

ANALYSIS OF THE ELLIPTIC INTEGRABLE NON-LINEAR SYSTEM IN IOTA USING TRACKING OF A SINGLE ELECTRON*

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Abstract

Integrable nonlinear lattices that can be realized in practical accelerators are of great interest, as they offer the potential to support high-intensity beams via Landau damping of collective instabilities. One such system, based on an elliptic potential, has been extensively studied at the IOTA storage ring at Fermilab. The analysis of strongly nonlinear dynamics with multi-particle bunches is challenging due to the rapid decoherence of kicked beams. IOTA has the capability to track single electrons using linear multi-anode photomultiplier tubes for simultaneously measuring transverse coordinates and arrival times of synchrotron-radiation pulses. This technology enables the full reconstruction of turn-by-turn positions and momenta in all three planes for a single particle. Using this apparatus, we measured the dependence of small-amplitude tunes on the strength of the nonlinear magnet, as well as tunes dependence on oscillations amplitudes.

INTRODUCTION

This paper presents one of the first application cases of beam diagnostics based on single-electron tracking [1, 2] at the Integrable Optics Test Accelerator (IOTA, [3]). The ability to track an electron in all six phase-space dimensions allows researchers to avoid a fundamental problem of multi-particle bunches – the smearing of collective signals due to decoherence. Nonlinear systems, such as the elliptical integrable system studied at IOTA, exacerbate decoherence, reducing the useful number of observable turns after a beam kick to only a few tens when the nonlinear insert is strongly excited.

The IOTA lattice must satisfy several conditions: zero dispersion in the nonlinear insert, and an equivalent of an axially symmetric thin focusing lens with predefined strength for the section outside the nonlinear insert. The nominal tunes must lie on the coupling resonance; however, for diagnostic purposes, a controlled distortion was introduced in the case of a disabled nonlinear insert. This adjustment was necessary to distinguish between vertical and horizontal modes. The parameters of the bare lattice and the beam sizes are given in Table 1 and Fig. 1. A comparison of the beta functions for the lattice with the nonlinear insert disabled and with strength $t = 0.45$ is shown in Fig. 2.

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Table 1: IOTA Parameters During the Experiment

Parameter	Value
Perimeter	39.96 m
Momentum	150 MeV/c
Bunch intensity	1 e^-
RF frequency	30 MHz
RF voltage	350 V
Betatron tunes, (ν_x, ν_y)	(5.3, 5.3)
Synchrotron tune, ν_s	3.5×10^{-4}
Damping times, (τ_x, τ_y, τ_s)	(2.08, 0.65, 0.24) s
Horizontal emittance, ϵ_x	127 nm
Momentum spread, $\Delta p/p$, RMS	1.3×10^{-4}
Momentum compaction, α_p	0.083
Natural chromaticity C_x, C_y	-10.9, -9.4
Compensated chromaticity C_x, C_y	0.0, 0.0

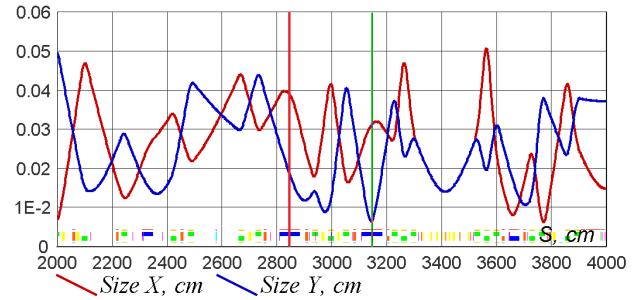


Figure 1: Horizontal and vertical beam sizes in the second half of IOTA. The vertical lines show locations of the detector at M3L (red) and M2L (green) dipoles.

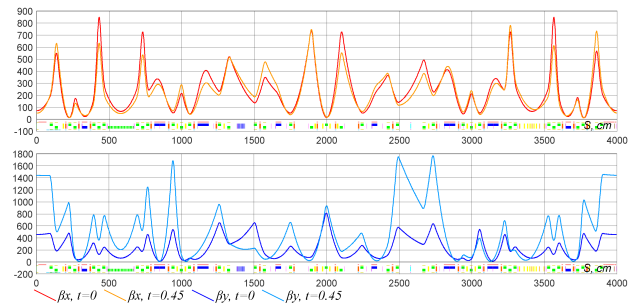


Figure 2: Horizontal and vertical beta-functions for the bare IOTA lattice and for $t=0.45$, which is close to the integer resonance in the vertical plane.

The first-order effect of the nonlinear insert is equivalent to that of a quadrupole. The dependence of the small-amplitude oscillation tunes on the insert's dimensionless scaling parameter t is given by [4]:

$$\nu_{x,y} = \nu_0 \sqrt{1 \pm 2t}. \quad (1)$$

For $t = 0.5$, the fractional part of the vertical tune becomes zero, placing the lattice on the integer resonance. In this case, the linear focusing component vanishes, yet the nonlinear contribution of the insert remains sufficient to keep the beam stable.

EXPERIMENTAL SETUP

All eight main dipoles in IOTA [3] are equipped with synchrotron-light stations mounted directly on top of the magnets. Photons emitted by an electron while traversing the dipole field are directed and focused into a dark box instrumented with customizable diagnostics [2, 5]. For this experiment, two such diagnostic stations were used, located at the M2L and M3L dipoles.

We employed the PML-16 detector from Becker & Hickl [6], which is based on a Hamamatsu multi-anode photomultiplier tube (PMT). The PML-16 features an active area of 16×16 mm containing 16 individual cathodes arranged in a linear array. To fully utilize this relatively large detection area, a defocusing lens was added to the optical system to match the beam image size to the detector's active area. The multi-anode PMT at the M2L station was used to record the horizontal position, while the one at M3L measured the vertical position.

Data were collected in 60-second intervals for a range of nonlinear magnet excitation strengths from $t = 0$ to $t = 0.45$ in steps of 0.05, and additionally for $t = 0.495$. At $t = 0$, the horizontal tune was shifted away from the vertical tune to a target value of $\nu_x = 0.295$.

RESULTS

The measured data consisted of three values for each photocount: the turn number, the arrival time relative to the most recent revolution marker, and the index of the PMT segment that detected the photon, providing a coarse estimate of the electron's position. With two detectors – one for the horizontal and one for the vertical plane – it is possible to reconstruct an electron's oscillations in 6D phase space for near-harmonic motion [2]. Applying this technique in the presence of a strong nonlinear magnet is feasible only for relatively small amplitudes, where nonlinear effects can be treated as perturbations whose primary impact is the dependence of the betatron tunes on oscillation amplitude.

The random, quantized nature of synchrotron radiation induces continual variations in the horizontal and longitudinal oscillations. Vertical oscillations, being well decoupled from the other degrees of freedom, typically have amplitude below the resolution limit. While scattering on residual gas atoms can excite all three oscillation amplitudes [1], the vacuum quality in this experiment was too high to rely on this mechanism. Therefore, to scan vertical amplitudes, the electron was given a kick every 6 seconds.

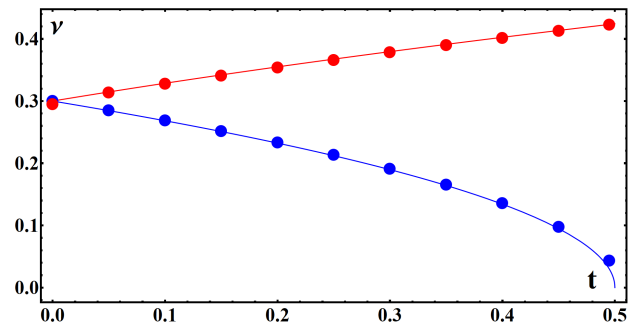


Figure 3: Dependence of the small-amplitude tunes on the strength of the nonlinear insert. Horizontal tunes are shown in red and vertical tunes in blue. The solid lines represent the model predictions, while the dots indicate the measured values.

Analysis of the collected data yielded measurements of the betatron tune dependence on the nonlinear insert strength, showing good agreement with the model (Fig. 3).

One notable observation was a reversal in the sign of the horizontal tune's dependence on vertical amplitude. Figure 4 shows the tune footprints for $t = 0.05$ and $t = 0.15$, illustrating this effect. A modeling study is underway to investigate the source of the unexpected additional nonlinearities introduced by the IOTA ring.

Another significant finding was electron capture by the sextupole resonance $2\nu_y = \nu_x$ at a nonlinear magnet's set-point of $t = 0.3$. Figure 5 presents the tune footprint with a clear capture along the corresponding resonance line. Figure 6 shows the natural evolution of electron amplitudes and tunes in the transverse planes over 2 million turns (0.26 s). The observed jumps in vertical tune correspond to the electron being captured and released from the resonance. While captured, vertical oscillations are driven through coupling with the horizontal plane. Although resonance capture typically occurs at higher amplitudes, in some cases it was observed at relatively small excitations. Improved amplitude measurement accuracy will be essential for a deeper understanding of the underlying dynamics.

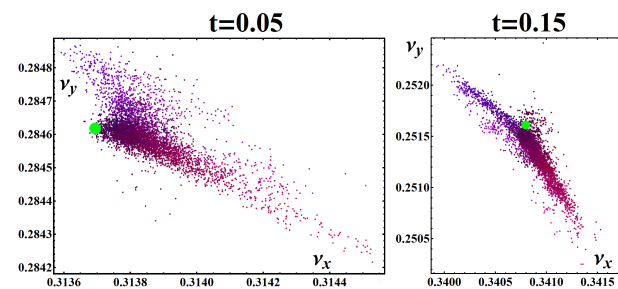


Figure 4: Betatron tunes footprints for $t = 0.05$ and $t = 0.015$. Red and blue components of the dots' colors are proportional to the corresponding horizontal and vertical amplitudes. Green dots show reconstructed zero-amplitude working point.

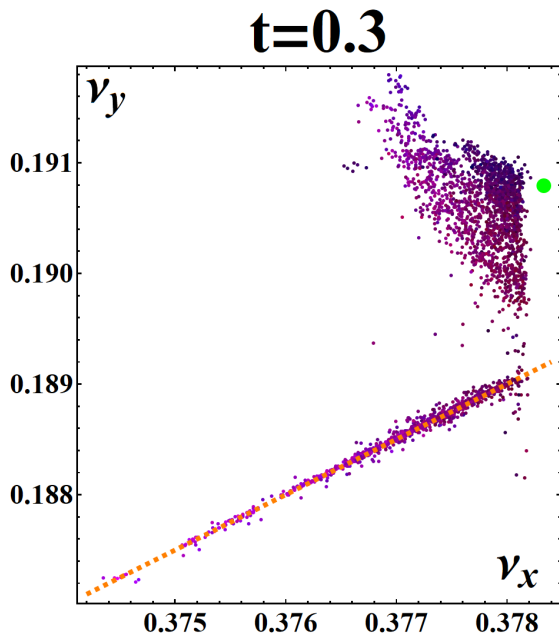


Figure 5: Betatron tunes footprint for $t = 0.3$. Red and blue components of the dots' colors are proportional to the corresponding horizontal and vertical amplitudes. Green dot shows reconstructed zero-amplitude working point. Capture at a sextupole resonance $2\nu_y = \nu_x$ (orange line) is clearly present.

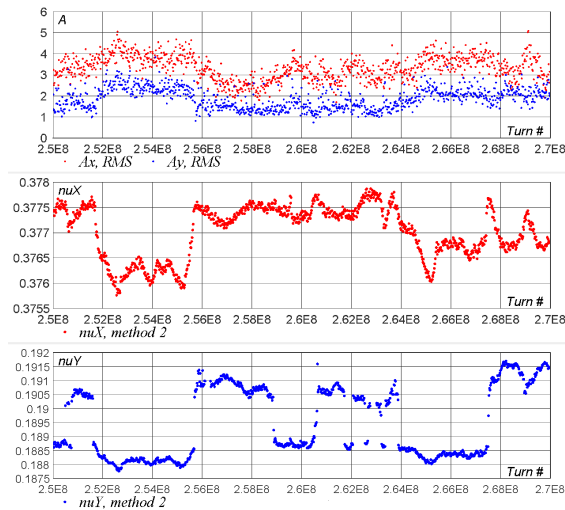


Figure 6: Evolution of electron's oscillations amplitudes (top plot) and betatron tunes in horizontal (red) and vertical (blue) planes. The strength of the non-linear magnet is $t = 0.3$. Each point is based on data from 15000 turns.

Each sequence of reconstructed tunes and amplitudes was used to fit the dependence of the betatron tunes on the oscillation amplitudes:

$$\nu = \nu_0 + c_x A_x + c_y A_y. \quad (2)$$

Figure 7 presents the linear coefficients for both planes as functions of the nonlinear magnet strength t .

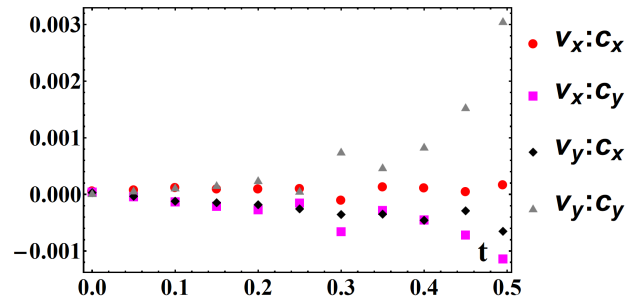


Figure 7: Linear terms of the tunes dependencies on amplitudes versus non-linear magnet strength.

SUMMARY

The presented results demonstrate the capabilities of single-particle diagnostics. The next stage of analysis will focus on improving the accuracy of betatron amplitude reconstruction. In addition, calibration of the scaling factors for both detectors will be carried out by comparing measured and modeled natural emittances, as well as by analyzing vertical oscillation amplitudes following kicks. This will be followed by a quantitative comparison of the measured and modeled tune dependence on amplitudes for various nonlinear magnet strengths.

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