SUBJECT 1.
STUDENT 1. GRAB ANDREI

choosing bases: $b_1 = 7$, $b_2 = 16$

choosing numbers: $x = 365164$, $y = 62464$, $z = BFAC07$, $f = C$

$365164_{(7)} + 62464_{(7)} = 460661_{(7)}$

```
  1 1 0 1 1 0
  3 6 5 1 6 4(7) +
  0 6 2 4 6 4(7)
-----------------
  4 6 0 6 6 1(7)
```

iterations:

i). $4_{(7)} + 4_{(7)} + 0_{(7)} = 8_{(10)}$

$8 \bmod 7 = 1$, $8 \operatorname{div} 7 = 1$

ii). $6_{(7)} + 6_{(7)} + 1_{(7)} = 13_{(10)}$

$13 \bmod 7 = 6$, $13 \operatorname{div} 7 = 1$

iii). $4_{(7)} + 1_{(7)} + 1_{(7)} = 6_{(10)}$

$6 \bmod 7 = 6$, $6 \operatorname{div} 7 = 0$

iv). $5_{(7)} + 2_{(7)} + 0_{(7)} = 7_{(10)}$

$7 \bmod 7 = 0$, $7 \operatorname{div} 7 = 1$

v). $6_{(7)} + 6_{(7)} + 1_{(7)} = 13_{(10)}$

$13 \bmod 7 = 6$, $13 \operatorname{div} 7 = 1$

$BFAC07_{(16)} \cdot C_{(16)} = 8FC1022_{(16)}$

```
  8 B 8 9 0 5 0
  0 B F A C 0 7(16) ·
              C(16)
-------------------
  8 F C 1 0 5 4
```

iterations:

i). $C_{(16)} \cdot 7_{(16)} + 0_{(16)} = 54_{(10)}$

$84 \bmod 16 = 4$, $84 \operatorname{div} 16 = 5$

ii). $C_{(16)} \cdot 0_{(16)} + 5_{(16)} = 5_{(10)}$

$5 \bmod 16 = 5$, $5 \operatorname{div} 16 = 0$

iii). $C_{(16)} \cdot C_{(16)} + 0_{(16)} = 144_{(10)}$

$144 \bmod 16 = 0$, $144 \operatorname{div} 16 = 9$

iv). $C_{(16)} \cdot A_{(16)} + 9_{(16)} = 129_{(10)} = 128 + 1$

$129 \bmod 16 = 1$, $129 \operatorname{div} 16 = 8$

v). $C_{(16)} \cdot F_{(16)} + 8_{(16)} = 188_{(10)}$

$188 \bmod 16 = 12 = C_{(16)}$

$188 \operatorname{div} 16 = 11 = B_{(16)}$

vi). $C_{(16)} \cdot B_{(16)} + B_{(16)} = 143_{(10)}$

$143 \bmod 16 = 15$, $143 \operatorname{div} 16 = 8$

$15 = F_{(16)}$

```
188 | 16
 16 | 11
----
 28
 16
 12

 11 ·
 12
---
 22
 11
---
132 +
 11
---
143
```

Subject 1

STUDENT 2. GROZA IULIA-DIANA

S = 460661

y = 62464

p = 8FC1022

g = C

$S_{(b1)} - y_{(b1)} = 460661_{(7)} - 62464_{(7)} = 365164_{(7)} = x_{(b1)}$

b: 0 -1 -1 0 -1 -1 0

4 6 0 6 6 1 (7) −

0 6 2 4 6 4 (7)

3 6 5 1 6 4 (7)

it1: 0+1-4 = -3 < 0 ⇒ -3+7 = 4, b = -1

it2: -1+6-6 = -1 < 0 ⇒ -1+7 = 6, b = -1

it3: -1+6-4 = 1 ≥ 0 ⇒ b = 0

it4: 0+0-2 = -2 < 0 ⇒ -2+7 = 5, b = -1

it5: -1+6-6 = -1 < 0 ⇒ -1+7 = 6, b = -1

it6: -1+4-0 = 3 ≥ 0 ⇒ b = 0

$P_{(b2)} : g_{(b2)} = 8FC1054_{(16)} : C_{(16)}$

r: 0

8FC1054 (16) | C (16)

8F

BC

81

90

5

54

0 r

BFACA7

2

it1: 08 = 0*16+8 = 8

8 div 12 = 0; 8 mod 12 = 8

it2: 8F = 8*16+F = 128+15 = 143

143 div 12 = $11_{(10)}$ = B; 143 mod 12 = $11_{(10)}$ = B

it3: BC = B*16+C = 11*16+12 = 188

188 div 12 = $15_{(10)}$ = F; 188 mod 12 = 8

it4: 81 = 8*16+1 = 129

129 div 12 = $10_{(10)}$ = A; 129 mod 12 = 9

it5: 90 = 9*16+0 = 144

144 div 12 = $12_{(10)}$ = C; 144 mod 12 = 0

it6: 54 = 5*16+4 = 84

84 div 12 = 7; 84 mod 12 = 0

Subject 2

STUDENT 2. GROZA IULIA-DIANA

$b=4$

$h=7$

$X_{(b)} = 12033.301_{(4)} = 1*4^4 + 2*4^3 + 0*4^2 + 3*4^1 + 3*4^0 + 3*4^{-1} + 0*4^{-2} + 1*4^{-3}$

$1_{(4)}=1_{(7)}$; $2_{(4)}=2_{(7)}$; $0_{(4)}=0_{(7)}$; $3_{(4)}=3_{(7)}$; $4=4_{(7)}$

$1_{(7)}*4_{(7)}^4 + 2_{(7)}*4_{(7)}^3 + 0_{(7)}*4_{(7)}^2 + 3_{(7)}*4_{(7)}^1 + 3_{(7)}*4_{(7)}^0 + 3_{(7)}*4_{(7)}^{-1} + 0_{(7)}*4_{(7)}^{-2} + 1_{(7)}*4_{(7)}^{-3} = 4_{(7)}^4 + 2_{(7)}*4_{(7)}^3 + 3_{(7)}*4_{(7)} + 3_{(7)} + 3_{(7)}*4_{(7)}^{-1} + 4_{(7)}^{-3}$

c: 20
$4_{(7)}$*
$4_{(7)}$
c: 10
$22_{(7)}$*
$4_{(7)}$
c: 100
$121_{(7)}$*
$4_{(7)}$
$514_{(7)}$
c: 1000
$121_{(7)}$*
$2_{(7)}$
$242_{(7)}$
c: 1000
$3_{(7)}$*
$4_{(7)}$
$15_{(7)}$

it 1 = $4_{(7)}*4_{(7)}+0_{(7)}=16$
16 div 7 = 2; 16 mod 7 = 2
it 1: $2_{(7)}*4_{(7)}+0_{(7)}=8$
8 div 7 = 1; 8 mod 7 = 1
it 2: $2_{(7)}*4_{(7)}+1_{(7)}=9$
9 div 7 = 1; 9 mod 7 = 2
it 1: $1_{(7)}*4_{(7)}+0_{(7)}=4$
it 2: $2_{(7)}*4_{(7)}+0_{(7)}=8$
8 div 7 = 1; 8 mod 7 = 1
it 3: $1_{(7)}*4_{(7)}+1_{(7)}=5$
5 div 7 = 0; 5 mod 5 = 5

c: 1000
$514_{(7)}$ +
$242_{(7)}$
$1056_{(7)}$ +
$54_{(7)}$
$1143_{(7)}$

$12033_{(4)} = 514_{(7)} + 242_{(7)} + 15_{(7)} + 3_{(7)} = 1056_{(7)} + 21_{(7)} = 1110_{(7)}$

$1_{(7)} \mid 4_{(7)}$

$10 \mid 0.(15)_{(7)}$

30

10

it1: $1*7+0=7$

$7 \text{ div } 4 = 1$; $7 \bmod 4 = 3$

it2: $3*7+0=25$

$25 \text{ div } 4 = 5$; $25 \bmod 4 = 1$

C: 00 200

$0.151_{(7)}$ *

$3_{(7)}$

$0.513_{(7)}$

C: 00 010

$0.513_{(7)}$

$0.005_{(7)}$

$0.521_{(7)}$

$0.151_{(7)} \mid 4_{(7)}$

$1 \mid 0.030_{(7)}$

15

1

it1: $1*7+5=12$

$12 \text{ div } 4 = 3$; $12 \bmod 4 = 0$

$0.030_{(7)} \mid 4_{(7)}$

$0 \mid 0.005_{(7)}$

3

30

1

$0.301_{(4)} = 0.513_{(7)} + 0.005_{(7)} \simeq 0.521_{(7)}$

$x_{(5)} = 1203.301_{(4)} = 1110.0521_{(7)} = y_{(7)}$

$y_{(7)} = 1110.521_{(7)}$

SUBIECT 2.
STUDENT 2. GRAB ANDREI

$h = 7,\ b = 4,\ [y_h] = 1110_{(7)},\ [x_b] = 12033_{(4)},$ ~~$\{y_h\} = 301_{(7)},\ \{x_b\} = 521$~~
$\{y_h\} = 521,\ \{x_b\} = 301$

convert y_h into base b: $1110_{(7)} = ?_{(4)}$

$01110_{(7)} \mid 4_{(7)}$
0201

1
11
1
10
3

iterations:
i). $1_{(7)} = 1_{(10)}$
$1 \text{ div } 4 = 0,\ 1 \bmod 4 = 1$
ii). $11_{(7)} = 8_{(10)}$
$8 \text{ div } 4 = 2,\ 8 \bmod 4 = 0$
iii). $1_{(7)} = 1_{(10)}$
$1 \text{ div } 4 = 0,\ 1 \bmod 4 = 1$
iv). $10_{(7)} = 7_{(10)}$
$7 \text{ div } 4 = 1,\ 7 \bmod 4 = 3$

$0201_{(7)} \mid 4_{(7)}$
033

20
21
3

iterations:
i). $2_{(7)} = 2_{(10)}$
$2 \text{ div } 4 = 0,\ 2 \bmod 4 = 2$
ii). $20_{(7)} = 2 \cdot 7^1 + 0 \cdot 7^0 = 14_{(10)}$
$14 \text{ div } 4 = 3,\ 14 \bmod 4 = 2$
iii) $21_{(7)} = 15_{(10)}$
$15 \text{ div } 4 = 3,\ 15 \bmod 4 = 3$

$033_{(7)} \mid 4_{(7)}$
06

33
1
0

iterations:
i). $3_{(7)} = 3_{(10)}$
$3 \text{ div } 4 = 0,\ 3 \bmod 4 = 3$
ii). $33_{(7)} = 3 \cdot 7 + 3 \cdot 1 = 24$
$24 \text{ div } 4 = 6,\ 24 \bmod 4 = 0$

~~out.~~

$06_{(7)} \mid 4_{(7)}$
1

2

iterations:
i). $6_{(7)} = 6_{(10)}$
$6 \text{ div } 4 = 1,\ 6 \bmod 4 = 2$

$1_{(7)} \mid 4_{(7)}$
0

1

iterations:
i). $1_{(7)} = 1_{(10)}$
$1 \text{ div } 4 = 0,\ 1 \bmod 4 = 1$

we stop the repeated division when the quotient is 0

$[x_b] = 12033_{(7)}$

3 1 0 0
$0,521_{(7)} \cdot 4_{(7)}$
3014

iterations:

i). $4_{(7)} \cdot 1_{(7)} + 0_{(7)} = 4_{(10)}$
$4 \bmod 7 = 4, \quad 4 \operatorname{div} 7 = 0$

ii). $4_{(7)} \cdot 2_{(7)} + 0_{(7)} = 8_{(10)}$
$8 \bmod 7 = 1, \quad 8 \operatorname{div} 7 = 1$

iii). $4_{(7)} \cdot 5_{(7)} + 1_{(7)} = 21_{(10)}$
$21 \bmod 7 = 0, \quad 21 \operatorname{div} 7 = 3$

iv). $4_{(7)} \cdot 0_{(7)} + 3_{(7)} = 3$
$3 \bmod 7 = 3, \quad 3 \operatorname{div} 7 = 0$

$0,521_{(7)} \; 4_{(7)} = 3,014_{(7)}$

2 0
$014_{(7)} \cdot 4_{(7)}$
62

iterations:

i). $4_{(7)} \cdot 4_{(7)} + 0_{(7)} = 16_{(10)}$
$16 \bmod 7 = 2, \quad 16 \operatorname{div} 7 = 2$

ii). $4_{(7)} \cdot 1_{(7)} + 2_{(7)} = 6_{(10)}$
$6 \bmod 7 = 6, \quad 6 \operatorname{div} 7 = 0$

$3,014_{(7)} \cdot 4_{(7)} = 0,062_{(7)}$

3 1 0
$062_{(7)} \cdot 4_{(7)}$
341

iterations:

i). $4_{(7)} \cdot 2_{(7)} + 0_{(7)} = 8_{(7)}$
$8 \bmod 7 = 1, \quad 8 \operatorname{div} 7 = 1$

ii). $4_{(7)} \cdot 6_{(7)} + 1_{(7)} = 25_{(7)}$
$25 \bmod 7 = 4, \quad 25 \operatorname{div} 7 = 3$

$0,062_{(7)} \cdot 4_{(7)} = 0,341$

~~$4y4 = 300$~~

~~$\Rightarrow y4 = 120\,33,300$~~

$4 \times_6 4 = 300$

$x_6 = 120\,33,300$

SUBJECT 3. OPTION 2.
STUDENT 1. GRAB ANDREI

choosing the numbers: $x = 0.237 < y = 0.876 < z = 0.995$

$x = 0.237_{(16)} = 0.3CAC_{(2)} = 0.0011111001010110$

$0.237 \cdot 16 = 3.792$
$0.792 \cdot 16 = 12.672$
$0.672 \cdot 16 = 10.752$
$0.752 \cdot 16 = 12.032$

237 · 16 = 1422 + 237 = 3792
792 · 16 = 4752 + 792 = 12672
672 · 16 = 4032 + 672 = 10752
752 · 16 = 4512 + 752 = 12032

$[X]_D = [X]_i = [X]_c = \overset{s}{0}|0011111001010110$ (on 16 bits)

$[-X]_D = \overset{s}{1}|0011111001010110$
$[-X]_i = \overset{s}{1}|1100000110101001$
$[-X]_c = \overset{s}{1}|1100000110101010$

$y = 0.876_{(16)} = 0.E041_{(2)} = 0.1110000001000001$

$0.876 \cdot 16 = 14.016$
$0.016 \cdot 16 = 0.256$
$0.256 \cdot 16 = 4.096$
$0.096 \cdot 16 = 1536$

876 · 16 = 5256 + 876 = 14016
16 · 16 = 96 + 16 = 256
256 · 16 = 1536 + 256 = 4096
96 · 16 = 576 + 96 = 1536

$[y]_D = [y]_i = [y]_c = \overset{s}{0}|1110000001000000$

$[-y]_D = \overset{s}{1}|1110000001000000$
$[-y]_i = \overset{s}{1}|0001111110111111$
$[-y]_c = \overset{s}{1}|0001111111000000$

$z = 0.995_{(16)} = 0.FEA8_{(2)} = 0.1111111010111000$

$0.995 \cdot 16 = 15.920$
$0.920 \cdot 16 = 14.720$
~~$0.520 \cdot 16 = 8.320$~~
~~$0.320 \cdot 16 = 5.120$~~
$0.720 \cdot 16 = 11.520$
$0.520 \cdot 16 = 8.320$

995 · 16 = 5970 + 995 = 15920
920 · 16 = 5520 + 920 = 14720
520 · 16 = 3120 + 520 = 8320
320 · 16 = 1920 + 320 = 5120
720 · 16 = 4320 + 720 = 11520

$[z]_D = [z]_i = [z]_c = \overset{S}{0}|1111111010111100$

$[z]_D = \overset{S}{1}|1111111010111100$

$[-z]_i = \overset{S}{1}|0000000101000011$

$[-z]_c = \overset{S}{1}|0000000101000100$

Subject 3. Option 2

STUDENT 2. GROZA IULIA-DIANA

$[x]_{compl} = [0.237]_{compl} = 0^S|001\ 1110\ 0101\ 0110$

$[-x]_{compl} = [-0.237]_{compl} = 1^S|110\ 0001\ 1010\ 1010$

$[y]_{compl} = [0.876]_{compl} = 0^S|111\ 0000\ 0010\ 0000$

$[-y]_{compl} = [-0.876]_{compl} = 1^S|000\ 1111\ 1110\ 0000$

$[z]_{compl} = [0.995]_{compl} = 0^S|111\ 1111\ 0101\ 1100$

$[-z]_{compl} = [-0.995]_{compl} = 1^S|000\ 0000\ 1010\ 0100$

$[x+y]_{compl} = [x]_{compl} \oplus$ c: 0 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0

$0^S|001\ 1110\ 0101\ 0110 \oplus$

$[y]_{compl}$ $0^S|111\ 0000\ 0010\ 0000$

$1^S|000\ 1110\ 0111\ 0110$

-overflow (operands are positive, result is negative)

$[x-y]_{compl} = [x]_{compl} \oplus$ c: 0 0 0 1 1 1 1 1 1 0 0 0 0 0 0 0

$0^S|001\ 1110\ 0101\ 0110 \oplus$

$[-y]_{compl}$ $1^S|000\ 1111\ 1110\ 0000$

$[-0.639]_{compl}$ $1^S|010\ 1110\ 0011\ 0110$

-correct result

$[z-x]_{compl} = [z]_{compl} \oplus$ c: 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 0

$0^S|111\ 1111\ 0101\ 1100 \oplus$

$[-x]_{compl}$ $1^S|110\ 0001\ 1010\ 1010$

$[0.758]_{compl} * 0^S|110\ 0001\ 0000\ 0110$

-correct result (F2)

$[-z-x]_{compl} = [-z]_{compl} \oplus$ c: 1 0 0 0 0 0 0 0 1 1 0 1 0 1 0 0 0

$1^S|000\ 0000\ 1010\ 0100 \oplus$

$[-x]_{compl}$ $1^S|110\ 0001\ 1010\ 1010$

$* 0^S|110\ 0010\ 0100\ 1110$

-overflow (operands are positive, result is negative)

$[x+y]_{compl} = 1|000\ 1110\ 0111\ 0110$ cannot be converted to decimal, because $x \oplus y$ resulted an overflow

$[x-y]_{compl} = 1|0101\ 1100\ 0110\ 110\Gamma$ (negative)

$0.1010\ 0011\ 1001\ 001\ 1 +$

$0.1010\ 0011\ 1001\ 0100$

A 3 9 4

$0.1010\ 0011\ 1001\ 010_{(2)} = 0.A394_{(16)} = 0 * 16^{0}_{(10)} + A * 16^{-1} + 3 * 16^{-2} + 9 * 16^{-3} + 4 * 16^{-4}$

$0_{(16)} = 0_{(10)}$; $A_{(16)} = 10_{(10)}$; $3_{(16)} = 3_{(10)}$; $9_{(16)} = 9_{(10)}$; $4_{(16)} = 4_{(10)}$

$16 = 16_{(10)}$

$\Rightarrow 0.A394_{(16)} = 0_{(10)} * 16^{0}_{(10)} + 10_{(10)} * 16^{-1}_{(10)} + 3_{(10)} * 16^{-2}_{(10)} + 9_{(10)} * 16^{-3}_{(10)} + 4_{(10)} * 16^{-4}_{(10)} = 0 + 10/16 + 3/16^2 + 9/16^3 + 4/16^4 =$

$= 0 + 0.625 + 0.011 + 0.002 + 0.000 = 0.6389 \simeq 0.639$

$[x-y]_{compl} < 0 \Rightarrow [-0.639]_{compl}$

$[z-x]_{compl} = 0|110\ 0001\ 0000\ 0110$ (positive)

$0.1100\ 0010\ 0000\ 110\Gamma$

C 2 0 C

$0.1100\ 0010\ 0000\ 0110_{(2)} = 0.C20C_{(16)} = 0 * 16^{0} + C * 16^{-1} + 2 * 16^{-2} + 0 * 16^{-3} + C * 16^{-4}$

$0_{(16)} = 0_{(10)}$; $C_{(16)} = 12_{(10)}$; $2_{(16)} = 2_{(10)}$; $16 = 16_{(10)}$

$0.C20C_{(16)} = 12_{(10)} * 16^{-1}_{(10)} + 2_{(10)} * 16^{-2}_{(10)} + 12_{(10)} * 16^{-4}_{(10)} =$

$= 12/16 + 2/16^2 + 12/16^4 = 0.750 + 0.007 + 0.001 = 0.756 \simeq 0.758$

$[z-x]_{compl} > 0 \Rightarrow [0.758]_{compl}$

$[-z-x]_{compl} = 0\,|\,110\ 0010\ 0100\ 1110$ cannot be converted to decimal, because $x \oplus y$ resulted an overflow