Break Even Analysis

Logic & Problem Solving

Lecture Week 25

Break Even Analysis

Agenda:

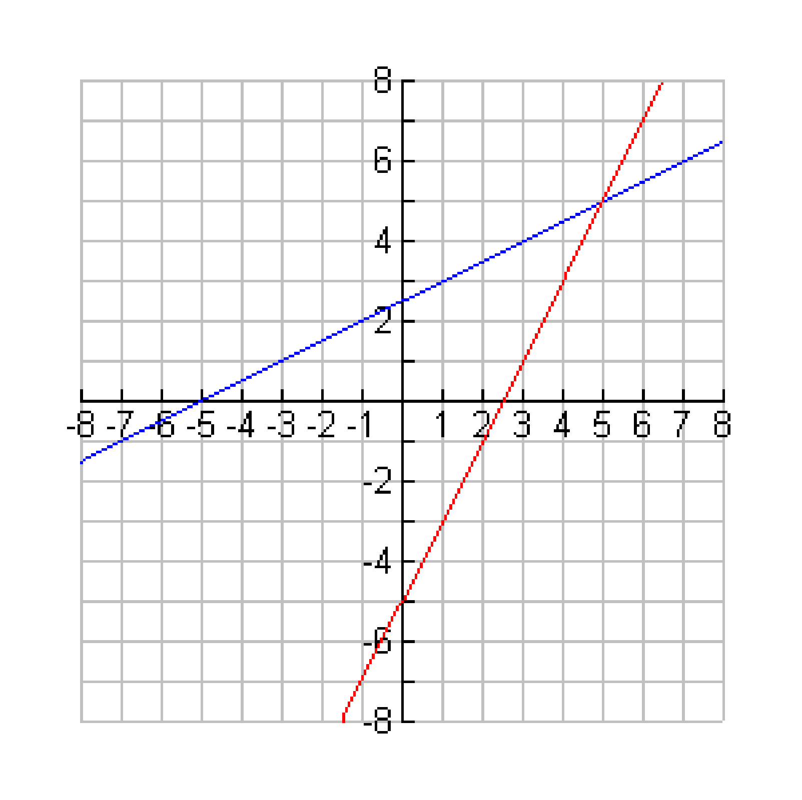
- **Week 25 lecture coverage**
- Break Even Analysis problems
- Solving Break Even Analysis problems with Graph
- Finding the output level for profit maximization

Nonparallel Lines

- We know that **parallel lines have equal slope**.
- For example, $y = 2x - 3$ and $y = 2x + 8$ are parallel because they have the same slope $m = 2$.

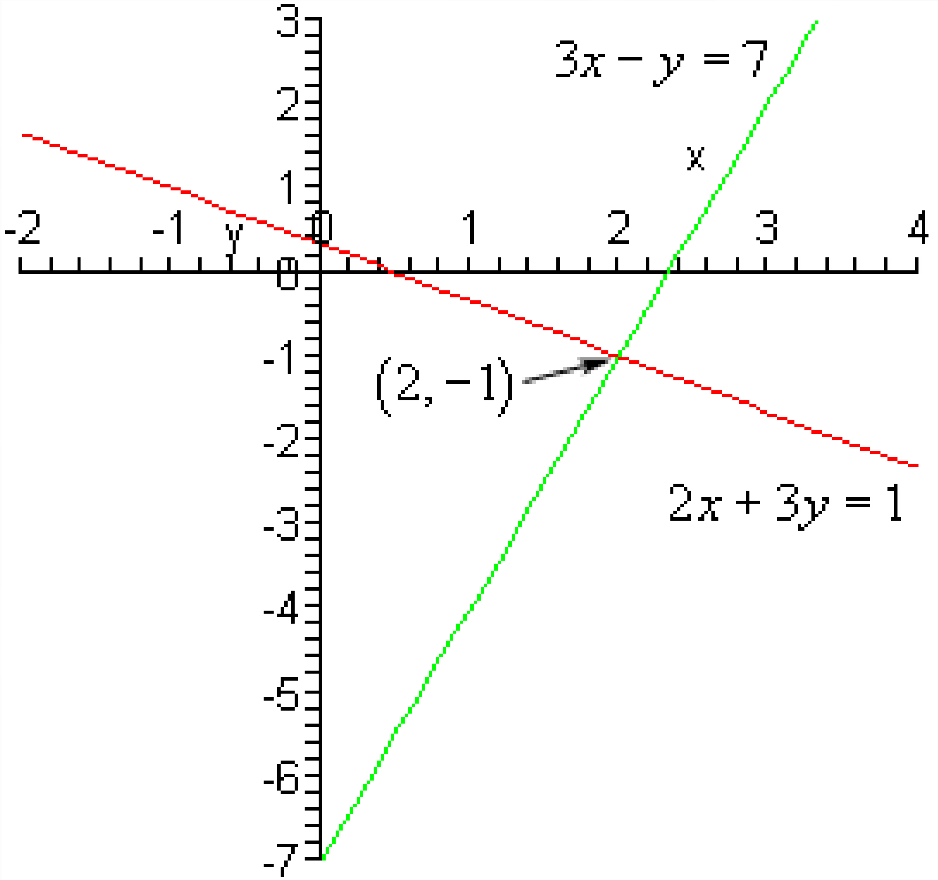
- Nonparallel lines have **different slopes**.

  ⇒ They will meet or intersect at a point.



Point of Intersection

- Consider line $3x - y = 7$ and line $2x + 3y = 1$



- We can draw the two lines and find the **point of intersection** graphically.
- The intersection point is (2, -1).
- Can we find the **point of intersection without drawing?**

Point of intersection

- We want to find where the line $3x - y = 7$ and line $2x + 3y = 1$ meet.
- Arrange into standard form:

Line 1: $3x - y = 7 \Rightarrow y = 3x - 7$

Line 2: $2x + 3y = 1 \Rightarrow y = (-2/3)x + 1/3$

- Where they meet, they have the same y and the same x.
- In other words, we find x and y that satisfy both equations. $\Rightarrow$ **solving simultaneous equations**

Point of Intersection

- Solve $y = 3x - 7$ (1)

  $y = (-2/3)x + 1/3$ (2)

- We can equate (1) to (2):

$$3x - 7 = (-2/3)x + 1/3$$

$$(11/3)x = 22/3$$

$$11x = 22$$

$$x = 2$$

- Substitute $x = 2$ into (1): $y = 3(2) - 7 = -1$
- **Intersection point is (2, -1)**

Examples

- **Example:** Find point of intersection between line $2x = 4 - y$ and line $y - 1 = x$ graphically.
- **Rearrange:**

  Line 1: $2x = 4 - y \Rightarrow y = (-1/2)x + 4$

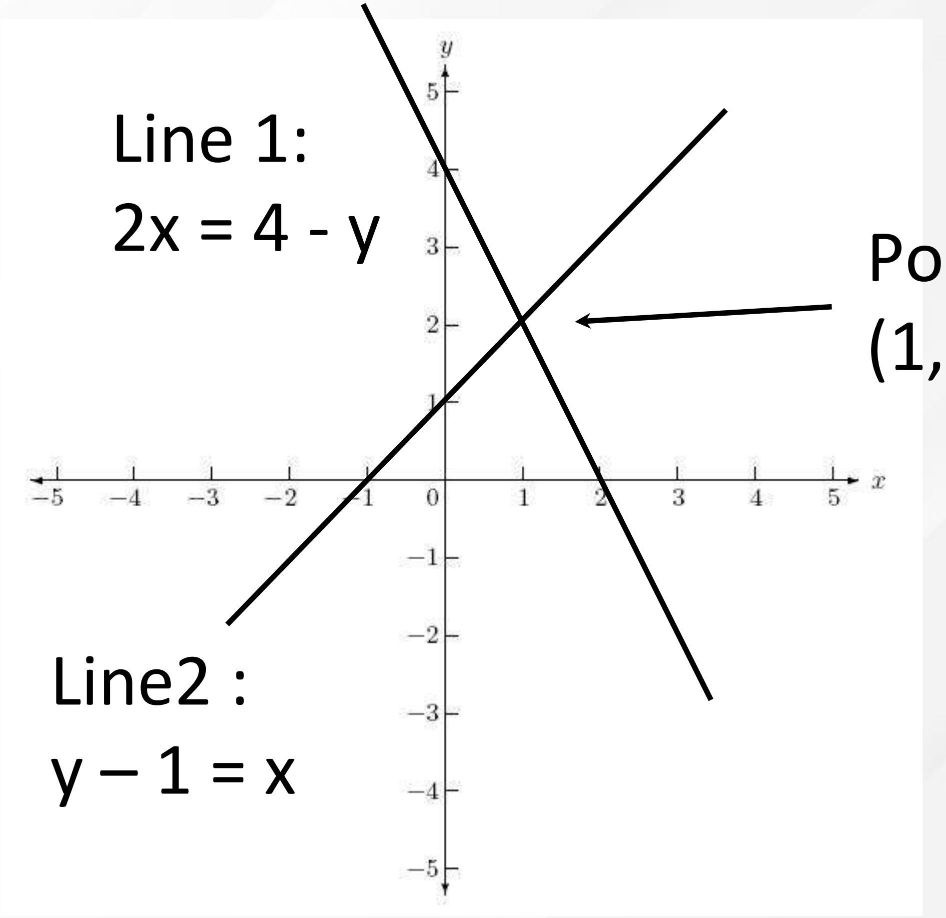
  Line 2: $y - 1 = x \Rightarrow y = x + 1$
- **Find intercepts:**

  Line 1 has intercepts at (0, 4) and (2, 0)

  Line 2 has intercepts at (0, 1) and (-1, 0)

Examples

- Draw line 1 and line 2 and find point of intersection



Point of intersection
(1, 2)

Examples

- **<u>Example:</u>** Find the point of intersection of two lines: $3x - 4y = 5$ and $2x + 3y = 1$.
- **Rearrange:**

$$3x - 4y = 5 \quad \Rightarrow \quad y = (3/4)x - 5/4 \qquad (1)$$

$$2x + 3y = 1 \quad \Rightarrow \quad y = (-2/3)x + 1/3 \qquad (2)$$

- Solve simultaneous equations by equating:

$$\begin{aligned} (3/4)x - 5/4 &= (-2/3)x + 1/3 \\ (17/12)x &= 19/12 \\ x &= 19/17 \end{aligned}$$

Examples

- Substitute $x = 19/17$ into (1)

$$\Rightarrow \quad y = (3/4)(19/17) - 5/4$$
$$= 57/68 - 5/4$$
$$= -28/68$$
$$= -7/17$$

- **Point of intersection is (19/17, -7/17)**

Examples

- **Example:** Find the point of intersection of two lines: $2x + 4y = 5$ and $x = 6 - 2y$.
- **Rearrange:**

$$2x + 4y = 5 \quad \Rightarrow \quad y = (-1/2)x + 5/4$$

$$x = 6 - 2y \quad \Rightarrow \quad y = (-1/2)x + 3$$

- Note that both lines have the **same slope**.

$\Rightarrow$ They must be parallel.

$\Rightarrow$ **No point of intersection.**

- **Always check the slopes of the lines.**

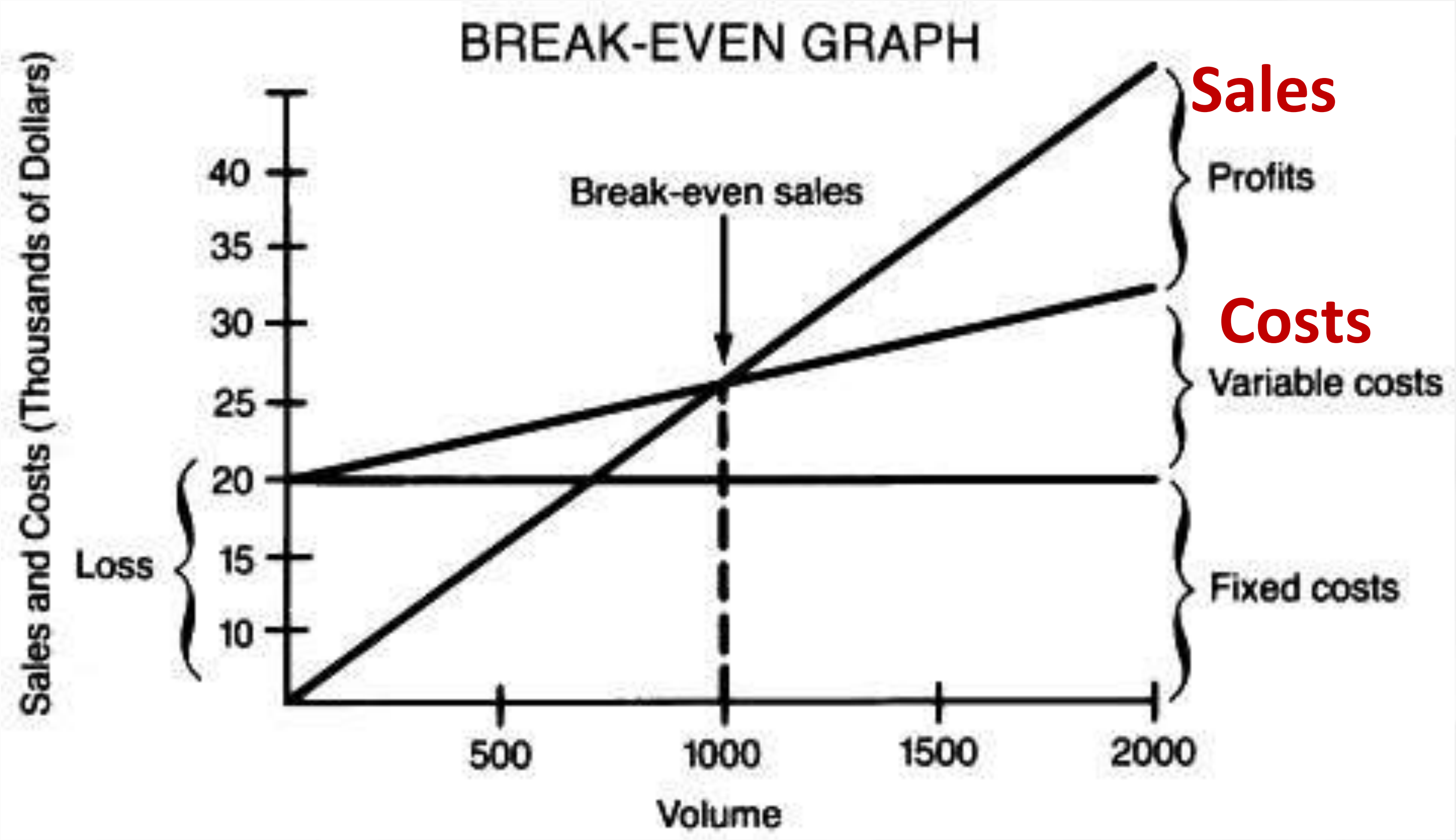
Introduction: Break-Even Point

Basics Terms :

- Cost Price
- Selling Price
- Fixed costs
- Variable costs
- Revenue
- Profit = Revenue – Total cost

Introduction: Break-Even Point

- Consider a break-even graph



Introduction: Break-Even Point (3)

- The sales increase as the volume of production increases.
- There are fixed costs (at y-intercept) and variable costs on top of the fixed costs.
- Graphs of sales and costs meet at a point called **break-even point.**
- Before this point, sales less than costs ⇒ loss
- After this point, sales greater than costs ⇒ profit
- To know when we are going to start making profit, we need find the break-even point. How?

⇒ **Find intersection point of the two graphs**

Break-Even Point Problem

- A company that manufactures bicycles has a fixed cost of \$100,000. It costs \$100 to produce each bicycle. The selling price per bike is \$300. Find the break-even point.
- Let x represent the number of bicycles sold.

Costs: $C = 100x + 100{,}000$

Revenue : $R = 300x$

At Break even **Cost = Revenue**

$$100x + 100{,}000 = 300x$$

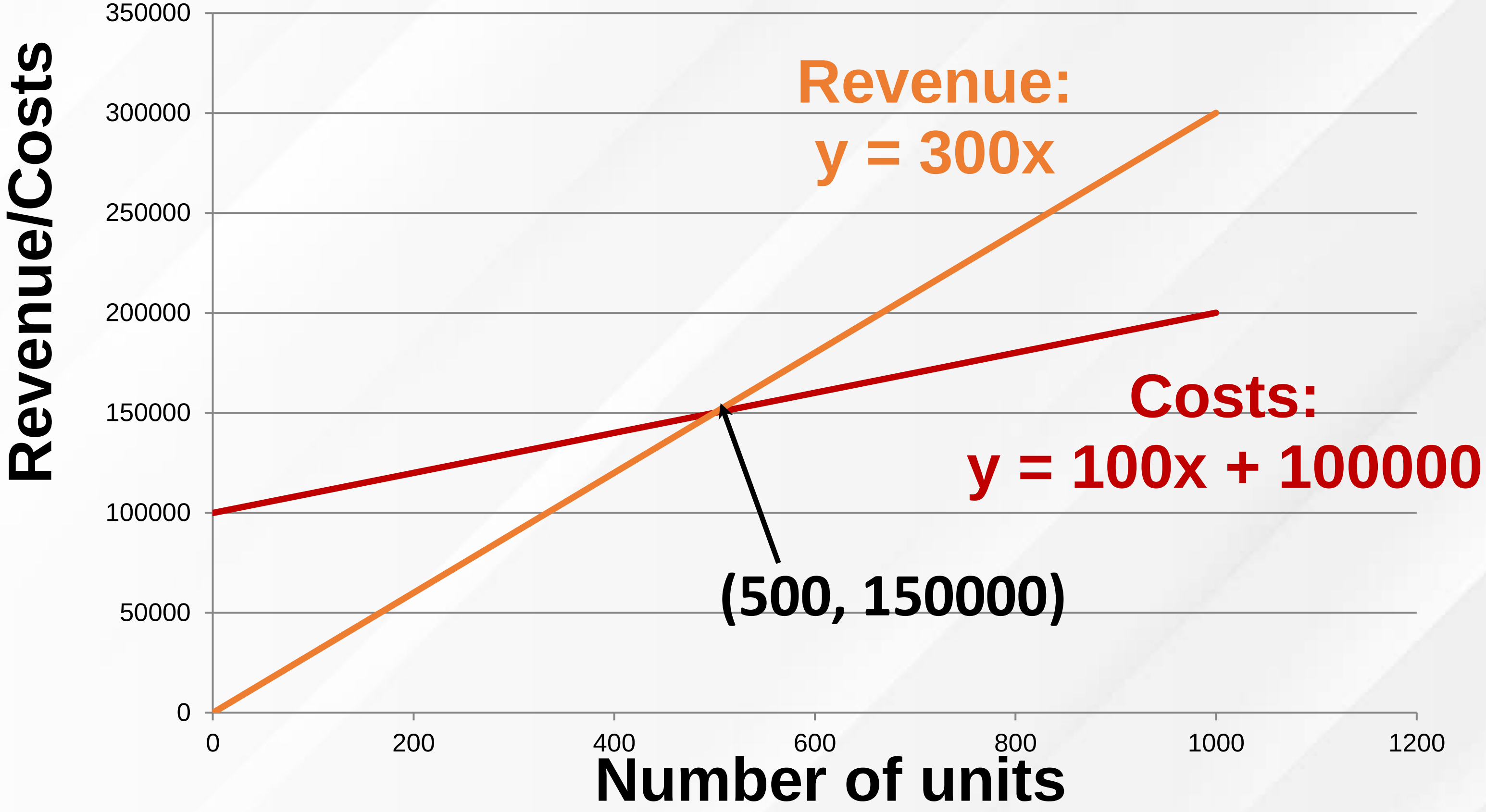
$$x = 500$$

Break-even point (500, 150000)

They must sell more than 500 bicycles to make profits

Break-Even Point Problem

- Illustrate by graphs



Maximum Profit…

- Applications of quadratic functions involve finding maximum values of function

$$y = f(x) = ax^2 + bx + c$$

- To have a maximum value of y, the function f(x) must describe downward parabola ($a < 0$).

- The maximum value of $y = c - b^2/4a$

  and it happens when $x = -b/2a$

- Problems involved range from finding maximum income of a company to the largest possible area of a farm.

Maximum Profit

- **<u>Example:</u>** The weekly profit function for a product is given by

$$\mathbf{P(x) \quad = \quad -\,0.0001x^2 + 3x - 12{,}500,}$$

where *x* is the number of units produced per week, and *P*(*x*) is the profit (in dollars).

- What is the **maximum weekly profit?** How many units should be produced for this profit?

Maximum Profit

- The function to be maximized is

$$P(x) \quad = \quad -\,0.0001x^2 + 3x - 12{,}500$$

- We note that

$$a = -0.0001, \quad b = 3 \quad \text{and} \quad c = -12{,}500$$

- **Maximum weekly profit is**

$$P \quad = \quad c - b^2/4a = \quad -12{,}500 + 22{,}500$$

$$= \quad 10{,}000 \text{ dollars}$$

- **The number of units produced is**

$$x \quad = \quad -b/2a \quad = \quad 15{,}000 \text{ units}$$

Practice Problems

Question 1 :

A company that produces calculators determines that its fixed cost is \$8820 per month. The variable cost is \$70 per calculator: the revenue is \$105 per calculator. The cost and revenue equations, respectively, are given by

$C = 70x + 8820$ and $R = 105x$ .Find the number of calculators the company must produce and sell to break even.

Practice Problems

Question 2 :

A publisher finds that the fixed cost associated with a new paperback is \$18,000. Each book costs \$2 to produce and will sell for \$6.50. Find the publisher's break-even point.

Practice Problems

Question 3 :

A firm manufactures a product that sells for $12 per unit. Variable cost per unit is $8 and fixed cost per period is $1200. Capacity per period is 1000 units.

a) Graph the revenue and cost functions.

b) Find the number of units sold and the revenue amount ($) at break-even point .

Practice Problems

Question 4 :

ABC company manufactures a product. The total fixed costs are \$750 and the variable cost per unit is \$25. The total revenue function is given **R = 150x – x^2** where x is the quantity produced and sold.

- Find the **total cost function C**
- Find the **Profit Function P**
- Find the **break even point** by plotting R and C functions
- Find the **maximum profit** and output level for maximum profit.

Thank you