Benchmark Optimization Functions Using Genetic Algorithms

Student Assignment Report

June 5, 2025

1 Introduction

This report presents a comprehensive analysis of Genetic Algorithm (GA) performance on benchmark optimization functions. We implemented and tested multiple GA configurations on two multimodal functions to evaluate their optimization capabilities and conduct statistical performance comparisons.

2 Benchmark Functions

We selected two widely used multimodal benchmark functions for optimization:

2.1 Function Definitions

Rastrigin Function:

$$f_1 : [-5.12, 5.12] \times [-5.12, 5.12] \to R \tag{1}$$

$$f_1(x, y) = 20 + (x^2 - 10\cos(2\pi x)) + (y^2 - 10\cos(2\pi y)) \tag{2}$$

The Rastrigin function is a non-convex function with many local minima, commonly used to evaluate global optimization algorithms. Its global minimum is at $(0, 0)$ with a value of 0.

Ackley Function:

$$f_2 : [-32.768, 32.768] \times [-32.768, 32.768] \to R \tag{3}$$

$$f_2(x, y) = -20\exp\left(-0.2\sqrt{0.5(x^2 + y^2)}\right) - \exp\left(0.5(\cos(2\pi x) + \cos(2\pi y)\right) \tag{4}$$

The Ackley function is a multimodal function characterized by a flat outer region and a central peak. The global minimum is also at $(0, 0)$ with a value of 0.

3 Genetic Algorithm Configuration

We implemented a configurable Genetic Algorithm with the following specifications:

3.1 Algorithm Parameters

- **Population Size**: 100
- **Number of Generations**: 100 (equivalent to 10,000 fitness evaluations)
- **Mutation Rate**: 0.1
- **Crossover Rate**: 0.8
- **Selection Method**: Tournament selection (size = 3)
- **Number of Independent Runs**: 30 per configuration

3.2 Encoding and Crossover Configurations

We tested four different GA configurations:

Binary Encoding:

- **1-Point Crossover**: Single crossover point divides parent chromosomes
- **2-Point Crossover**: Two crossover points create three segments for exchange
- **Bit String Length**: 16 bits per variable (32 bits total)

Real-Valued Encoding:

- **Arithmetic Crossover**: Generates offspring by linearly combining parent genes: $child_1 = \alpha \cdot parent_1 + (1 - \alpha) \cdot parent_2$
- **BLX-α Crossover**: Generates offspring by sampling from an extended interval around the parents with $\alpha = 0.5$

4 Experimental Results

4.1 Rastrigin Function Results

Table 1 presents the summary statistics for all GA configurations on the Rastrigin function over 30 independent runs.

Table 1: Summary Statistics for Rastrigin Function Optimization

Configuration	Count	Mean	Std Dev	Min	Max	Median
Binary 1-Point	30	0.061265	0.056677	0.001581	0.278085	0.050812
Binary 2-Point	30	0.081691	0.098293	0.000041	0.362103	0.030296
Real Arithmetic	30	0.287085	0.444257	0.000000	1.087601	0.000005
Real BLX-α	30	0.176351	0.349764	0.000000	0.994959	0.000000

4.2 Statistical Significance Analysis

We performed comprehensive statistical testing to determine if performance differences between configurations are statistically significant. The results are presented in Table 2.

Table 2: Statistical Test Results for Rastrigin Function (p-values)

Test Type	Configuration Comparisons			
	Binary 1P vs 2P	Binary vs Real Arith	Binary vs Real BLX	Real Arith vs BLX
Mann-Whitney U	0.807	0.137/0.096	0.016/0.010	0.011
Independent t-test	0.328	0.008/0.016	0.080/0.159	0.288
Wilcoxon Rank-sum	0.802	0.135/0.095	0.017/0.011	0.012
ANOVA Test Results: F-statistic: 3.851, p-value: 0.011				

4.3 Performance Analysis

Key Findings for Rastrigin Function:

- **Best Overall Performance**: Binary 1-Point crossover achieved the most consistent results with the lowest mean fitness (0.061) and standard deviation (0.057)
- **Best Single Solution**: Real-valued encodings (both arithmetic and BLX-α) found the global optimum (0.000000) multiple times
- **Statistical Significance**: ANOVA test reveals significant differences between configurations (p = 0.011)
- **Significant Pairwise Differences**:
  - Real BLX-α vs all other configurations (Mann-Whitney U test, p ¡ 0.05)
  - Binary configurations vs Real Arithmetic (t-test, p ¡ 0.05)

5 Discussion

5.1 Encoding Comparison

The results reveal interesting performance differences between binary and real-valued encodings:

Binary Encoding Advantages:

- More consistent performance with lower variance

- Better average convergence on the Rastrigin function
- Less susceptible to premature convergence

Real-Valued Encoding Advantages:

- Capable of finding exact global optima
- More direct representation of the search space
- Potential for faster convergence when successful

5.2 Crossover Strategy Impact

The choice of crossover operator significantly affects performance:

- **1-Point vs 2-Point (Binary)**: No statistically significant difference, but 1-point showed slightly more consistent results
- **Arithmetic vs BLX-α (Real)**: BLX-α showed better exploration capabilities but higher variance

5.3 Function Characteristics Impact

The Rastrigin function's highly multimodal landscape with many local optima particularly challenges optimization algorithms. The results suggest that:

- Binary encodings may provide better balance between exploration and exploitation
- Real-valued encodings can achieve perfect solutions but with lower reliability
- The choice of representation significantly impacts algorithm robustness

6 Experimental Methodology

6.1 Fair Comparison Protocol

To ensure fair comparison across all configurations:

- Fixed number of fitness evaluations: 10,000 per run
- Identical random seed handling across runs
- Same selection pressure (tournament size = 3)
- Consistent mutation and crossover rates

6.2 Statistical Analysis Protocol

We employed multiple statistical tests to ensure robust conclusions:

- **ANOVA**: Overall significance testing across all configurations
- **Mann-Whitney U**: Non-parametric pairwise comparisons
- **Independent t-test**: Parametric pairwise comparisons
- **Wilcoxon Rank-sum**: Alternative non-parametric test

7 Conclusions

Based on our comprehensive experimental analysis:

1. **Configuration Performance**: Binary encoding with 1-point crossover provided the most reliable performance on the Rastrigin function, achieving consistent near-optimal solutions.
2. **Statistical Significance**: Significant performance differences exist between GA configurations (ANOVA $p = 0.011$), particularly between real BLX-α and other methods.
3. **Encoding Trade-offs**: While real-valued encodings can find exact global optima, binary encodings offer more robust and consistent performance across multiple runs.
4. **Algorithm Design Implications**: The choice of encoding and crossover strategy should consider the balance between solution quality and algorithm reliability.
5. **Function-Specific Behavior**: The highly multimodal nature of the Rastrigin function favors algorithms with good exploration-exploitation balance, which binary encodings seemed to provide better.

8 References

1. Rastrigin, L. A. (1974). *Systems of Extremal Control*. Mir Publishers.
2. Ackley, D. H. (1987). *A Connectionist Machine for Genetic Hillclimbing*. Springer.
3. Goldberg, D. E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*. Addison-Wesley.
4. Deb, K. (2001). *Multi-Objective Optimization Using Evolutionary Algorithms*. John Wiley & Sons.