Project 1 (0.2 points)

- *Input*: non-zero natural number n
- *Output*:
  1. the number of partitions on a set $A = \{a_1, \ldots, a_n\}$
  2. the partitions on a set $A = \{a_1, \ldots, a_n\}$ and their corresponding equivalence relations (for $n \leq 8$)

Example:

- *Input*: $n = 3$
- *Output*:
  1. the number of partitions on a set $A = \{a_1, a_2, a_3\}$ is 5
  2. using the notation $\Delta_A = \{(a_1, a_1), (a_2, a_2), (a_3, a_3)\}$, the partitions on a set $A = \{a_1, a_2, a_3\}$ and their corresponding equivalence relations are:

$$\{a_1\}, \{a_2\}, \{a_3\}\} \rightsquigarrow \Delta_A$$
$$\{a_1, a_2\}, \{a_3\}\} \rightsquigarrow \Delta_A \cup \{(a_1, a_2), (a_2, a_1)\}$$
$$\{a_1, a_3\}, \{a_2\}\} \rightsquigarrow \Delta_A \cup \{(a_1, a_3), (a_3, a_1)\}$$
$$\{a_2, a_3\}, \{a_1\}\} \rightsquigarrow \Delta_A \cup \{(a_2, a_3), (a_3, a_2)\}$$
$$\{\{a_1, a_2, a_3\}\} \rightsquigarrow A \times A$$

Project 2 (0.2 points)

- *Input*: non-zero natural number n
- *Output*:
  1. the number of associative operations on a set $A = \{a_1, \ldots, a_n\}$
  2. the operation table of each associative operation (for $n \leq 4$)

Example:

- *Input*: $n = 2$
- *Output*:
  1. the number of associative operations on a set $A = \{a_1, a_2\}$ is 8
  2. identifying an operation table $\begin{array}{c|cc} & a_1 & a_2 \\ \hline a_1 & x & y \\ a_2 & z & t \end{array}$ by the matrix $\begin{pmatrix} x & y \\ z & t \end{pmatrix} \in M_2(A)$, the operation tables of the associative operations on $A = \{a_1, a_2\}$ are given by the matrices:

$$\begin{pmatrix} a_1 & a_1 \\ a_1 & a_1 \end{pmatrix}, \begin{pmatrix} a_1 & a_1 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_1 \\ a_2 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_2 & a_1 \end{pmatrix}, \begin{pmatrix} a_1 & a_2 \\ a_2 & a_2 \end{pmatrix}, \begin{pmatrix} a_2 & a_1 \\ a_1 & a_2 \end{pmatrix}, \begin{pmatrix} a_2 & a_2 \\ a_2 & a_2 \end{pmatrix}.$$