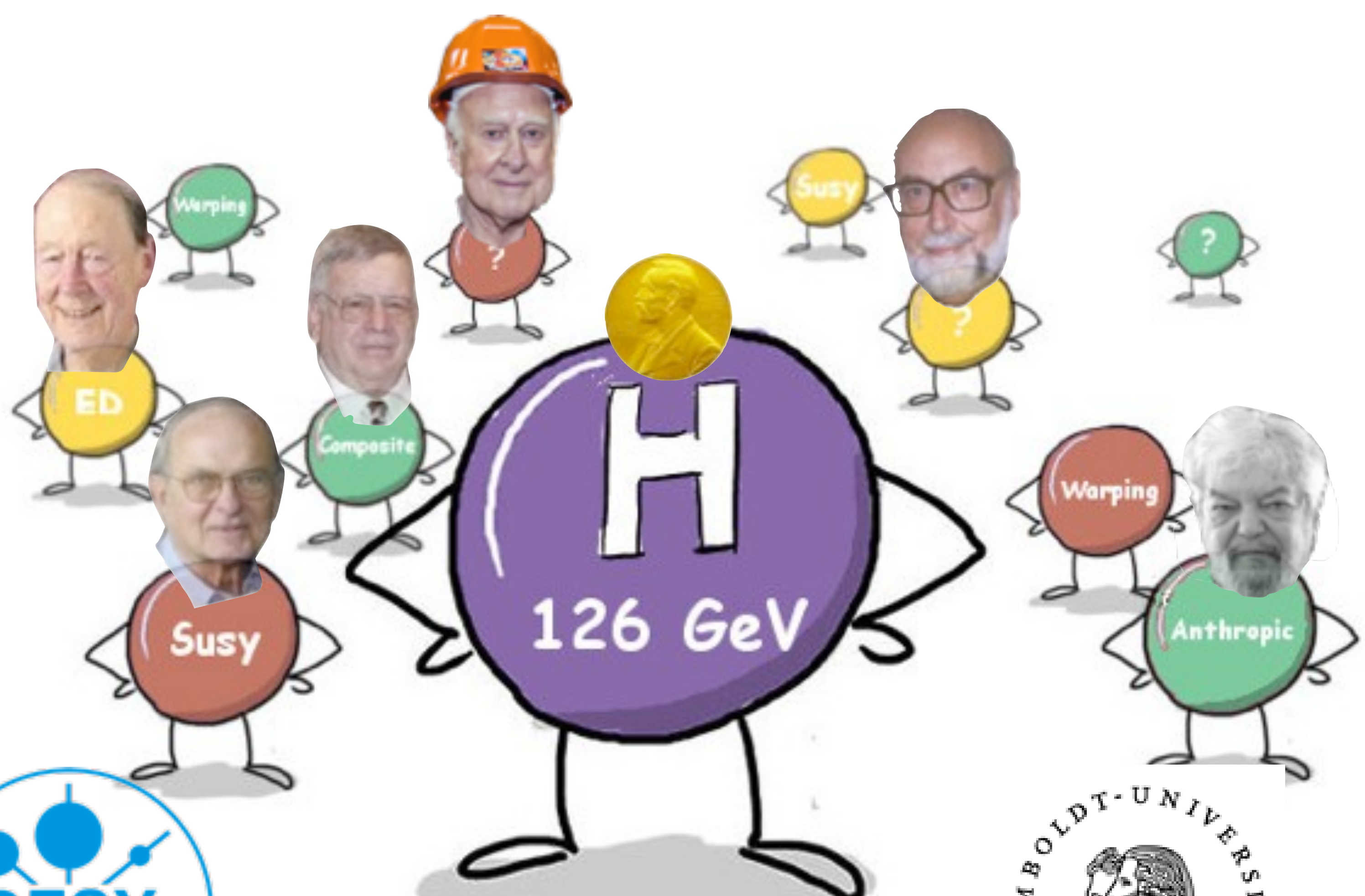
Introduction HEP Theory

DESY summer student lectures 2017



Lectures 1+2/6

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Outline

1. Friday

   Quantum field theory, dimensional analysis, symmetries (space-time and internal gauge symmetries, continuous, global), group theory

2. Friday

   Local symmetries and gauge invariance principle, QED, Standard Model

3. Monday

   Spontaneous symmetry breaking, Goldstone theorem, Higgs mechanism

4. Monday

   Electroweak precision test, stability of the EW vacuum, the hierarchy problem

5. Tuesday

   Field quantization, S-matrix, Feynman rules, scattering, cross sections, decay rates, calculation tricks, sample calculation

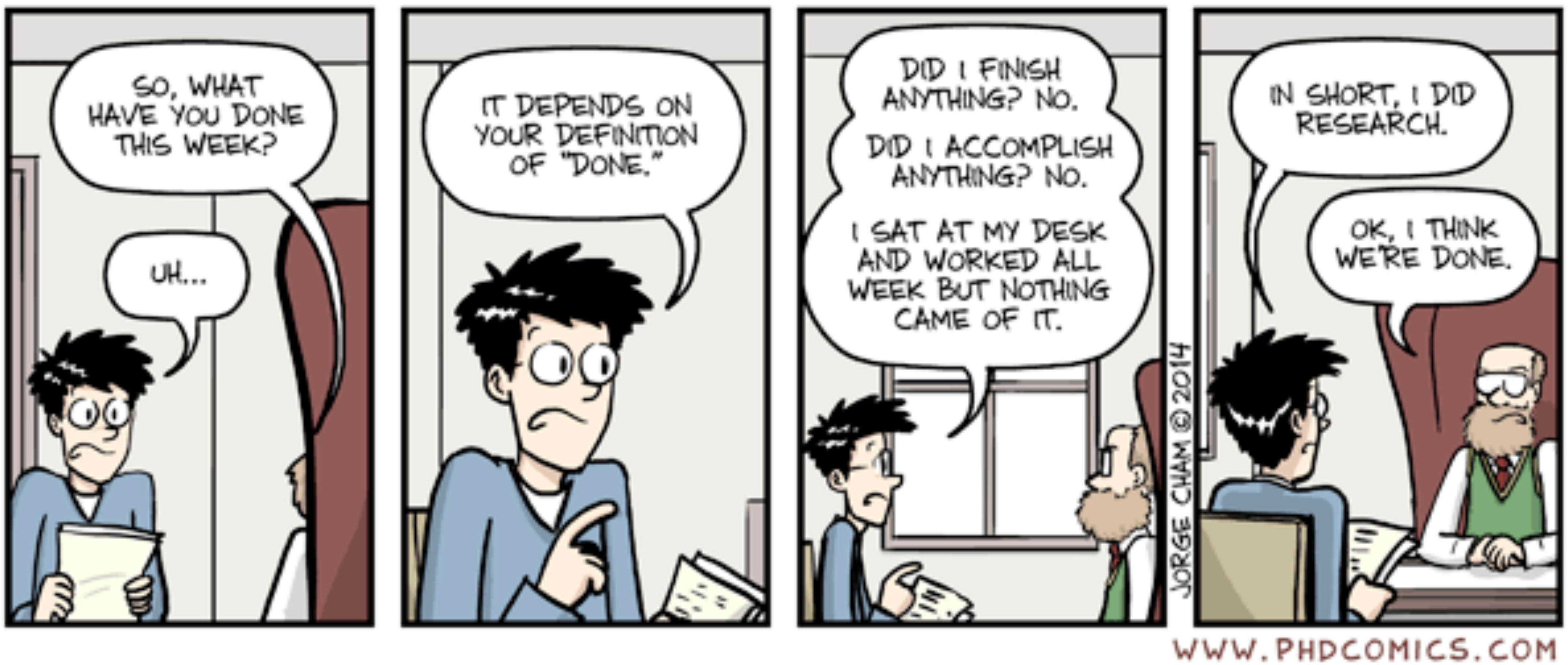
6. Tuesday

   Non-abelian gauge theories, Standard Model Lagrangian and its phenomenological properties, observables

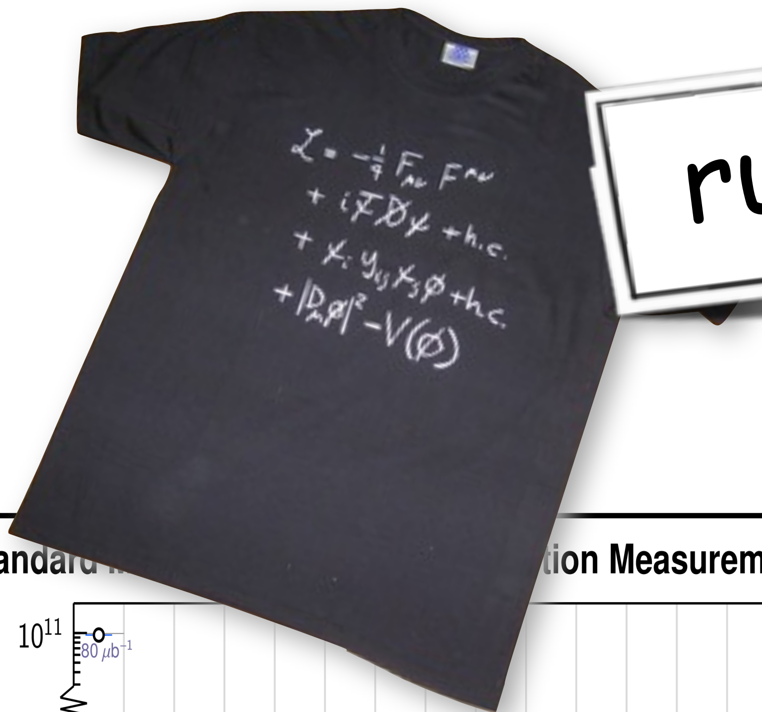
the order will probably change

Ask questions

Your work, as students, is to question all what you are listening during the lectures...

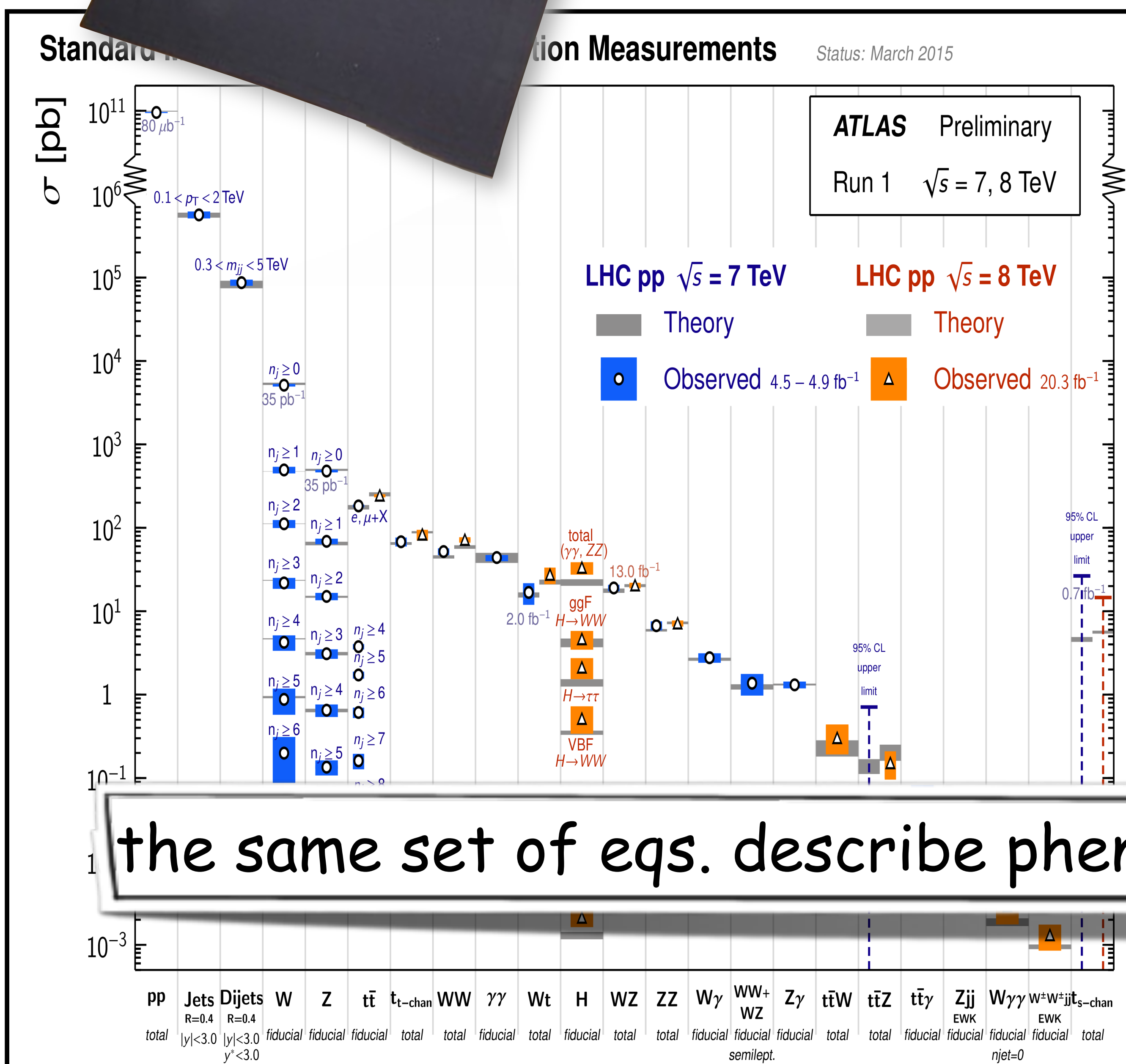


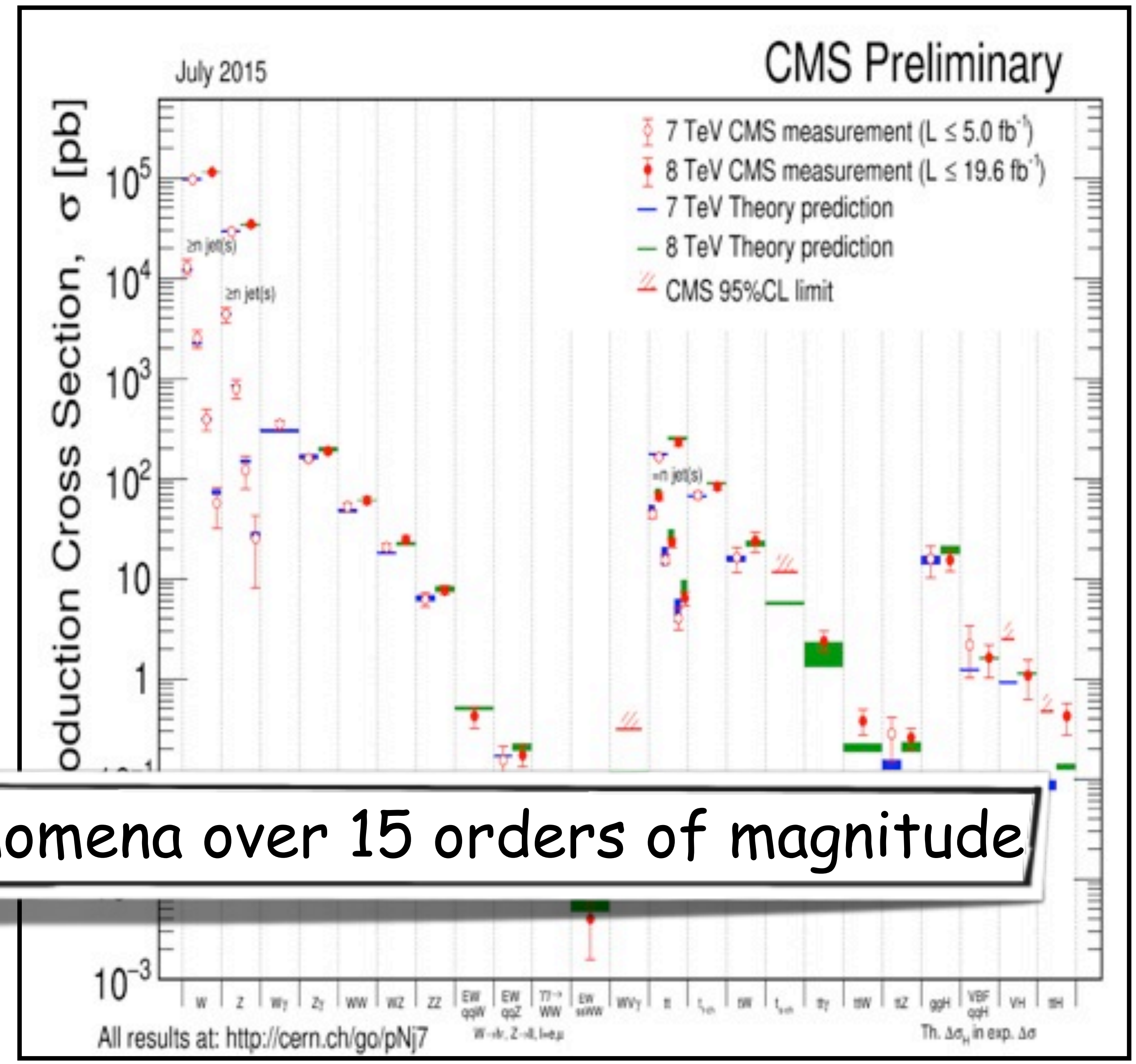
The SM and... the LHC data so far



rules the world!

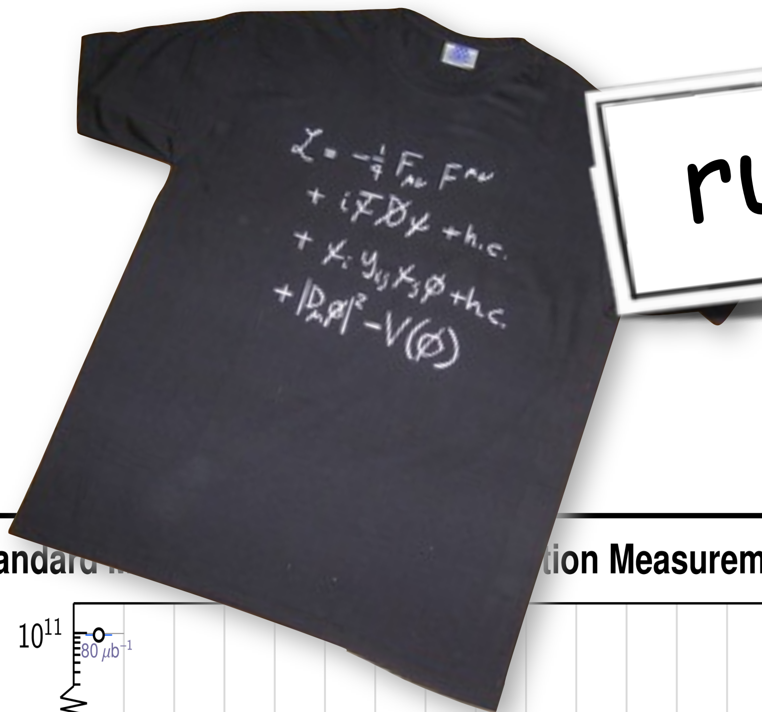
[and we, HEP practitioners, are all entitled for some royalties!]





the same set of eqs. describe phenomena over 15 orders of magnitude

The SM and... the LHC data so far



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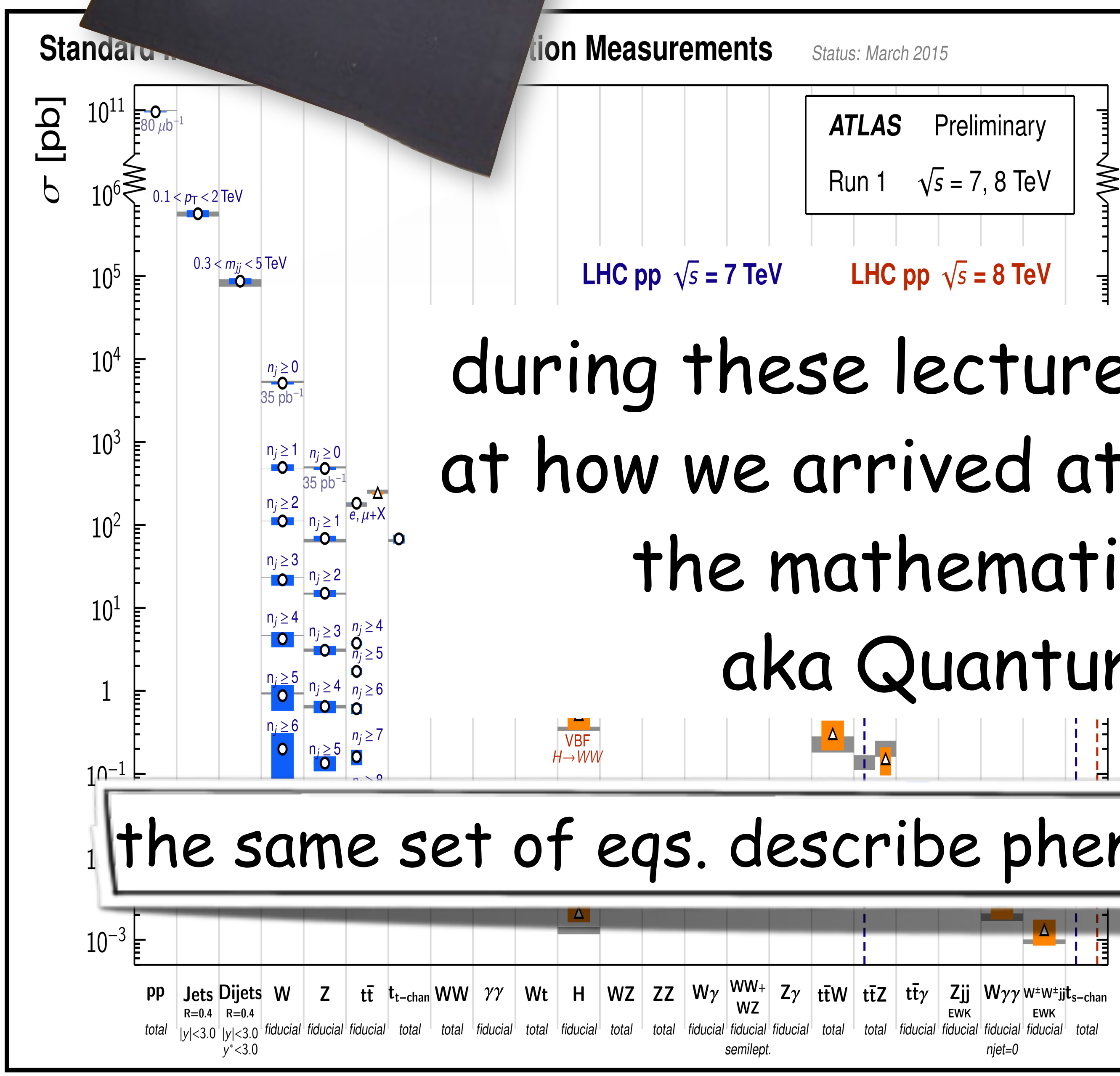
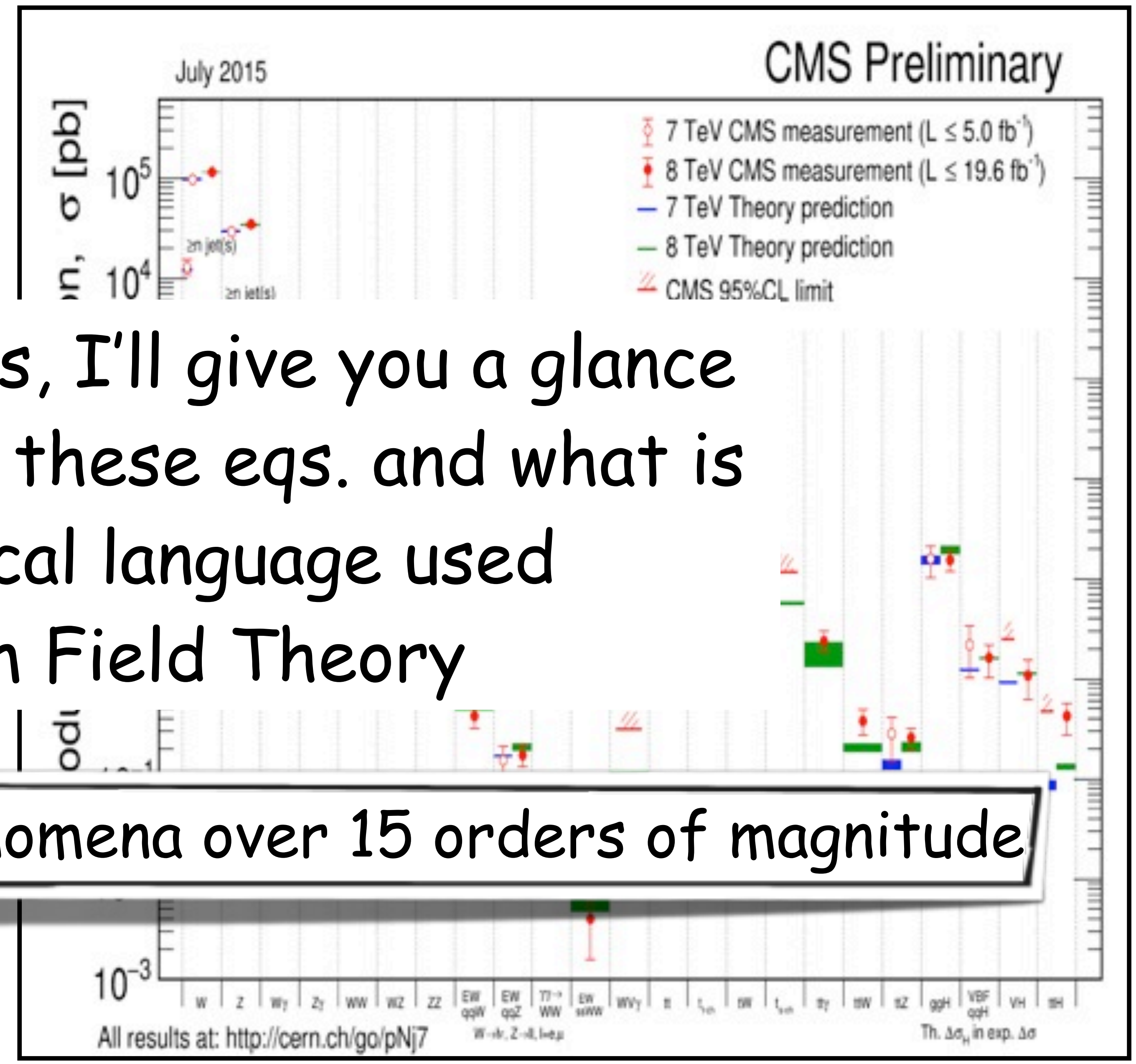




during these lectures, I'll give you a glance
at how we arrived at these eqs. and what is
the mathematical language used
aka Quantum Field Theory

the same set of eqs. describe phenomena over 15 orders of magnitude

The elementary particles

What is a particle?

A small, quantic and fast-moving object

Quantum Mechanics

➡ duality wave-particle

➡ Heisenberg inequalities

energy non-conservation
on time intervals Δt energy fluctuations ΔE

$$\Delta t \times \Delta E \sim \hbar$$

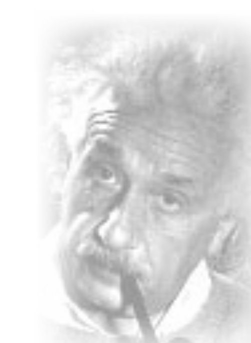
Special Relativity

➡ space-time

constant speed of light
ex: how long does it take for a photon to travel from your feet to your brain?

➡ energy = mass

number of particles
is not constant



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ex: how long does it take for a photon to travel from your feet to your brain?

➡ energy = mass

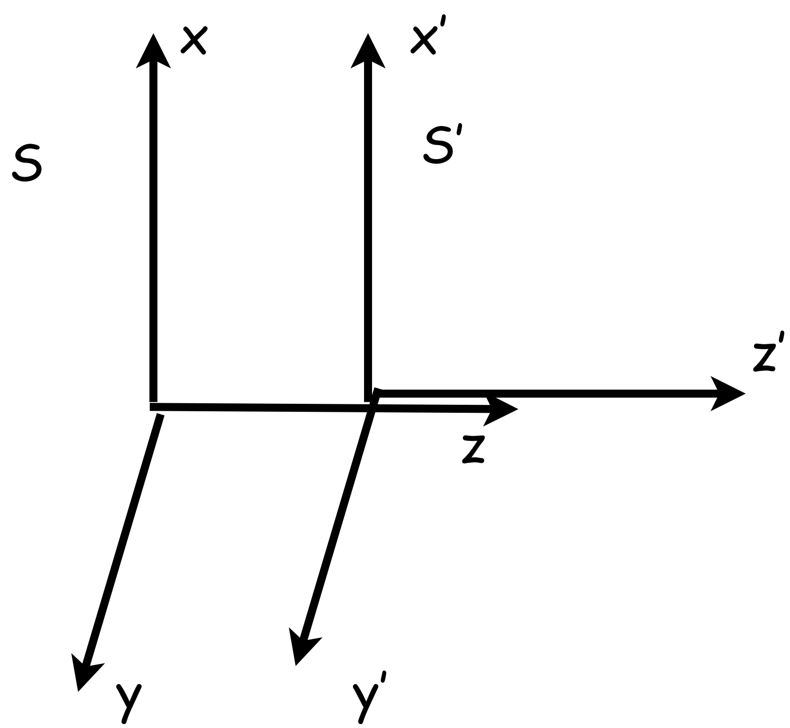
try to localize a particle $\Delta x \ll 1/m \Rightarrow \Delta k \gg m$ ie $\Delta E \gg m$

ultrarelativistic particles
particle creations

Special relativity

The two postulates of Special relativity

- Speed of light is the same in all reference frames
- Causality



Look for coordinate transformations that satisfy these requirements

Unique choice:
Lorentz transformations

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix}$$

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Lorentz transformations keep the lightcone fixed

Special relativity

consider two systems of coordinates

S S'

$$\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} \to \begin{pmatrix} ct' = \gamma\,(ct - \beta x) \\ x' = \gamma\,(-\beta ct + x) \\ y' = y \\ z' = z \end{pmatrix}$$

$$\beta = \frac{v}{c} \quad \text{and} \quad \gamma = \frac{1}{\sqrt{1-\beta^2}}$$

consider two events E_1 and E_2 characterized by their space-time coordinates

E_1

$$t_1 = 0 \qquad t'_1 = 0$$
$$x_1 = 0 \qquad x'_1 = 0$$

E_2

$$t_2 > 0 \qquad ct'_2 = \gamma\,(ct_2 - \beta x_2)$$
$$x_2 > 0 \qquad x'_2 = \gamma\,(-\beta ct_2 + x_2)$$

t'_2 can be positive or negative
causality ≠ time ordering

$$\Delta'^2 = (ct'_2)^2 - (x'_2)^2 = (ct_2)^2 - x_2^2 = \Delta^2$$

Lorentz transformations

$$\begin{pmatrix} ct' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ z \end{pmatrix} \qquad \beta = \frac{v}{c} \qquad \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Time dilation

consider time interval $\tau = t'_2 - t'_1$ in S′, the rest frame of a particle located at $z'_1 = z'_2 = 0$.

then in frame S where the particle is moving: $t_2 - t_1 = \gamma\tau$

-->The observed lifetime of a particle is $\gamma \times \tau$

so it can travel over a distance $\beta c \gamma \tau$

-->muons which have a lifetime $\tau \sim 2 \times 10^{-6}$ s produced by reaction of cosmic rays with atmosphere at 15-20 km altitude can reach the surface

ex: estimate the energy of the cosmic rays

length contraction

an object at rest in S′ has length $L_0 = z'_2 - z'_1$

It measures in S $z_2 - z_1 = L_0/\gamma$

--> densities increase $\rho_0 = \Delta n/(\Delta x' \Delta y' \Delta z')$ $\rho = \Delta n/(\Delta x \Delta y \Delta z) = \gamma \rho_0$

$\Delta x \Delta y \Delta z \Delta t$ is invariant

Four vectors

Time and space get mixed-up under Lorentz transformations. They are considered as different components of a single object, a four-component spacetime vector:

$$x^\mu = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 \\ x^1 \\ x^2 \\ x^3 \end{pmatrix} = \begin{pmatrix} x^0 \\ \vec{x} \end{pmatrix}$$

By construction: $x_\mu x^\mu = \mathbf{x}^2 = {x^0}^2 - \vec{x}^2$ is invariant

$$d\tau = \sqrt{dt^2 - d\vec{x}^2}$$ is invariant

Lorentz invariant action built with the proper time $d\tau$

$$\boxed{S = -m \int d\tau = \int \mathcal{L} dt}$$

$$\left.\begin{aligned} \mathcal{L} &= -m\sqrt{1-\dot{x}^2} \\ \vec{p} &= \frac{\partial \mathcal{L}}{\partial \dot{x}} = m\gamma\vec{\beta} \\ E &= \vec{p}.\dot{x} - \mathcal{L} = m\gamma \end{aligned}\right\} \quad \vec{p} = E\vec{\beta}$$

Energy momentum

so we find: $$m^2 = E^2 - \vec{p}^2$$

This suggests to define the four-vector $$p^\mu = (\frac{E}{c}, p_x, p_y, p_z)$$

$$m = 0 \to \beta = 1 \to \gamma = \infty \to \tau = \infty$$

a massless particle cannot decay

Conservation energy-momentum

Consider collision between A and B

Define center of mass (CM) frame as where

$$\vec{p}_A + \vec{p}_B = 0$$

Energy available in center of mass frame is an invariant: $\sqrt{s} = E_* = E_A + E_B$

$$\mathbf{p}_{tot}^2 = E_*^2$$

1) Collision on fixed target

B is at rest in lab frame, $E_B = m_B$ and E_A is energy of incident particle

$$E_*^2 = m_A^2 + m_B^2 + 2m_B E_A$$

2) Colliding beams

A and B travel in opposite directions

$$E_*^2 = m_A^2 + m_B^2 + 2(E_A E_B + |p_A||p_B|) \approx 4E_A E_B$$

if $m_A, m_B \ll E_A, E_B$

So for fixed target machine $E_* \sim \sqrt{2m_B E_A}$

While for colliding beam accelerators $E_* \sim 2E$

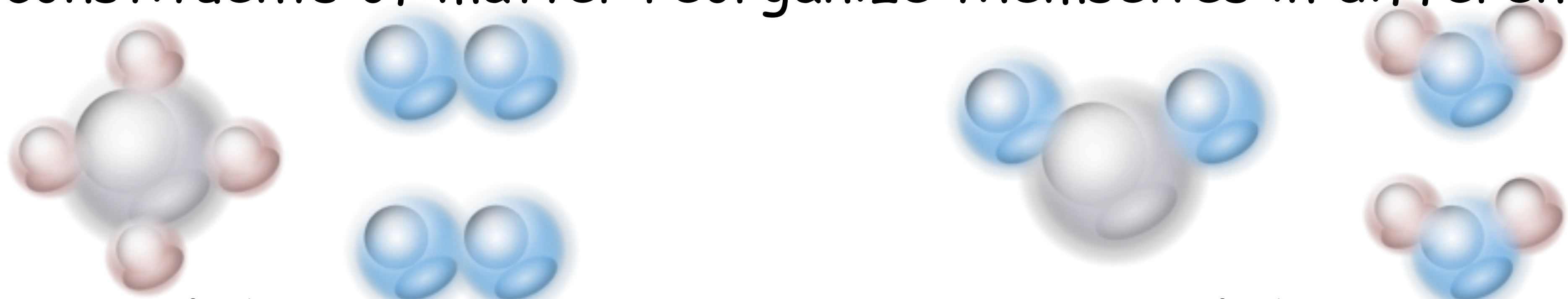
To obtain 2 TeV in the CM with a fixed proton target accelerator the energy of a proton beam would need to be 2000 TeV!

The elementary particles

Particle physics is special

- ***Chemistry: reorganization of matter***

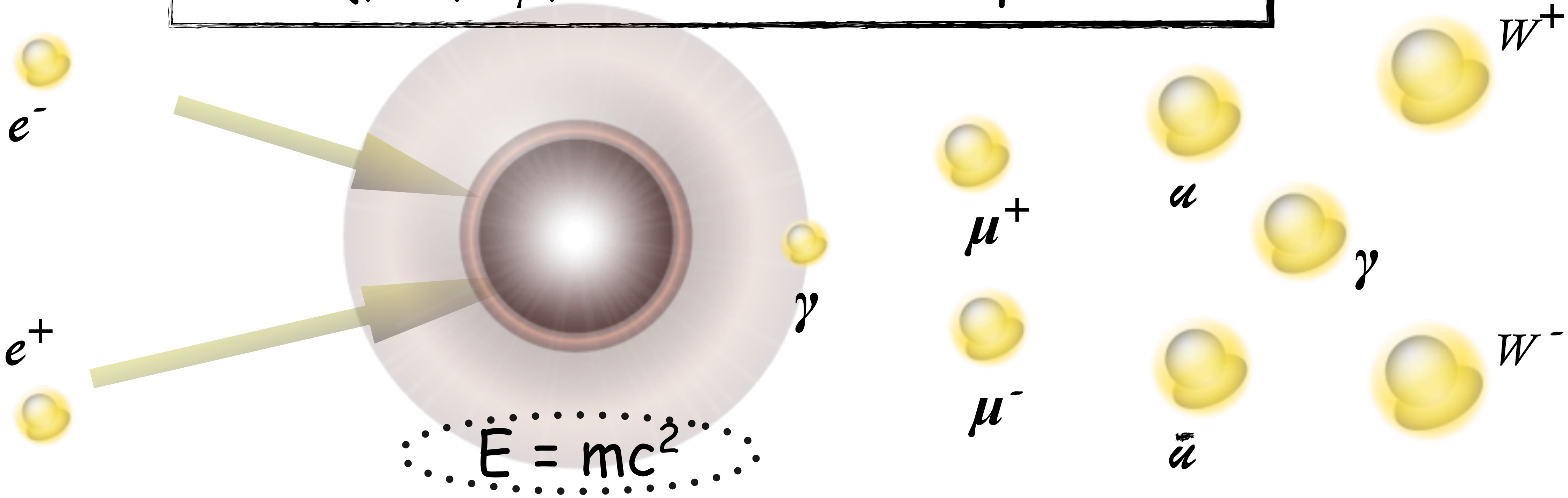
the various constituents of matter reorganize themselves in different structures



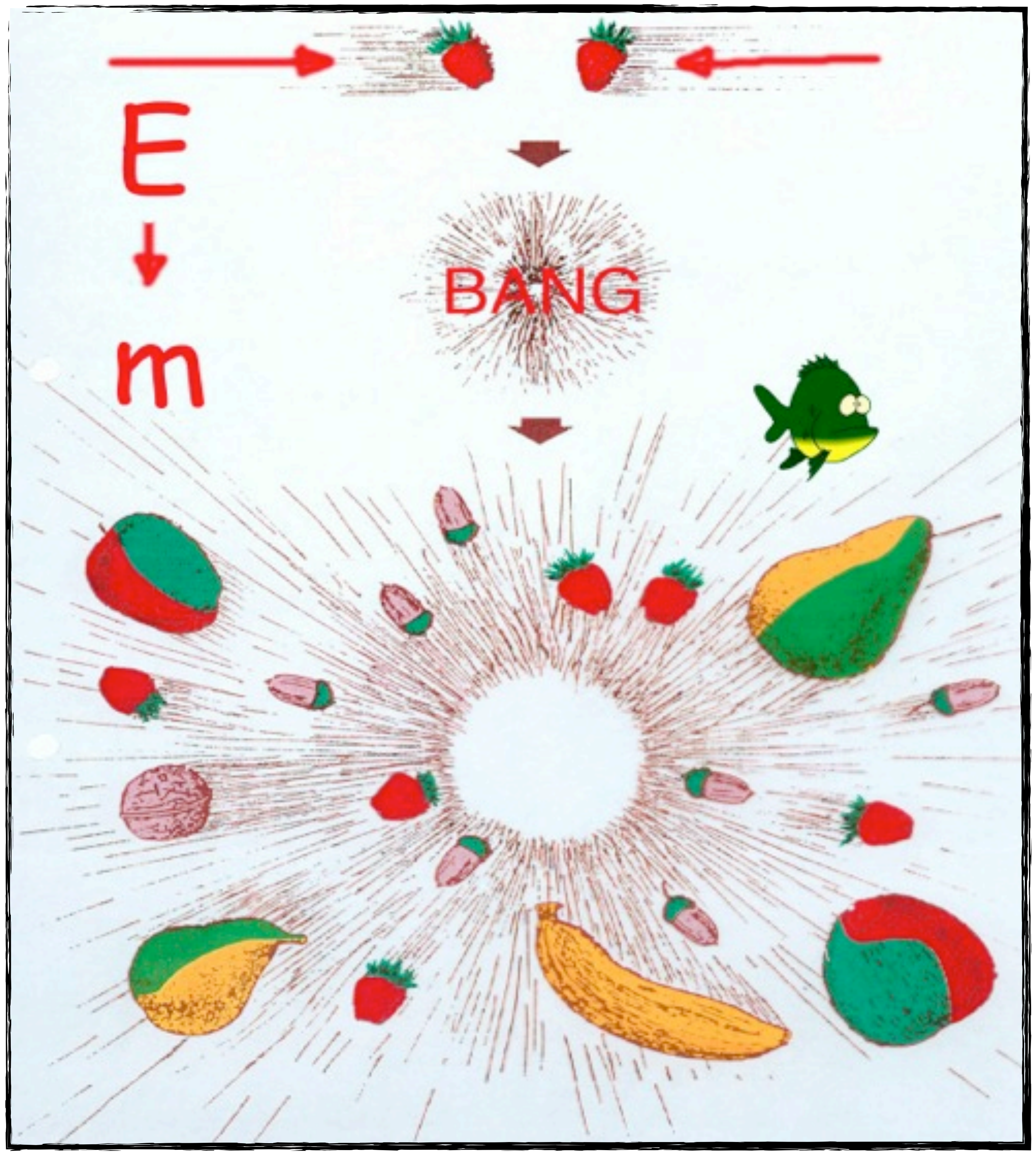
$$CH_4 + 2\,O_2 \rightarrow CO_2 + 2\,H_2O$$

- ***Particle physics: transformation matter $\leftrightarrow$ energy***

$\boxed{\text{NB: } Q, L_e, L_\mu, B = \text{conserved quantities}}$



Classical vs. Quantum Collisions



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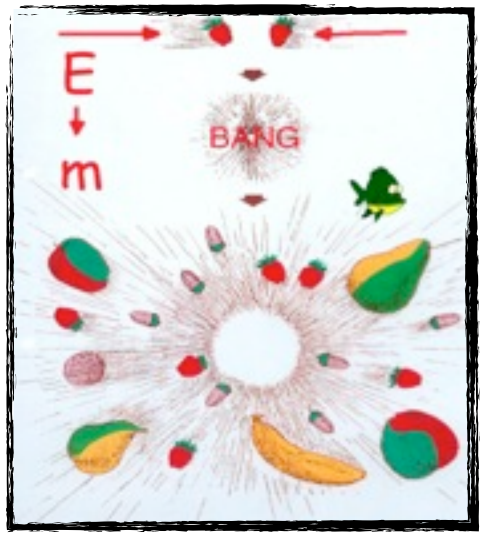
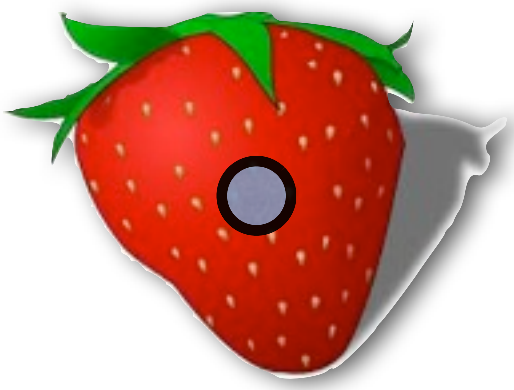


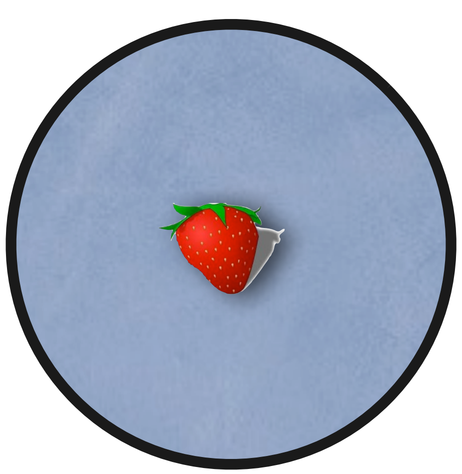
Compton wavelength $\lambda = \frac{\hbar}{mc}$



strawberry : m~30 g ~10^{25} GeV/c^2 → λ~10^{-40} m

classical : $\lambda \ll R$

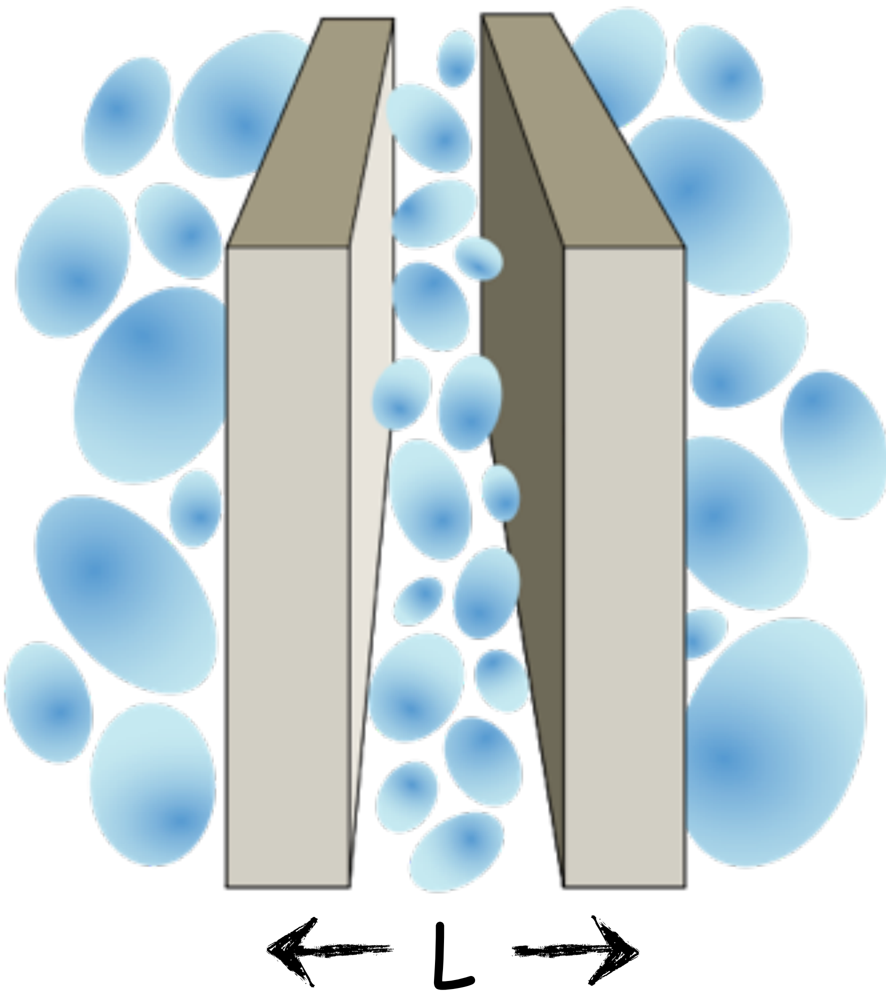
quantum : $\lambda \gg R$



e^- : m~9.1×10^{-31} kg ~0.5 MeV/c^2 → λ~10^{-13} m

p : m~1.6×10^{-27} kg ~1 GeV/c^2 → λ~10^{-16} m

Vacuum fluctuations



Casimir (1948)

($S \gg L^2$: no boundary effects)

attractive force between two neutral plates

ideal conductors (electric conductivity=∞) and uncharged

E=energy of virtual photons between the plates

E ↘ when L ↘ ➡ attractive force

Force per unit area

QM Special Relativity

$$\boxed{\frac{dF}{dS} = -\frac{\pi^2}{240}\frac{\hbar c}{L^4}}$$

non-trivial coefficient

dim. analysis

numerically: pressure of ~ 1 atm for a 10nm separation

The quantum vacuum is not empty

Energy Scales of Particle Physics

e^- e^- ⟶

v=0 v~700 km/s

$1\ \text{TeV} = 10^{12}\ \text{eV}$

1eV=energy of an electron accelerated
by a potential difference of 1 volt

$1\ \text{eV} = 1.6\times10^{-19}$ Joule

1 kg sugar = 4000 kCalories= 17 millions of Joule

but 1 kg sugar ≈ 10^{27} protons

0.1 eV / protons

If one wanted to accelerate each protons contained in 1kg of sugar to 14 TeV, (s)he would need the caloric energy contained in 10^{14}kg of sugar* or 1% of the total energy produced yearly

*yearly worldwide production of sugar=150 millions of tons≈10^{11} kg

The LHC: some numbers

- total amount of matter accelerated by the LHC during 10 years:

  ~ 5 μg (hydrogen nuclei)

- yearly energy consumption:

  ~800,000 MWh (~2×10^9 x the energy stored in the battery of my laptop)

- beam energy (~ protons) :

  350 MJ ≈ E_{kin} (TGV) can melt 500 kg of copper

- energy stored in the magnets

  10 GJ

- amount of information produced :

  40 M collisions/s, 1 collision=1MB

  trigger: only 100 collisions/s are recorded

  15 PB data/year

  (1 TB=yearly book production, 1000 PB=total information produced on earth in one year)

- CERN budget :

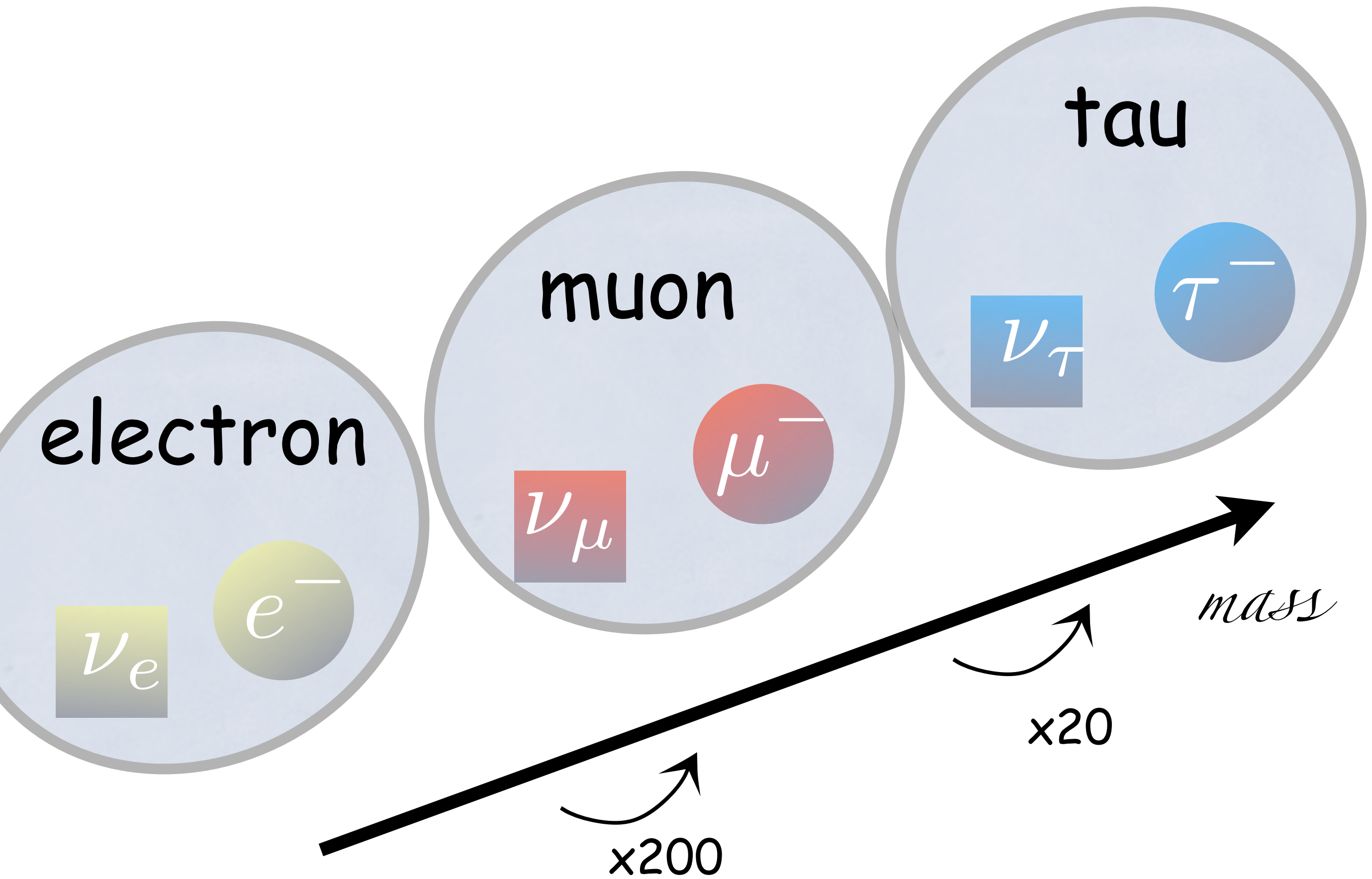
  ~ 1 Bn CHF/year (worldwide military expenses: 1200 Bn $)

The Standard Model: matter

the genetic code of matter

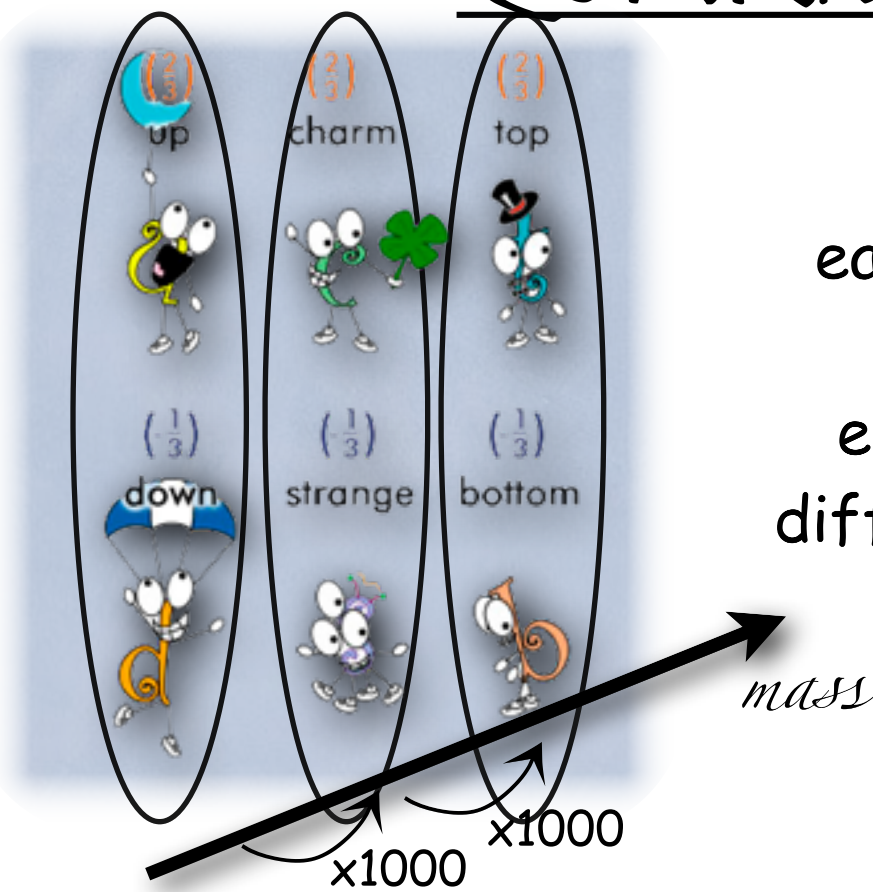
the elementary particles all the other particles are made of

LEPTONS



no composite states
made of leptons

QUARKS



each of the 6 quarks exist with 3 different colors

composite states (white object)

- baryons
  - proton p=uud
  - neutron n=udd
- mésons

+ antiparticles

Classical/Quantum EM & Antimatter

an electron makes an electric field which carries an energy

$$\Delta E_{\text{Coulomb}}(r) = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

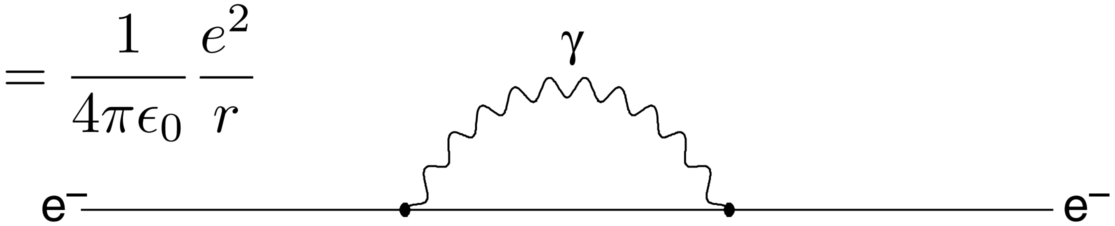
and interacts back to the electron and contributes to its mass $\delta m c^2 = \Delta E$

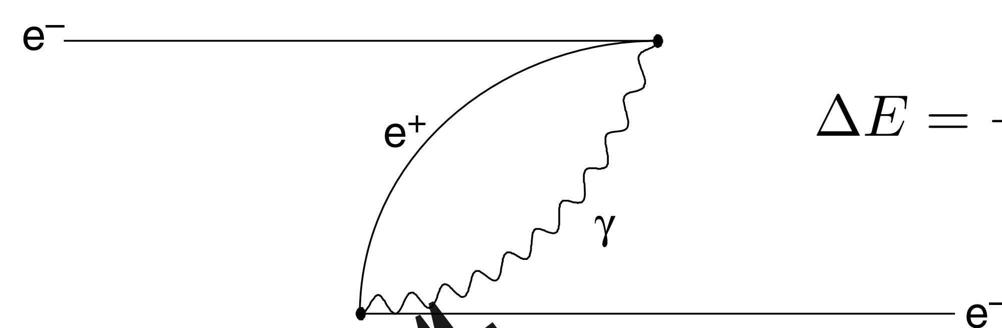
$$\delta m < m_e \quad \Rightarrow \quad r > r_e \equiv \frac{e^2}{4\pi\epsilon_0 m_e c^2} \sim 10^{-13}\ \text{m} \quad \text{i.e.} \quad E < \frac{\hbar c}{r_e} \sim 5\,\text{MeV}$$

At shortest distances or larger energies, classical EM breaks down

Quantum EM

$$\Delta E = \frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$





$$\Delta E = -\frac{1}{4\pi\epsilon_0}\frac{e^2}{r}$$

$$\Delta E = \frac{3\alpha}{4\pi} m_e c^2 \log\frac{\hbar}{r m_e c}$$

Weisskopf '39

new states ≈ softer high-energy (UV) behavior: $\delta m < 0.1 m_e \Rightarrow E < 10^{21}\,\text{GeV}$

Antimatter and Dirac equation

Schrödinger's equation (1926) is non-relativistic

(cannot account for creation/annihilation of particles)

Schrödinger Equation (1926): $$\left(i\hbar\frac{\partial}{\partial t}+\frac{\hbar^2}{2m}\Delta - V\right)\Phi = 0$$

$$E = \frac{p^2}{2m}+V$$

classical ↔ quantum correspondance

$$E \to i\hbar\frac{\partial}{\partial t} \quad \& \quad p \to i\hbar\frac{\partial}{\partial x}$$

Klein-Gordon Equation (1927): $$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \Delta + \frac{m^2c^2}{\hbar^2}\right)\Phi = 0$$

$$\frac{E^2}{c^2} = p^2 + m^2c^2$$

Dirac Equation (1928): $$\left(i\gamma^\mu\partial_\mu - \frac{mc}{\hbar}\right)\Psi = 0$$

$$E = \begin{cases} +\sqrt{p^2c^2+m^2c^4} & \text{matter} \\ -\sqrt{p^2c^2+m^2c^4} & \text{antimatter} \end{cases}$$

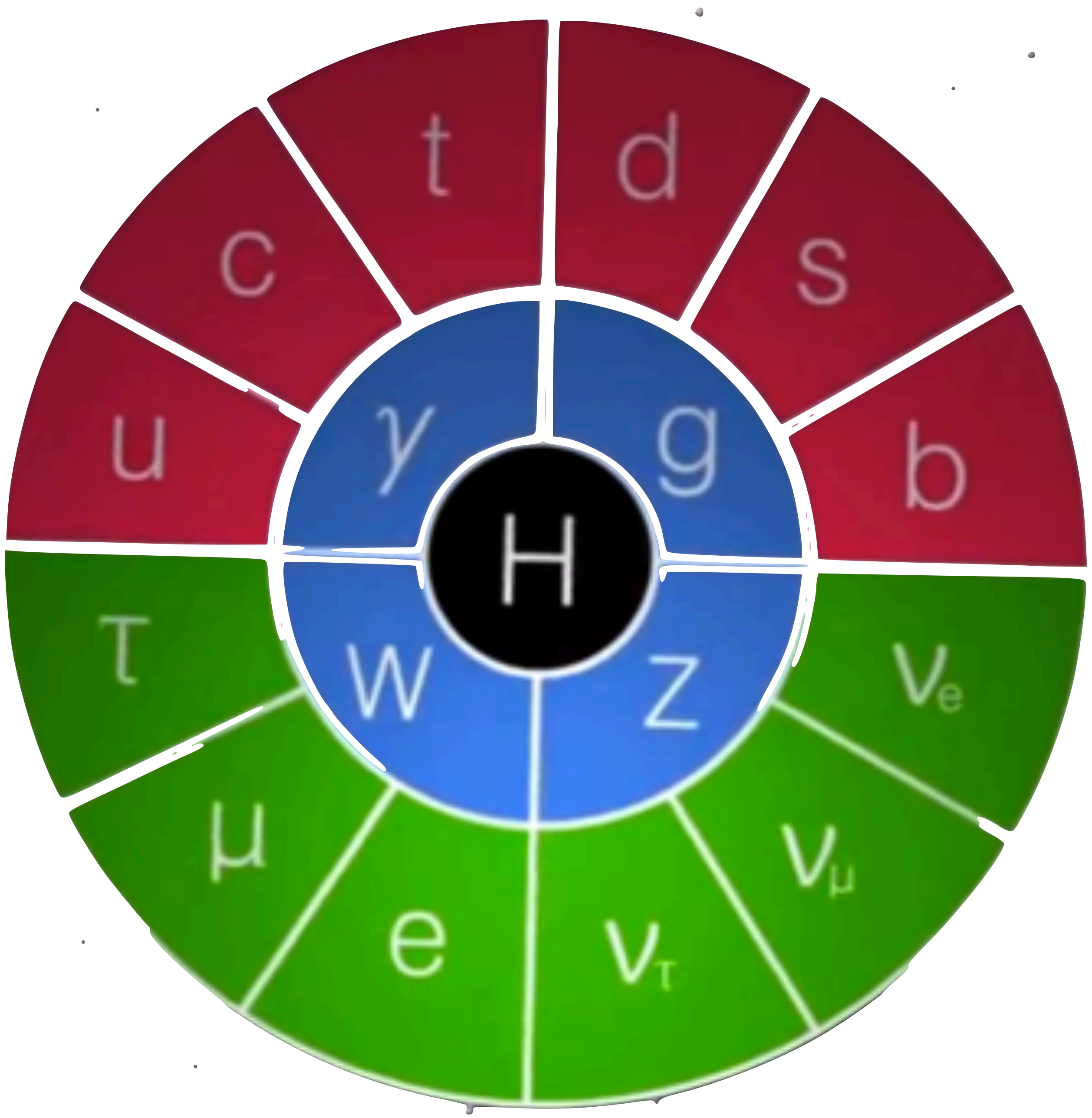
$$E = \vec{\alpha}\vec{p}\,c + \beta mc^2$$

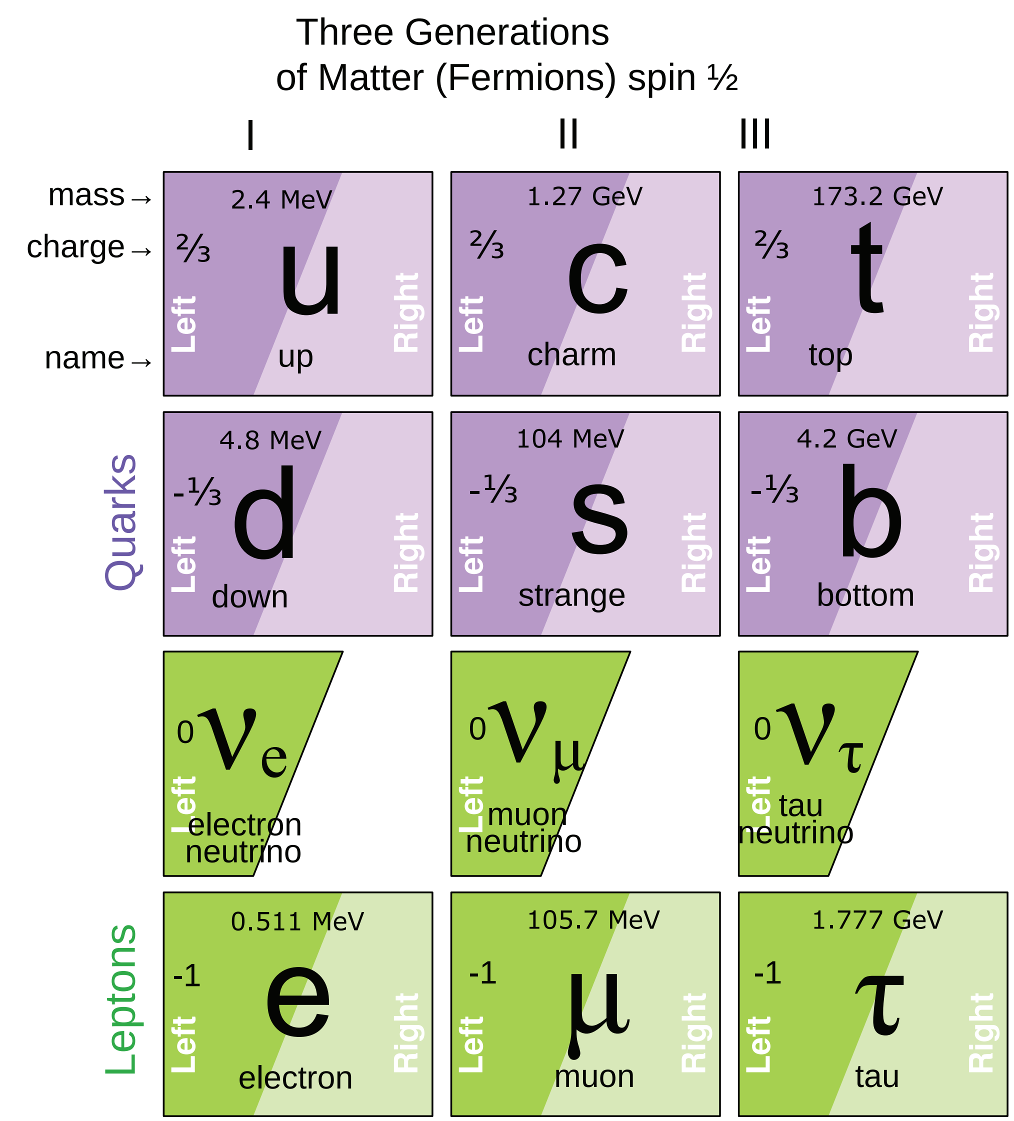
$$\gamma^0 = \beta,\ \gamma^i = \beta\alpha^i,\ \{\gamma^\mu,\gamma^\nu\} = 2\eta^{\mu\nu}$$

positron (e^+) discovered by C. Anderson in 1932

The Standard Model: Matter

How many quarks and leptons?





The Standard Model: Matter

How many quarks and leptons?

$6+6=12?$

$6\times3+6=24?$

shouldn't we count different color states?

$6\times3\times2+3\times2+3=45?$

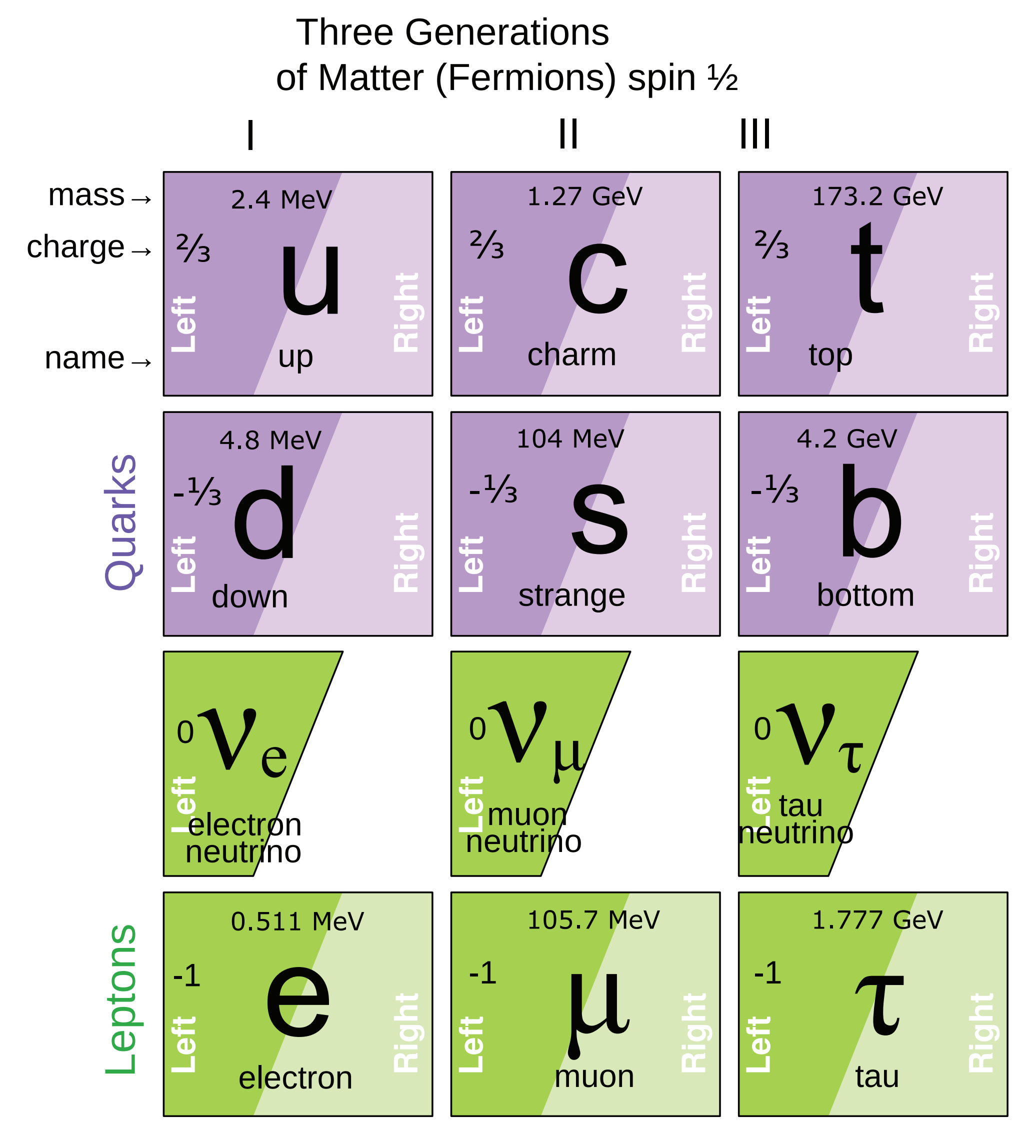
it is an accident that $e_L \sim e_R$ for QED

SM is a chiral theory: $e_L \neq e_R$

$6\times3\times2+6\times2=48?$

are there ν_R ?

are they part of the SM?



The Standard Model: Matter

How many quarks and leptons?

Is the SM theoretically consistent?

SM = theory based on (chiral) gauge symmetries

a symmetry is consistent with QM

iff the "sum" of the charges of the different fermions vanishes

$$Q = T_L^3 - Y$$

Particles	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$L_L^i = \begin{cases} N^i = (\nu^i, \tilde{\nu}^i) \\ E_L^i = (l_L^i, \tilde{l}_L^i) \end{cases}$	1	2	1/2
$E^i = (CP(l_R^i), CP(\tilde{l}_R^i))$	1	1	-1
$Q_L^i = \begin{cases} U_L^i = (u_L^i, \tilde{u}_L^i) \\ D_L^i = (d_L^i, \tilde{d}_L^i) \end{cases}$	3	2	$-1/6$
$U_R^i = (CP(u_R^i), CP(\tilde{u}_R^i))$	$\bar{3}$	1	2/3
$D_R^i = (CP(d_R^i), CP(\tilde{d}_R^i))$	$\bar{3}$	1	$-1/3$

Exercise 1: within the SM, check that

(1) $\mathrm{Tr}_L Y - \mathrm{Tr}_R Y = 0$

(2) $\mathrm{Tr}_L Y^3 - \mathrm{Tr}_R Y^3 = 0$

note that there was a priori no guarantee to find a solution to this system of non-linear equations.

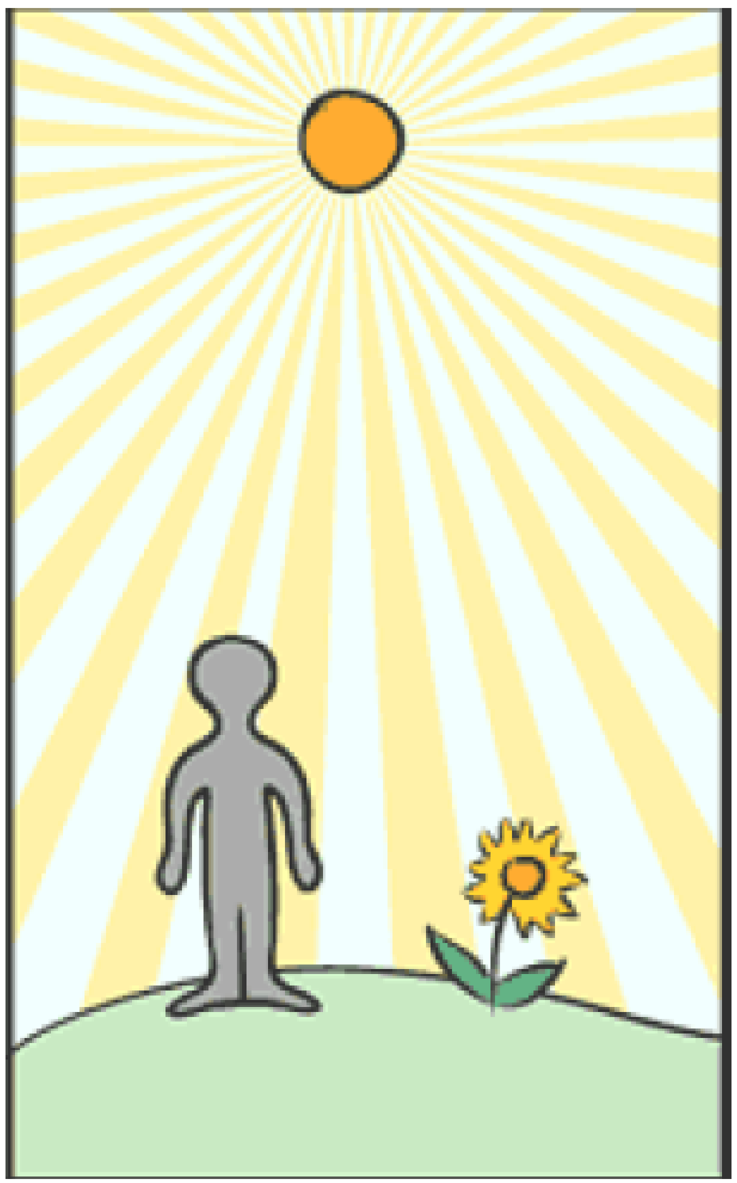
It works because EM is a vector-like theory

Exercise 2: Within the SM, the anomaly cancelation fixes the relative electric charges of the leptons and quarks. Show that with the addition of a right-handed neutrino, this ratio of electric charges is free. Still the cancelation of the anomaly imposes that the proton is electrically neutral

The fundamental interactions

Interactions between Particles

The very observation that the sun is shining for several millennia tells us that there are various mechanisms of energy production.



Sun = gigantic source of energy

- 1 cm^3 of ice under the sun (in the summer) melts in ~ 40mn
- an ice cap 1 cm thick and 300 million km of diameter centered around the sun will melt in 40mn

energy produced by burning 10^{19} liters of oil
(~ volume Sun-Mercury/1000)

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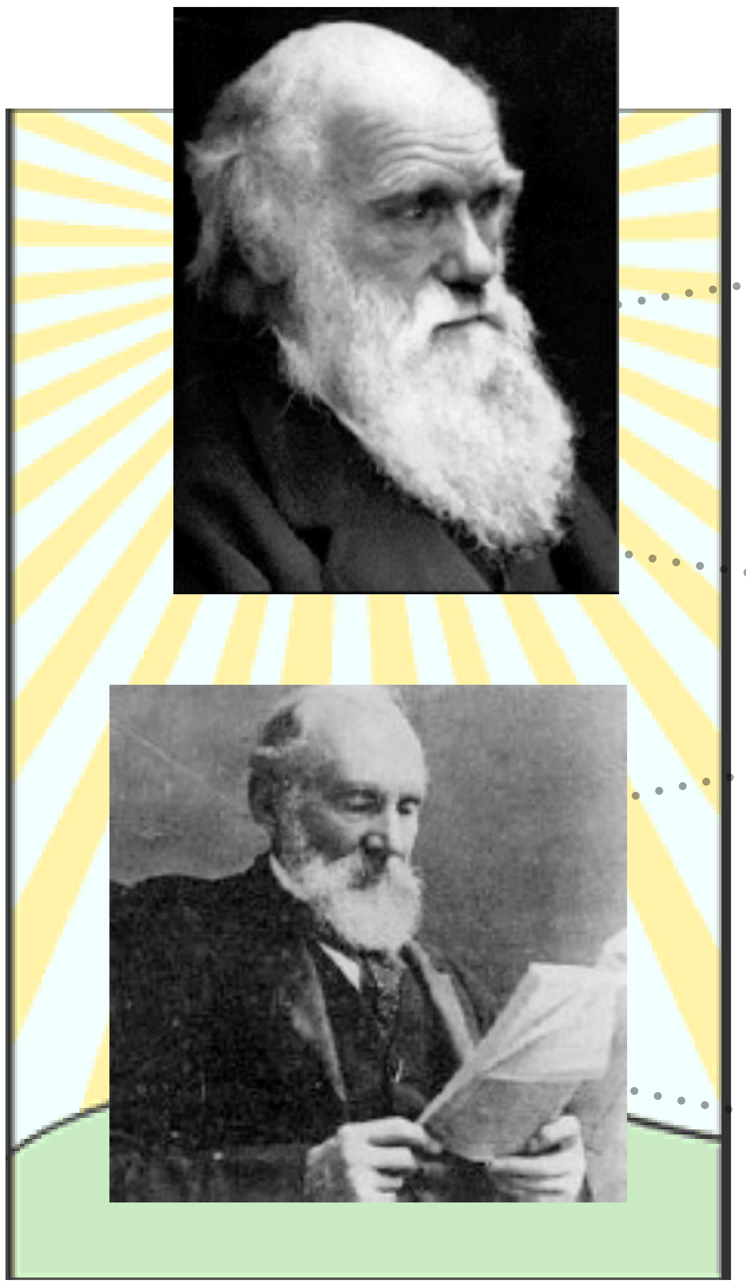


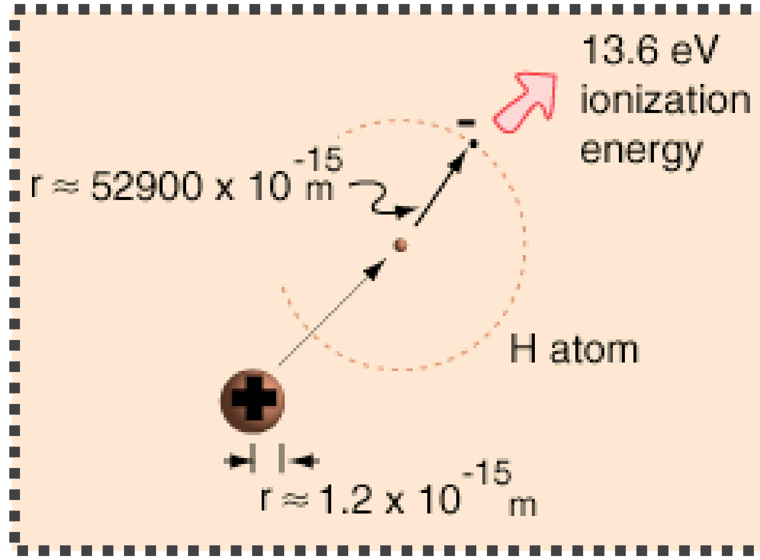
Sun = gigantic source of energy

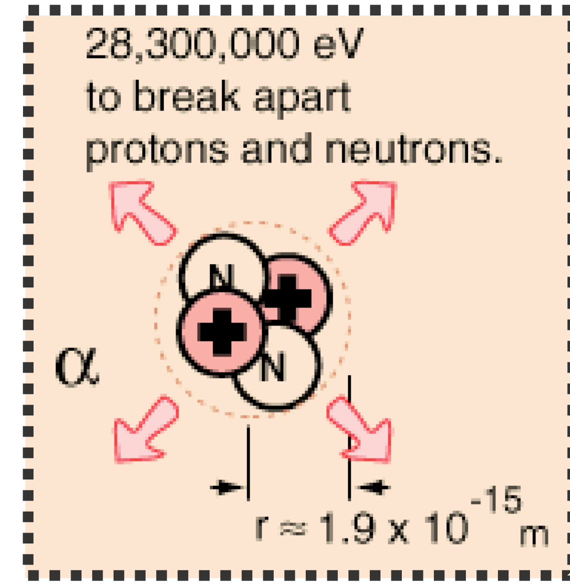
Darwin ("*On the origin of species by means of natural selection*", 1st edition, 1859) estimates that the age of the Earth, and thus the age of the Sun also, has to be larger than 300 millions of years to account for the erosion of hills in South England.

Thompson, Lord Kelvin, computes the gravitational energy of the Sun and with the assumption that it is entirely converted in heat, concludes that the Sun cannot be older than 20 million years (chemical energy would allow the Sun to shine for at most 3000 years)

We know today that the Sun is more than 4.5 billion years

Different Interactions





- ***Atomic Physics***

  mass of an atom = mass of nucleus + masses of electrons

  <u>example</u> : hydrogen atom, mass ~ 1 GeV, binding energy ~ 13 eV ⇨ 10^{-8}

- ***Nuclear Physics***

  mass of a nucleus < Σ masses of protons and neutrons

  <u>example</u> : Helium nucleus, mass ~ 4 GeV, binding energy ~ 28 MeV ⇨ 10^{-2}

- ***Particle Physics***

  mass of a proton or a neutron $\gg$ Σ masses of quarks

  proton mass ~ 1 GeV, constituent quarks masses~ 12 MeV ⇨ 10^{2}

The Standard Model: Interactions

Even though EM is way stronger than gravity, it was unnoticed until ~ 300 years because 1-1=0

- ***gravity*** — 10^{-40}

- ***electromagnetic interactions***

  (1873, Maxwell)

  tested with an accuracy of 10^{-8}

  light, atoms, molecules — 10^{-5}

- ***weak interactions***

  (1933, Fermi)

  tested with an accuracy of 10^{-3}

  β decay

  $$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

  10^{-2}

- ***strong interactions***

  tested with an accuracy of 10^{-1}

  atomic nuclei

  α decay

  $${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

  1

(1911, Rutherford ; 1921, Chadwick and Biesler)

strength

Gauge Theories

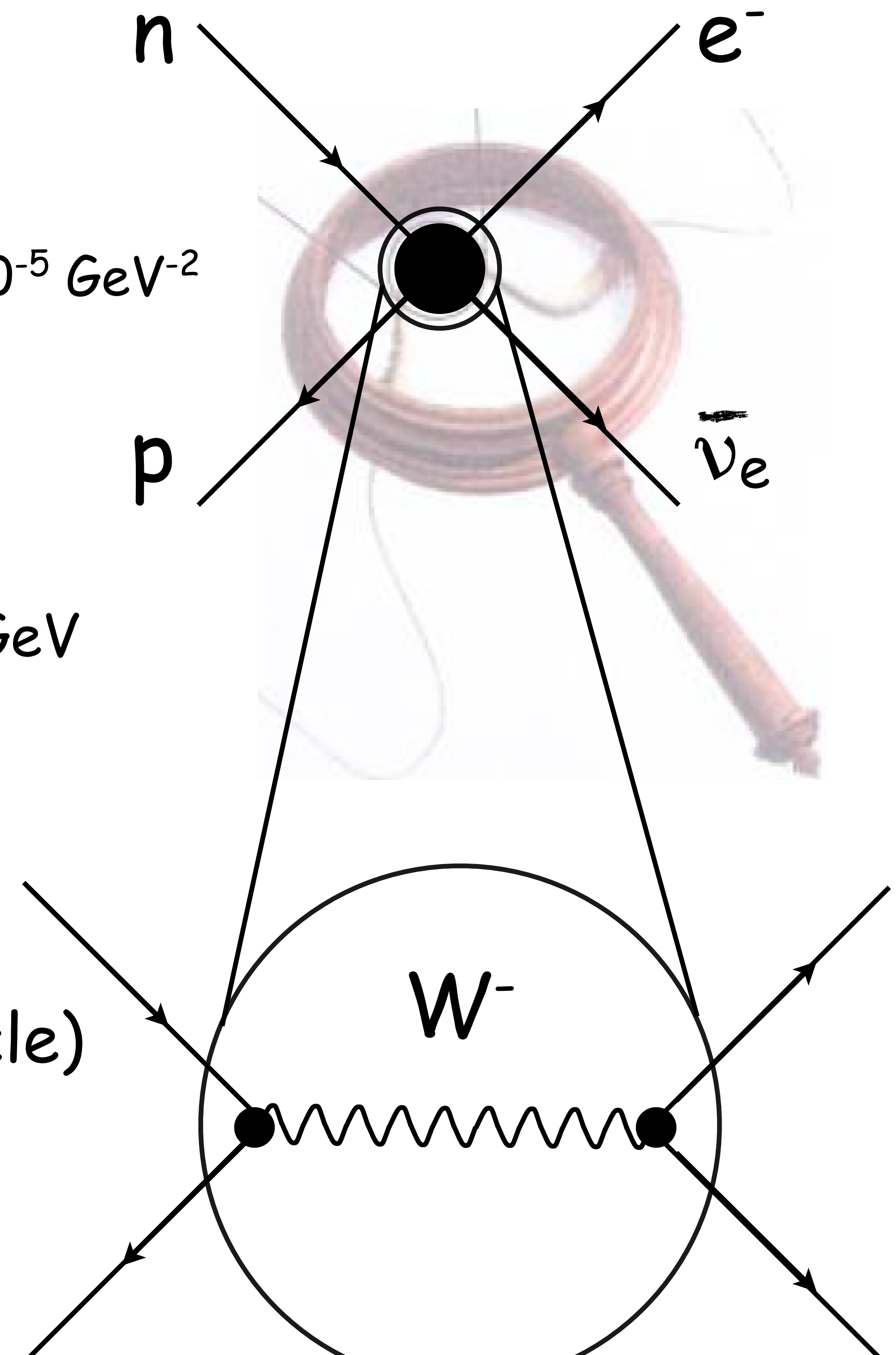
Fermi Theory

(paper rejected by Nature: declared too speculative !)

$$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

exp: $G_F = 1.166 \times 10^{-5}$ GeV^{-2}

$$\mathcal{L} = G_{\mathcal{F}} (\bar{n} p)(\bar{\nu}_e e)$$



$$\mathcal{A} \propto G_{\mathcal{F}} E^2$$

- no continuous limit
- inconsistent above 300 GeV

Gauge theory

microscopic theory

(exchange of a massive spin 1 particle)

$$G_{\mathcal{F}} = \frac{\sqrt{2} g^2}{8 m_W^2}$$

exp: m_W=80.4 GeV

➡ $g \approx 0.6$, ie, same order as e=0.3

unification EM & weak interactions

Gauge Theories: EM & Yang-Mills

EM U(1) $\qquad \phi \to e^{i\alpha}\phi \qquad$ but $\qquad \partial_\mu\phi \to e^{i\alpha}\left(\partial_\mu\phi\right) + \underbrace{i(\partial_\mu\alpha)}_{\neq 0 \text{ if local transformations}}\phi$

EM field and covariant derivative $\qquad \partial_\mu\phi + ieA_\mu\phi \to e^{i\alpha}(\partial_\mu\phi + ieA_\mu\phi)$

if $\quad A_\mu \to A_\mu - \frac{1}{e}\partial_\mu\alpha$

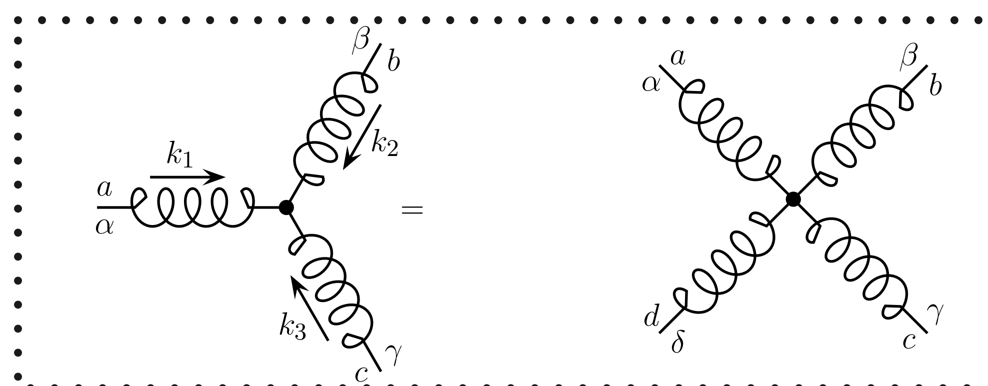
the EM field keeps track of the phase in different points of the space-time

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

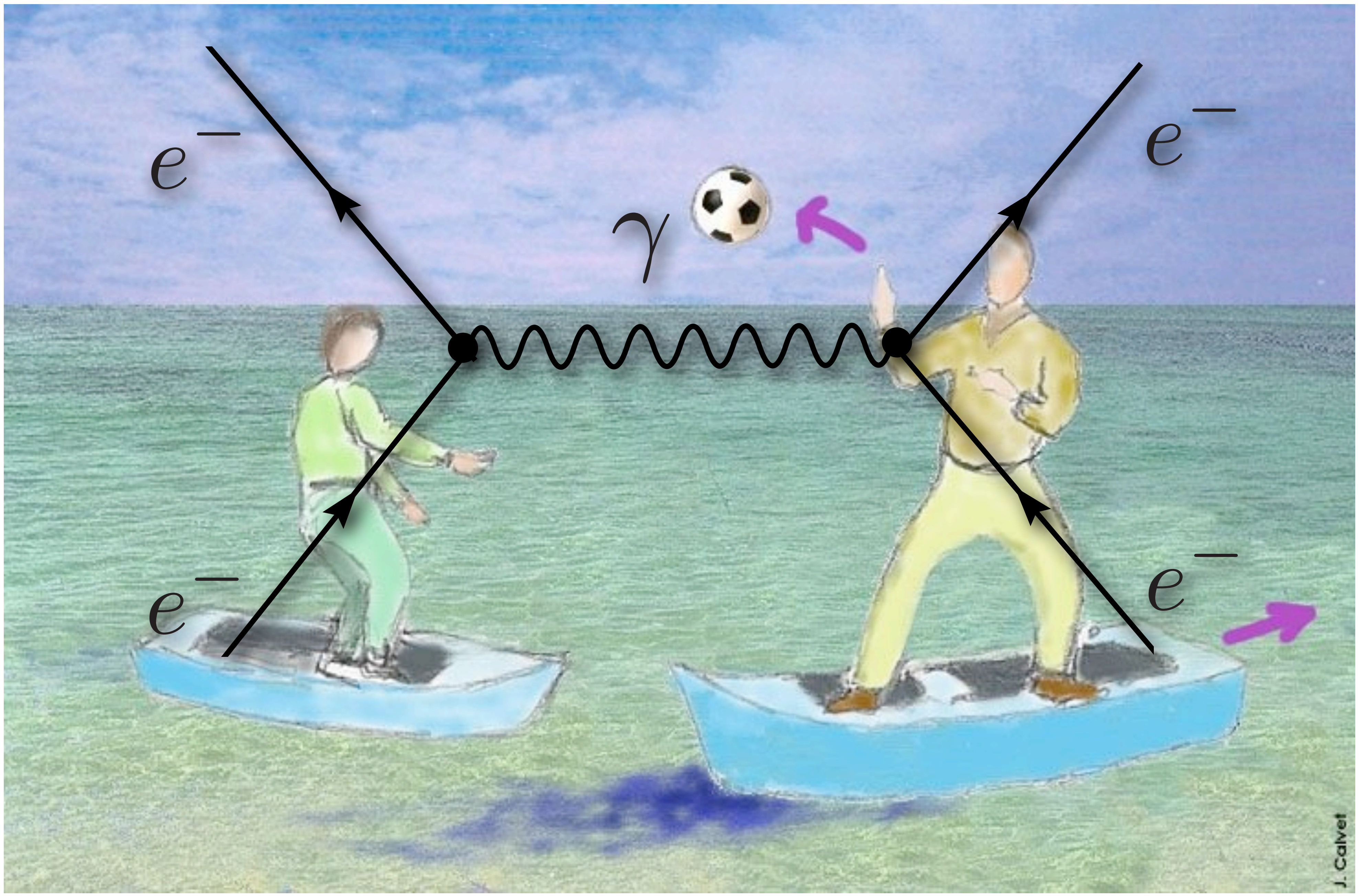
Yang-Mills : non-abelian transformations $\qquad \phi \to U\phi$

$\partial_\mu\phi + igA_\mu\phi \to U(\partial_\mu\phi + igA_\mu\phi) \qquad$ if $\qquad A_\mu \to UA_\mu U^{-1} - \frac{i}{g}U\partial_\mu U^{-1}$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \underbrace{ig[A_\mu, A_\nu]}_{\text{non-abelian int.}}$$



Interactions between Particles



Elementary particles interact on each other
by the exchange of gauge bosons

The Standard Model: Interactions

- $U(1)_Y$ ***electromagnetic interactions***

  Photon γ

  light, atoms, molecules — 10^{-5}

- $SU(2)_L$ ***weak interactions***

  bosons $W^\pm, Z^0$

  β decay

  $$n \xrightarrow{W^\pm} p + e^- + \bar{\nu}_e$$

  $$e^+ + e^- \xrightarrow{Z^0} D^+_{(c\bar{s})} + D^-_{(\bar{c}s)}$$

  10^{-2}

- $SU(3)_c$ ***strong interactions***

  gluons g^a

  atomic nuclei

  α decay

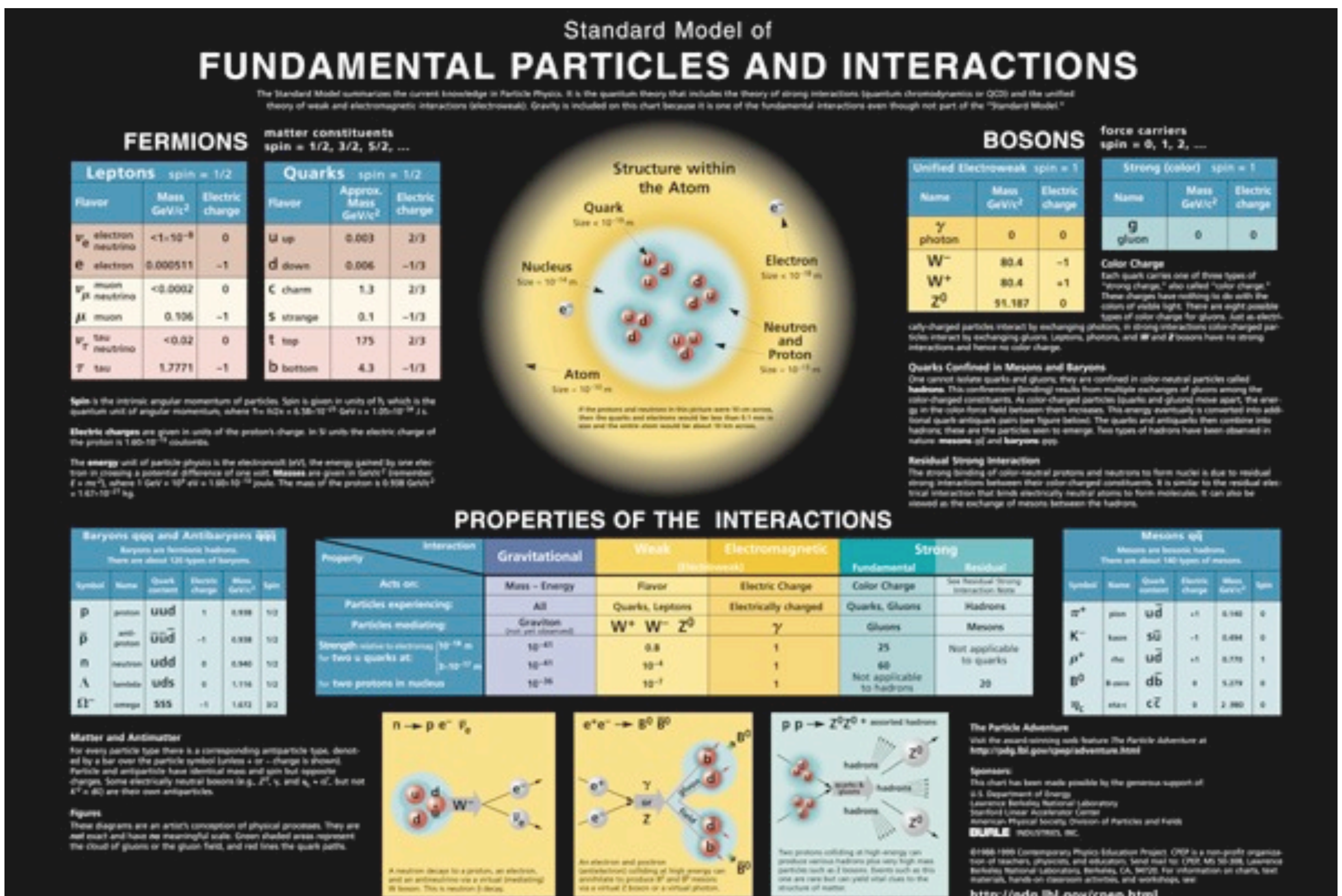
  $${}^{238}_{92}\mathrm{U} \longrightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He}$$

  1

strength

The Standard Model

Nobel Prize '79



description of all elementary particles and their interactions

The underlying principles of the SM

The beauty of the SM comes from the the identification of
a unique dynamical principle describing the different interactions
that seem so different from each others

gauge theory = spin-1

at the same time a particular and predictive structure that
still leaves room for a rich variety of phenomena
(long range interaction, spontaneous symmetry breaking, confinement)

gravitation = general relativity= spin-2

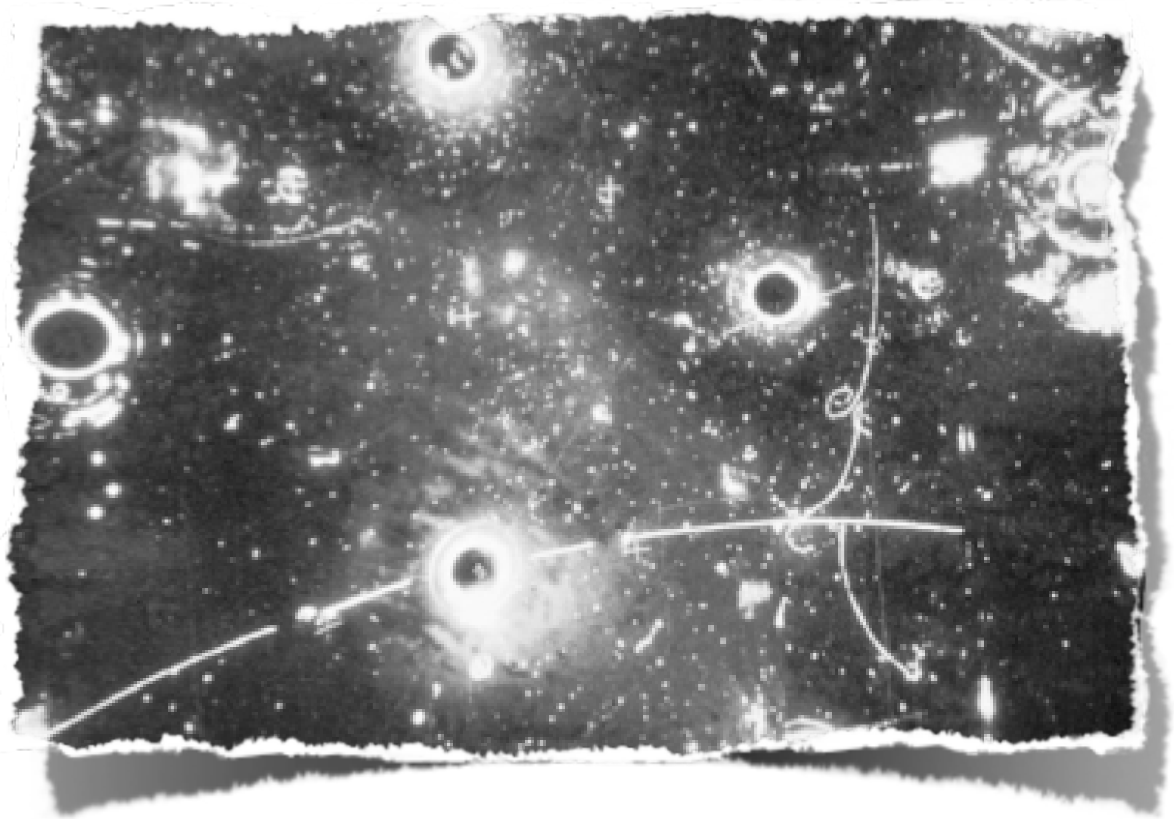
much more rigid theory = unique theory

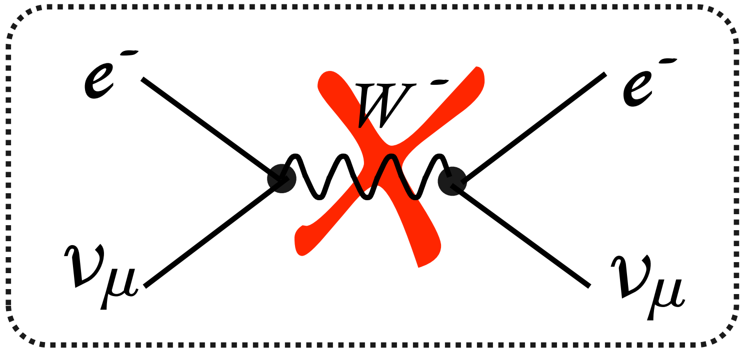
The Standard Model

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$\nu_\mu e^- \to \nu_\mu e^-$





[Gargamelle collaboration, '73]

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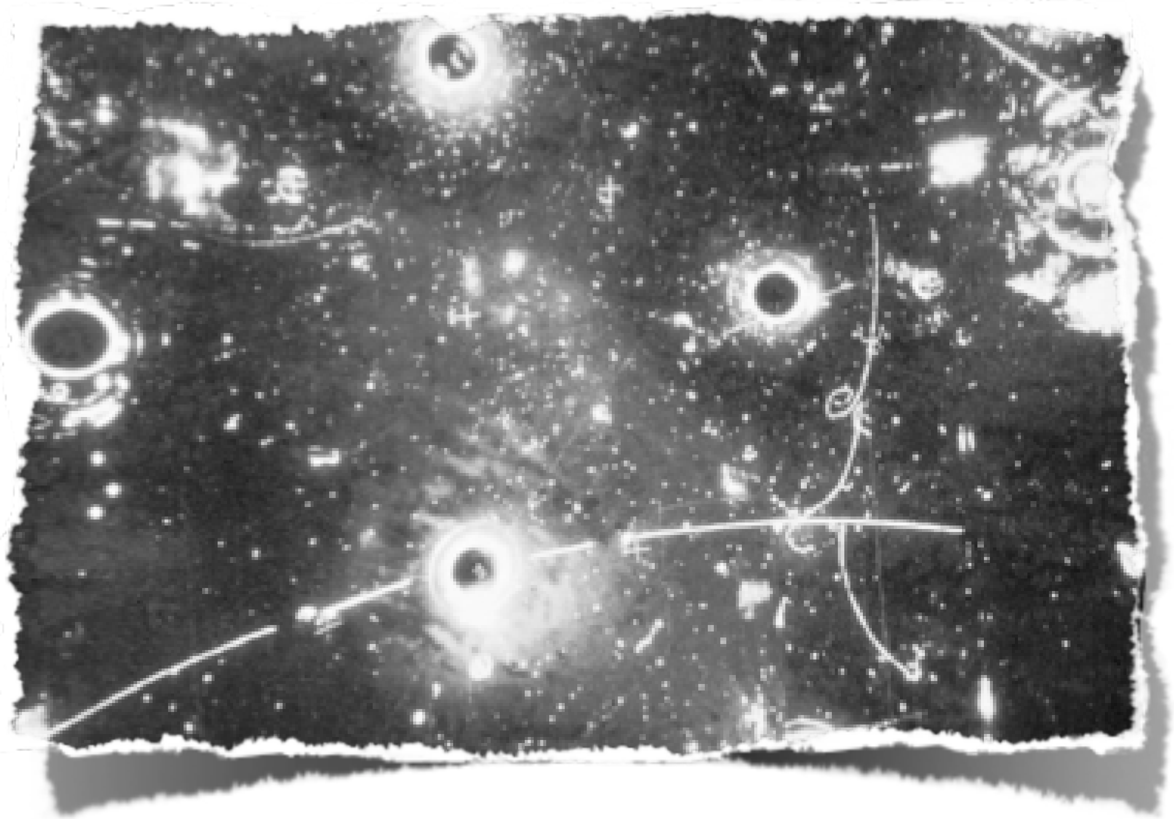

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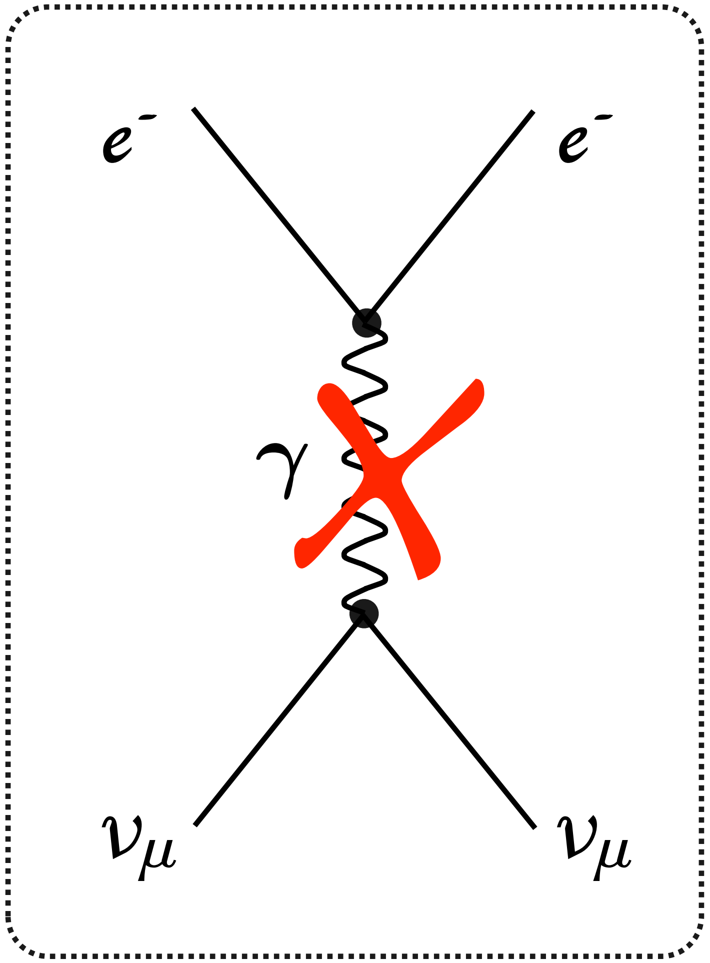
$\nu_\mu\, e^- \to \nu_\mu\, e^-$



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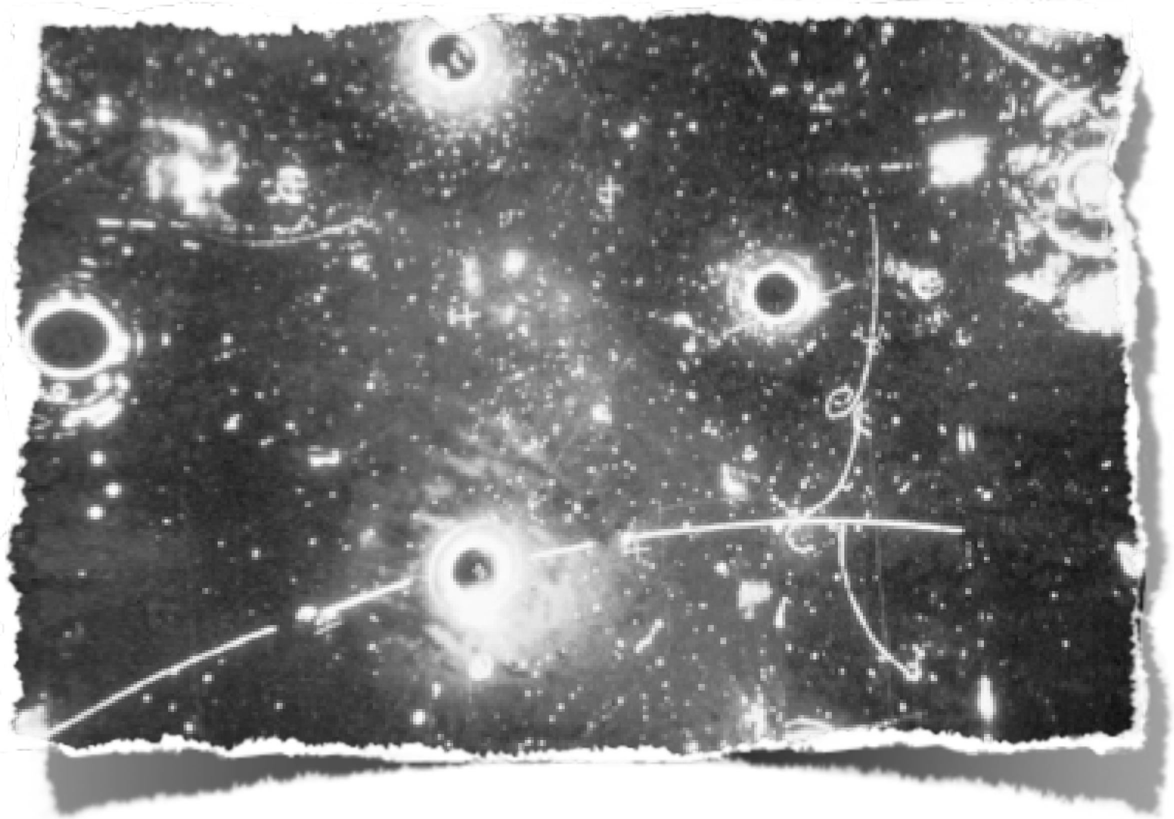

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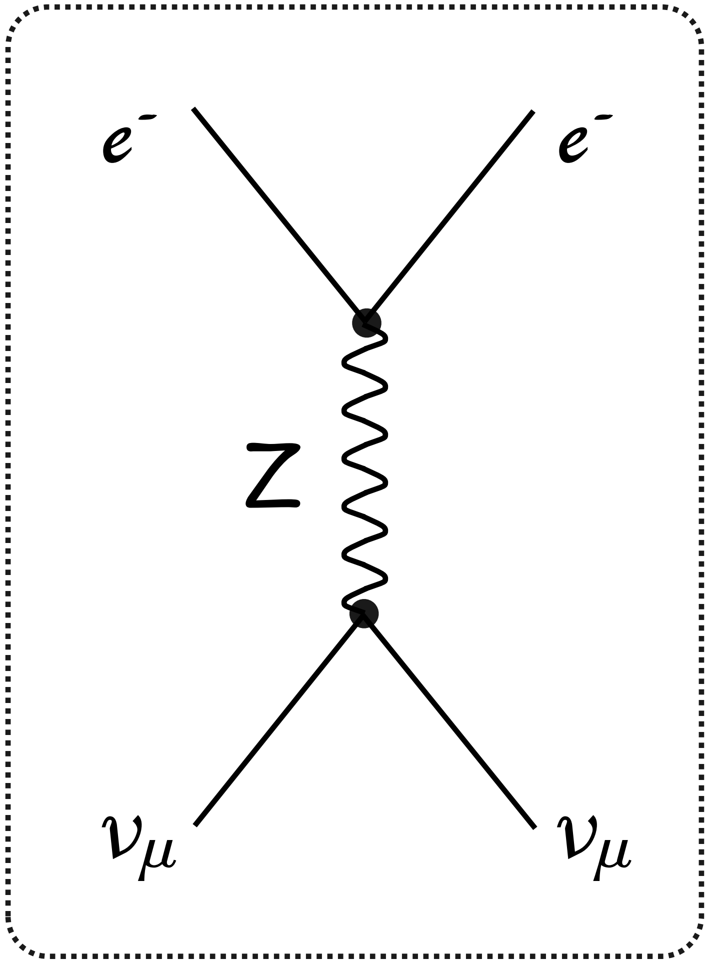
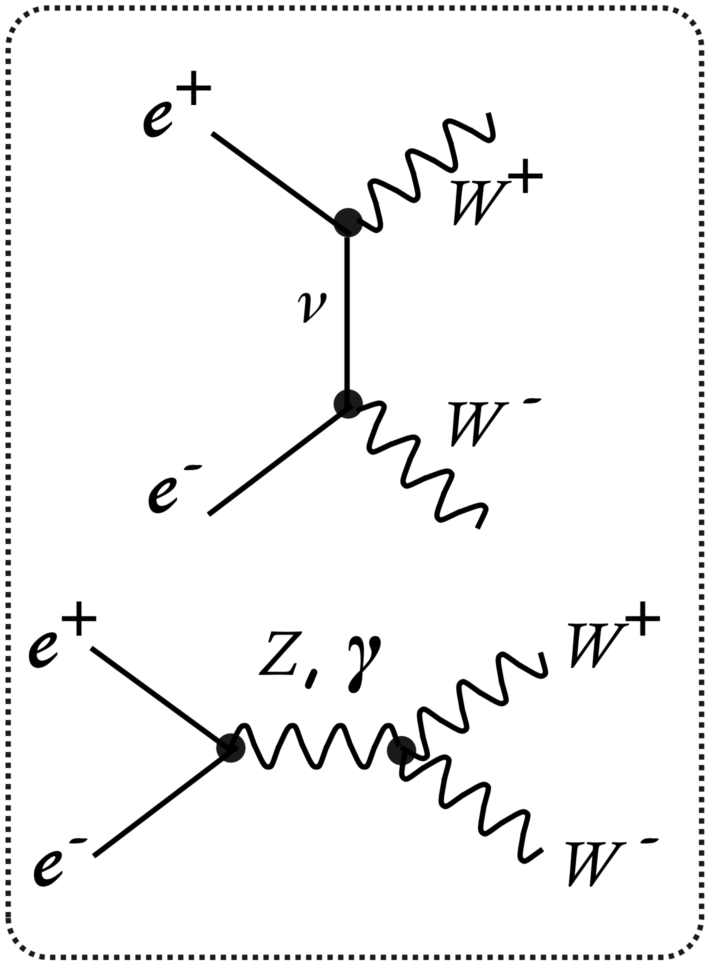


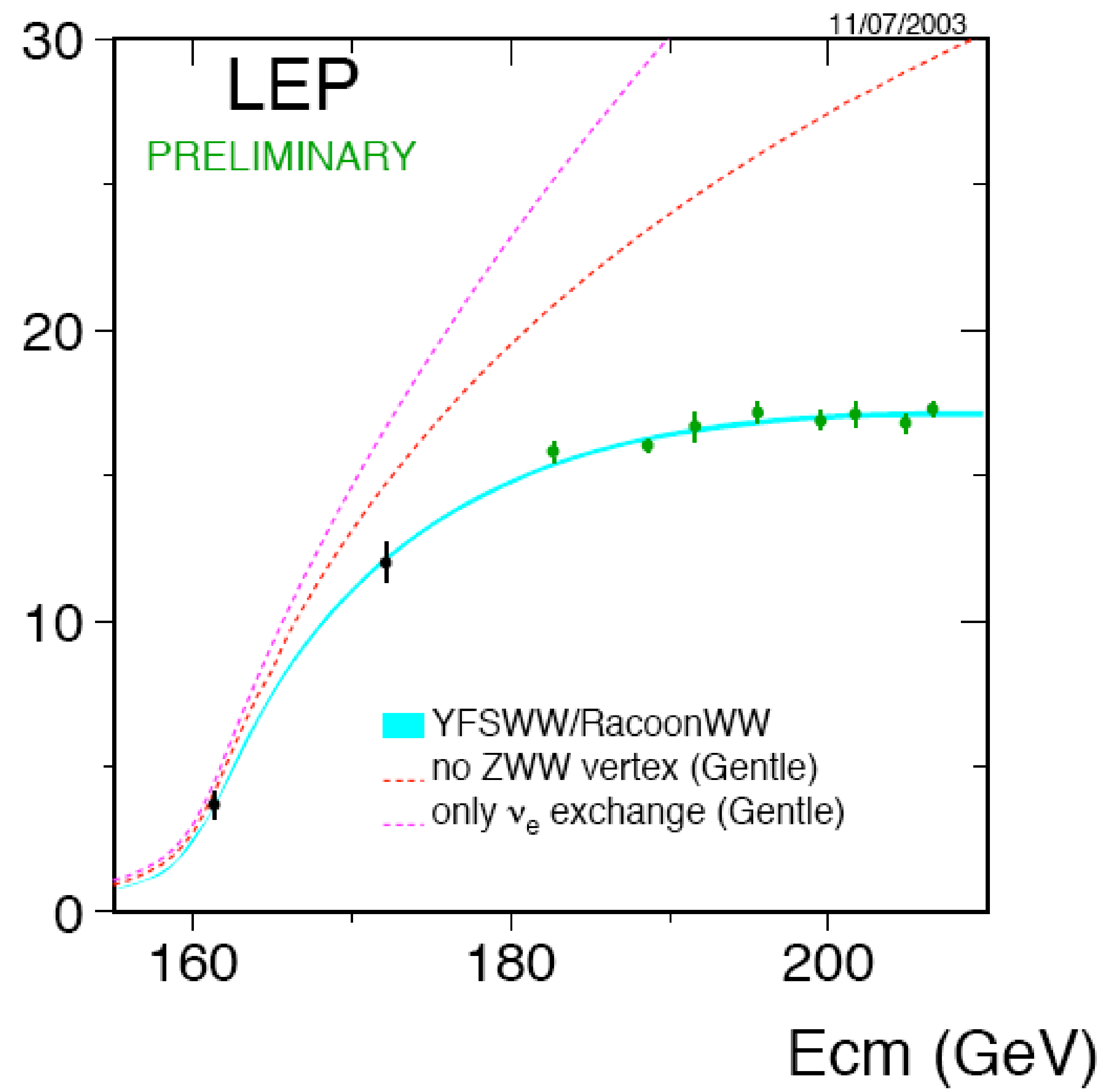
Gauge Theory as a Dynamical Principle

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

$e^+e^- \to W^+W^-$





The Standard Model and the Mass Problem

the strong, weak and electromagnetic interactions of the elementary particles are described by gauge interactions

$SU(3)_C \times SU(2)_L \times U(1)_Y$

the masses of the quarks, leptons and gauge bosons don't obey the full gauge invariance

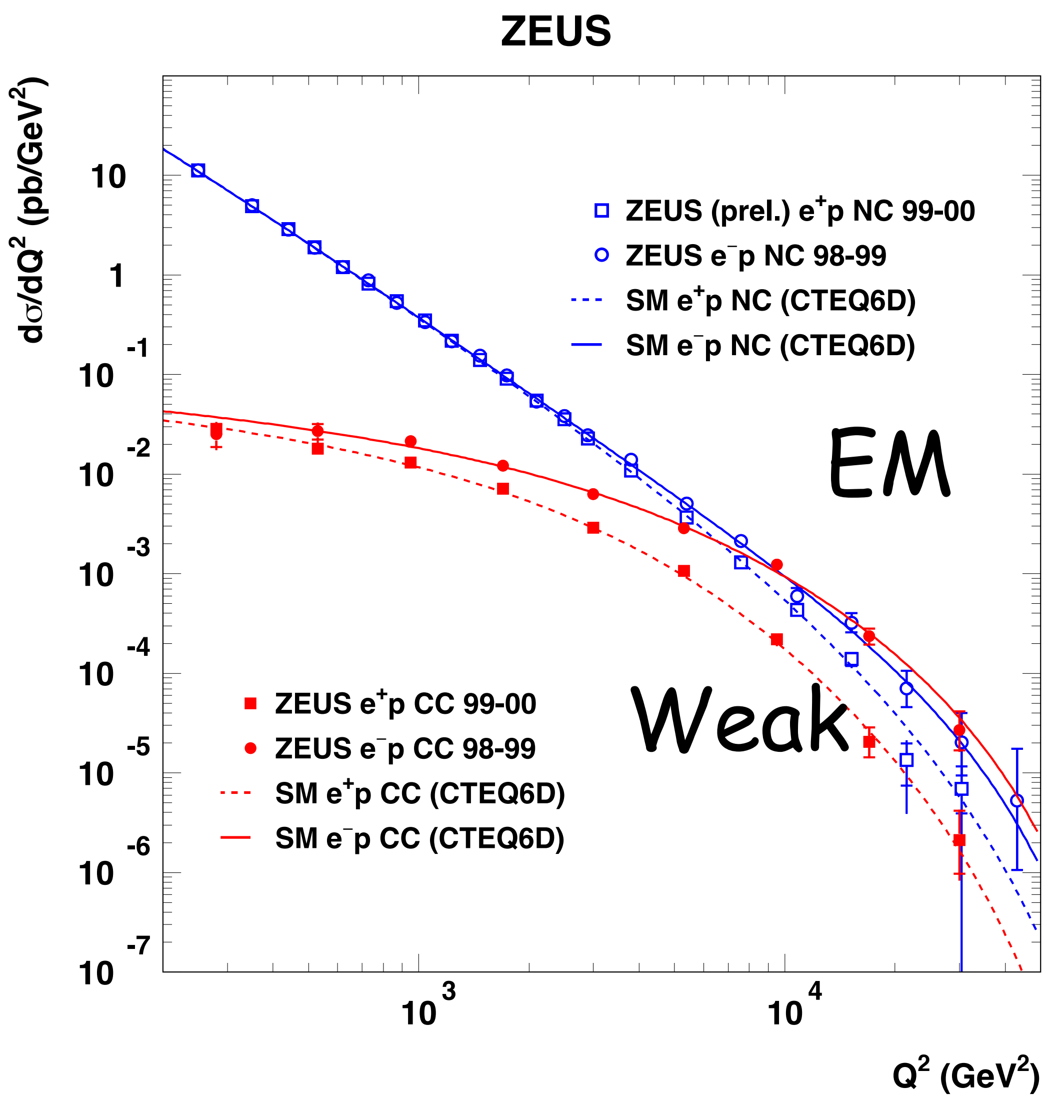
- $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ is a doublet of $SU(2)_L$ but $m_{\nu_e} \ll m_e$

- a mass term for the gauge field isn't invariant under gauge transformation $\delta A^a_\mu = \partial_\mu \epsilon^a + g f^{abc} A^b_\mu \epsilon^c$

⇒ spontaneous breaking of gauge symmetry ⇐

Electroweak Unification

High energy (~ 100 GeV)



Low energy

This room is full of photons
but no W/Z
The symmetry between W, Z and γ
is broken at large distances

EM forces ≈ long ranges

Weak forces ≈ short range

$$m_\gamma < 6 \times 10^{-17}\ \mathrm{eV}$$
$$m_{W^\pm} = 80.425 \pm 0.038\ \mathrm{GeV}$$
$$m_{Z^0} = 91.1876 \pm 0.0021\ \mathrm{GeV}$$

Classical field theory

classical mechanics & lagrangian formalism

a system is described by $S = \int dt \mathcal{L}(q, \dot{q})$

position momentum

action principle determines classical trajectory:

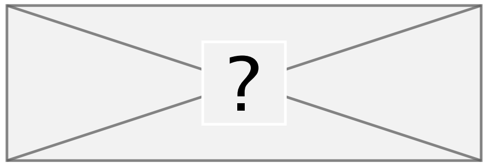
$\delta S = 0$ --> Euler-Lagrange equations $\dfrac{\partial \mathcal{L}}{\partial q_i} - \dfrac{\partial}{\partial t}\dfrac{\partial \mathcal{L}}{\partial \dot{q}_i} = 0$

conjugate momenta $p_i = \dfrac{\partial \mathcal{L}}{\partial \dot{q}_i}$ hamiltonian $H(p,q) = \sum_i p_i \dot{q}_i - \mathcal{L}$

extend lagrangian formalism to dynamics of fields

$$S = \int d^4x \mathcal{L}(\varphi, \partial_\mu \varphi)$$

$$\partial_\mu = \frac{\partial}{\partial x^\mu}$$



$\delta S = 0$ --> $\dfrac{\partial \mathcal{L}}{\partial \varphi_i} - \partial_\mu \dfrac{\partial \mathcal{L}}{\partial(\partial_\mu \varphi_i)} = 0$

conjugate momenta $\Pi_i = \dfrac{\partial \mathcal{L}}{\partial(\partial_0 \varphi_i)}$ hamiltonian $H(x) = \sum_i \Pi_i(x) \partial_0 \varphi_i(x) - \mathcal{L}$

Noether theorem

Invariance of action under continuous global transformation ---> There is a conserved current/charge

$$\partial_\mu j^\mu = 0 \qquad Q = \int d^3x j^0(x,t)$$

example of transformation: $\boxed{\varphi \to \varphi e^{i\alpha}}$ (*)

if small increment $\alpha \ll 1$ $\varphi \to \varphi + i\alpha\varphi$

$$\delta\varphi = i\alpha\varphi$$
$$\delta\varphi' = i\alpha\varphi'$$

invariance of $\mathcal{L}$ under (*): $\delta\mathcal{L} = 0 = i\alpha\left(\frac{\partial\mathcal{L}}{\partial\varphi}\varphi + \frac{\partial\mathcal{L}}{\partial\varphi'}\varphi'\right)$

Euler-Lagrange equations: $\frac{\partial}{\partial x}\left(\frac{\partial\mathcal{L}}{\partial\varphi'}\right) - \frac{\partial\mathcal{L}}{\partial\varphi} = 0$

$$\boxed{\frac{\partial}{\partial x}(\underbrace{\varphi\frac{\partial\mathcal{L}}{\partial\varphi'}}_{\equiv J}) = 0}$$

conserved current

Symmetries and conservation laws

Noether's theorem (from classical field theory) :

A continuous symmetry of the system -> a conserved quantity

I- Continuous global space-time symmetries:

translation invariance in space -> momentum conservation

translation invariance in time -> energy conservation

rotational invariance -> angular momentum conservation

Fields are classified according to their transformation properties under Lorentz group:

$$x^\mu \to x'^\mu = \Lambda^\mu_\nu x^\nu \qquad \phi(x) \to \phi'(x')$$

$$\phi'(x) = \phi(x)$$ scalar

$$V^\mu \to \Lambda^\mu_\nu V^\nu$$ vector

The true meaning of spin arises in the context of a fully Lorentz-invariant theory (while it is introduced adhoc in non-relativistic quantum mechanics)

Symmetries and conservation laws

I- Continuous global space-time (Poincaré) symmetries all particles have (m, s)

-> energy, momentum, angular momentum conserved

II- Global (continuous) internal symmetries -> B, L conserved

(accidental symmetries)

III- Local or gauge internal symmetries -> color, electric charge conserved

$$SU(3)_c \times SU(2)_L \times U(1)_Y$$

IV- Discrete symmetries -> CPT

Group theory