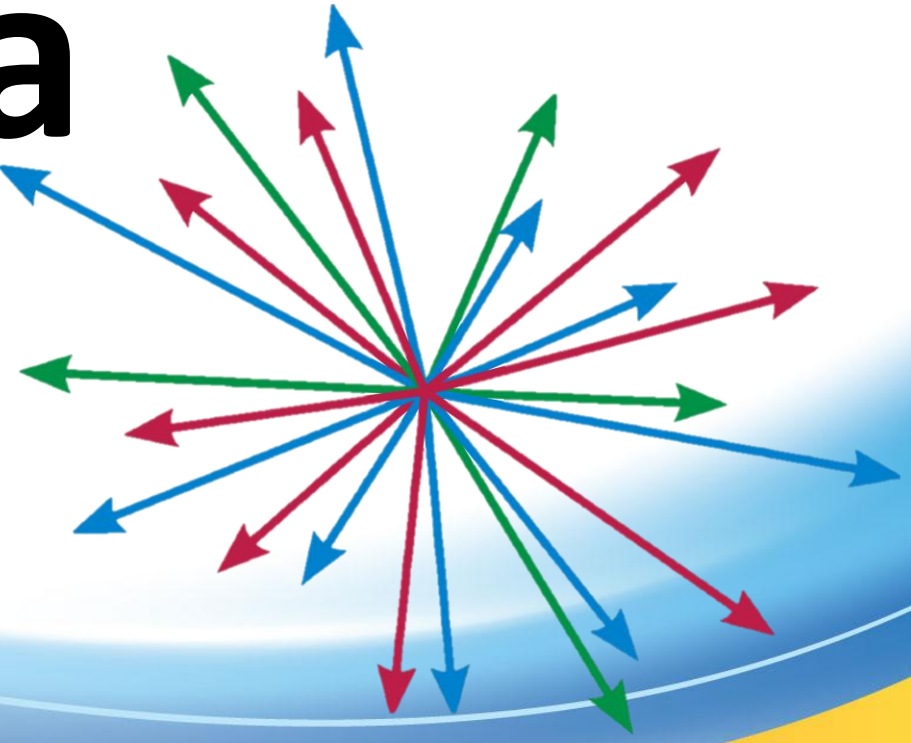
Undergraduate Course in Mathematics

Linear Algebra



Topic: System of Linear Equations

Conducted By

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Equation, Variables, Unknowns, Parameters and Constants

$2x+5 = 7x-3y$

$f(x) = 2x+3$, $2x+3y=5$

$2x - b = c$

$\Rightarrow x = \frac{b+c}{2}$

Linear Equation

- power of unknown = 1
- no product of unknowns.

$2x+3y=5$

$6x-7y=9$

$6x^2-5y=7$

$6x-7xy=8$

Linear Equation in 2 unknowns

$$ax + by = c$$

Ex. $2x - 3y = 5$

$6x + 7y = 8$

Which of the following are linear equations

a) $x + 5y = 1$

b) $\sqrt{2}\,x - 3y = 0$

c) $2\sqrt{x} + 3y = 9$ $\Rightarrow$ $2 \cdot x^{\frac{1}{2}} + 3y = 9$

d) $x + xy = 5$ $\Rightarrow$

e) $5x + \frac{1}{y} = 6$ $\Rightarrow$ $5x + y^{-1} = 6$

f) $7x - x^{\frac{2}{3}} = 0$

g) $\sin(x) + 3y = 5$

h) $\sin(2) \cdot x + \ln(3) \cdot y = \tan(5)$

$ax + by = e$

Linear Equation in 3 unknowns

$$ax + by + cz = d$$

$$2x - 3y + 6z = 13$$ ✓

Which of the following are linear equations

a) $x + 5y - \sqrt{2}\,z = 1$

b) $x + 5y - 2\sqrt{z} = 1$

c) $x_1 + 3x_2 + x_1x_3 = 2$

d) $x = -7y + 8z$

e) $x^{-2} + y + 8w = 9$

f) $x_1^{3/2} - 2x_2 + x_3 = 5$

g) $\pi x - \sqrt{2}\,y + \frac{1}{3}z = 7^{\frac{1}{3}}$

Linear Equation in n Unknowns

x

x, y

x, y, z

$x_1 \quad x_2 \quad x_3 \quad x_4 \quad - - \quad . \quad - \quad x_n$

$$a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + \cdots + a_nx_n = b$$

System of Linear Equations

An arbitrary system of **m** linear equations in **n** unknowns can be written as

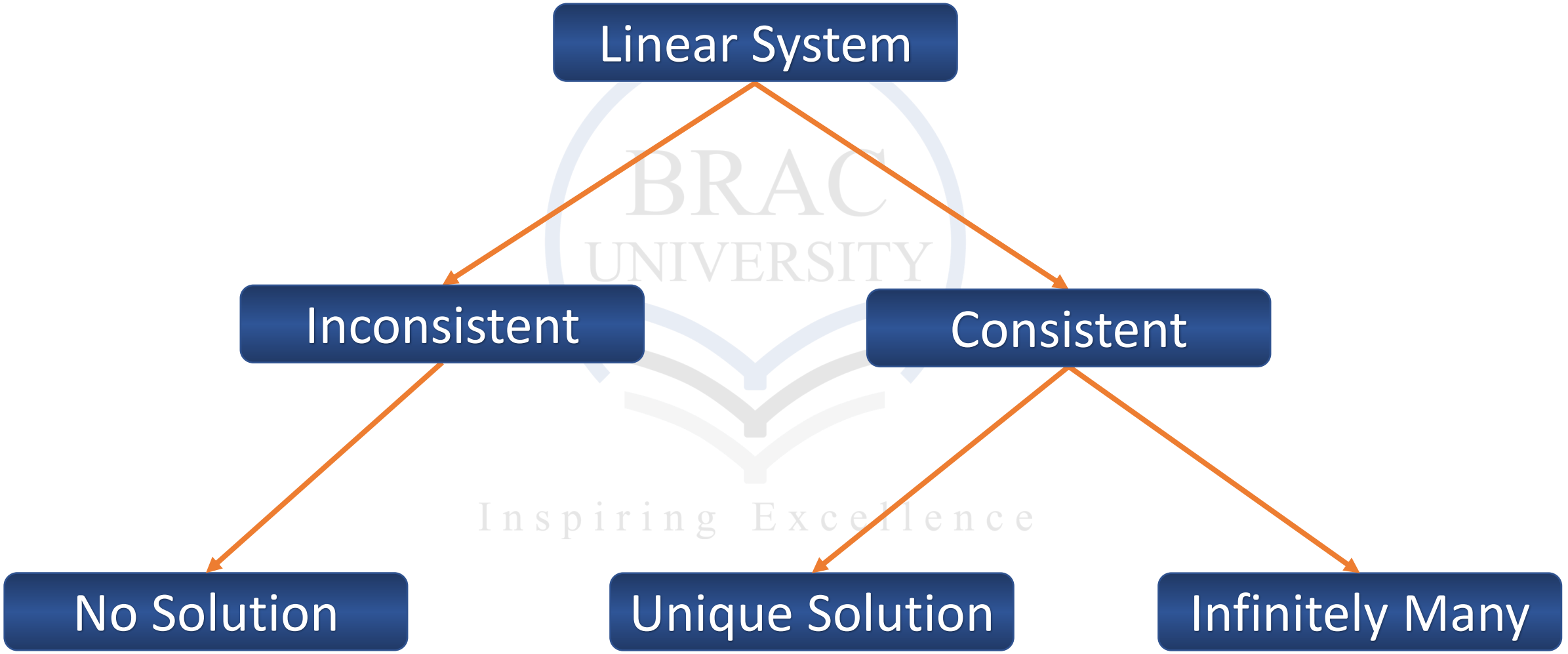
$$
\begin{aligned}
a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\
a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\
\vdots \quad\quad \vdots \quad\quad \vdots \quad\quad & \quad\quad \vdots \quad\quad \vdots \\
a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m
\end{aligned}
$$

$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$

$a_1x_1 + a_2x_2 + a_3x_3 + \cdots + a_nx_n = b$

Where $x_1, x_2, \ldots x_n$ are the unknowns and the subscripted a's and b's denote constants.

Types of Solution



Solve The system

~~$2x + 3y = 1$~~

$$x - y = 3$$

Solve the equation $x - y = 3$

$$(x, y) = (5, 2), (4, 1), (3, 0), (2, -1), - - -$$

Solve The system

$2x + 3y = 1$ — (1)

$x - y = 3$ — (2)

$\Rightarrow y = x - 3$

$2x + 3(x-3) = 1$

$\Rightarrow 2x + 3x - 9 = 1$

$\Rightarrow 5x = 10$

$\Rightarrow \boxed{x = 2}$

$\therefore y = x - 3$

$= 2 - 3$

$= -1$

$(x, y) = (2, -1)$

Solution

unique soln

Solve The system

$$2x + 3y = 1$$

$$4x + 6y = 1$$

$6y = 1 - 4x \Rightarrow y = \frac{1-4x}{6}$

$2x + 3 \cdot \frac{1-4x}{6} = 1$

$\Rightarrow 2x + \frac{1-4x}{2} = 1$

$\Rightarrow 4x + 1 - 4x = 2 \Longrightarrow \underline{1 = 2}$

No Solution

Solve The system

$$2x + 3y = 1$$

$$4x + 6y = 2$$

$$3y = 1 - 2x \Rightarrow y = \frac{1-2x}{3}$$

$$4x + 6y = 2$$

$$\Rightarrow 4x + 6 \cdot \frac{1-2x}{3} = 2$$

$$\Rightarrow 4x + 2(1-2x) = 2$$

$$\Rightarrow 4x + 2 - 4x = 2$$

$$\Rightarrow 2 = 2$$

many

$$\Rightarrow 0 = 0$$

Solve The system

$$2x + 3y = 1$$

$$4x + 6y = 2$$

$(x, y) = (0, \frac{1}{3}), (\frac{1}{2}, 0), (-0.4, 0.6), \ldots$

Types of Solution

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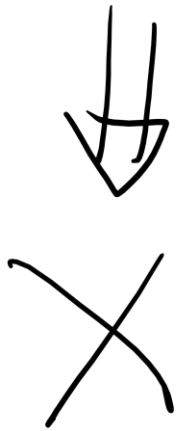
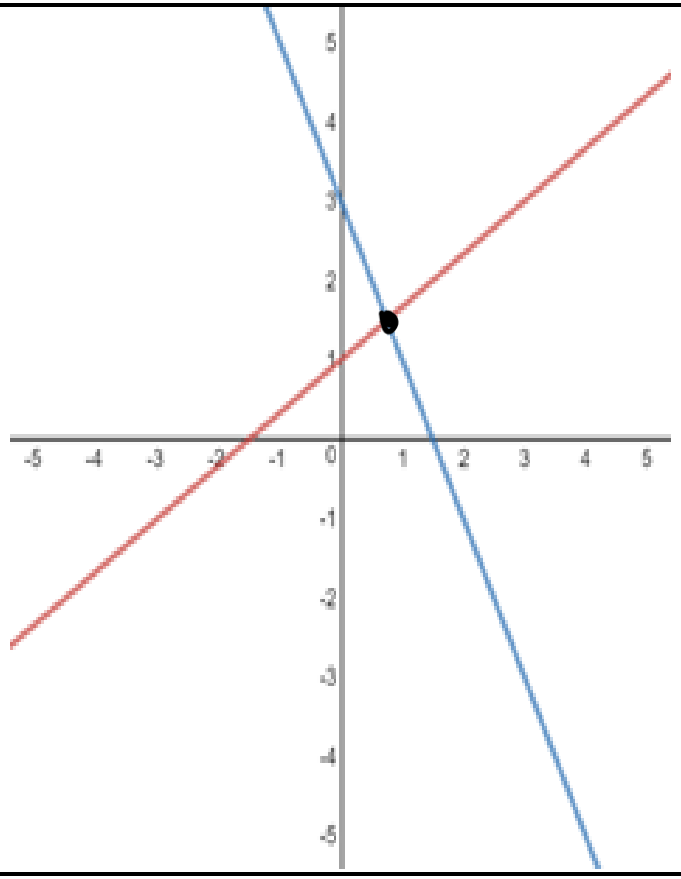
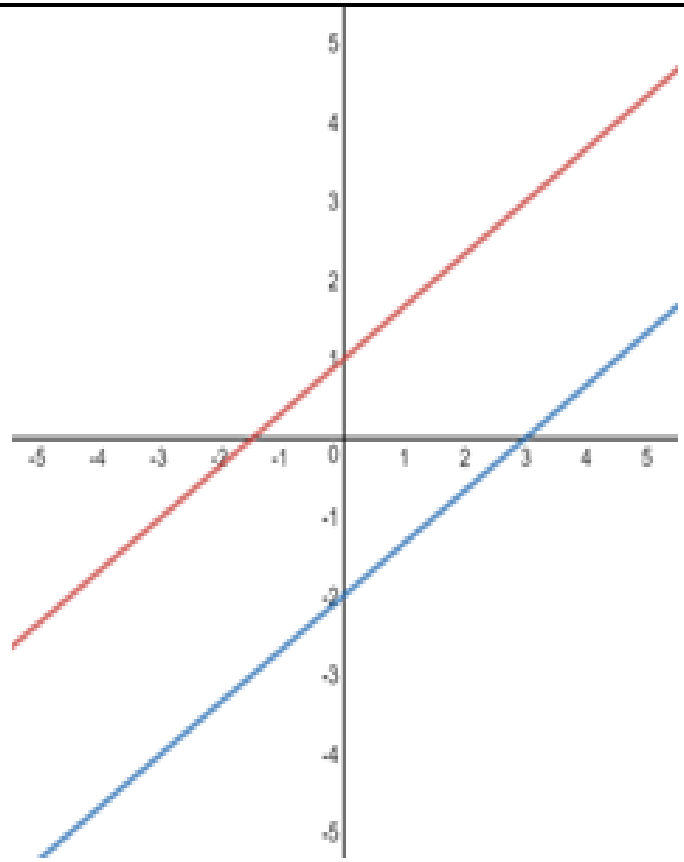
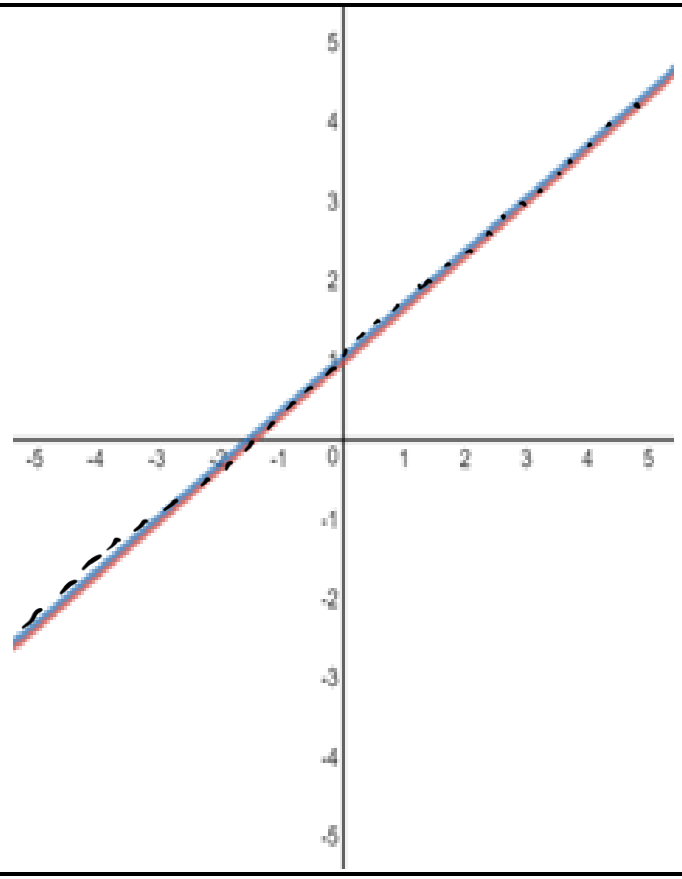
No Solution



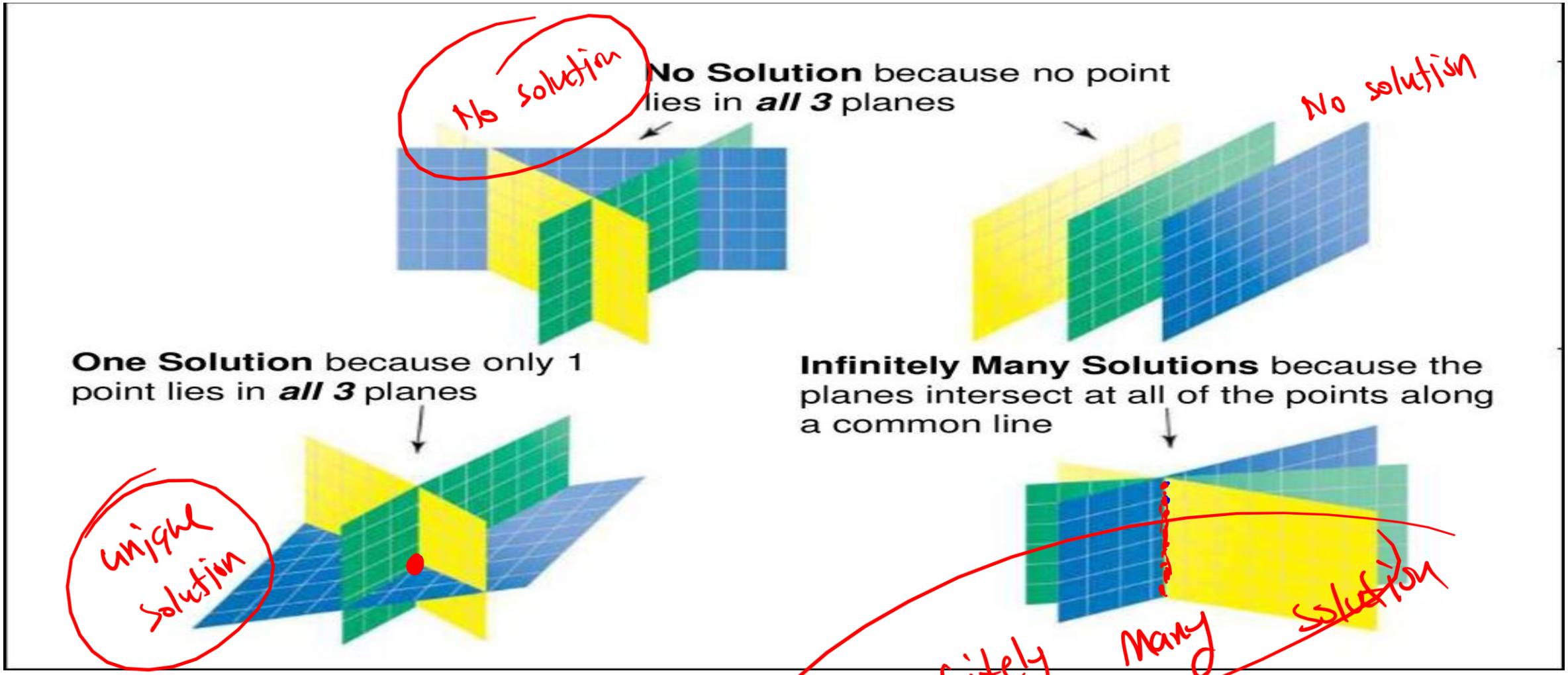
Unique Solution

Infinitely Many

In 2D

Case 1: one solution *(different slope)*	Case 2: no solutions *(same slope, different y-intercept)*	Case 3: infinite solutions *(same slope, same y-intercept)*
		

In 3D


No Solution because no point lies in all 3 planes
No solution
No solution
One Solution because only 1 point lies in all 3 planes
unique solution
Infinitely Many Solutions because the planes intersect at all of the points along a common line
infinitely Many solution

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