

# YZV231E - Prob. & Stat. for Data Science

## 2023/2024 Fall

### Homework 2

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- Cheating is highly discouraged. It will be punished by a negative grade. Also disciplinary actions will be taken. Please do your homework on your own. Team work is not allowed. Pattern of your **solutions must belong only to you**.
- Upload your solutions through **Ninova**. Homeworks sent via e-mail and late submissions **will not be accepted**.
- You should write all your codes in **Python (3.8+)** language using **Jupyter notebook**, unless stated otherwise in the question. You can install Jupyter Notebook by following these steps on [this documentation](#). If you are not familiar with Jupyter Notebook, you can check [this tutorial](#). You may also use [Google Colab](#).
- Prepare a **report** using Latex including all your solutions, **codes** and their results.
- You **should use the provided Latex** template for the report. You may want to use [mathpix](#). You **cannot embed your handwritten solutions** into the document unless it's stated so. **Handwritten solutions will not be evaluated**.
- Mathematical calculations must be done **step-by-step**.
- This homework has a total of **100 points**. Each question has **specified** number of points.

## 1 Problems (70 Points)

### Question 1 (10 pts)

A random variable  $X$  has the pdf:

$$f_X(x) = \begin{cases} \frac{-x}{4}, & -2 \leq x \leq 0 \\ x, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

- a) Find  $E[X]$ .
- b) Find and plot  $F_X(x)$ , cdf of  $X$ .  
**Note:** You may use Latex **TikZ library** or **handwritten** plots for this part.
- c) Find  $P(X > 0)$ .
- d) Let the event  $A$  denote ' $X > 0$ '. Then, find the conditional pdf  $f_{X|A}(x | A)$ .

### Question 2 (10 pts)

The probability of heads of a random coin is a r.v.  $P$  uniform in the interval  $(0.3, 0.7)$ .

- a) Find the probability that at the next tossing of the coin heads will show.
- b)** The coin is tossed 10 times and heads shows 7 times. Find the probability that at the next tossing heads will show. (You have to write all integrals, but you are not obliged to evaluate them).

### Question 3 (10 pts)

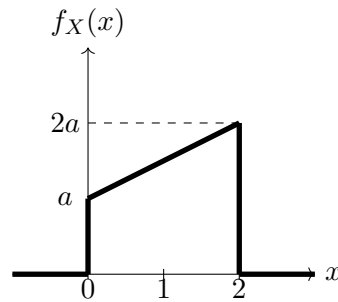
In a party the host organizes a game with 99 empty boxes and 1 box with a prize. Each box is numbered 1 to 100 and there are 50 guests present in the party. Each guest takes a random box, checks if it has the prize and puts it back. If any of the guests finds the prize, the game finishes and the guests win, otherwise they lose.

- a) What is the probability that guests win the game?
- b) Guests decide to unify and make a strategy beforehand. Each of them selects a number before the game starts and they make sure that no two guests select the same number. What is the probability that guests win the game with this strategy?
- c) Let empty boxes have the value of 0, and the prize have the value of 100. Let's say in the first strategy, whoever finds the prize, keeps it. For second strategy (part b) guests decide that they will split the prize equally. What are the expected values for a single player, for the first player, and for the last player in these two different strategies? Compare them.

**Question 4 (10 pts)**

There are three highways in the county. The number of daily accidents that occur on these highways are Poisson random variables with respective parameters .3, .5, and .7.

- Find the expected number of accidents that will happen on any of these highways today.
- What is the probability that at least one accident will occur in any of these three roads today?

**Question 5 (15 pts)**

The probability density function of r.v.  $X$  is given above.

- Find the constant  $a$ .
- Find  $E[X]$ .
- Find  $\sigma_X^2$ .
- Let the event  $A$  denote ' $X > 1$ '. Then, find the conditional pdf  $f_{X|A}(x | A)$ .

**Question 6 (15 pts)**

Assume that a drone moves within the region  $S$  bounded by the  $x$  axis, the line  $x = 1$ , and the line  $y = \ln(x + 1)$  in such a way that if  $(X, Y)$  denotes the position of the drone at a given time, the joint density of  $X$  and  $Y$  is given by

$$f(x, y) = \begin{cases} cxe^y & (x, y) \in S \\ 0 & (x, y) \notin S \end{cases}$$

for some constant  $c$ .

- Find constant  $c$ .
- Find marginal densities of  $X$  and  $Y$ .
- Find  $P(X > 0.5 \text{ and } Y < 0.5)$

## 2 Coding (30 Points)

For this part, please download the Jupyter notebook and code the necessary parts.

## 3 Submission

You need to prepare a report for all the parts including coding part and its results.

You need to submit the files shown below as zip file named with your number (i.e 123123123.zip).

- |                                   |                               |
|-----------------------------------|-------------------------------|
| 1. Homework 2 - Coding Part.ipynb | # solution of the coding part |
| 2. report.pdf                     | # report file as pdf format   |
| 3. report.tex                     | # report file as tex format   |