BITS Pilani
Hyderabad Campus

Data Structures and Algorithms Design

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CONTACT SESSION 6 -PLAN

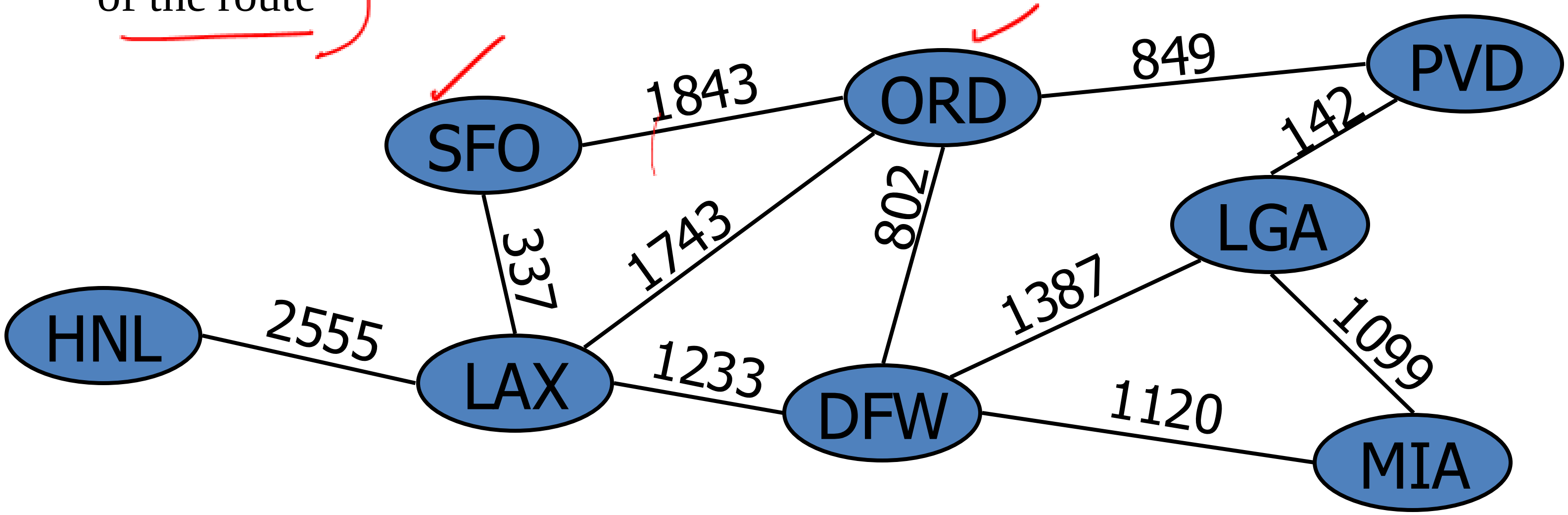
Contact Sessions(#)	List of Topic Title	Text/Ref Book/external resource
6	Graphs - Terms and Definitions, Properties, Representations (Edge List, Adjacency list, Adjacency Matrix), Graph Traversals (Depth First and Breadth First Search)	T1: 6.1, 6.2, 6.3

Graphs

- Graphs
  - Definition
  - Applications
  - Terminology
  - Properties
  - ADT
- Data structures for graphs
  - Edge list structure
  - Adjacency list structure
  - Adjacency matrix structure

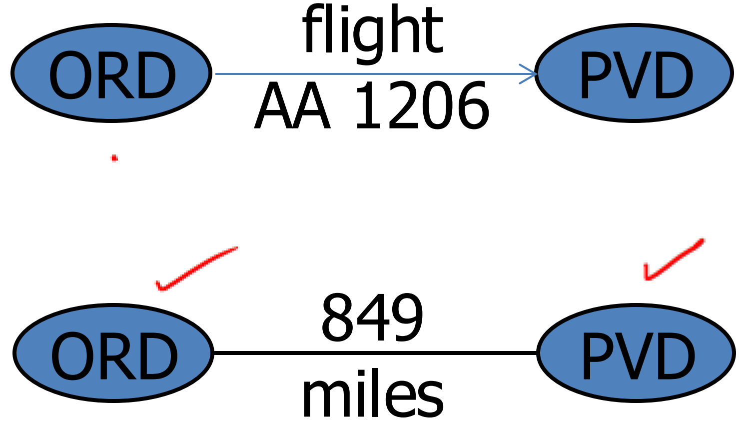
Graphs

- A graph is a pair (***V, E***), where
  - ***V*** is a set of nodes, called vertices
  - ***E*** is a collection of pairs of vertices, called edges
  - Vertices and edges are **positions** and store elements
- Example:
  - A vertex represents an airport and stores the three-letter airport code
  - An edge represents a flight route between two airports and stores the mileage of the route



Graphs

- Edge Types
- Directed edge
  - ordered pair of vertices (***u***,***v***)
  - first vertex ***u*** is the origin
  - second vertex ***v*** is the destination
  - e.g., a flight
- Undirected edge
  - unordered pair of vertices (***u***,***v***)
  - e.g., a flight route
- Directed graph
  - all the edges are directed
  - e.g., flight network
- Undirected graph
  - all the edges are undirected
  - e.g., route network

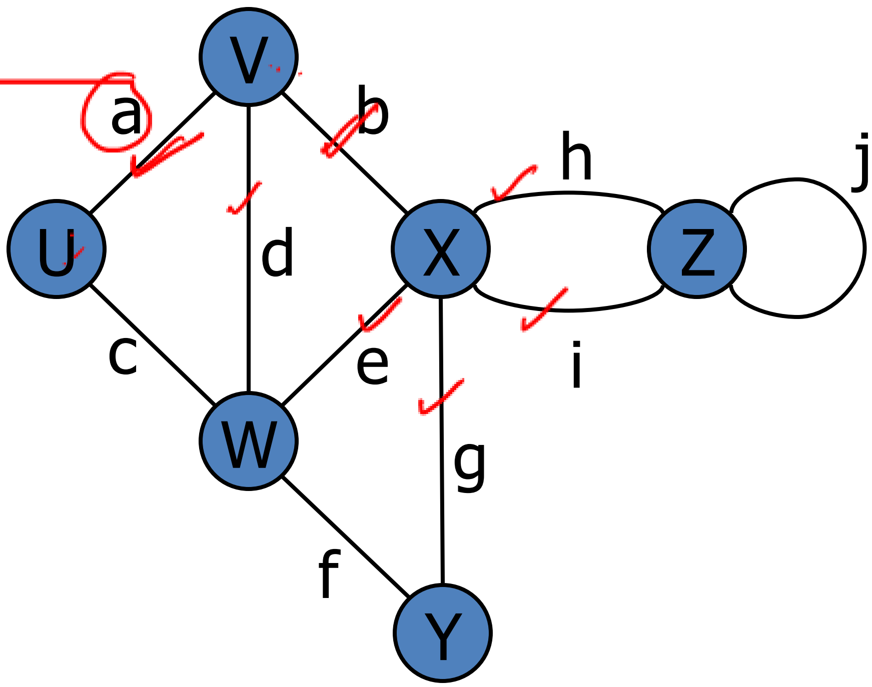


Graphs-Applications

- Electronic circuits
  - Printed circuit board
- Transportation networks
  - Highway network
  - Flight network
- Computer networks
  - Local area network
  - Internet
- Databases
  - Entity-relationship diagram

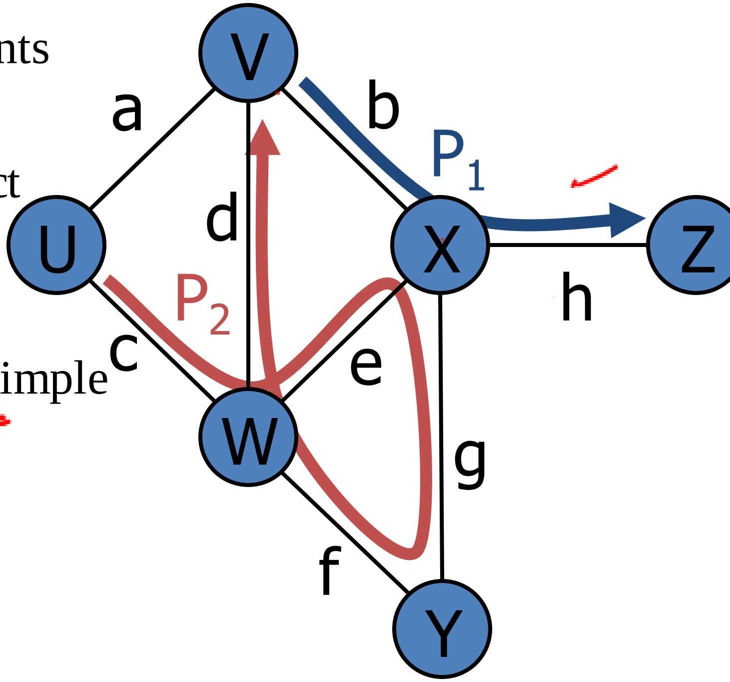
Graphs-Terminology

- End vertices (or endpoints) of an edge
  - U and V are the endpoints of a
- Edges incident on a vertex
  - a, d, and b are incident on V
- Adjacent vertices
  - U and V are adjacent
- Degree of a vertex
  - X has degree 5
- Parallel edges
  - h and i are parallel edges
- Self-loop
  - j is a self-loop

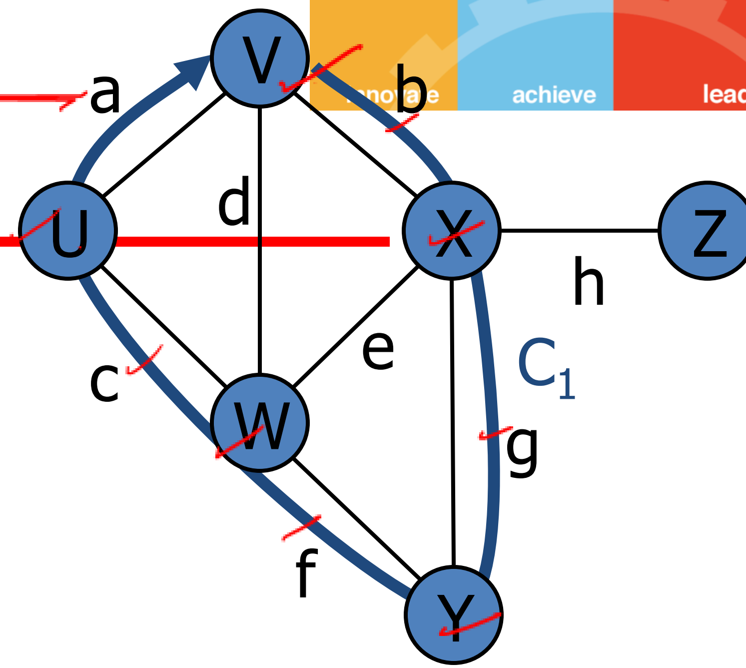


Graphs-Terminology

- Path
  - **sequence of alternating vertices and edges**
  - begins with a vertex
  - ends with a vertex
  - each edge is preceded and followed by its endpoints
- Simple path
  - path such that all its vertices and edges are distinct
- Examples
  - P_1=(V,b,X,h,Z) is a simple path
  - P_2=(U,c,W,e,X,g,Y,f,W,d,V) is a path that is not simple

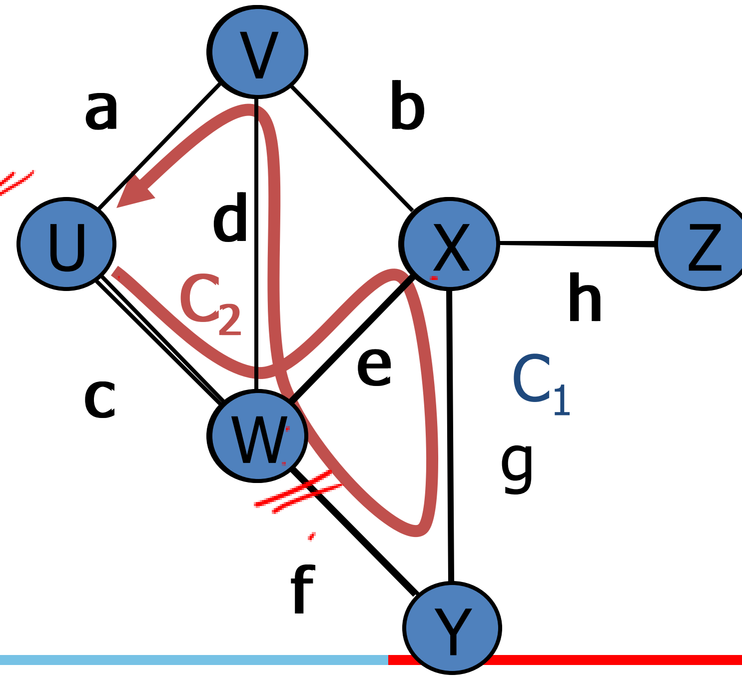


Graphs-Terminology



- Cycle
  - circular sequence of alternating vertices and edges
  - each edge is preceded and followed by its endpoints
- Simple cycle
  - cycle such that all its vertices and edges are distinct
- Examples
  - C_1=(V,b,X,g,Y,f,W,c,U,a) is a simple cycle
  - C_2=(U,c,W,e,X,g,Y,f,W,d,V,a)

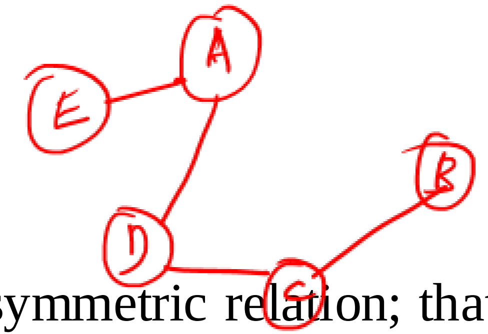
  is a cycle that is not simple



Graph-Example 1

We can visualize collaborations among the researchers of a certain discipline by constructing a graph whose vertices are associated with the researchers themselves, and whose edges connect pairs of vertices associated with researchers who have coauthored a paper or book.

Is it a directed or undirected graph?



Such edges are **undirected** because coauthorship is a symmetric relation; that is, if A has coauthored something with B, then B necessarily has coauthored something with A .

Graph-Example 2

- We can associate with an object-oriented program a graph whose vertices represent the classes defined in the program, and whose edges indicate inheritance between classes. There is an edge from a vertex v to a vertex u if the class for v extends the class for u.
- Is it a directed or undirected graph?

- Such edges are directed because the inheritance relation only goes in one direction (that is, it is asymmetric).

Graph-Example 3

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- A city map can be modelled by a graph whose vertices are intersections or dead ends, and whose edges are stretches of streets without intersections.
- Directed or undirected?

- This graph has **both undirected edges**, which correspond to stretches of two-way streets, **and directed edges**, which correspond to stretches of one-way streets. Thus, a graph modelling a city map is a mixed graph

Graph-Example 4

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- Physical examples of graphs are present in the electrical wiring and plumbing networks of a building.
- Such networks can be modelled as graphs, where each connector, fixture, or outlet is viewed as a vertex, and each uninterrupted stretch of wire or pipe is viewed as an edge.
- Such graphs are actually components of much larger graphs, namely the local power and water distribution networks.
- Depending on the specific aspects of these graphs that we are interested in, we may consider their edges as undirected or directed, for, in principle, water can flow in a pipe and current can flow in a wire in either direction.

Graph-Example 5-Path and Cycle

Given a graph G representing a city map, we can model a couple driving from their home to dinner at a recommended restaurant as traversing a path though G.

If they know the way, and don 't accidentally go through the same intersection twice, then they traverse a simple path in G.

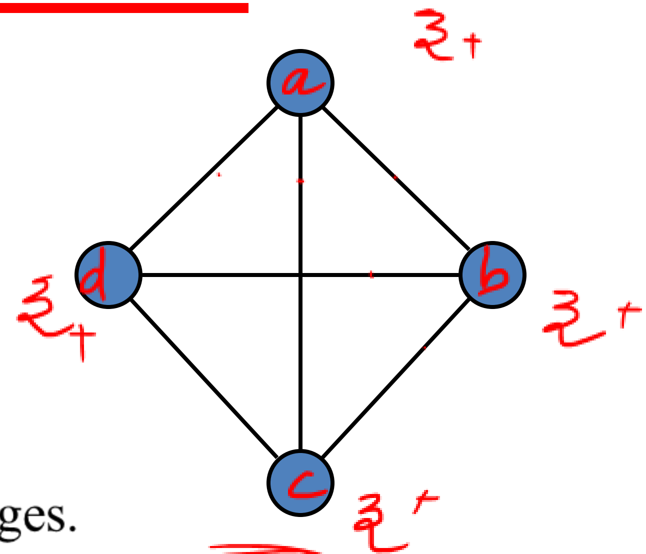
Likewise, we can model the entire trip the couple takes, from their home to the restaurant and back, as a cycle.

If they go home from the restaurant in a completely different way than how they went, not even going through the same intersection twice, then their entire round trip is a simple cycle.

Finally, if they travel along one-way streets for their entire trip, then we can model their night out as a directed cycle.

Graphs-Properties

m = 6   2m = 12



Property 1

If G is a graph with m edges, then

$$\sum_v \mathbf{deg}(v) = \mathbf{2}m$$

Proof: each edge is counted twice

Property 2

Let G be a simple graph with n vertices and m edges.

In an undirected graph with no self-loops and no multiple edges

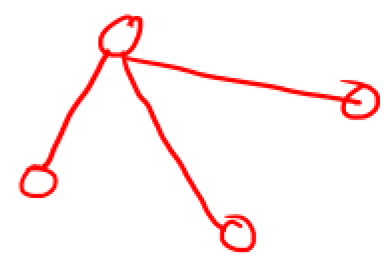
$$m \le n\,(n-1)/2 \text{ is } O(n^2)$$

Proof: each vertex has degree at most $(n-1)$

Property 3

If G is a directed graph with m edges, then

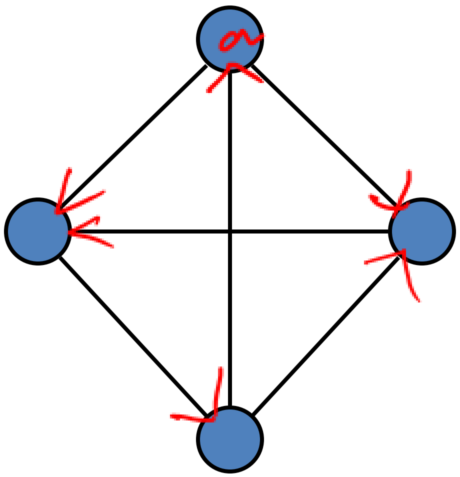
$$\sum_{v\in G} indeg(v) = \sum_{v\in G} outdeg(v) = m$$



Graphs-Properties

Example

- $n = 4$
- $m = 6$
- $\deg(v) = 3$

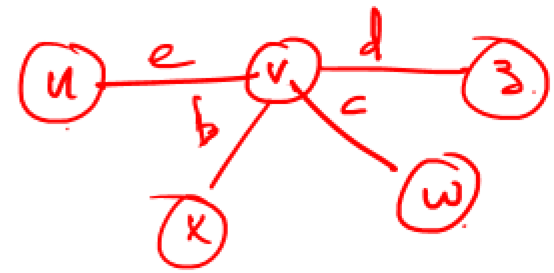


indegree(a) = 1

outdeg(a) = 2

Graphs-ADT

- A graph is a positional container of elements that are stored at the graph's vertices and edges
- Vertices and edges
  - are positions
  - store elements



- **Accessor methods** **[Complexity]**
  - **incidentEdges(v)**:Return an iterator of the edges incident upon v
  - endVertices(e):Return an array of size 2 storing the end vertices of e.
  - degree(v):
  - adjacentVertices(v):Return an iterator of the vertices adjacent to v.
  - opposite(v, e):Return the endpoint of edge e distinct from v.
  - **areAdjacent(v, w):**Return whether vertices v and w are adjacent

Graphs-ADT

Methods Dealing with Directed Edges

- directed Edges(): Return an iterator of all directed edges.
- undirected Edges(): Return an iterator of all undirected edges.
- destination(e): Return the destination of the directed edge e.
- origin (e): Return the origin of the directed edge e.
- isDirected(e): Return true if and only if the edge e is directed.

Graphs-ADT

Update methods

- **insertVertex(o):**Insert and return a new vertex storing the object o
- **insertEdge(v, w, o):**Insert and return an undirected edge between vertices v and w, storing the object o.
- insertDirectedEdge(v, w, o)
- **removeVertex(v)**:Remove vertex v and all its incident edges.
- **removeEdge(e)**
- Generic methods
  - numVertices()
  - numEdges()
  - vertices():Return an iterator of the vertices of G.
  - edges()

Graphs-ADT

Also supports

- size()
- isEmpty ( )
- elements ( )
- positions()
- replaceElement(p, o)
- swapElements (p , q)

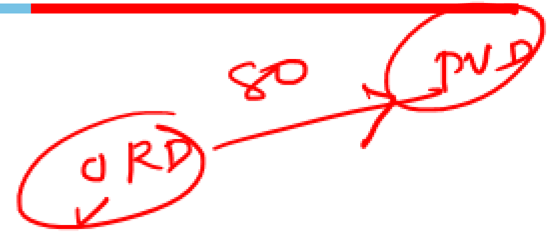
where p and q denote positions, and o denotes an object (that is, an element)

Data Structure for Graphs

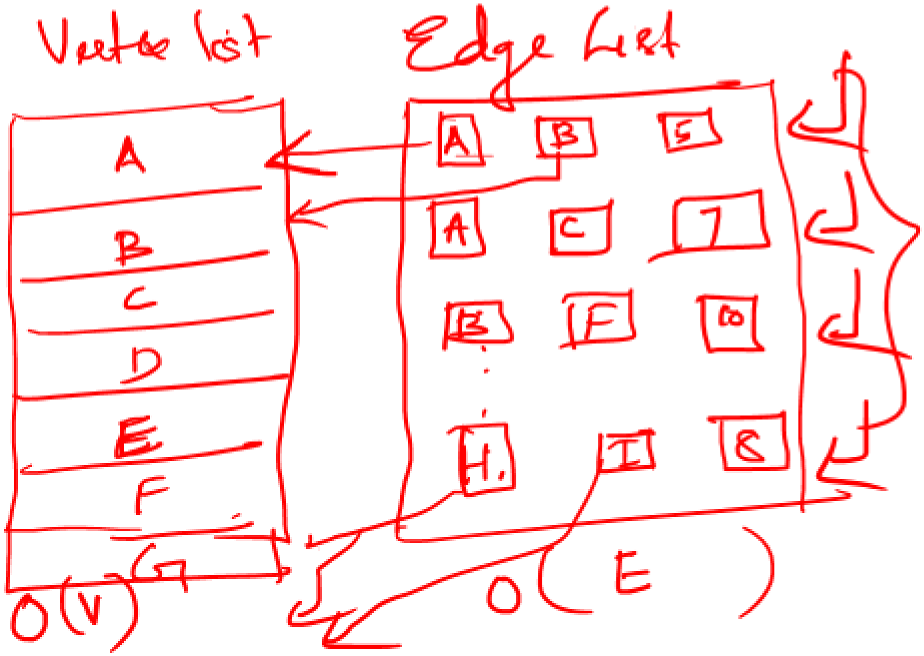
- Edge list structure
- Adjacency list structure
- Adjacency matrix

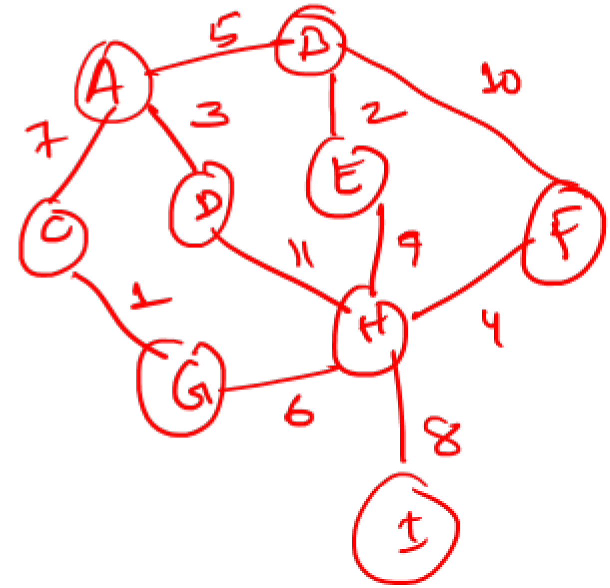
Edge List Structure

- Vertex object
  - **Element, o**
- Edge object
  - **element**
  - **origin vertex object**
  - **destination vertex object**



- Vertex sequence
  - sequence of vertex objects
- Edge sequence
  - sequence of edge objects


Vertex list
Edge List
A
B
C
D
E
F
G
O(V)
O(E)

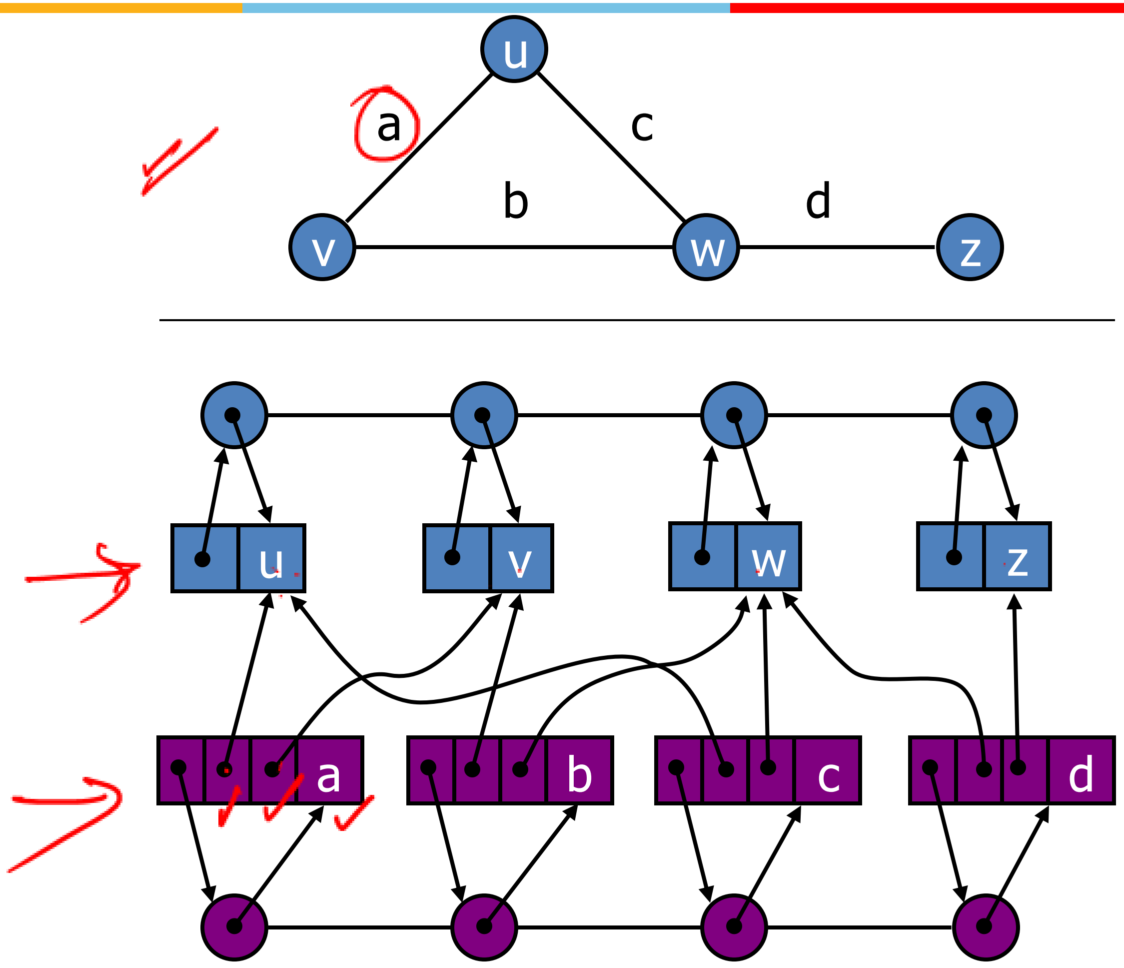

A
B
C
D
E
F
G
H
t
5
10
7
3
2
11
9
1
4
6
8

$O(V+E)$

Time Complexity

Methods	Edge List- Time Complexity
incidentEdges(v)	O(m)
areAdjacent(v, w)	O(m)
insertVertex(o)	O(1)
insertEdge(v, w, o)	O(1)
removeVertex(v)	O(m)
removeEdge(e)	O(1)

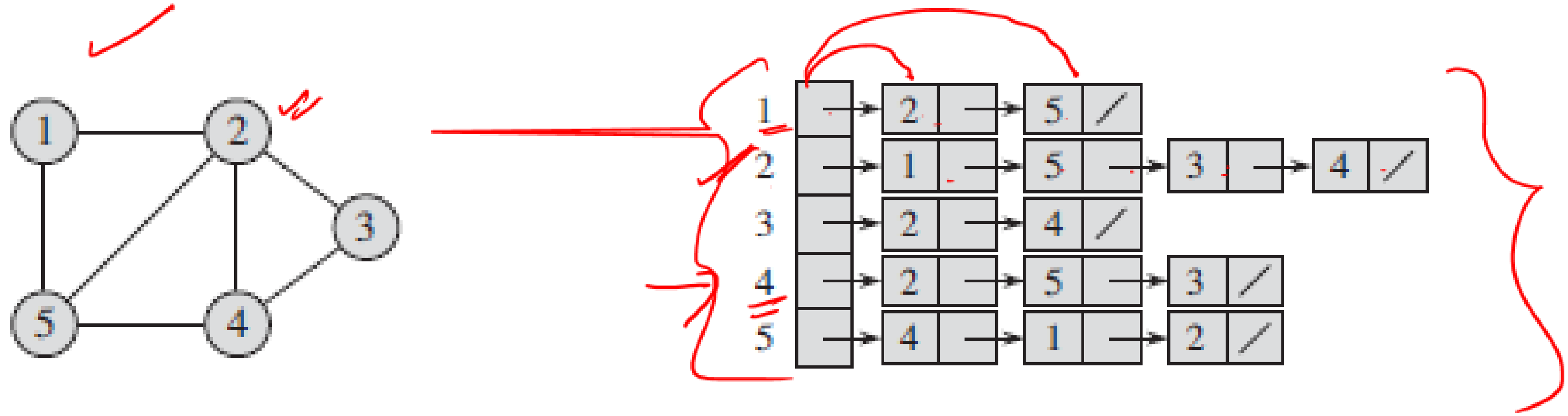
Edge List Structure

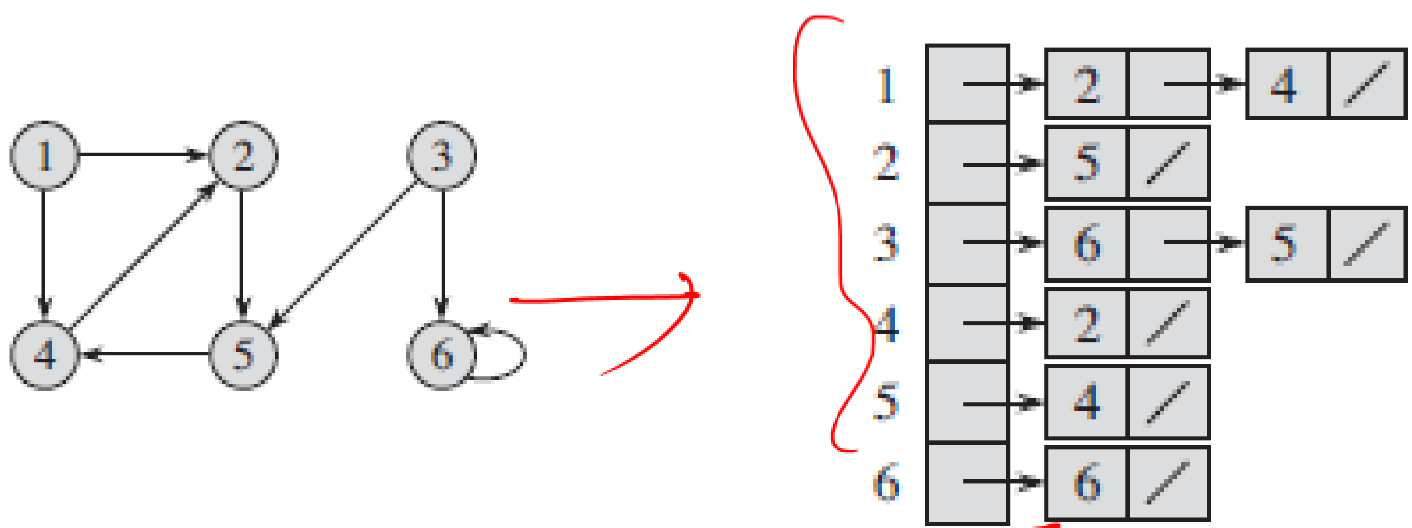


Adjacency List Structure

- Extends edge list structure
- Add extra information that supports direct access to the incident edges (and thus to the adjacent vertices) of each vertex
- For each vertex v, store a reference to a list of the vertices adjacent to it.

Adjacency List





Time Complexity

	Edge List	Adjacency List
incidentEdges(v)	m	deg(v)
areAdjacent(v, w)	m	min(deg(v),deg(w))
insertVertex(o)	1	1
insertEdge(v, w, o)	1	1
removeVertex(v)	m	deg(v)
removeEdge(e)	1	1

Adjacency Matrix Structure

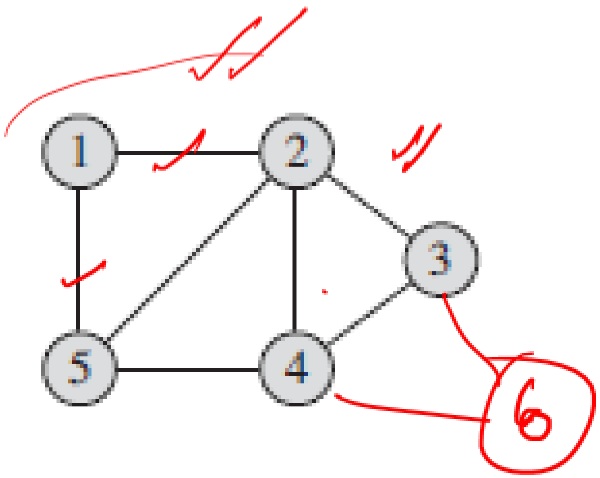
- The ***adjacency-matrix representation*** of a graph G =(V,E), the vertices are numbered 1,2,…,|V| in some arbitrary manner. Then the adjacency-matrix representation of a graph G consists of a |V| X |V| matrix

n x n

$$A = (a_{ij}) \text{ such that}$$

$$a_{ij} = \begin{cases} 1 & \text{if } (i, j) \in E \,, \\ 0 & \text{otherwise} \,. \end{cases}$$

Adjacency Matrix



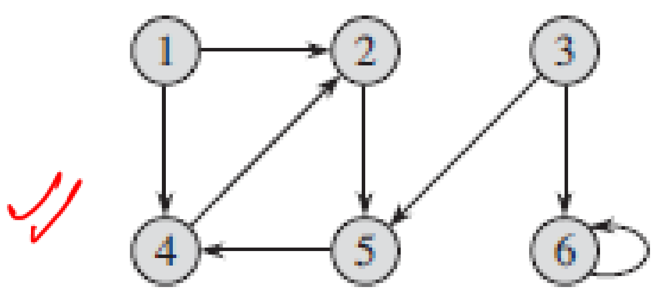
n

	1	2	3	4	5	6
1	0	1	0	0	1	0
2	1	0	1	1	1	0
3	0	1	0	1	0	1
4	0	1	1	0	1	1
5	1	1	0	1	0	0
6	0	0	1	1	0	0

n

A[4][3]

= ~~0~~ 1



	1	2	3	4	5	6
1	0	1	0	1	0	0
2	0	0	0	0	1	0
3	0	0	0	0	1	1
4	0	1	0	0	0	0
5	0	0	0	1	0	0
6	0	0	0	0	0	1

Asymptotic Performance

	Edge List	Adjacency List	Adjacency Matrix
incidentEdges(v)	m	deg(v)	n
areAdjacent(v, w)	m	min(deg(v),deg(w))	1
insertVertex(o)	1	1	n^2
insertEdge(v, w, o)	1	1	1
removeVertex(v)	m	deg(v)	n^2
removeEdge(e)	1	1	1

Graph ADT

THANK YOU!

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