```
In []:
1 %pylab inline

Today
• List comprehensions
• Mental models (math fundamentals)
• Algorithm analysis
• Big-O notation

Modifying a list

In [2]:
1 my_list = [1, 3, 5, 7, 9]

In [3]:
1 new_list = []
2 for element in my list:
```

```
1  new_list = []
2  for element in my_list:
3    new_element = element + 1
4    new_list.append(new_element)
```

```
In [5]:
```

```
my_list = [1, 3, 5, 7, 9]
new_list = []
for element in my_list:
    new_element = element * 2
    new_list.append(new_element)
print(new_list)
```

```
[2, 6, 10, 14, 18]
```

```
What is the difference between these two programs?

my_list = [1, 3, 5, 7, 9]
new_list = []
for element in my_list:
    new_element = element + 1
    new_list.append(new_element)

my_list = [1, 3, 5, 7, 9]
new_list = []
for element in my_list:
    new_element = element * 2
    new_list.append(new_element)
Introducing list comprehensions
```

In pseudo-code we want something like:

```
# For every element in a list:
# Perform some operation on the element
# Give me a new list
```

```
# Perform operation on x every time we have an x in a list
```

```
# Perform operation on x for x in my_list
```

```
# x + 1 for x in my_list
```

```
In [ ]:
```

```
1  my_list = [1, 3, 5, 7, 9]
2  new_list = []
3  for x in my_list:
4    new_element = x + 1
5    new_list.append(new_element)
```

```
In [6]:
```

```
1 my_list = [1, 3, 5, 7, 9]
2 new_list = [x + 1 for x in my_list]
```

```
[2, 4, 6, 8, 10]
In [11]:

1  my_list = [1, 3, 5, 7, 9]
2  new_list = [x ** 2 for x in my_list]
3  print(new_list)

[1, 9, 25, 49, 81]
```

### List comprehension exercise:

This is the pseudo-code:

1 | print(new\_list)

In [7]:

```
\# x + 1 for x in my list
```

Using the list [1, 3, 5, 7, 9], do the following:

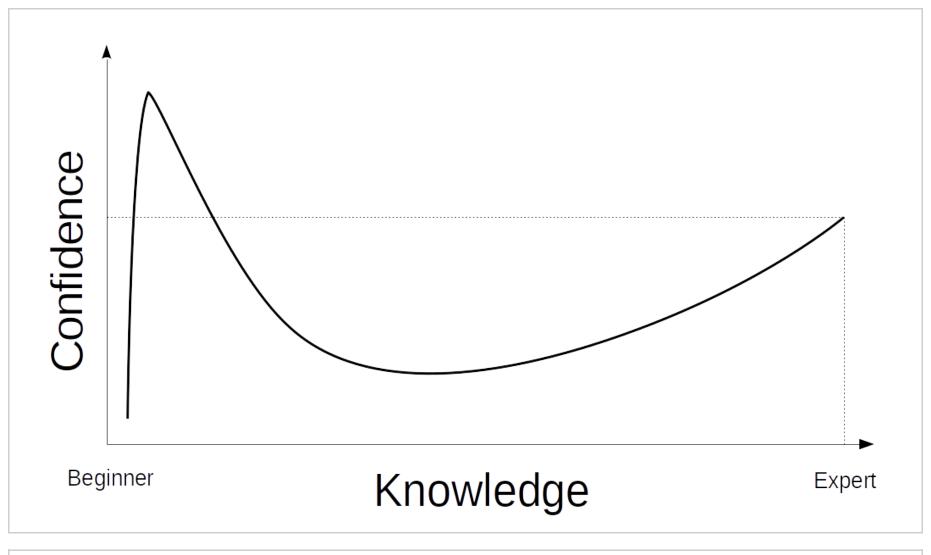
- Increase all elements by 3
- Subtract 10 from all elements
- Multiply all elements by themselves

### Understanding the world through models

- The world is complex
  - We can never understand all of it
- ... So we think in models
  - "I heard that americans are stupid, therefore all americans I meet are stupid"
- Machines do this too:
  - "... algorithms are ... wholly dependent on the data it is given"
     (https://medium.com/codait/cognitive-bias-in-machine-learning-d287838eeb4b)
  - Pretty dangerous in <u>for instance criminal prediction models</u>
     (<u>https://www.propublica.org/article/machine-bias-risk-assessments-in-criminal-sentencing</u>)

### No models are true

... but some are useful



```
Linear f(x) = x
```

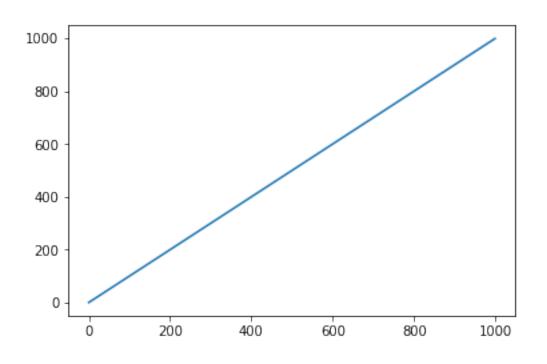
```
In [13]:
```

```
import matplotlib.pyplot as plt

xs = range(0, 1000)
ys = [x for x in xs]
plt.plot(xs, ys)
```

### Out[13]:

[<matplotlib.lines.Line2D at 0x7f51f803bdd8>]



- Running time -> calories burned
- Fuel in car -> distance it can drive

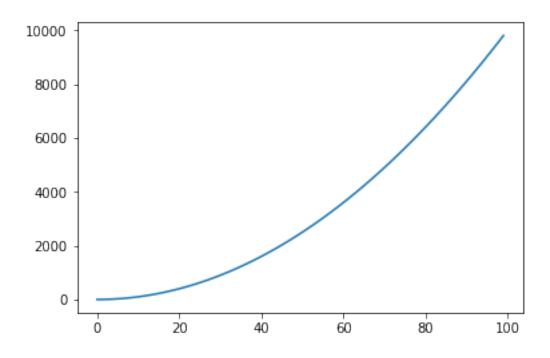
# Quadratic $f(x) = x^2$

```
In [14]:
```

```
1  xs = range(0, 100)
2  ys = [x * x for x in xs]
3  plt.plot(xs, ys)
```

#### Out[14]:

[<matplotlib.lines.Line2D at 0x7f51b4e0dba8>]



- Heat -> Ice cream sales
- Size of a square -> area

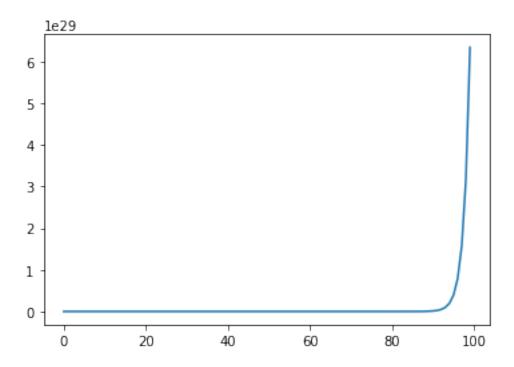
# **Exponential** $f(x) = 2^x$

### In [15]:

```
1  xs = range(0, 100)
2  ys = [2 ** x for x in xs]
3  plt.plot(xs, ys)
```

### Out[15]:

[<matplotlib.lines.Line2D at 0x7f51b4de7f98>]



- Loans work like this (compound interest:  $Pe^{rt}$ )
- Radioactive decay  $(Ne^{-t/\tau})$
- Population growth

# Logarithmic $f(x) = log_2(x)$

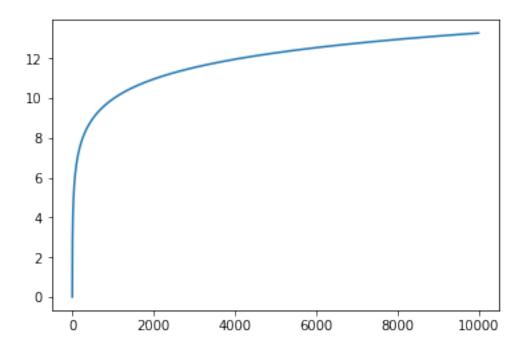
```
In [17]:
```

```
import math

xs = range(1, 10000)
ys = [math.log2(x) for x in xs]
plt.plot(xs, ys)
```

### Out[17]:

[<matplotlib.lines.Line2D at 0x7f51b4ce16d8>]



- Money -> Happiness
- Decibel and richter scale

## Logartihm examples

```
In [18]:
    import math

In [19]:
    print(math.log10(1))

0.0

In [20]:
    print(math.log10(10))

1.0
```

```
In [21]:
    print(math.log10(100))
2.0
In [22]:
    print(math.log10(1000))
3.0
In [ ]:
 print(math.log10(20))
Log<sub>2</sub>
In [23]:
    print(math.log2(1))
0.0
In [24]:
    print(math.log2(2))
1.0
In [25]:
    print(math.log2(4))
2.0
In [26]:
    print(math.log2(8))
3.0
In [27]:
 1 print(math.log2(16))
4.0
In [28]:
    print(math.log2(3))
1.584962500721156
```

### **Logarithm definition**

Answers: What is the exponent m that I need to raise the base b with to get x?

•  $b^m = x$ 

Question: What is the exponent m that I need to raise the base 10 with to get 100?

Answer:  $10^m = 100$ 

```
In [ ]:
```

```
1 math.log10(100)
```

Question: What is the exponent m that I need to raise the base 2 with to get 4?

Answer:  $2^m = 4$ 

```
In [ ]:
```

```
1 math.log2(4)
```

```
In [ ]:
```

```
1 2 ** 2
```

### Order of growth

Given 1 increase in input, how much output do we get?

• Linear: f(x) = x

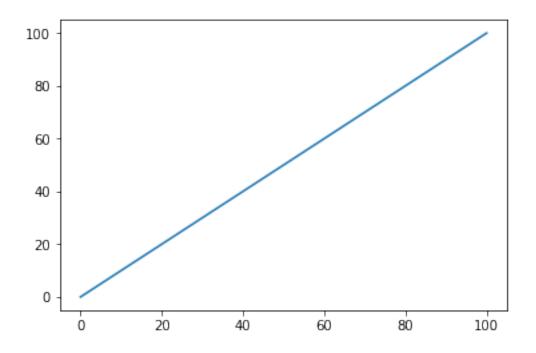
### In [29]:

```
import matplotlib.pyplot as plt

xs = [0, 100]
ys = [0, 100]
plt.plot(xs, ys)
```

### Out[29]:

[<matplotlib.lines.Line2D at 0x7f51b4cc53c8>]

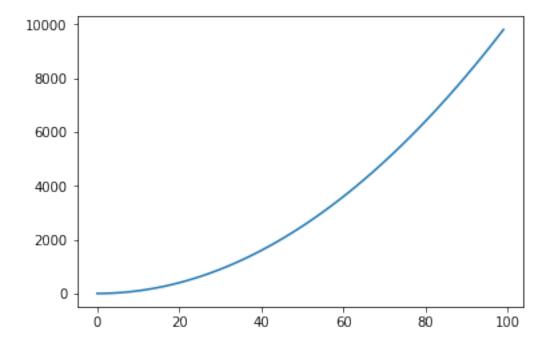


• Quadratic:  $f(x) = x^2$ 

#### In [30]:

### Out[30]:

[<matplotlib.lines.Line2D at 0x7f51b4c5f710>]



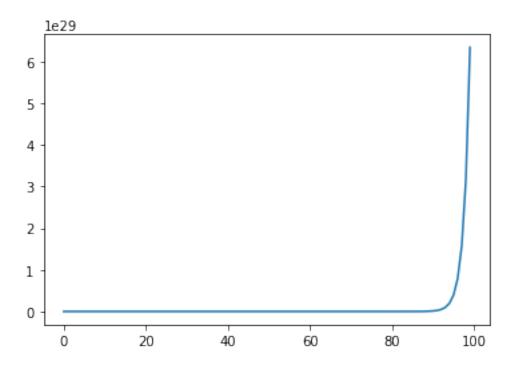
• Exponential:  $f(x) = 2^x$ 

```
In [31]:
```

```
1  xs = range(0, 100)
2  ys = [2 ** x for x in xs]
3  plt.plot(xs, ys)
```

### Out[31]:

[<matplotlib.lines.Line2D at 0x7f51b4c35b70>]



• Logarithm:  $f(x) = log_2(x)$ 

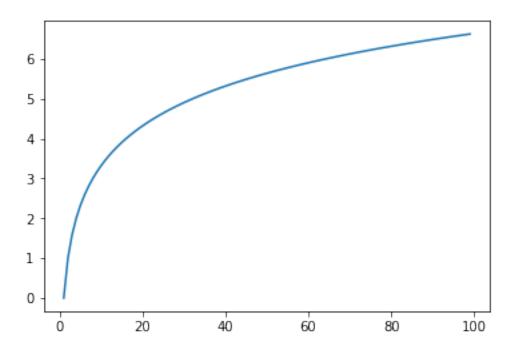
### In [32]:

```
import math

xs = range(1, 100)
ys = [math.log2(x) for x in xs]
plt.plot(xs, ys)
```

### Out[32]:

[<matplotlib.lines.Line2D at 0x7f51b4b990b8>]



## Viewing plots in Mu

To run programs that plot something:

```
import matplotlib.pyplot as plt

xs = range(100)
ys = [x for x in xs]

plt.plot(xs, ys)
plt.show()
```

- If you run into problems, run the programs from the terminal using pythonw instead of python:
  - \$ pythonw my\_python\_file.py