



Figure 1: Node Graph

Q1. Complete Enumeration

A1.1 For $P(+D)$, we need to calculate all possibilities related to other variables.

$$\begin{aligned}
 P(+D) &= P(+D, A, B, C, E) = \sum_{A, B, C, E} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C) \\
 &= P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C) + P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) \\
 &= P(A) * P(B|A) * P(-C|A) * P(D|B, -C) * P(E|-C) + P(A) * P(B|A) * P(-C|A) * P(D|B, -C) * P(-E|-C) \\
 &= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C) + P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) \\
 &= P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(E|-C) + P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(-E|-C) \\
 &= P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(E|C) + P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(-E|C) \\
 &= P(-A) * P(B|-A) * P(-C|-A) * P(D|B, -C) * P(E|-C) + P(-A) * P(B|-A) * P(-C|-A) * P(D|B, -C) * P(-E|-C) \\
 &= P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(E|C) + P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(-E|C) \\
 &= P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(E|-C) + P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(-E|-C)
 \end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.8) * (0.2) * (0.8) * (0.8) + (0.2) * (0.8) * (0.2) * (0.8) * (0.2) \\
&= (0.2) * (0.8) * (0.8) * (0.8) * (0.6) + (0.2) * (0.8) * (0.8) * (0.8) * (0.8) \\
&= (0.2) * (0.2) * (0.2) * (0.8) * (0.8) + (0.2) * (0.2) * (0.2) * (0.8) * (0.2) \\
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.6) + (0.2) * (0.2) * (0.2) * (0.8) * (0.2) \\
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.6) + (0.8) * (0.2) * (0.05) * (0.8) * (0.2) \\
&= (0.8) * (0.2) * (0.95) * (0.8) * (0.6) + (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
&= (0.8) * (0.8) * (0.05) * (0.8) * (0.8) + (0.8) * (0.8) * (0.05) * (0.8) * (0.2) \\
&= (0.8) * (0.8) * (0.95) * (0.05) * (0.6) + (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
&= \mathbf{0.32} \quad \checkmark\checkmark
\end{aligned}$$

A1.2 For $P(+D, -A)$, we need to calculate all possibilities related to other variables.

$$\begin{aligned}
P(+D) &= P(+D, -A, B, C, E) = \sum_{B, C, E} = P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(E|C) \\
&= P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(E|C) + P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(-E|C) \\
&= P(-A) * P(B|-A) * P(-C|-A) * P(D|B, -C) * P(E|-C) + P(-A) * P(B|-A) * P(-C|-A) * P(D|B, -C) * P(-E|-C) \\
&= P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(E|C) + P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(-E|C) \\
&= P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(E|-C) + P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(-E|-C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.6) + (0.8) * (0.2) * (0.05) * (0.8) * (0.2) \\
&= (0.8) * (0.2) * (0.95) * (0.8) * (0.6) + (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
&= (0.8) * (0.8) * (0.05) * (0.8) * (0.8) + (0.8) * (0.8) * (0.05) * (0.8) * (0.2) \\
&= (0.8) * (0.8) * (0.95) * (0.05) * (0.6) + (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
&= \mathbf{0.184} \quad \checkmark\checkmark
\end{aligned}$$

A1.3 For $P(+E|-B)$, we need to calculate all possibilities related to other variables. However first, we should use joint probability formula as below.

$$P(E|-B) = \frac{P(E, -B)}{P(-B)} = \frac{P(E, -B)}{P(+E, -B) + P(-E, -B)}$$

$$\begin{aligned} P(+E, -B) &= P(+E, -B, A, C, D) = \sum_{A, C, D} = P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C) \\ &= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C) + P(A) * P(-B|A) * P(C|A) * P(-D|-B, C) * P(E|C) \\ &= P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(E|-C) + P(A) * P(-B|A) * P(-C|A) * P(-D|-B, -C) * P(E|-C) \\ &= P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(E|C) + P(-A) * P(-B|-A) * P(C|-A) * P(-D|-B, C) * P(E|C) \\ &= P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(E|-C) + P(-A) * P(-B|-A) * P(-C|-A) * P(-D|-B, -C) * P(E|-C) \end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned} &= (0.2) * (0.2) * (0.2) * (0.8) * (0.8) + (0.2) * (0.2) * (0.2) * (0.2) * (0.8) \\ &= (0.2) * (0.2) * (0.8) * (0.05) * (0.6) + (0.2) * (0.2) * (0.8) * (0.95) * (0.6) \\ &= (0.8) * (0.8) * (0.05) * (0.8) * (0.8) + (0.8) * (0.8) * (0.05) * (0.2) * (0.8) \\ &= (0.8) * (0.8) * (0.95) * (0.05) * (0.6) + (0.8) * (0.8) * (0.95) * (0.95) * (0.6) \\ &= \mathbf{0.416} \end{aligned}$$

$$\begin{aligned} P(-E, -B) &= P(-E, -B, A, C, D) = \sum_{A, C, D} = P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) \\ &= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) + P(A) * P(-B|A) * P(C|A) * P(-D|-B, C) * P(-E|C) \\ &= P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(-E|-C) + P(A) * P(-B|A) * P(-C|A) * P(-D|-B, -C) * P(-E|-C) \\ &= P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(-E|C) + P(-A) * P(-B|-A) * P(C|-A) * P(-D|-B, C) * P(-E|C) \\ &= P(-A) * P(-B|-A) * P(-C|-A) * P(D|-B, -C) * P(-E|-C) + P(-A) * P(-B|-A) * P(-C|-A) * P(-D|-B, -C) * P(-E|-C) \end{aligned}$$

Now, we can place numbers corresponding them.

$$= (0.2) * (0.2) * (0.2) * (0.8) * (0.2) + (0.2) * (0.2) * (0.2) * (0.2) * (0.2)$$

$$\begin{aligned}
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.4) + (0.2) * (0.2) * (0.8) * (0.95) * (0.4) \\
&= (0.8) * (0.8) * (0.05) * (0.8) * (0.2) + (0.8) * (0.8) * (0.05) * (0.2) * (0.2) \\
&= (0.8) * (0.8) * (0.95) * (0.05) * (0.4) + (0.8) * (0.8) * (0.95) * (0.95) * (0.4) \\
&= \mathbf{0.264}
\end{aligned}$$

$$P(E|-B) = \frac{P(E,-B)}{P(-B)} = \frac{P(E,-B)}{P(+E,-B) + P(-E,-B)} = \frac{0.416}{0.416 + 0.264} = \mathbf{0.612} \checkmark \checkmark$$

A1.4 For $P(+A | +D, -E)$, we need to calculate all possibilities related to other variables. However first, we should use joint probability formula as below.

$$P(A|D, -E) = \frac{P(A, D, -E)}{P(D, -E)} = \frac{P(A, D, -E)}{P(A, D, -E) + P(-A, D, -E)}$$

$$\begin{aligned}
P(A, D, -E) &= P(A, D, -E, B, C) = \sum_{B, C} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) \\
&= P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) + P(A) * P(B|A) * P(-C|A) * P(-D|B, -C) * P(-E|C) \\
&= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) + P(A) * P(-B|A) * P(-C|A) * P(-D|-B, -C) * P(-E|C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.8) * (0.2) * (0.8) * (0.2) + (0.2) * (0.8) * (0.8) * (0.8) * (0.4) \\
&= (0.2) * (0.2) * (0.2) * (0.8) * (0.2) + (0.2) * (0.2) * (0.8) * (0.05) * (0.4) \\
&= \mathbf{0.048}
\end{aligned}$$

$$\begin{aligned}
P(-A, D, -E) &= P(-A, D, -E, B, C) = \sum_{B, C} = P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(-E|C) \\
&= P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(-E|C) + P(-A) * P(B|-A) * P(-C|-A) * P(-D|B, -C) * P(-E|C) \\
&= P(-A) * P(-B|-A) * P(C|-A) * P(D|-B, C) * P(-E|C) + P(-A) * P(-B|-A) * P(-C|-A) * P(-D|-B, -C) * P(-E|C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
 &= (0.8) * (0.2) * (0.05) * (0.8) * (0.2) + (0.8) * (0.2) * (0.95) * (0.8) * (0.4) \\
 &= (0.8) * (0.8) * (0.05) * (0.8) * (0.2) + (0.8) * (0.8) * (0.95) * (0.05) * (0.4) \\
 &= \mathbf{0.0672}
 \end{aligned}$$

$$P(A|D, -E) = \frac{P(A, D, -E)}{P(D, -E)} = \frac{P(A, D, -E)}{P(A, D, -E) + P(-A, D, -E)} = \frac{0.048}{0.048 + 0.0672} = \mathbf{0.417} \quad \checkmark \checkmark$$

A1.5 For $P(+B, -E | A)$, we need to calculate all possibilities related to other variables. However first, we should use joint probability formula as below.

$$P(B, -E|A) = \frac{P(B, -E, A)}{P(A)} = \frac{P(B, -E, A)}{P(B, E, A) + P(B, -E, A) + P(-B, E, A) + P(-B, -E, A)}$$

$$\begin{aligned}
 P(B, E, A) &= P(B, E, A, C, D) = \sum_{C, D} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C) \\
 &= P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C) + P(A) * P(B|A) * P(C|A) * P(-D|B, C) * P(E|C) \\
 &= P(A) * P(B|A) * P(-C|A) * P(D|B, -C) * P(E|-C) + P(A) * P(B|A) * P(-C|A) * P(-D|B, -C) * P(E|-C)
 \end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
 &= (0.2) * (0.8) * (0.2) * (0.8) * (0.8) + (0.2) * (0.8) * (0.2) * (0.2) * (0.8) \\
 &= (0.2) * (0.8) * (0.8) * (0.8) * (0.6) + (0.2) * (0.8) * (0.8) * (0.2) * (0.6) \\
 &= \mathbf{0.1024}
 \end{aligned}$$

$$P(B, -E, A) = P(B, -E, A, C, D) = \sum_{C, D} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C)$$

$$\begin{aligned}
&= P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C) + P(A) * P(B|A) * P(C|A) * P(-D|B, C) * P(-E|C) \\
&= P(A) * P(B|A) * P(-C|A) * P(D|B, -C) * P(-E|-C) + P(A) * P(B|A) * P(-C|A) * P(-D|B, -C) * P(-E|-C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.8) * (0.2) * (0.8) * (0.2) + (0.2) * (0.8) * (0.2) * (0.2) * (0.2) \\
&= (0.2) * (0.8) * (0.8) * (0.8) * (0.4) + (0.2) * (0.8) * (0.8) * (0.2) * (0.4) \\
&= \mathbf{0.0576}
\end{aligned}$$

$$\begin{aligned}
P(-B, E, A) &= P(-B, E, A, C, D) = \sum_{C,D} = P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C) \\
&= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C) + P(A) * P(-B|A) * P(C|A) * P(-D|-B, C) * P(E|C) \\
&= P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(E|-C) + P(A) * P(-B|A) * P(-C|A) * P(-D|-B, -C) * P(E|-C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.2) * (0.2) * (0.8) * (0.8) + (0.2) * (0.2) * (0.2) * (0.2) * (0.8) \\
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.6) + (0.2) * (0.2) * (0.8) * (0.95) * (0.6) \\
&= \mathbf{0.0256}
\end{aligned}$$

$$\begin{aligned}
P(-B, -E, A) &= P(-B, -E, A, C, D) = \sum_{C,D} = P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) \\
&= P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(-E|C) + P(A) * P(-B|A) * P(C|A) * P(-D|-B, C) * P(-E|C) \\
&= P(A) * P(-B|A) * P(-C|A) * P(D|-B, -C) * P(-E|-C) + P(A) * P(-B|A) * P(-C|A) * P(-D|-B, -C) * P(-E|-C)
\end{aligned}$$

Now, we can place numbers corresponding them.

$$\begin{aligned}
&= (0.2) * (0.2) * (0.2) * (0.8) * (0.2) + (0.2) * (0.2) * (0.2) * (0.2) * (0.2) \\
&= (0.2) * (0.2) * (0.8) * (0.05) * (0.4) + (0.2) * (0.2) * (0.8) * (0.95) * (0.4) \\
&= \mathbf{0.0144}
\end{aligned}$$

$$\begin{aligned}
 P(B, -E|A) &= \frac{P(B, -E, A)}{P(A)} = \frac{P(B, -E, A)}{P(B, E, A) + P(B, -E, A) + P(-B, E, A) + P(-B, -E, A)} \\
 &= \frac{0.0576}{0.1024 + 0.0576 + 0.0256 + 0.0144} = \mathbf{0.288} \checkmark\checkmark
 \end{aligned}$$

Q2. Variable Elimination

A2.1 For $P(+D)$, After determining parents of it, we can eliminate variables if it is possible.

$$P(+D) = P(+D, A, B, C, E) = \sum_{A, B, C, E} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C)$$

Suppose that $Y(B, C)$ is $\sum_A = P(A) * P(B|A) * P(C|A)$

$Y(B, C)$	$P(A) * P(B A) * P(C A)$	$P(-A) * P(B -A) * P(C -A)$
B,C	$0.2 * 0.8 * 0.2$	$0.8 * 0.2 * 0.05$
B,-C	$0.2 * 0.8 * 0.8$	$0.8 * 0.2 * 0.95$
-B,C	$0.2 * 0.2 * 0.2$	$0.8 * 0.8 * 0.05$
-B,-C	$0.2 * 0.2 * 0.8$	$0.8 * 0.8 * 0.95$

$$Y(B, C) = 0.032 + 0.008 = 0.04$$

$$Y(-B, C) = 0.008 + 0.032 = 0.04$$

$$Y(B, -C) = 0.128 + 0.152 = 0.28$$

$$Y(-B, -C) = 0.032 + 0.608 = 0.64$$

Now, our equation is $P(+D) = \sum_{B, C, E} = Y(B, C) * P(D|B, C) * P(E|C)$

Suppose that $Z(C)$ is $\sum_B = Y(B, C) * P(D|B, C)$

$Z(C)$	$Y(B, C) * P(D B, C)$	$Y(-B, C) * P(D -B, C)$
C	$0.04 * 0.8$	$0.04 * 0.8$
-C	$0.28 * 0.8$	$0.64 * 0.05$

$$Z(C) = 0.032 + 0.032 = 0.064$$

$$Z(-C) = 0.224 + 0.032 = 0.256$$

Now, our equation is $P(+D) = \sum_{C,E} Z(C) * P(E|C)$

Suppose that $X(E)$ is $\sum_C Z(C) * P(E|C)$

$X(E)$	$Z(C) * P(E C)$	$Z(-C) * P(E -C)$
E	$0.064 * 0.8$	$0.256 * 0.6$
-E	$0.064 * 0.2$	$0.256 * 0.4$

$$X(E) = 0.0512 + 0.01536 = 0.2048$$

$$X(-E) = 0.224 + 0.1024 = 0.1152$$

Now, our equation is $P(+D) = \sum_E X(E) = X(E) + X(-E) = \mathbf{0.32} \checkmark \checkmark$

A2.2 For $P(+D|-A)$, After determining parents of it, we can eliminate variables if it is possible.

$$P(+D) = P(+D, -A, B, C, E) = \sum_{B,C,E} = P(-A) * P(B|-A) * P(C|-A) * P(D|B, C) * P(E|C)$$

Suppose that $Y'(E, C)$ is $\sum_{E,C} P(E|C)$

$Y'(E, C)$	$P(E C)$
E, C	0.8
E, -C	0.6
-E, C	0.2
-E, -C	0.4

$$Y(E, C) = 0.8 + 0.2 = 1$$

$$Y(E, -C) = 0.6 + 0.4 = 1$$

Suppose that $Z'(-A,B,C,D)$ is $\sum_{B,C} = P(C|-A) * Y(C) * P(D|B,C)$

$Z'(-A,B,C,D)$	$P(C -A) * Y(C) * P(D B,C)$
-A,B,C,D	0.05* 0.8
-A,B,C,D	0.95* 0.8
-A,B,C,D	0.05*0.8
-A,B,C,D	0.95*0.05

$$Z(-A, B, D) = 0.76$$

$$Z(-A, -B, D) = 0,04$$

Suppose that $X'(-A,B,D)$ is $\sum_B = Z(-A, B, D) * P(B|-A)$

$X'(-A,B,D)$	$Z(-A, B, D) * P(B -A)$
-A,B,D	0.2*0.76
-A,-B,D	0.8*0.04

$$X(-A, D) = 0.152 * 0.032 = 0.184$$

$$P(D, -A) = \mathbf{0.184} \quad \checkmark \checkmark$$

A2.3 For $P(+E|-B)$, After determining parents of it, we can eliminate variables if it is possible.

$$P(E|-B) = \frac{P(E, -B)}{P(-B)} = \frac{P(E, -B)}{P(+E, -B) + P(-E, -B)}$$

$$P(+E|-B) = P(+E, -B, A, C, D) = \sum_{A,C,D} = P(A) * P(-B|A) * P(C|A) * P(D|-B, C) * P(E|C)$$

Suppose that $Y'(A,-B,C)$ is $\sum_{A,C} = P(A) * P(-B|A) * P(C|A)$

$Y(A,-B,C)$	$P(A) * P(-B A) * P(C A)$
A,-B,C	$0.2*0.2*0.2$
A,-B,-C	$0.2*0.2*0.8$
-A,-B,C	$0.8*0.8*0.05$
-A,-B,-C	$0.8*0.8*0.95$

$$Y(-B, C) = 0.008 + 0.032 = 0.04$$

$$Y(-B, -C) = 0.032 + 0.608 = 0.064$$

Suppose that $Z'(-B,C,D)$ is $\sum_{C,D} = P(D|-B,C)$

$Z'(-B,C,D)$	$P(D -B,C)$
A,-B,C	0.8
A,-B,-C	0.2
-A,-B,C	0.05
-A,-B,-C	0.95

$$Z(-B, C) = 0.8 + 0.2 = 1$$

$$Z(-B, -C) = 0.95 + 0.05 = 1$$

Suppose that $X'(-B,C,E)$ is $\sum_{A,C} = Y(-B,C) * Z(-B,C) * P(E|C)$

$X'(-B,C,E)$	$Y(-B,C) * Z(-B,C) * P(E C)$
A,-B,C	$0.04*1*0.8$
A,-B,-C	$0.64*1*0.6$
-A,-B,C	$0.04*1*0.2$
-A,-B,-C	$0.64*1*0.4$

$$X(-B, E) = 0.032 + 0.384 = 0.416$$

$$X(-B, -E) = 0.008 + 0.256 = 0.264$$

$$P(E|-B) = \frac{P(E,-B)}{P(-B)} = \frac{P(E,-B)}{P(+E,-B) + P(-E,-B)} = \frac{0,416}{0,264 + 0,416} = \mathbf{0,612} \checkmark\checkmark$$

A2.4 For $P(+A | +D, -E)$, After determining parents of it, we can eliminate variables if it is possible.

$$P(A, D, -E) = P(A, D, -E, B, C) = \sum_{B, C} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(-E|C)$$

Suppose that $Y'(A, B, C, D)$ is $\sum_{B, C} = P(-B|A) * P(D|B, C)$

$Y'(A, B, C, D)$	$P(-B A) * P(D B, C)$
A, B, C, D	0.8*0.8
A, B, -C, D	0.8*0.8
A, -B, C, D	0.2*0.8
A, -B, -C, D	0.2*0.05
-A, B, C, D	0.2*0.8
-A, B, -C, D	0.2*0.8
-A, -B, C, D	0.8*0.8
-A, -B, -C, D	0.8*0.05

$$Y(A, C, D) = 0.64 + 0.16 = 0.8$$

$$Y(A, -C, D) = 0.64 + 0.01 = 0.65$$

$$Y(A, C, D) = 0.16 + 0.64 = 0.8$$

$$Y(A, -C, D) = 0.16 + 0.04 = 0.25$$

Suppose that $X'(A, C, D, -E)$ is $\sum_C = P(C|A) * P(-E|C) * Y(A, C, D)$

$X'(A, C, D, -E)$	$P(C A) * P(-E C) * Y(A, C, D)$
A, C, D, -E	0.2*0.2*0.8
A, -C, D, -E	0.8*0.4*0.208
-A, C, D, -E	0.05*0.2*0.8
-A, -C, D, -E	0.05*0.4*0.2

$$X(A, D, -E) = 0.032 + 0.208 = 0.24$$

$$X(-A, D, -E) = 0.008 + 0.076 = 0.084$$

$$P(A, D, -E) = \sum_{B, C} = P(A) * X(A, D, -E) = 0.2 * 0.24 = 0.048$$

$$P(-A, D, -E) = \sum_{B,C} = P(A) * X(A, D, -E) = 0.8 * 0.084 = 0.0672$$

$$P(A|D, -E) = \frac{P(A, D, -E)}{P(D, -E)} = \frac{P(A, D, -E)}{P(A, D, -E) + P(-A, D, -E)} = \frac{0.048}{0.048 + 0.067} = \mathbf{0.417} \checkmark\checkmark$$

A2.5 For $P(+B, -E | +A)$, After determining parents of it, we can eliminate variables if it is possible.

$$P(B, E, A) = P(B, E, A, C, D) = \sum_{C,D} = P(A) * P(B|A) * P(C|A) * P(D|B, C) * P(E|C)$$

Suppose that $Y'(A, B, C, D, -E)$ is $\sum_{C,D} = P(C|A) * P(-E|C) * P(D|B, C)$

$Y'(A, B, C, D, -E)$	$P(C A) * P(-E C) * P(D B, C)$
A, B, C, D, -E	$0.2 * 0.2 * 0.8$
A, B, C, -D, -E	$0.2 * 0.2 * 0.2$
A, B, -C, D, -E	$0.8 * 0.4 * 0.8$
A, B, -C, -D, -E	$0.8 * 0.4 * 0.2$

$$Y(A, B, D, E) = 0.032 + 0.256 = 0.288$$

$$Y(A, B, -D, E) = 0.08 + 0.064 = 0.072$$

Suppose that $X'(A, B, D, -E)$ is $\sum_C = P(C|A) * P(-E|C) * P(D|B, C)$

$X'(A, B, D, -E)$	$P(C A) * P(-E C) * P(D B, C)$
A, B, D, -E	0.288
A, B, -D, -E	0.072

$$X(A, B, -E) = 0.288 + 0.072 = 0.36$$

$$P(B, E, A) = P(B, E, A, C, D) = \sum_{C,D} = P(A) * P(B|A) * X(A, B, -E) = 0.2 * 0.8 * 0.36 = 0.0576$$

$$P(B, -E|A) = \frac{P(B, -E, A)}{P(A)} = \frac{P(B, -E, A)}{P(B, E, A) + P(B, -E, A) + P(-B, E, A) + P(-B, -E, A)} = \frac{0.0576}{0.2} = \mathbf{0.288} \checkmark\checkmark$$