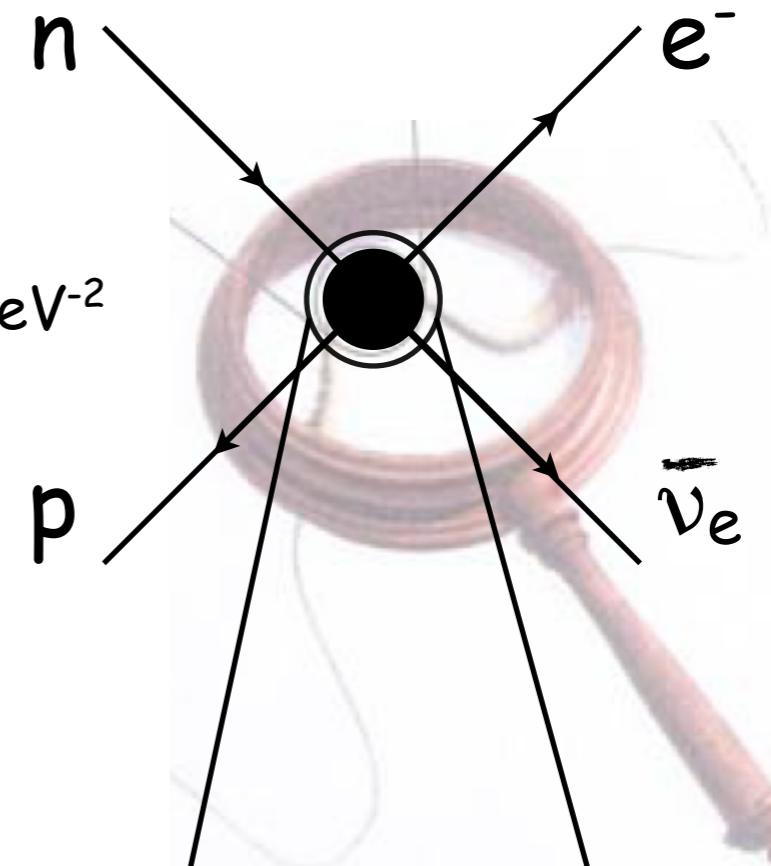
$$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

$$\mathcal{L} = G_F (\bar{n} p) (\bar{\nu}_e e)$$

$$A \propto G_F E^2$$

- no continuous limit
- inconsistent above 300 GeV

(paper rejected by Nature: declared too speculative !)



## Gauge theory

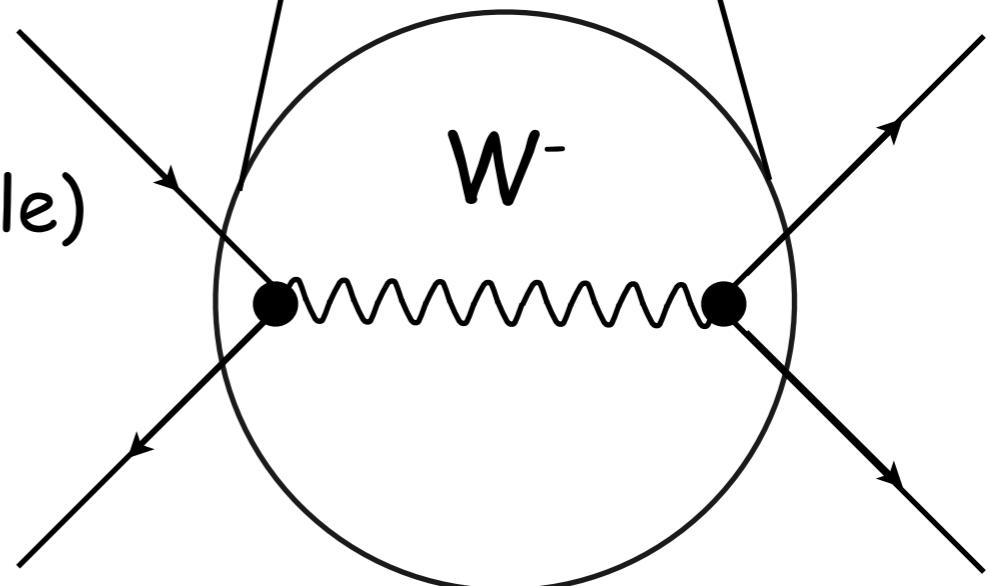
### microscopic theory

(exchange of a massive spin 1 particle)

$$G_F = \frac{\sqrt{2}g^2}{8m_W^2}$$

exp:  $m_W = 80.4$  GeV

➡  $g \approx 0.6$ , ie, same order as  $e=0.3$   
unification EM & weak interactions



# Gauge Theories: EM & Yang-Mills

EM U(1)       $\phi \rightarrow e^{i\alpha} \phi$       but       $\partial_\mu \phi \rightarrow e^{i\alpha} (\partial_\mu \phi) + i(\partial_\mu \alpha) \phi$

$\neq 0$  if local transformations

EM field and covariant derivative       $\partial_\mu \phi + ieA_\mu \phi \rightarrow e^{i\alpha}(\partial_\mu \phi + ieA_\mu \phi)$

if       $A_\mu \rightarrow A_\mu - \frac{1}{e}\partial_\mu \alpha$

the EM field keeps track of the phase in different points of the space-time

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

Yang-Mills : non-abelian transformations

$$\phi \rightarrow U\phi$$

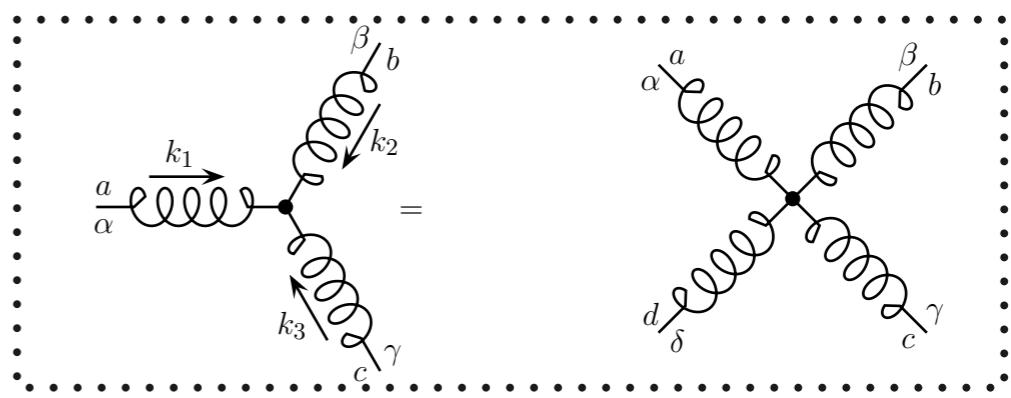
$$\partial_\mu \phi + igA_\mu \phi \rightarrow U(\partial_\mu \phi + igA_\mu \phi)$$

if

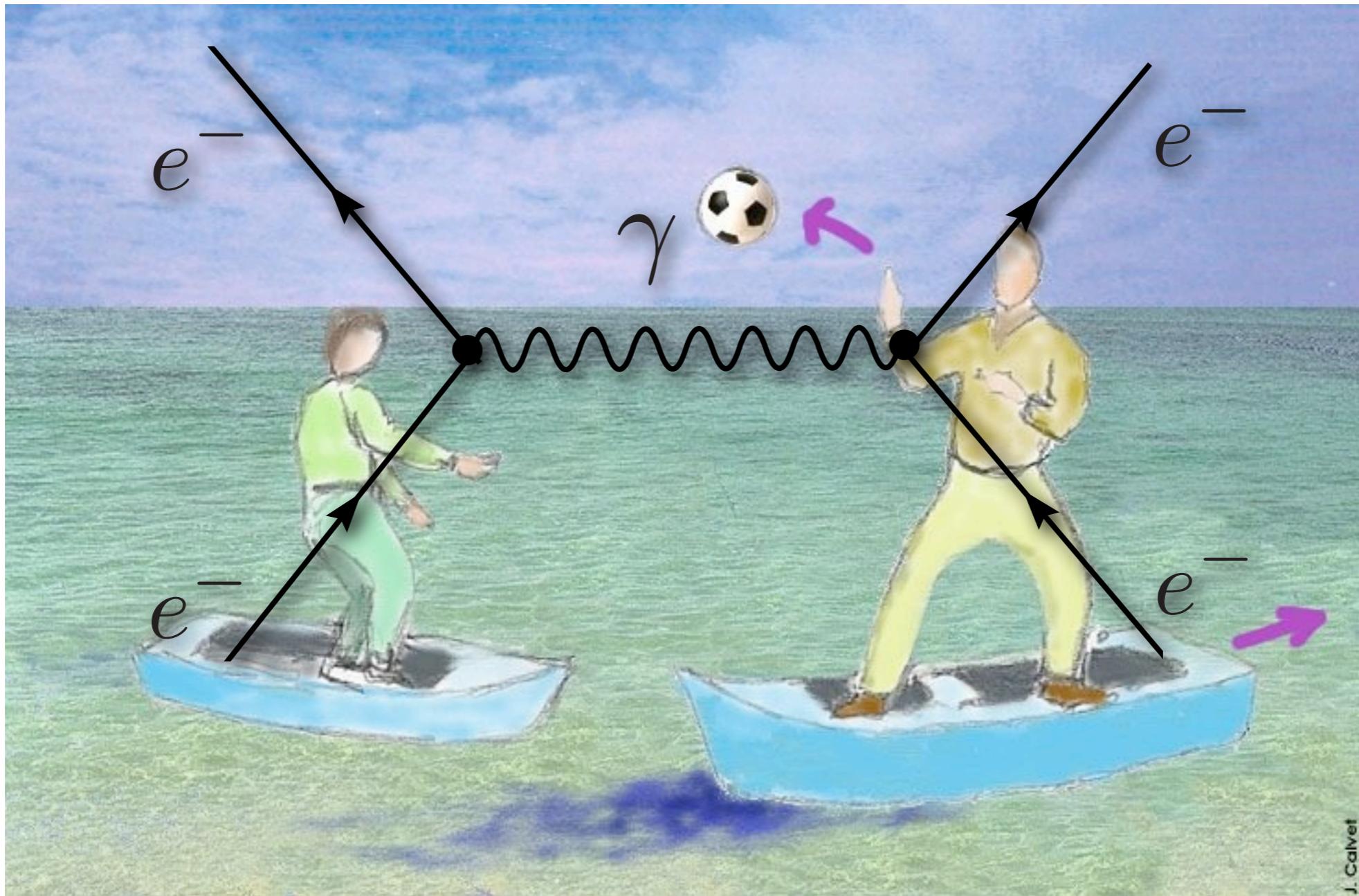
$$A_\mu \rightarrow UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + ig[A_\mu, A_\nu]$$

non-abelian int.



# Interactions between Particles



Elementary particles interact on each other  
by the exchange of gauge bosons

# The Standard Model: Interactions

- $U(1)_Y$  electromagnetic interactions

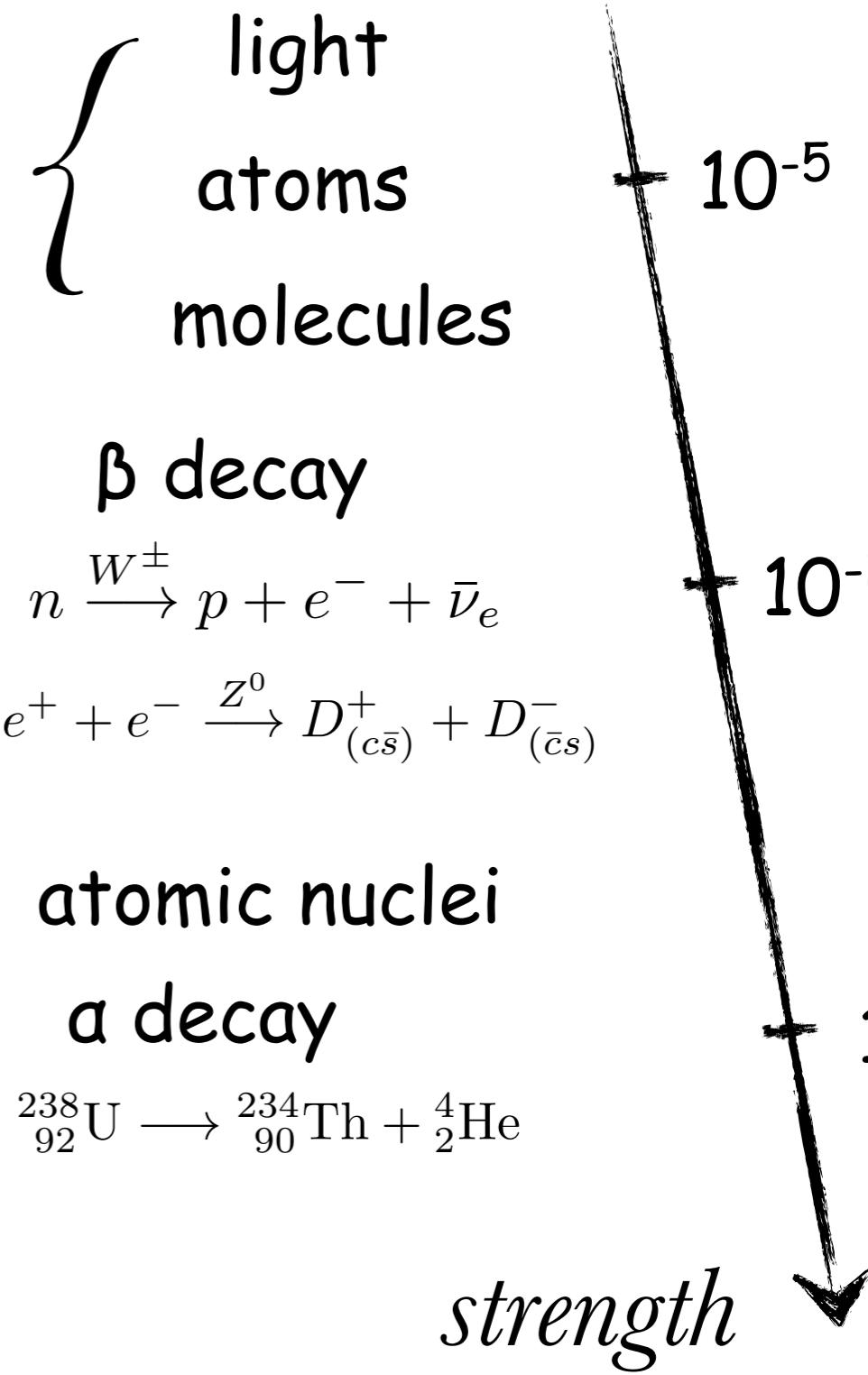
Photon  $\gamma$

- $SU(2)_L$  weak interactions

bosons  $W^\pm, Z^0$

- $SU(3)_c$  strong interactions

gluons  $g^a$



# The Standard Model

Nobel Prize '79



## Standard Model of FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model summarizes the current knowledge in Particle Physics. It is the quantum theory that includes the theory of strong interactions (quantum chromodynamics or QCD) and the unified theory of weak and electromagnetic interactions (electroweak). Gravity is included on this chart because it is one of the fundamental interactions even though not part of the "Standard Model."

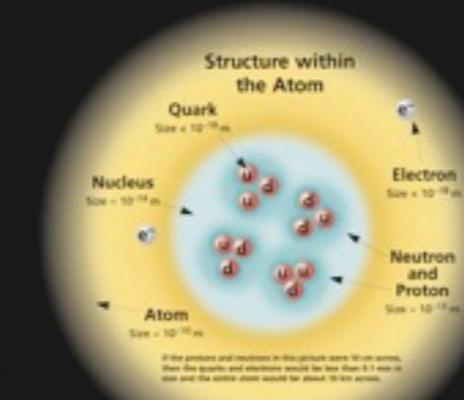
### FERMIONS

Leptons spin = 1/2		
Flavor	Mass GeV/c <sup>2</sup>	Electric charge
$\nu_e$ neutrino	<1.50 <sup>-8</sup>	0
e electron	0.000511	-1
$\nu_\mu$ muon	<0.0002	0
$\mu$ muon	0.106	-1
$\nu_\tau$ tau neutrino	<0.02	0
T tau	1.7771	-1

Spin is the intrinsic angular momentum of particles. Spin is given in units of  $\hbar$ , which is the quantum unit of angular momentum, where  $\hbar/2e = 6.58 \cdot 10^{-27}$  GeV/c =  $1.05 \cdot 10^{-34}$  J.s.

Electric charges are given in units of the proton's charge. In Si units the electric charge of the proton is  $1.60 \cdot 10^{-19}$  coulombs.

The energy unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. Masses are given in GeV/c<sup>2</sup> (Gevres).  $1 \text{ GeV} = 10^9 \text{ eV} = 1.60 \cdot 10^{-10}$  Joule. The mass of the proton is 0.938 GeV/c<sup>2</sup>. The mass of the photon is  $1.67 \cdot 10^{-37}$  kg.



### BOSONS

Unified Electroweak spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge
$\gamma$ photon	0	0
$W^-$	80.4	-1
$W^+$	80.4	+1
$Z^0$	91.187	0

**Color Charge**  
Each quark carries one of three types of "color charges," also called "color charge." These charges have nothing to do with the colors of visible light. There are eight possible types of color charge for gluons. Just as electrically charged particles interact by exchanging photons, in strong interactions color-charged particles interact by exchanging gluons. Leptons, photons, and W and Z bosons have no strong interactions and hence no color charge.

**Quarks Confined in Mesons and Baryons**  
One cannot isolate quarks or gluons; they are confined in color-neutral particles called hadrons. This is due to the strong interactions between the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color force field between them increases. This energy eventually is converted into additional quark-antiquark pairs (see figure below). The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge. Two types of hadrons have been observed in nature: mesons ( $\pi$ ) and baryons ( $\Lambda$ ).

**Residual Strong Interaction**  
The strong binding of color-neutral protons and neutrons to form nuclei is due to residual strong interactions between their color-charged constituents. It is similar to the residual gravitational interaction that binds electrically neutral atoms to form molecules. It can also be viewed as the exchange of mesons between the hadrons.

### PROPERTIES OF THE INTERACTIONS

Mesons qq					
Mesons are baryons, hadrons. There are about 140 types of mesons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
p	proton	uud	+1	0.938	1/2
$\bar{p}$	antiproton	$\bar{u}\bar{d}$	-1	0.938	1/2
n	neutron	udd	0	0.946	1/2
$\Lambda$	baryon	uds	0	1.116	1/2
$\Xi^+$	omega	sss	-1	1.672	3/2

**Matter and Antimatter**  
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g.,  $Z^0$ ,  $\gamma$ , and  $\eta_c \rightarrow c\bar{c}$ , but not  $K^0 \rightarrow \bar{K}^0$ ) are their own antiparticles.

**Figures**  
These diagrams are an artist's conception of physical processes. They are not meant to have meaningful scale. Green shaded areas represent the cloud of gluons or the gluon field, and red lines the quark paths.

**Hadron Production**  
 $n \rightarrow p + e^- + \bar{\nu}_e$   
 $e^+e^- \rightarrow B^0\bar{B}^0$   
 $p + p \rightarrow Z^0Z^0 \rightarrow \text{hadrons}$

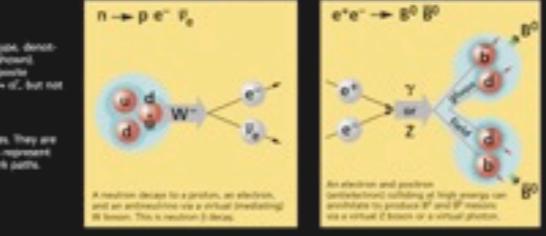
**The Particle Adventure**  
Visit the award-winning web feature The Particle Adventure at <http://pdg.lbl.gov/pa/pa.html>

**Spins**  
This chart has been made possible by the generous support of U.S. Department of Energy, Lawrence Berkeley National Laboratory, Fermilab, Jefferson Lab, and American Physical Society, Division of Particles and Fields, DURAC INDUSTRIES, INC.

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description of all  
elementary particles and  
their interactions

Baryons qqq and Antibaryons $\bar{q}\bar{q}\bar{q}$					
Baryons are baryons, hadrons. There are about 130 types of baryons.					
Symbol	Name	Quark content	Electric charge	Mass GeV/c <sup>2</sup>	Spin
p	proton	uud	+1	0.938	1/2
$\bar{p}$	antiproton	$\bar{u}\bar{d}$	-1	0.938	1/2
n	neutron	udd	0	0.946	1/2
$\Lambda$	baryon	uds	0	1.116	1/2
$\Xi^+$	omega	sss	-1	1.672	3/2



# The underlying principles of the SM

The beauty of the SM comes from the identification of a unique dynamical principle describing the different interactions that seem so different from each others

.....  
gauge theory = spin-1  
.....

at the same time a particular and predictive structure that still leaves room for a rich variety of phenomena  
(long range interaction, spontaneous symmetry breaking, confinement )

.....  
gravitation = general relativity= spin-2  
.....

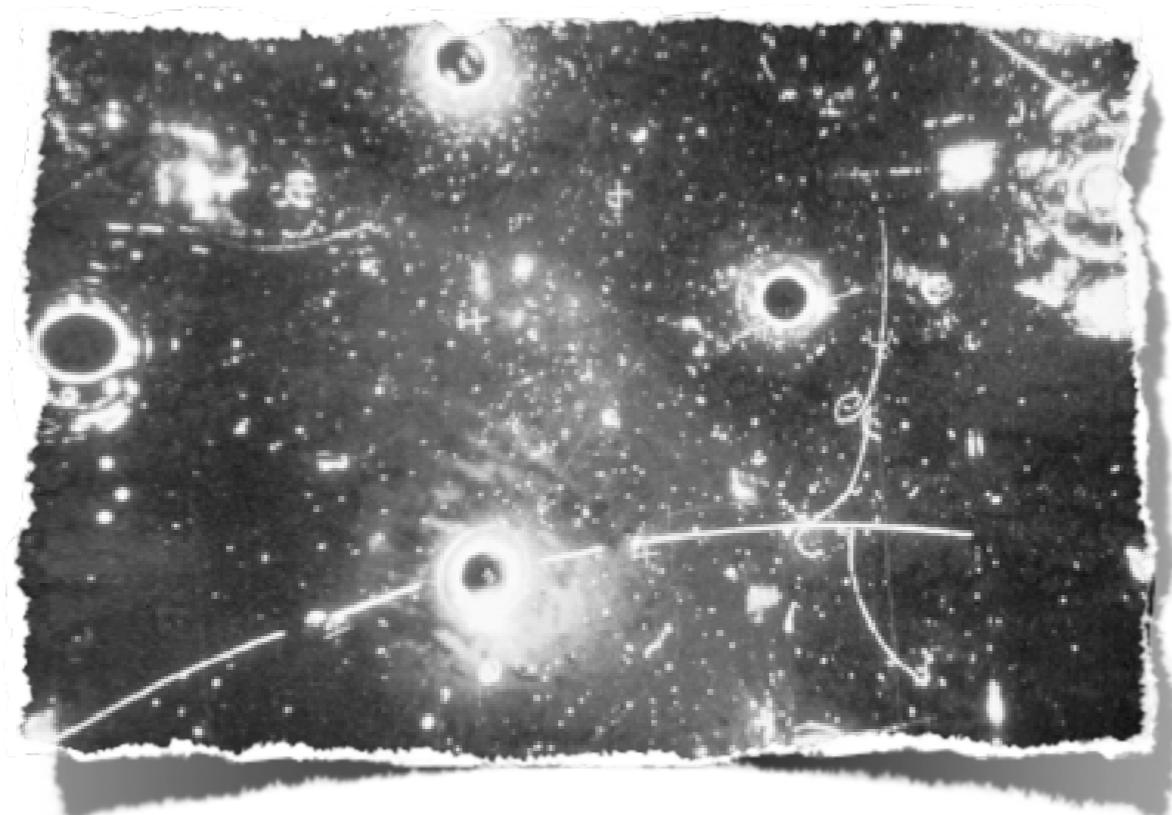
much more rigid theory = unique theory

# The Standard Model

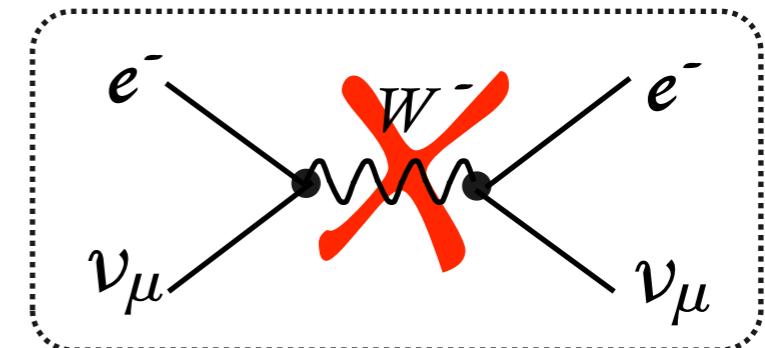
the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$



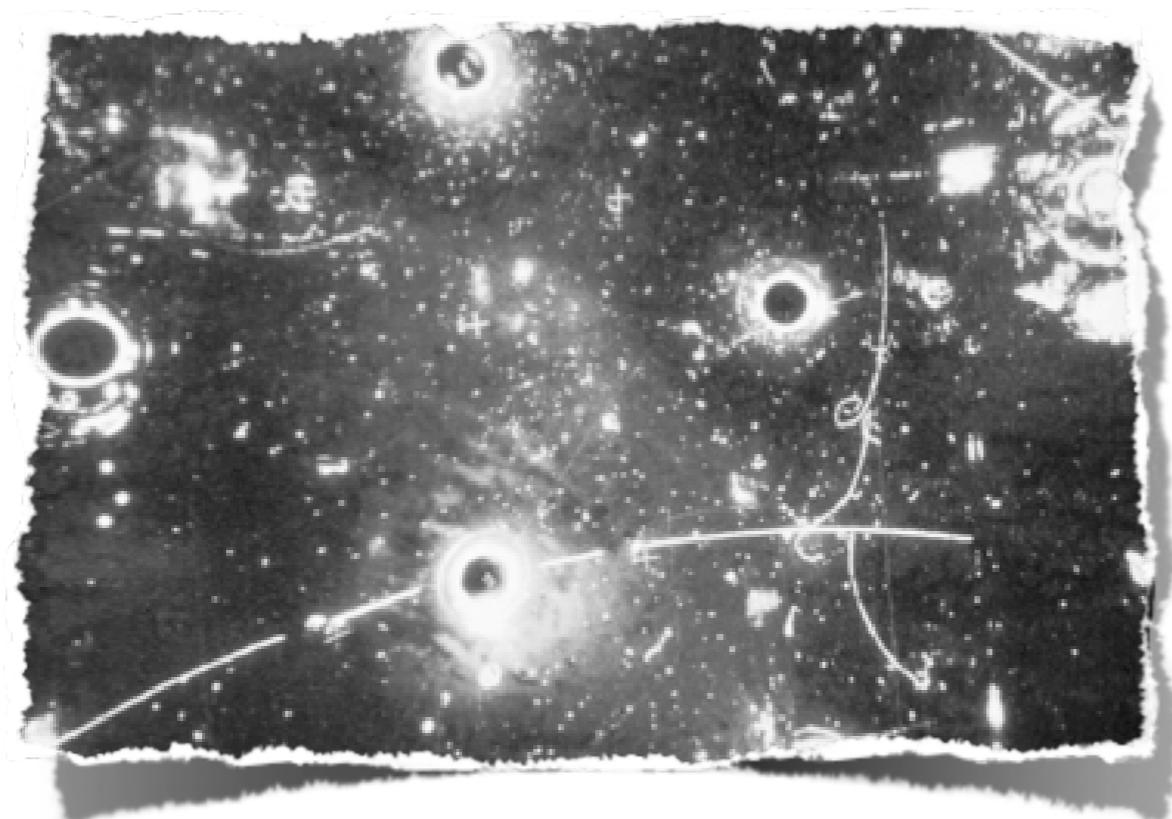
[Gargamelle collaboration, '73]



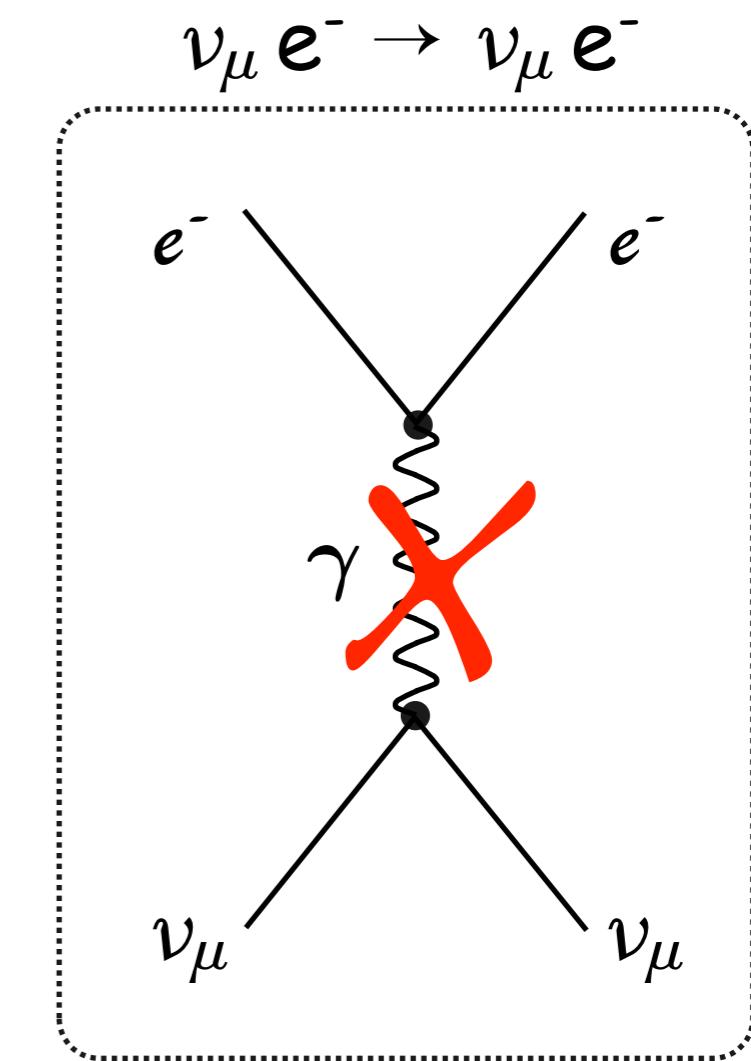
# The Standard Model

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$



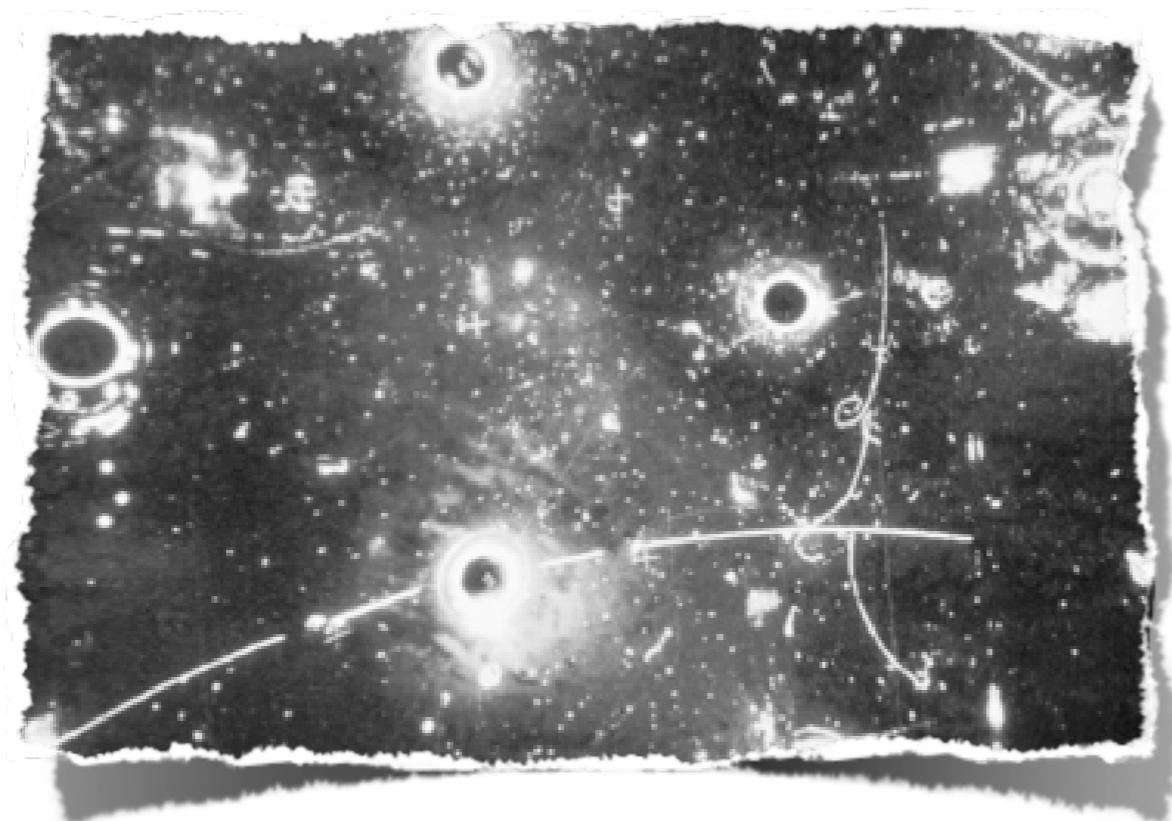
[Gargamelle collaboration, '73]



# The Standard Model

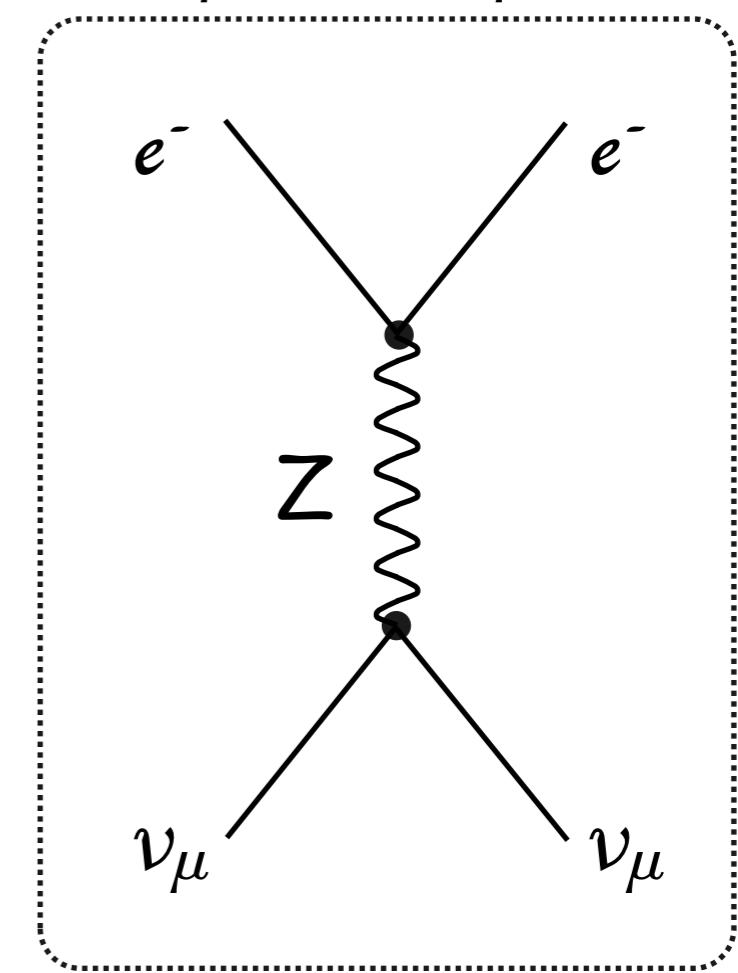
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[Gargamelle collaboration, '73]

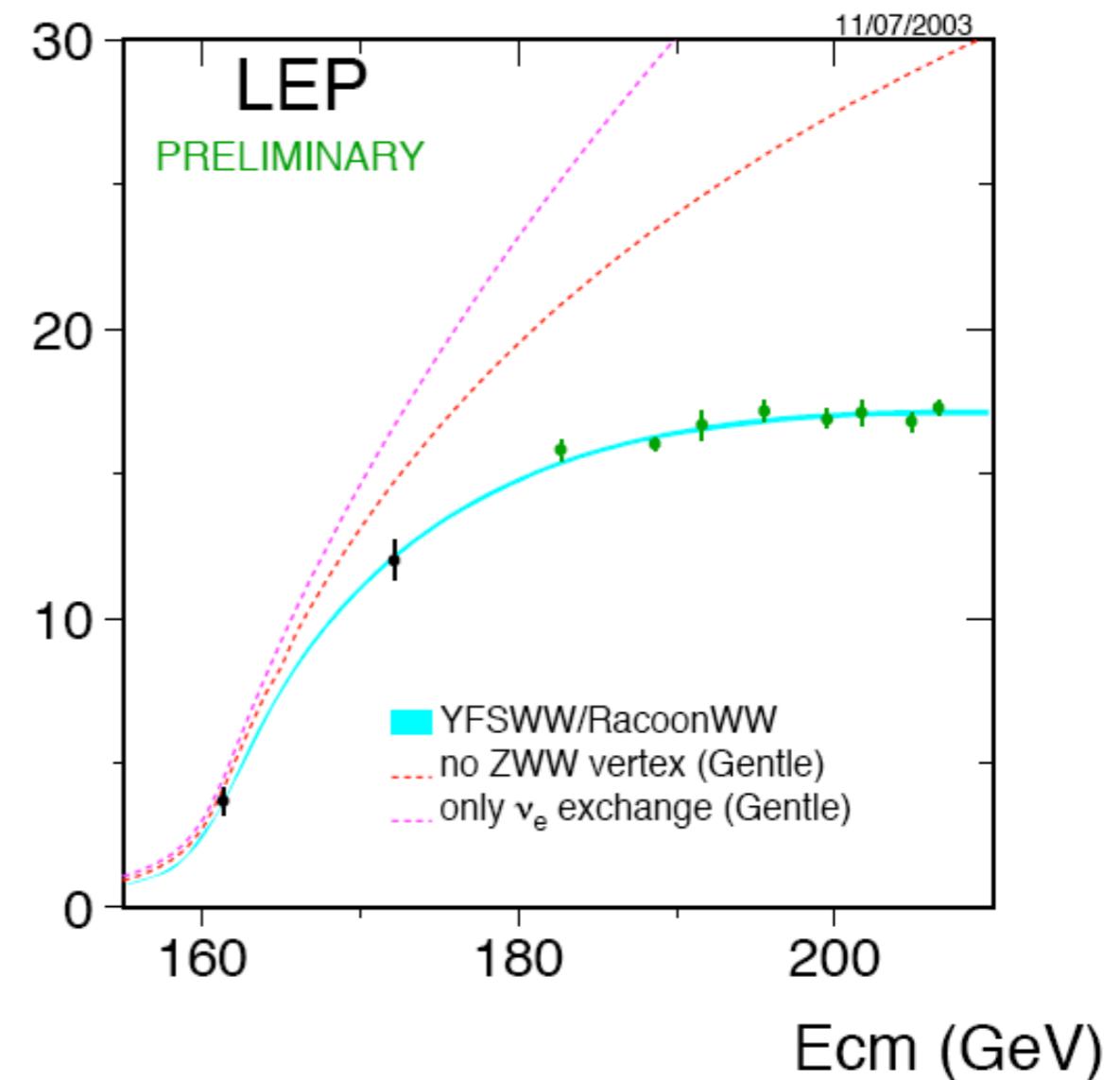
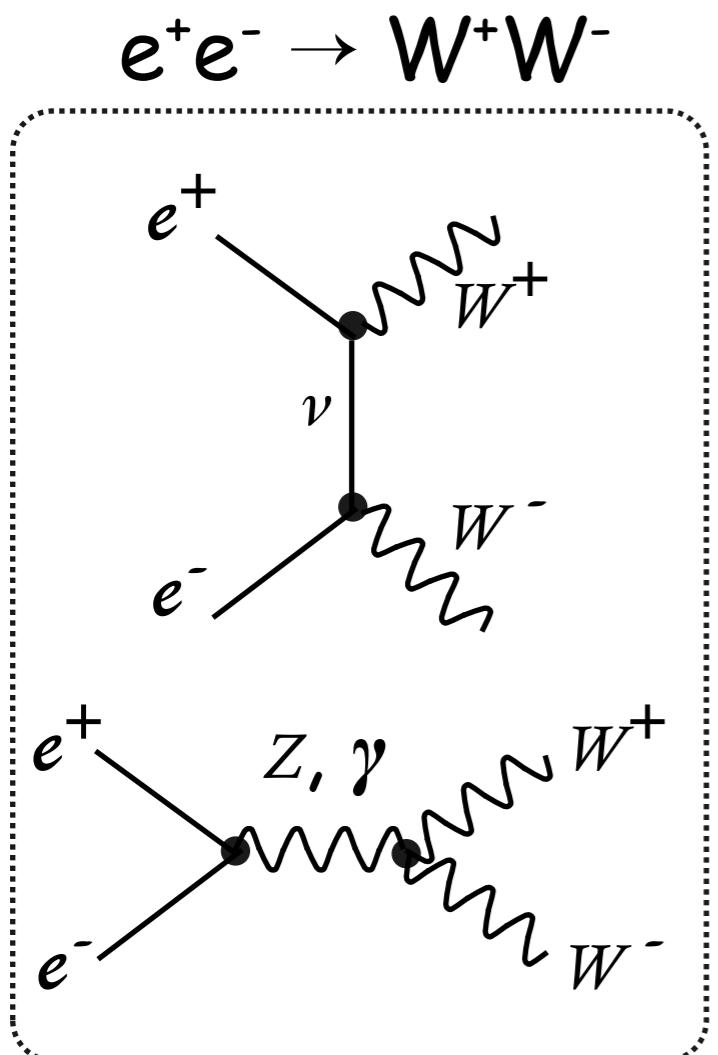
$$\nu_\mu e^- \rightarrow \nu_\mu e^-$$



# Gauge Theory as a Dynamical Principle

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$



# The Standard Model and the Mass Problem

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

the masses of the quarks, leptons and gauge bosons  
don't obey the full gauge invariance

- $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$  is a doublet of  $SU(2)_L$  but  $m_{\nu_e} \ll m_e$
- a mass term for the gauge field isn't invariant under gauge transformation

$$\delta A_\mu^a = \partial_\mu \epsilon^a + g f^{abc} A_\mu^b \epsilon^c$$

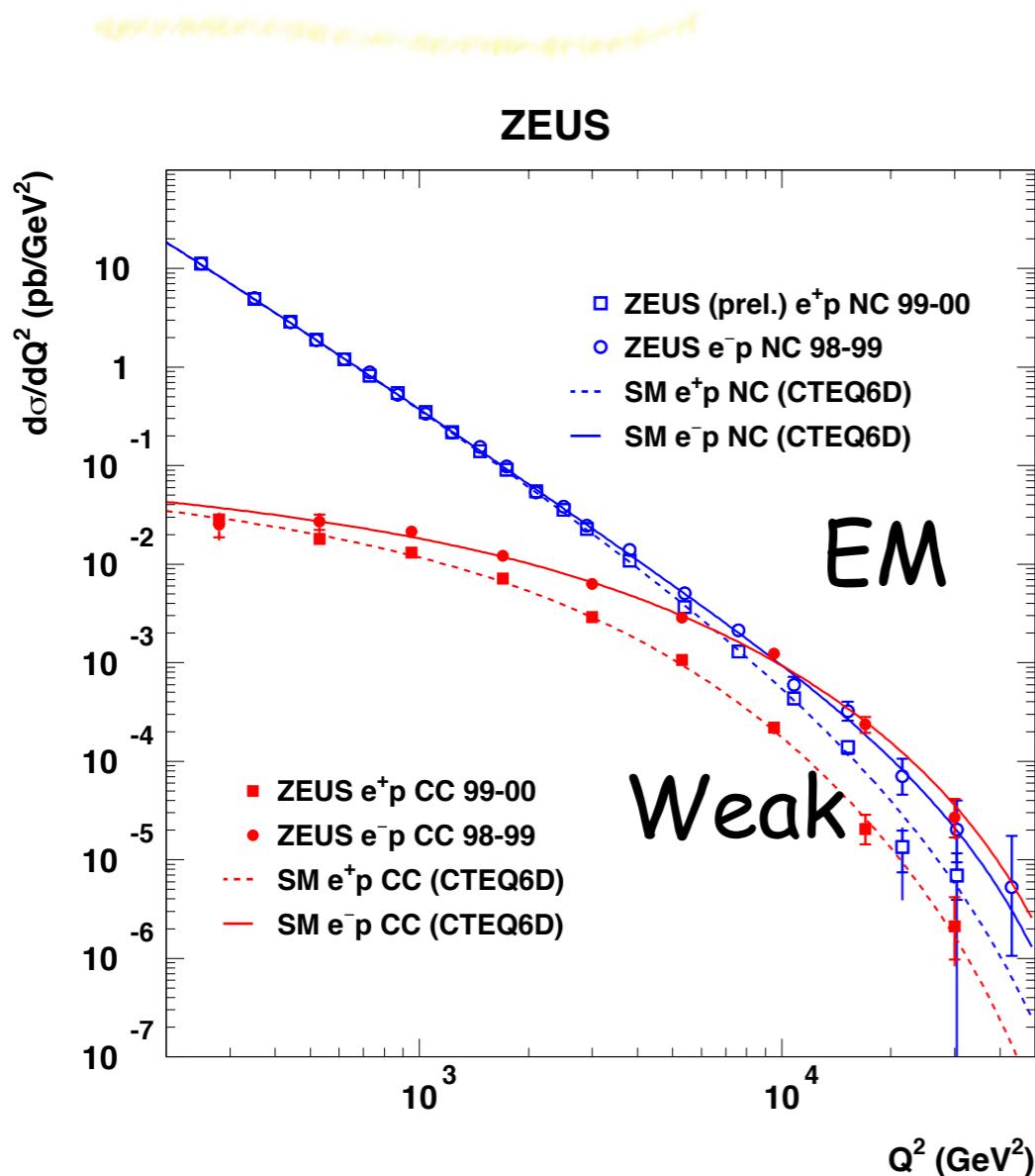


spontaneous breaking of gauge symmetry



# Electroweak Unification

High energy ( $\sim 100$  GeV)



Low energy

This room is full of photons  
but no  $W/Z$

The symmetry between  $W$ ,  $Z$  and  $\gamma$   
is broken at large distances

EM forces  $\approx$  long ranges

Weak forces  $\approx$  short range

$$m_\gamma < 6 \times 10^{-17} \text{ eV}$$

$$m_{W^\pm} = 80.425 \pm 0.038 \text{ GeV}$$

$$m_{Z^0} = 91.1876 \pm 0.0021 \text{ GeV}$$

# Classical field theory

classical mechanics & lagrangian formalism

action principle  
determines classical  
trajectory:

a system is described by

$$S = \int dt \mathcal{L}(q, \dot{q})$$

$\delta S = 0 \rightarrow$  Euler-Lagrange equations

$$\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\partial}{\partial t} \frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$$

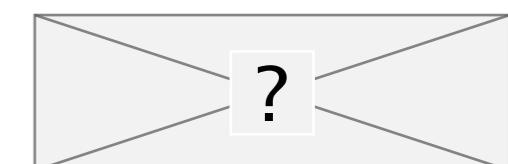
conjugate momenta     $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$     hamiltonian     $H(p, q) = \sum_i p_i \dot{q}_i - \mathcal{L}$

extend lagrangian formalism to  
dynamics of fields

$$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$

$$\delta S = 0 \rightarrow \frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial (\partial_\mu \varphi_i)} = 0$$



conjugate momenta     $\Pi_i = \frac{\partial \mathcal{L}}{\partial (\partial_0 \varphi_i)}$

hamiltonian

$$H(x) = \sum_i \Pi_i(x) \partial_0 \varphi_i(x) - \mathcal{L}$$

# Noether theorem

Invariance of action under  
continuous global transformation

--->

There is a conserved current/charge

$$\partial_\mu j^\mu = 0 \quad Q = \int d^3x j^0(x, t)$$

example of  
transformation:

$$\varphi \rightarrow \varphi e^{i\alpha} \quad (*)$$

if small increment  $\alpha \ll 1$   $\varphi \rightarrow \varphi + i\alpha\varphi$

$$\delta\varphi = i\alpha\varphi$$

$$\delta\varphi' = i\alpha\varphi'$$

invariance of  $\mathcal{L}$  under (\*):  $\delta\mathcal{L} = 0 = i\alpha\left(\frac{\partial\mathcal{L}}{\partial\varphi}\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\varphi'\right)$

Euler-Lagrange equations:  $\frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\right) - \frac{\partial\mathcal{L}}{\partial\varphi} = 0$

$$\left. \frac{\partial}{\partial x}\left(\varphi \frac{\partial\mathcal{L}}{\partial\varphi'}\right) = 0 \right\} \equiv J$$

conserved current

# Symmetries and conservation laws

Noether's theorem (from classical field theory) :  
A continuous symmetry of the system  $\rightarrow$  a conserved quantity

## I- Continuous global space-time symmetries:

translation invariance in space  $\rightarrow$  momentum conservation

translation invariance in time  $\rightarrow$  energy conservation

rotational invariance  $\rightarrow$  angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \rightarrow x'^\mu = \Lambda_\nu^\mu x^\nu \quad \phi(x) \rightarrow \phi'(x')$$

$$\phi'(x) = \phi(x) \quad \text{scalar}$$

$$V^\mu \rightarrow \Lambda_\nu^\mu V^\nu \quad \text{vector}$$

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)

# Symmetries and conservation laws

I- Continuous global space-time (Poincaré) symmetries

all particles have (m, s)

-> energy, momentum, angular momentum conserved

II- Global (continuous) internal symmetries

-> B, L conserved

(accidental symmetries)

III- Local or gauge internal symmetries

-> color, electric charge conserved

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

IV- Discrete symmetries

-> CPT

# Group theory