Applied Machine Learning

BITS Pilani
Pilani Campus

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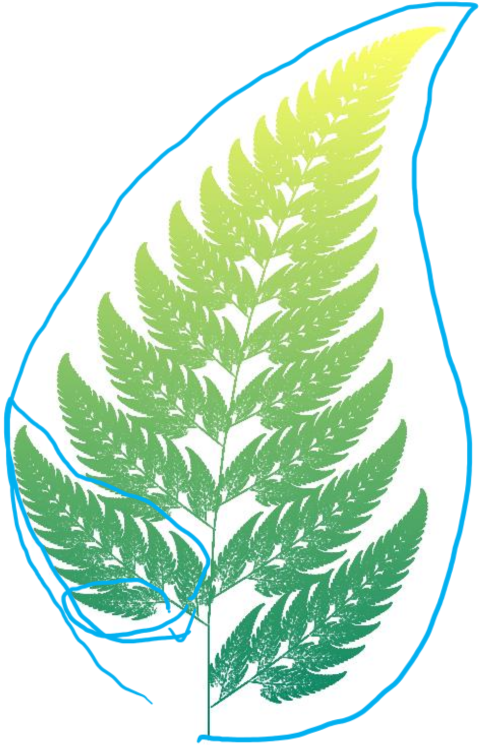
Computer Science and Information Systems

SE ZG568 / SS ZG568,
Applied Machine Learning
Lecture No. 3 [04- Feb-2025]

Recap

‘Learning’ in Machine Learning
Supervised Learning Setup
k-NN
Training, Crossvalidation, Testing
Performance Metric
Curse of Dimensionality
Basics of Linear Algebra, Linear Regression, PCA

Why should I learn Linear Algebra?



Barnsley Fern Leaf

Fractals using Iterated Function Systems

Why should I learn Linear Algebra?

Barnsley Fern Leaf

Are you curious to **write a computer program** to generate this fern leaf?

w	a	b	c	d	e	f	p	Portion generated
f_1	0	0	0	0.16	0	0	0.01	Stem
f_2	0.85	0.04	−0.04	0.85	0	1.60	0.85	Successively smaller leaflets
f_3	0.20	−0.26	0.23	0.22	0	1.60	0.07	Largest left-hand leaflet
f_4	−0.15	0.28	0.26	0.24	0	0.44	0.07	Largest right-hand leaflet

These correspond to the following transformations:

$$f_1(x,y) = \begin{bmatrix} 0.00 & 0.00 \\ 0.00 & 0.16 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \longrightarrow \text{Linear Transform}$$

$$f_2(x,y) = \begin{bmatrix} 0.85 & 0.04 \\ -0.04 & 0.85 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

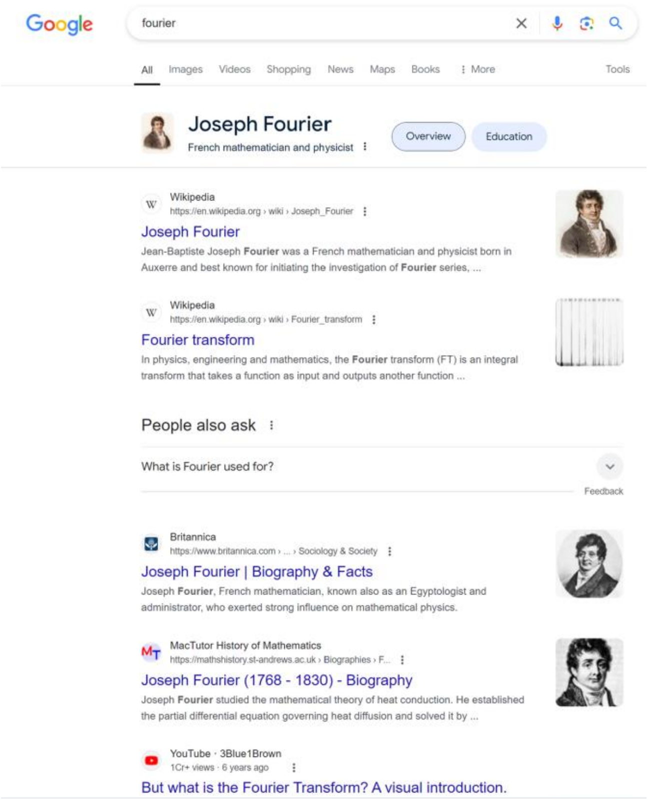
$$f_3(x,y) = \begin{bmatrix} 0.20 & -0.26 \\ 0.23 & 0.22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 1.60 \end{bmatrix}$$

$$f_4(x,y) = \begin{bmatrix} -0.15 & 0.28 \\ 0.26 & 0.24 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0.00 \\ 0.44 \end{bmatrix}$$

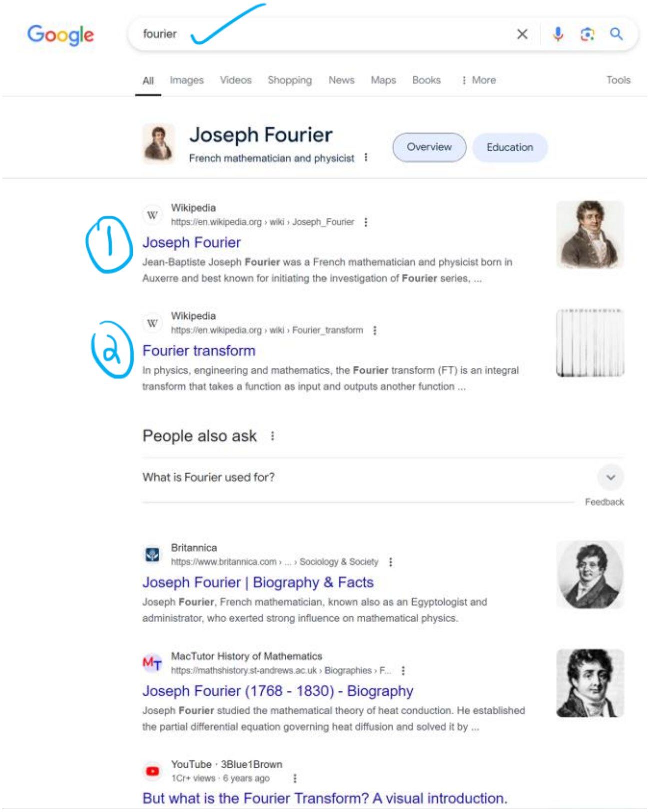
Affine Transform

Source of Equations: Wikipedia: https://en.wikipedia.org/wiki/Barnsley_fern

Why should I learn Linear Algebra?



Why should I learn Linear Algebra?



Page Rank

How does Google Search Engine works? The secret lies in linear algebra

Page Rank Algorithm

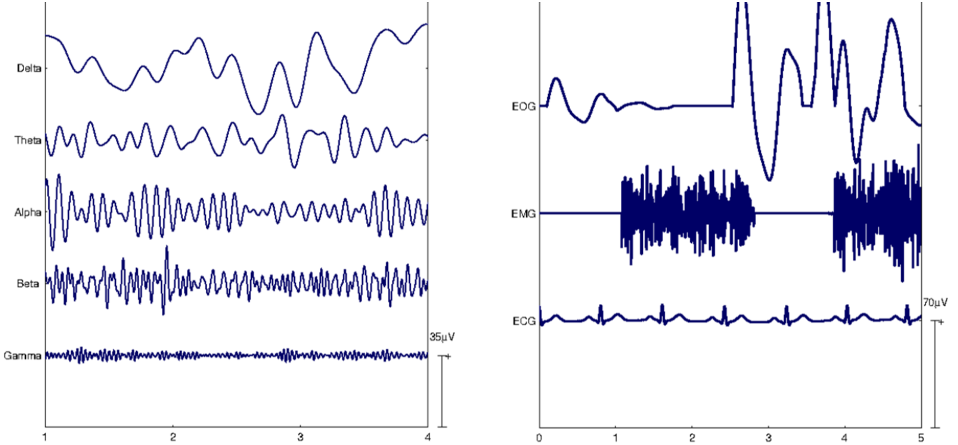
Larry Page and Sergey Brin

Image Source: https://www.theverge.com/2019/12/4/20994361/google-alphabet-larry-page-sergey-brin-sundar-pichai-co-founders-ceo-timeline

Why should I learn Linear Algebra?

Ever measured the electrical activity of your brain?

Independent Component Analysis (ICA), Principal Component Analysis (PCA) to remove EEG artifacts



(a) Brain Rhythms (b) Artifacts

Figure 1. (a) *Five normal brain rhythms, from low to high frequencies.* Delta, Theta, Alpha, Beta and Gamma rhythms comprise the background EEG spectrum. (b) *Three different types of artifacts.* Ocular, muscular and cardiac artifacts are the most frequent physiological contaminants in the literature on EEG artifact removal.

1. Reference: Urigüen, J. A., & Garcia-Zapirain, B. (2015). EEG artifact removal—state-of-the-art and guidelines. *Journal of neural engineering*, *12*(3), 031001.

Goal of Linear Algebra

- The central problem of Linear Algebra is to **understand** a system of linear equations.

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$$\left.\begin{aligned} -x + y &= 0 \\ 2x + y &= 3 \end{aligned}\right\} \text{System of Linear Equations}$$

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$$\boxed{\text{Does this system of linear equations has a solution?}}$$

$$\Downarrow$$

$$\boxed{\text{What is the value of the } \textbf{unknown variables } \boldsymbol{x} \textbf{ and } \boldsymbol{y} \text{ that satisfies this system of linear equations?}}$$

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Does this system of linear equations has a solution?

What is the value of the **unknown variables *x* and *y*** that satisfies this system of linear equations?

- **Understanding involves**
  - Insights about row picture and column picture.
  - Explore the existence of solution to the system of linear equations.
  - Insights about column space, row space, right null space, left null space.
  - **What new can we say about the system?**

Matrix- vector multiplication as dot product

A

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11} \times x + a_{12} \times y \\ a_{21} \times x + a_{22} \times y \end{bmatrix}$$

$-x + y = 0$

$2x + y = 3$

$$\begin{array}{ccc} & \text{Col 1} & \text{Col 2} \\ \text{Row 1} & \left[\begin{matrix} -1 \\ 2 \end{matrix}\right. & \left.\begin{matrix} 1 \\ 1 \end{matrix}\right] \\ \text{Row 2} & & \end{array} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

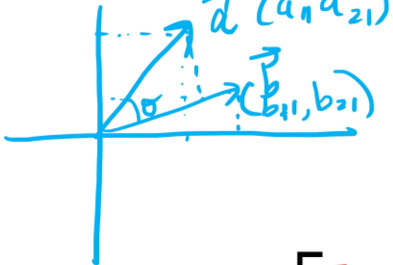
2×2 2×1 2×1

No of Rows   No of cols

$A_{m \times n} \vec{z}_{n \times k} = j_{m \times k}$

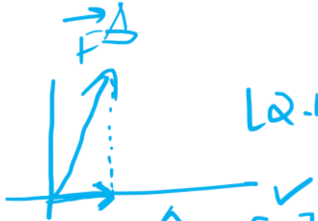
$a^T_{1 \times 2} \; b_{2 \times 1}$

Dot Product



$$a = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix} \quad b = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$$

$\boxed{\|F\|_2 \cos\theta} \to \|\hat{i}\|_2$



L2-norm

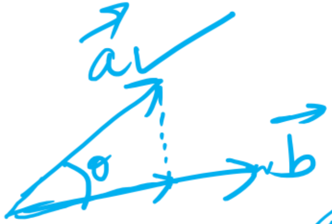
$\hat{u} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\|\hat{i}\|_2 = \sqrt{1^2 + 0^2} = 1$

$a = \begin{bmatrix} a_{11} \\ a_{21} \end{bmatrix}$ 2×1 $b = \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix}$

$F = m \cdot \vec{a}$

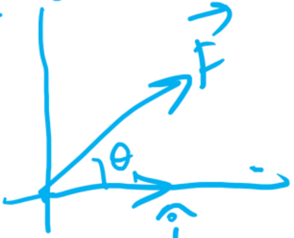
$m \in \mathbb{R}^1$ $a \in \mathbb{R}^1$



$\boxed{a \cdot b = \|a\|_2 \|b\|_2 \cos\theta}$

dot product

$$a \cdot b = a^T b$$



$\|F\|_2 \underbrace{\|\hat{i}\|_2}_{1} \cos\theta$

$a \cdot b = a^T b$

$[a_{11}\ a_{21}]_{1\times 2} \begin{bmatrix} b_{11} \\ b_{21} \end{bmatrix} = \boxed{a_{11}b_{11} + a_{21}b_{21}}$

Matrix- vector multiplication as dot product

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a_{11} \times x + a_{12} \times y \\ a_{21} \times x + a_{22} \times y \end{bmatrix}$$

Geometric Interpretation of Matrix Vector Multiplication

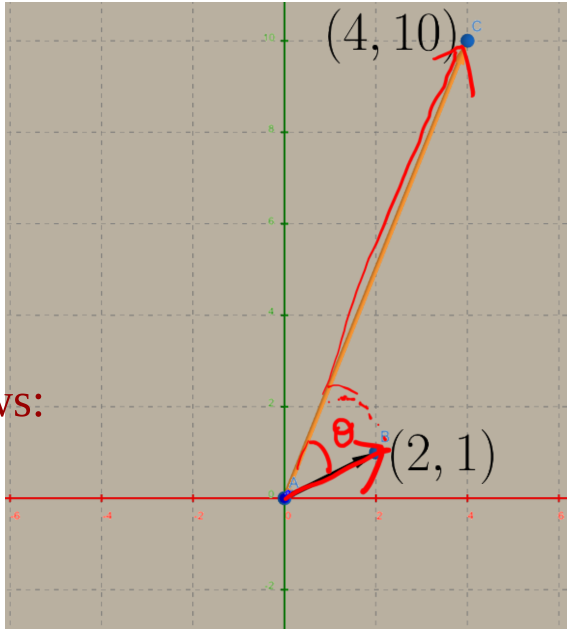
Intuition for Matrix vector multiplication for Square Matrices

$$\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$$

2×2

$1 \times 2 + 2 \times 1 = 4$

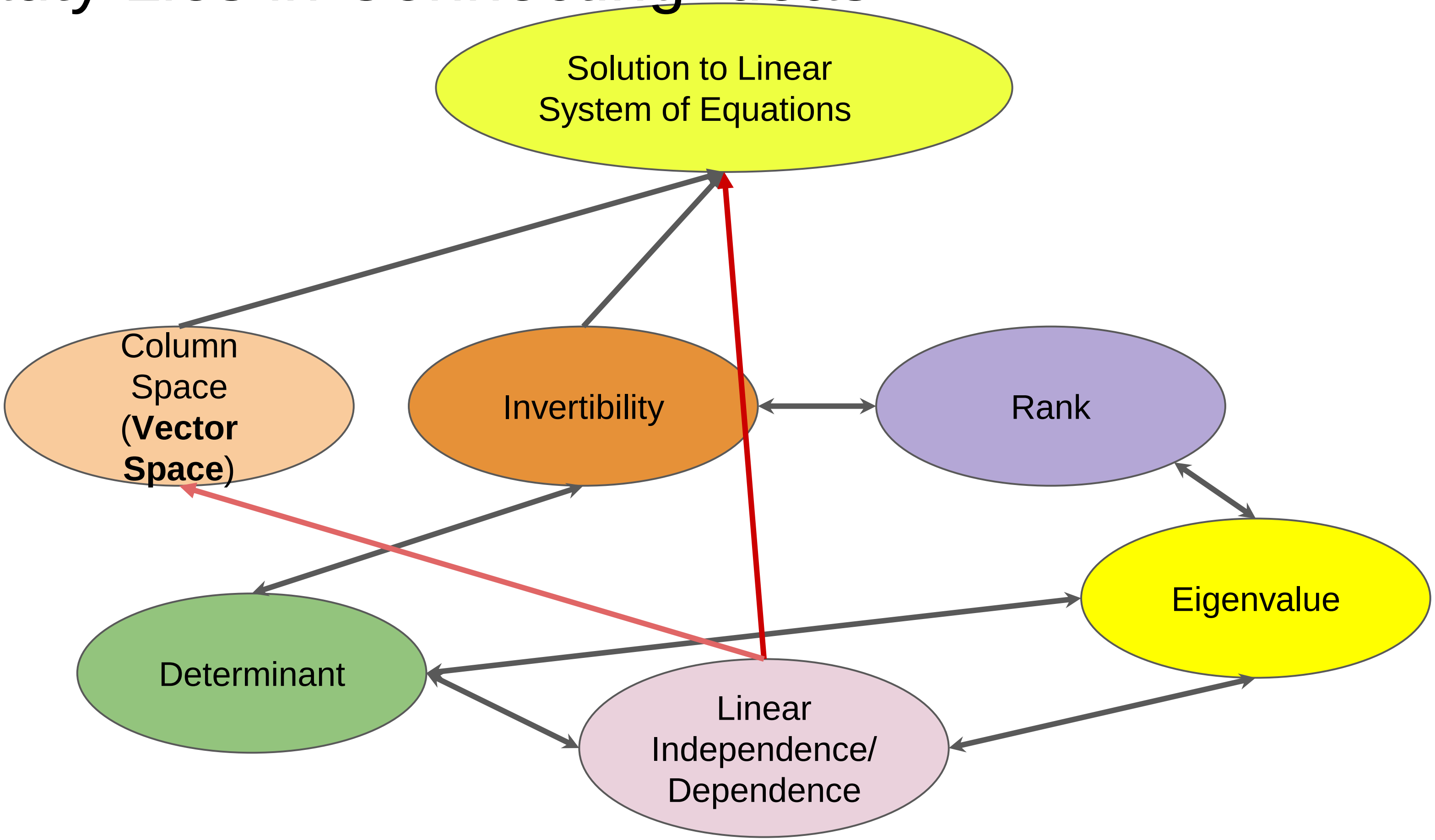
$3 \times 2 + 4 \times 1 = 10$



Matrix(Square Matrix) vector multiplication can be seen as follows:

- Rotation
- Stretching or Shrinking

Beauty Lies in Connecting Ideas

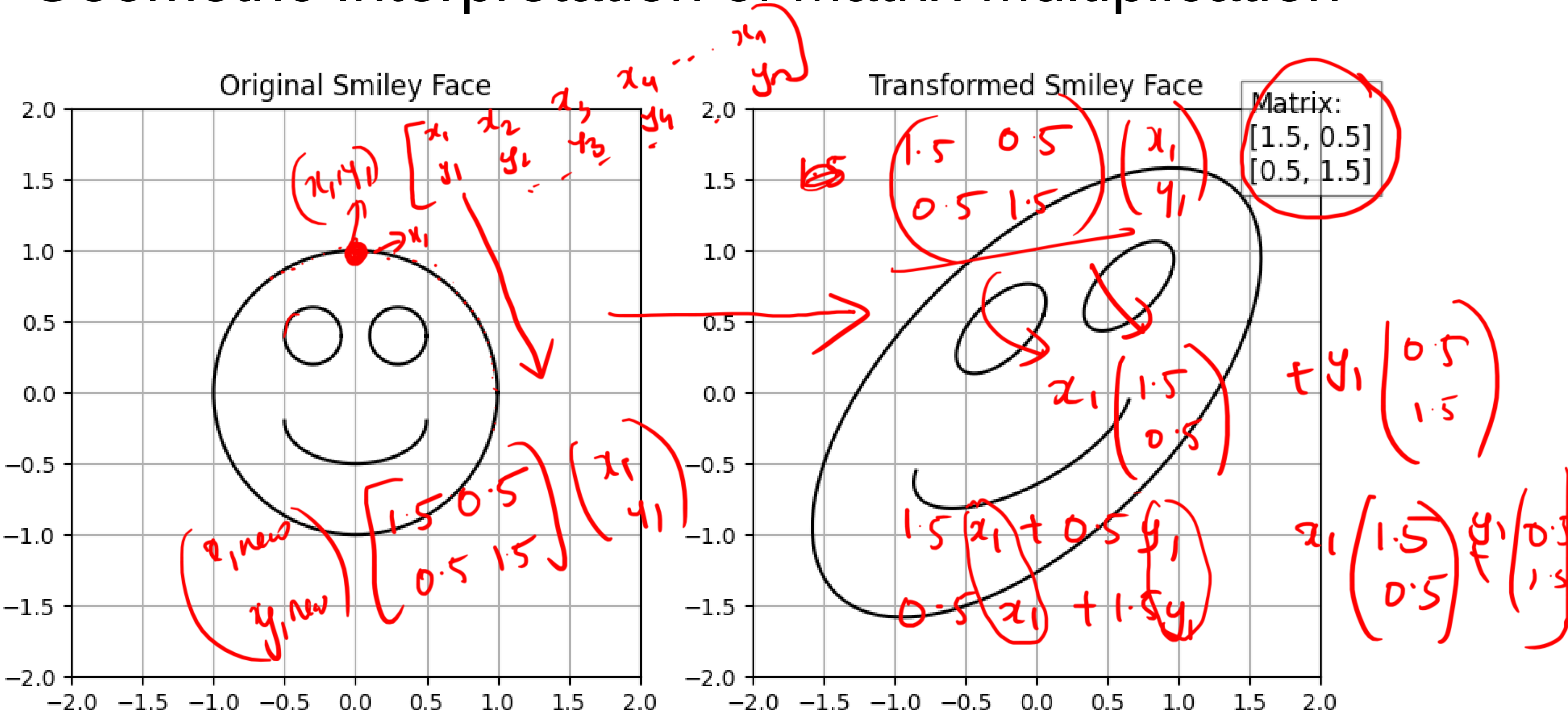


Matrix- vector multiplication

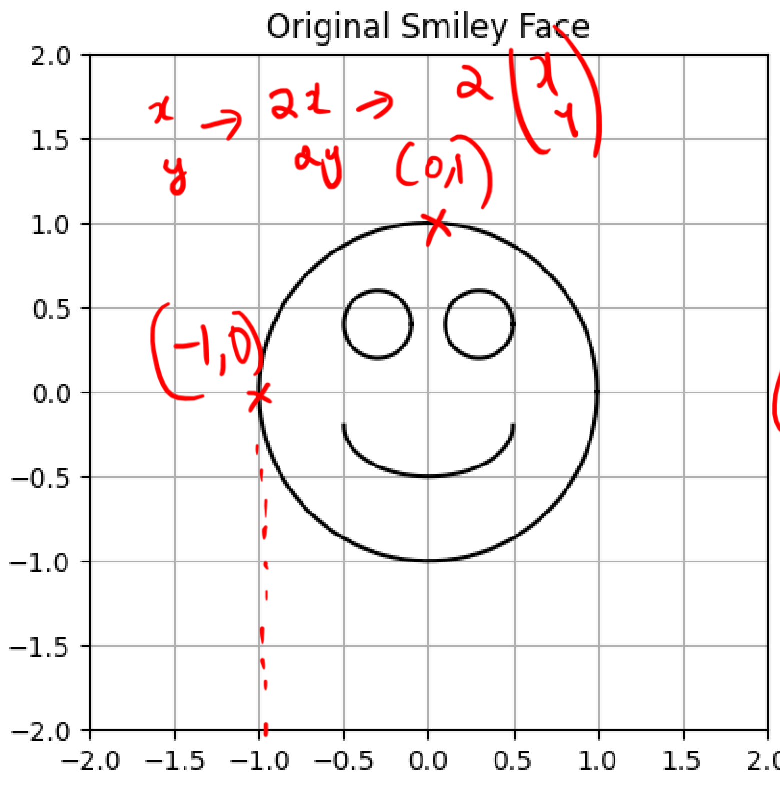
$$
\begin{aligned}
-x + y &= 0 \\
2x + y &= 3
\end{aligned}
$$

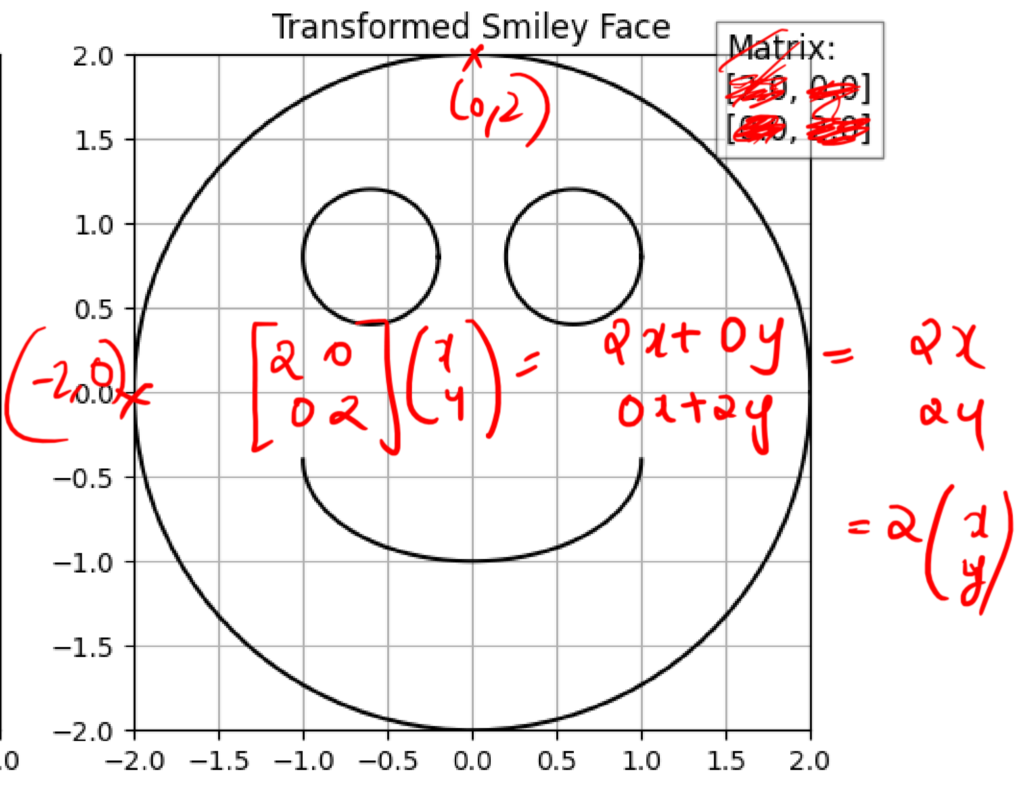
Dot Product

Geometric Interpretation of Matrix Multiplication



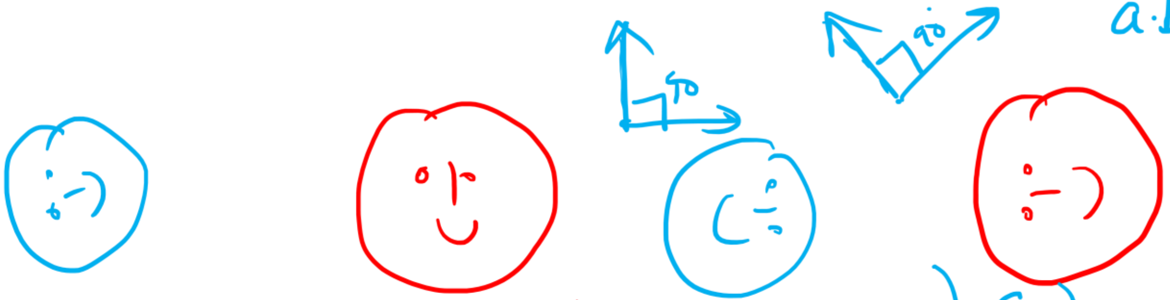
Geometric Interpretation of Matrix Multiplication





What should I do if I want to rotate the smiley by $\boxed{90}$ degree?

$a \cdot b \quad a^T b = 0$



$$\begin{bmatrix} 1/2 & 0 \\ 0 & 1/2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \ \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{matrix} 0 \times x - 1 \times y = -y \\ -1 \times x \quad 0 \times y \quad -x \end{matrix}$$

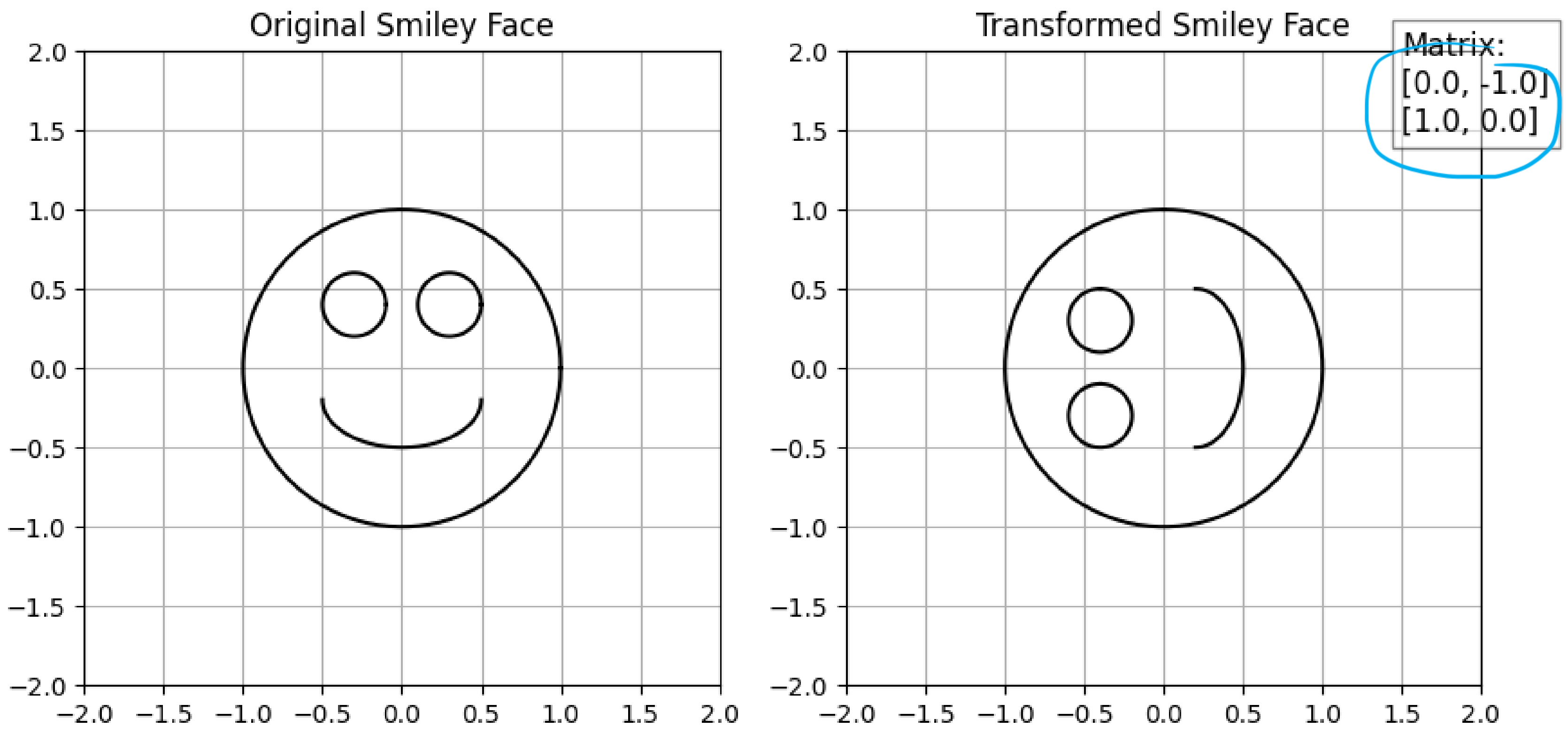
$-xy - yx \Rightarrow -2xy$

$$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{pmatrix} -y \\ x \end{pmatrix}$$

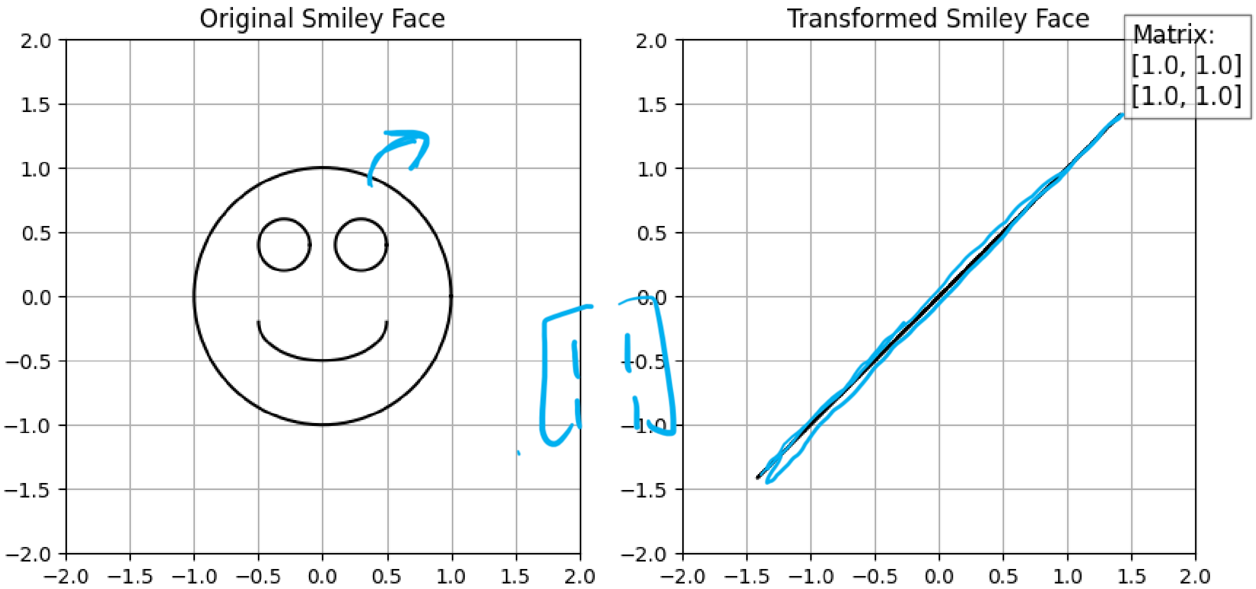
$$(x \ y) \begin{bmatrix} -y \\ -x \end{bmatrix} =$$

$$[x \ y] \begin{pmatrix} -y \\ x \end{pmatrix} = \underline{\underline{-xy + yx = 0}}$$

What should I do if I want to rotate the smiley by 90 degree?



What can you say about the following transformation?



$$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}x+y\\ x+y\end{pmatrix}$$

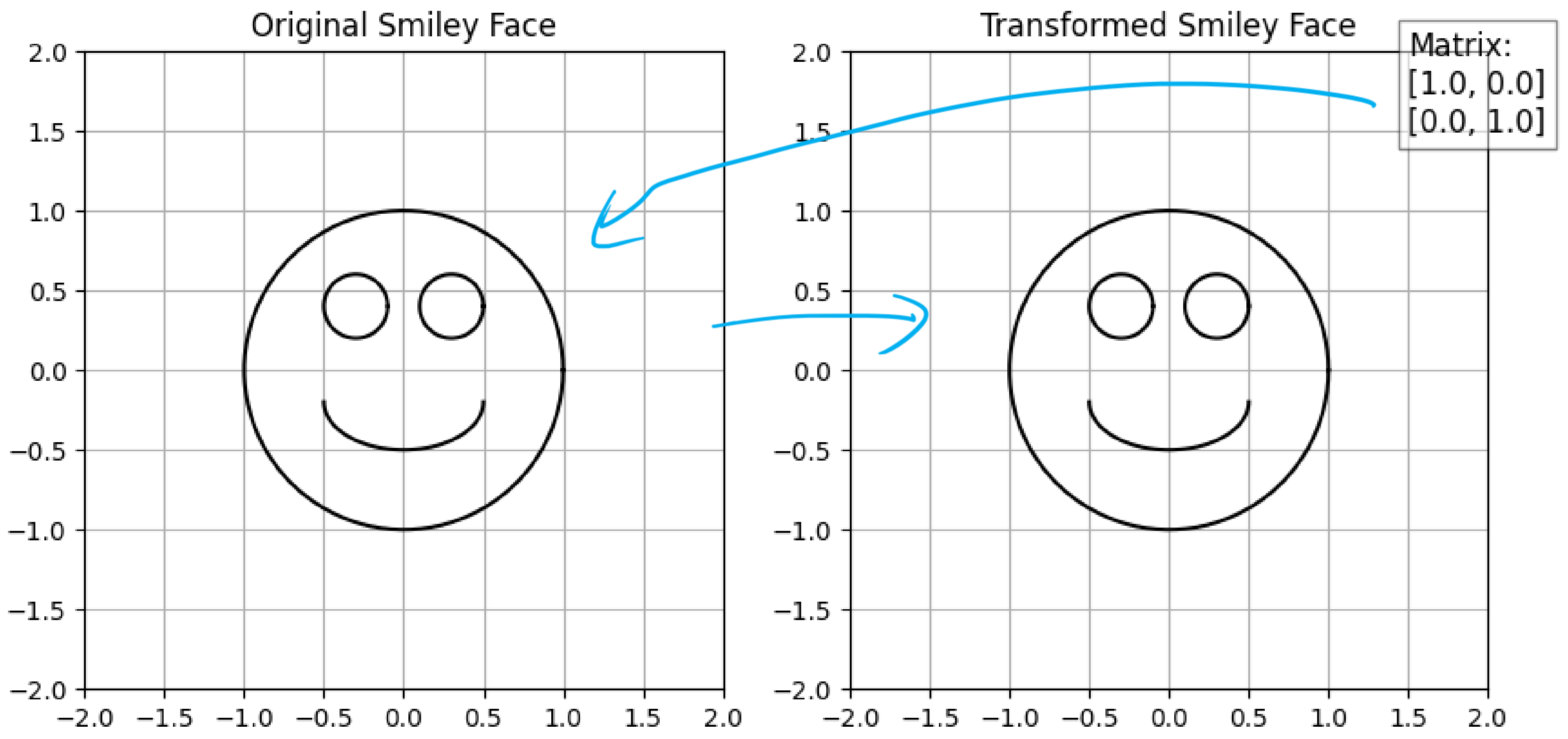
$$x\begin{pmatrix}1\\ 1\end{pmatrix} + y\begin{pmatrix}1\\ 1\end{pmatrix}$$

$$\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix}$$

Figure out the transformation (matrix) that will preserve the smiley as it is!

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Figure out the transformation (matrix) that will preserve the smiley as it is!



Identity Matrix

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{matrix} x + 0y \\ 0x + 1y \end{matrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

Inverse of a Matrix

If a matrix is invertible

A^{-1}

$$\boxed{A_{n\times n} A^{-1}_{n\times n} = A^{-1}_{n\times n} A_{n\times n} = I_{n\times n}}$$

$$AA^{-1} = A^{-1}A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2}$$

If $A_{n\times n}$ is invertible $A^{-1}_{n\times n}$

$$AA^{-1} = A^{-1}A = I_{n\times n}$$

$$\begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & 1 \\ & & & & \ddots \end{bmatrix}_{n\times n}$$

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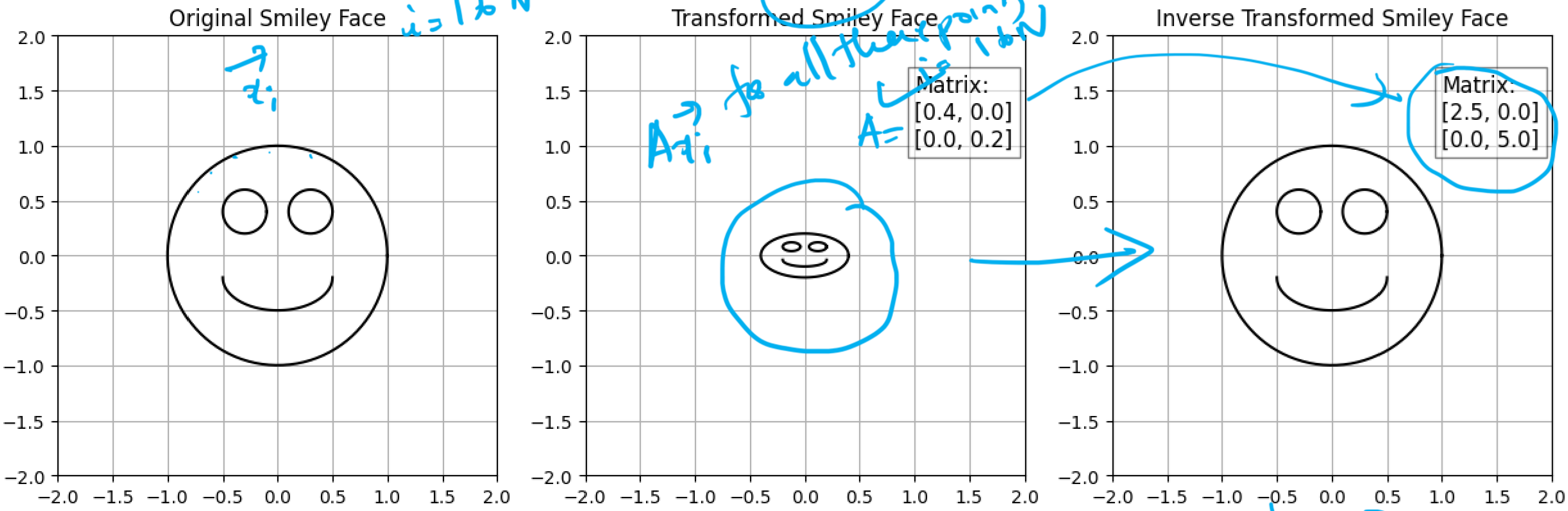
$$\overset{A}{\begin{bmatrix}0.4 & 0\\ 0 & 0.2\end{bmatrix}}\overset{B}{\begin{bmatrix}2.5 & 0\\ 0 & 5\end{bmatrix}} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} I$$

How do we understand matrix inverse geometrically?

$B = A^{-1}$

$i = 1$ to N

$A\vec{x}_i$ for all the points $i = 1$ to N



$\vec{x}_i$ $\quad$ $A\vec{x}_i$ $\quad$ $\underbrace{A^{-1}A}_{I}\vec{x}_i$

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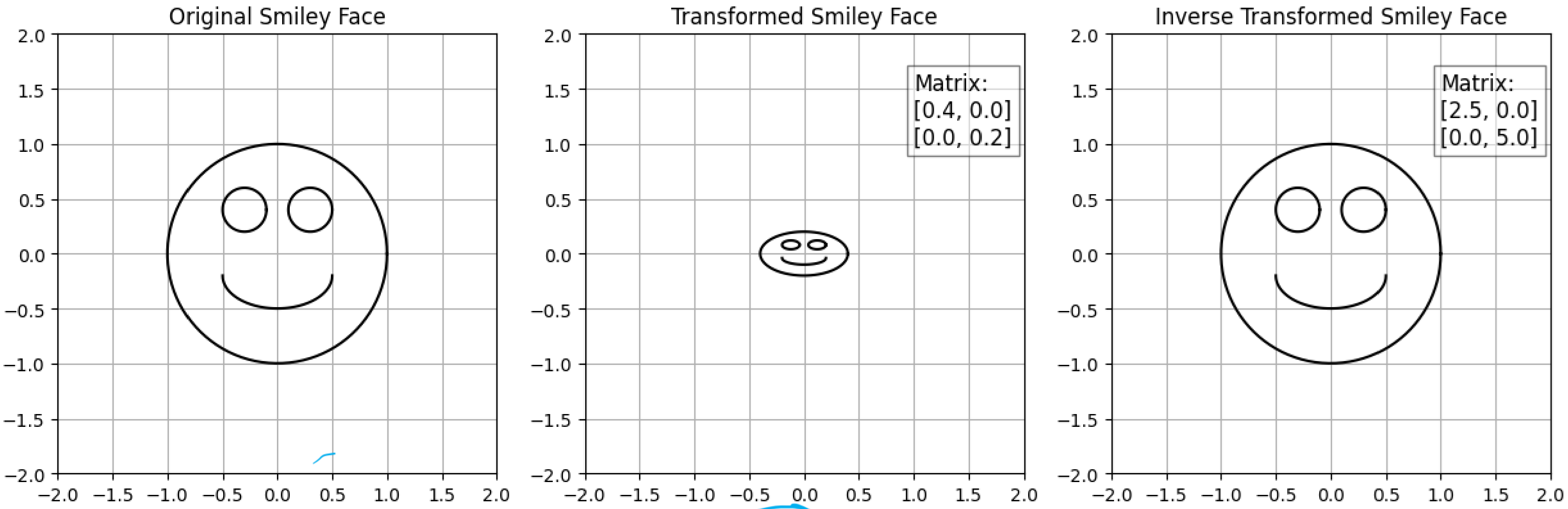
In a Nutshell



$\vec{x_i}$ $\quad$ $A\vec{x_i}$ $\quad$ $A^{-1}A\vec{x_i}$

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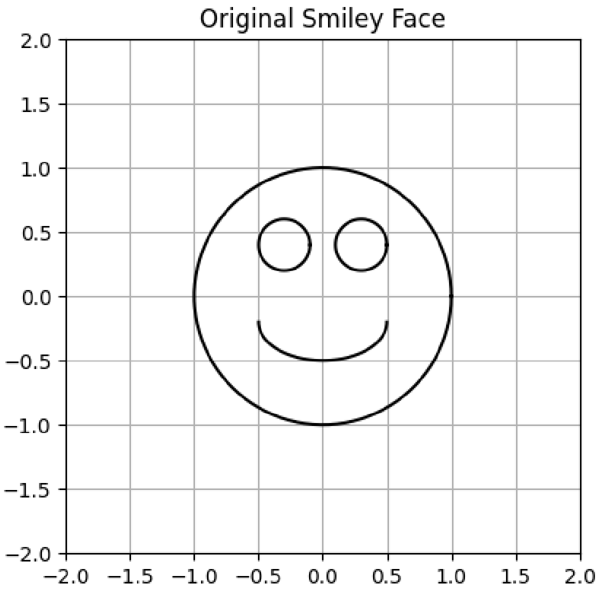
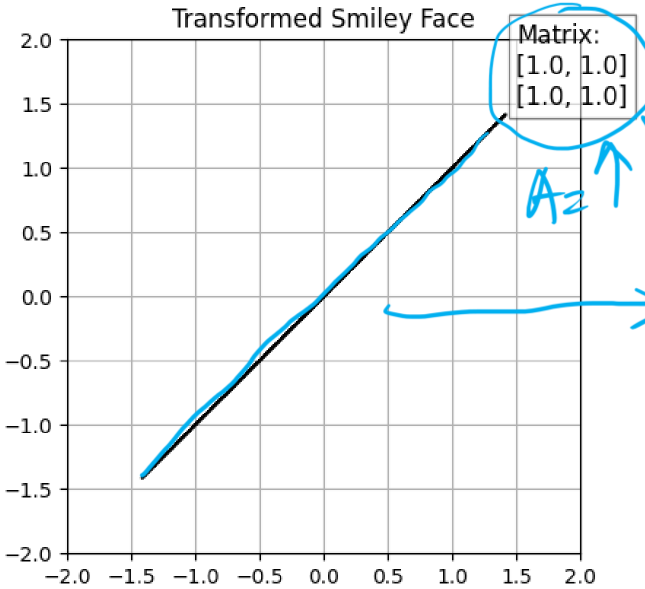
Can you get back the original smiley?





A_2

$\times \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

?

$\vec{x_i}$

$A\vec{x_i}$

$\underbrace{A^{-1}A}\vec{x_i}$

Solutions to System of Linear Equations

$$-x + y = 0$$
$$2x + y = 3$$

Algebraic Interpretation

Row Picture

Column Picture

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Two Equations & Two Unknowns - Algebraic Interpretation

$$-x + y = 0 \quad -(1)$$
$$2x + y = 3 \quad -(2)$$

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Gaussian elimination

Two Equations and Two Unknowns-
Algebraic Interpretation

$$\begin{aligned} -x + y &= 0 \\ 2x + y &= 3 \end{aligned}$$

What is the value of the **unknown variables *x* and *y*** that satisfies this system of linear equations?

$$\begin{bmatrix} x_1 & x_4 & x_7 \\ x_2 & x_4 & x_8 \\ x_3 & x_6 & x_9 \end{bmatrix} \rightarrow \begin{bmatrix} z_1 & z_4 & z_7 \\ 0 & z_5 & z_8 \\ 0 & 0 & z_9 \end{bmatrix}$$

3×3

$$\begin{bmatrix} -2 & 2 \\ 0 & 3 \end{bmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$0x + 3y = 3$

Elimination

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 2 & 1 & 3 \end{array}\right] \; R_1 \rightarrow 2R_1 \; \left[\begin{array}{cc|c} -2 & 2 & 0 \\ 2 & 1 & 3 \end{array}\right] \; \boxed{R_2 \rightarrow R_1 + R_2} \; \left[\begin{array}{cc|c} -2 & 2 & 0 \\ 0 & 3 & 3 \end{array}\right]$$

$Row_1 \rightarrow 2Row_1$

$R_2 \rightarrow R_1 + R_2$

$\boxed{y=1}$

$$\begin{aligned} 3y &= 3 \\ y &= 1 \end{aligned}$$

Sub. *y* = *1* in Equation 1, we get: ***x* = *1***

$$\text{Solution: } x = 1, \text{and } y = 1$$

$-2x + 2y = 0$

$-2x + 2 = 0$

$-2x = -2$

$x = 1$

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Two Equations and Two Unknowns-

Geometric Interpretation

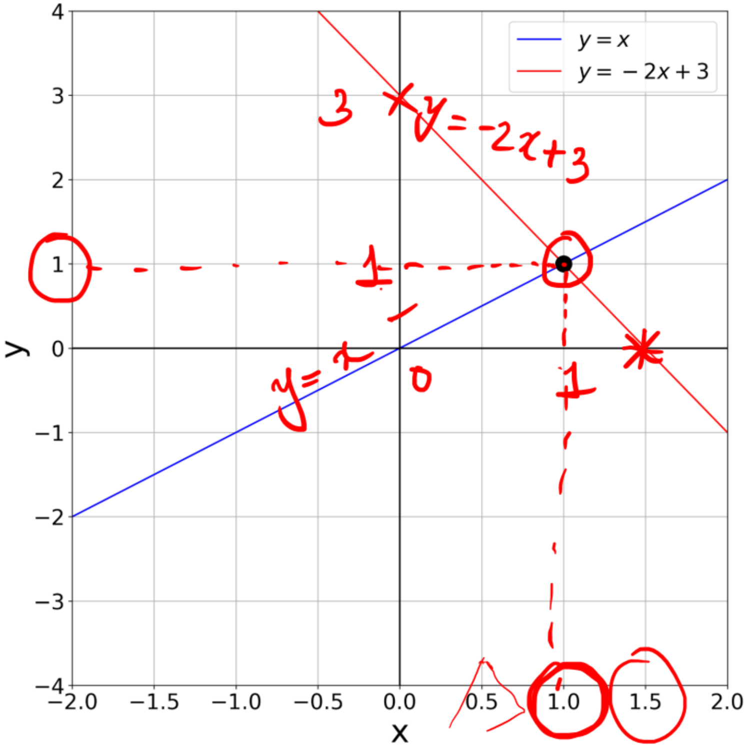
$$-x + y = 0$$
$$2x + y = 3$$

Row Picture

$$y = x$$
$$y = -2x + 3$$

$0 = -2x + 3$

$y = x$
$y = x + c$



Two Equations and Two Unknowns-
Geometric Interpretation

$$-x + y = 0$$
$$2x + y = 3$$

direction

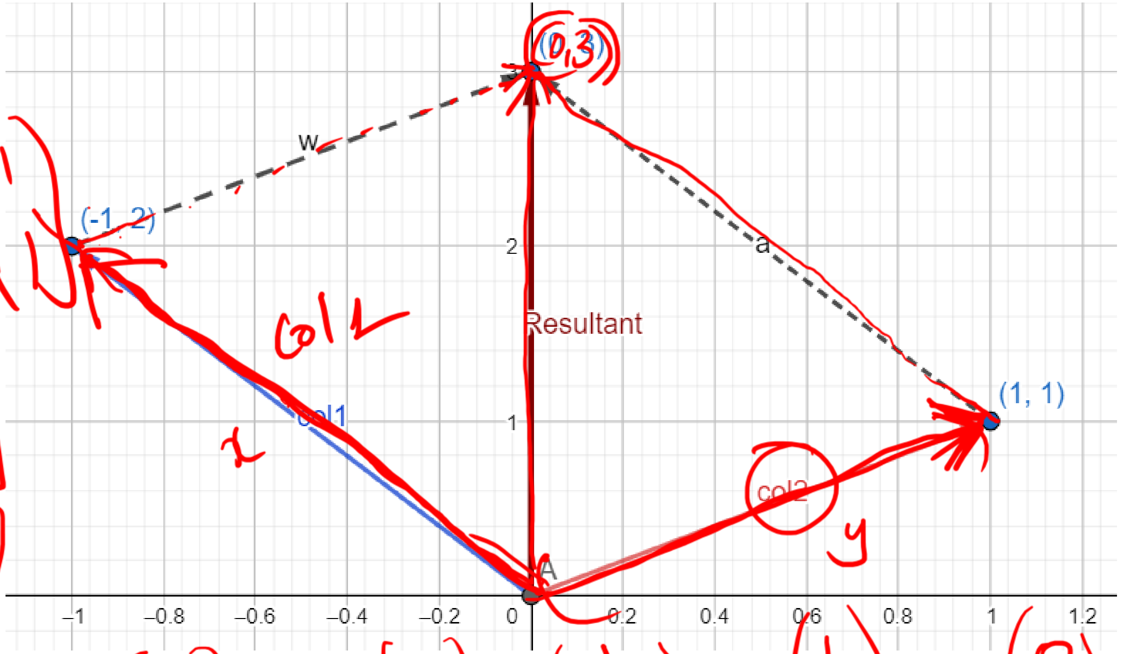
$$x\begin{bmatrix}-1\\2\end{bmatrix}+y\begin{pmatrix}1\\1\end{pmatrix}$$

Column Picture

col 1 col 2 $= \begin{pmatrix}0\\3\end{pmatrix}$

$$x\begin{bmatrix}-1\\2\end{bmatrix}+y\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}0\\3\end{bmatrix}$$

1 1



$$1\cdot\begin{bmatrix}-1\\2\end{bmatrix}+1\cdot\begin{bmatrix}1\\1\end{bmatrix}=\begin{pmatrix}-1\\2\end{pmatrix}+\begin{pmatrix}1\\1\end{pmatrix}=\begin{pmatrix}0\\3\end{pmatrix}$$

Two Equations and Two Unknowns-
Some Observations

$$\vec{x} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\begin{aligned} -x + y &= 0 \\ 2x + y &= 3 \end{aligned}$$

$$x \begin{pmatrix} -1 \\ 2 \end{pmatrix} + y \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$$

$$\underbrace{\begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x \\ y \end{bmatrix}}_{\vec{x}} = \underbrace{\begin{bmatrix} 0 \\ 3 \end{bmatrix}}_{b} \Leftrightarrow \boxed{A\vec{x} = b}$$

$$\Updownarrow$$

$$x \begin{bmatrix} -1 \\ 2 \end{bmatrix} + y \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

$A\vec{x} = b$ is the weighted linear combinations of columns of A

Two Equations and Two Unknowns-
Determinant

$$\begin{aligned} -x + y &= 0 \\ 2x + y &= 3 \end{aligned}$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

$$|A| = -3$$

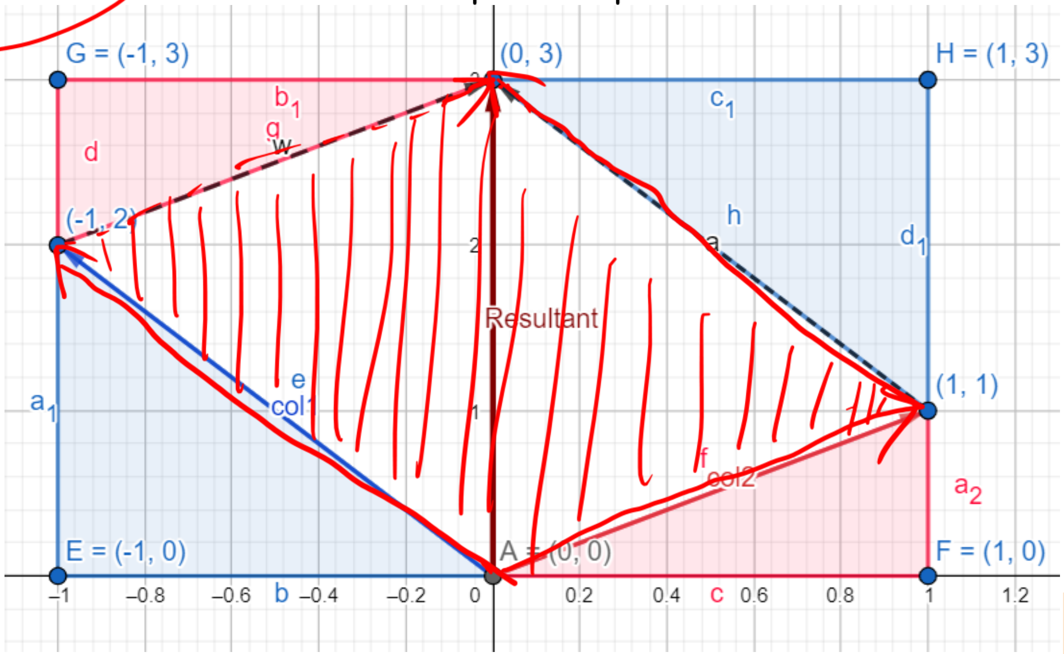
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$|A| =$

$|A| = ad - bc$

$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$|A| = 0$



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Two Equations and Two Unknowns-
Invertibility

$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$|A| = 0$

$$\boxed{A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}} \qquad A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad AA^{-1} = A^{-1}A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

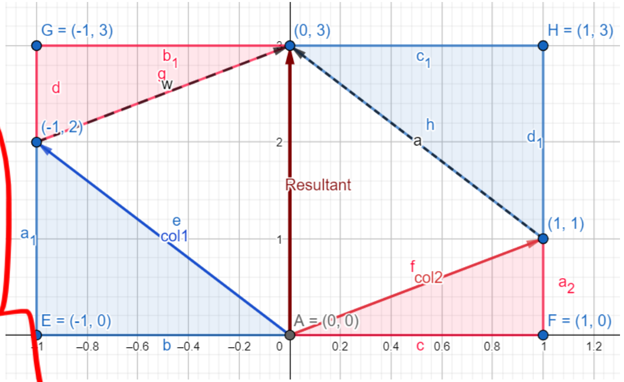
$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

$$-x + y = 0$$
$$2x + y = 3$$

$$A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$$

$$|A| = -3$$

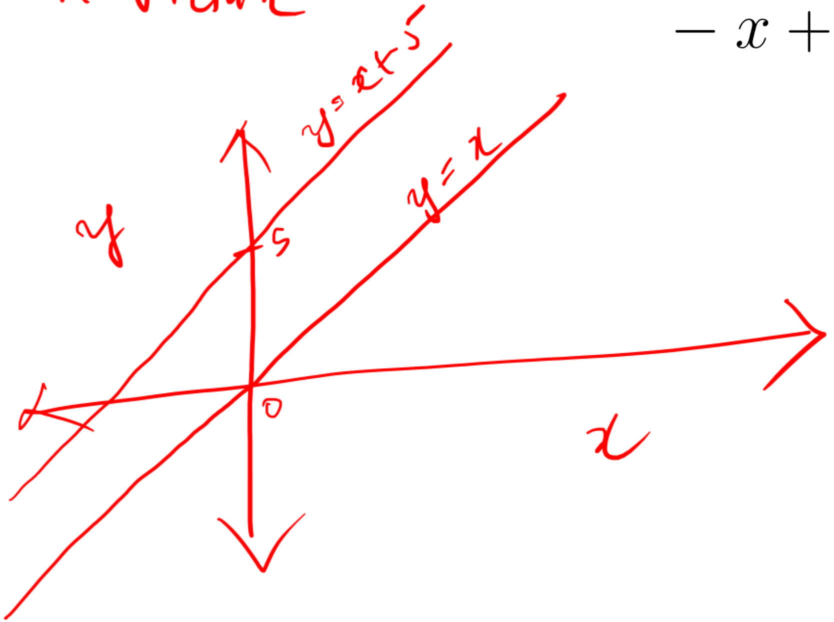
$$\frac{1}{ad-bc}\begin{bmatrix} ad-bc & bd-bd \\ -ac+ac & -bc+ad \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$



Two Equations and Two Unknowns-
Contd..

$$-x + y = 0$$
$$-x + y = 5$$

Row Picture



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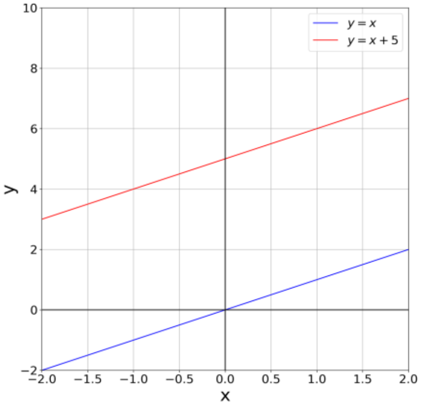
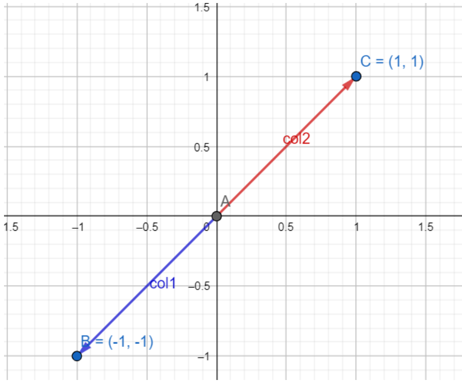
Permanent Breakdown of Elimination

$$-x + y = 0$$
$$-x + y = 5$$

$$\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \end{bmatrix}$$



$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ -1 & 1 & 5 \end{array}\right] \quad R_2 \to R_2 - R_1 \quad \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 5 \end{array}\right]$$



$R_2 \to R_2 - R_1$

$$\boxed{0y = 5}$$

$y = 5/0$

Permanent Breakdown of elimination (NO SOLUTION)

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Two Equations and Two Unknowns-
Contd..

$$\begin{array}{ccc} \downarrow col_1 & \downarrow col_2 & \downarrow col_3 \\ \end{array}\begin{bmatrix} & & \\ & & \end{bmatrix}$$

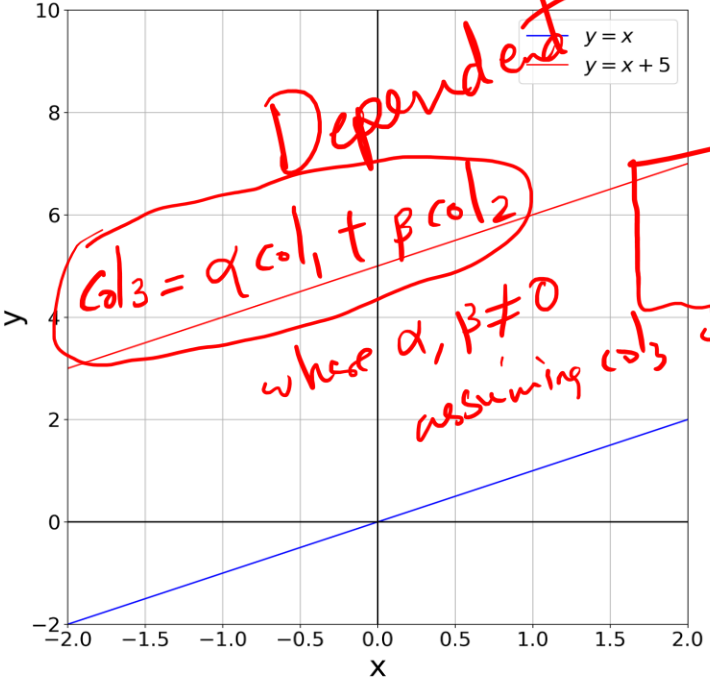
$$\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$$

Row Picture

$$-x + y = 0$$
$$-x + y = 5$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Column Picture



Dependent

$col_3 = \alpha\, col_1 + \beta\, col_2$

where $\alpha, \beta \neq 0$

assuming col_3 is non zero vector

$$\begin{array}{cc} & \downarrow col_2 \\ \end{array}\begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

$|A| = 0$

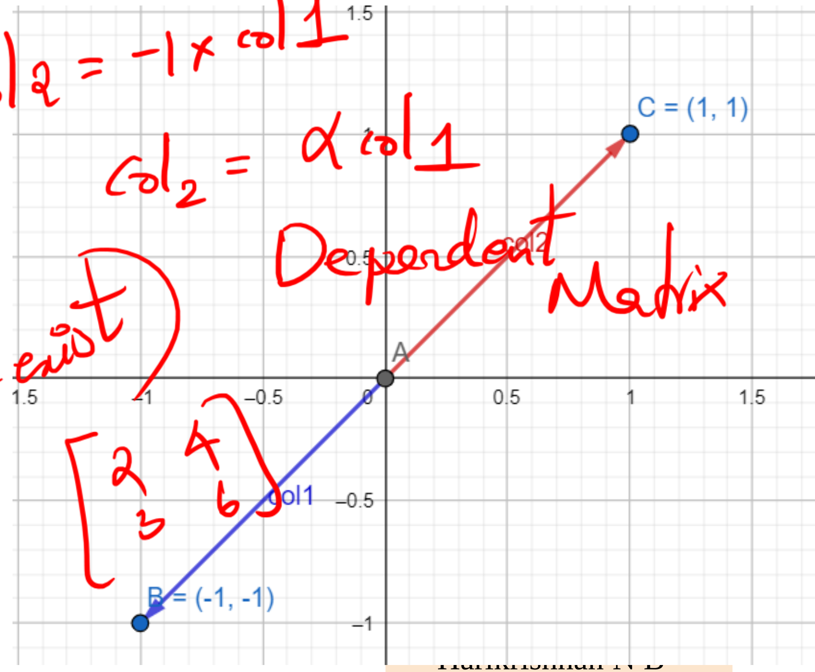
(A^{-1} does not exist)

$col_2 = -1 \times col_1$

$col_2 = \alpha\, col_1$

Dependent Matrix

$$\begin{bmatrix} 2 & 4 \\ 3 & 6 \end{bmatrix}$$



Two Equations and Two Unknowns-

Invertibility

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad AA^{-1} = A^{-1}A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

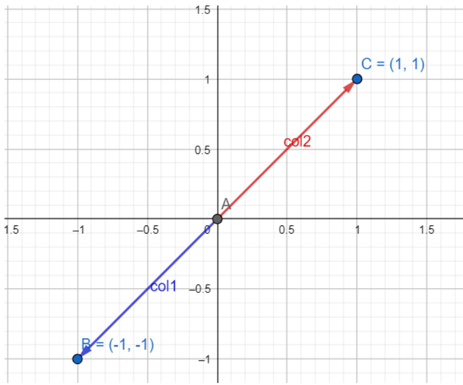
$$-x + y = 0$$
$$-x + y = 5$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

$$A = \begin{bmatrix} -1 & 1 \\ -1 & 1 \end{bmatrix}$$

NOT INVERTIBLE

$$|A| = 0$$



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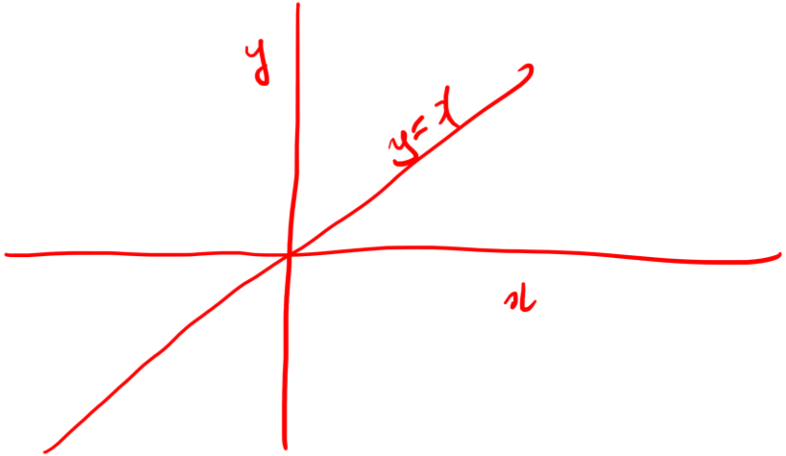
Two Equations and Two Unknowns-Contd..

$$-x + y = 0$$
$$-2x + 2y = 0$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Infinitely many Solutions

$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$$

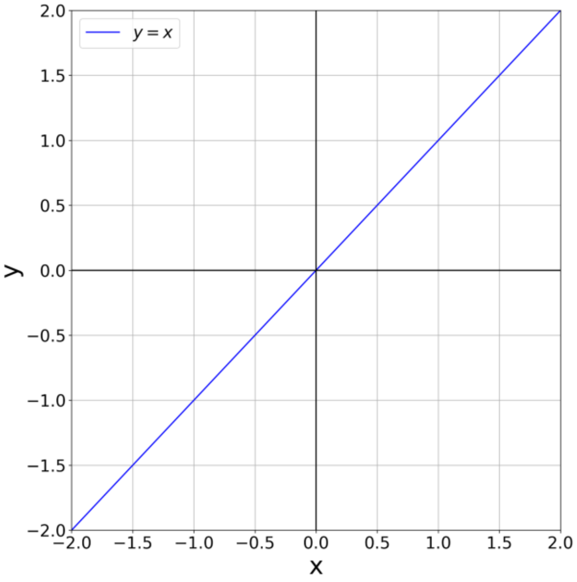


Two Equations and Two Unknowns- Contd..

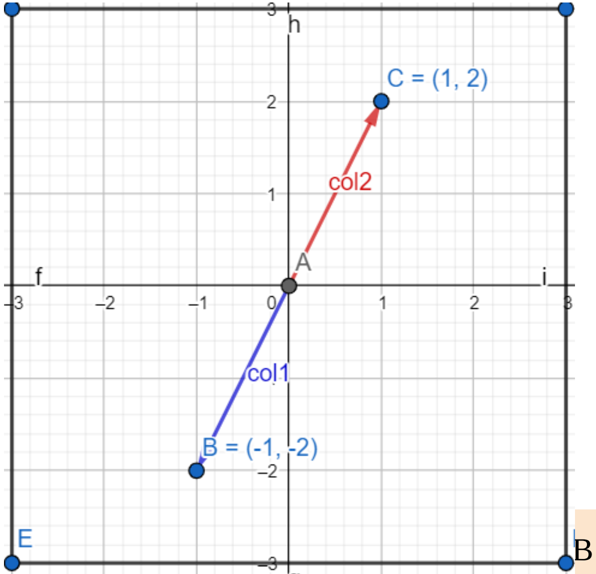
$$-x + y = 0$$
$$-2x + 2y = 0$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Row Picture



Column Picture



Two Equations and Two Unknowns-Contd..

$$-x + y = 0$$
$$-2x + 2y = 0$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$Ax = b$ has solution only when b lies in the "col space" of A

Temporary Breakdown of Elimination

$$-x + y = 0$$
$$-2x + 2y = 0$$

$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} -1 & 1 & 0 \\ -2 & 2 & 0 \end{array}\right] R_2 \rightarrow R_2 - 2R_1 \left[\begin{array}{cc|c} -1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

y can take any value

nonzero number

0

Permanent

$$\boxed{0y = 0}$$

0/0 know

***y* can take any value (Infinitely many solutions)**

y takes the value k

$$\boxed{y = k}$$

$-x + y = 0$

$-x + k = 0$

$-x = -k$

$\boxed{x = k}$

$-x + y = 0$

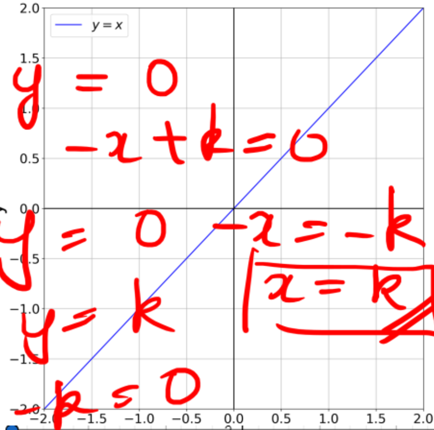
$y = k$

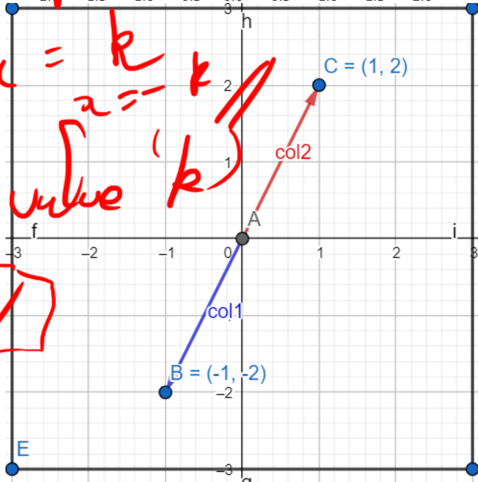
$-x - k = 0$

$-x = k$

$x = -k$

(k)





Two Equations and Two Unknowns-
Invertibility

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \qquad A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \qquad AA^{-1} = A^{-1}A = I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

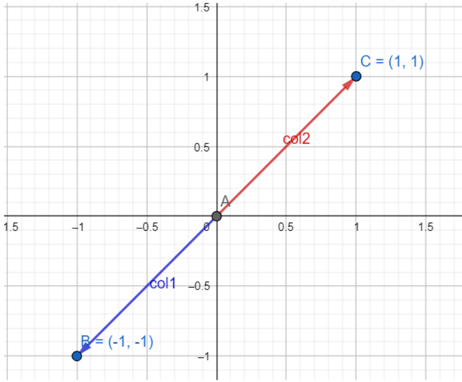
$$-x + y = 0$$
$$-2x + 2y = 0$$

$$|A| = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (ad - bc)$$

$$A = \begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}$$

NOT INVERTIBLE!!!

$$|A| = 0$$



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