

New Syllabus



First Year B.C.A.

(1st Semester)

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South Gujarat University

Mathematics

Syllabus Prescribed for First Year B.C.A. Effective from June, 2020

Credit 3

Total Hrs/Week : 3

Aim : Objective is to provide develop Mathematical Abilities relevant to Computer Science

Prerequisite : School Mathematics

1. Set Theory

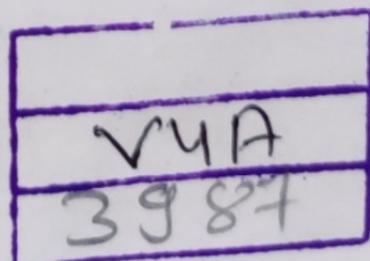
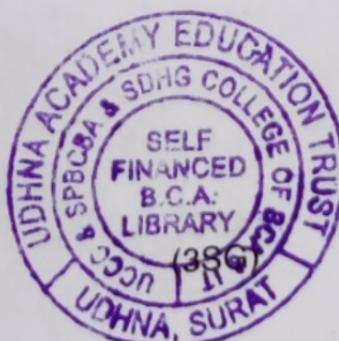
- 1.1 Introduction
- 1.2 Representation
- 1.3 Operation and its properties
- 1.4 Venn Diagram
- 1.5 Cartesian product and graph

2. Functions

- 2.1 Definition
- 2.2 Types - Domain and Range
- 2.3 Construction and functions

3. Mathematical Logic & Boolean Algebra

- 3.1 Introduction to logic
- 3.2 Truth Table
- 3.3 Definition & Examples of Boolean Algebra
- 3.4 Boolean Functions
- 3.5 Representation and minimization of Boolean Functions
- 3.6 Design example using Boolean algebra.



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4. Matrices and Determinants

- 4.1 Matrices of order M * N
- 4.2 Row and Column transformation
- 4.3 Addition, Subtraction and multiplication of Matrices
- 4.4 Computation of Inverse
- 4.5 Cramer's Rule
- 4.6 Business Application of Matrices

: Reference Books :

- 1. Co-ordinate Geometry – Shanti Narayan
- 2. Linear Algebra – Sushoma Verma
- 3. Advanced Mathematics – B.S. Shah & Co.
- 4. Schaum's Outline of Boolean algebra and swathing circuits – Ell Mendelson

■ Work is often the father of pleasure. ■

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|---|---|
| <ol style="list-style-type: none"> 1. Introduction : Concept of a set 2. Methods of representing sets 3. Types of sets 4. Some important number sets 5. Venn diagrams 6. Operations on sets <ol style="list-style-type: none"> (i) Intersection of sets (ii) Union of sets 7. Distributive law of union over intersection 8. Distributive law of intersection over union | <ol style="list-style-type: none"> 9. Complement of a set 10. De Morgan's law for union 11. De Morgan's law for intersection 12. Difference of two sets 13. Cartesian product of two sets 14. Number of elements in finite sets 15. Exercise.
University Questions |
|---|---|

: SYLLABUS :

Set Theory : Introduction, Representation, Operation and its properties, Ven- Diagram, Cartesian product and graph.

1. Introduction : Concept of a Set :

The students of your class is a set of students and you are a member of that set. The college cricket team is a set of players and the captain of that team is a member of that set. Thus, a set is well defined collection of distinct objects and there should exist a rule with the help of which we should be in a position to tell whether a particular object belongs to that collection or not. The objects forming a set are known as elements or members of the set. The followings are some examples of sets :

- (i) The set of ministers in Gujarat state.
- (ii) The set of alphabates of English language.
- (iii) The set of professors of your college.

The sets are generally denoted by capital letters A, B, C, D, X, Y, Z etc. and the elements of the set are denoted by small letters a, b, c, x, y, z . If a is a member of a set A, then we write $a \in A$ and read it as `a belongs to A' or a is a member of A. On the other hand if y does not belong to set A, we write $y \notin A$.

2. Methods of Representing Sets :

(i) Tabular form : In this method we write the members of the set in parentheses and separate them by putting commas between them. e.g., The set of even numbers between 1 and 13 can be represented as

$$\{2, 4, 6, 8, 10, 12\}.$$

Similarly the set of roots of the quadratic equation, $x^2 - 5x + 6 = 0$ $\{2, 3\}$, $\{1, 2, 3\}$ and $\{3, 2, 2, 1, 1\}$ represent the same set.

(ii) Set-builder form : In this method we write the properties which all the elements of the set must satisfy and write an element x to represent all the elements of the set.

e.g., $A = \{x : x \text{ is a vowel in English alphabates}\}$

$$B = \{x : x^2 - 3x - 4 = 0\}$$

$$C = \{x : x \text{ is an integer and } 5 < x < 15\}$$

3. Types of Sets :

(i) Finite Set : A set having finite number of elements is called a finite set.

e.g., $A = \{1, 2, 3, 4, 5\}$

$$B = \{x : x \text{ is minister of Gujarat state}\}$$

(ii) Infinite Set : A set in which the number of elements is not finite is called an infinite set.

e.g., $A = \{1, 2, 3, 4, 5, 6, 7, \dots\}$

$$B = \{x : x = n^2 \text{ and } n \text{ is a natural number}\}$$

(iii) Singleton Set : A set having only one element is known as a singleton set.

e.g., $A = \{2\}$

(iv) Empty Set or Null Set : A set having no element is called an empty set or a null set and it is denoted by \emptyset .

e.g., $A = \{x : x \text{ is an integer divisible by 7 between 8 and 13}\}$ is a null set.

(v) Equal Sets :* Two sets A and B are said to be equal if all the elements of A are the elements of B and also all the elements of B are elements of A. Symbolically we write $A = B$.

i.e., if for each $x \in A \Rightarrow x \in B$ and

$$x \in B \Rightarrow x \in A, \text{ then } A = B$$

* Define : Equal sets

(Oct./Nov. 09 - Nov. 10, Dec., 2010)

e.g. $\{1, 2, 3\}$; $B = \{3, 2, 1\}$ are equal sets.

Similarly $A = \{x : x^2 - 5x + 6 = 0\}$, and $B = \{2, 3\}$ are equal sets.

(vi) Equivalent Sets :* If the elements of one set can be put into one-to-one correspondence with the elements of another set then the two sets are called equivalent sets. Symbolically we can write $A \equiv B$.

e. g. $A = \{1, 2, 3, 4\}$

$B = \{1, 4, 9, 16\}$

Here $A \equiv B$

(vii) Subset of a Set :@ If all the elements of a set A are the elements of a set B then A is said to be a subset of B. Symbolically we write,

$A \subseteq B$. i.e, if $x \in A \Rightarrow x \in B$ then $A \subset B$.

e.g., If $A = \{1, 2, 3\}$; $B = \{1, 2, 3, 4, 5\}$, then $A \subseteq B$.

It should be noted that if $A \subseteq B$ and $B \subseteq A$, then A and B are equal sets i.e. $A = B$.

(1) The empty set ϕ is a subset of every set.

(2) Every set is a subset of itself.

(viii) Proper Sub-sets :**#

If all elements of set A are the elements of set - B and at least one element of superset B is not an element of set A, then set A is called proper subset of superset B.

Symbolically we write $A \subset B$.

e.g., $A = \{2, 4, 6\}$ $B = \{2, 4, 6, 8, 10\}$

Here $A \subset B$.

(ix) Power Set :*** The family of all the subsets of a given set is called its power set.

e.g., If $A = \{1, 2, 3\}$, then its power set denoted by $P(A)$ can be given by,

$P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$

The set having n elements will have 2^n subsets in its power set.

e.g., for $A = \{3, 5\}$ the number of sets in its power set $= 2^2 = 4$

and for $A = \{a, b, c, d\}$ the number of sets in its power set $= 2^4 = 16$.

* Define : Equivalent Sets (Nov./Dec. 2008, Dec., 15)

** How do you separate proper and improper subset of a set ?

Give illustration (Nov./Dec., 2008, April, 2010; Mar, 15)

Explain proper subset of non empty sets with illustration. (Dec., 14)

*** Define power set with illustration. (Mar., 15)

@ Define subset of a set with illustration. (Dec., 11, 16)

(x) Universal Set : A parent set from which all different subsets are considered is known as an Universal set for that particular situation. Generally universal set is denoted by U.

$$\text{e.g., } U = \{1, 2, 3, 4, 5, 6, 7, \dots\}$$

$$A = \{1, 3, 5, 7, \dots\}$$

$$B = \{2, 4, 6, 8, \dots\}$$

$$C = \{1, 4, 9, 16, \dots\}.$$

Here A, B and C are the subsets of universal set U or U is a universal set of the sets A, B and C.

The set of all the members of a family can be considered as an universal set and set of brothers, set of sisters etc. are its subsets.

4. Some Important Number Sets :

We are familiar with the following number sets

1. Set of natural numbers $N = \{1, 2, 3, 4, \dots\}$
2. Set of integers $Z = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
3. Set of rational numbers

$$Q = \left\{ \frac{a}{b} \mid a, b \in Z \text{ and } b \neq 0 \right\}$$

Q is a set of rational numbers. Rational numbers include all positive and negative integers, positive and negative fractions and zero.

4. Set of real numbers

R = Set of real numbers, which includes all rational numbers and all irrational numbers like $\sqrt{2}, \sqrt{3}, \dots$

clearly, $N \subset Z \subset Q \subset R$

5. Venn–Diagrams :

The representation of sets can be done by diagrams, popularly known as Venn–diagrams after the name of English logician John Venn. For the universal set a rectangle is drawn and one or more subsets of the universal set are shown by circles within the rectangle. Two such circles of two sets intersect each other if they have common elements. If they are disjoint sets i.e., if there is no common element in them then they are shown as separate circles within the rectangle. The set operations of intersection, union, complementation etc. can be effectively illustrated by such diagrams.

6. Operation on Sets :

(i) Intersection of sets :*

The intersection of two sets A and B is the set of all elements which belong to both A and B and it is denoted by $A \cap B$.

$$\text{i.e. } A \cap B = \{x/x \in A, \text{ and } x \in B\}$$

□ Properties of intersection :

$$(i) (A \cap B) \subseteq A \text{ and } (A \cap B) \subseteq B$$

$$(ii) A \cap \phi = \phi$$

$$(iii) A \cap A = A$$

$$(iv) A \cap B = B \cap A \text{ (Commutative property)}$$

$$(v) (A \cap B) \cap C = A \cap (B \cap C) \text{ (Associative property)}$$

Illustration 1 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 9, 11\}$, $C = \{2, 11, 18, 22\}$

find : $A \cap B$, $B \cap C$, $C \cap A$, $A \cap B \cap C$.

Also verify that : $(A \cap B) \cap C = A \cap (B \cap C)$,

Ans :

$$\text{Here } A \cap B = \{1, 2, 3, 4\} \cap \{3, 4, 9, 11\}$$

$$\therefore A \cap B = \{3, 4\}$$

$$B \cap C = \{11\}$$

$$C \cap A = \{2\}$$

$$A \cap B \cap C = \phi$$

Now we want to verify that $(A \cap B) \cap C = A \cap (B \cap C)$

$$A \cap B = \{3, 4\}, C = \{2, 11, 18, 22\}$$

$$\therefore (A \cap B) \cap C = \phi$$

$$\text{Also } A = \{1, 2, 3, 4\}, B \cap C = \{11\}$$

$$\therefore A \cap (B \cap C) = \phi$$

$$\therefore (A \cap B) \cap C = A \cap (B \cap C)$$

Illustration 2 : If $A = \{x/x \in N, x \leq 5\}$ $B = \{x/x \in N, 2 \leq x \leq 8\}$
 $C = \{x/x \in N, x \leq 3\}$, find $A \cap B$, $B \cap C$ and $C \cap A$

Ans : Here

$$A = \{1, 2, 3, 4, 5\}$$

$$B = \{2, 3, 4, 5, 6, 7, 8\}$$

$$C = \{1, 2, 3\}$$

$$\therefore A \cap B = \{2, 3, 4, 5\}$$

$$B \cap C = \{2, 3\}$$

$$C \cap A = \{1, 2, 3\}$$

* Define : Intersection of two sets

(Sep./Oct. 2009 - Old Course)

R1 Prove that $(A \cap B) \cap C = A \cap (B \cap C)^*$

In order to prove $(A \cap B) \cap C = A \cap (B \cap C)$

We shall have to prove that

$$(i) \quad (A \cap B) \cap C \subseteq A \cap (B \cap C) \text{ and}$$

$$(ii) \quad A \cap (B \cap C) \subseteq (A \cap B) \cap C$$

For (i) Let x be any element of $(A \cap B) \cap C$

$$\Rightarrow x \in (A \cap B) \cap C$$

$$\Rightarrow x \in (A \cap B) \text{ and } x \in C$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ and } x \in C$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ and } x \in C)$$

$$\Rightarrow x \in A \text{ and } x \in (B \cap C)$$

$$\Rightarrow x \in A \cap (B \cap C)$$

Thus every element x of $(A \cap B) \cap C$ is also an element of $A \cap (B \cap C)$

$$\therefore (A \cap B) \cap C \subseteq A \cap (B \cap C) \quad \dots \text{(i)}$$

For (ii) Let y be any element of $A \cap (B \cap C)$

$$\Rightarrow y \in A \cap (B \cap C)$$

$$\Rightarrow y \in A \text{ and } y \in (B \cap C)$$

$$\Rightarrow y \in A \text{ and } (y \in B \text{ and } y \in C)$$

$$\Rightarrow (y \in A \text{ and } y \in B) \text{ and } y \in C$$

$$\Rightarrow y \in (A \cap B) \text{ and } y \in C$$

$$\Rightarrow y \in (A \cap B) \cap C.$$

Thus every element y of $A \cap (B \cap C)$ is also an element of $(A \cap B) \cap C$

$$\therefore A \cap (B \cap C) \subseteq (A \cap B) \cap C \quad \dots \text{(ii)}$$

From (i) and (ii) We have

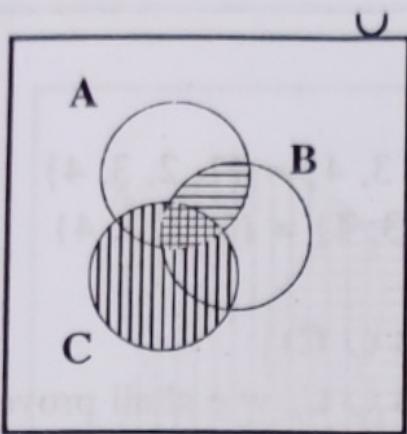
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Illustration 3 : Show that $(A \cap B) \cap C = A \cap (B \cap C)$ by Venn diagrams.

Ans.

We are required to verify $(A \cap B) \cap C = A \cap (B \cap C)$

* Prove that $(A \cap B) \cap C = A \cap (B \cap C)$ (March/April, 2007, 2008)

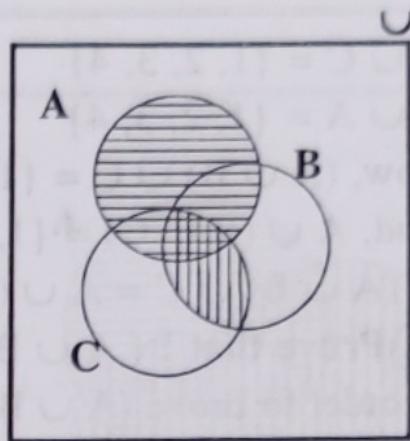


$$(A \cap B) \cap C$$

$$(A \cap B) = \boxed{}$$

$$C = \boxed{ }$$

$$(A \cap B) \cap C = \boxed{ }$$



$$A \cap (B \cap C)$$

$$A = \boxed{}$$

$$B \cap C = \boxed{ }$$

$$A \cap (B \cap C) = \boxed{ }$$

(ii) Union of Sets*

The union of two sets A and B is the set of all elements which belong to either A or B or both and it is denoted by $A \cup B$.

$$\text{i.e. } A \cup B = \{x/x \in A \text{ or } x \in B \text{ or } x \in \text{both } A \text{ and } B\}$$

Properties of union of Sets

$$(i) \quad A \subseteq (A \cup B) \text{ and } B \subseteq (A \cup B)$$

$$(ii) \quad A \cup \phi = A$$

$$(iii) \quad A \cup A = A$$

$$(iv) \quad A \cup B = B \cup A \text{ (Commutative property)}$$

$$(v) \quad (A \cup B) \cup C = A \cup (B \cup C) \text{ (Associative property)}$$

$$(vi) \quad A \cup B = \phi \Rightarrow A = \phi \text{ and } B = \phi$$

$$(vii) \quad A \cap B \subset A \subset A \cup B$$

Illustration 4 : $A = \{x/x \in \mathbb{N}, x^2 < 10\}$,

$$B = \{x/x \in \mathbb{N}, x \leq 1\},$$

$$C = \{x/x \in \mathbb{N}, 1 \leq x < 5\}$$

Find $A \cup B$, $B \cup C$, $C \cup A$ and verify that

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Ans. :

$$A = \{1, 2, 3\}, B = \{1\}, C = \{1, 2, 3, 4\},$$

$$A \cup B = \{1, 2, 3\}$$

Define Union of two sets with illustration.

(March/April 2007, Sept./Oct. 2009 - Old Course)

$$B \cup C = \{1, 2, 3, 4\}$$

$$C \cup A = \{1, 2, 3, 4\}$$

$$\text{Now, } (A \cup B) \cup C = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$$

$$\text{And, } A \cup (B \cup C) = \{1, 2, 3\} \cup \{1, 2, 3, 4\} = \{1, 2, 3, 4\}$$

$$\therefore (A \cup B) \cup C = A \cup (B \cup C)$$

R₂ Prove that : $(A \cup B) \cup C = A \cup (B \cup C)$

In order to prove $(A \cup B) \cup C = A \cup (B \cup C)$ we shall prove that

$$(i) (A \cup B) \cup C \subseteq A \cup (B \cup C)$$

$$(ii) A \cup (B \cup C) \subseteq (A \cup B) \cup C$$

(i) Let x be any element of $(A \cup B) \cup C$

$$\Rightarrow x \in (A \cup B) \cup C$$

$$\Rightarrow x \in (A \cup B) \text{ or } x \in C$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ or } x \in C$$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ or } x \in C)$$

$$\Rightarrow x \in A \text{ or } (x \in B \cup C)$$

$$\Rightarrow x \in A \cup (B \cup C)$$

Thus, every element of $(A \cup B) \cup C$ is also an element of $A \cup (B \cup C)$

$$\therefore (A \cup B) \cup C \subseteq A \cup (B \cup C) \quad \dots (i)$$

(ii) Let y be any element of $A \cup (B \cup C)$.

$$\Rightarrow y \in A \cup (B \cup C)$$

$$\Rightarrow y \in A \text{ or } y \in (B \cup C)$$

$$\Rightarrow y \in A \text{ or } y \in B \text{ or } y \in C$$

$$\Rightarrow (y \in A \text{ or } y \in B) \text{ or } y \in C$$

$$\Rightarrow y \in (A \cup B) \text{ or } y \in C$$

$$\Rightarrow y \in (A \cup B) \cup C$$

Thus, every element of $A \cup (B \cup C)$ is also an element of $(A \cup B) \cup C$

$$\therefore A \cup (B \cup C) \subseteq (A \cup B) \cup C \quad \dots (i)$$

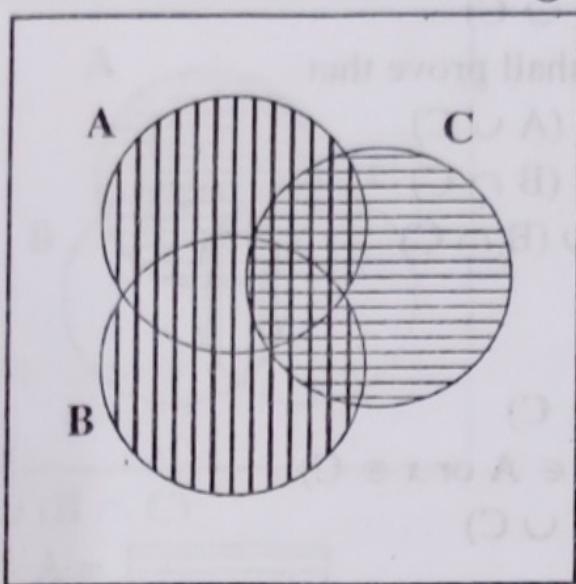
From (i) & (ii) by the definition of equality of sets, we have

$$(A \cup B) \cup C = A \cup (B \cup C)$$

Illustration 5 : Verify that $(A \cup B) \cup C = A \cup (B \cup C)$ by Venn diagrams.

Ans.

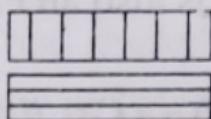
We are required to verify $(A \cup B) \cup C = A \cup (B \cup C)$



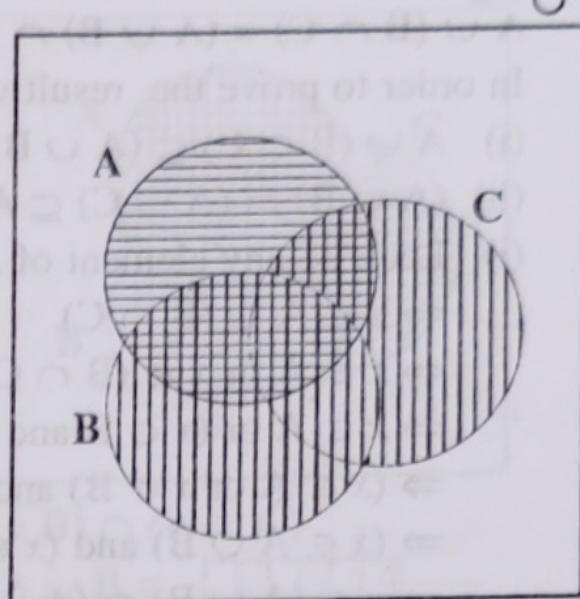
$$(A \cup B) \cup C$$

$$A \cup B =$$

C =



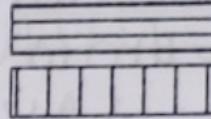
$(A \cup B) \cup C =$ Total shaded region.



$$A \cup (B \cup C)$$

$$A =$$

B ∪ C =



$A \cup (B \cup C) =$ Total shaded region.

7. Distributive Law of Union over Intersection

Union is distributive over intersection

$$\text{i.e. } A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Illustration 6 : If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 5\}$ verify that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans.

$$\text{Here, } A = \{1, 2, 3, 4\}$$

$$B = \{3, 4, 5\}$$

$$C = \{1, 3, 5\}$$

$$\text{L.H.S.} = A \cup (B \cap C)$$

$$= \{1, 2, 3, 4\} \cup \{3, 5\}$$

$$= \{1, 2, 3, 4, 5\}$$

$$\text{R.H.S.} = (A \cup B) \cap (A \cup C)$$

$$= \{1, 2, 3, 4, 5\} \cap \{1, 2, 3, 4, 5\}$$

$$= \{1, 2, 3, 4, 5\}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

R₃ If A, B and C are any three sets prove that*

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

In order to prove the result we shall prove that

$$(i) A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$$

$$(ii) (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C)$$

(i) Let x be any element of $A \cup (B \cap C)$

$$\Rightarrow x \in A \cup (B \cap C)$$

$$\Rightarrow x \in A \text{ or } x \in (B \cap C)$$

$$\Rightarrow x \in A \text{ or } (x \in B \text{ and } x \in C)$$

$$\Rightarrow (x \in A \text{ or } x \in B) \text{ and } (x \in A \text{ or } x \in C)$$

$$\Rightarrow (x \in A \cup B) \text{ and } (x \in A \cup C)$$

$$\Rightarrow x \in (A \cup B) \cap (A \cup C)$$

Thus, every element of $A \cup (B \cap C)$ is also an element of

$$(A \cup B) \cap (A \cup C)$$

$$\therefore A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C) \quad \dots\dots (i)$$

(ii) Let y be any element of $(A \cup B) \cap (A \cup C)$

$$\Rightarrow y \in (A \cup B) \cap (A \cup C)$$

$$\Rightarrow y \in (A \cup B) \text{ and } y \in (A \cup C)$$

$$\Rightarrow (y \in A \text{ or } y \in B) \text{ and } (y \in A \text{ or } y \in C)$$

$$\Rightarrow y \in A \text{ or } y \in B \text{ and } y \in C$$

$$\Rightarrow y \in A \text{ or } y \in (B \cap C)$$

$$\Rightarrow y \in A \cup (B \cap C)$$

Thus, each element of $(A \cup B) \cap (A \cup C)$ is also an element of $A \cup (B \cap C)$

$$\therefore (A \cup B) \cap (A \cup C) \subseteq A \cup (B \cap C) \quad \dots\dots (ii)$$

From (i) & (ii) by the definition of equality of sets we have

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

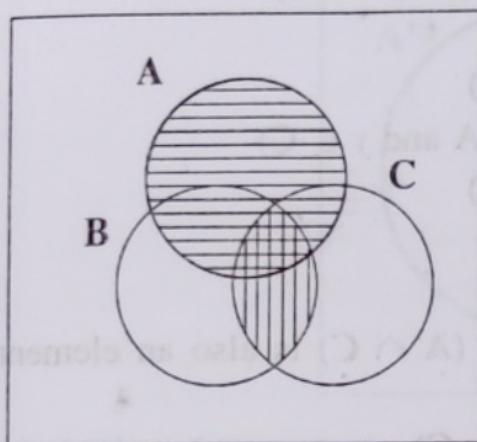
Illustration 7: Verify $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$, using Venn diagram.

* State and prove distributive law of union over intersection.

(Dec, 2007, 10)

* In usual notations, prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(April 2010, Oct./Nov. 2009)

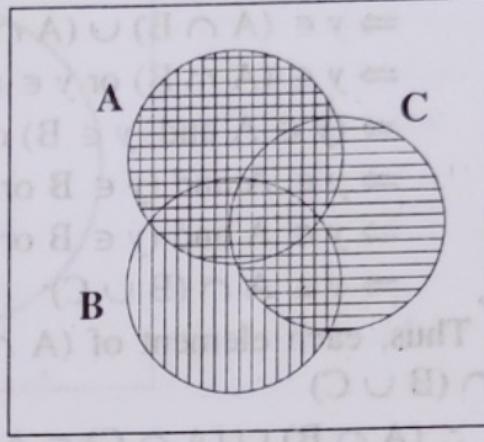


$$A \cup (B \cap C)$$

$$A = \boxed{\text{_____}}$$

$$B \cap C = \boxed{\text{_____}}$$

$$A \cup (B \cap C) = \text{Total shaded area} (A \cup B) \cap (A \cup C) \boxed{\text{_____}}$$



$$(A \cup B) \cap (A \cup C)$$

$$A \cup B = \boxed{\text{_____}}$$

$$A \cup C = \boxed{\text{_____}}$$

8. Distributive Law of Intersection over Union Intersection is distributive over union

i. e. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

R₄ If A, B, & C are any three sets, prove that*

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

In order to prove the given result we shall prove that

(i) $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$

(ii) $(A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C)$

(i) Let x be any element of $A \cap (B \cup C)$

$$\Rightarrow x \in A \cap (B \cup C)$$

$$\Rightarrow x \in A \text{ and } x \in (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (x \in B \text{ or } x \in C)$$

$$\Rightarrow (x \in A \text{ and } x \in B) \text{ or } (x \in A \text{ and } x \in C)$$

$$\Rightarrow x \in (A \cap B) \text{ or } x \in (A \cap C)$$

$$\Rightarrow x \in (A \cap B) \cup (A \cap C)$$

Thus, every element of $A \cap (B \cup C)$ is also an element of

$$(A \cap B) \cup (A \cap C)$$

$$\therefore A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C) \quad \dots(i)$$

* Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(Sep./Oct. 2009 – Old Course)

* State & prove distributive law for intersection over union.

(Dec., 14, March/April, 14)

(ii) Let y be any element of $(A \cap B) \cup (A \cap C)$

$$\Rightarrow y \in (A \cap B) \cup (A \cap C)$$

$$\Rightarrow y \in (A \cap B) \text{ or } y \in (A \cap C)$$

$$\Rightarrow (y \in A \text{ and } y \in B) \text{ or } (y \in A \text{ and } y \in C)$$

$$\Rightarrow y \in A \text{ and } (y \in B \text{ or } y \in C)$$

$$\Rightarrow y \in A \text{ and } (y \in B \text{ or } y \in C)$$

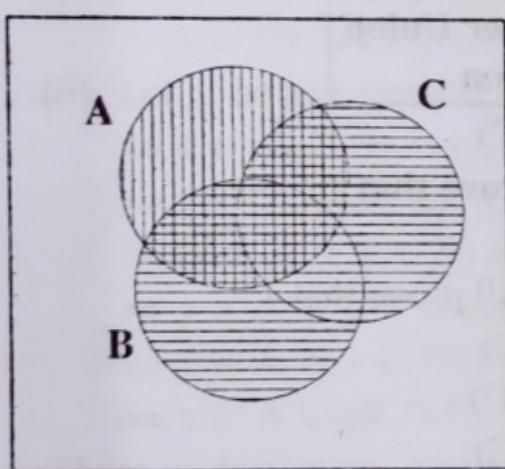
$$\Rightarrow y \in A \cap (B \cup C)$$

Thus, each element of $(A \cap B) \cup (A \cap C)$ is also an element of $A \cap (B \cup C)$

$$\therefore (A \cap B) \cup (A \cap C) \subseteq A \cap (B \cup C) \quad \dots\dots (ii)$$

$$\text{From (i) and (ii)} A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Illustration 8 : Verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ with the help of Venn - diagram.

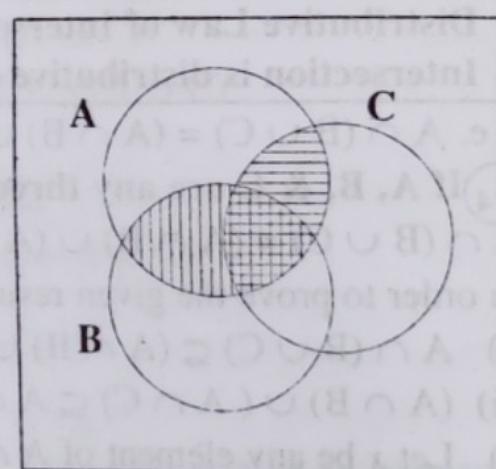


$$A \cap (B \cup C)$$

$$A = \boxed{\square \square \square}$$

$$B \cup C = \boxed{ }$$

$$A \cap (B \cup C) = \boxed{ }$$



$$(A \cap B) \cup (A \cap C)$$

$$A \cap B = \boxed{\square \square \square}$$

$$A \cap C = \boxed{ }$$

$$(A \cap B) \cup (A \cap C) = \text{Shaded region}$$

9. Complement of a Set *

Complement of a set is always with respect to universal set. The complement of a set A is a set of all elements which do not belong to set A but belong to the universal set. The complement of a set A is denoted by i.e., $A' = U - A = \{x/x \in U \text{ but } x \notin A\}$

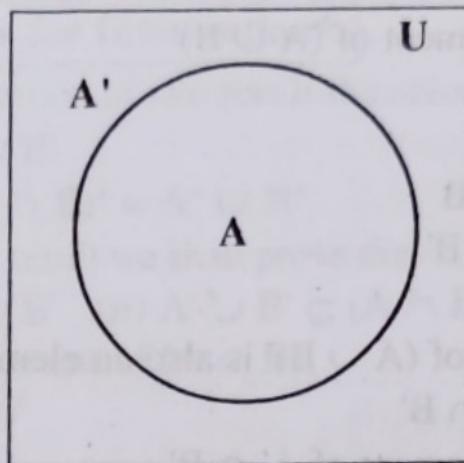
e.g. $U = \{1, 2, 3, 4, 5, 6\}$

$$A = \{2, 4\}$$

$$A' = U - A = \{1, 3, 5, 6\}$$

* Define complement of a set.

(Nov/Dec., 2008, Dec., 2007, J



□ Properties of Complement

- (i) $A \cap A' = \emptyset$
- (ii) $A \cup A' = U$
- (iii) $U' = \emptyset$ and $\emptyset' = U$
- (iv) $(A')' = A$
- (v) If $A \subset B$ then $B' \subset A'$
- (vi) $(A \cap B) \cup (A \cap B') = A$
- (vii) $(A \cup B) \cap (A \cup B') = A$

10. De Morgan's Law for Union

Complement of Union of two sets is the intersection of their complements

$$\text{i.e., } (A \cup B)' = A' \cap B'$$

Illustration 9 : If $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 6\}$, $B = \{3, 5, 6\}$ then verify that (i) $(A \cup B)' = A' \cap B'$

$$\text{Ans. (i)} \quad A \cup B = \{2, 3, 5, 6\}$$

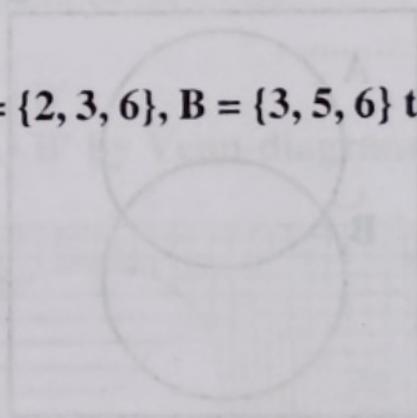
$$(A \cup B)' = \{1, 4\}$$

$$A' = U - A = \{1, 4, 5\}$$

$$B' = U - B = \{1, 2, 4\}$$

$$A' \cap B' = \{1, 4\}$$

$$\therefore (A \cup B)' = A' \cap B'$$



R₅ If A and B be any two sets prove that $(A \cup B)' = A' \cap B'^*$

To prove $(A \cup B)' = A' \cap B'$. We shall prove that

- (i) $(A \cup B)' \subseteq A' \cap B'$
- (ii) $A' \cap B' \subseteq (A \cup B)'$

State and prove De Morgans Law for union. (Dec. – 2007, Mar. 14)

Prove that $(A \cup B)' = A' \cap B'$

(Nov./Dec. - 2008)

14

(i) Let x be any element of $(A \cup B)'$

$$\Rightarrow x \in (A \cup B)'$$

$$\Rightarrow x \notin (A \cup B)$$

$$\Rightarrow x \notin A \text{ and } x \in B$$

$$\Rightarrow x \in A' \text{ and } x \in B'$$

$$\Rightarrow x \in (A' \cap B')$$

i.e., every element of $(A \cup B)'$ is also an element of $A' \cap B'$

$$\therefore (A \cup B)' \subseteq A' \cap B'$$

(ii) Let y be any element of $A' \cap B'$

$$\Rightarrow y \in (A' \cap B')$$

$$\Rightarrow y \in A' \text{ and } y \in B'$$

$$\Rightarrow y \notin A \text{ and } y \notin B$$

$$\Rightarrow y \notin (A \cup B)$$

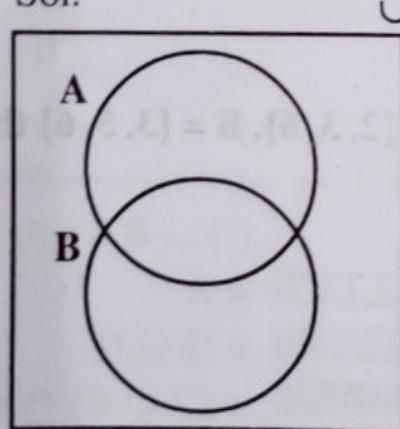
$$\Rightarrow y \in (A \cup B)'$$

i.e., each element of $(A' \cap B')$ is also an element of $(A \cup B)'$

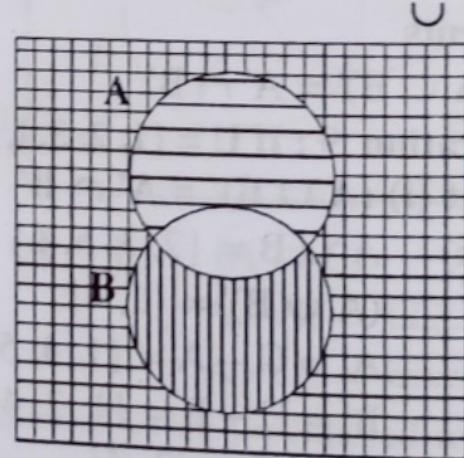
$$\therefore A' \cap B' \subseteq (A \cup B)'$$

From (i) & (ii), we conclude that $(A \cup B)' = (A' \cap B)'$ **Illustration 10 : Verify $(A \cup B)' = A' \cap B'$ with the help of Venn diagram**

Sol.



$$(A \cup B)' = \boxed{\text{_____}}$$



$$A' = \boxed{\text{_____}}$$

$$B' = \boxed{\text{_____}}$$

$$A' \cap B' = \boxed{\text{_____}}$$

11. De Morgan Law for Intersection*

Complement of intersection of two sets is the union of their complements.

$$\text{i.e } (A \cap B)' = A' \cup B'$$

R₆ Prove that $(A \cap B)' = A' \cup B'$

In order to prove the result we shall prove that

$$(i) (A \cap B)' \subseteq A' \cup B' \quad (ii) A' \cup B' \subseteq (A \cap B)'$$

(i) Let x be any element of $(A \cap B)'$

$$\Rightarrow x \in (A \cap B)'$$

$$\Rightarrow x \notin (A \cap B)$$

$$\Rightarrow x \notin A \text{ or } x \notin B$$

$$\Rightarrow x \in A' \text{ or } x \in B'$$

$$\Rightarrow x \in A' \cup B'$$

So, each element of $(A \cap B)'$ is also an elements of $A' \cup B'$

$$\therefore (A \cap B)' \subseteq (A' \cup B') \quad \dots \dots \dots (i)$$

(ii) Let y be any element of $A' \cup B'$

$$\therefore y \in (A' \cup B')$$

$$\Rightarrow y \in A' \text{ or } y \in B'$$

$$\Rightarrow y \notin A \text{ or } y \notin B$$

$$\Rightarrow y \notin A \cap B$$

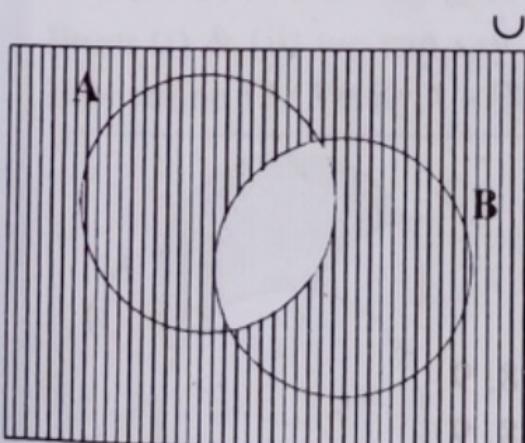
$$\Rightarrow y \in (A \cap B)'$$

So each element of $A' \cup B'$ is also an element of $(A \cap B)'$

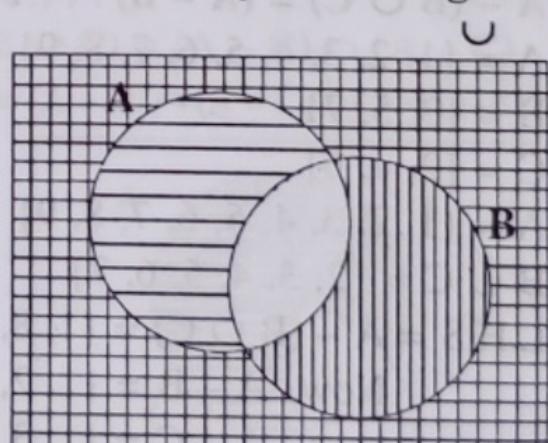
$$\therefore A' \cup B' \subseteq (A \cap B)' \quad \dots \dots \dots (ii)$$

From (i) and (ii) we have $(A \cap B)' = A' \cup B'$

Illustration 11 : Verify $(A \cap B)' = A' \cup B'$ by Venn-diagram



$$(A \cap B)' = \boxed{ }$$



$$A' = \boxed{ }$$

$$B' = \boxed{ }$$

$$A' \cup B' = \text{All shaded area}$$

State and Prove De Morgan's Law for intersection.

(March/April 2009, April 2010)

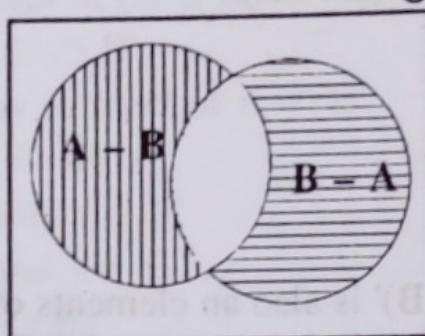
12. Difference of Two Sets*

The difference of two sets A and B is the set of all elements which belong to A but not to B. It is denoted by $A - B$

$$\text{i.e. } A - B = \{x/x \in A \text{ but } x \notin B\}$$

$$B - A = \{x/x \in B \text{ but } x \notin A\}$$

By Venn. diagram



$$A - B = \boxed{\quad \quad \quad}$$

$$B - A = \boxed{\quad \quad \quad}$$

□ Properties of Difference of two sets

- (i) $A - B, A \cap B, B - A$ are mutually disjoint sets,
- (ii) $A - (A - B) = A \cap B$ and $B - (B - A) = A \cap B$
- (iii) $A - B \subseteq A$ and $B - A \subseteq B$

De Morgan's law on difference of Sets :

If A, B, C, are any three sets, then $A - (B \cup C) = (A - B) \cap (A - C)$

Illustration 12 : If $A = \{x/x \leq 9, x \in \mathbb{N}\}$,

$B = \{y / 3 \leq y \leq 7, \text{ and } y \text{ is odd number}\}$

$C = \{z / 1 < z < 7, \text{ and } z \text{ is even number}\}$ then prove that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B = \{3, 5, 7\}$$

$$C = \{2, 4, 6\}$$

$$A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$B \cup C = \{2, 3, 4, 5, 6, 7\}$$

$$L.H.S = A - (B \cup C) = \{1, 8, 9\} \quad \dots \dots \dots \text{(i)}$$

$$\text{Now, } A - B = \{1, 2, 4, 6, 8, 9\}$$

$$A - C = \{1, 3, 5, 7, 8, 9\}$$

$$R.H.S = (A - B) \cap (A - C) = \{1, 8, 9\} \quad \dots \dots \dots \text{(ii)}$$

From (i) and (ii) We have $A - (B \cup C) = (A - B) \cap (A - C)$

* Define : difference of two sets

(March/April 2009; March,

R₇) prove that $A - (B \cup C) = (A - B) \cap (A - C)$

To prove the result we shall prove that

$$(i) A - (B \cup C) \subseteq (A - B) \cap (A - C)$$

$$(ii) (A - B) \cap (A - C) \subseteq A - (B \cup C)$$

(i) Let x be any element of $A - (B \cup C)$

$$\Rightarrow x \in A - (B \cup C)$$

$$\Rightarrow x \in A \text{ but } x \notin (B \cup C)$$

$$\Rightarrow x \in A \text{ but } (x \notin B \text{ and } x \notin C)$$

$$\Rightarrow x \in (A \text{ but } x \notin B) \text{ and } (x \in A \text{ but } x \notin C)$$

$$\Rightarrow x \in (A - B) \text{ and } x \in (A - C)$$

$$\Rightarrow x \in (A - B) \cap (A - C)$$

Thus, each element x of $A - (B \cup C)$ is also an element of

$$(A - B) \cap (A - C)$$

$$\therefore A - (B \cup C) \subseteq (A - B) \cap (A - C) \quad \dots \dots \dots (i)$$

(ii) Let y be any element of $(A - B) \cap (A - C)$

$$\Rightarrow y \in (A - B) \cap (A - C)$$

$$\Rightarrow y \in (A - B) \text{ and } y \in (A - C)$$

$$\Rightarrow (y \in A \text{ but } y \notin B) \text{ and } (y \in A \text{ but } y \notin C)$$

$$\Rightarrow y \in A \text{ but } (y \notin B \text{ and } y \notin C)$$

$$\Rightarrow y \in A \text{ but } y \notin (B \cup C)$$

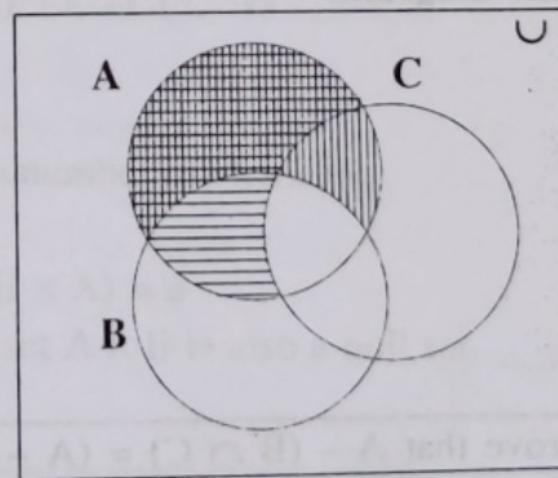
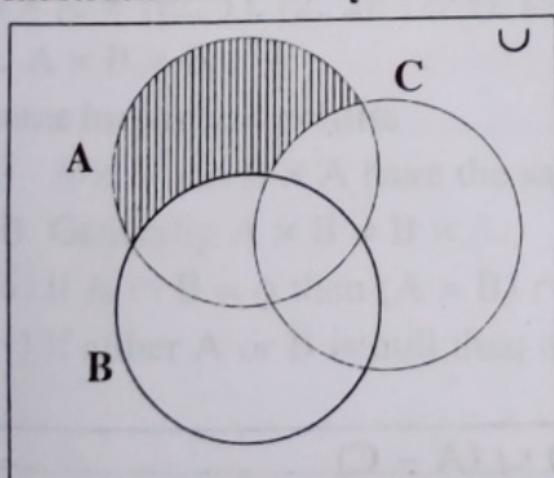
$$\Rightarrow y \in A - (B \cup C)$$

Thus, each element y of $(A - B) \cap (A - C)$ is also an element of $A - (B \cup C)$

$$\therefore (A - B) \cap (A - C) \subseteq A - (B \cup C) \quad \dots \dots \dots (ii)$$

From (i) & (ii) we can say $A - (B \cup C) = (A - B) \cap (A - C)$

Illustration 13 : Verify $A - (B \cup C) = (A - B) \cap (A - C)$ by Venn-diagram



* Prove that $A - (B \cup C) = (A - B) \cap (A - C)$

(March/April 2007, March, 2016)

$$\begin{aligned}
 A - (B \cup C) &= \boxed{} \\
 A - B &= \boxed{} \\
 A - C &= \boxed{} \\
 (A - B) \cap (A - C) &= \boxed{}
 \end{aligned}$$

(R) If A, B, & C be any three sets then prove that*

$$A - (B \cap C) = (A - B) \cup (A - C)$$

In order to prove the result we shall prove that

$$(i) A - (B \cap C) \subseteq (A - B) \cup (A - C) \quad (ii) (A - B) \cup (A - C) \subseteq A - (B \cap C)$$

(i) Let x be any element of $A - (B \cap C)$

$$\Rightarrow x \in A - (B \cap C)$$

$$\Rightarrow x \in A \text{ but } x \notin (B \cap C)$$

$$\Rightarrow x \in A \text{ but } x \in (B \text{ or } C)$$

$$\Rightarrow (x \in A \text{ but } x \notin B) \text{ or } (x \in A \text{ but } x \notin C)$$

$$\Rightarrow x \in (A - B) \text{ or } x \in (A - C)$$

$$\Rightarrow x \in (A - B) \cup (A - C)$$

$$\therefore A - (B \cap C) \subseteq (A - B) \cup (A - C) \quad \dots \text{(i)}$$

(ii) Let y be any element of $(A - B) \cup (A - C)$

$$\Rightarrow y \in (A - B) \cup (A - C)$$

$$\Rightarrow y \in (A - B) \text{ or } y \in (A - C)$$

$$\Rightarrow (y \in A \text{ but } y \notin B) \text{ or } (y \in A \text{ but } y \notin C)$$

$$\Rightarrow y \in A \text{ but } (y \notin B \text{ and } y \notin C)$$

$$\Rightarrow y \in A \text{ but } y \notin (B \cap C)$$

$$\Rightarrow y \in A - (B \cap C)$$

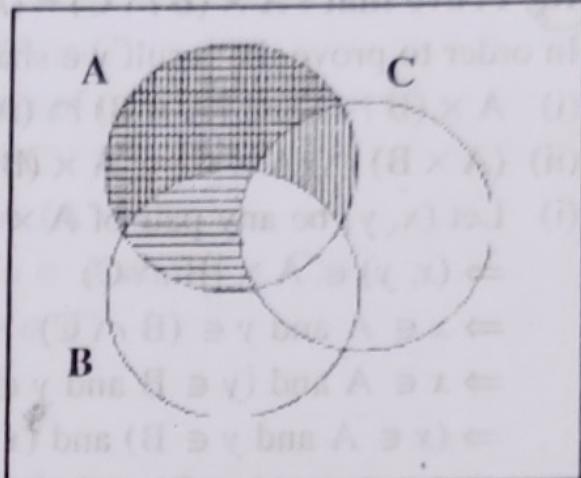
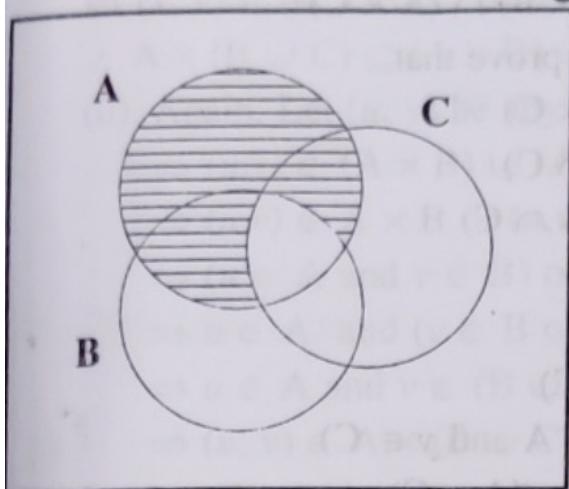
$$\therefore (A - B) \cup (A - C) \subseteq A - (B \cap C) \quad \dots \text{(ii)}$$

From (i) & (ii) we can say that $A - (B \cap C) = (A - B) \cup (A - C)$

Illustration 14 : Verify that $A - (B \cap C) = (A - B) \cup (A - C)$ by Venn-diagram.

* Prove that $A - (B \cap C) = (A - B) \cup (A - C)$

(Nov./Dec. 2008, Dec. 200



$$A - B = \boxed{000}$$

$$A - C = \boxed{0}$$

$$A - (B \cap C) = \boxed{0} \quad (A - B) \cup (A - C) \text{ All shaded area}$$

13. Cartesian Product of Two Sets *

If A and B be are two sets then the set of all ordered pairs whose first element belongs to set A and second element belongs to set B is called the cartesian product of A and B in that order and denoted by $A \times B$. read as A cross B.

In other words if A, B are two sets then the set of all ordered pairs like (x, y) where $x \in A$ and $y \in B$ is called the cartesian product of the sets A and B.

Symbolically,

$$A \times B = \{(x, y) / x \in A \text{ and } y \in B\}$$

$$\text{e.g. } A = \{1, 2, 3\}, B = \{2, 3\}$$

$$A \times B = \{(1, 2), (1, 3), (2, 2), (2, 3), (3, 2), (3, 3)\} \text{ and}$$

$$B \times A = \{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

$$\therefore A \times B \neq B \times A$$

Some important results

(i) $A \times B$ and $B \times A$ have the same number of elements.

(ii) Generally $A \times B \neq B \times A$.

(iii) If $A \cap B = \emptyset$ then $(A \times B) \cap (B \times A) = \emptyset$

(iv) If either A or B is null then the set $A \times B$ is also a null set.

Explain cartesian product of two non empty sets with illustration.

(March, 2015, Dec., 2016)

R₈ Prove that : $A \times (B \cap C) = (A \times B) \cap (A \times C)^*$

In order to prove the result we shall prove that

$$(i) A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$$

$$(ii) (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$$

(i) Let (x, y) be any pair of $A \times (B \cap C)$

$$\Rightarrow (x, y) \in A \times (B \cap C)$$

$$\Rightarrow x \in A \text{ and } y \in (B \cap C)$$

$$\Rightarrow x \in A \text{ and } (y \in B \text{ and } y \in C)$$

$$\Rightarrow (x \in A \text{ and } y \in B) \text{ and } (x \in A \text{ and } y \in C)$$

$$\Rightarrow (x, y) \in (A \times B) \text{ and } (x, y) \in (A \times C)$$

$$\Rightarrow (x, y) \in (A \times B) \cap (A \times C)$$

$$\therefore A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$$

(ii) Let (u, v) be any pair of $(A \times B) \cap (A \times C)$

$$\Rightarrow (u, v) \in (A \times B) \cap (A \times C)$$

$$\Rightarrow (u, v) \in (A \times B) \text{ and } (u, v) \in (A \times C)$$

$$\Rightarrow (u \in A \text{ and } v \in B) \text{ and } (u \in A \text{ and } v \in C)$$

$$\Rightarrow u \in A \text{ and } (v \in B \text{ and } v \in C)$$

$$\Rightarrow u \in A \text{ and } v \in (B \cap C)$$

$$\Rightarrow (u, v) \in A \times (B \cap C)$$

$$\therefore (A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$$

From (i) & (ii) by equality of sets we can say

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

R₉ Prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)^{\#}$

In order to prove the result we shall prove that

$$(i) A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$$

$$(ii) (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$$

(i) Let (x, y) be any pair of $A \times (B \cup C)$

$$\Rightarrow (x, y) \in A \times (B \cup C)$$

$$\Rightarrow x \in A \text{ and } y \in (B \cup C)$$

$$\Rightarrow x \in A \text{ and } (y \in B \text{ or } y \in C)$$

$$\Rightarrow (x \in A \text{ and } y \in B) \text{ or } (x \in A \text{ and } y \in C)$$

$$\Rightarrow (x, y) \in (A \times B) \text{ or } (x, y) \in (A \times C)$$

* Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(March 12, 13, 15, Dec.

In usual notations, prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(Dec., 15)

$$\Rightarrow (x, y) \in (A \times B) \cup (A \times C)$$

$$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C) \quad \dots\dots(i)$$

(ii) Again, Let (u, v) be any pair of $(A \times B) \cup (A \times C)$

$$\Rightarrow (u, v) \in (A \times B) \cup (A \times C)$$

$$\Rightarrow (u, v) \in A \times B \text{ or } (u, v) \in (A \times C)$$

$$\Rightarrow (u \in A \text{ and } v \in B) \text{ or } (u \in A \text{ and } v \in C)$$

$$\Rightarrow u \in A \text{ and } (v \in B \text{ or } v \in C)$$

$$\Rightarrow u \in A \text{ and } v \in (B \cup C)$$

$$\Rightarrow (u, v) \in A \times (B \cup C)$$

$$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C) \quad \dots\dots(ii)$$

From (i) & (ii) we can say that $A \times (B \cup C) = (A \times B) \cup (A \times C)$

14. Number of Elements in Finite Sets :

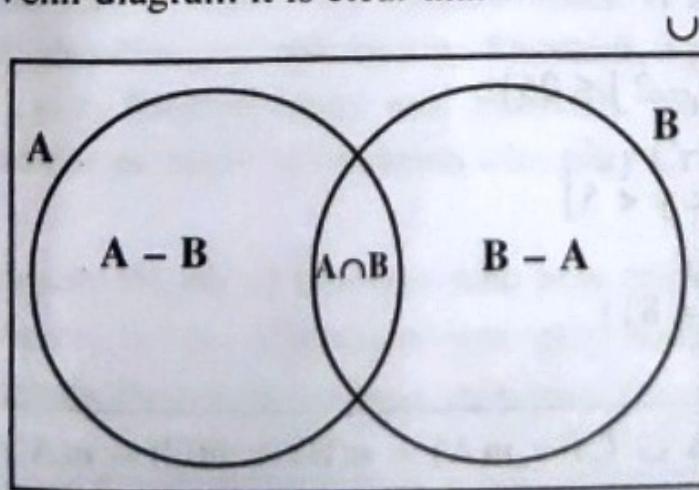
Let A and B be two finite sets. Let $n(A)$, $n(B)$ denote the number of elements in finite sets A and B respectively.

- (1) If A and B are disjoint sets then the number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B)$
- (2) If A and B are not disjoint, sets then the number of elements in $A \cup B$ is $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- (3) If A , B and C are disjoint sets, then the number of elements in $A \cup B \cup C$ is $n(A \cup B \cup C) = n(A) + n(B) + n(C)$
- (4) If A , B and C are not disjoint sets, then the number of elements in $A \cup B \cup C$ is $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Prove that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Let A & B be any two not disjoint sets

from Venn-diagram it is clear that



$(A - B)$, $(A \cap B)$ and $(B - A)$ are disjoint sets.

$$A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$$

$$\therefore n(A \cup B) = n(A - B) + n(A \cap B) + n(B - A)$$

From the venn-diagram it is seen that

$$n(A - B) = n(A) - n(A \cap B)$$

$$n(B - A) = n(B) - n(A \cap B)$$

$$\therefore n(A \cup B) = n(A) - n(A \cap B) + n(A \cap B) + n(B) - n(A \cap B)$$

$$\therefore n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

R₁₀ Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

Let A, B and C are the three finite sets and let D = $(A \cup B)$

$$\therefore n(A \cup B \cup C) = n(D \cup C)$$

$$= n(D) + n(C) - n(D \cap C)$$

$$\text{Also, } n(D) = n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$\begin{aligned} n(A \cup B \cup C) &= n(A) + n(B) - n(A \cap B) + n(C) - n[(A \cup B) \cap C] \\ &= n(A) + n(B) - n(A \cap B) + n(C) - [n(A \cap C) \cup n(B \cap C)] \\ &= n(A) + n(B) - n(A \cap B) + n(C) - \{n(A \cap C) + n(B \cap C) \\ &\quad - n[(A \cap C) \cap (B \cap C)]\} \\ &= n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) \\ &\quad + n(A \cap B \cap C) \end{aligned}$$

$$\therefore n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$$

Illustration 15 : If $A = \{x / x \in N, |x^3 - 2| \leq 25\}$,

$$B = \{y / y \in N, 1 \leq y \leq 5\}$$

$$C = \{z / z \in N, z^4 = 81\}$$

Verify that $= A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

Ans :

$$A = \{x / x \in N, |x^3 - 2| \leq 25\}$$

$$\text{i.e. } A = \{1, 2, 3\}$$

$$B = \{y / y \in N, 1 \leq y \leq 5\}$$

$$\text{i.e. } B = \{2, 3, 4\}$$

$$C = \{z / z \in N, z^4 = 81\}$$

$$\text{i.e. } C = \{3\}$$

* Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$ (Dec., 201)

$$\begin{aligned}
 \text{Now } A &= \{1, 2, 3\} & A \cup B &= \{1, 2, 3, 4\} \\
 B \cap C &= \{3\} & A \cup C &= \{1, 2, 3\} \\
 L.H.S. &= A \cup (B \cap C) = \{1, 2, 3\} \\
 R.H.S. &= (A \cup B) \cap (A \cup C) \\
 &= \{1, 2, 3\} \\
 \therefore L.H.S. &= R.H.S
 \end{aligned}$$

Illustration 16 : If $A = \{a / a^2 - 1 \leq 10, a \in \mathbf{Z}\}$,
 $B = \{b / |b - 1| \leq 2, b \in \mathbf{N}\}$
 $C = \{c / |c| \leq 1, c \in \mathbf{Z}\}$
Prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Ans :

$$\begin{aligned}
 \text{Here } A &= \{-3, -2, -1, 0, 1, 2, 3\}, \\
 B &= \{1, 2\} \\
 C &= \{-1, 0, 1\}
 \end{aligned}$$

$$\text{Now } B \cap C = \{1\}$$

$$L.H.S. = A \times (B \cap C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$$

$$A \times B = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1), (-3, 2), (-2, 2), (-1, 2), (0, 2), (1, 2), (2, 2), (3, 2)\}$$

$$A \times C = \{(-3, -1), (-2, -1), (-1, -1), (0, -1), (1, -1), (2, -1), (3, -1), (-3, 0), (-2, 0), (-1, 0), (0, 0), (1, 0), (2, 0), (3, 0), (-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$$

$$R.H.S. = (A \times B) \cap (A \times C) = \{(-3, 1), (-2, 1), (-1, 1), (0, 1), (1, 1), (2, 1), (3, 1)\}$$

$$\therefore L.H.S. = R.H.S$$

Illustration 17 : In a class of 42 students, each play atleast one of the three games Cricket, Hockey and Football. It is found that 14 play Cricket, 20 play Hockey and 24 play Football, 3 play both Cricket and Football, 2 play both Hockey and Football. None play all the three games. Find the number of students who play Cricket but not Hockey.

Ans.

Let C denote the set of students who play cricket $\Rightarrow n(C) = 14$.

Let H denote the set of students who play Hockey $\Rightarrow n(H) = 20$

Let F denote the set of students who play Football $\Rightarrow n(F) = 24$

Also, $n(C \cup H \cup F) = 42$

$C \cap F = \{\text{students who play both Cricket \& Football}\} \Rightarrow n(C \cap F) = 2$
 $H \cap F = \{\text{students who play Hockey \& Football}\} \Rightarrow n(H \cap F) = 2$
 $C \cap H \cap F = \{\text{students who play Cricket, Hockey \& Football}\}$
 $\Rightarrow n(C \cap H \cap F) = 0$

Here, we are required to find the number of students who play cricket but not Hockey i.e. $n(C \cap H')$

$$\begin{aligned} n(C \cup H \cup F) &= n(C) + n(H) + n(F) - n(C \cap H) \\ &\quad - n(H \cap F) + n(F \cap C) + n(C \cap H \cap F) \\ \therefore 42 &= 14 + 20 + 24 - n(C \cap H) - 2 - 3 + 0 \\ \therefore 42 &= 53 - n(C \cap H) \\ \therefore n(C \cap H) &= 11 \end{aligned}$$

$$\text{Now } n(C) = n(C \cap H') + n(C \cap H)$$

$$\therefore 14 = n(C \cap H') + 11$$

$$\therefore n(C \cap H') = 3$$

$\therefore 3$ students play Cricket but not Hockey.

Illustration 18 : If $U = \{x / x \in N, x \leq 10\}$,

$$A = \{x / x \in N, x^2 \leq 10\}$$

$$B = \{2, 4, 6\}$$

$$C = \{x / x^3 - 3x^2 - 4x = 0\}$$

Verify that, (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$

$$(ii) A' - B' = B - A$$

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$$

$$A = \{1, 2, 3\}, B = \{2, 4, 6\}, C = \{0, -1, 4\}$$

$$[\therefore x^3 - 3x^2 - 4x = 0,$$

$$x(x^2 - 3x - 4) = 0,$$

$$x(x-4)(x+1) = 0,$$

$$x = 0 \text{ or } x = 4 \text{ or } x = -1]$$

$$(i) B - C = \{2, 6\}$$

$$\therefore A \cap (B - C) = \{2\} \text{(i)}$$

$$\text{Now } A \cap B = \{2\}, A \cap C = \{\},$$

$$(A \cap B) - (A \cap C) = \{2\} \text{(ii)}$$

From (i) and (ii)

$$\therefore A \cap (B - C) = (A \cap B) - (A \cap C)$$

$$(ii) A' = \{4, 5, 6, 7, 8, 9, 10\}$$

$$B' = \{1, 3, 5, 7, 8, 9, 10\}$$

$$\text{L.H.S} = A' - B' = \{4, 6\}$$

$$\text{R.H.S} = B - A = \{4, 6\}$$

$$\therefore \text{LHS} = \text{RHS}$$

Illustration 19 : Examine the validity of the following statements and justify your answer :

- (i) If $A = \{x / x^3 - 5x^2 + 6x = 0\}$
 $B = \{x / x \in \mathbb{Z}, |x| < 1\}$ then $A \cap B = \emptyset$
- (ii) If $A - B = A$, Then $A \cap B = A$
- (iii) If $A = \{\emptyset, a\}$ Then $\{\emptyset\} \in P(A)$
- (iv) If $x \notin (A \cup B)$ Then $x \notin A$ or $x \notin B$
- (v) If $A = \{a, b, c\}$, $B = \{2, 5, 7\}$
then $B \times A = \{2a, 5b, 7c\}$

Ans. :

- (i) **False** : Here $A = \{0, 2, 3\}$, $B = \{0\}$

$$\therefore A \cap B = \{0\}, \text{ and not } \emptyset$$

\therefore Statement is false

- (ii) **False** : $A - B = A$

i.e. No element of B is in A Hence $A \cap B = \emptyset$. The statement is therefore false.

- (iii) **True** : because $P(A) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b, c\}\}$

Thus $\{\emptyset\} \in P(A)$ is true

- (iv) **False** : because If $x \notin (A \cup B)$

$$\Rightarrow x \notin A \text{ and } x \notin B.$$

- (v) **False** : because

$$A = \{a, b, c\}, B = \{2, 5, 7\}$$

Then $B \times A = \{(2, a), (2, b), (2, c), (5, a), (5, b), (5, c), (7, a), (7, b), (7, c)\}$

Illustration 20 : If $A = [1, 3, a, \{1\}, \{1, a\}]$, state whether the following statements are true or false.

(i) $1 \in A$,

(ii) $\{1\} \in A$,

(iii) $\emptyset \in A$,

(iv) $\{1, a\} \subset A$

(v) $\emptyset \subset A$

(vi) $\{1, a\} \in A$

Ans. :

- (i) $1 \in A$, True
- (ii) $\{1\} \in A$, True
- (iii) $\emptyset \in A$, False
- (iv) $\{1, a\} \subset A$, True,
- (v) $\emptyset \subset A$, True
- (vi) $\{1, a\} \in A$, True

Illustration 21 : Prove that $(A')' = A$

We shall prove that

- (i) $(A')' \subseteq A$ (ii) $A \subseteq (A')'$
- (i) Let x be any element of $(A')'$ $\Rightarrow x \in (A')'$
 $\Rightarrow x \notin A'$
 $\Rightarrow x \in A$
 $\Rightarrow (A')' \subseteq A$ (i)
- (ii) Let y be any element of A $\Rightarrow y \in A$
 $\Rightarrow y \notin A'$
 $\Rightarrow y \in (A')'$
 $\Rightarrow A \subseteq (A')'$ (ii)

'From (i) & (ii), $(A')' = A$ **Illustration 22 : Prove that : $A - (A - B) = A \cap B^{\#}$** We shall prove that (i) $A - (A - B) \subseteq (A \cap B)$

(ii) $A \cap B \subseteq A - (A - B)$

- (i) Let x be any element of $A - (A - B)$
 $\Rightarrow x \in A - (A - B)$
 $\Rightarrow x \in A$ but $x \notin (A - B)$
 $\Rightarrow x \in A$ but ($x \notin A$ and $x \in B$)
 $\Rightarrow x \in A$ and $x \in B$
 $\Rightarrow x \in A \cap B$
 $\Rightarrow A - (A - B) \subseteq A \cap B$ (i)
- (ii) Let y be any element of $A \cap B$
 $\Rightarrow y \in (A \cap B)$
 $\Rightarrow y \in A$ and $y \in B$
 $\Rightarrow y \in A$ but ($y \notin A$ and $y \in B$)
 $\Rightarrow y \in A$ but $y \notin (A - B)$

* Prove that $(A')' = A$

(March, 2011, Dec., 2011)

Prove that $A - (A - B) = A \cap B$

(Dec., 2011)

$$\Rightarrow y \in A - (A - B)$$

$$\Rightarrow A \cap B \subseteq (A - (A - B))$$

.....(ii)

From (i) & (ii)

$$A - (A - B) = (A \cap B)$$

EXERCISE

1. Define the following and give one example of each.
 - (1) Set (2) Enumeration method (3) Set builder method (4) Finite set
 - (5) Infinite set (6) Empty set (7) Singleton set (8) Equal sets
 - (9) Equivalent sets (10) Disjoint sets (11) Subsets (12) Proper subsets
 - (13) Power set (14) Universal set (15) Union of sets
 - (16) Intersection of sets (17) Complement of a set (18) Difference of sets (19) Catesian product of two sets.
2. If $A = \{2, 3, 1\}$, $U = \{0, 1, 2, 3, 4\}$ State whether following statements are correct or incorrect giving reasons.
 - (i) $\{0\} \in A'$ (ii) $\phi \in A'$ (iii) $\{0\} \subset A'$ (iv) $0 \in A'$ (v) $0 \subset A'$.
3. Let $A = \{x / x \text{ is a letter in English alphabets}\}$ be the universal set.
 $V = \{x / x \text{ is a vowel}\}$, $C = \{x / x \text{ is a consonant}\}$
 $N = \{x / x \text{ is a letter in your full name}\}$
 - (a) Describe the above four sets by listing the elements of each set.
 - (b) List the elements of the following sets.
 - (i) C' (ii) $C \cap N'$ (iii) $N \cap V'$ (iv) $V \cup C$ (v) $N \cap C$ (vi) $N \cup V$.
4. If $A = \{a, b, c, d, e, f\}$, $B = \{a, e, i, o, u\}$, $C = \{m, n, o, p, q, r, s, t, u\}$ find,
 - (i) $A \cup B$ (ii) $A \cup C$ (iii) $B \cup C$ (iv) $A - B$ (v) $A \cap B$
 - (vi) $B \cap C$ (vii) $A \cup (B - C)$ (viii) $A \cup B \cup C$ (ix) $A \cap B \cap C$

[Ans : (i) {a, b, c, d, e, f, i, o, u}
(ii) {a, b, c, d, e, f, m, n, o, p, q, r, s, t, u}
(iii) {a, e, i, o, u, m, n, p, q, r, s, t}
(iv) {b, c, d, f}
(v) {a, e}
(vi) {o, u}
(vii) {a, b, c, d, e, f, i}
(viii) {a, c, d, e, f, i, o, u, m, n, p, q, r, s, t}
(ix) {ϕ}]

5. If the Universal set $U = \{x / x \text{ is a positive integer } < 25\}$, $A = \{2, 6, 8, 14, 22\}$, $B = \{4, 8, 10, 14\}$, $C = \{6, 10, 12, 14, 18, 20\}$ Verify the relations.
 (i) $(A \cap B)' = A' \cup B'$ (ii) $(B' \cap C) \cup (A' \cap C) = C \cap (A' \cup B')$
6. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4, 5\}$ and $C = \{2, 4, 6, 8\}$ verify
 (i) $A \cup B = (A - B) \cup B$ (ii) $A - (A - B) = A \cap B$
 (iii) $A \cap (B - C) = (A \cap B) - (A \cap C)$
7. If A has 32 elements, B has 42 elements and $A \cup B$ has 62 elements find the number of elements in $A \cap B$. [Ans : 12]
8. A town has a total population of 50000 persons and of them 28000 read, "Gujarat Samachar" and 23000 read "Sandesh" while 4000 read both the papers. Indicate how many read neither "Gujarat Samachar" nor "Sandesh"? [Ans : 3000]
9. If $A = \{x / x^2 - 17x + 60 = 0\}$
 $B = \{x / x^2 - 7x + 12 = 0\}$ find $A \cup B$ and $A \cap B$.
 [Ans : {3, 4, 5, 12}, \emptyset]
10. In a college, there are 500 girls and of them 300 have taken Economics and 250 have taken Mathematics. How many of them have taken both the subjects? All girls have taken atleast one of these two subjects. [Ans : 50]
11. Let the universal set $U = \{x / 3 \leq x \leq 13, x \in \mathbb{N}\}$
 $A = \{y / 2 < y < 7, y \in \mathbb{N}\}$
 $B = \{3, 5, 7, 9\}$, find
 (1) A' (2) B' (3) $(A \cup B)'$
 [Ans.: (1) {7, 8, 9, 10, 11, 12, 13}
 (2) {4, 6, 8, 10, 11, 12, 13}
 (3) {8, 10, 11, 12, 13}]
12. If $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$ and $C = \{3, 4, 7, 8, 11, 12\}$ then show that.
 (1) $(A \cup B) \cup C = A \cup (B \cup C)$ (2) $(A \cap B) \cap C = A \cap (B \cap C)$
13. If $A = \{2, 3, 4\}$, $B = \{1, 3, 4\}$, $S = \{1, 2, 3\}$, $T = \{1, 3, 5\}$ verify that, $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$
14. If $A = \{2, 3, 4\}$, $B = \{3, 4, 5, 6\}$, $C = \{2, 4, 6, 8\}$ verify that
 (i) $A \cup B = (A - B) \cup B$ (ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$
15. If $A = \{x / x \in \mathbb{N}, 2 < x < 6\}$, $B = \{x / x \in \mathbb{N}, x^2 < 5x\}$ prove that
 (i) $(A \cup B)' = A' \cap B'$ (ii) $(A \cap B)' = (A' \cup B')$

16. If $A = \{5, 6, 7\}$, $B = \{7, 8\}$, $C = \{5, 8\}$ verify that (i) $A \times (B - C) = (A \times B) - (A \times C)$ (ii) $A \times (B \cap C) = (A \times B) \cap (A \times C)$
17. If A & B any two sets then prove that $A \cup B = (A - B) \cup B$
18. If $A = \{a, c\}$, $B = \{b, d\}$, $C = \{b, c, e\}$ then verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$
19. Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$
20. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$, $D = \{2, 4, 5\}$ verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
21. If $U = \{a, b, c, 1, 2, 3\}$ is a universal set, $A = \{a, b, c\}$, $B = \{1, 2, 3\}$, $C = \{a, 1, 2\}$, $D = \{a, b, 3\}$ then find the following sets (i) $(A - B) \cap (B - A)$ (ii) $(C \cup D)'$ (iii) $(A - D) \cap (A \cap D)$
[Ans : (i) \emptyset (ii) {c} (iii) \emptyset]
22. Prove the $(A \cup B) \cup (A \cup B)' = A$
23. If $A = [1, 4]$, $B = \{2, 3\}$, $C = \{3, 5\}$,
 Prove that $A \times B \neq B \times A$, Also find $(A \times B) \cap (A \times C)$.
24. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ $A = \{2, 4, 6, 8\}$, $B = \{3, 6, 9\}$ prove that, $(A \cap B)' = A' \cup B'$.
25. If $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 5\}$ then prove that,
 $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
26. If $A = \{2, 3\}$ then find A^2
[Ans : {(2, 2), (2, 3), (3, 2), (3, 3)}
27. Prove that $(A \cap B) \cup (A \cap B)' = A$
28. If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $S = \{1, 3, 4\}$, $T = \{2, 4, 5\}$ verify that, $(A \times B) \cap (S \times T) = (A \cap S) \times (B \cap T)$
29. Prove that $A \cap (B - C) = (A \cap B) - (A \cap C)$,
30. If the universal set is $X = \{x / x \in N, 1 \leq x \leq 12\}$ and $A = \{1, 9, 10\}$, $B = \{3, 4, 6, 11, 12\}$, $C = \{2, 5, 6\}$ are subsets of X, find the sets $A \cup (B \cap C)$ and $(A \cup B) \cap (A \cup C)$.
[Ans : {1, 6, 9, 10}]
31. If $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, $C = \{1, 3, 1\}$
 prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.
32. If $A = \{2, 4\}$, $B = \{2, 4, 6\}$ find $A \times B$, $A \times A$ and $B \times B$
[Ans : $A \times B = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6)\}$
 $A \times A = \{(2, 2), (2, 4), (4, 2), (4, 4)\}$
 $B \times B = \{(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)\}$

33. $U = \{1, 2, 3, 4, 5, 6\}$, $A = \{2, 3, 6\}$, $B = \{3, 5, 6\}$

Prove that $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

34. N is the universal set $S = \{x / x > 40\}$ and $T = \{x / 40 < x < 50\}$
then find $S \cup T$, $S \cap T$, and T'

$$[Ans : S \cup T = \{41, 42, \dots, 48, 49, \dots\}]$$

$$S \cap T = \{41, 42, \dots, 49\}$$

$$T' = \{1, 2, 3, \dots, 40, 50, 51, 52, \dots\}$$

University Questions

Part I : Theory Questions

- (1) State and Prove De Morgan's Law for intersection

(March/April 2009, April, 2010)

- (2) In usual notation prove that $(A \cap B) \cap C = A \cap (B \cap C)$

(March/April 2007 and 2009)

- (3) In usual notations prove that

$$A - (B \cap C) = (A - B) \cup (A - C)$$

(Nov./Dec. 2008) (Dec. 2009)

- (4) Define complement of set and prove that if A, B be any two sets then $(A \cup B)' = A' \cap B'$

(Nov./Dec. 2008)

- (5) In usual notations prove that

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(April 2010; Oct./Nov. 2009)

- (6) In usual notations prove that

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

(Sep./Oct. 2009 – Old Course)

- (7) Prove that for any set A, B and C

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

(Mar. 12, 13, 15, Dec., 16)

- (8) State and prove distributive law of Union over intersection.

(Dec. 2007, 2010)

- (9) State and prove De Morgan's Law for Union.

(Dec. 2007; Mar. 2010)

- (10) In usual notations prove that

$$A - (B \cup C) = (A - B) \cap (A - C)$$

- (March/April 2007, 2009)

- (11) State and prove distribution law for intersection over union.

(Dec., 14, March 15)

(12) In usual notation, prove that $A \times (B \cup C) = (A \times B) \cup (A \times C)$
(Dec., 14, 15)

(13) Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$ *(Dec., 12)*

University Question

Part II : Problems

- (1) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$ then verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$. *(March/April 2009)*
- (2) Let $\cup = \{x/2 < x < 14, x \in \mathbb{N}\}$, $A = \{z/3 \leq z < 8; z \in \mathbb{N}\}$ and $B = \{m/m \text{ is odd integer between 1 and 11}\}$ then find
 (i) A' (ii) B' (iii) $(A \cup B)'$. *(March/April 2009, Nov./Dec. 2008)*
 [Ans. : (i) $A' = \{3, 8, 9, 10, 11, 12, 13, 14\}$
 (ii) $B' = \{4, 6, 8, 10, 11, 12, 13, 14\}$
 (iii) $(A \cup B)' = \{8, 10, 11, 12, 13, 14\}$]
- (3) If $A = \{x/x \leq 3; x \in \mathbb{N}\}$, $B = \{x/1 < x \leq 4, x \in \mathbb{N}\}$ and $C = \{1, 3, 4\}$ then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. *(Nov./Dec. 2008)*
- (4) If $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 4\}$ and $C = \{2, 4, 6, 8\}$ then verify
 (i) $A \cup B = (A - B) \cup B$
 (ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$ *(Nov./Dec. 2008)*
- (5) If $A = \{x/2 \leq x < 8, x \text{ is an odd integer}\}$
 $B = \{x/2 < x \leq 6, x \text{ is an even integer}\}$ then find $A \times B$ and $B \times A$ *(Nov./Dec. 2008)*
 [Ans. : $A \times B = \{\{3, 4\}, \{3, 6\}, \{5, 4\}, \{5, 6\}, \{7, 4\}, \{7, 6\}\}$
 $B \times A = \{\{4, 3\}, \{4, 5\}, \{4, 7\}, \{6, 3\}, \{6, 5\}, \{6, 7\}\}$]
- (6) If $A = \{a, d, e\}$, $B = \{e, f\}$, $C = \{a, f, g\}$ then verify that $A \times (B - C) = (A \times B) - (A \times C)$ *(April, 2010)*
- (7) If $A = \{x/x^2 - 17x + 16 = 0\}$, $B = \{x/x^2 - 7x + 12 = 0\}$ then find $(A \cup B) - (A \cap B)$ *(April, 2010)*
 [Ans. : $\{1, 3, 4, 16\}$]
- (8) If $A = \{x/x \in \mathbb{N}; 2 < x < 6\}$,
 $B = \{x/x \in \mathbb{N}; x^2 < 5x\}$
 $U = \{x/x \in \mathbb{N}; x \leq 10\}$ then prove that
 $(A \cup B)' = A' \cap B'$ *(April 2010, Oct./Nov. 2009 –)*,
(March/April 2007)

- (9) If $A = \{x/x \leq 3; x \in N\}$, $B = \{x/1 < x \leq 5, x \in N\}$ and $C = \{x/x \text{ is an even positive integer less than } 10\}$ then verify $A \cap (B - C) = (A \cap B) - (A \cap C)$ (April 2010)
- (10) If $A = \{1, 3\}$, $B = \{3, 5\}$ and $C = \{3, 5, 6\}$ then verify $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (Oct. Nov./2009)
- (11) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$ and $D = \{2, 4, 5\}$ then find
 (i) $(A - B) \cap (C - D)$ (ii) $A \cup (B \cap C) \cup D$
 (Oct./Nov. 2009 –
 [Ans. : (i) {1} (ii) {1, 2, 3, 4, 5}]
- (12) Let $U = \{x/1 \leq x \leq 15, x \in N\}$, $A = \{y/3 < y < 7; y \in N\}$ and $B = \{2, 4, 6, 8\}$ then find (i) $A' \cup B'$ (ii) $(A \cap B)'$ (iii) $A' - B'$. (Oct./Nov. 2009 – New Course)
 [Ans. : (i) {1, 2, 3, 5, 7, 8....15}
 (ii) {1, 2, 3, 5, 7, 8.....15}
 (iii) {2}]
- (13) If $A = \{x/2 < x < 8, x \text{ is an odd number}\}$ and $B = \{x/2 \leq x \leq 6, x \text{ is an even number}\}$ then find $A \times B$ and $B \times A$. (Sept./Oct. 2009 –
 [Ans. : $A \times B = \{\{3, 2\}, \{3, 4\}, \{3, 6\}, \{5, 2\}, \{5, 4\}, \{5, 6\}, \{7, 2\}, \{7, 4\}, \{7, 6\}\}$
 $B \times A = \{\{2, 3\}, \{2, 5\}, \{2, 7\}, \{4, 3\}, \{4, 5\}, \{4, 7\}, \{6, 3\}, \{6, 5\}, \{6, 7\}\}$]
- (14) If $U = \{x/x \in N; x \leq 10\}$, $A = \{x/x \in N; x^2 < 10\}$ and $B = \{x/x \in N; 2 \leq x \leq 5\}$ then prove that $(A \cup B)' = A' \cap B'$ (Sep./Oct. 2009 –
- (15) If $A = \{x/x^2 - 5x + 4 = 0\}$ and $B = \{x/x^2 - 4x + 3 = 0\}$ then find $A \cup B$ and $A \cap B$ (Sep./Oct. 2009 –
 [Ans. : $A \cup B = \{1, 3, 4\}$ and $A \cap B = \{1\}$]
- (16) If $A = \{x/x \leq 10, x \in N\}$, $B = \{1, 3, 5, 7\}$ and $C = \{z/2 \leq z \leq 8, z \text{ is an even number}\}$ then prove that $A - (B \cup C) = (A - B) \cap (A - C)$. (Dec. 2007)
- (17) If $A = \{5, 6, 7\}$, $B = \{7, 8\}$, $C = \{5, 8\}$ then verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$ (Dec. 2007)
- (18) If $A = \{x/x^2 - 17x + 60 = 0\}$, $B = \{x/x^2 - 7x + 12 = 0\}$ then find $A \cup B$ and $A \cap B$. (Dec. 2007)
 [Ans. : $A \cup B = \{3, 4, 5, 12\}$, $A \cap B = \emptyset$]

- (19) If $U = \{x/x \text{ is a positive integer}\}$, $A = \{2, 6, 8, 14, 22\}$,
 $B = \{4, 8, 10, 14\}$, $C = \{6, 10, 12, 14, 18, 20\}$ then verify that
 $(B' \cap C) \cup (A' \cap C) = C \cap (A' \cap B')$ (Dec. 2007)
- (20) If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$, $C = \{1, 3, 4\}$, $D = \{2, 4, 5\}$ then
verify that $(A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D)$
(March/April 2007)
- (21) Let the universal set $U = \{x/3 \leq x \leq 13, x \in N\}$,
 $A = \{y/2 < y < 7, y \in N\}$, $B = \{3, 5, 7, 9\}$ find
(1) A' (2) B' (March/April 2007)
[Ans. : (1) $A' = \{7, 8, 9, \dots, 13\}$
(2) $B' = \{4, 6, 8, 10, 11, 12, 13\}$]
- (22) If $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{3, 5\}$ then prove that
 $A \times (B \cup C) = (A \times B) \cup (A \times C)$ (March/April 2007)
- (23) If $A = \{x/x^2 - 17x + 60 = 0\}$ and $B = \{x/x^2 - 7x + 12 = 0\}$
then find $(A \cup B) - (A \cap B)$. (Oct./Nov. 2009 – New Course)
[Ans. : {3, 4, 5, 12}]
- (24) If $A = \{1, 2, 3\}$, $B = \{2, 3, 5, 4\}$ and $C = \{2, 4, 6, 8\}$ then verify
(i) $A \cup B = (A - B) \cup B$
(ii) $A \cap (B - C) = (A \cap B) - (A \cap C)$.
(Nov.Dec. 2008 – New Course)
- (25) If $A = \{x/x \leq 5; x \in N\}$, $B = \{x/x^2 \leq 49; x \in Z\}$ and $C = \{x/-1 \leq x \leq 4; x \in N\}$ then verify $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$.
(March 15)
- (26) If $A = \{x/x \leq 3; x \in N\}$, $B = \{x/-1 \leq x \leq 2; x \in Z\}$ and $C = \{x/x^2 - 5x + 6 = 0; x \in R\}$ considering $U = R$, verify De Morgan's law for intersection.
(March 15)
- (27) If $A = \{x/x \leq 3, x \in N\}$, $B = \{x/2 \leq x \leq 4, x \in N\}$, and $C = \{1, 3, 4\}$
then prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (March 15)
- (28) In a college there are 500 girls and of them 300 have taken Economics and 250 have taken Mathematics. How many of them have taken both the subjects ? All girls have taken at least one of these two subjects. (March 15)
- (29) If $A = \{a/a^2 - 1 < 10; a \in N\}$, $B = \{b/b - 1 < 2; b \in N\}$ and $C = \{c/|c| \leq 1; c \in Z\}$ that verify that $A \times (B \cup C) = (A \times B) \cup (A \times C)$.
(Dec. 15)

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- (30) If $U = \{x/x \leq 10; x \in N\}$, $A = \{x/x^2 < 10; x \in N\}$, $B = \{2, 4, 6\}$ and $C = \{x/x^3 - 3x^2 - 4x = 0; x \in R\}$ Then verify that
 (i) $A \cap (B - C) = (A \cap B) - (A \cap C)$.
 (ii) $A' - B' = B - A$. (Dec. 15)
- (31) Prove that
 (i) $A - (A - B) = A \cap B$
 (ii) $(A')' = A$. (Dec. 15)
- (32) A town has a total population of 50,000 persons and of them 28,000 read 'Gujarat Samachar' and 23000 read 'Sandesh' while 4000 read both the papers. Prove that there are 3000 persons who read neither of both. (Dec. 15)

University Questions

Part III : Objective/Short answer questions.

- (1) Define difference of two sets (March/April 2009)
 (2) How do you separate proper and improper subset of a set ? Give illustration. (Nov./Dec. 2008)
 (3) Define equivalent Sets. (Nov./Dec. 2008, Dec. 2015)
 (4) Explain proper and improper subsets (April 2010)
 (5) If $A = \{2, 4, 6, 8\}$, $B = \{1, 3, 4, 5, 6\}$ then find $(A - B) \cup (B - A)$. (April 2010)
[Ans. : {1, 2, 3, 5, 8}]
- (6) Define : equal sets (Oct./Nov. 09 – New Course)
 (7) If $A = \{1, 2, 3, 4\}$, $B = \{1, 2, 5, 9\}$ then find $(A - B) \cap (B - A)$ (Oct./Nov. 09 – New Course)
[Ans. : \emptyset]
- (8) Define union and intersection of two sets (Sep./Oct. 2009 – Old Course)
 (9) Define complement of a set with illustration. (Dec. 2007)
 (10) If $A = \{1, 4\}$, $B = \{2, 3\}$ then find $(A \times B)$. (Dec. 2007)
[Ans. : $A \times B = \{(1, 2), (1, 3), (4, 2), (4, 3)\}$]
- (11) Define union of two sets with illustration. (March/April 2007)
 (12) If $A = \{1, 2, 3\}$ then write the power set of A. (March/April 2007)
[Ans. : $P(A) = \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$]
- (13) Explain Cartisian product of two non-empty sets with illustration. (March 2015)