In [ ]:

```
%pylab inline
```

Big-O Notation

Recap

- We want to analyse the runtime and performance of our code
- Logarithmic, linear, quadratic, qubic and exponential times

Turtle example

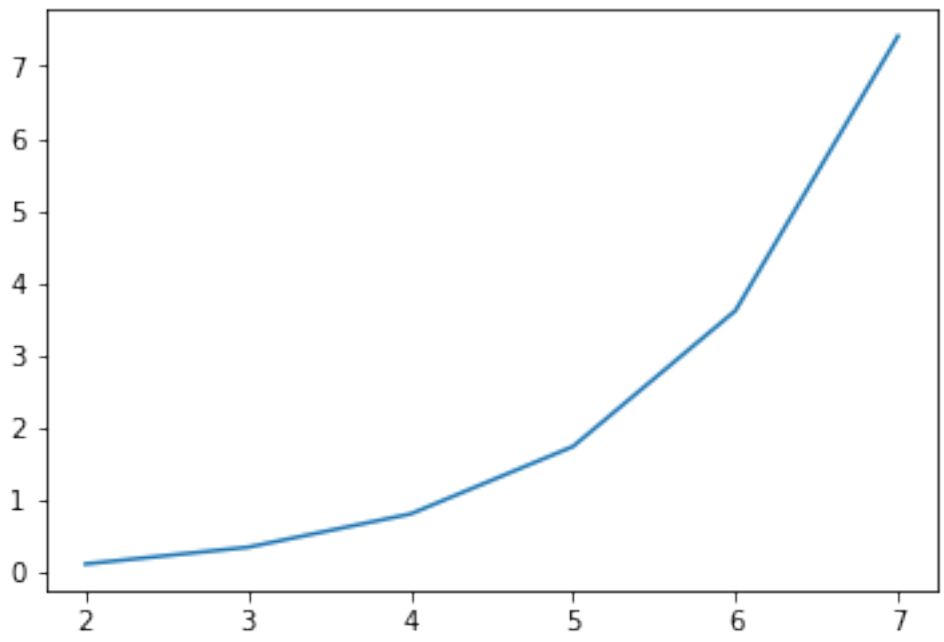
In [ ]:

```
from timeit import default_timer


def do_something():
    start = default_timer()
    # Magic happens here
    end = default_timer()
    return end - start

xs = [do_someting(x) for x in range(4, 8)]
```

lv	Running time
2	0.11665843200171366
3	0.3481470539991278
4	0.8079697409993969
5	1.7410687139999936
6	3.619540038998821
7	7.414633906002564

In [11]:

```
1  import matplotlib.pyplot as plt
2
3  xs = [2, 3, 4, 5, 6, 7]
4  ys = [0.11665843200171366, 0.3481470539991278, 0.8079697409993969, 1.74106871
5
6  plt.plot(xs, ys)
7  plt.show()
```



Objectives

- To understand why algorithm analysis is important.
- To be able to use "Big-O" to describe execution time.
- To understand the "Big-O" execution time of common operations on Python lists and dictionaries.
- To understand how the implementation of Python data impacts algorithm analysis.
- To understand how to benchmark simple Python programs.

General Scope

- In general, we are interested in *the performance* of the programs that we write
  - That is, how many seconds/minutes/hours/days is the code going to run?

- This is very dependent on the computer we run the program on
  - Can we find something more independent of the actual computer?U

Finding the position of the smallest number in a list

Let's solve the following problem:

We want to find a certain element in a list and return it's position.

- Input: Given a list of numbers in ascending order.
- That is, write a function `find_element_in_a_list(data, element)` with the following behavior:

```
>>> find_element_in_a_list([1, 3, 5], 3)
1
>>> find_element_in_a_list([2 * x for x in range(1000)], 550)
275
```

Defining the data

In [ ]:

```
data_list = list(range(1000))
element_to_find = 999
```

In [ ]:

```
def find_element_in_a_list(data_list, element):
    for idx, el in enumerate(data_list):
        if el == element:
            return idx
```

In [ ]:

```
%time find_element_in_a_list(data_list, element_to_find)
```

How much is a microsecond?

1 μs = 0.000001 s = $\frac{\frac{1s}{1000}}{1000} = \frac{1s}{1000^2}$

Big-O Notation

Describes the **the limiting behaviour of a function** (Wikipedia)

Big-O ignores everything except the **limiting** part of a piece of code. Examples from math:

$f(x) = 2x \qquad (x)$

$f(x) = x^{1000} + 100000000 \qquad (x^{1000})$

$f(x) = x^4 + \frac{x}{9} + 2005 * \frac{x^9}{1000} \qquad (\frac{x^9}{1000} \approx x^9)$

Big-O in code

- What is the running time of this?

```
statement 1
```

- 1 (constant!)
  - $O(1)$

- What is the running time of this?

```
if (cond):
    block 1 #sequence of statements
else:
    block 2 #sequence of statements
```

- 1 (constant!)
  - $O(1)$

- What is the running time of this?

```
statement 1
statement 2
...
statement n
```

```
total_time = statement 1 + statement 2 + ... + statement n
```

- 1 (constant!)
  - It does **not** depend on the input size. The amount of statements is constant
  - $O(1)$

- What is the running time of this?

```
for x in range(0, n):
    block 1
```

- Linear
  - The runtime depends on exactly the input (once for each element)
  - $O(n)$

- What is the running time of this?

```
for x in range(0, n):
    for y in range(0, n):
        block 1
```

- Quadratic
  - Everytime we have one n we need to go through all other n
  - $O(n^2)$ or $(n * n)$

- What is the running time of this?

```
for x in range(0, n):
    for y in range(0, m):
        block 1
```

- Quadratic
  - But no longer only depending on n
  - $O(n * m)$

- Is the runtime complexity of `find_element_in_a_list` and `find_element_double_loop` different?

```python
def find_element_in_a_list(data_list, element):
    for idx, el in enumerate(data_list):
        if el == element:
            return idx


def find_element_double_loop(data_list, element):
    for idx, el in enumerate(data_list):
        if el == element:
            first_result_idx = idx
            break
    for idx, el in enumerate(data_list):
        if el == element and idx == first_result_idx:
            return idx, el
```

Why do we not say $O(2n)$?

- Because only n matters when the size grows very big in $2n$, the 2 does not matter anymore:
  - $10 \not\approx 20$
  - $1'000'000 \approx 2'000'000$

Let's say one elementary operation takes 10 nanoseconds, then we get

	n							
$t_{\mathcal{A}}(n)$	10	100	1000	10^4	10^5	10^6	10^7	10^8
$\log n$	33ns	66ns	0.1μs	0.1μs	0.2μs	0.2μs	0.2μs	0.3μs
$\sqrt{n}$	32ns	0.1μs	0.3μs	1μs	3.1μs	10μs	31μs	0.1ms
n	100ns	1μs	10μs	0.1ms	1ms	10ms	0.1s	1s
$n \log n$	0.3μs	6.6μs	0.1ms	1.3ms	16ms	0.2s	2.3s	27s
$n^{3/2}$	0.3μs	10μs	0.3ms	10ms	0.3s	10s	5.2m	2.7h
n^2	1μs	0.1ms	10ms	1s	1.7m	2.8h	11d	3.2y
n^3	10μs	10ms	10s	2.8h	115d	317y	$3.2{\cdot}10^5$y	
1.1^n	26ns	0.1ms	$7.8{\cdot}10^{25}$y					
2^n	10μs	$4.0{\cdot}10^{14}$y						
$n!$	36ms	$3.0{\cdot}10^{142}$y						
n^n	1.7m	$3.2{\cdot}10^{184}$y						

Big-O as an approximation

Describes the **the limiting behaviour of a function** (Wikipedia)

Big-O ignores everything except the **limiting** part of a piece of code.

Can be used to know the complexity of your code **without running it**!

What is the worst-case runtime of this?

```
def find_element_in_a_list(data_list, element):
    for idx, el in enumerate(data_list):
        if el == element:
            return idx, el
```

How often do we iterate over the elements of the list in the worst case?

What is the worst-case runtime of this?

```
def find_element_double_loop(data_list, element):
    for idx, el in enumerate(data_list):
        if el == element:
            first_result_idx = idx
            break
    for idx, el in enumerate(data_list):
        if el == element and idx == first_result_idx:
            return idx, el
```

How often do we iterate over the elements of the list in the worst case?

How often do we iterate over the elements of the list in the worst case?

What is the worst-case runtime of this?

```
def find_element_nested_loop(data_list, element):
    for idx, el in enumerate(data_list):
        for idx2, el2 in enumerate(data_list):
            if el == el2 == element and idx == idx2:
                return idx, el
```

How often do we iterate over the elements of the list in the worst case?

What is the worst-case runtime of this?

```
def find_element_recursive(data_list, element):
    half_idx = len(data_list) // 2

    if element == data_list[half_idx]:
        return half_idx
    elif element < data_list[half_idx]:
        lower_half_list = list(data_list[:half_idx])
        return find_element_recursive(lower_half_list, element)
    elif element > data_list[half_idx]:
        upper_half_list = list(data_list[half_idx:])
        return find_element_recursive(upper_half_list, element)
```

How often do we iterate over the elements of the list in the worst case?

How to count and *not* count...

When asked 'count the number of occurrences of all words in a list'. Assuming we have a list of words in `words`, here are two solutions:

In [12]:

```
words = ['seldom', 'heard', 'him', 'mention', 'her', 'under', 'any',
         'other', 'name', 'In', 'his', 'eyes', 'she', 'eclipses',
         'and', 'predominates', 'the', 'whole', 'of', 'her', 'sex',
         'It', 'was', 'not', 'that', 'he', 'felt', 'any', 'emotion',
         'akin', 'to', 'love', 'for', 'Irene', 'Adler', 'All',
         'emotions', 'and', 'that', 'one', 'particularly', 'were',
         'abhorrent', 'to', 'his', 'cold', 'precise', 'but',
         'admirably', 'balanced', 'mind', 'He', 'was', 'I', 'take',
         'it', 'the', 'most', 'perfect', 'reasoning', 'and',
         'observing', 'machine', 'that', 'the', 'world', 'has',
         'seen', 'but', 'as', 'a', 'lover', 'he', 'would', 'have',
         'placed', 'himself', 'in', 'a', 'false', 'position', 'He',
         'never', 'spoke', 'of', 'the', 'softer', 'passions', 'save',
         'with', 'a', 'gibe', 'and', 'a', 'sneer']
```

How to count and *not* count...

When asked 'count the number of occurrences of all words in a list'. Assuming we have a list of words in `words`, here are two solutions:

In [ ]:

```
word_counts = {}
for word in words:
    word_counts.setdefault(word, 0)
    word_counts[word] += 1
print(word_counts)
```

In [ ]:

```
word_counts = {}
for word in words:
    counts = words.count(word)
    word_counts[word] = counts
print(word_counts)
```

In [14]:

```
print(help([].count))
```

```
Help on built-in function count:

count(value, /) method of builtins.list instance
    Return number of occurrences of value.

None
```

- What is the running time complexity of each solution?
- Which one is thus preferable?