Lab9.

Let g_h be the minimal size of an AVL tree of height h. Prove $g_h = F_{h+2} - 1$.

Base case: $g_0 = 0 \Rightarrow F_{0+2} - 1 = 0$

$g_1 = 1 \Rightarrow F_{1+2} - 1 = 1$

$g_2 = 2 \Rightarrow F_{2+2} - 1 = 2$

$g_3 = 4 \Rightarrow F_{3+2} - 1 = 4$

$\vdots$

Inductive Case.

$g_h = F_{h+2} - 1$.

$\therefore g_h = g_{h-1} - 1 + g_{h-2} + 1 \Rightarrow F_{h+2} - 1$

$g_{h-1} = F_{h+1} - 1$ $\quad g_{h-2} = F_h - 1$.

$g_h = \underbrace{g_{h-1} + g_{h-2}}_{\text{two recursive}} + 1$.

$g_h = F_{h+1} - 1 + F_h - 1 + 1 = F_{h+1} + F_h - 1$

$F_n = F_{n-1} + F_{n-2}$ $\quad$ Let $n = h+2$

$F_{h+2} = F_{h+1} + F_h$ $\leftarrow$

$\therefore g_h = F_{h+2} - 1$.