# Listing Maximal $k$-Plexes in Large Real-World Graphs

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1. Background
2. Algorithms
3. Implementation Techniques
4. Experiments

## Finding Cohesive Groups

Finding ***cohesive groups*** (or ***communities***) has received attention from various areas.

- In the WWW, identify clients sharing similar interests and serve them with a common proxy to reduce network traffic.
- In social networks, discover closely related individuals.
- In biological networks, predict the structure and function of protein.

# Clique Model

Naturally, cohesive groups can be modeled with ***Cliques***.
A clique is a subgraph where vertices are pairwise connected, i.e., a complete graph.

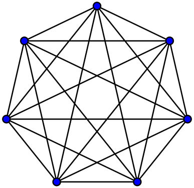

Figure 1: $K_7$

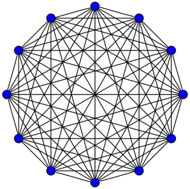

Figure 2: $K_{12}$

# $k$-Plex Model

Due to various reasons like *data noise*, communities rarely appear as cliques.
***k-Plexes*** allow every vertex missing at most $k-1$ links to other vertices.

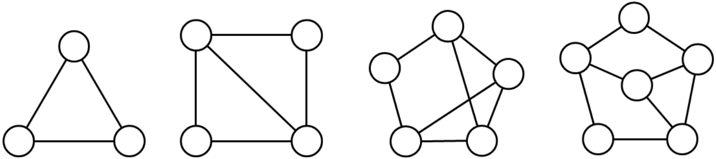

Figure 3: 1, 2, 3, 4-plex

# Properties

## Lemma 1 (Hereditary Property)

- *Any induced subgraph of a k-plex is still a k-plex.*
- *A k-plex is maximal if it is not a subgraph of any larger k-plex.*

## Lemma 2 (Distance Property)

- *Any k-plex with at least* $2k - 1$ *vertices has its diameter at most 2.*
- *A k-plex with at most* $2k - 2$ *vertices may be unconnected.*

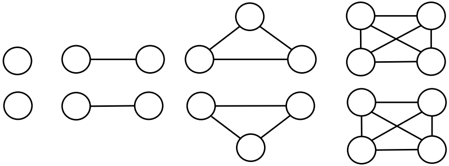

Figure 4: unconnected $2, 3, 4, 5$-plex

# Translated Problems

We model cohesive groups with ***maximal k-plexes***.

## Problem 1 (Listing maximal $k$-plexes)

*Given a graph $G$, a positive integer $k$, list all maximal $k$-plexes from $G$.*

## Problem 2 (Listing large maximal $k$-plexes)

*Given a graph $G$, two positive integers $k$ and $l$ where $l \geq 2k - 1$, list all maximal $k$-plexes with at least $l$ vertices.*

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## The Bron-Kerbosch Algorithm

Our algorithm stems from the Bron-Kerbosch algorithm, say *BKPlex*.
BKPlex accepts three sets $P$, $C$ and $X$ as parameters,

- $P$: ***plex vertices*** of the growing $k$-plex,
- $C$: ***candidate vertices*** for further branching,
- $X$: ***excluded vertices*** to avoid non-maximal solutions.

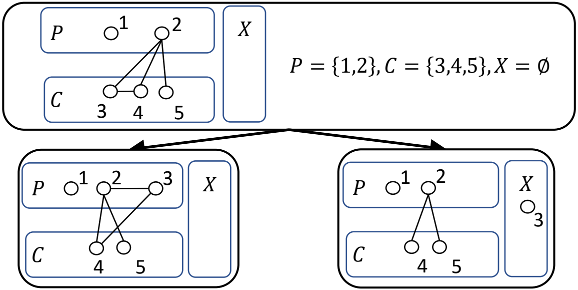
then lists maximal $k$-plexes $G[P']$ with three properties:

- $P \subseteq P'$
- $P' \subseteq P \cup C$
- $\forall v \in X$, the subgraph $G[\{v\} \cup P']$ is not a $k$-plex.

***Hereditary Prop.*** Any induced subgraph of a $k$-plex is still a $k$-plex.

# The Bron-Kerbosch Algorithm

BKPlex branches by doing bipartition recursively. [1]



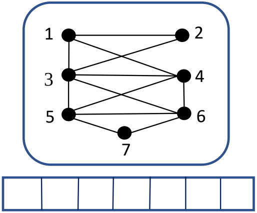

| Vtx | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| Br1 | P | P | **P** | **C** | **C** |
| Br2 | P | P | **X** | **C** | **C** |

Figure 5: An example of BKPlex.

[1] Simplified for clarity. In fact, a variant of bipartition.

# Degeneracy Ordering

- $\eta = v_1, \ldots, v_n$ is called *degeneracy ordering* (*core ordering*) if each vertex $v_i$ has the minimum degree in the induced subgraph $G[\{v_i, \ldots, v_n\}]$.
- Given a degeneracy ordering $\eta = v_1, \ldots, v_n$, the degree of $v_i$ in $G[\{v_i, \ldots, v_n\}]$ is called the *core number* of $v_i$.
- For any degeneracy ordering of the same graph, the largest core number among all vertices is a constant $D$ called *degeneracy*.
- Due to the sparsity of many real-world graphs, $D \ll \Delta \ll n$ where $\Delta$ is the maximum degree.

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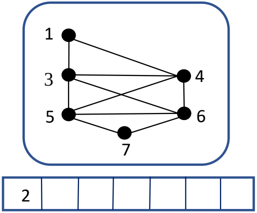


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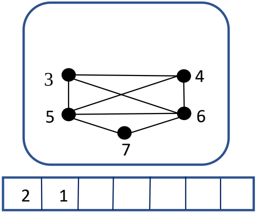


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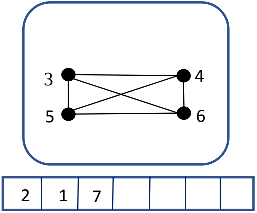


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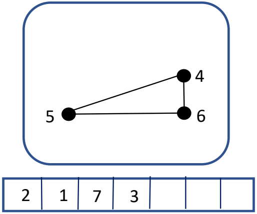


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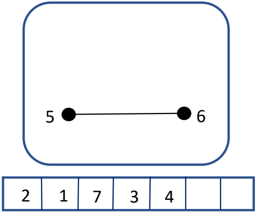


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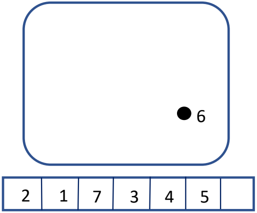


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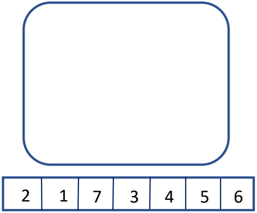


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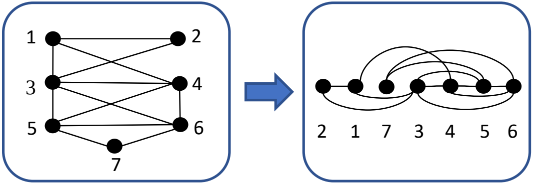


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## Recent Algorithms [3]

***Pivot Heuristic*** Zhou et al. (2020) proposed *BKPivot* with a *pivot* heuristic in the branch scheme of BKPlex, and improved its running time from $O^*(2^n)$ to $O^*(\gamma_k^n)$, where $\gamma_k < 2$. [2]

***Graph Decomposition*** Conte et al. (2018) proposed *D2K*, a decomposition-based algorithm for listing $k$-plexes with the diameter at most 2.
For each vertex, D2K builds a local subgraph and then adopts BKPlex to list maximal $k$-plexes locally which runs in $O^*(2^{D\Delta})$.

***Distance Prop.*** Any $k$-plex with at least $2k-1$ vertices has the diameter at most 2.

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***Distance Prop.*** Any $k$-plex with at least $2k - 1$ vertices has the diameter at most 2.

We combine them and propose *ListPlex* running in $O^*(\gamma_k^D)$.

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[3] Simplified for clarity.

## ListPlex

### Listing All Maximal $k$-Plexes

Based on Distance Property, ListPlex divides its task into two parts.

- *Part I*: k-plexes with size at most $2k-2$. (Solved directly by BKPlex)
- ***Part II***: k-plexes with size at least $2k-1$.

***Distance Prop.*** Any $k$-plex with at least $2k-1$ vertices has the diameter at most 2.

# ListPlex

## Part II

***Procedure***

1) Sort $V$ by a degeneracy ordering $\eta = v_1 \dots v_n$.

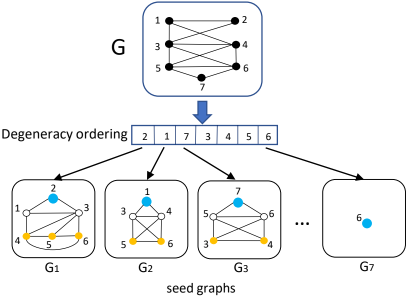
- $N^k(v)$ denotes $k$-hop neighbors of $v$.
- $N^k_{\succ}(v_i)$ denotes $N^k(v_i) \cap \{v_{i+1}, ..., v_n\}$, say ***forward*** $k$-hop neighbors of $v_i$

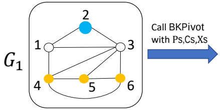
2) From $v_1$ to $v_n$, in the $i$-th iteration, list maximal $k$-plexes with $i$ as the minimum index of vertices.

- Build ***seed graph*** $G_i = G[\{v_i\} \cup N_{\succ}(v_i) \cup N^2_{\succ}(v_i)]$.
- Call BKPivot with given combinations of $N^2_{\succ}(v_i)$, say ***seed set*** $S$.
- Validate maximality in $G$ for $k$-plexes generated in $G_i$.

# ListPlex

## Part II

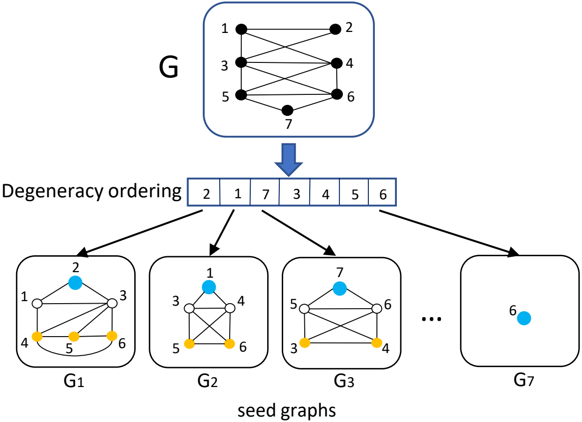



| S | Ps | Cs | Xs |
|---|---|---|---|
| ∅ | {2} | {1,3} | {4,5,6} |
| {4} | {2,4} | {1,3} | {5,6} |
| {5} | {2,5} | {1,3} | {4,6} |
| {6} | {2,6} | {1,3} | {4,5} |
| {4,5} | {2,4,5} | {1,3} | {6} |
| {4,6} | {2,4,6} | {1,3} | {5} |
| {5,6} | {2,5,6} | {1,3} | {4} |

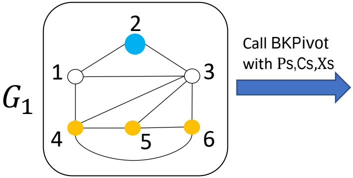
Figure 7: An example of ListPlex's Part II.
(L) sort $V$ in degeneracy ordering $\eta$ and induce seed graphs $G_i$ for each $v_i \in \eta$.
(R) enumerate $S \subseteq N^2_{\succ}(v_i)$ with bound $|S| \leq k-1$ ($k=3$) and call BKPivot with $P_s, C_s, X_s$.

# ListPlex
## Graph Decompostion




- Cohesive groups appear locally in large real-world graphs,
- Parallelism,
- Smaller scale and better locality,
- $D \ll \Delta$.

# ListPlex
## Seed Set

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- At most $k-1$ vertices come from $N^2_{\succ}(v_i)$,
- Reducing candidates fast,
- $|N^2_{\succ}(v_i)|$ is potentially $D\Delta$,
- Pruning rules.

## ListPlex
### Listing Large Maximal k-Plexes

With lower bound $l$, a related lower bound $l'$ can be derived.

**Lemma 3**

*Assume $|P| \geq l$, for any vertex pair $u, v \in P$, $|N(u) \cap N(v) \cap P| \geq l'$*

Removing unfruitful candidates from $G_i$ reduces the scale of $G_i$.

- Consider vertex pair $(v_i, u)$, $u \in V_i$.

Dropping unfruitful seed sets $S$ saves the forthcoming exponential search.

- Consider vertex pair $(u, v)$ from some $S$.

## Reducing cache misses

Validating maximality in $G$ for maximal $k$-plex of $G_i$ suffers a high amount of cache misses.

Alleviation:

- Build a bipartite graph $B_i$ for each $G_i$, which serves as a cache of currently useful data of large $G$,
- Pruning bipartite graph just like seed graph.

## Parallelization

ListPlex owns appealing parallel features.

How To:

- Parallelize searches of maximal $k$-plexes on each $G_i$, say $T_i$,
- Split $T_i$ when some cores are idle for better load balance,
- Construct degeneracy ordering and perform generated $T_i$ in parallel.

# Dataset

All graphs are taken from SNAP and LAW [4].

Table 1: Considered networks and their properties

| Network | n | m | Δ | D |
|---|---|---|---|---|
| jazz | 198 | 2742 | 100 | 29 |
| ca-grqc | 5241 | 14484 | 81 | 43 |
| gnutella08 | 6301 | 41554 | 97 | 10 |
| wiki-vote | 7116 | 100763 | 1065 | 53 |
| lastfm | 7624 | 55612 | 216 | 20 |
| as-caida | 26475 | 53381 | 2628 | 22 |
| soc-epinions | 75888 | 405739 | 3044 | 67 |
| soc-slashdot | 82144 | 500480 | 2548 | 54 |
| email-euall | 265214 | 365569 | 7636 | 37 |
| amazon0505 | 410236 | 2439436 | 2760 | 10 |
| in-2004 | 1353703 | 13126172 | 21869 | 488 |
| soc-pokec | 1632803 | 22301964 | 14854 | 47 |
| as-skitter | 1696415 | 11095298 | 35455 | 111 |
| soc-livejournal | 4847571 | 68993773 | 14815 | 360 |
| arabic-2005 | 22744080 | 639999458 | 575628 | 3247 |
| uk-2005 | 39459925 | 936364282 | 1372171 | 584 |
| it-2004 | 41291594 | 1150725436 | 1243927 | 3209 |
| webbase-2001 | 118142155 | 1019903190 | 816127 | 1506 |

[4] http://law.di.unimi.it/

# Listing All Maximal $k$-Plexes

Table 2: Listing all maximal $k$-plexes in small graphs

| Network | $k$ | $\#k$-plexes | The running time (s) | | | | Speedup |
|---|---|---|---|---|---|---|---|
| | | | BKPlex | BKPivot | ListPlex | ListPlex(16) | |
| jazz | 2 | 35214 | 648.864 | 0.29 | **0.086** | 0.408 | 0.211 |
| jazz | 3 | 3602575 | 772.826 | 17.55 | **6.477** | 0.832 | 7.785 |
| jazz | 4 | 193056583 | 3226.746 | 829.40 | **417.646** | 26.187 | 15.949 |
| ca-grqc | 2 | 13718439 | OOT | 1858.02 | **649.985** | 40.880 | 15.899 |
| gnutella08 | 2 | 19866959 | 1500.208 | 3627.57 | **1117.858** | 70.207 | 15.922 |
| wiki-vote | 2 | 66193264 | 10356.553 | 10671.92 | **1526.884** | 95.656 | 15.962 |
| lastfm | 2 | 29086855 | 2643.394 | 6676.89 | **1989.701** | 124.525 | 15.978 |

# Listing Large Maximal $k$-Plexes

Table 3: The running time of listing large maximal $k$-plexes from small and medium graphs by CommuPlex[5], D2K[6] and ListPlex.

| Graph $(\|V\|, \|E\|)$ | $k$ | $l$ | #$k$-plexes | The running time (s) CommuPlex | D2K | ListPlex |
|---|---|---|---|---|---|---|
| jazz (198, 2742) | 4 | 12 | 2745953 | 25.218 | 33.054 | **4.498** |
| lastfm (7624, 55612) | 4 | 12 | 1827337 | 20.724 | 23.991 | **4.586** |
| as-caida (26475, 53381) | 3 | 12 | 281251 | 5.684 | 13.421 | **0.867** |
| | 4 | 12 | 15939891 | 300.388 | 785.506 | **47.98** |
| amazon0505 (410236, 2439436) | 2 | 12 | 376 | 1.825 | 0.641 | **0.137** |
| | 3 | 12 | 6347 | 11.359 | 0.77 | **0.286** |
| | 4 | 12 | 105649 | 47.049 | 5.338 | **1.171** |
| email-euall (265214, 365569) | 2 | 12 | 412779 | 8.793 | 11.199 | **1.946** |
| | 3 | 12 | 32639016 | 619.384 | 1043.266 | **91.62** |
| | | 20 | 2637 | 10.754 | 53.691 | **0.429** |
| | 4 | 20 | 1707177 | 825.126 | 3800.889 | **24.089** |
| soc-slashdot (82144, 500480) | 2 | 12 | 27208777 | 376.071 | 213.141 | **59.42** |
| | | 20 | 11411028 | 227.016 | 137.159 | **32.988** |
| | | 30 | 453 | 10.77 | 16.481 | **0.688** |
| | 3 | 12 | 2807943240 | OOT | 26029.006 | **7813.045** |
| | | 20 | 1303148522 | 28361.707 | 15308.777 | **4538.022** |
| | | 30 | 1679468 | 699.876 | 2066.598 | **51.364** |
| | 4 | 30 | 502699966 | OOT | OOT | **6680.261** |
| as-skitter (1696415, 11095298) | 2 | 50 | 47969775 | OOT | OOT | **520.884** |
| | 2 | 100 | 0 | 1.793 | 2.951 | **0.716** |
| | 3 | 50 | 21070497438 | OOT | OOT | OOT |
| | 3 | 100 | 0 | 2.37 | 3.285 | **0.718** |
| in-2004 (1353703, 13126172) | 2 | 50 | 25855779 | 7663.843 | 576.06 | **150.212** |
| | 2 | 100 | 9978037 | 5899.638 | 256.225 | **72.063** |
| | 3 | 50 | 29045783792 | OOT | OOT | OOT |
| | 3 | 100 | 4257410159 | OOT | OOT | **28384.76** |

| Graph $(\|V\|, \|E\|)$ | $k$ | $l$ | #$k$-plexes | The running time (s) CommuPlex | D2K | ListPlex |
|---|---|---|---|---|---|---|
| wiki-vote (7116, 100763) | 2 | 12 | 2919931 | 75.871 | 115.757 | **17.653** |
| | | 20 | 52 | 4.52 | 11.289 | **0.591** |
| | | 30 | 0 | 1.033 | **0.027** | 0.091 |
| | 3 | 12 | 458153397 | OOT | OOT | **2185.598** |
| | | 20 | 156727 | 595.636 | 1852.186 | **9.384** |
| | | 30 | 0 | 1.072 | **0.029** | 0.1 |
| | 4 | 20 | 46729532 | OOT | OOT | **1174.2** |
| | | 30 | 0 | 9.17 | 3.627 | **0.112** |
| soc-pokec (1632803, 22301964) | 2 | 12 | 7679906 | 1537.506 | 172.987 | **47.475** |
| | | 20 | 94184 | 1064.371 | 20.03 | **15.161** |
| | | 30 | 3 | 662.64 | **8.637** | 9.557 |
| | 3 | 12 | 520888893 | OOT | OOT | **1607.285** |
| | | 20 | 5911456 | 1470.536 | 856.393 | **46.262** |
| | | 30 | 5 | 717.425 | **9.993** | 10.127 |
| | 4 | 20 | 318035938 | 34048.155 | OOT | **1825.216** |
| | | 30 | 4515 | 1140.117 | 111.987 | **11.211** |
| soc-epinions (75888, 405739) | 2 | 12 | 49823056 | 843.9 | 735.589 | **193.307** |
| | | 20 | 3322167 | 137.427 | 180.061 | **19.382** |
| | | 30 | 0 | 8.995 | 12.109 | **0.492** |
| | 3 | 20 | 548634119 | 27037.614 | 35525.693 | **3072.267** |
| | | 30 | 16066 | 546.69 | 2591.439 | **6.123** |
| | 4 | 30 | 13172906 | OOT | OOT | **661.103** |
| com-livejournal (4847571, 68993773) | 2 | 340 | 650322 | 2284.435 | OOT | **109.382** |
| | 2 | 345 | 0 | 57.548 | 13589.487 | **6.914** |
| | 3 | 340 | 555718694 | OOT | OOT | **22863.467** |
| | 3 | 345 | 3963139 | 24861.871 | OOT | **826.183** |

[5] Zhou et al. (2020)
[6] Conte et al. (2018)

# Listing Large Maximal $k$-Plexes

Table 4: The parallel running time of large networks by ListPlex and D2K with 16 threads.

| Graph $(\|V\|, \|E\|)$ | $k$ | $l$ | #$k$-plexes | The running time (s) D2K(16) | ListPlex(16) |
|---|---|---|---|---|---|
| arabic-2005 (22744080, 639999458) | 2 | 800 | 224870903 | 2195.272 | **714.159** |
| | 2 | 1000 | 236897 | 151.328 | **40.202** |
| | 3 | 800 | >25062182205 | OOT | OOT |
| | 3 | 1000 | 34155502 | 587.967 | **128.737** |
| uk-2005 (39459925, 936364282) | 2 | 250 | 106243475 | OOT | **355.855** |
| | 2 | 500 | 256406 | 318.118 | **35.001** |
| | 3 | 250 | >18336111409 | OOT | OOT |
| | 3 | 500 | 28199814 | 9506.661 | **121.726** |
| it-2004 (41291594, 1150725436) | 2 | 2000 | 675111 | 340.904 | **41.983** |
| | 2 | 3000 | 675111 | 307.735 | **38.468** |
| | 3 | 2000 | 197679229 | 4254.456 | **724.979** |
| | 3 | 3000 | 197679229 | 4235.389 | **715.002** |
| webbase-2001 (118142155, 1019903190) | 2 | 800 | 1599005 | 374.134 | **54.19** |
| | 2 | 1000 | 1164383 | 346.393 | **53.651** |
| | 3 | 800 | 1785341050 | 36116.817 | **5521.386** |
| | 3 | 1000 | 1484341137 | 35005.343 | **6960.816** |

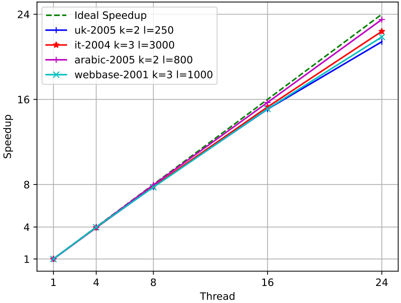


Figure 8: The speedup of ListPlex for the large graphs with different parameters.

# Excluding unfruitful seed sets

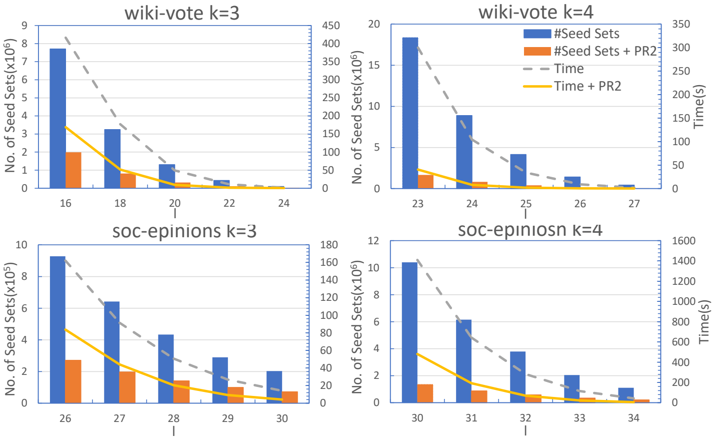


Figure 9: The number of seed sets and running time with and without Prune Rule 2.

# Reducing cache misses

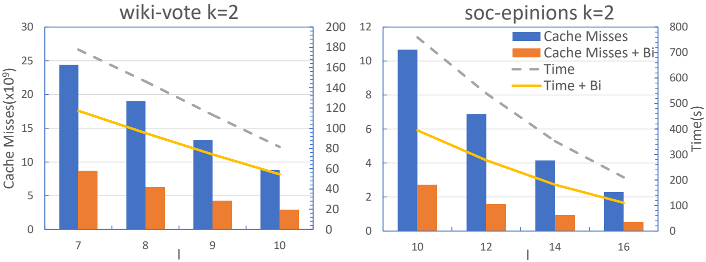


Figure 10: The total number of data cache misses and the running time with and without using bipartite graph $B_i$.

# Q&A

# Thanks!