

## Vagueness

I chose the following rules, which define a method to estimate a hotel's review score based on two ratings: one for cleanliness and one for location.

Rules:

If cleanliness is poor or location is bad, then the review is low.

If cleanliness is decent, then the review is moderate.

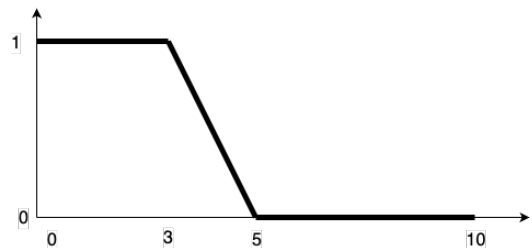
If cleanliness is excellent or location is good, then the review is high.

Questions: Rate the cleanliness and location on a 10-point scale.

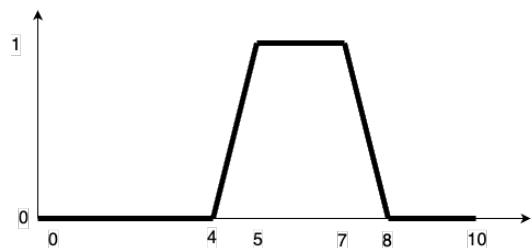
Since the predicates are vague, they correspond to functions that define their interpretation. Cleanliness and location are described on a scale from 0 to 10, while the review score is represented as a value between 1 and 5. For each predicate, I defined a degree curve.

To describe cleanliness, there are 3 predicates: *poor*, *decent*, and *excellent*, defined as:

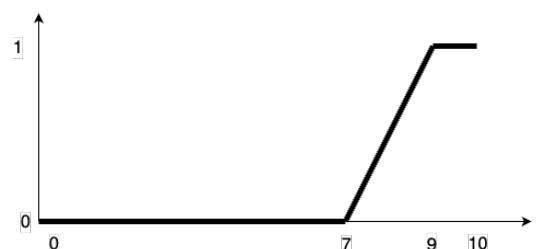
$$\text{poor}(x) = \begin{cases} 1, & 0 \leq x \leq 3 \\ \frac{5-x}{2}, & 3 < x \leq 5 \\ 0, & 5 < x \leq 10 \end{cases}$$



$$\text{decent}(x) = \begin{cases} 0, & x \leq 4 \\ x-4, & 4 < x \leq 5 \\ 1, & 5 < x \leq 7 \\ 8-x, & 7 < x \leq 8 \\ 0, & 8 < x \leq 10 \end{cases}$$

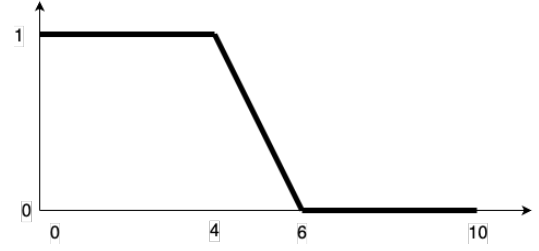


$$\text{excellent}(x) = \begin{cases} 0, & x \leq 7 \\ \frac{x-7}{2}, & 7 < x \leq 9 \\ 1, & 9 < x \leq 10 \end{cases}$$

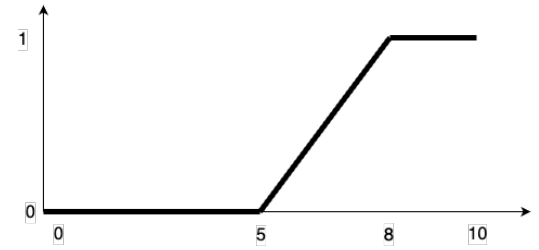


Location has 2 predicates, *good* and *bad*, which correspond to the following functions:

$$\text{bad}(x) = \begin{cases} 1, & 0 \leq x \leq 4 \\ \frac{6-x}{2}, & 4 < x \leq 6 \\ 0, & 6 < x \leq 10 \end{cases}$$

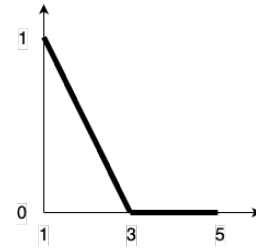


$$\text{good}(x) = \begin{cases} 0, & x \leq 5 \\ \frac{x-5}{3}, & 5 < x \leq 8 \\ 1, & 8 < x \leq 10 \end{cases}$$

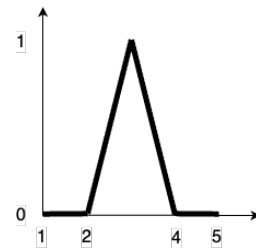


Finally, the review scores can be classified as low, moderate, or high, defined as:

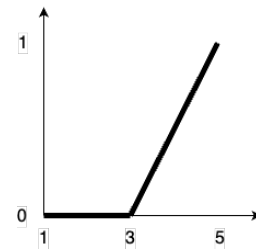
$$\text{low}(x) = \begin{cases} \frac{3-x}{2}, & 1 \leq x < 3 \\ 0, & x \geq 3 \end{cases}$$



$$\text{moderate}(x) = \begin{cases} 0, & 1 \leq x < 2 \\ x-2, & 2 \leq x \leq 3 \\ 4-x, & 3 < x \leq 4 \\ 0, & 4 < x \leq 5 \end{cases}$$

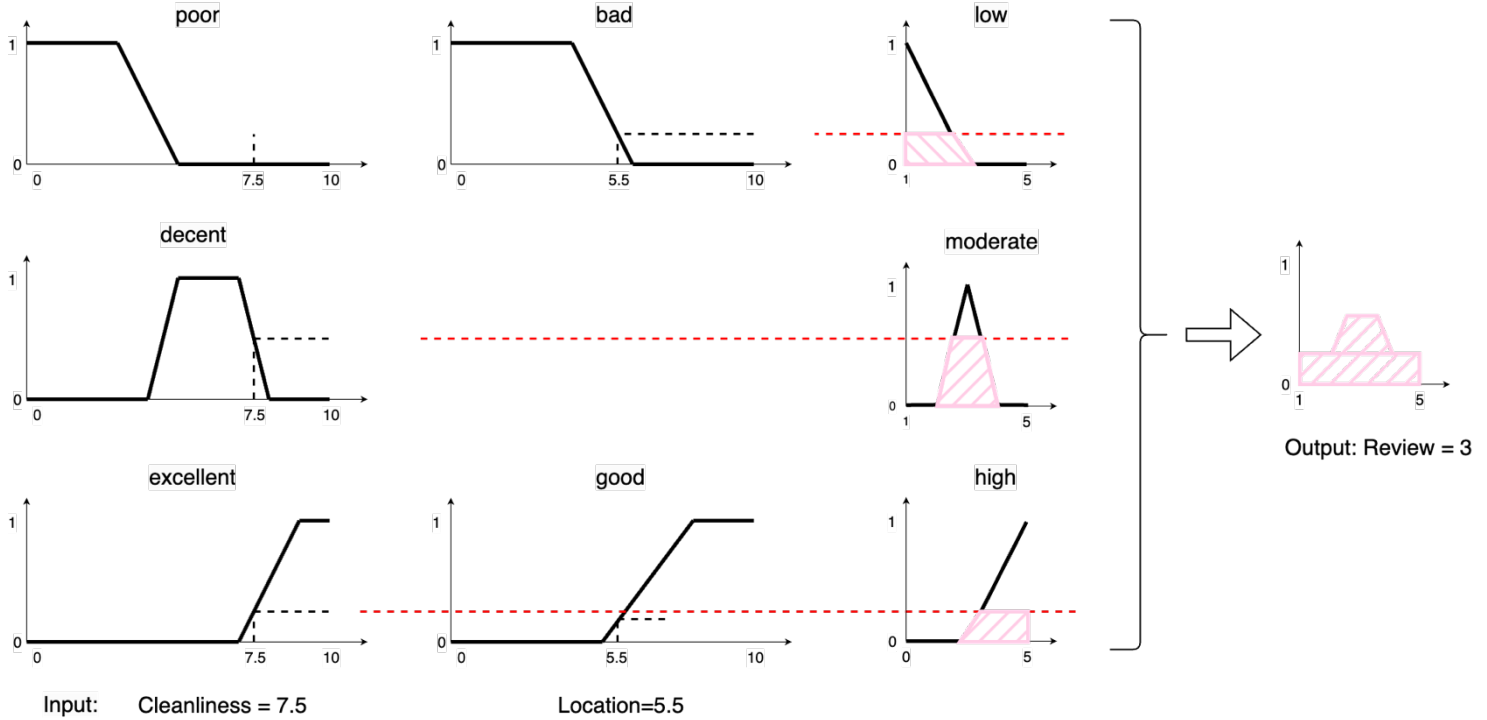


$$\text{high}(x) = \begin{cases} 0, & 1 \leq x < 3 \\ \frac{x-3}{2}, & 3 \leq x \leq 5 \end{cases}$$



Based on the input, the degrees of each predicate are calculated and combined at the rule level. The consequents are then aggregated into a single degree curve for the review score, which is used to generate the output. For example, with scores of 7.5 for *cleanliness* and 5.5 for *location*, the review score is computed as:

At the final step, the aggregated degree curve is discrete (computed only for the inputs 1,2,...5)



by sampling each point in the output domain and taking the maximum across all review functions.

Defuzzification is then carried out by summing  $x \times \mu_{agg}(x)$  over all sampled points and dividing by the sum of  $\mu_{agg}(x)$ . For example, we obtain the following discrete points for the variable review:

$$(1, 0.25), (2, 0.25), (3, 0.5), (4, 0.25), (5, 0.25)$$

So the final estimation of the review is:

$$\text{Review} = \frac{1 \times 0.25 + 2 \times 0.25 + 3 \times 0.5 + 4 \times 0.25 + 5 \times 0.25}{0.25 + 0.25 + 0.5 + 0.25 + 0.25} = 3$$