Question 1

(a) If Ali wins:

BBB $\rightarrow \frac{3!}{3!} = 1$

BBBW $\rightarrow \frac{4!}{3! \cdot 1!} = 4$

BBBWW $\rightarrow \frac{5!}{3! \cdot 2!} = \frac{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \overset{2}{\cancel{4}} \cdot 5}{\cancel{1} \cdot \cancel{2} \cdot \cancel{3} \cdot \cancel{2}} = 10$

If Ahmed wins:

WWW $\rightarrow \frac{3!}{3!} = 1$

WWWB $\rightarrow \frac{4!}{3! \cdot 1!} = 4$

WWWBB $\rightarrow \frac{5!}{3! \cdot 2!} = \frac{\overset{2}{\cancel{4}} \cdot 5}{\cancel{2}} = 10$

Ω = { BBB, BBBW, BBWB, BWBB, WBBB,
BBBWW, BBWWB, BWWBB, WWBBB,
WBWBB, WBBWB, WBBBW, BWBWB,
BWBBW, BBWBW
WWW, WWWB, WWBW, WBWW, BWWW,
WWWBB, WWBBW, WBBWW, BBWWW,
BWBWW, BWWBW, BWWWB, WBWBW,
WBWWB, WWBWB }

$|\Omega| = 30$

(b) P(Ali) → probability that Ali wins the game

E_{Ali}: Ali wins = { BBB, BBBW, BBWB, BWBB, WBBB
BBBWW, BBWWB, BWWBB, WWBBB,
WBWBB, WBBWB, WBBBW, BWBWB,
BWBBW, BBWBW }

$$P(Ali) = \frac{|E_{Ali}|}{|\Omega|} = \frac{15}{30} = \frac{1}{2}$$

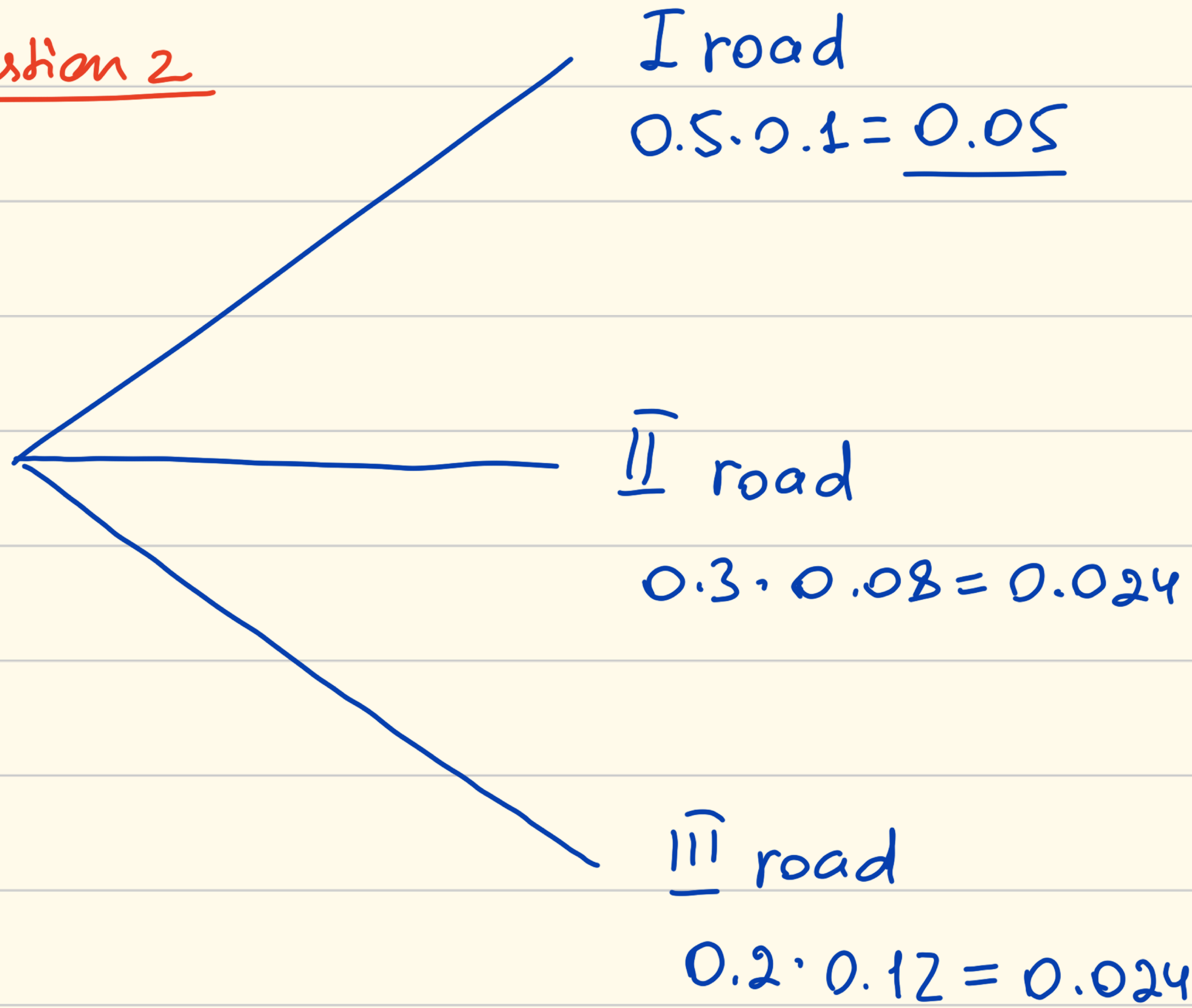
(c) P(AliBlack) = probability that Ali wins the game, given that the first car is black

$E_{AliBlack}$: Ali wins

$E_{AliBlack}$ = { BBB, BBBW, BBWB, BWBB,
BBWWW, BBWWB, BWWBB, BWBWB,
BWBBW, BBWBW }

$$P(E_{AliBlack}) = \frac{|E_{AliBlack}|}{|\Omega|} = \frac{10}{30} = \frac{1}{3}$$

Question 2



ⓐ P(first rode) : probability of choosing first rode

P(no traffic | first road) : probability of not getting stuck in traffic in first road

P(first road)P(no traffic | first road) =

$\Rightarrow 0.5 \cdot 0.1 = \underline{\underline{0.05}}$

ⓑ P(second rode) : probability of choosing second road

P(no traffic | second road) : probability of not getting stuck in traffic in second road

$\Rightarrow 0.3 \cdot 0.08 = \underline{\underline{0.024}}$

Ⓒ P(third rode) : probability of choosing third road

P(no traffic | third road) : probability of not getting stuck in traffic in third road

$\Rightarrow 0.2 \cdot 0.12 = 0.024$

Question 4

n couples
n prizes

each person can win only one prize

choosing a prize from n prizes for each couple :

1st couple: $\binom{n}{1}$ → $\dfrac{n!}{\cancel{(n-1)!}\,1!}$ ·

2nd couple: $\binom{n-1}{1}$ → $\dfrac{\cancel{(n-1)!}}{\cancel{(n-2)!}\,1!}$ ·

3rd couple: $\binom{n-2}{1}$ → $\dfrac{\cancel{(n-2)!}}{\cancel{(n-3)!}\cdot 1!}$ · ... ·

... nth couple: $\binom{1}{1}$ → $\underbrace{\dfrac{\cancel{1!}}{\underbrace{0!}_{1}\cdot 1!}}_{1}$

n! → choosing a prize from n prizes for each couple:

then, we should take into account that we have 2 person choices for each couple → we have n couples

so totally we have 2^n person choices.

$|E| = n! \cdot 2^n$

Ω : our samplespace is choosing n people from 2n people

$|\Omega| = \binom{2n}{n}$

so, our probability is: $P(E) = \frac{|E|}{|\Omega|} = \boxed{\frac{n! \cdot 2^n}{\binom{2n}{n}}}$

answer

Question 5

ⓐ $P(T=t) = \binom{t-1}{r-1} \cdot p^r \cdot (1-p)^{t-r}$

t: the number of trials

r: the number of successes we want (5 toys for ages 3-8)

p is the probability of success (0.3)

1-p is the probability of failure (0.7)

Given that $P(T=t)$ represents the probability of finding 5 toys suitable for ages 3-8 on the t^{th} trial, we can calculate this PMF for $t > 14$

$$p_T(t) = \binom{t-1}{5-1} \times (0.3)^5 \times (1-0.3)^{t-5}$$

ⓑ The conditional probability $P(T=t \mid T>14)$ is the probability of needing t trials given that we haven't found the 5 toys even after 14 trials

- $P(T=t \cap T>14)$: the probability of requiring t trials and it being greater than 14
- $P(T>14)$: the probability that the number of trials needed is more than 14

$$P(T>14) = \sum_{t=15}^{\infty} P(T=t)$$

$$P(T>14) = 1-\sum_{t=0}^{14} P(T=t)$$

$$P(T>14) = 1-\sum_{t=0}^{14} \binom{t-1}{5-1} \times 0.3^5 \times (1-0.3)^{t-5}$$

Question 6

(a) $P(i \geq k) = \sum_{i=k}^{N} \binom{N}{i} \times p_1^i \times (1-p_1)^{N-i}$

↳ the formula for calculating the probability of at least k students present in the 1st week

(b) $P(n \text{ students in week } w) = \binom{N}{n} \times (p_w)^n \times (1-p_w)^{N-n}$

$$P(\text{current week } w) = \frac{1}{W_{max}}$$

$P(\text{current week } w \mid n \text{ students}) =$

$$= P(n \text{ students in week } w) \times \frac{1}{W_{max}}$$

Question 3

(a) $P(k \text{ errors}) = \binom{n}{k} \times p^k \times (1-p)^{n-k}$

$n = 64$ (number of bits)

$p = 0.008$ (probability of error for a single bit)

k : the number of errors

the probability of having 0, 1, or 2 errors is:

$$P(0 \text{ error}) = \binom{64}{0} \times 0.008^0 \times (1-0.008)^{64} \approx 0.2131$$

$$P(1 \text{ error}) = \binom{64}{1} \times 0.008^1 \times (1-0.008)^{63} \approx 0.3457$$

$$P(2 \text{ errors}) = \binom{64}{2} \times 0.008^2 \times (1-0.008)^{62} \approx 0.2571$$

$$P(\text{accepted}) = P(0 \text{ error}) + P(1 \text{ error}) + P(2 \text{ errors})$$

$\downarrow$

$$P(\text{accepted}) \approx 0.2131 + 0.3457 + 0.2571 \approx \underline{\underline{0.8159}}$$

answer

$P(\text{accepted}) = P(\text{no retransmission})$

(b) $\underline{(P(\text{no retransmission}))}^5 \to$ the probability that 5 blocks are transmitted without any retransmission

$P(\text{5 blocks without retransmission}) = (P(\text{no retransmission}))^5 \approx$
$\approx (0.8159)^5 \approx \underline{\underline{0.3615}}$ answer