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ONE-TO-ONE MAPPING BETWEEN THE ELECTROMAGNETIC MODES OF CYLINDRICAL AND COAXIAL HALF-WAVE CAVITIES*

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Abstract

Design of radio frequency (RF) couplers and diagnostics require a good understanding of the electromagnetic mode patterns of RF cavities. This study investigates the adiabatic transformation of transverse magnetic (TM) modes of a cylindrical cavity into transverse electromagnetic (TEM) modes of a coaxial cavity by gradually introducing an inner conductor. Using CST Studio Suite, we simulate the eigenmode evolution as the geometry transforms from a pure cylindrical to a coaxial configuration. We track the behavior of TM_{010} through TM_{014} modes to observe the continuous evolution into the corresponding TEM_0 through TEM_4 modes of the coaxial cavity. The process is governed by the evolution of the electric field orientation as the geometry shifts, enabling the axial TM fields to reorient into the radial electric field configuration of TEM modes. Field patterns, eigen-frequencies, and mode identities are analyzed throughout the transition. The results provide simulation-based evidence that TM to TEM conversion occurs without generation of newer eigenmodes, offering a valuable insight into the design of transition regions in superconducting RF (SRF) systems and provides a foundation for experimental validation.

INTRODUCTION

Transitions between different RF cavity geometries are common in superconducting radio frequency (SRF) systems, particularly at interfaces involving input couplers, pickup probes, and diagnostic ports. Cylindrical cavities typically support transverse magnetic (TM) modes, while coaxial structures support transverse electromagnetic (TEM) modes [1]. Ensuring a smooth transition between these structures without introducing additional resonant modes is essential for maintaining mode purity and minimizing power reflection.

A common example occurs is SRF input couplers, where RF power is delivered through a coaxial line into a cavity resonating in a TM mode, such as TM_{010}. To avoid unwanted coupling or mode conversion losses, the coaxial-to-cavity interface must be carefully designed to ensure that the field structures match. Similar transitions are found in pickup probes used to monitor cavity fields, where coaxial lines extract signals from TM modes. In these applications, the introduction of an inner conductor near the cavity wall must not excite higher-order or spurious modes.

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In this work, we use CST Studio Suite to take a closer look at how the TM modes of a cylindrical cavity transform into TEM modes as we gradually introduce an inner conductor. We simulate how TM_{010} through TM_{014} modes evolve into their corresponding TEM_0 through TEM_4 modes in a coaxial structure. Our main goal is to show that this transition happens without the generation of any new modes. The electric field reorients smoothly from being primarily axial in the TM modes to radial in the TEM modes, indicating an adiabatic transformation. This one-to-one mapping gives us confidence that such transitions can be used reliably in SRF systems without introducing unwanted resonances.

CAVITY GEOMETRY

The model geometry consists of a cylindrical RF half-wave cavity with a fixed length that is dimensioned to ensure that its resonant frequencies fall within the operational range of modern accelerator systems.The outer radius is chosen to shift the cutoff frequencies of transverse electric (TE) modes away from the operating range of the TEM modes.A central coaxial gap is introduced symmetrically from both ends by inserting an inner conductor. The dimensions of the geometry is given in Table 1.

Table 1: Cavity Dimensions

Parameters	Value [mm]
Cavity length (L)	460
Outer conductor radius	101
Inner conductor radius	50

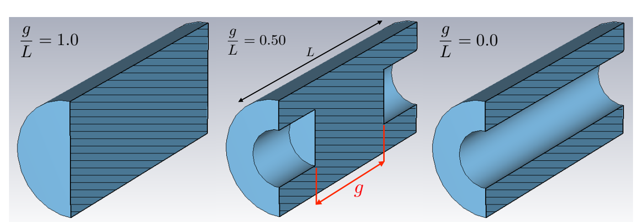

Figure 1: Geometry transformation from cylindrical cavity (left) to coaxial cavity (right) structure.

The gap parameter g represents the remaining central length after subtracting the equal-length coaxial recesses introduced from each end of the cavity, such that g = L − 2d, where L is the total cavity length and d is the depth of each cut from both ends. By varying the value of g,

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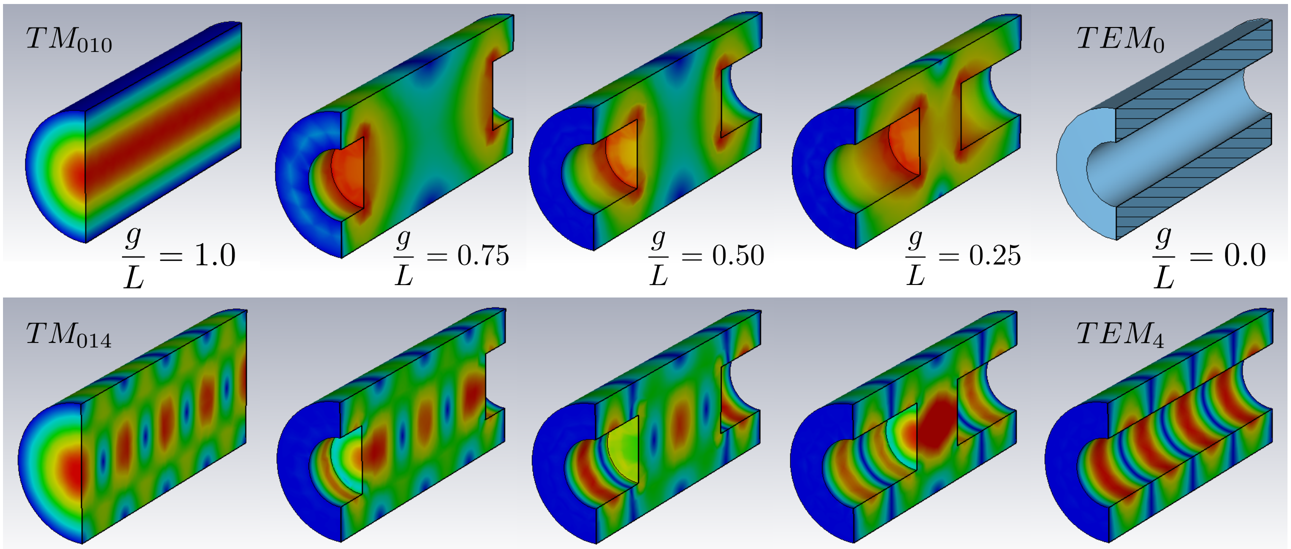

Figure 2: Cavity showing electric field distribution of $\mathrm{TM_{010}}$ to $\mathrm{TEM_0}$ (top) and $\mathrm{TM_{014}}$ to $\mathrm{TEM_4}$ (bottom) transitions for $\frac{g}{L}$ ratio of 1 to 0 (left to right).

the cavity geometry transitions continuously from a pure cylindrical structure (g = L) to a fully coaxial cavity (g = 0). An illustration of the cylindrical, intermediate and coaxial geometry is shown in Figure 1.

EM FIELD ANALYSIS

Simulation Setup

The simulations are carried out using the Eigenmode Solver in CST Studio Suite. For each geometry defined by a specific value of g, a high resolution mesh was generated using adaptive tetrahedral meshing, with refined regions near the inner conductor. A total of 25 geometries are simulated, with $\frac{g}{L}$ ratio decreasing from 1 for cylindrical cavity to 0 for coaxial cavity. The structure is modeled using perfect electric conductor (PEC) boundaries with electric walls at the cavity ends.

EM Field Evolution

For each configuration, the first 20 eigenmodes are extracted. The modal frequencies, field distributions, and mode symmetries are analyzed to verify continuous evolution. Table 2 illustrates the frequencies and geometrical factors of the $\mathrm{TM_{01p}}$ of cylindrical cavity and $\mathrm{TEM_p}$ of the coaxial cavity where p = 0, 1, 2, 3, 4 denotes full cycle variation of the field along cavity axis, z.

Particular attention is paid to the orientation of the electric field vectors: in cylindrical cavities, the TM modes exhibit dominant axial fields, whereas in the coaxial geometry, the fields reorient into radial configurations consistent with TEM modes. This gradual reorientation provides evidence of an adiabatic mode transition driven by geometry rather than mode excitation or boundary effects.

Figure 2 illustrates the evolution of the electric field for two representative modes, $\mathrm{TEM_0}$ and $\mathrm{TEM_4}$ as the cavity geometry transitions from fully cylindrical ($\frac{g}{L}$ = 1) to fully coaxial ($\frac{g}{L}$=0). As shown, the electric field undergoes a smooth reorientation from axial to radial direction. The intermediate steps ($\frac{g}{L} = 0.75, 0.50, 0.25$) highlight the continuous and adiabatic nature of the transformation, confirming that each $\mathrm{TM_{01p}}$ mode evolves cleanly into a corresponding $\mathrm{TEM_p}$ mode without mode mixing or distortion.

Table 2: Mode Frequency

Mode	Frequency [MHz]	Geometric factor [Ω]
Cylindrical cavity		
$\mathrm{TM_{010}}$	1136.16	306.95
$\mathrm{TM_{011}}$	1181.96	318.96
$\mathrm{TM_{012}}$	1309.78	351.71
$\mathrm{TM_{013}}$	1498.84	398.98
$\mathrm{TM_{014}}$	1729.21	454.55
Coaxial cavity		
Mode	Frequency [MHz]	Geometric factor [Ω]
$\mathrm{TEM_0}$	0	0
$\mathrm{TEM_1}$	325.84	51.34
$\mathrm{TEM_2}$	651.69	102.50
$\mathrm{TEM_3}$	977.54	153.12
$\mathrm{TEM_4}$	1303.41	202.96

In the cylindrical configuration, the magnetic field exhibits azimuthal loop pattern characteristic of TM modes, with well defined axial periodicity. In Figure 3, as the coaxial inner conductor is introduced and extended, the magnetic field gradually transforms into the transverse configuration of TEM modes, where the field loops around the central conductor in a mode typical of coaxial lines. This transition highlights the continuous and adiabatic evolution of the

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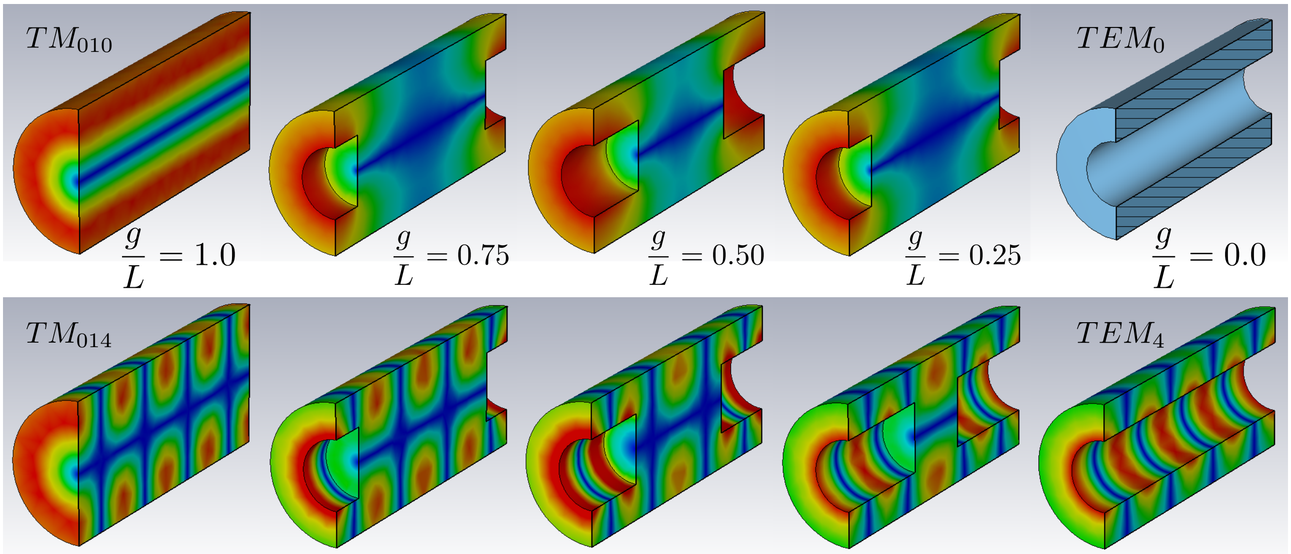

Figure 3: Magnetic field transition from TM_{010} to TEM_0 (top) and from TM_{014} to TEM_4 (bottom).

magnetic field structure, supporting the one-to-one mode mapping from TM to TEM.

MODE MAPPING

Using CST eigenmode simulations, we tracked the behavior of the first 11 resonant modes, TM_{01p}, TE_{11p} for p = 0, 1, 2, 3, 4, and TE_{21p} for p = 1, 2 as a function of the geometry parameter $\frac{g}{L}$, enabling a continuous scan of intermediate geometries. Figure 4 presents the mode tracking across all 25 steps of transition.

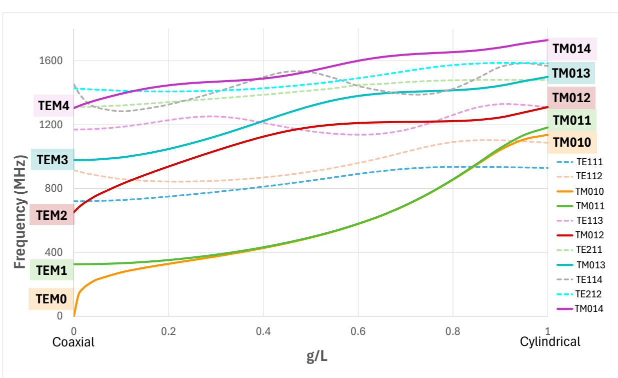

Figure 4: Eigenfrequencies of the first 11 modes plotted as a function of the geometry parameter $\frac{g}{L}$.

The TM to TEM transition curves evolve smoothly with no mode crossings, discontinuities, indicating that the transformation is adiabatic. Although some eigen-frequency curves intersect in the spectrum, these are true mode crossings between non-interacting modes with different field symmetries. The primary TM_{01p} to TEM_p mode evolution remains uninterrupted and continuous, confirming a clean one-to-one mapping throughout the transition. There is no evidence of mode mixing, sudden frequency changes, or extra resonances, which shows that each mode remains isolated throughout the transition. These results provide strong numerical evidence that smooth geometric changes can preserve mode structure and that coaxial transitions can be designed to avoid exciting unwanted modes.

CONCLUSION

This study confirms that TM modes in a cylindrical cavity transform smoothly into TEM modes in a coaxial geometry through a one-to-one mapping. CST simulations show that each mode evolves continuously without generating new modes. The results validate that the TM-to-TEM transition is adiabatic and preserves mode identity across the geometry change.

In particular, the TM_{010} mode in the cylindrical cavity evolves into the TEM_0 mode of the coaxial cavity. The frequency of this mode is 0, DC current flow along the surface, the magnetic field is static, and no electric field is present. For very small gap g, the frequency of this mode is proportional to $\sqrt{g}$ [2].

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