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# Simple Gaze Tracker

SIGB Assignment 1

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by

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## Introduction

This is the first mandatory assignment for the course SIGB F2013. In this assignment we'll implement a simple gaze tracker. This will be done in the programming language python with help from the opencv and numpy libraries.

This report is an attempt to document what has been done to make this gaze tracker.

The basic structure for each section will be a short introduction of the goal of the section, followed by the theory behind our approach ending with a description of our actual implementation with visual aids used for documentation. Additionally we will accompany the report with captured videos demonstrating the usage of the eye tracker

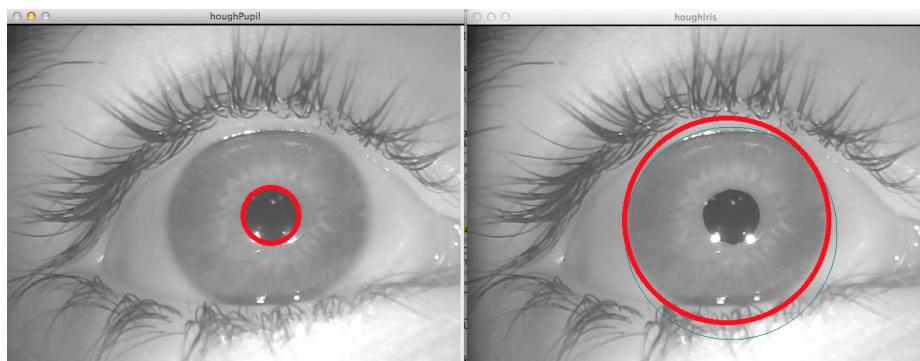


Figure 1: Eye located with hough

# Pupil detection

## Overall rationale and theory

The eye consists of several distinct features such as pupil, iris, limbus and sclera. Furthermore several features are present in images as result of the light conditions when a picture is taken. The position of these can aid in correctly determining the gaze. Detecting these features is therefore a good starting for a gaze tracker implementation, but it poses some challenges in correctly identifying each component, and filtering away noise.

Of these features one of the easiest to recognize is the pupil, as it is the darkest part of the eye. Furthermore it's a good starting point, as it is surrounded by the other eye components. The challenges/noise in locating the pupil is mostly due to glints/reflections of light, but these also aids in locating the pupil because we know that the pupil will reflect light in a certain way.

As mentioned, the pupil is the darkest part of the eye. Furthermore a pupil will reflect two “glints” of light. A good way to find the pupil is to find an intensity value which can separate it from the background. This is called thresholding, and will be explained in the following sections. Thresholding can be applied to both find pupil candidates as well as glints used for more robustly picking the best pupil candidate. When thresholding has been performed we will analyse certain features of the BLOBs in the image to see which are good candidates for pupil and glints. To aid in selecting a proper threshold we will use a form of pixel classification to automatically set a threshold. This method will be described last in this section.

## Thresholding

Thresholding is a form of point processing used for separating areas of an image based on intensity in these areas. The desired end result of this method is a binary image with the foreground (object) in one color and the background (everything else) in a different color. This is usually black and white respectively. This will effectively separate the foreground from the background for us, leaving us with only the contours. In that sense we lose information and granularity in the image, but since we're only interested in position we haven't lost anything important.

## Theory

As with all point processing methods the basic theory behind thresholding is applying a calculation to every pixel in the image, effectively changing the value of that pixel. We know that the end result is a binary image, following this, each pixel will be transformed into one of two values. We've so far operated on

byte images so the max value is 255 and the min value is of course 0. For clarity we will assign each pixel either the min or the max value. Thus thresholding can be expressed as the following with T being the assigned Threshold value and f being the function applied to each pixel:

$$\text{if } f(x, y) \leq T \text{ then } g(x, y) = 0 \quad \text{and} \quad \text{if } f(x, y) < T \text{ then } g(x, y) = 255$$

Performing these operations will leave us with an image where only certain BLOBs are visible. We can then analyse the properties of these BLOBs to find which one is most likely to be a pupil, and which are most likely to be glints. Namely we will look at the area and the circularity of blobs. Area is simply a count of all pixels in the blob.

Finding the circularity is a bit more complex. Used here is “Heywoods circularity factor” which is derived from the perimeter and area of the BLOB. Perimeter is the count of pixels on the rim of the contour. A “cheaper” approximation can be found by taking the perimeter of the bounding box of a BLOB. The bounding box can be found simply by finding the lowest and highest values of x and y respectively within a BLOB.

Once we have the perimeter and the area heywoods method can be applied. It’s defined as follows:

$$\text{Circularity} = \frac{\text{perimeter}}{2 * \sqrt{\pi * \text{area}}}$$

The area and circularity will be used to identify the best candidates

## Our implementation

The usefulness of thresholding relies on having a total binary image with the foreground easily distinguishable from the background. The foreground is the pupil. The pupil is a dark circle-like object surrounded by a lighter circle (the iris). So we’re looking for a threshold value that is lower than the surrounding iris. In an ideal world the pupil in it’s entirety will have only one intensity throughout, and the ideal threshold value will be that. However, the world is rarely so black and white (literally) and this also isn’t the case here. Although the pupil is the darkest spot it’s not completely dark. So a threshold value of 1 would be far to low in most circumstances. It also requires very specialized lighting conditions to achieve a pupil with the same intensity throughout, so there is no “silver bullet” threshold value which will perfectly cover the entire pupil.

What we’re looking for is a value that is close enough to every part of the pupil and far enough away from every part of the iris. The way of finding this perfect value is mostly trial and error in this stage. Luckily we had a slider to play around with when searching for this value. For the thresholding itself, a built in cv2 method was used.

```
val,binI =cv2.threshold(gray, thr, 255, cv2.THRESH_BINARY_INV)
```

Where:

gray is our (grayscale) image

thr is our selected threshold value

255 is the maximum value

the last argument is a constant indicating that output is a binary image

We quickly saw that a good value for most of the sequences floats around 100. An example of this can be seen in figure 2:

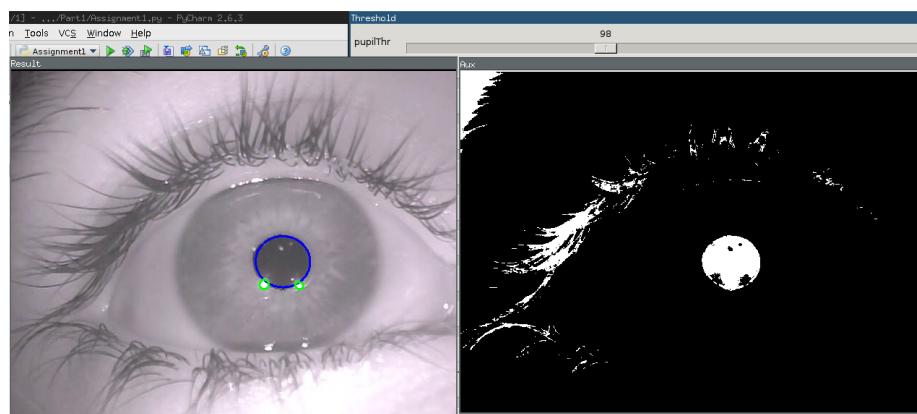


Figure 2: Good threshold

This value is not robust in all cases, and too high or too low values will yield to few or two many results respectively, seen in figures 3 and 4

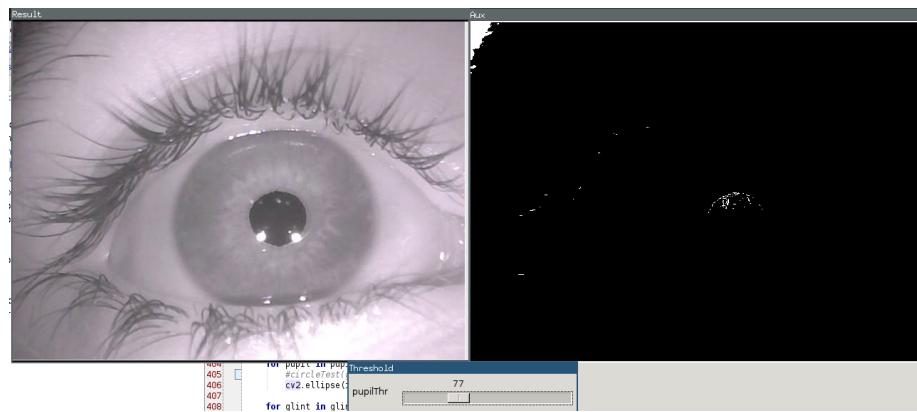


Figure 3: Low threshold

Another aspect of this is that further analysis is needed on the contours. So if a distorted figure is all we have it will be difficult or impossible to correctly

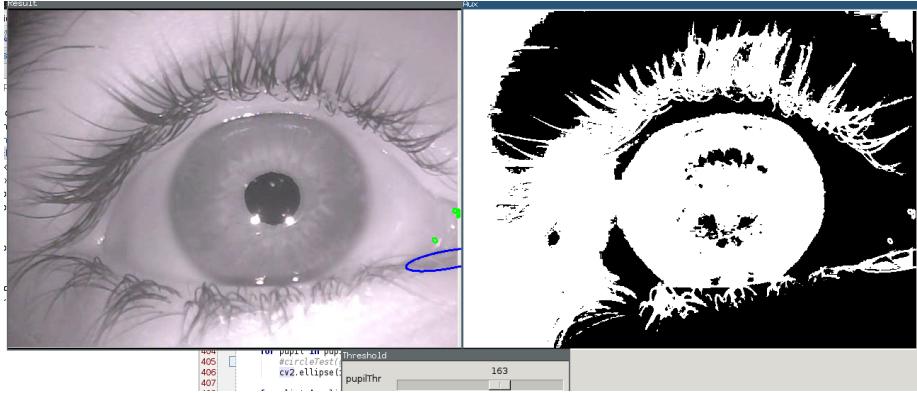


Figure 4: High threshold

determine the nature of the contour. An example of this false classification is seen in figure 5

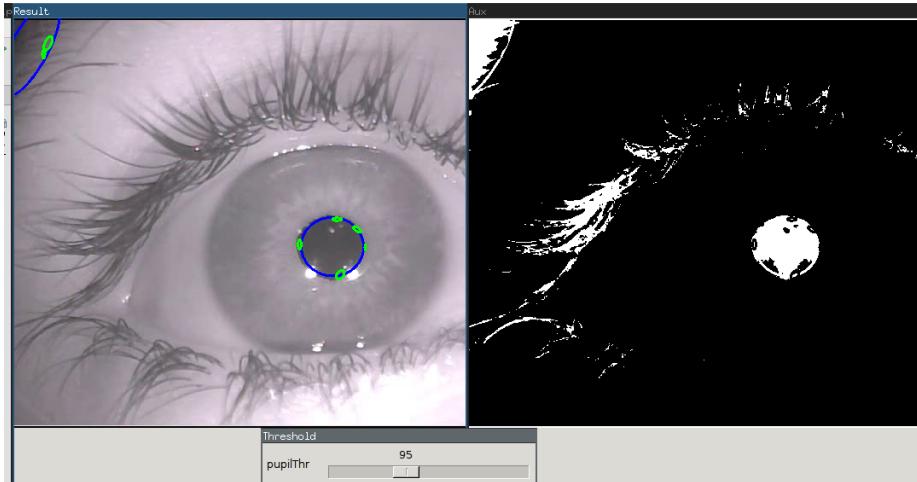


Figure 5: Contours found with threshold

In the happy path scenario analysis of the blobs will be performed as described in the theory section above. First the area of the BLOB is found:

This will be found using a cv2 methods. Presumably it's naively implemented and has linear complexity, but this is speculation as we don't have the implementation. Usage is as follows:

```
a = cv2.contourArea(con)
```

Where con is a contour

Next the perimeters is found. Again opencv has an implementation for this, given a closed contour:

```
p = cv2.arcLength(con, True)
```

With these variables in hand the results can be filtered in a simple imperative manner:

```
if(a==0 or a<minArea or a>maxArea):
    continue
p = cv2.arcLength(con, True)
m = p/(2.0*math.sqrt(math.pi * a))
if (m<1.7):
    if(len(con)>=5):
        ellips = cv2.fitEllipse(con)
        matches.append(ellips)
```

The min and max area as well as the circularity value of 1.7 are found through trial and error. The area parameters can be adjusted using sliders as needed for each sequence. The constraints of the length of the con is because we need 5 parameters for an ellipse

Last step is to redo this with the glints.

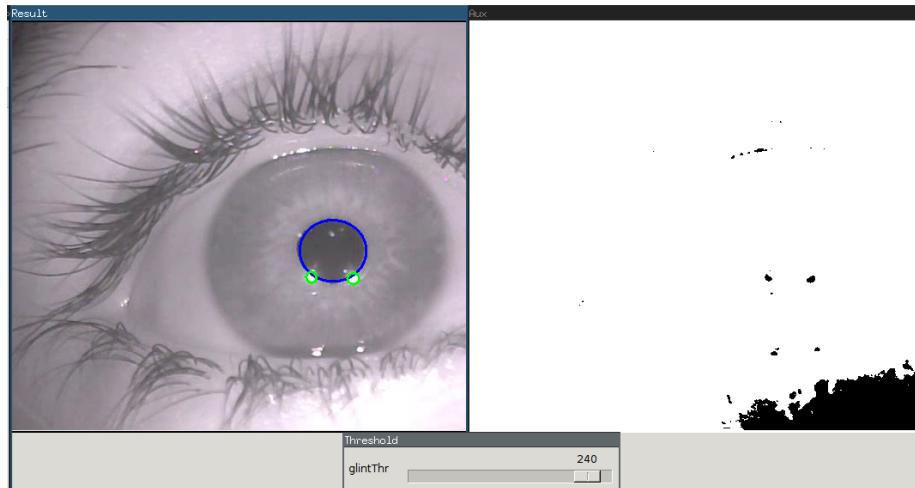


Figure 6: Glints with threshold

When we have the two glints of the pupil we can filter the data further based on these newly found features. Our algorithm for this is very straight forward and imperative:

```

for candA in glints:
    for candB in glints:
        #only accepting points with a certain distance to each other.
        if (Distance(candA[0],candB[0])> sliderVals['glintMinDist'] and Distance(candA[0],candB[0])< sliderVals['glintMaxDist']):
            glintList.append(candA)

#run through the remaining glints, keeping those that are close to the pupil candidates.
for glintCand in glintList:
    for pupCand in pupils:
        if(Distance(glintCand[0],pupCand[0])>sliderVals['glint&pubMINDist'] and Distance(glintCand[0],pupCand[0])< sliderVals['glint&pubMAXDist']):
            glintList1.append(glintCand)

#run through the pupil candidates keeping those that are close to the final glints list
for candP in pupils:
    for glintCand in glintList1:
        if(Distance(candP[0],glintCand[0])>sliderVals['glint&pubMINDist'] and Distance(candP[0],glintCand[0])< sliderVals['glint&pubMAXDist']):
            pupilList.append(candP)

#sort out the pupils too far away from the found glints.
return (set(glintList1),set(pupilList))

```

One critique of this approach could be that the pupil and glints depend on each other “both ways”. That is, we first filter the glint candidates based on the pupil candidates and then the reverse is done. However, the results seem to be fairly precise

## Pixel classification

In this section it will be demonstrated how a form of machine learning is applied to perform supervised classification in order to semi-automatically set a correct thresholding value. Correct is defined as a value which will allow us to perform the steps described in the previous section

Clustering is the practice of grouping a set of elements into several smaller groups of elements with similar features. It has a wide range of applications within datamining and different types of analysis. Obviously what will be demonstrated here is its use within image analysis. Clustering isn't a specific algorithm. Rather it's the task we wish to perform in order to achieve our goal. In this assignment we've used the k-means algorithm for this

K-means is a clustering algorithm which can group a number of observations/data points into k number of clusters based on their nearest mean value.

The properties of the k-means algorithm (k groups based on mean intensity) combined with an existing knowledge of the properties of the eye(the pupil is

the darkest part) makes it possible to use k-means for setting a threshold value automatically

## Theory

The basic idea behind k-means clustering is to iteratively run through a dataset assigning points in their correct cluster based on previously selected values. Initially  $k$  points are selected and denoted as a center for its cluster,  $c_1, \dots, c_k$ . These points can be selected on random or based on some guessed distribution. On each run through the dataset every point is examined. For each point the closest  $c$  is found, and the point is marked as to belong to this cluster. Once all points have been examined and placed in a cluster, the mean value of each cluster is calculated as  $c_{i\text{val}}$ . Compare the mean value for the cluster with the previously recorded value of  $c_{i\text{val}}$ . If it has changed, another run through is performed. This continues until a desired level of precision is achieved or amount of runs have taken place

When we have done this we have  $k$  different threshold values to choose from, given our knowledge of the pupil, we will choose the one with the lowest mean value ( $c_{i\text{val}}$ ).

## Our implementation

There are a couple of possible caveats for this approach. Firstly there is an element of uncertainty in exactly how the clusters will be distributed. We need therefore to have a high enough  $k$  value to be sure to get the right cluster. There is also some uncertainty about whether the pupil always belongs to the darkest cluster. If for instance our  $k$ -value is too high, and there exists a darker region in the image (dark spot on the skin for instance) then the value of this cluster will be chosen as a threshold value, and we might miss the pupil

Through trials it was found that 8 is a good value for  $k$  in the sense that it often proved to segment the picture enough to allow the intensity value of the pupil to exist in one of the clusters. The other parameter is a constant that we apply so that the distance between the points has less importance by dividing each point with this constant. For this 15 was chosen as a good value.

The first step is performance optimization by resizing the image. This saves a lot of computations:

```
smallI = cv2.resize(gray, reSize)
```

where  $\text{reSize}$  is a  $1 \times 2$  vector (30x30 was used)

When this is done we operate on column vectors

```

M,N = smallI.shape
X,Y = np.meshgrid(range(M),range(N))

z = smallI.flatten()
x = X.flatten()
y = Y.flatten()
O = len(x)

```

From this we can create a “feature” matrix with values corresponding to the intensity value of each pixel. Then the k-means algorithm is applied to this feature-matrix to produce the centroids of each cluster. The specific k-means implementation comes from scipy. The same library also provides a vector quantization which gives us the label for each feature. Combining this we’re left with a label image with intensities. From these we pick the darkest and take that value as to mean the intensity of the pupil. In reality this could probably be analysed further to more robustly identify which cluster contains the pupil. For instance we could analyse the original image and verify that there exists an area with this intensity which looks sufficiently like a pupil. If not, the second darkest area could be explored. This will however further increase the computational complexity.

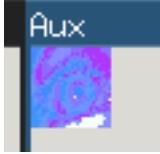


Figure 7: K-Means

### Usefulness of the k-Means algorithm

K-Means seems very robust and definitely useful in applications like this. A huge downside to this however is that it’s computationally expensive. Our implementation is at least. To overcome this we’ve used a downsized image. As can be seen in figure ???. This also makes it very hard to visualize the method, as 30x30 pictures doesn’t look to great. If the downsizing isn’t done, then the sequence can’t really run. One could argue that the lighting conditions of a picture rarely changes on each frame. So doing k-means every time the frame is updated might be overkill, and some calculations could be saved.

## Template Matching

*What do we want to achieve?*

We want to describe how a successful match is found for a template when matching it with an image. We want to explain the main theories behind the methods, and test our implementation under various circumstances.

## Main Theory: Use filtering methods

### Correlation

All the implementations of template matching (that we use) utilizes variations of correlation, to compare templates with image segments. The technique is the same as filtering with various predetermined kernels, like gaussian, sobel, box-filter etc. , to gain various filter effects, but instead we use an actual image template as a kernel and measure the difference between this and a section of the image we want to look through.

Normal (un-normalized) correlation is mathematically described as:

$$h[m, n] = \sum g[k, i]f[m + k, n + l]$$

Using this method for matching, is prone to a lot of “false” positives, as the result is dependent of the actual pixel values, not determined by the degree of match between the filter and the image segment. The sum of the multiplied pixel values, will be large in bright areas of the image and vice versa in the dark areas. To make sure the correlation also considers the overall brightness of the segment, a normalized version of the formula is introduced.

If you step out one level of abstraction and view the template H and image segment F as two vectors (achieved by joining each row or column after each other), the correlation can be viewed as finding the dot product between the two vectors. H dot F. (Just take a second and realize that the dot product is found the same way as we calculate correlation). As we know a vector can be normalized by dividing it by its length, we can also normalize the dot product by dividing it by the multiplication of the length of the two vectors.

the relationship is described in the formula:

$$H \cdot F = \|H\| * \|F\| * \cos V$$

$\leftrightarrow$

$$\cos V = (H \cdot F) / (\|H\| * \|F\|)$$

The normalised expression will always give a result between -1 and 1 as  $\cos V \in [-1, 1]$ , and since the image vectors only contains positive numbers the result will be between zero and one.

The length of each the two vectors are found with the formula: (here with a vector in 3 dimensions)

$$\|a\| = \sqrt{a_1^2 + a_2^2 + a_3^2}$$

Translating back from this abstraction level and looking at correlation again the formula is translated into:

Normalized Cross Correlation:

$$(x, y) = \sum g[k, l]f[m + k, n + l]$$

This can be refined further by subtracting the means for both template and image patch. This gives us the zero mean normalized cross-correlation, AKA correlation coefficient:

$$h[m, n] = \frac{\sum k, l(g[k, l] - \bar{g})(f[m + k, n + l] - \bar{f}_{m, n})}{((\sum k, l g[k, l] - \bar{g})^2 \sum k, l (f[m + k, n + l] - \bar{f}_{m, n})^2)^{0.5}}$$

This gives a very accurate result, but is pretty slow as well.

#### **Another implementations using correlation:**

Sum of squared difference:

$$(u, v) = \sum x, y f(x, y) - t(x - u, y - v)^2$$

s compares each corresponding pixels describing the difference between them with a calculated positive number. (because of the squared) The closer to zero the final result is, the better the match. The image viewed (where white equals good results) are calculated with: image (x,y) = 1 - sqrt(result of above expression).

Our Implementation:

```
def GetEyeCorners(img, leftTemplate, rightTemplate, pupilPosition=None):
    #The method parameters are: the image, templates and optional pupil position.
    sliderVals = getSliderVals()

    #Enable adjustment from slidervalues for match threshold.
    matchLeft = cv2.matchTemplate(img, leftTemplate, cv2.TM_CCOEFF_NORMED)
    matchRight = cv2.matchTemplate(img, rightTemplate, cv2.TM_CCOEFF_NORMED)

    #The openCV library allows use of different matching methods. Here we use COOEFF_NORMED
    #Matching both for left template and right template.

    if (pupilPosition != None):
        #If the pupil position is set, slice the image in two halfs at the pupil position.
        pupX, pupY = pupilPosition
        matchRight = matchRight[:, pupX:]
        matchLeft = matchLeft[:, :pupX]
    matchListRight = np.nonzero(matchRight > (sliderVals['templateThr']*0.01))
    matchListLeft = np.nonzero(matchLeft > (sliderVals['templateThr']*0.01))

    #Sort out the results with values below the match threshold.
```

```
matchList = (matchListLeft,matchListRight)
return matchList
```

### Our Results: (Also see video)

We found that, when using the normalize cross corellation, a good starting threshold is 0.85. That doesn't draw to many detections from the start and, provides a good starting point for further adjustments. All the tests runs utilizes the pupil position, allowing matches for the left template only in the left side and opposite for the right template matches.

#### Test 1 - Unknown Subject.

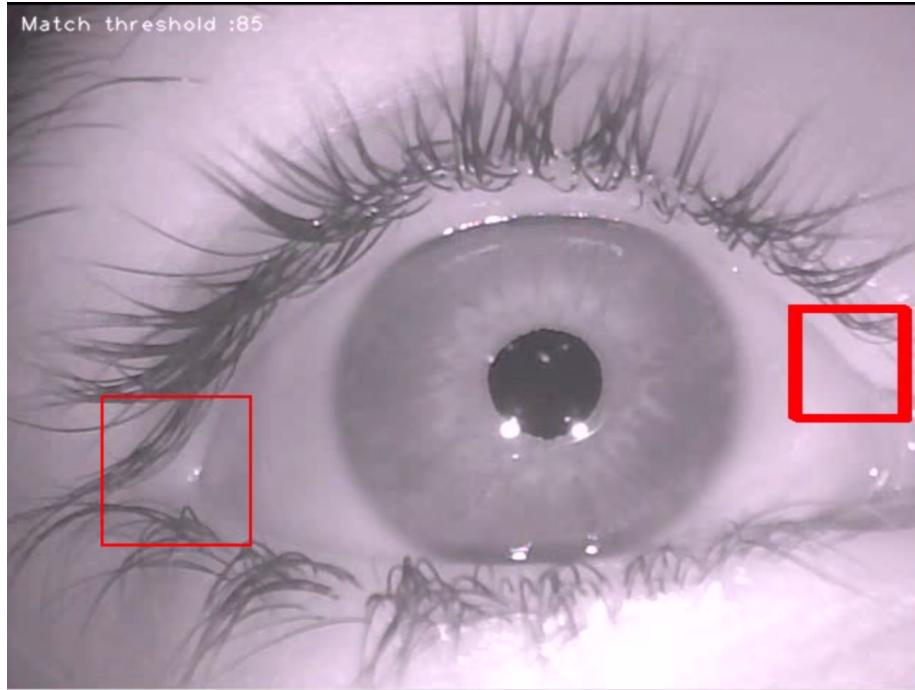


Figure 8: Threshold 85

As we see in figure 8, with at threshold of 0.85, things work out fine, when there are not too much movement in the image. In controlled light, controlled subjects, the correlations methods works like a charm.

Seen in figure 9. As soon as the subject moves too much, the threshold has to be lowered to find any results. It quickly becomes a problem having a shared thresh-



Figure 9: Threshold 66

old for both template matchings (left & right), as the circumstances around the two places in the image are quite different.

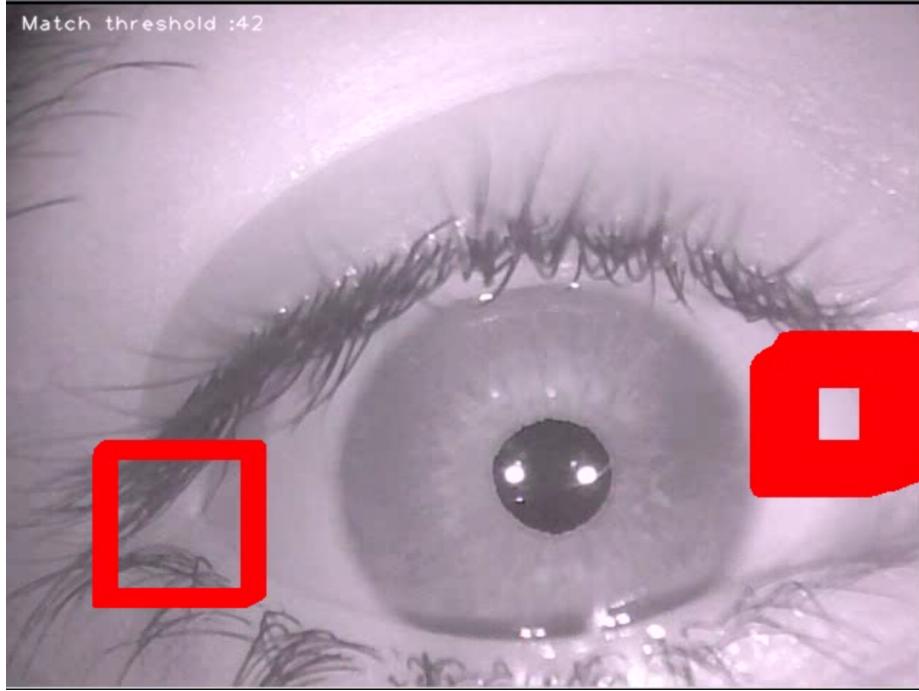


Figure 10: Threshold 42

Seen in figure 10. Lowering the threshold even more, allows the subject to move more freely and still finding matches in both sides. Again we see a big difference in which sides that struggle and which that finds multiple matches.

#### Test 2 - Young Master Ghurt - recorded tuesday 12/03/13

In figure 11. Starting out with a high threshold of 0.92 seems to work quite well. When the template contains enough contrasts/diversity, and the subject remains still, the errors are few

In figure 12 we see it doesn't take a lot of change before the threshold has to be lowered to find a result. Again there is a huge difference on the sides, and we can conclude that a shared threshold wouldn't be suitable in an industrial/finished solution.

In figure 13 we see as we continue the path downwards lowering the threshold to find matches at the sides, a lot of false results now appear, as the match no longer have to fit perfectly. This is not a bad setup for further development,

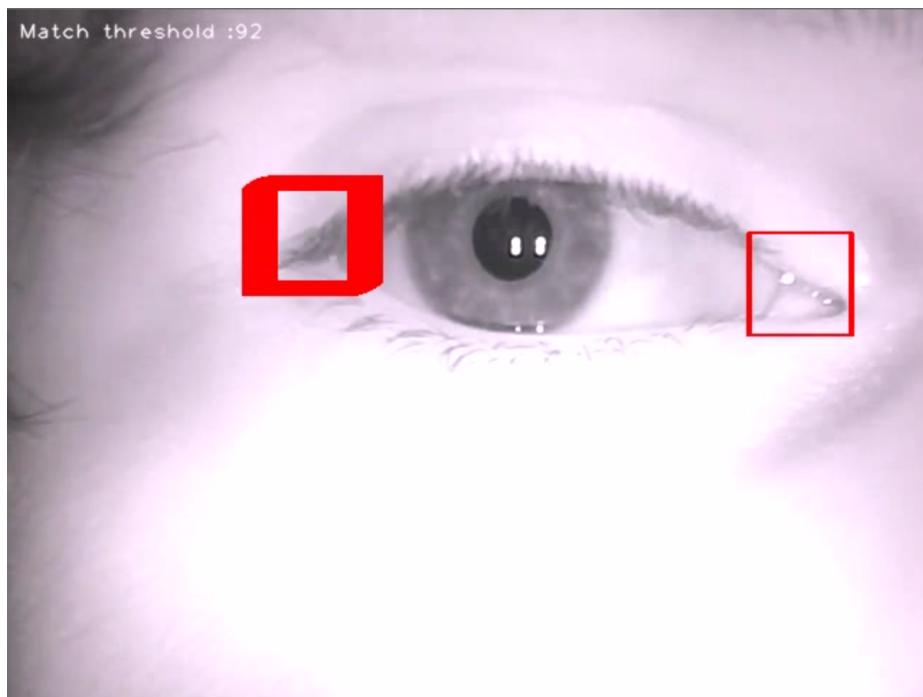


Figure 11: Threshold 92

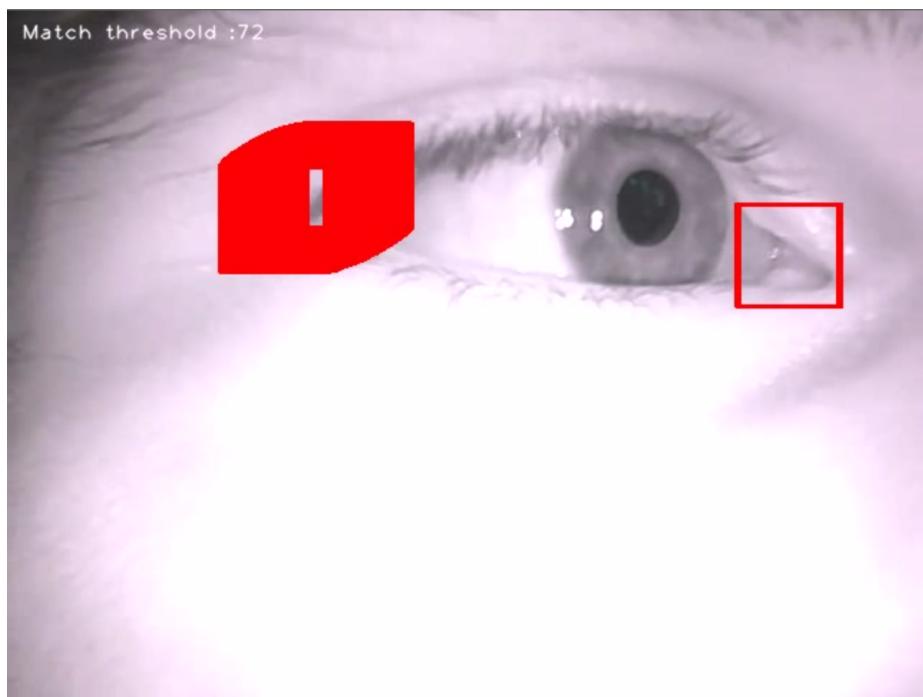


Figure 12: Threshold 72

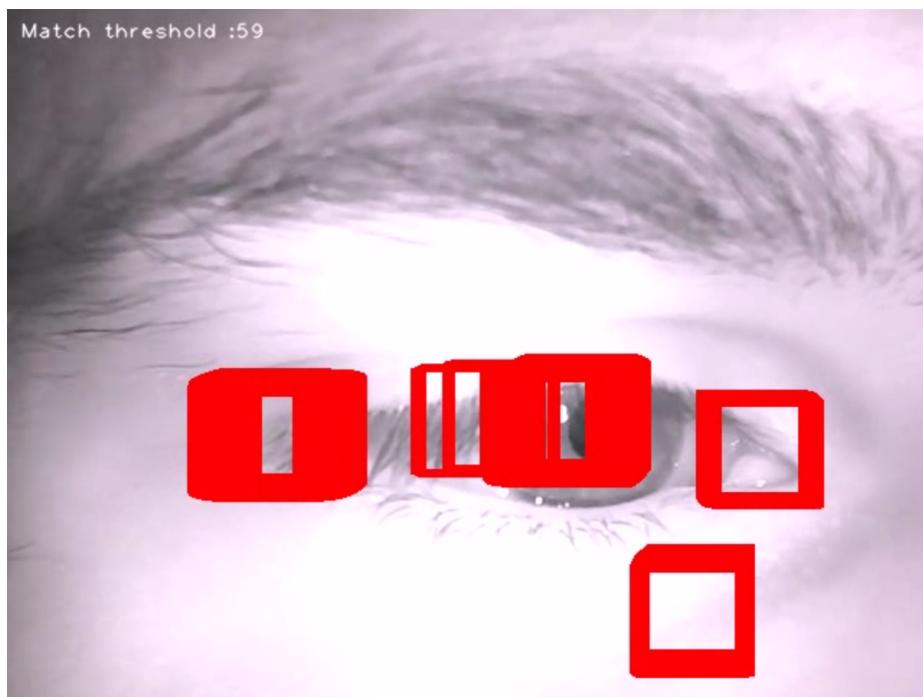


Figure 13: Threshold 59

as the “bad” results easily could be filtered away. (in the sense that too many matches are better than none at all).

### **Concluding on the results and improving the method:**

It is obvious that our implementation can be improved. As in all the different techniques mentioned in this report, controlling the circumstances are vital for success. Out of the box, cross correlation proves to be a pretty solid way of finding matches, but it requires the subject to stand still and keep the same angle towards the camera so the match area doesn’t change completely. The method doesn’t provide blazing speeds as it is now. But this could be improved by performing some of the work in parallel processes. Instead of matching twice with different templates, you could use one common template, that would find enough matches on each side and then filter out bad results even more. As of now, results are only restricted in that they have to appear on the correct side of a found pupil location. This could be even more improved, utilizing the distance from each other, or placement relative to other facial features. As with the other techniques it proves impossible to find parameters to fit a sequence of images where the subject changes/moves. It’s far better to find many matches and remove “bad” results afterwards.

## Edges and Gradients

### Overall rationale and theory

Extending on the previously described means of detecting the pupil, image intensity gradients provide a method for finding the edge between pupil and iris, as well as the limbus (the edge between iris and sclera). The problem of finding edges in an image can be reduced to the problem of finding high magnitudes in the image's gradients. In practice:

- Noise reduction is performed,
- The image gradients are found, and
- A subset of the gradients are scanned to determine the location of edges in the original image.



Figure 14: An attempt at using binary thresholding to locate the iris. while the iris is not a continuous blob, it is starting to converge with the sclera.

This solution is explored partly as an alternative to finding the limbus via thresholding. , illustrates the difficulty of finding a good threshold for the iris.

### Using gradient magnitudes

A bilateral filter was chosen for noise reduction. In contrast to blurs like gaussian filters, bilateral filters weigh the inputs according to both euclidean distance to the output pixel, and the intensities in the source patch. The consideration of intensity differences makes bilateral filters edge preserving which, not surprisingly, is crucial for edge detection.

The image gradients are found by applying the sobel operator on the image. In effect; finding approximated partial derivatives for the vertical and horizontal

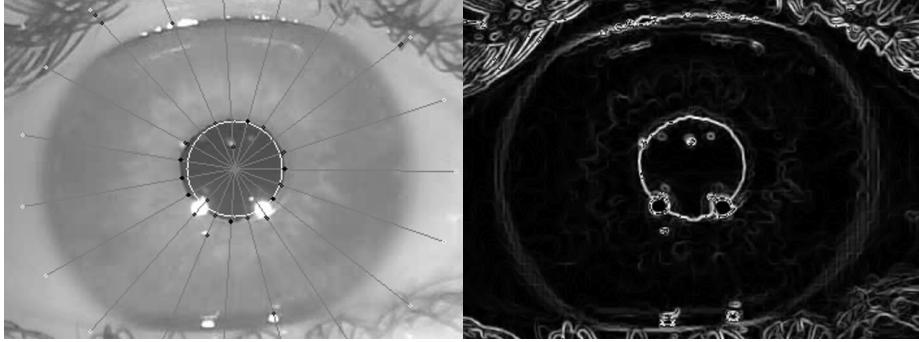


Figure 15: **Left:** Black dots are the highest gradient magnitudes found on the grey lines (here, 2 per line). The white circle is the initial estimate of the pupils location. **Right:** The set of gradient magnitudes represented as an image.

axes respectively. Together, the two derived images form a set of vectors, each describing the gradient for its pixel location. The length of one of these vectors is the gradient magnitude of a certain point in the original image. That is; the rate of change in intensity in a specific pixel. An "edge" is a continuous path of high gradient magnitudes. The gradient magnitude in the point  $(i, j)$  is found by

$$\sqrt{x^2 + y^2}$$

where

$$x = \frac{\delta z}{\delta i}, \quad y = \frac{\delta z}{\delta j}$$

and  $z$  denotes pixel intensity.

The scanned lines are chosen as the normals of a previously found estimate of pupil, here scaled to 4.5 times the estimate's radius, and spaced with 0.314 radians. In principle this method should provide 20 points on the periphery of the pupil, and should be extensible to finding the limbus. In practice, however, a few problems might arise.

## Using gradient orientation

When examining the above two images of the eye, it becomes obvious that the pupils edge might not be have the highest gradient magnitude. For example, the eyelashes and glints might give a larger intensity change, than the black-to-grey change between pupil and iris. Luckily this problem can be bettered using information already available, namely the image gradients' orientations. By considering the gradient's angle to the pupil normal, some of the high gradients can be discarded. This works on the assumption that the pupil is fairly circular, and that estimated pupil location is somewhat correct. Thus, the angle between

normal and gradient on the pupil edge will be small. The gradients' angles to the positive x-axis is found and compared to the normals angle to the positive x-axis. The angle between a vector and the positive x-axis can be found by applying the arctangens2 operator to its lengths:  $\text{atan2}(x, y)$ .

The use of gradient orientation presents a new problem; performance. The use of the atan2 operator proves to be heavy enough that it became necessary to reduce the size of the input image. As this method assumes a previous estimate of the pupil's location, it might be preferable to select an image patch around the eye, rather than actually downsampling the input image. Though the latter will be less robust to change in eye size or distance to the camera.

The described method is applicable to limbus detection, but in practice the iris is much harder to detect. This is probably due to the smaller contrast between iris and sclera, as well as to the edge between them being 'jagged' (this is most visible in the right side of iris in the above gradient image).

## Hough Transformation

### Main Theory:

Hough transform is a voting technique that is used to determine the "most possible" lines in the image.

Each line in a 2d image can be represented by:

$$y = mx + b, \text{ where } (x, y) \text{ is a point in the image space.}$$

Each line in image space can also be represented by a point  $(m, b)$  in the Hough space, where:

$$b = -mx + Y$$

A single point  $(x, y)$  in image space maps to all solutions of  $b = -mx + y$  which corresponds to a line in the hough space.

If we have two points  $(x_0, y_0)$  and  $(x_1, y_1)$  they'll each correspond to lines in the hough space. As each point in hough space is a line in the image space, the point where the lines intersect in hough space, will be the line in the image space, that contains both points  $(x_0, y_0)$  and  $(x_1, y_1)$ . The solution to:

$$b = -mX_0 + y_0 \text{ and } b = -m x_1 + y_1 \text{ (intersection between)}$$

### How does it work?

As input to a Hough algorithm we use a thresholded gradient magnitude image. (see Fig. 18) This is usually computed by first doing some soft smoothing, then using a Sobel filter (fig. 17) for both vertical and horizontal directions.

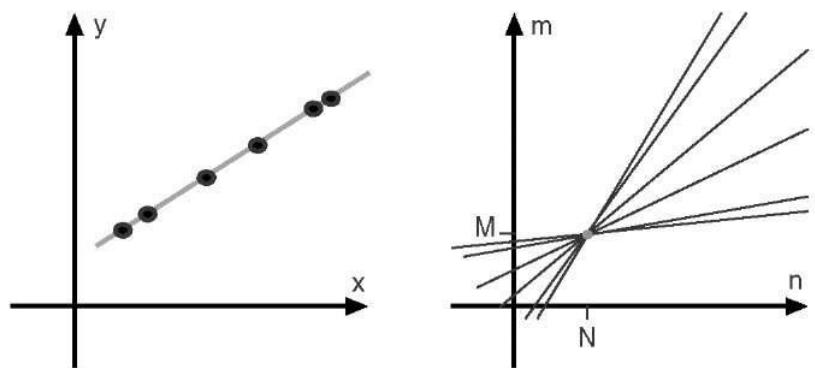


Figure 16: Hough space

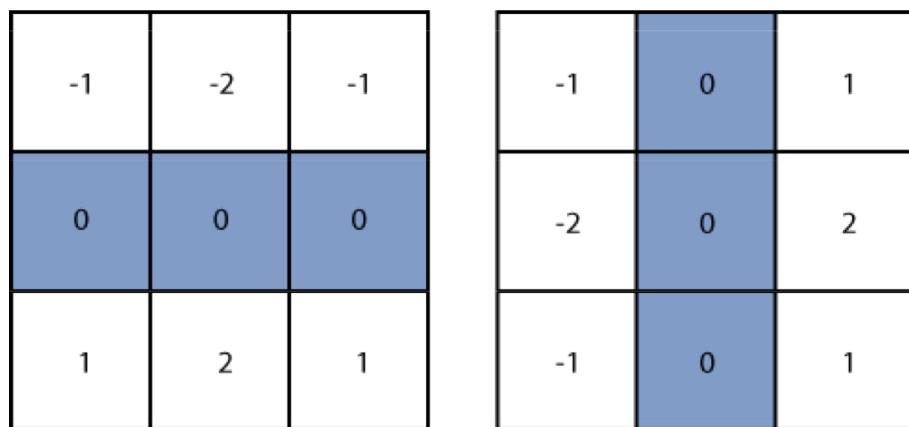


Figure 17: Sobel kernel

From this image the edge magnitude image is computed, where:

$$g(x, y) = \sqrt{g_1^2(x, y) + g_2^2(x, y)}$$
 is the gradient norm for in the point  $(x, y)$



Figure 18: Our calculated edge magnitude image for a closeup image of an eye. Brighter spots equals a longer gradient which means a larger contrast

Thresholding this image will (if we exclude all below the threshold) give us a set of  $n$  pixels, that with high probability is part of a edge/line in the image. Each of these pixels will produce an infinite amount of lines in the Hough space. Where a lot of these lines intersect is a point corresponding to a line in the image space. Each point in image space, will produce a lot of “bad” results, and to filter out these results the hough algorithm uses a voting system.

The hough space is quantized into small cells. Each time a line in the hough space passes through a cell, the value of the cell is increased by one. We say that the line “votes” for that cell. After calculating all possible votes in the hough space, we’ll end up with an accumulated image, where several points have voted for certain points, thereby voting for the existence of a line in the image space.

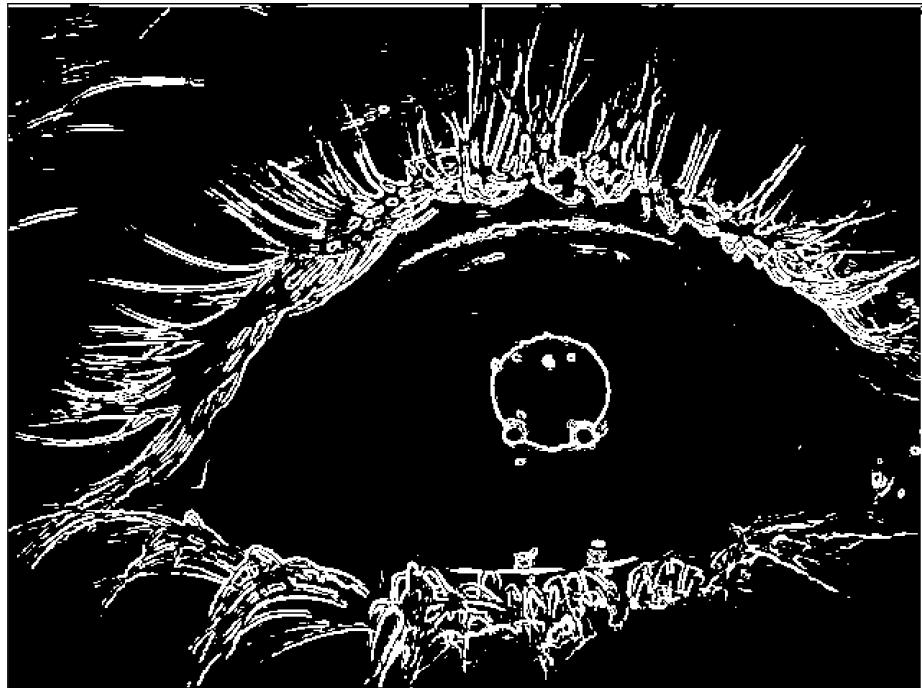


Figure 19: Thresholded magnitude image

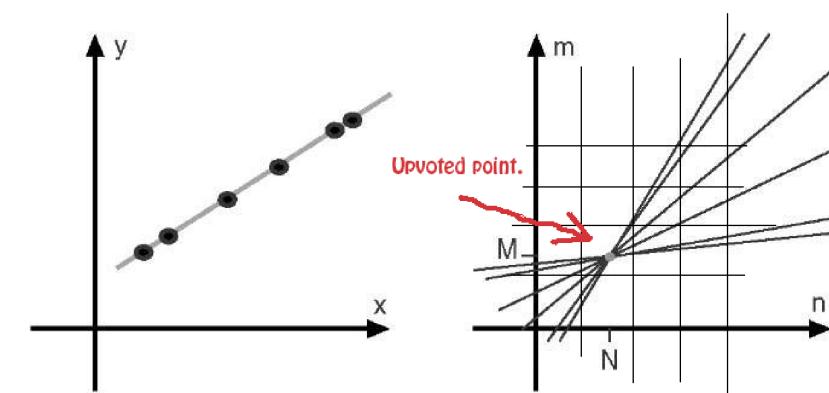


Figure 20: Upvoted point

## Polar representation

Using  $b = -mx + y$  as representation in Hough space can be problematic when dealing with vertical lines, so instead we use the polar representation where each point in image space  $(x,y)$  corresponds to points on the sinusoid curve  $x_i \cos \theta + y_i \sin \theta = \rho$  in Hough space. The technique is the same, where each point  $(\rho, \theta)$  on the curve will up-vote points in the accumulated Hough-image  $H[\rho, \theta]$



Figure 21: Hough space sinusoid curve. (Here rather illustrative but alas not very informative, as it is without coordinates.)

## Hough Circle Transform:

In the line detection case, a line was defined by two parameters. In the circle case, we need three parameters to define a circle.(From OpenCV documentation.)

$$C : (X_{center}, Y_{center}, radius)$$

The hough-space is therefore also increased by one dimension, to accommodate the third “unknown”. Should we wish to find more sophisticated shapes, the hough space can theoretically be increased by n dimensions. Although the calculations performed in the voting algorithm, are expensive and the accumulated cost rise dramatically with each dimension added.

## Open CV implementation:

The openCV method first performs a canny edge detection, and then uses a bit more complex voting method using the gradient directions to guide the Hough voting process. (Presumably using the methods we described earlier , sorting out voting results, where the angle between the gradient, and the circle normal within a defined threshold. Although the OpenCv documentation is not quite clear on this point).

### Our Implementation:

```
def detectPupilHough(gray):
    #The method takes a gray-scale image.
    blur = cv2.GaussianBlur(gray, (81,81), 11)
    #The image is blurred to remove noise, that could potentially create gradients where the
    #slidervals = getSliderVals()
    #This is so we can adjust the min/max radius for accepted circles.
    dp = 6; minDist = 30
    #This determines the resolution of the Hough-space and the minimum distance between found
    #highThr = 20
    #As the Cv2 uses canny edge detection, this sets the threshold for accepting "line" pixels.
    accThr = 150;
    #Determines how many votes that a circle should receive to be accepted (and drawn).
    minRadius = slidervals['Hough pupil size']-7;
    maxRadius = slidervals['Hough pupil size']+7;
    #Sets the size of the accepted circles.
    circles = cv2.HoughCircles(blur, cv2.cv.CV_HOUGH_GRADIENT, dp,minDist,
    #The actual cv2 method that uses all the parameters just explained.
    #The rest is just drawing the circles (drawing a red circle for the circle with most votes).
    gColor = cv2.cvtColor(gray, cv2.COLOR_GRAY2BGR)
    if (circles !=None):
        all_circles = circles[0]
        M,N = all_circles.shape
        k=1
        for c in all_circles:
            cv2.circle(gColor, (int(c[0]),int(c[1])),c[2], (int(k*255/M),k*128,0))
            K=k+1
            c=all_circles[0,:]
            cv2.circle(gColor, (int(c[0]),int(c[1])),c[2], (0,0,255),5)
    cv2.imshow("houghPupil",gColor)
```

### Our Results

Finding the pupil: As with all the methods in this report, the parameters of the cv2 method had to be adjusted to fit the resolution and lighting of each video.

**Test 1. Unknown subject closeup.**

figure 22 Test settings: Smoothing: gaussian blur, kernel size (11,11). Circle Threshold: 200.

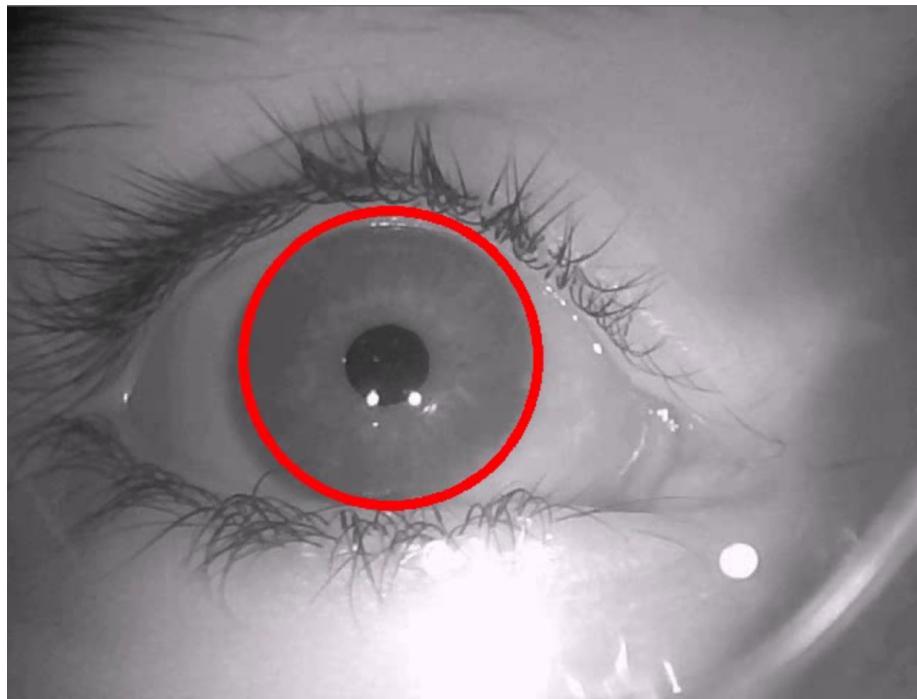


Figure 22: These circumstances are ideal for detecting the iris, as the iris itself is highly visible and a lot of high contrast circle points help determine the circle placement. The top of the circle may be detected on the eyelid rather than the iris though, as the contrast there is larger than on the actual iris.

**Test 2. Young Master Roed recorded 12/03/13 Pupil Detection.**

Test settings: GaussianBlur(gray, (11,11), 11) Threshold : 50 Min/Max size circle: 10-17

**Test 3. Young Master Roed recorded 12/03/13 Iris Detection.**

Test settings: GaussianBlur(gray, (21,21), 11) Threshold : 120 Min/Max size circle: 30-37

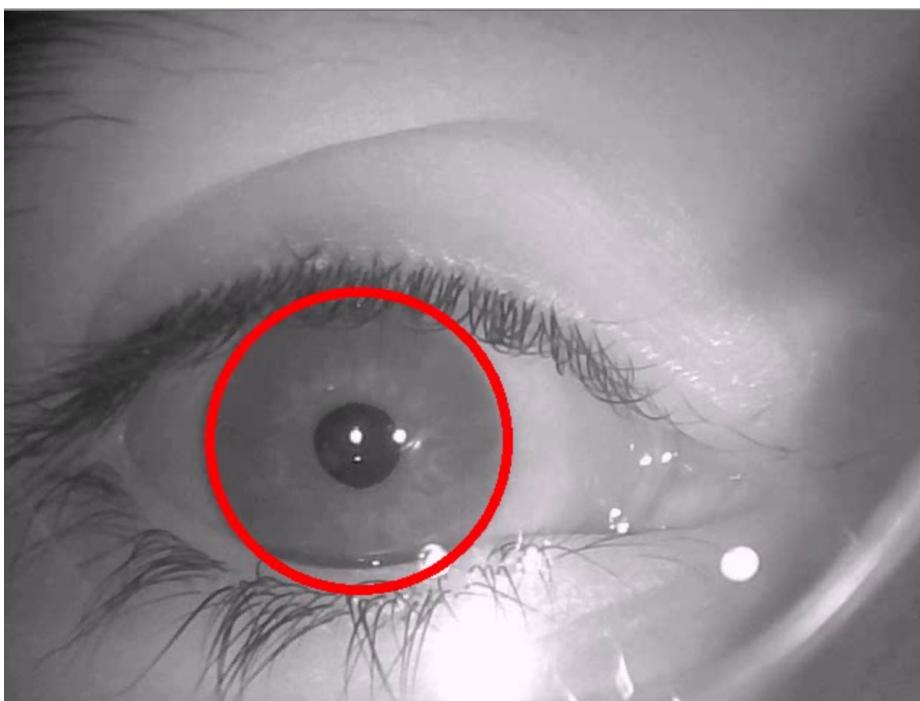


Figure 23: On this image we see why the hough transformation is really reliable. The circle is detected even though the eyelids top and bottom cover some of the iris. The points at the sides make up for that and casts enough votes on there being a full circle where the iris is, therefore overcomming the problem that some of the iris is not visible.

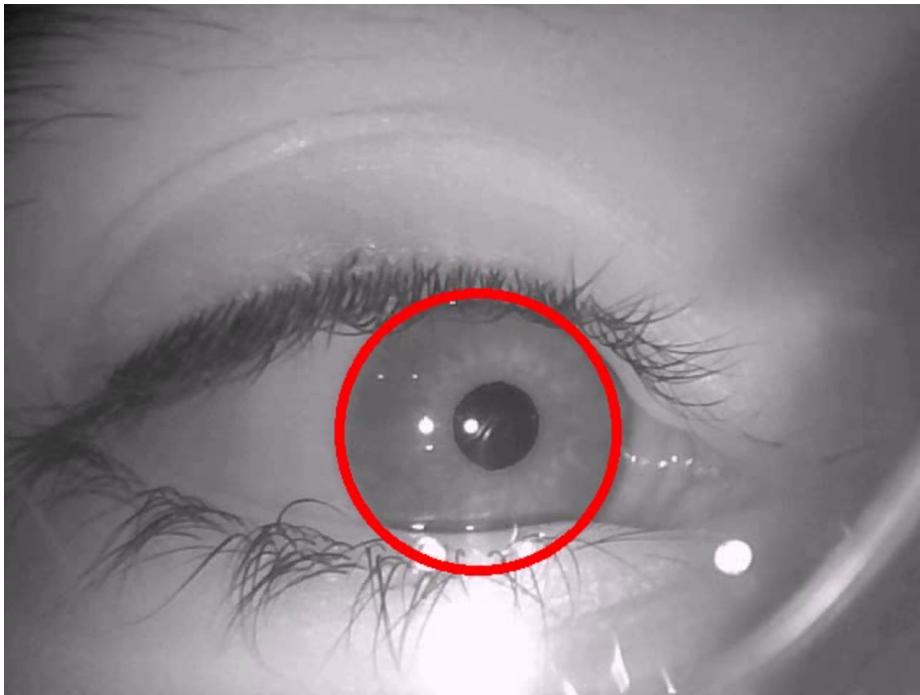


Figure 24: Even at the sides the method detects a circle, through movement and other distractions. The circle jumps a lot, as more than one circle is detected around the iris and they each take turns getting the most votes, but overall a good result in these ideal circumstances.

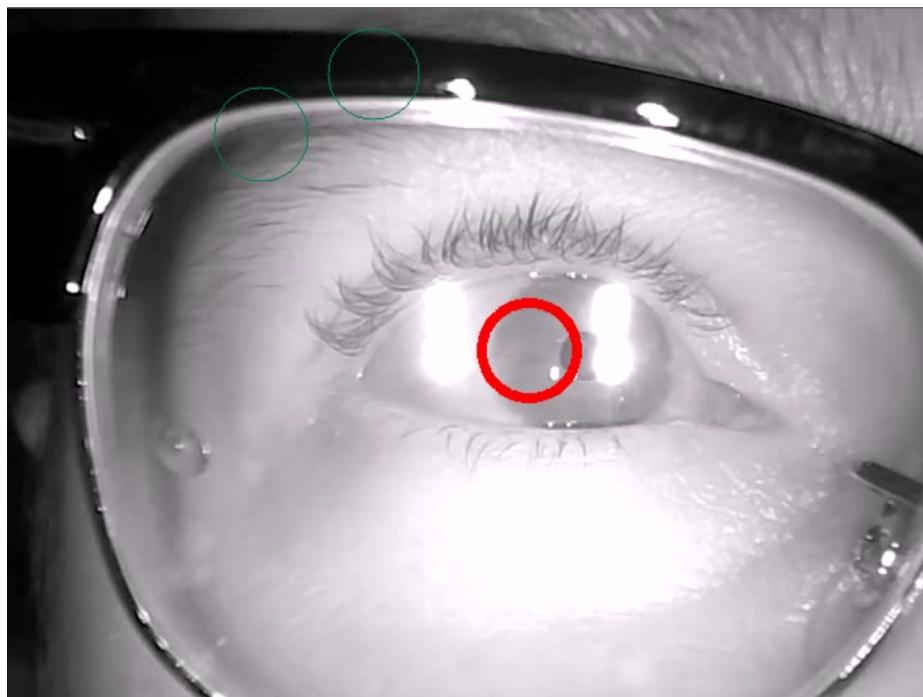


Figure 25: This image contains many distractions. The infrared lights made a massive four reflections in the glasses covering the pupil. Instead of finding the pupil, a part of the iris is detected as a circle edge.

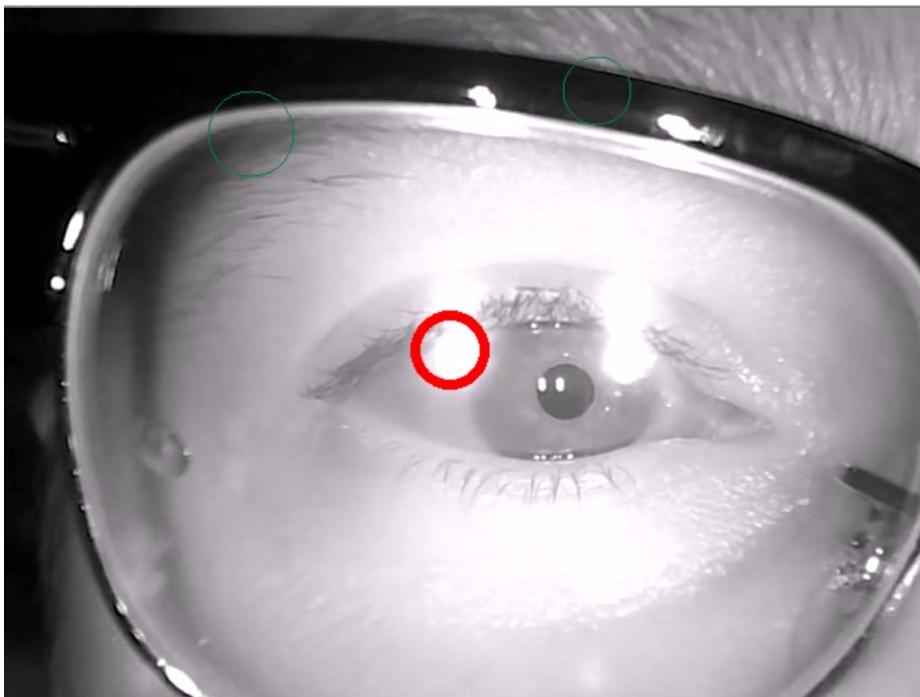


Figure 26: The distractions keep having an influence on the result, as on this image, where the best circle match is the circular reflection of the infrared light. This could be compensated for by lowering the threshold and filtering the bad results afterwards.

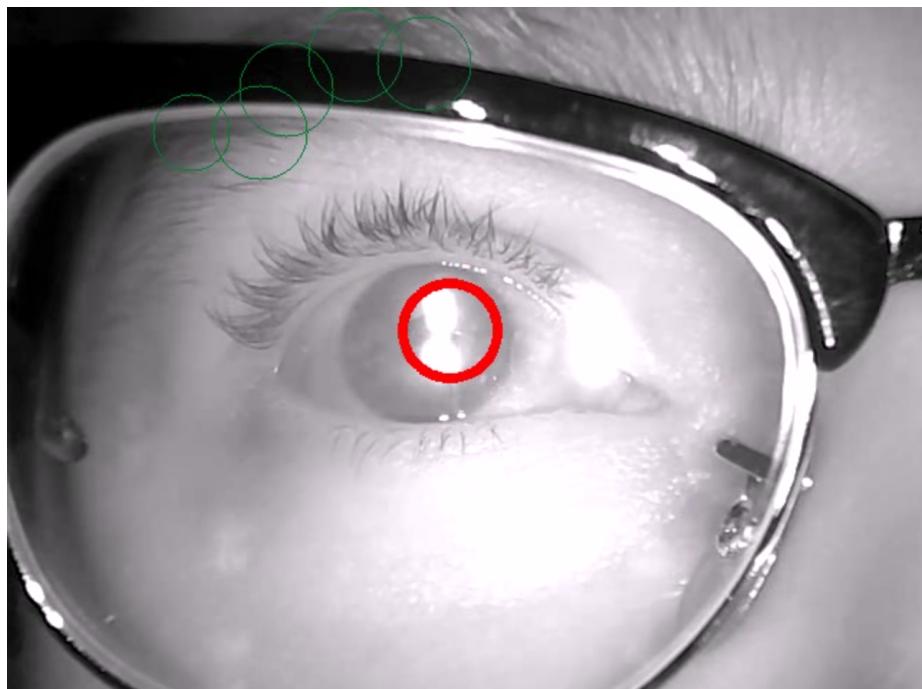


Figure 27: The distractions just proved to be too large an obstacle, as the eye movements keeps concealing most of the pupil. Even though the Hough method is pretty stable, it still requires a bit more controlled environment.

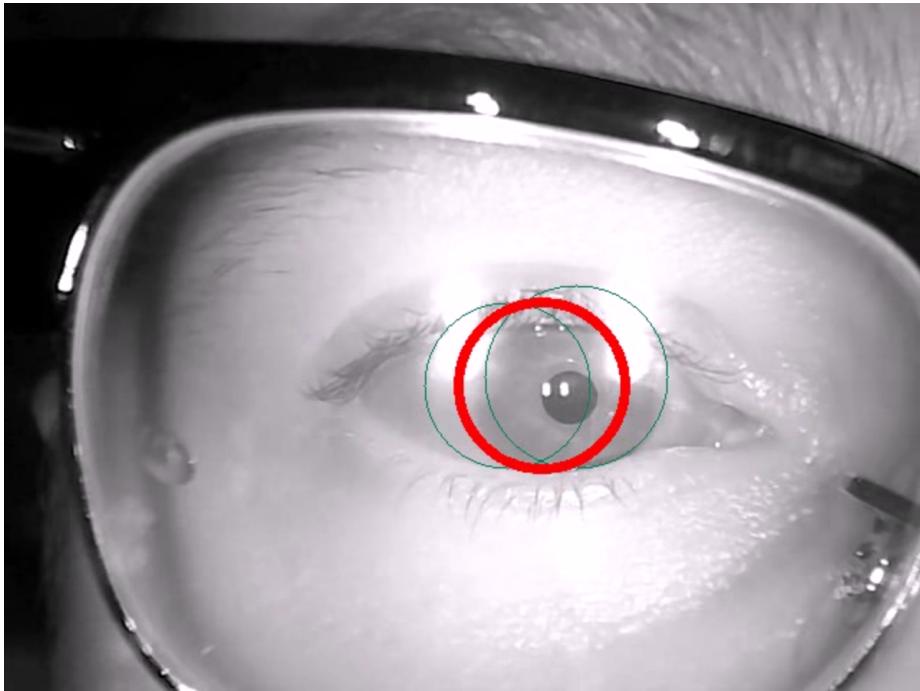


Figure 28: Finding the pupil proved a bit easier. This is probably because the iris size is much bigger than the distraction reflection circles.



Figure 29: Looking to the sides will make the circle jump around a lot, as it can be seen at the video. A lot of the votes still end up pat the correct spot though.



Figure 30: In this fram, there's just not enough circle edge points visable to determine the presence of a circle. The right side of the iris could help, but its just not enough without lowering the threshold even further.

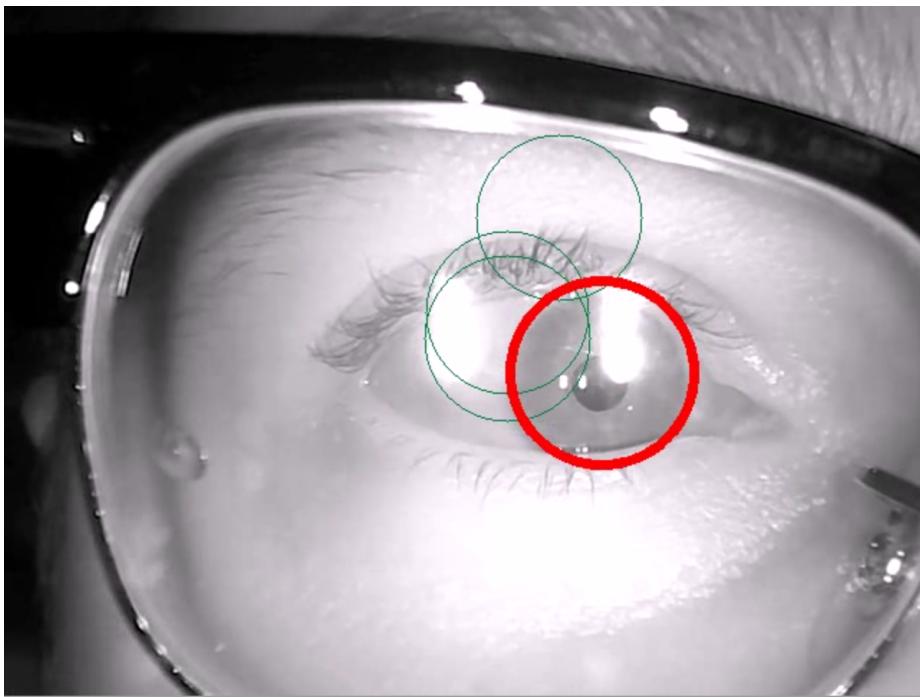


Figure 31: The threshold has been lowered even further here to 90 and other circles appear. The most votes are still placed on the correct circle though.

## **Our Impressions of the technique:**

### **When does it work?**

In an environment, where you can control the distance, resolution and lighting of the video/image sequence, the Hough transform is very solid. The method shines when trying to detect circles that are not complete. As long as enough points are visible so that the circle-votes pass the defined threshold, the method will draw the circle. This puts the method ahead of the competition. When compared to other methods like the thresholding method, that depends on the circularity of the found blob, and when looking at solely using the norms for detection, being more prone to error caused by other high gradients placed on the norm lines, the Hough transform seems like the way to go for pupil and Iris detection.

### **When does it not work?**

It can be hard to set the correct threshold for the canny edge detection as well as the minimum votes threshold, to get the correct amount of circles. If you set it to strict it may look well with optimal conditions, and then jump to drawing nothing as soon as the conditions deteriorate. On the other hand, if you are less rigid with the thresholding and min/max radius, it may look well in one image and then jump to thousands of found circles in the next image, slowing down the whole process. When detecting the iris we ran into some problems as well, as the iris is not so well defined as the pupil. As it can be seen in the video, the found iris-circle jumps a lot around. This is probably caused by the lack of iris edge-pixels visible in the image for the circle. The circle is found only with votes from the sides of the iris, as the shape of the iris in reality resembles an ellipse, not a circle.

## **Improving the method:**

If we were to improve the method, we would combine it with more of the other techniques mentioned in this report. We would: Automatically determining the pupil size with k-mean clustering. Sort out “wrong” results with positioning I relation to other features. Use the last known position to filter out wrong results. (as the video is about 30 frames per second we can estimate the max movement per frame, and filter out results that lie outside this range.)

## **Conclusion:**

Out of the box, Hough transform works quite well for pupil/iris detection, its robust even if the image contains noise (salt and pepper, not large distractions),

and when considering that the method allows for searching for multiple times appearance of a shape in a single pass its both fast and reliable.