

# THE PULSED ION REFLEX KLYSTRON: A NEW ACCELERATOR FOR HIGH EFFICIENCY VOLTAGE CONVERSION

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## Abstract

Beam Alpha has developed a kilowatt-scale fusion microreactor that directly converts nuclear energy to electrical energy with an output of 1.6 MV DC. The “Pulsed Ion Reflex Klystron” has been developed to transform this to useful electrical power. This tube aims to achieve high efficiencies by directing negative ions through a re-entrant resonant cavity hundreds of times to gradually transfer energy from the moving particles to said cavity. Ideal axial and radial fields for achieving this particle motion have been derived and simulated for two different modes of operation, one involving isochronous bunches and one involving repeated harmonic switching.

## INTRODUCTION

In this paper, two different tube topologies for efficiently implementing the conversion from 1.6 MV DC to a transformable RF wave [1] will be presented and compared. This RF wave can then be rectified into an intermediate ~400V DC bus, allowing for easy conversion to other useful power levels. The first topology is an isochronous design, where particle bunches oscillate back and forth inside the tube [2] with equal frequency regardless of how much energy has already been coupled out of them. The second design utilizes harmonic switching, so that a bunch’s period of oscillation through the tube decreases by one cavity oscillation period per pass through the cavity.

## IDEAL ISOCHRONOUS POTENTIAL

A parabolic axial potential causes nonrelativistic particles to take the same amount of time to accelerate to  $z=0$  regardless of initial rest position. Axial voltage is:

$$V(z) = -\frac{2\pi^2 m_i f^2}{q} z^2 \quad (1)$$

Where  $m_i$  is the ion mass,  $f$  is the frequency of axial particle oscillation and  $q$  is the elementary charge. A resonant cavity is then placed at  $z = 0$  to couple energy out of ion bunches.

This potential is problematic because of its inherent defocusing nature in a cylindrical geometry, where radial electric field at a small distance from the axis is:

$$E_r = -\frac{r}{2} \frac{dE_z}{dz} \quad (2)$$

Where  $E_r$  is radial electric field,  $E_z$  is axial electric field, and  $r$  is radial position. [3] Thus, a parabolic potential as described above will produce a defocusing force on negative ions resulting in exponential outward motion.

To combat this, a perturbation must be applied to the potential to produce strong focusing. The perturbation must be quasi-periodic over voltage increments of double the average cavity deceleration voltage, so particle bunches experience a specially-tuned temporary inward force each

time they slow down to a halt. The basic waveform and amplitude of this perturbation can be fully derived simply from the desire to maximize stable initial ion pulse length and the fact that axial and radial bunching must occur in order to keep ions inside of the tube. Once a desired cavity oscillation frequency and voltage are decided on, there are no other significant free parameters. The perturbation is removed at voltages below a threshold value, under which ions will be ejected from the tube.

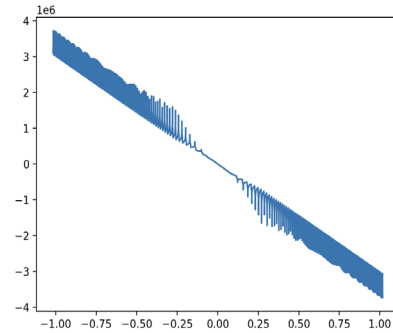


Figure 1: Ideal  $E_z$  over  $z$  for a 1.6 MV, 2.778 MHz Isochronous Pulsed H- Ion Reflex Klystron with 8.101 kV peak cavity voltage, perturbation threshold 108 kV.

For a 1.6 MV, 2.78 MHz hydrogen ion tube with resonant cavity voltage of 8.101 kV, the amplitude of this perturbation ranges from about 300-900 volts, resulting in a 2 M long tube with electric field p-p perturbation of about 600 kV/M throughout (Fig. 1). Lowering operation frequency results in the same voltage curve, just stretched, and thus electric field is directly proportional to frequency.

## HARMONIC SWITCHING POTENTIAL

When the axial potential is changed to be proportional to  $z^{\frac{2}{3}}$ , the time necessary for a bunch to accelerate from rest to the center of the tube is now linearly proportional to initial potential energy. In the non-relativistic case:

$$V(z) = -\frac{(3\pi\sqrt{2})^{\frac{2}{3}}}{4} \left(\frac{m_i}{q}\right)^{\frac{1}{3}} \left(\frac{V_0}{t_0}\right)^{\frac{2}{3}} |z|^{\frac{2}{3}} \quad (3)$$

Where  $t_0$  is the time necessary to accelerate to  $z=0$  from  $V(z) = V_0$ . This curve has a positive second derivative, meaning that with the exception at  $z=0$ , the potential will be radially focusing instead of defocusing. As such, no significant perturbation is needed for radial focusing (Fig. 2).

A potential like this decelerates ion bunches by utilizing harmonic switching each time the bunch goes back and forth through the tube. The last time the beam passes through the cavity it will have been  $5/2$  periods of cavity oscillation since the second-to last pass. The third-to-last pass will also be  $5/2$  periods before the second, but ion motion will occur on the other side of the tube. The next two

tube traversal times will then be  $7/2$  periods, then  $9/2$ ,  $11/2$  and so on.

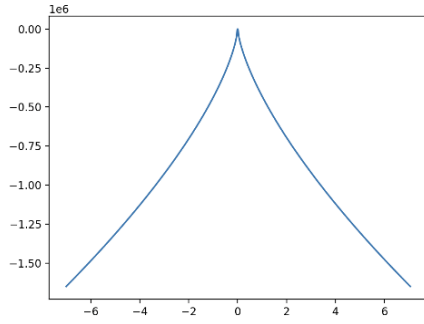


Figure 2: Potential vs  $z$  for a 1.6 MV Harmonic Switching Pulsed I- Ion Reflex Klystron with a 5 MHz cavity with peak voltage 9 kV.

There are two corrections in this potential that must be made for ideal operation. First, as  $z$  approaches zero, the derivative of  $|z|^{3/2}$  approaches infinity. If the tube traversal time is switched to constant at low voltages, the potential there will become parabolic and thus be practically realizable. This portion of the tube will be defocusing, and thus this will also allow low energy ions to be ejected. Secondly, due to high harmonic switching, traversal time between the two cavity electrodes can become significant. This is corrected by slightly adjusting the voltage levels at which half-tube traversal times are odd multiples of half cavity period and causes the deceleration voltage to slightly decrease near  $z=0$ . A computer program can then be used to solve for the target potential by solving the relativistic equation:

$$t = \left| \int_{z_0}^{z_1} \frac{V(z) - V(z_0) + \frac{m_i c^2}{q}}{3 \cdot 10^8 \sqrt{(V(z) - V(z_0))(V(z) - V(z_0) + 2 \frac{m_i c^2}{q})}} dz \right| \quad (4)$$

## AXIAL BUNCHING

Axial position and velocity can be transformed into a normalized phase space in both topologies described above. This can be used to maximize regions of stability and discern maximum possible ion pulse lengths. This 2-dimensional phase space is made up of “net energy,” and “phase delay”. Net energy is expressed as:

$$E_n = \frac{E_k + E_p - E_{pi}}{q V_d} + N \quad (5)$$

where  $E_k$  and  $E_p$  are kinetic and potential electron energy,  $E_{pi}$  is the initial potential energy of the ion bunch on average,  $V_d$  is average bunch deceleration voltage during one pass through the resonant cavity, and  $N$  is the number of passes through the cavity that the bunch has already undergone. Phase delay is expressed as:

$$p_d = 2\pi \left( \frac{t(v, z) + t}{p} - \frac{N^*}{2} - \frac{1}{4} \right) \quad (6)$$

Where  $t(v, z)$  is the time it will take to get to the resonant cavity next given current electron velocity and position,  $t$  is the time that has passed since the start of beam injection into the tube, and  $N^*$  is the total number of periods of cavity oscillation that should have occurred between injection and the last cavity crossing. In the isochronous case,  $N^*=N$ . Using these quantities with an optimal potential, an

ideal bunch will have both a net energy and phase delay of zero for its entire path.

By understanding the time-varying nature of the cavity voltage and the relation between initial ion energy and tube traversal time, a particle's position in phase space can be calculated after each pass through the cavity. Voltage curvatures for both the isochronous and harmonic switching case have been optimized to maximize the region of stability of axial ion motion using this method.

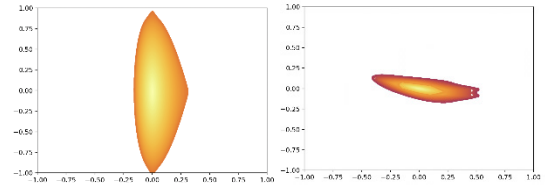


Figure 3: Region of axial stability at beam injection in isochronous (left) and harmonic switching (right) case.

It was found that if a uniform initial energy bunch was injected into a 1.6MV tube with target deceleration voltage of about 8kV, maximum stable pulse length was theoretically .46 radians for the isochronous case, and .82 radians for the harmonic switching case (Fig. 3).

## SIMULATION IN LTSPICE

To verify the curves described in the previous section, a simulation in LTSpice was generated to predict particle motion in the Pulsed Ion Reflex Klystron. This simulation is capable of analyzing point charges inside of the tube. Fundamentally, this simulation has two important parts: a module that simulates axial motion of the particle, and a module that uses axial motion to solve for radial motion.

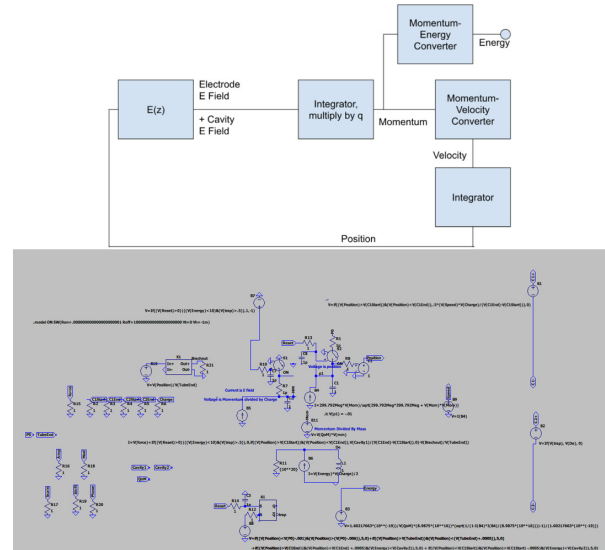


Figure 4: Block diagram of simulation methodology (top), corresponding LTSpice circuit (bottom).

The axial motion of an ion in a cylindrical geometry is generated by integrating the axial electric field as a function of time to get momentum, using momentum to find velocity, and then using velocity to produce a position. This electric field consists of two components: the ideal electric field as described in the previous section, and a simulated

resonant cavity electric field in the center, modeled as a constant-over-space sinusoidally varying electric field which integrates over space to the total cavity voltage at a particular point in time (Fig. 4).

To calculate the radial motion of a point charge, a lookup table is used to map the axial position of the particle to a particular , and radial acceleration is derived using equation 2. Doubly integrating this produces radial position, defining a differential equation (Fig. 5).

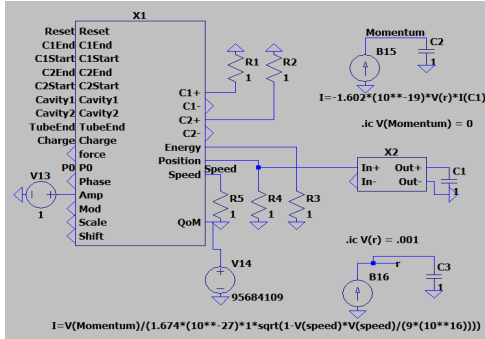


Figure 5: LTSpice Circuit diagram of radial focusing model. X1 is a container for the circuit in Fig. 4.

## RESULTS AND COMPARISON

Simulation was then done for both a 6 M long .926 MHz isochronous tube utilizing H- ions with a peak cavity voltage of 8.101 kV, and a 14 M long 5 MHz harmonic switching tube utilizing I- ions with a peak cavity voltage of 9 kV. Both were shown to maintain axial and radial stability for the entirety of particle lifetime, indicating at least 93% and 96% beam to cavity efficiency for the isochronous and harmonic switched case respectively. Maximum stable pulse length was found to be at least 64 ns for the isochronous tube, and at least 22 ns for the harmonic switching tube, approaching the theoretical maximum values of 80 ns and 26 ns respectively.

Both simulations also demonstrated satisfactory radial focusing, keeping radial test particle position within reasonable bounds until the particles lost most of their energy, at which point the particles were ejected (Fig. 6).

The biggest differences in the practical implications of these two topologies are with operating cavity frequency (as a function of ion mass and tube length), and the rate of change of the axial electric fields. A harmonic switching tube will have a frequency of  $\frac{2V_i}{3V_d}$  times that of an isochronous tube of the same length, where  $V_i$  is initial particle voltage and  $V_d$  is average deceleration voltage per pass through the cavity. For 200 passes, this is a factor of  $\sim 133$ . As such, heavier ions must be used in the harmonic switching case to keep pulse lengths and tube dimensions manageable. This is why the isochronous design utilizes H- ions and the harmonic switching design utilizes I-.

In addition, the isochronous design has an electric field that is much more spatially variant. This makes it more challenging to reproduce with a physical electrode configuration, as multiple electrodes are needed per period of perturbation to accurately produce the target waveform. The harmonic switching tube has a much smoother voltage

curve, which should translate to less electrodes. Another important point is that in the isochronous case, the factor that specifies the required voltage curve accuracy is radial focusing, whether in the harmonic switching case, axial bunching is limiting. This is because although radial focusing is basically trivial in the harmonic switching case, maximum stable axial emittance is lower, and small errors in actual potential can more easily cause bunch instability.

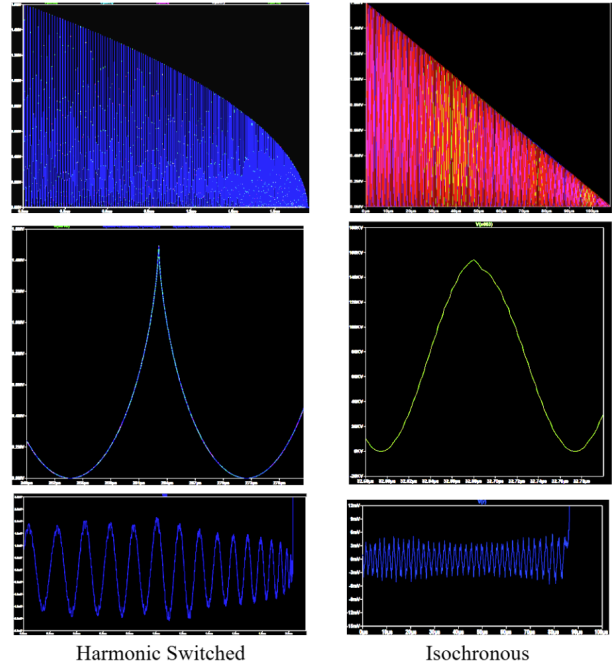


Figure 6: Pulsed Ion Reflex Klystron waveforms: Top: bunch energies (eV) vs time (s) over particle lifetime, Middle: bunch energies over single tube traversal, Bottom: radial position (m) vs time (s).

## CONCLUSION

Two topologies for the Pulsed Ion Reflex Klystron have been proposed and simulated using LTSpice. These tubes are meant to take 1.6 MV DC and convert to a RF wave that can be easily transformed and rectified to an intermediate DC bus. Simulations have demonstrated that both the ideal isochronous and harmonic switching potentials have high beam to cavity conversion efficiencies. Beam Alpha plans to build a prototype of one of these topologies in the near future.

## REFERENCES

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