ClassNotes

UNIT 10

SIMPLE HARMONIC MOTION AND WAVES

After studying this unit, the students will be able to:

a. State the conditions necessary for an object to oscillate with SHM.

b. Explain SHM with simple pendulum ball and bowl examples.

c. Draw forces acting on a displaced pendulum.

d. Solve problems by using the formula $T = 2\pi\sqrt{L/g}$ for simple pendulum.

e. Understand that damping progressively reduces the amplitude of oscillation.

f. Describe wave motion as illustrated by vibrations in rope, slinky spring and by experiments with water waves.

g. Describe that waves are means of energy transfer without transfer of matter.

h. Distinguish between mechanical and electromagnetic waves.

i. Identify transverse and longitudinal waves in mechanical media stinky and springs.

j. define the terms speed (v), frequency (f), wavelength (Z), time period (T) amplitude crest, trough, cycle, wave front, compression and rarefaction.

k. Derive equation $v = f\lambda$.

l. Solve problems by applying the relation $f = \frac{1}{t}$ and $v = f\lambda$.

m. Describe properties of waves such as reflection, refraction and diffraction with the help of ripple tank.

Q1. What is simple harmonic motion? What are the necessary conditions for a body to execute simple harmonic motion?

Answer

Simple harmonic motion

Definition:

The motion in which the acceleration of a body is directly proportional to the displacement from the mean position and always directed towards the mean position is called simple harmonic motion.

Necessary conditions for a body to execute simple harmonic motion.

i) A body executing SUM always vibrates about a fixed position.

ii) Its acceleration is always directed towards the mean position.

iii) The magnitude of acceleration is always directly proportional to its displacement from the mean position i.e. acceleration will be zero at the mean position while it will be maximum at the extreme position

iv) Its velocity is maximum at the mean position and zero on the extreme position

Q.2 Prove that the motion of mass attached to g spring is SHM?

Answer

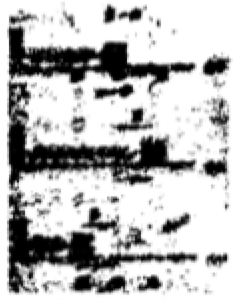


Fig.10.1 SHM of a mass-spring system

One of the simplest types of oscillatory motion is that 'of horizontal mass spring system.

If the spring is stretched or compressed through a small 'x' from its mean position, it exerts a force 'F' on the mass.

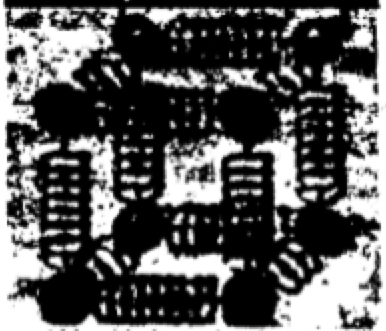
According to, Hook's law this force is directly proportional to the change in length 'x' of the spring.

i.e.

$$F = -kx..........(i)$$

Where 'x' is the displacement of the mass from its mean position 'O', and 'K' is a spring constant defined as

For your Information



Although the molecules in a solid are held together, they can still move and vibrate, as though connected by springs.

The value of 'k' is a measure of the stiffness of the spring. Stiff springs have large 'k.' values and soft springs have small 'k' values.

The negative. sign in equation (1) means that the force exerted by the spring' is always directed opposite to the displacement of the mass. Because the spring force always acts towards the mean position.

It is sometimes called a restoring force.

Restoring force:

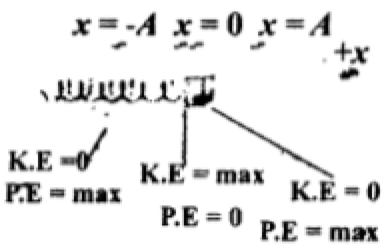
A restoring force always pushes or pulls the object performing oscillatory motion towards the mean position.

Explanation:

Initially the mass 'm' is at rest in mean position '0' and the resultant force on the mass is zero. Suppose the mass is pulled through a distance 'x' up to extreme position 'A' and then released.

The restoring force exerted by the spring on the mass will pull it towards the mean position '0. Due to the restoring force the mass moves back towards the mean position 00 the magnitude of the restoring force decreases with the distance from the mean position and becomes zero at 0. However, the mass gains speed as it moves towards the mean position and becomes maximum at O. Due to inertia the mass does not stop at the mean position 'O' but continues its motion and reaches the extreme position 'B'. As the mass moves

For your information



Kinetic and potential energy at different position in a mass-spring system from the mean position to the extreme position 'B', the restoring force acting … towards the mean position steadily increases in strength. Hence the speed of mass decreases as it moves towards the extreme position 'B'. The mass finally comes briefly to rest at the extreme position. Ultimately the mass returns to the position due to the restoring force.

This process is repeated, and the mass continues to oscillate back and forth about the mean position '0'.

Such motion of a mass attached to a spring on horizontal frictionless surface is known as simple harmonic motion. The time period T of the simple harmonic motion of a mass 'm' attached to a spring is,

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Q.3 Does the system of ball and bowl execute simple harmonic motion? Explain.

Answer

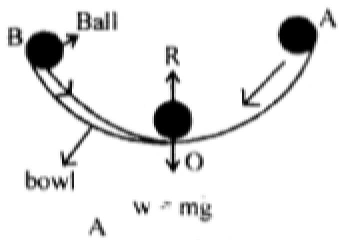


Fig. 10.2 When a ball Is gently displaced from the entre of a bowl it starts oscillating about the centre due to force of gravity which acts as a restoring force.

The motion of a ball placed in a bowl is an example of simple harmonic motion.

When the ball is at the mean position '0' that is, at the center of the bowl, net force acting on the ball is zero. In this position weight of the ball acts downward and is equal to the upward normal force of the surface of the bowl.

Hence there is no motion. Now if webring the ball to position 'A' and then release it, the ball will start moving towards the mean position '0' due to the restoring force caused by its weight At position '0' the ball gets maximum speed and due to inertia it moves towards the extreme position ‘8’. While going towards the position ‘B‘ the speed of the ball decreases due to the restoring force which acts towards the mean position. At the position ‘B’ the ball stops for a while and then again moves towards the mean position ‘O’ all its energy is lost due to friction. Thus, the to and fro motion of the 'ball about a mean position placed in' a bowl is an example of simple harmonic motion.

Q.4 Is the motion of a simple pendulum a simple harmonic motion. Write in detail.

For your information

The period of a pendulum is independent of its mass and amplitude.

Answer

A simple pendulum exhibits SHM. It consists of a small bob of mass ‘m’ suspended from a light string of length ‘L’ fixed at its upper end in the equilibrium position ‘0‘the net force on the bob is zero and the bob is stationary. Now if we bring the bob to extreme position 'A' the net force is not zero. There is no force acting along the string as the tension in the string cancels the component of the weight mg Cos θ. Hence there is no motion along this direction. The component of the weight mg Sine is directed towards the mean position and acts as a restoring force. Due to this force the bob starts moving towards the mean position ‘0’. At '0' the bob has gat the maximum velocity and due to inertia 'it does not stop at ‘0‘rather it continues to move towards the extreme position ‘B’.

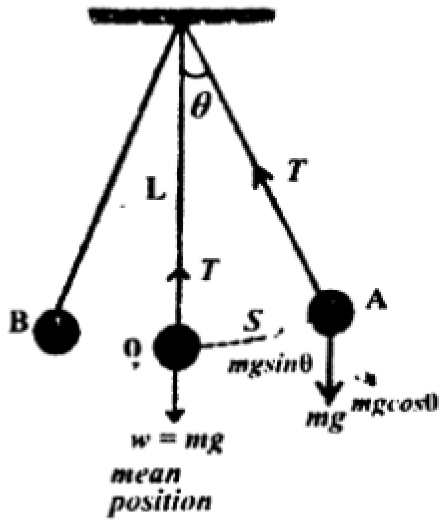


Fig. 10.3: Force acting on a displaced pendulum. The restoring force that causes the pendulum to undergo simple harmonic motion is the component of gravitational force mg sin tangent to the path of motion.

During its motion towards point ‘B’, the velocity of the bob decreases due to restoring force. The velocity of the bob becomes zero as it reaches the point ‘B’. The restoring force. ‘mg Sinθ’ still acts towards the mean position ‘0’ and due to this force, the bob again starts moving towards the mean position ‘0’. In this way, the bob continues its to and fro motion about the mean position '0’.

It shows that the speed of the bob increases while moving from point ‘A‘ to‘0‘due to the restoring force which acts towards ‘0’. Therefore, acceleration of the bob is also directed towards ‘0‘.

Similarly, when the bob moves from ‘0’ to ‘8'. Its speed decreases due to restoring force which again acts towards ‘0‘. Therefore, acceleration of the bob is always directed towards ‘0‘. It follows that the acceleration of the bob is always directed towards the mean position ‘0’. Hence the motion of a simple pendulum is SHM.

The formula for the time period of simple pendulum is:

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Q.5 Define the following terms which characterize simple harmonic motion?

Vibration, Time period, Frequency, Amplitude.

Answer

Vibration:

One complete round trip of a vibrating body about its mean position is called one vibration.

Time period:

The time taken by a vibrating body to complete one vibration is called time period.

Frequency:

The number of vibrations per cycle of a vibrating in one second is called its frequency.

Amplitude:

The maximum displacement of a vibrating body on either side from its mean position is called its amplitude.

$$F = \frac{1}{T}$$

Q.6 What are damped oscillations? How dumbing progressively reduces the amplitude of oscillation?

Answer

Damped oscillations

Definition:

The oscillations of a system in the presence of some resistive force are damped oscillations.

Explanation:

Fig. 10.4 Variation of amplitude with time of damping system

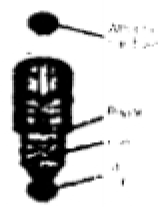


Fig. 10.5: shock absorber

Vibratory motion of Ideal systems in the absence of any friction or resistance continuous indefinitely under the action of a restoring force. Practically, in all systems the force of friction retards the motion, so the system do not oscillate indefinitely.

The friction reduces the mechanical energy of the system as time passes and the motion is said to be damped. This damping progressively reduces the amplitude of the motion as shown in the figure. Shock absorbers in automobiles are one practical application of damped motion. The shock absorber consists of a piston moving through a liquid such as oil. The upper part of the shock absorber is firmly attached to the body of the car.

When the car travels over a hump on the road, the car may vibrate violently. The shock absorbers damp these vibrations and convert their energy into heat energy of the 'oil.

Q.7 Define the terms "wave" and "wave motion"? Write different activities to describe the generation and propagation of wave?

Answer

Wave (definition):

A wave is a disturbance that propagates away from the source (travels) through any medium usually by transference of energy.

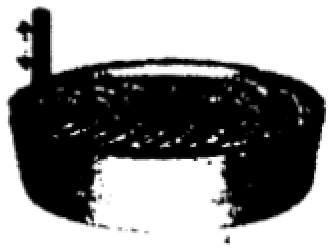


Fig. 10.6: Waves produced by dipping a pencil in a water tub

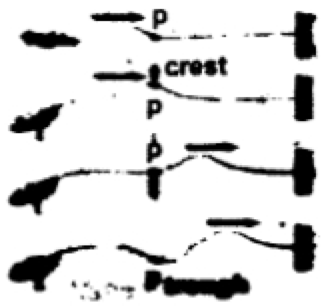


Fig. 10.7: Waves produced on a string

Wave motion (definition):

The process by which a disturbance at one with no net transport of the particles of the medium is called a wave motion.

Waves are carrier of energy and information over large distances.

The production and generation of different waves with the help of vibratory motion of object are demonstrated by the activities below.

Activity 1:

Dip one end of a pencil into a tub of water, and move it up and down vertically. The disturbance in the form of ripples produces water waves, which move away from the source. When the wave reaches a small piece of cork floating near the disturbance, it moves up and down but its original position while the wave will travel outwards. The net displacement of the cork is zero. The cork repeats its vibratory motion about its mean position.

Activity 2:

Take a rope and mark a point ‘P’ on it. Tie one end of the rope with a hook and stretch the string by holding its other end in’ your hand. Now flipping the rope up and down regularly set up a wave in the rope which will travel towards the fixed end.

The point ‘P’ on the rope will start vibrating up and down as the wave passes across it.

The motion of point ‘P’ will be perpendicular to the direction of the motion of wave.

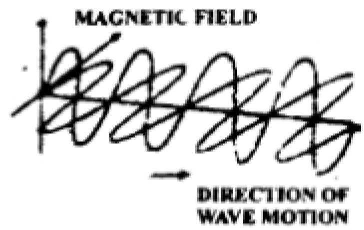
Q.8 Describe the types of waves?

Answer

There are three types of waves:

1. Mechanical waves
2. Electromagnetic waves
3. Matter waves

For your information



Electromagnetic waves consist of electric and magnetic fields oscillating perpendicular to each other.

1) Mechanical waves:

Waves which require any medium for their propagation are called mechanical waves. In such types of waves, the particles of the medium vibrate about their respective mean position and propagate disturbance in the forward direction.

Examples of mechanical waves are water waves sound waves and waves produced on the springs or

2) Electromagnetic waves:

Waves which do not require any medium for their propagation are called electromagnetic waves.

Such Waves are consisted of electric and magnetic other fields.

Radio waves, television waves, x-rays, heat and light waves are some examples of electromagnetic waves.

3) Matter waves:

Such waves in which the particles of the medium actually move from one position to other by carrying energy.

Examples are ocean waves and electron waves when electrons move at a very high-speed approaching speed of light.

Q.9 Distinguish between longitudinal and transverse waves with suitable examples?

Answer

Depending upon the direction of displacement of medium with respect to the direction of motion of the wave itself, mechanical waves may be classified as longitudinal or transverse.

Longitudinal (Compressional) waves:

Longitudinal waves can be produced on a spring (slinky) placed on a smooth floor or along bench. Fix one end of the slinky with a rigid support and hold another end into your hand. Now give it a regular push and pull quickly in the direction of its length.

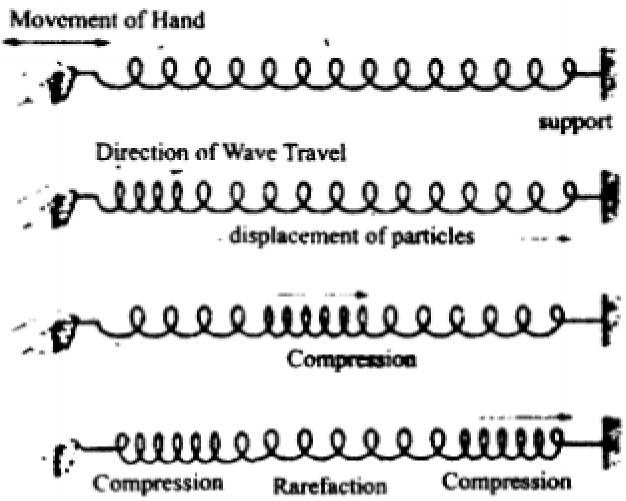


Fig.10.8: Longitudinal wave on a slinky

A series of disturbances in the form of waves will start moving along the length of the slinky. Such a wave consists of regions called "compressions", where the loops of the spring are close together, alternating with regions called "rarefactions" (expansions), where the coils are spaced apart.

In the regions of compressions, particles of the medium are closer together while in the regions of rarefaction particles of the medium are spaced apart.

The compression and rarefactions move back and forth along the direction of motion of the wave.

Definition:

In longitudinal or compressional waves, the particles of the medium move back and forth along the direction of propagation of Wave.

Transverse waves:

Transverse waves can be produced with the help of slinky (flexible spring). Stretch out a slinky along a smooth floor or a long bench with one end fixed. Grasp the other end of the slinky ‘and move it up and down quickly. A wave in the form of alternative crests and troughs will start travelling towards the fixed end.

The crests are the highest points while the troughs are the lowest points of the particles of the medium from the mean position.

The crests and troughs move perpendicular to the direction of the wave.

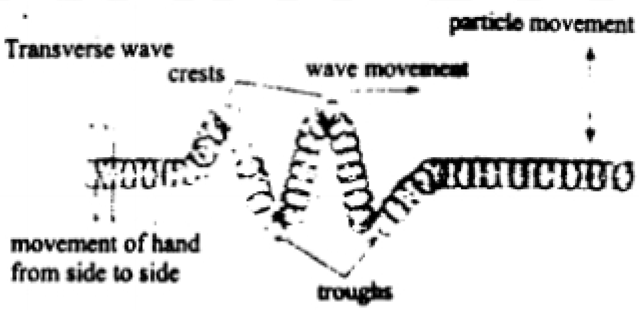


Fig. 10.9: Transverse wave on a slinky

Definition:

In the case of transverse waves, the motion of particles of the medium is perpendicular to the motion of wave.

Q.10 Waves are the means of energy transfer without transfer of matter. Justify this statement with the help of a simple experiment.

Answer

Energy can be transferred from one place to another through waves.

For example; when we shake the stretched spring up and down, we provide our muscular energy to the string. As a result, a set of waves can be seen travelling along the string. The vibrating force from the hand disturbs the particles of the string and sets them in motion.

These particles then transfer their energy to the adjacent particles in the string. Energy is thus transferred from one place of the medium to the other in the form of wave.

The amount of energy Carried by the wave depends upon the distance of the stretched string from its rest position. That is, the energy in a wave depends on the wave amplitude of the wave.

If we shake the string faster, we give more energy per second to produce wave of higher frequency and the wave delivers more energy per second to the particles of the string as it moves forward. Water waves also transfer energy from one place to another as explained below.

Activity:

Drop a stone into a pond of water. Water waves will be produced on the surface of water and will travel outwards as shown in the fig. Place a cork at some distance from the falling stone, when waves reach the cork. it will move up and down along with the motion of the water particles by getting energy from the wave.

This activity shows that water waves like other waves transfer energy from one place to other without transferring matter, i.e. water.

Q.11 Derive a relationship between speed, frequency and wave length of a wave. Write a formula relating speed of a wave to its time period and wave length?

Answer

The relation between the velocity, frequency and wave length of the wave is known as wave equation.

Wave is infecting a disturbance in a medium which travel from one place to another and hence have a specific velocity of travelling. This is called the velocity of wave which is defined by,

$$velocity = \frac{dis\tan ce}{time}$$
$$v = \frac{d}{t}$$

If time taken by the wave in moving from one point to another is equal to the time period then the distance covered by the wave will be equal to one Wave length, hence we can write;

But time period ‘T’ is reciprocal of the frequency ‘P, i.e.,

ClassNotes, Inc. © 2020

$$v = \frac{\lambda}{T}$$

$$t = \frac{1}{f}$$

Therefore, $$v = f\lambda$$

This equation is called the wave equation which is true for all type of waves, _i.e. longitudinal, transverse etc.

Q.12 What is a ripple tank? Write its construction and use to study the wave properties?

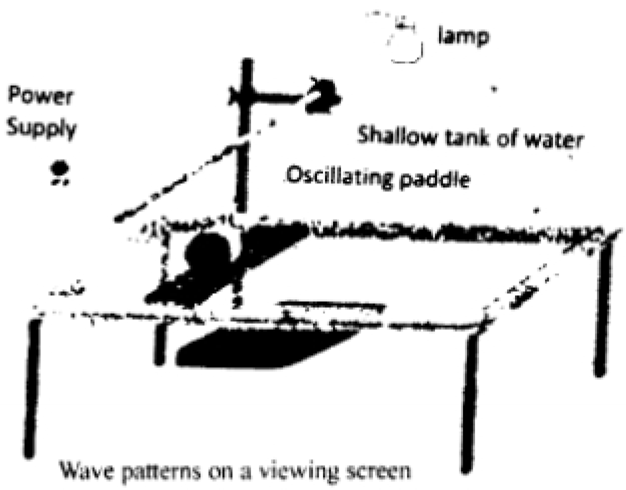
Answer

Ripple Tank:

Ripple tank is a device to produce water waves and to study then characteristics.

Construction & Use:

Ripple tank consists of a rectangular tray having glass bottom and Is placed nearly half meter above the surface of a table.



Waves can be produced on- the surface of water present in the tray by means vibrator (paddle).

This vibrator is on oscillating electric motor fixed on a wooden plate over the tray such that its lower surface just touches the surface of water. On setting the vibrator ON, this wooden plate starts vibrating to generate plane water waves.

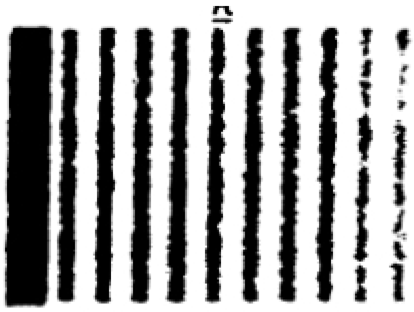


Fig 10.12 plane waves

An electric bulb is hung above the tray to observe the image of water waves-on the paper or screen.

The crests and troughs of the waves appear as bright and dark lines, on the screen.

Q.13 Explain the following properties of waves with reference to ripple tank experiment?

a) Reflection b) Refraction c) Diffraction

Answer

a) Reflection:

To study the reflection of water waves, place a barrier in the ripple tank. The water waves will reflect from the barrier. If the barrier is placed at an angle to the wave front, the reflected waves can be seen to obey the law of reflection i.e. the angle of the incident wave along the normal 'will be equal to the angle of the reflected wave as shown in fig 10.13.

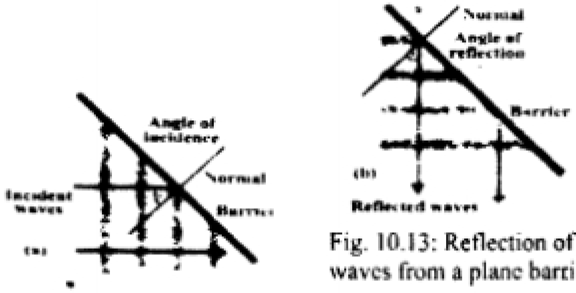


Fig. 10.13: Reflection of water waves from a plane barrier.

Thus, we define reflection as:

"When waves moving in one medium fall on the surface of another medium, they bounce back into the first medium such that the angle of incidence is equal to the angle of reflection. The phenomenon is called reflection of waves."

The speed of a wave in Water depends on the depth of water. If a block is submerged in the ripple tank, the depth of water in the tank will be shallower over the block than elsewhere.

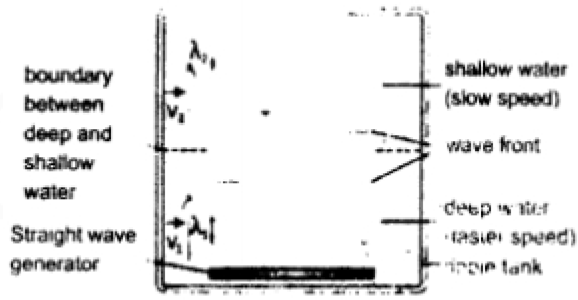


Fig 10.14

When water waves enter the region of shallow water their wave length decreases. But the frequency of the water waves remains the same in _both parts of water because it is equal to the frequency of the vibrator.

b) Refraction:

For the observation of refraction of water waves, we repeat this experiment such that the boundary between the depend the shallower water is at some angle to the wave. front as shown in fig. 10.15. -Now we will observe that in addition to the change in wave length the waves change their direction of propagation as well.

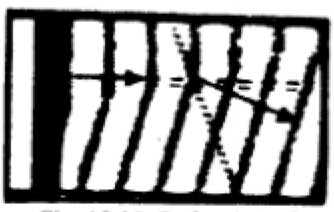


Fig. 10.15: Refraction of water waves

The direction of propagation is always normal to the wave fronts.

This change of path of water waves while passing from a region of deep water to that of shallower is called refraction which is defined as,

"When waves from one medium enter in the second medium at some angle their direction of travel may change. This phenomenon is called refraction of water waves."

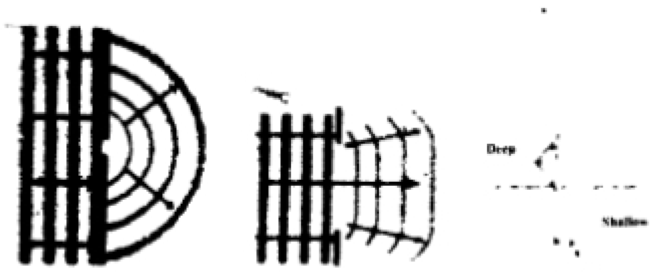


Fig 10.16. Diffraction of water waves through a small slit. Fig 10.17. Diffraction of water waves through a large slit. Fig 10.8

c) Diffraction:

To observe the phenomena of diffraction of water waves, generate plane waves in a ripple tank and place two obstacles in line in such a way that separation between them is equal to the wave length of, water waves. After passing through a small slit between the two obstacles the waves will spread in every direction and change into almost semicircular pattern.

Diffraction of waves can only be observed clearly if the size-of the obstacle is comparable with the wave length of the Wave Fig 10.17 shows the diffraction of waves while passing through a slit with size larger than the wave length of the wave.

Only a small diffraction occurs near the corners of the obstacle.

"'The bending or spreading of waves around 'the sharp edges or comers of obstacles is called diffraction."