

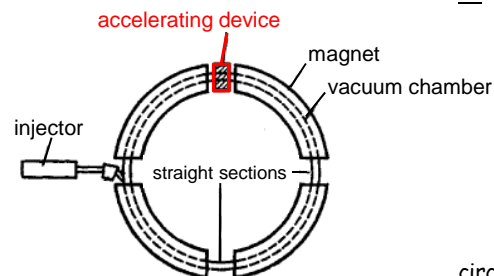
# Introduction to Accelerator Physics

## Part 3

Pedro Castro / Accelerator Physics Group (MPY)  
Introduction to Accelerator Physics  
DESY, 25th July 2017



linear accelerator (linac)



circular accelerator: synchrotron



## Motion in electric and magnetic fields

Equation of motion under Lorentz Force

$$\frac{d\vec{p}}{dt} = \vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$\frac{d\vec{p}}{dt}$  → momentum  
 $q$  → charge  
 $\vec{v}$  → velocity  
 $\vec{E}$  → electric field  
 $\vec{B}$  → magnetic field  
 of the particle

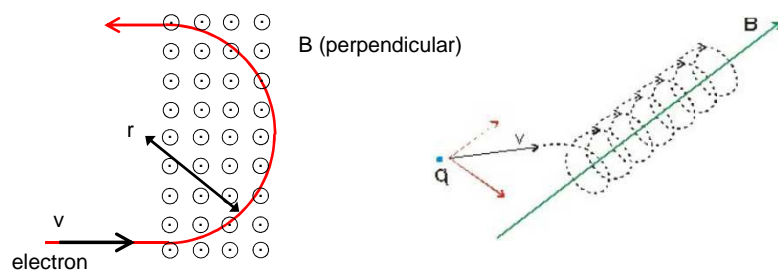
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## Motion in magnetic fields

if the electric field is zero ( $E=0$ ), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B} \rightarrow \vec{F} \perp \vec{v}$$



Magnetic fields do not change the particles energy, only electric fields do !

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## Motion in magnetic fields

if the electric field is zero ( $E=0$ ), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

$$E^2 = \vec{p}^2 c^2 + E_0^2$$

energy-momentum relation in special relativity

total energy  $\nearrow$   $\nwarrow$  momentum  $\nwarrow$  energy at rest

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## Motion in magnetic fields

if the electric field is zero ( $E=0$ ), then

$$\vec{F} = \frac{d\vec{p}}{dt} = q \cdot \vec{v} \times \vec{B}$$

$$E^2 = \vec{p}^2 c^2 + E_0^2$$

$$E \frac{dE}{dt} = c^2 \vec{p} \frac{d\vec{p}}{dt} = c^2 q \vec{p} (\vec{v} \times \vec{B}) = c^2 q |\vec{p}| |\vec{v} \times \vec{B}| \cos \phi = 0$$

since  $\vec{v} \times \vec{B} \perp \vec{v} \rightarrow \phi = 90^\circ$

Magnetic fields do not change the particles energy, only electric fields do !

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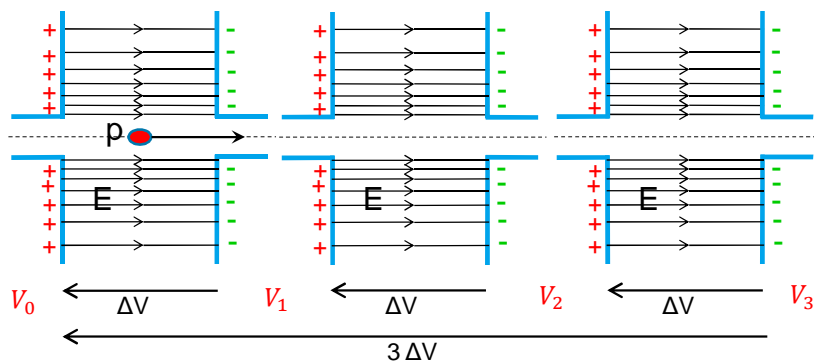


In general:

- Static magnetic fields → to guide (bend + focus) particle beams



### acceleration with DC electric fields



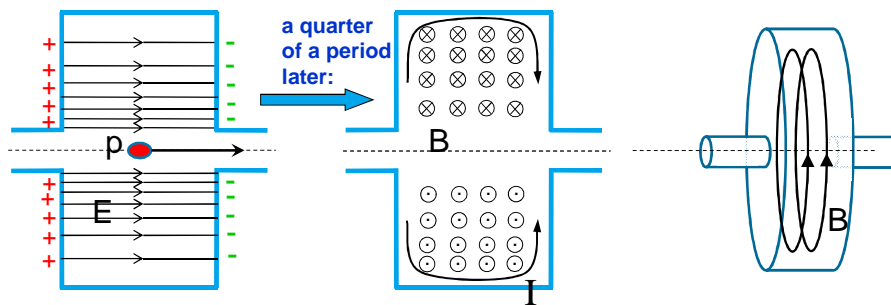
In general:

- Static magnetic fields  $\rightarrow$  to guide (bend + focus) particle beams
- Static electric fields  $\rightarrow$  accelerate particle beams (low energy)
- Radio-frequency EM fields  $\rightarrow$  accelerate particle beams (high E)

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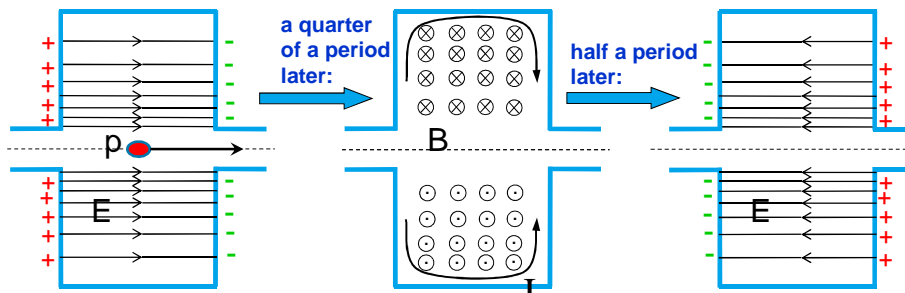
### RF cavity basics: the pill box cavity



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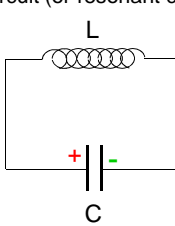
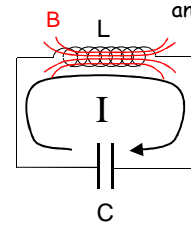
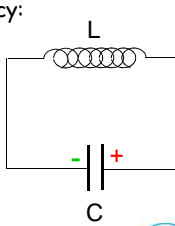


### RF cavity basics: the pill box cavity




a quarter of a period later:      half a period later:

LC circuit (or resonant circuit) analogy:

angular frequency:  $\omega = \frac{1}{\sqrt{LC}}$

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### Maxwell's equations (differential formulation in SI units)

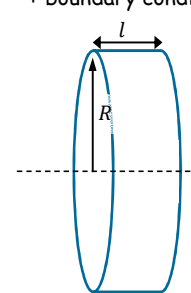
$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

+ boundary conditions



$R$  : cavity radius

$l$  : cavity length

⇒

set of solutions with  $B_z = 0$  (that is,  $\vec{B}$  is transverse)

set of solutions with  $E_z = 0$  (that is,  $\vec{E}$  is transverse)

**TM modes**  
(transverse magnetic modes)

~~**TE modes**~~  
(transverse electric modes)

**Maxwell's equations**  
(differential formulation in SI units)

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

+ boundary conditions  
 $R$  : cavity radius  
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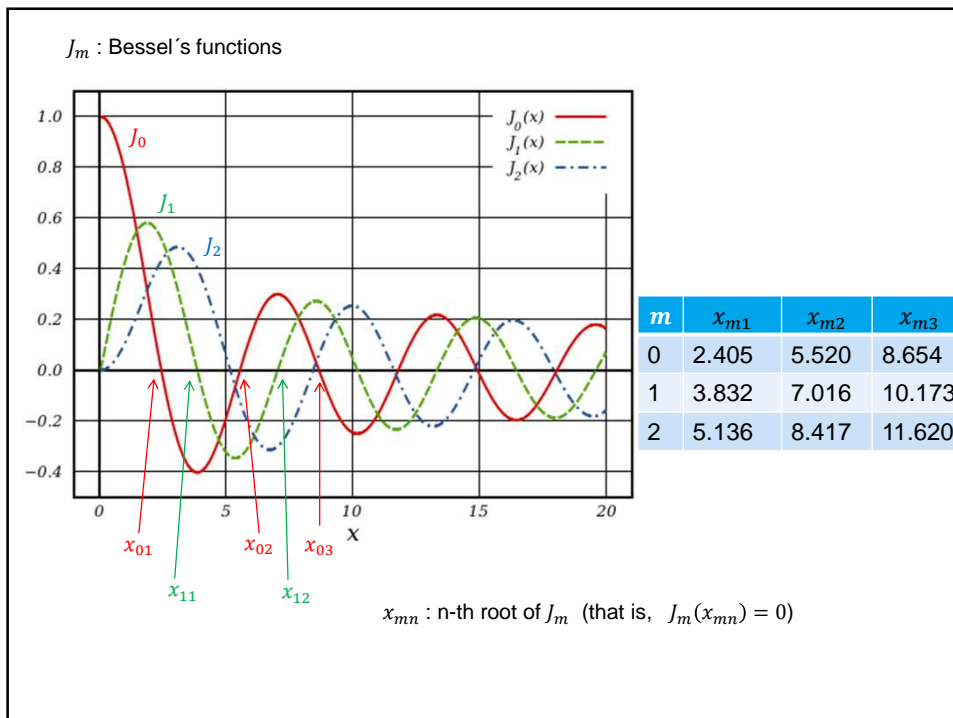
set of solutions with  $B_z = 0$  (that is,  $\vec{B}$  is transverse)

$$\begin{cases} E_z = E_0 J_m \left( x_{mn} \frac{r}{R} \right) \cos m\theta \cos \left( \frac{p\pi}{l} z \right) e^{j\omega t} \\ E_r = -\frac{p\pi}{l} \frac{R}{x_{mn}} E_0 J'_m \left( x_{mn} \frac{r}{R} \right) \cos m\theta \sin \left( \frac{p\pi}{l} z \right) e^{j\omega t} \\ E_\theta = -\frac{p\pi}{l} \frac{mR^2}{x_{mn}^2 r} E_0 J_m \left( x_{mn} \frac{r}{R} \right) \sin m\theta \sin \left( \frac{p\pi}{l} z \right) e^{j\omega t} \\ B_z = 0 \\ B_r = -j\omega \frac{mR^2}{x_{mn}^2 r c^2} E_0 J_m \left( x_{mn} \frac{r}{R} \right) \sin m\theta \cos \left( \frac{p\pi}{l} z \right) e^{j\omega t} \\ B_\theta = -j\omega \frac{R}{x_{mn} c^2} E_0 J'_m \left( x_{mn} \frac{r}{R} \right) \cos m\theta \cos \left( \frac{p\pi}{l} z \right) e^{j\omega t} \end{cases}$$

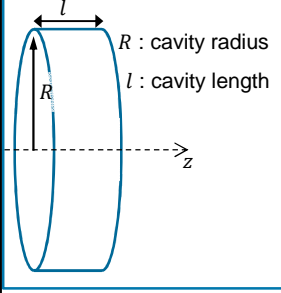
indices:  
 $m = 0, 1, 2, \dots$  : number of full period variations in  $\theta$  of the fields  
 $n = 1, 2, \dots$  : number of zeros of the axial field component in  $\vec{r}$   
 $p = 0, 1, 2, \dots$  : number of half period variations in  $z$  of the fields

$J_m$  : Bessel's functions                       $x_{mn}$  : n-th root of  $J_m$  (that is,  $J_m(x_{mn}) = 0$ )  
 $J'_m$  : derivative of the Bessel's functions

angular frequency :  $\omega = c \sqrt{\left( \frac{x_{mn}}{R} \right)^2 + \left( \frac{p\pi}{l} \right)^2}$



**boundary conditions**



$R$  : cavity radius  
 $l$  : cavity length

**fundamental solution with  $B_z = 0$  (that is,  $\vec{B}$  is transverse)**

$$E_z = E_0 J_0 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$

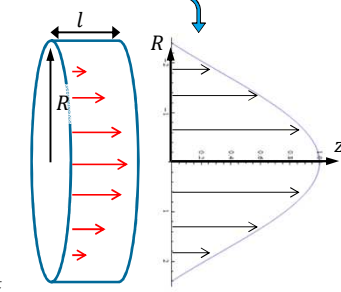
$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega \frac{R}{x_{01} c^2} E_0 J_1 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$



**$m = 0$  : rotation symmetry of the fields**  
 **$n = 1$  : no zeros of the axial field component in  $\vec{r}$**   
 **$p = 0$  : no variation in  $z$  of the fields**

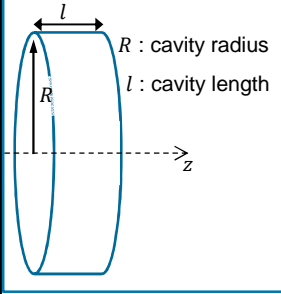
$J_m$  : Bessel's functions

$J'_m$  : derivative of the Bessel's functions

angular frequency :  $\omega = c \frac{x_{01}}{R}$

$x_{01} = 2.405$

**boundary conditions**



$R$  : cavity radius  
 $l$  : cavity length

**fundamental solution with  $B_z = 0$  (that is,  $\vec{B}$  is transverse)**

$$E_z = E_0 J_0 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$

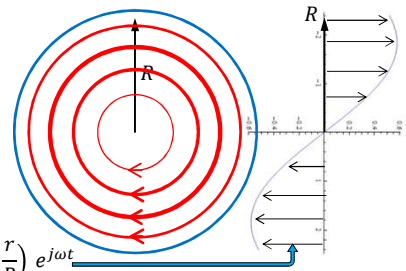
$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega \frac{R}{x_{01} c^2} E_0 J_1 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$



**$m = 0$  : rotation symmetry of the fields**  
 **$n = 1$  : no zeros of the axial field component in  $\vec{r}$**   
 **$p = 0$  : no variation in  $z$  of the fields**

$J_m$  : Bessel's functions

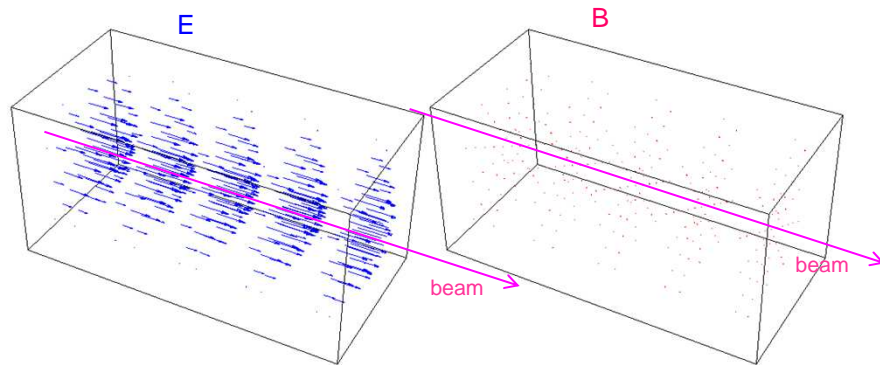
$J'_m$  : derivative of the Bessel's functions

angular frequency :  $\omega = c \frac{x_{01}}{R}$

$x_{01} = 2.405$



## Pill box cavity: 3D visualisation of E and B

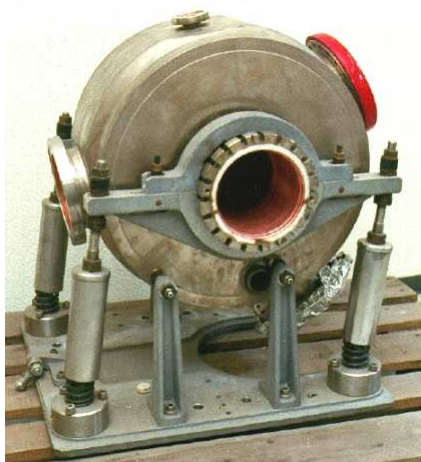


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## Examples of pill box cavities

DESY cavity (pill box)



ADONE cavity 51 MHz (pill box)  
Frascati lab, Italy



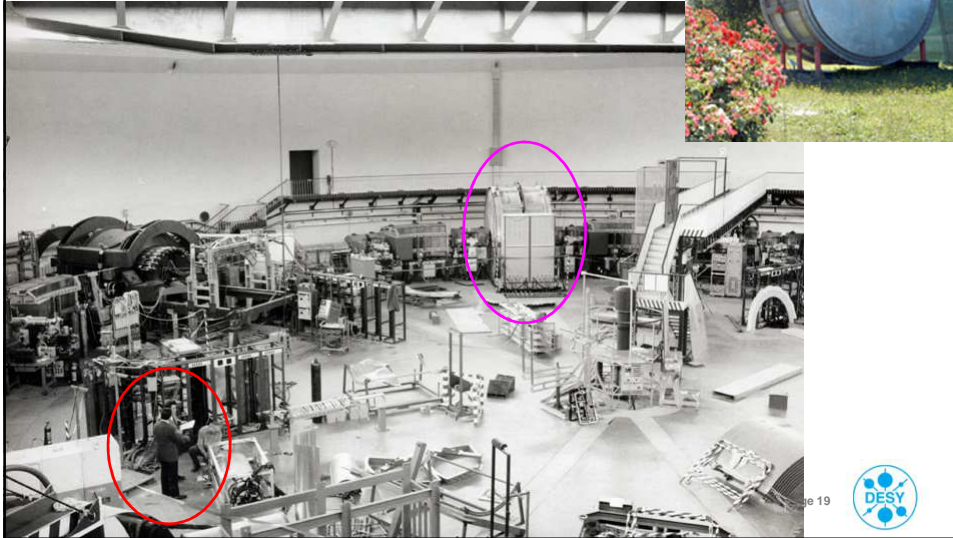
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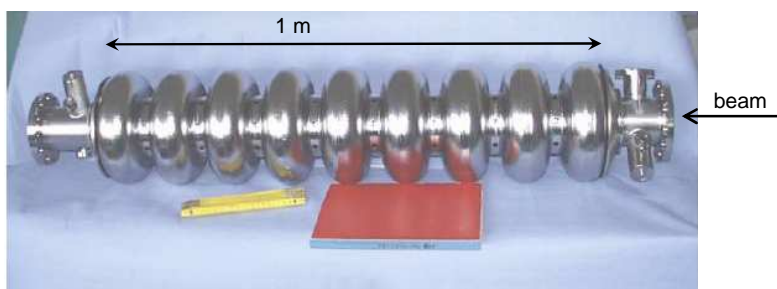
## Examples of pill box cavities

ADONE cavity 51 MHz (pill box)  
Frascati lab, Italy

ADONE in 1963, Laboratori Nazionali di Frascati, Italy



## Superconducting cavity used at DESY



material: pure Niobium

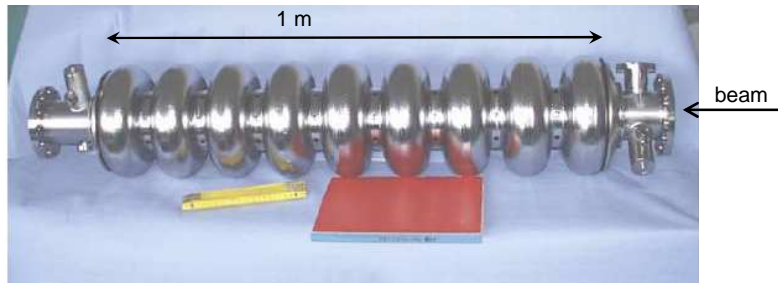
operating temperature: 2 K

accelerating field gradient: up to 35 MV/m



## Superconducting cavity used at DESY

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)

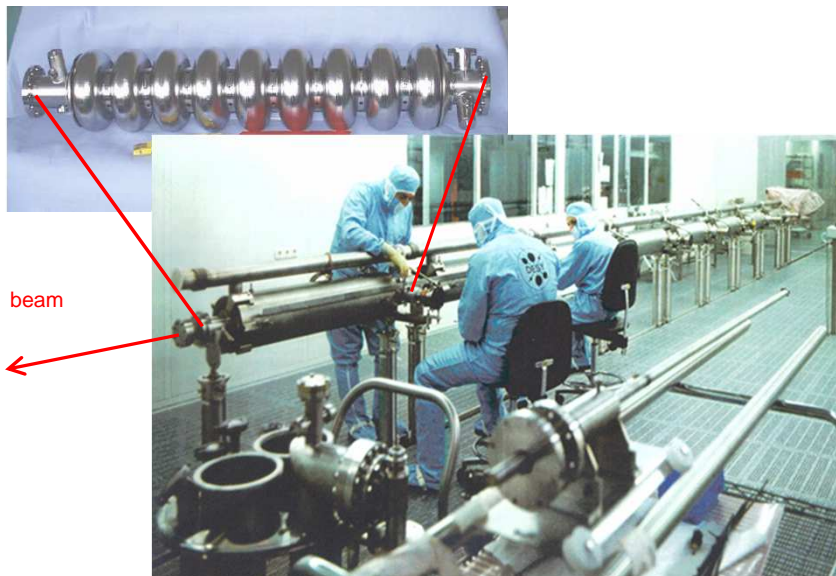


Free-electron <u>L</u> ASer in <u>H</u> amburg	0.3 km	DESY	2004-	?	e-	1.2 GeV
European <u>X</u> -ray Free-Electron <u>L</u> aser	3 km	DESY	2016-	?	e-	17.5 GeV
<u>I</u> nternational <u>L</u> inear <u>C</u> ollider	30 km	?	?		e-/e+	2x250 GeV

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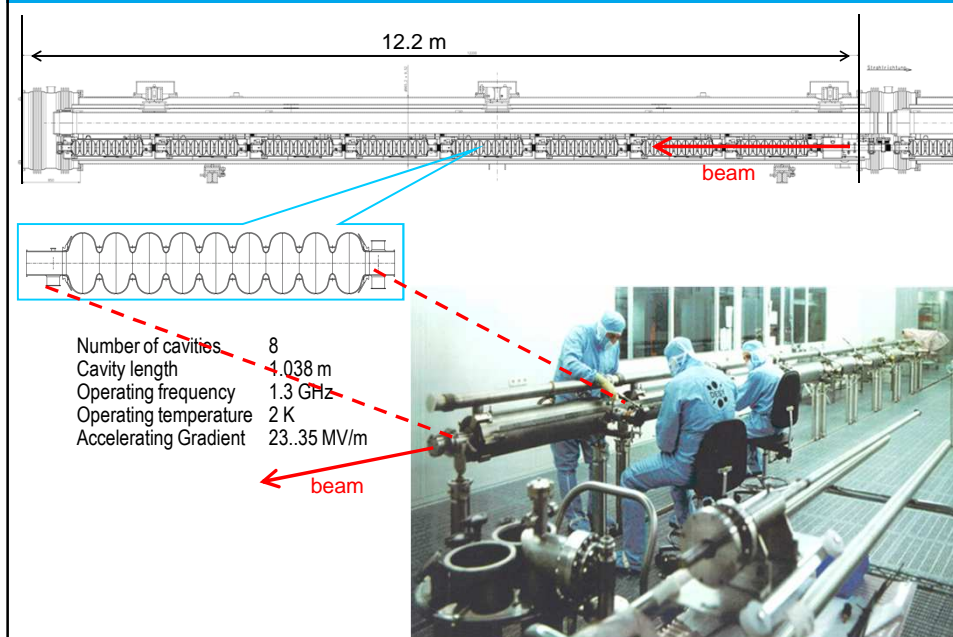
## Cavities inside a cryostat



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### Cavities inside a cryostat



### Cavities inside a cryostat





## Cavities inside an accelerator module (cryostat)

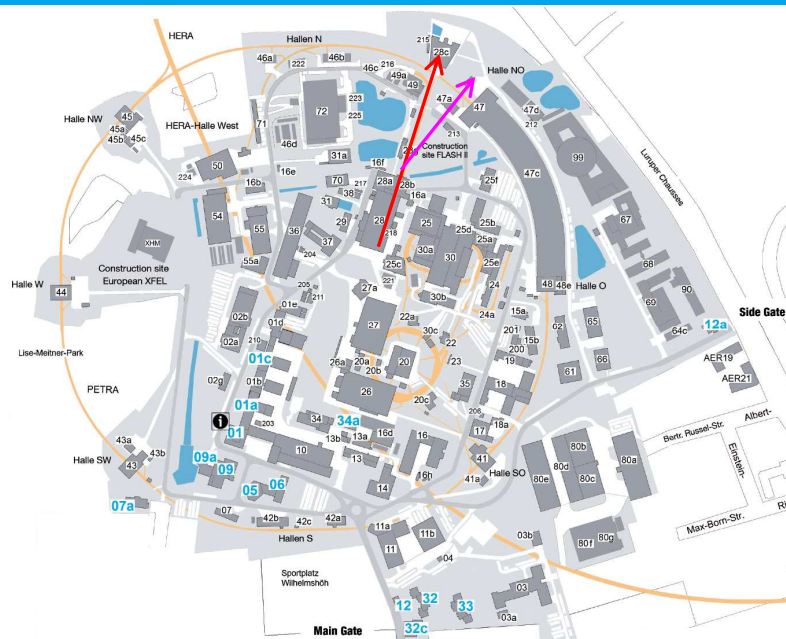


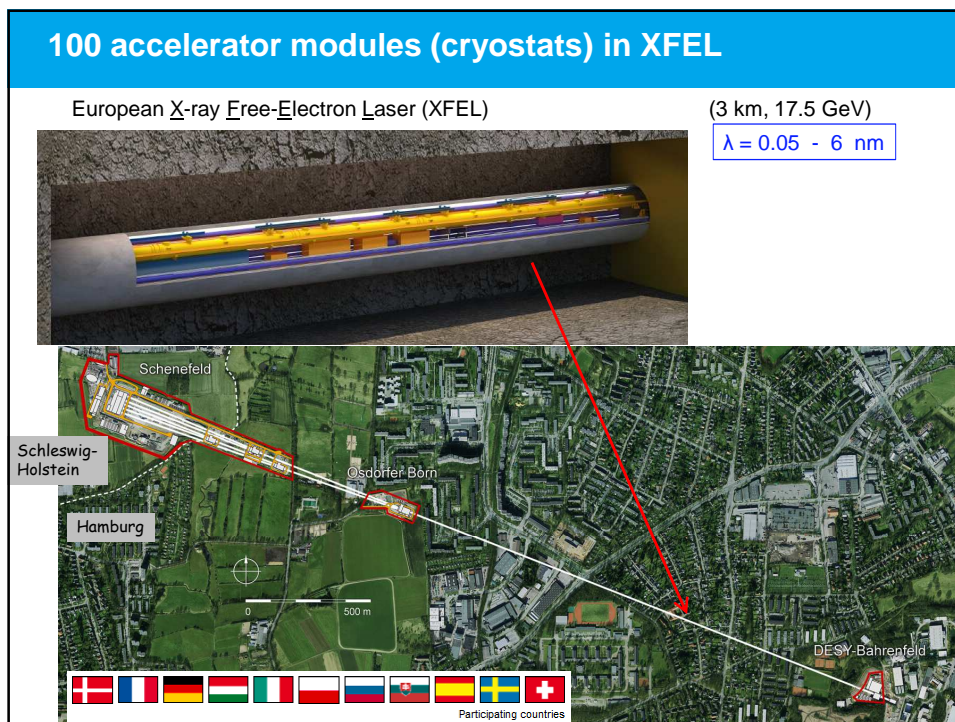
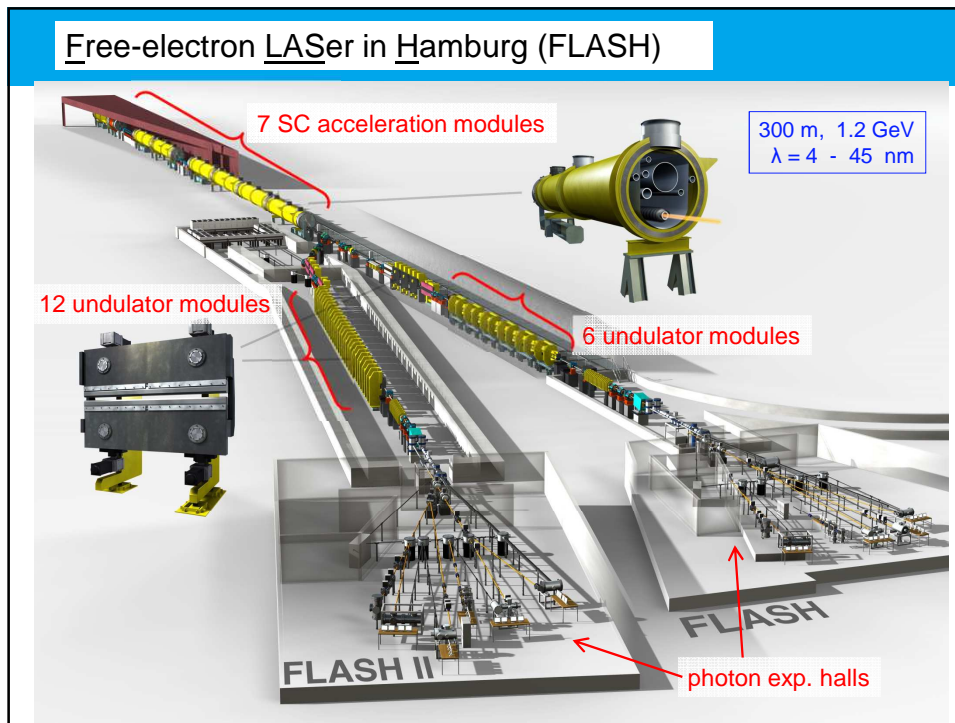
module installation in FLASH (2004)

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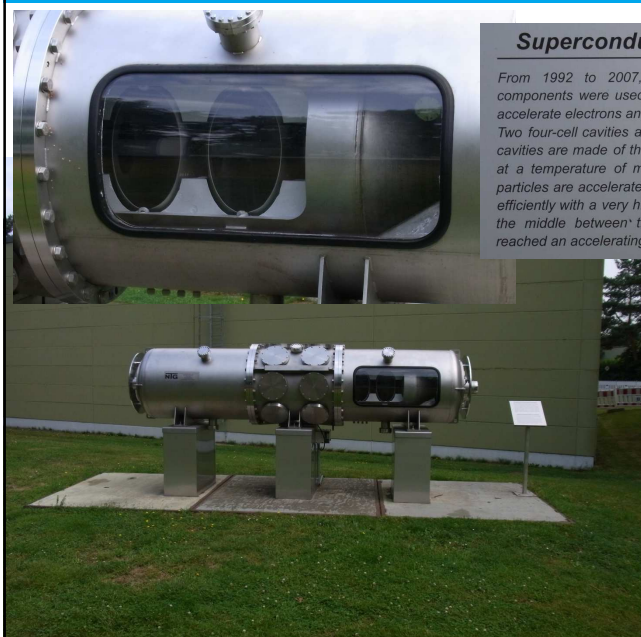


## Free-electron LASer in Hamburg (FLASH)





## Superconducting cavities at HERA




**Superconducting Particle Accelerator**

From 1992 to 2007, eight of these superconducting accelerator components were used in the 6.3-kilometre long storage ring HERA to accelerate electrons and their antiparticles, positrons.

Two four-cell cavities are arranged in one thermal vessel (cryostat). The cavities are made of the metal niobium which becomes superconducting at a temperature of minus 269 degrees Celsius. At this temperature, particles are accelerated almost without electric resistance and thus very efficiently with a very high electric alternating voltage which is injected in the middle between the cavities. During HERA operation, this cavity reached an accelerating gradient of 5 million volts per metre.

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## Other accelerators using superconducting cavities

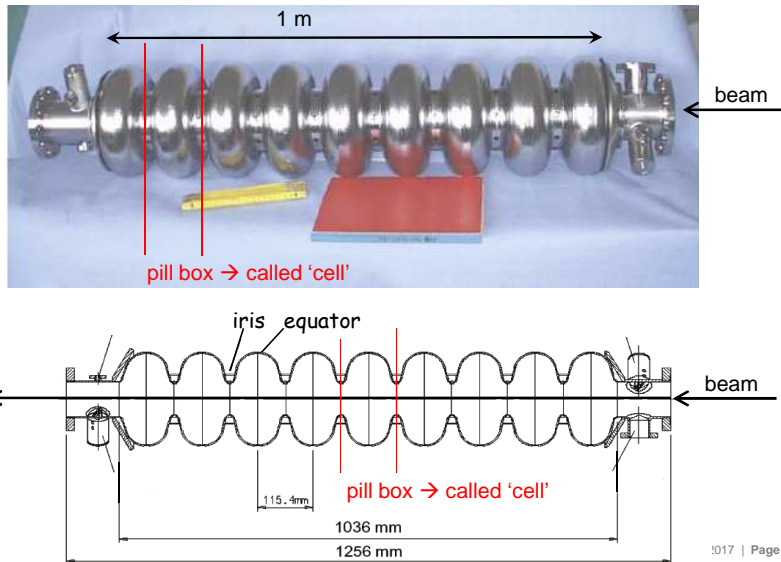
- 5 de-commissioned
- 11 in operation
- 4 in construction
- 9 in design phase

Total = 29

full list: [http://tesla-new.desy.de/sites/site\\_tesla/content/e163749/e163751/infoboxContent163765/SRFAccelerators.pdf](http://tesla-new.desy.de/sites/site_tesla/content/e163749/e163751/infoboxContent163765/SRFAccelerators.pdf)

## Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



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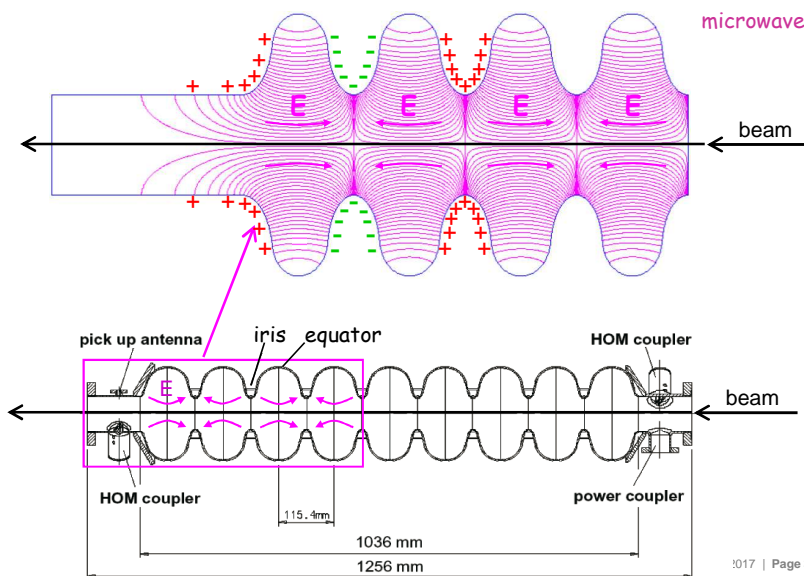


## Accelerating field map

Simulation of the fundamental mode: electric field lines

$f_{RF} = 1.3 \text{ GHz}$

microwaves: (L-band)



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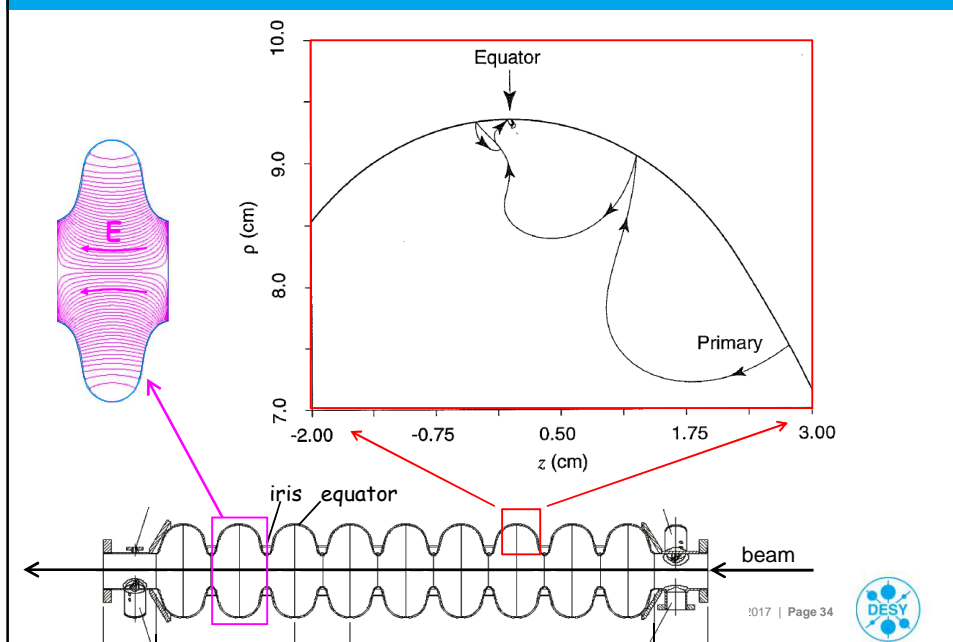
## Frequently Asked Questions

- 1) Why this shape?
- 2)
- 3)
- 4)
- 5)

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## Multipacting mitigation in superconducting cavities



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1) Why this shape? ..... to reduce/avoid multipacting

2) How to feed  $\vec{E}$  in?

3)

4)

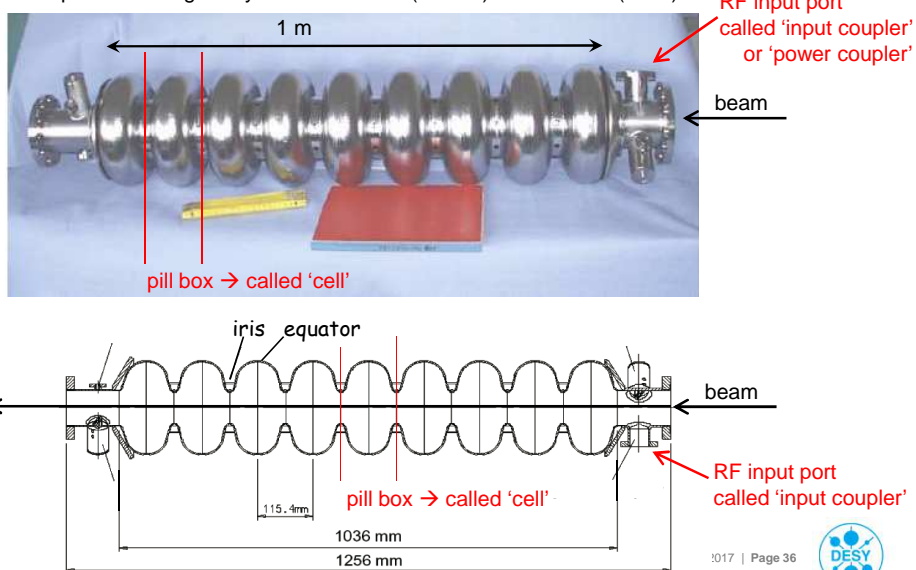
5)

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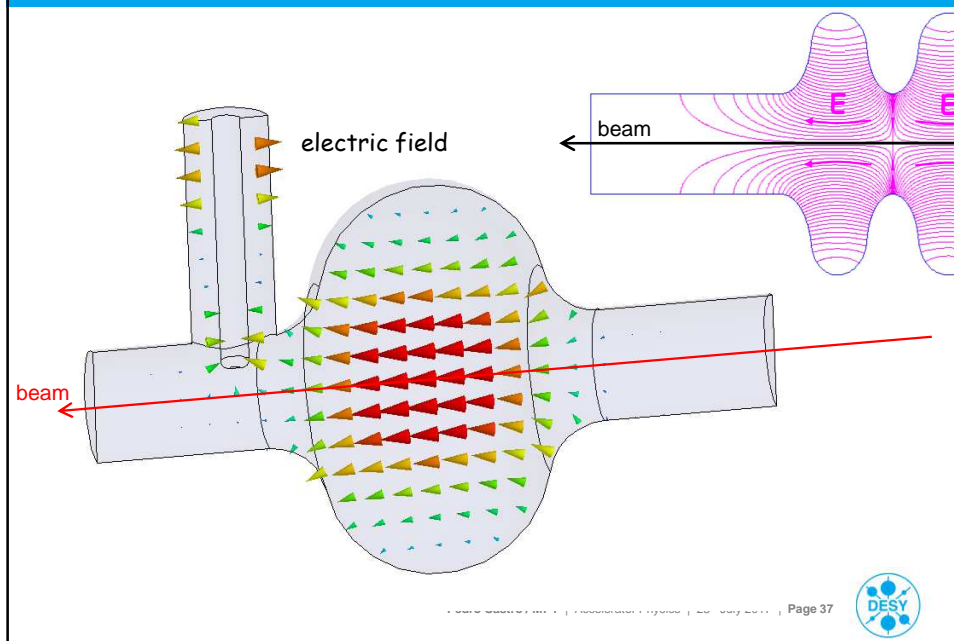


## Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



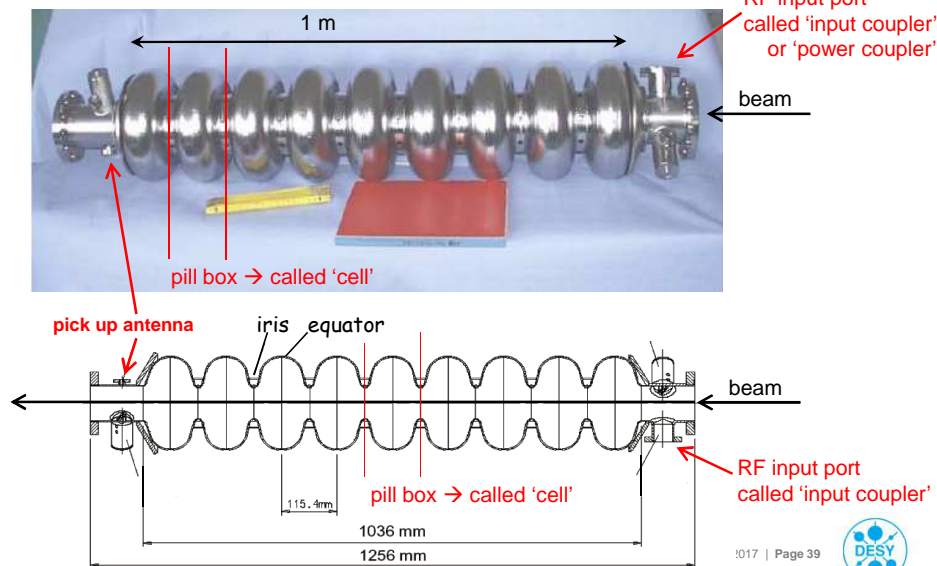
## Fundamental mode coupler (input coupler)



- 1) Why this shape? ..... to reduce/avoid multipacting
- 2) How to feed  $\vec{E}$  in? ..... with input couplers
- 3) How to measure  $\vec{E}$  ?
- 4)
- 5)

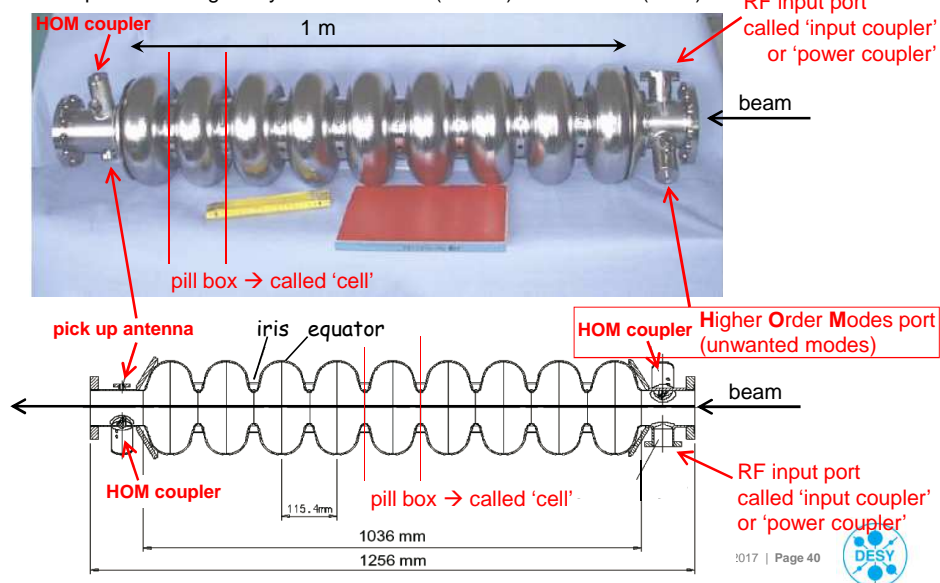
## Superconducting cavity used in FLASH and in XFEL

Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



## Superconducting cavity used in FLASH and in XFEL

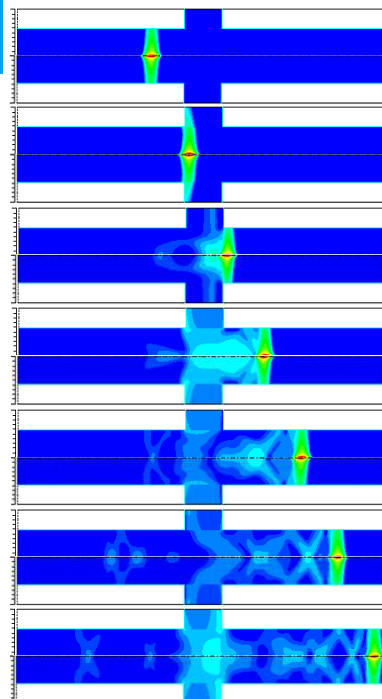
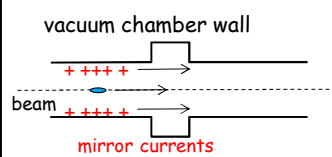
Superconducting cavity used in FLASH (0.3 km) and in XFEL (3 km)



- 1) Why this shape? ..... to reduce/avoid multipacting
- 2) How to feed  $\vec{E}$  in? ..... with input couplers
- 3) How to measure  $\vec{E}$  ? ..... with pick up antennas
- 4) What are HOM couplers for?
- 5)



## Wakefields



- 1) Why this shape? ..... to reduce/avoid multipacting
- 2) How to feed  $\vec{E}$  in? ..... with input couplers
- 3) How to measure  $\vec{E}$  ? ..... with pick up antennas
- 4) What are HOM couplers for? ..... to reduce HOM (wakefields)
- 5) Is there a net acceleration?

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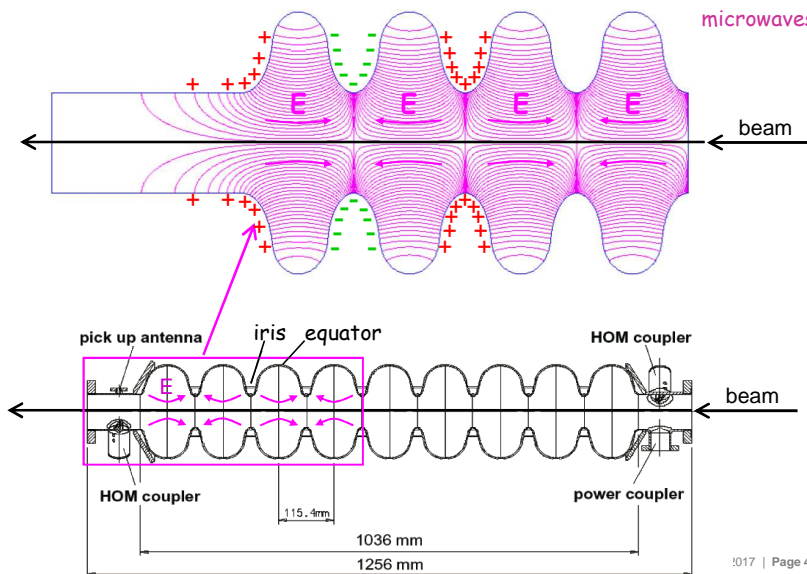


## Accelerating field map

Simulation of the fundamental mode: electric field lines

$f_{RF} = 1.3 \text{ GHz}$

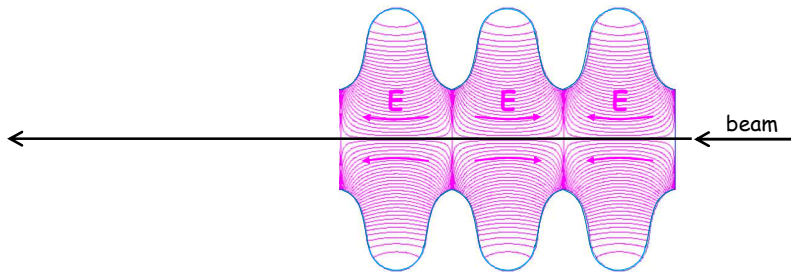
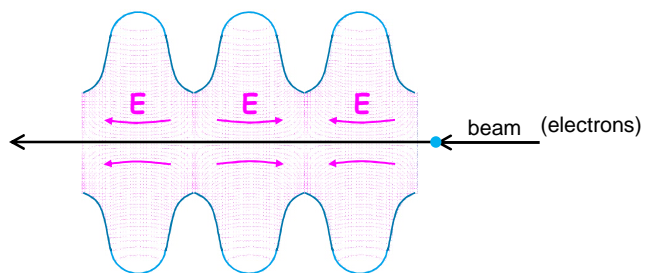
microwaves: (L-band)

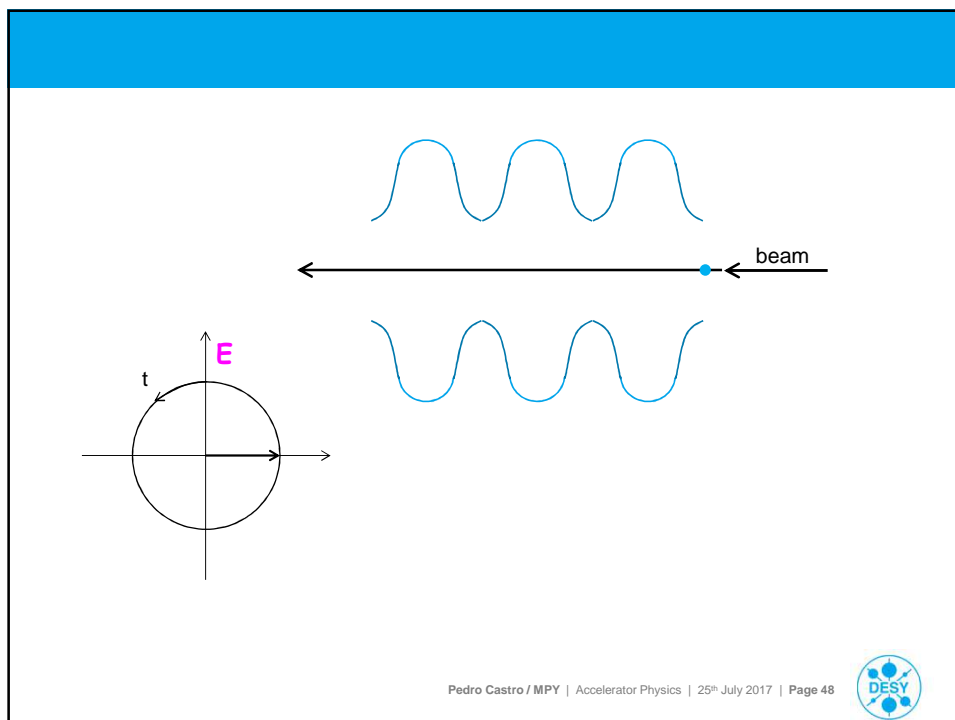
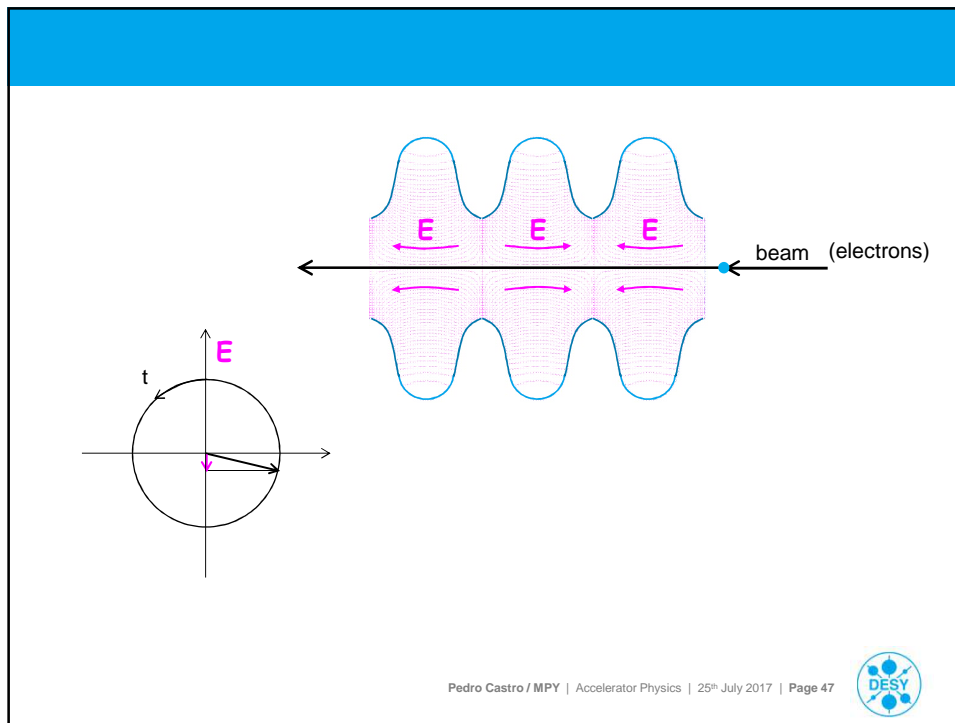


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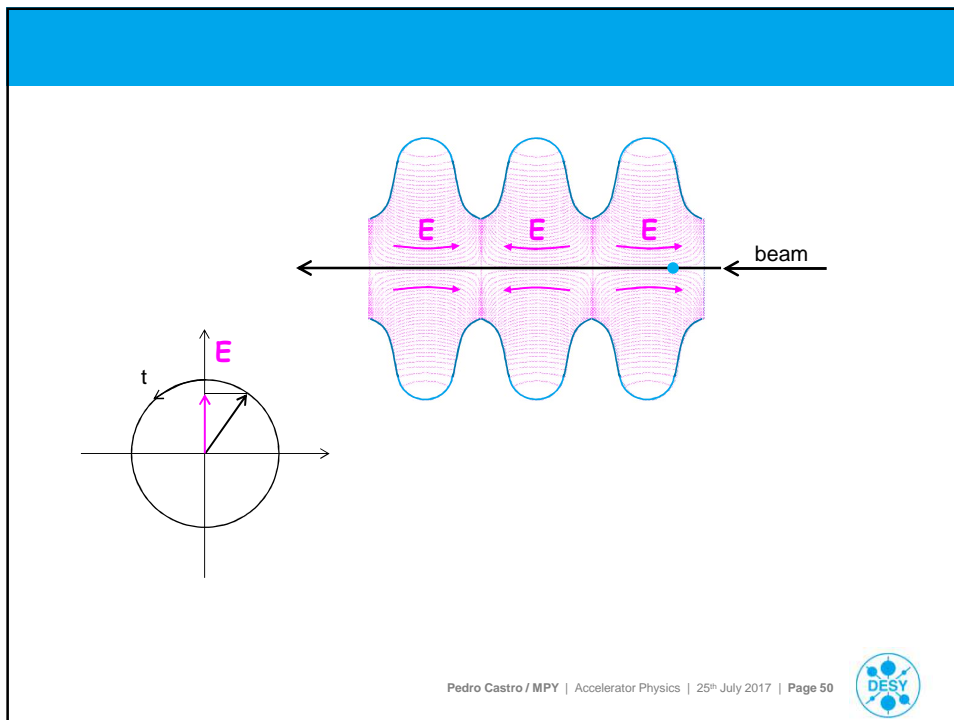
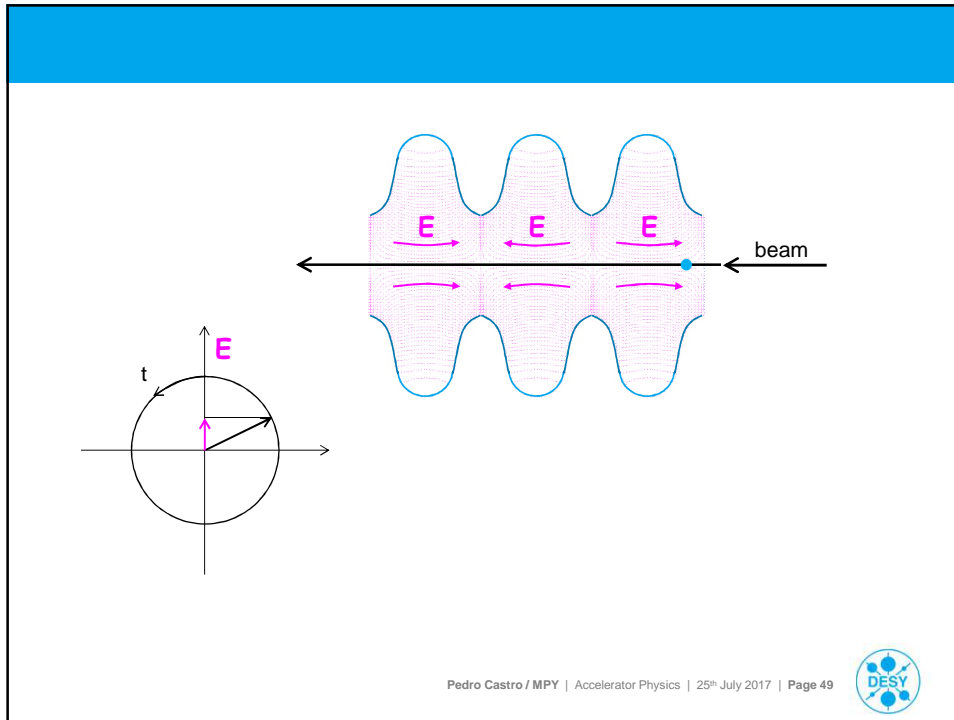


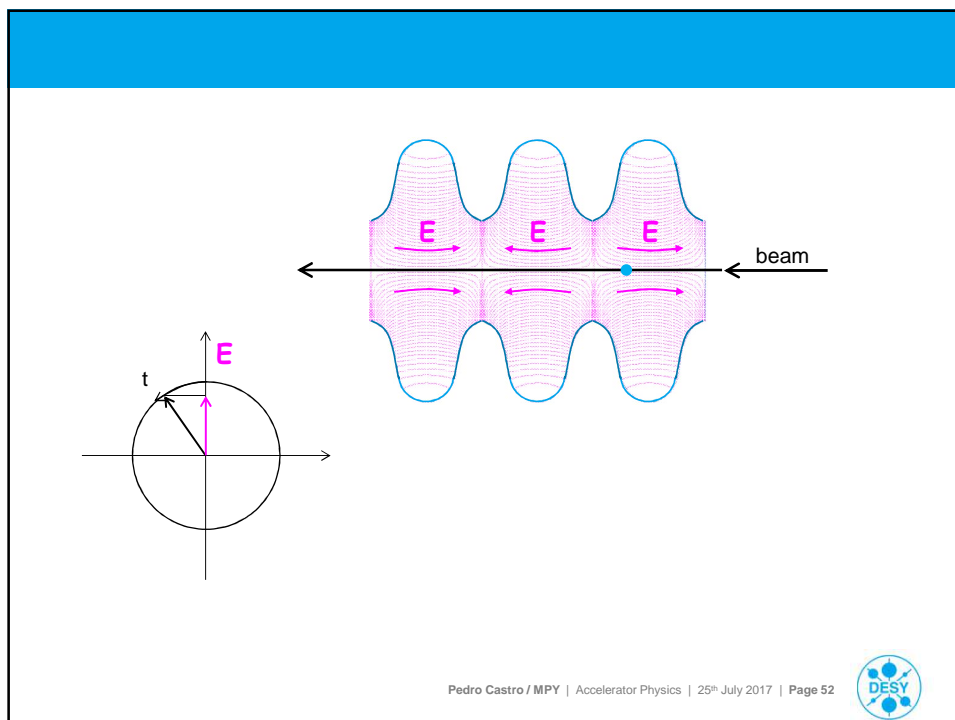
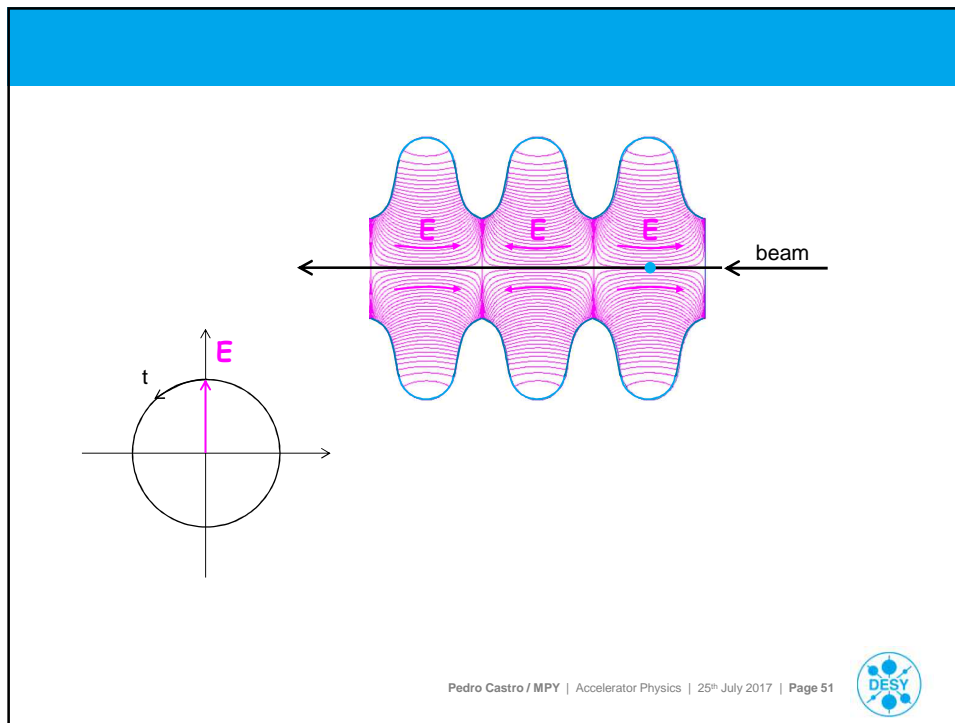
## Simulation of the fundamental mode: electric field lines

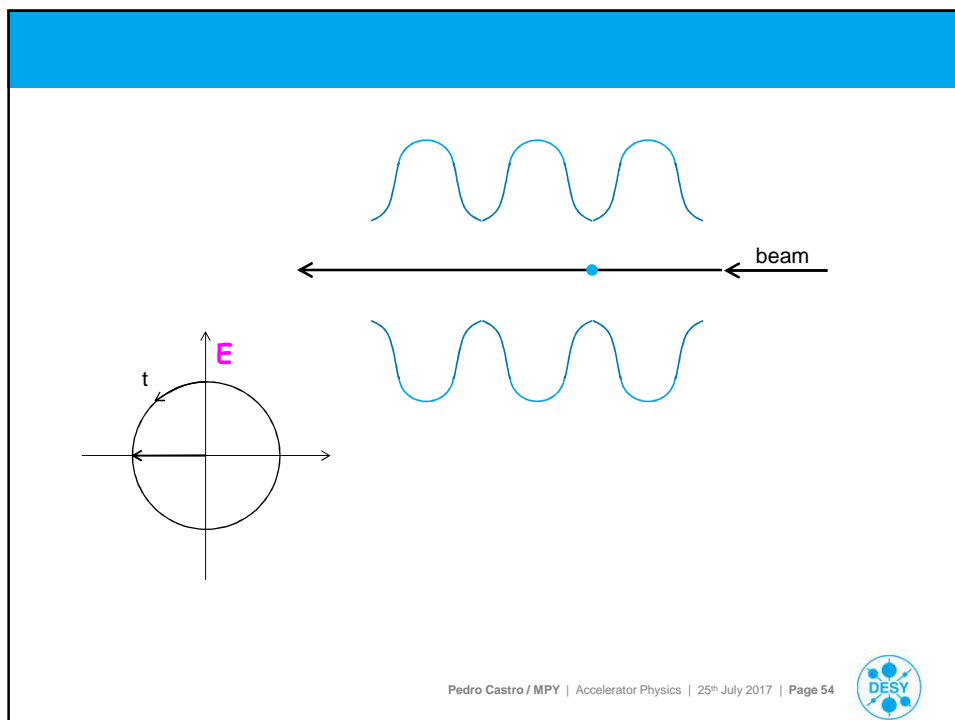
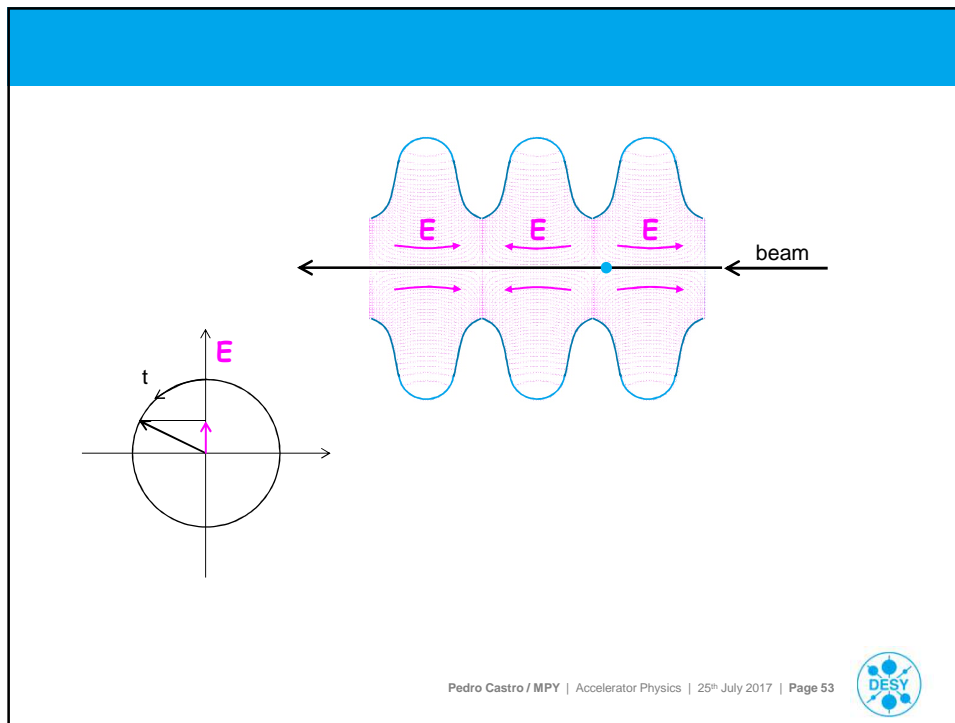
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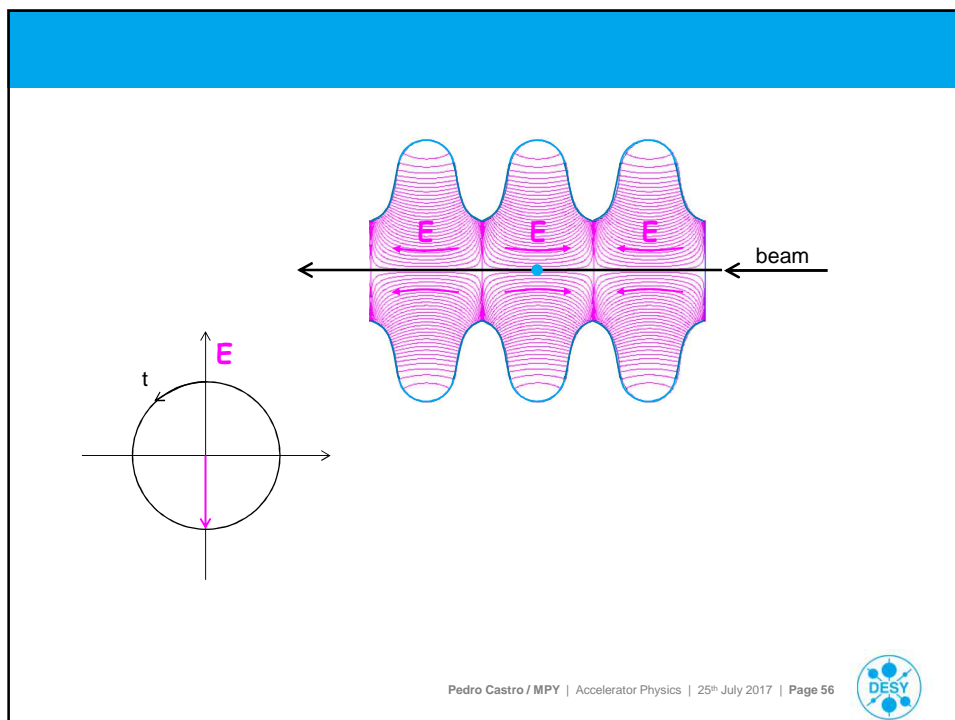
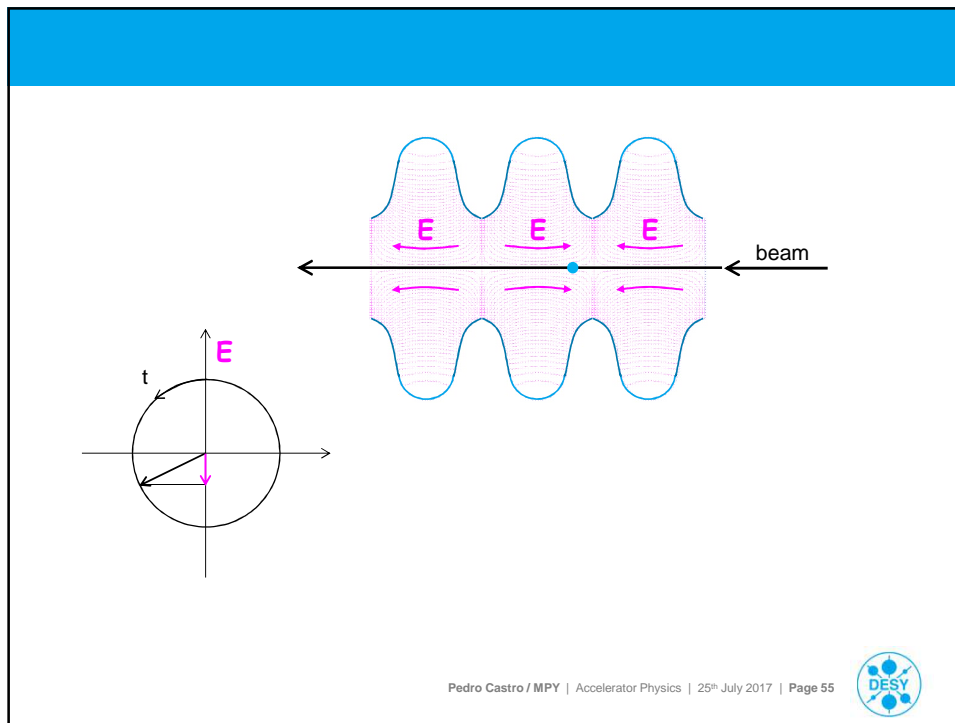


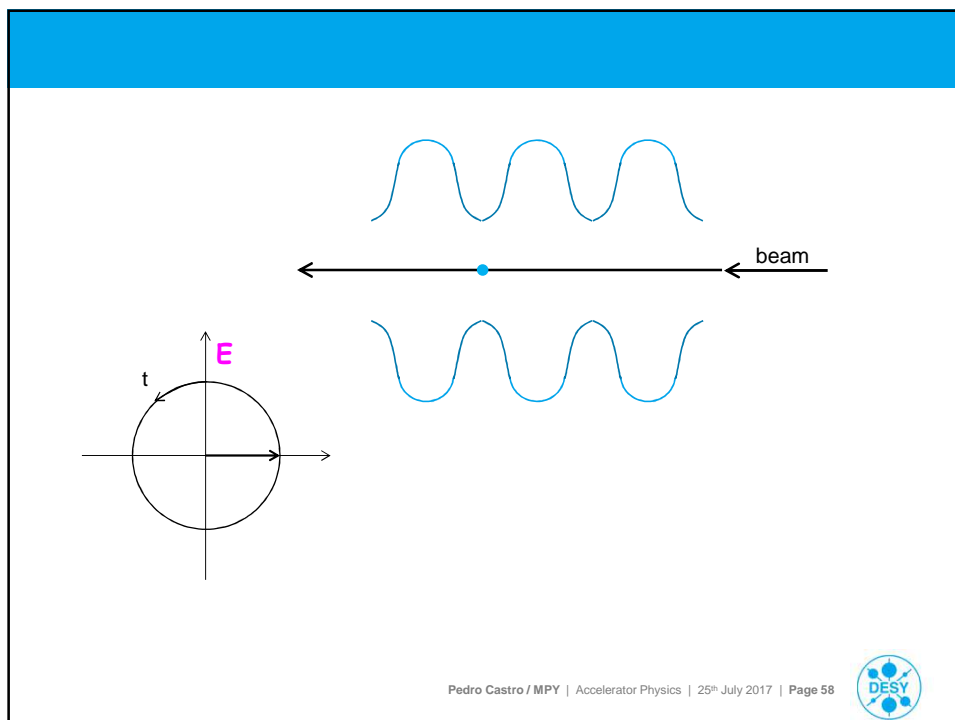
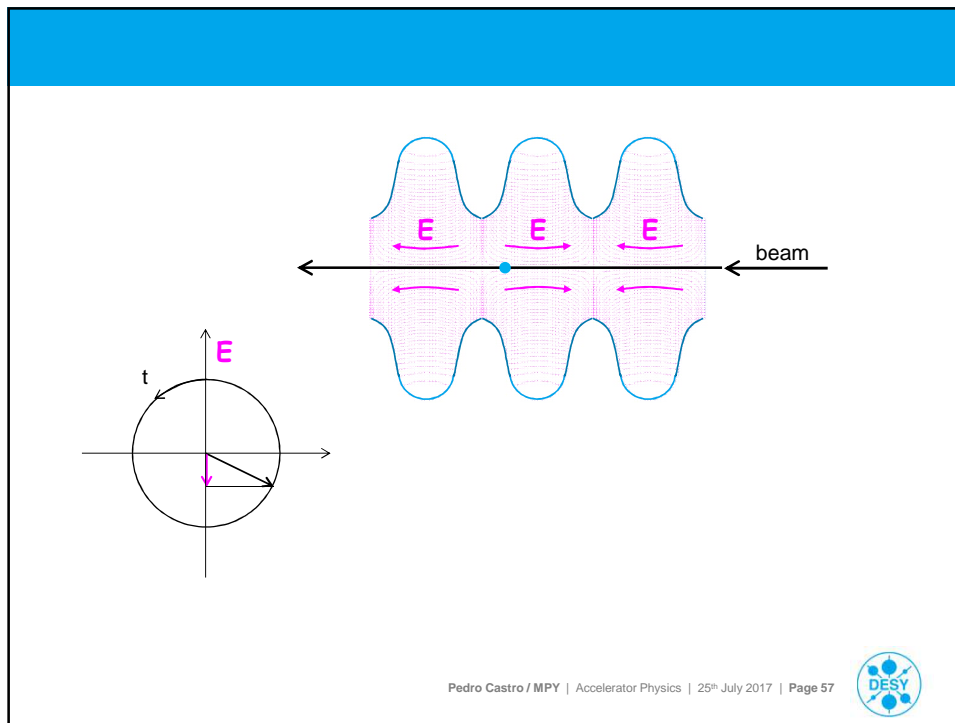


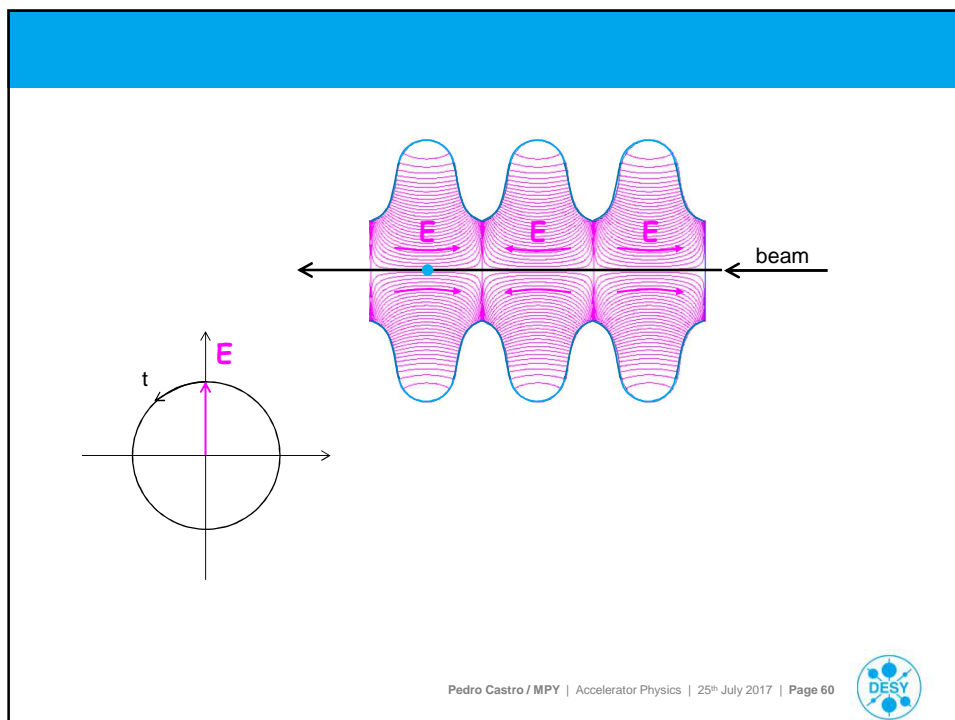
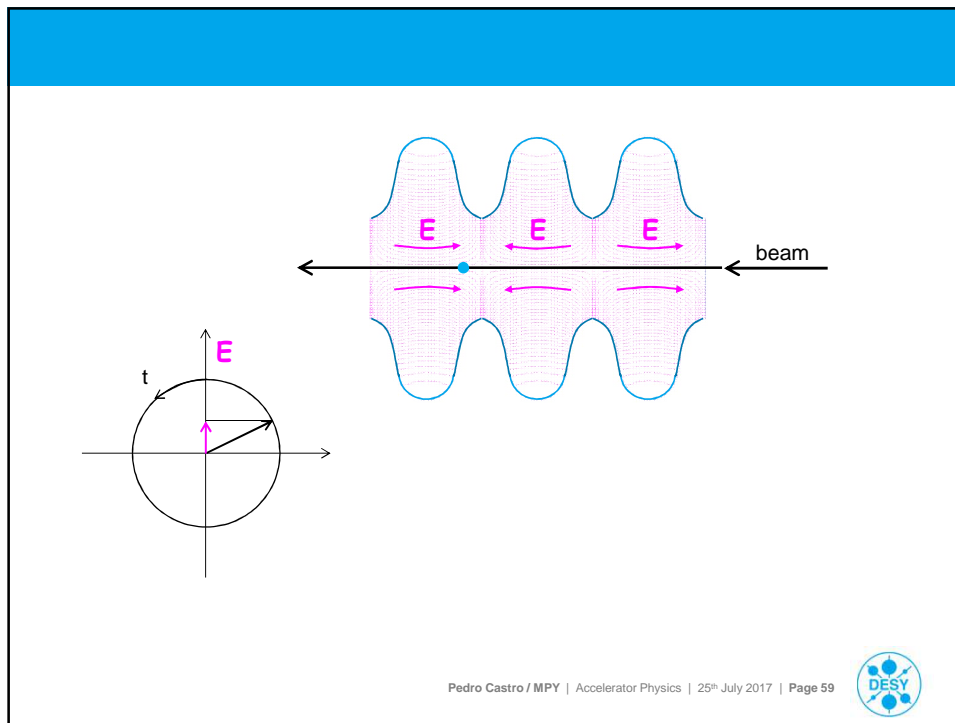


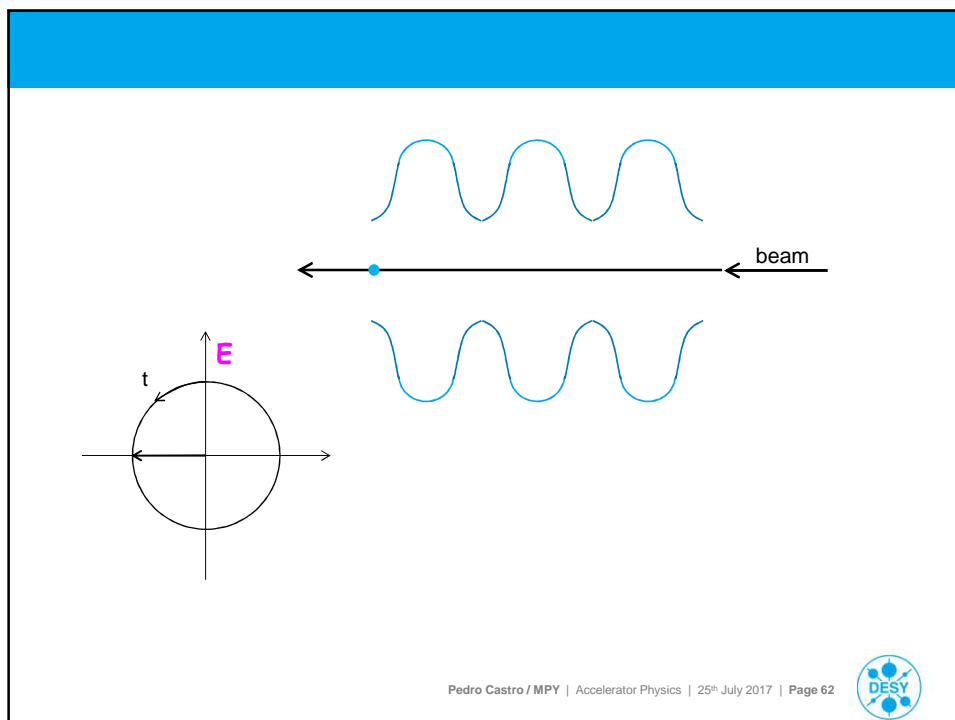
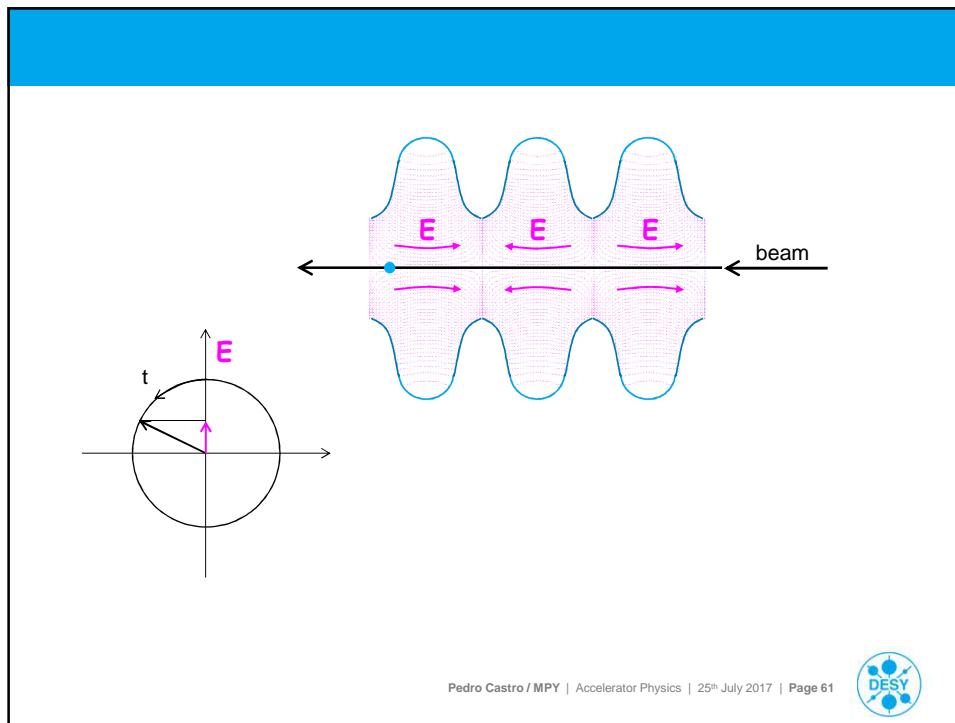












- 1) Why this shape? ..... to reduce/avoid multipacting
- 2) How to feed  $\vec{E}$  in? ..... with input couplers
- 3) How to measure  $\vec{E}$ ? ..... with pick up antennas
- 4) What are HOM couplers for? ..... to reduce HOM (wakefields)
- 5) Is there a net acceleration? ..... timing is the key

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## Summing-up of this part

Particle acceleration using radio-frequency fields:

basic cavity:  
pill box

{  
 analogy to an LC circuit  
 infinite number of solutions for  $\vec{E}$  and  $\vec{B}$   
 eq. for the fundamental solution for  $\vec{E}$  and  $\vec{B}$

superconducting  
cavity

{  
 multipacting mitigation  
 RF couplers and antennas  
 wakefields and HOMs  
 FLASH and XFEL

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## Exercise

Calculate the resonant frequency of the fundamental mode in a 'coca-cola' tin



assume a cylindrical shape  
with a diameter of 6.4 cm and a height of 12.1 cm

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## Exercise

Calculate the diameter of the ADONE pill box cavity for a fundamental frequency of 51 MHz

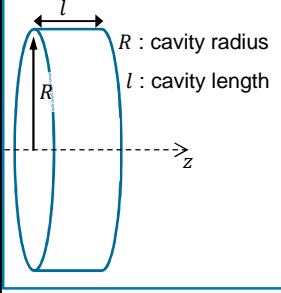
ADONE cavity 51 MHz (pill box)  
Frascati lab, Italy



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**boundary conditions**



$R$  : cavity radius  
 $l$  : cavity length

**fundamental solution with  $B_z = 0$  (that is,  $\vec{B}$  is transverse)**

$$E_z = E_0 J_0 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$

$$E_r = 0$$

$$E_\theta = 0$$

$$B_z = 0$$

$$B_r = 0$$

$$B_\theta = j\omega \frac{R}{x_{01} c^2} E_0 J_1 \left( x_{01} \frac{r}{R} \right) e^{j\omega t}$$

**$m = 0$  : rotation symmetry of the fields**  
 **$n = 1$  : no zeros of the axial field component in  $\vec{r}$**   
 **$p = 0$  : no variation in  $z$  of the fields**

$J_m$  : Bessel's functions  
 $J'_m$  : derivative of the Bessel's functions

angular frequency :  $\omega = c \frac{x_{01}}{R}$        $x_{01} = 2.405$

## Exercise

Calculate the resonant frequency of the fundamental mode in a 'coca-cola' tin



assume a cylindrical shape  
with a diameter of 6.4 cm and a height of 12.1 cm

$$\omega = c \frac{x_{01}}{R} = 3 \cdot 10^8 \frac{2.405}{0.032} = 2.25 \cdot 10^{10} \text{ rad} \cdot \text{s}^{-1}$$

$$f = \frac{\omega}{2\pi} = 3.6 \text{ GHz}$$

Microwave frequency bands

Letter Designation	Frequency range	Wavelength range	Typical uses
L band	1 to 2 GHz	15 cm to 30 cm	military telemetry, GPS, mobile phones (GSM), amateur radio
S band	2 to 4 GHz	7.5 cm to 15 cm	weather radar, surface ship radar, and some communications satellites (microwave ovens, microwave devices/communications, radio astronomy, mobile phones, wireless LAN, Bluetooth, ZigBee, GPS, amateur radio)
C band	4 to 8 GHz	3.75 cm to 7.5 cm	long-distance radio telecommunications
X band	8 to 12 GHz	25 mm to 37.5 mm	satellite communications, radar, terrestrial broadband, space communications, amateur radio
K <sub>u</sub> band	12 to 18 GHz	16.7 mm to 25 mm	satellite communications

<http://en.wikipedia.org/wiki/Microwave>

## Exercise

Calculate the diameter of the ADONE pill box cavity for a fundamental frequency of 51 MHz

ADONE cavity 51 MHz (pill box)  
Frascati lab, Italy



$$2\pi f = \omega = c \frac{x_{01}}{R}$$

$$R = c \frac{x_{01}}{2\pi f} = 3 \cdot 10^8 \frac{2.405}{2\pi \cdot 51 \cdot 10^6} = 2.25 \text{ m}$$

$$d = 2R = 4.5 \text{ m}$$

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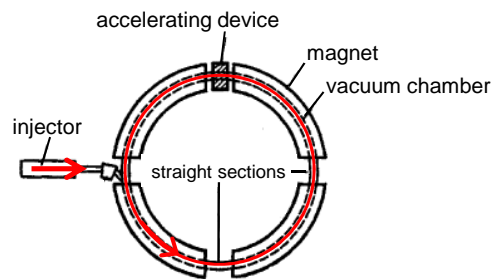
## Introduction to Accelerator Physics

### Part 4

Pedro Castro / Accelerator Physics Group (MPY)  
Introduction to Accelerator Physics  
DESY, 25th July 2017



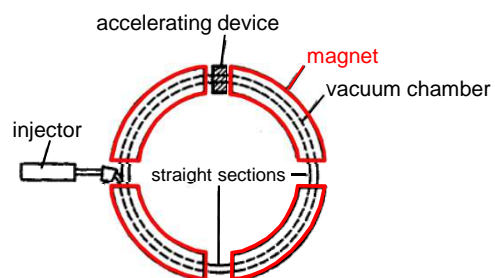
## Circular accelerators: the synchrotron



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## Circular accelerators: the synchrotron

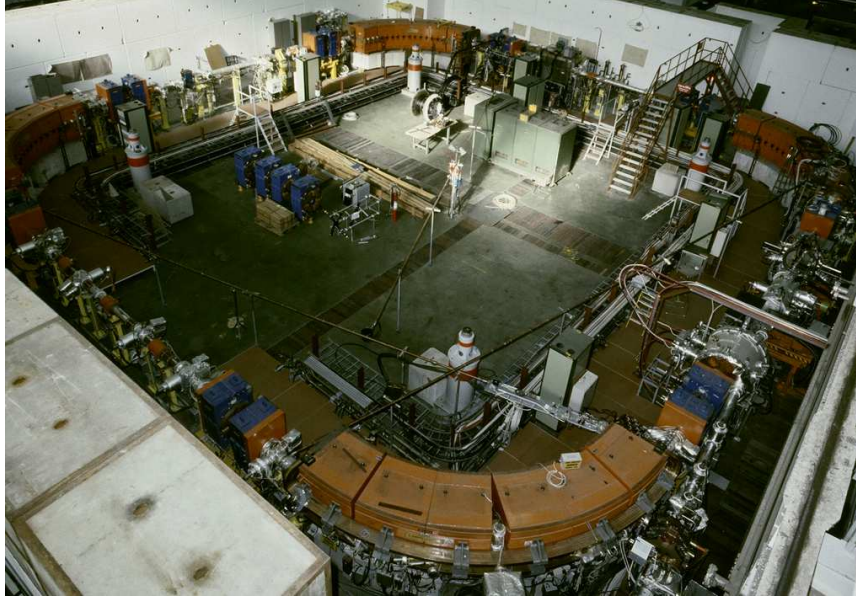


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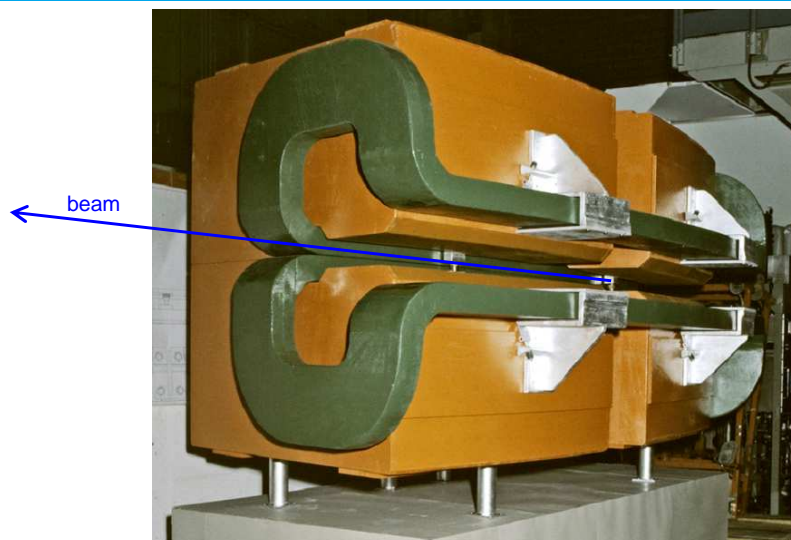


## Circular accelerators: the synchrotron

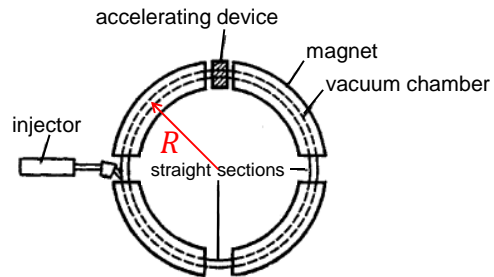
Low Energy Antiproton Ring (LEAR) at CERN



## Dipole magnet



## Circular accelerators: the synchrotron

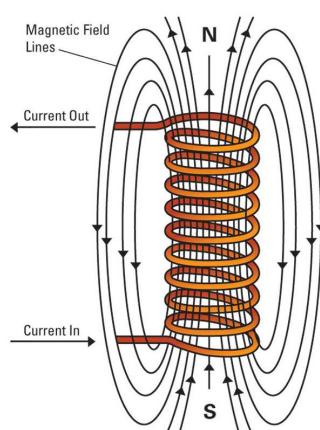


$$\left. \begin{aligned} \vec{B} \perp \vec{v} &\rightarrow F = qvB \\ \vec{F} \perp \vec{v} &\rightarrow F = m \frac{v^2}{R} \quad (\text{circular motion}) \end{aligned} \right\} qB = \frac{mv}{R} \rightarrow R = \frac{(mv)_{\max}}{qB_{\max}} = \text{constant}$$

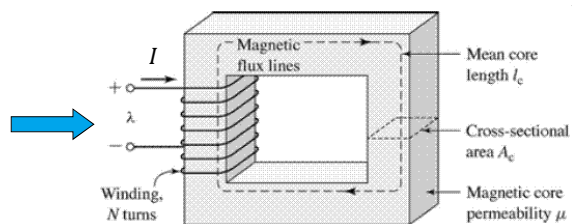
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## Electromagnet



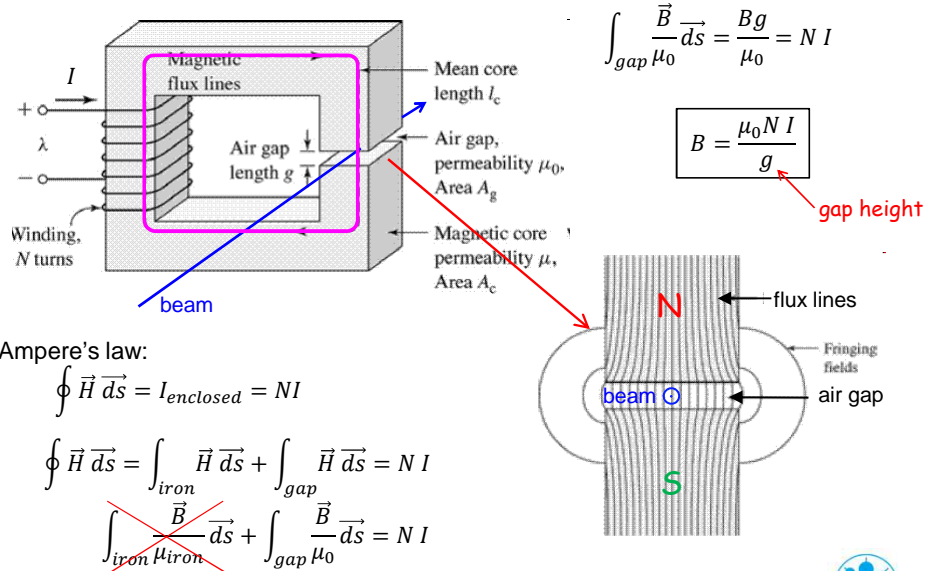
permeability of iron = 300...10000 larger than air



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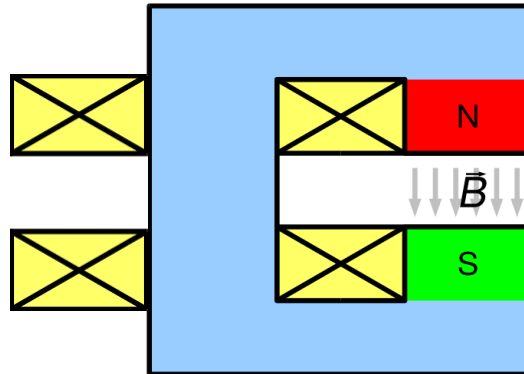
## Dipole magnet



## Dipole magnet cross section

increase  $B \rightarrow$  increase current, but power dissipated  $P = R \cdot I^2$   
 $\rightarrow$  large conductor cables

## Dipole magnet cross section

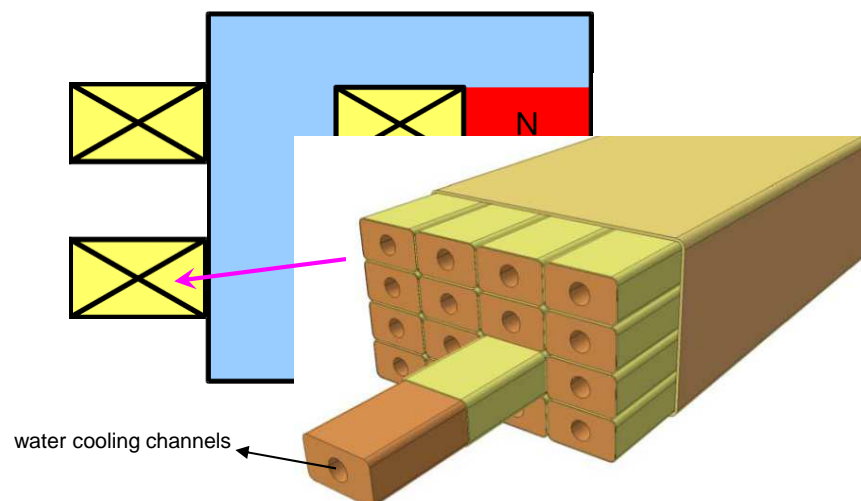


increase  $B \rightarrow$  increase current, but power dissipated  $P = R \cdot I^2$   
 $\rightarrow$  large conductor cables

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## Dipole magnet cross section

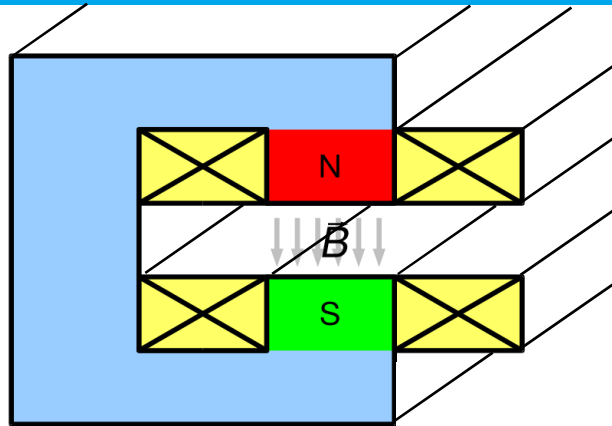


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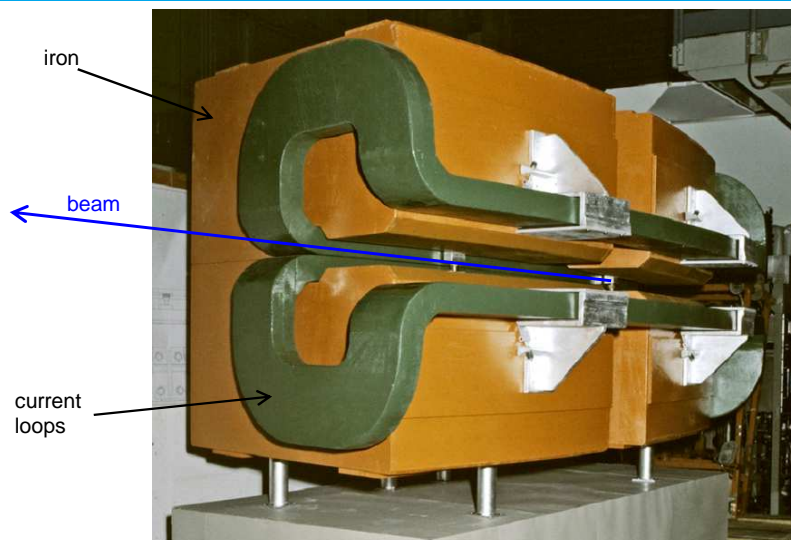
## Dipole magnet cross section



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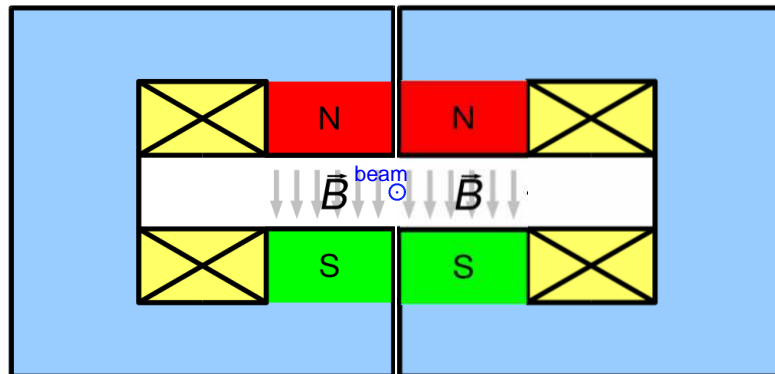
## Dipole magnet



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## Dipole magnet cross section

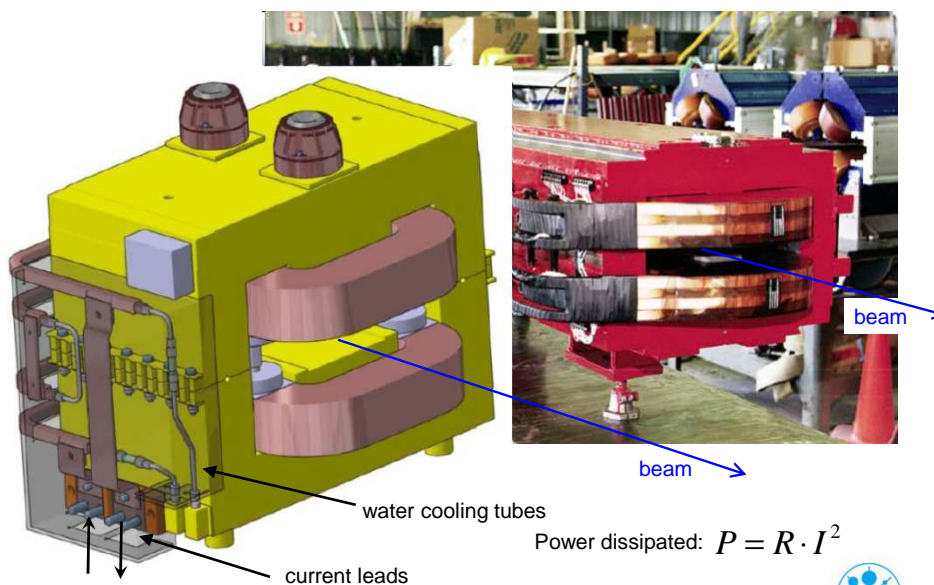


C magnet + C magnet = H magnet

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## Dipole magnet cross section (another design)

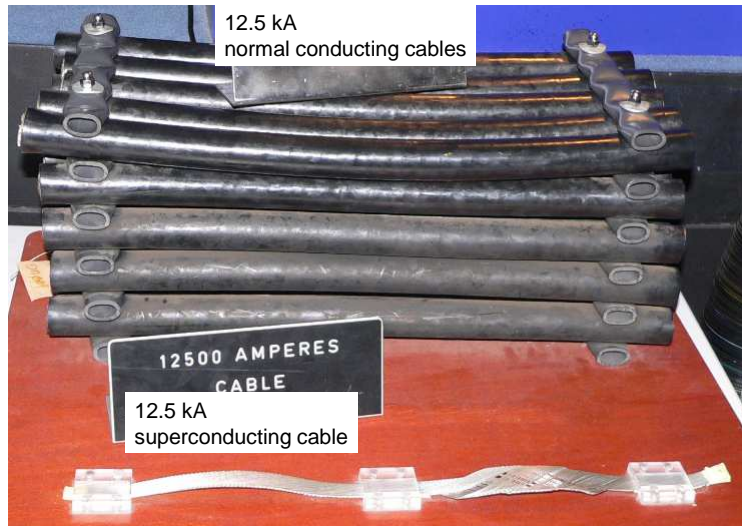


Power dissipated:  $P = R \cdot I^2$

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## Superconductivity

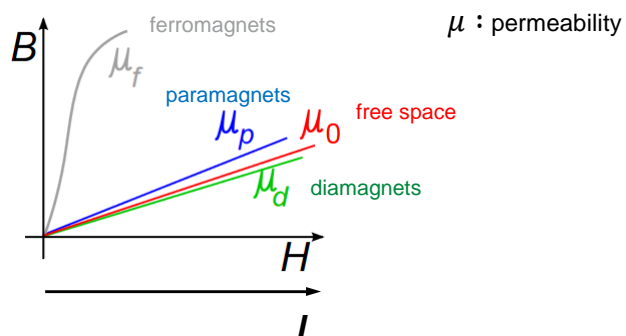


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## Saturation of iron: 1.6 – 2 T

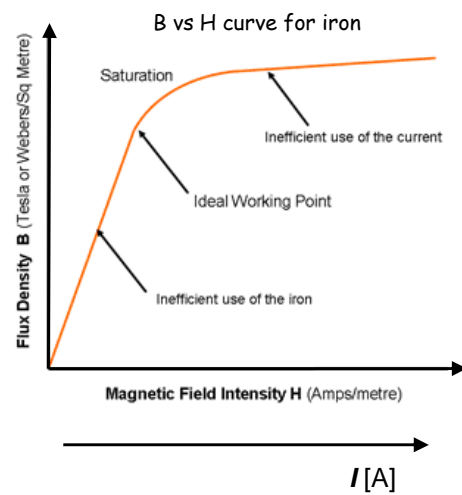
increase  $B \rightarrow$  increase current, but power dissipated  $P = R \cdot I^2$   
 $\rightarrow$  large conductor cables  
 $\rightarrow$  saturation effects



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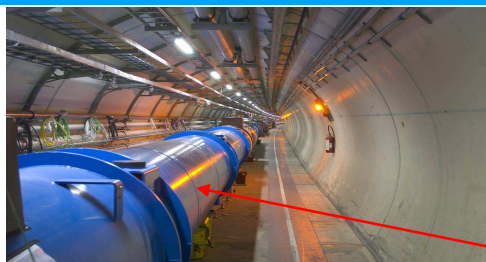
## Saturation of iron: 1.6 – 2 T



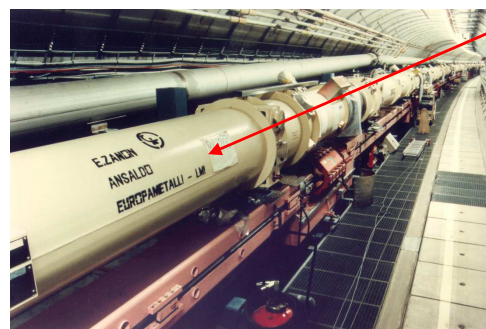
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## Superconducting dipole magnets



LHC




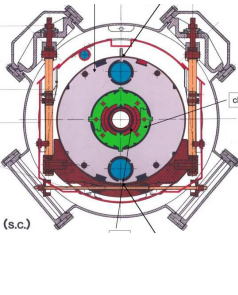
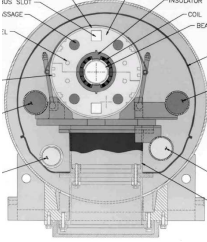
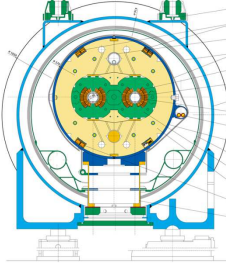
HERA

superconducting dipoles

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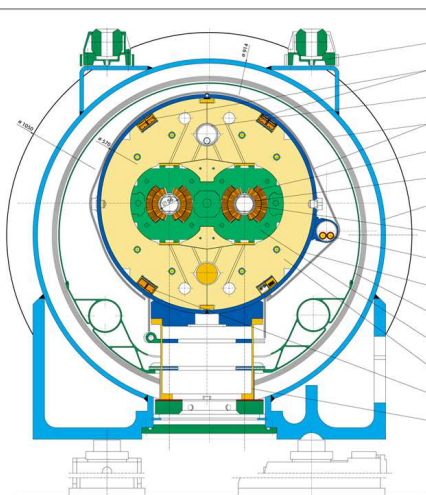
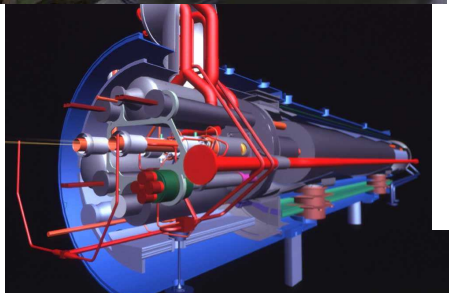
## Superconducting dipole magnets: cross section

Tevatron	HERA	RHIC	LHC
Fermilab Chicago (USA)	DESY Hamburg (Germany)	Brookhaven Long Island (USA)	CERN Geneva (Switzerland)
4.5 T	5.3 T	3.5 T	8.3 T
			

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## Superconducting dipole magnets



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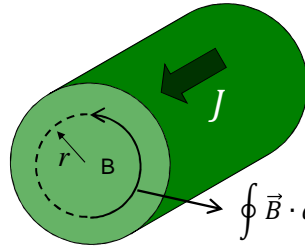


## Dipole field inside 1 conductor

$J$ : uniform current density

Ampere's law:

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 I$$



$$\oint \vec{B} \cdot d\vec{s} = \oint B ds = 2\pi r B = \mu_0 \pi r^2 J$$

$$B = \frac{\mu_0 J}{2} r$$

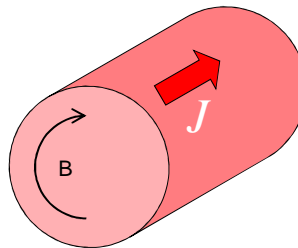
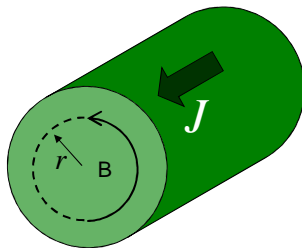
$$\left\{ \begin{array}{l} B_x = -\frac{\mu_0 J}{2} r \sin \theta \\ B_y = \frac{\mu_0 J}{2} r \cos \theta \end{array} \right.$$

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## Dipole field inside 2 conductors

$J$  = uniform current density



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## Dipole field inside 2 conductors

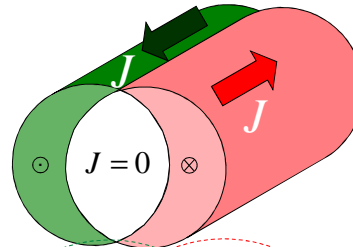
$J$  = uniform current density

one conductor: 
$$\begin{cases} B_x = -\frac{\mu_0 J}{2} r \sin \theta \\ B_y = \frac{\mu_0 J}{2} r \cos \theta \end{cases}$$

superposition:

$$B_x = \frac{\mu_0 J}{2} (-r_1 \sin \theta_1 + r_2 \sin \theta_2)$$

$$B_y = \frac{\mu_0 J}{2} (r_1 \cos \theta_1 - r_2 \cos \theta_2)$$



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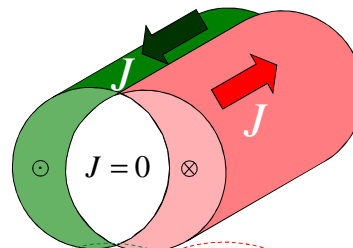
## Dipole field inside 2 conductors

$J$  = uniform current density

one conductor: 
$$\begin{cases} B_x = -\frac{\mu_0 J}{2} r \sin \theta \\ B_y = \frac{\mu_0 J}{2} r \cos \theta \end{cases}$$

$$B_x = \frac{\mu_0 J}{2} (-r_1 \sin \theta_1 + r_2 \sin \theta_2) = 0$$

$$B_y = \frac{\mu_0 J}{2} (r_1 \cos \theta_1 - r_2 \cos \theta_2)$$



$$h = r_1 \sin \theta_1 = r_2 \sin \theta_2$$

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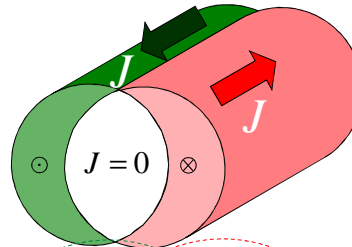




## Dipole field inside 2 conductors

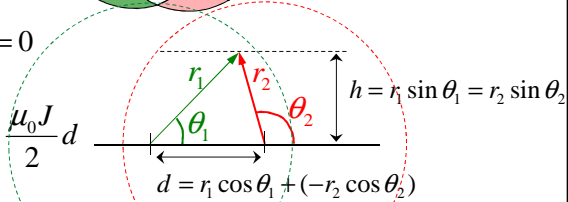
$J$  = uniform current density

one conductor: 
$$\begin{cases} B_x = -\frac{\mu_0 J}{2} r \sin \theta \\ B_y = \frac{\mu_0 J}{2} r \cos \theta \end{cases}$$



$$B_x = \frac{\mu_0 J}{2} (-r_1 \sin \theta_1 + r_2 \sin \theta_2) = 0$$

$$B_y = \frac{\mu_0 J}{2} (r_1 \cos \theta_1 - r_2 \cos \theta_2) = \frac{\mu_0 J}{2} d$$

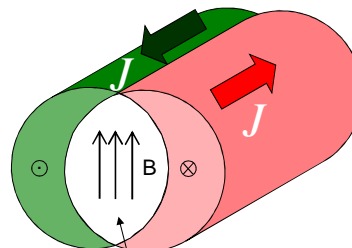


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## Dipole field inside 2 conductors

$J$  = uniform current density



$$B_x = \frac{\mu_0 J}{2} (r_1 \sin \theta_1 - r_2 \sin \theta_2) = 0$$

$$B_y = \frac{\mu_0 J}{2} (r_1 \cos \theta_1 - r_2 \cos \theta_2) = \frac{\mu_0 J}{2} d$$

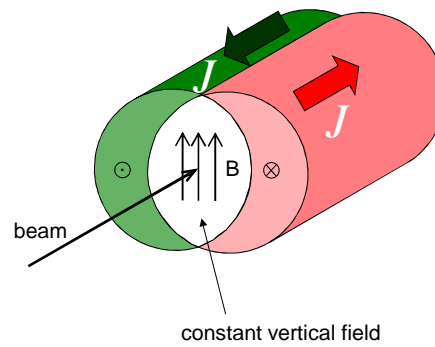
constant vertical field

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## Dipole field inside 2 conductors

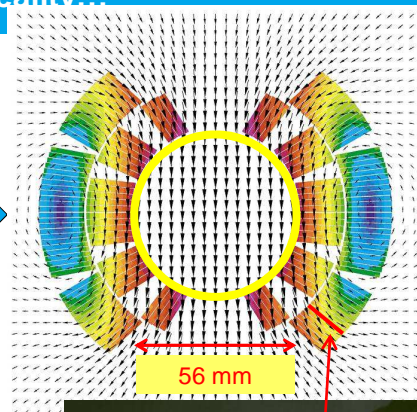
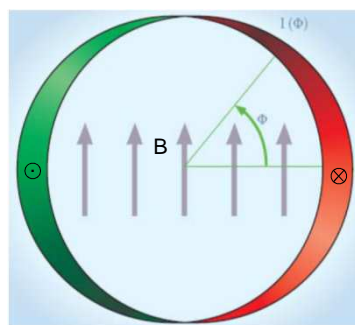


$$B_y = \frac{\mu_0 J}{2} d$$

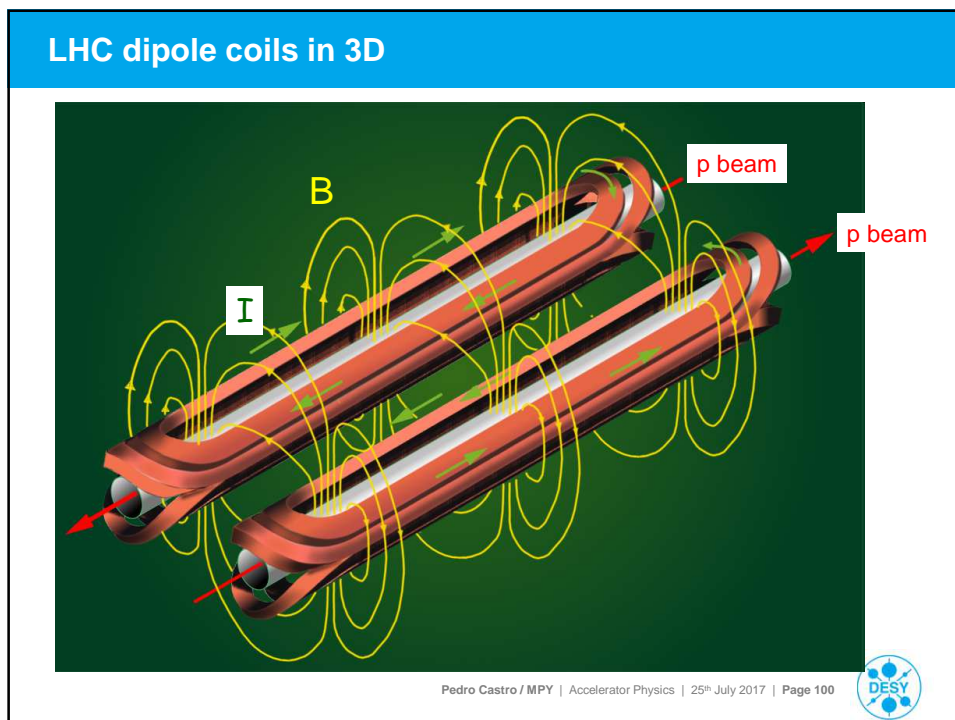
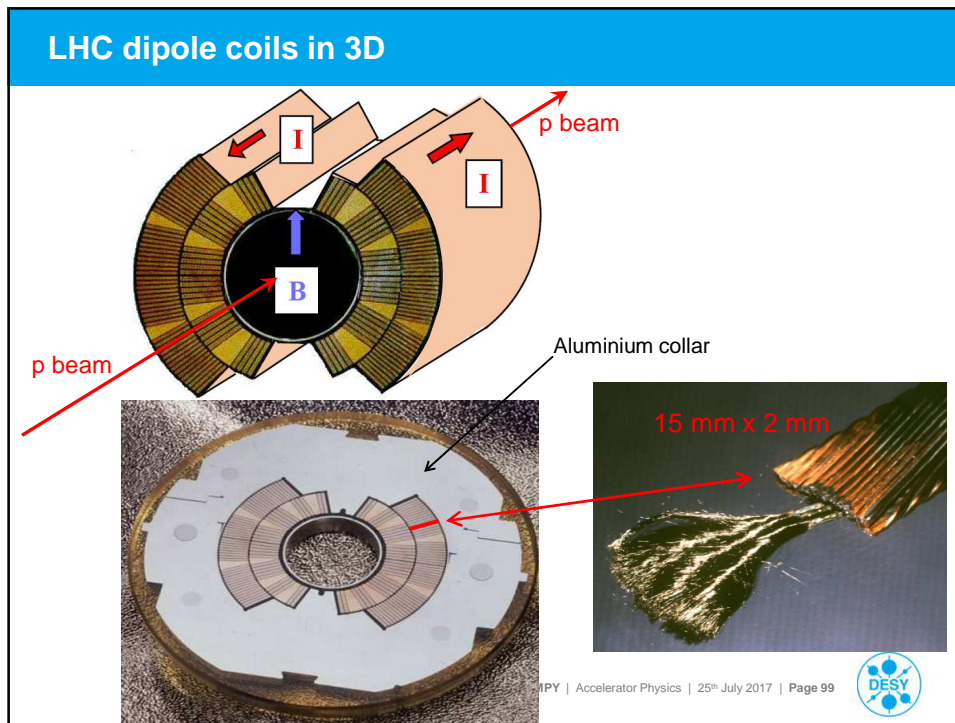
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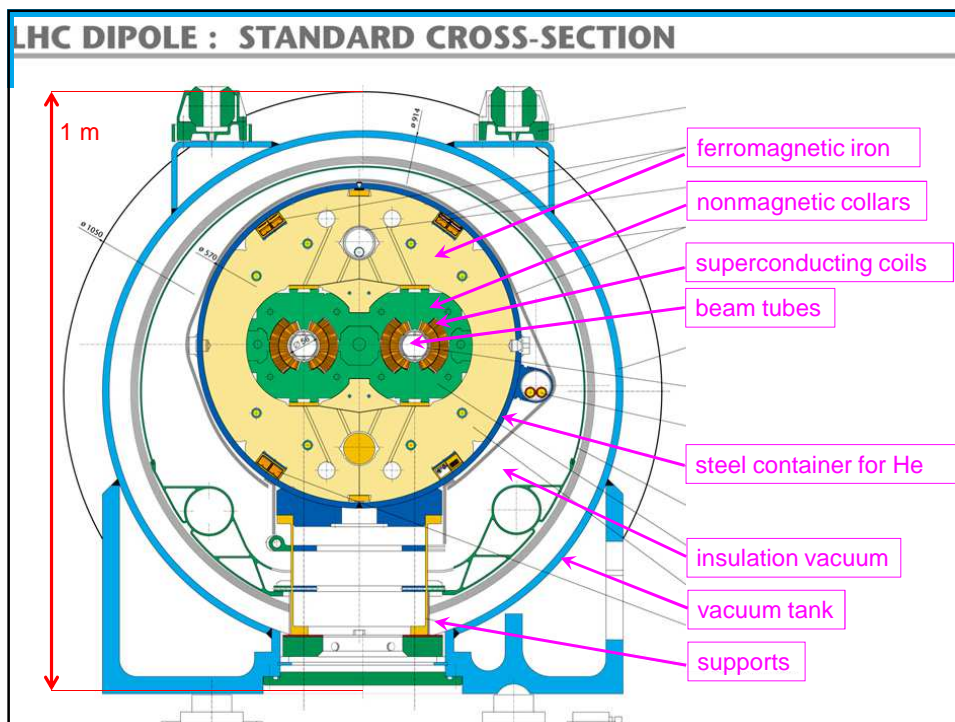
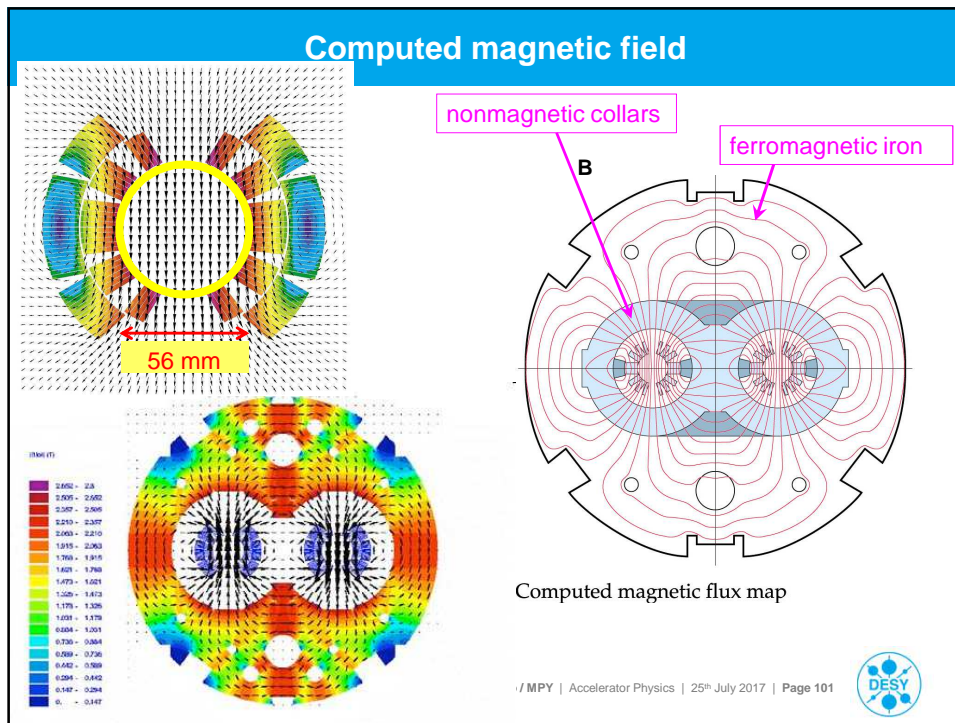


## From the principle ... to the reality...



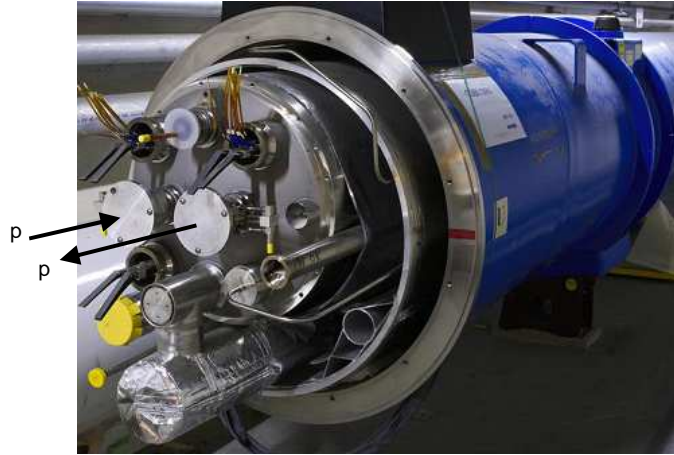
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## Superconducting dipole magnets

LHC dipole magnet interconnection:

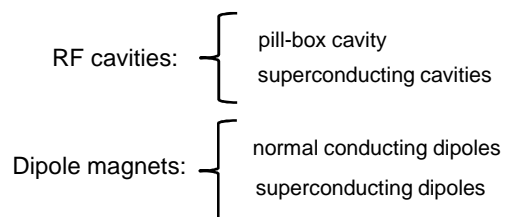


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## Summing-up of this part

Circular accelerators: the synchrotron



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Thank you for your attention

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