

# BEAM-BEAM LIMITATION TOWARD $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ LUMINOSITY FOR ELECTRON-ION COLLIDER\*

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## Abstract

Achieving the design luminosity of in the Electron-Ion Collider (EIC) requires a deep understanding of beam-beam interaction limits in a weak-strong collision regime with flat hadron beams. This contribution presents recent studies of synchro-betatron resonances — particularly the coupling resonance  $2\nu_x - 2\nu_y + p\nu_z = 0$  — that can induce emittance transfer between the horizontal and vertical planes and limit luminosity performance. We identify the hourglass effect, caused by the proton bunch length being comparable to the vertical beta functions of both beams at the interaction point (IP), as the dominant source driving this resonance. We further investigate how physical noise, such as intra-beam scattering and fluctuations in the electron orbit and beam size, couples with the beam-beam interaction to amplify emittance growth and reduce luminosity. Mitigation strategies are also discussed. These studies provide important guidance for future EIC operation and inform strategies for potential upgrades.

## INTRODUCTION

The Electron-Ion Collider (EIC) is poised to operate in a novel regime of beam dynamics that pushes beyond current collider experience. Achieving high luminosity is a fundamental objective for all modern colliders, as it directly determines the event rate for physics processes of interest. At the EIC, this goal must be balanced against complex beam dynamics effects, particularly those arising from intense beam-beam interactions and the geometric configurations imposed by the collider design.

The beam-beam interaction, a nonlinear electromagnetic force exerted between colliding bunches, is intrinsically six-dimensional and strongly influences beam quality. In lepton colliders, a well-defined empirical beam-beam limit has been established, with typical beam-beam parameters on the order of  $0.10 \sim 0.15$  [1–3]. In contrast, hadron colliders operate with beam-beam parameters approximately one order of magnitude lower due to the absence of radiation damping [4]. The EIC enters uncharted territory by simultaneously operating with beam-beam parameters comparable to those achieved in high-performance lepton and hadron colliders — a combination never before realized in a single machine.

To accommodate detector constraints and suppress parasitic collisions, the EIC employs a large crossing angle at the interaction point (IP). This geometry is compensated by crab cavities, which restore effective head-on collision through transverse RF kicks that tilt the bunches in the

horizontal-longitudinal plane. Crab crossing was successfully demonstrated in KEKB through collisions of crabbing electron and positron bunches, leading to record luminosities [5]. Crabbing of proton bunches has been experimentally demonstrated at CERN’s SPS [6]. As a result, crab cavities have become an essential component in the design of next-generation colliders, including the HL-LHC [7] and the EIC [8].

A key innovation in the EIC design is the use of flat hadron beams, in which the vertical emittance is an order of magnitude smaller than the horizontal [9]. While flat beams are naturally produced in electron storage rings, they are nontrivial to achieve in hadron storage rings due to the lack of damping mechanisms. Given the flatness at the IP characterized by the aspect ratio  $\kappa = \sigma_y^*/\sigma_x^*$ , the luminosity and beam-beam parameters scales as:

$$L \propto \frac{1}{\kappa}, \quad \xi_x \propto \frac{\beta_x^*}{1 + \kappa}, \quad \xi_y \propto \frac{\beta_y^*}{\kappa(1 + \kappa)} \quad (1)$$

By adjusting the beta functions  $\beta_x^*$  and  $\beta_y^*$  accordingly, one can boost luminosity without increasing the beam-beam parameters beyond acceptable limits.

Together, these design features, strong beam-beam interactions, crab crossing, and flat hadron beams, define a challenging yet promising regime for collider design. They also necessitate detailed simulation studies to understand their interplay and identify potential limitations in integrated luminosity, which this work seeks to address.

## SYNCHRO-BETATRON RESONANCE

In the presence of crab cavities, the finite bunch length introduces a significant coupling between the transverse and longitudinal dynamics. The transverse kick from the crab cavity depends sinusoidally on the longitudinal position  $z$ , and for the hadron bunch parameters of the EIC, the condition  $|k_c z| \sim 1$  is met, where  $k_c$  is the wave number of the crab cavity. This leads to nonlinear crabbing, where the induced offset at the IP contains cubic terms in  $z$ . This nonlinearity couples the transverse betatron motion with the synchrotron oscillation, giving rise to synchro-betatron resonances.

Frequency map analysis reveals two families of resonances:

$$mv_x + p\nu_z = 1, \quad (2)$$

$$2\nu_x - 2\nu_y + p\nu_z = 1 \quad (3)$$

where  $m$  and  $p$  are integers, and  $\nu_{x,y,z}$  are horizontal, vertical, and longitudinal tunes, respectively.

While the first family appears only in the presence of crab crossing, the second exists even for head-on collisions due

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to non-zero chromaticity and the hourglass effect. Previous simulations [10] show that luminosity degradation is significantly faster in the crab crossing case.

To minimize the impact of finite bunch length effects, the hadron bunch length must be kept short. This design choice increases the hadron synchrotron tune, which, when combined with the large tune spread induced by beam-beam interactions, makes the second family of synchro-betatron resonances unavoidable. Nevertheless, through careful optimization of the working point and the inclusion of a second-order harmonic crab cavity, the first family of resonances can be effectively suppressed. As a result, weak-strong simulations show negligible emittance growth in the hadron beam under these optimized conditions [11].

However, when external physical noise is introduced, the second family resonance amplifies particle diffusion and drives emittance transfer between the horizontal and vertical planes [12]. This effect has been clearly identified in the presence of representative noise sources, including electron orbit ripple [13], electron beam size fluctuations [14], and hadron intra-beam scattering (IBS) [15]. These physical perturbations lead to irreversible emittance flow from the horizontally “hotter” plane to the vertically “colder” one, which can degrade the flat-beam condition and reduce luminosity over time.

## MITIGATION STRATEGIES

Unequal transverse emittances, synchro-betatron resonance, and physical fluctuations collectively form the essential ingredients that drive emittance transfer. Among these, the emittance asymmetry is fundamental to achieving high luminosity in flat-beam collisions and cannot be sacrificed. On the contrary, maintaining the transverse emittance ratio is a core design objective, as it underpins the luminosity enhancement in the EIC.

One straightforward mitigation strategy is to reduce the amplitudes of known fluctuation sources [16]. While effective in principle, this approach often incurs higher cost in accelerator subsystems and may even demand technologies beyond the current state of the art. Although dedicated feedback systems could help relax such constraints, unknown or unpredictable noise sources during machine operation remain a persistent challenge to beam stability.

Another strategy to mitigate emittance transfer is working point optimization. Since the emittance transfer is driven by synchro-betatron resonances, carefully choosing the betatron tunes can help suppress the associated diffusion. This approach has proven effective in simulations, where specific working points significantly reduce vertical emittance growth.

However, the flexibility of working point selection is constrained by several practical factors. First, the working point must be compatible with dynamic aperture requirements. Although our simplified model allows the identification of an optimal tune, operational experience at RHIC indicates that the hadron beam can be sensitive to resonances of or-

der ten and higher [17]. As a result, the viable tune space is narrow. Second, the interplay between beam-beam effects and collective wakefields further restricts the allowable tune range. In certain tune regimes, coherent beam-beam modes may couple with wakefields, potentially triggering beam instabilities [18]. These considerations underscore the challenge of finding a working point that simultaneously avoids high-order resonances, maintains dynamic aperture, and mitigates collective effects.

These considerations highlight the need for an active strategy to suppress emittance transfer. To this end, we propose a dynamic focusing scheme that directly targets the underlying resonance mechanism. By combining crab cavities with sextupoles in a carefully phased lattice configuration, this approach introduces a time-dependent focusing that counteracts the drift space between the IP and collision point (CP). Unlike passive strategies such as noise suppression or working point optimization, dynamic focusing modifies the beam optics to cancel the resonance-driving terms at their origin. This suppression mechanism is demonstrated in Fig. 1, and its effectiveness in improving noise tolerance is quantified in Table 1.

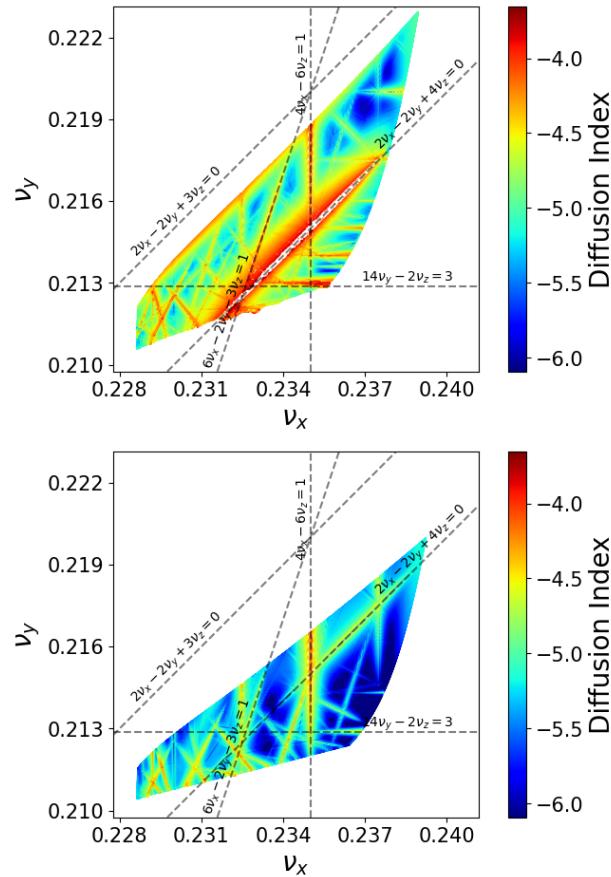


Figure 1: Frequency maps for particles launched at  $z_0 = 3\sigma_z$ , top: EIC baseline, bottom: dynamic focusing. The dynamic focusing scheme clearly suppresses the synchro-betatron resonance.

Table 1: Comparison of Tolerances Under 20%/h Vertical Emittance Growth for the EIC Baseline and Dynamic Focusing Scheme (Note That a Smaller IBS Time Means a Larger Threshold of IBS Diffusion)

Parameter	Unit	EIC Baseline	Dynamic Focusing
$\beta_{y,e}^*$	cm	5.6	7.2
Luminosity	$10^{34} \text{ cm}^{-2} \text{s}^{-1}$	0.93	0.97
Electron orbit ripple	$\sigma_{x,y}^*$	2.5%	5.0%
Electron size ripple	$\sigma_{x,y}^*$	0.3%	1.0%
Hadron IBS time	hour	4.5	1.5

Dynamic focusing decouples hadron vertical beta function from the hadron bunch length, allowing further reduction of vertical beta while keeping the vertical beam size constant. This simultaneously improves the emittance ratio, lowers the vertical beam-beam parameter, and reduces the strength of resonance-driving terms. Simulation results, shown in [12], demonstrate that with a reduced vertical beta, the emittance transfer is completely suppressed.

## LUMINOSITY MODEL

To evaluate the long-term impact of emittance transfer on collider performance, we model the luminosity evolution over a time scale of hours. However, direct tracking of individual beam-beam kicks over such durations is computationally prohibitive. Instead, we incorporate the effect of emittance transfer into a simplified model based on macroscopic emittance growth rates.

The modeling follows the framework developed in Ref. [19], where long-term beam evolution is tracked using simplified lattice elements and stochastic heating models. Instead of simulating individual beam-beam interactions turn-by-turn, emittance growth is incorporated through parametrized kicks derived from simulation benchmarks. The model applies Gaussian-distributed momentum kicks to macroparticles at each timestep, scaled to match the expected IBS and beam-beam induced diffusion rates. By simulating multiple real turns within a single model step, the cumulative effects of noise and diffusion are captured efficiently.

Rather than assuming a constant vertical emittance growth rate, we approximate it as linearly proportional to the horizontal emittance growth caused by the IBS. This approximation captures the observation in weak-strong simulation results.

Let  $g_x$  denote the horizontal IBS growth rate obtained from analytical formulas. The modified evolution equations for the horizontal and vertical emittances are then expressed as:

$$\frac{1}{\epsilon_x} \frac{d\epsilon_x}{dt} = g_x \left(1 - \frac{k\epsilon_y}{\epsilon_x}\right), \quad \frac{1}{\epsilon_y} \frac{d\epsilon_y}{dt} = g_x k \quad (4)$$

where  $k$  is a dimensionless proportionality constant. This model captures the emittance transfer effect in a simple and interpretable form. While the assumption of constant  $k$  is clearly an idealization, it serves as a first-order approximation. More realistic modeling of  $k$  — incorporating its dependence on beam parameters — will be developed in future work.

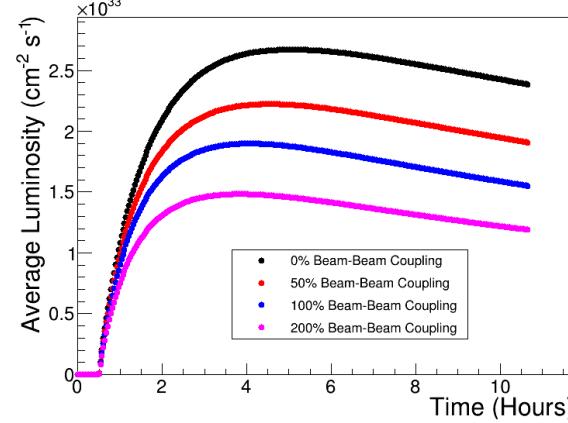


Figure 2: Average luminosity for different emittance transfer coefficients  $k$ . The averages are calculated including 2 hours of dead-time between hadron stores. Additionally, the luminosity is zero during first 30 minutes while we fill the electrons and turn on the detector.

Figure 2 presents the luminosity evolution curves for different values of the emittance transfer coefficient  $k$ . The average luminosity, computed over a long timescale, decreases as  $k$  increases. When  $k = 0$ , corresponding to the absence of emittance transfer, the average luminosity reaches  $2.7 \times 10^{33} \text{ cm}^{-2} \text{s}^{-1}$ . In this regime, the luminosity degradation is dominated by horizontal and longitudinal IBS growth, which prevents the system from reaching the design goal of  $10^{34} \text{ cm}^{-2} \text{s}^{-1}$ .

As  $k$  increases to 0.5, 1.0, and 2.0, the average luminosity declines to 2.2, 1.9, and  $1.5 \times 10^{33} \text{ cm}^{-2} \text{s}^{-1}$ , respectively. These results underscore the critical importance of mitigating vertical emittance growth due to emittance transfer in order to preserve luminosity performance in flat-beam operation.

## CONCLUSION

This paper summarizes recent studies of emittance growth in flat hadron beam collisions at the EIC. Synchro-betatron resonances amplify physical noise and drive vertical emittance growth and degrade luminosity. Mitigation strategies include noise reduction, working point optimization, and a dynamic focusing scheme that suppresses resonance-driving terms.

A simplified luminosity model incorporating emittance transfer shows that average luminosity drops appreciably when vertical growth is present. Dynamic focusing eliminates this growth. However, the achieved luminosity is still limited by horizontal and longitudinal IBS growth. To reach the design goal of  $10^{34} \text{ cm}^{-2} \text{s}^{-1}$ , the effective high energy cooling is necessary.

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