MTH 161 A1 Exam 3

University of Miami

Spring semester, 2025

Name: ______________________________

Points Distribution

Question:	1	2	3	4	5	6	7	8	Total
Points:	12	10	10	10	16	24	8	10	100
Score:									

Instructions:

1. You have **75 minutes** to complete the examination.
2. Write all your work and answers in this booklet.
3. No calculators are allowed on this exam.
4. Please sign the Honor Code statement:

Honor Code, I certify that I have neither given nor received any aid on this examination.

Signature: ______________________________

Good luck!
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1. (12 points)

   (a) (3 points) Write down the distance function between two points (x_0, y_0) and (x_1, y_1) on the xy-plane.

   $D =$ ______________________________

   (b) (1 points) To minimize or maximize D, it suffices to minimize or maximize D^2. Briefly explain why.

   ____

   (c) (8 points) Use part (a) and (b), find the point(s) on the curve

   $$y = x^2 - 13$$

   that are **closest** to the point $\big(0,\ 3.5\big)$. It suffices to give either x or y coordinate(s).

2. (10 points) Suppose an object is moving along a straight line. The acceleration function of the car with respect to time is given by

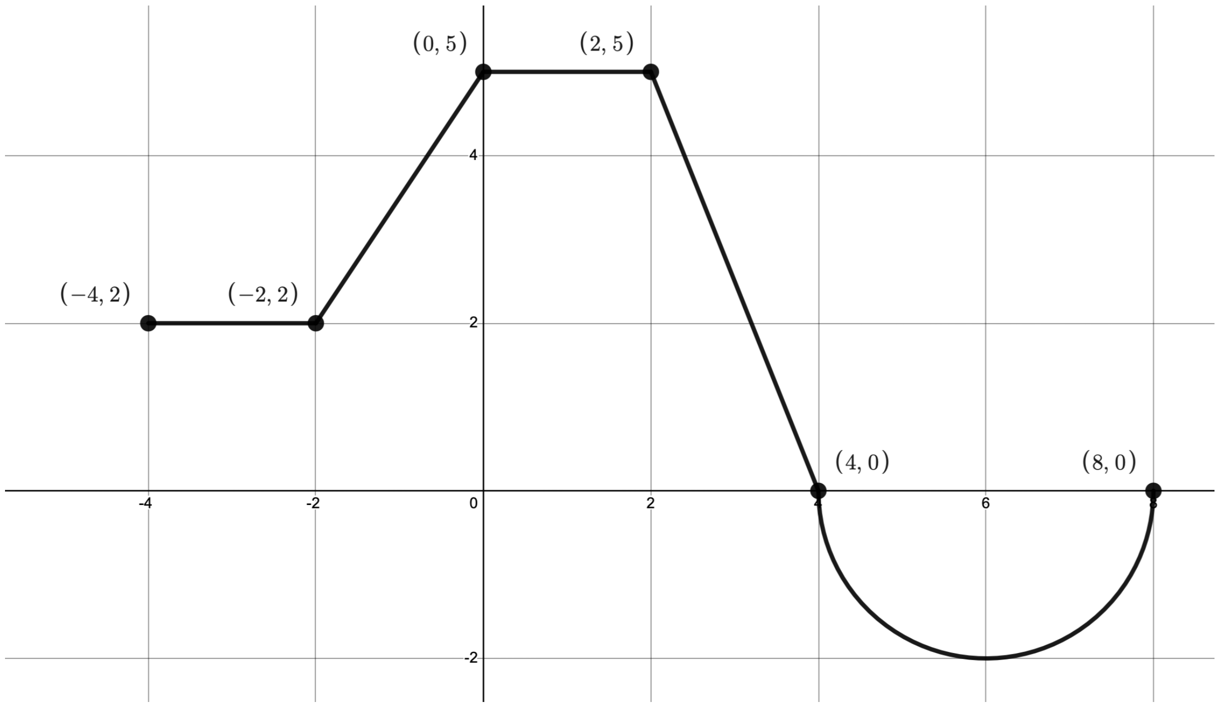
$$a(t) = 4t - 3.$$

You can assume the domain of function in this question to be $t \in [0, \infty)$.

(a) (4 points) Find the velocity function $v(t)$ with the initial condition $v(0) = -9$.

(b) (6 points) Write down the expression that calculate the **distance** travelled by the object during $[0, 6]$ **using definite integral**. You do not need to evaluate the expression, but you need to express it so that **no absolute value function appears**.

Reminder: you don't have to evaluate the expression!



3. (10 points) The graph of f defined on $[-4, 8]$ is given above. Define

$$g(x) = \int_2^x f(t)dt.$$

(a) (2 points) Find $g(6)$.

Answer: ______________________

(b) (2 points) Find $g(-2)$.

Answer: ______________________

(c) (3 points) On which interval(s) is g increasing?

Answer: ______________________

(d) (3 points) On which interval(s) is g' increasing?

Answer: ______________________

4. (10 points) Use **Fundamental Theorem of Calculus** to evaluate the $\boxed{\textbf{derivative}}$ of the function

$$y = \int_{x^3}^{-x} \sin(\sqrt[3]{t}) + t \; dt$$

5. (16 points) Evaluate the following definite integrals.

(a) (8 points) $\int_0^3 x\sqrt{3-x}\,dx$, it's okay to stop at the last evaluating step (i.e. it suffices to write down the antiderivative and the corresponding upper and lower bounds).

(b) (8 points) $\int_0^{\pi/4} \frac{1-\cos^2(\theta)}{\cos^2(\theta)}\,d\theta$

6. (24 points) Evaluate the following indefinite integrals.

(a) (8 points) $\int \csc^2(x) \cdot \cot^7(x)\, dx$

(b) (8 points) $\int \frac{\cos(\pi/x)}{x^2}\, dx$

(c) (8 points) $\displaystyle\int \left(x + \frac{1}{x}\right)^2 \, dx$

7. (8 points) Let $f(x) = (x-3)^{-2}$. Is there a value c on $(1,4)$ that satisfies

$$f'(c) = \frac{f(4) - f(1)}{4 - 1} \quad ?$$

If your answer is no, why does this not contradict the Mean Value Theorem?

8. (10 points) We have learnt quite a few methods to evaluate an integral. One interesting result involves both geometry and algebra.

   Answer 2 short questions and use them to evaluate an integral.

   (a) (2 points) What does it mean for a function f to be an **odd** function? Examples of odd function: $\sin(x),\ x^3,\ \cos(5x) \cdot x^5$.

   A function is odd if its graph is symmetric about ______________________.

   (b) (2 points) If f is an odd function and integrable, then $\displaystyle\int_{-a}^{a} f(x)\, dx =$ ____.

   (c) (6 points) Evaluate

   $$\int_{-5}^{5} x^{2025} \cdot \cos(3x) + |x - 3|\, dx$$

   using part (a) and (b).

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