

# DYNAMIC APERTURE STUDIES AT THE FERMILAB RECYCLER RING

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## Abstract

As part of the Proton Improvement Plan II (PIP-II), Fermilab aims to increase beam intensity delivered to neutrino experiments. In this context, higher intensity injection into the Recycler Ring enhances space charge effects, pushing operations closer to third-order resonances. These resonances reduce the Dynamic Aperture (DA), leading to increased beam loss. This study presents simulations of DA as a function of tune in the Recycler Ring, incorporating chaos indicators such as the Reversibility Error Method (REM). The effectiveness of existing resonance mitigation strategies is evaluated by quantifying their ability to delay DA degradation. Additionally, the study examines how space charge detuning and DA limitations dictate viable operational tune points for the Recycler Ring.

## INTRODUCTION

The dynamic aperture (DA) is defined as the region of phase space where stable motion occurs. Third-order betatron resonances reduce the Dynamic Aperture (DA) of circular accelerators, particularly when the machine operates near resonance lines in tune space. In high-intensity regimes, this leads to increased beam losses and decreased transmission efficiency. As part of the Proton Improvement Plan II (PIP-II) [1], the Fermilab Recycler Ring is expected to handle higher intensity beams, which makes understanding and mitigating these resonances critical to successful operation.

The Recycler Ring (RR) is a permanent magnet storage ring located in the Fermilab Accelerator Complex. RR is a machine operating at a fixed momentum of 8.835 GeV/c, equivalent to an energy of 8 GeV. Figure 1 shows the approximate nominal tunes for the bare machine, i.e., with all tune control magnets turned off [2]. As the Recycler Ring starts to deal with higher intensities and larger tune spreads, it is important to study how its Dynamic Aperture (DA) behaves close to neighboring third order resonances. Figure 1 also shows approximate space charge tune shifts for nominal PIP-II parameters.

The following work explores how third order resonances decrease available dynamic aperture in the Fermilab Recycler Ring. This is done through tracking simulations using Xsuite [3]. In particular, the nominal transverse tunes in the lattice are swept while recording Dynamic Aperture region and Reversibility Error Method (REM) chaos indicators. Furthermore, this work investigates how dynamic aperture behaves along the space charge detuning direction.

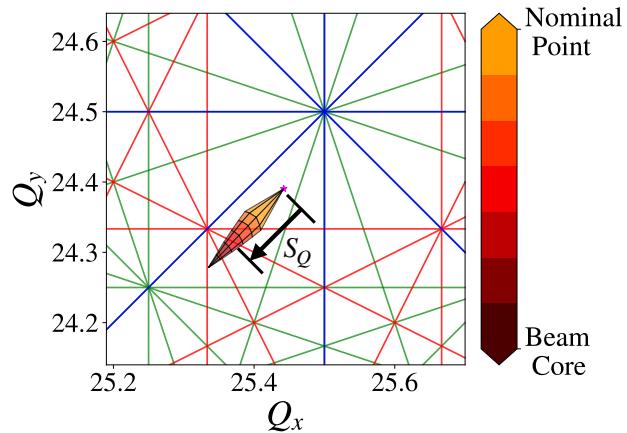


Figure 1: Tune diagram for the Recycler Ring with approximate tune spread due to space charge forces from nominal set point to beam core calculated using PySCRDT [4, 5]. The space charge detuning  $S_Q$  is illustrated.

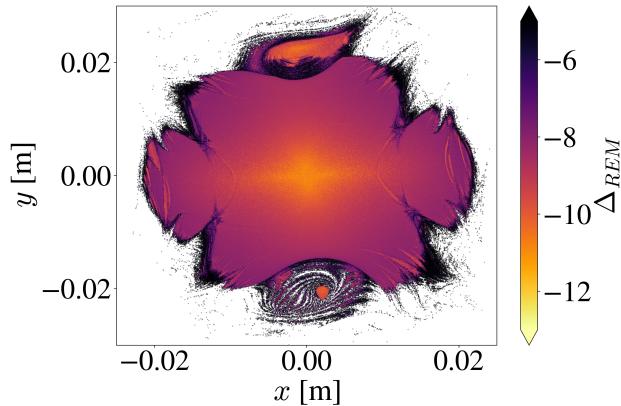


Figure 2: Dynamic aperture (DA) of the Fermilab Recycler Ring for the bare machine, using the Reversibility Error Method (REM) as a chaos indicator.

## DYNAMIC APERTURE SIMULATIONS

In order to get dynamic aperture calculations, tracking simulations through the lattice model were done using Xsuite [3]. Xsuite is a modular, Python-based accelerator simulation tool developed at CERN, capable of high-performance multi-particle tracking on GPU contexts. For this study, the full Recycler Ring lattice was imported from MAD-X into Xsuite, and  $2.5 \times 10^6$  particles were tracked over 2048 turns for a grid of initial transverse amplitudes. A physical aperture ten times the size of the real aperture was introduced in order to capture the entire region of stability.

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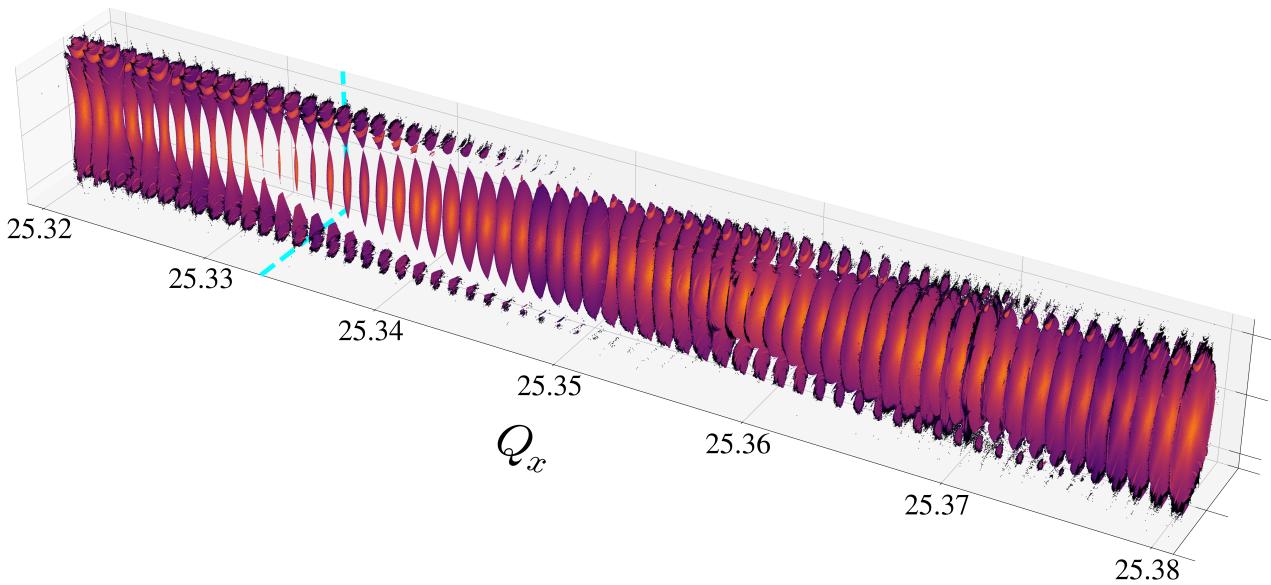


Figure 3: Dynamic aperture scan in both planes as a function of horizontal tune as the  $3Q_x = 76$  resonance is crossed for the bare machine. The resonance line is illustrated with cyan dashed line.

Tracking was performed in 4D phase space, ensuring longitudinal coordinates were frozen. Forward- and back-tracking were performed in order to calculate the REM chaotic indicator.

Figure 2 shows the dynamic aperture (DA) of the Fermilab Recycler Ring at its bare machine tunes, using REM as a chaos indicator. The color represents the value of  $\Delta_{REM}$ , a log-scale indicator of trajectory reversibility, with darker colors indicating higher chaos. The stable region (DA) appears as a complex, star-shaped area in the center, enclosed by sharp boundaries. The dark outer region corresponds to particles lost or strongly chaotic—they exceed the dynamic aperture. The lower part of the plot shows a prominent island structure, suggesting a nonlinear resonance island trapping some stable orbits.

### DA Loss at Third Order Resonance

To quantify the impact of the  $3Q_x = 76$  resonance on beam stability, we performed a horizontal tune scan for the bare Recycler lattice. The scan was carried out by incrementally varying  $Q_x$  while keeping  $Q_y = 24.44$  constant, and tracking particle motion for a grid of initial transverse amplitudes with longitudinal coordinates frozen for tracking.

Figure 3 shows the dynamic aperture in both planes as a function of  $Q_x$  across the resonance. A 3-dimensional visualization is used. The cyan dashed line indicates the resonance location in tune space. A pronounced reduction in the horizontal DA is observed as the nominal tunes approach the resonance, with losses occurring first at large horizontal amplitudes.

Taking slices from Fig. 3 one can build the 2-dimensional projection shown in Fig. 4. In particular, Fig. 4 shows horizontal slices of the original 3-dimensional structure at  $y = 0$ . The loss of dynamic aperture close to the third order reso-

nance has a sharp slope. This particular plot from Fig. 4 is to be compared with the fish-bone diagrams from Ref. [6].

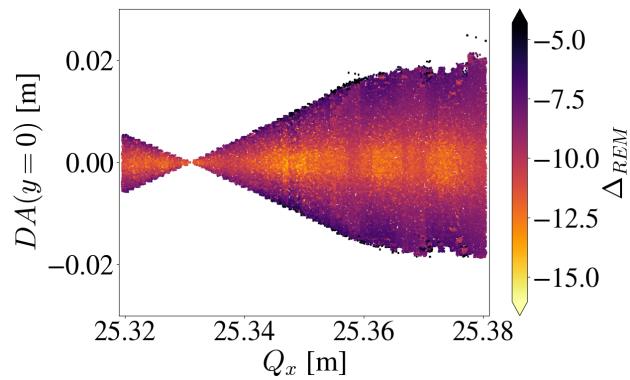


Figure 4: Horizontal dynamic aperture slices at  $y = 0$  as a function of tune for the bare machine, using the Reversibility Error Method (REM) as a chaos indicator.

### DA with Compensation Scheme

The Recycler Ring currently employs a resonance driving term (RDT) compensation scheme based on strategically placed sextupole families [7–9]. This scheme was activated in the model to assess its effectiveness in preserving DA near the  $3Q_x = 76$  resonance.

As shown in Fig. 5, the compensation mitigates DA loss, extending the stable region deeper into the resonance region. The onset of chaos is delayed, and the DA reduction slope as a function of tune becomes shallower compared to the bare lattice case. Although the compensation does not eliminate the resonance completely, it substantially improves stability margins, suggesting what is seen operationally—losses

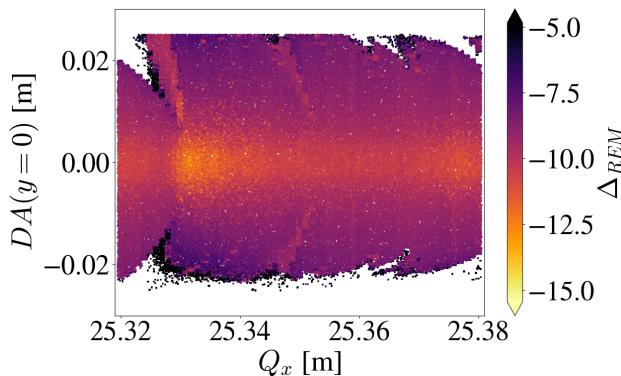


Figure 5: Horizontal dynamic aperture slices at  $y = 0$  as a function of tune for the machine with compensation scheme turned on, using the Reversibility Error Method (REM) as a chaos indicator.

across the  $3Q_x$  line are reduced when compensation scheme is turned on.

### DA Loss and Space Charge Detuning

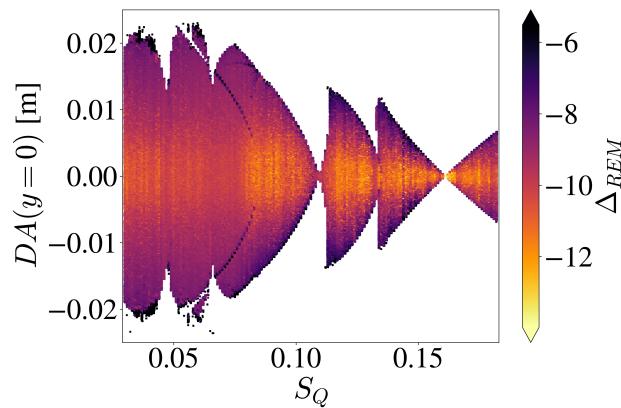


Figure 6: Horizontal dynamic aperture slices at  $y = 0$  as a function of space charge detuning  $S_Q$  for the bare machine, using the Reversibility Error Method (REM) as a chaos indicator.

Space charge will cause particles to experience an incoherent detuning. Each particle depending on its position along the bunch will have a different transverse tune. While all the simulations showed herein do not include any space charge module for tracking, this subsection tries to tackle this phenomena. In particular, this study changed the lattice nominal tune in the direction where space charge tune shift would occur. For this purpose, the detuning parameter  $S_Q$ , defined as

$$S_Q = \sqrt{\Delta Q_x + \Delta Q_y}, \quad (1)$$

is the effective tune distance from the bare machine tunes on the one-to-one diagonal direction. Figure 1 illustrates this  $S_Q$  parameter.

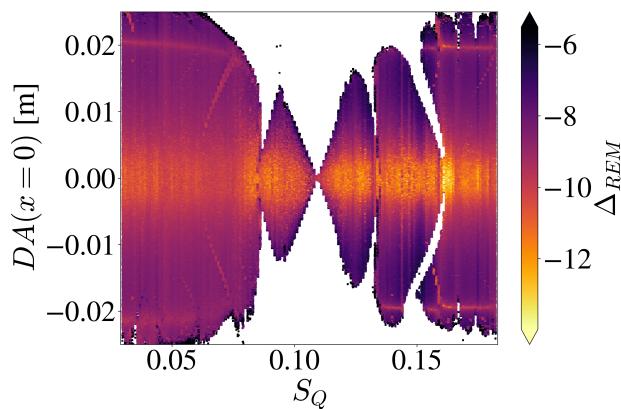


Figure 7: Vertical dynamic aperture slices at  $x = 0$  as a function of space charge detuning  $S_Q$  for the bare machine.

Although the present simulations do not include direct space charge tracking, the effect of incoherent tune spread was modeled by shifting the lattice tunes along the  $S_Q$  direction defined in Eq. (1). This mimics the detuning pattern expected from space charge forces, where particles in the bunch core experience the largest tune shifts.

Figure 6 shows the horizontal DA at  $y = 0$  as a function of  $S_Q$  for the bare machine. The results indicate a gradual DA degradation as  $S_Q$  increases, with more severe losses when the detuning direction moves the footprint toward resonance lines. In particular, three big zero nodes are observed in Fig. 6 corresponding to the three resonance lines that would affect the horizontal dynamic aperture. On the other side, Fig. 7 shows the vertical equivalent for vertical loss of DA where the four resonance lines can be seen. For this case, the  $3Q_x$  resonance is so strong that combined with some residual coupling, it can also affect the vertical DA.

## CONCLUSIONS AND FUTURE WORK

High-resolution Xsuite tracking with the Reversibility Error Method (REM) showed that the bare Recycler lattice suffers significant DA loss well before the  $3Q_x = 76$  resonance, with rapid growth of chaotic regions. Activating the existing RDT compensation scheme delays this degradation and extends the usable tune space, though resonance effects remain. Future work should explore how the resonance compensation scheme affects the DA loss in the  $S_Q$  direction, i.e., in the space charge detuning direction. Future work should also look at how to incorporate space charge tracking modules and in order to look at its relationship with DA.

## ACKNOWLEDGMENTS

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