Logic & Problem Solving

Lecture week 23

Linear Programming Using Graphical Method

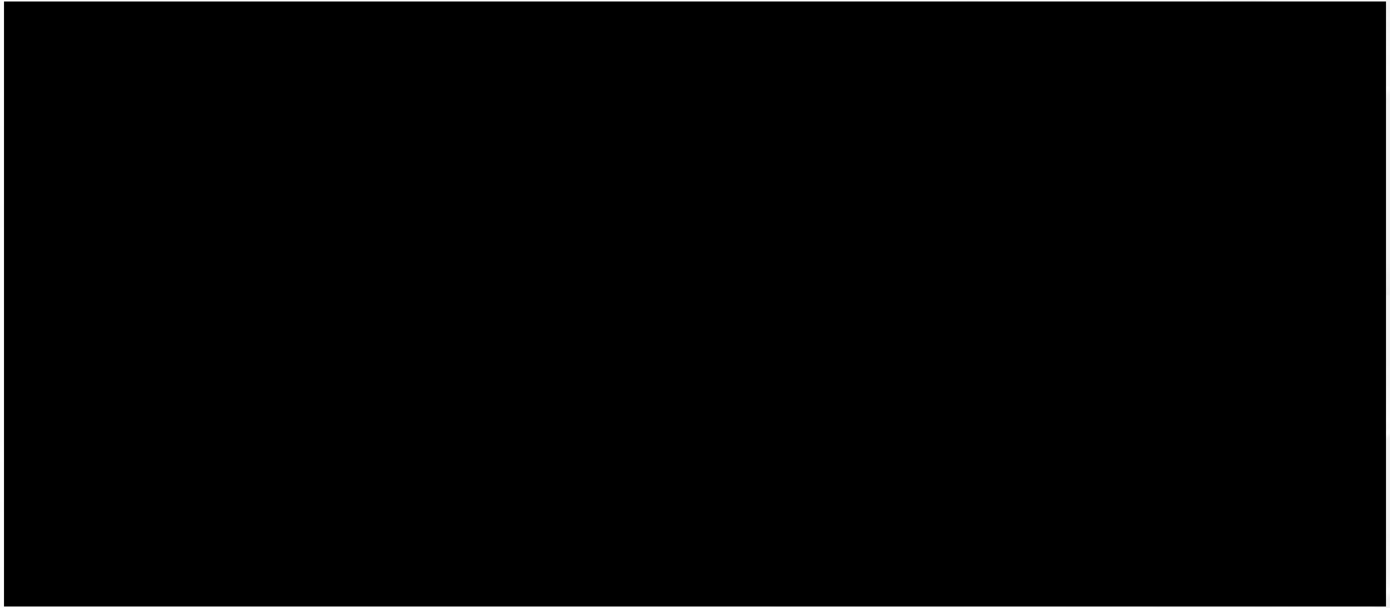
Agenda:

- **Week 23 lecture coverage**
  - Introduction to LPP
  - Mathematical Formulation of LPP
  - Solving LPP graphically

Important Notice:

Please bring scale, pencil and A4 graph paper on your tutorial classes .

Unexpected Goals in Football....



Group Course Work …

Lets talk about group course work ….

Group Course Work….

This coursework accounts for **50%** of your total module marks.

Group Course Work....

You are supposed to form a group of **3 students** and do the group coursework .

Group Course Work....

Every group will be given with **3 questions** (same for all groups) and groups are supposed to answer all the question.

Group Course Work….

Warning:

London Metropolitan University and Islington College takes Plagiarism seriously. Offenders will be dealt with sternly.

Topics to be covered for GW…

- Linear Programming Problem
- Break - even Analysis Problem
- How to use excel for solving Linear Programming Problems?
- How to write a procedure in MS -excel to calculate taxes?

Any Questions?

Linear Programming Problem:

- Linear programming has nothing to do with computer programming.
- The use of the word “programming” here means “choosing a course of action.”
- Linear programming involves choosing a course of action when the mathematical model of the problem contains only linear functions.

Linear Programming Problem:

A **linear programming** is a mathematical techniques which is used to solve **optimization problems** (Maximization/Minimization).

The linear programming model consists of the following components:

- ❖ **Decision Variables**
- ❖ **Objective Function**
- ❖ **Constraints**

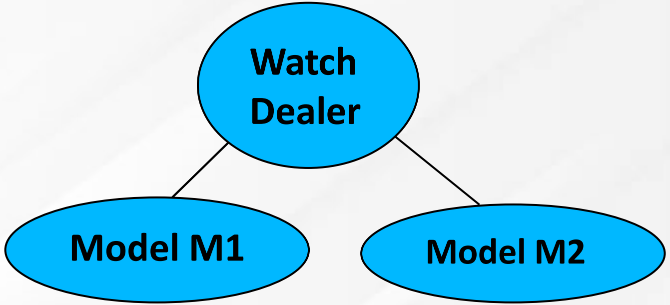
LPP Example :

Problem 1:

A watch dealer wishes to buy new watches and has two models **M1 and M2** to choose from. Model M1 costs **$100** and M2 costs **$200**.In view of the showcase of the dealer, he wants to buy watches **not more than 30** and can spend up to **$4000**. The watch dealer can make a profit of **$20** in M1 and **$50** in M2.How Many of each model should he buy to obtain **maximum profit**?

LPP Example :

Problem analysis:



Cost Price	$100 for M1	$200 for M2
Profit	$20 from M1	$50 from M2
Showcase constraints	Not more than 30 Watches	
Investment Constraints	Can Spend only up to $4000	

LPP Example :

Mathematical Formulation of LPP:

For Decision Variables:

Let **x and y** be the number of watches of model M1 and M2 the dealer should purchase in order to maximize his profit .

For Objective Function:

Total profit =$ (20x + 50y)

Let Z = 20x + 50y

Maximize Z = 20x + 50y

LPP Example :

For Constraints:

$x + y \leq 30$	(Showcase Constraints)
$100x + 200y \leq 4000$	(Investment Constraints)
$x , y \geq 0$	(Non negativity Constraints)

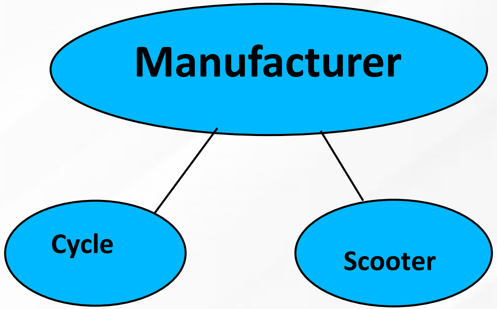
LPP Example :

Problem 2:

A manufacturer produces cycles and scooters, each of which must be processed through two machines A and B. Machine A has a maximum of **120 hours** available and machine B has a maximum of **180 hours** available. Manufacturing a cycle requires **6 hours in machine A** and **3 hours in machine B** . Manufacturing a scooter requires **4 hours in machine A** and **10 hours in machine B** .If profits are **$45** for cycle and **$55** for a scooter ,formulate a mathematical model for the **maximization** of the profits .

LPP Example :

Problem analysis:



For cycle	6 hours in Machine A	3 hours in machine B
For Scooter	4 hours in Machine A	10 hours in machine B
Profit	$45 from cycle	$55 from scooter
Machine A constraints	Can operate Maximum 120 hours	
Machine B constraints	Can operate Maximum 180 hours	

LPP Example :

Mathematical Formulation of LPP:

For Decision Variables:

Let **x and y** be the number of cycle and scooter the manufacturer should produce in order to maximize the profit .

For Objective Function:

Total profit =$ (45x + 55y)

Let Z = 45x + 55y

Maximize Z = 45x + 55y

LPP Example :

For Constraints:

$6x + 4y \leq 120$ (Machine A Constraints)

$3x + 10y \leq 180$ (Machine B Constraints)

$x , y \geq 0$ (Non negativity Constraints)

Any Questions?

LPP Example :

Problem 3:

A manufacturer of furniture's makes two products - chairs and tables. Processing of these products is done on two machines A and B . A chair requires **2 hours** on machine A and **6 hours** on machine B. A table requires **5 hours** on machine A and requires **no time** in machine B. There are **16 hours** of time per day available on machine A and **30 hours** on machine B. Profit gained by the manufacturer from a chair and a table is **$ 1 and $ 5** respectively. Formulate the mathematical model to **maximize the profit**.

LPP Example :

Problem 4:

Food X contains **5 units** of vitamin A and **6 units** of vitamin B per gram and cost **20 paisa/gram**. Food Y contains **8 units** of vitamin A and **10 units** of vitamin B per gram and costs **30 paisa/gram**. The minimum daily requirements of A and B are **80 units and 100 units** respectively. Formulate the above as a L.P. problem to **minimize the cost**.

LPP Example :

Problem 5:

Suppose that **8, 12 and 9** units of protein, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains **2, 6 and 1 units** of protein, carbohydrate and fat respectively per kg. , food B contains **1, 1 and 3** units respectively per kg. and food C contains **2 ,3 and 2 units** respectively per kg. If A costs **$ 85 per kg** , B costs **$ 40 per kg** and C costs **$30 per kg**; how many kgs of each should he buy per week to minimize his cost and still meet his minimum requirements? Formulate the above problem as a linear programming problem.

LPP Using Graphical Method :

Steps for graphical method :

1. Formulate the mathematical model of the given LPP.
2. Change the inequalities involved in the constraints to equality.
3. Plot each equation on the graph paper finding at least two points and also do the origin test.
4. Find the corners of feasible region or solution area and get the solution to the given LPP.

Example – Graphical Solution…

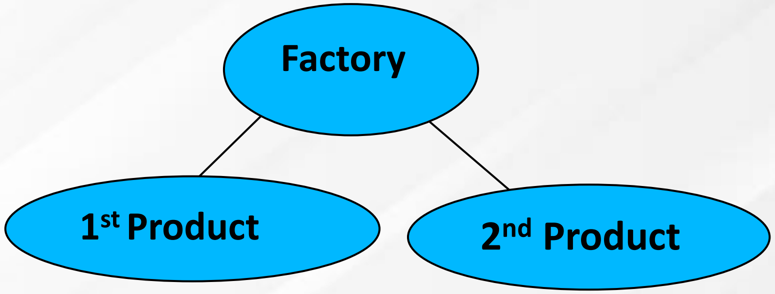
Problem :

A factory uses three different resources for the manufacture of two different products, **20 units of resource A**, **12 units of resource B** and **16 units of C** being available. One unit of first product requires **2, 2 and 4 units** of the respective resources and 1 unit of the second product requires **4, 2 and 0 units** of the respective resources. It is known that the first product gives a profit of **$ 2** per unit and the second **$3**. Formulate the linear programming problem to find the number of units of each product that should be manufactured for **maximizing the profit**.

Solve it graphically.

Example – Graphical Solution…

Problem analysis:



Resource			Available
A	2	4	20
B	2	2	12
C	4	0	16
Profit	\$2	\$3	

Example – Graphical Solution…

Mathematical formulation :

- **For Decision variables:**

  Let **x and y** be the numbers of units of products 1st and 2nd should be produced in order to maximize the profit and meet the requirements.

- **For Objective function:**

  Total profit = $2x + 3y$

  Let $Z = 2x + 3y$

  Maximize $Z = 2x + 3y$

- **For Constraints:**

  $2x + 4y \leq 20$ (Resource A constraints)

  $2x + 2y \leq 12$ (Resource B constraints)

  $4x + 0y \leq 16$ (Resource C constraints)

  $x, y \geq 0$ (Nonnegative constraints)

Example – Graphical Solution…

Graphical Solution :

<u>For Graphical solution:</u>

Changing the above inequalities to equalities, we get,

$2x + 4y = 20$ …… i

$2x + 2y = 12$ ….. ii

$4x + 0y = 16$ …..iii

For graphing equation (i)

$2x + 4y = 20$

Put $x = 0$, $y = 5$

Put $y = 0$, $x = 10$

Equation (i) passes through (0 , 5) and (10 , 0)

Origin test :

Put $x = 0$ and $y = 0$ in the $2x + 4y \leq 20$

$0 \leq 20$ which is **true**, so the origin lies inside equation (i)

Example – Graphical Solution…

For graphing equation (ii)

$2x + 2y = 12$

Put $x = 0$, $y = 6$

Put $y = 0$, $x = 6$

Equation (ii) passes through (0 , 6) and (6 , 0)

Origin test:

Put $x = 0$ and $y = 0$ in the $2x + 2y \leq 12$

$0 \leq 12$ which is **true**, so the origin lies inside equation (ii)

For graphing equation (iii)

$4x + 0y = 16$

This is a straight line perpendicular to x axis i.e $x = 4$

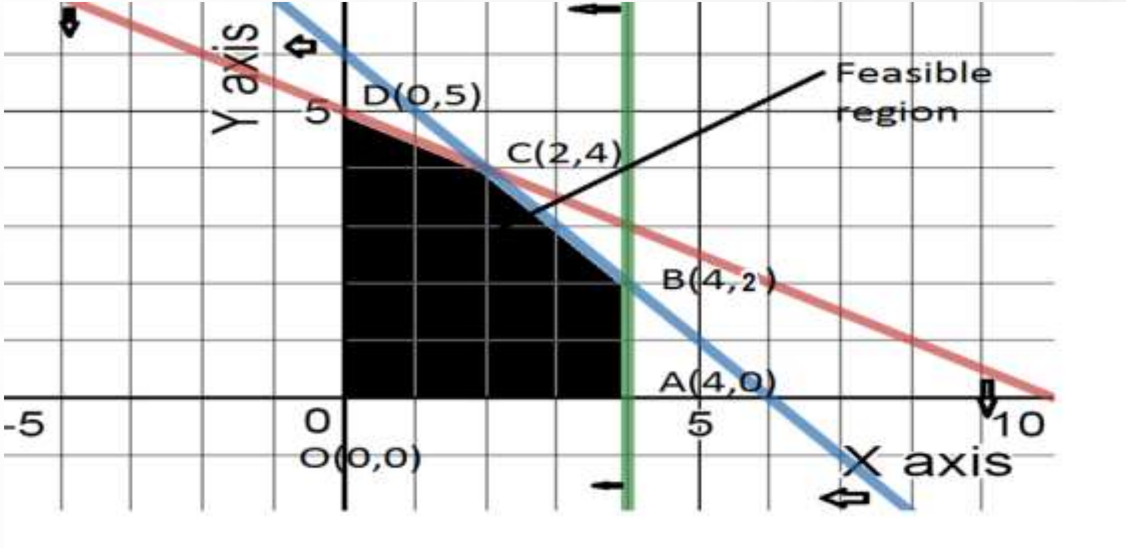
Origin test :

Put $x = 0$ and $y = 0$ in the $4x + 0y \leq 16$

$0 \leq 16$ which is **true**, so the origin lies inside equation (iii)

Example – Graphical Solution…

- **Graphical representation of equations:**



Example – Graphical Solution…

- **From Graph OABCD Is the feasible region:**

Vertex	x	y	Z = 2x + 3y
O(0,0)	0	0	0
A(4,0)	4	0	8
B(4,2)	4	2	14
C(2,4)	2	4	16 Maximum value
D(0,5)	0	5	15

Conclusion:

Hence the maximum value of Z will be 16 when x = 2 and y = 4, therefore the factory should produce 2 units of 1st product and 4 units of 2nd product in order to get the **maximum profit of $ 16.**

Summary:

- **Linear Programming problem**
- **Decision variables**
- **Objective functions**
- **Constraints**
- **Graphical solution for LPP**

What to Expect: Week 23 Tutorials

- **Review** and **practice** Linear Programming Problems through in-class assignments to acquire them.
- **Practice problems** to know how concept of LPP can be useful in solving various mathematical problems.

Thank you