Q1. **Prove that if any diagonal element of**

$$M = \begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$$

is zero than the rows are linearly dependent.

A1. If any element of diagonal matrix is zero, then determinant of this diagonal matrix is zero.

For example: $\begin{bmatrix} 0 & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$ ==> det $\begin{pmatrix} 0 & b & c \\ 0 & d & e \\ 0 & 0 & f \end{pmatrix} = 0\ x\ d\ x\ f\ = 0$

so, according to **WRONSKİAN THERORЕМ**, if determinant of any matrices is zero, then rows are lineary dependent.

Q2. **Find two different bases for the subspace of all vectors in R3 whose first two components are equal.**

A2. If we have any set of three vectors that are linearly independent in $\mathbb{R}^3$, then our set of vectors is basis for $\mathbb{R}^3$.

$$B_1 = \left\{ \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} \right\}, \ \mathrm{B}_2 = \left\{ \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ 25 \\ 7 \end{bmatrix} \right\}$$

These two different basis for the subspace of all vectors in $\mathbb{R}^3$ whose first two components are equal.

We must check whether all vectors of basis are linearly independent or not.

$$B_1 = a_1 \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$For\ B_1, \left[\begin{array}{ccc|c} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 4 & 0 \end{array}\right] \qquad a_1 = a_2 = a_3 = 0$$

$$B_2 = a_1 \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix} + a_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + a_3 \begin{bmatrix} 10 \\ 25 \\ 7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$For\ B_2, \left[\begin{array}{ccc|c} 5 & 0 & 10 & 0 \\ 0 & 5 & 25 & 0 \\ 0 & 0 & 7 & 0 \end{array}\right] ==> R_3/7 => R_3, \left[\begin{array}{ccc|c} 5 & 0 & 10 & 0 \\ 0 & 5 & 25 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] ==> R_1/5 => R_1, \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 5 & 25 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$R_2/5 => R_2, \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] ==> -5R_3{+}R_2 => R_2, \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] ==> -2R_3{+}R_1 => R_1, \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \qquad a_1 = a_2 = a_3 = 0$$

So, B_1 and B_2 are bases that first two components are equal for the subspace of all vectors in $\mathbb{R}^3$

Q3. Which pairs are orthogonal among the vectors and why?

$$\vec{v_1}= \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix} \quad , \quad \vec{v_2}= \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix} \quad , \quad \vec{v_3}= \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$$

A3.

Being orthogonal, production of two vector must be equal to zero.

$$\vec{v_1} \ . \ \vec{v_3} = \begin{bmatrix} 1 \\ 2 \\ -2 \\ 1 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & -2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = 1 + (-2) + 2 + (-1) = 0$$

$$\vec{v_2} \ . \ \vec{v_3} = \begin{bmatrix} 4 \\ 0 \\ 4 \\ 0 \end{bmatrix}^T \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 4 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix} = 4 + 0 + (-4) + 0 = 0$$

Q4. If we have a format that matches the IEEE 754 standard but has 1 bit for the sign, 4 bits for the exponent, and 5 bits for the significand parts, then

a. What is the smallest positive subnormal number representable in this format?

b. What is the largest positive subnormal number representable in this format?

c. What is the smallest positive normalized number representable in this format?

d. What is the largest positive normalized number representable in this format?

e. What is the smallest number that is greater than 1?

f. What is the largest number that is greater than 1?

A4. a. 0 0000 00001

b. 0 0000 11111

c. 0 0001 00000

d. 0 1110 11111

e. 0 0111 00001

f. 0 0110 11111