GROZA IULIA DIANA

Question 2

$A \in M_2(\mathbb{R})$, $\det(2A^2 + 6i_2) = 0 \Rightarrow \det A = ?$.

$\det(2A^2 + 6i_2) = 0 \quad (\Leftrightarrow)$

$(\Leftrightarrow) \; 4\det(A^2 + 3i_2) = 0_2 \quad (\Leftrightarrow)$

$(\Leftrightarrow) \; \det(A^2 - Tr(A)A + \det(A)i_2 + 3i_2 + Tr(A)A -$

$- \det(A)i_2) = 0 \quad (\Leftrightarrow)$

$(\Leftrightarrow) \; \det(3i_2 + Tr(A)A - \det(A)i_2) = 0 \quad (\Leftrightarrow)$

$(\Leftrightarrow) \; \det(Tr(A)A + (3 - \det(A))i_2) = 0 \quad ①$

$\det(xA + yi_2) = \begin{vmatrix} xa_{11} + y & xa_{12} \\ xa_{21} & xa_{22} + y \end{vmatrix} =$

$= (xa_{11} + y)(xa_{22} + y) - x^2 a_{12} a_{21} =$

$= x^2 a_{11} a_{22} + xy a_{22} + xy a_{11} + y^2 - x^2 a_{12} a_{21} =$

$= x^2(a_{11} a_{22} - a_{12} a_{21}) + xy(a_{11} + a_{22}) + y^2 =$

$= x^2 \det A + xy \, Tr(A) + y^2 \quad ②$

Using ② in ①, we obtain:

$\det(Tr(A)A + (3 - \det(A))i_2) = 0 \quad (\Leftrightarrow)$

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$(\Rightarrow) Tr^2(A) \det A + Tr(A)(3 - \det A) + (3 - \det A)^2 = 0 (\Rightarrow$

$(\Rightarrow) 3Tr^2(A) + 9 + \det^2(A) - 6\det(A) = 0 (\Rightarrow)$

$(\Rightarrow)$ The equation should have just one

solution $\Rightarrow \Delta = 0 (\Rightarrow) 36 - 4(3Tr^2(A) + 9) = 0 \Rightarrow)$

$(\Rightarrow) 36 - 12Tr^2(A) - 36 = 0 \Rightarrow Tr(A) = 0.$

The equation becomes:

$\swarrow \det^2(A) - 6\det(A) + 9 = 0 (\Rightarrow)$

$(\Rightarrow) (\det(A) - 3)^2 = 0 \Rightarrow \det(A) = 3$