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SINGLE-BUNCH INSTABILITIES AT THE FERMILAB RECYCLER RING

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Abstract

Understanding and characterizing collective instabilities is critical for high-intensity operation at the Fermilab Recycler Ring. This work presents an application of the the Nested Head-Tail (NHT) formalism for modeling single-bunch transverse instabilities, incorporating analytical solutions to the resistive wall impedances in the absence of space charge. Predicted growth rates and mode structures are benchmarked against PyHEADTAIL simulations and ongoing experimental measurements. The experimental program includes studies of bare-machine instabilities at different machine parameters. These results support the understanding of naturally-occurring instabilities in the Recycler and contribute to the development of predictive tools for beam stability in future high-intensity configurations.

INTRODUCTION

As intensity is increased for the machines in the Fermilab Accelerator Complex, there are several physics phenomena that need to be better understood. This work focuses on one accelerator in the complex which is the Recycler Ring (RR). The Recycler Ring is permanent-magnet circular accelerator operating at an energy of 8 GeV. Specifically for RR, single bunch instabilities generated by collective effects in the bunch need to be better understood.

A single bunch instability refers to collective processes that affect an individual particle bunch within a particle accelerator. These instabilities arise from the interaction of the particles with the electromagnetic wakefields produced when the bunch interacts with the surrounding environment [1]. All along the accelerator, including the beam pipe itself, there will be impedance elements that will interact with the beam and create such wakefields. This type of instabilities exhibit an exponential growth of the beam oscillation, just at it is shown Fig. 1. Single bunch instabilities are a significant concern in high-intensity beam operations as they can lead to deterioration of beam quality, including emittance growth and particle loss, and can limit the achievable beam intensity.

The following work compares theoretical predictions and simulations against experimental data of single bunch instabilities. The theoretical predictions are done under the Nested Head-Tail (NHT) framework [2]. On the other hand, the simulations were done using PyHEADTAIL. In both cases, no space charge was modeled. Table 1 shows the nominal parameters used in the calculations.

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Table 1: Nominal Recycler Ring properties for beam used in instability experiments, theoretical calculations and PyHEADTAIL simulations.

Parameter	Value	Unit
Circumference C	3319.4	m
Machine Mean Radius R_0	528.30	m
Momentum	8.835	GeV/c
Relativistic Factor γ	9.47	
Revolution Period T_0	11.1	µs
Revolution Frequency f_0	89.8	kHz
RF Frequency	2.5	MHz
RF Voltage	80	kV
Synchrotron Tune Q_s	0.000595	
Slip Factor η	-8.6×10^{-3}	
Superperiodicity	2	
Horizontal Tune Q_x	25.4601	
Vertical Tune Q_y	24.412	
Horizontal Chromaticity ξ_x	0	
Vertical Chromaticity ξ_y	0	
95% Normalized Emittance	15	π mm mrad
Bunch Length σ_z	9	m
95% Longitudinal Emittance	3.64	eV·s
Beam Pipe Major Axis a	47.625	mm
Beam Pipe Minor Axis b	22.225	mm
Conductivity (SS) σ	1.4×10^6	S/m
Dipole Yokoya X G_{1x}	0.4501	
Dipole Yokoya Y G_{1y}	0.8369	

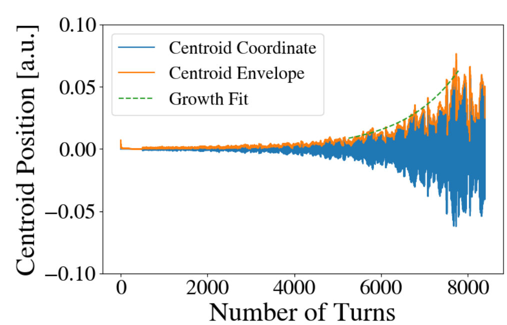

Figure 1: Measurement of a single bunch instability in the Recycler Ring with corresponding envelope used for fitting exponential trend.

NHT THEORY

The Nested Head-Tail (NHT) model, developed by A. Burov [2], is a Vlasov solver used to study transverse oscillations in bunched beams, taking into account factors such as impedance, feedback, and beam-beam interactions without taking into account any space-charge effects.

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The main source of impedance for the Recycler Ring is the resistive wall impedance [3]. The impedance for a resistive wall interaction inside a beam pipe is approximated as [1]:

$$Z_{\perp}^{RW}(\omega) = R_W \times \begin{cases} \frac{1-i}{\omega^{1/2}} & \text{for } \omega > 0 \\ \frac{1+i}{-|\omega|^{1/2}} & \text{for } \omega < 0\,, \end{cases} \tag{1}$$

where R_W can be calculated from the parameters in Table 1:

$$R_W = G_{1y} \frac{C}{\pi b^3} \sqrt{\frac{2\pi}{\sigma}} \approx 1.77083 \times 10^{-4}\ \mathrm{cm}^{-2}\ \mathrm{s}^{1/2}. \tag{2}$$

The impedance operator for a resistive wall wake is:

$$\begin{gathered} \hat{Z}^{RW}_{lm\alpha\beta} = i^{l-m} \frac{\kappa}{n_r} R_W \Phi^{(1/2)}_{lm\alpha\beta} \times \\ \left[(1-i) - (1+i)(-1)^{l+m}\right], \end{gathered} \tag{3}$$

where we have defined an intermediate integral $\Phi^{(1/2)}_{lm\alpha\beta}$, that can be calculated from this analytical form:

$$\begin{gathered} \Phi^{(\lambda)}_{\mu\nu ab} = \int_0^\infty \frac{J_\mu(at) J_\nu(bt)}{t^\lambda} dt = \\ \frac{a^\mu \Gamma\left(\frac{1}{2}\nu + \frac{1}{2}\mu - \frac{1}{2}\lambda + \frac{1}{2}\right)}{2^\lambda b^{\mu-\lambda+1} \Gamma\left(\frac{1}{2}\nu - \frac{1}{2}\mu + \frac{1}{2}\lambda + \frac{1}{2}\right)} \times \\ {}_2F_1\left(\tfrac{1}{2}(\mu-\nu-\lambda+1), \tfrac{1}{2}(\mu+\nu-\lambda+1); \mu+1; \frac{a^2}{b^2}\right). \end{gathered} \tag{4}$$

Figure 2 shows the solution to the eigenvalue equation proposed by the NHT model, with Eq. 3 as the input impedance operator. In this case, the intensity is scanned and the real and imaginary part of the eigenvalues Ω are saved. An instability arises when two modes couple and the imaginary part of these modes mode becomes larger than zero, i.e., $\mathrm{Im}\Omega > 0$. This is known as a Transverse Mode Coupling Instability (TMCI). For the nominal parameters outlined in Table 1, Fig. 3 shows the TMCI threshold to be somewhere close to 2.5×10^{11} protons per bunch (ppb).

PyHEADTAIL SIMULATIONS

PyHEADTAIL is a macroparticle tracking code specifically designed to simulate collective effects in circular accelerators [4]. The simulation setup for our problem included

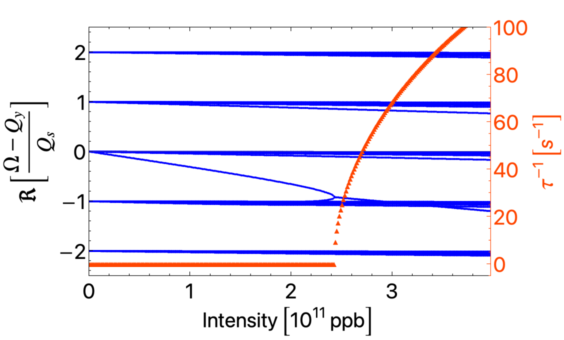

Figure 2: Solution to the NHT eigenvalue equation using a resistive wall wake as a function of protons per bunch (ppb).

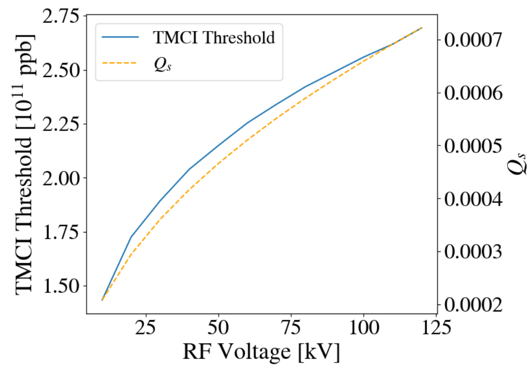

Figure 3: TMCI threshold as a function of RF voltage. The calculated synchrotron tune Q_s is also plotted on twin axis.

a linear segment with tunes Q_x, Q_y and Q_s and a resistive wall wake kick. The longitudinal dynamics is modeled with a nonlinear RF kick with the slip factor η defined in Table 1. The initial particle distribution was matched in the transverse planes, but set to an airbag distribution in the longitudinal phase space. The simulations ran for 50000 turns and on every turn the centroid position coordinates were saved.

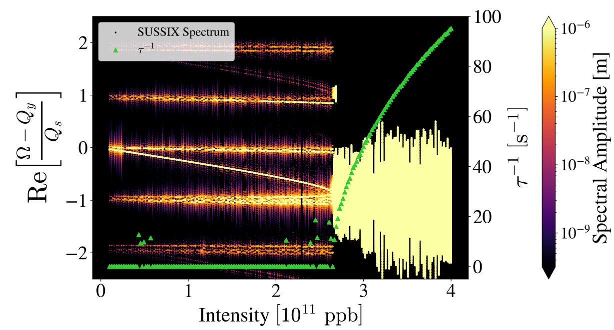

Figure 4: Spectrum and instability growth rates for PyHEADTAIL simulations and SUSSIX calculations.

For these particular scans, the number of particles per bunch was scanned while holding everything else constant. The post-simulation analysis included running the transverse centroid data through SUSSIX [5, 6] in order to get the frequency spectrum. The vertical tune Q_y was subtracted from the frequencies and normalized by the synchrotron tune Q_s. Additionally, once the beam would cross the TMCI threshold, the instability growth rate τ^{-1} would be calculated from an exponential fit. Figure 4, shows an intensity scan of spectral data and growth rate. One can clearly see the TMCI threshold is close to 2.5×10^{11} protons per bunch (ppb)—similar value obtained with NHT calculations. This is a good benchmark for the NHT analytical solution presented in Eq. 3. It is worth pointing out that this PyHEADTAIL scan takes around 3 days on the Fermilab computer cluster while the NHT calculations take around 10 seconds on a personal laptop.

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EXPERIMENTAL DATA

The following experiments follow up on previous attempts to characterize TMCI at the Recycler Ring, similar to those described in Ref. [7]. The nominal parameters for these experiments are those highlighted in Table 1. In addition to this, the Recycler Ring transverse dampers were turned off. A long stripline pick-up was used in order to capture the dipole motion of the beam. Figure 1 shows an instability measured with this setup. In particular, using relatively long bunches with the 2.5 MHz RF station allows us to look at intrabunch motion.

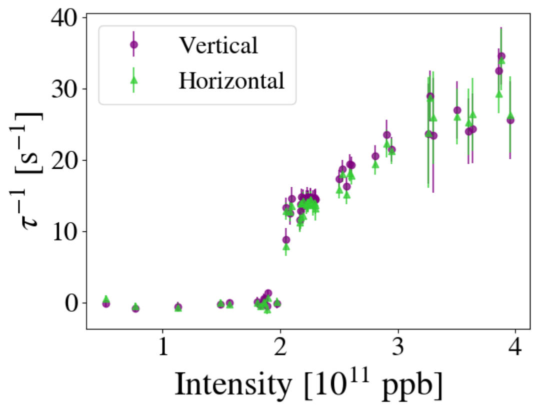

Figure 5: Experimental data for instability growth rate versus intensity in both planes.

Three experiments are herein presented: 1.) instability growth rate versus intensity at constant tune and RF voltage (Fig. 5), 2.) growth rate versus intensity at two RF voltages (Fig. 6), and 3.) growth rate versus horizontal tune with constant vertical tune and intensity (Fig. 7).

Figure 5 shows how the instability growth rates, for both the horizontal and vertical plane, depend on the number of protons per bunch. There is a clear intensity threshold for when the instability starts to develop in both planes—around 2.0×10^{11} protons per bunch. NHT model calculations and simulations predict the beam should only go unstable in one plane.

Figure 6 shows the experimental instability threshold increases as the RF voltage is decreased. While the RF voltage changed by approximately 30%, the intensity threshold doubled. This goes in contradiction to what is shown in Fig. 3 from theory. The scaling factor for the instability threshold and the RF voltage is not as large for NHT predictions.

Figure 7 shows a sharp dependence between the instability growth rate and the horizontal tune for a fixed vertical tune and intensity. It also shows that when the machine is coupled on the $Q_x - Q_y = -1$ line, an instability can be measured. Otherwise, when the working tunes were away from the coupling line no instability was observed. This was true for all available intensities. If there is a TMCI threshold for the uncoupled experimental case it is higher than 5×10^{11} ppb.

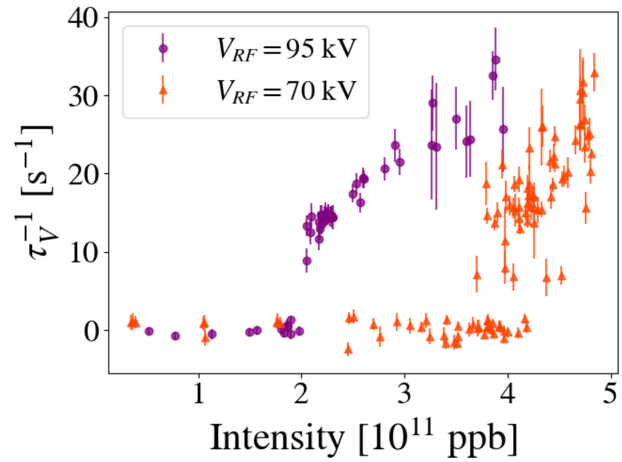

Figure 6: Two experimental datasets at different RF voltages for vertical instability growth rate versus intensity.

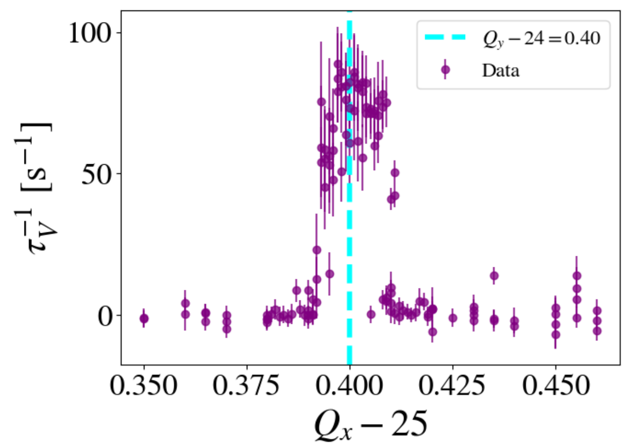

Figure 7: Experiment for vertical growth rate versus horizontal tune with constant vertical tune and constant intensity.

CONCLUSIONS & FUTURE WORK

Experiments show a strong dependence of the instability growth rate against linear coupling in the machine, which is not predicted by theory/simulations. Current theoretical and computational models do not include any space charge or linear coupling components, which the experiments suggest to also play an important role. Future work will look to integrate space charge and linear coupling into the theoretical calculations and PyHEADTAIL simulations. Additionally, head-tail amplification and spectral information still needs to be calculated from the experimental datasets.

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