Exercise 34 - Groza Iulia-Diana

$\int (\sin(x))^4 dx = \int (\sin^2(x))^2 dx$ ①

$\cos(2x) = 1 - 2\sin^2(x) \Rightarrow \sin^2(x) = \frac{1}{2}(1 - \cos(2x))$ ②

Using ② in ①, we obtain: $\int (\sin(x))^4 dx = \int \left(\frac{1}{2}(1-\cos(2x))\right)^2 dx =$

$= \frac{1}{4}\int (1-\cos(2x))^2 dx = \frac{1}{4}\int (1 - 2\cos(2x) + (\cos(2x))^2) dx$ ③

$\cos(2x) = 2\cos^2(x) - 1 \Rightarrow \cos^2(2x) = \frac{1}{2}(1+\cos(4x))$ ④

Using ④ in ③, we obtain: $\int (\sin(x))^4 dx = \frac{1}{4}\int (1 - 2\cos(2x) + \frac{1}{2} + \frac{1}{2}\cos(4x)) dx =$

$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos(2x) + \frac{1}{2}\cos(4x)\right) dx = \frac{1}{4}\left(\frac{3}{2}\int dx - 2\int \cos(2x) dx + \frac{1}{2}\int \cos(4x) dx\right) =$

$= \frac{1}{4}\left(\frac{3}{2}x - \frac{2}{2}\sin(2x) + \frac{1}{2}\cdot\frac{1}{4}\sin(4x)\right) = \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$

$\Rightarrow \int (\sin(x))^4 dx = \frac{3}{8}x - \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x) + C$

$\int (\cos(x))^5 dx = \int (\cos(x))^4 \cdot \cos(x) dx = \int (\cos^2(x))^2 \cdot \cos(x) dx$ ⑤

$\sin^2(x) + \cos^2(x) = 1 \Rightarrow \cos^2(x) = 1 - \sin^2(x)$ ⑥

Using ⑥ in ⑤, we obtain: $\int (\cos(x))^5 dx = \int (1 - \sin^2(x))^2 \cos(x) dx$

$t \overset{not}{=} \sin(x) \Leftrightarrow t' dt = (\sin(x))' dx \Leftrightarrow dt = \cos(x) dx$

$\Rightarrow \int (\cos(x))^5 dx = \int (1-t^2)^2 dt = \int (1 - 2t^2 + t^4) dt = \int dt - 2\int t^2 dt + \int t^4 dt =$

$= t - \frac{2}{3}t^3 + \frac{1}{5}t^5 = \sin(x) - \frac{2}{3}(\sin(x))^3 + \frac{1}{5}(\sin(x))^5 + C$

$\Rightarrow \int (\cos(x))^5 dx = \sin(x) - \frac{2}{3}(\sin(x))^3 + \frac{1}{5}(\sin(x))^5 + C$