Empirical Evaluation of Regularized Covariance Estimators in High-Dimensional Portfolio Optimization

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Abstract

This paper investigates the out-of-sample performance of various covariance matrix estimators in the context of Global Minimum Variance (GMV) portfolios. Using 100 Fama-French portfolios from 1990 to 2025, we evaluate the Sample Covariance Matrix (SCM) against Ledoit-Wolf Shrinkage and a 3-Factor structural model. While the implementation of long-only constraints significantly mitigates the estimation error inherent in the SCM, our results indicate that Ledoit-Wolf Shrinkage provides superior operational stability. Specifically, the shrinkage approach reduced quarterly portfolio turnover by approximately 2% compared to the SCM. These findings suggest that while constraints provide a safety net for risk, regularized estimators remain essential for reducing transaction costs and improving the numerical stability of large-scale asset allocation.

1 Introduction

Modern Portfolio Theory (MPT), as introduced by Markowitz (1952), relies heavily on the accurate estimation of the covariance matrix Σ. However, in high-dimensional settings where the number of assets N is large relative to the number of observations T, the Sample Covariance Matrix (SCM) becomes an unreliable estimator. This "curse of dimensionality" leads to extreme eigenvalues, causing optimization algorithms to assign erratic weights to assets based on noise rather than signal.

This paper provides an empirical comparison of three covariance estimation techniques: the classical SCM, the Ledoit-Wolf shrinkage estimator, and a structural factor model based on the Fama-French three-factor framework. We test these estimators using a Global Minimum Variance (GMV) strategy applied to the Fama-French 100 Portfolios, evaluating them based on realized volatility, maximum drawdown, and portfolio turnover.

2 Methodology

The primary challenge in portfolio optimization is the accurate estimation of Σ. We define R_t as the $N \times 1$ vector of excess returns at time t.

2.1 The Global Minimum Variance Portfolio

The GMV portfolio is the optimal testbed for covariance estimators because it minimizes risk without requiring estimates of expected returns. The optimization problem is defined as:

$$\min_{w} \sigma_p^2 = w^T \Sigma w \tag{1}$$

Subject to:

- **Budget Constraint:** $\sum_{i=1}^{N} w_i = 1$
- **Long-Only Constraint:** $w_i \geq 0 \quad \forall i$

2.2 Estimator Specifications

2.2.1 Sample Covariance Matrix (SCM)

The SCM is the maximum likelihood estimator under the assumption of normality:

$$\hat{\Sigma}_{SCM} = \frac{1}{T-1} \sum_{t=1}^{T} (R_t - \bar{R})(R_t - \bar{R})^T \tag{2}$$

2.2.2 Ledoit-Wolf Linear Shrinkage

The Ledoit-Wolf (2004) estimator shrinks the SCM towards a target matrix F (the scaled identity matrix) to reduce the estimation error:

$$\hat{\Sigma}_{LW} = \delta F + (1 - \delta)\hat{\Sigma}_{SCM} \quad (3)$$

where $\delta \in [0, 1]$ is the shrinkage intensity determined by a quadratic loss function.

2.2.3 Factor-Based Structural Model

Utilizing the Fama-French 3-Factor model, we assume returns are driven by common factors f_t:

$$R_t = \alpha + \beta f_t + \epsilon_t \quad (4)$$

The covariance matrix is reconstructed as:

$$\hat{\Sigma}_{Factor} = \beta \Sigma_F \beta^T + \Delta \quad (5)$$

where Σ_F is the factor covariance matrix and Δ is the diagonal matrix of residual variances.

2.3 Backtesting Protocol

We employ a rolling-window approach:

- **Estimation Window:** 60 months.
- **Rebalancing Frequency:** Quarterly (3 months).
- **Testing Period:** 1990 – 2025.

3 Data and Empirical Results

3.1 Data Description

The dataset comprises the Fama-French 100 Portfolios, sorted by Size and Book-to-Market ratio, spanning from January 1990 to the present. For the factor-based estimator, the Fama-French 3-Factor model (Market, SMB, and HML) is utilized. The high dimensionality ($N = 100$) relative to the estimation window ($T = 60$) creates a challenging environment for the standard Sample Covariance Matrix.

3.2 Risk and Performance Metrics

Table 1 summarizes the out-of-sample performance of the Global Minimum Variance (GMV) portfolio under the three competing estimators.

Table 1: Out-of-Sample Performance Summary (Quarterly Rebalancing)

Estimator	**Ann. Volatility (%)**	**Sharpe Ratio**	**Max Drawdown (%)**
SCM	14.09	1.02	-26.37
Shrinkage (LW)	14.04	1.02	-26.36
Factor Model	14.13	1.01	-27.34

The results indicate that realized volatility is remarkably consistent across all three models. This convergence is attributed to the long-only constraint ($w \geq 0$), which acts as a robust regularizer, effectively "capping" the estimation error of the SCM.

3.3 Portfolio Stability and Turnover

While risk metrics are similar, the operational efficiency of the estimators differs significantly. Table 2 presents the average quarterly turnover, representing the stability of the portfolio weights.

Table 2: Average Quarterly Portfolio Turnover

Estimator	**Average Quarterly Turnover (%)**
SCM	30.27
Shrinkage (LW)	28.36
Factor Model	30.58

The **Ledoit-Wolf Shrinkage** estimator achieved the highest stability, reducing turnover by approximately 2% per quarter compared to the SCM and Factor models. Over a long-term investment horizon, this reduction implies significantly lower transaction costs.

4 Conclusion

This study evaluated the performance of advanced covariance estimators in a high-dimensional equity universe. Our findings confirm that while long-only constraints provide a "natural" floor for risk reduction, the choice of estimator remains crucial for portfolio stability. The Ledoit-Wolf shrinkage method provides the most efficient balance between risk reduction and operational stability, making it the most suitable choice for large-scale, constrained portfolio management.