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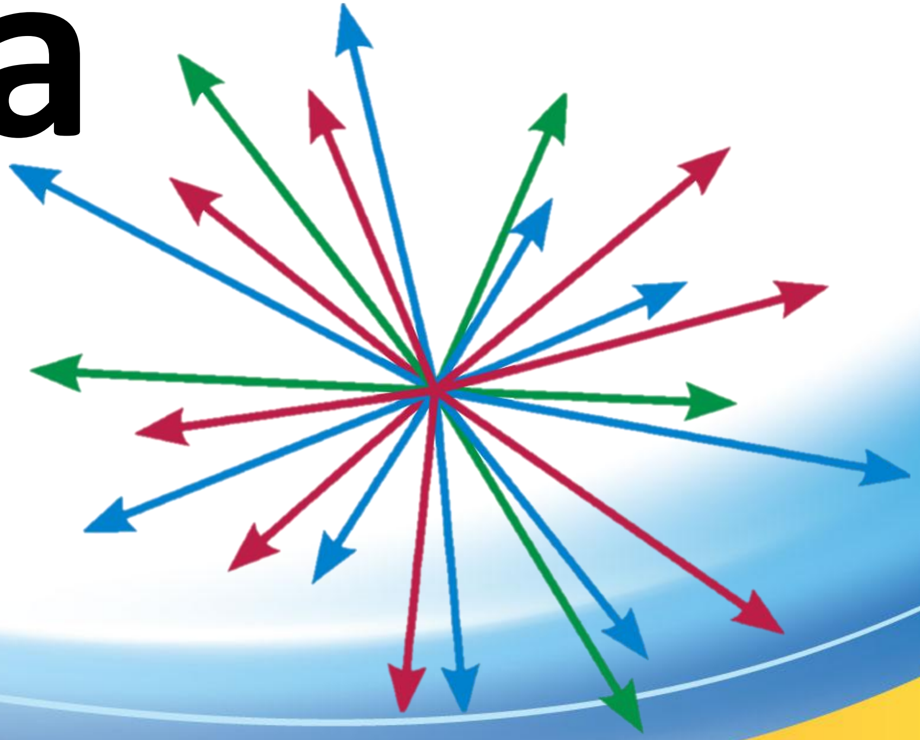
Undergraduate Course in Mathematics

Linear Algebra

Topic: Eigenvector



Conducted By

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Eigenvalue and Eigenvector of a Matrix

Let $\boldsymbol{A}$ be a square matrix of size $n \times n$. A scalar number $\boldsymbol{\lambda}$ is called an **eigenvalue** of $\boldsymbol{A}$ if there exists a vector $\vec{v}$ such that $\vec{v} \neq 0$ with

$$A\vec{v} = \lambda\vec{v}$$

Here the vector $\vec{v}$ is called an eigenvector of the eigenvalue λ.

Finding Eigenvalue

Let $\boldsymbol{A}$ be a square matrix of size $n \times n$. A scalar number $\boldsymbol{\lambda}$ is called an **eigenvalue** of $\boldsymbol{A}$ if and only if

$$\boxed{\det(A - \lambda I) = 0}$$

$$A\vec{v} = \lambda \vec{v}$$

$$S \cdot \vec{x} = 0$$

$$\Rightarrow A\vec{v} - \lambda \vec{v} = 0$$

$$\Rightarrow \boxed{(A - \lambda I)\vec{v} = 0}$$

solve

Null space of $A - \lambda I$

Finding Eigenvector

Let A be a square matrix of size $n \times n$. If λ is an eigenvalue of A then a non trivial solutions $\vec{v}$ of

$$(A - \lambda I)\vec{v} = 0$$

is a eigenvectors of A.

Some Facts of Eigenvectors

- There exists infinitely many eigenvectors for each eigenvalue.
- If $\vec{v}$ is an eigenvector then any multiple of $\vec{v}$ is also an eigenvector.

Find the eigenvalues and eigenvectors of A.

$$A = \begin{pmatrix} 3 & -2 \\ 2 & -2 \end{pmatrix}$$

let λ be an eigenvalue of A.

$$A - \lambda I = \begin{pmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{vmatrix} = 0$$

$$(3-\lambda)\cdot(-2-\lambda) - (2)\cdot(-2) = 0$$

$$-6 - 3\lambda + 2\lambda + \lambda^2 + 4 = 0$$

$$\lambda^2 - \lambda + 2 = 0$$

$$(\lambda + 1)(\lambda - 2) = 0$$

$$\therefore \lambda = -1, 2$$

For $\lambda = -1$, let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{pmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & -2 \\ 2 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{cc|c} 4 & -2 & 0 \\ 2 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 4 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad R_2' = 2R_2 - R_1$$

$$\left(\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

$$\Rightarrow x_1 - \frac{1}{2}x_2 = 0$$

free variable $x_2 = t$

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{2}t \\ t \end{pmatrix} = t\begin{pmatrix} \frac{1}{2} \\ 1 \end{pmatrix} \quad \text{ev}$$

$$= \frac{1}{2}t\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$\therefore$ For $\lambda = -1$, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is an eigenvector

For $\lambda = 2$, let $\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{pmatrix} 3-\lambda & -2 \\ 2 & -2-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -2 \\ 2 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 2 & -4 & 0 \end{array}\right)$$

$$\left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right) \quad R_2' = R_2 - 2R_1$$

$\Rightarrow x_1 - 2x_2 = 0$

x_2 is free, let $x_2 = t$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 2t \\ t \end{pmatrix} = t\underbrace{\begin{pmatrix} 2 \\ 1 \end{pmatrix}}$$

$\therefore$ for $\lambda = 2$, $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigenvector

Find the eigenvalues and eigenvectors of A

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{pmatrix}$$

Let λ be an eigenvalue

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{pmatrix}.$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{vmatrix} = 0$$

$$(1-\lambda) \cdot \begin{vmatrix} 2-\lambda & 2 \\ 1 & 3-\lambda \end{vmatrix} - (1) \cdot \begin{vmatrix} 0 & 2 \\ -1 & 3-\lambda \end{vmatrix} + (2) \cdot \begin{vmatrix} 0 & 2-\lambda \\ -1 & 1 \end{vmatrix} = 0$$

$$(1-\lambda) \cdot [(2-\lambda)(3-\lambda) - 2] - 1 \cdot [0 - (-1)(2)] + 2 \cdot [0 - (-1)(2-\lambda)] = 0$$

$$(1-\lambda) \cdot [4 - 5\lambda + \lambda^2] - 1 \cdot [2] + 2 \cdot [(2-\lambda)] = 0$$

$$4 - 5\lambda + \lambda^2 - 4\lambda + 5\lambda^2 - \lambda^3 - 2 + 4 - 2\lambda = 0$$

$$-\lambda^3 + 6\lambda^2 - 11\lambda + 6 = 0$$

$$\boxed{\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0}$$

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$(\lambda - 1)(\lambda - 2)(\lambda - 3) = 0$$

$$\lambda = 1, 2, 3$$

For $\lambda = 1$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = \vec{0}$

$$\begin{pmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 2 \\ 0 & 1 & 2 \\ -1 & 1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ -1 & 1 & 2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right) \quad R_1 \leftrightarrow R_3$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad R_3' = R_3 - R_2$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad R_1' = R_1 - R_2$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad R_1' = -R_1$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = 0$

$x_2 + 2x_3 = 0$

x_3 is free

let $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -2t \\ t \end{pmatrix} = t \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

$\therefore$ For $\lambda = 1$, $\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$ is an eigenvector.

For $\lambda = 2$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & 2 \\ 0 & 0 & 2 \\ -1 & 1 & 1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ -1 & 1 & 1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 2 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - x_2 = 0$

$x_3 = 0$

free x_2.

let $x_2 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} t \\ t \\ 0 \end{pmatrix} = t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

For $\lambda = 2$, $\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$ is an eigenvector.

For $\lambda = 3$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 1-\lambda & 1 & 2 \\ 0 & 2-\lambda & 2 \\ -1 & 1 & 3-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 1 & 2 \\ 0 & -1 & 2 \\ -1 & 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 1 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 2 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - 2x_3 = 0$

$x_2 - 2x_3 = 0$

x_3 is free

$\therefore$ Let $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 2t \\ 2t \\ t \end{pmatrix} = t \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

$\therefore$ For $\lambda = 3$, $\begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector.

Find the eigenvalues and eigenvectors of A

$$A = \begin{pmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} 4-\lambda & 1 & -1 \\ 2 & 5-\lambda & -2 \\ 1 & 1 & 2-\lambda \end{pmatrix}$$

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} 4-\lambda & 1 & -1 \\ 2 & 5-\lambda & -2 \\ 1 & 1 & 2-\lambda \end{vmatrix} = 0$$

$$(4-\lambda)\cdot\begin{vmatrix} 5-\lambda & -2 \\ 1 & 2-\lambda \end{vmatrix} - (1)\cdot\begin{vmatrix} 2 & -2 \\ 1 & 2-\lambda \end{vmatrix} + (-1)\cdot\begin{vmatrix} 2 & 5-\lambda \\ 1 & 1 \end{vmatrix} = 0$$

$$(4-\lambda)\cdot[(5-\lambda)(2-\lambda)-(-2)] - 1\cdot[2(2-\lambda)-(-2)] + (-1)\cdot[2-(5-\lambda)] = 0$$

$$(4-\lambda)\cdot[12-7\lambda+\lambda^2] - 1\cdot[6-2\lambda] + (-1)\cdot[-3+\lambda] = 0$$

$$48-28\lambda+4\lambda^2-12\lambda+7\lambda^2-\lambda^3-6+2\lambda+3-\lambda = 0$$

$$-\lambda^3+11\lambda^2-39\lambda+45 = 0$$

$$\lambda^3-11\lambda^2+39\lambda-45 = 0$$

$$\lambda^3 - 11\lambda^2 + 39\lambda - 45 = 0$$

$$(\lambda - 5)(\lambda - 3)(\lambda - 3) = 0$$

$\lambda = \underline{5}, \underline{3}, 3$

For $\lambda = 5$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 4-\lambda & 1 & -1 \\ 2 & 5-\lambda & -2 \\ 1 & 1 & 2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 1 & -1 \\ 2 & 0 & -2 \\ 1 & 1 & -3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 2 & 0 & -2 & 0 \\ 1 & 1 & -3 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 2 & -4 & 0 \\ 0 & 2 & -4 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -2 & 0 & 2 & 0 \\ 0 & 2 & -4 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 - x_3 = 0$

$x_2 - 2x_3 = 0$

free x_3.

$\therefore$ Let $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} t \\ 2t \\ t \end{pmatrix} = t \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$$

For $\lambda = 5$, $\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$ is an eigenvector.

For $\lambda = 3$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 4-\lambda & 1 & -1 \\ 2 & 5-\lambda & -2 \\ 1 & 1 & 2-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 & -1 \\ 2 & 2 & -2 \\ 1 & 1 & -1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 2 & 2 & -2 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right) \quad \begin{array}{l} R_2' = R_2 - 2R_1 \\ R_3' = R_3 - R_1 \end{array}$$

$\Rightarrow x_1 + x_2 - x_3 = 0$

x_2 and x_3 free

let $x_2 = t_1$ and $x_3 = t_2$.

$x_1 = -x_2 + x_3 = -t_1 + t_2$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -t_1 + t_2 \\ t_1 \\ t_2 \end{pmatrix} = t_1 \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + t_2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

$\therefore$ For $\lambda = 3$, $\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ are linearly independent eigenvectors.

Find the eigenvalues and eigenvectors of A

$$A = \begin{pmatrix} 0 & -6 & -4 \\ 5 & -11 & -6 \\ -6 & 9 & 4 \end{pmatrix}$$

$$A - \lambda I = \begin{pmatrix} -\lambda & -6 & -4 \\ 5 & -11-\lambda & -6 \\ -6 & 9 & 4-\lambda \end{pmatrix}$$

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 0-\lambda & -6 & -4 \\ 5 & -11-\lambda & -6 \\ -6 & 9 & 4-\lambda \end{vmatrix} = 0$$

$$(0-\lambda)\cdot\begin{vmatrix} -11-\lambda & -6 \\ 9 & 4-\lambda \end{vmatrix} - (-6)\cdot\begin{vmatrix} 5 & -6 \\ -6 & 4-\lambda \end{vmatrix} + (-4)\cdot\begin{vmatrix} 5 & -11-\lambda \\ -6 & 9 \end{vmatrix} = 0$$

$$(-\lambda)\cdot[(-11-\lambda)(4-\lambda)+54] + 6\cdot[5(4-\lambda)-36] - 4\cdot[54-(-6)(-11-\lambda)] = 0$$

$$(-\lambda)\cdot[10+7\lambda+\lambda^2] + 6\cdot[-16-5\lambda] - 4\cdot[-21-6\lambda] = 0$$

$$-10\lambda - 7\lambda^2 - \lambda^3 - 96 - 30\lambda + 84 + 24\lambda = 0$$

$$-\lambda^3 - 7\lambda^2 - 16\lambda - 12 = 0$$

$$\lambda^3 + 7\lambda^2 + 16\lambda + 12 = 0$$

$$\lambda^3 + 7\lambda^2 + 16\lambda + 12 = 0$$

$$(\lambda + 3)(\lambda + 2)(\lambda + 2) = 0$$

$$\lambda = -3,\ -2,\ -2$$

For $\lambda = -3$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 0-\lambda & -6 & -4 \\ 5 & -11-\lambda & -6 \\ -6 & 9 & 4-\lambda \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -6 & -4 \\ 5 & -8 & -6 \\ -6 & 9 & 7 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} 3 & -6 & -4 & 0 \\ 5 & -8 & -6 & 0 \\ -6 & 9 & 7 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & -6 & -4 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & -3 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & -6 & -4 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 3 & 0 & -2 & 0 \\ 0 & 6 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & -2/3 & 0 \\ 0 & 1 & 1/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$x_1 - \frac{2}{3}x_3 = 0$$

$$x_2 + \frac{1}{3}x_3 = 0$$

x_3 is free

let $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{2}{3}t \\ -\frac{1}{3}t \\ t \end{pmatrix} = t \cdot \begin{pmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ 1 \end{pmatrix} = \frac{1}{3}t \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$$

$\therefore$ For $\lambda = -3$, $\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$ is an eigenvector.

For $\lambda = -2$, let $\vec{v} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ be an eigenvector

So $(A - \lambda I)\vec{v} = 0$

$$\begin{pmatrix} 0-\lambda & -6 & -4 \\ 5 & -11-\lambda & -6 \\ -6 & 9 & 4-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -6 & -4 \\ 5 & -9 & -6 \\ -6 & 9 & 6 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

The Augmented System

$$\left(\begin{array}{ccc|c} 2 & -6 & -4 & 0 \\ 5 & -9 & -6 & 0 \\ -6 & 9 & 6 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 2 & -6 & -4 & 0 \\ 0 & 12 & 8 & 0 \\ 0 & -9 & -6 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 2 & -6 & -4 & 0 \\ 0 & 12 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 4 & 0 & 0 & 0 \\ 0 & 12 & 8 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 2/3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$x_1 = 0$

$x_2 + \frac{2}{3}x_3 = 0$

x_3 free

let $x_3 = t$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ -\frac{2}{3}t \\ t \end{pmatrix} = \frac{1}{3}t \begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$$

$\therefore$ For $\lambda = -2$, $\begin{pmatrix} 0 \\ -2 \\ 3 \end{pmatrix}$ is an eigenvector.

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