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PRELIMINARY STUDY OF SPACE CHARGE AND BEAM-BEAM INTERPLAY IN A COLLIDER RING *

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Abstract

Hadron Collider Rings offer unprecedented opportunities to address fundamental scientific questions in particle and nuclear physics. To achieve these ambitious goals, the colliders must deliver exceptionally high levels of luminosity, hence require high intensity hadron beam in the ring, which leads to high beam-beam parameter, as well as comparable space charge effects. This study focuses on nonlinear effects that impact the beam dynamics within the hadron accelerator ring, including weak-strong beam-beam interactions and their interplay with space charge effects. Accurately predicting these non-linearities, particularly resonances arising during multi-turn acceleration, is critical for long beam lifetime and optimal accelerator performance. The current work presents an initial attempt to develop an optimized approach that integrates space charge effects across the entire ring length while incorporating localized beam-beam interactions at specific interaction points.

INTRODUCTION

It is well established that space charge forces, arising from the mutual Coulomb repulsion between charged particles in a high-intensity beam, must be carefully managed and, when necessary, actively compensated. The nonlinear nature of this force can lead to emittance growth, halo formation, and beam instabilities. These effects become particularly pronounced in scenarios where additional sources of nonlinearity—such as beam–beam interactions in colliders or synchrotron radiation damping and excitation in storage rings—are also present, potentially aggravating the degradation of beam quality and limiting machine performance [1, 2].

Therefore, recent literature highlights the critical need to accurately model nonlinear effects for predicting beam behavior, optimizing accelerator performance, and mitigating potential sources of instability.

This work is organized into two sections. The first presents the theory behind the transfer map that models momentum variation due to the space-charge force. The second part describes the implementation of this model in a weak-strong beam-beam tracking code. In this code, one hundred thousand macroparticles representing the proton beam are tracked over several thousand turns. The parameters used correspond to those of the Electron-Ion Collider (EIC) at Brookhaven National Laboratory.

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TRANSVERSE MOMENTUM VARIATION FROM SPACE CHARGE EFFECTS

In high-intensity beams, space-charge forces often dominate the collective dynamics, significantly influencing beam stability and quality. In the beam's rest frame, each particle experiences the electromagnetic field produced by the surrounding charge distribution. This self-field imparts transverse momentum kicks, leading to betatron tune shifts and amplitude modulation. Under the 2D approach, the bunch is considered infinitely long and uniformly charged along the z-direction, reducing the problem to solving a purely transverse Poisson equation for the scalar potential. However, this approach neglects synchro-betatron coupling effects. To address this, a 2.5D model is adopted in this work, in which the longitudinal charge density follows a Gaussian distribution along the longitudinal direction (as shown in Eq. (1)), allowing the study and prediction of potential coupling resonances.

$$\lambda(z) = \frac{e^{-\frac{z^2}{2\sigma_z^2}}}{\sqrt{2\pi}\sigma_z} \tag{1}$$

The solution to the Poisson equation ($E_{x,y}$in Eq. (2)) is expressed using the Bassetti–Erskine formula [3]. The resulting transverse momentum kick is given by Eq. (2), where K denotes the perveance factor – defined as $K = 2r_0N/\beta^3/\gamma^3$ – in terms of classical radius r_0 and number of particles in the bunch N.

$$dp_{x,y} = \sum_{i=1}^{n} \lambda(z_i) \cdot K \cdot E_{x,y}(x_i, y_i; \sigma_x, \sigma_y) \cdot ds \tag{2}$$

The kick is applied at specific locations or interaction points (IPs) along the ring, which is divided into multiple segments. This segmentation enables the integration of the space-charge force over the length of each segment ds in Eq. (2). Moreover, the *frozen beam size* technique was applied in this preliminary study to reduce numerical errors [4]. As a result, the beam size is not directly influenced by the space-charge force but depends on the average value at the interaction point (IP).

IMPACT OF SPACE CHARGE AND BEAM–BEAM INTERACTIONS

The parameters used for the EIC simulations can be found in [5]. Synchro-betatron coupling resonances were studied using Frequency Map Analysis (FMA), performed with the NAFFlib Python library. The resulting color map illustrates the tune diffusion rate [6].

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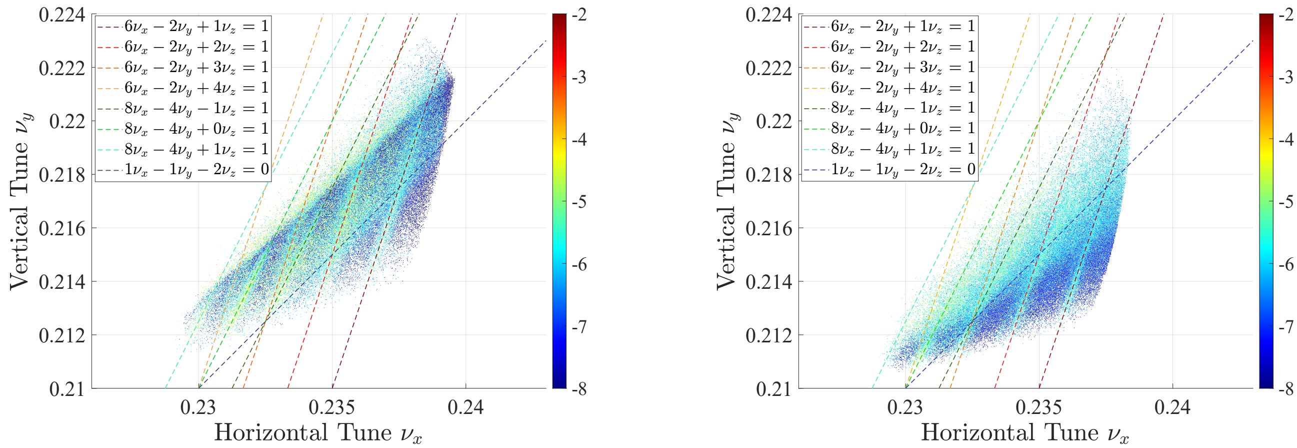

(a) (b)

Figure 1: (a) Frequency map analysis resulting from beam-beam interaction. (b) Frequency map analysis resulting from interplay of beam-beam and space charge forces.

Beam-Beam Interaction Induced Resonances

Beam-beam collisions occur at the interaction point (IP), where the symplectic transfer map is used to efficiently model the transverse and longitudinal kicks in the code EPIC [7,8]. The tune diagram in Fig. 1a displays the positive tune shifting of approximately 0.01 from the tune working point ($Q_x = 0.228, Q_y = 0.21$). The fourth order non-linear resonance is evident on the tune footprint, as are other weaker coupling resonances that satisfy the resonance condition.

Space Charge Induced Resonances

It is worth noting that the force behaves linearly at the *core beam*; while, nonlinear forces act at larger transverse distances. This results in a greater tune shift for particles near the beam core. Nevertheless, particles near the *beam-edge* maintain the original working point tune.

The space charge tune footprint highlights the negative tune shift induced by space-charge force, in contrast to the positive shift typically caused by beam–beam interactions, as shown in Fig. 2.

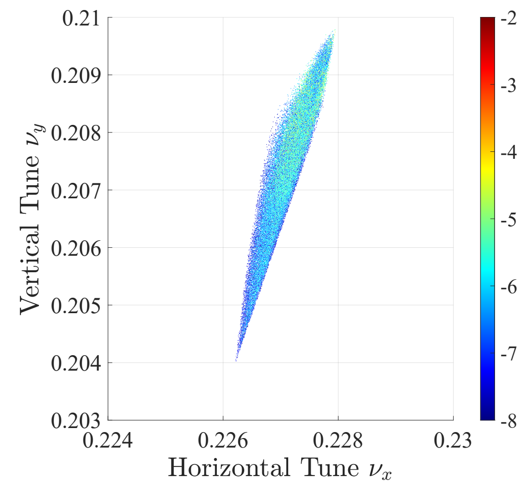

Figure 2: Frequency map analysis resulting from space charge force.

Space Charge and Beam-Beam Interplay

When the space charge effects are included along the ring section length of the collider, the tune diagram (Fig. 1b) shows a downward shift of about $\Delta\nu_x = 0.003$ and $\Delta\nu_y = 0.005$.

Consequently, although the space charge force is not of equivalent magnitude to the beam-beam force, its presence throughout the ring can still affect the overall tune shift.

Therefore in order to investigate the interplay between two collective forces of equivalent magnitude, an analysis is conducted, in which a stronger space-charge force is assumed. As illustrated in Fig. 3a, the space charge tune shift extends up to 0.01 and 0.03 respectively in the x and y directions, crossing other high-order resonances. Thus, the footprint resulting from the space charge and beam–beam interaction − when accounting for the integrated space-charge force, exhibits a larger tune shift −, leading to the crossing of several resonances — most notably a fifth-order resonance and the related synchrotron sidebands, as illustrated in Fig. 3b. The nonlinearity in the current model is derived solely from space charge and beam-beam effects. Both of these effects diminish for particles with large amplitudes. However, magnet nonlinearities must be considered due to the possibility of low-order resonances caused by multipolar magnetic components, specifically for large-amplitude particle trajectories.

2.5D SYMPLECTIC SPACE CHARGE MAP

The model under discussion is not symplectic, as the energy variation does not depend on the transverse momenta. Symplecticity is essential to ensure self-consistency in long-term tracking over many thousands of turns [9, 10]. Therefore, the Hamiltonian formalism can be used to derive the energy variation via Eq. (3), [11].

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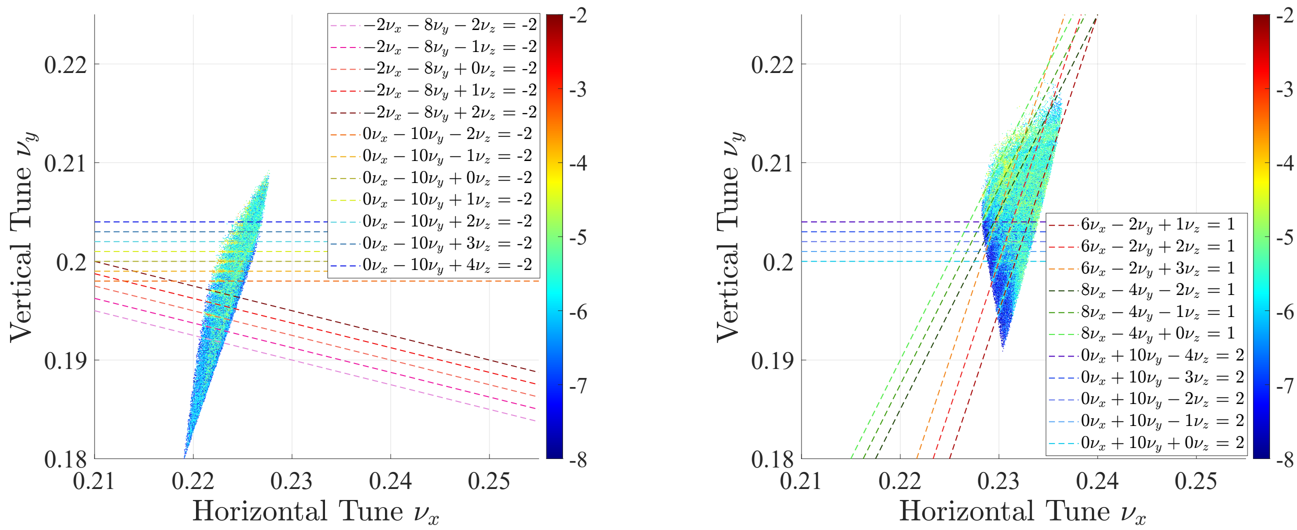

(a) (b)

Figure 3: (a) Frequency map analysis resulting from space charge force with force magnitude comparable to beam-beam force. (b) Frequency map analysis resulting from the interplay of beam-beam and space charge forces of comparable magnitude.

$$\frac{dp_z}{ds} = -\frac{\partial H(x,y,z)}{\partial z} \tag{3}$$

Assuming the space charge Hamiltonian is of the form $H = K \cdot \varphi_0(x,y,z)$, where K is a constant depending on the beam perveance and $\varphi_0(x,y,z)$ is the zeroth-order potential solution, the total energy-momentum variation can be expressed as:

$$dp_z = -K\sum_i^n \left(\varphi(x_i,y_i) + \varphi_{00}\right)\frac{\partial \lambda_z(z_i)}{\partial z}\cdot ds \tag{4}$$

Here, λ_z denotes the longitudinal charge density and $\varphi_0(x,y,z)$ is evaluated assuming the 2.5D Gaussian distribution, i.e., $\varphi_0(x,y,z) = (\varphi(x,y) + \varphi_{00})\lambda_z(z)$, which is valid under the condition $\frac{\partial^2\phi}{\partial x,y^2} \gg \frac{1}{\gamma^2}\frac{\partial^2\phi}{\partial z^2}$.

Therefore, the energy variation depends on the transverse and longitudinal dimensions. Fig. 4 illustrates such z-dependence.

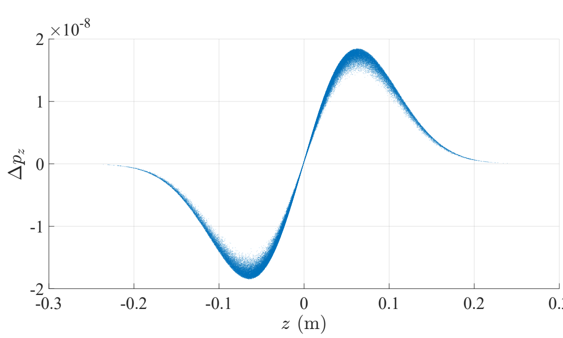

Figure 4: Energy variation due to 2.5 D symplectic map.

The space charge tune diagram, reported in Fig. 5, correctly matches the previous model.

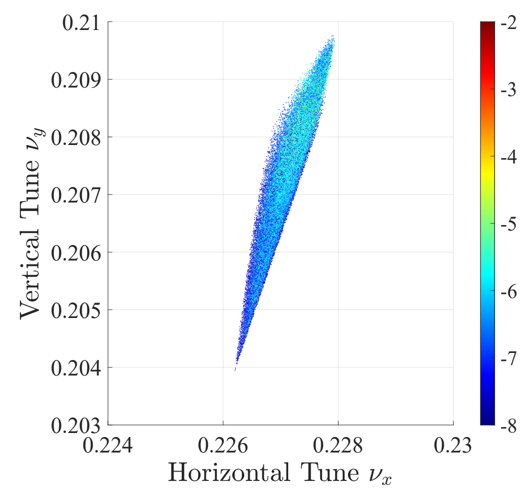

Figure 5: Tune diagram resulting from space charge 2.5 D symplectic model.

CONCLUSION

This work presents an alternative method for calculating the space charge force in colliders and hadron rings. The model has been implemented in the Electron-Proton/Ion Collision code to analyze the interplay between two nonlinear forces: beam-beam interactions and space charge. The 2.5D approach enables the study of synchro-betatron couplings, which play a crucial role in the long-term stability of the beam in high-intensity machines. These couplings can drive particles into resonant conditions, potentially leading to emittance growth and beam losses. A deep understanding of these effects, along with effective mitigation strategies, is essential for ensuring beam stability and maximizing luminosity.

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