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FIRST OBSERVATIONS OF DISPERSIVE SHOCK WAVES FROM THE KDV EQUATION IN AN ELECTRON BEAM

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Abstract

We present the first experimental observation of dispersive shock waves (DSWs) in an electron beam. Using the University of Maryland Electron Ring (UMER), a 10 keV electron beam was perturbed with a localized beam velocity dip. The resulting current profiles evolve into oscillatory structures consistent with dispersive shock behavior governed by the Korteweg–de Vries (KdV) equation. These results are predicted by particle-in-cell simulations using the WARP code. The observed dynamics open new avenues for studying nonlinear longitudinal phenomena in intense beam systems.

INTRODUCTION

Dispersive shock waves (DSWs) are nonlinear wave structures characterized by an expanding train of oscillations trailing a soliton-like leading edge. They arise when nonlinear wave steepening is balanced by dispersion, a phenomenon observed in a variety of physical contexts such as tidal bores and atmospheric "morning glory" waves [1], as well as laboratory experiments with plasmas [2–5], nonlinear optics [1, 6–8], classical fluids [9, 10], and superfluids [11, 12]. Despite the differing media, the defining feature remains consistent: nonlinear steepening modulated by dispersion leads to the formation of a characteristic wave train.

The Korteweg–de Vries (KdV) equation models the dynamics of weakly nonlinear, dispersive waves and admits solitary and cnoidal wave solutions [13]. For step-like initial conditions, it predicts the formation of a DSW, as demonstrated by Gurevich and Pitaevskii [14] and confirmed numerically by Fornberg and Whitham [15].

In charged particle beams, similar wave dynamics arise from the interplay of nonlinear space-charge forces and dispersive effects due to self-consistent electric fields. Under the assumptions of cold, intense beams with negligible longitudinal emittance, the beam fluid model reduces to a KdV-like equation [16]:

$$\frac{\partial u}{\partial t} + \alpha u \frac{\partial u}{\partial z} + \beta \frac{\partial^3 u}{\partial z^3} = 0, \quad (1)$$

where $u(z,t)$ represents a velocity or current perturbation, and α, β are coefficients describing nonlinearity and dispersion, respectively. Previous work has demonstrated soliton formation in beams triggered by positive perturbations, exhibiting characteristic linear relationships between velocity and amplitude as well as amplitude and width squared [16, 17].

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In this study, we report the first experimental observation of DSWs in a recirculating electron beam using the University of Maryland Electron Ring (UMER). By introducing localized velocity perturbations, we observe expanding nonlinear structures consistent with DSWs, providing a controlled environment to validate theoretical predictions and simulations over multiple passes.

THEORY

The KdV equation captures the evolution of nonlinear waves in dispersive media. When initialized with a localized negative perturbation, the solution does not form a solitary wave but instead develops a modulated wave train characteristic of a dispersive shock wave.

Figure 1 shows a numerical solution of the KdV equation evolved from such a negative initial perturbation. Nonlinear steepening sharpens the wavefront until a gradient catastrophe occurs; subsequently, dispersion dominates, generating oscillations. The leading edge travels fastest and possesses the largest amplitude, a defining signature of a DSW.

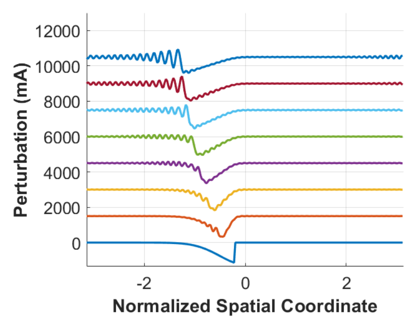

Figure 1: Numerical solution of the KdV equation on a periodic domain $x \in [-\pi, \pi]$, evolved using a pseudo-spectral method in space and a fourth-order Runge–Kutta scheme in time. Time increases vertically in steps of 100 µs. A negative initial perturbation develops into a dispersive wave train.

This theoretical evolution provides a template for the beam experiment: an initial negative velocity perturbation steepens and evolves into a dispersive shock structure with a leading soliton followed by dispersing oscillations, consistent with KdV dynamics.

PARTICLE-IN-CELL SIMULATIONS

To investigate DSW formation in an intense electron beam, we performed numerical modeling using the WARP particle-in-cell (PIC) code [18] in cylindrical (R–Z) geometry. The

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beam was initialized with uniform density and velocity, matching the experimental parameters of the University of Maryland Electron Ring (UMER). The simulations tracked 16 million macroparticles with a time step of 1 ns, using 64 radial and 2048 axial cells. The domain extended 0.0254 m radially, bounded by a conducting wall, and 11.52 m axially with periodic boundary conditions to replicate the ring geometry. The beam energy was set to 10 keV. All numerical parameters were verified for convergence, and prior studies have shown excellent agreement between this model and experiment [16].

A one-time, localized longitudinal electric field perturbation was applied in the beam frame to mimic the experimental induction-cell kick. This perturbation was centered in the axial domain and shaped as a narrow Gaussian pulse in velocity space, with a width corresponding to approximately 10 ns of beam travel time. After one pass through the simulation domain, the perturbation was removed, allowing the beam to evolve self-consistently under its own space-charge forces and the imposed boundary conditions. This approach emulates a transient energy modulation and enables analysis of the subsequent dispersive shock formation and relaxation.

The initial velocity perturbation generates two oppositely signed current perturbations. These separate into a fast wave and a slow wave, propagating in opposite directions in the beam frame [19, 20]. After several turns, the faster wave develops into a DSW, with the leading edge forming a sharp, high-amplitude front followed by a decaying oscillatory wake. This behavior is observed in Figure 2.

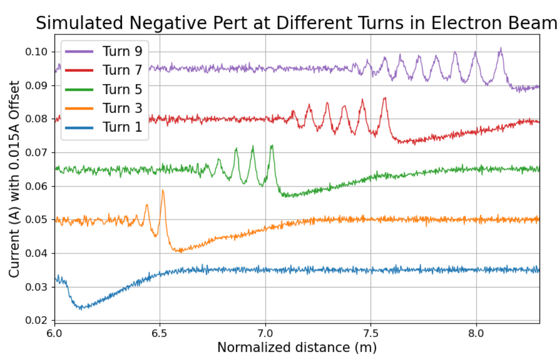

Figure 2: WARP PIC simulation of DSW formation from a negative velocity perturbation in a UMER-like electron beam. Shown here is the fast wave over turns 1-9. Every 2 turns is shown with a 0.015 A offset between turns.

EXPERIMENTAL SETUP

The experimental studies were performed using the University of Maryland Electron Ring (UMER), an 11.52 m circumference storage ring used for investigations of space-charge-dominated beam dynamics. For this work, a 10 keV electron beam was injected as a single, long, rectangular bunch with a duration of 100 ns. The beam current was measured once per turn using a wall current monitor (WCM) located 7.67 m downstream from the injection point. During the experiments, the peak beam current was varied between 20 mA and 50 mA.

To generate a localized velocity perturbation, we employed an induction cell located between the injection point and the WCM. This device was driven by a pulsed voltage timed to coincide with the passage of a specific beam segment. As the beam traversed the induction cell, the applied voltage pulse imparted a controlled energy deviation to the targeted segment without significantly affecting the transverse beam dynamics. By adjusting the pulse amplitude and duration, we could control both the magnitude and the longitudinal extent of the velocity perturbation [21]. The pulsed voltage loop couples to the beam via the cell gap, creating the localized longitudinal electric field that produces the desired perturbation.

RESULTS

The evolution of the velocity perturbation in the experiment was measured turn-by-turn using the WCM. To highlight the perturbation dynamics, the signal from an unperturbed reference beam was subtracted from each turn's waveform. Figure 3 shows the resulting difference signals for turns 9 through 16, revealing the development of a sharp leading edge followed by a decaying oscillatory structure.

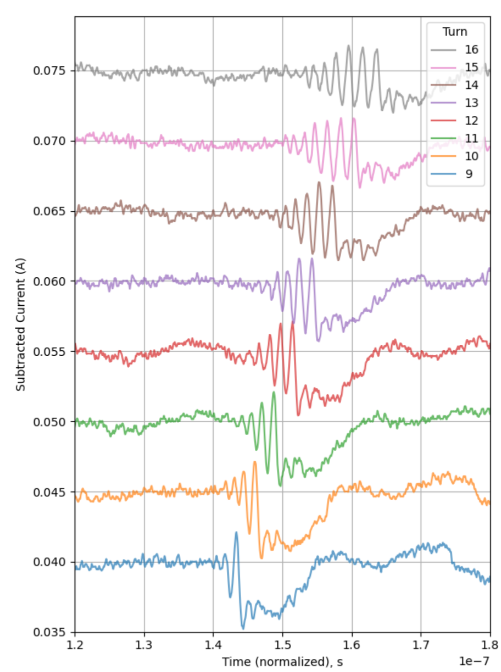

Figure 3: Experimental WCM measurements with the unperturbed beam signal subtracted, showing the evolution of the velocity perturbation from turns 9-16 with 0.005 A offset between each turn.

To further characterize the leading edge of the structure, we measured its amplitude, width, and velocity as it evolved. The results are compared to the linear relations expected for a soliton-like wave governed by the KdV equation.

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Figure 4 presents the experimentally measured velocity of the leading edge plotted against its amplitude. The data closely follow the expected linear trend with a reduced chi-square value of 0.43.

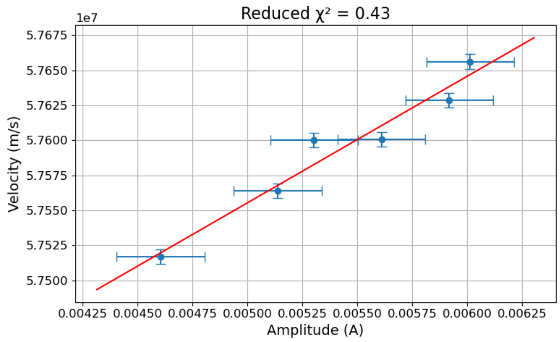

Figure 4: Propagation velocity of the DSW leading edge versus its amplitude with error bars from experimental systematics. The linear trend is fit with a reduced chi-square of 0.43.

A second soliton property predicted by the KdV equation is an inverse-square relationship between amplitude and pulse width. Figure 5 shows the experimentally measured amplitude of the leading edge plotted against the square of its width. The data follow the expected linear trend, with a reduced chi-squared value of 4.46 for the linear fit.

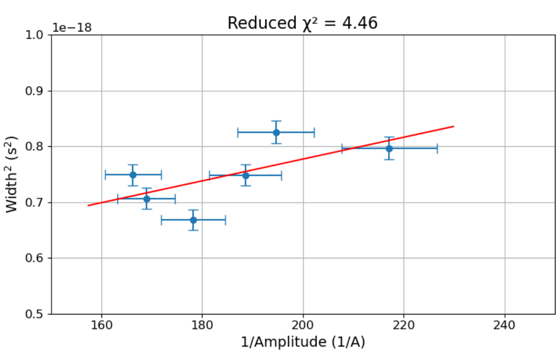

Figure 5: Amplitude of the DSW leading edge versus the square of its width with error bars from experimental systematics. The linear fit yields a reduced chi-squared value of 4.46.

In addition to the leading-edge scalings, we analyzed the overall DSW width. Figure 6 shows the measured DSW width increases approximately linearly during the early turns, with a reduced chi-squared value of 2.04 for the linear fit. Deviations from this trend at later turns are attributed to beam loss.

DISCUSSION

The leading edge of a dispersive shock wave is expected to behave like a soliton, with velocity–amplitude and

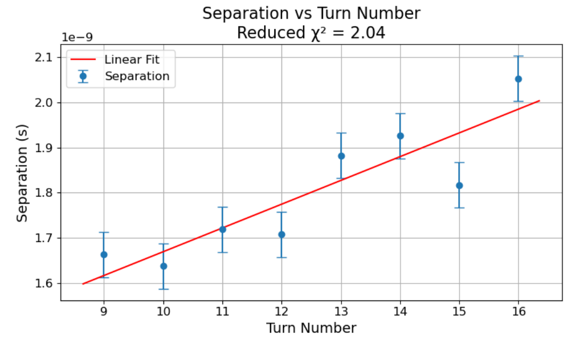

Figure 6: Measured DSW width versus time with error bars from experimental systematics. The width is defined as the distance between the center of the leading soliton-like peak and the peak of the second oscillation. An approximately linear increase is observed at early times, with deviations at later turns.

amplitude–width^2 scalings characteristic of KdV soliton solutions. Turn-by-turn WCM measurements show a sharp leading edge with trailing oscillations, matching the KdV description of a DSW. The measured leading-edge scalings follow the predicted linear relationships for solitons. These results strongly support the interpretation of the observed structure as a dispersive shock wave. Additionally, an initial look at the overall DSW width found a linear growth in time.

CONCLUSION

We have made the first experimental observation of dispersive shock waves in a charged particle beam. Using UMER, a controlled negative velocity perturbation produced a soliton-like leading edge and dispersive oscillations in agreement with KdV predictions and PIC simulations.

Building on these results, we plan to extend DSW studies to proton beams at the FAST/IOTA facility, enabling exploration of space-charge-driven nonlinear dynamics in hadron systems. These future investigations will expand our understanding of nonlinear wave phenomena in accelerators and may inform the design and operation of next-generation high-brightness machines.

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