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# JUTRACK, A JULIA-BASED AUTO-DIFFERENTIABLE ACCELERATOR SIMULATION CODE FOR ADVANCED DYNAMICS, SCIENTIFIC MACHINE LEARNING AND OPTIMIZATION*

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*Abstract*

JuTrack is a high-performance accelerator modeling and particle tracking package developed in the Julia programming language. With compiler-level automatic differentiation (AD), JuTrack enables fast and precise derivative computations for arbitrary differentiable simulation functions. It supports efficient modeling of complex collective effects such as space-charge forces, wakefields, and beam-beam interactions. Beyond conventional tracking simulations, JuTrack also incorporates a machine learning–based module for self-field modeling and convenience interface that brings data-driven and physics-driven model together. In this paper, we demonstrate the capability and performance of JuTrack through a broad set of beam dynamics applications and optimization across various accelerator types, including a synchrotron light source, a heavy-ion linear accelerator, and the colliders. Built on Julia's high-performance architecture and user-friendly syntax, JuTrack provides a powerful tool for beam dynamics studies and accelerator design optimization.

## INTRODUCTION

Automatic differentiation (AD) is a computational technique for evaluating derivatives of functions implemented in computer programs. By decomposing a function into elementary operations and applying the chain rule sequentially, AD delivers machine-precision derivatives while avoiding the complexity and overhead of symbolic differentiation. It has long been a foundational tool in fields such as machine learning and numerical optimization.

Recently, AD has attracted growing interest in the accelerator physics community. AD-enabled simulation codes, such as JuTrack [1], Cheetah [2], BMAD [3], and MAD-NG [4], have emerged. JuTrack is a high-performance accelerator modeling and tracking package written in Julia. With compiler-level AD, it enables rapid and accurate derivative evaluations for arbitrary differentiable functions. In this paper, we present the capability and performance of JuTrack in a broad set of beam dynamics and optimization tasks, including collective effects studies based on machine learning, gradient-based optimization for a synchrotron light source, multicharge state particle tracking in a heavy-ion linac, and dynamic aperture analysis of a collider using the convergence map method.

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## NONLINEAR BEAM DYNAMICS OPTIMIZATION

One of the primary motivations for integrating AD into accelerator modeling is to optimize the design parameters to meet specific beam objectives, such as minimizing natural emittance, maximizing the dynamic aperture, and improving overall stability. With derivatives, many complex optimization tasks become straightforward gradient descent problems.

As an example, for the Stanford Positron Electron Asymmetric Ring (SPEAR3), a third-generation synchrotron light source, its geometric resonance driving terms (RDTs) such as $h_{21000}$, $h_{10110}$, $h_{30000}$, $h_{10200}$, and $h_{10020}$, can be evaluated with JuTrack. These RDTs are key indicators of dynamic aperture and reducing them tends to enlarge the dynamic aperture.

To minimize the RDTs, a simple test is performed in which two sextupole families in the ring are varied. Here we just tune the two sextupole families to demonstrate the effectiveness of the AD gradients in beam dynamics optimization. In real optimization processes, additional constraints, such as preserving the chromaticity of the ring, should be considered. Figure 1 shows that after roughly 500 iterations, the gradient-descent algorithm converges to a local minimum, with the gradients for both sextupole strengths approaching zero. This shows that one can effectively suppress the RDTs with the AD gradients.

## MULTI-CHARGE-STATE PARTICLE TRACKING

Most tracking codes handle only a single species of charged beam, while the Facility for Rare Isotope Beams (FRIB) accelerates multiple charge states simultaneously after stripping, creating a distinctive challenge for particle tracking simulation.

To address this specific requirement, JuTrack has been extended to track multiple charge states while preserving full differentiability. Figure 2 shows the concurrent tracking of three charge states, $^{124}Xe^{49+}$, $^{124}Xe^{50+}$ and $^{124}Xe^{51+}$. The JuTrack tracking results were compared with FLAME, a fast envelope tracking code used for daily operation at FRIB. With the same lattice configurations and initial conditions, the JuTrack results show great agreement with FLAME, suggesting the precision of the multi-charge state tracking capability [5].

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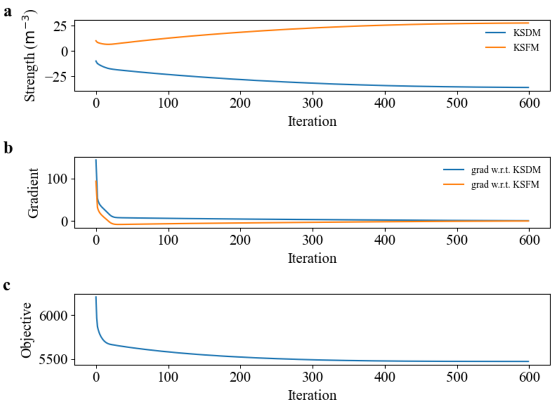


Figure 1: Optimization process of the RDTs for the SPEAR3 ring. (a) shows the evolution of the strengths of two sextupole families. (b) shows the evolution of the derivatives of the objective function with respect to the sextupole strengths. (c) is the change of the objective function, which is the summation of five geometric RDTs.

## SELF-FIELD EVALUATION AND MACHINE LEARNING-BASED SURROGATE MODEL

Assuming perfectly conducting rectangular pipe boundary conditions, the space-charge Hamiltonian $H_2$ can be approximated with a gridless spectral method (see details in Ref. [6]),

$$H_2 = 4\pi \frac{K}{ab} \frac{1}{N_p} \sum_{i=1}^{N_P} \sum_{j=1}^{N_P} \sum_{l=1}^{N_l} \sum_{m=1}^{N_m} \frac{1}{\gamma_{lm}^2} \sin(\alpha_l x_j) \sin(\beta_m y_j) \times \sin(\alpha_l x_i) \sin(\beta_m y_i) \tag{1}$$

where $N_l$ and $N_m$ are the number of modes in the horizontal and vertical plane, respectively. $a$ is the horizontal width of the pipe and $b$ is the vertical width of the pipe. $\gamma_{lm}^2 = \alpha_l^2 + \beta_m^2$. With the approximated $H_2$, the symplectic space-charge kick is obtained by differentiation. JuTrack implements transverse space-charge effects in particle tracking using this approach.

In addition to the physical model, a machine learning-based space-charge model trained on a large dataset is also developed with JuTrack [7]. This surrogate model delivers space-charge kicks roughly an order of magnitude faster than the physical model.

Figure 3 presents the simulated tune diagram for a 1 GeV proton beam passing 20,000 periodic FODO cells. The beam current is 20 A and the normalized emittance is 1 µm. The difference between the gridless spectral model and the ML model is around 1%, indicating the high precision of the ML-based space-charge model.

## DYNAMIC APERTURE ANALYSIS WITH CONVERGENCE MAP

For a dynamical system described by the canonical variable $x$ and momentum $p$, one can introduce the complex Courant-Snyder variable $z = x - ip$ and its conjugate $z^*$. The set $\{z, z^*, z^2, zz^*, z^{*2}, z^3 ...\}$ forms a new column $Z$, where the one-turn map can be described by a square matrix $M$ [8],

$$Z = MZ_0, \tag{2}$$

and the Jordan transformation,

$$UM = e^{i\mu I + \tau} U. \tag{3}$$

A transformation $W \equiv UZ$ was proposed in Ref. [8] based on the Jordan transformation of the square matrix. Each row of $W$, e.g., $w_{x0}, w_{x1}, ..., w_{y0}, w_{y1}, ...$, can be used as a set of approximated action-angle variables. The details of the transformation are given in Ref. [8]. The trajectories of these action-angle variables form circles with small deviations from a rigid rotation. A convergent series solution can be used as an indicator to the existence of the quasi-periodic solution. If the convergence map blows up, it suggests that the trajectory has a large amplitude or is close to a resonance line.

JuTrack supports Truncated Power Series Algebra (TPSA), enabling the computation of the convergence map. To illustrate its effectiveness for nonlinear beam dynamics analysis, we use a crab cavity model in which synchro-betatron coupling is introduced by

$$\delta p_x = -\frac{\tan\theta_{cc} \sin(k_c z)}{k_c \sqrt{\beta_{cc}\beta_{IP}}} + b_3(x^2 - y^2) \sin(k_c z) \tag{4}$$

$$\delta p_y = -2b_3 xy \sin(k_c z) \tag{5}$$

$$\delta p_z = -\frac{x \tan\theta_{cc} \cos(k_c z)}{\sqrt{\beta_{cc}\beta_{IP}}} + \frac{b_3 k_c}{3}(x^3 - 3xy^2) \cos(k_c z), \tag{6}$$

where $\theta_{cc}$ and $k_c$ are the crossing angle and the wave number of the cavity, respectively, and $b_3$ represent the high-order components in the cavity. The remainder of the periodic map is a pure rotation.

Figure 4 shows the convergence map of the crab cavity model and the comparison with the frequency map. The convergence metric in the plot is defined as the minimal normalized distance of the action-angle transformation to a rigid rotation. The convergence map captures all resonances identified by the frequency map, confirming its precision and effectiveness in characterizing 6-D nonlinear dynamics in this system. By extracting information directly from the one-turn map, the convergence map reveals the resonance structure that frequency map analysis typically requires thousands of tracking turns, offering an efficient tool for nonlinear beam dynamics studies and dynamic aperture evaluation.

## CONCLUSION

We have presented JuTrack, a Julia-based simulation code that combines high-performance particle tracking with

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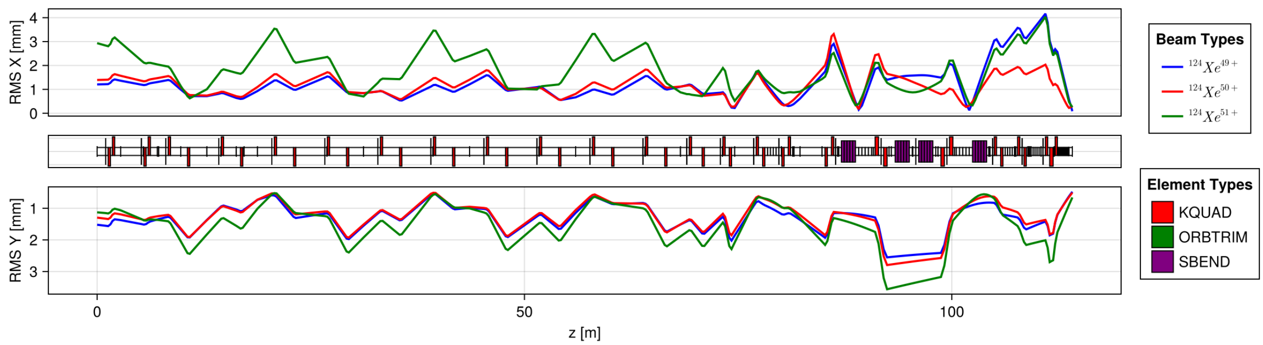


Figure 2: Concurrent tracking of three charge states at post-stripper section of FRIB.

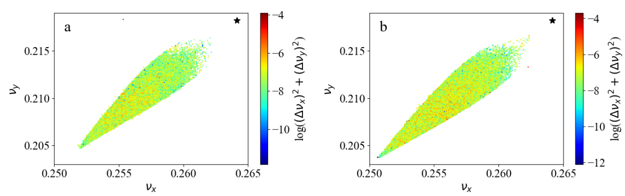

Figure 3: Tune diagram with space-charge effects. The left plot represents results with the gridless spectral method and the right plot represents the results of the machine learning model. The black start represents the nominal tune without space-charge effects.

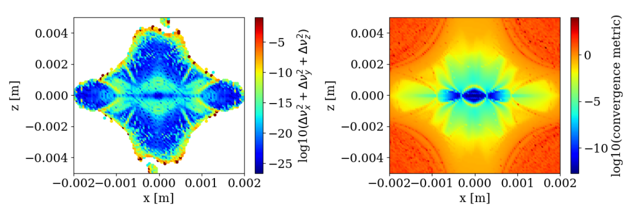

Figure 4: Analysis results of using frequency map (left) and convergence map (right) to a 3-D toy map.

compiler-level AD. Taking advantage of Julia's just-in-time compilation and advanced AD capabilities, JuTrack achieves both computational efficiency and flexibility for accelerator modeling. We demonstrated its effectiveness and capabilities for advance beam dynamics study across diverse applications, including nonlinear beam dynamics optimization, multi-charge state tracking, space-charge effect simulations, and dynamic aperture analysis through convergence maps.

## ACKNOWLEDGEMENTS

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