

Solution to Tutorial Week 24

1. Solve the following LPP using simplex Method.

- a) Maximize $Z = 3x_1 + 4x_2$
 Subjected to,
 $x_1 + x_2 \leq 20$
 $2x_1 + 3x_2 \leq 50$
 $x_1, x_2 \geq 0$

Solution,

Let S_1 and S_2 be the slack variables.

Now,

$$\begin{aligned} x_1 + x_2 + S_1 &= 20 \\ 2x_1 + 3x_2 + S_2 &= 50 \end{aligned}$$

$$x_1, x_2, S_1, S_2 \geq 0$$

Standard equation for simplex table:

$$\begin{aligned} 1. Z - 3x_1 - 4x_2 + 0S_1 + 0S_2 &= 0 \\ 0.Z + 1.x_1 + 1.x_2 + 1.S_1 + 0.S_2 &= 20 \\ 0.Z + 2.x_1 + 3.x_2 + 0.S_1 + 1.S_2 &= 50 \end{aligned}$$

Simplex Table 1:

	Z	x_1	x_2	S_1	S_2	constant	Ratio
R_0	1	-3	-4	0	0	0	-
R_1	0	1	1	1	0	20	$20/1=20$
R_2	0	2	3	0	1	50	$50/3=16.67$

Highest Negative: Key Column

Minimum positive ratio: Key Row

Key Element

Here R_2 is the key row so we should update it first.

$$\text{New } R_2 = \frac{\text{Old } R_2}{\text{Key element}} = \frac{\text{Old } R_2}{3}$$

$$0, \quad \frac{2}{3}, \quad 1, \quad 0, \quad \frac{1}{3}, \quad \frac{50}{3}$$

Similarly, we should update R_0 and R_1 using the formula,

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$$New R_0 = Old R_0 + 4 \times New R_2$$

Old R0 +	4 * New R2	New R0
1	0	1
-3	8/3	-1/3
-4	4	0
0	0	0
0	4/3	4/3
0	200/3	200/3

$$New R_1 = Old R_1 - 1 \times New R_2$$

Old R1 -	1 * New R2	New R1
0	0	0
1	2/3	1/3
1	1	0
1	0	1
0	1/3	-1/3
20	50/3	10/3

Simplex table 2:

	Z	X1	X2	S1	S2	constant	Ratio
R0	1	-1/3	0	0	4/3	200/3	-
R1	0	1/3	0	1	-1/3	10/3	10
R2	0	2/3	1	0	1/3	50/3	25

Highest Negative: Key Column

Minimum positive ratio: Key Row

Key Element

Here R_1 is the key row and need to updated first using the formula

$$New R_1 = \frac{Old R_1}{key\ element} = Old R_1 \times 3$$

$$0, 1, 0, 3, -1, 10$$

Similarly, we should update R_0 and R_2 using the formula,

$$New R_0 = Old R_0 + \frac{1}{3} \times New R_1$$

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Old R0 +	1/3 * New R1	New R0
1	0	1
-1/3	1/3	0
0	0	0
0	1	1
4/3	-1/3	1
200/3	10/3	70

$$\text{New } R_2 = \text{Old } R_2 - \frac{2}{3} \times \text{New } R_1$$

Old R2 -	2/3 * New R1	New R2
0	0	0
2/3	2/3	0
1	0	1
0	2	-2
1/3	-2/3	1
50/3	20/3	10

Simplex Table 3:

	Z	X_1	X_2	S_1	S_2	Constant
R_0	1	0	0	1	1	70
R_1	0	1	0	3	-1	10
R_2	0	0	1	-2	1	10

Here all the coefficient of variables in R_0 row are all ≥ 0 so we reached the optimum solution.

Hence,

Maximum $Z = 70$

$$X_1 = 10$$

$$X_2 = 10.$$

b) Maximize $Z = 2x_1 + 3x_2$

Subjected to,

$$10x_1 + 20x_2 \leq 175$$

$$x_1 + 2x_2 \geq 50$$

$$x_1, x_2 \geq 0$$

(Try this on your own)

Answer is $Z = 35$, $X_1 = 17.5$, $X_2 = 0$

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- c) Minimize $Z = 3x_1 + 4x_2 + 5x_3$
 Subjected to,
 $x_1 + x_2 + x_3 \geq 30$
 $10x_1 + 15x_2 + 20x_3 \leq 600$
 $x_1, x_2, x_3 \geq 0$

(Note: S1 means S_1 and so on)

Solution,

Let S1 be the surplus variable, S2 be the slack variable and let A1 be the artificial variable.

Now,

$$\begin{aligned} x_1 + x_2 + x_3 - S_1 + A_1 &= 30 \\ 10x_1 + 15x_2 + 20x_3 + S_2 &= 600 \\ x_1, x_2, x_3, S_1, S_2, A_1 &\geq 0 \end{aligned}$$

Standard equation for simplex table:

$$\begin{aligned} 1Z - 3x_1 - 4x_2 - 5x_3 + 0S_1 + 0S_2 - 10A_1 &= 0 \\ 0Z + 1x_1 + 1x_2 + 1x_3 - 1S_1 + 0S_2 + 1A_1 &= 30 \\ 0Z + 10x_1 + 15x_2 + 20x_3 + 0S_1 + 1S_2 + 0A_1 &= 600 \end{aligned}$$

Simplex table 1:

	Z	X1	X2	X3	S1	S2	A1	Constant
R0	1	-3	-4	-5	0	0	-10	0
R1	0	1	1	1	-1	0	1	30
R2	0	10	15	20	0	1	0	600

For identity matrix:

$$\text{New R0} = \text{OldR0} + 10 * \text{R1}$$

Old R0 +	10*R1	New R0
1	0	1
-3	10	7
-4	10	6
-5	10	5
0	-10	-10
0	0	0
-10	10	0
0	300	300

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Simplex Table 2:

	Z	X1	X2	X3	S1	S2	A1	Constant	Ratio
R0	1	7	6	5	-10	0	0	300	-
R1	0	1	1	1	-1	0	1	30	30
R2	0	10	15	20	0	1	0	600	60

Highest Positive: Key Column

Minimum Positive: Ratio Key Row

Key element

Here R1 is the key row and should be updated first using formula

New R1 = Old R1 / Key element = (Old R1) / 1

0, 1, 1, 1, -1, 0, 1, 30

Similarly, we should update R0 and R2 using the formula,

New R0 = Old R0 – 7 * New R1

New R2 = Old R2 – 10 * New R1

Old R0 -	7*New R1	New R0
1	0	1
7	7	0
6	7	-1
5	7	-2
-10	-7	-3
0	0	0
0	7	-7
300	210	90

Old R2 -	10*New R1	New R2
0	0	0
10	10	0
15	10	5
20	10	10
0	-10	10
1	0	1
0	10	-10
600	300	300

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Simplex Table 3:

	Z	X1	X2	X3	S1	S2	A1	Constant
R0	1	0	-1	-2	-3	0	-7	90
R1	0	1	1	1	-1	0	1	30
R2	0	0	5	10	10	1	-10	300

Here all the coefficient of variables in R0 row are all ≤ 0 so we reached the optimum solution.

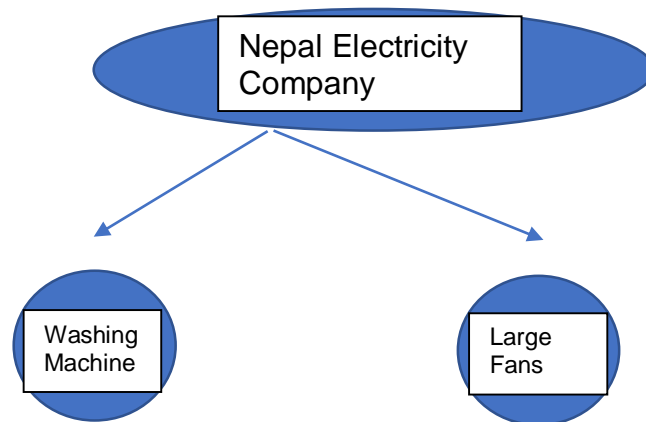
Hence,

$$\begin{aligned} \text{Minimum } Z &= 90 \\ X1 &= 30 \\ X2 &= 0 \\ X3 &= 0 \end{aligned}$$

Write conclusion here:

2. Nepal Electric Corporation manufacturer's two electrical products: washing machine and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each washing machine takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the production period 240 hours of wiring time and up to 140 hours of drilling time may be used. Each washing machine sold yields a profit of Rs 5250. Each fan assembled may be sold for a Rs.2150 profit. Formulate and solve this LPP using simplex method.

Solution, **Problem analysis:**



Machines			Time Limit
Wiring	3 hours	2 hours	240 hours
Drilling	2 hours	1 hour	120 hours
Profit	Rs 5250	Rs 2150	

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Mathematical Formulation:

For decision variable: Let x and y be the number of washing machine and fans the NEC should manufacture in order to maximize the profit.

For Objective Function:

Total profit = $5250x + 2150y$

Let $Z = 5250x + 2150y$

Maximize $Z = 5250x + 2150y$

For Constraints:

$3x + 2y \leq 240$ (Wiring Machine Constraints)

$2x + 1y \leq 120$ (Drilling machine constraints)

$x, y \geq 0$ (Non Negativity Constraints)

Let S_1 and S_2 be the slack variables.

Now,

$3x + 2y + S_1 = 240$

$2x + 1y + S_2 = 120$

$x, y, S_1, S_2 \geq 0$

Standard equation for simplex table:

$1Z - 5250x - 2150y + S_1 + S_2 = 0$

$0Z + 3x + 2y + 1S_1 + 0S_2 = 240$

$0Z + 2x + 1y + 0S_1 + 1S_2 = 120$

Simplex Table 1:

	Z	x	y	S1	S2	Constant	Ratio
R0	1	-5250	-2150	0	0	0	-
R1	0	3	2	1	0	240	80
R2	0	2	1	0	1	140	70

Highest Negative: Key Column

Minimum positive Ratio: Key Row

Key Element

Here R2 is a key row so it should be updated first using the formula

New R2 = Old R2 / Key Elements = OldR2 / 2

0, 1, 1/2, 0, 1/2, 70

Similarly, we should update R0 and R1 using the formula,

New R0 = Old R0 - (-5250) * New R2

New R1 = Old R1 - 3 * New R2

Old R0 +	5250*New R2	New R0
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1	0	1
-5250	5250	0
-2150	2625	475
0	0	0
0	2625	2650
0	367500	367500

Old R1 -	3*New R2	New R1
0	0	0
3	3	0
2	3/2	1/2
1	0	1
0	3/2	-3/2
240	210	30

Simplex Table 2:

	Z	x	y	S1	S2	Constant
R0	1	0	475	0	2625	367500
R1	0	0	1/2	1	-3/2	30
R2	0	1	1/2	0	1/2	70

Here all the coefficient of variables in R0 row are all ≥ 0 so we reached the optimum solution.

Here, Maximum $Z = 367500$, $x = 70$, $y = 0$.

Conclusion:

Hence NEC should manufacture 70 units of washing machine and no fans in order to make the maximum profit of Rs.367500.

- A manufacturer makes two products P1 and P2 using two machines M1 and M2. Product P1 requires 5 hours on machine M1 and no time on machine M2, product P2 requires 1 hour on machine M1 and 3 hour on machine M2. There are 16 hours of time per day available on machine M1 and 30 hours on M2. Profit margin from P1 and P2 is Rs. 2 and Rs. 10 per unit respectively. What should be the daily production mix to maximize profit? Use Simplex Method.

(Its similar to question no 2. Try yourself)

(Answer is Maximum $Z = \text{Rs.}102$, $x = 1$ and $y = 10$ We rounded down $x = 1.2$ to 1 as number of products can't be in decimal.)

- A dealer wishes to purchase a number of fans and electric iron. He has only Rs. 5760 to invest and has space for at most 20 items. A fan costs him Rs. 360 and a electric iron Rs. 240. His expectation is that he can sell a fan at a profit of Rs. 22 and electric iron at a profit of Rs. 18. Assuming that he can sell all the items that he can

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buy, how should he invest his money in order to maximize his profit? Use Simplex Method.

(Its similar to question no 2. Try yourself)

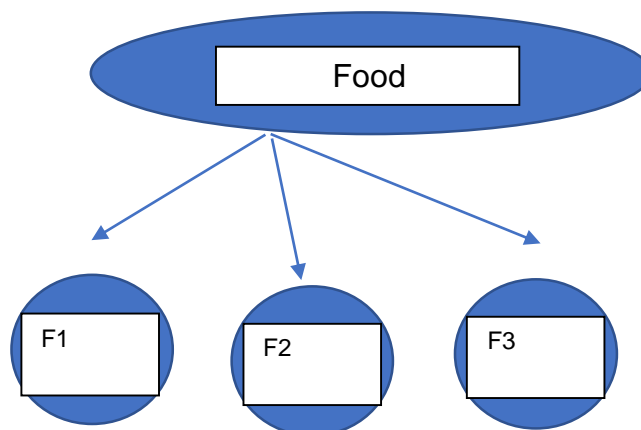
(Answer is Maximum Z= Rs.392, x= 8 and y =12)

5. A dietician in a teaching hospital is to arrange a special diet using three foods F1; F2 & F3. Each gm of Food F1 contains 20 units of calcium, 10 units of iron and 10 units of vitamin A, and 20 units of cholesterol. Each gm of food F2 contains 10 units of calcium, 10 units of iron, 20 units of vitamin A and 24 units of cholesterol. Each gm of food F3 contains 10 units of calcium, 10 units of iron, 10 units of vitamin A and 18 units of cholesterol. If the minimum daily requirements are 300 units of calcium, 200 units of iron, 240 units of vitamin A. How many grams of each food should be used to meet the minimum requirements at the same time minimizing the cholesterol intake. Use simplex method.

Solution,

(The constraints have the greater than or equal to sign (\geq) because we want to ensure that the diet provides at least the minimum daily requirements of each nutrient. Using the greater than or equal to sign ensures that the amount of each nutrient in the diet is not less than the minimum requirement.)

Problem Analysis:



	F1	F2	F3	Min.Requirement
Calcium	20	10	10	300
Iron	10	10	10	200
Vitamin A	10	20	10	240
Cholesterol	20	24	18	

Mathematical Formulation:

For Decision Variable:

Let x gm, y gm and z gm of food F1, F2 and F3 should be mixed and given to the patients in order to **minimize the cholesterol intake** and meet the requirements.

For Objective Function:

$$\text{Total cholesterol intake} = 20x + 24y + 18z$$

$$\text{Let } C = 20x + 24y + 18z$$

$$\text{Minimize } C = 20x + 24y + 18z$$

$$\text{Minimize } C = 4(5x + 6y + \frac{9}{2}z)$$

$$\text{Minimize } C = 4 * \text{Minimize } C'$$

$$\text{Let, Minimize } C' = 5x + 6y + 9/2z$$

For Constraints:

$$20x + 10y + 10z \geq 300$$

$$2x + 1y + 1z \geq 30 \text{ (Calcium Constraints)}$$

$$10x + 10y + 10z \geq 200$$

$$1x + 1y + 1z \geq 20 \text{ (Iron Constraints)}$$

$$10x + 20y + 10z \geq 240$$

$$1x + 2y + 1z \geq 24 \text{ (Vitamin A Constraints)}$$

Let S_1, S_2, S_3 be the surplus variables and let A_1, A_2, A_3 be the artificial variables.

Now,

$$2x + 1y + 1z - S_1 + A_1 = 30$$

$$1x + 1y + 1z - S_2 + A_2 = 20$$

$$1x + 2y + 1z - S_3 + A_3 = 24$$

$$x, y, z, S_1, S_2, S_3, A_1, A_2, A_3 \geq 0$$

Standard equation for simplex table:

$$1C' - 5x - 6y - \frac{9}{2}z + 0S_1 + 0S_2 + 0S_3 - 10A_1 - 10A_2 - 10A_3 = 0$$

$$0C' + 2x + 1y + 1z - 1S_1 + 0S_2 + 0S_3 + 1A_1 + 0A_2 + 0A_3 = 30$$

$$0C' + 1x + 1y + 1z + 0S_1 - 1S_2 + 0S_3 + 0A_1 + 1A_2 + 0A_3 = 20$$

$$0C' + 1x + 2y + 1z + 0S_1 + 0S_2 - 1S_3 + 0A_1 + 0A_2 + 1A_3 = 24$$

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Simplex table1:

	C'	x	y	z	S_1	S_2	S_3	A_1	A_2	A_3	Constant
R_0	1	-5	-6	-9/2	0	0	0	-10	-10	-10	0
R_1	0	2	1	1	-1	0	0	1	0	0	30
R_2	0	1	1	1	0	-1	0	0	1	0	20
R_3	0	1	2	1	0	0	-1	0	0	1	24

For Identity Matrix:

$$\text{New } R_0 = \text{Old } R_0 + 10 \times (R_1 + R_2 + R_3)$$

Old R_0 +	$10 \times (R_1 + R_2 + R_3)$	New R_0
1	0	1
-5	40	35
-6	40	34
-9/2	30	$\frac{51}{2}$
0	-10	-10
0	-10	-10
0	-10	-10
-10	10	0
-10	10	0
-10	10	0
0	740	740

Simplex table 2:

Highest Positive: Key Column

	C'	x	y	z	S_1	S_2	S_3	A_1	A_2	A_3	Constant	Ratio
R_0	1	35	34	51/2	-10	-10	-10	0	0	0	740	-
R_1	0	2	1	1	-1	0	0	1	0	0	30	15
R_2	0	1	1	1	0	-1	0	0	1	0	20	20
R_3	0	1	2	1	0	0	-1	0	0	1	24	24

Minimum Positive Ratio: Key Row

Key Element

Here R_1 is the key row so it should be updated first using the formula,

$$\text{New } R_1 = \frac{\text{Old } R_1}{\text{Key Element}} = \frac{\text{Old } R_1}{2}$$

$$0, \quad 1, \quad \frac{1}{2}, \quad \frac{1}{2}, \quad -\frac{1}{2}, \quad 0, \quad 0, \quad \frac{1}{2}, \quad 0, \quad 0, \quad 15$$

Similarly, we should update R_0, R_2 and R_3 using the formula,

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$$New R_0 = Old R_0 - 35 \times New R_1$$

$$New R_2 = Old R_2 - 1 \times New R_1$$

$$New R_3 = Old R_3 - 1 \times New R_1$$

<i>Old R₀ -</i>	<i>35 × New R₁</i>	<i>New R₀</i>
1	0	1
35	35	0
34	35/2	33/2
51/2	35/2	8
-10	-35/2	15/2
-10	0	-10
-10	0	-10
0	35/2	-35/2
0	0	0
0	0	0
740	525	215

<i>Old R₂ -</i>	<i>1 * New R₁</i>	<i>New R₂</i>
0	0	0
1	1	0
1	1/2	1/2
1	1/2	1/2
0	-1/2	1/2
-1	0	-1
0	0	0
0	1/2	-1/2
1	0	1
0	0	0
20	15	5

<i>Old R₃ -</i>	<i>1 * New R₁</i>	<i>New R₃</i>
0	0	0
1	1	0
2	1/2	3/2
1	1/2	1/2
0	-1/2	1/2
0	0	0
-1	0	-1
0	1/2	-1/2
0	0	0
1	0	1
24	15	9

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Simplex table 3:

	C'	x	y	Z	S_1	S_2	S_3	A_1	A_2	A_3	Const ant	Ratio
R_0	1	0	33/2	8	15/2	-10	-10	-35/2	0	0	215	-
R_1	0	1	1/2	1/2	-1/2	0	0	1/2	0	0	15	30
R_2	0	0	1/2	1/2	1/2	-1	0	-1/2	1	0	5	10
R_3	0	0	3/2	1/2	1/2	0	-1	-1/2	0	1	9	6

Highest Positive: Key Column

Minimum Positive Ratio: Key Row

Key Element

Here R_3 is the key row and it need to updated first,

$$\text{New } R_3 = \frac{\text{Old } R_2}{\frac{3}{2}} = \text{Old } R_2 \times \frac{2}{3}$$

$$0, \quad 0, \quad 1, \quad \frac{1}{3}, \quad \frac{1}{3}, \quad 0, \quad -\frac{2}{3}, \quad -\frac{1}{3}, \quad 0, \quad \frac{2}{3}, \quad 6$$

Similarly, we should update R_0, R_1 and R_2 using the formula,

$$\text{New } R_0 = \text{Old } R_0 - \frac{33}{2} \times \text{New } R_3$$

$$\text{New } R_1 = \text{Old } R_1 - \frac{1}{2} \times \text{New } R_3$$

$$\text{New } R_2 = \text{Old } R_2 - \frac{2}{3} \times \text{New } R_3$$

Old R_0 -	$\frac{33}{2} \times \text{New } R_3$	$\text{New } R_0$
1	0	1
0	0	0
33/2	33/2	0
8	33/6	5/2
15/2	33/6	2
-10	0	-10
-10	-11	1
-35/2	-33/6	-12
0	0	0
0	11	-11
215	99	116
Old R_1 -	$\frac{1}{2} \times \text{New } R_3$	$\text{New } R_1$
0	0	0
1	0	1

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1/2	1/2	0
1/2	1/6	1/3
-1/2	1/6	-2/3
0	0	0
0	-1/3	1/3
1/2	-1/6	2/3
0	0	0
0	1/3	-1/3
15	3	12
<i>Old R₂ -</i>	<i>$\frac{1}{2} * \text{New R}_3$</i>	<i>New R₂</i>
0	0	0
0	0	0
1/2	1/2	0
1/2	1/6	1/3
1/2	1/6	1/3
-1	0	-1
0	-1/3	1/3
-1/2	-1/6	-1/3
1	0	1
0	1/3	-1/3
5	3	2

Simplex table 4:

	C'	x	Y	z	S_1	S_2	S_3	A_1	A_2	A_3	Constant	Ratio
R_0	1	0	0	5/2	2	-10	1	-12	0	-11	116	-
R_1	0	1	0	1/3	-2/3	0	1/3	2/3	0	-1/3	12	36
R_2	0	0	0	1/3	1/3	-1	1/3	-1/3	1	-1/3	2	6
R_3	0	0	1	1/3	1/3	0	-2/3	-1/3	0	2/3	6	18

Highest Positive: Key Column

Minimum Positive Ratio: Key Row

Key Element

Here R_2 is the key row and should be updated first,

$$\text{New } R_2 = \frac{\text{Old } R_2}{\frac{1}{3}} = \text{Old } R_2 \times 3$$

0, 0, 0, 1, 1, -3, 1, -1, 3, -1, 6

Similarly, we should update R_0, R_1 and R_3 using the formula,

$$\text{New } R_0 = \text{Old } R_0 - \frac{5}{2} \times \text{New } R_2$$

$$\text{New } R_1 = \text{Old } R_1 - \frac{1}{3} \times \text{New } R_2$$

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$$New R_3 = Old R_3 - \frac{1}{3} \times New R_2$$

<i>Old R₀ –</i>	<i>$\frac{5}{2} * New R_2$</i>	<i>New R₀</i>
1	0	1
0	0	0
0	0	0
5/2	5/2	0
2	5/2	-1/2
-10	-15/2	-5/2
1	5/2	-3/2
-12	-5/2	-19/2
0	15/2	-15/2
-11	-5/2	-17/2
116	15	101
<i>Old R₁ –</i>	<i>$\frac{1}{3} * New R_2$</i>	<i>New R₁</i>
0	0	0
1	0	1
0	0	0
1/3	1/3	0
-2/3	1/3	-1
0	-1	1
1/3	1/3	0
2/3	-1/3	1
0	1	-1
-1/3	-1/3	0
12	2	10
<i>Old R₃ –</i>	<i>$\frac{3}{2} * New R_2$</i>	<i>New R₃</i>
0	0	0
0	0	0
1	0	1
1/3	1/3	0
1/3	1/3	0
0	-1	1
-2/3	1/3	-1
-1/3	-1/3	-1
0	1	-1
2/3	-1/3	1
6	2	4

Simplex table 5:

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	C'	x	Y	z	S_1	S_2	S_3	A_1	A_2	A_3	Constant
R_0	1	0	0	0	-1/2	-5/2	-3/2	-19/2	-15/2	-17/2	101
R_1	0	1	0	0	-1	1	0	1	-1	0	10
R_2	0	0	0	1	1	-3	1	-1	3	-1	6
R_3	0	0	1	0	0	1	-1	0	-1	1	4

Here all the coefficient of variables in R_0 row are all ≤ 0 so we reached the optimum solution.

Hence,

Minimum $C = 4 \times 101 = 404$ (We have divided the objective function by 4 earlier)

$$x = 10$$

$$y = 4$$

$$z = 6$$

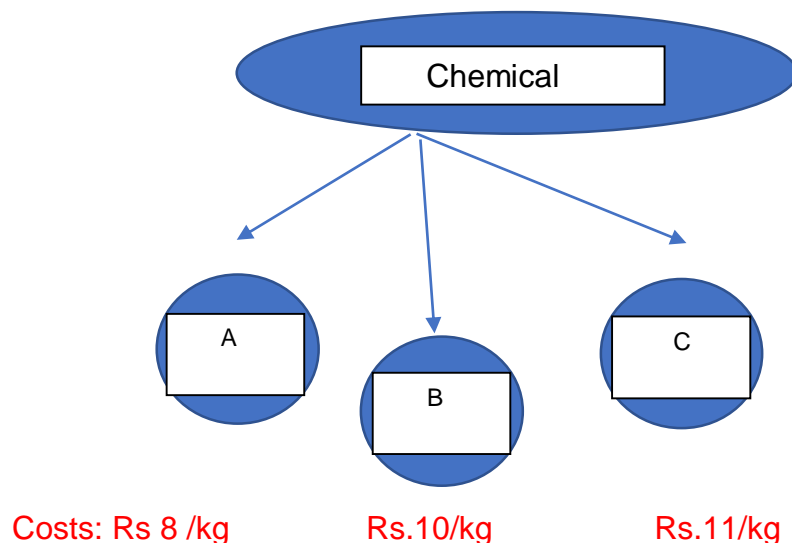
Conclusion:

Hence the dietician should mix 10gm of food F1, 4 gm of food F2 and 6gm of food F3 and give it to the patient so that he can minimize the cholesterol intake to 404 units.

6. Chemicals Ltd. must produce 10,000 kgs. of a special mixture for a customer. The mix consists of ingredients A, B and C. A Costs Rs. 8 per kg., B costs Rs. 10 per kg. and C costs Rs. 11 per kg. No more than 35000 kgs of A can be used and at least 1,500 kgs. of B must be used. Also, at least 2,000 kgs of C is required. Calculate the number of kgs. For each ingredient to use in order to minimize total costs for 10,000 kgs.

Solution,

Problem Analysis:



Solution to Tutorial Week 24

No more than 35000 kg of A

At least 1500 kg of B

At least 2000 kg of C

Must Produce 10000 kg

Mathematical Formulation:

For Decision Variables: Let x kg of A, y kg of B and z kg of C should be used to make the mixture of 10000 kg and to minimize the cost.

For Objective function:

$$\text{Total Cost} = 8x + 10y + 11z$$

$$\text{Let } C = 8x + 10y + 11z$$

$$\text{Minimize } C = 8x + 10y + 11z$$

For Constraints:

$$x \leq 35000 \text{ (A constraints)}$$

$$y \geq 1500 \text{ (B constraints)}$$

$$z \geq 2000 \text{ (C constraints)}$$

$$x + y + z = 10000 \text{ (Total mixture constraints)}$$

$$x, y, z \geq 0 \text{ (Non Negative Constraints)}$$

Let S1 be the slack variable, Let S2 and S3 be the surplus variables and let A1, A2 and A3 be the artificial variables.

Now,

$$x + S1 = 35000$$

$$y - S2 + A1 = 1500$$

$$z - S3 + A2 = 2000$$

$$x + y + z + A3 = 10000$$

$$x, y, z, S1, S2, S3, A1, A2, A3 \geq 0$$

Standard equation for simplex table:

$$1C - 8x - 10y - 11z + 0S1 + 0S2 + 0S3 - 100A1 - 100A2 - 100A3 = 0$$

$$0C + 1x + 0y + 0z + 1S1 + 0S2 + 0S3 + 0A1 + 0A2 + 0A3 = 35000$$

$$0C + 0x + 1y + 0z + 0S1 - 1S2 + 0S3 + 1A1 + 0A2 + 0A3 = 1500$$

$$0C + 0x + 0y + 1z + 0S1 + 0S2 - 1S3 + 0A1 + 1A2 + 0A3 = 2000$$

$$0C + 1x + 1y + 1z + 0S1 + 0S2 + 0S3 + 0A1 + 0A2 + 1A3 = 10000$$

Simplex Table 1:

	C	x	y	z	S1	S2	S3	A1	A2	A3	Constant
R0	1	-8	-10	-11	0	0	0	-100	-100	-100	0
R1	0	1	0	0	1	0	0	0	0	0	35000
R2	0	0	1	0	0	-1	0	1	0	0	1500
R3	0	0	0	1	0	0	-1	0	1	0	2000
R4	0	1	1	1	0	0	0	0	0	1	10000

For Identity Matrix:

$$\text{New R0} = \text{Old R0} + 100 * (R2 + R3 + R4)$$

OldR0 +	100 * (R2+R3+R4)	New R0
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Solution to Tutorial Week 24

1	0	1
-8	100	92
-10	200	190
-11	200	189
0	0	0
0	-100	-100
0	-100	-100
-100	100	0
-100	100	0
-100	100	0
0	1350000	1350000

Simplex Table 2:

	C	x	y	z	S1	S2	S3	A1	A2	A3	Constant	Ratio
R0	1	92	190	189	0	-100	-100	0	0	0	1350000	-
R1	0	1	0	0	1	0	0	0	0	0	35000	∞
R2	0	0	1	0	0	-1	0	1	0	0	1500	1500
R3	0	0	0	1	0	0	-1	0	1	0	2000	∞
R4	0	1	1	1	0	0	0	0	0	1	10000	10000

Highest Positive: Key column

Minimum Positive Ratio: Key Row

Key Element

Here R2 is the key row and need to update it first,
 New R2 = Old R2 / 1
 0,0,1,0,0,-1,0,1,0,0,1500
 Similarly we need to update R0,R1,R3 and R4 using ,
 New R0 = Old R0 -190 *New R2
 New R1 = Old R1-0*New R2
 New R3 = Old R3 – 0* New R2
 New R4 = Old R4 – 1*New R2

OldR0 -	190 * New R2	New R0
1	0	1
92	0	92
190	190	0
189	0	189
0	0	0
-100	-190	90

Solution to Tutorial Week 24

-100	0	-100
0	190	-190
0	0	0
0	0	0
1350000	285000	1065000

No change in R1 and R3.

Old R4 -	1 * New R2	New R4
0	0	0
1	0	1
1	1	0
1	0	1
0	0	0
0	-1	1
0	0	0
0	1	-1
0	0	0
1	0	1
10000	1500	8500

Simplex Table 3:

Highest Positive: Key column

	C	x	y	z	S1	S2	S3	A1	A2	A3	Constant	Ratio
R0	1	92	0	189	0	90	-100	-190	0	0	1065000	-
R1	0	1	0	0	1	0	0	0	0	0	35000	∞
R2	0	0	1	0	0	-1	0	1	0	0	1500	∞
R3	0	0	0	1	0	0	-1	0	1	0	2000	2000
R4	0	1	0	1	0	1	0	-1	0	1	8500	8500

Minimum Positive Ratio: Key Row

Key Element

Here R3 is the key row so need to be updated first,

New R3 = Old R3 / 1

0,0,0,1,0,0,-1,0,1,0,2000

Solution to Tutorial Week 24

Similarly we need to update R0,R1,R2 and R4 using ,

New R0 = Old R0 -189 *New R3

New R1 = Old R1-0*New R3

New R2 = Old R2 – 0* New R3

New R4 = Old R4 – 1*New R3

OldR0 -	189 * New R3	New R0
1	0	1
92	0	92
0	0	0
189	189	0
0	0	0
90	0	90
-100	-189	89
-190	0	-190
0	189	-189
0	0	0
1065000	378000	687000

N0 change in R1 and R2

Old R4 -	1 * New R3	New R4
0	0	0
1	0	1
0	0	0
1	1	0
0	0	0
1	0	1
0	1	1
-1	0	-1
0	1	-1
1	0	1
8500	2000	6500

Solution to Tutorial Week 24

Simplex Table 4:

	C	x	y	z	S1	S2	S3	A1	A2	A3	Constant	Ratio
R0	1	92	0	0	0	90	89	-190	-189	0	687000	-
R1	0	1	0	0	1	0	0	0	0	0	35000	35000
R2	0	0	1	0	0	-1	0	1	0	0	1500	∞
R3	0	0	0	1	0	0	-1	0	1	0	2000	∞
R4	0	1	0	0	0	1	1	-1	-1	1	6500	6500

Highest Positive: Key column

Minimum Positive Ratio: Key Row

Key Element

Here R4 is the key row so it needed to be updated first ,

New R4 = Old R4 /1

0,1,0,0,0,1,1,-1,-1,1,6500

Similarly we need to update R0,R1,R2 and R3 using ,

New R0 = Old R0 -92 *New R4

New R1 = Old R1-1*New R4

New R2 = Old R2 – 0* New R4

New R3 = Old R3 – 0*New R4

OldR0 -	92 * New R4	New R0
1	0	1
92	92	0
0	0	0
0	0	0
0	0	0
90	92	-2
89	92	-3
-190	-92	-98
-189	-92	-97
0	92	-92
687000	598000	89000

No Change in R2 and R3.

OldR1 -	1 * New R4	New R1
0	0	0
1	1	0
0	0	0
0	0	0
1	0	1
0	1	-1
0	1	-1
0	1	1
0	1	1
0	1	-1
35000	6500	28500

Solution to Tutorial Week 24

Simplex Table 5:

	C	x	y	z	S1	S2	S3	A1	A2	A3	Constant
R0	1	0	0	0	0	-2	-3	-98	-97	-92	89000
R1	0	0	0	0	1	-1	-1	1	1	-1	28500
R2	0	0	1	0	0	-1	0	1	0	0	1500
R3	0	0	0	1	0	0	-1	0	1	0	2000
R4	0	1	0	0	0	1	1	-1	-1	1	6500

Here all the coefficient of variables in R0 row are all ≤ 0 so we reached the optimum solution.

Hence,

Minimum C = 89000

x = 6500

y = 1500

z = 2000

Conclusion:

Hence the chemical company should mix 6500kg of chemical A, 1500 kg of chemical B and 2000kg of chemical C and meet the requirements with the minimum cost of Rs.89000.