ISBN: 978-3-95450-261-5 ISSN: 2673-7000 doi: 10.18429/JACoW-NAPAC2025-THP059

ORTHOGONAL DIRECTIONS CONSTRAINED GRADIENT METHOD FOR BEAM OPTICS CORRECTION*

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Abstract

An Orthogonal Directions Constrained Gradient Method (ODCGM) has been developed and experimentally used for optimization and correction of H^- optical beam parameters for laser assisted charge exchange injection (LACE) experiments. LACE experiment requires precise tune up of H^- beam parameters for high efficiency stripping. High precision tuning of beam parameters cannot be done in one step due to miscellaneous errors of power supplies and other factors. Then, subsequent application of the optic correction method considered here can do fine tune up of the beam. A simple experimental demonstration of ODCGM is presented.

INTRODUCTION

In this paper we consider an Orthogonal Directions Constrained Gradient Method (ODCGM) method for precise tune-up of H^- beam parameters for Laser Assisted Charge Exchange injection (LACE) experiments [1,2]. This method has been studied [3] in math and computation sciences for general optimization problems. Here we adopted this method for the practical problem of ion beam optics optimization.

The LACE project considers interaction of H^-/H^0 bunch with a powerful laser for excitation and stripping of hydrogen beam (see Fig. 1). High efficiency excitation and stripping of a hydrogen beam requires extremely precise adjustment of beam parameters at the interaction point. The ion beam must be strongly focused at the interaction point. Also, other parameters such as dispersion must be adjusted precisely for crab-crossing scheme of laser-bunch interaction [4].

During normal SNS operation, we are not required to tune up the beam with high precision in order to provide beam transport. Unfortunately, this approach will not work for high-accuracy beam tune up required by LACE due to the errors of magnet fields and input H^- beam parameters. Also, the model of beam propagation used for beam tune-up has its own error. H^- beam parameters at the focal point of interaction are very sensitive to all listed errors. Standard tune-up methods can be used for preliminary optics setup for LACE at the interaction point. Then, we can apply the

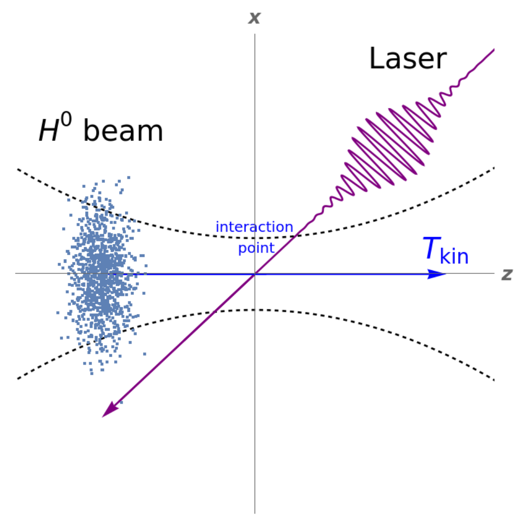

Figure 1: Interaction of H^0 bunch with laser.

ODCGM method for beam optics correction and precise tune-up of the desired parameters at the focal point. The ODCGM algorithm calculates the quadrupole magnet corrections based on correction of one selected beam parameter to the desired accuracy while keeping other selected optical beam parameters unchanged. The method is very similar to the beam orbit correction technique [5] used at the SNS. The orbit correction method uses corrector magnets and a linear accelerator model to correct the orbit position. The beam optics correction method considered here uses quadrupole magnets to correct beam optics parameters instead of orbit. For this purpose, the ODCGM algorithm adopted and developed here is based on local gradient mechanism of search for optimal beam correction. The algorithm of the method itself can also be applied to other problems in accelerator physics, not only beam optics tune-up.

In this paper, we first describe the LACE optimization problem using the ODCGM algorithm.

BEAM OPTICS CORRECTION ALGORITHM

Beam optics is normally characterized by the Twiss parameters α, β, ϵ for the vertical and horizontal directions. Dispersion parameters $D, \partial D/\partial z$ that play important role in LACE beam tune up are also considered. For simplicity, we will denote all beam parameters as p_i. Figure 2 schematically represents the beam along the z axis with beam parameters $p_1 ... p_n$ that can generally vary with z_i. In our problem, we consider the tune-up parameters only at one location of laser-bunch interaction. Using the input Twiss parameters and

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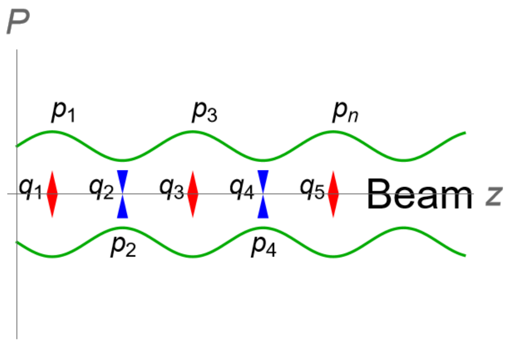

Figure 2: Schematic representation of beam with miscellaneous parameters p_i depending on quadrupole fields q_j.

the beam propagation physics model, we can calculate the parameters p_i as functions of quadrupole fields q_j:

$$\begin{cases} p_0 = f_0(q_1, q_2, ...q_m) \\ p_1 = f_1(q_1, q_2, ...q_m) \\ p_2 = f_2(q_1, q_2, ...q_m) \\ ... \\ p_n = f_n(q_1, q_2, ...q_m) \end{cases} \tag{1}$$

In general, the required beam parameters, p_i, can be calculated and adjusted by using Eq. (1) but it is hard to do with high precision due to experimental errors of quadrupole fields q_j, input beam parameters and model error. The ultimate goal of this problem is to find quadrupole corrections, Δq_j, that will perform correction of desired parameter Δp_0 into the goal value while keeping all other parameters $\Delta p_i = 0, i > 0$ unchanged. To solve this problem, we first define the gradient of the i^{th} parameter p_i with respect to the quadrupole strengths q_j:

$$\vec{g}_i = \nabla p_i = \frac{\partial f_i}{\partial q_j}. \tag{2}$$

This can be performed numerically in the case of a numerically calculated model. Now we need to find the vector $\vec{n}$ that is orthogonal to all constraining gradient parameters to be unchanged while maximizing the change of corrected parameter p_0:

$$\begin{gathered} \vec{n} \cdot \vec{g}_0 \to max \\ \vec{n} \cdot \vec{g}_i = 0, \quad i = 1...n \end{gathered} \tag{3}$$

This is a linear algebra problem that gives a single solution for $\vec{n}$ if the number of quadrupoles, m, is equal to or greater than the number of beam parameters, n+1: $m \geq n + 1$. The more quadrupoles involved in the beam tune up, the better solution can be obtained. Figure 3 schematically represents the problem of beam optics correction for one selected parameter p_0 keeping all other parameters p_j unchanged. Correction starts from initial quadrupole settings $\vec{Q}_0 = (q_1, q_2, ...q_m)$. Then using a gradient descent δq moving along vector $\vec{n}$ we can achieve desired correction of parameter Δp_0. By setting up the small step of the curve length δq the change of beam parameter δp_0 can be calculated as the scalar product of the unit vector and the gradient of the parameter:

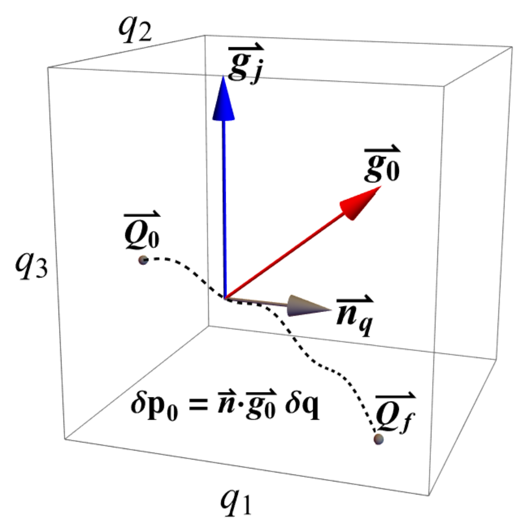

Figure 3: Beam optics correction algorithm. Red vector, g_0, represents gradient of tunable beam parameter. Blue vector, g_j, represents set of gradients for beam parameters to be unchanged. The gray vector, $\vec{n}_q$, represents optimal quadrupole correction within all variable quadrupoles.

$$\delta p_0 = \vec{n} \cdot \vec{g}_0 \quad \delta q. \tag{4}$$

The beam parameter change, Δp_0, and the corresponding quadrupole setting, $\vec{Q}_f$, are calculated through numerical integration:

$$\begin{gathered} \Delta p_0 = \sum \delta p_0 = \sum \vec{n} \cdot \vec{g}_0 \quad \delta q, \\ \vec{Q}_f = \vec{Q}_0 + \sum \vec{n} \delta q. \end{gathered} \tag{5}$$

We can find the vector $\vec{n}$ from Eq. (3) by solving system of linear equations in order to find orthogonal space of vectors. Then we can calculate projection n_q of vector g_0 onto the orthogonal space.

Computation Algorithm

Here we will represent final solution for any number of n and m. We will define some matrices and vectors as sub-matrices of Jacobian matrix $g_{ij} = \partial f_i / \partial q_j, i = 0...n, j = 1...m$. Vector $\mathbf{\vec{g}}_0 = (g_{0,1}, g_{0,2}, ..., g_{0,m})$ is the gradient of the tunable beam parameter. Matrices $\mathbf{M}_1, \mathbf{M}_2$ are defined as the following sub-matrices consisting of orthogonal gradient components:

$$\begin{gathered} \mathbf{M}_1 = \begin{pmatrix} g_{1,1} & \cdots & g_{1,m-n} \\ \vdots & \ddots & \vdots \\ g_{n,1} & \cdots & g_{n,m-n} \end{pmatrix}, \\ \mathbf{M}_2 = \begin{pmatrix} g_{1,m-n+1} & \cdots & g_{1,m} \\ \vdots & \ddots & \vdots \\ g_{n,m-n+1} & \cdots & g_{n,m} \end{pmatrix}. \end{gathered} \tag{6}$$

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Then the solution $\vec{n} = \vec{s}/|\vec{s}|$ of the problem (3) can be calculated as:

$$\vec{\mathbf{s}} = (\mathbf{I}\,|\,\mathbf{M})^{\mathbf{T}}(\mathbf{M}\mathbf{M}^{\mathbf{T}} + \mathbf{I})^{-1}(\mathbf{I}\,|\,\mathbf{M})\vec{\mathbf{g}}_0$$
$$\mathbf{M} = -(\mathbf{M}_2^{-1}\mathbf{M}_1)^{\mathbf{T}} \quad . \tag{7}$$

Here, $(\mathbf{I} \mid \mathbf{M})$ represents concatenation (combining) of $(m-n)$-dimensional identity matrix $\mathbf{I}$ and matrix $\mathbf{M}$. Upper symbols "$\mathbf{T}$" and "-1" denote transposed and inverse matrix transformations correspondingly. Expression (7) represents an algorithm for fast computation of beam parameter correction using linear algebra operations.

EXPERIMENTAL BEAM TUNE UP

We applied the method for current LACE experiments at the SNS. The H^- bunch must be focused vertically as much as possible at the interaction point with the laser. Figure 4 represents problem of beam tune up for LACE. The blue

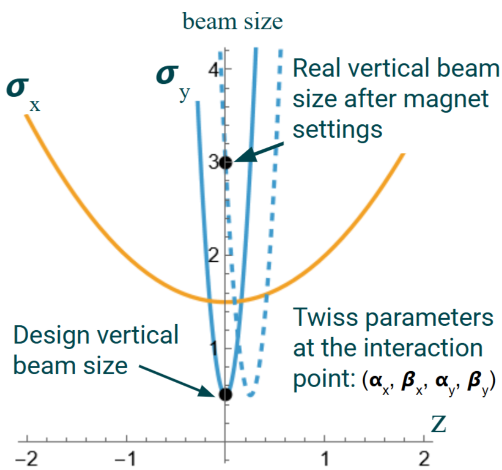

Figure 4: Schematic representation of vertical beam focus tune up for LACE.

solid line schematically represents design and goal vertical beam size that must be strongly focused at the interaction point. After design quadrupoles are set, the real beam size (dashed blue line) will have bigger size due to focus position error. In this way we need to correct the vertical focus while keeping the horizontal beam size unchanged.

In a simple case we consider only 4 Twiss parameters at the interaction point that depend on few quadrupoles (minimum 4) that we use to fix the focus: $(\alpha_x, \beta_x, \alpha_y, \beta_y) = \mathbf{F}(q_1, q_2, ..., q_m)$. To perform this correction we need to lower vertical β function to the design value and α to zero at the interaction point simultaneously. Instead of doing this, we will redefine parameters α_y, β_y at the interaction point into parameters of focus position $\Delta z_y = \alpha_y\beta_y/(1+\alpha_y^2)$ relative to the interaction point and minimum beta function at that position assuming parabolic approximation: $\beta_y(min) = \beta_y/(1+\alpha_y^2)$. After this transformation we can correct only one parameter Δz_y keeping all other 3 parameters unchanged. Then the problem (7) will be defined as

$$\begin{cases} \Delta z_y = \alpha_y\beta_y/(1+\alpha_y^2) = f_0(q_1, q_2, ..., q_m) \\ \beta_y(min) = \beta_y/(1+\alpha_y^2) = f_1(q_1, q_2, ..., q_m) \\ \alpha_x = f_2(q_1, q_2, ..., q_m) \\ \beta_x = f_3(q_1, q_2, ..., q_m) \end{cases} . \tag{8}$$

Figure 5 shows experimental results on vertical beam focus correction in LACE experiments. Because Δz_y is not a

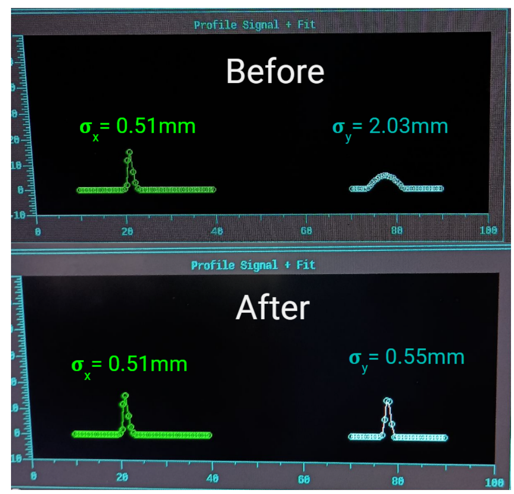

Figure 5: Measured wire scanner vertical and horizontal beam profiles before vertical focus correction and after correction

measurable parameter we perform correction by a few iterations until the minimum vertical beam size is achieved and measured.

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