

# SAFE EXTREMUM SEEKING FOR REAL-TIME ADAPTIVE ACCELERATOR CONTROL\*

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## Abstract

This paper presents Safe Extremum Seeking (Safe ES), a robust n-dimensional adaptive control method, applied to automatic tuning and optimization tasks of accelerators while guaranteeing that the system remains within safe operating conditions. A key strength of Safe ES is its ability to handle safety measures that are analytically unknown. For instance, the algorithm can adaptively tune quadrupole magnets to achieve a desired beam profile while maintaining safety by keeping beam losses below a user-defined threshold.

The presentation provides a comprehensive technical overview of this optimization technique, highlighting its theoretical foundations and practical applications. It includes simulation studies demonstrating its effectiveness for accelerator systems, as well as results from an in-hardware demonstration conducted on the LANSCE accelerator at LANL. Also, the talk explores how Safe ES can be integrated with generative machine learning models to enable adaptive machine learning.

## INTRODUCTION

Particle accelerators present a uniquely difficult challenge for both control and estimation. On the control side, performance must be optimized in real time on systems with thousands of coupled components, all subject to slow drift, high dimensionality, and unknown dynamics. At the same time, safety is critical—beam loss not only reduces performance, but can cause irreversible damage to equipment and create hazardous conditions for maintenance. On the estimation side, diagnostic measurements that fully characterize the six-dimensional (6D) phase space of charged particle beams are either invasive (destructive to the beam) or so slow as to be unusable in feedback. These two challenges—fast, safe control and non-invasive high-dimensional inference—demand adaptive methods that can operate directly on measured signals, without relying on analytical models or full state access.

*Extremum seeking* (ES) methods, see Ref. [1] for a general review, particularly in their recent constrained forms, offer a powerful solution to this dual problem. A constrained ES algorithm, Safe ES, uses a control barrier function (CBF)-inspired law to enforce safety constraints while optimizing unknown objectives in real time, using only measurements [2–4]. These methods have been applied experimentally on live accelerator systems at LANSCE, performing multivariable tuning of magnets while maintaining beam cur-

rent constraints. Beyond control, Safe ES has also inspired new methods for estimation, including a physics-informed neural networks (PINNs) [5] method and an adaptive latent space tuning method for generative models [6], utilizing a bounded form of ES [7]. Recent advances even extend this paradigm to diffusion-based generative AI, using conditional latent diffusion to infer full 2D projections of a beam’s 6D phase space from simple non-invasive diagnostics like beam current monitors [8]. This paper surveys reviews advances in constrained forms of ES and ES-inspired methods for accelerator control and machine learning, highlighting theory and experiment.

A variety of model-free, online optimization methods have been successfully applied to real particle accelerators. These include Bayesian optimization approaches for maximizing beam performance using surrogate models [9], as well as safe variants designed to operate under strict machine protection constraints [10]. Evolutionary algorithms, such as genetic algorithms combined with dimensionality reduction, have been used for multi-objective tuning in storage rings [11], while local direct-search methods like robust conjugate direction search (RCDS) have been deployed for real-time luminosity optimization [12]. More recently, reinforcement learning has been explored for autonomous accelerator control, with agents trained in high-fidelity simulations and targeted for deployment on hardware [13]. These methods represent a rich and growing body of work focused on real-time. Our approach contributes to this area by introducing a safety-aware, model-free optimization technique that operates entirely on live measurements. We also show use cases in which this technique can be combined with physics-informed models and generative machine learning for estimation.

## SAFE EXTREMUM SEEKING

We begin our discussion with the work of Ref. [2]. This work, termed *Safe ES*, introduces an ES algorithm that solves the constrained optimization problem:

$$\min_{\theta \in \mathbb{R}^n} J(\theta) \quad \text{subject to} \quad h(\theta) \geq 0, \quad (1)$$

where both the objective  $J(\theta) : \mathbb{R}^n \rightarrow \mathbb{R}$  and the CBF  $h(\theta) : \mathbb{R}^n \rightarrow \mathbb{R}$ , are unknown and only measurable, with unknown gradients. The parameters available to tune are given by  $\theta$ .

To enforce safety during optimization, the algorithm incorporates a quadratic program (QP)-style CBF filter into the parameter update law. The dynamics of the idealized

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(exact-gradient) algorithm are:

$$\dot{\theta} = -\nabla J(\theta) + \frac{\nabla h(\theta)}{\|\nabla h(\theta)\|^2} \max \{ \nabla J(\theta)^T \nabla h(\theta) - ch(\theta), 0 \}, \quad (2)$$

where  $c > 0$  is a design parameter governing the rate of attraction to the safe set  $C = \{\theta \in \mathbb{R}^n : h(\theta) \geq 0\}$ .

Since gradients are not available, Safe ES approximates Eq. (2) using online estimation via sinusoidal perturbations. The Safe ES algorithm seeks to minimize an unknown function  $J(\theta)$  while keeping a measured but unknown safety signal  $h(\theta)$  non-negative. The update dynamics for the tuning parameter vector  $\theta$  can be decomposed as:

$$\begin{aligned} \text{Gradient Ascent of the Safety} \\ \dot{\theta} = k\omega_f \underbrace{(-G_J + AG_h)}_{\text{Gradient Descent of the Objective}} + \underbrace{\dot{S}(t)}_{\text{Exploration Signal}}, \end{aligned} \quad (3)$$

where  $G_J$  and  $G_h$  are real-time estimates of  $\nabla J(\theta)$  and  $\nabla h(\theta)$ , respectively, obtained via demodulation of periodic perturbations  $S(t)$ . The scalar  $A$  is a state-dependent gain that activates the safety correction term when necessary:

$$\begin{aligned} A = \min \{ \|G_h\|^{-2}, M^+ \} \cdot \max \{ G_J^T G_h - c\eta_h, 0 \} \geq 0, \quad (4) \\ \approx \frac{\max \{ \nabla J(\theta)^T \nabla h(\theta) - ch(\theta), 0 \}}{\|\nabla h(\theta)\|^2}. \end{aligned}$$

The term  $AG_h$  emulates the action of a CBF-based safety filter (on vanilla gradient descent dynamics), increasing its influence in order to satisfy a rate constraint on  $h$  (see Refs. [2, 14] for background). This term can be demonstrated to be the solution a quadratic program (QP) which is the smallest such term such that safe set invariance (for trajectories starting inside the safe set) and convergence (for trajectories starting outside the safe set) is guaranteed. The perturbation signal  $\dot{S}(t)$  is used to probe the unknown maps to enable gradient estimation. The design constants  $k$ ,  $\omega_f$ , and  $a$  govern learning rate, filter speed, and perturbation amplitude.

The Safe ES algorithm ensures semiglobal practical asymptotic (SPA) stability of the constrained optimum to the solution  $\theta_c^*$  and guarantees *practical safety*—meaning trajectories remain within a user-adjustable margin of the safe set. If  $h$  is initialized with a negative value then trajectories approach positive values of  $h$  — while minimizing the function  $J$ . The main results are summarized from Ref. [2]:

**Theorem 1** Let the unknown functions  $J$  and  $h$  satisfy smoothness and geometric assumptions in [2]. Then, for any desired bounds  $v, \delta > 0$  and any bounded initial condition, there exist design parameters  $(a, k, \omega_f)$  such that the Safe ES trajectory satisfies:

$$\|\hat{\theta}(t) - \theta_c^*\| \leq \beta_\theta (\|\hat{\theta}(0) - \theta_c^*\|, k \cdot \omega_f \cdot t) + v, \quad (5)$$

$$h(\theta(t)) \geq h(\theta(0)) e^{-ck\omega_f t} + O(\delta), \quad (6)$$

where  $\beta_\theta \in \mathcal{KL}$ , and the second inequality guarantees practical safety and attractivity from unsafe initial conditions—the safety constraint is satisfied up to a user-chosen tolerance  $\delta$ .

A Lyapunov function of the form

$$V_\alpha(\theta) = \max \{ -\alpha h(\theta), 0 \} + \max \{ J(\theta) - J^*, 0 \}$$

is used to prove the result, employing nonsmooth analysis and generalized gradients, along with the singular perturbation and averaging results of Ref. [15, 16]. This Lyapunov function penalizes both constraint violation and suboptimality, and is shown to decrease along trajectories of the Safe ES algorithm.

Safe ES proves to be a useful tool directly in the tuning of accelerators [3]. The Safe ES algorithm addresses a key challenge in particle accelerator operation: the need to optimize system performance (e.g., beam transmission, spot size) while maintaining safety (beam loss, power draw) in the presence of unknown and time-varying system dynamics. Traditional tuning methods often rely on expert intuition or offline models that can quickly become outdated due to component drift, thermal variation, or noise. Safe ES, by contrast, performs real-time optimization using only live measurements of performance and safety metrics—such as beam current or loss monitors—without requiring an analytical model — a near impossibility given a complex accelerator system involving thousands of components. In the most dire cases, extreme beam loss can cause catastrophic damage to components in inside of the beam line.

## DIRECT TUNING OF ACCELERATORS

In a simulation example in Ref. [3], Safe ES is used to tune quadrupole magnet strengths for the pRad beamline to achieve a desired beam spot size while maintaining safe beam loss levels. The objective is to minimize the squared error between the simulated beam spot size and a 2.5 cm target size, while ensuring that at least 80% of the beam remains. Classical ES was compared to Safe ES on 100 randomized initial conditions. Figure 2 shows a single representative trajectory comparing the error and beam remaining over time. While Classical ES achieves lower error in some cases, it does so at the expense of safety. Safe ES maintains beam survival above the 80% threshold throughout.

Another simulation example shown in Fig. 1 demonstrates the transient effect of the parameter  $c$ , which can be specified by the user to control the rate at which the constraint measurement approaches the threshold value (in this case also corresponding to 80% of beam remaining or 20% beam loss). Note that a more conservative value of  $c = 0.1$  naturally corresponds to slightly slower convergence rate in the primary objective or error function in lower half of the figure.

In an experimental demonstration at the Los Alamos Neutron Science Center (LANSCE), Safe ES was deployed to tune magnet strengths in the low-energy beam transport (LEBT) line. The objective was to maximize the downstream beam current  $I_b$  while maintaining the upstream beam loss at a collimator, measured by  $I_c$ , within safe limits—where safety is defined by the condition  $h(\theta) \geq 0$  corresponding

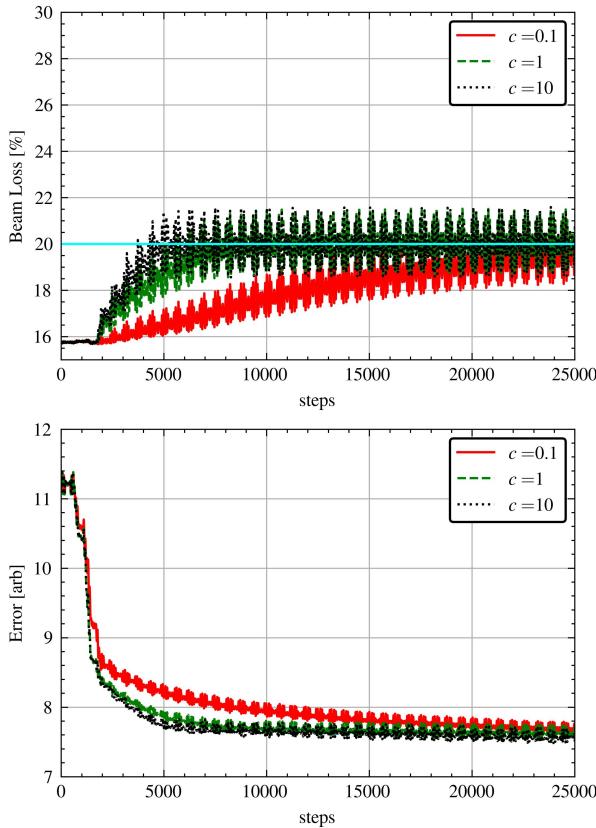


Figure 1: Percentage of beam loss and error for Safe ES with different values of  $c$ .

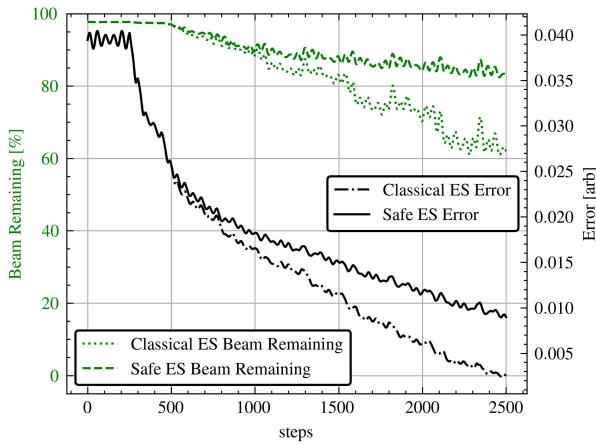


Figure 2: Percentage of beam remaining and spot size error for Safe ES vs. Classical ES over one trajectory. Safe ES maintains beam safety throughout [3].

to  $I_c \geq -0.5$  mA. Starting from an intentionally unsafe condition, Safe ES successfully recovered both performance and safety without manual intervention. Over the course of the first 100 steps, the algorithm adaptively improved beam current while ensuring the safety metric remained within acceptable bounds, as shown in Fig. 3. This experiment demonstrates Safe ES as a viable tool for real-time,

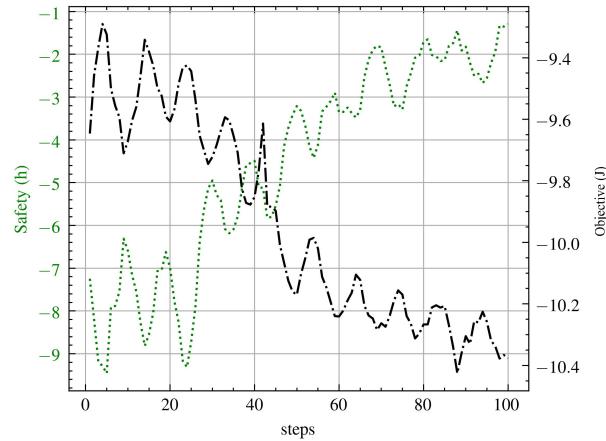


Figure 3: Experimental Safe ES: Safety and objective function evolution during the first 100 steps. Beam loss (safety metric  $h$ ) improves while maximizing beam current, equivalent to minimizing  $J$  [3].

autonomous, model-free control in high-risk, large-scale accelerator environments [3].

We also highlight the Assignably Safe ES (ASfES) algorithm, introduced in Ref. [4], which presents a variant of Safe ES: it enables explicit user control over the rate at which the system approaches the boundary of the safe set. While the semiglobal Safe ES algorithm in Ref. [2] provides practical safety guarantees by ensuring  $h(\theta(t)) \geq h(\theta(0))e^{-ck\omega_f t} + O(\delta)$ , the true exponential approach to the safe boundary is implicitly determined by other tuning parameters  $k, \omega_f$ , which may be small. ASfES improves on this by allowing this rate to be directly specified by the user with a bound

$$h(\theta(t)) \geq h(\theta(0))e^{-ct} + O(\delta), \quad (7)$$

granting greater flexibility in controlling safety aggressiveness and responsiveness. Notably, the formulation in Ref. [4] is more conservative than prior Safe ES designs because the smoothed  $\max_\delta$  always overestimates the corrective safety term, thus biasing trajectories further from unsafe regions. A Newton-based extension (NB-ASfES) is also presented in Ref. [4], where both the safety convergence rate and the parameter convergence rate can be independently assigned. ASfES highlights a tradeoff between global convergence (as in semiglobal Safe ES) and local but highly tunable safety responsiveness, making it a useful addition to the safe optimization toolbox, particularly when local precision and explicit safety dynamics are critical.

## MACHINE LEARNING APPLICATIONS

### QP Based PINNs Training

PINNs [17] aim to approximate solutions to partial differential equations (PDEs) by embedding physical laws directly into the loss function [17]. The method involves minimizing a composite loss function that balances a PDE residual loss (e.g., the residual loss in Laplace's equation

$L_{PDE} = \|\nabla^2 \phi(x, y)\|^2$  with data and boundary condition losses. The power of the PINNs framework is due to the automatic differentiation that enables the PDE residual to be directly computed. However, when data are noisy or scarce, conventional PINNs may overfit to unreliable measurements at the cost of violating known physics. To address this, PINN training is framed as constrained optimization problem [5]: the total loss  $f(\theta)$ , containing the PDE residual and boundary terms, is minimized subject to the constraint  $g(\theta) = l_{\text{DATA}}(\theta)^2 - z^2 \delta^2 \leq 0$ , where  $l_{\text{DATA}}$  is a data loss (e.g., root-mean-square error),  $\delta$  reflects known uncertainty, and  $z$  is a tunable tolerance. The QPGD update law (see Ref. [5] for details) is essentially a discretized version of the law in Eq. (2):

$$\theta^{(t+1)} = \theta^{(t)} - \gamma \left( \nabla f(\theta^{(t)}) + \alpha(\theta^{(t)}) \nabla g(\theta^{(t)}) \right), \quad (8)$$

with  $\alpha(\theta^{(t)})$ , containing design parameters  $c > 0$  and  $\epsilon_\alpha$ , enforcing constraint satisfaction using a CBF-inspired term similar to the term  $A$  in Eq. (4). Here we use  $g$  to denote the inequality constraint instead of  $h$  to fit nonlinear programming conventions. This ensures the PDE loss is minimized only after the data is fit sufficiently well, avoiding overfitting to noise. In the capacitor case study governed by Laplace's equation  $\nabla^2 \phi = 0$ , the QPGD-based PINN accurately infers the unknown plate voltage and reconstructs the potential field throughout the domain (see Fig. 4). Compared to a standard PINN solution, QPGD achieved lower error and a more accurate estimate of the unknown voltage, illustrating the advantage of incorporating constraint-aware optimization into scientific ML [5].

The QPGD scheme in Eq. (8) can be interpreted as a discretization of the exact Safe ES law in Eq. (2) under known gradients, where  $f$  is the objective function,  $g$  defines the constraint. In the QPGD approach, these dynamics are discretized into a gradient descent update law. The main theoretical result, shown below [5], says that this update law guarantees semiglobal asymptotic convergence to the unique solution of the constrained optimization problem  $\min_\theta f(\theta)$  subject to  $g(\theta) \leq 0$ .

**Theorem 2** Suppose  $f$  and  $g$  satisfy smoothness and geometric assumptions in [5]. There exists an  $\epsilon_\alpha^*$  such that for any  $\epsilon_\alpha \in (0, \epsilon_\alpha^*)$  and for any initial parameter  $\theta^{(0)} \in \mathbb{R}^n$  there exists a learning rate  $\gamma^* > 0$  such that for all  $\gamma \in (0, \gamma^*)$  the sequence  $\{\theta^{(t)}\}$  asymptotically converges to  $\theta^*$  as  $t \rightarrow \infty$  given the the update law Eq. (8).

If multiple constraints exist (inequality and equality), the work [18] studies this problem, and generalizes the control law in Eq. (2), using a similar Lyapunov argument to show local stability — albeit in continuous time. Ongoing work is focused on generalizing the Safe ES scheme for multiple constraints (both inequality and equality).

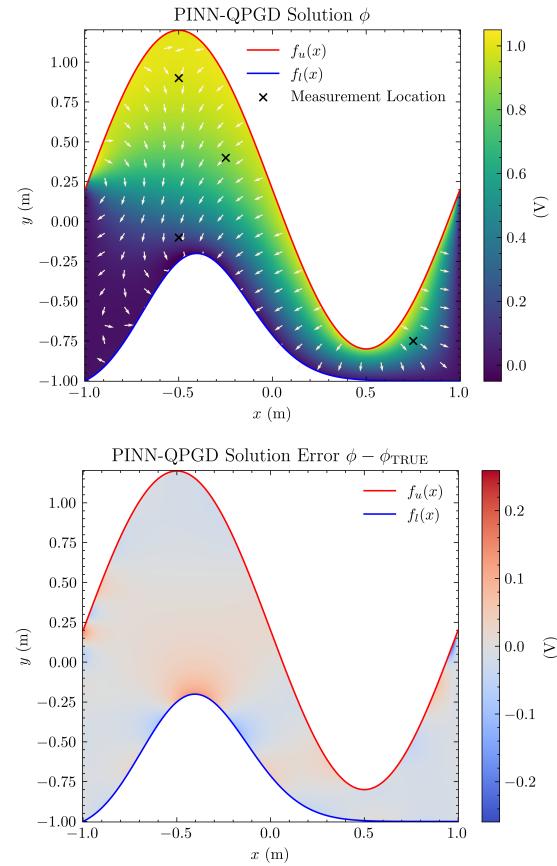


Figure 4: *Top:* Potential field solution with four noisy measurements. Because the solution of the electric field  $\phi(x, y)$  is parameterized with a neural network, automatic differentiation is used to compute the electric field gradients ( $\partial \phi / \partial x, \partial \phi / \partial y$ ) — the arrows represent the normalized, estimated electric field directions. *Bottom:* Error between the true and estimated potential field throughout the domain of the PDE [5].

### Latent Space Tuning of Generative Neural Networks

Next we discuss so called “latent space tuning” for generative models [6], developed to improve prediction power neural networks in the modeling of time varying systems. This paper introduces a novel adaptive latent space tuning method to improve the robustness of machine learning (ML) models — specifically encoder-decoder convolutional neural networks (CNNs) — when applied to time-varying systems like ultrafast electron diffraction (UED) accelerators [6, 19]. The method avoids re-training by adjusting a low-dimensional latent space representation based on real-time feedback from limited, partial measurements. It is demonstrated on the HiRES accelerator to reconstruct full 6D phase space diagnostics (including all 15 2D projections) using only a small subset of data, such as  $(x, y)$  and  $(z, E)$  projections, as shown in Fig. 5. The adaptive feedback dynamically perturbs the 2D latent vector to minimize a cost function based on dis-

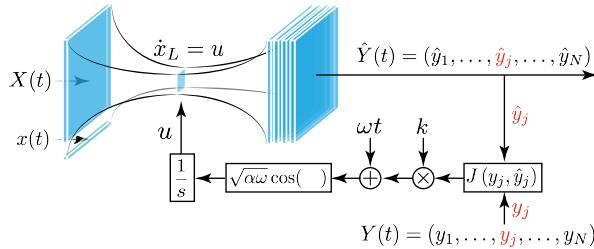


Figure 5: Bounded ES used for real-time adaptive latent space tuning to track all 15 projections of a charged particle beam’s 6D phase space density using only 1 available 2D projection for measurement as developed in Refs. [6, 19]. In this approach, after the generative model has been trained, we assume that we no longer have access to input beam image measurements  $X(t)$  and also to beam parameters  $x(t)$ , but that we can measure just a single 2D projection of the beam downstream  $y_j(t)$ . We always generate all 15 unique 2D projections of the beam’s phase space  $\hat{Y}(t) = (\hat{y}_1(t), \dots, \hat{y}_{15}(t))$ , and then create a cost function based on the one projection that we can measure  $J(y_j(t), \hat{y}_j(t))$ , such as the mean squared error between those two images. That cost function is then minimized by Bounded ES which acts directly on the low-dimensional latent space of the generative model to track the time-varying beam without access to the time-varying  $X(t)$  and  $x(t)$ .

crepancies between predicted and observed projections using Bounded ES. The approach enables tracking of unknown and time-varying input beams and accelerator parameters far beyond the span of the original training data. The model also includes a physics-informed uncertainty quantification (UQ) method based on redundancy across projections. Results show significant improvements in prediction accuracy and robustness over standard CNNs, particularly when faced with distribution shift or unseen conditions, making the approach highly promising for online diagnostics in accelerators and other complex physical systems.

Another work [20], demonstrates a practical framework in which the key contribution is a fusion of latent space tuning [6] with a discretized Safe ES controller, enabling safe control based on this virtual diagnostic. The method is trained on simulated phase space data, with a variational autoencoder (VAE) used to encode high-resolution ( $256 \times 256$ ) phase space images into a compact latent representation. The latent vector is mapped to both a predicted beam loss and a phase space estimate. Then, using true beam loss measurements, the latent vector is tuned iteratively (with a fast loop) by minimizing a loss between true and predicted beam loss in order to generate a “best” estimate of the phase space  $\hat{y}_{ps}$ . Once this optimization is solved, the discretized Safe ES algorithm adjusts the accelerator parameters (with a slow loop), taking a single discrete step, based on a tracking error between  $\hat{y}_{ps}$  and  $y_{ps,des}$ , the desired phase space image — then  $\hat{y}_{ps}$  is re-estimated, and so on. This approach is illustrated in Fig. 6.

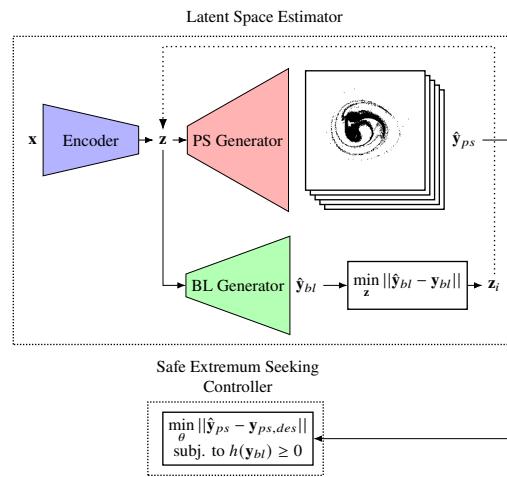


Figure 6: Diagram of the estimation (using latent space tuning) and the discretized Safe ES scheme used for safe control. The phase space (PS) generator and the beam loss (BL) generator are networks which provide estimates of the hard-to-measure phase space of the beam, and the easy-to-measure beam loss. The value  $y_{bl}$  is the true measured beam loss which provides an error for the latent space tuner. The total loss, a component of  $y_{bl}$ , is used by the discretized Safe ES controller [20].

## CONCLUSION

This work introduces Safe Extremum Seeking (Safe ES), a real-time, model-free optimization framework that ensures safety during adaptive control of particle accelerators, with demonstrated effectiveness both in simulation and on hardware at LANSCE. We further explored extensions to Safe ES, including Assignably Safe variants, and demonstrated applications in constrained training of physics-informed neural networks and latent space tuning of generative models. Ongoing research focuses on multi-constraint generalizations of Safe ES, faster discretized implementations, and broader integration with adaptive machine learning for virtual diagnostics and control.

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