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SIMULATIONS OF IBS THROUGH ELECTRIC FIELD FLUCTUATIONS*

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Abstract

We present a study of intra-beam scattering (IBS) in high-brightness electron beams, incorporating a recent theory that accounts for enhanced temporal correlations of electric field fluctuations. These correlations, absent in conventional binary-collision models, arise from the periodic betatron motion of particles within the beam. To enable direct verification of the theoretical calculations, we perform simulations in a computer code specifically written for that purpose. In the code, the particle distribution is preserved over time, ensuring conditions compatible with theoretical assumptions, and the IBS is neatly separated from the conventional Space Charge (SC) effect.

The simulations, benchmarked against an exactly solvable case of an infinite isotropic uniform plasma, show good agreement with both uncorrelated models, such as Piwinski's, and the new correlation-based theory, across various bunch distributions and dynamical regimes. This validates the simulation approach and highlights the role of time-correlated fields in accurate IBS modeling.

INTRODUCTION

Space Charge (SC) is the effect of Coulomb's forces acting on the particles in a bunch. These forces are considered to be produced from a smooth distribution function, not taking the discreetness of particles into account. The effect of the difference between the full Coulomb's fields and the fields from a smooth function is called the Intra Beam Scattering (IBS). The IBS has been first studied by Piwinski [1] and by Bjorken and Mtingwa [2]. Estimates of IBS for an x-ray FEL driver can be found in Ref. [3].

All these papers consider the IBS as a result of binary collisions between the particles in the beam, averaged by time. Recently, a new treatment was proposed in Ref. [4] that derives the IBS from the interaction of particles with the fluctuating electromagnetic fields in the beam on the microscopic scales. For perfect periodic orbits with no longitudinal motion, it was obtained that the energy spread squared σ_η^2 grows quadratically with time. And if the longitudinal motion is included, the IBS growth rate was shown to saturate to a value higher than the Piwinski predictions.

In this paper, we carry out simulations of IBS within the framework of the fluctuating fields using a computer code written for that purpose. To test the code, we compare the results of our simulations for a uniform plasma with the plasma collision integral and find a good agreement. We also compare our simulations with the Piwinski formula, and the new theory from Ref. [4].

THEORY

In this paper, we study only the axially symmetrical beams of electrons and positrons, and are interested only in the energy spread growth $\mathrm{d}\sigma_\eta^2/\mathrm{d}t$. Here η denotes the relative energy deviation of the particle and σ_η is the rms energy spread in the beam. For simplicity, in this paper we present all the results in the beam frame, using the longitudinal velocity spread $\sigma'_v = c\,\sigma_\eta$, where prime means the parameter in the moving frame and v is the longitudinal velocity. The primes are omitted in what follows. The theory results are transformed accordingly. The conventional Piwinski theory for a three dimensional gaussian spatial distribution and two dimensional gaussian velocity distribution (with no longitudinal velocity spread at $t = 0$) gives [3]

$$\sigma_v^2(t) = t\,\frac{r_e^2\,c^4\,N\,\Lambda}{4\,\sigma_x^2\,\sigma_z\,\sigma_{v_x}}, \tag{1}$$

where N is the number of particles in the bunch, r_e is the classical electron radius, σ_x and σ_z are the rms bunch transverse and longitudinal sizes, σ_{v_x} is the rms transverse velocity spread, and $\Lambda = \ln\left(r_{\max}/r_{\min}\right)$ is the Coulomb's logarithm. The $r_{\max}$ and $r_{\min}$ are the maximum and minimum impact parameters in a single scattering event [1].

For simplicity of the analysis and comparison with simulations we define a dimensionless parameter A with the meaning of the average growth rate across the time period t:

$$\sigma_v^2(t) = t\,A(t)\,\frac{r_e^2\,c^4\,N}{\sigma_x^2\,\sigma_z\,\sigma_{v_x}}. \tag{2}$$

For the case defined above, $A(t) = \frac{1}{4}\Lambda$. It is straightforward to see that the function $A(t)$ is a constant for all the averaging theories. For the same bunch, but with uniform longitudinal distribution instead of gaussian, $A(t) = (\sqrt{\pi}/4\sqrt{3})\,\Lambda$ [5].

Betatron Motion

Equation (35) in Ref. [4] gives a general answer for a beam with gaussian distributions by x, v_x, y, v_y and v (in this paper it is longitudinal velocity), and infinite uniform distribution by z. Particles execute transverse betatron motion (constant focusing) and uniform longitudinal motion.

$$A(\phi) = \int_0^\infty \mathrm{d}\kappa \int_0^\infty \mathrm{d}\kappa' \int_0^\phi \mathrm{d}\phi_2 \int_0^{\phi_2} \mathrm{d}\phi_1 \times \int_{-\infty}^{\infty} \mathrm{d}q_z F(\kappa,\kappa',\phi_1,q_z), \tag{3}$$

with

$$F = \frac{2\kappa\kappa'}{\pi\phi\sqrt{3}} \exp\left[-(\kappa^2+\kappa'^2)^2\right] I_0(2\kappa\kappa'\cos\phi_1) \times \frac{q_z^2 \exp\left[-\tau\, q_z^2\, \phi^2\right]}{(\kappa^2+q_z^2)(\kappa'^2+q_z^2)},$$

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where $I_0(z)$ is the modified Bessel function of the first kind, and $\tau = \sigma_v^2/2\,\sigma_{v_x}^2$. Here we transformed the result to the beam frame, changed the integration variables to the betatron phase ϕ, and derived $A(\phi)$ from $\mathrm{d}\sigma_v^2/\mathrm{d}t$.

SIMULATIONS

We simulate IBS by directly computing the electric field from all particles in the bunch and evaluating its effect on each particle. The longitudinal velocity spread after the interaction region is calculated as

$$\begin{aligned}\sigma_v^2(t) &= \left\langle v_i(t)^2\right\rangle - \left\langle v_i(t)\right\rangle_i^2 \\ &= \left\langle \left(v_i(0) + \frac{e}{m}\int_0^t E_i(t_1)\,\mathrm{d}t_1\right)^2\right\rangle_i - \left\langle v_i(t)\right\rangle_i^2, \end{aligned}\tag{4}$$

where $E_i(t_1)$ is the z component of the electric field E at the position of the i-particle due to all other particles at time t_1. In this paper, we always use scalar E meaning the z component of the 3d field, and not the total amplitude of the field. The notation $\langle f_i\rangle_i$ denotes an averaging over index i (over all particles).

To isolate IBS from the average space-charge (SC) field, we write $E(t) = E_0(t) + \delta E(t)$, where $E_0(t)$ is the field from the smooth averaged distribution and $\delta E(t)$ is the fluctuation. The IBS-induced growth, assuming no initial longitudinal velocity spread, is then

$$\Delta\sigma_v^2(t) = \left(\frac{e}{m}\right)^2 \int_0^t \mathrm{d}t_1 \int_0^t \mathrm{d}t_2\, \langle \delta E_i(t_1)\, \delta E_i(t_2)\rangle_i. \tag{5}$$

We omit δ in what follows.

In the simulations, we repeat this procedure for discrete time steps. We generate N electrons, define their trajectories (the theories do not take the IBS influence on the bunch into account), and calculate the electric fields at each point of time at each particle position. Eq. (5) is then computed with the substitution $\int_0^t \mathrm{d}t_1 f(t_1) \to t_s \sum_{i=1}^M f(t_i)$ and compared with a suitable theory. Here t_s is the time step, and M is the number of steps in the simulation.

All theories assume a constant particle density in time, which is supported by either constrained motion (for example, betatron oscillations), or infinite extent by one or more coordinates . Infinities are not supported in simulations, so the particle trajectories are altered accordingly.

To suppress the average field E_0 without altering fluctuations, half the bunch is assigned positive charge (positrons) and half negative (electrons). This leaves the second moment unaffected while canceling the mean field, thereby eliminating the SC contribution. The comparison with the average growth rate A from Eq. (2) gives:

$$\frac{t_s}{e^2 M}\sum_{i,j=1}^{M} \langle E_l(t_i) E_l(t_j)\rangle_l = \frac{A\,N}{\sigma_x^2\,\sigma_{v_x}\,\sigma_z}. \tag{6}$$

UNIFORM PLASMA

We first verify the most simple (in terms of theory) case: a uniform plasma, where an infinite space is uniformly filled with particles. The field is damped on a characteristic distance a (otherwise the IBS growth rate diverges due to the infinite sizes of the plasma), and is constant at small distances $r < d$, where d is a parameter. The longitudinal component of the field between is then

$$E(r)/e = \begin{cases} \frac{z}{r^2}\left(\frac{1}{r} + \frac{1}{a}\right) e^{-r/a} & \text{if } r > d, \\ \frac{z}{r\,d}\left(\frac{1}{d} + \frac{1}{a}\right) e^{-d/a} & \text{if } r < d, \end{cases} \tag{7}$$

where z is the longitudinal component of the radius-vector between the particles r. Our theoretical calculations, which will be presented elsewhere for a two-dimensional gaussian distribution of velocities, give:

$$\frac{\mathrm{d}\sigma_v^2}{\mathrm{d}t} = \frac{I\,r_e^2\,c^4}{2\,\pi}\frac{n}{\sigma_{v_x}}\Lambda, \tag{8}$$

where $I \approx 35$, n is the particle density, and Λ, although denoted with the same symbol as the Coulomb's logarithm, is a specific function, that is very close to the Piwinski Λ. In this case, A is not strictly defined, because the bunch sizes, as well as the number of particles, are infinite. We can substitute the infinite space with a large but finite volume V, with characteristic size $r \gg a$, with keeping the dynamics and results mostly intact. After this, we can define A:

$$A = \frac{I\,\sigma_x^2\,\sigma_z}{\pi\,V}\Lambda, \tag{9}$$

where σ_x and σ_z are the rms sizes of the volume V.

In this paper, we choose a sphere as the volume. The electric field E is exactly the same as assumed in the theory. To simulate the constant particle density in time, we redirect particles back to the volume as soon as they cross its border (flip the sign of the velocity).

A comparison of A from the theory and simulations is presented in Fig. 1. The values are shown for different integration times, emphasizing the saturation of the energy spread growth rate. The bunch parameters on the plot are: $a = 0.05\,r$, $d = 1\times10^{-3}\,r$, where r is the radius of the sphere. The red dashed line is the theoretical prediction, and the black line is obtained from the simulation.

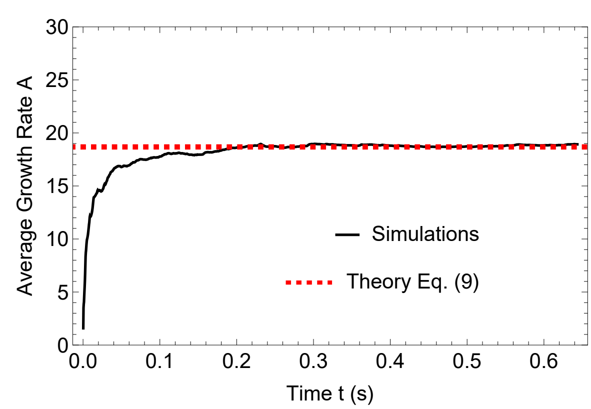

Figure 1: Uniform plasma case. A dependence on time t. Comparison of the simulation results with the theoretical expectations for three dimensional isotropic Gaussian distribution of velocities.

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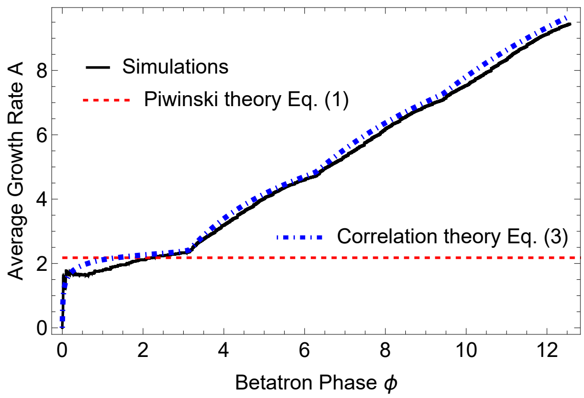

Figure 2: Same as in Fig. 1, but for a bunch with betatron motion and Gaussian density distribution in transverse plane, and uniform density distribution by z. $\tau = 0$, $d = \sigma_x/5000$, $\sigma_x = \sigma_y = L$.

BETATRON MOTION

We can see from Eq. (5) that the IBS growth rate depends on the electric field *correlations* in time. Therefore, if the particle motion is periodic, the growth rate is higher because the fields depend on the particle distribution.

The average velocity squared growth rate is given by Eq. (3). The dimensionless wavenumber $\kappa = 2\,\pi\,\sigma_x/\lambda$ is the inverse of the bunch density modulation wavelength in units of the bunch transverse size. The highest κ is determined by the smallest allowed impact parameter d ($\kappa_{\max} = \sigma_x/d$), and the lowest κ - by the longest modulation wavelength by z axis. The truncation by $\kappa_{\max}$ is required in theory to avoid divergence, and in simulations—to avoid infinite fields. Truncation by $\kappa_{\min}$ is not required if the longitudinal size of the bunch is infinite, but necessary when it is finite. When $\sigma_z = \sigma_x$, we can set $\kappa_{\min} = 1$.

For large κ, the function under the integral is peaked at betatron phases $\phi = \pi\,n$. The conventional Piwinski theory takes only the first peak at $\phi = 0$ into account.

The electric field between two particles is the pure Coulomb's field, with a constant cutoff at small distances:

$$E(r) = \begin{cases} \frac{z}{r^3} & \text{if } r > d \\ \frac{z}{r\,d^2} & \text{if } r < d \end{cases} \quad (10)$$

The x and y planes are not coupled, and particle trajectories in x and y phase spaces are perfect circles. The beta-function is taken to be $\beta = 1$ m to avoid oscillations of the beam size.

A comparison with the theories for a two-dimensional Gaussian distribution of velocities ($\tau = 0$), Gaussian density distribution in transverse plane, and uniform longitudinal, is presented in Fig. 2. Black is simulations, red is Piwinski theory, and blue is the new correlations theory. The minimal interaction distance is $d = \sigma_x/5000$. The bunch is truncated longitudinally at $z = -L$ and $z = L$, with $L = \sigma_x$.

It is clearly seen that the Piwinski theory is the limit of the simulations and correlations theory in the absence of the peaks at $\phi = \pi\,n$, $n = 1, 2, 3....$ When the particles make

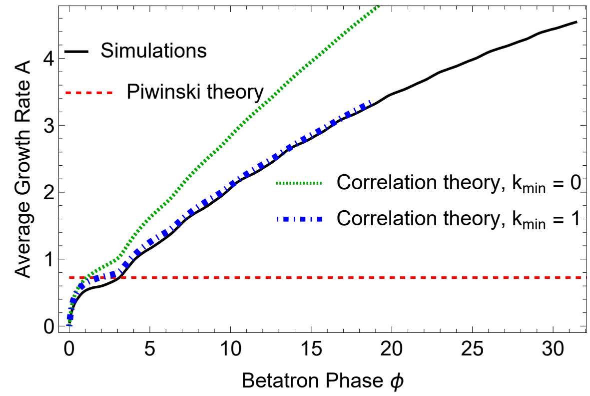

Figure 3: Same as in Fig. 2, but with different parameters. $\tau = 0.5 \times 10^{-4}$, $d = \sigma_x/17$. Two different choices of $\kappa_{\min}$ are made for theoretical predictions.

half the betatron revolution, the growth rate increases due to the correlations.

When an **initial energy spread** is present, the particles have to be confined in the longitudinal direction. For that, we make the boundaries at $z = -L$ and $z = L$ reflective. When a particle with velocity v_0 leaves the cylinder, we introduce a new particle with $v_1 = -v_0$ on the boundary to simulate constant density. The betatron motion parameters (J_x, J_y, ϕ_x, ϕ_y) are randomly generated from the corresponding distribution functions.

A comparison with the theories for the same bunch as in Fig. 2 but with $\tau = 0.5 \times 10^{-4}$ and $d = \sigma_x/17$ is presented in Fig. 3. The growth rate increases by less and less amounts at $\phi = \pi\,n$, $n = 1, 2, 3...$ peaks, eventually saturating at a characteristic phase $\phi_{\text{sat}} = 1/k_{\max}\,\tau$.

SUMMARY

A new approach to IBS calculation, incorporating temporal correlations from periodic betatron motion, predicts an enhanced energy spread growth beyond conventional models such as Piwinski's. In this paper, we presented the IBS simulations based on the direct computation of the electric field fluctuations. Our results show a good agreement with various models, including the exactly solvable uniform plasma case and beams with betatron motion. This agreement validates both the simulation method and the correlation-based theory within their range of applicability, highlighting the importance of time-correlated field effects in accurate IBS modeling. These simulations are now suitable for application to real-world accelerator systems.

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