# Linear Programming Using Simplex Method

Logic & Problem Solving

**Lecture Week 24**

**Linear Programming**

# Agenda:

- **Week 24 lecture coverage**
  - Introduction to LPP
  - Mathematical Formulation of LPP
  - Solving LPP using simplex method

# Amazing Magic Tricks …

# LPP Using Graphical Method :

**Steps for graphical method :**

1. **Formulate** the **mathematical model** of the given LPP.
2. Change the **inequalities** involved in the constraints to **equality**.
3. **Plot** each equation on the **graph** paper finding at least two points and also do the **origin test**.
4. Find the corners of **feasible region** or solution area and get the solution to the given LPP.

# LPP Using Simplex Method :

Graphical method can solve LPP problems involving only **two decision variables** .

We must use **Simplex method** to solve LPP problems involving **Three or more decision variables** .

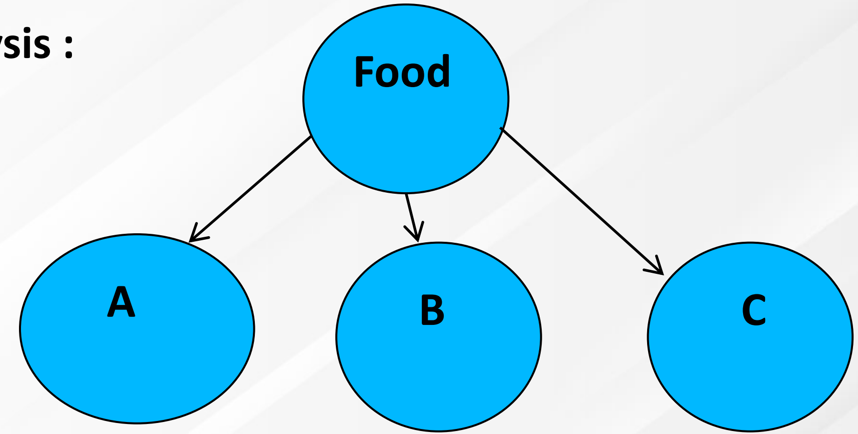
# LPP Using Simplex Method :

**For Example :**

Suppose that **8, 12 and 9 units** of protein, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains **2, 6 and 1** units of protein, carbohydrate and fat respectively per kg. , food B contains **1, 1 and 3** units respectively per kg. and food C contains **2 ,3 and 2** units respectively per kg. If A costs **$ 85 per kg** , B costs **$ 40 per kg** and C costs **$30 per kg**; how many kg of each should he buy per week to minimize his cost and still meet his **minimum** requirements? Formulate the above problem as a linear programming problem.

# LPP Using Simplex Method :

**Problem Analysis :**




Here there will be 3 decision variables x , y and z to represent quantity of Food A , Food B and Food C to minimize the cost and meet the protein requirements .This problem can't be solved graphically hence we have to use **Simplex method** .

# LPP Using Simplex Method :

## Variables used for Simplex Method

1. **Slack Variables :** This variable is used to change the $\leq$ inequality constraints to equality.
2. **Surplus variables:** This variable is used to change the $\geq$ inequality constraints to equality.
3. **Artificial variables:** This variable is used to change the $\geq$ and $=$ inequality constraints to equality.

# LPP Using Simplex Method :

❖ **<u>FOR CONSTRAINTS WITH ≤ SIGN:</u>**

Use **Slack variable** to change it into equality.

❖ **<u>FOR CONSTRAINTS WITH ≥ SIGN</u>**:

Use **surplus variable with negative sign** and an **artificial variable** to change it into equality.

❖ **<u>FOR CONSTRAINTS WITH = SIGN:</u>**

Use an **artificial variable** to change it into equality .

# LPP Using Simplex Method :

**For Example :**

**Maximize** $Z = 2x + 3y + 2z$

**Subjected to the constraints,**

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

How many **slack ,surplus and artificial variables** are needed to solve the above LPP?

# LPP Using Simplex Method :

**Maximize** $Z = 2x + 3y + 2z$

**Subjected to the constraints,**

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

How many slack ,surplus and artificial variables are needed to solve the above LPP?

**Answer:**

We need **2 slack variables** S1 and S2 for two ≤ inequalities . We also need **1 surplus variable** S3 and **1 artificial variable** A1 for ≥ inequality.

# LPP Using Simplex Method :

## Question:

**Maximize** Z = 2x + 3y + 2z

**Subjected to the constraints,**

$x + y + z \leq 60$

$2x + 3y + 7z \leq 150$

$3x + 6y + 4z \geq 200$

$x, y, z \geq 0$

**Solution,**

Let $S_1$ and $S_2$ be the slack variables , let $S_3$ be the surplus variable and let $A_1$ be the artificial variable.

$$\mathbf{x + y + z + S_1 = 60}$$

$$\mathbf{2x + 3y + 7z + S_2 = 150}$$

$$\mathbf{3x + 6y + 4z - S_3 + A_1 = 200}$$

$$\mathbf{x, y, z, S_1, S_2, S_3, A_1 \geq 0}$$

# LPP Using Simplex Method :(Try it….)

**Question :**

**Maximize** $Z = 4x + 7y + 2z$

**Subjected to the constraints,**

$$x + y + z \leq 60$$

$$2x + 3y + 7z \geq 150$$

$$3x + 6y + 4z = 200$$

$$x, y, z \geq 0$$

Write the equation with all the variables required for simplex method .

# LPP Using Simplex Method :

**Question :**
**Maximize** $Z = 4x + 7y + 2z$
**Subjected to the constraints,**

$$x + y + z \leq 60$$
$$2x + 3y + 7z \geq 150$$
$$3x + 6y + 4z = 200$$
$$x, y, z \geq 0$$

Write the equation with all the variables required for simplex method .

**Solution,**
Let S1 be the slack variable , S2 be the surplus variable and let A1 and A2 be the artificial variables.

$$x + y + z + S1 = 60$$
$$2x + 3y + 7z - S2 + A1 = 150$$
$$3x + 6y + 4z + A2 = 200$$
$$x, y, z, S1, S2, A1, A2 \geq 0$$

# LPP Using Simplex Method : (Try it…)

**Question :**

**Minimize** $Z = 10x + 7y - 6z$

**Subjected to the constraints,**

$4x - y + 3z \geq 60$

$6x + 3y - 7z = 150$

$3x + 6y + 4z = 200$

$x, y, z \geq 0$

Write the equation with all the variables required for simplex method .

# LPP Using Simplex Method :

**Question :**

**Minimize** $Z = 10x + 7y - 6z$

**Subjected to the constraints,**

$4x - y + 3z \geq 60$

$6x + 3y - 7z = 150$

$3x + 6y + 4z = 200$

$x, y, z \geq 0$

Write the equation with all the variables required for simplex method .

**Solution,**

Let S1 be the surplus variable and A1, A2 and A3 be the artificial variables.

$\mathbf{4x - y + 3z - S1 + A1 = 60}$

$\mathbf{6x + 3y - 7z + A2 = 150}$

$\mathbf{3x + 6y + 4z + A3 = 200}$

$\mathbf{x, y, z, S1, A1, A2, A3 \geq 0}$

# LPP Using Simplex Method :

## Standard Equation for simplex table.

After defining all the variables and making the inequalities to equalities we need to write the **standard equations** required for the simplex table .

The standard equations **consists of all the variables** .

# LPP Using Simplex Method :

**Write the standard equation for the simplex table to solve the following LPP .**

**Maximize** $Z' = 2x + 3y + 2z$

**Subjected to the constraints,**

$$x + y + z \leq 60$$
$$2x + 3y + 7z \leq 150$$
$$3x + 6y + 4z \geq 200$$
$$x, y, z \geq 0$$

**Solution,**

Let $S_1$ and $S_2$ be the slack variables and let $S_3$ be surplus variable.

Let $A_1$ be the artificial variables.

Now,

$$x + y + z + S_1 = 60$$
$$2x + 3y + 7z + S_2 = 150$$
$$3x + 6y + 4z - S_3 + A_1 = 200$$

# LPP Using Simplex Method :

**Maximize** $Z' = 2x + 3y + 2z$

$x + y + z + S_1 = 60$

$2x + 3y + 7z + S_2 = 150$

$3x + 6y + 4z - S_3 + A_1 = 200$

## Standard equation for simplex table :

$1Z' - 2x - 3y - 2z + 0S_1 + 0S_2 + 0S_3 + \mathbf{10\ A_1} = 0$

$0Z' + 1x + 1y + 1z + 1S_1 + 0S_2 + 0S_3 + 0A_1 = 60$

$0Z' + 2x + 3y + 7z + 0S_1 + 1S_2 + 0S_3 + 0A_1 = 150$

$0Z' + 3x + 6y + 4z + 0S_1 + 0S_2 - 1S_3 + 1A_1 = 200$

$x, y, z, S_1, S_2, S_3 \ \& \ A_1 \geq 0$

**Note that all 4 equations contains all the variables.**

# LPP Using Simplex Method :

**Standard Equation :**

$1Z' - 2x - 3y - 2z + 0S_1 + 0S_2 + 0S_3 + \mathbf{10\ A_1} = 0$

$0Z' + 1x + 1y + 1z + 1S_1 + 0S_2 + 0S_3 + 0A_1 = 60$

$0Z' + 2x + 3y + 7z + 0S_1 + 1S_2 + 0S_3 + 0A_1 = 150$

$0Z' + 3x + 6y + 4z + 0S_1 + 0S_2 - 1S_3 + 1A_1 = 200$

$x, y, z, S_1, S_2, S_3 \;\&\; A_1 \geq 0$

The first equation is created from objective function by shifting all to left-hand side and making it equal to zero. Take note that in the first equation the coefficient of artificial variable is **+10** because for maximization problem we will always take Positive sign for artificial variable and the value 10 is assigned by seeing the coefficient of x, y and z in objective which are all single figure digit. If any one of the coefficient was double digit, then we must have to take 100 for the coefficient of artificial variable.

# LPP Using Simplex Method :

**Write the standard equation for the simplex table to solve the following LPP .**

Minimize $Z' = 5x + 12y + 2z$

Subjected to the constraints,

$$7x + 3y - z \leq 220$$

$$12x - 6y + 8z \geq 1500$$

$$8x + 16y - 4z = 2000$$

$$x, y, z \geq 0$$

# LPP Using Simplex Method :

**Solution,**

Let $S_1$ and $S_2$ be the slack variable and surplus variable respectively and let $A_1$ and $A_2$ be the artificial variables.

Minimize $Z' = 5x + 12y + 2z$

$7x + 3y - z + S_1 = 220$

$12x - 6y + 8z - S_2 + A_1 = 1500$

$8x + 16y - 4z + A_2 = 2000$

$x, y, z, S_1, S_2, A_1 \& A_2 \geq 0$

**Standard equation for simplex table:**

$1Z' - 5x - 12y - 2z + 0S_1 + 0S_2 \mathbf{- 100\, A_1 - 100\, A_2} = 0$

$0Z' + 7x + 3y - 1z + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 220$

$0Z' + 12x - 6y + 8z + 0S_1 - 1S_2 + 1A_1 + 0A_2 = 1500$

$0Z' + 8x + 16y - 4z + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 2000$

$x, y, z, S_1, S_2, A_1 \& A_2 \geq 0$

**Note that all 4 equations contains all the variables.**

# LPP Using Simplex Method :

**Standard equation for simplex table:**

$$1Z' - 5x - 12y - 2z + 0S_1 + 0S_2 - \mathbf{100\,A_1} - \mathbf{100\,A_2} = 0$$
$$0Z' + 7x + 3y - 1z + 1S_1 + 0S_2 + 0A_1 + 0A_2 = 220$$
$$0Z' + 12x - 6y + 8z + 0S_1 - 1S_2 + 1A_1 + 0A_2 = 1500$$
$$0Z' + 8x + 16y - 4z + 0S_1 + 0S_2 + 0A_1 + 1A_2 = 2000$$
$$x, y, z, S_1, S_2, A_1 \,\&\, A_2 \geq 0$$

The first equation is created from objective function by shifting all to left-hand side and making it equal to zero .Take note that in the first equation the coefficient of artificial variables $A_1$ & $A_2$ are **-100** because for minimization problem we will always take negative sign for artificial variable and the value 100 is assigned by seeing the coefficient of x , y and z in objective function in which y has double digit coefficient. If any one of the coefficient was triple digit, then we must have to take 1000 for the coefficient of artificial variable.

# LPP Using Simplex Method :

**Write the standard equation for each of the following LPP defining all the necessary variables .**

a. Maximize $Z = 3x_1 + 4x_2$
   Subjected to ,
   $x_1 + x_2 \leq 20$
   $2x_1 + 3x_2 \leq 50$
   $x_1, x_2 \geq 0$

b. Maximize $Z = 2x_1 + 3x_2$
   Subjected to ,
   $x_1 + 2x_2 \geq 50$
   $10x_1 + 20x_2 \leq 175$
   $x_1, x_2 \geq 0$

c. Minimize $Z = 3x_1 + 4x_2 + 5x_3$
   Subjected to,
   $x_1 + x_2 + x_3 \geq 30$
   $10x_1 + 15x_2 + 20x_3 \leq 600$
   $x_1, x_2, x_3 \geq 0$

# LPP Using Simplex Method :

## Question d:

Suppose that 8, 12 and 9 units of protein, carbohydrates and fats respectively are the minimum weekly requirements for a person. Food A contains 2, 6 and 1 units of protein, carbohydrate and fat respectively per kg. , food B contains 1, 1 and 3 units respectively per kg. and food C contains 2 ,3 and 2 units respectively per kg. If A costs $ 85 per kg , B costs $ 40 per kg and C costs $30 per kg; how many kg of each should he buy per week to minimize his cost and still meet his minimum requirements? **Write down the standard equations needed to solve the given LPP.**

# LPP Using Simplex Method (Complete Solution) :

**Question :**

Solve the following LPP using simplex method.

**Maximize** $Z = 90x + 70y$

**Subjected to ,**

$125x + 100y \leq 25000$

$20x + 30y \leq 6000$

$x, y \geq 0$

# LPP Using Simplex Method :

**Question :**

Solve the following LPP using simplex method.

Maximize $Z = 90x + 70y$

Subjected to ,

$125x + 100y \leq 25000$

$20x + 30y \leq 6000$

$x, y \geq 0$

**Solution ,**

Let $S_1$ and $S_2$ be the slack variables .

Now ,

$125x + 100y + S_1 = 25000$

$20x + 30y + S_2 = 6000$

$x, y, S_1, S_2 \geq 0$

# LPP Using Simplex Method :

**Standard equation for simplex table:**

$$1Z - 90x - 70y + 0S_1 + 0S_2 = 0$$
$$0Z + 125x + 100y + 1S_1 + 0S_2 = 25000$$
$$0Z + 20x + 30y + 0S_1 + 1S_2 = 6000$$
$$x, y, S_1, S_2 \geq 0$$

**Simplex table 1 :**

Highest Negative (Key column)

| Row | Z | X | Y | $S_1$ | $S_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | -90 | -70 | 0 | 0 | 0 | _ |
| $R_1$ | 0 | 125 | 100 | 1 | 0 | 25000 | 25000/125 = 200 |
| $R_2$ | 0 | 20 | 30 | 0 | 1 | 6000 | 6000/20 =300 |

Minimum positive ratio ( Key Row)

Key Element

# LPP Using Simplex Method :

| Row | Z | X | Y | $S_1$ | $S_2$ | Constant |
|---|---|---|---|---|---|---|
| $R_0$ | 1 | -90 | -70 | 0 | 0 | 0 |
| $R_1$ | 0 | 125 | 100 | 1 | 0 | 25000 |
| $R_2$ | 0 | 20 | 30 | 0 | 1 | 6000 |

Here X is the **key column** , $R_1$ is the **key row** and 125 is the **key element**.

Now, we must update key row ($R_1$) first using the formula,

**New $R_1$ = Old $R_1$/ Key elements = Old $R_1$ / 125**

0 , 1 ,4/5 ,1/125 ,0,200

**New $R_0$ = Old R0 – (-90) x New $R_1$**

**New $R_2$ = Old $R_2$ – 20 x New $R_1$**

**We need to update rows until all the elements of $R_0$ are $\geq 0$ for Maximization problems.**

# LPP Using Simplex Method :

Updating $R_0$ and $R_2$ using the formula :

**New $R_0$ = Old $R_0$ – (-90) x New $R_1$**

**New $R_2$ = Old R2 – 20 x New $R_1$**

| Old $R_0$ - | (-90) x New $R_1$ | New $R_0$ | Old $R_2$ - | 20 x New $R_1$ | New $R_2$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| -90 | -90 | 0 | 20 | 20 | 0 |
| -70 | -72 | 2 | 30 | 16 | 14 |
| 0 | -18/25 | 18/25 | 0 | 4/25 | -4/25 |
| 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | -18000 | 18000 | 6000 | 4000 | 2000 |

# LPP Using Simplex Method :

## Simplex Table 2:

| Row | Z | X | Y | $S_1$ | $S_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|
| $R_0$ | **1** | **0** | **2** | **18/25** | **0** | **18000** | |
| $R_1$ | 0 | 1 | 4/5 | 1/125 | 0 | 200 | |
| $R_2$ | 0 | 0 | 14 | -4/25 | 1 | 2000 | |

**Here all the coefficient of variables in $R_0$ row are $\geq 0$, so we reached the optimum solution .**
**Hence ,**
**Maximum Z= 18000**
**X= 200**
**Y = 0**

# LPP Using Simplex Method (Another Problem) :

**Question :**

Solve the following LPP using simplex method.

Minimize $Z = 4x + 6y$

Subjected to ,

$$x + 2y \geq 80$$
$$3x + y \geq 75$$
$$x, y \geq 0$$

**Solution ,**

**Let $S_1$ and $S_2$ be the surplus variables and let $A_1$ and $A_2$ be the artificial variables.**

**Now ,**

$x + 2y - S_1 + A_1 = 80$

$3x + y - S_2 + A_2 = 75$

$S_1, S_2, A_1$ & $A_2 \geq 0$

# LPP Using Simplex Method :

**Standard equation for simplex table:**

$1Z - 4x - 6y + 0S_1 + 0S_2 - 10A_1 - 10A_2 = 0$

$0Z + 1x + 2y - 1S_1 + 0S_2 + 1A_1 + 0A_2 = 80$

$0Z + 3x + 1y + 0S_1 - 1S_2 + 0A_1 + 1A_2 = 75$

$S_1, S_2, A_1 \ \& \ A_2 \geq 0$

**Simplex Table 1 :**

| Row | Z | x | y | $S_1$ | $S_2$ | $A_1$ | $A_2$ | Constant |
|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | -4 | -6 | 0 | 0 | -10 | -10 | 0 |
| $R_1$ | 0 | 1 | 2 | -1 | 0 | 1 | 0 | 80 |
| $R_2$ | 0 | 3 | 1 | 0 | -1 | 0 | 1 | 75 |

# LPP Using Simplex Method :

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant |
|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | -4 | -6 | 0 | 0 | **-10** | **-10** | 0 |
| $R_1$ | 0 | 1 | 2 | -1 | 0 | 1 | 0 | 80 |
| $R_2$ | 0 | 3 | 1 | 0 | -1 | 0 | 1 | 75 |

**For Identity Matrix,**

$$\textbf{New } R_0 = \textbf{Old } R_0 + 10\,(R_1+R_2)$$

# LPP Using Simplex Method :

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant |
|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | -4 | -6 | 0 | 0 | -10 | -10 | 0 |
| $R_1$ | 0 | 1 | 2 | -1 | 0 | 1 | 0 | 80 |
| $R_2$ | 0 | 3 | 1 | 0 | -1 | 0 | 1 | 75 |

**New $R_0$ = Old $R_0$ + 10 $(R_1+R_2)$**

| Old $R_0$ + | 10 $(R_1 + R_2)$ | New $R_0$ |
|---|---|---|
| 1 | 0 | 1 |
| -4 | 40 | 36 |
| -6 | 30 | 24 |
| 0 | -10 | -10 |
| 0 | -10 | -10 |
| -10 | 10 | 0 |
| -10 | 10 | 0 |
| 0 | 1550 | 1550 |

# LPP Using Simplex Method :

Key Column (highest Positive)

## Simplex Table 2 :

| Row | Z | x | y | $S_1$ | $S_2$ | $A_1$ | $A_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | 36 | 24 | -10 | -10 | 0 | 0 | 1550 | - |
| $R_1$ | 0 | 1 | 2 | -1 | 0 | 1 | 0 | 80 | 80/1 = 80 |
| $R_2$ | 0 | 3 | 1 | 0 | -1 | 0 | 1 | 75 | 75/3 = 25 |

Key Element

Key Row (Minimum positive ratio)

Here X is the **key column** ,$R_2$ is the **key row** and 3 is the **key element**.

**New $R_2$ = Old $R_2$ / Key element = Old $R_2$ /3**

0 , 1 , 1/3 , 0 , -1/3 , 0 , 1/3 ,25

**New $R_0$ = Old $R_0$ – 36 x New $R_2$**

**New $R_1$ = Old $R_1$ – 1 x New $R_2$**

**We Need to update Rows until all the elements of $R_0 \leq 0$ .**

# LPP Using Simplex Method :

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant |
|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | 36 | 24 | -10 | -10 | 0 | 0 | 1550 |
| $R_1$ | 0 | 1 | 2 | -1 | 0 | 1 | 0 | 80 |
| $R_2$ | 0 | 3 | 1 | 0 | -1 | 0 | 1 | 75 |

**New $R_0$ = Old $R_0$ – 36 x New $R_2$ , New $R_1$ = Old $R_1$ – 1 x New $R_2$**

| Old $R_0$ - | 36 x New $R_2$ | New $R_0$ | Old $R_1$ - | 1 x New $R_2$ | New $R_1$ |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 36 | 36 | 0 | 1 | 1 | 0 |
| 24 | 12 | 12 | 2 | 1/3 | 5/3 |
| -10 | 0 | -10 | -1 | 0 | -1 |
| -10 | -12 | 2 | 0 | -1/3 | 1/3 |
| 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 12 | -12 | 0 | 1/3 | -1/3 |
| 1550 | 900 | 650 | 80 | 25 | 55 |

# LPP Using Simplex Method :

**Simplex Table 3 :**

Key Column

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | 0 | 12 | -10 | 2 | 0 | -12 | 650 | - |
| $R_1$ | 0 | 0 | 5/3 | -1 | 1/3 | 1 | 1/3 | 55 | 55/5*3 =33 |
| $R_2$ | 0 | 1 | 1/3 | 0 | -1/3 | 0 | 1/3 | 25 | 25/1*3 =75 |

Key Row

Key Element

Here Y is the **key column** , $R_1$ is the **key row** and 5/3 is the **key element**

Updating Key row using the formula ,

**New$R_1$ = Old $R_1$ / Key elements = Old $R_1$/(5/3)**

0 , 0 , 1 , -3/5 , 1/5 , 3/5 ,1/5 ,33

**New $R_0$ = Old R0 – 12 x New $R_1$**

**New $R_2$ = Old $R_2$ – 1/3 x New $R_1$**

# LPP Using Simplex Method :

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | 0 | 12 | -10 | 2 | 0 | -12 | 650 | - |
| $R_1$ | 0 | 0 | 5/3 | -1 | 1/3 | 1 | 1/3 | 55 | 55/5*3 =33 |
| $R_2$ | 0 | 1 | 1/3 | 0 | -1/3 | 0 | 1/3 | 25 | 25/1*3 =75 |

**New $R_0$ = Old $R_0$ – 12 x New$R_1$**   **New $R_2$ = Old $R_2$ – 1/3 x New$R_1$**

| Old R0 - | 12 x NewR1 | New R0 | Old R2 - | 1/3 x New R1 | New R2 |
|---|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 0 | 1 |
| 12 | 12 | 0 | 1/3 | 1/3 | 0 |
| -10 | -36/5 | -14/5 | 0 | -1/5 | 1/5 |
| 2 | 12/5 | -2/5 | -1/3 | 1/15 | -2/5 |
| 0 | 36/5 | -36/5 | 0 | 1/5 | -1/5 |
| -12 | 12/5 | -72/5 | 1/3 | 1/15 | 4/15 |
| 650 | 396 | 254 | 25 | 11 | 14 |

# LPP Using Simplex Method :

**Simplex Table 4 :**

| Row | Z | x | y | $S_1$ | $S_2$ | $A_3$ | $A_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | 0 | 0 | -14/5 | -2/5 | -36/5 | -72/5 | 254 | |
| $R_1$ | 0 | 0 | 1 | -3/5 | 1/5 | 3/5 | 1/5 | 33 | |
| $R_2$ | 0 | 1 | 0 | 1/5 | -2/5 | -1/5 | 4/15 | 14 | |

Here all the coefficient of variables in $\mathbf{R_0 \leq 0}$ ,so we reached the optimum solution .

**Hence,**
**Minimum Z = 254**
**x = 14**
**y = 33**

# LPP Using Simplex Method :

**Question : (Past Year Course Work Question)**

Martin and Son's company wants to manufacture a mixture containing three contents X, Y and Z. The cost of X, Y and Z are $5, $4 and $3 respectively. The company prepares the mixture to meet out the demand of the costumers in the following manner.

The quantity of X cannot be more than 200 kgs in the mixtures.

The quantity of Y used should be at least 300 kgs.

The content of Z cannot be more than 400 kgs.

Find the optimal combination of the three contents for a mixture of 1000 kgs, so that the total cost is minimum.

# LPP Using Simplex Method :

## Question :

Martin and Son's company wants to manufacture a mixture containing three contents X, Y and Z. The cost of X, Y and Z are \$5, \$4 and \$3 respectively. The company prepares the mixture to meet out the demand of the costumers in the following manner.

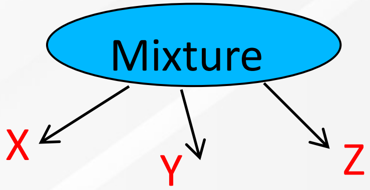
The quantity of X cannot be more than 200 kgs in the mixtures.

The quantity of Y used should be at least 300 kgs.

The content of Z cannot be more than 400 kgs.

Find the optimal combination of the three contents for a mixture of 1000 kgs, so that the total cost is minimum.

## Problem Analysis:




Cost : \$ 5 \$4 \$3

X cannot be more than 200 kg ( $X \leq 200$)

Y should be at least 300 kg ( $Y \geq 300$)

Z cannot be more that 400 kg ($Z \leq 400$)

Total mixture must be 1000 kg ($X + Y + Z = 1000$)

# LPP Using Simplex Method :

## Mathematical Formulation :

**For Decision Variables**

Let x kg ,y kg and z kg of mixture X,Y and Z should be purchased and mix together to form 1000 kg of mixture in order to minimize the cost.

**For Objective Function**

Total cost = $5x + 4y + 3z$

Let $C = 5x + 4y + 3z$

**Minimize $C = 5x + 4y + 3z$**

**For Constraints**

**$x \leq 200$** ( Mixture X constraint)

**$y \geq 300$** ( Mixture Y constraint)

**$z \leq 400$** ( Mixture Z constraint)

**$x + y + z = 1000$** ( Total Mixture constraint)

**$x, y, z \geq 0$**

# LPP Using Simplex Method :

**Simplex Method Solution :**

Let $S_1$ and $S_2$ be slack variables ,let $S_3$ be surplus variable and let $A_1$ and $A_2$ be artificial variables.

Now ,

$x + S_1 = 200$

$y - S_3 + A_1 = 300$

$z + S_2 = 400$

$x + y + z + A_2 = 1000$

**Standard Equation for Simplex Table**

$1C - 5x - 4y - 3z + 0S_1 + 0S_2 + 0S_3 - 10A_1 - 10A_2 = 0$

$0C + 1x + 0y + 0z + 1S_1 + 0S_2 + 0S_3 + 0A_1 + 0A_2 = 200$

$0C + 0x + 1y + 0z + 0S_1 + 0S_2 - 1S_3 + 1A_1 + 0A_2 = 300$

$0C + 0x + 0y + 1z + 0S_1 + 1S_2 + 0S_3 + 0A_1 + 0A_2 = 400$

$0C + 1x + 1y + 1z + 0S_1 + 0S_2 + 0S_3 + 0A_1 + 1A_2 = 1000$

# LPP Using Simplex Method :

## Simplex Table 1 :

| Row | C | x | y | z | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | Constant |
|---|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 1 | -5 | -4 | -3 | 0 | 0 | 0 | **-10** | **-10** | 0 |
| $R_1$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 200 |
| $R_2$ | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | 300 |
| $R_3$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 400 |
| $R_4$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1000 |

**For identity Matrix**

$$\textbf{New } R_0 = \textbf{Old } R_0 + 10\,(R_2 + R_4)$$

# LPP Using Simplex Method :

## For identity Matrix : New $R_0$ = Old $R_0$ + 10 ($R_2$ +$R_4$)

| Old R0 + | 10(R2 +R4) | New R0 |
|---|---|---|
| 1 | 0 | 0 |
| -5 | 10 | 5 |
| -4 | 20 | 16 |
| -3 | 10 | 7 |
| 0 | 0 | 0 |
| 0 | 0 | 0 |
| 0 | -10 | -10 |
| -10 | 10 | 0 |
| -10 | 10 | 0 |
| 0 | 13000 | 13000 |

# LPP Using Simplex Method :

## Simplex Table 2 :

Key Column

| Row | C | x | y | z | $S_1$ | $S_2$ | $S_3$ | $A_1$ | $A_2$ | Constant | Ratio |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $R_0$ | 0 | 5 | 16 | 7 | 0 | 0 | -10 | 0 | 0 | 13000 | - |
| $R_1$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 200 | $\infty$ |
| $R_2$ | 0 | 0 | 1 | 0 | 0 | 0 | -1 | 1 | 0 | 300 | 300 |
| $R_3$ | 0 | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 400 | $\infty$ |
| $R_4$ | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 1 | 1000 | 1000 |

Key Row

Key Element

**Here we need to update $R_2$ first using : New $R_2$ = Old $R_2$ / 1**

**0 , 0 , 1 , 0 , 0 , 0 , -1 ,1 , 0 , 300**

**New $R_0$ = Old $R_0$ – 16 x New $R_2$**

**New $R_1$ = Old $R_1$ – 0 x New $R_2$**

**New $R_3$ = Old $R_3$ – 0 x New $R_2$**

**New $R_4$ = Old $R_4$ – 1 x New $R_2$**

**We need to update rows until all the coefficient of variables in $R_0$ are $\leq 0$.**

# LPP Using Simplex Method :

## <u>Differences between Maximization and Minimization Problem</u>

| **Maximization Problem** | **Minimization Problem** |
|---|---|
| **Artificial variable** is written with +ve sign in standard equation . **$1Z - 2x - 3y - 4z + 0S_1 + 0S_2 + 10A_1 + 10A_2 = 0$** | **Artificial variable** is written with -ve sign in standard equation . **$1Z - 2x - 3y - 4z + 0S_1 + 0S_2 - 10A_1 - 10A_2 = 0$** |
| For **Key Column** we need to see **highest Negative** value in $R_0$ row . | For **Key Column** we need to see **highest positive** value in $R_0$ row . |
| For **Optimal Solution** all the coefficient of Variables in $R_0$ row should be $\geq$ **0** . | For **Optimal Solution** all the coefficient of Variables in $R_0$ row should be $\leq$ **0** . |

# *Thank you*