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MINIMIZING DISPERSION THROUGH RESONANT EXTRACTION FOR BNL'S NSRL

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Abstract

Simulations, analysis, and measurements are performed on the BNL Booster's third integer resonance extraction to the NSRL line, which uses a constant optics slow extraction method. In this method, ring dipoles and quadrupoles are changed synchronously for a coasting beam, which aids in maintaining a fixed separatrix orientation through the spill. Simulations show that the outgoing beam has a very small dispersion, independent of the periodic dispersion value at the septum. We show using a first-order normal form approximation that transforms to the Kobayashi Hamiltonian, how the dynamics of such a spill lead to a dispersion-free outgoing beam, which is critical to the uniformity requirements of the NSRL. Finally, we measure the dispersion of the beam by varying the flattop energy of the coasting beam in the booster before engaging the spill and show that the magnitude of dispersion is reduced by over a factor of 5 from the periodic value in the ring.

INTRODUCTION

BNL's Booster ring uses a third-integer resonance to perform slow extraction of a coasting beam into the NSRL transfer line. Typical third-integer resonant extraction schemes manipulate tune or sextupole strength to shrink the stable area and drive particles into the unstable separatrix. In contrast, the Booster-to-NSRL extraction uses a constant optics scheme in which the working point remains fixed during the extraction [1]. Instead, the extraction ramp is driven by a slow reduction of dipole fields, leading to an adiabatic decrease in reference momentum $(B\rho)$. Through large negative chromaticity, this reduces the average tune of the beam, bringing it into resonance. The key insight is that while the periodic dispersion at the septum may be nonzero, the outgoing particles—those that cross the separatrix—do so in such a way that their final trajectories are nearly independent of momentum deviation. This apparent cancellation of dispersion is central to meeting NSRL's stringent requirements on beam uniformity and position stability during slow spill delivery.

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FIRST-ORDER RESONANT NORMAL FORM

Elliptical Fixed Point

To interpret the dynamics analytically, we simplify the motion by only focusing on the horizontal phase space, and perform a first-order normal form transformation around the dispersive *elliptical* fixed point of the third-integer resonance. The power of normal form transformations of truncated power series are enhanced when the map is close to the identity, so in the vicinity of $Q_x \approx m/3$ we transform to the co-moving reference frame that is the composition of 3 one-turn maps. The transformed tune is $\omega \equiv 6\pi\Delta Q_x$, where $3\Delta Q_x = 3Q_x - m$. After normalizing phase space coordinates into $(\tilde{x}, \tilde{p}_x)$ via the Twiss parameters, we write the phasor basis coordinates:

$$z = \sqrt{2J}e^{i\phi_x}, \quad 2J = z\bar{z} = \tilde{x}^2 + \tilde{p}_x^2, \quad \phi_x = \text{phase advance.}$$

While the RF cavities are not entirely off, the debunched beam is coasting with approximately constant energy. Thus, the individual particle tunes Q_x are parametrized through chromaticity by the fractional deviation between particle momentum p and the reference momentum p_0 of the dipole fields $\delta = \frac{p-p_0}{p_0}$. The amplitude-dependent tune shift is negligible in this ring since the highest multipoles are sextupoles whose effective tune shift only has nonlinear dependence on amplitude. Then to first order, $\mathbf{Q}(\delta) = \mathbf{Q}_0 + \delta\,\boldsymbol{\xi} + \mathcal{O}(\delta^2)$ and $\omega = 2\pi(3Q_{0,x} - m) + 6\pi\delta\xi_x$.

Continuing, we can write the transformed Hamiltonian to first order in sextupole strength a la Kobayashi [2, 3]:

$$H = \omega J - \frac{1}{8}\left(S_v z^3 + \bar{S}_v \bar{z}^3\right),$$

where S_v is the virtual sextupole/resonance strength [4]:

$$S_v = \frac{1}{2}\sum_j \left[k_2 l\, \beta^{\frac{3}{2}}\, e^{3i\phi_x}\right]_j.$$

Here, k_{2j}, l_j, β_j, $\phi_{x,j}$ are the normalized strength, length, beta function, and phase advance at each sextupole location. This sum over sextupoles defines an effective coherent strength that governs the shape and orientation of the separatrix, which is given by the effective phase ψ of the virtual sextupole $3\psi \equiv \arg(S_v)$.

When $H = \frac{8\omega^3}{27|S_v|^2}$, the Hamiltonian can be factored into a product of three lines forming an equilateral triangular

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separatrix in the normalized $(\tilde{x}, \tilde{p}_x)$ plane. For $S_\mathrm{v} \in \mathbb{R}^+$, the lines defining the separatrix are:

$$\tilde{x} = -\frac{2\omega}{3|S_\mathrm{v}|},$$

$$\tilde{x} \pm \sqrt{3}\tilde{p}_x = \frac{4\omega}{3|S_\mathrm{v}|}.$$

For general $S_\mathrm{v} \in \mathbb{C}$, the appropriate separatrix is gotten by rotating the preceding equations by ψ.

Transforming back to physical coordinates (x, p_x) via the Twiss parameter transformation:

$$\begin{pmatrix} x \\ p_x \end{pmatrix} = \begin{pmatrix} \sqrt{\beta_x} & 0 \\ -\frac{\alpha_x}{\sqrt{\beta_x}} & \frac{1}{\sqrt{\beta_x}} \end{pmatrix} \begin{pmatrix} \tilde{x} \\ \tilde{p}_x \end{pmatrix},$$

maps the equilateral triangle in normalized phase space to a distorted triangle in real phase space. Because β_x and α_x are constant during the spill, this mapping does not rotate the separatrix as the tune changes, keeping its unstable arms fixed.

For a fixed S_v, we can find the area of stable beam motion within the separatrix from the triangle. By Liouville's theorem, this must be equal to $2\pi J$ for a particle on the separatrix. Thus the border of stability is determined by:

$$2\pi J = \frac{8\omega^2}{3\sqrt{3}|S_\mathrm{v}|^2}.$$

Extracted Beam Distribution

The dominant contribution to the extracted beam width comes from the range of amplitudes at which particles cross their separatrices, effectively reflecting the initial beam amplitude distribution—since instability occurs when the separatrix narrows below a fixed fraction of a particle's ellipse. The separatrix position at crossing is largely momentum-independent, as instability is triggered by betatron amplitude and δ and the tune shift from sextupoles is weak and second-order. Although dispersion $\eta(s)$ exists at the septum, the final escape position does not follow $\Delta x = \eta\delta$; instead, particles exit along a common unstable manifold defined by their amplitude, resulting in a notably flat transverse beam profile. Figure 1 compares the predicted separatrix from first-order normal form theory, a symplectically tracked particle trajectory, and the extracted bunch distribution, showing excellent agreement.

MODEL SIMULATION

The simulation framework tracks particles through a full Booster lattice using a 3rd-order resonance model with chromaticity, synchrotron motion turned off (coasting), and a time-dependent ramp on the magnetic rigidity. The main power supplies are ramped slowly (adiabatically) downward with primary windings around dipoles and secondary windings on quadrupoles which preserves the linear optics (beta

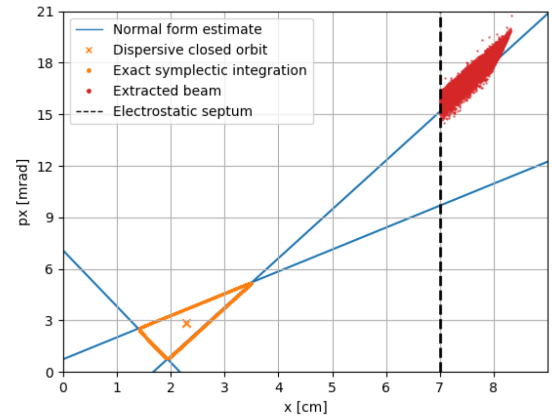

Figure 1: Comparison of normal form separatrix prediction for a particle with $\delta = 0.35\%$ and $J = 8.45\ \mu$m with symplectic tracking through the exact s-dependent lattice Hamiltonian shows remarkable agreement. Also shown is the extracted bunch distribution. The lattice parameters are $\beta_x = 8.02$ m, $\alpha_x = -1.31$, $Q_x = 4.364$ $\xi_x = -7.6$, and $S_\mathrm{v} = 9.27 + 8.63i$.

functions and phase advance), ensuring approximately constant optics throughout the spill.

The Booster lattice is initialized at a working point of $\mathbf{Q}_0 = (4.36, 4.82)$ to avoid interference with the $\mathrm{Q}_y = \{\frac{19}{4}, \frac{24}{5}\}$ resonances, and a reference kinetic energy of 1 GeV/u. A de-bunched Gaussian beam with normalized emittances of $\epsilon_{x,y,\mathrm{norm}} = 3\ \mu$m and a momentum spread of $\sigma_\delta = 0.1\%$ is matched to the periodic Twiss parameters and populated with particles along the reference closed orbit.

Two pairs of drive sextupoles are used to stimulate the third-integer horizontal resonance with minimal effect on chromaticity, and the two families of chromaticity-correcting sextupoles are turned off to facilitate chromatic slow extraction. With natural chromaticities $\boldsymbol{\xi} = (-7.6, -2.1)$, this setup brings about an unstable separatrix which is parametrized by δ, and its orientation in (x, p_x) phase space remains approximately fixed as the reference momentum is slowly swept through the resonance due to the constant optics.

Particles are initialized just within the border of stability. As the tune decreases due to chromaticity and the energy ramp, particles slowly drift into the unstable region. After crossing the separatrix, particles become unstable and quickly spiral out and away from their stable dispersive fixed point. Those that cross over the electrostatic septum are extracted, while those that collide with the septum cross-section or length are lost. A key diagnostic tracks the final transverse position of each extracted particle as a function of their relative momentum $\Delta p/\langle p\rangle$. This shows remarkably weak dependence on δ, indicating suppressed outgoing dispersion, as shown in Fig. 2.

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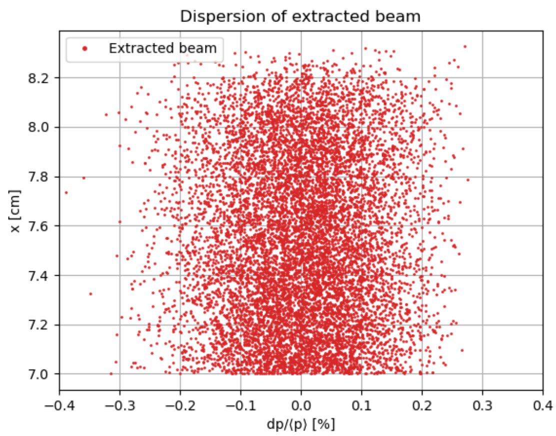

Figure 2: Extracted particles horizontal position vs momentum deviation relative to bunch average at the D3 electrostatic septum shows almost zero correlation, indicating zero beam dispersion at that point.

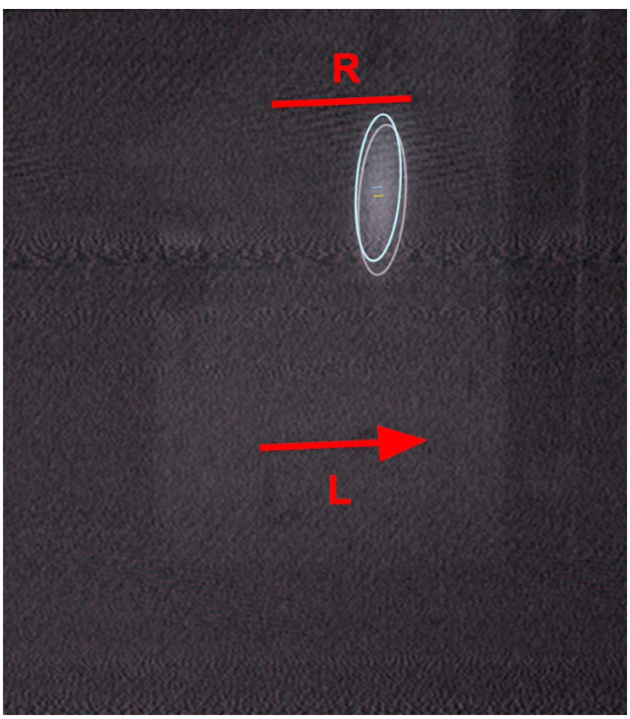

Figure 3: D6 flag showing two overlaid beam spots corresponding to radial offsets of $dR = -13.5$ mm (blue) and $dR = -6.5$ mm (grey). The red "L" and "R" lines are 1.11″ apart.

MEASUREMENT

To quantify the residual output dispersion, we varied the beam momentum at the flattop to two values prior to initiating the extraction spill via the radial loop in the RF system before disengaging the RF. The radial offsets we compared were −13.5 mm and −6.5 mm, which correspond to a relative momentum deviation of 0.93‰ from one another given that $\gamma_{tr} = 4.33$ and $\rho = 13.866$ m. We then inserted a flag into the D6 extraction vacuum chamber which is just over one FODO period away from the initial electrostatic septum at D3. By observing the spot position for the two momenta and fitting Gaussians to them, we can compare the relative position to the known width of the flag, which is 1.11″ = 2.819 cm. In Fig. 3, we show an overlaid images of the two spots on the flag, including the position markers "L" and "R". The measured difference is approximately $\Delta x \sim 0.7 \pm 0.5$ mm, which we conclude corresponds to a dispersion of approximately $D_x \approx 8 \pm 5$ cm. Compared to the periodic dispersion function at that point $\eta_x = 2.93$ m, this is 1-2 orders of magnitude smaller, consistent with the expectation of vanishing dispersion.

CONCLUSION

We have demonstrated through simulation, theoretical analysis, and measurement that BNL Booster's constant optics resonant slow extraction to NSRL enables extraction of a nearly dispersion-free beam. The combination of a fixed resonance separatrix, induced tune shift via chromaticity, and adiabatic energy ramping allows particles to escape along a fixed geometric path that is nearly independent of energy deviation. This suppresses the usual $\eta\delta$-driven dispersion and results in a beam suitable for sensitive biological and materials experiments at NSRL. Further refinements in chromaticity control and septum placement may further improve extraction uniformity and reduce sensitivity to machine drifts.

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