

# PASSIVE HIGHER-HARMONIC RF CAVITY SIMULATIONS FOR BUNCH LENGTHENING IN THE NSLSII-U

Aamna Khan\*, Gabriele Bassi, Victor Smaluk  
BNL, NSLS-II, Upton, NY, USA

## Abstract

Higher-harmonic cavities (HHCs) are commonly employed in modern synchrotron light sources to lengthen electron bunches without increasing the natural energy spread. This approach effectively reduces intrabeam scattering and enhances Touschek lifetime. Accurate simulations are essential for predicting beam dynamics, particularly the impact of cavity detuning on beam stability and performance. Simulations are performed with the particle-tracking code ELEGANT, and key uniform-fill results are verified with the Vlasov–Fokker–Planck solver SPACE.

## INTRODUCTION

Achieving extremely low electron beam emittance is critical for advancing the capabilities of modern synchrotron light sources. However, ultra-low emittance operation often results in high particle density, which implies shorter bunch lengths and enhances collective effects such as intra-beam scattering (IBS) and impedance-induced effects [1, 2]. This leads to emittance growth and reduced beam lifetime, particularly at higher beam currents. To mitigate these effects, passive higher-harmonic cavities (HHCs) are employed to lengthen the bunch while preserving the natural energy spread. This reduces the strength of IBS and provides Landau damping for suppressing longitudinal instabilities [3].

The NSLSII-U is a proposed upgrade to NSLS-II, designed to operate at 4 GeV with a complex-bend achromat lattice [4]. As part of its intensity and stability enhancement strategy, the design includes a passive superconducting third-harmonic cavity (3HC). Table 1 summarizes the primary design parameters. In passive mode, the 3HC is detuned from the third harmonic of the main RF frequency, enabling beam-induced voltage to flatten the total RF waveform and lengthen the bunch. Although a multi-harmonic RF system combining multiple cavity orders is under consideration for NSLSII-U to further increase bunch lengthening and improve beam stability [5], this study focuses on the impact of a single 3HC. Uniform-fill simulations were carried out with the particle-tracking code ELEGANT; key results were cross-validated against the Vlasov–Fokker–Planck solver SPACE to confirm consistency.

## THEORETICAL MODEL

For passive HHC operation, the total RF voltage experienced by a particle at longitudinal coordinate  $\tau$  is the combination of the main RF cavity voltage and the beam-induced

Table 1: NSLSII-U High-Brightness Storage Ring Parameters for Complex-Bend Lattice

Parameters	Symbol	Value	Unit
Circumference	C	791.72	m
Beam energy	E	4	GeV
Average current	$I_{av}$	400	mA
Momentum compaction	$\alpha$	6.67	$10^{-5}$
Natural energy spread	$\sigma_\delta$	1.10	$10^{-3}$
RF Voltage	$V_{rf}$	4.8	MV
RF frequency	$f_{rf}$	499.8	MHz
Harmonic number	h	1320	
Bunch length	$\sigma_{\tau 0}$	7.87	ps
Energy loss per turn	$U_0$	1940	keV
Main RF shunt impedance	$R_{s,main}$	33375	M $\Omega$
Main RF quality factor	$Q_{main}$	$7.5 \times 10^8$	—
HHC shunt impedance	$R_H$	8800	M $\Omega$
HHC quality factor	$Q_H$	$1.0 \times 10^8$	—
Detuning scan range	$\Delta f$	31–35	kHz

voltage in the harmonic cavity [3]:

$$V(\tau) = V_{rf} \sin(\omega_{rf}\tau + \phi_s) - i_m R_H \cos \psi_m \cos(m\omega_{rf}\tau + \psi_m), \quad (1)$$

where  $V_{rf}$  and  $\omega_{rf}$  are the amplitude and angular frequency of the main RF cavity, respectively,  $\phi_s$  is the synchronous phase,  $m$  is the harmonic number of the HHC,  $R_H$  is the shunt impedance of the HHC, and  $\psi_m$  is the cavity detuning angle. The beam-induced current component at the harmonic frequency  $m\omega_{rf}$  for an approx. Gaussian bunch of rms length  $\sigma_\tau$  is given by  $i_m = 2I_0 \exp\left[-\frac{1}{2}(m\omega_{rf}\sigma_\tau)^2\right]$ , where  $I_0$  is the average stored beam current.

The detuning angle  $\psi_m$  is determined by the cavity's frequency offset from the exact harmonic multiple of the main RF frequency as:

$$\tan \psi_m = 2Q_H \frac{\omega_m - m\omega_{rf}}{m\omega_{rf}}, \quad (2)$$

with  $Q_H$  and  $\omega_m$  being the quality factor and resonant angular frequency of the superconducting harmonic cavity, respectively.

For superconducting HHC operation in the NSLSII-U storage ring, the cavity shunt impedance  $R_H$  is significantly higher than the ideal value required for passive operation. This large impedance value forces the cavity to operate at a detuning angle  $\psi_m$  close to  $\pi/2$ . Under these conditions, the optimal operating points satisfy  $V(0) = V'(0) = 0$ , leading to the relationships:

$$\sin \phi_s = \sin \phi_{s0}, \quad \cos \psi_m = -\frac{V_{rf} \cos \phi_s}{i_m m R_H}, \quad (3)$$

\* akhan1@bnl.gov

where  $\phi_{s0}$  is the synchronous phase without the presence of the harmonic cavity.

To achieve maximum bunch lengthening while preserving beam stability, the superconducting harmonic cavity must be detuned by an optimal frequency offset  $\Delta\omega_H = \omega_m - m\omega_{rf}$ , given by:

$$\Delta\omega_H = -\frac{m^2\omega_{rf}i_m R_H}{2Q_H V_{rf} \cos\phi_s}. \quad (4)$$

Using the NSLSII-U storage ring parameters summarized in Table 1, this detuning frequency offset evaluates to approximately 35 kHz.

## NUMERICAL SIMULATIONS

We begin by loading the NSLSII-U lattice into each code and defining the RF systems in beam-loaded mode. In ELEGANT, the main RF cavity is modeled with the built-in beam-loading `rfmode` element, and a second `rfmode` element is configured in passive mode to represent the 3HC. The shunt impedance of the harmonic cavity is gradually ramped up over the first 20,000 turns to suppress transient effects, after which it remains constant during detuning frequency scans. During each run, we track all bunches using a particle distribution file and periodically record the bunch length and centroid using histogram and watch elements.

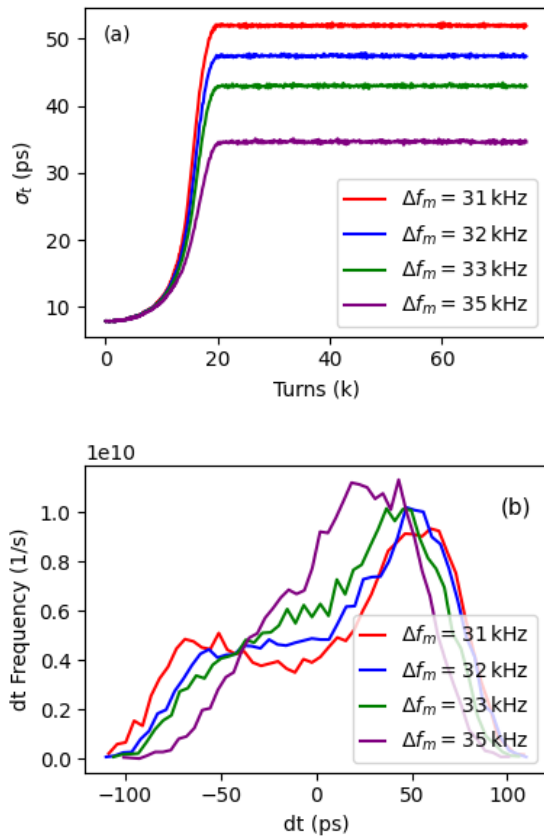


Figure 1: Simulated (a) rms bunch length and (b) bunch profiles for the uniformly filled NSLSII-U lattice with a passive 3HC, as computed by ELEGANT.

Figure 1 shows the simulated steady-state longitudinal beam parameters for the uniformly filled NSLSII-U lattice with a passive 3HC using ELEGANT. The initial rms bunch length without the harmonic cavity is  $\sigma_{t,0} = 7.87$  ps. With a detuning frequency of  $\Delta f = 35$  kHz, the steady-state bunch length is approximately 35 ps (bunch-lengthening factor  $u! \approx 4.4$ ). The corresponding longitudinal density develops a broad, shallow top characteristic of a quartic RF potential, although a perfectly flat profile is not reached because the large shunt impedance of the superconducting cavity limits the achievable compensation of the sinusoidal curvature. At  $\Delta f = 32$  kHz the distribution broadens further and a shallow double-bump appears, signalling an over-flattened RF potential that is still acceptable for uniform bunch lengthening.

In SPACE, we represent each of the 1320 bunches with 30k macroparticles and solve the Vlasov–Fokker–Planck equation self-consistently, including radiation damping, quantum excitation, and beam-induced voltages in both cavities. The simulations run until the cavities and beam reach equilibrium, after which we extract steady-state bunch lengths and centroids for comparison to the results obtained from ELEGANT.

Figure 2 presents a comparative analysis of steady-state bunch density distributions and rms bunch lengths as calculated by SPACE and ELEGANT. The density plots clearly illustrate uniform bunch lengthening across the entire bunch train. For the optimal 35 kHz detuning frequency, there is a small but noticeable difference of approximately 3 ps between the two codes. This discrepancy arises primarily from the distinct numerical methods: particle tracking with transient modulation in ELEGANT and self-consistent solution of the Vlasov–Fokker–Planck equation in SPACE. Despite this minor difference, both simulation approaches validate the effectiveness of passive 3HC operation for achieving significant bunch lengthening in NSLSII-U.

In normal NSLS-II operations, the storage ring is typically filled with 1200 bunches, leaving a gap of 120 RF buckets. To examine the impact of this non-uniform filling pattern, simulations were performed using ELEGANT for two different detuning frequencies of the harmonic cavity ( $\Delta f_H = 32$  kHz and  $\Delta f_H = 35$  kHz). As shown in Fig. 3, the bunch length ( $\sigma_t$ ) exhibits a clear transient behavior along the bunch train. Bunches near the head of the train have shorter lengths, which gradually increase toward the tail, reaching a maximum before slightly decreasing near the end. A corresponding shift in the bunch centroid positions ( $\Delta c_t$ ) is also observed, starting with positive values at the head and transitioning to negative values toward the tail. Such transient effects in bunch lengthening and centroid shifts due to filling gaps have been previously reported and are consistent with other studies [6]. These preliminary simulations highlight the importance of careful consideration of filling patterns in the operation of passive harmonic cavities. Further comparisons, including SPACE simulations, are underway to fully resolve observed discrepancies and ensure robustness of these observations.

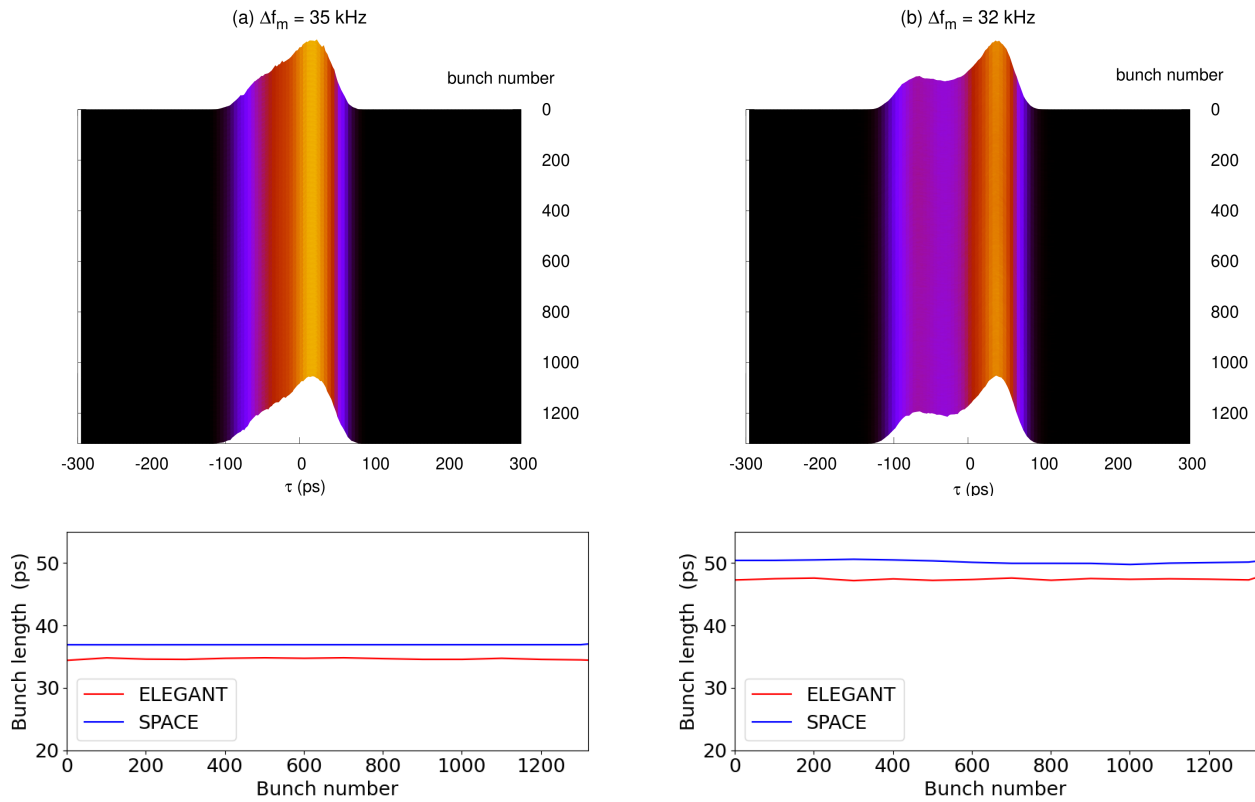


Figure 2: Comparison of steady-state longitudinal density distributions (a, b) computed with SPACE and bunch lengths (c, d) obtained from SPACE and ELEGANT simulations. Both simulations yield consistent results, with a small bunch length difference of approximately 3 ps.

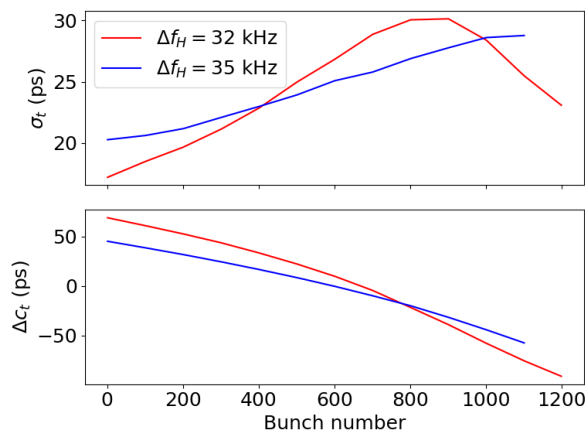


Figure 3: Simulated bunch length  $\sigma_t$  (top) and centroid shift  $\Delta c_t$  (bottom) along a 1200-bunch train with a 120-bucket gap, calculated using ELEGANT for two harmonic cavity detuning frequencies,  $\Delta f_H = 32$  kHz (red) and  $\Delta f_H = 35$  kHz (blue).

## CONCLUSION

Passive third-harmonic cavities lengthen NSLSII-U bunches by a factor of 4–6 under uniform filling, but superconducting R/Q limits prevent the perfectly flat profile predicted for an ideal quartic potential. Introducing the 120-bucket ion gap used in routine operations reduces the attain-

able lengthening from  $u \approx 4.4$  to  $\approx 3.5$  at 35 kHz detuning. This sensitivity to filling pattern underscores the need for multi-harmonic RF schemes [7], which are now being evaluated and will be benchmarked with both ELEGANT and SPACE in future work.

## ACKNOWLEDGMENT

This work has been supported by DOE under Contract No. DE-SC0012704.

## REFERENCES

- [1] A. Khan, G. Bassi, and V. Smaluk, “Study of the combined effect of intrabeam scattering and impedance in a low-emittance ring”, in *Proc. IPAC’23*, Venice, Italy, May 2023, pp. 3436–3438. doi:10.18429/JACoW-IPAC2023-WEPL141
- [2] A. Khan *et al.*, “Simulation and measurement of beam-induced heating of ceramic vacuum chambers”, *Phys. Rev. Accel. Beams*, vol. 27, no. 8, p. 084501, 2024. doi:10.1103/PhysRevAccelBeams.27.084501
- [3] G. Bassi and J. Tagger, “Longitudinal beam dynamics with a higher-harmonic cavity for bunch lengthening”, *Int. J. Mod. Phys. A*, vol. 34, no. 36, p. 1942040, 2019. doi:10.1142/S0217751X19420405
- [4] G. Wang *et al.*, “Complex bend: strong-focusing magnet for low-emittance synchrotrons”, *Phys. Rev. Accel. Beams*, vol. 21, no. 10, p. 100703, 2018. doi:10.1103/PhysRevAccelBeams.21.100703

- [5] G. Bassi, A. Khan, and V. Smaluk, “Bunch lengthening induced by a combination of higher-harmonic cavities of different order in low-emittance rings”, in *Proc. IPAC’24*, Nashville, TN, USA, May 2024, pp. 2952–2955.  
doi:10.18429/JACoW-IPAC2024-THBD2
- [6] G. Bassi, “Bunch lengthening by a third-harmonic cavity in a low-emittance ring”, in *Proc. IPAC’23*, Venice, Italy, May 2023, pp. 3432–3435.  
doi:10.18429/JACoW-IPAC2023-WEPL140
- [7] G. Bassi, A. Khan, F. Gao, J. Rose, and V. Smaluk, “A multi-harmonic cavity system for bunch lengthening for the NSLS-II upgrade”, presented at NAPAC’25, Sacramento, CA, USA, Aug. 2025, paper TUP007, this conference.