COL783: Digital Image Processing

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Wavelet: Recap

- Image Pyramid
- Wavelets can perform multi-resolution analysis of images.
- Wavelet analysis performs what is known as space-frequency localization.
- Continuous Wavelet Transform
- Discrete Wavelet Transform
- Haar Wavelets

Wavelet

Multi Resolution Analysis

Image Pyramid

Gaussian Pyramid →

Laplacian Pyramid →

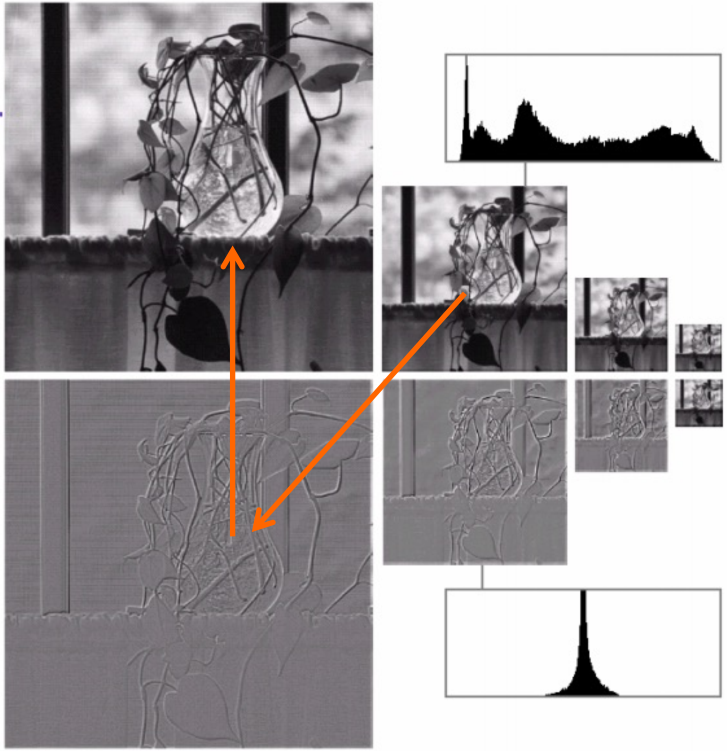
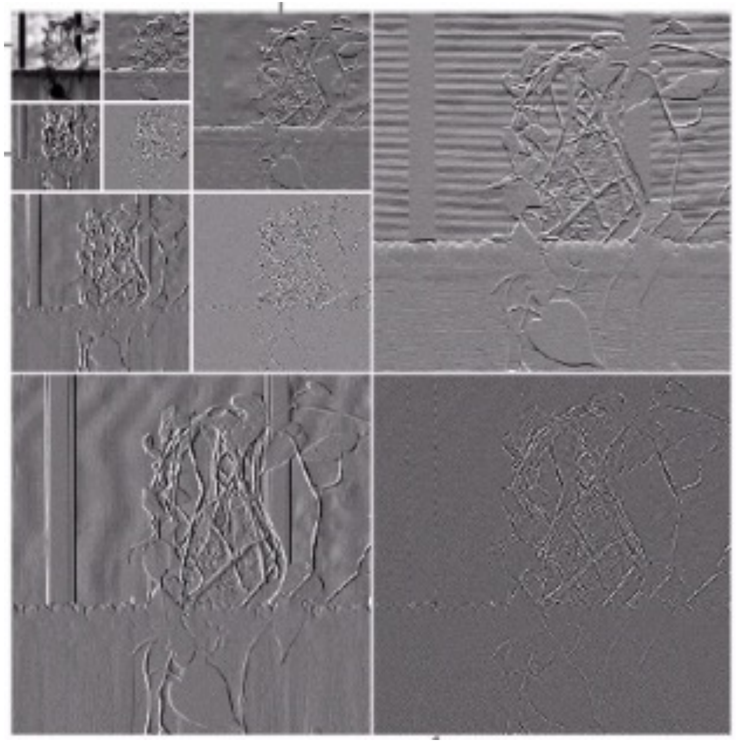


FIGURE 7.3 Two image pyramids and their statistics: (a) a Gaussian (approximation) pyramid and (b) a Laplacian (prediction residual) pyramid.

Wavelet

Multi Resolution Analysis

Multiresolution representation facilitates efficient compression by exploiting the redundancies across the resolutions.



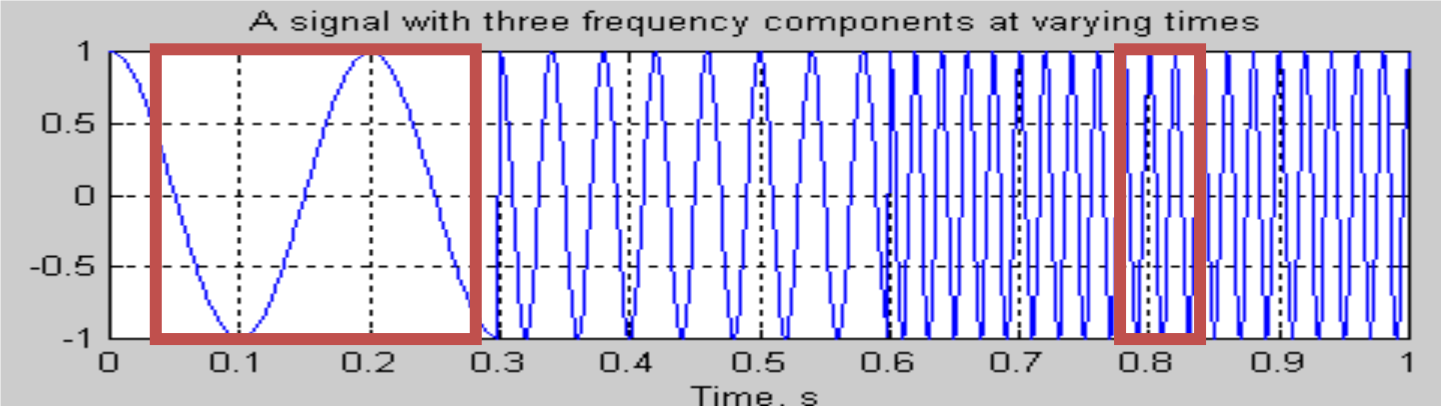
Wavelet

Localization in time/space and frequency

Uses a variable length window, e.g.:

Narrower windows are more appropriate at high frequencies

Wider windows are more appropriate at low frequencies



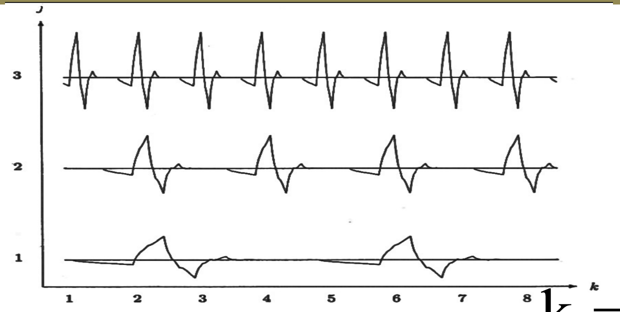
Wavelet

- It is convenient to take special values for s and τ in defining the wavelet basis: **s = 2^{-j}** and **τ = k . 2^{-j}**

$$\psi(s,\tau,t) = \frac{1}{\sqrt{s}}\psi\left(\frac{t-\tau}{s}\right) \quad = \frac{1}{\sqrt{2^{-j}}}\,\psi\left(\frac{t-k.2^{-j}}{2^{-j}}\right) = 2^{\frac{j}{2}}\psi(2^{j}t-\mathrm{k})$$

scale =1/2^{j}
(1/frequency)

j



k

Haar Wavelets

- Suppose we are given a 1D "image" with a resolution of 4 pixels:

$$[9\ 7\ 3\ 5]$$

- The Haar wavelet transform is the following:

$$[6 \quad 2 \quad 1 \quad -1] \qquad \text{(with sub-sampling)}$$

$$L_0\ D_1\ D_2\ D_3$$

Haar Wavelets

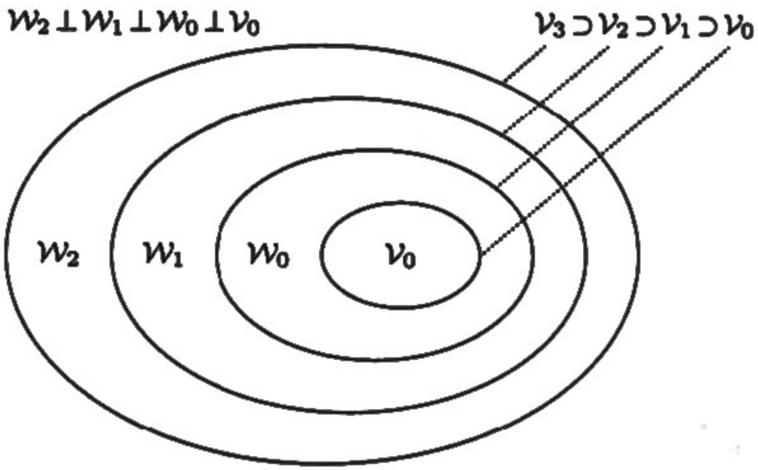
Multi Resolution

- Let W_j be the orthogonal complement of V_j in V_{j+1}

$$V_{j+1} = V_j + W_j$$

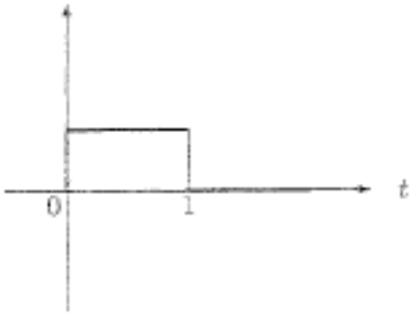
$$f(t) = \sum_k c_k \varphi(2^{j+1} t - k) \qquad V_{j+1}$$

$$f(t) = \underbrace{\sum_k c_k \varphi(2^j t - k)}_{V_j} + \underbrace{\sum_k d_{jk} \psi(2^j t - k)}_{W_j}$$



Haar Wavelets

$$\phi(x) = \begin{cases} 1 & \text{if } 0 \le x < 1 \\ 0 & \textit{otherwise} \end{cases}$$

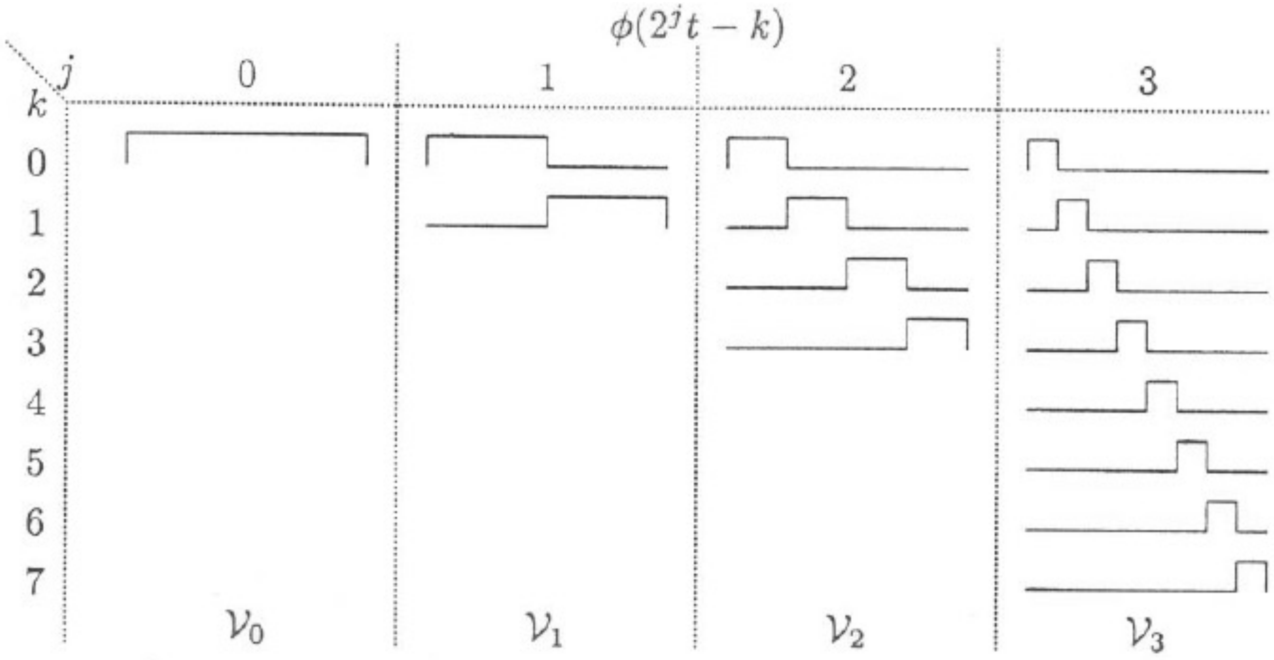


(a) $\phi(t)$

$$\phi_k^j(x) := \phi(2^j x - k), \quad k = 0, 1, \ldots, 2^j - 1$$

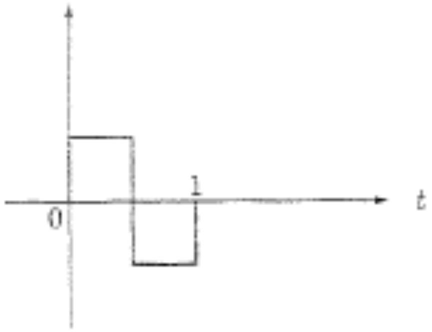
(scaled and translated versions of the box function below)

Haar Wavelets



Haar Wavelets

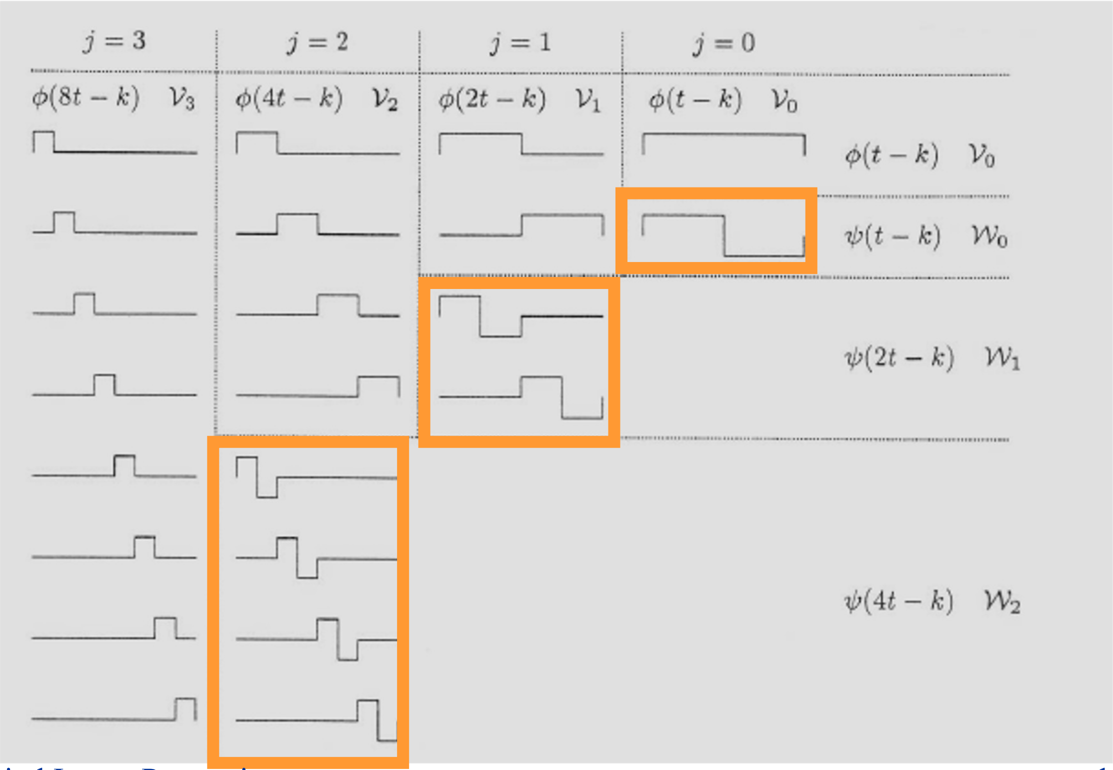
$$\psi(x) = \begin{cases} 1 & \text{if } 0 \le x < 1/2 \\ -1 & \text{if } 1/2 \le x < 1 \\ 0 & \text{otherwise} \end{cases}$$



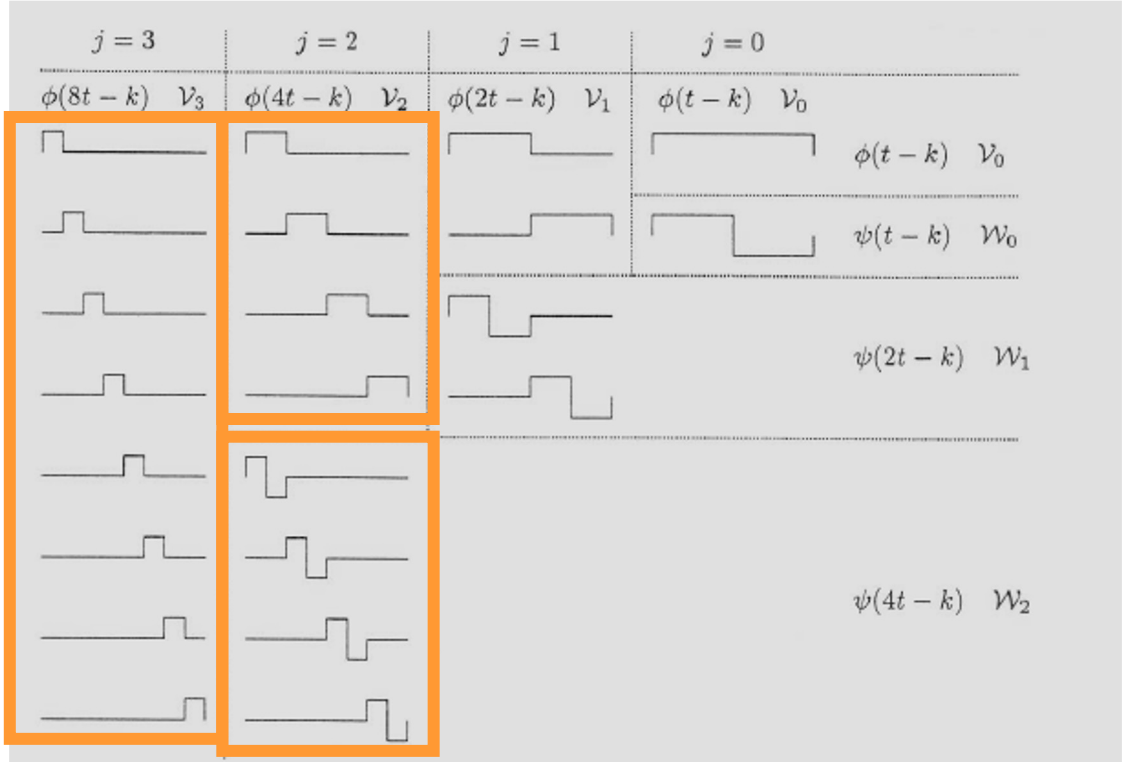
(b) $\psi(t)$

$$\psi_k^j(x) := \psi(2^j x - k), \quad k = 0, 1, \ldots, 2^j - 1$$

Haar Wavelets



Haar Wavelets



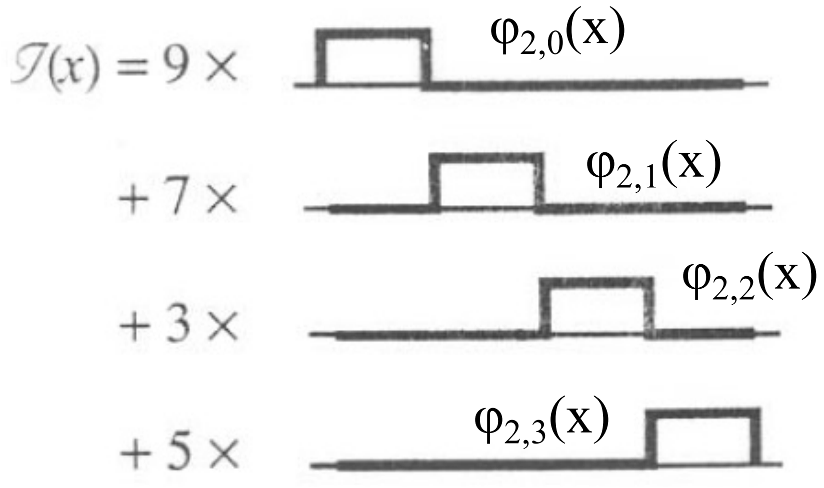
Haar Wavelets

using the basis functions in V^2

$$f(x) = c_0^2\phi_0^2(x) + c_1^2\phi_1^2(x) + c_2^2\phi_2^2(x) + c_3^2\phi_3^2(x)$$

$$f(x) = [9 \;\; 7 \;\; 3 \;\; 5]$$

V_2



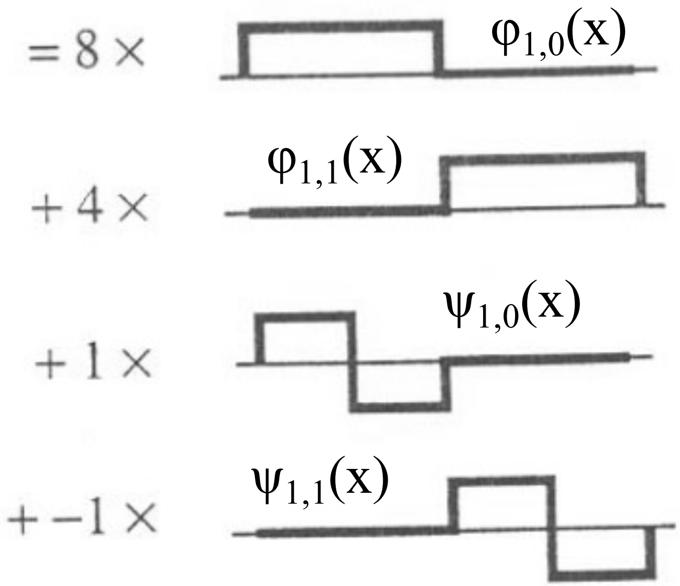
Haar Wavelets

using the basis functions in V^1 and W^1

$$V^2 = V^1 + W^1$$

$$f(x) = c_0^1\phi_0^1(x) + c_1^1\phi_1^1(x) + d_0^1\psi_0^1(x) + d_1^1\psi_1^1(x)$$

Resolution	*Averages*	*Detail Coefficients*
4	[9 7 3 5]	[]
2	[8 4]	[1 – 1]
4	[6]	[2]



Haar Wavelets

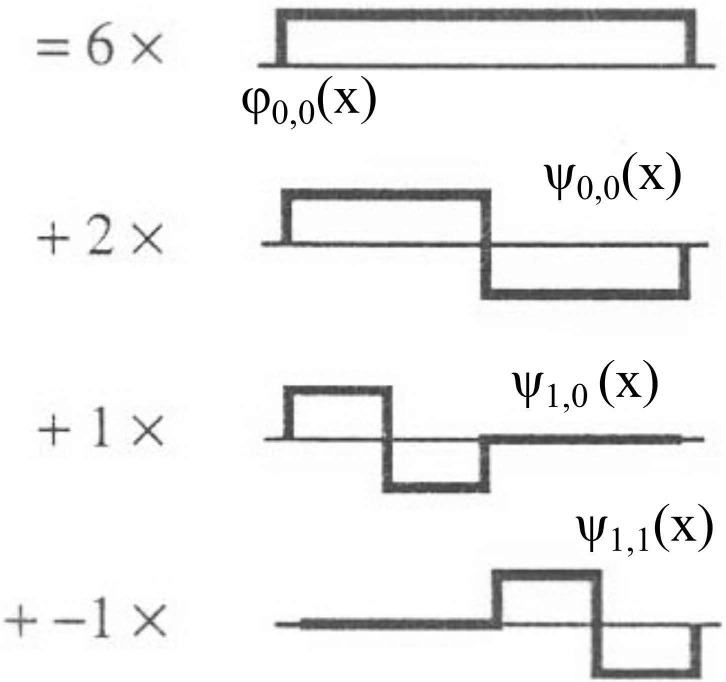
using the basis functions in V^0, W^0 and W^1

$$V^2 = V^1 + W^1 = V^0 + W^0 + W^1$$

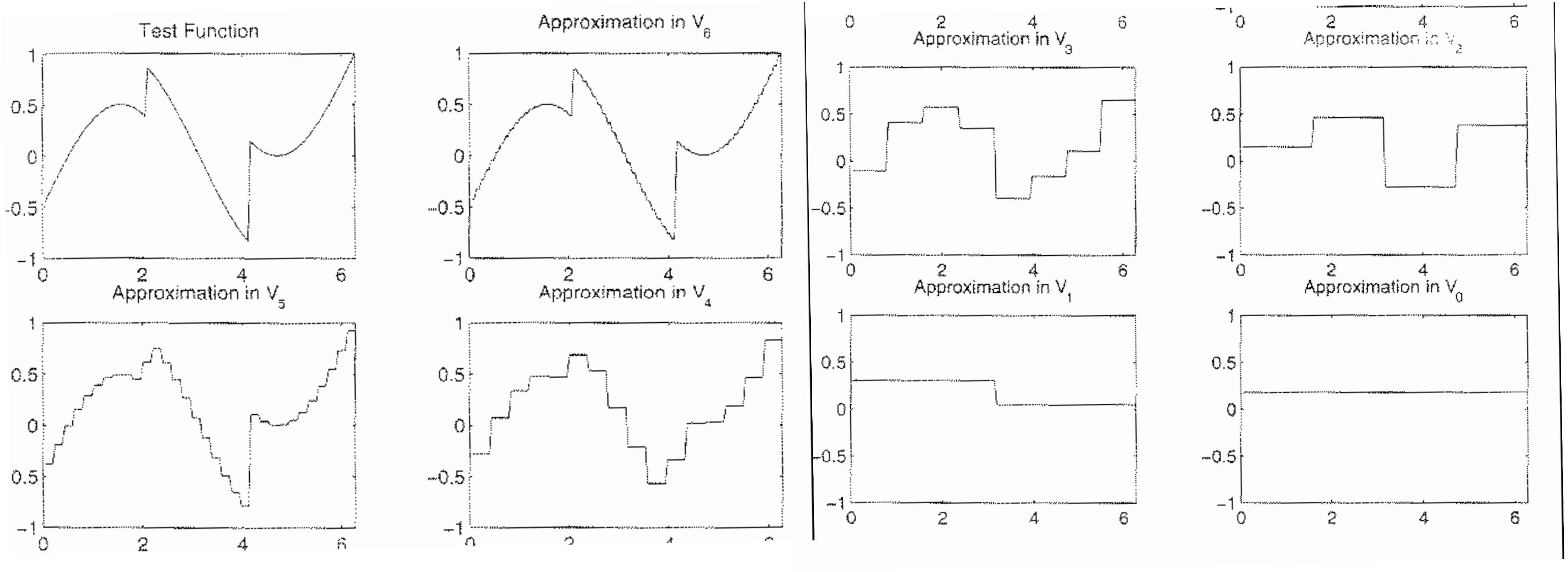
$$f(x) = c_0^0\phi_0^0(x) + d_0^0\psi_0^0(x) + d_0^1\psi_0^1(x) + d_1^1\psi_1^1(x)$$

$$f(t) = \sum_k c_k \varphi(t-k) + \sum_k \sum_j d_{jk} \psi(2^j t - k)$$

scaling function   wavelet function

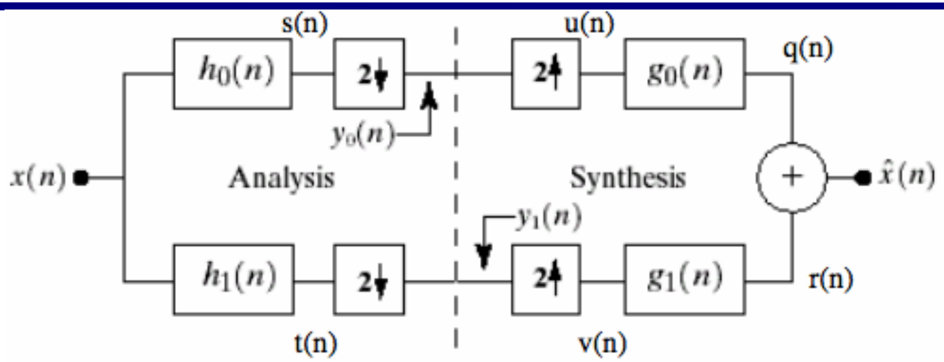


Haar Wavelets



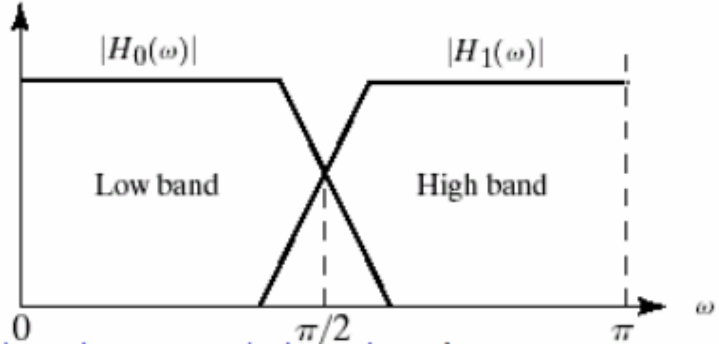
Wavelet

Multi Resolution Analysis



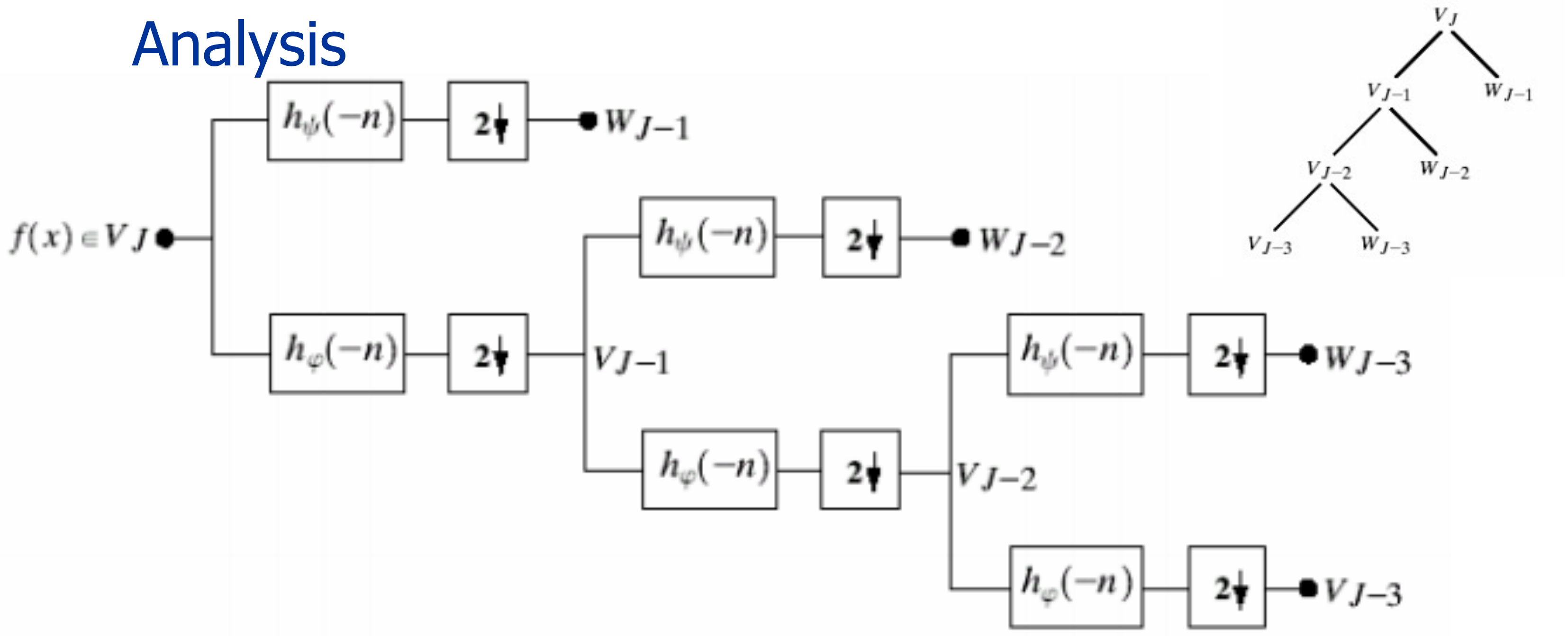
Sub bands

Filter banks



Wavelet

Multi Resolution Analysis



Haar Wavelet

Example (Revisit)

$$[9\ 7\ 3\ 5]$$

low-pass, down-sampling

high-pass, down-sampling

$$[8\quad 4] \qquad [1\quad -1]$$

(9+7)/2 (3+5)/2 (9-7)/2 (3-5)/2

Haar Wavelet

Example (Revisit)

[9 7 3 5]

[8 4] [1 −1]

low-pass,
down-sampling

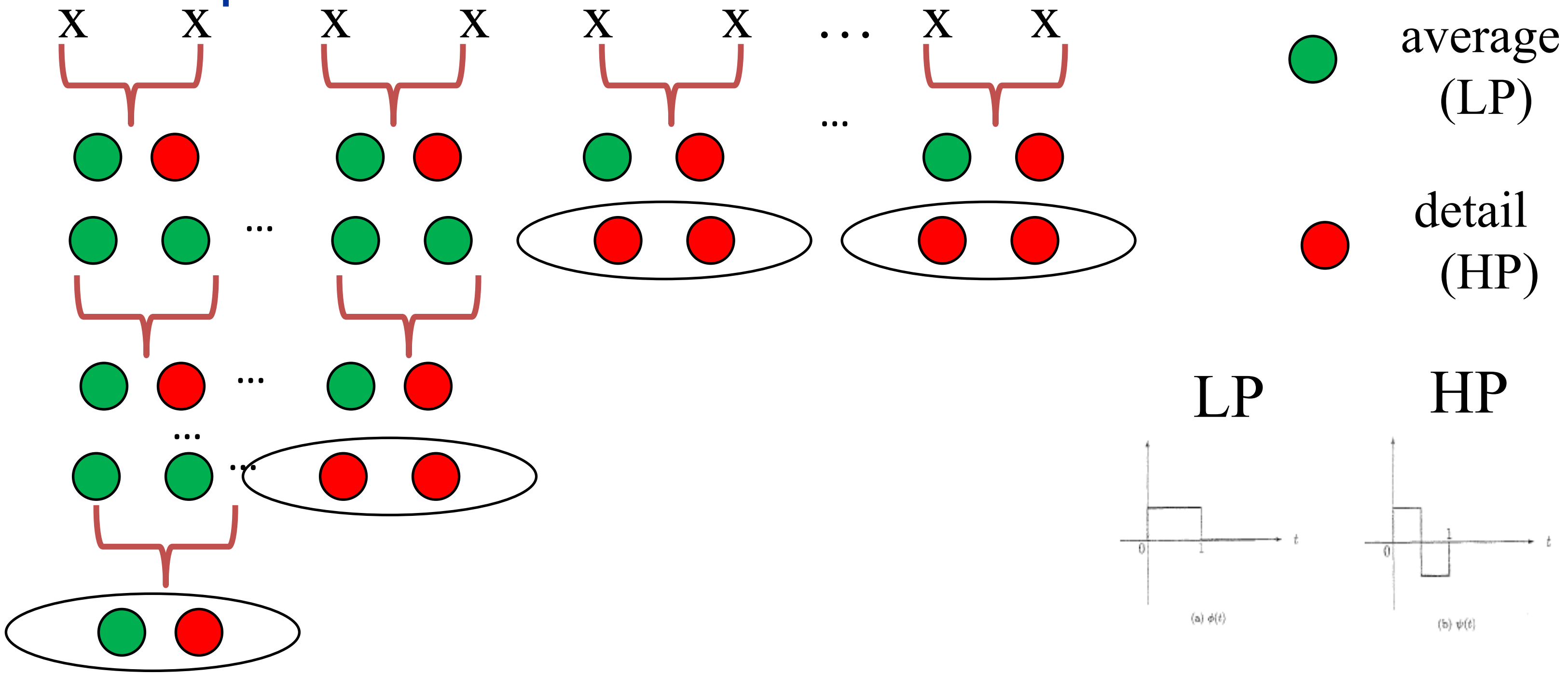
high-pass,
down-sampling

[6] [2]

(8+4)/2 (8-4)/2

Haar Wavelet

Decomposition



Haar Wavelet

2D Decomposition

The 2D Haar wavelet decomposition can be computed using 1D Haar wavelet decompositions.

i.e., 2D Haar wavelet basis is separable

Two different decompositions:

Standard decomposition

Non-standard decomposition

Haar Wavelet

Standard Decomposition

Steps:

(1) Compute 1D Haar wavelet decomposition of each row of the original pixel values.

(2) Compute 1D Haar wavelet decomposition of each column of the row-transformed pixels.

Haar Wavelet

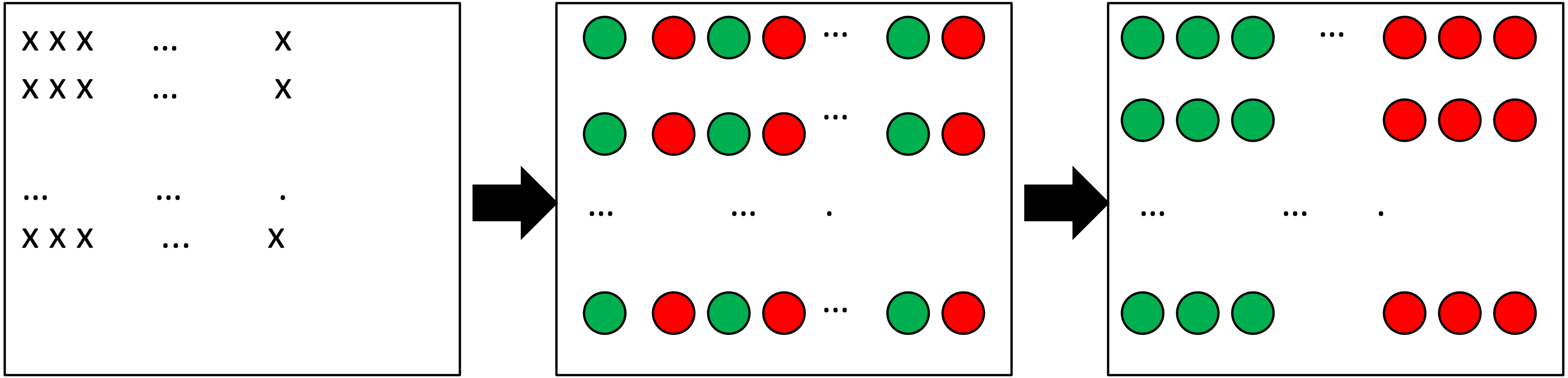
Standard Decomposition

average

detail

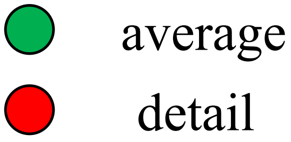
(1) row-wise Haar decomposition:

re-arrange terms



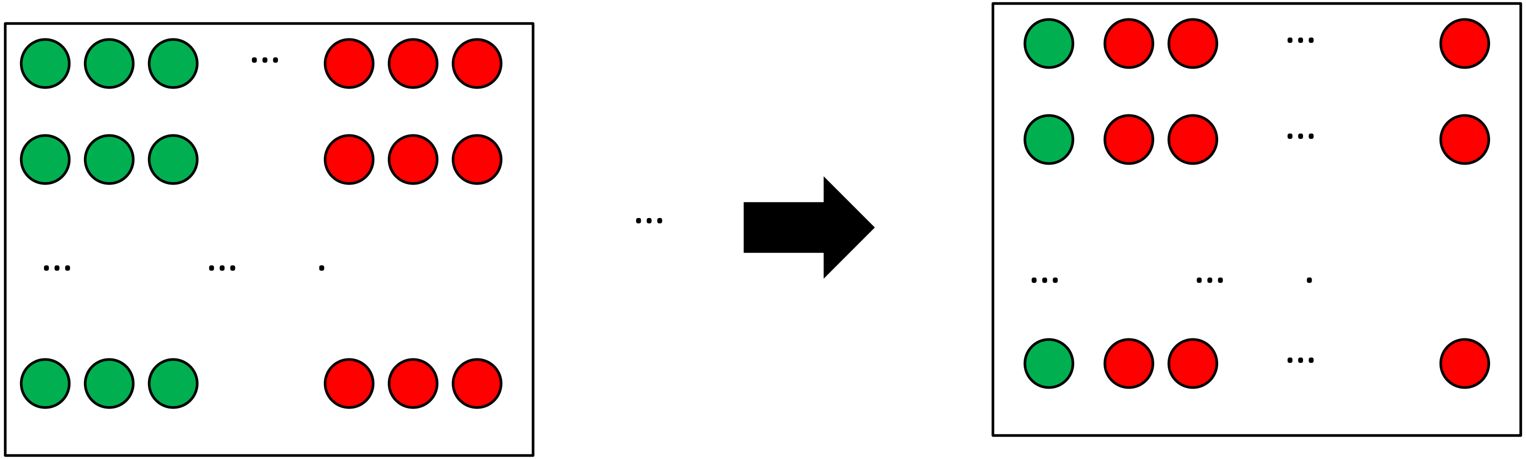
Haar Wavelet

Standard Decomposition



(1) row-wise Haar decomposition:

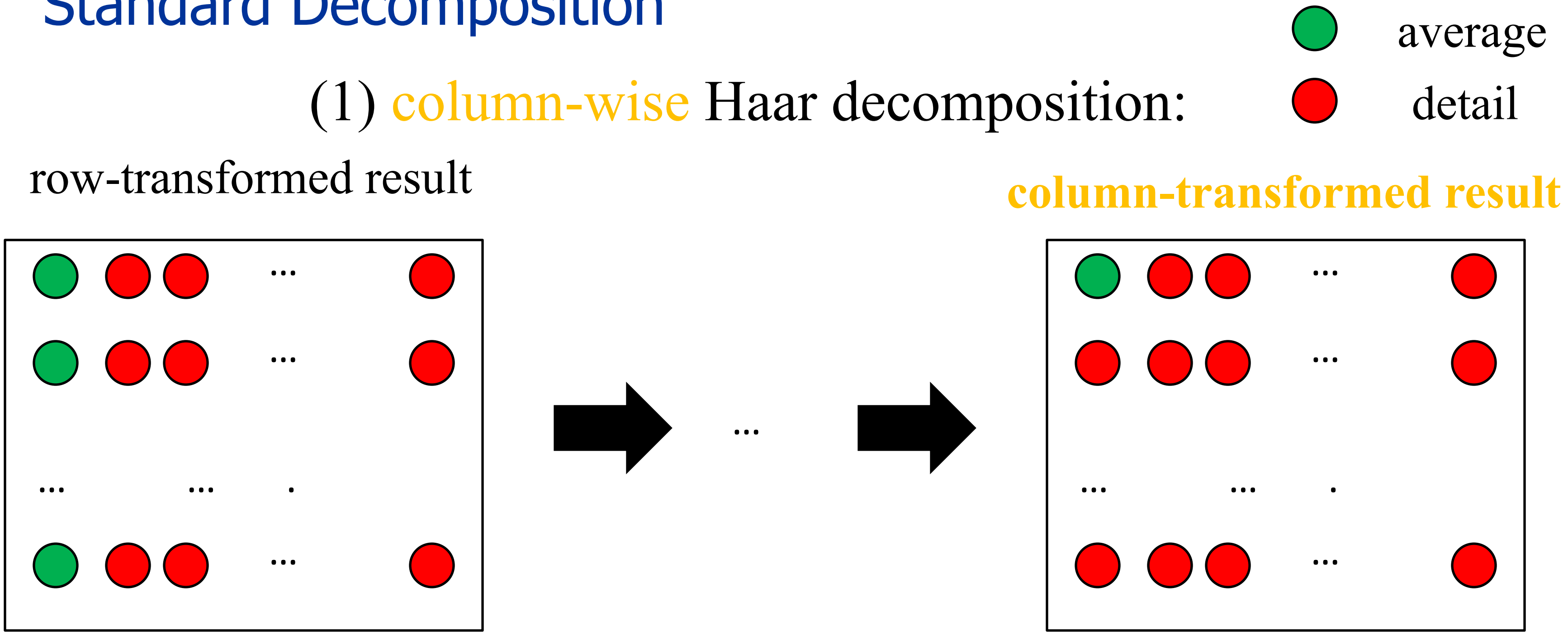
row-transformed result



Haar Wavelet

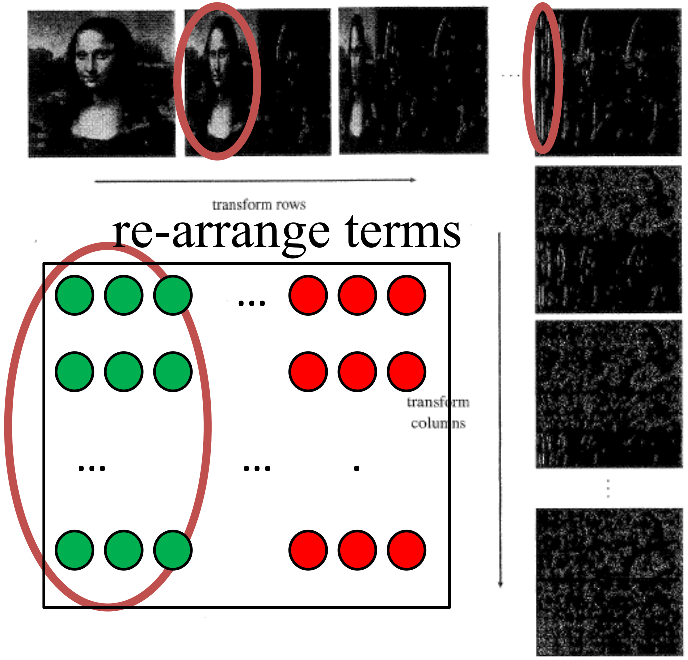
Standard Decomposition

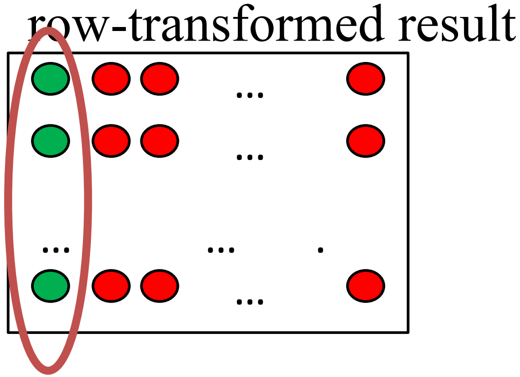
(1) column-wise Haar decomposition:



Haar Wavelet

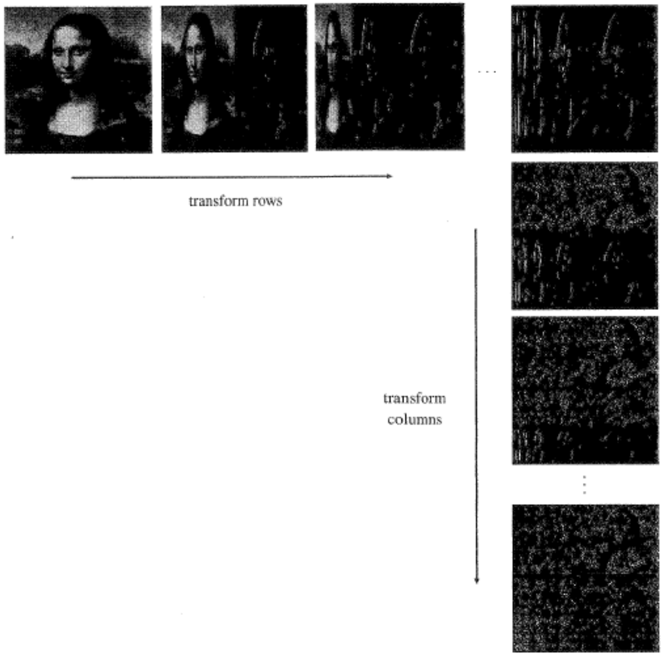
Standard Decomposition



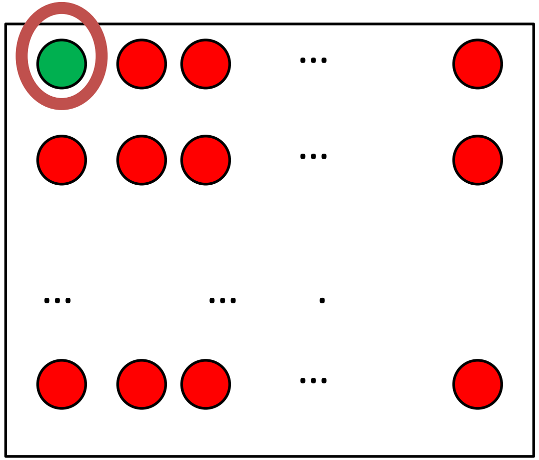


Haar Wavelet

Standard Decomposition

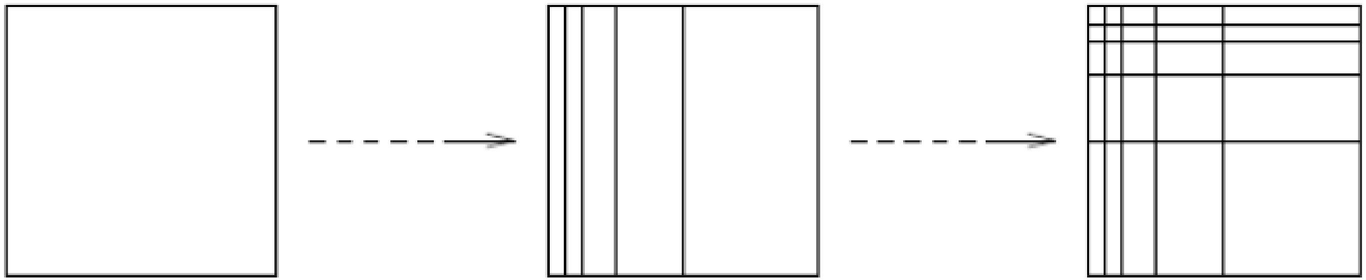


column-transformed result



Haar Wavelet

Standard Decomposition

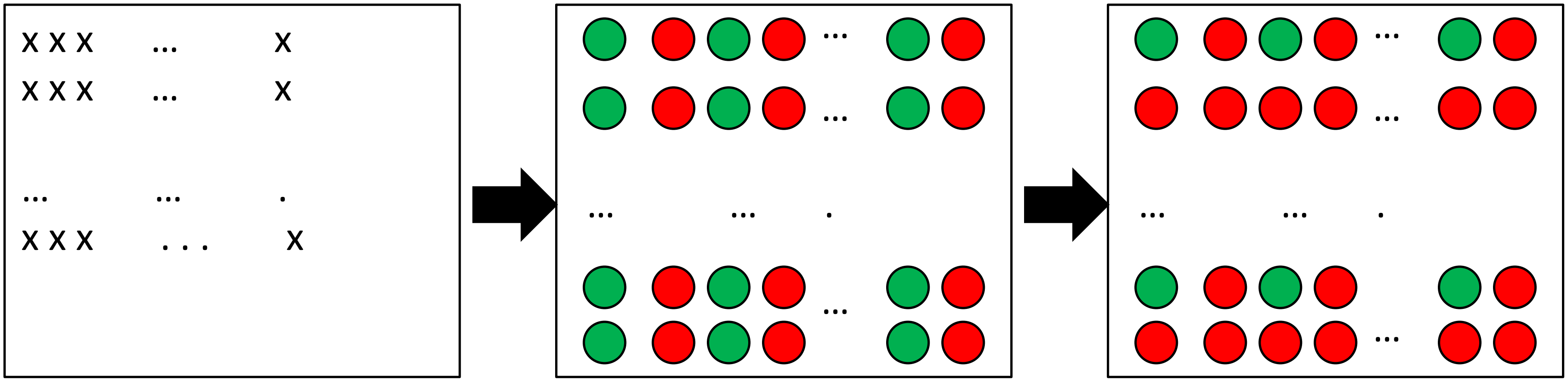


Haar Wavelet

Non Standard Decomposition

one level, horizontal Haar decomposition:

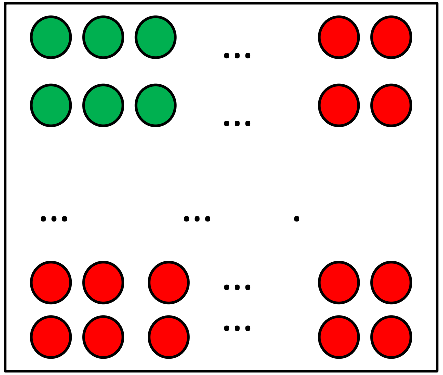
one level, vertical Haar decomposition:



Haar Wavelet

Non Standard Decomposition

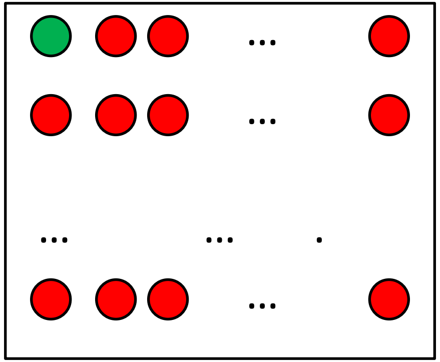
re-arrange terms



one level, horizontal Haar decomposition on "green" quadrant

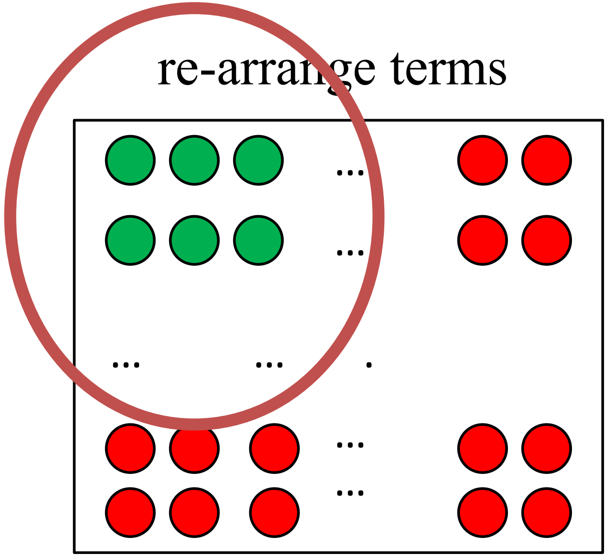
one level, vertical Haar decomposition on "green" quadrant

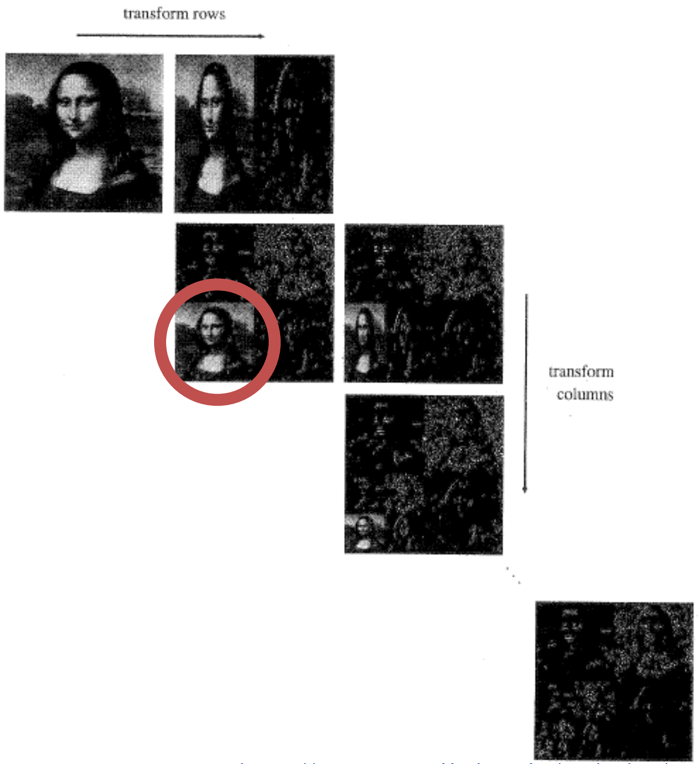
…



Haar Wavelet

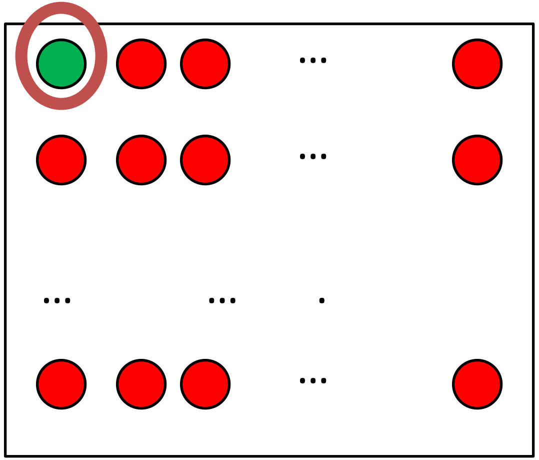
Non Standard Decomposition

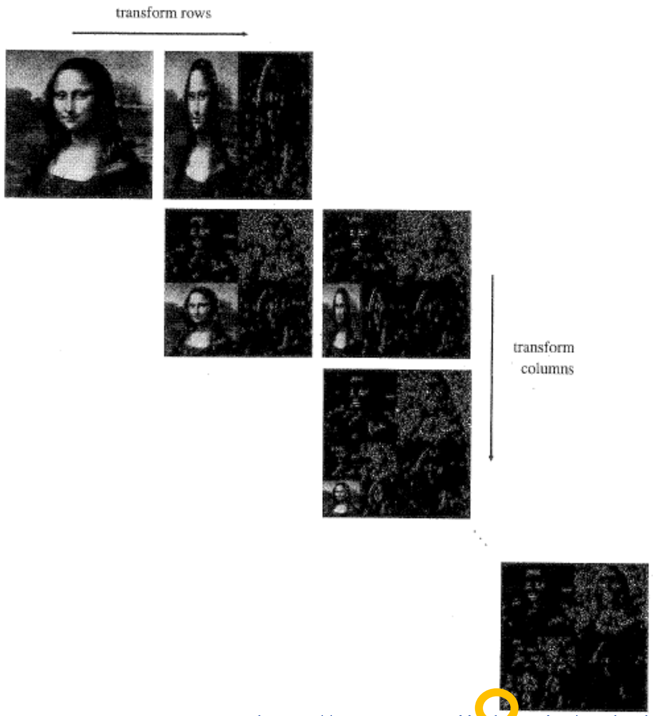




Haar Wavelet

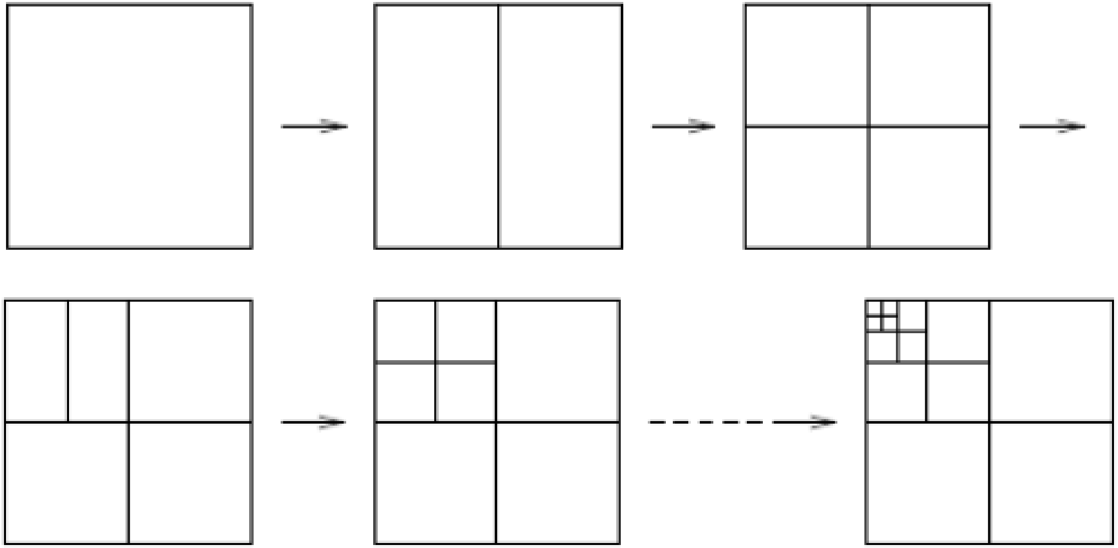
Non Standard Decomposition





Haar Wavelet

Non Standard Decomposition



Haar Wavelet

Sub band image coding

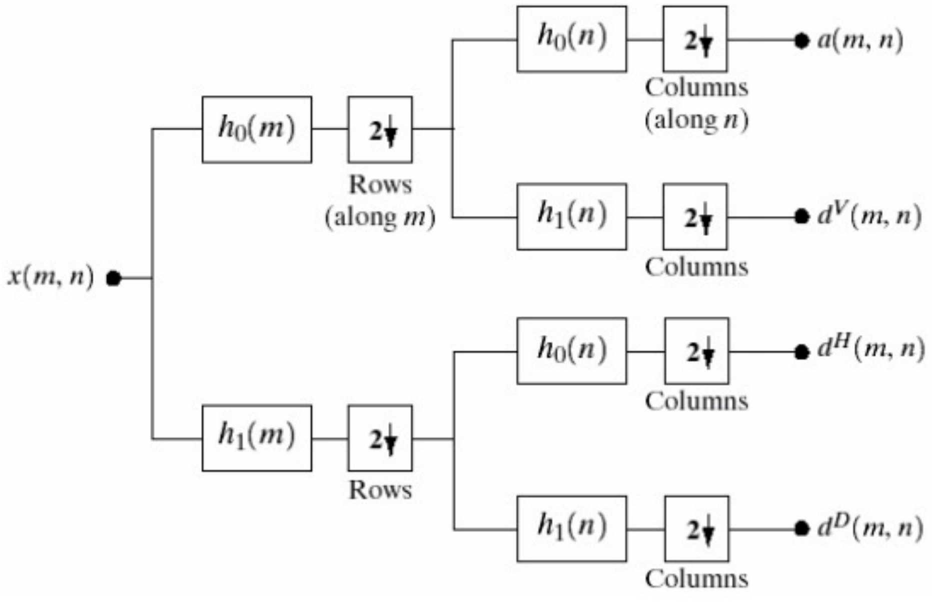


FIGURE 7.5 A two-dimensional, four-band filter bank for subband image coding.

Wavelet

Comparison with DCT

(a) (b) (c)

Original DCT Wavelet