Brief Look at the Basic Consepts of Particle Physics

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Outline:

- Lorentz Transformations
- 4-Vector
- Invariance
- Energy & Momentum
- Rest Mass
- Jets

RELATIVISTIC KINEMATICS

LORENTZ TRANSFORMATIONS

- in special relativity, laws of physics equally hold for all inertial reference systems.

  - inertial system: Newton's 1st law

    objets in motion stay in motion,
objects at rest stay at rest } unless a force exerted.

  - any system moving with constant v with respect to an inertial system is also inertial.

  - for S, S' two inertial frames,

    $S' \rightarrow \bar{v}$ with respect to S
$S \rightarrow -\bar{v}$ with respect to S'

- Transformations:

$$x' = \gamma(x - vt)$$

$$y' = y$$

$$z' = z$$

$$t' = \gamma\left(t - \frac{v}{c^2}x\right)$$

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

- Inverse transformations:

$$x = \gamma(x' + vt')$$

$$y = y'$$

$$z = z'$$

$$t = \gamma\left(t' + \frac{v}{c^2}x'\right)$$

- consequences of the transformations:

1. relativity of simultoneity

2 events occuring at S, in the same time but at different places don't happen at the same time in S'.

..

$$t_A = t_B$$

$$t'_A = t'_B + \frac{\gamma v}{c^2}(x_B - x_A)$$

events that are simultaneous in one inertial system are not simultaneous in others.

2. Lorentz Contraction

a moving object gets shortened by a factor of γ, compared to its length in the system at rest.

it only applies in the direction of the motion. perpendicular directions are not affected.

$$x' = L' \longrightarrow x' = L'/\gamma$$

3. Time Dilation

a difference of elapsed time between 2 events measured by observers either moving relative to each other, or differently situated from a gravitational mass.

clocks in S tick longer $(T = \gamma T')$

moving clocks run slow.

moving particles last longer than they would while at rest.

4. Velocity Addition

particle moving in x-dir with speed u', wrt S'. then its speed u wrt S would be:

$$\Delta x = \gamma(\Delta x' + v\Delta t') \qquad \Delta t = \gamma\left[\Delta t' + (v/c^2)\Delta x'\right]$$

$$\frac{\Delta x}{\Delta t} = \frac{\Delta x' + v\Delta t'}{\Delta t' + (v/c^2)\Delta x'} = \frac{(\Delta x'/\Delta t') + v}{1 + (v/c^2)\Delta x'/\Delta t'} = u$$

$$\frac{\Delta x'}{\Delta t'} = u'$$

$$u = \frac{u' + v}{1 + (u'v/c^2)}$$

FOUR-VECTORS

- position-time 4-vector

$$x^\mu, \quad \mu = 0, 1, 2, 3$$

- lorentz transformations:

  or in compact form:

$$x^{\mu\prime} = \sum_{0}^{3} \Lambda^\mu_\nu x^\nu \qquad (\mu = 0, 1, 2, 3)$$

$$\Lambda = \begin{bmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Lambda^0_0 = \Lambda^1_1 = \gamma$$

$$\Lambda^1_0 = \Lambda^0_1 = -\gamma\beta$$

$$\Lambda^2_2 = \Lambda^3_3 = 1$$

- ex: $x^{0\prime} = \Lambda^0_0 x^0 + \Lambda^0_1 x^1 + \Lambda^0_2 x^2 + \Lambda^0_3 x^3 = \gamma x^0 - \gamma\beta x^1$

$$= \gamma(x^0 - \beta x^1)$$

- invariance:

$$I \equiv (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 = (x^{0'})^2 - (x^{1'})^2 - (x^{2'})^2 - (x^{3'})^2$$

can be thought as the rotational invariance of $r^2 = x^2 + y^2 + z^2$

- g metric $g_{\mu\nu}$

$$g = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

- now I can be written as:

$$I = \sum_{\mu=0} \sum_{\nu=0} g_{\mu\nu} x^{\mu} x^{\nu} = g_{\mu\nu} x^{\mu\nu}$$

$$\longrightarrow I = x_{\mu} x^{\mu}$$

- covariant 4-vector: $x_{\mu} = g_{\mu\nu} x^{\nu}$

- controvariant 4-vector: $x^{\mu} = g^{\mu\nu} a_{\nu}$

- a^{μ}, b^{μ} any 4 vectors

$$a^{\mu} b_{\mu} = a_{\mu} b^{\mu} = a^0 b^0 - a^1 b^1 - a^2 b^2 - a^3 b^3 \rightarrow \text{invariant.}$$

- $a \cdot b \equiv a_\mu b^\mu$, but don't confuse this with $\underbrace{\vec{a} \cdot \vec{b}}_{\text{3 vector}}$, the scalar prod.

$$a \cdot b = a^0 b^0 - \vec{a} \cdot \vec{b}$$

$$a^2 \equiv a \cdot a = (a^0)^2 - \vec{a}^2$$

here, a^2 can be

$a^2 > 0$ $\quad a^\mu \longrightarrow$ timelike : 2 events same place, different time

$a^2 < 0$ $\quad a^\mu \longrightarrow$ spacelike : 2 events same time, different places

$a^2 = 0$ $\quad a^\mu \longrightarrow$ lightlike : BC. light can travel in this interval

Energy and Momentum

- moving clock runs slow, ground time (maybe observer time?) changes infinitesimally by $d\tau$

$$d\tau = \frac{dt}{\gamma}$$

- at "normal" speeds $\gamma \to 1$, $d\tau = dt$

- <u>BUT</u> in particle physics, there is a difference between lab time and particle time. we can transform and get both from one another but it's not convenient.

- Best thing to do is work with "proper time" where all observes can read the particle's watch at any given time and agree on the value. They own watches may be different.

- velocity of the particle means the distance it travels (lab frame) divided by the time it takes. (lab clock)

$$v = \frac{dx}{dt}$$

- proper velocity $\rightarrow \eta \longrightarrow$ distance travelled divided by proper time

$$\eta \equiv \frac{dx}{d\tau}$$

- two velocities are related by a factor of γ

$$\eta = \gamma v$$

- proper velocity as 4-vector:

$$\eta^\mu = \frac{dx^\mu}{d\tau} = \gamma(c, v_x, v_y, v_z)$$

$$\eta^0 = \frac{dx^0}{d\tau} = \frac{d(ct)}{(1/\gamma)dt} = \gamma c$$

- invariant = $\eta_\mu \eta^\mu = \gamma^2(c^2 - v_x^2 - v_y^2 - v_z^2) = \gamma^2 c^2(1 - v^2/c^2) = c^2$

- Now, we wanna define momentum relativisticly. It's velocity * mass, but should we use regular, or proper velocity?

regular v = mv = conservation of momentum $\longrightarrow$ if holds for one inertial system wouldn't hold in others.

proper v = $m\eta$ = conservation of momentum $\longrightarrow$ holds for one system, holds for every inertial system.

- CAREFUL $\longrightarrow$ this does not Guarantee that the momentum IS conserved. That depends on the experiments. It means that IF you're hoping to extend momentum cons. to the relativistic domain, you better use $m\eta$

- $p^\mu = m\eta^\mu$

- relativistic momentum three vector:

$$\bar{p} = \gamma m \bar{v} = \frac{m\bar{v}}{\sqrt{1 - v^2/c^2}}$$

- Relativistic energy:

$$E = \gamma m c^2 = \frac{mc^2}{\sqrt{1 - v^2/c^2}}$$

- so p^0, 0^{th} component of $p^\mu = \frac{E}{c}$

$\longrightarrow$ $p^\mu = \left(\frac{E}{c}, p_x, p_y, p_z\right)$

$\longrightarrow$ $\eta_\mu \eta^\mu = \gamma^2 (c^2 - v_x^2 - v_y^2 - v_z^2) = \gamma^2 c^2 (1 - v^2/c^2) = c^2$

$p^\mu = m\eta^\mu \qquad p_\mu = m\eta_\mu$

$$p^\mu p_\mu = m^2 \eta^\mu \eta_\mu = m^2 c^2 = \frac{E^2}{c} - \underbrace{p_x^2 - p_y^2 - p_z^2}_{\bar{p}^2}$$

- Expand the energy using Taylor

$$E = mc^2\left(1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots\right)$$

$$= \underbrace{mc^2}_{\text{cnst.}} + \underbrace{\frac{1}{2}mv^2}_{\substack{\text{classical}\\ \text{K.E}}} + \frac{3}{8}m\frac{v^4}{c^2} + \cdots$$

$E = mc^2$ (or R) $\longrightarrow$ rest energy rest mass / invariant mass

- according to special relativity, relative to the observer, mass of an object increases with its velocity.

- but objects at rest tends to have "normal" rest that resists applied force. It is the minimum mass that object can have.

- Another explanation is that: the mass the object or system has that is independent of the motion of the system.

- system's total energy and momentum that remains same in all reference frames.

$$m_0^2 c^2 = \left(\frac{E}{c}\right)^2 - |p|^2$$

for $c = 1 \longrightarrow m_0^2 = E^2 - p^2$

- In center of momentum frame, inv. mass = total mass in rest frame

Ex

Invariant mass of 2-particle collision

$$m_0^2 = (E_1 + E_2)^2 - |\bar{p}_1 + \bar{p}_2|^2$$

Jets & Fat Jets

- hadronization: forming $\boxed{\text{hadrons}}$ out of quarks and gluons.

$$\text{hadrons} \begin{cases} \text{mesons} & q\bar{q} \\ \text{baryons} & qqq \end{cases}$$

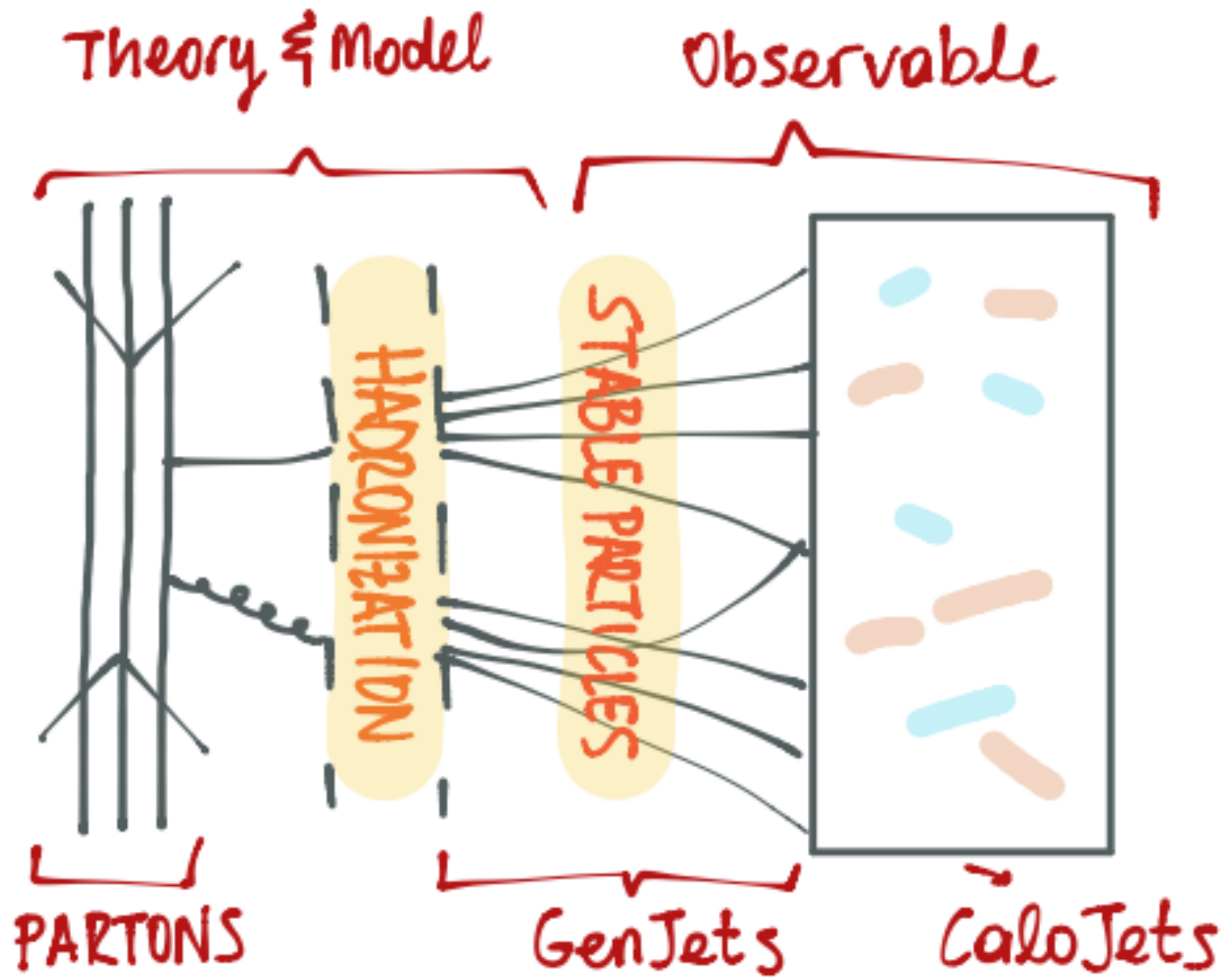
— Gluon & all quarks (except t) hadronize before decay. Top quark can not hadronize, because its half life is not long enough for it.

- jet: when a high energy quark transforms into a spray of hadrons.

— again, for the top quark it's different. t decays to W and b before a jet can form.

— jets measured in hadron colliders produced in the $\boxed{\text{hard interaction.}}$

→ the scattering between partons.



- GenJets : created from stable simulated particles.
- CaloJets : created using the calorimeter output