YAP 618 – Homework Assignment 2

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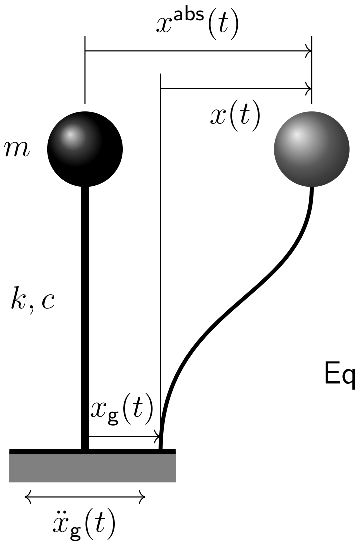
1 Numerical Solution of Equations of Motion

1.1 SDOF Systems

Consider the single-degree-of-freedom (SDOF) system shown below. Develop a MATLAB code that reads a text file of ground motion acceleration data of an historical earthquake and solves the equation of motion. The program should find and plot $x(t)$, $\dot{x}(t)$, $\ddot{x}(t)$, inertia force and forces generated by stiffness and damping. Solve a sample system using the program written and program NONLIN for the same ground acceleration. Compare the results. (Due 4 March 2016, 5:00pm)



$x(t)$: relative displacement of mass m
$x^{\mathrm{abs}}(t)$: absolute displacement of mass m
$x_{\mathrm{g}}(t)$: ground displacement
$\ddot{x}_{\mathrm{g}}(t)$: ground acceleration
m : mass
k : stiffness
c : damping constant

Equation of motion : $m\ddot{x}(t) + c\dot{x}(t) + kx(t) = -m\ddot{x}_{\mathrm{g}}(t)$

1.2 MDOF Systems

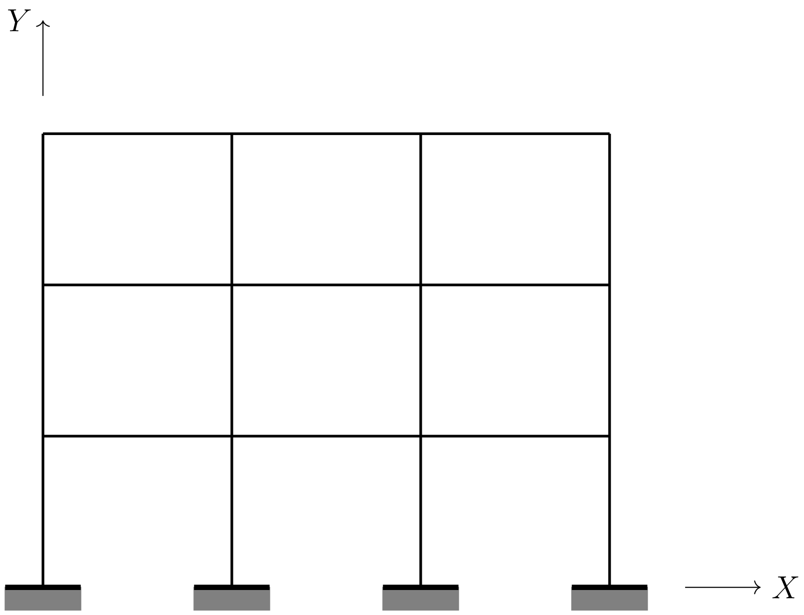
Consider the general form of equation of motion for a multi-degree-of-freedom (MDOF) system given below:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{r}\mathbf{a}_{\mathrm{g}}(t)$$

where $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ are mass, damping, and stiffness matrices; $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ are acceleration, velocity and displacement vectors and $\mathbf{a}_{\mathrm{g}}(t)$ is the ground acceleration vector. Develop a MATLAB code that solves the equation of motion for given system matrices, $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$ and $\mathbf{r}$, and given ground acceleration data, $\mathbf{a}_{\mathrm{g}}(t)$. Consider that ground accelearation vector has horizontal and vertical components, $\mathbf{a}_{\mathrm{g}}(t) = \left[\begin{array}{cc} \mathbf{a}_{\mathrm{g,X}}(t), & \mathbf{a}_{\mathrm{g,Y}}(t) \end{array}\right]^{\mathrm{T}}$. The code should plot $\ddot{\mathbf{x}}(t)$, $\dot{\mathbf{x}}(t)$ and $\mathbf{x}(t)$ for given degrees of fredoom. (Due 11 March 2016, 5:00pm)

2 Derivation of Equation of Motion for Frames

Consider a general frame as shown below. Develop an algorithm that can derive the mass and stiffness matrices ($\mathbf{M}$, and $\mathbf{K}$) that will appear in the equation of motion of the structure. Consider that ground accelearation vector has horizontal and vertical components, $\mathbf{a}_{\mathrm{g}}(t) = \left[\ a_{\mathrm{g,X}}(t),\quad a_{\mathrm{g,Y}}(t)\ \right]^{\mathrm{T}}$. The algorithm does not need to be in MATLAB m-file syntax.



Hints:

1. Assume that displacements of the restrained degrees-of-freedom will be the corresponding ground displacements. For example, for a fixed support, absolute global displacement vector will be $\mathbf{D}_{\mathrm{abs}}(t) = \left[\ x_{\mathrm{g,X}}(t),\quad x_{\mathrm{g,Y}}(t),\quad 0\ \right]^{\mathrm{T}}$, where $x_{\mathrm{g,X}}(t)$ and $x_{\mathrm{g,Y}}(t)$ are the ground displacements in the $X-$ and $Y-$directions respectively x'.
2. Assume that displacements of the unrestrained degrees-of-freedom will be the summation of the displacements relative to the ground and corresponding ground displacements. For example for a fully unrestrained joint i, the displacement vector will be $\mathbf{D}_{\mathrm{abs}}(t) = \left[\ X_i + x_{\mathrm{g,X}}(t),\quad Y_i + x_{\mathrm{g,Y}}(t),\quad \theta_i\ \right]^{\mathrm{T}}$, where X_i and Y_i are the global degrees-of-freedom in the $X-$ and $Y-$directions, and θ_i is the rotational degree-of-freedom of joint i.
3. Assume that there are no external static forces applied to the structure.
4. Consider that mass of structure is lumped at the joints. For the sake of this problem assume that all frame elemants has a mass per length, $\bar{m}$ (*i.e.*, total mass of a frame element with length L is $\bar{m}L$.
5. For the components of the mass matrix that correspond to the rotational degrees-of-freedom, you can either use the mass value that corresponds to the translational degrees-of-freedom or a consistent mass value (derivation of mass matrices are generally well-explained in structural dynamics books, *see e.g.* the book by Clough and Penzien)

(Due 4 March 2016, 5:00pm)