MTH 161 A1 Exam 2

University of Miami

Spring semester, 2025

Name: ______________________________

Points Distribution

Question:	1	2	3	4	5	6	7	Total
Points:	24	12	12	12	12	12	16	100
Score:								

Instructions:

1. You have **75 minutes** to complete the examination.
2. Write all your work and answers in this booklet.
3. **NO CALCULATORS ARE ALLOWED ON THIS EXAM.**
4. Please sign the Honor Code statement:

Honor Code, I certify that I have neither given nor received any aid on this examination.

Signature: ______________________________

Good luck!

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1. (24 points) Find derivatives.

   (a) $f(x) = \sin(\tan(1 - x))$

   (b) $g(x) = \dfrac{\cos(x)}{4x^3 + 3}$

   (c) $h(x) = \csc\left(x^2 \cdot \sin(x)\right)$

2. (12 points) Let

$$f(x) = (x-4)^{1/3} \cdot (2x+1)^2.$$

Find the **two** x-values such that the tangent line to the graph of f at x is horizontal.

Hint: To compare an expression with zero, you should fully factor the expression.

3. (12 points) Suppose an object is moving along a straight line. The position function of the car with respect to time is given by

$$s(t) = \frac{2}{3}x^3 - \frac{1}{2}x^2 - 6x + 4.$$

We assume the domain of function in this question is $t \in [0, \infty)$.

(a) Find the velocity function $v(t)$ and acceleration function $a(t)$ of the object.

$v(t) =$ ____________________, $a(t) =$ ____________________.

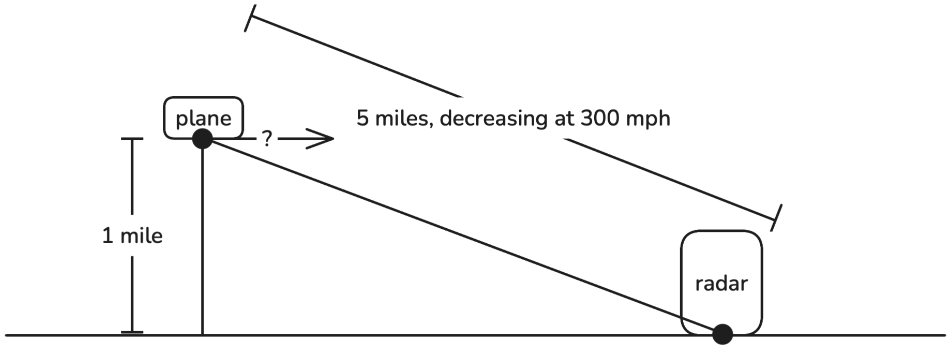
(b) An object is said to be <u>speeding up</u> if its velocity and acceleration have the same sign (i.e. both positive or both negative). Otherwise, the object is said to be <u>slowing down</u>.
Find the time interval(s) where the object is speeding up/slowing down.

(c) Multiple choice: select **all** that apply. In general, over certain period of time,

- ○ displacement of an object can be negative.
- ○ distance traveled of an object can be negative.
- ○ displacement of an object can be larger than its distance traveled.
- ○ distance traveled of an object can be larger than its displacement.

4. (12 points) Suppose y depends on x, find y' (or $\dfrac{dy}{dx}$) if

$$\tan(\sqrt{y}) = 1 + x^2y^2.$$



5. (12 points) A plane is flying at an altitude of 1 mile above ground on a flight path that will take it directly over a radar station. If the diagonal distance between the plane and the station is decreasing at a rate of 300 miles per hour when it is 5 miles, how fast is the plane traveling?

6. (12 points) Recall that you can find **absolute extremums** of a function defined on an closed interval by comparing the values at critical points.

   Find the critical points of

   $$f(x) = \frac{x^4}{4} + \frac{x^3}{3} - x^2,\ x \in [-2, 3],$$

   and find the absolute minimum and maximum points.

7. (16 points) Let $f(x) = x^{2/3}\left(\frac{5}{2} - x\right)$ so that $f'(x) = \frac{5(1-x)}{3x^{1/3}}$ and $f''(x) = \frac{5(-2x-1)}{9x^{4/3}}$. Answer the following questions.

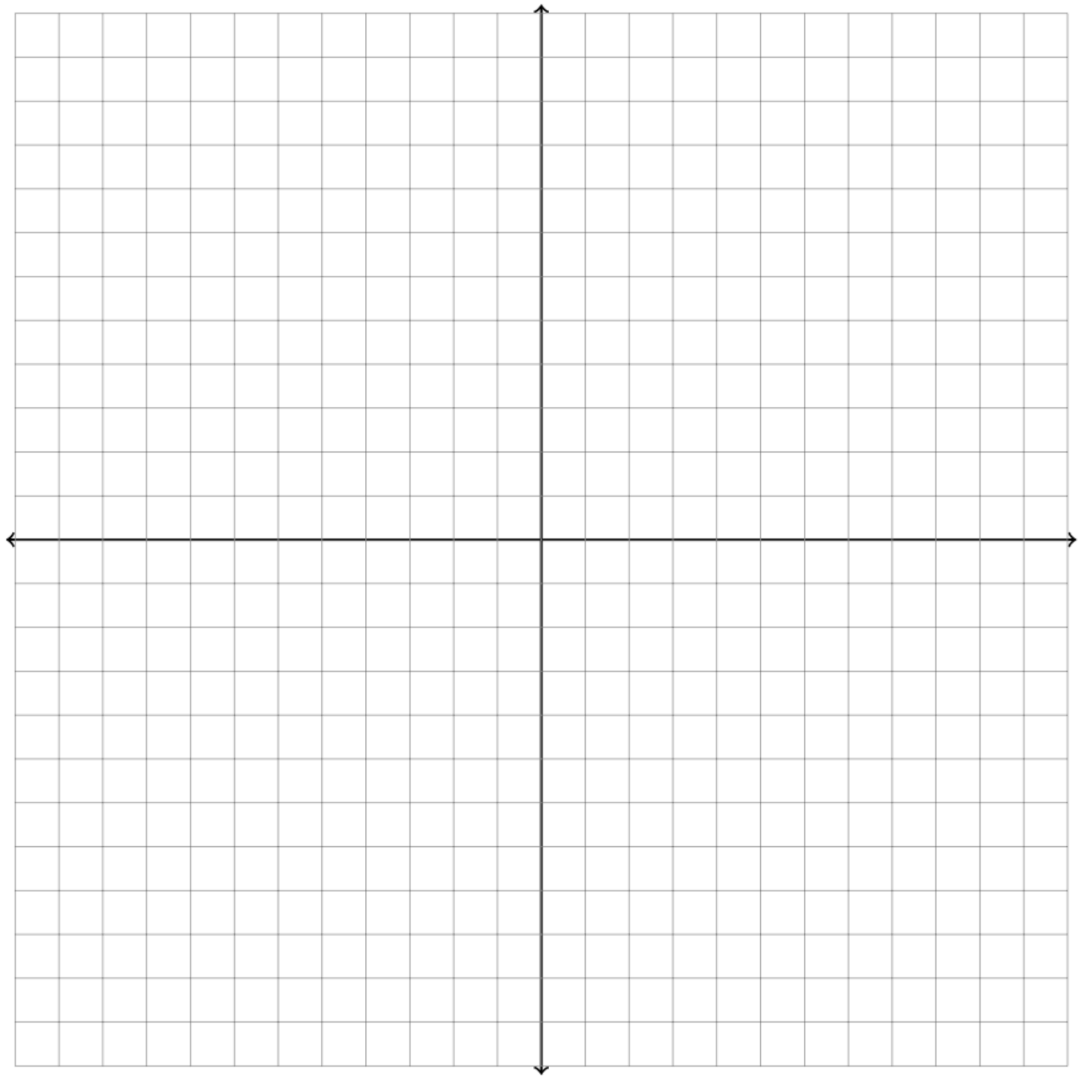
(a) What is the domain of f?

(b) What are the x and y-intercept(s) of f? **Write your answer in (x, y)-coordinates.**

(c) Find the interval(s) of increasing or decreasing of f.

(d) Find the interval(s) of concaving upward or downward of f.

(e) Find local maximum(s) and minimum(s) of f, if any. **Write your answer in** (x, y)**-coordinates.**



(g) Plot the graph of f by connecting the points it passes you calculated from above and using the increasing/decreasing/concavity information. You shall decide the size of the plot, **but you should label the plot properly.**

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