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VERTICAL EMITTANCE GROWTH FROM IBS DIFFUSION VIA BEAM-BEAM COUPLING IN THE EIC*

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Abstract

We investigate vertical emittance growth in the Electron-Ion Collider (EIC) arising from intrabeam scattering (IBS) diffusion through beam-beam interactions. Using weak-strong simulations, we demonstrate that when horizontal noise is introduced, vertical emittance increases even in the absence of direct vertical diffusion. This behavior is attributed to resonance streaming around the synchro-betatron resonance $2\nu_x - 2\nu_y + p\nu_z = 0$, which enables unidirectional emittance transfer from the horizontal to the vertical plane. We further show that horizontal cooling alone cannot suppress the vertical growth, confirming that dedicated vertical cooling is essential for preserving the flat beam profile in high-luminosity EIC operation.

INTRODUCTION

Intra-beam scattering (IBS) refers to a diffusion process arising from multiple small-angle Coulomb scatterings between particles within the same beam [1]. While small emittances are essential for achieving high luminosity, the high phase space density of hadron beams makes them particularly susceptible to IBS-induced emittance growth. As a result, IBS is a major performance limitation in hadron colliders.

The Electron-Ion Collider (EIC), currently under construction at Brookhaven National Laboratory (BNL), is designed to probe the internal structure of protons and nuclei with high precision [2]. To maximize luminosity, the EIC will be the first collider to employ flat hadron beams, where the vertical emittance is ten times smaller than the horizontal [3]. However, this flat-beam configuration presents new challenges. The EIC will operate above transition energy, where IBS drives unbounded growth in all transverse and longitudinal emittances in the absence of effective cooling. This makes it difficult to preserve the large aspect ratio between vertical and horizontal emittance.

High energy cooling is the most direct method to counteract IBS-induced diffusion. However, practical implementation of cooling for high energy hadron beams remains under development. Consequently, top energy cooling is not included in the current EIC baseline design.

According to existing IBS formalisms for Gaussian beams, diffusion primarily occurs in the horizontal and longitudinal planes due to the presence of horizontal dispersion. In contrast, vertical emittance growth is typically much smaller, as vertical dispersion is absent in a planar storage ring [4].

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This paper focuses on the additional vertical emittance growth induced by the beam-beam interaction in the presence of IBS diffusion. The beam-beam force is highly nonlinear, and the hourglass effect introduces a dependence on the longitudinal position, making synchro-betatron resonances unavoidable [5]. We study how horizontal and longitudinal diffusion driven by IBS can couple into the vertical plane through these resonances, leading to vertical emittance growth and potential degradation of the flat-beam configuration.

IBS MODELING

We employ a custom-developed weak strong simulation code to study the interplay between beam-beam interaction and IBS diffusion. The strong electron beam is modeled as a rigid Gaussian distribution, while the weak proton beam is represented by dynamically evolving macro-particles subject to beam-beam kicks and external perturbations. The beam parameters used in the simulation are consistent with the EIC design values in [2]. Further details of the beam-beam simulation framework are described in [6].

In our simulations, particles are tracked for one million turns, corresponding to approximately 12.8 seconds. Over this timescale, it is reasonable to assume that the IBS growth and cooling times remain constant. Therefore, the IBS diffusion and potential cooling effects are modeled using a lumped element located at the interaction point (IP), analogous to the radiation damping and excitation models used in other beam beam simulation codes [7].

$$\lambda_u = \exp\left(-\frac{-T_{\mathrm{rev}}}{\tau_u}\right), \tag{1}$$

$$u_{n+1} = \lambda_u u_n + R_u \sigma_u \sqrt{1-\lambda_u^2}. \tag{2}$$

By turning off cooling, the diffusion equations simplify further:

$$u_{n+1} = u_n + R_u \sigma_u \sqrt{1-\lambda_u^2} \approx u_n + \frac{T_{\mathrm{rev}}}{\tau_u}\sigma_u R_u, \tag{3}$$

where $u = x, p_x, y, p_y, z, p_z$ represents the phase space coordinate, T_{rev} the revolution time, τ_u the characteristics time of IBS diffusion, σ_u the RMS size of corresponding dimension, and R_u the random number following standard Gaussian distribution.

Alternatively, the IBS kick can be applied solely to the momentum components of the particles, with the kick strength scaled proportionally to the local particle density, as proposed in [8]. This density-dependent model offers a more localized description of diffusion and will be benchmarked in future studies.

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EMITTANCE GROWTH WITH IBS DIFFUSION OR ARTIFICIAL COOLING

Figure 1 shows that vertical emittance growth occurs when horizontal diffusion is present. Specifically, a horizontal IBS growth time of $\tau_x = 1$ h drives a vertical emittance growth rate of 100%/h, requiring vertical cooling time as short as 0.5 h to suppress this substantial growth. However, implementing such strong vertical cooling is not part of the current EIC design. In contrast, longitudinal diffusion does not induce any horizontal or vertical emittance growth.

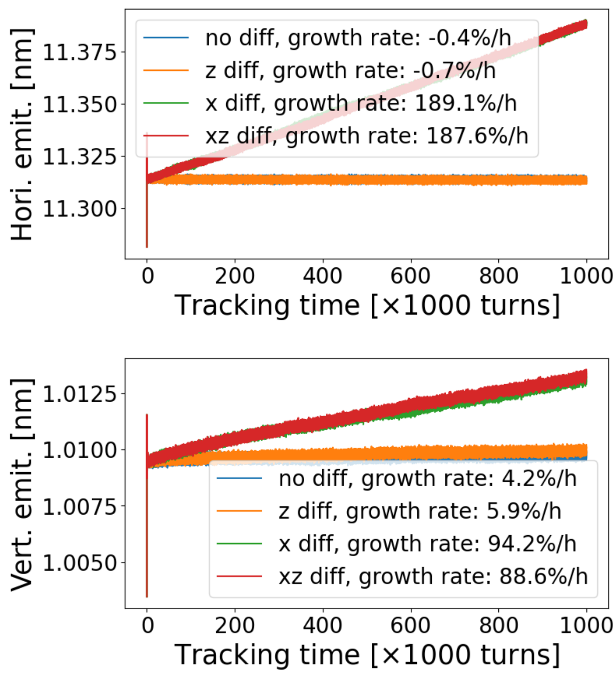

Figure 1: Proton emittance evolution for different horizontal ($\tau_x = 1$ h) or longitudinal ($\tau_z = 2$ h) IBS growth time: top (horizontal), bottom (vertical). The growth rates are linear fitted from the last 60% tracking data. Here τ_x and τ_z are IBS growth time in terms of beam size, as shown by Eq. (3). The blue curve, shadowed by other curves, serves as a reference, showing no emittance growth in both planes.

The effective cooling technique for high energy hadron beams are still under development. Here, we assume artificial cooling in the simulation. Figure 2 presents simulation results including both IBS growth and artificial cooling. In the absence of beam beam interaction, IBS growth is fully balanced by cooling, as expected. When the beam beam force is included, horizontal and longitudinal emittance growth remain suppressed, indicating successful mitigation in these planes. However, vertical emittance continues to grow due to horizontal IBS diffusion, even with horizontal cooling applied. This result demonstrates that horizontal cooling alone is insufficient to suppress vertical emittance growth driven by horizontal diffusion through the beam beam interaction. Additional mitigation strategies are required to control vertical emittance growth in this regime.

The vertical emittance growth observed in this study presents a significant challenge to high energy cooling. Un-

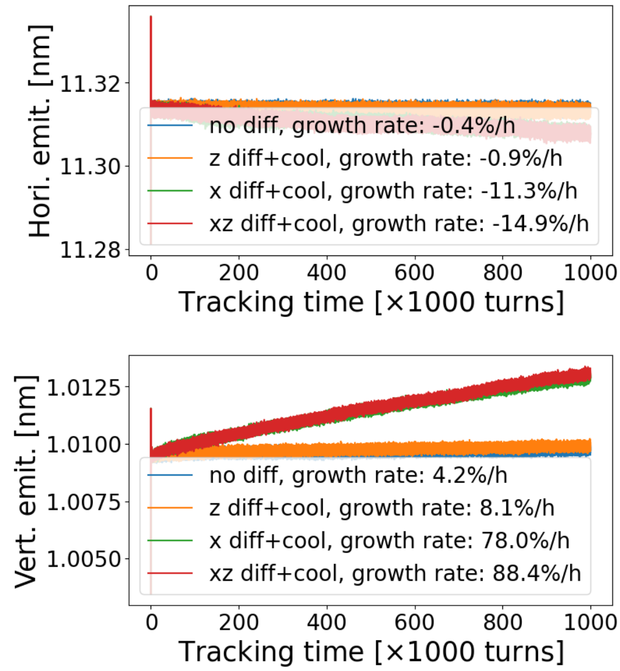

Figure 2: Proton emittance evolution for different horizontal ($\tau_x = 1$ h) or longitudinal ($\tau_z = 2$ h) IBS/cooling time: top (horizontal), bottom (vertical). Without beam-beam, the IBS growth is balanced by cooling.

like dispersion-based mechanisms that allow diffusion or damping to redistribute between planes, this phenomenon induces a unidirectional emittance transfer from the horizontal to the vertical plane, without enabling cross-plane cooling. As a result, dedicated vertical cooling is necessary. To preserve the flat hadron beam profile, the cooling system must be sufficiently strong in all three planes to counteract IBS-driven emittance growth.

UNDERLYING MECHANISM TO DRIVE VERTICAL EMITTANCE GROWTH

The similar growth in vertical plane has also been observed in the study of electron orbit [9] and size fluctuations [10]. The frequency map shows that the synchro-betatron resonance $2\nu_x - 2\nu_y + p\nu_z = 0$ crossing the beam-beam footprint [11]. This resonance amplifies the external noise and drive the emittance flows from the horizontal into the vertical plane.

The tune scan is performed to confirm that the vertical emittance growth due to the horizontal IBS diffusion also relates to this resonance. Figure 3 presents the tune scan results with horizontal IBS diffusion $\tau_x = 1$ h . A clear diagonal pattern is observed. Minimal vertical emittance growth is observed at $\nu_x - \nu_y \approx 0.011$ or 0.016. The vertical emittance growth rate is highly sensitive to the working point, as demonstrated in the bottom plot of Fig. 3, where the growth rate ranges from 25%/h to 250%/h. This sensitivity suggests that optimizing the working point could be an effective strategy for mitigating vertical emittance growth.

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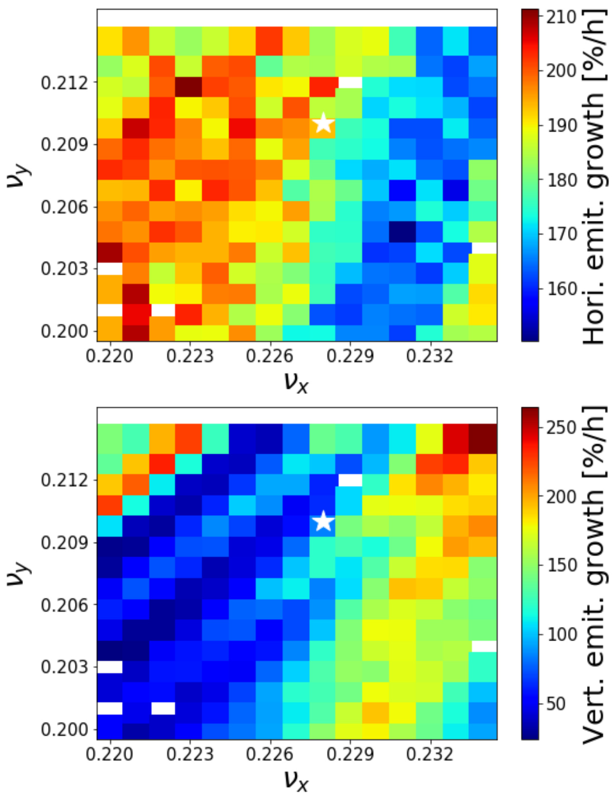

Figure 3: Proton emittance growth rates at different working points with horizontal IBS diffusion $\tau_x = 1$ h: top (horizontal), bottom (vertical). The pentagram shows the nominal working point in the EIC CDR [2].

The strength of the beam-beam interaction is directly proportional to the bunch intensities. As the electron bunch intensity decreases, the beam-beam kick received by the proton beam is reduced, which mitigates or even eliminates vertical emittance growth. Figure 4 shows the dependence of vertical emittance growth rate on the electron bunch intensity under fixed horizontal IBS diffusion with $\tau_x = 1$ h. Notably, the vertical growth rate does not scale linearly with electron intensity. When the intensity is sufficiently low, the vertical emittance growth is negligible. However, once the intensity exceeds a certain threshold, the growth rate increases rapidly, exhibiting an approximately exponential dependence on the electron bunch intensity.

This behavior has important implications for early EIC operation, where the electron bunch intensity is expected to be reduced to one fourth of the nominal design value. In this regime, the vertical emittance growth is negligible.

Another key observation is that the vertical growth rate is nearly identical in both head-on and crab-crossing collision scenarios. This is because the underlying synchro-betatron resonance is driven by the hourglass effect, which introduces a longitudinal dependence into the beam-beam force. In the EIC design, a second-order harmonic crab cavity is employed to compensate for the nonlinear crabbing [12], effectively restoring the overlap area of colliding beams.

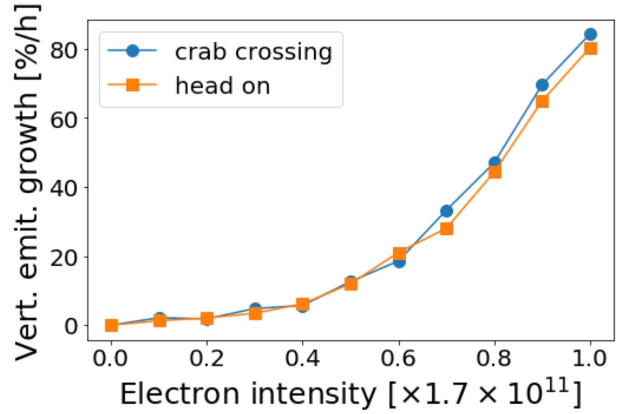

Figure 4: Proton vertical emittance growth rates for different electron intensities, with fixed proton horizontal IBS diffusion $\tau_x = 1$ h.

Reducing the longitudinal action effectively weakens the synchro betatron resonance strength, as demonstrated in Fig. 5. Among the longitudinal parameters, reducing the bunch length is more effective than reducing the momentum spread. This is consistent with the earlier observation that the synchro betatron coupling originates from the hourglass effect. A shorter bunch length mitigates the hourglass effect directly, thereby suppressing the resonance that drives vertical emittance growth.

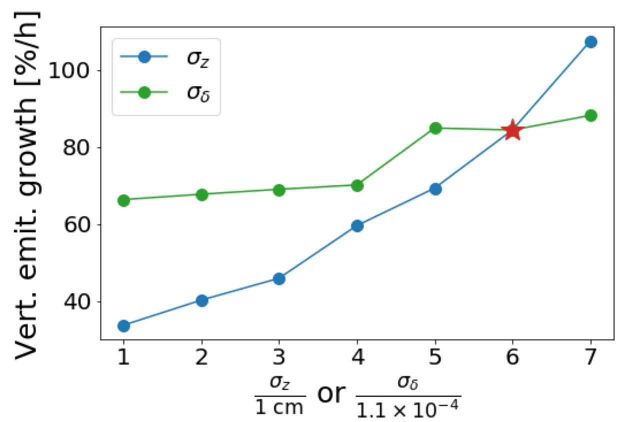

Figure 5: Proton vertical emittance growth rates for different bunch lengths and momentum spreads.

CONCLUSION

Weak strong simulations show that horizontal IBS diffusion can drive vertical emittance growth in the EIC flat beam collision. Horizontal or longitudinal cooling alone cannot suppress this effect. The effective mitigation methods include: working point optimization, IBS diffusion control, and a novel dynamic focusing scheme to suppress the synchro-betatron resonance [13].

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