Introduction

Computational geometry (CG) is a branch of computer science which focuses on algorithms that can be stated in terms of geometry. The concerned branch of Computational Geometry; **<u>the Combinatorial Computational Geometry, also known as “algorithmic geometry”,</u>** and it deals with geometric shapes as discrete objects.

An example would be of a simple representation of a road by using nodes and connecting one node to the other. These nodes would be data points on an x-y coordinate and their connections, intersections, distance between then and so on could be found using geometrical computations. An example is shown below.

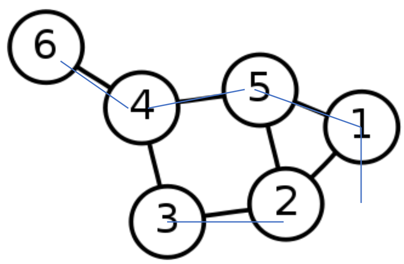


Fig 1. 1 Simple representation of road in algorithmic geometry

<u>1.1 Points, Lines, Line Segment, Rays, angles, and Planes</u>

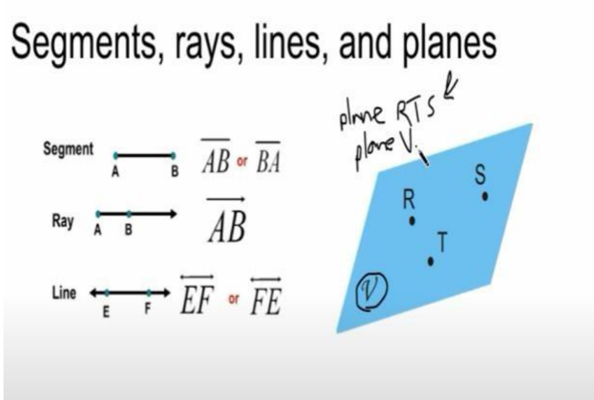
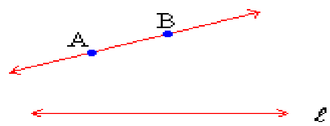


Fig 1. 2 Simple representation of segments, rays, planes, and points

Point: The most fundamental geometric form is a point. It is represented as a dot with a capital alphabet which is its name

Points

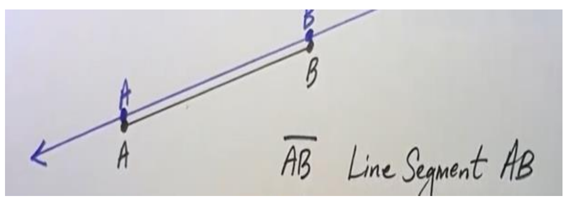
Line: A line is a set of points and it extends in opposite directions up to infinity. It is represented by two points on the line and a double headed arrow or a single alphabet in the lower case



Lines

$\overleftrightarrow{AB} = \overleftrightarrow{BA}$

Line Segment: A line segment is a part of a line. It has a fixed length and consequently two end points. They are used to name the line segment

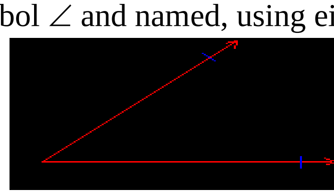


Ray: A ray has one end point (first point) and extends in the other direction up to infinity. It is represented by naming the end point and any other point on the ray with the symbol→.

Ray $\overrightarrow{JK}$

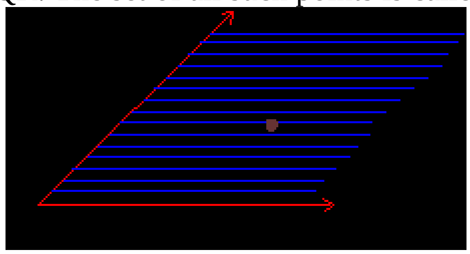
J is the end point of a ray and K is a point on it. This ray is represented as $\overrightarrow{JK}$. A ray can extend in any one direction only.

Angles: Two rays going in different directions, but having a common end point, form an angle. The common end point is called the vertex of the angle and the rays are called its sides or arms. An angle is represented by the symbol ∠ and named, using either both the rays or just the vertex

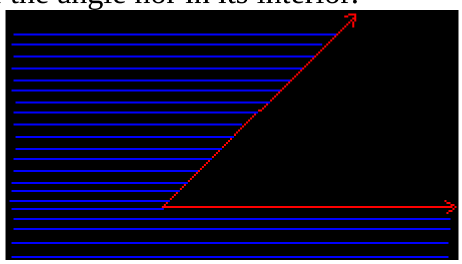


∠ XYZ or ∠ Y

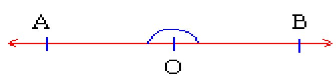
- **<u>Interior and Exterior of an angle:</u> The interior** of ∠ PQR is the shaded region in the following figure. **S** is a point in the interior of ∠Q because it lies on the R- side of ray PQ and the P - side of ray QR. The set of all such points is called the interior of ∠PQR.



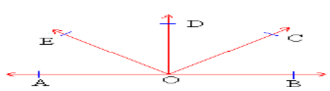
The exterior of an angle is defined as the set of points in the plane of a given angle which is neither on the sides of the angle nor in its interior.



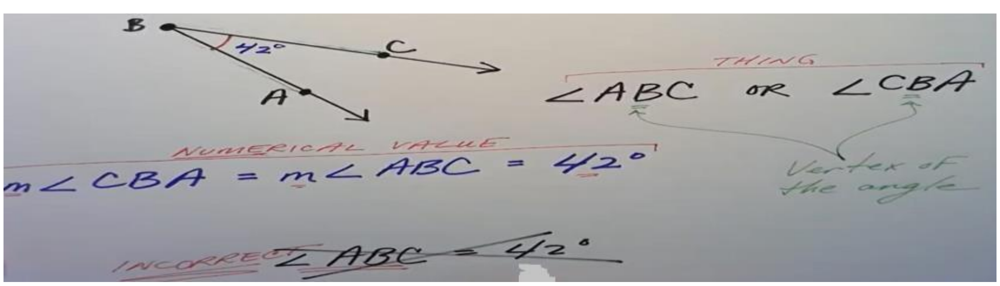
- **<u>Measure of an angle:</u>** Every angle has a measure. It is measured in degrees from 0^0 to 180^0 and is represented as m ∠. A line is also an angle because it satisfies the definition of having two rays going in different (in this case opposite) directions with a common end point



$\overleftrightarrow{AB}$ is also ∠ AOB and m ∠ AOB = 180^0



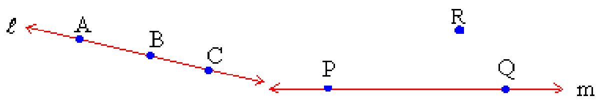
All rays starting from O and going above line AB form angles with $\overrightarrow{OB}$ such that their measure is between 0^0 and 180^0. E.g. ∠ COB, ∠ DOB and ∠ EOB.

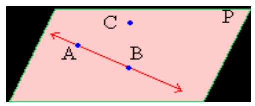


➢

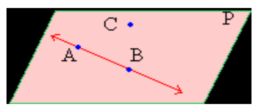
<u>Plane:</u> A plane has obviously no size and definitely no shape. However, it is represented as a quadrangle and a single capital letter

<u>Collinear points</u> Three or more points are said to be **<u>collinear</u>** if a single line contains all of them. Otherwise, they are said to be **<u>non collinear</u>**.



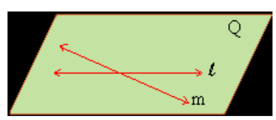


<u>Axiom 1.1:</u> Through any three non collinear points there can be one and only one plane. By this way we can represent the plane by the three non collinear points.



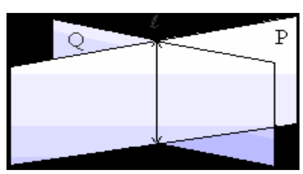
Plane ABC or Plane P.

<u>Axiom 1.2:</u> If two lines intersect, exactly one plane passes through both of them



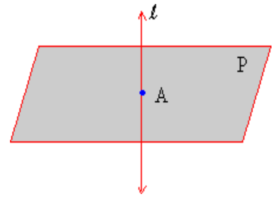
Plane Q contains intersecting lines *l* and m.

<u>Axiom 1.3:</u> If two planes intersect their intersection is exactly one line



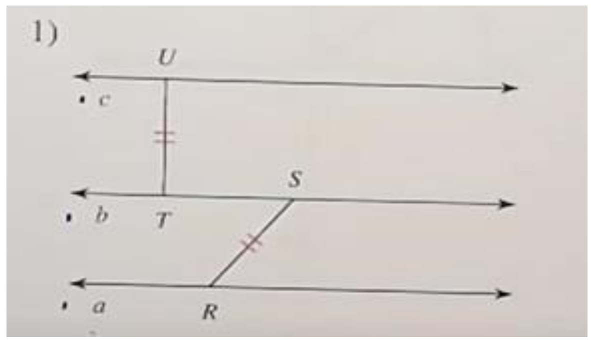
Planes P and Q intersect and their intersection is line *l*.

<u>Axiom 1.4:</u> If a line does not lie in a plane but intersects it, their intersection is a point



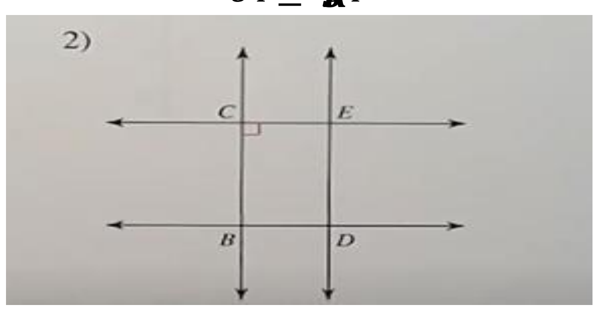
Point A is the intersection point of line *l* and plane P.

<u>List all information given by the marks on the diagram:</u>



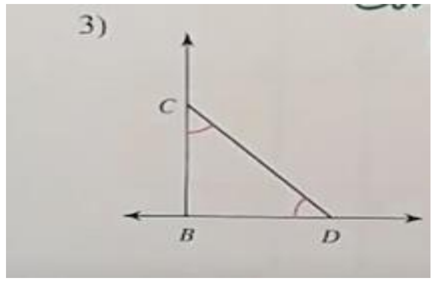
The two dashes on the line segments $\overline{UT}$ and $\overline{SR}$ means that these line segments are **<u>congruent- “equal”</u>** i.e. they have the same length and properties and denoted by

$$\overline{UT} \cong \overline{SR}$$



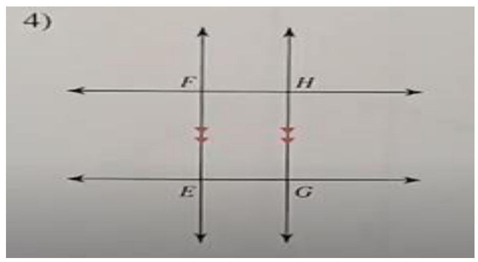
The line $\overleftrightarrow{CB}$ perpendicular on the line $\overleftrightarrow{CE}$ and denoted by

$$\overleftrightarrow{CB} \perp \overleftrightarrow{CE}$$



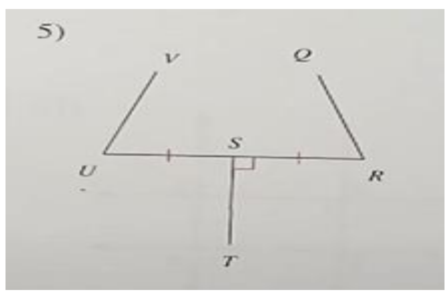
< BCD and < BDC are congruent, this mean that the angles have same measures and same characteristics and denoted by

$$< BCD \cong < BDC$$



The two lines $\overleftrightarrow{FE}$ and $\overleftrightarrow{HG}$ are parallel and denoted by

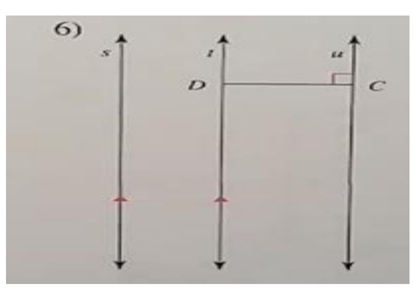
$$\overleftrightarrow{FE} \parallel \overleftrightarrow{HG}$$



$\overline{ST} \perp \overline{RU}$

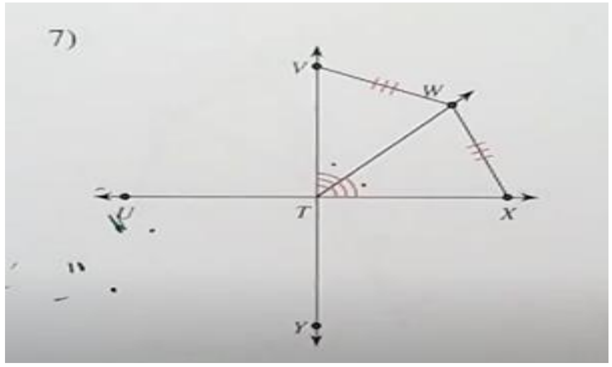
and

$\overline{US} \cong \overline{SR}$



$\overline{DC} \perp n$

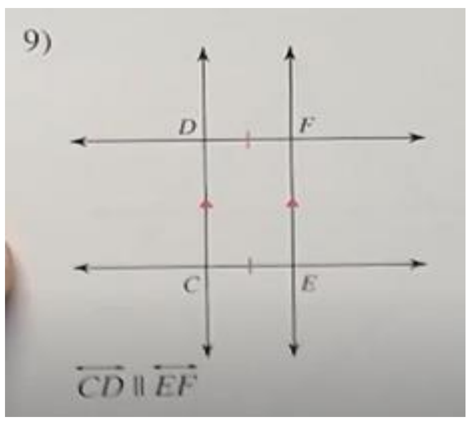
s ‖ t



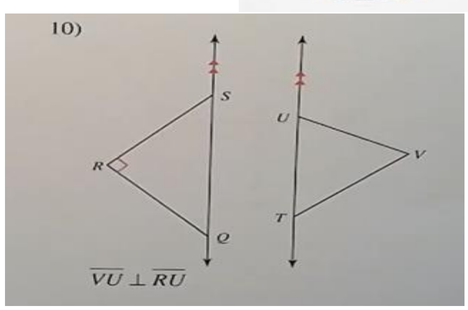
$\overline{VW} \cong \overline{WX}$

$< VTX \cong < WTX$

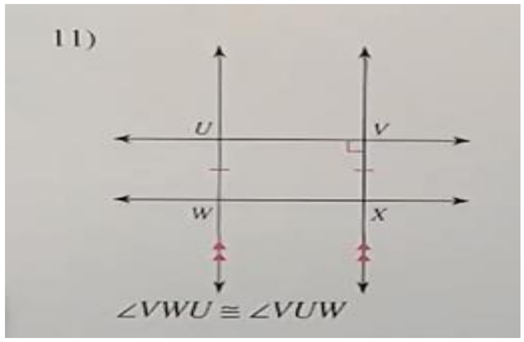
<u>Write if the statement given is indicated by the marks on the diagram:</u>



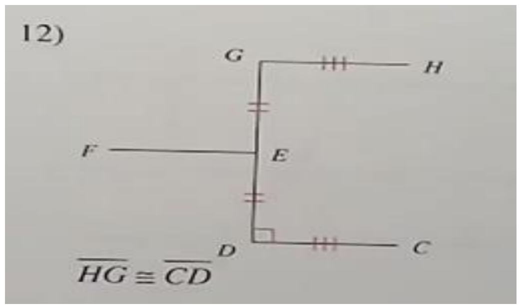
<u>True</u>



<u>False</u>



False



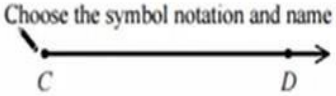
True

Choose the symbol notation and name for the geometric figure.

[A] $\overleftrightarrow{AB}$, line [B] $\overrightarrow{AB}$, ray

[C] $\overline{AB}$, segment [D] $\overrightarrow{BA}$; ray

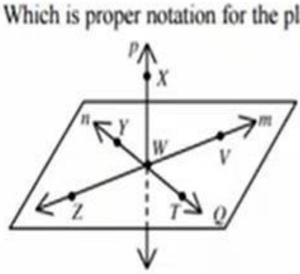
Choose the symbol notation and name for the geometric figure.



[A] $\overrightarrow{DC}$, ray [B] $\overleftrightarrow{CD}$, line

[C] $\overrightarrow{CD}$, ray [D] $\overline{CD}$, segment

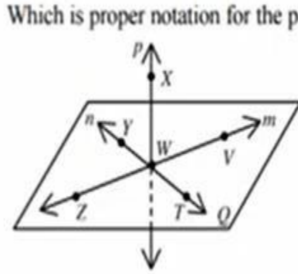
Which is proper notation for the plane in the figure?



[A] plane Q [B] plane YWT

[C] plane $TWYZ$ [D] plane m

Which is proper notation for the plane in the figure?



[A] plane ZYV [B] plane WZV

[C] plane $YWTV$ [D] plane pW

Answer:

Choose the symbol notation and name for the geometric figure.

[A] $\overleftrightarrow{AB}$, line [B] $\overrightarrow{AB}$, ray

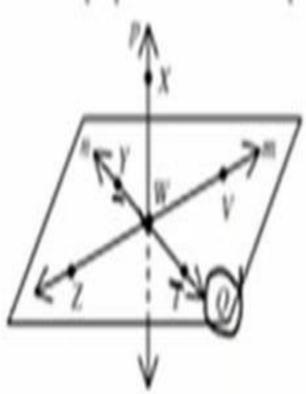
[C] $\overline{AB}$, segment [D] $\overrightarrow{BA}$, ray

Choose the symbol notation and name for the geometric figure.

[A] ~~$\overrightarrow{DC}$, ray~~ [B] ~~$\overleftrightarrow{CD}$, line~~

[C] $\overrightarrow{CD}$, ray [D] ~~$\overline{CD}$, segment~~

Which is proper notation for the plane in the figure?



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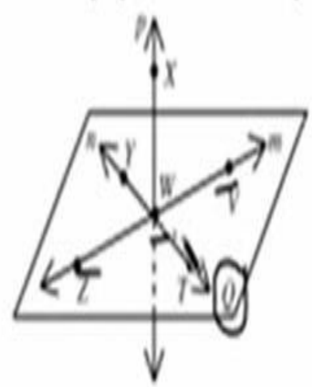
[C] plane $TWYZ$ [D] plane m

Which is proper notation for the plane in the figure?



[A] plane ZYV [B] plane WZV

[C] ~~plane WTV~~ [D] ~~plane nW~~

Which of the following statements is false?

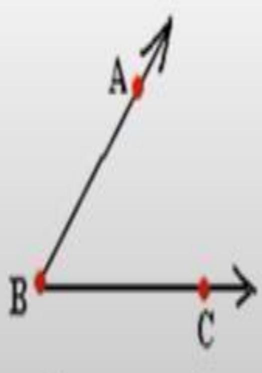
[A] If A is the midpoint of $\overline{PQ}$, then $\overline{PA} \cong \overline{AQ}$. [B] $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are equivalent notations.

[C] AB means the measure of segment $\overline{AB}$. [D] $\overleftrightarrow{AB}$ is a line, $\overrightarrow{AB}$ is a ray, and $\overline{AB}$ is a segment.

Which of the following statements is false?

[A] If A bisects $\overline{PQ}$, then A is the midpoint of $\overline{PQ}$. [B] $\overleftrightarrow{AB}$ is equivalent to $\overleftrightarrow{BA}$.

[C] $\angle ABC$ is equivalent to $\angle BCA$. [D] $\angle ABC$ is equivalent to $\angle CBA$.



$\angle ABC$ (or) $\angle CBA$

Answer:

Which of the following statements is false?

T [A] If A is the midpoint of $\overline{PQ}$, then $\overline{PA} \cong \overline{AQ}$.

F [B] $\overrightarrow{AB}$ and $\overrightarrow{BA}$ are equivalent notations.

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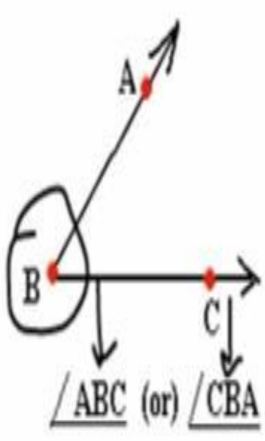
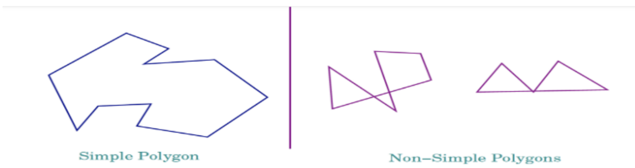
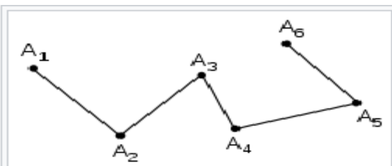


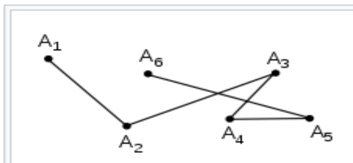
a polygon

- In geometry, a polygon is a plane figure that is described by
- A finite chain of line segments.
- Line segments called edges, their endpoints called vertices.
- A simple polygon is a closed polygonal curve without self-intersection.

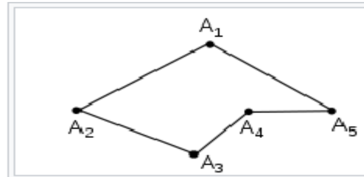




A simple polygonal chain



A self-intersecting polygonal chain



A closed polygonal chain

- We will concern with closed polygon chain

Terminology

- The segments of a polygonal circuit are called its ***edges*** or ***sides***.
- The points where two edges meet are the polygon's ***vertices*** or ***corners***.
- The interior of a solid polygon is sometimes called its ***body***.
- An ***n*-gon** is a polygon with *n* sides; for example, a **triangle** is a 3-gon.
- Consecutive sides : Consecutive sides are those which have a vertex in common.
- Diagonals : Diagonals are segments joining non-consecutive vertices.

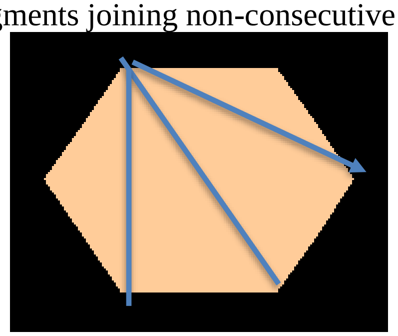
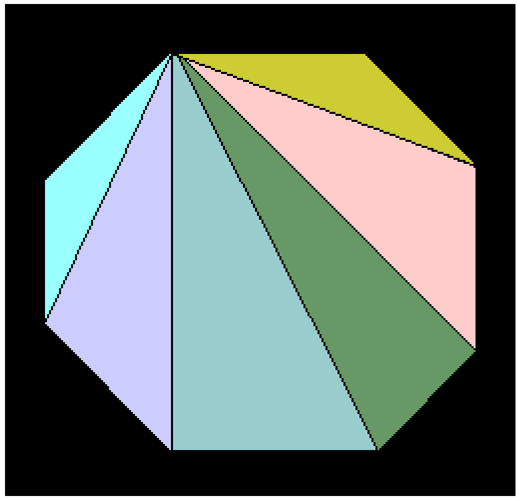


Figure 3.2

- In figure 3.2 A, B, C, D, E & F are vertices. AB has two consecutive sides BC and AF. Similarly two consecutive sides exist for the rest of the sides.
- Segments joining A to all vertices except B & F are diagonals. Similarly, diagonals can be drawn from all the other vertices.

Sum of Interior angles of a Polygon

- 

- Figure 3.3

Figure 3.3 shows an octagon. Five diagonals can be drawn from A. This gives rise to six triangles.

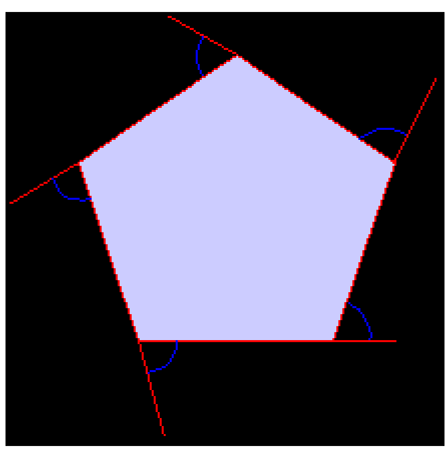
Since the sum of all internal angles of a triangle is 180^0, the sum all the internal angles of this polygon is $6 \times 180^0 = 1080^0$.

This can be generalized as:

For any n sided polygon the sum of its internal angles is
$\underline{(n - 2) \times 180}$.

Sum of exterior angles of a Polygon

Consider the following pentagon

- 

- Figure 3.4

Its external angles are named from a to e. The aim is to find the sum of these five angles.

It is known that the sum of internal angles of a Pentagon is

$$(5 - 2) \times 180^0$$

$$= 3 \times 180^0$$

$$= 540^0$$

each interior angle of the pentagon measures

$$540^0 / 5 = 108^0$$

Each exterior angle measures $180^0 - 108^0 = 72^0$

Sum of five exterior angles = $5 \times 72 = 360^0$

<u>Prove that the sum of the exterior angles for any polygon is 360 $^{0.}$</u>

Sum of interior angles of an n sided polygon = $(n - 2)\, 180^0$.

$\therefore$ Measure of each internal angle = $\dfrac{(n-2)180^0}{n}$

$\therefore$ Each exterior angle = $180 - \dfrac{(n-2)180^0}{n}$

$\therefore$ Sum of n exterior angles = $n\left[180 - \dfrac{(n-2)180}{n}\right]^0$

$= 180n - 180n + 360^0$

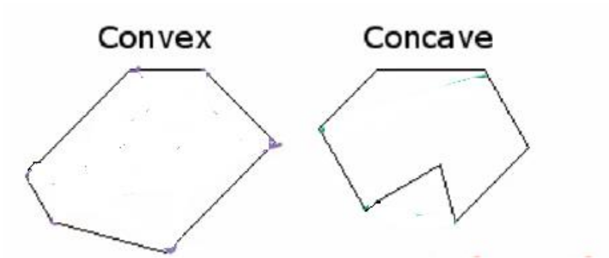
$= 360^0$

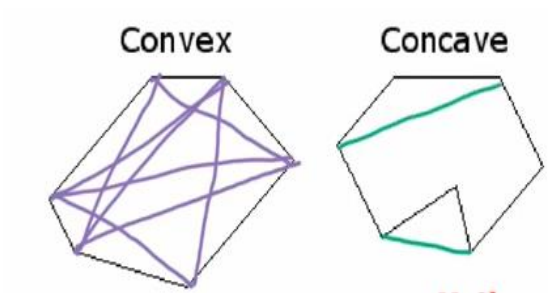
Conclusion : The sum of interior angles of a polygon is dependent on the number of sides but the sum of the exterior angles is always 360^0.

<u>Examples of Differences between Convex and Concave Polygons</u>

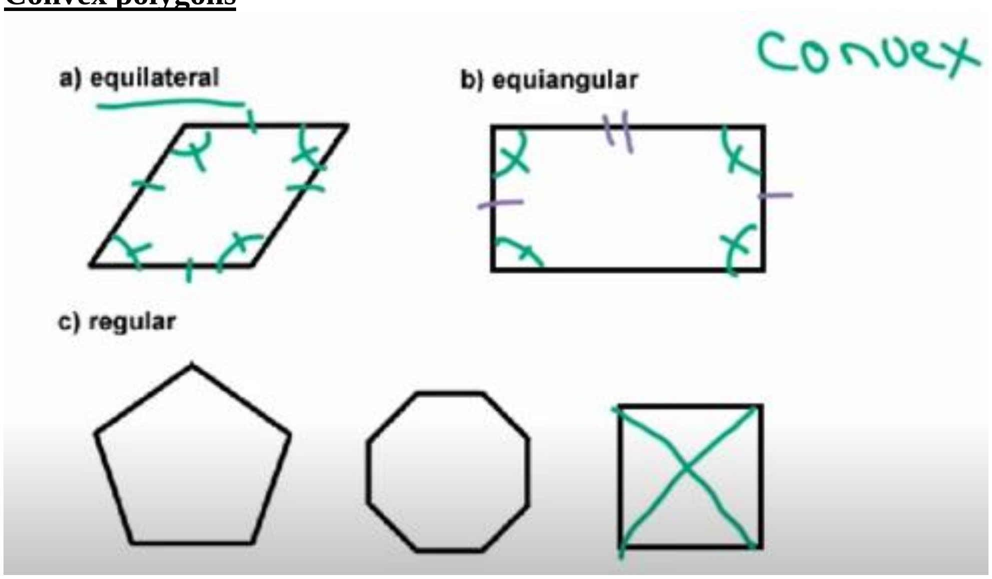
<u>Convex Polygons</u>: all diagonals are onside

<u>Concave Polygons:</u> one or more diagonal are outside

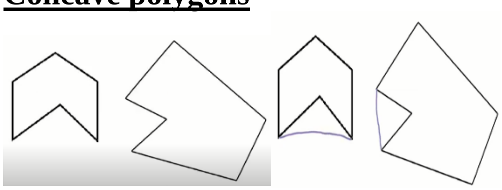




<u>Convex polygons</u>



<u>Concave polygons</u>



<u>Which of the following are concave or convex polygon</u>