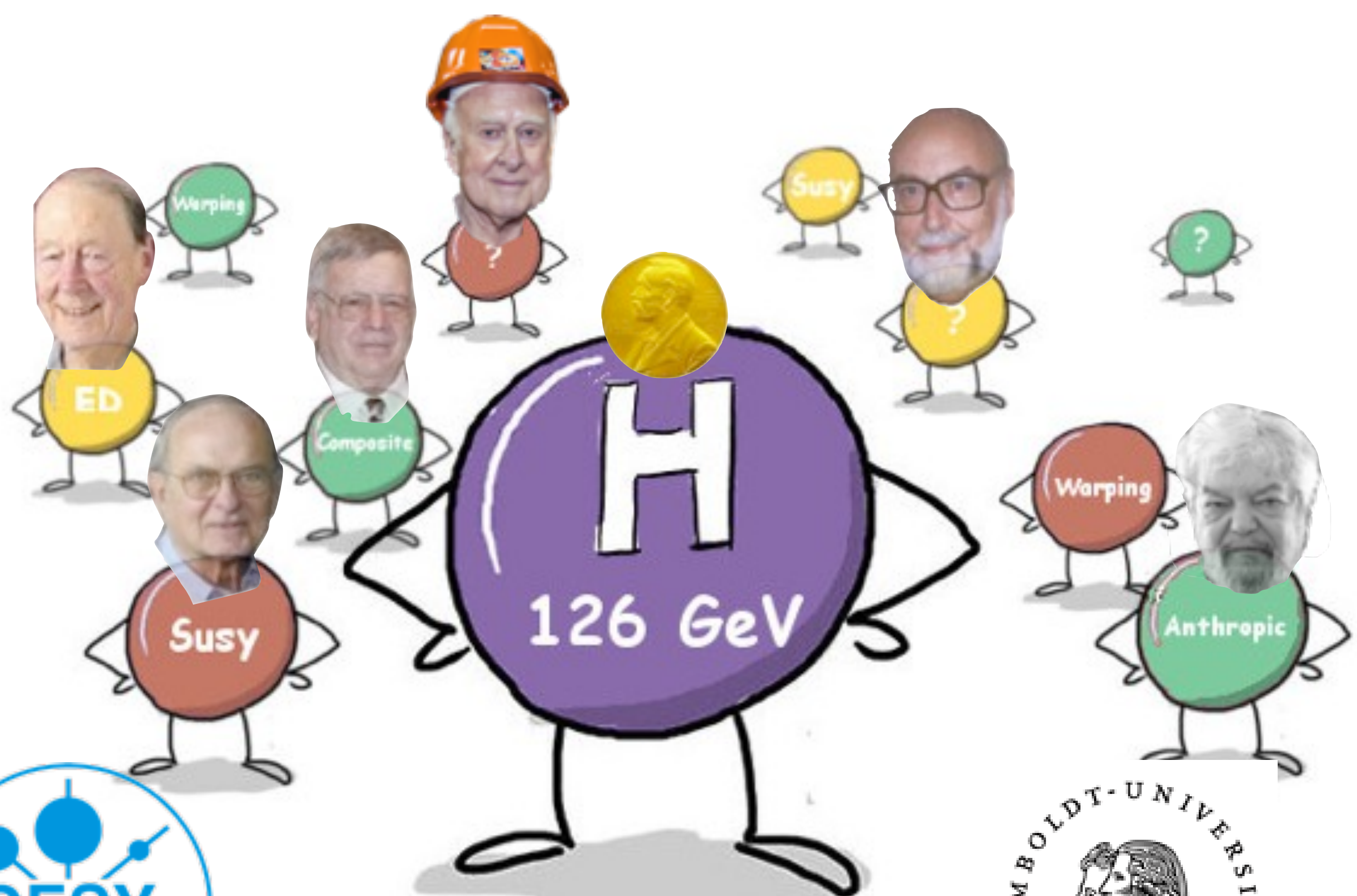
Introduction
HEP Theory

DESY summer student lectures 2017



Lectures 3+4/6

Christophe Grojean
DESY (Hamburg)
Humboldt University (Berlin)
(christophe.grojean@desy.de)

Outline (v0)

1. Friday

   Quantum field theory, dimensional analysis, symmetries (space-time and internal gauge symmetries, continuous, global), group theory

2. Friday

   Local symmetries and gauge invariance principle, QED, Standard Model

3. Monday

   Spontaneous symmetry breaking, Goldstone theorem, Higgs mechanism

4. Monday

   Electroweak precision test, stability of the EW vacuum, the hierarchy problem

5. Tuesday

   Field quantization, S-matrix, Feynman rules, scattering, cross sections, decay rates, calculation tricks, sample calculation

6. Tuesday

   Non-abelian gauge theories, Standard Model Lagrangian and its phenomenological properties, observables

the order will probably change

Outline (v1)

1. Friday

Quantum field theory, dimensional analysis

2. Friday

elementary particles, different fundamental interactions, symmetries (space-time and internal gauge symmetries, continuous, global), group theory (not yet!)

3. Monday

Fermi theory, effective theory, gauge symmetry, QED, non-abelian gauge symmetries, Standard Model

4. Monday

Spontaneous symmetry breaking, Goldstone theorem, Higgs mechanism

Electroweak precision test, stability of the EW vacuum, the hierarchy problem

5. Tuesday

Field quantization, S-matrix, Feynman rules, scattering, cross sections, decay rates, calculation tricks, sample calculation

6. Tuesday

Non-abelian gauge theories, Standard Model Lagrangian and its phenomenological properties, observables

Recap

1. QM+Special relativity:

   i. uncertainty principle + mass energy↔energy: particle creation

   ii. antiparticle: logarithmic corrections to e^- mass: running of α

2. Finite number of elementary particles & 3 families

3. There are different fundamental forces among the elementary particles

4. Non-trivial (quantum) consistency of particle content

   ↔ electric neutrality of the atoms

Homework

1. How long does it take for a photon to travel from your feet to your brain?
   What is the maximal frequency at which the brain can function?
   When is AI going to take over humans? (all because of finite light speed and human size!)

2. Estimate the energy of the cosmic rays

3. Distance between the plates for the Casimir pressure to be of the order of 1 atm

4. Volume of oil to burn a 1cm-thick ice cap surrounding the sun at a distance of 150 million kms

5. Check that in the SM (1) $Tr_L Y - Tr_R Y = 0$ and (2) $Tr_L Y^3 - Tr_R Y^3 = 0$

6. Check that in the presence of right handed neutrinos, the ratio of the electric charges of the electron vs the up quark is not fixed by the absence of anomaly, but still the proton and the electron have to have opposite electric charges.

Gauge Theories

Fermi Theory

(paper rejected by Nature: declared too speculative !)

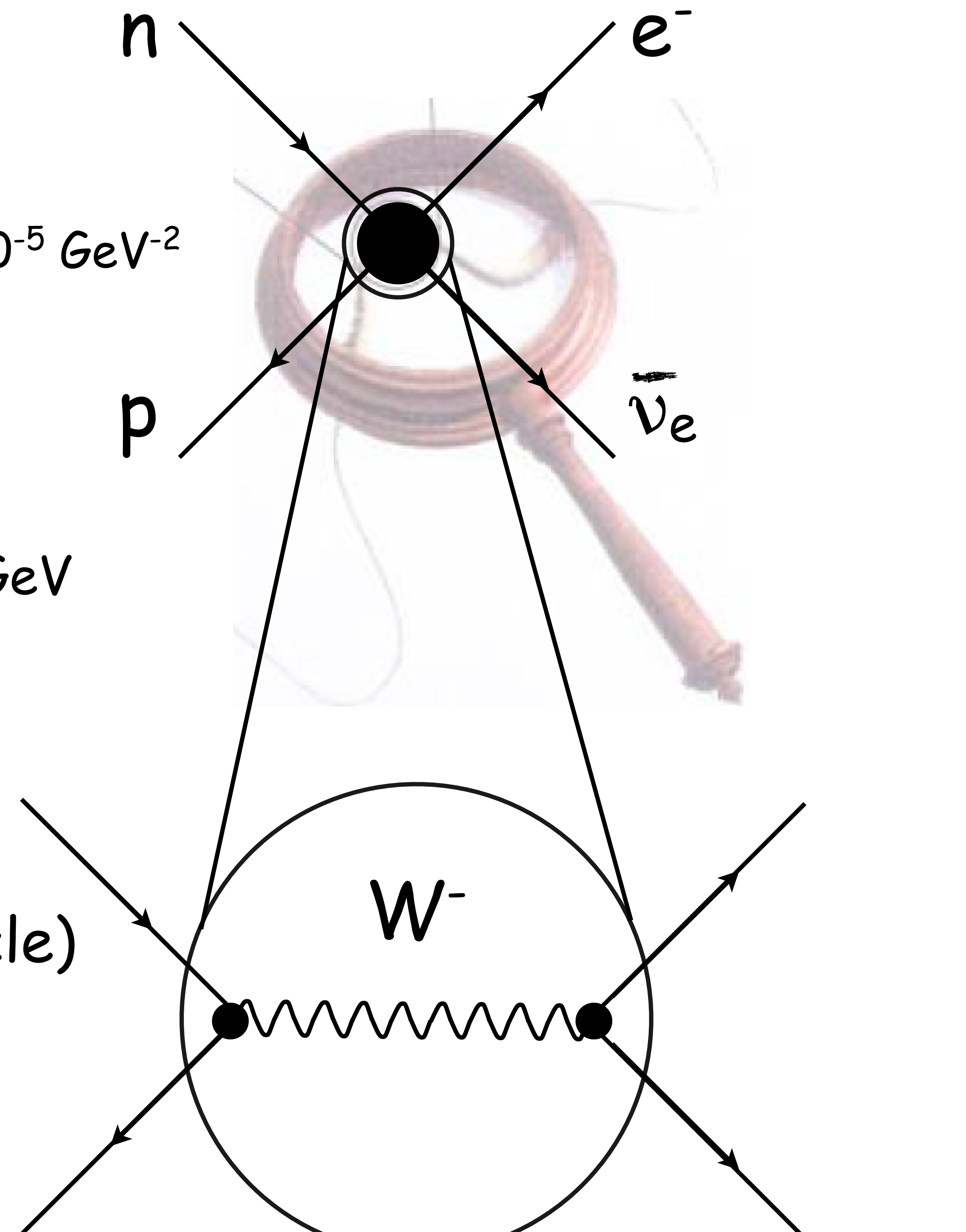
$$n \longrightarrow p + e^- + \bar{\nu}_e$$

exp: G_F=1.166x10^{-5} GeV^{-2}

$$\mathcal{L} = G_{\mathcal{F}} (\bar{n} p)(\bar{\nu}_e e)$$

$$\mathcal{A} \propto G_{\mathcal{F}} E^2$$

- no continuous limit
- inconsistent above 300 GeV



Gauge theory

microscopic theory

(exchange of a massive spin 1 particle)

$$G_{\mathcal{F}} = \frac{\sqrt{2} g^2}{8 m_W^2}$$

exp: m_W=80.4 GeV

➡ g ≈0.6, ie, same order as e=0.3

unification EM & weak interactions

Why Gauge Theories?

How are we sure that muon and neutron decays proceed via the same interactions?

$\tau_\mu \approx 10^{-6}$s vs. $\tau_{neutron} \approx 900$s

$$\underbrace{\mathcal{L}}_{[\text{mass}]^4} = \underbrace{G_F}_{[\text{mass}]^{-2}}\, \underbrace{\psi^4}_{[\text{mass}]^{3/2\times 4}} \quad \Longrightarrow \quad \Gamma \propto G_F^2 \underbrace{m}_{[\text{mass}]}^5$$

for the muon, the relevant mass scale is the muon mass m_μ=105MeV: $\Gamma_\mu = \frac{G_F^2 m_\mu^5}{192\pi^3} \sim 10^{-19}\,\text{GeV}$

for the neutron, the relevant mass scale is $(m_n-m_p)\approx 1.29$MeV: $\Gamma_n = \mathcal{O}(1)\frac{G_F^2 \Delta m^5}{\pi^3} \sim 10^{-28}\,\text{GeV}$

ex: what about π decay τ_π? Why $\frac{\Gamma(\pi^- \to e^- \bar{\nu}_e)}{\Gamma(\pi^- \to \mu^- \bar{\nu}_\mu)} \sim 10^{-4}$?

What about weak scattering process, e.g. $e\nu_e \to e\nu_e$?

$$\underbrace{\sigma}_{[\text{mass}]^{-2}} \propto \underbrace{G_F^2}_{[\text{mass}]^{-2\times 2}} \underbrace{E^2}_{[\text{mass}]^2} \quad \Longrightarrow$$

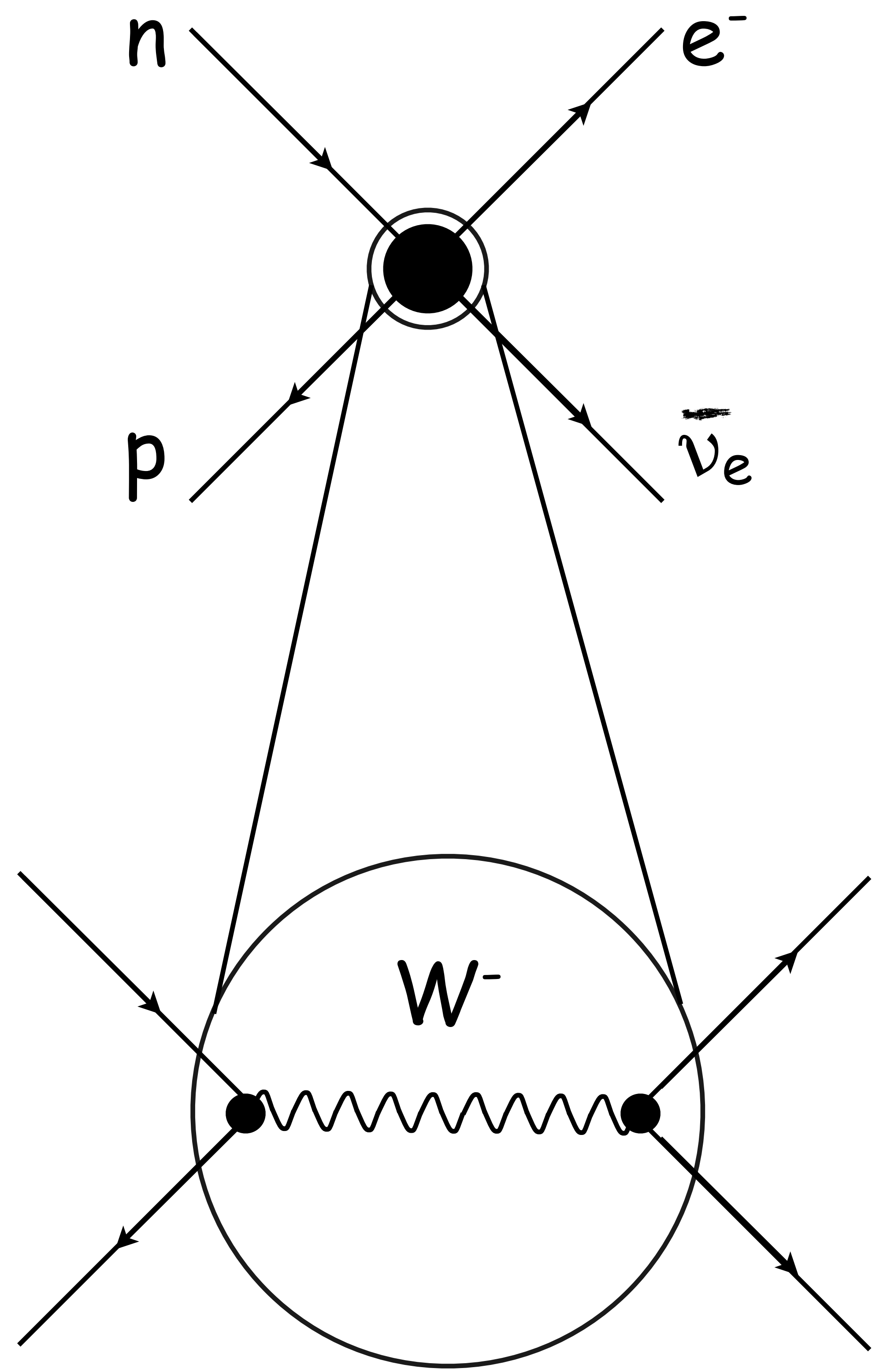
non conservation of probability
(non-unitary theory)
inconsistent at energy above 300GeV

Why Gauge Theories?

What about weak scattering process, e.g. $e\nu_e \to e\nu_e$?

$$\sigma \propto G_F^2 E^2$$

$[\text{mass}]^{-2}$ $[\text{mass}]^{-2\times 2}$ $[\text{mass}]^2$



Gauge theory

$$\sigma \propto g^4 \frac{E^2}{m_W^2\,(E^2 + m_W^2)}$$

- match with Fermi theory at low energy $G_F \propto \frac{g^2}{m_W^2}$
  (we say that the Fermi theory is an **effective theory** of the weak gauge theory at low energy)
- good high energy behavior

Why non-abelian Gauge Theories?

EM = exchange of photon = U(1) gauge symmetry

EM U(1) $\phi \to e^{i\alpha}\phi$ but $\partial_\mu\phi \to e^{i\alpha}\,(\partial_\mu\phi) + \underbrace{i(\partial_\mu\alpha)}_{}\,\phi$

≠0 if local transformations

EM field and covariant derivative $\partial_\mu\phi + ieA_\mu\phi \to e^{i\alpha}(\partial_\mu\phi + ieA_\mu\phi)$

if $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$

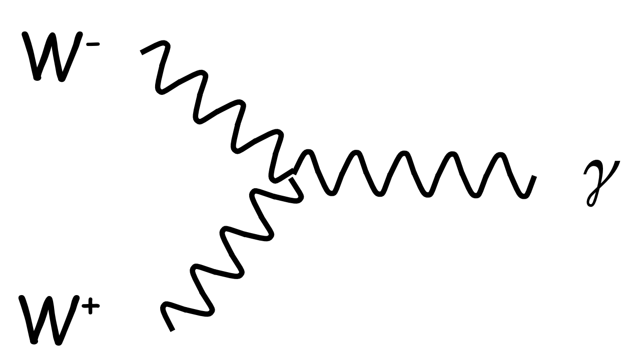
the EM field keeps track of the phase in different points of the space-time

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad \to F_{\mu\nu}$

photon do not interact with itself because it doesn't carry an electric charge

W carries an electric charge since it mediates charged current interactions

W interacts with the photon ☛ non-abelian interactions



Gauge Theories: EM & Yang-Mills

EM U(1) $\phi \to e^{i\alpha}\phi$ but $\partial_\mu \phi \to e^{i\alpha}(\partial_\mu\phi) + \underbrace{i(\partial_\mu\alpha)}_{\neq 0 \text{ if local transformations}}\phi$

EM field and covariant derivative $\partial_\mu\phi + ieA_\mu\phi \to e^{i\alpha}(\partial_\mu\phi + ieA_\mu\phi)$

if $A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$

the EM field keeps track of the phase in different points of the space-time

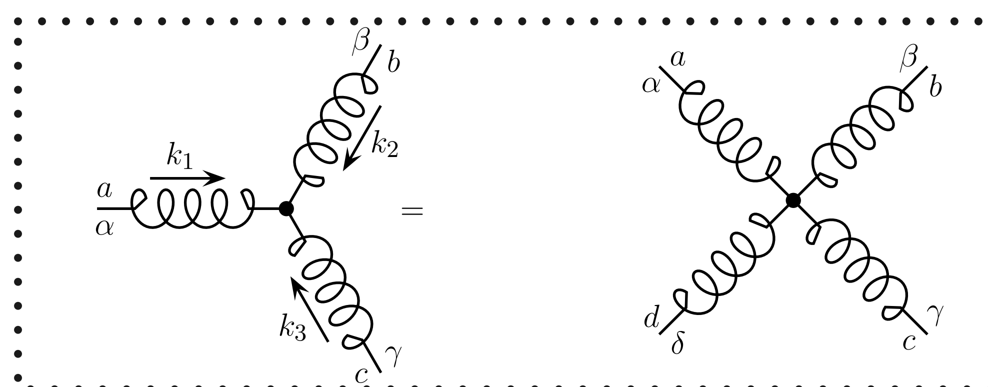
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \to F_{\mu\nu}$

Yang-Mills : non-abelian transformations $\phi \to U\phi$

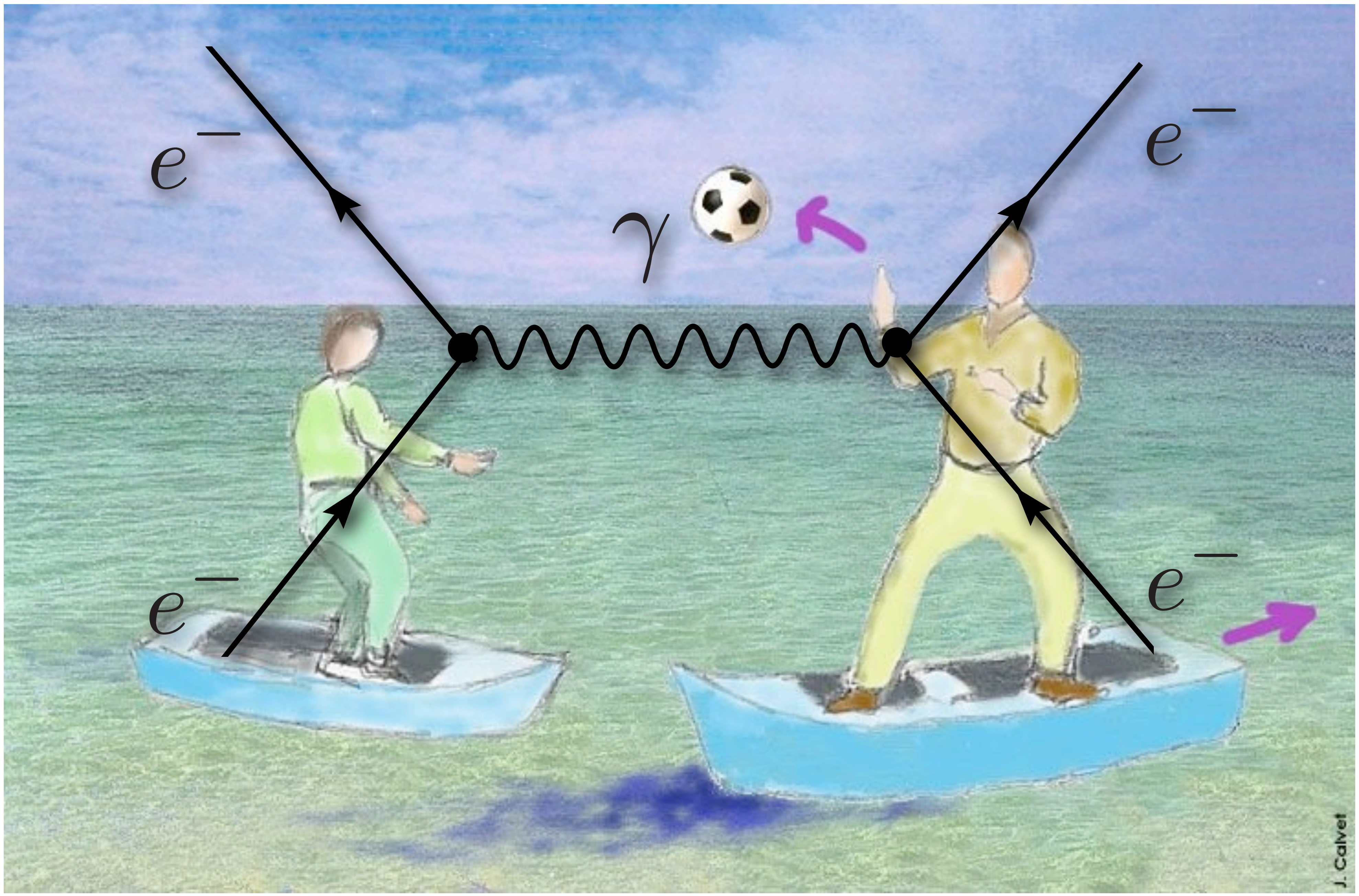
$\partial_\mu\phi + igA_\mu\phi \to U(\partial_\mu\phi + igA_\mu\phi)$ if $A_\mu \to UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1}$

ex

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{ig[A_\mu, A_\nu]}_{\text{non-abelian int.}} \to UF_{\mu\nu}U^{-1}$



Interactions between Particles



Elementary particles interact on each other
by the exchange of gauge bosons

The Standard Model: Interactions

- U(1)$_Y$ <u>*electromagnetic interactions*</u>

  ➔ Photon γ

  light, atoms, molecules — 10^{-5}

- SU(2)$_L$ <u>*weak interactions*</u>

  ➔ bosons $W^\pm, Z^0$

  β decay

  $$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

  10^{-2}

- SU(3)$_c$ <u>*strong interactions*</u>

  ➔ gluons g^a

  atomic nuclei

  α decay

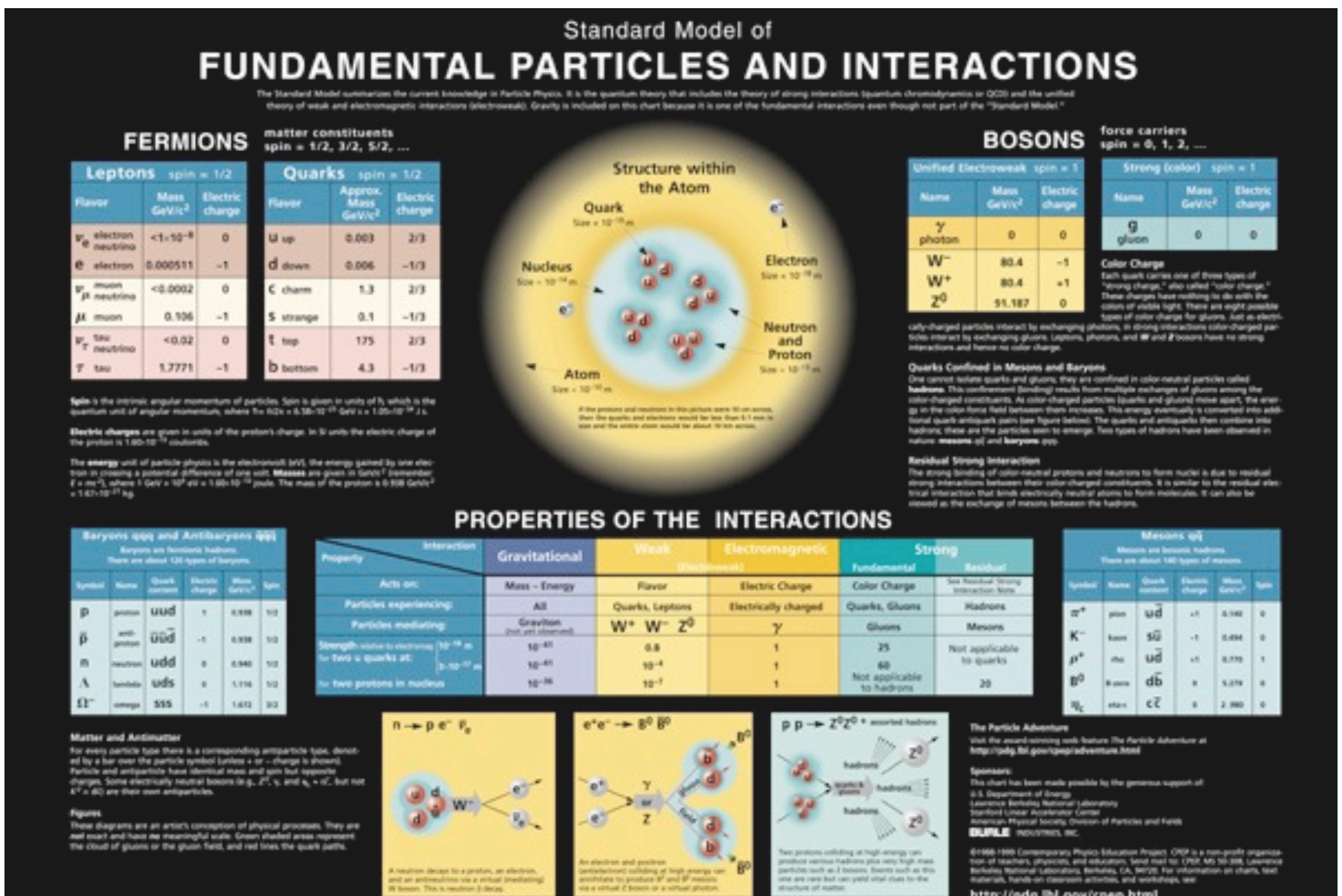
  $$^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

  1

strength

The Standard Model

Nobel Prize '79



description of all elementary particles and their interactions

The underlying principles of the SM

The beauty of the SM comes from the the identification of
a unique dynamical principle describing the different interactions
that seem so different from each others

gauge theory = spin-1

at the same time a particular and predictive structure that
still leaves room for a rich variety of phenomena
(long range interaction, spontaneous symmetry breaking, confinement)

gravitation = general relativity= spin-2

much more rigid theory = unique theory

Classical field theory

classical mechanics & lagrangian formalism

a system is described by $S = \int dt \mathcal{L}(q, \dot{q})$

position momentum

action principle determines classical trajectory:

$\delta S = 0$ --> Euler-Lagrange equations $\boxed{\frac{\partial \mathcal{L}}{\partial q_i} - \frac{\partial}{\partial t}\frac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0}$

conjugate momenta $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ hamiltonian $H(p, q) = \sum_i p_i \dot{q}_i - \mathcal{L}$

extend lagrangian formalism to dynamics of fields

$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$ $\partial_\mu = \frac{\partial}{\partial x^\mu}$

$\delta S = 0$ --> $\boxed{\frac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} = 0}$

conjugate momenta $\Pi_i = \frac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_i)}$ hamiltonian $H(x) = \sum_i \Pi_i(x)\partial_0\varphi_i(x) - \mathcal{L}$

Noether theorem

Invariance of action under continuous global transformation ---> There is a conserved current/charge

$$\partial_\mu j^\mu = 0 \qquad Q = \int d^3x j^0(x,t)$$

example of transformation: $\boxed{\varphi \to \varphi e^{i\alpha}}$ (*)

if small increment $\alpha \ll 1$ $\varphi \to \varphi + i\alpha\varphi$

$$\delta\varphi = i\alpha\varphi$$
$$\delta\varphi' = i\alpha\varphi'$$

invariance of $\mathcal{L}$ under (*): $\delta\mathcal{L} = 0 = i\alpha(\frac{\partial\mathcal{L}}{\partial\varphi}\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\varphi')$

Euler-Lagrange equations: $\frac{\partial}{\partial x}(\frac{\partial\mathcal{L}}{\partial\varphi'}) - \frac{\partial\mathcal{L}}{\partial\varphi} = 0$

exercise

$$\boxed{\frac{\partial}{\partial x}(\underbrace{\varphi\frac{\partial\mathcal{L}}{\partial\varphi'}}_{\equiv J}) = 0}$$

conserved current

Symmetries and conservation laws

Noether's theorem (from classical field theory) :

A continuous symmetry of the system -> a conserved quantity

I- Continuous global space-time symmetries:

translation invariance in space -> momentum conservation

translation invariance in time -> energy conservation

rotational invariance -> angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu \qquad \phi(x) \to \phi'(x')$$

$$\phi'(x) = \phi(x) \qquad \text{scalar}$$

$$V^\mu \to \Lambda^\mu_\nu V^\nu \qquad \text{vector}$$

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)

Symmetries and conservation laws

I- Continuous global space-time (Poincaré) symmetries

all particles have (m, s)

-> energy, momentum, angular momentum conserved

II- Global (continuous) internal symmetries

-> B, L conserved

(accidental symmetries)

III- Local or gauge internal symmetries

-> color, electric charge conserved

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

IV- Discrete symmetries

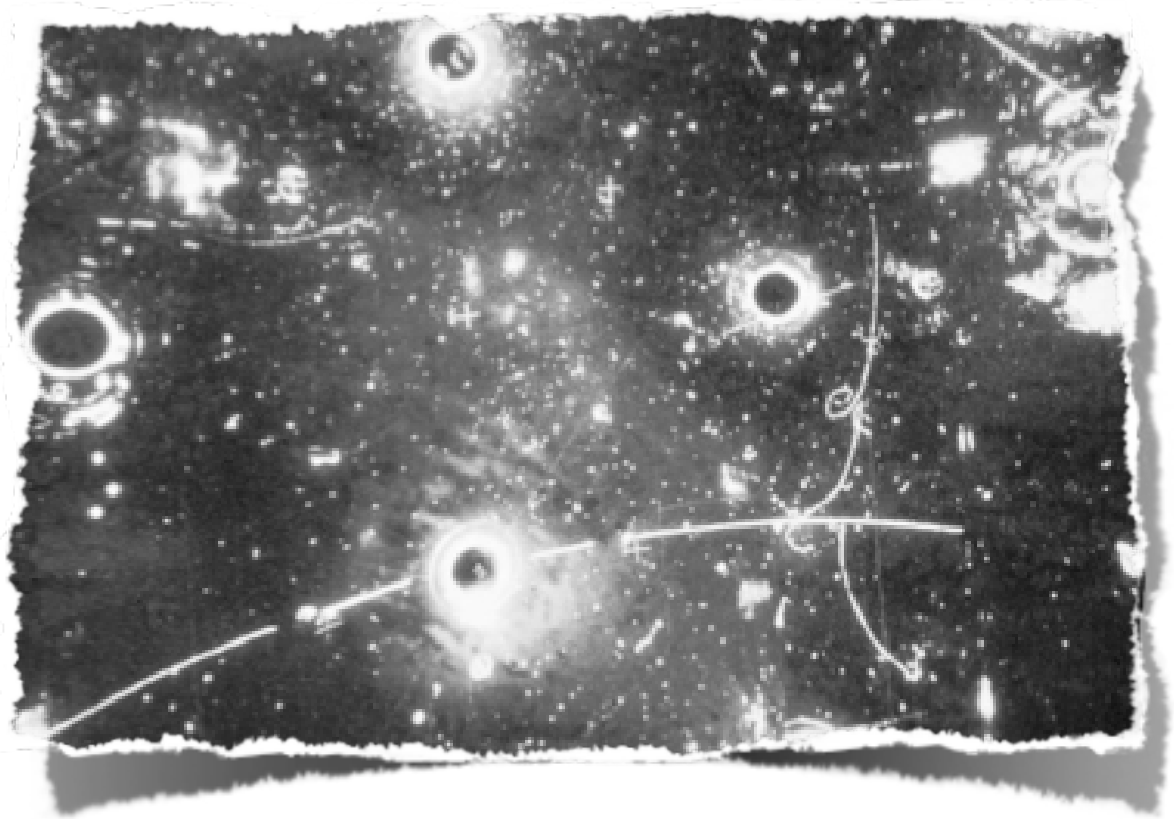
-> CPT

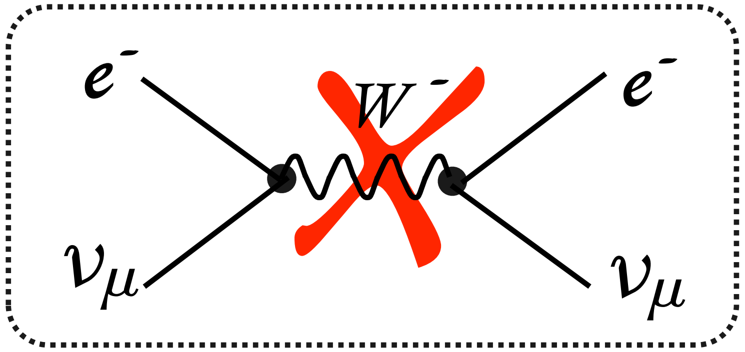
The Standard Model

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$\nu_\mu e^- \to \nu_\mu e^-$





[Gargamelle collaboration, '73]

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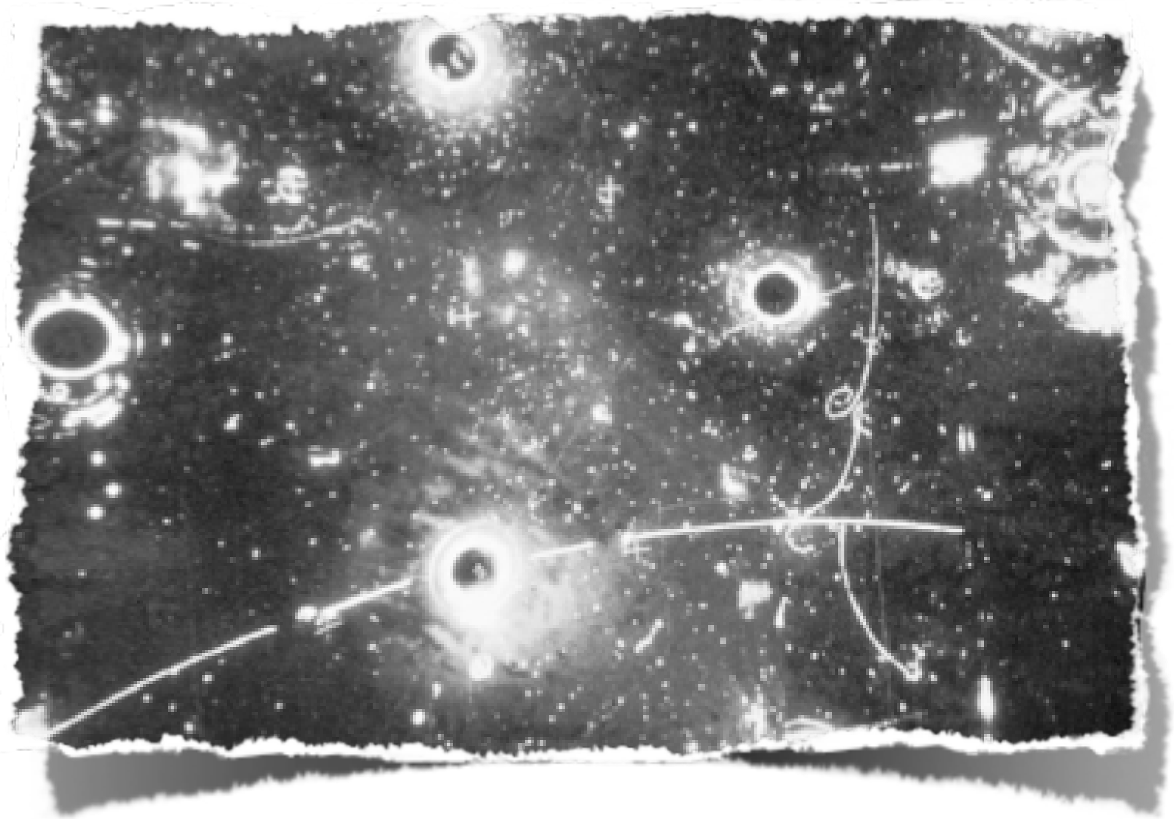

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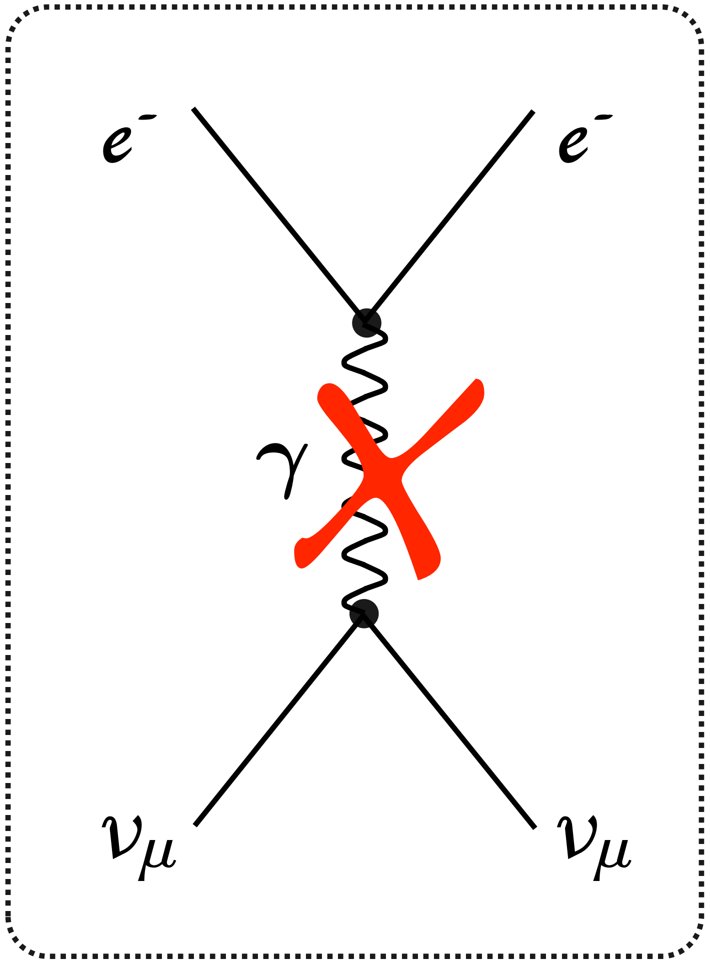

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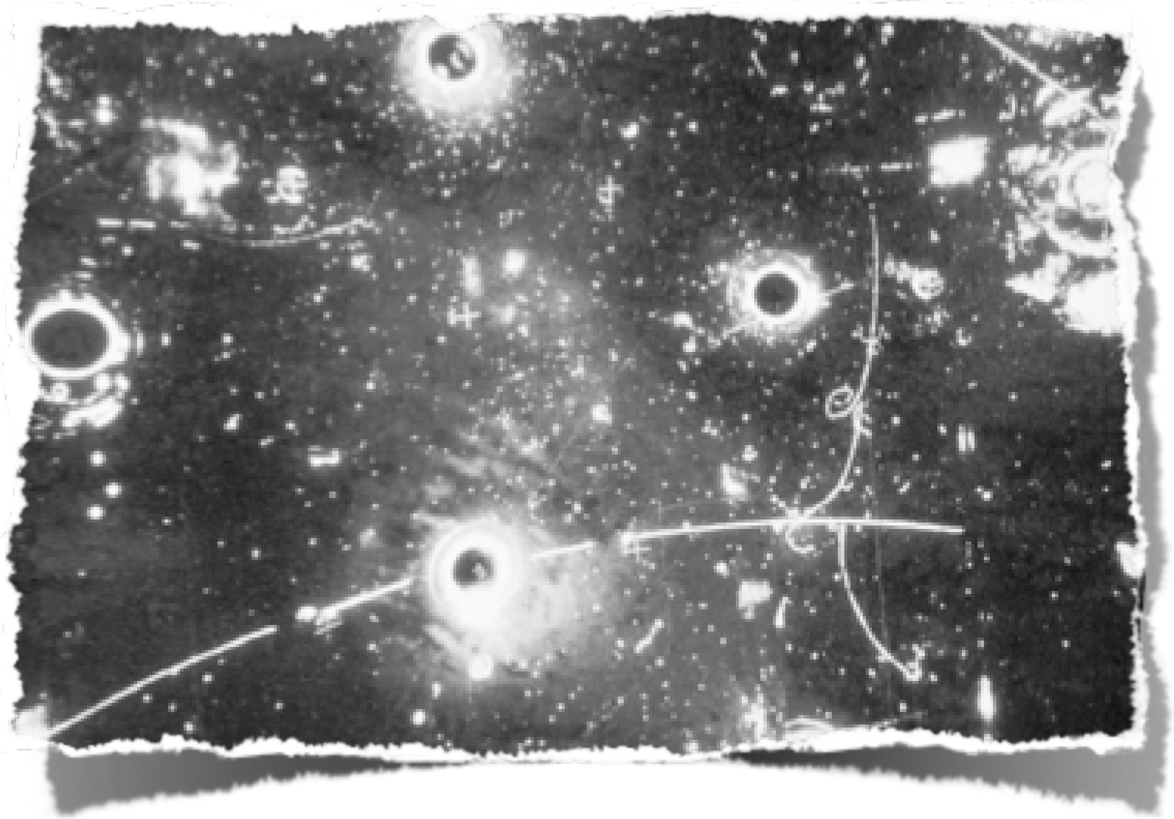

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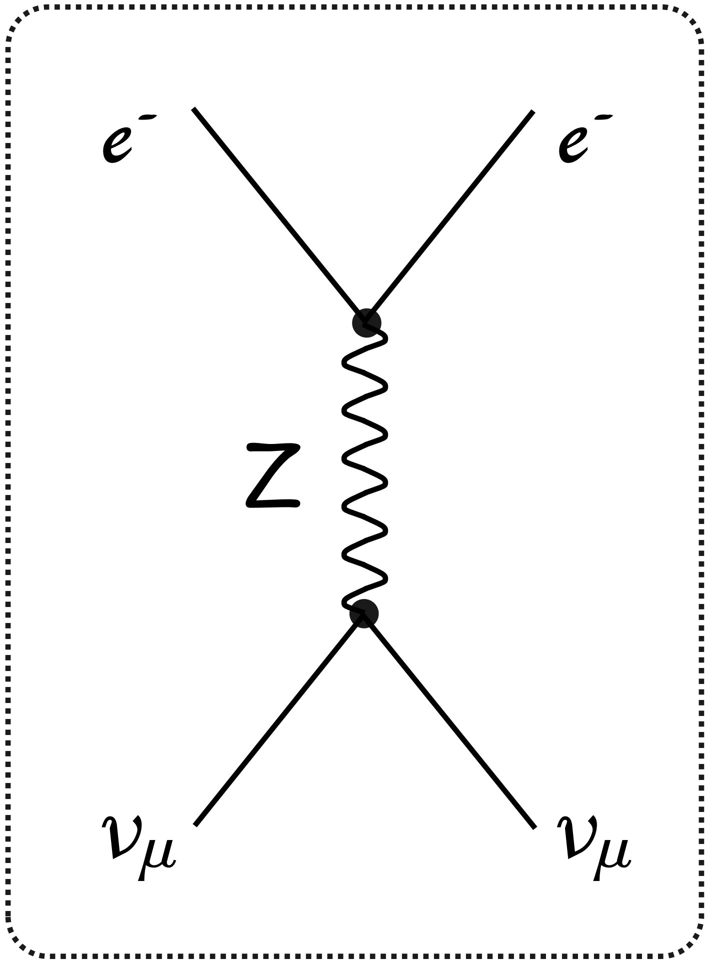
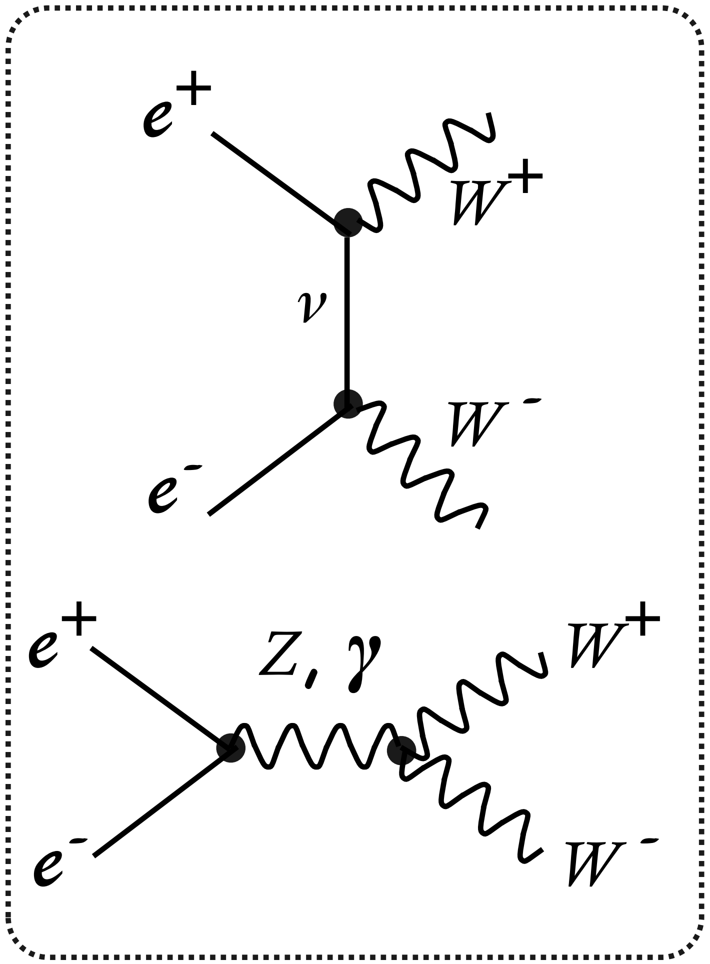


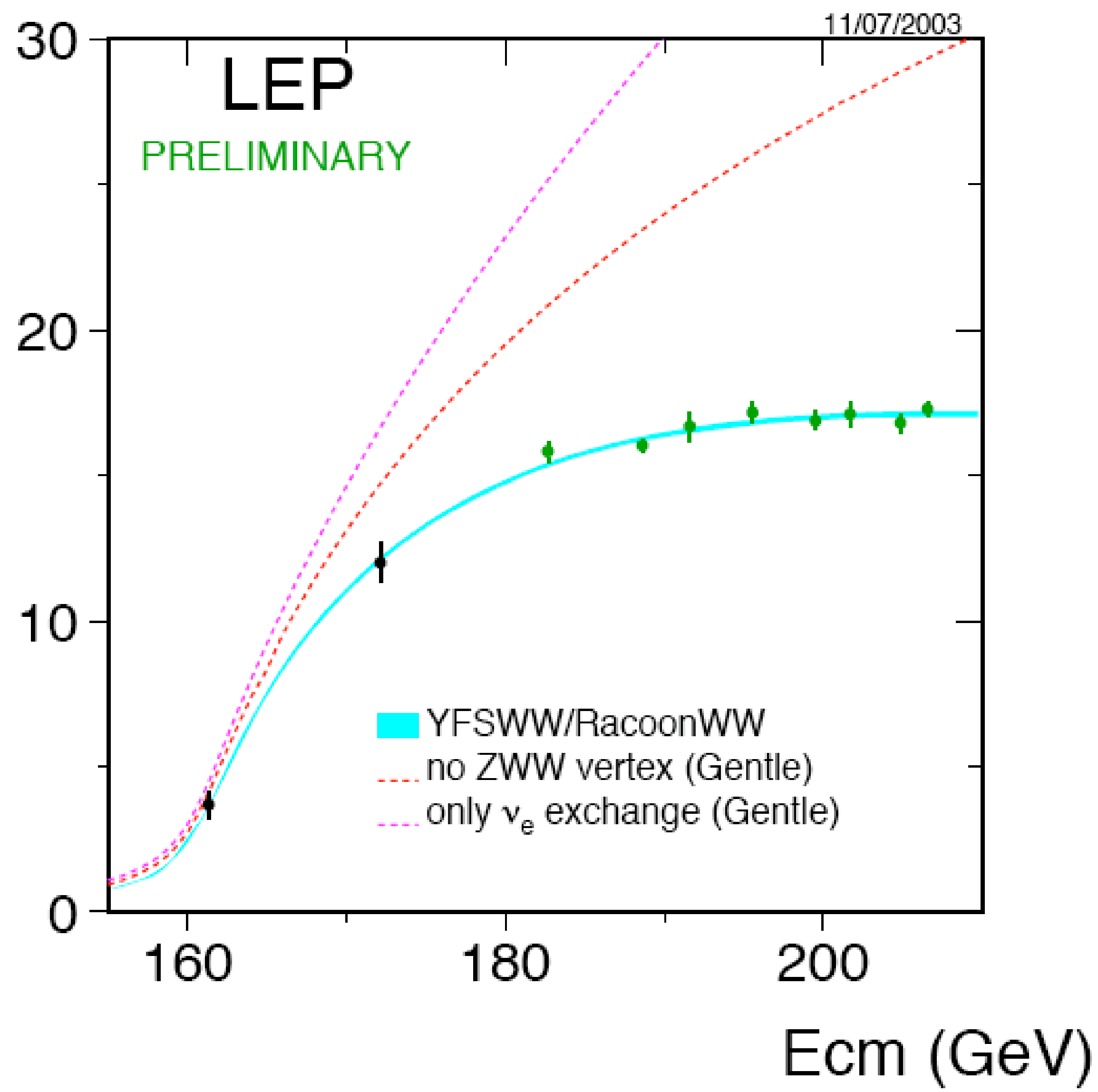
Gauge Theory as a Dynamical Principle

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$e^+e^- \to W^+W^-$





The Standard Model and the Mass Problem

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions
$SU(3)_C \times SU(2)_L \times U(1)_Y$

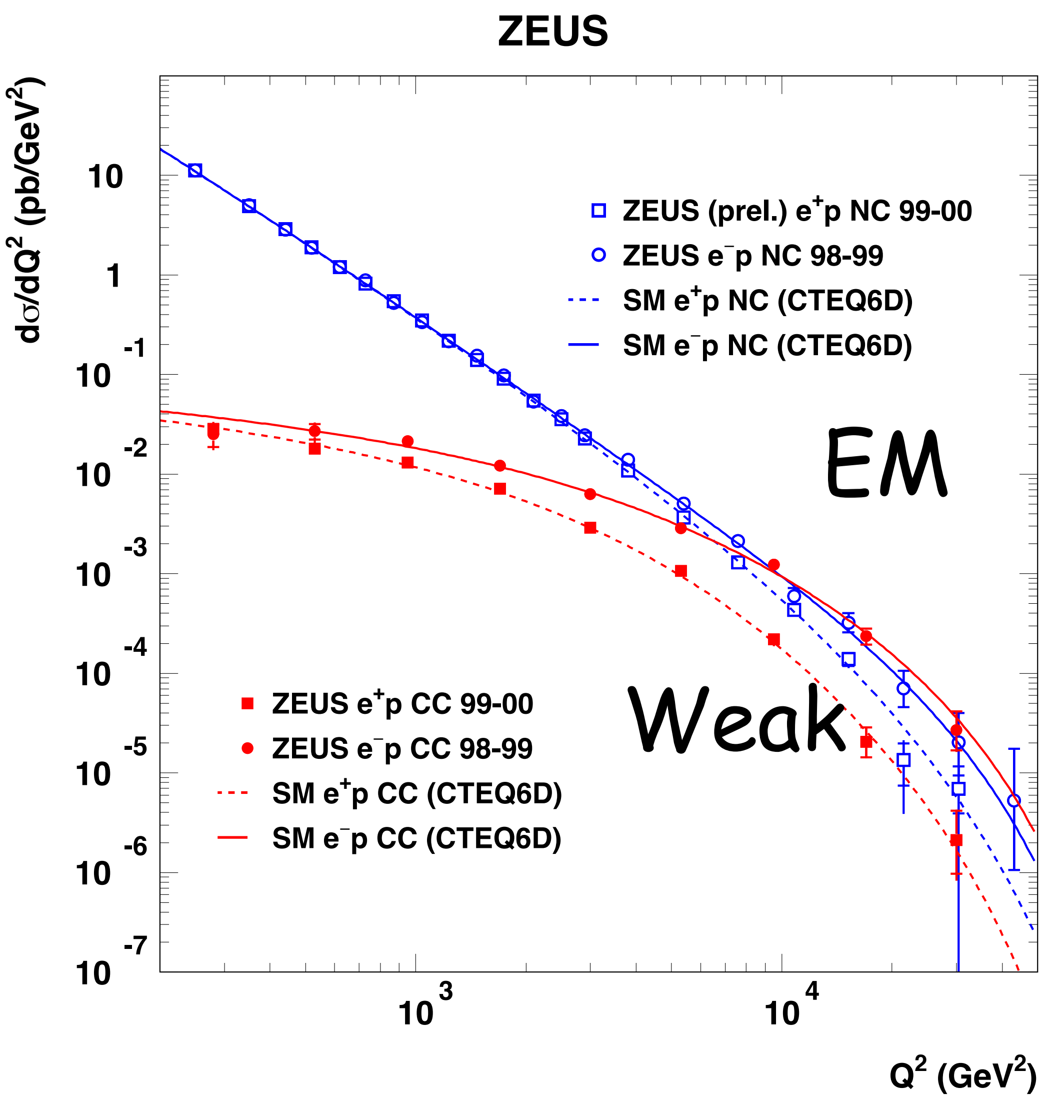
the masses of the quarks, leptons and gauge bosons don't obey the full gauge invariance

- $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ is a doublet of $SU(2)_L$ but $m_{\nu_e} \ll m_e$
- a mass term for the gauge field isn't invariant under gauge transformation $\delta A^a_\mu = \partial_\mu \epsilon^a + g f^{abc} A^b_\mu \epsilon^c$

spontaneous breaking of gauge symmetry

Electroweak Unification

High energy (~ 100 GeV)



Low energy

This room is full of photons
but no W/Z

The symmetry between W, Z and γ
is broken at large distances

EM forces ≈ long ranges

Weak forces ≈ short range

$$m_\gamma < 6 \times 10^{-17}\ \text{eV}$$
$$m_{W^\pm} = 80.425 \pm 0.038\ \text{GeV}$$
$$m_{Z^0} = 91.1876 \pm 0.0021\ \text{GeV}$$

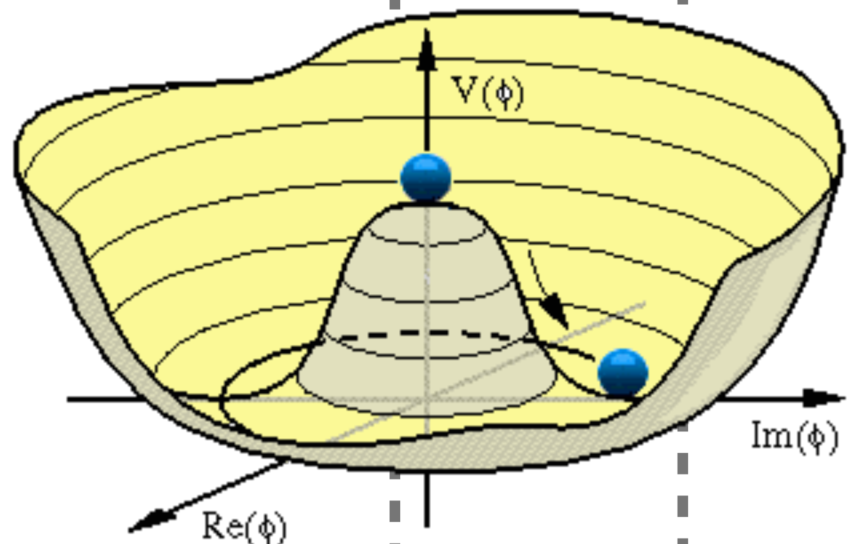
Higgs Mechanism

Symmetry of the Lagrangian

$$SU(2)_L \times U(1)_Y$$

Higgs Doublet

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$



Symmetry of the Vacuum

$$U(1)_{e.m.}$$

Vacuum Expectation Value

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \quad \text{with} \quad v \approx 246 \text{ GeV}$$

$$D_\mu H = \partial_\mu H - \frac{i}{2} \begin{pmatrix} g W^3_\mu + g' B_\mu & \sqrt{2} g W^+_\mu \\ \sqrt{2} g W^-_\mu & -g W^3_\mu + g' B_\mu \end{pmatrix} H \quad \text{with} \quad \mathrm{W}^\pm_\mu = \frac{1}{\sqrt{2}} (\mathrm{W}^1_\mu \mp \mathrm{W}^2_\mu)$$

$$|D_\mu H|^2 = \frac{1}{4} g^2 v^2 W^+_\mu W^{-\,\mu} + \frac{1}{8} (W^3_\mu \; B_\mu) \begin{pmatrix} g^2 v^2 & -g g' v^2 \\ -g g' v^2 & g'^2 v^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

- Gauge boson spectrum
  - electrically charged bosons $M_W^2 = \frac{1}{4} g^2 v^2$
  - electrically neutral bosons $Z_\mu = c W^3_\mu - s B_\mu$, $\gamma_\mu = s W^3_\mu + c B_\mu$

Weak mixing angle

$$c = \frac{g}{\sqrt{g^2 + g'^2}} \qquad s = \frac{g'}{\sqrt{g^2 + g'^2}}$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2) v^2$$

$$M_\gamma = 0$$

Interactions Fermions-Gauge Bosons

Gauge invariance says:

$$\mathcal{L} = gW^3_\mu \left(\sum_i T_{3L\,i}\ \bar\psi_i \bar\sigma^\mu \psi_i \right) + g' B_\mu \left(\sum_i y_i\ \bar\psi_i \bar\sigma^\mu \psi_i \right)$$

Going to the mass eigenstate basis:

$$Z_\mu = cW^3_\mu - sB_\mu \qquad\qquad c = \frac{g}{\sqrt{g^2+g'^2}}$$

with

$$\gamma_\mu = sW^3_\mu + cB_\mu \qquad\qquad s = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$Q = T_{3L} + Y$$

$$\mathcal{L} = \sqrt{g^2+g'^2} Z_\mu \left(\sum_i (T_{3L\,i} - s^2 Q_i)\ \bar\psi_i \bar\sigma^\mu \psi_i \right) + \frac{gg'}{\sqrt{g^2+g'^2}} \gamma_\mu \left(\sum_i Q_i\ \bar\psi_i \bar\sigma^\mu \psi_i \right)$$

not protected by gauge invariance
corrected by radiative corrections + new physics

protected by $U(1)_{em}$ gauge invariance
⇒ no correction

electric charge

$$e = \frac{gg'}{\sqrt{g^2+g'^2}} = sg = cg'$$

Custodial Symmetry

- Rho parameter

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_w} = \frac{\frac{1}{4}g^2v^2}{\frac{1}{4}(g^2+g'^2)v^2\frac{g^2}{g^2+g'^2}} = 1$$

- Consequence of an approximate global symmetry of the Higgs sector

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$ Higgs doublet = 4 real scalar fields

$$V(H) = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2$$ is invariant under the rotation of the four real components

$$SO(4) \sim SU(2)_L \times SU(2)_R$$

$$SU(2)_R$$

$$SU(2)_L \Rightarrow \begin{pmatrix} i\sigma^2 H^\star & H \end{pmatrix} = \Phi$$

2x2 matrix

$$\Phi^\dagger\Phi = H^\dagger H \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$V(H) = \frac{\lambda}{4}\left(\mathrm{tr}\Phi^\dagger\Phi - \mathrm{v}^2\right)^2$$

explicitly invariant under $SU(2)_L \times SU(2)_R$

Custodial Symmetry

Higgs vev

$$\langle H \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \qquad \langle \Phi \rangle = \frac{v}{\sqrt{2}} \begin{pmatrix} 1 & \\ & 1 \end{pmatrix}$$

$$SU(2)_L \times SU(2)_R \to SU(2)_V$$

unbroken symmetry in the broken phase

$$(W_\mu^1, W_\mu^2, W_\mu^3) \quad \text{transforms as a triplet}$$

$$(Z_\mu \, \gamma_\mu) \begin{pmatrix} M_Z^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ \gamma^\mu \end{pmatrix} = (W_\mu^3 \, B_\mu) \begin{pmatrix} c^2 M_Z^2 & -csM_Z^2 \\ -csM_Z^2 & s^2 M_Z^2 \end{pmatrix} \begin{pmatrix} W^{3\,\mu} \\ B^\mu \end{pmatrix}$$

The $SU(2)_V$ symmetry imposes the same mass term for all W^i thus $c^2 M_Z^2 = M_W^2$

$\rho = 1$

The hypercharge gauge coupling and the Yukawa couplings break the custodial SU(2)$_V$, which will generate a (small) deviation to $\rho = 1$ at the quantum level.

Fermion Masses

SM is a chiral theory (≠ QED that is vector-like)

$m_e \bar{e}_L e_R + h.c.$ is not gauge invariant

The SM Lagrangian doesn't not contain fermion mass terms
fermion masses are emergent quantities
that originate from interactions with Higgs vev

$$y_{ij} \bar{f}_{L_i} H f_{R_j} = \frac{y_{ij} v}{\sqrt{2}} \bar{f}_{L_i} f_{R_j} + \frac{y_{ij}}{\sqrt{2}} h \bar{f}_{L_i} f_{R_j}$$

Fermion Masses

In SM, the Yukawa interactions are the only source of the fermion masses

$$y_{ij}\bar{f}_{L_i}Hf_{R_j} = \underbrace{\frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i}f_{R_j}}_{\text{mass}} + \underbrace{\frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}}_{\text{higgs-fermion interactions}}$$

both matrices are simultaneously diagonalizable

⇩ ⇩ ⇩

no tree-level Flavor Changing Current induced by the Higgs

Not true anymore if the SM fermions mix with vector-like partners(*) or for non-SM Yukawa

$$y_{ij}\left(1+c_{ij}\frac{|H|^2}{f^2}\right)\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\left(1+c_{ij}\frac{v^2}{2f^2}\right)\bar{f}_{L_i}f_{R_j} + \left(1+3c_{ij}\frac{v^2}{2f^2}\right)\frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

Look for SM forbidden Flavor Violating decays $h \to \mu\tau$ and $h \to e\tau$

(look also at $t \to hc$ ATLAS '14)

- weak indirect constrained by flavor data ($\mu \to e\gamma$): BR<10%
- ATLAS and CMS have the sensitivity to set bounds O(1%)
- ILC/CLIC/FCC-ee can certainly do much better

Blankenburg, Ellis, Isidori '12

Harnik et al '12
Davidson, Verdier '12
CMS-PAS-HIG-2014-005

(*) e.g. Buras, Grojean, Pokorski, Ziegler '11

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$$y_{ij}\bar{f}_{L_i}Hf_{R_j} = \frac{y_{ij}v}{\sqrt{2}}\bar{f}_{L_i}f_{R_j} + \frac{y_{ij}}{\sqrt{2}}h\bar{f}_{L_i}f_{R_j}$$

mass higgs-fermion interactions

both matrices are simultaneously diagonalizable

no tree-level Flavor Changing Current induced by the Higgs

Quark mixings

$$\mathcal{L}_{Yuk} \quad = \quad \lambda^L_{ij}(\bar{L}^i_L\phi^c)l^j_R + \lambda^U_{ij}(\bar{Q}^i_{L,\alpha}\phi)u^j_{R,\alpha} + \lambda^D_{ij}(\bar{Q}^i_{L,\alpha}\phi^c)d^j_{R,\alpha} + cc$$

$$\mathcal{L}_L^\dagger\left(\frac{v}{\sqrt{2}}\lambda^L\right)\mathcal{L}_R = \begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}$$

$$\mathcal{U}_L^\dagger\left(\frac{-v}{\sqrt{2}}\lambda^U\right)\mathcal{U}_R = \begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}$$

$$\mathcal{D}_L^\dagger\left(\frac{v}{\sqrt{2}}\lambda^D\right)\mathcal{D}_R = \begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}$$

$$\begin{aligned}\mathcal{L}_{Yuk\,quad} = & - \left(\bar{e}_L, \bar{\mu}_L, \bar{\tau}_L\right)\begin{pmatrix} m_e & & \\ & m_\mu & \\ & & m_\tau \end{pmatrix}\begin{pmatrix} e_R \\ \mu_R \\ \tau_R \end{pmatrix} \\ & - \left(\bar{u}_{L,\alpha}, \bar{c}_{L,\alpha}, \bar{t}_{L,\alpha}\right)\begin{pmatrix} m_u & & \\ & m_c & \\ & & m_t \end{pmatrix}\begin{pmatrix} u_{R,\alpha} \\ c_{R,\alpha} \\ t_{R,\alpha} \end{pmatrix} \\ & - \left(\bar{d}_{L,\alpha}, \bar{s}_{L,\alpha}, \bar{b}_{L,\alpha}\right)\mathcal{V}_{KM}^\dagger\begin{pmatrix} m_d & & \\ & m_s & \\ & & m_b \end{pmatrix}\begin{pmatrix} d_{R,\alpha} \\ s_{R,\alpha} \\ b_{R,\alpha} \end{pmatrix} \\ & + \quad cc\end{aligned}$$

$$\mathcal{V}_{KM} = \mathcal{D}_L^\dagger\mathcal{U}_L$$

Goldstone Theorem

Goldstone's theorem [edit]

Goldstone's theorem examines a generic continuous symmetry which is spontaneously broken; i.e., its currents are conserved, but the ground state is not invariant under the action of the corresponding charges. Then, necessarily, new massless (or light, if the symmetry is not exact) scalar particles appear in the spectrum of possible excitations. There is one scalar particle—called a Nambu–Goldstone boson—for each generator of the symmetry that is broken, i.e., that does not preserve the ground state. The Nambu–Goldstone mode is a long-wavelength fluctuation of the corresponding order parameter.

By virtue of their special properties in coupling to the vacuum of the respective symmetry-broken theory, vanishing momentum ("soft") Goldstone bosons involved in field-theoretic amplitudes make such amplitudes vanish ("Adler zeros").

In theories with gauge symmetry, the Goldstone bosons are "eaten" by the gauge bosons. The latter become massive and their new, longitudinal polarization is provided by the Goldstone boson.

QCD example:

For two light quarks, *u* and *d*, the symmetry of the QCD Lagrangian called *chiral symmetry*, and denoted as $U(2)_L \times U(2)_R$, can be decomposed into

$$SU(2)_L \times SU(2)_R \times U(1)_V \times U(1)_A \, .$$

The quark condensate spontaneously breaks the $SU(2)_L \times SU(2)_R$ down to the diagonal vector subgroup *SU(2)*$_V$, known as isospin. The resulting effective theory of baryon bound states of QCD (which describes protons and neutrons), then, has mass terms for these, disallowed by the original linear realization of the chiral symmetry, but allowed by the nonlinear (spontaneously broken) realization thus achieved as a result of the strong interactions.[4]

The Nambu-Goldstone bosons corresponding to the three broken generators are the three pions, charged and neutral. More precisely, because of small quark masses which make this chiral symmetry only approximate, the pions are **Pseudo-Goldstone bosons** instead, with a nonzero, but still atypically small mass,[5] $m_\pi \approx \sqrt{v\, m_q} \,/\, f_\pi$.

sic!

Goldstone Boson

$U(1)$

$$\phi \to e^{i\alpha}\phi$$

$$\mathcal{L} = \partial_\mu \phi^\dagger \partial^\mu \phi - \lambda \left(\left|\phi^2\right|^2 - \frac{f^2}{2} \right)^2$$

$$\phi = \frac{1}{\sqrt{2}} \left(f + h(x) \right) e^{i\theta(x)/f}$$

$$h \to h$$

$$\theta \to \theta + \alpha f$$

$U(1)$ non-linearly realized

shift symmetry forbids any mass term for θ

$$\mathcal{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\left(\frac{f+h}{f}\right)^2 \partial_\mu \theta \partial^\mu \theta - \lambda \left(f^2 h^2 + f h^3 + \frac{1}{4} h^4 \right)$$

If the U(1) symmetry is gauged, the Goldstone boson is eaten and it becomes the longitudinal component of the massive gauge boson

Example of Uneaten Goldstone Bosons

$SU(N) \to SU(N-1)$ $\quad (N^2-1)-((N-1)^2-1) = 2N-1$ Goldstone bosons

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix} \qquad \phi = \exp\left(\frac{i}{f} \left(\begin{array}{ccc|c} -\pi_0 & & & \pi_1 \\ & \ddots & & \vdots \\ & & -\pi_0 & \pi_{N-1} \\ \hline \pi_1^\star & \dots & \pi_{N-1}^\star & (N-1)\pi_0 \end{array} \right) \right) \begin{pmatrix} 0 \\ \vdots \\ 0 \\ f \end{pmatrix}$$

$$\phi = e^{i\pi} \phi_0$$

(N-1) complex, $\vec{\pi}$, and 1 real, π_0, scalars

Let us assume that only SU(N-1) is gauged: then the Goldstone are uneaten.

$SU(N-1)$

$$\phi \to U_{N-1}\phi = U_{N-1} e^{i\pi} U_{N-1}^\dagger U_{N-1} \phi_0 = e^{i U_{N-1} \pi U_{N-1}^\dagger} \phi_0$$

$$\pi \to \left(\begin{array}{c|c} U_{N-1} & \\ \hline & 1 \end{array} \right) \left(\begin{array}{c|c} \pi_0 & \pi \\ \hline \pi^\dagger & \pi_0 \end{array} \right) \left(\begin{array}{c|c} U_{N-1}^\dagger & \\ \hline & 1 \end{array} \right) = \left(\begin{array}{c|c} \pi_0 & U_{N-1}\pi \\ \hline \pi^\dagger U_{N-1}^\dagger & \pi_0 \end{array} \right)$$

linear transformations

$\dfrac{SU(N)}{SU(N-1)}$

$$\phi \to \exp\left(i \left(\begin{array}{c|c} & \vec{\alpha} \\ \hline \vec{\alpha}^\dagger & \end{array} \right) \right) \exp\left(i \left(\begin{array}{c|c} & \vec{\pi} \\ \hline \vec{\pi}^\dagger & \end{array} \right) \right) \phi_0 \approx \exp\left(i \left(\begin{array}{c|c} & \vec{\pi}+\vec{\alpha} \\ \hline \vec{\pi}^\dagger + \vec{\alpha}^\dagger & \end{array} \right) \right) \phi_0$$

non-linear transformations

Counting the degrees of freedom

$$SU(2)_L \times U(1)_Y \to U(1)_{\mathrm{EM}}$$

3 broken gauge directions = 3 eaten Goldstone bosons

$$H = \begin{pmatrix} h^+ \\ h^0 \end{pmatrix}$$

Higgs doublet = 4 real scalar fields

3 eaten Goldstone bosons

One physical degree of freedom the Higgs boson

$$H = e^{i\pi^a T^a} \begin{pmatrix} 0 \\ \frac{v+h}{\sqrt{2}} \end{pmatrix}$$

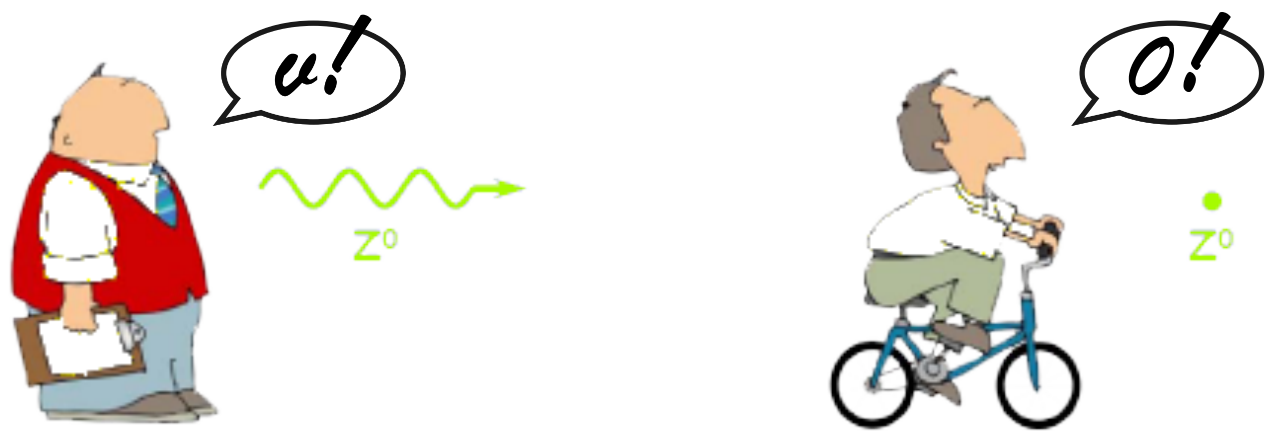
In the unitary gauge, the π^a are non-physical.

$$V(H) = \lambda \left(H^\dagger H - \frac{v^2}{2} \right)^2 = \underbrace{\frac{m_h^2}{2} h^2}_{\text{mass term}} + \underbrace{\frac{m_h^2}{2v} h^3 + \frac{m_h^2}{8v^2} h^4}_{\text{self-couplings}}$$

with $m_h^2 = 2\lambda v^2$

The longitudinal polarization of massive W, Z

a massless particle is never at rest: always possible to distinguish (and eliminate!) the longitudinal polarization



the longitudinal polarization is physical for a massive spin-1 particle

(pictures: courtesy of G. Giudice)

symmetry breaking: new phase with more degrees of freedom

$$\epsilon_{\|} = \left(\frac{|\vec{p}|}{M}, \frac{E}{M} \frac{\vec{p}}{|\vec{p}|} \right)$$ polarization vector grows with the energy

The longitudinal polarization of massive W, Z

a massless particle is never at rest: cl o distinguish
(and elimin n

3=2+1

Guralnik et al '64

the longitudinal polarization is physical for a massive spin-1 particle

(pictures: courtesy of G. Giudice)

symmetry breaking: new phase with more degrees of freedom

$$\epsilon_{\|} = \left(\frac{|\vec{p}|}{M}, \frac{E}{M} \frac{\vec{p}}{|\vec{p}|} \right)$$ polarization vector grows with the energy

Longitudinal polarization of a massive spin 1

a massive spin 1 particle has 3 physical polarizations:

$$A_\mu = \epsilon_\mu \; e^{ik_\mu x^\mu}$$

$$\epsilon^\mu \epsilon_\mu = -1 \qquad k^\mu \epsilon_\mu = 0$$

$$k^\mu = (E, 0, 0, k)$$

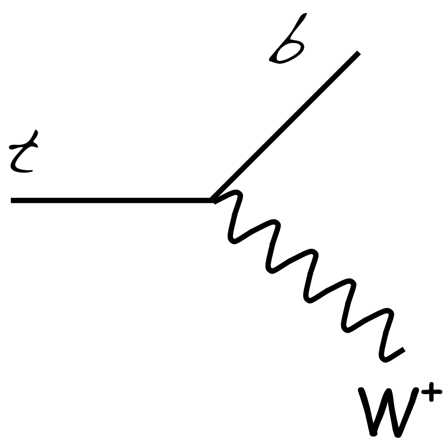
with $k_\mu k^\mu = E^2 - k^2 = M^2$

- 2 transverse: $\begin{cases} \epsilon_1^\mu = (0,1,0,0) \\ \epsilon_2^\mu = (0,0,1,0) \end{cases}$
- 1 longitudinal: $\epsilon_{\|}^\mu = (\frac{k}{M}, 0, 0, \frac{E}{M}) \approx \frac{k^\mu}{M} + \mathcal{O}(\frac{E}{M})$

(in the R-ξ gauge, the time-like polarization ($\epsilon^\mu \epsilon_\mu = 1 \quad k^\mu \epsilon_\mu = M$) is arbitrarily massive and decouple)

The BEH mechanism: "V_L=Goldstone bosons"

At high energy, the physics of the gauge bosons becomes simple



$$\Gamma(t \to bW_L) = \frac{g^2}{64\pi}\frac{m_t^2}{m_W^2}\frac{(m_t^2 - m_W^2)^2}{m_t^3}$$

$$\Gamma(t \to bW_T) = \frac{g^2}{64\pi}\frac{2(m_t^2 - m_W^2)^2}{m_t^3}$$

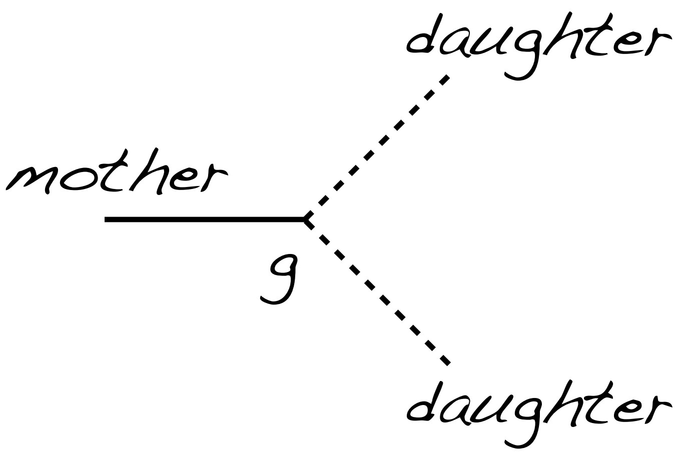
- at threshold ($m_t \sim m_W$) democratic decay
- at high energy ($m_t \gg m_W$) W_L dominates the decay

At high energy, the dominant degrees of freedom are W_L

The BEH mechanism: "V_L=Goldstone bosons"

At high energy, the physics of the gauge bosons becomes simple

~~ why you should be stunned by this result: ~~



we expect: (dimensional analysis) $\Gamma \sim g^2\, m_{\text{mother}}$

instead $\Gamma \propto m^3_{\text{mother}}$ means $g \propto m$ like the Higgs couplings!

very efficient way to suck up energy from the mother particle

$$\tau \ll \tau_{\text{naive}}$$

Goldstone equivalence theorem
$W^{\pm}_L, Z_L \approx SO(4)/SO(3)$

LEP already established the BEH mechanism
The pending question was: how is it realized?
Via a fundamental EW doublet? A la technicolor?
Is there a Higgs boson in addition to the 3 Goldstone bosons?

In other words, LEP established a simple description of the electroweak sector for E >> m_W.

This description is valid for $m_W \ll E \ll 4\pi v = \dfrac{8\pi m_W}{g}$

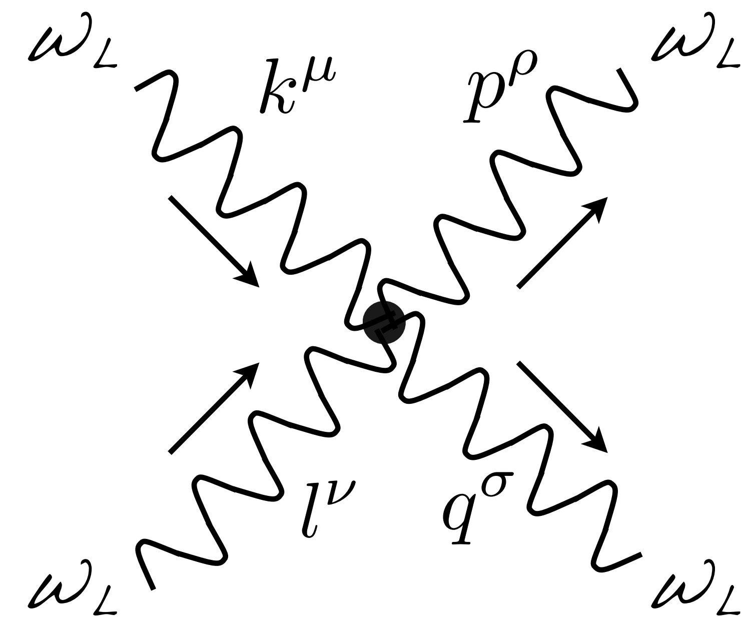
The goal of the LHC was/is to understand what comes next

Call for extra degrees of freedom

NO LOSE THEOREM

Bad high-energy behavior for the scattering of the longitudinal polarizations

$$\mathcal{A} = \epsilon_{\parallel}^{\mu}(k)\epsilon_{\parallel}^{\nu}(l)g^2\left(2\eta_{\mu\rho}\eta_{\nu\sigma} - \eta_{\mu\nu}\eta_{\rho\sigma} - \eta_{\mu\sigma}\eta_{\nu\rho}\right)\epsilon_{\parallel}^{\rho}(p)\epsilon_{\parallel}^{\sigma}(q)$$



$$\boxed{\mathcal{A} = g^2 \frac{E^4}{4M_W^4}}$$

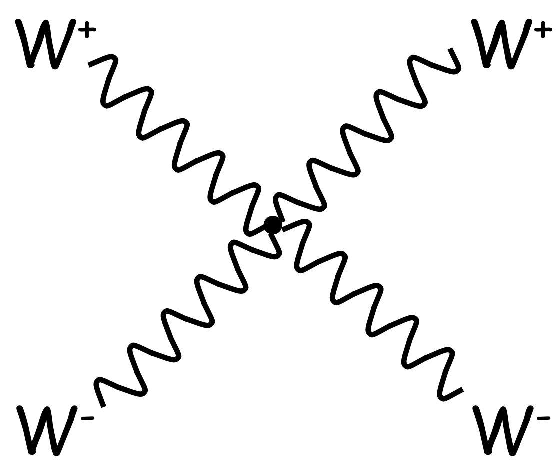
violations of perturbative unitarity around E ~ M/√g (actually M/g)

Extra degrees of freedom are needed to have a good description of the W and Z masses at higher energies

numerically: E ~ 3 TeV ⬅ the LHC was sure to discover something!

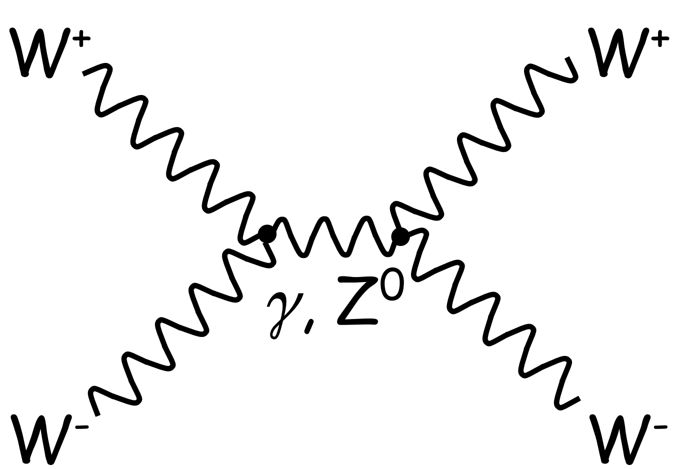
$M_W/\sqrt{(g/4\pi)} \sim 500\,\text{GeV}$ or $M_W/(g/4\pi) \sim 3\,\text{TeV}$?

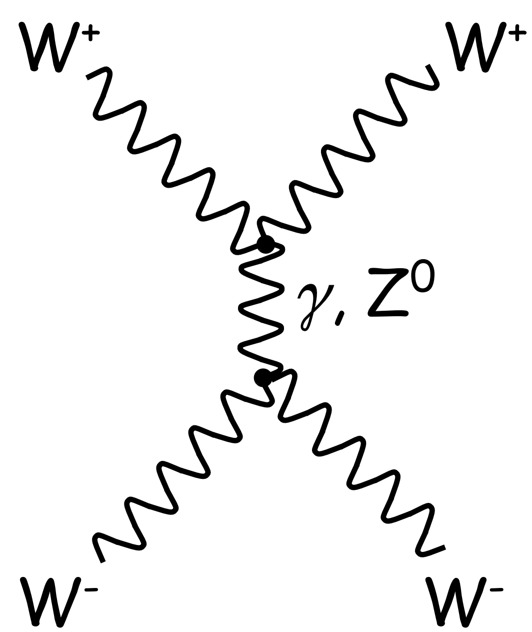
Lewellyn Smith '73
Dicus, Mathur '73
Cornwall, Levin, Tiktopoulos '73



$$\mathcal{A} = g^2 \left(\frac{E}{M_W}\right)^4$$

+





+

$$\mathcal{A} = -g^2 \left(\frac{E}{M_W}\right)^4$$

$$\mathcal{A} = g^2 \left(\frac{E}{M_W}\right)^2$$

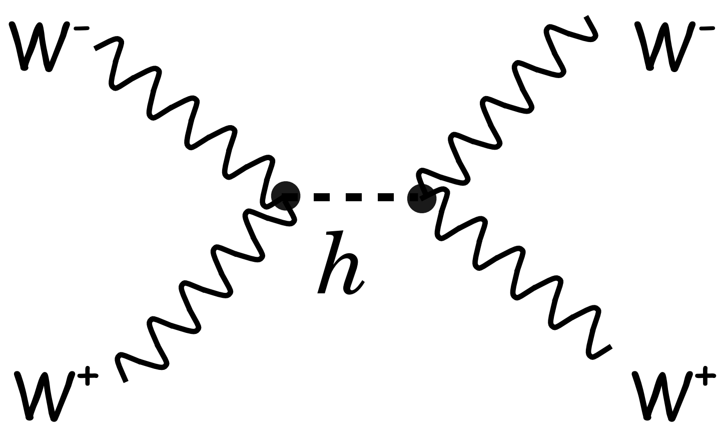
impossible to further cancel the amplitude
without introducing new degrees of freedom

What is the SM Higgs?

A single scalar degree of freedom that couples to the mass of the particles

$$\mathcal{L}_{\rm EWSB} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

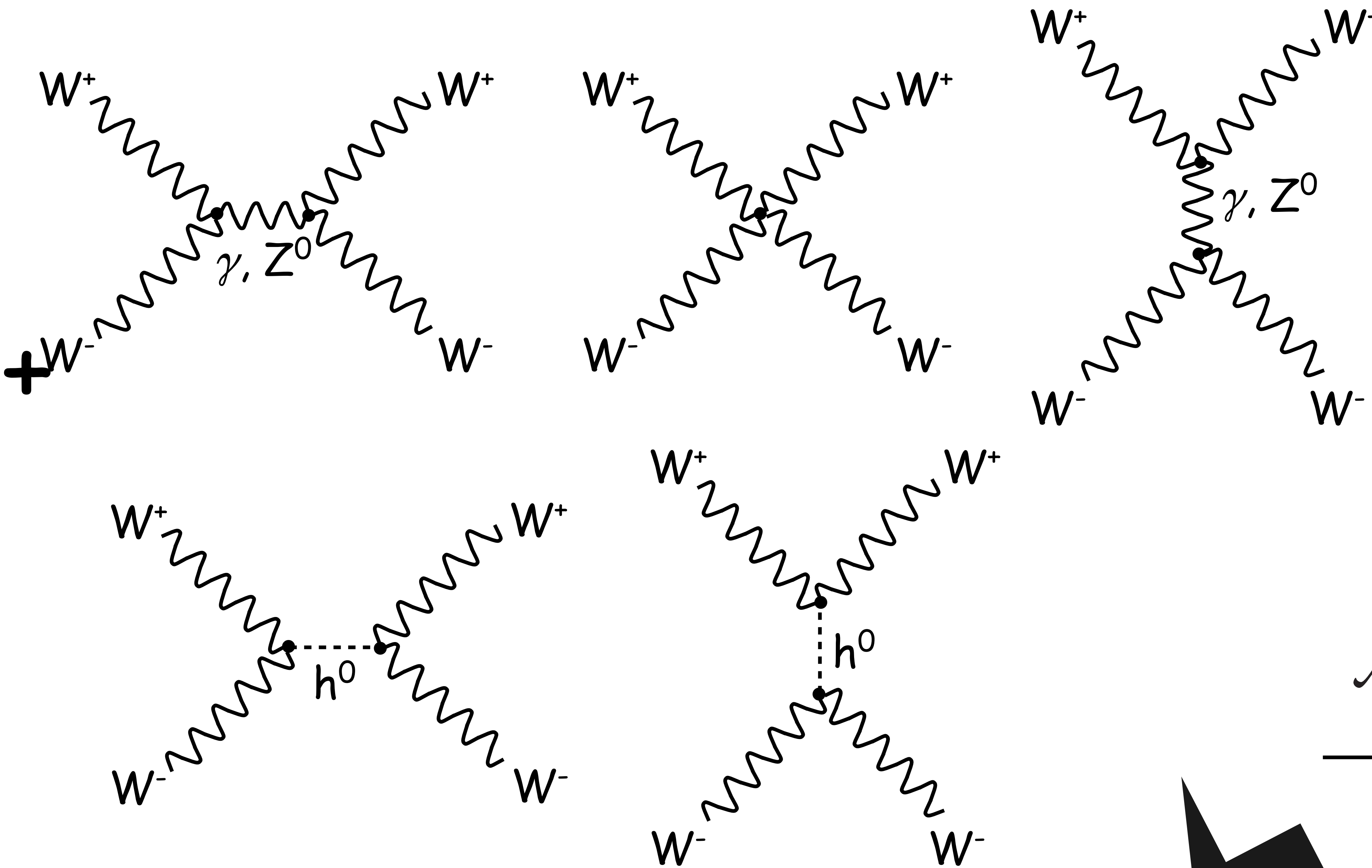
'a', 'b' and 'c' are arbitrary free couplings



$$\mathcal{A} = \frac{1}{v^2}\left(s - \frac{a^2 s^2}{s - m_h^2}\right)$$

growth cancelled for
a = 1
restoration of
perturbative unitarity

What is the SM Higgs?



$$\mathcal{A} = g^2 \left(\frac{E}{M_W}\right)^2$$

+

$$\mathcal{A} = -g^2 \left(\frac{E}{M_W}\right)^2$$

$$\mathcal{A} = g^2 \left(\frac{M_H}{2M_W}\right)^2$$

The Higgs boson unitarizes the W scattering
(if its mass is below ~ 1 TeV)

Lee, Quigg, Thacker '77

What is the Higgs the name of?

A single scalar degree of freedom that couples to the mass of the particles

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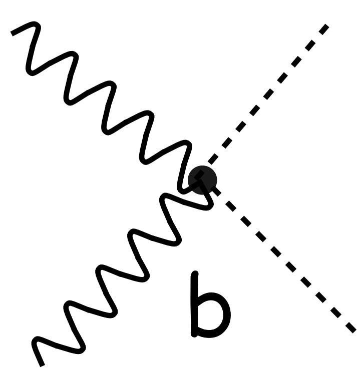
'a', 'b' and 'c' are arbitrary free couplings

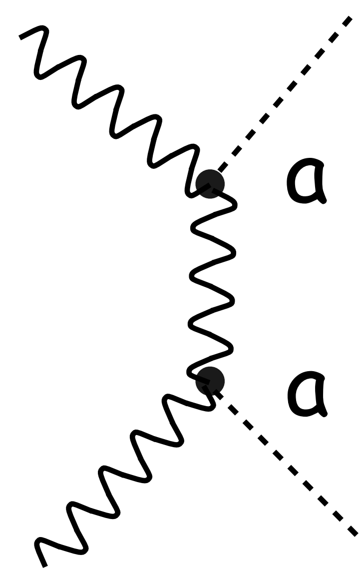
For a=1: perturbative unitarity in elastic channels WW $\to$ WW

For b = a^2: perturbative unitarity in inelastic channels WW $\to$ hh

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10





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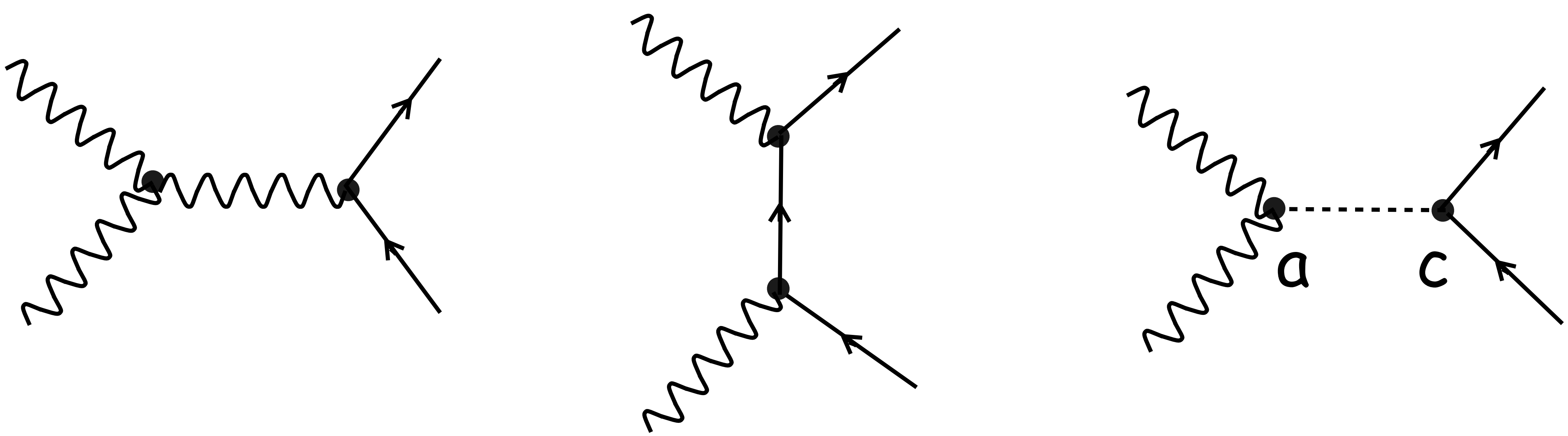
For b = a^2: perturbative unitarity in inelastic channels WW $\to$ hh

For ac=1: perturbative unitarity in inelastic WW $\to$ $\psi\,\psi$

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



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$$\mathcal{L}_{\mathrm{EWSB}} = m_W^2 W_\mu^+ W_\mu^+ \left(1 + 2a\frac{h}{v} + b\frac{h^2}{v^2}\right) - m_\psi \bar{\psi}_L \psi_R \left(1 + c\frac{h}{v}\right)$$

'a', 'b' and 'c' are arbitrary free couplings

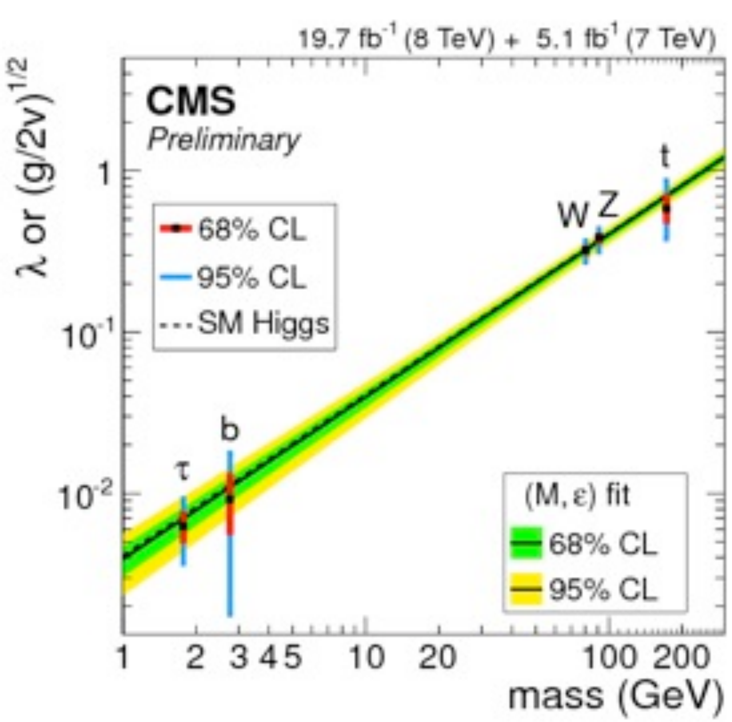
For a=1: perturbative unitarity in elastic channels WW $\to$ WW

For b = a^2: perturbative unitarity in inelastic channels WW $\to$ hh

For ac=1: perturbative unitarity in inelastic WW $\to$ ψ ψ

Cornwall, Levin, Tiktopoulos '73

Contino, Grojean, Moretti, Piccinini, Rattazzi '10



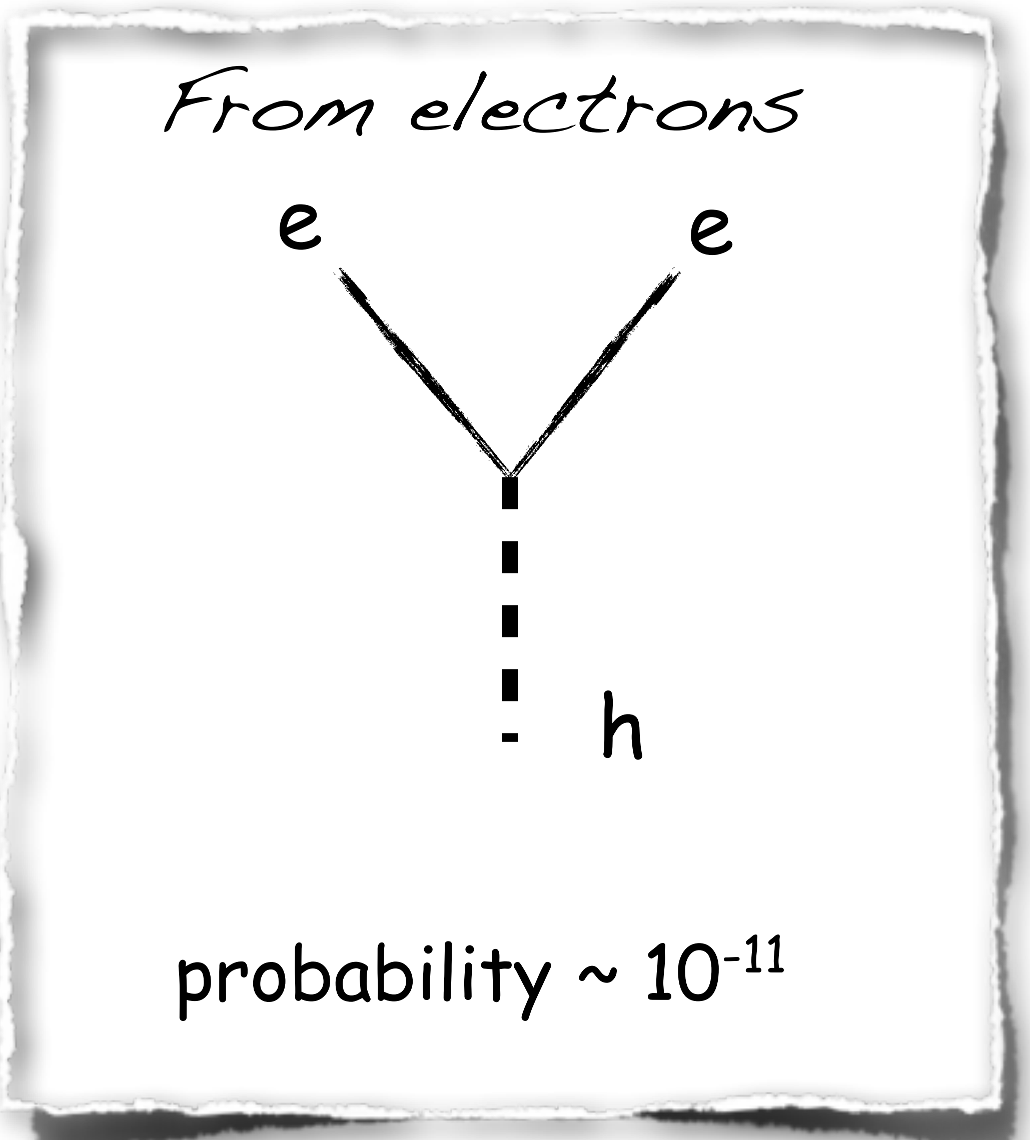
CMS PAS-HIG-14-009

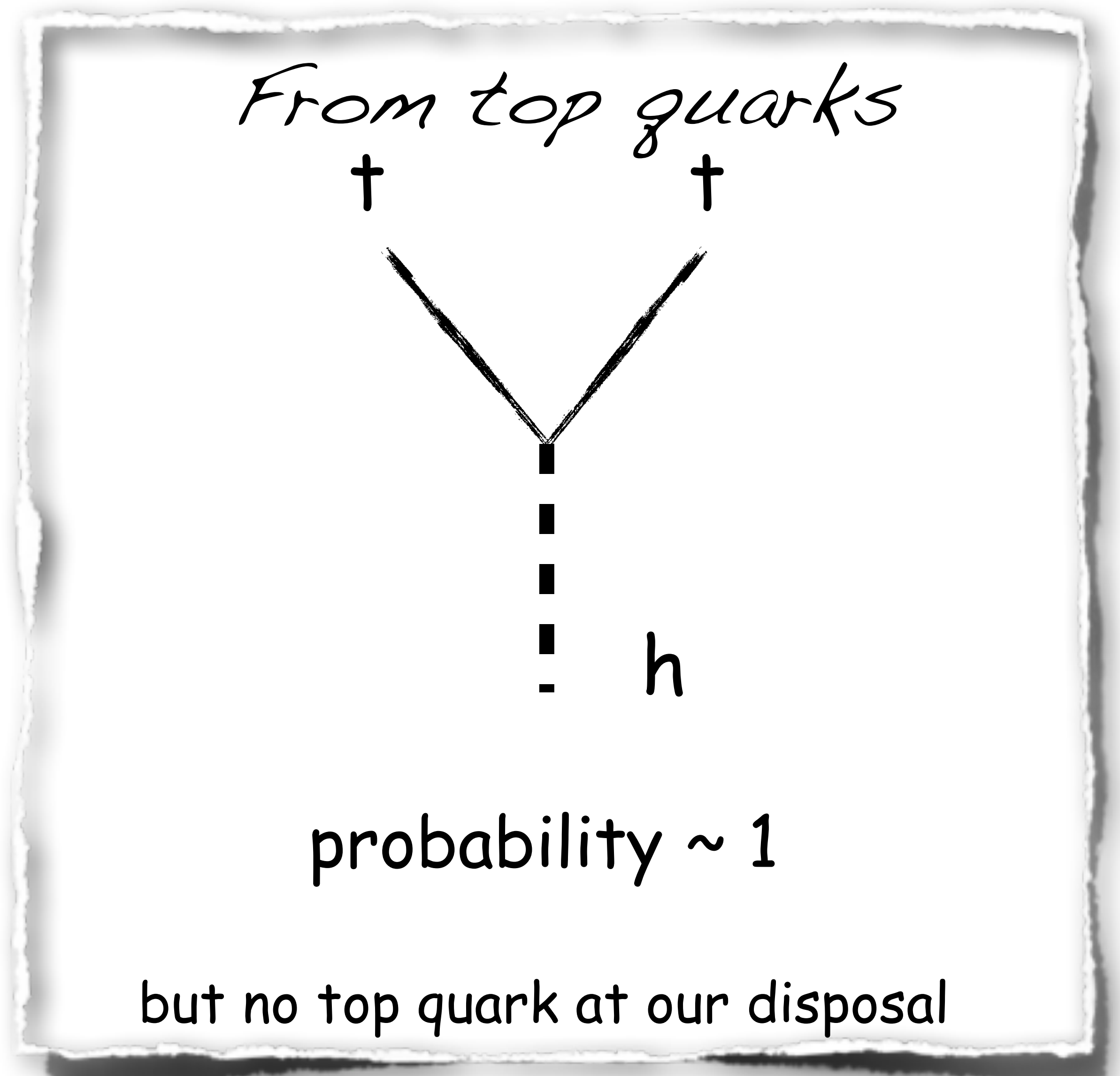
Higgs couplings are proportional to the masses of the particles

$$\lambda_\psi \propto \frac{m_\psi}{v}, \qquad \lambda_V^2 \equiv \frac{g_{VVh}}{2v} \propto \frac{m_V^2}{v^2}$$

Higgs boson at the LHC

producing a Higgs boson is a rare phenomenon
since its interactions with particles are proportional to masses
and ordinary matter is made of light elementary particles
NB: the proton is not an elementary particle,
its mass doesn't measure its interaction with the Higgs substance



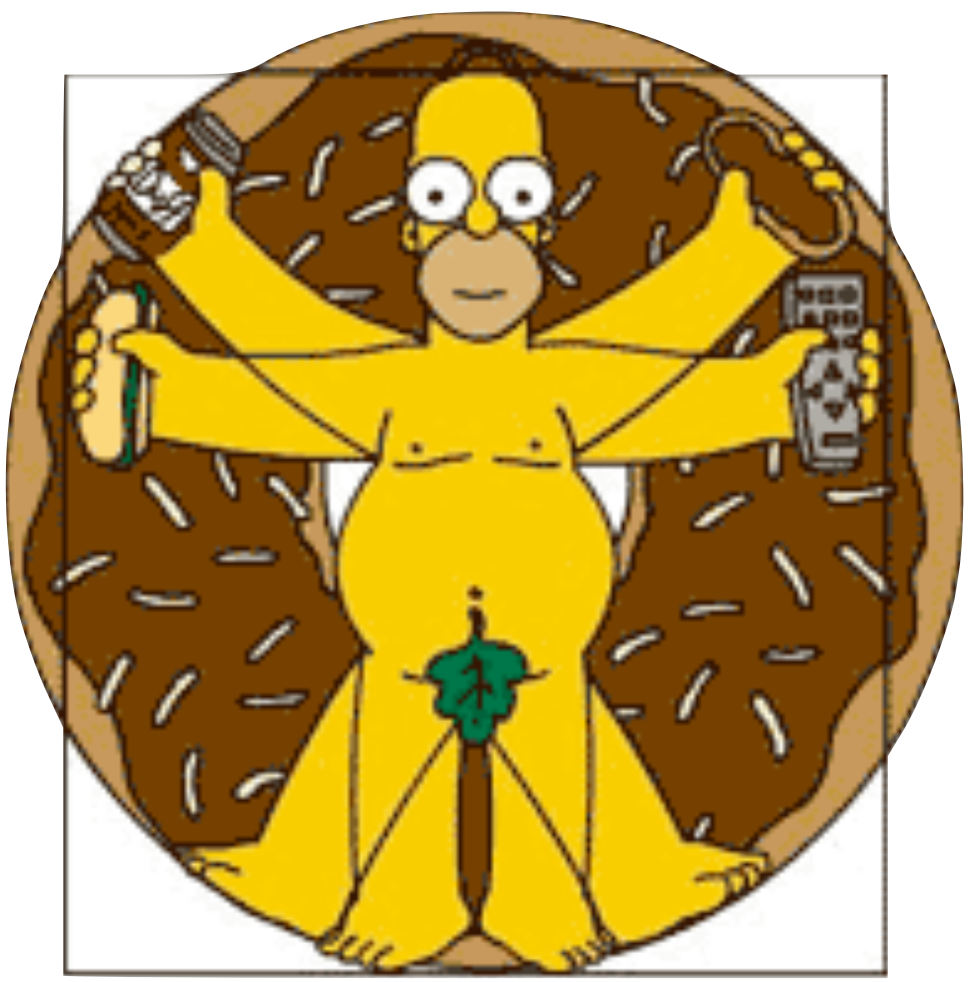


Higgs boson at the LHC

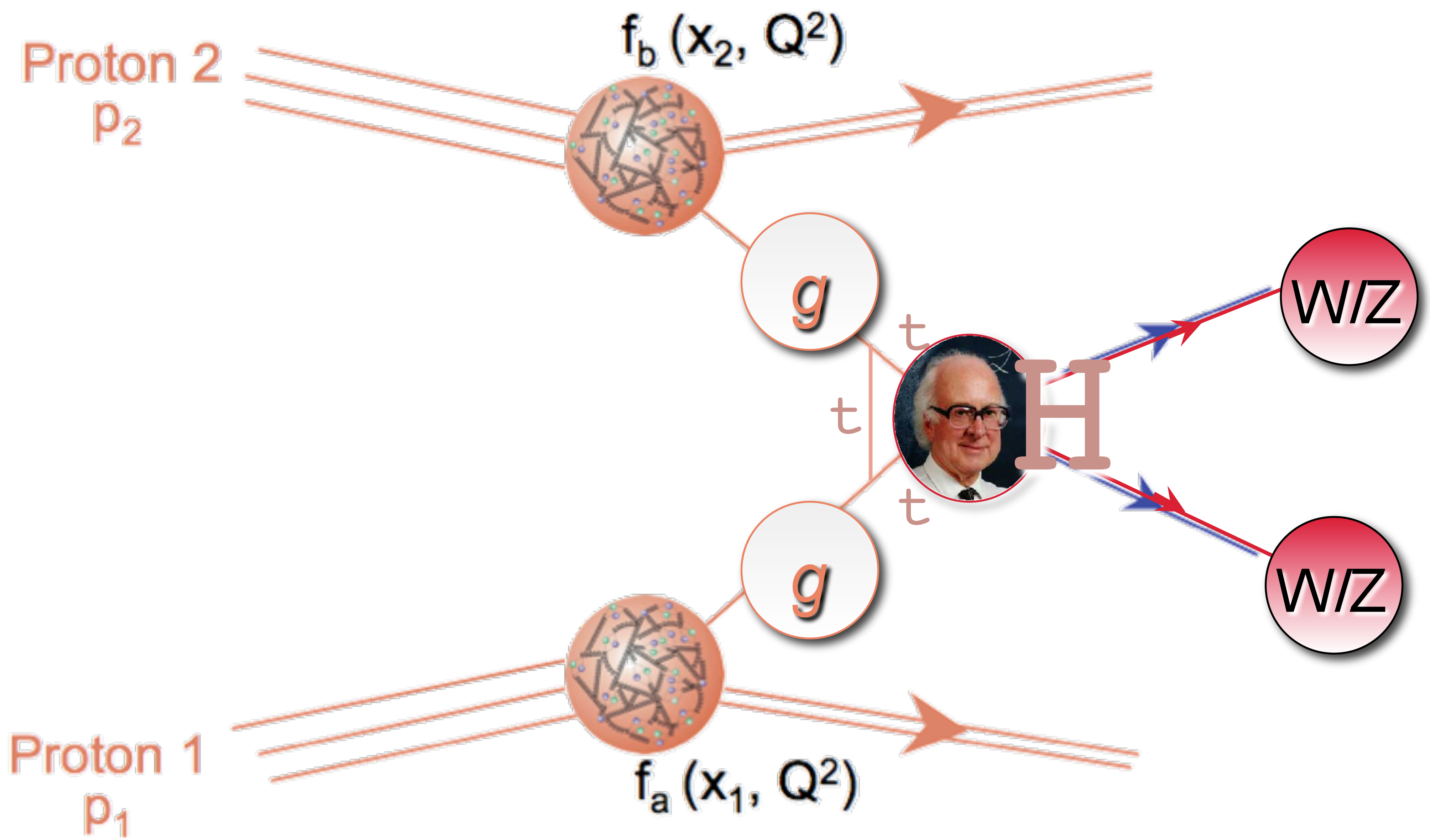
Difficult task

Homer Simpson's principle of life:

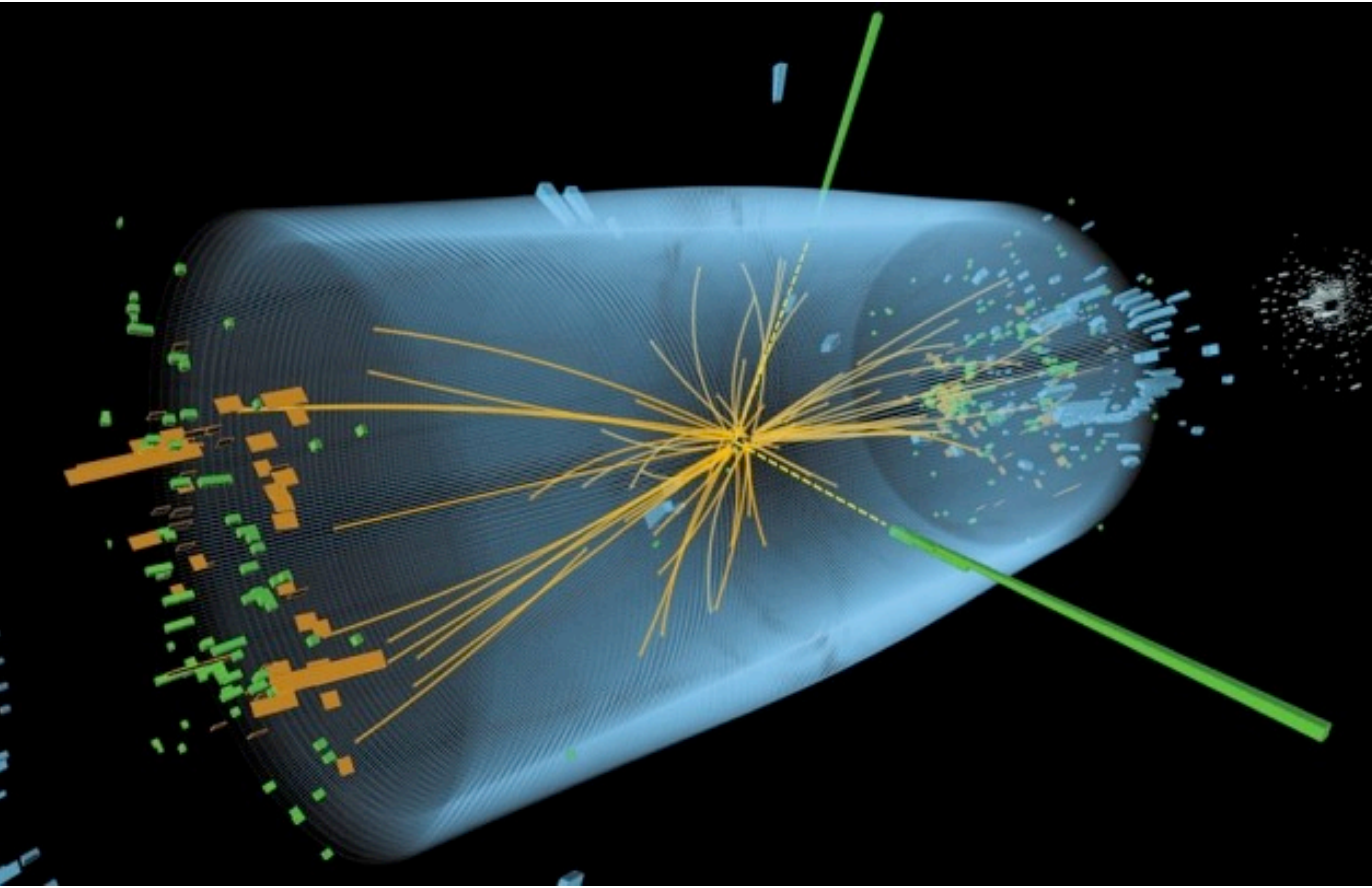
If something's hard to do, is it worth doing?



Higgs boson at the LHC



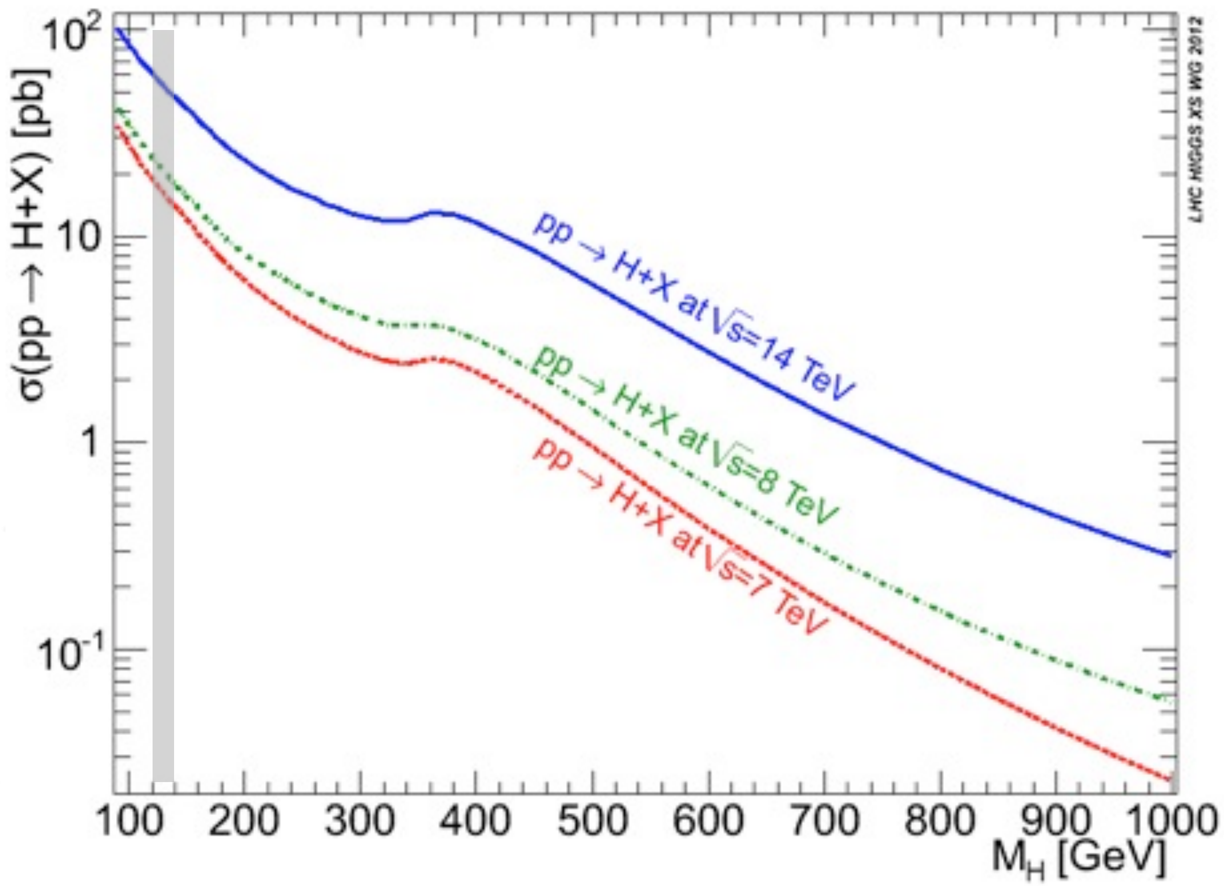
Higgs boson at the LHC



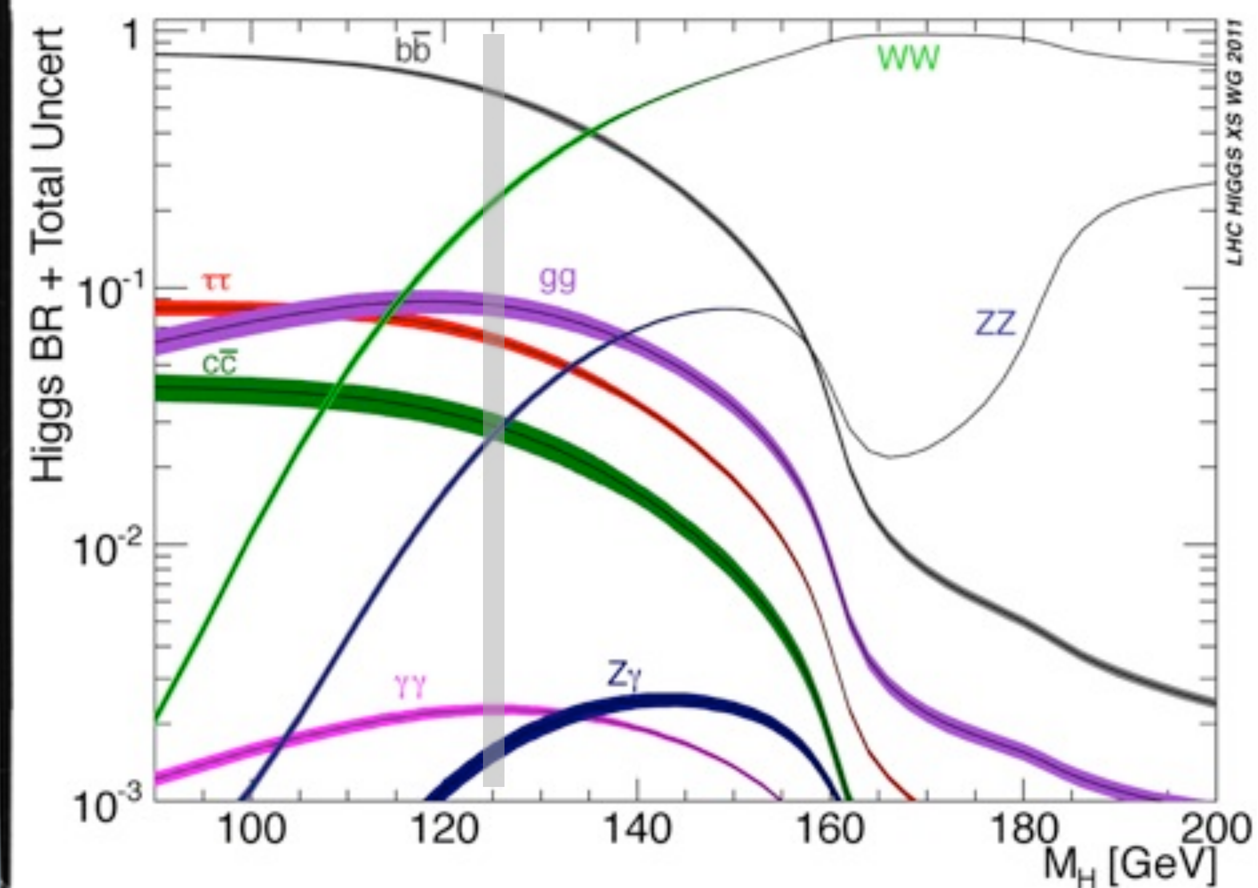
Higgs boson at the LHC

$\sigma \sim 10$ pb $\Leftrightarrow 10^5$ events for L=10 fb^{-1}

Higgs production



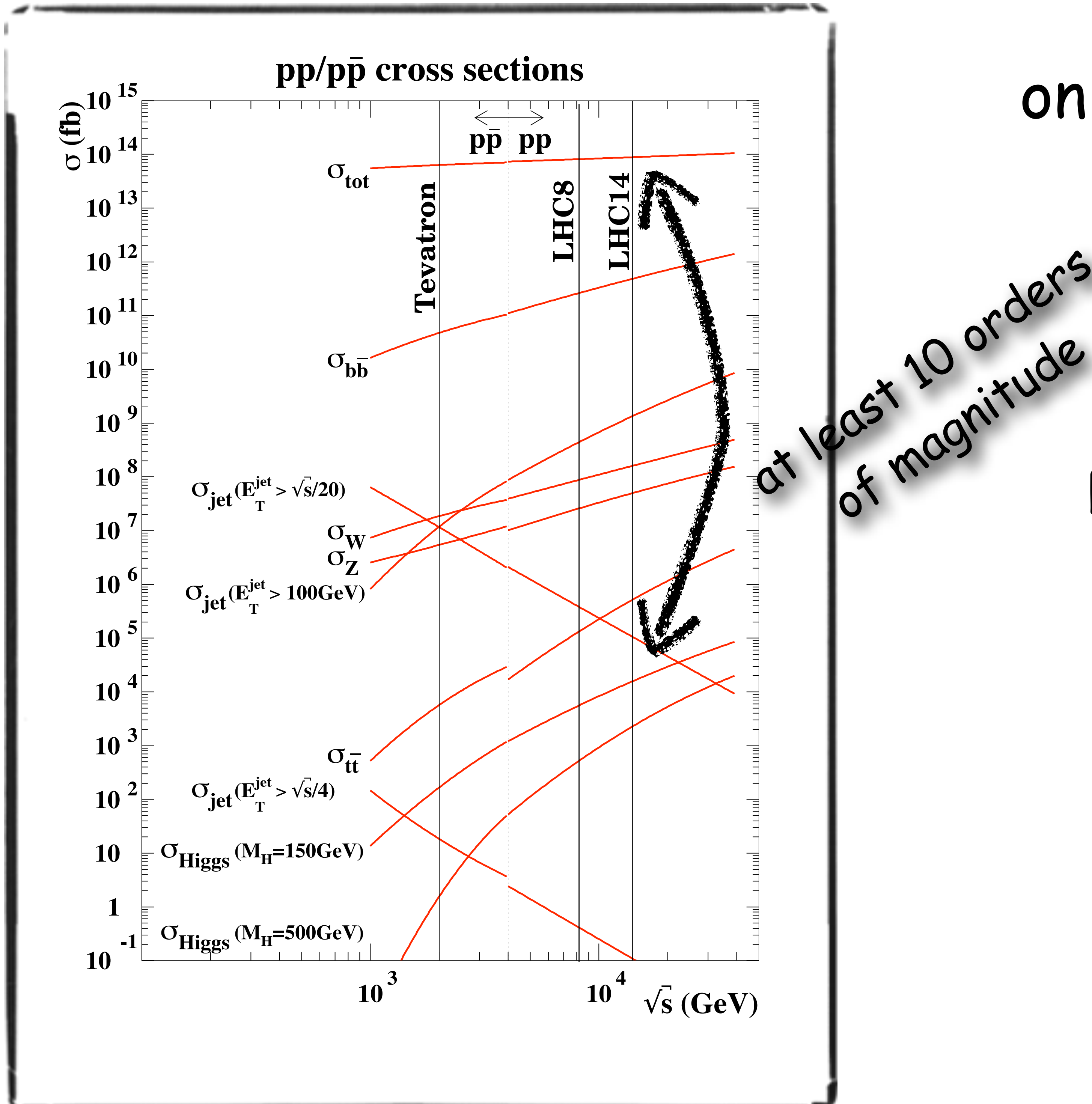
Higgs decay



The LHC has produced 10^5 Higgs bosons out of 10^{16} pp collisions

SM Higgs @ LHC

The production of a Higgs is wiped out by QCD background



only 1 out of 100 billions events are "interesting"

(for comparison, Shakespeare's 43 works contain only 884,429 words in total)

furthermore many of the background events furiously look like signal events

SM Higgs @ LHC

The production of a Higgs is wiped out by QCD background

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... like finding the paper you are looking for in (10^8 copies of) John Ellis' office