

Islington College
Module: MA4001NI Logic and Problem Solving

1. Answer the following questions:

- Why is $R = \{(1, 1), (2, 2), (3, 3)\}$ not reflexive on $\{1, 2, 3, 4\}$?
- Why is $R = \{(1, 2), (2, 1), (3, 1)\}$ not symmetric on $\{1, 2, 3, 4\}$?
- Why is $R = \{(1, 2), (2, 3), (1, 3), (2, 1)\}$ not transitive on $\{1, 2, 3, 4\}$?
- Is $\{(1, 1), (2, 2), (3, 3)\}$; Reflexive? Symmetric? Transitive on $\{1, 2, 3, 4\}$?

2. Let B is a set equals to $\{11, 12, 13, 4, 5\}$ and R be the relation as

$$R = \{(11, 11), (11, 12), (11, 13), (11, 4), (11, 5), (12, 12), (12, 4), (12, 5), (13, 13), (13, 4), (13, 5), (4, 4), (4, 5), (5, 5)\}.$$

Determine, if the relation is reflexive, symmetric and antisymmetric or not.

3. If $A = \{2, 3, 4, 6, 8, 9, 12, 18\}$ for all a, b in A , aRb iff a divides b .

List the element of R as a set of order pairs.

4. Let set $A = \{a, b, c, d\}$, and R be a relation defined on A . List the elements represented by the given matrix and check all the properties of the relation.

$$M = [1 \ 1 \ 0 \ 1 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 0 \ 1 \ 0 \ 1]$$

- Given $f(x) = x^2 - 5x + 1$ and $g(x) = 2x + 1$, find $(f \circ g)(2)$ and $(g \circ f)(-2)$.
- Given $f(x) = \frac{x}{x+1}$ and $g(x) = 9x - 3$, find $f^{-1}(x)$, $g^{-1}(x)$, $(f \circ g)(x)$ and $(g \circ f)(x)$.
- Let $f(x) = x^2 + 7x - 18$ and $h(x) = 2(x^{\frac{1}{2}} + 1)$. Evaluate $(f \circ h)(4)$
- Find $g(x)$ when $(f \circ g)(x) = 5x - 3$ and $f(x) = x - 1$
- M&M plain candies come in various colours. The distribution of colours for plain M&M candies is

Colour	Purple	Yellow	Red	Orange	Green	Blue	Brown
Percentage	20	20	20	10	10	10	10

Suppose you have a large bag of plain M&M candies and you choose one candy at random. Find

- $P(\text{green candy or blue candy})$. Are these outcomes mutually exclusive? Why?
 - $P(\text{yellow candy or red candy})$. Are these outcomes mutually exclusive? Why?
 - $P(\text{not purple candy})$
10. You roll two fair dice, one green and one red.
- Are the outcomes on the dice independent?

- b. Find $P(5 \text{ on green die and } 3 \text{ on red die})$.
 - c. Find $P(3 \text{ on green die and } 5 \text{ on red die})$
 - d. Find $P((5 \text{ on green die and } 3 \text{ on red die}) \text{ or } (3 \text{ on green die and } 5 \text{ on red die}))$
11. You roll two fair dice, one green and one red.
- a. What is the probability of getting a sum of 6?
 - b. What is the probability of getting a sum of 4?
 - c. What is the probability of getting a sum of 6 or 4? Are these outcomes mutually exclusive?
12. You draw two cards from a standard deck of 52 cards without replacing the first one before drawing the second.
- a. Are the outcomes on the two cards independent? Why?
 - b. Find $P(\text{ace on 1}^{\text{st}} \text{ card and king on 2}^{\text{nd}})$
 - c. Find $P(\text{king on 1}^{\text{st}} \text{ card and ace on 2}^{\text{nd}})$
 - d. Find the probability of drawing an ace and a king in either order.
13. You draw two cards from a standard deck of 52 cards, but before you draw the second card, you put the first one back and reshuffle the deck.
- a. Are the outcomes on the two cards independent? Why?
 - b. Find $P(\text{ace on 1}^{\text{st}} \text{ card and king on 2}^{\text{nd}})$
 - c. Find $P(\text{king on 1}^{\text{st}} \text{ card and ace on 2}^{\text{nd}})$
 - d. Find the probability of drawing an ace and a king in either order.
14. Compute following:
- a. $P_{5,2}$
 - b. $C_{7,7}$
15. There are three nursing positions to be filled at Lilly hospital. Position 1 is the day nursing supervisor; position 2 is the night nursing supervisor; and position 3 is the nursing coordinator position. There are 15 candidates qualified for all three of the positions. Determine the number of different ways the positions can be filled by these applications.
16. The ski club with ten members is to choose three officers captain, co-captain & secretary, how many ways can those offices be filled?
17. The company Sea Esta has ten members on its board of directors. In how many different ways can it elect a president, vice-president, secretary and treasurer?

18. Suppose you are asked to list, in order of preference, the three best movies you have seen this year. If you saw 10 movies during the year, in how many ways can the three best be chosen and ranked?