YZV231E - Prob. & Stat. for Data Science
2023/2024 Fall
Homework 1

Professor: Murat Okatan
Res. Asst.: Batuhan Cengiz
cengiz16@itu.edu.tr

- Cheating is highly discouraged. It will be punished by a negative grade. Also disciplinary actions will be taken. Please do your homework on your own. Team work is not allowed. Pattern of your **solutions must belong only to you**.
- Upload your solutions through **Ninova**. Homeworks sent via e-mail and late submissions **will not be accepted**.
- You should write all your codes in **Python (3.8+)** language using **Jupyter notebook**, unless stated otherwise in the question. You can install Jupyter Notebook by following these steps on this documentation. If you are not familiar with Jupyter Notebook, you can check this tutorial. You may also use Google Colab.
- Prepare a **report** using Latex including all your solutions, **codes** and their results.
- You **should use the provided Latex** template for the report. You may want to use mathpix. You **cannot embed your handwritten solutions** into the document unless it's stated so. **Handwritten solutions will not be evaluated.**
- Mathematical calculations must be done **step-by-step**.
- This homework has a total of **100 points.** Each question has **specified** number of points.

1 Problems (70 Points)

Question 1 (10 pts)
Two kids, Ali and Ahmed, are playing a game on the street. They observe the cars passing through the street, and Ali earns a point when a black (B) car passes, whereas Ahmed earns a point when a white (W) car passes. The first to collect 3 points wins. Assume that half of the cars in this city are black and the rest are white, and each passing car is independent of the others.
Define an outcome of the game as the order of cars observed when the game is finished. For instance, BWBB is an outcome resulting in a win for Ali, whereas WBW is not an outcome since the game is not complete yet.

a) How many different outcomes are possible? Please show sample space Ω of all possible outcomes.

b) What is the probability that Ali wins the game?

c) What is the probability that Ali wins the game, given that the first car is black?

Question 2 (10 pts)
There are 3 main highways in a city. % 50 of the cars in the city prefer to use first road while % 30 prefers the second one. During the rush hour, there is a risk to get stuck in traffic by % 90 for first, % 92 for second and % 88 for the last roads. Knowing that a person goes home at rush hour without getting stuck in traffic, what is the probability s/he took

a) first road.

b) second road.

c) third road.

Question 3 (10 pts)
In a binary communication channel, a block of 64 bits is transmitted with a probability of bit error $p = 0.008$.

a) The receiver accepts the block if the block has 2 or fewer errors. Find the probability that the block is accepted.

b) The block is re-transmitted if the block has more than 2 errors. What is the probability that 5 blocks are sequentially transmitted without any retransmission?

Question 4 (10 pts)
There are n couples competing in a dancing competition. Suppose there are n prizes available in total and each person can win only one prize. What is the probability that exactly one person from each couple wins a prize?

Question 5 (15 pts)
Suppose we pick toys from a production line. Each toy is suitable for either

- children who are aged between 3 and 8, or
- children who are over 8 years old.

Assume that there is a 0.3 probability of a toy being suitable for ages 3-8, and we need toys for 5 such children. Let T represent the number of toys we have to check to find these 5 toys.
Consider the condition $A = \{T > 14\}$.

a) Find the probability mass function (PMF) $p_T(t)$

b) Find the conditional PMF of T given that we have checked 14 toys and still have not found all of the needed 5 toys, $p_{T|A}(t)$.

Question 6 (15 pts)
A probability course has N students enrolled. There are W_{max} weeks in each semester. Assume that, each student has a probability p_w to be present in class at week w.

a) What is the probability that at least k students are present in the first week, with $p_w = p_1$?

b) Given that there are n students present in class, what is the probability of the current week being week w?

2 Coding (30 Points)

For this part, please download the Jupyter notebook and code the necessary parts.

3 Submission

You need to prepare a report for all the parts including coding part and its results.
You need to submit the files shown below as zip file named with your number (i.e 123123123.zip).

1. Homework 1 - Coding Part.ipynb # solution of the coding part
2. report.pdf # report file as pdf format
3. report.tex # report file as tex format