Project 4 (0.2 points)

- *Input*: non-zero natural numbers k and n with $k \leq n$
- *Output*:
  1. the number of k-dimensional subspaces of the vector space $\mathbb{Z}_2^n$ over $\mathbb{Z}_2$
  2. a basis of each such subspace (for $1 \leq k \leq n \leq 6$)

Example: The vector space $\mathbb{Z}_2^3$ over $\mathbb{Z}_2$ has 8 vectors, namely $(0,0,0)$, $(0,0,1)$, $(0,1,0)$, $(0,1,1)$, $(1,0,0)$, $(1,0,1)$, $(1,1,0)$, $(1,1,1)$. Any 2-dimensional subspace has a basis with two vectors. There are $C_7^2 = 21$ possibilities to choose 2 vectors out of the 8 vectors of $\mathbb{Z}_2^3$, but some of them will generate the same subspace. Only 7 choices will generate different subspaces.

- *Input*: $k = 2$, $n = 3$
- *Output*:
  1. the number of 2-dimensional subspaces of the vector space $\mathbb{Z}_2^3$ over $\mathbb{Z}_2$ is 7
  2. a basis of each such subspace is:

((0,0,1),(0,1,0))
((0,0,1),(1,0,0))
((0,0,1),(1,1,0))
((0,1,0),(1,0,0))
((0,1,0),(1,0,1))
((0,1,1),(1,0,0))
((0,1,1),(1,0,1))

Project 5 (0.2 points)

- *Input*: natural numbers $m, n \geq 2$
- *Output*:
  1. the number of matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in *reduced* (that is, each column containing a leading 1 has zeros everywhere else) echelon form
  2. the matrices $A \in M_{m,n}(\mathbb{Z}_2)$ in reduced echelon form (for $2 \leq m, n \leq 5$)

Example:

- *Input*: $m = 2$, $n = 3$
- *Output*:
  1. the number of matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form is 15
  2. the matrices $A \in M_{2,3}(\mathbb{Z}_2)$ in reduced echelon form are (the leading 1's are framed):

$$\begin{pmatrix} \boxed{1} & 1 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 1 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 1 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & \boxed{1} & 0 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & \boxed{1} & 0 \end{pmatrix} \Bigg| \begin{pmatrix} 0 & \boxed{1} & 1 \\ 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{pmatrix} 0 & 0 & \boxed{1} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\begin{pmatrix} \boxed{1} & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Bigg| \begin{pmatrix} \boxed{1} & 0 & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \Bigg| \begin{pmatrix} 0 & \boxed{1} & 0 \\ 0 & 0 & \boxed{1} \end{pmatrix} \Bigg| \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$