MTH 161 A1 Exam 1

University of Miami

Spring semester, 2025

Name: _____

Points Distribution

Question:	1	2	3	4	5	6	Total
Points:	10	48	10	12	10	10	100
Score:							

Instructions:

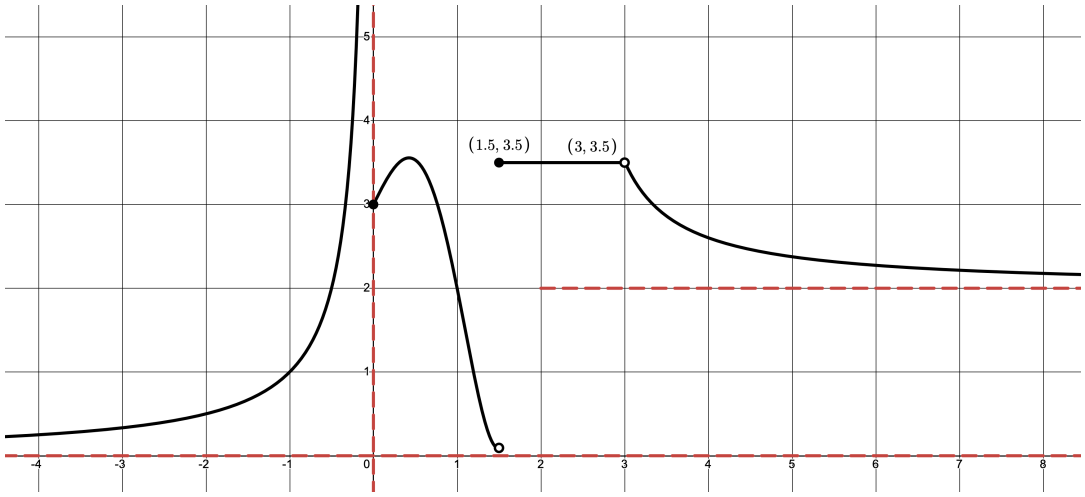
1. You have **75 minutes** to complete the examination.
2. Write all your work and answers in this booklet.
3. **NO CALCULATORS ARE ALLOWED ON THIS EXAM.**
4. Please sign the Honor Code statement:

Honor Code, I certify that I have neither given nor received any aid on this examination.
Signature: _____

Good luck!

The page has been left intentionally blank.

1. (10 points) The graph of a function f has been given, where the dotted lines indicate the asymptotes. Answer the following questions.



(a) $\lim_{x \to 0} f(x) =$ _____

(b) $f(0) =$ _____

(c) $\lim_{x \to 1.5^+} f(x) =$ _____

(d) $\lim_{x \to 3} f(x) =$ _____

(e) $f(3) =$ _____

2. (48 points) Evaluate the following limits. Show all work that lead to your answer. Using L'Hospital's rule or a table of values will not be considered supporting work.

   (a) $\lim_{x\to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$

Answer: _____

   (b) $\lim_{x\to 16} \dfrac{4 - \sqrt{x}}{16x - x^2}$

Answer: _____

(c) $\lim_{x \to -6^-} \dfrac{3x + 18}{|x + 6|}$

Answer: _____

(d) $\lim_{x \to 0} \dfrac{\sin(3x)}{4x^2 + 7x}$

Answer: _____

(e) $\lim_{x\to\infty} 3x - \sqrt{9x^2 + x}$

Answer: _____

(f) $\lim_{x\to\infty} \dfrac{-x\sqrt{x} + x}{x^{3/2} + 3x + 3}$

Answer: _____

3. (10 points) Recall that the output of sine function always lies between -1 and 1, that is,

$$-1 \leq \sin(u) \leq 1, \text{for any } u \in \mathbb{R}.$$

Use this and the Squeeze Theorem to evaluate

$$\lim_{x \to 0} x^2 \cdot \sin(\frac{1}{x}).$$

Answer: _____

4. (12 points) Suppose $f(x) = \dfrac{1}{x^2}$.

   (a) Evaluate $f(2)$.

   Answer: _____

   (b) Write down the expression $f(h+2)$.

   Answer: _____

   (c) Put (a) and (b) together, what is $\dfrac{f(2+h)-f(2)}{h}$?

   Answer: _____

   (d) Evaluate the same expression but now passing to the limit $h \to 0$, that is, evaluate

$$\lim_{h\to 0} \frac{f(h+2)-f(2)}{h}.$$

Answer: _____

5. (10 points) Suppose

$$g(x) = \begin{cases} -3x^2 + 3, & \text{if } x < 0, \\ (x+1)^2 + 2, & \text{if } 0 \leq x \leq 2, \\ 13 - x, & \text{if } 2 < x. \end{cases}$$

Determine if g is a continuous function or not.

6. (10 points) Suppose

$$f(x) = \begin{cases} (x+c)^2 - 4, & \text{if } x < 0, \\ 2 - 3x + c, & \text{if } 0 \leq x < 6. \end{cases}$$

Use the limit definition of continuity to find all real numbers c that makes f continuous at 0, or explain if such number does not exist.