## Project 3 (0.2 points)

- Input: non-zero natural number n
- Output:
  - 1. the number of bases of the vector space  $\mathbb{Z}_2^n$  over  $\mathbb{Z}_2$
  - 2. the vectors of each such basis (for  $n \leq 4$ )

Example: The vector space  $\mathbb{Z}_2^2$  over  $\mathbb{Z}_2$  has 4 vectors, namely (0,0),(0,1),(1,0),(1,1). Its dimension is 2, so every basis has two vectors. The first vector  $v_1$  may be chosen in 3 ways  $(v_1 \neq 0)$ , while the second vector  $v_2$  may be chosen in 2 ways  $(v_2 \neq (0,0))$  and  $v_2$  different of any linear combination of other non-zero vectors, which in this case means  $v_2 \neq v_1$ . Hence there are  $3 \cdot 2 = 6$  bases.

- Input: n=2
- Output:
  - 1. the number of bases of the vector space  $\mathbb{Z}_2^2$  over  $\mathbb{Z}_2$  is 6
  - 2. the vectors of each such basis are:

$$\begin{array}{c} ((0,1),(1,0)) \\ ((0,1),(1,1)) \\ ((1,0),(0,1)) \end{array} \qquad \qquad \begin{array}{c} ((1,0),(1,1)) \\ ((1,1),(0,1)) \\ ((1,1),(1,0)) \end{array}$$