Comparison between **Particle Swarm Optimization (PSO)** and **Ant Colony Optimization (ACO)** across several key dimensions:

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## 📊 PSO vs ACO: Comparison Table

| Feature | PSO | ACO |
|---|---|---|
| **Inspiration** | Bird flocking / fish schooling | Ant foraging behavior |
| **Solution Representation** | Particles (vectors in continuous space) | Paths (sequences of discrete decisions) |
| **Search Space** | Typically continuous | Typically discrete (e.g., graphs, permutations) |
| **Memory Usage** | Each particle remembers its best position | Pheromone trails encode collective memory |
| **Update Mechanism** | Velocity and position updates based on pBest and gBest | Pheromone update and probabilistic path selection |
| **Exploration vs Exploitation** | Controlled via inertia weight and coefficients | Controlled via pheromone evaporation and reinforcement |
| **Convergence Behavior** | Fast convergence, risk of premature convergence | Slower but more robust to local optima |
| **Parameter Sensitivity** | Sensitive to inertia, cognitive/social coefficients | Sensitive to pheromone decay rate and influence parameters |
| **Best For** | Continuous optimization problems | Combinatorial problems like TSP, routing, scheduling |
| **Parallelism** | Naturally parallelizable | Also parallelizable, especially in multi-ant simulations |

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## 🧠 Summary

- **PSO** is ideal for **continuous optimization** problems like function minimization or neural network training.
- **ACO** excels in **discrete optimization** problems like the **Traveling Salesman Problem (TSP)**, **network routing**, and **job scheduling**.

The **Particle Swarm Optimization (PSO)** algorithm is a **population-based stochastic optimization technique** inspired by the social behavior of birds flocking or fish schooling. It was introduced by **James Kennedy and Russell Eberhart in 1995**.

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## 🧠 Core Concept

Each solution in PSO is called a **particle**. These particles "fly" through the solution space, adjusting their positions based on:

- Their own best-known position (**personal best**, or `pBest`)
- The best-known position of the entire swarm (**global best**, or `gBest`)

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## 🔧 Key Components

- **Particles**: Represent potential solutions.
- **Velocity**: Determines how a particle moves in the search space.
- **Position**: The current solution represented by the particle.
- **Fitness Function**: Evaluates how good a solution is.

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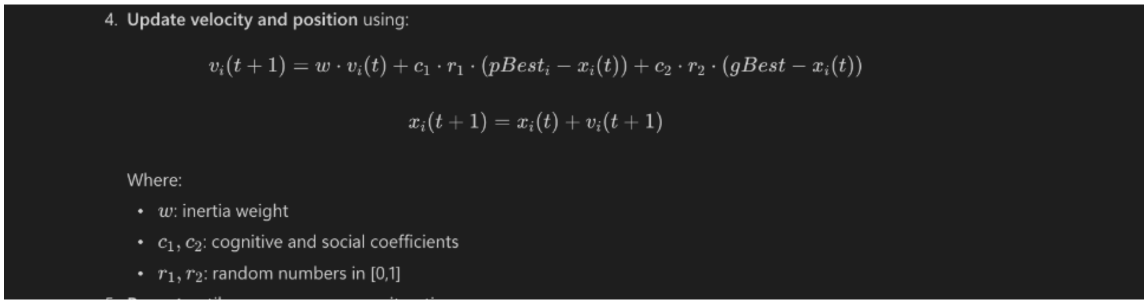
## 🔁 Algorithm Steps

1. **Initialize** a swarm of particles with random positions and velocities.
2. **Evaluate** the fitness of each particle.
3. Update each particle's:
   - **Personal best** if the current position is better.
   - **Global best** if any particle has a better position than the current global best.
4. **Update velocity and position** using:



4. **Update velocity and position** using:

$$v_i(t+1) = w \cdot v_i(t) + c_1 \cdot r_1 \cdot (pBest_i - x_i(t)) + c_2 \cdot r_2 \cdot (gBest - x_i(t))$$

$$x_i(t+1) = x_i(t) + v_i(t+1)$$

Where:

- $w$: inertia weight
- $c_1, c_2$: cognitive and social coefficients
- $r_1, r_2$: random numbers in [0,1]

Where:

    - ( w ): inertia weight
    - ( c*1, c*2 ): cognitive and social coefficients
    - ( r*1, r*2 ): random numbers in [0,1]
5. **Repeat** until convergence or max iterations.

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## 📦 Applications

- Function optimization
- Neural network training
- Feature selection
- Scheduling
- Robotics path planning

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## 🧪 Example Use Case

In **feature selection for machine learning**, each particle can represent a binary string where 1 means a feature is selected. The fitness function could be model accuracy or F1-score.

The **Ant Optimization Algorithm**, more formally known as **Ant Colony Optimization (ACO)**, is a nature-inspired metaheuristic algorithm used to solve **combinatorial optimization problems**. It mimics the behavior of real ants searching for the shortest path between their colony and food sources.

## 🐜 Core Idea

Ants deposit **pheromones** on paths they travel. Over time, shorter paths accumulate more pheromones because they are traversed more frequently. This positive feedback loop helps ants collectively find optimal paths.

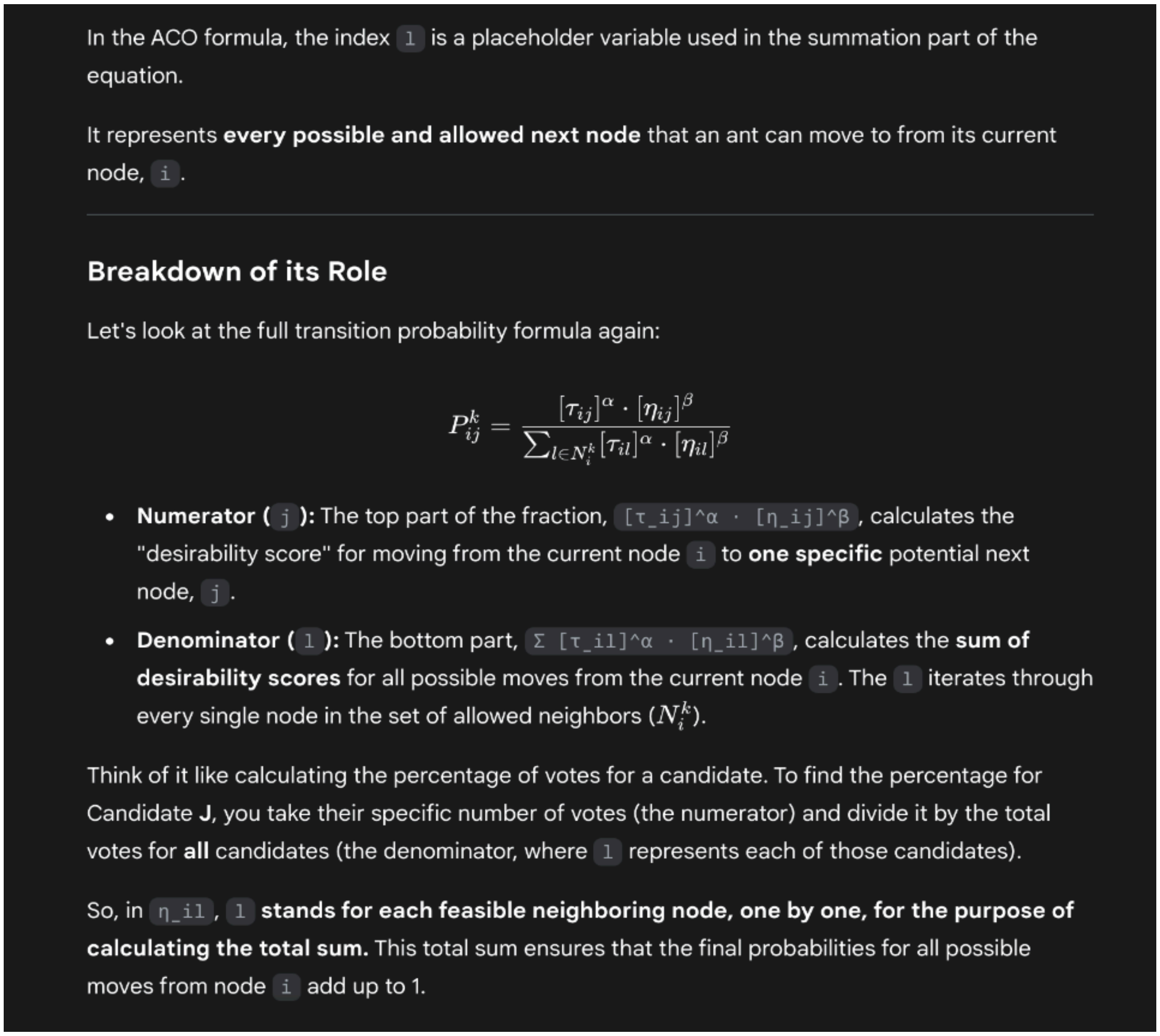
## 🔧 Key Components

1. **Artificial Ants**: Simulated agents that construct solutions step-by-step.
2. **Pheromone Trails**: Numerical values associated with solution components, guiding the ants.
3. **Heuristic Information**: Problem-specific data that helps ants make decisions (e.g., distance between cities in TSP).
4. **Pheromone Update Rules**:

- **Evaporation**: Reduces pheromone intensity to avoid convergence to suboptimal paths.
- **Deposition**: Ants deposit pheromones based on the quality of the solution.

## 🧠 Algorithm Steps

1. **Initialization**: Set initial pheromone levels.
2. **Solution Construction**: Each ant builds a solution using pheromone and heuristic info.
3. **Pheromone Update**: Evaporate old pheromones and deposit new ones.
4. **Daemon Actions (optional)**: Apply local search or update best solution.
5. **Termination**: Repeat until convergence or max iterations.



In the ACO formula, the index `l` is a placeholder variable used in the summation part of the equation.

It represents **every possible and allowed next node** that an ant can move to from its current node, `i`.

### Breakdown of its Role

Let's look at the full transition probability formula again:

$$P_{ij}^k = \frac{[\tau_{ij}]^\alpha \cdot [\eta_{ij}]^\beta}{\sum_{l \in N_i^k} [\tau_{il}]^\alpha \cdot [\eta_{il}]^\beta}$$

- **Numerator ( `j` ):** The top part of the fraction, `[τ_ij]^α · [η_ij]^β`, calculates the "desirability score" for moving from the current node `i` to **one specific** potential next node, `j`.
- **Denominator ( `l` ):** The bottom part, `Σ [τ_il]^α · [η_il]^β`, calculates the **sum of desirability scores** for all possible moves from the current node `i`. The `l` iterates through every single node in the set of allowed neighbors ($N_i^k$).

Think of it like calculating the percentage of votes for a candidate. To find the percentage for Candidate **J**, you take their specific number of votes (the numerator) and divide it by the total votes for **all** candidates (the denominator, where `l` represents each of those candidates).

So, in `η_il`, `l` **stands for each feasible neighboring node, one by one, for the purpose of calculating the total sum.** This total sum ensures that the final probabilities for all possible moves from node `i` add up to 1.

## 🧭 Applications

- **Traveling Salesman Problem (TSP)**
- **Vehicle Routing**
- **Scheduling**
- **Network Routing**
- **Feature Selection in ML**

## 🧪 Example: TSP

In TSP, ants build tours by choosing the next city based on:

- **Pheromone level** on the edge
- **Inverse of distance** (heuristic)
- **Probability formula** combining both

The **Genetic Algorithm (GA)** is a powerful optimization technique inspired by the process of **natural selection and genetics**. It's widely used for solving both **continuous and discrete optimization problems**, especially when the search space is large, complex, or poorly understood.

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## 🧬 Core Concepts of Genetic Algorithm

1. **Population**: A set of candidate solutions (called chromosomes).
2. **Chromosome**: A representation of a solution (often a binary string or vector).
3. **Fitness Function**: Evaluates how good a solution is.
4. **Selection**: Chooses the fittest individuals for reproduction.
5. **Crossover (Recombination)**: Combines two parent chromosomes to produce offspring.
6. **Mutation**: Randomly alters parts of a chromosome to maintain diversity.
7. **Elitism**: Preserves the best individuals into the next generation.

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## 🔁 Algorithm Steps

1. **Initialize** a population of random solutions.
2. **Evaluate** each solution using the fitness function.
3. **Select** parents based on fitness.
4. **Apply crossover** to produce offspring.

5. **Apply mutation** to introduce variation.
6. **Form new population** and repeat until convergence or max generations.

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## 🧠 Mathematical Insight

- **Selection**: Often uses methods like roulette wheel, tournament, or rank selection.
- **Crossover**: Single-point, multi-point, or uniform crossover.
- **Mutation**: Bit-flip (binary), Gaussian noise (real-valued), etc.

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## 📦 Applications

- Function optimization
- Feature selection
- Scheduling problems
- Game strategy evolution
- Neural network architecture search
- Design and engineering problems

Here are some strong **alternatives to Particle Swarm Optimization (PSO)** and **Ant Colony Optimization (ACO)**, depending on the nature of the problem you're solving:

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## 🔄 1. Genetic Algorithm (GA)

- **Inspired by**: Natural selection and genetics
- **Best for**: Both continuous and discrete problems
- **Strengths**: Good global search capability, flexible representation
- **Weaknesses**: Can be slow to converge, sensitive to parameter tuning

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## 🧠 2. Differential Evolution (DE)

- **Inspired by**: Evolutionary strategies
- **Best for**: Continuous optimization

- **Strengths**: Simple, robust, and effective for high-dimensional problems
- **Weaknesses**: May require careful parameter tuning

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## 🧮 3. Simulated Annealing (SA)

- **Inspired by**: Thermodynamics and annealing in metallurgy
- **Best for**: Discrete and combinatorial problems
- **Strengths**: Escapes local optima well
- **Weaknesses**: Slow convergence, sensitive to cooling schedule

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## 🧬 4. Evolution Strategies (ES)

- **Inspired by**: Biological evolution
- **Best for**: Continuous optimization
- **Strengths**: Strong in noisy environments, scalable
- **Weaknesses**: Less intuitive than GA or PSO

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## 🧭 5. Artificial Bee Colony (ABC)

- **Inspired by**: Foraging behavior of honey bees
- **Best for**: Continuous and discrete optimization
- **Strengths**: Good balance of exploration and exploitation
- **Weaknesses**: May struggle with complex constraints

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## 🧪 6. Grey Wolf Optimizer (GWO)

- **Inspired by**: Leadership hierarchy and hunting behavior of grey wolves
- **Best for**: Continuous optimization
- **Strengths**: Simple and effective, fewer parameters
- **Weaknesses**: May converge prematurely

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## 📦 7. Firefly Algorithm

- **Inspired by**: Flashing behavior of fireflies
- **Best for**: Continuous optimization

- **Strengths**: Good for multimodal problems
- **Weaknesses**: Can be slow for large-scale problems

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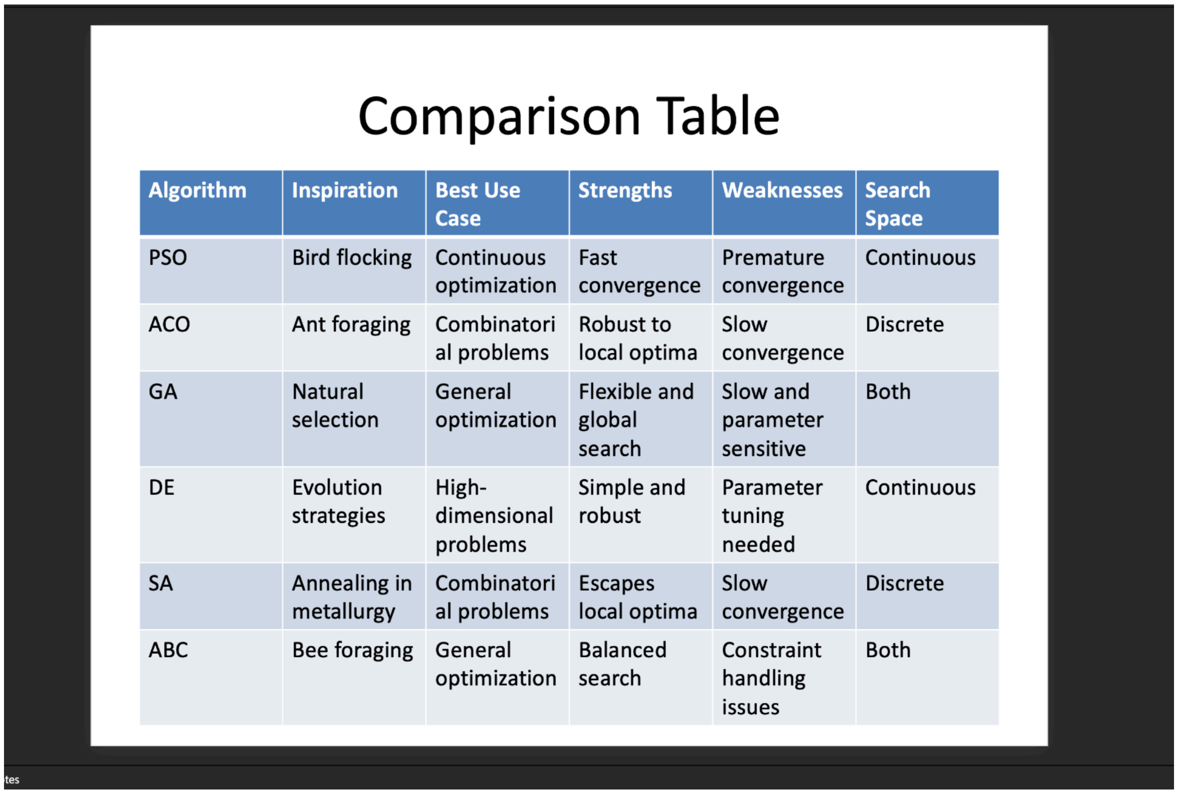

## Comparison Table

| Algorithm | Inspiration | Best Use Case | Strengths | Weaknesses | Search Space |
|---|---|---|---|---|---|
| PSO | Bird flocking | Continuous optimization | Fast convergence | Premature convergence | Continuous |
| ACO | Ant foraging | Combinatorial problems | Robust to local optima | Slow convergence | Discrete |
| GA | Natural selection | General optimization | Flexible and global search | Slow and parameter sensitive | Both |
| DE | Evolution strategies | High-dimensional problems | Simple and robust | Parameter tuning needed | Continuous |
| SA | Annealing in metallurgy | Combinatorial problems | Escapes local optima | Slow convergence | Discrete |
| ABC | Bee foraging | General optimization | Balanced search | Constraint handling issues | Both |

- Pijk is the probability of ant *k* moving from node *i* to node *j*.
- τij represents the amount of pheromone on the edge between node *i* and node *j*. A higher value indicates that this path has been frequently used by other ants that found good solutions.
- ηij is the heuristic information, which is a measure of the desirability of moving from node *i* to node *j*. For many problems, like the Traveling Salesperson Problem (TSP), this is the inverse of the distance between the two nodes (1/dij), meaning shorter paths are more desirable.
- α and β are parameters that control the relative influence of the pheromone trail and the heuristic information, respectively.
    - If α=0, the choice is purely based on the heuristic information (a greedy approach).
    - If β=0, the choice is solely based on the pheromone levels, ignoring the heuristic desirability.

- Nik is the set of feasible nodes that ant *k* can move to from its current node *i*. This prevents the ant from visiting the same node twice in a single tour.

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## Step-by-Step Calculation

Here's how an ant decides its next move:

1. **Identify Feasible Moves:** From its current location (node *i*), the ant identifies all the possible next nodes it can visit. This typically excludes nodes it has already visited in its current tour.
2. **Calculate the Numerator for Each Feasible Move:** For each potential next node *j*, the ant calculates the product of the pheromone level on the path to it and the heuristic value of that path, each raised to their respective importance parameters (α and β).
    - This step combines the "learned" information from other ants (pheromones) with the inherent "attractiveness" of the path (heuristics).
3. **Calculate the Denominator:** The ant then sums up the results from step 2 for all possible next nodes. This sum represents the total desirability of all feasible moves from the current node.
4. **Determine the Probability:** Finally, to get the probability of moving to a specific node *j*, the value calculated in step 2 for that particular node is divided by the total sum calculated in step 3.
5. **Probabilistic Selection:** The ant then uses a mechanism like a roulette wheel selection to choose its next node based on these calculated probabilities. A higher probability for a particular path means it is more likely to be chosen, but it doesn't guarantee it will be. This probabilistic nature allows for exploration of different paths.

You have been asked to solve below linear equation problem with multiple variables:

$$2a + 7b - 5c + d = 0$$

where a, b,c, and d are integers in the range [-20,20] (equation can have more than one solution)
You have decided to solve this question using genetic algorithm. Show all the steps involved in solving this using Genetic Algorithm. Number each of your steps, and provide appropriate title to those steps. (No need to get the final solution, just the first iteration of each step involved in the algorithm)

Most likely representation of chromosome could be a list with 4 integers with each integer representing variables a,b,c or d. For example [3,2,3,1] would imply a=3,b=2,c=3,d=1.

Generate random population (list of lists)

Fitness function - use a fitness function (most likely f(x) = 2a + 7b -5c +d) and calculate the fitness of each individual

Apply some selection criteria based on fitness value (could be roulette or the best fit etc.) to come up with initial population

Apply crossover and mutation (how students chose to incorporate crossover and mutation may differ but it should be consistent)

Of course. This document outlines the process of using a **Genetic Algorithm (GA)** to find integer solutions for the linear equation $2a + 7b - 5c + d = 0$.

A Genetic Algorithm is an optimization technique inspired by Charles Darwin's theory of natural evolution. It mimics processes like selection, crossover, and mutation to "evolve" better solutions to a problem over several generations.

Here's an explanation of each step for the first iteration of the algorithm.

## Step 1: Chromosome Representation

First, we need a way to represent a potential solution to the problem. In GA terminology, a single solution is called a **chromosome**.

- **What it is:** For this problem, a solution consists of four integer variables: $a$, $b$, $c$, and $d$. A logical way to represent this is as a list of four integers.
- **Example:** The chromosome [3, 2, 3, 1] represents the potential solution where $a=3$, $b=2$, $c=3$, and $d=1$.

## Step 2: Generate Initial Population

The algorithm doesn't start with just one guess; it starts with a collection of many random guesses. This initial set of chromosomes is called the **population**.

- **What it is:** We create a list containing multiple, randomly generated chromosomes. Each integer within each chromosome is randomly chosen from the specified range of -20 to 20.
- **Example:** A small initial population might look like this (a list of lists):

```
[
  [3, 2, 3, 1],
  [-10, 5, 8, -4],
  [15, -2, -1, 20],
  [...and so on for many more chromosomes]
]
```

## Step 3: Fitness Function

Next, we need a way to evaluate how "good" each chromosome (potential solution) is. This is done using a **fitness function**. A "fitter" chromosome is a better solution.

- **What it is:** The goal is to make the equation 2a + 7b - 5c + d equal to **0**. A solution is perfect when the result of this expression is exactly 0. Therefore, the fitness of a chromosome is higher the closer the expression's result is to 0.
- **How it works:** We can't just use f(x) = 2a + 7b - 5c + d directly, because a result of -100 would be considered less fit than a result of 10. A better fitness function would be:
  Fitness=1+∣(2a+7b−5c+d)∣1
  In this formula, |...| denotes the absolute value.
  1. If a chromosome is a **perfect solution**, the expression 2a + 7b - 5c + d equals 0. The fitness will be 1 / (1 + 0) = 1, which is the maximum possible score.
  2. If a chromosome is a **poor solution**, the expression will be a large positive or negative number. Its absolute value will be large, making the denominator large and the fitness score very close to 0.
- **Example Calculation:** For the chromosome [3, 2, 3, 1]:
  1. Calculate the expression: 2(3) + 7(2) - 5(3) + 1 = 6 + 14 - 15 + 1 = 6
  2. Calculate fitness: Fitness = 1 / (1 + |6|) = 1/7 ≈ 0.14
- The algorithm performs this calculation for every chromosome in the population.

## Step 4: Selection

Now we select the "fittest" chromosomes from the population to be "parents" for the next generation. This mimics the "survival of the fittest" principle in nature.

- **What it is:** Chromosomes with higher fitness scores have a higher probability of being chosen to pass on their "genes" (the integer values) to the next generation.
- **How it works:** A common method is **Roulette Wheel Selection**. Imagine a roulette wheel where each chromosome gets a slice proportional to its fitness score. A fitter chromosome gets a larger slice and is therefore more likely to be selected when the wheel is spun. This process is repeated to create a "mating pool" of parent chromosomes.

## Step 5: Crossover and Mutation

This is how new solutions (offspring) are created from the selected parents, introducing new combinations and variations.

**Crossover**

- **What it is:** Two parent chromosomes are chosen from the mating pool and their genetic material is combined to create one or more new offspring.
- **How it works (Single-Point Crossover):**
  - Choose a random "crossover point" in the chromosome.
  - The offspring is created by taking the genes from the first parent up to the crossover point and the genes from the second parent after that point.
- **Example:**
  - **Parent 1:** [-10, 5, | 8, -4]
  - **Parent 2:** [15, -2, | -1, 20]
  - Let's say the crossover point is after the 2nd gene.
  - **Offspring 1:** [-10, 5, -1, 20]

**Mutation**

- **What it is:** To ensure genetic diversity and prevent the algorithm from getting stuck on a suboptimal solution, we introduce small, random changes into the new offspring's chromosomes.
- **How it works:** With a very small probability, one gene (integer) in the chromosome is randomly changed to a new value within the allowed range [-20, 20].
- **Example:**
  - **Offspring before mutation:** [-10, 5, -1, 20]
  - A mutation might occur on the 3rd gene.
  - **Offspring after mutation:** [-10, 5, 18, 20]

After these steps, we have a new generation of solutions (the offspring). This entire cycle—calculating fitness, selecting parents, and creating new offspring through crossover and mutation—is then repeated for many generations. Over time, the average fitness of the population tends to increase, leading the algorithm to converge on a good or perfect solution.

ACO Prob

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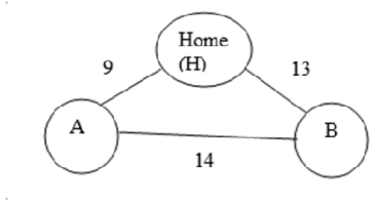
a) Abhay is on a tour. He starts from home and visits two cities and has to reach back home. The cost of each route between cities and between his home and cities is given below in the form of graph. Determine the shortest path through Ant Colony Optimization. Use the following information regarding the various parameters for ACO.



Pheromone matrix is as given below:

$$\begin{array}{c|ccc} & H & A & B \\ H & 0 & 0.15 & 0.26 \\ A & 0.15 & 0 & 0.48 \\ B & 0.26 & 0.48 & 0 \end{array}$$

Rate of evaporation = 0.1; Q = 90; The relative importance of pheromone is 0.3 and the relative importance of distance is 0.4;

You provided an ACO problem with:

- A graph consisting of 3 nodes: **Home (H), A, B**
- Distances between nodes:
  - H ↔ A: 9
  - H ↔ B: 13
  - A ↔ B: 14

Pheromone levels matrix:

| | H | A | B |
|---|---|---|---|
| H | 0 | 0.15 | 0.26 |
| A | 0.15 | 0 | 0.48 |
| B | 0.26 | 0.48 | 0 |

We'll solve a **Traveling Salesman Problem (TSP)** using **ACO**, starting from **Home**, and visiting each city once before returning home.

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## ✅ Step-by-Step Ant Colony Optimization (ACO)

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### Step 1: Initialization

We define:

- **α (alpha)** = 1 (pheromone importance)
- **β (beta)** = 2 (visibility importance)
- **Number of ants** = 1 (for simplicity)

**Visibility** (η) is the inverse of distance:
ηij=1dij\eta_{ij} = \frac{1}{d_{ij}}
So, the visibility matrix becomes:

| | H | A | B |
|---|---|---|---|
| H | -- | 1/9 ≈ 0.111 | 1/13 ≈ 0.077 |
| A | 1/9 | -- | 1/14 ≈ 0.071 |
| B | 1/13 | 1/14 | -- |

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### Step 2: Transition Probability Formula

An ant at node ii will choose the next node jj based on the formula:

Pij=[τij]α[ηij]β∑k∈Ni[τik]α[ηik]βP_{ij} = \frac{[\tau_{ij}]^{\alpha} [\eta_{ij}]^{\beta}}{\sum_{k \in N_i} [\tau_{ik}]^{\alpha} [\eta_{ik}]^{\beta}}

Where:

- τij\tau_{ij}: pheromone on edge i→ji \rightarrow j
- ηij\eta_{ij}: visibility (inverse of distance)
- NiN_i: unvisited neighbors of node ii

## Step 3: Starting at Home (H)

Unvisited nodes = {A, B}

Calculate probabilities from H to A and B.

- τHA=0.15\tau_{HA} = 0.15, ηHA=1/9≈0.111\eta_{HA} = 1/9 \approx 0.111
- τHB=0.26\tau_{HB} = 0.26, ηHB=1/13≈0.077\eta_{HB} = 1/13 \approx 0.077

Now compute:

- Numerator(H→A):
  0.151 · (1/9)2=0.15 · 0.0123=0.001850.15^1 \cdot (1/9)^2 = 0.15 \cdot 0.0123 = 0.00185
- Numerator(H→B):
  0.261 · (1/13)2=0.26 · 0.0059=0.001540.26^1 \cdot (1/13)^2 = 0.26 \cdot 0.0059 = 0.00154
- Denominator = 0.00185 + 0.00154 = 0.00339
- Probabilities:
  - PHA=0.00185/0.00339≈0.546P_{HA} = 0.00185 / 0.00339 ≈ 0.546
  - PHB=0.00154/0.00339≈0.454P_{HB} = 0.00154 / 0.00339 ≈ 0.454

✅ **Choose A** (random draw with weighted probability)

## Step 4: From A → ?

Unvisited = {B}

Only option: **A → B**

So path now is:
**H → A → B**

## Step 5: From B → H

Only option: Return to home.

Final path:
**H → A → B → H**

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## Step 6: Calculate Tour Length

- H → A = 9
- A → B = 14
- B → H = 13

**Total Distance = 9 + 14 + 13 = 36**

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## Step 7: Pheromone Update

Let:

- **Q = 100** (pheromone constant)
- Deposit: Δτij=Q/L\Delta\tau_{ij} = Q / L, where L=L = tour length

Δτ=100/36≈2.78\Delta\tau = 100 / 36 ≈ 2.78

Now, update pheromone on each edge used:

- H–A:
  New τHA=0.15+2.78=2.93\tau_{HA} = 0.15 + 2.78 = 2.93
- A–B:
  New τAB=0.48+2.78=3.26\tau_{AB} = 0.48 + 2.78 = 3.26
- B–H:
  New τBH=0.26+2.78=3.04\tau_{BH} = 0.26 + 2.78 = 3.04

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### Step 8: Evaporation (Optional)

If using evaporation rate ρ=0.1\rho = 0.1:

τij=(1−ρ)⋅τij+Δτij\tau_{ij} = (1 - \rho) \cdot \tau_{ij} + \Delta\tau_{ij}

But since you didn't specify, we'll assume no evaporation for this step.

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### ✅ Final Result after 1 iteration:

**Path taken:**
**H → A → B → H**

**Total Distance:**
**36 units**

**Updated Pheromone Matrix:**

| | H | A | B |
|---|---|---|---|
| H | 0 | 2.93 | 0.26 |
| A | 2.93 | 0 | 3.26 |
| B | 3.04 | 3.26 | 0 |