Q&A

Sunday, June 5, 2022 4:22 PM

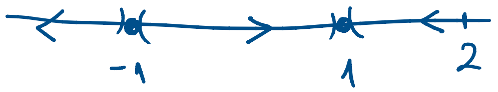
$\dot{x} = 1 - x^2$

$f(x) = 1 - x^2$ eq: $f(x) = 0 \iff x \in \{-1, 1\}$ $\eta_1^* = -1, \ \eta_2^* = 1$

$f'(x) = -2x$

$f'(\eta_1^*) = f'(-1) = (-2)\cdot(-1) = 2 > 0 \Rightarrow \eta_1^*$ is a repeller

$f'(\eta_2^*) = f'(1) = -2 < 0 \Rightarrow \eta_2^*$ is an attractor



$\varphi(t,1)$ the sol. of the IVP $\begin{cases} \dot{x} = 1 - x^2 \\ x(0) = 1 \end{cases}$

$\varphi(t,1) = 1, \ \forall t \in \mathbb{R}$

(η^* is an equil. point if $\varphi(t,\eta^*) = \eta^*, \ \forall t \in \mathbb{R}$)

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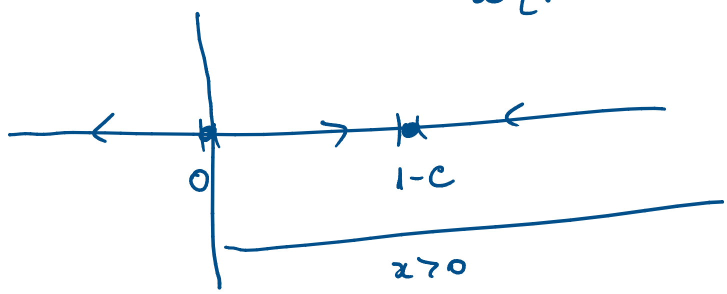
2. $0 < c < 1$ $\dot{x} = x(1-x) - cx$

$f(x) = x(1-x) - cx = x - x^2 - cx = (1-c)x - x^2$

eq: $f(x) = 0$ $(1-c)x - x^2 = 0 \iff x_1 = 0 \quad x_2 = 1-c$

$x[1-c-x] = 0$ $\eta_1^* = 0 \quad \eta_2^* = 1-c > 0$

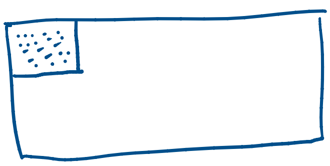


$f'(x) = 1 - c - 2x$

$f'(0) = 1-c > 0 \Rightarrow \eta_1^* = 0$ is a repeller

$f'(1-c) = (1-c) - 2(1-c) = -(1-c) < 0 \Rightarrow$

$\Rightarrow \eta_2^* = 1-c$ is an attractor



$x(t)$ is density of fish in a lake

$c > 0$ the fishing rate

if the initial density $x(0)$ is smaller than $1-c$, then the density increases, but till the value $1-c$

if the initial density $x(0)$ is smaller than $1-c$, ...

increases, but till the value $1-c$

if $x(0) > 1-c$, then the density decreases, but till $1-c$.

~~if~~ Also, the equilibrium value, $1-c$ is bigger if the fishing rate is smaller.

$\varphi(t,\eta)$ the sol. of the IVP $\begin{cases} x' = x(1-x) - cx \\ x(0) = \eta \end{cases}$

if $0 < \eta < 1-c \Rightarrow \varphi(t,\eta)$ is increasing and $\lim\limits_{t\to\infty} \varphi(t,\eta) = 1-c$

if $\eta > 1-c \Rightarrow$ — " — decreasing and $\lim\limits_{t\to\infty} \varphi(t,\eta) = 1-c$

the eq. $\eta^* = 1-c$ increases when the parameter c decreases.

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37. $\begin{cases} y' = 1 + xy^2, & x \in [0,1] \\ y(0) = 0 \end{cases}$ $h = 0.02$ stepsize

Denote by $\varphi(x)$ the exact solution.

write the Euler's numerical formula.

$h \; h$

$0 \; x_1 \; x_2 \; \dots \dots \; x_n = 1$

$x_0 = 0$ $x_n = 1$

$n = \frac{1}{h} = \frac{1}{0.02} = \frac{1}{\frac{2}{100}} = \frac{100}{2} = 50$

50 the number of steps to arrive to 1. , $x_{50} = 1$

$\boxed{x_k = kh, \quad k = \overline{0,50}}$

$y_k = ?$ $y_k \approx \varphi(x_k)$

$y_{k+1} = y_k + h f(x_k, y_k), \quad k = \overline{0,50}$

$y'(x) = f(x, y(x))$

$f(x,y) = 1 + xy^2$

$\boxed{y_{k+1} = y_k + h\left(1 + x_k \cdot y_k^2\right), \quad k = \overline{0,50}}$

$\boxed{y_{k+1} = y_k + h\left(1 + k \cdot h \cdot y_k^2\right), \quad k = \overline{0,50} \qquad h = 0.02}$

$x_k = k \cdot h, \quad k = \overline{0,50}$ $\qquad \varphi(0.5) \qquad x_k = 0.5 \qquad k \cdot h = 0.5 \Rightarrow$

$$\Rightarrow \quad k = \frac{0.5}{0.02} = \frac{5}{1\cancel{0}} \cdot \frac{\overset{5}{\cancel{10}\cancel{0}}}{\cancel{2}} = 25 \text{ steps.}$$

$\varphi(1) \qquad 50$ steps.