

An Introduction to Wavelet Analysis with Applications to Image and JPEG 2000

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Image processing based on continuous or discrete transforms are classic topics for researchers, for it is widely used in telecommunications, information and electronics. Nowadays, the wavelet theories make up very popular methods of the image compression, denoising, etc. Considering that other literature focus more on the pure mathematics, this paper presents an overview of the wavelet, including the continuous wavelet transform and the discrete wavelet transform and discusses about its application in image processing, consisting of the multiresolution analysis. Then, the Joint Photographic Experts Group 2000 (JPEG 2000) - a new standard for image compression is paid much attention to. Finally, we use Matlab to verify the theories before and implement some basic image processing function, like: image compression, enhancement and passivation.

CCS Concepts: • Theory of computation → Theory and algorithms for application domains;

KEYWORDS: Wavelets, Haar Transform, image processing, JPEG 2000, multiresolution

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1 INTRODUCTION

The computer has developed rapidly the field of multi-media, so that it requires the high performance and low memory space. Nowadays, the image processing and analysis based on continuous or discrete transforms are the classic processing techniques [1], which is widely used in telecommunication, information and electronics [2]. And in the signal processing, the commonly used transforms include Fourier Transform, Sine Transform and Walsh Transform [3]. All these functions are orthogonal, and their transforms only involve the additions and subtraction, which is easy to implement on computers.

Over the past decades, development of the theory of wavelets has been accomplished and researchers continue to find the new application domains. In addition, wavelet is widely used in many areas, such as: signal processing, image compression and enhancement. A variety of powerful and sophisticated schemes based on wavelet for image compression were developed and implemented. And it will be talked about later. Also, wavelet gives many advantages, which are applied in the JPEG-2000 standard as wavelet-based algorithm [4]. Generally, wavelets were intended to apply the concept to some practical applications [5]. For instance, the discrete wavelet transform can be applied in many areas, including the image compression and coding based on haar functions. Thus, it is seen as the most promising technology by many researchers.

Fourier methods are not always effective in recapturing the signal, for it is highly non- smooth. In order to reconstruct the signal, too much Fourier transform is needed. Thus, the appearance of wavelet provides a solution to this problem by using a completely modulated window. The window can be shifted along the signal and the spectrum can be calculated for every position. And this process is repeated with a window with small changes (shorter or longer) at every new cycle and produces a collection of time-frequency signals finally. Because of this collection of representations, we can talk about a multiresolution analysis [6]. However, in the most of literature nowadays, the discussion about the wavelet requires the strong mathematical background and coding ability, which is not helpful for beginners. The purpose of this paper is to provide an overview of the wavelet and its application in the image processing [7,8].

2 WAVELET TRANSFORM

2.1 Overview of wavelet transform

Wavelets are literally “little waves”, small oscillating waveforms that begin from zero, swell to a maximum, and then quickly decay to zero again. They can be contrasted to, for example, sine or cosine waves, which go on “forever” [9], repeating out to positive and negative infinity. Like sin() and cos() functions used in the Fourier Transform, wavelets can define a set of basis functions $\psi_{j,k}(t)$. The goal is to decompose a given function f as:

$$f(t) = \sum_k \sum_j a_{j,k} \psi_{j,k}(t) \quad (1)$$

The basis can be constructed by applying translations represented by b and scalings represented by a on the ‘mother’ wavelet $\psi(t)$ shown in formula 2.

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right) \quad (2)$$

And the choice of basis function should satisfy the following requirements: orthogonality, size of support, vanishing moments which satisfy the following formula 3:

$$\int_{-\infty}^{\infty} t^k \psi(t) dt = 0 \mid \text{for } 0 \leq k < p, \quad (3)$$

symmetry and regularity.

2.2 Continuous wavelet transform

Basically, wavelet transforms are of two categories: the continuous wavelet transforms (CWT) and the discrete wavelet transforms (DWT). Whereas the CWT is useful for extracting features, the DWT is mainly used for noise reduction and data compression. Analysis of co-movements (the image processing) in this paper are done with the CWT with the package (WaveletComp) developed by Roesch and Schmidbauer [10]. The continuous wavelet transform of a function $x(t)$ at a scale $(a > 0, a \in \mathbb{R}^+)$ and translational value $b \in \mathbb{R}$ is shown by the formula 4 and 5.

Forward:

$$W_f f(t), \psi_{a,b}(t) = |a|^{-1/2} \int_{-\infty}^{\infty} f(t) \bar{\psi}\left(\frac{t-b}{a}\right) dt \quad (4)$$

where $\psi(t)$ is a continuous function in both the time and the frequency domain. To recover the original signal $x(t)$, the inverse continuous wavelet transform can be exploited.

Inverse:

$$f(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_0^{\infty} \frac{|a|^{-1/2}}{a^2} W_f(a, b) \tilde{\psi}\left(\frac{t-b}{a}\right) da db \quad (5)$$

where $\tilde{\psi}(t)$ is the dual function (if $f(f(x)) = x$, then x and $f(x)$ are dual) of $\psi(t)$.

$$C_\psi = \int_{-\infty}^{\infty} \frac{\bar{\psi}(w) \hat{\psi}(w)}{|w|} dw \quad (6)$$

C_ψ is admission constant. Traditionally, this constant is called wavelet admission constant. A wavelet whose admissible constant satisfies

$$0 < C_\psi < \infty \quad (7)$$

is called an admissible wavelet.

2.3 Discrete wavelet transform

A discrete wavelet transform is any wavelet transform for which the wavelets are discretely sampled. The first DWT was invented by Hungarian mathematician Alfred Haar [11]. Here, he haar function is taken as an example. Assume having an input represented by $2n$ numbers, the Haar wavelet transform may be considered to pair up input values, in order to save the difference and pass the sum. The process is repeated recursively, leading to $2n - 1$ differences and a final sum [12]. Thus, the formulas can be changed to the following:

$$\psi_{j,k}(t) = \sqrt{2^j} \psi(2^j t - k) \quad (8)$$

where $k = 0, 1, \dots, 2^j - 1$ and $j = 0, 1, \dots, \log_2 N - 1$

The constant $\sqrt{2^j}$ is selected so that the scale product $\psi_{j,k}, \psi_{j,k} = 1, \psi_{j,k} \in L^2(\mathbf{R})$. Think about the wavelet function on other intervals than $[0,1]$, the normalization constant changes accordingly. For example: $\psi_0^0 = \text{haar}(1, t)$, $\psi_0^1 = \text{haar}(2, t)$, $\psi_1^1 = \text{haar}(3, t)$. Generally, $\psi_{j,k} = \psi_k^j = \text{haar}(2^j + i, t)$. From this example follows that function ψ_k^j are orthogonal to one another. Thus, the work is proven to achieve the linear space of vector space $W^j = \text{span}\{\psi_k^j\}_{i=0, \dots, 2^j-1}$. A collection of linearly independent function $\{\psi_k^j\}_{i=0, \dots, 2^j-1}$ spanning W^j is called wavelets [11]. And their formulas are shown in 9 and 10.

Forward:

$$\psi_f(j, k) = f(n), \psi_{j,k}(n) = \sum_{n=-\infty}^{+\infty} f(n) \overline{\psi_{j,k}(n)} \quad (9)$$

Inverse:

$$f(t) = \sum_{j,k} f, \psi_{j,k} \overline{\psi_{j,k}} \quad (10)$$

2.4 Multiresolution analysis (MRA)

A multiresolution analysis (MRA), in other words the multiscale approximation (MSA) is the method used for image compression based on discrete wavelet transform talked about before. It allows the construction of an orthonormal basis with dynamic adaptive resolution and systematic improvability [13].

A multiresolution analysis with l levels of a continuous signal $f \in L^2(\mathbf{R})$ with finite energy can be seen as a projection of f on a basis $\{\phi_{j,k}, \{\psi_{j,k}\}_{j \leq l, k \in \mathbf{Z}}\}$. Basis function $\phi_{j,k}(x) = 2^{-j/2} \phi(2^j x - k)$ results from translation and dilation of a same function $\phi(x)$ named scaling function, verifying $\int \phi(x) dx = 1$. The family $\{\phi_{j,k}\}_{k \in \mathbf{Z}}$ span a subspace $V_j \in L^2(\mathbf{R})$. The projection of f on V_j gives an approximation $\{a_{j,k} = f, \phi_{j,k}\}_{k \in \mathbf{Z}}$ of f at the scale 2^j [13].

Eventually, a multiresolution analysis with l levels yields the following decomposition of $f \in L^2(\mathbf{R})$

$$f(x) = \sum_k a_{j,k} \widetilde{\phi_{j,k}}(x) + \sum_{j \leq l} \sum_k w_{j,k} \widetilde{\psi_{j,k}}(x) \quad (11)$$

Functions $\tilde{\phi}(x)$ and $\tilde{\psi}(x)$ need to be dual to make sure that the reconstruction is performed well [14].

In figure 1, it shows how the multiresolution analysis is realized by the filters. Here, the S represents the original signal, the low pass filter can get the approximate value of the signal - A for only the low-frequency information can pass; while high pass filter gets the detailed information - D.

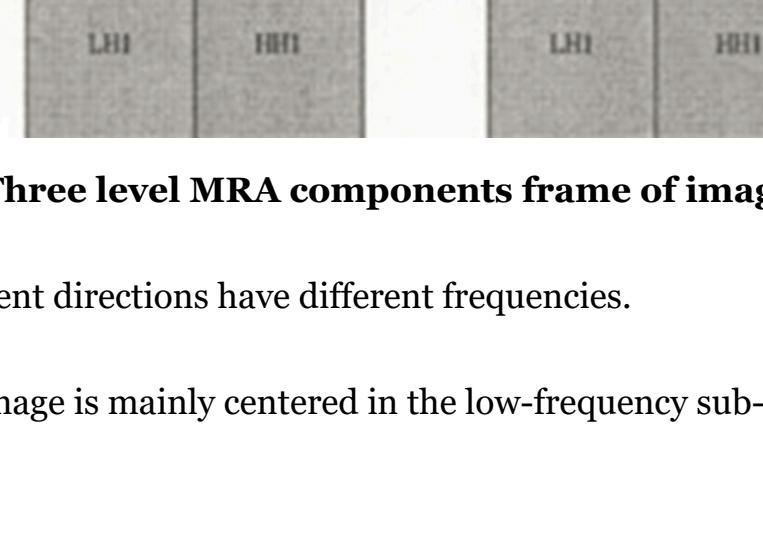


Figure 1: MRA pipeline

2.5 Application in image

Article I. Due to separability of the transform, two-dimensional wavelet transform can be achieved by one dimensional wavelet transforms along both the directions. Thus, if the wavelet is applied to the image processing, the result is figure 2 shown above. Here, every L means the part after the low filter while H represents the high filter. Thus, the work proves that a 2D wavelet transform of an image yields four components: Approximation coefficients (LL), Horizontal coefficients (HL), Vertical coefficients (LH) and Diagonal coefficients (HH). In addition, according to [15], some important characteristics of sub- image can be obtained:

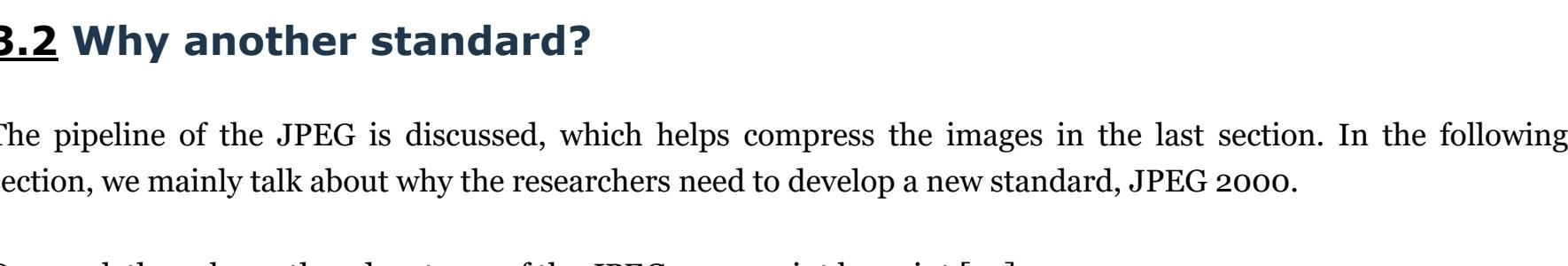


Figure 2: Three level MRA components frame of image wavelet [11]

Article II. Sub-images in different directions have different frequencies.

Article III. The energy of the image is mainly centered in the low-frequency sub-image.

3 JOINT PHOTOGRAPHIC EXPERTS GROUP 2000 (JPEG 2000)

3.1 Joint Photographic Experts Group (JPEG)

JPEG is a standard created for still image compression, which is created in 1986 by: International Organization for Standardization (ISO) and International Telecommunication Union (ITU). It is the first international standard in image compression. The compression technique employed by JPEG allows a large image file to be compressed down to a much smaller size while retaining a substantial amount of the integrity and quality of the image [16]. Architecture of standard can be divided into the following steps and the pipeline in shown in figure 3.

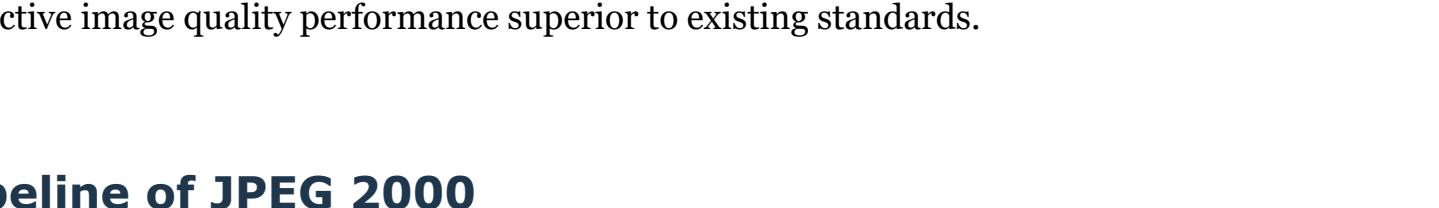


Figure 3: pipeline for JPEG [16]

There are three main steps in the process of JPEG compression: the Discrete Cosine Transform, quantization and Huffman coding. The aim of the Discrete Cosine Transform is to show the colors of the pixels by the form of matrix. And quantization helps discard some useless information, only keep the important information. Finally, Huffman coding helps compress the information.

3.2 Why another standard?

The pipeline of the JPEG is discussed, which helps compress the images in the last section. In the following section, we mainly talk about why the researchers need to develop a new standard, JPEG 2000.

Our work then shows the advantages of the JPEG 2000 point by point [17].

- Low bit-rate compression: At low bit- rates especially when below 0.25 bpp, the distortion in JPEG becomes unacceptable.
- Lossless and lossy compression: It provides two kinds of compression: lossless and lossy in one codestream.
- Large images: JPEG does not compress images greater than 64×64 K without tiling.

Thus, the researchers hope that the new standard can satisfy the following characteristics. First, it needs to have the capacity of dealing with more various images, for example, different kinds of still images (gray-level, color, ...), images with different characteristics (natural, scientific, ...) and different imaging models (client/server, real-time, ...). Furthermore, this coding system should support low bit-rate applications, exhibiting rate-distortion and have subjective image quality performance superior to existing standards.

3.3 Pipeline of JPEG 2000

In this section, the pipeline of JPEG2000 shown below is discussed. Similar to encoding of JPEG, JPEG 2000 also needs to experience the forward transform, quantization and entropy encoding to convert the source image data to compressed image data. And when it comes to the storage or the transmission, we need to change the compressed image data to the reconstructed data, which helps save the memory. While the decoding process is the inverse one of the coding, which means it will go through the entropy decoding, quantization and inverse transform successively shown in figure 4.

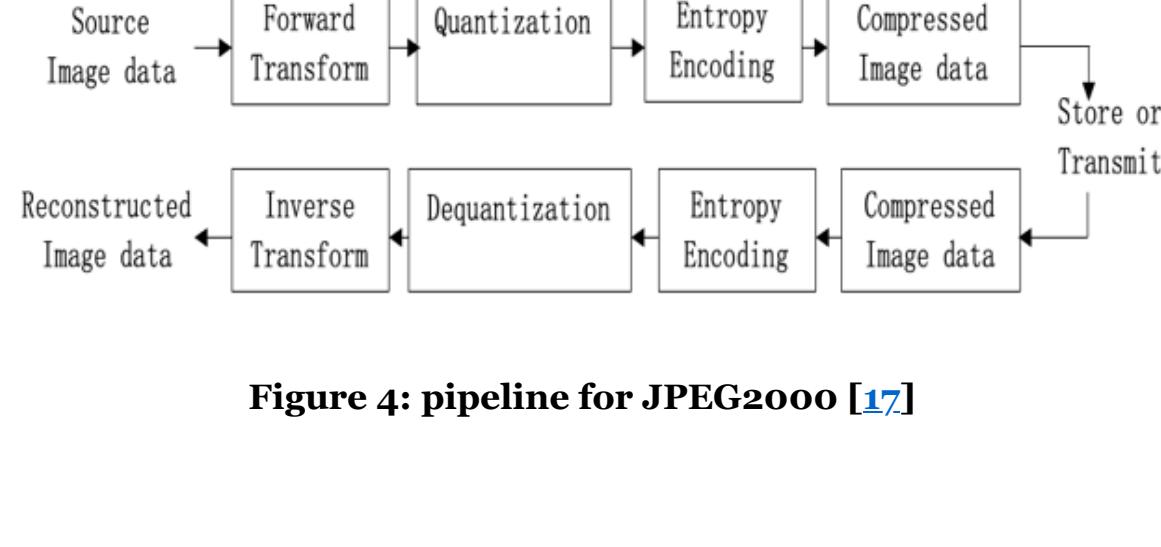


Figure 4: pipeline for JPEG2000 [17]

3.4 Pre-processing

In order to deal with the data successfully, we need to do some pre-processing. And the first step is the image tiling.

3.4.1 Imaging tiling: Image may be quite large in comparison to the amount of memory, thus dividing the original image into tiles will be needed, to ensure that images are compressed independently. An example about how to divide the image into tiles is shown in figure 5. In this case, it is obvious that we divide the image into four smaller parts, called the tiling [18].

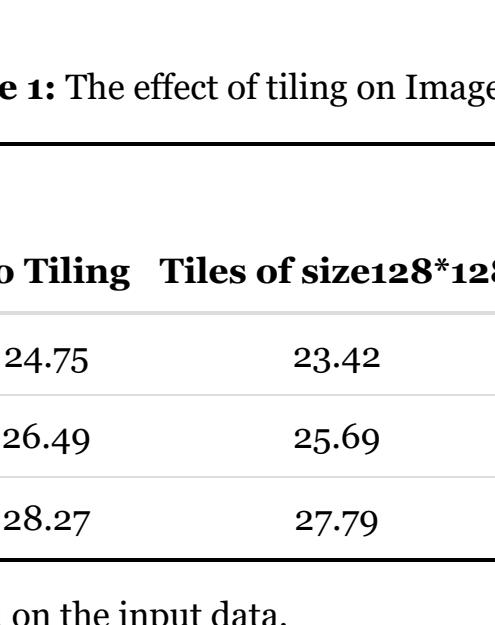


Figure 5: Divide Images into four tiles

All operations, including components mixing, wavelet transform, quantization and entropy coding are performed independently on the image tiles. In addition, tiling affects the image quality both subjectively and objectively. And smaller tiles create more tiling artifacts shown in the table 1.

Table 1: The effect of tiling on Image quality

Tiling	Bite Rate(b/p)	No Tiling	Tiles of size128*128	Tiles of size64*64
	0.125	24.75	23.42	20.07
	0.25	26.49	25.69	23.95
	0.5	28.27	27.79	26.80

Next, DC-level shifting will be processed on the input data.

3.4.2 DC-level shifting: The codec requires that its input sample data to be approximately centered about zero. Thus, it allows to have a shift on the original range. For example, the input sample data range is from 0 to 256. Obviously, it is not centered at 0. Thus, the work needs to move it to the -128–128, whose center is 0 and keeps the range length the same.

3.4.3 Components transformation: Lastly, because people's eyes are more sensitive to the light instead of color, we use the YCrCb / YUV color mode (Y: Luminance, Cr: the difference between red and luminance, Cb: the difference between blue and luminance.) rather than the RGB mode.

Maps data from RGB to YCrCb serves to reduce the correlation between components, leading to improved coding efficiency. There are two kinds of transforms - reversible and irreversible.

Irreversible component transformation(IFT) [19]:

It is a kind of floating point, for use with irreversible wavelet. And the transformation matrix is shown in formula 12.

$$\begin{pmatrix} Y \\ Cb \\ Cr \end{pmatrix} = \begin{pmatrix} 0.299 & 0.587 & 0.114 \\ -0.16875 & -0.33126 & 0.5 \\ 0.5 & -0.41869 & -0.08131 \end{pmatrix} \cdot \begin{pmatrix} R \\ G \\ B \end{pmatrix} \quad (12)$$

In the RGB mode, it separates the original mode to the red, green and blue, three parts. While to the YCrCb mode, the image is divided into luminance (only black and white) and chrominance. And figure 6 is one example of mode convert shown below. The left is the RGB mode, while the right is the YCrCb one.

Figure 6: Mode convert [12]

Reversible component transformation (RCT):

Different from ICT, RCT is a kind of integer approximation, for use with reversible wavelet. Shown in formula 13, the value of Y is approximation for taking the ceil function, and it is integer for R, G and B are integers.

$$\begin{pmatrix} Y \\ U \\ V \end{pmatrix} = \begin{pmatrix} \lceil \frac{R+2G+B}{4} \rceil \\ R-G \\ B-G \end{pmatrix} \quad (13)$$

Overall, component transformations improve compression and allow visually relevant quantization.

3.5 Wavelet transform

Then, after pre-processing, wavelet transform is applied. As discussed before, two different wavelet filters can be used.

Lossy compression

Different from 9/7 wavelet filter, 5/3 wavelet filter is applied for lossless coding. It is integer arithmetic and have low implementation complexity.

When it comes to the two dimensional signals, like images, we filter each row and column with a high pass and low pass filter respectively, followed by downsampling by two in order to keep the sample rate. Suppose have the original image below, then filter the row with a high pass and low pass filter, it will get an approximation on the left and detailed information on the right. Then to keep the sample rate, it needs to downsample the column by two. Similarly, filter the column and downsample. Thus, after one stage of DWT, it will get one approximation of the original image and three other detailed images like in figure 7.

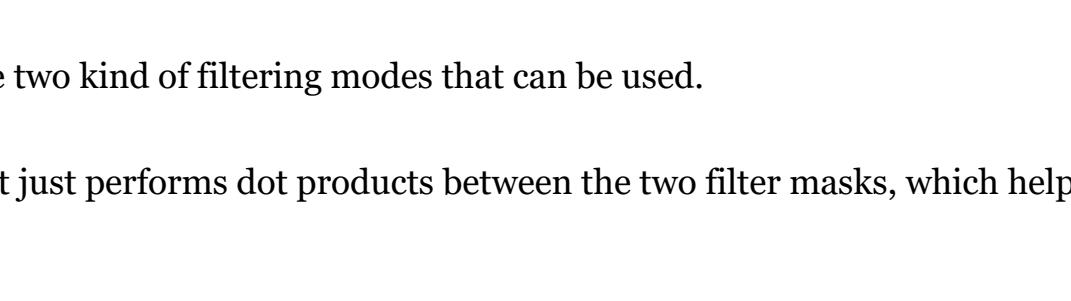


Figure 7: Stage one DWT process [11]

In addition, there are two kind of filtering modes that can be used.

Convolution based: It just performs dot products between the two filter masks, which helps compress the data.

Lifting based: Do simple filtering operations, which means alternately doing the up-dation of the odd sample values, then having a weighted sum of the even ones, and vice versa.

Shown in figure 8, assume that the original signal is $x(n)$, then classify it into the even and odd ones, do the z-transform and downsample by two respectively. Then apply the prediction process on the even ones, to get the new even values. Next, compare the new even values with the original odd ones, which will create an error. Finally, the error should be passed to the even ones to adjust. Thus, after prediction and update process, the new form of even and odd numbers will be obtained.

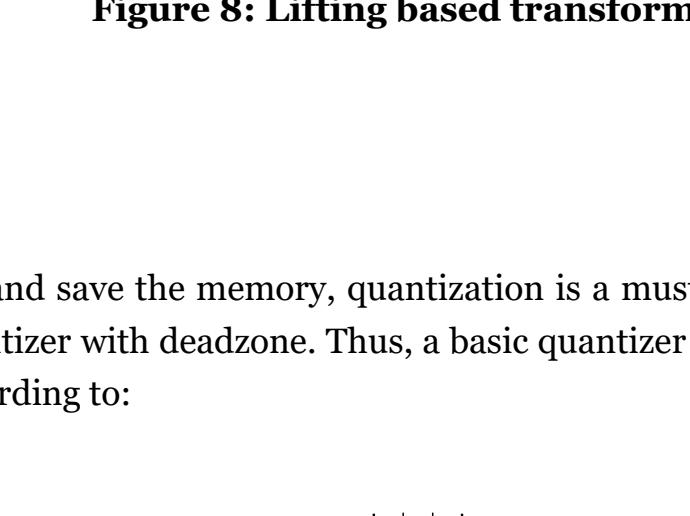


Figure 8: Lifting based transform

3.6 Quantization

In order to compress the data and save the memory, quantization is a must process. The wavelet coefficients are quantized using a uniform quantizer with deadzone. Thus, a basic quantizer step size Δb is used to quantize all the coefficients in the subband according to:

$$q = \text{sign}(y) \left\lfloor \frac{|y|}{\Delta b} \right\rfloor \quad (14)$$

Here, $\lfloor x \rfloor$ means nearest smaller integer. For instance, we assume that quantizer step is 10 and encoder input value is 21.82, then the quantizer index can be calculated as below:

In addition, the figure 9 demonstrates well how the compression works. For instance, the original input data ranging from 10 to 20 can be represented with one value- positive one, which helps compress the data on a large scale.

$$\text{Quantizer Index} = - \left\lfloor \frac{21.82}{10} \right\rfloor = -2$$

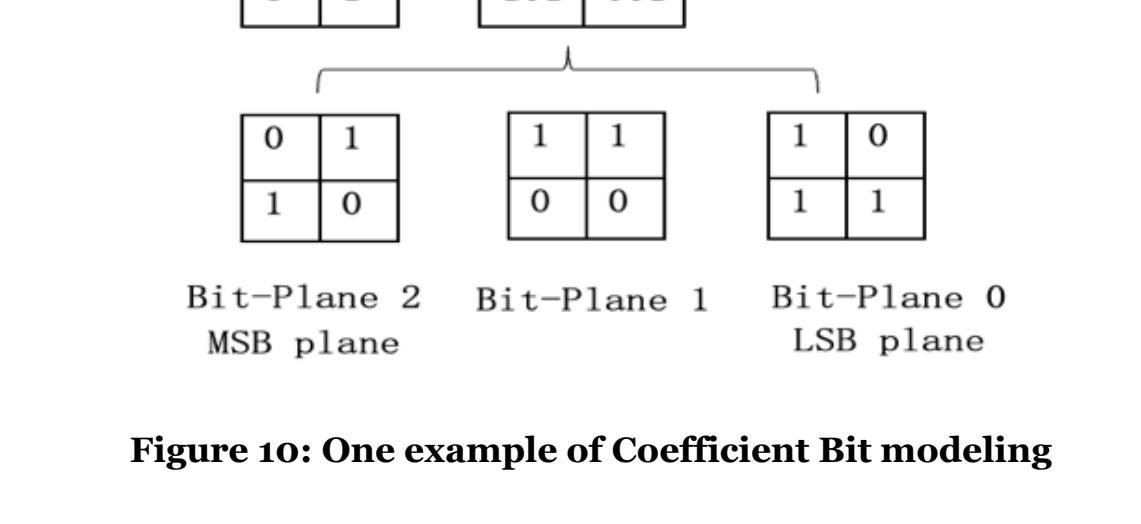


Figure 9: One quantization example

3.7 Coefficient Bit modeling

The coefficient bit modeling is performed on code-blocks, the rectangular blocks within each sub-band. The following figure shows one example about the division of subbands into code-blocks. And the operations, like pre-processing and coding can be done independently on the code-blocks like in figure 10. Code-blocks are coded a bit-plane at a time starting from the Most Significant Bit-plane to the Least Significant one. For example, have the code-blocks like figure 11 below. Then use two bits to represent the number respectively. For instance, $3 = 0*2^2 + 1*2^1 + 1*2^0$, thus, we can use 011 to represent 3. Similarly, 6, 5, 1 can be represented by 100, 101, 001 accordingly. And then place the numbers into different bit-planes, like putting the first number into bit plane 2, in other words, the Most Significant Bit-plane, for it decides whether it contains 22 - the largest number. Similarly, the bit plane 0 is named as the least significant.



Bit-Plane 2 Bit-Plane 1 Bit-Plane 0
MSB plane LSB plane

Figure 10: One example of Coefficient Bit modeling

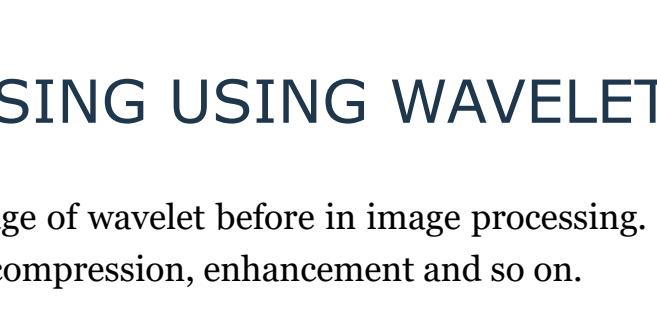


Figure 11: Coding Scanning pattern

For each bit-plane, a zigzag scan pattern shown in figure 12 is applied. Each coefficient bit in the bit-plane is coded in only one of the Three Coding Passes, including: the significance propagation, magnitude refinement and clean-up. And their detailed information can be found in [20].

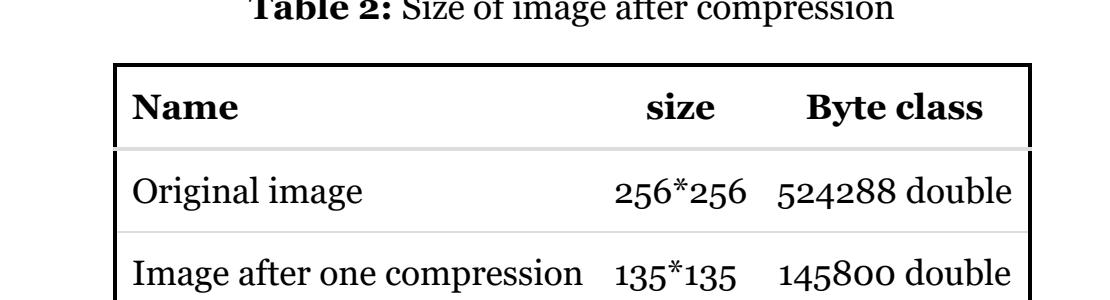


Figure 12: Image compression

4 IMAGE PROCESSING USING WAVELET

In this section, apply the knowledge of wavelet before in image processing. Our work uses Matlab to realize some image processing functions, like: compression, enhancement and so on.

4.1 Image compression

Discussed in the first section, the image can be divided into four parts after one wavelet transform. And the left top corner is the approximated image of the original one, thus only keep this part and ignore other detailed information [21]. And figure 12 shows the Matlab results of the image compression based on wavelet.

The following table 2 shows the size of the original and compressed images. The image will decrease in size and bytes on a large scale after compression.

Table 2: Size of image after compression

Name	size	Byte class
Original image	256*256	524288 double
Image after one compression	135*135	145800 double
Image after two compression	75*75	45000 double

4.2 Image enhancement

In order to get the enhanced image, we need to emphasize the high-frequency components and alleviate the low-frequency ones. In addition, we pick some of the code to show how it works. Here, the Haar wavelet is used to implement function. In order to get the enhancement, just multiply 1.2 by cA, which means lifting the low

frequency, and multiply 0.6 by other high frequency components [22]. And the Matlab results are shown in figure 13.

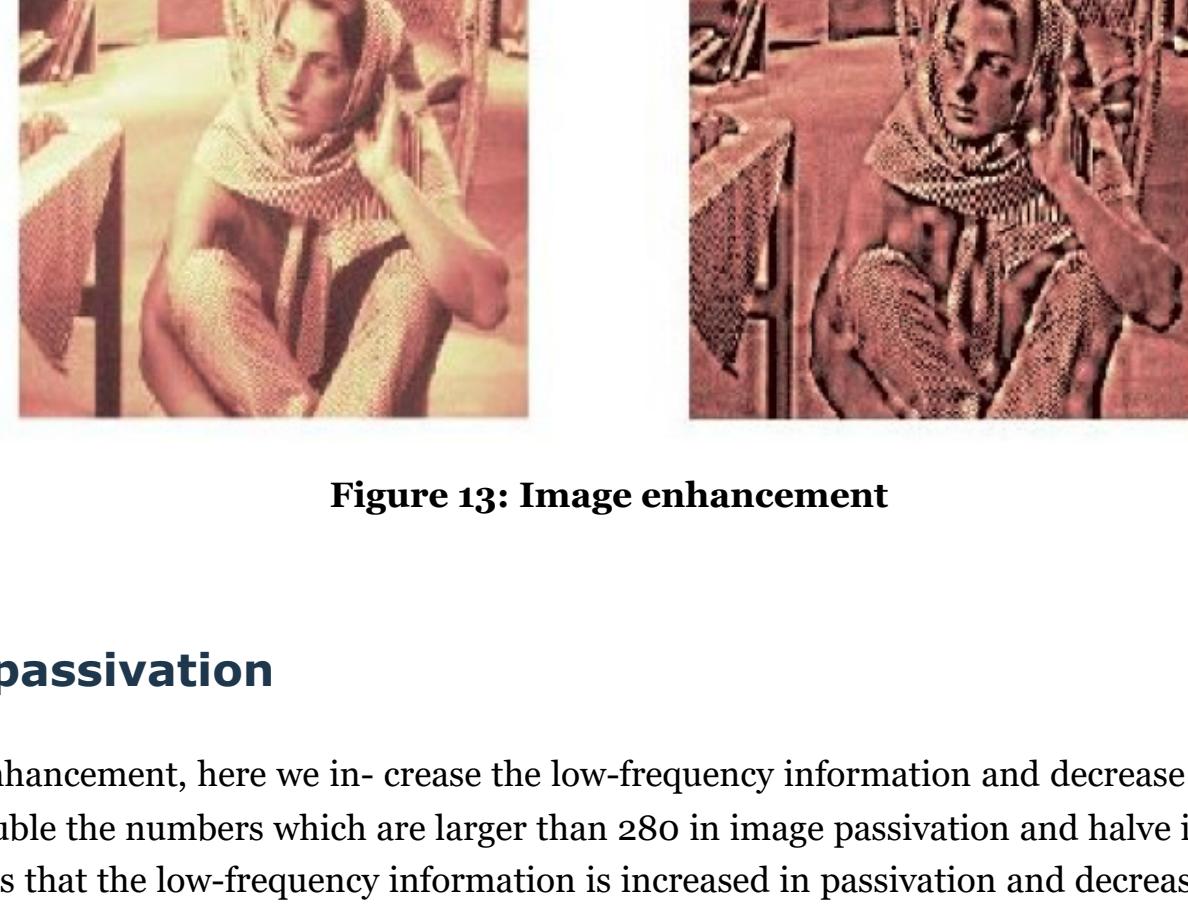


Figure 13: Image enhancement

4.3 Image passivation

Contrary to the enhancement, here we increase the low-frequency information and decrease the high-frequency. It supposed to double the numbers which are larger than 280 in image passivation and halve in the enhancement. And it corresponds that the low-frequency information is increased in passivation and decreased in enhancement. Similarly, when the numbers less than 280, it does the halving and doubling in passivation and enhancement respectively. Moreover, our work shows the Matlab result of the image passivation shown in figure 14 [23].

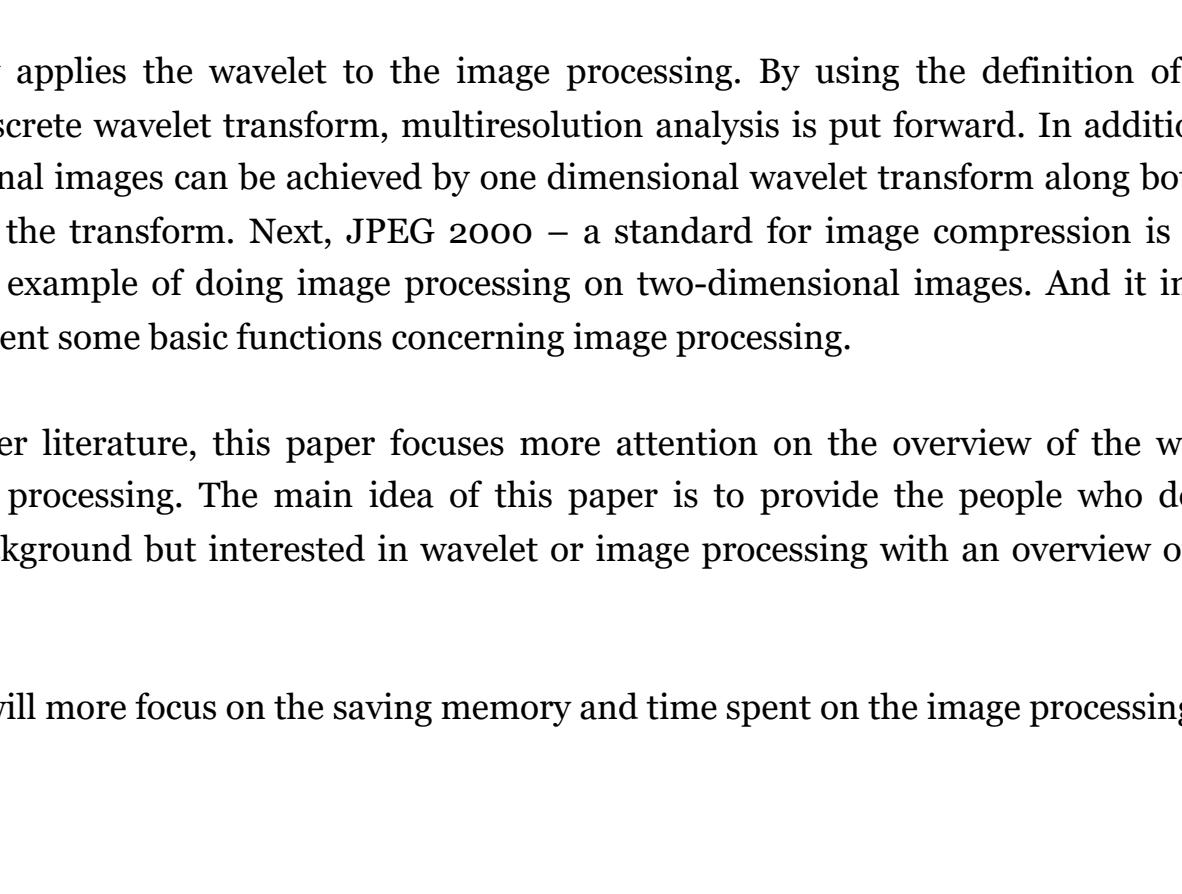


Figure 14: image passivation

5 CONCLUSION

Our work mainly applies the wavelet to the image processing. By using the definition of continuous wavelet transform and discrete wavelet transform, multiresolution analysis is put forward. In addition, the transform on the two-dimensional images can be achieved by one dimensional wavelet transform along both the directions due to separability of the transform. Next, JPEG 2000 – a standard for image compression is discussed in details, which shows one example of doing image processing on two-dimensional images. And it inspires us to use the Matlab to implement some basic functions concerning image processing.

Compared to other literature, this paper focuses more attention on the overview of the wavelet and how it is applied to image processing. The main idea of this paper is to provide the people who do not have so much mathematical background but interested in wavelet or image processing with an overview of the wavelet and its application.

The future work will more focus on the saving memory and time spent on the image processing based on wavelet.

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