Logistic Regression

Week 2

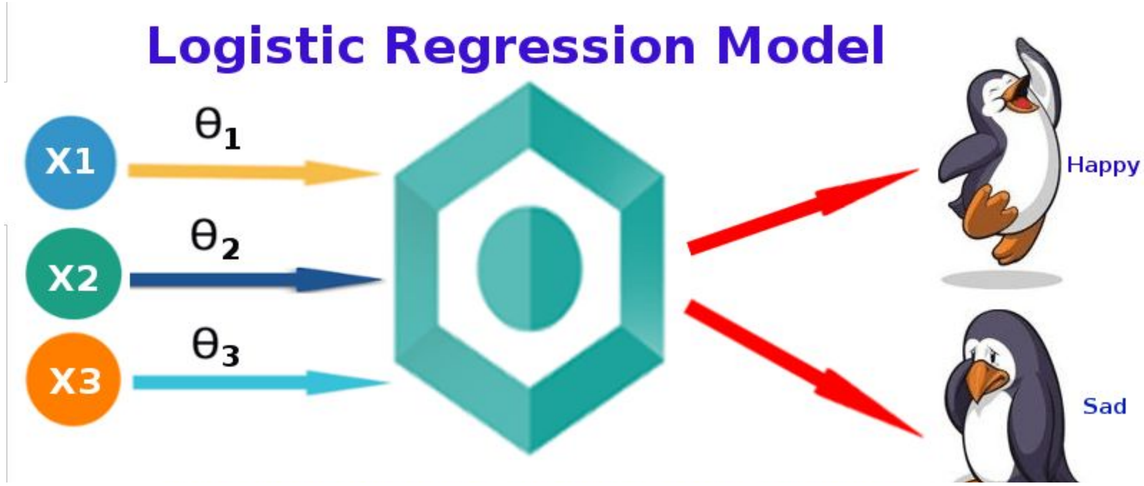
To be or not to be?

- Predict whether an email is a spam or not
- Predict whether a tumour is malignant or not
- Predict the animal in the picture cat or not?
- ...

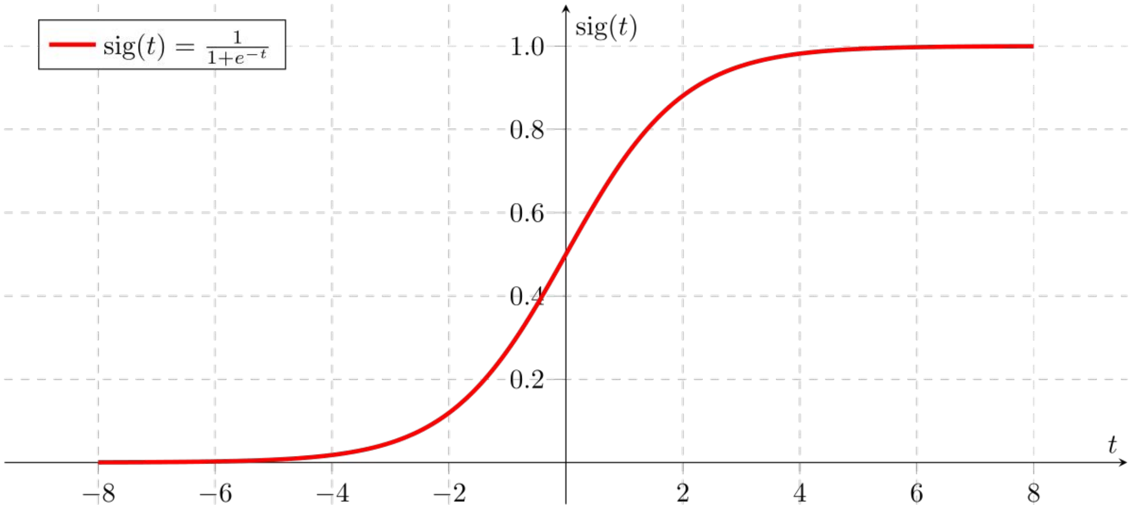
Hypothesis Function

- Probability of classified object [0, 1] (P(X) ~ 0) or (P(X) ~ 1)
- Hypothesis Function is equation that maps inputs to outputs;

$$Z = \mathbf{W} * X + \mathbf{b}$$



Sigmoid Function



$$\sigma(X) = \frac{1}{1+e^{-X}}$$

$h_{\Theta}(x)$ = P (Y=1|X; theta)

Probability that Y=1 given X which is parameterized by 'theta'.

P (Y=1|X; theta) + P (Y=0|X; theta) = 1

P (Y=0|X; theta) = 1 - P (Y=1|X; theta)

Cost and Loss function

$$-(y\log(p) + (1-y)\log(1-p))$$

Gradient Descent Algorithm

Gradient

$$\boxed{z = w_1x_1 + w_2x_2 + b} \rightarrow \boxed{\hat{y} = a = \sigma(z)} \rightarrow \boxed{L(\hat{y}, y)}$$

$\Leftrightarrow \boxed{a = \hat{y}}$

$$\boxed{w_1} \Rightarrow \frac{\partial(L)}{\partial w_1} = \frac{\partial L}{\partial a} \cdot \frac{\partial a}{\partial z} \cdot \frac{\partial(z)}{\partial w_1}$$

$$\frac{\partial L}{\partial a} = \frac{\partial}{\partial a}(-y\log a - (1-y)\log(1-a))$$

$$= -y\left(\frac{1}{a}\right) - (-1)\frac{(1-y)}{(1-a)}$$

$$\boxed{\frac{\partial L}{\partial a} = \left(\frac{-y}{a}\right) + \left(\frac{1-y}{1-a}\right)}$$

$$\boxed{\frac{\partial a}{\partial z} = a(1-a)}$$

$$\boxed{\frac{\partial z}{\partial w_1} = x_1}$$

$$\frac{\partial L}{\partial w_1} = \left(\left(\frac{-y}{a} + \frac{(1-y)}{1-a}\right) \cdot (a)(1-a)\right) \cdot x_1$$

$$= (a-y) \cdot x_1$$

Update for w_1,

$$\frac{\partial L}{\partial w_1} = (a-y) \cdot x_1$$

Here, $(a-y) = \frac{\partial L}{\partial z}$

$$\boxed{w_1 = w_1 - \alpha \frac{\partial L}{\partial w_1}}$$

Similarly, for all parameters

$$\boxed{w_i = w_i - \alpha \frac{\partial L}{\partial w_i}}$$

$i = 1, 2, \ldots, m$

m = no of parameters

$$\boxed{b = b - \alpha \frac{\partial L}{\partial b}}$$

Where, $\frac{\partial L}{\partial b} = (a-y)$