Analysis of beam-beam instability including longitudinal impedance

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Acknowledgements:
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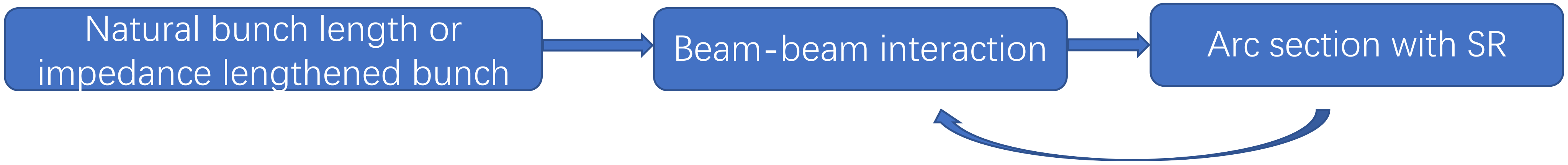
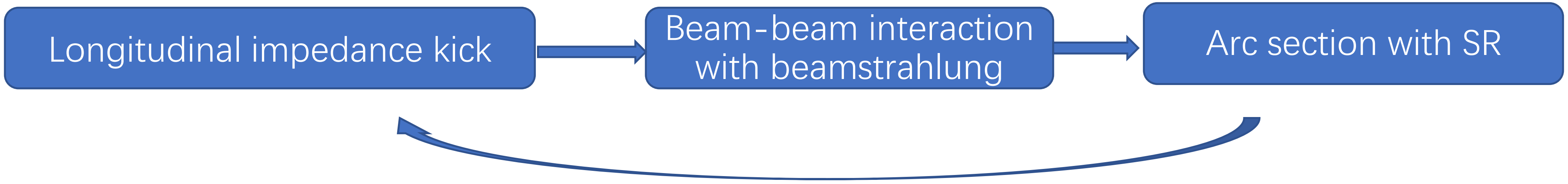
Beam-beam simulation with ZL



Conventional simulation



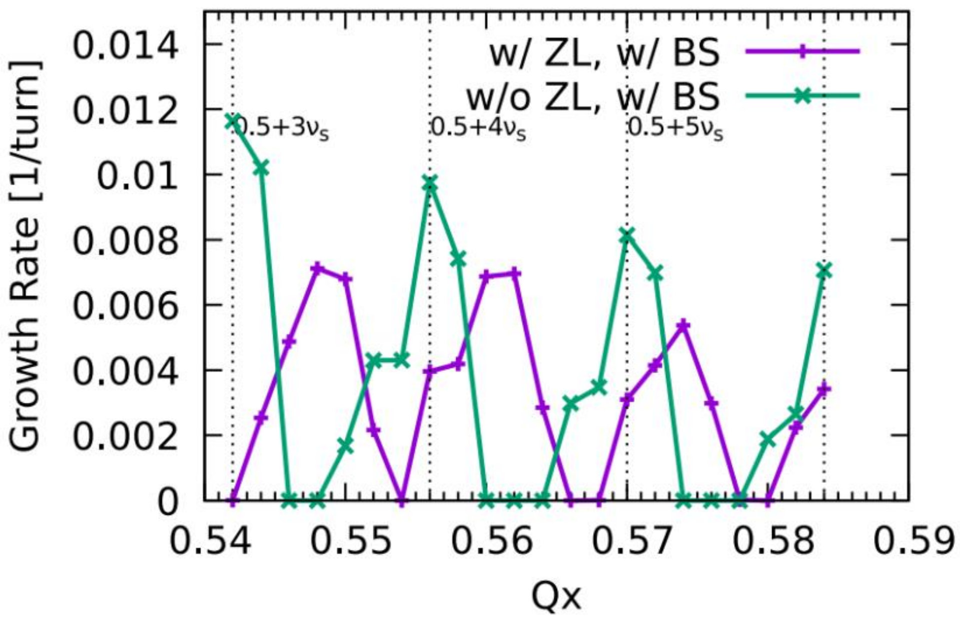
Self-consistent simulation

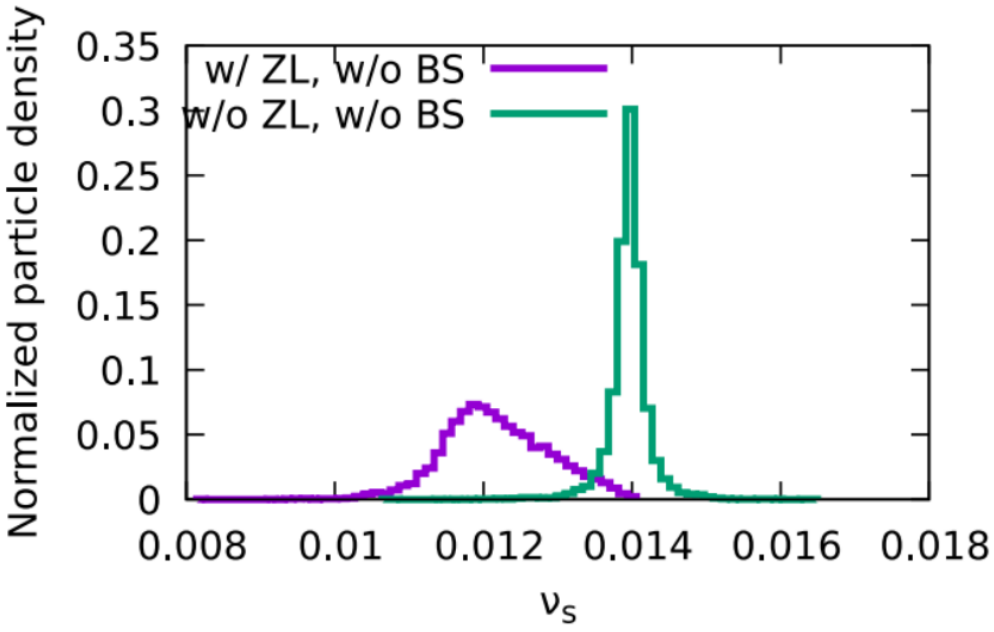
Intense beam
Low emittance
Small beta function
Large Piwinski angle
and so on

→

Beamstrahlung
x-z instability and other new effects

Beam-beam simulation with ZL





Simulation results[1] w/ and w/o ZL for CEPC-Z

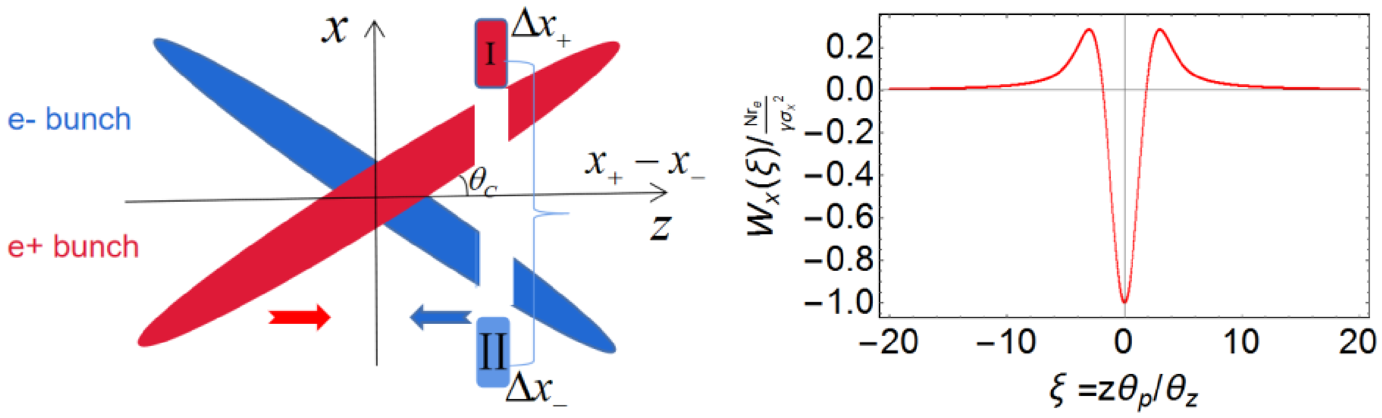
1. The shift of stable tune area
2. The squeeze of stable tune area
3. The decrease in growth rate

← Coherent synchrotron tune shift downward

← Synchrotron tune spread

[1]Y.Zhan et al., PRAB 23, 104402 (2020).

Beam-beam induced cross wake force



Evaluation of cross-wake force and cross-wake function

The "cross-wake force"[1] has been introduced to explain the coherent beam-beam instability with a large Piwinski angle without ZL.

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)\cdot x^{(+)}\left(z'\right)dz' \quad \text{(dipole)}$$

$$+\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)dz'\cdot x^{(-)}(z) \quad \text{(quadrupole)}$$

where $W_x^{(-)}(z)$ is cross-wake function induced by beam-beam interaction.

[1] K.Ohmi et al., **Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle**, **PRL (2017)**.

Transverse single bunch instability method

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} w_x^{(-)}\left(z - z'\right) \rho^{(+)}\left(z'\right) \cdot x^{(+)}\left(z'\right) dz' \quad \text{(dipole)}$$
$$+ \int_{-\infty}^{\infty} w_x^{(-)}\left(z - z'\right) \rho^{(+)}\left(z'\right) dz' \cdot x^{(-)}(z) \quad \text{(quadrupole)}$$

Beam-beam force is quite localized:

- Conventional transverse mode coupling instability (TMCI) theory for continuous wake force is not suitable[2].
- It requires localized treatment for the wake force[3][4][5].

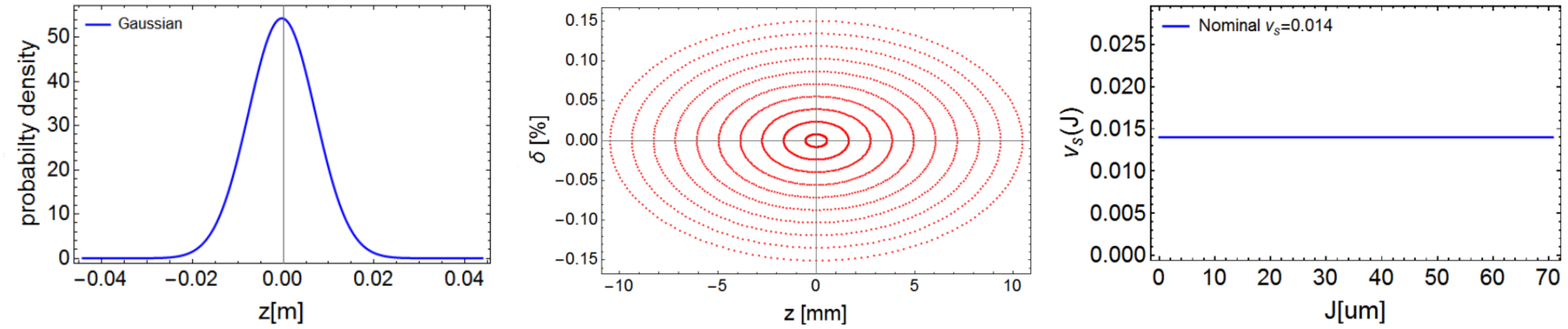
[2] A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators , New York, 1993.

[3] F. Ruggiero, Transverse mode coupling instability due to localized structure, Part. Accel. 20, 45 (1986).

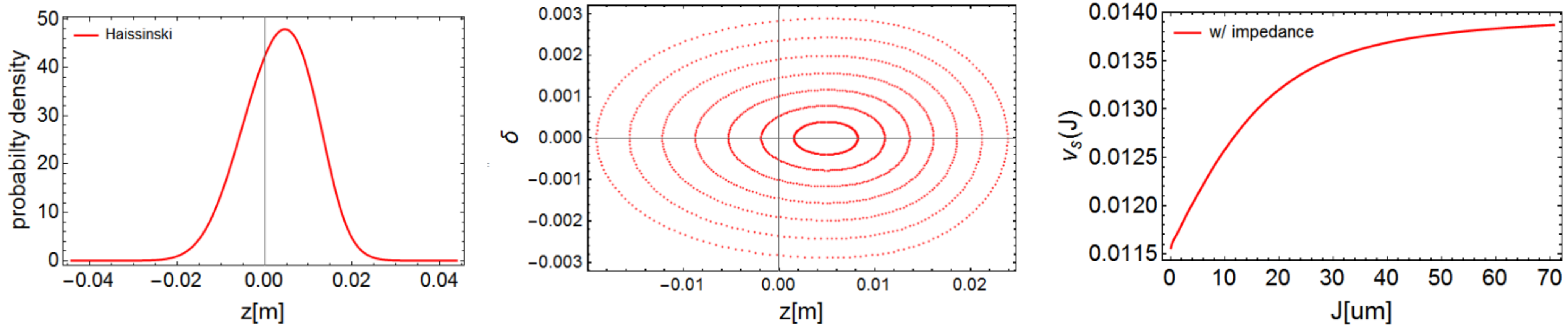
[4] K.Ohmi et al., Coherent Beam-Beam Instability in Collisions with a Large Crossing Angle, PRL (2017).

[5] K. Nami et al. PhysRevAccelBeams.21.031002.

Longitudinal beam dynamics



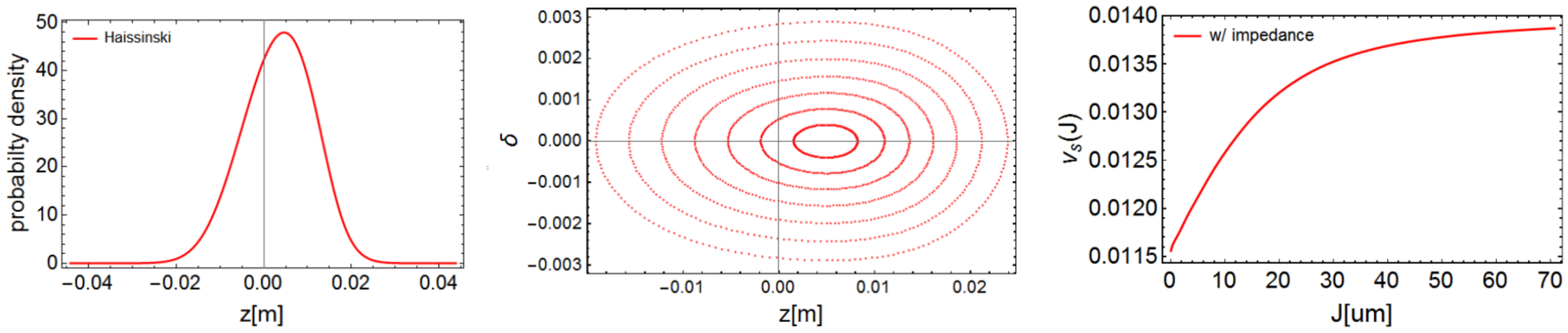
w/o zL



w/ zL

The distortion of longitudinal dynamics can come from other perturbation sources.

Beam-beam instability with ZL



Longitudinal beam dynamics with ZL

In the following ,we study the beam-beam instability including the effects of longitudinal impedance.

The main ideas of the method are:

- discretizing longitudinal action[6] J in longitudinal action-angle phase space (J, ϕ),
- transverse instability analysis method for localized wake[7]

Combining the two ideas to develop a transverse mode-coupling analysis method (action discretization method) which includes the effects of longitudinal impedance.

[6] K. Oide and K. Yokoya, KEK Report No. 90-10, 1990..

[7] K. Nami et al. PhysRevAccelBeams.21.031002.

Action discretization method

$\alpha = 0$, normalized coordinates (x, p_x)

Consider the transformation in longitudinal action-angle phase space (J, ϕ)

$J \equiv \frac{1}{2\pi} \oint \delta dz$ φ is the conjugate of J

Consider the horizontal dipole amplitude[8] x, p_x, in longitudinal phase, truncate l at $\pm l_{max}$,

$$x(J,\phi) = \sum_{l=-l_{max}}^{l_{max}} x_l(J) e^{il\phi}, \quad p_x(J,\phi) = \sum_{l=-l_{max}}^{l_{max}} p_l(J) e^{il\phi}$$

Amplitude – dependent tune

In the arc section, for electron bunch

$$\begin{pmatrix} x_l^{(-)}(J) \\ p_l^{(-)}(J) \end{pmatrix} = e^{-2\pi i l \underline{\nu_s^{(-)}(J)}} \begin{pmatrix} \cos\mu_x^{(-)} & \sin\mu_x^{(-)} \\ -\sin\mu_x^{(-)} & \cos\mu_x^{(-)} \end{pmatrix} \begin{pmatrix} x_l^{(-)}(J) \\ p_l^{(-)}(J) \end{pmatrix} \equiv M_\beta^{(-)} \begin{pmatrix} x_l(J) \\ p_l(J) \end{pmatrix}$$

Discretize and sample J

The ideal of action discretization is that we discretize J at $J_1, J_2, \ldots, J_{n_J}$,

$$\begin{pmatrix} x_l^{(-)}(J_i) \\ p_l^{(-)}(J_i) \end{pmatrix} = e^{-2\pi i l \nu_s^{(-)}(J_i)} \begin{pmatrix} \cos\mu_x^{(-)} & \sin\mu_x^{(-)} \\ -\sin\mu_x^{(-)} & \cos\mu_x^{(-)} \end{pmatrix} \begin{pmatrix} x_l^{(-)}(J_i) \\ p_l^{(-)}(J_i) \end{pmatrix} \equiv M_\beta^{(-)} \begin{pmatrix} x_l^{(-)}(J_i) \\ p_l^{(-)}(J_i) \end{pmatrix}$$

The same procedures are for positron bunch,

$$\begin{pmatrix} x_l^{(+)}(J_i) \\ p_l^{(+)}(J_i) \end{pmatrix} = M_\beta^{(+)} \begin{pmatrix} x_l^{(+)}(J_i) \\ p_l^{(+)}(J_i) \end{pmatrix}$$

We basically transform the dipole moment vector $(x_l^{(-)}(J_i), p_l^{(-)}(J_i), x_l^{(+)}(J_i), p_l^{(+)}(J_i))$, and finally we have the transfer matrix for the arc section,

$$M_\beta = \begin{pmatrix} M_\beta^{(-)} & 0 \\ 0 & M_\beta^{(+)} \end{pmatrix}$$

[8] C.Lin, K.Ohmi, and Y.Zhang, PhysRevAccelBeams.25.011001 (2022).

Action discretization method

At IP, the discretization of momentum change

$$\Delta p_l^{(\pm)}(J) = -\frac{\beta_x^{(\pm)}}{2\pi}\sum_{l'}\int dJ' W_{ll'}^{(\pm)}(J,J')\,\underline{\psi^{(\mp)}}(J')\,x_{l'}^{(\mp)}(J')$$

Longitudinal phase space distribution, by tracking or Haissinski solution. Assume the microwave instability do not happen.

can be expressed as,

$$\Delta p_l^{(\pm)}(J_i) = -\frac{\beta_x^{(\pm)}}{2\pi}\sum_{l'}\sum_{i'}\Delta J_{i'} W_{ll'}^{(\pm)}(J_i,J_{i'})\,\psi^{(\mp)}(J_{i'})\,x_{l'}^{(\mp)}(J_{i'}) \equiv \beta_x^{(\pm)} M_{lil'i'}^{(\pm)} x_{l'}^{(\mp)}(J_{i'})$$

or more consise form, for electron and positron bunch the momentum change is ,

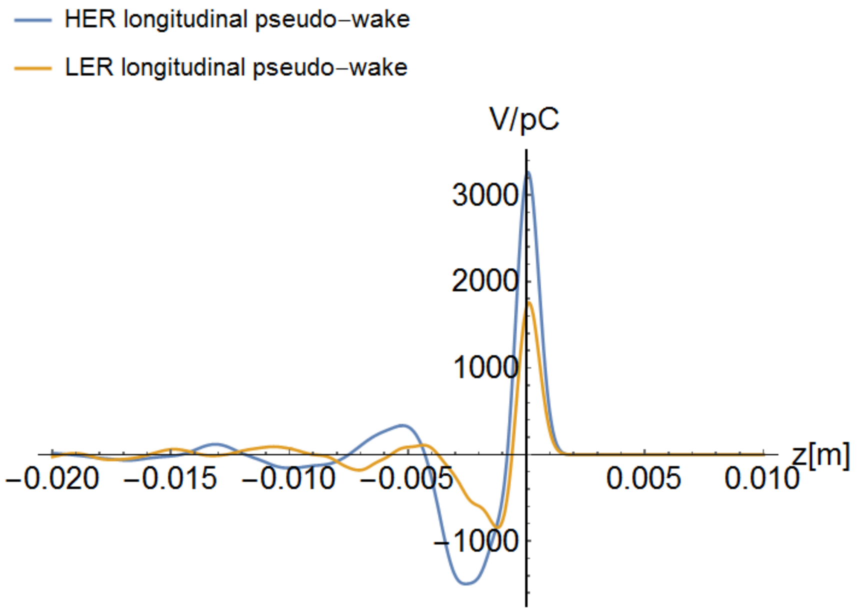
$$M_W = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \beta_x^{(-)} M_{lil'i'}^{(-)} & 0 \\ 0 & 0 & 1 & 0 \\ \beta_x^{(+)} M_{lil'i'}^{(+)} & 0 & 0 & 1 \end{pmatrix}$$

The dimension of the matrix M_β, M_W is $(2\times 2\times(2l_{\max}+1)\times n_J)^2$.

Finally, the stability of the colliding beams is determined by the eigenvalues $\lambda' s$ of the revolution matrix $M_\beta M_W$.

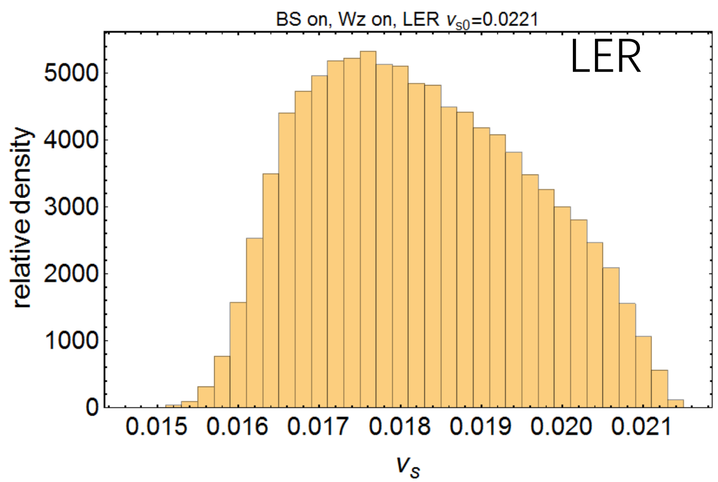
Asymmetric collision

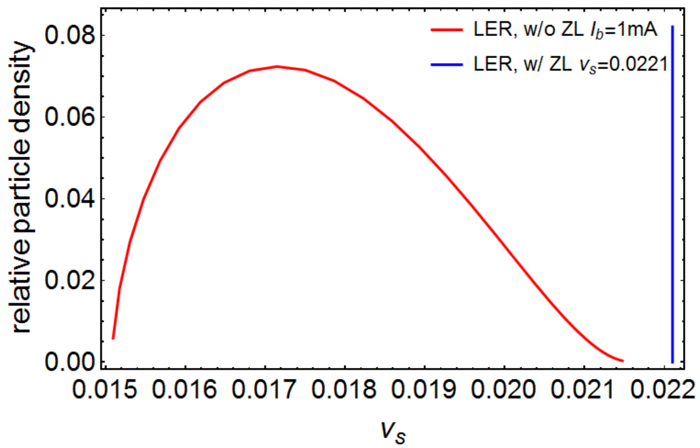
Parameters		HER	LER
Circumference	$L(m)$	3016.315	
Crossing angle	$\theta_c(mrad)$	0.0415×2	
Beam energy	$E(GeV)$	7	4
Synchrotron tune	ν_s	0.0272	0.0221
β at IP	$\beta_x^*(m)$	0.06	0.08
Horizontal emmitance	$\epsilon_x(nm)$	4.6	4.0
Longitudinal emmitance	$\epsilon_z(\mu m)$	3.182	3.642
Longitudinal β	$\beta_z(m)$	8.00	6.43
Single bunch population	$N_0(10^{10})$	6.275	5.9

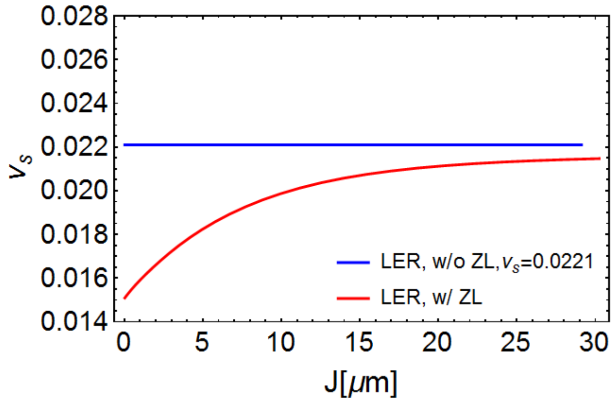


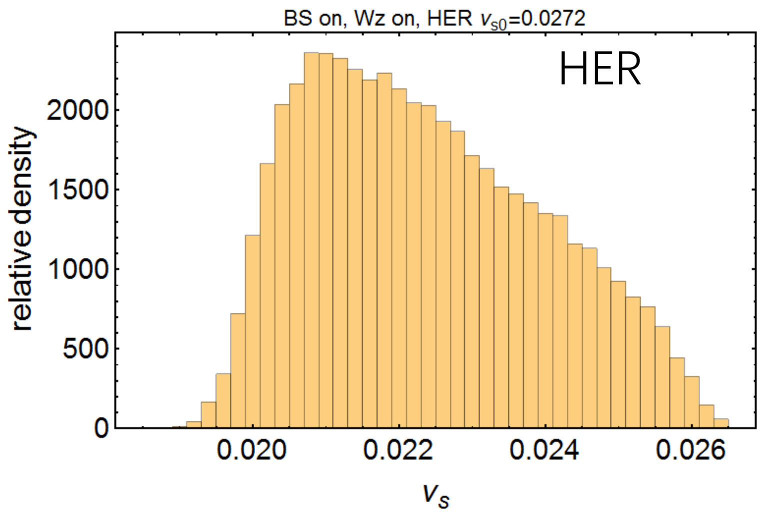
SuperKEKB main parameters and longitudinal wake function. [Courtesy of D.Zhou,KEK]

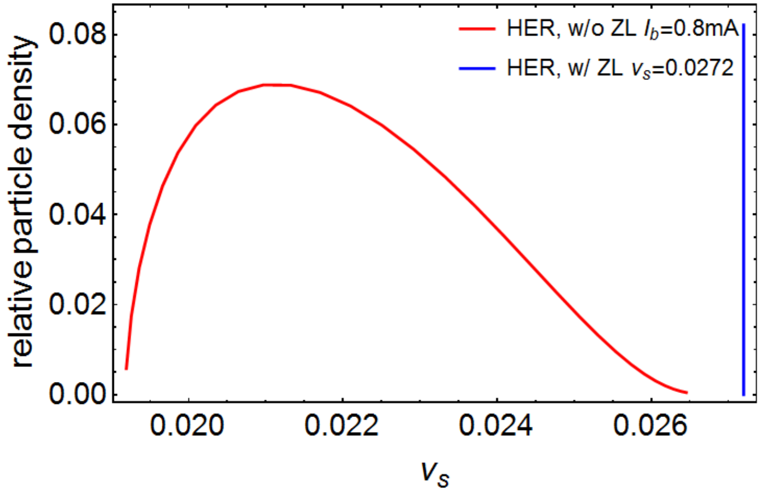
Synchrotron tune spread for HER/LER

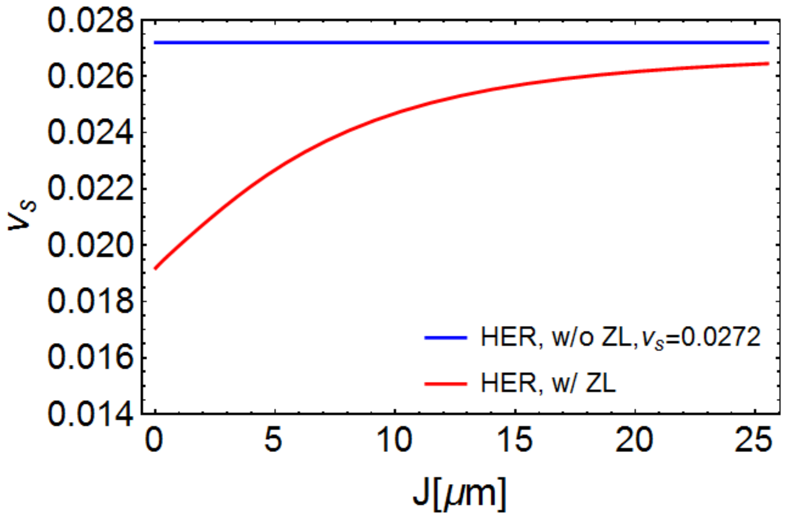












FFT based on particle tracking with longitudinal wake

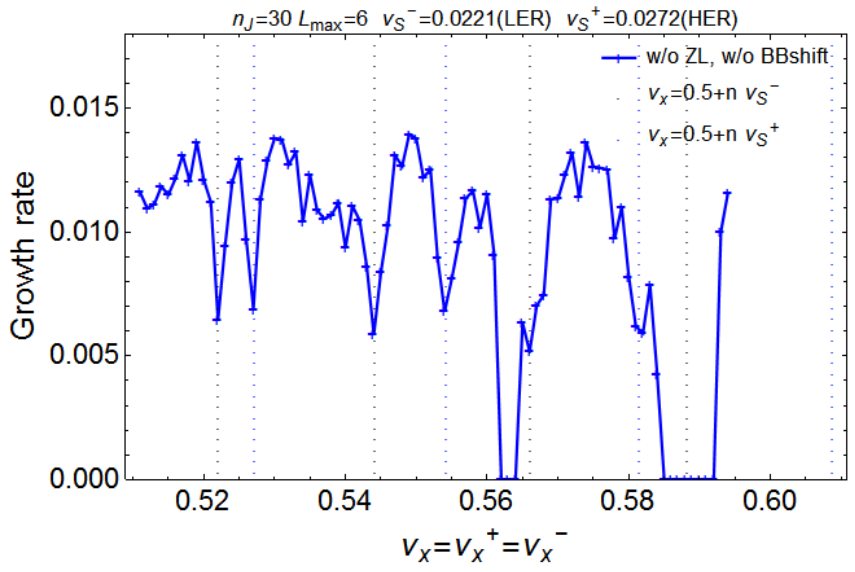
Based on Haissinski solution with longitudinal wake

Growth rate v.s. horizontal tune

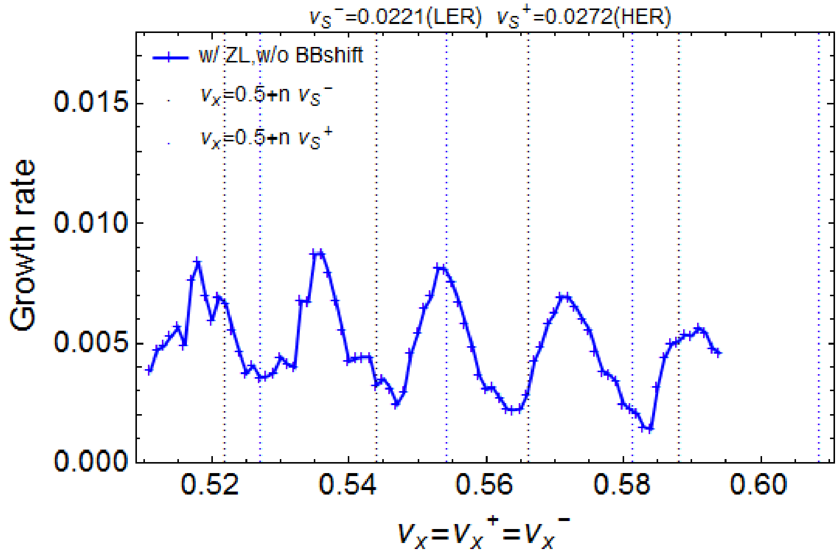
- Same tune between LER/HER.
- w/o and w/ ZL
- Only consider the dipole term

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)\cdot x^{(+)}\left(z'\right)dz' \quad \text{(dipole)}$$

$$+\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)dz'\cdot x^{(-)}(z) \quad \text{(quadrupole)}$$

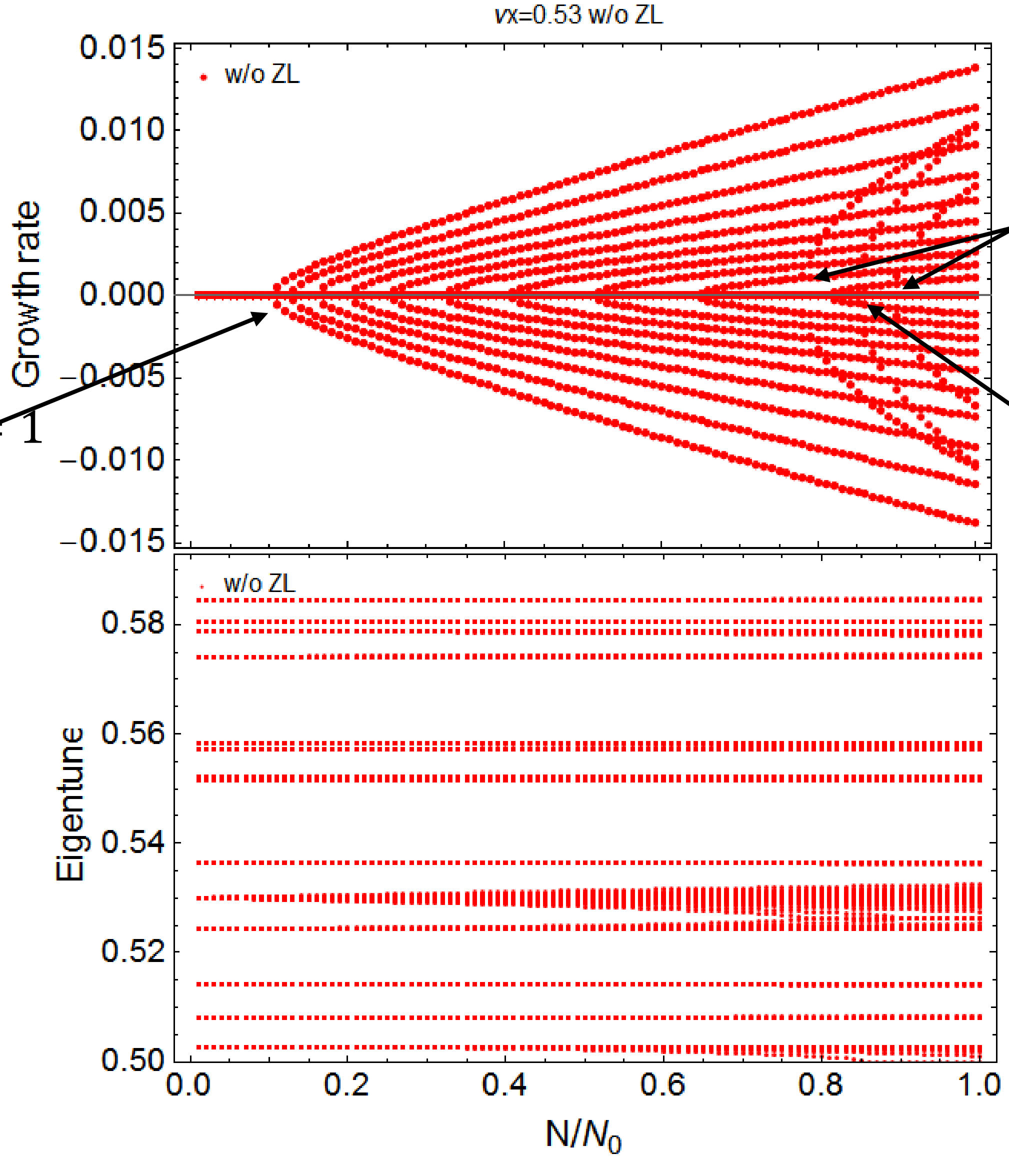


There is no stable horizontal tune near 0.5.
At large ν_x , there could have stable region.



The growth rate is reduced. But the stable tuen area is squeezed, and there is no stable region in this case. We see a decline tendency in the growth rate as the ν_x increases.

- Same tune between LER/HER
- w/ ZL , υ_x=0.530



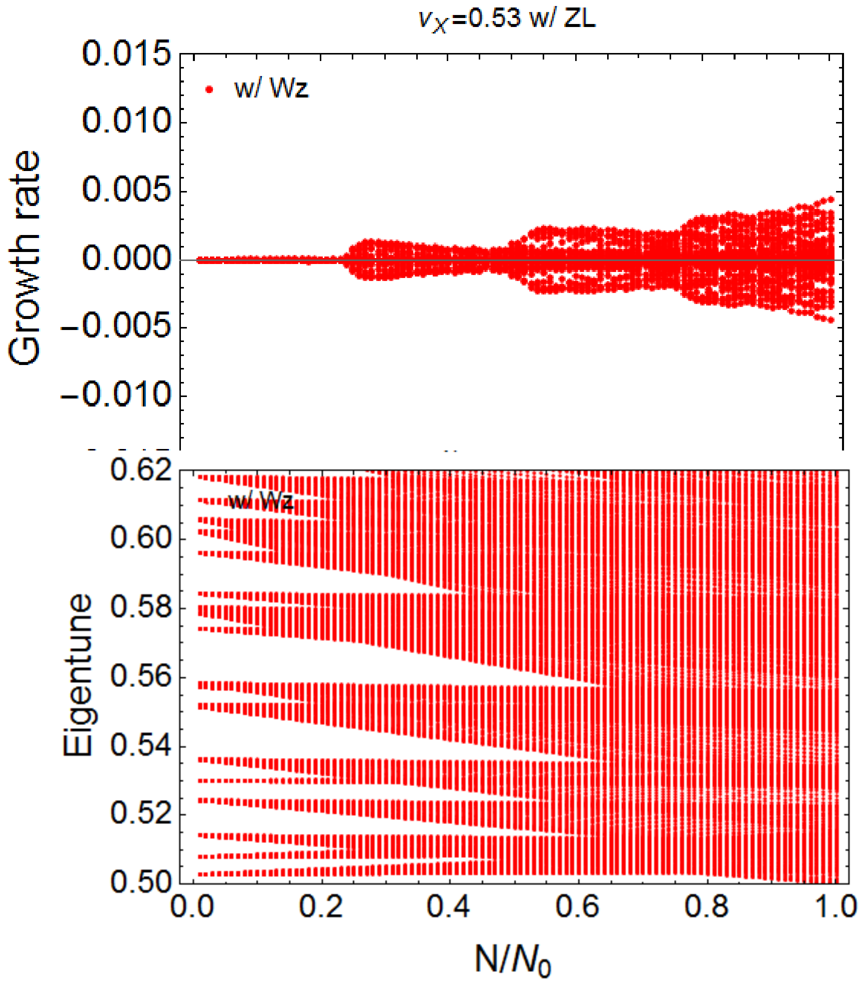
Mode-coupling only exists in same parity.

Bunch population scanning w/o ZL

Eigentune and growth rate as a function of bunch population. The lines start at $\nu=\nu_x+n\nu_s^+$, and $\nu=\nu_x+n\nu_s^-$. ν_s^+/ν_s^- are nominal synchrotron tunes for HER/LER.

- v_x=0.530
- Same tune between LER/HER.
- w/ ZL

All parity modes could couple



Bunch population scanning w/ ZL

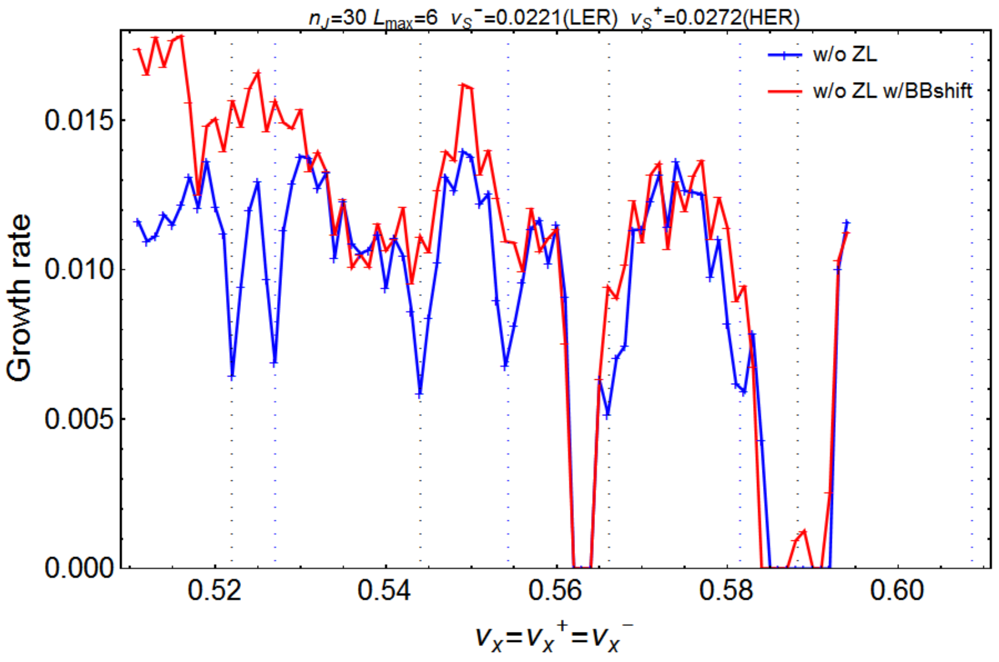
Eigentune and growth rate as a function of bunch population. The lines start at $\nu = \nu_x + n\nu_s^+$, $\nu = \nu_x + n\nu_s^-$.

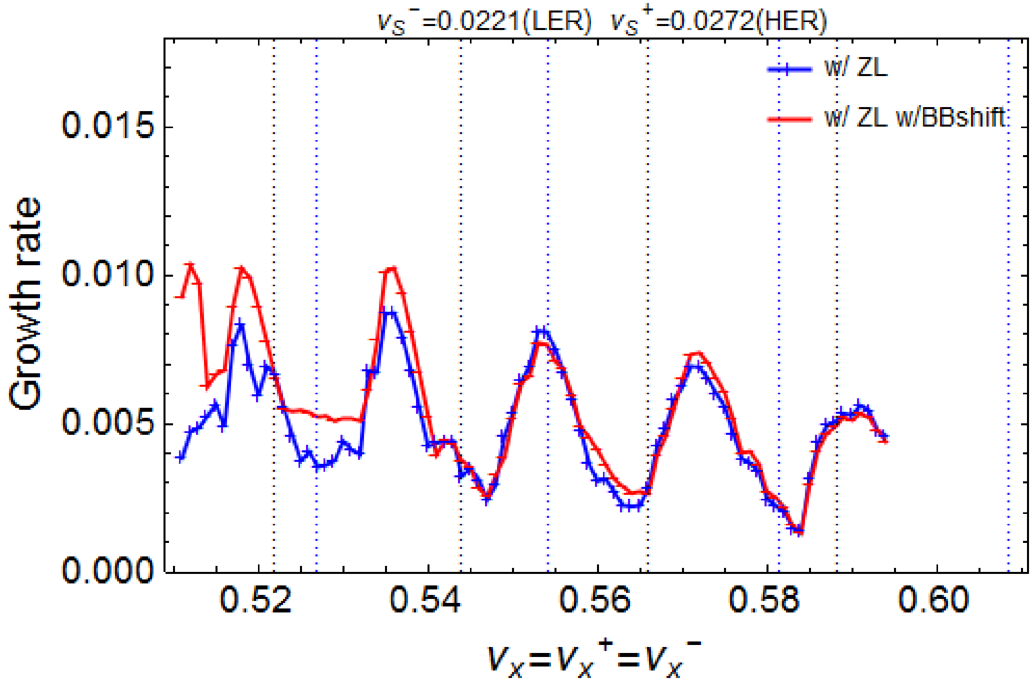
ν_s^+ / ν_s^- are synchrotron tunes for HER/LER. Due to the synchrotron tune spread, more modes are coupled, but the value of growth rate is reduced compared to the case without ZL.

Quadrupole (BB shift) effect

- Same tune between LER/HER.
- w/o and w/ ZL
- w/o and w/ quadrupole term (BB shift)

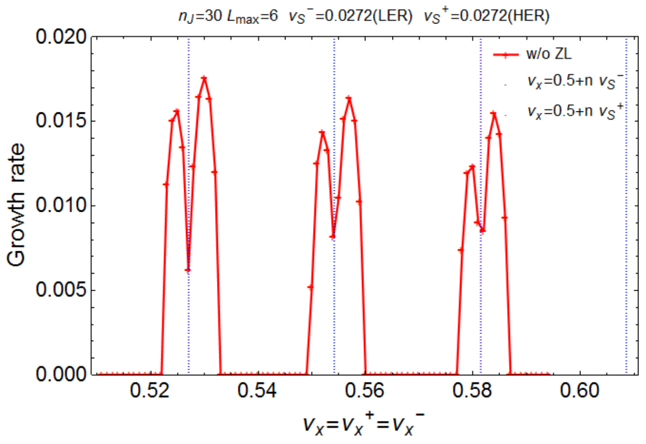
$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)\cdot x^{(+)}\left(z'\right)dz' \quad \text{(dipole)}$$
$$+\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)dz'\cdot x^{(-)}(z) \quad \text{(quadrupole)}$$



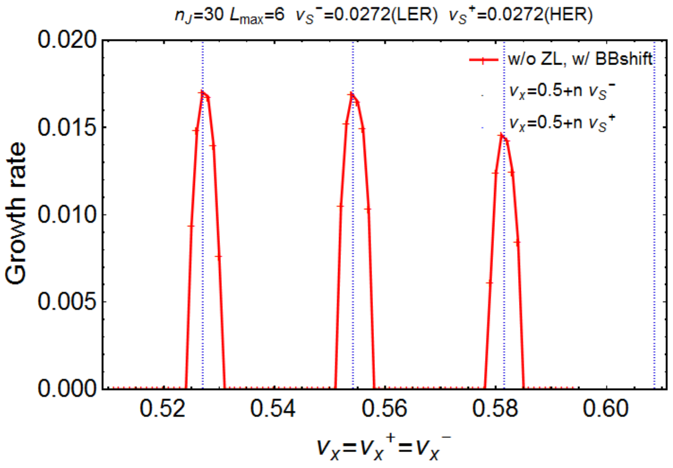


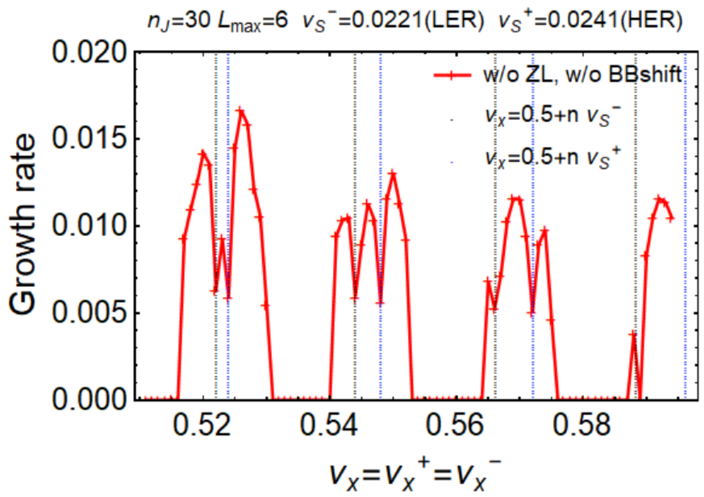
For asymmetric collision, the quadrupole term (BB shift) do not induce distinctive horizontal tune shift especially in case w/ ZL.

Quadrupole effect of different $\Delta\nu_s = \nu_s^+ - \nu_s^-$

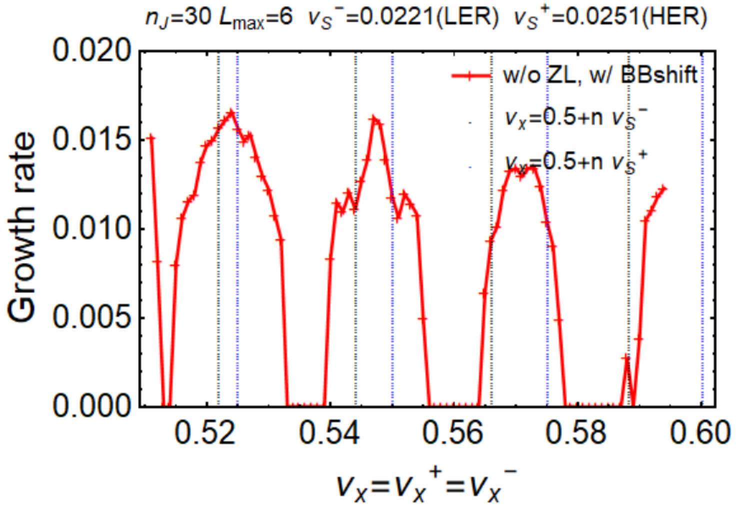


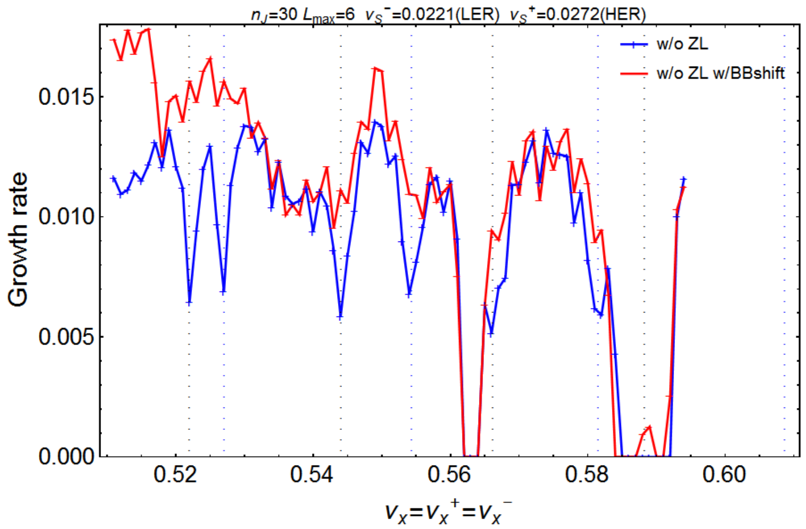
$\Delta\nu_s = 0$





$\Delta\nu_s = 0.002$

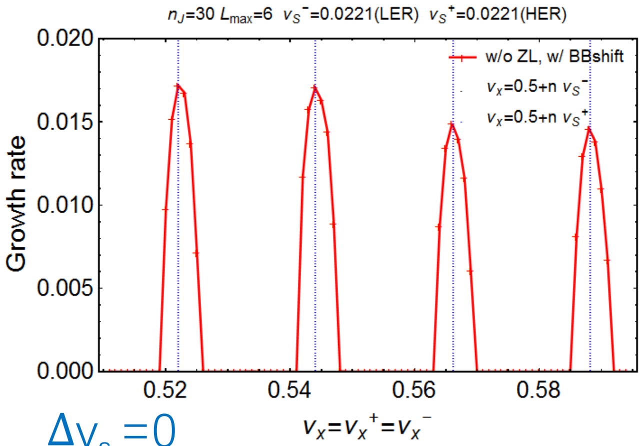




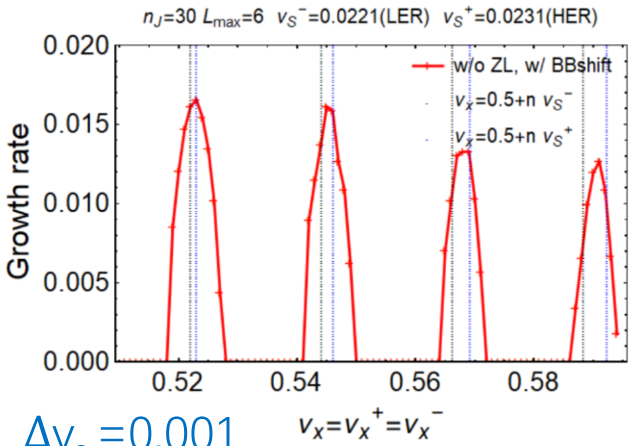
$\Delta\nu_s = 0.005$

As the difference $\Delta\nu_s = \nu_s^+ - \nu_s^-$ increases, the horizontal tune shift becomes less obvious

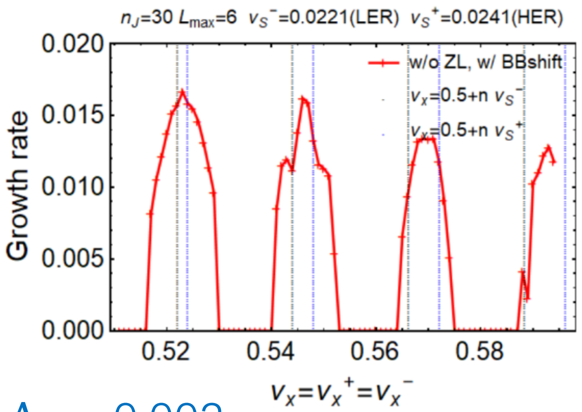
Width of stability region of different $\Delta\nu_s = \nu_s^+ - \nu_s^-$



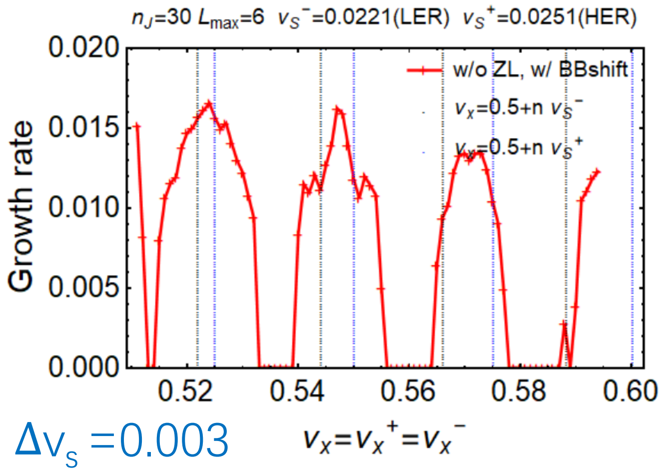
$\Delta\nu_s = 0$



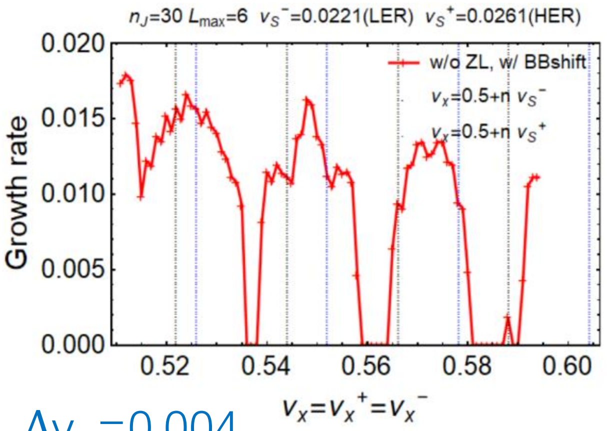
$\Delta\nu_s = 0.001$



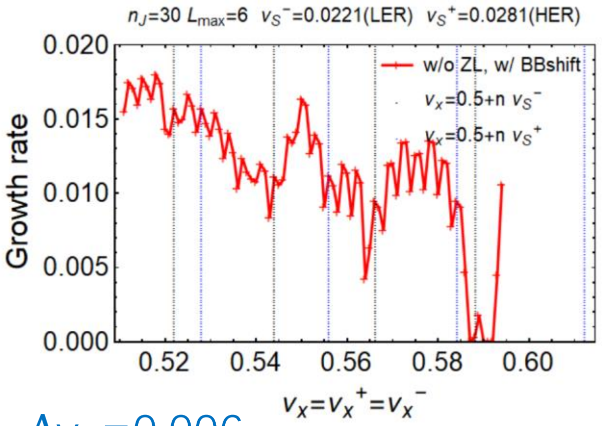
$\Delta\nu_s = 0.002$



$\Delta\nu_s = 0.003$



$\Delta\nu_s = 0.004$



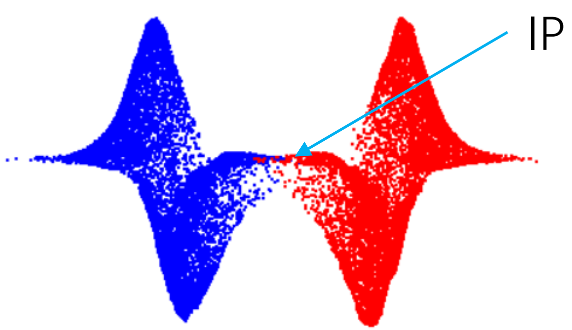
$\Delta\nu_s = 0.006$

As the difference $\Delta\nu_s = \nu_s^+ - \nu_s^-$ increases, the stable area is squeezed. We guess that the same synchrotron tune configuration for the two beam may help increase the stable area.

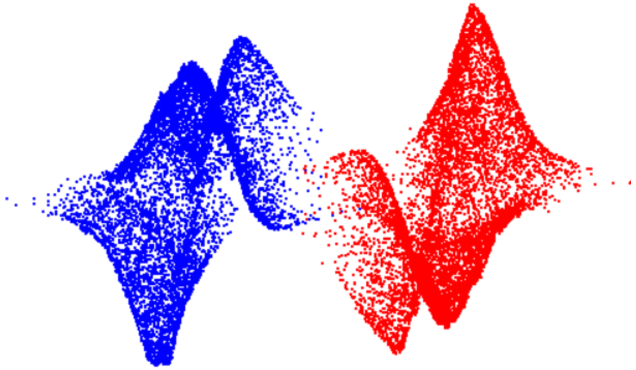
Symmetric collision

According to the simulation, for example CEPC, the two colliding bunches have a statistical relationship dependent on the horizontal tune.

σ mode



π mode



$$\sigma \text{ mode}: \quad \rho^{(+)}(z)x^{(+)}(z) = \rho^{(-)}(z)x^{(-)}(z)$$

$$\pi \text{ mode}: \quad \rho^{(+)}(z)x^{(+)}(z) = -\rho^{(-)}(z)x^{(-)}(z)$$

"-" for σ mode
"+" for π mode

Two beam problem is reduced to single beam, which is very similar to ordinary transverse wake force

$$\Delta p_x^{(-)}(z) = -\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)\cdot x^{(+)}\left(z'\right)dz' \quad \text{(dipole)}$$
$$+\int_{-\infty}^{\infty} w_x^{(-)}\left(z-z'\right)\rho^{(+)}\left(z'\right)dz'\cdot x^{(-)}(z) \quad \text{(quadrupole)}$$

$$\Rightarrow$$

$$\Delta p_x(z) = \mp\int_{-\infty}^{\infty} W_x\left(z-z'\right)\rho(z')x\left(z'\right)dz'$$
$$+\int_{-\infty}^{\infty} W_x\left(z-z'\right)\rho(z')dz'\cdot x\left(z\right)$$

Conventional treatment for the localized cross wake

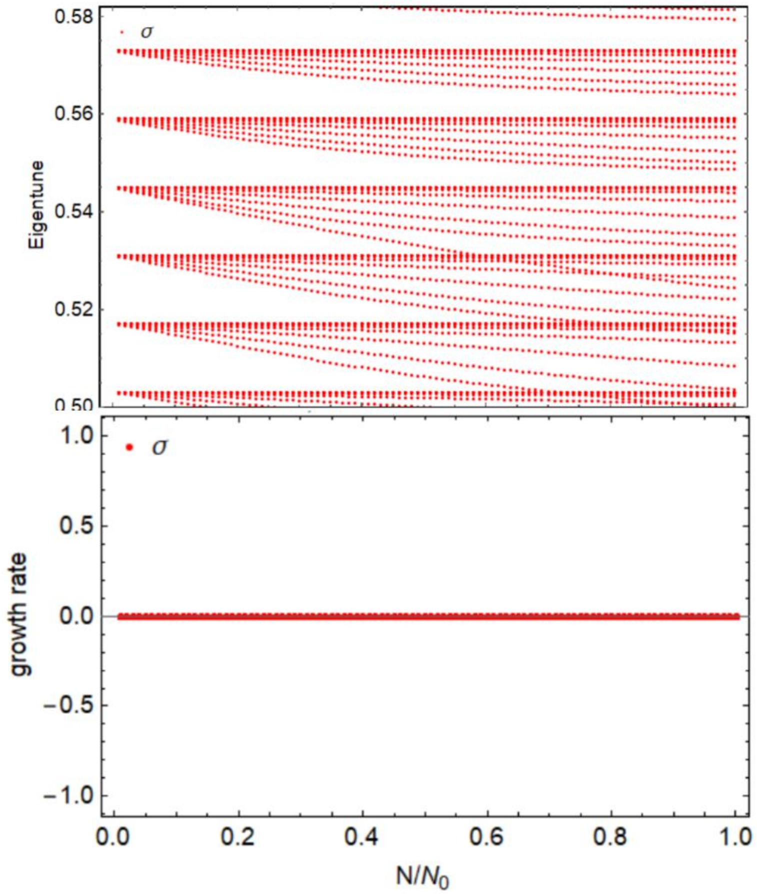
For CEPC-Z $\upsilon_x = 0.546$, the two beam exhibit σ mode,

Conventional TMCI theory[3] for this wake force. Wake force is continuously smeared around the ring.

$$\Delta p_x(z) = -\int_{-\infty}^{\infty} W_x\left(z - z'\right)\rho(z')x(z')dz'$$

w/o ZL, longitudinal Gaussian beam

Modes are coupling but no instability occur.



Growth rate and eigentune v.s. bunch population

[3] **A. Chao, Physics of Collective Beam Instabilities in High Energy Accelerators , New York, 1993.**

Discussion and Summary

A transverse instability analysis method for localized wake is developed to study beam-beam interaction including the effects of longitudinal impedance. This method gives us some physical interpretation of beam-beam interaction under the influence of longitudinal impedance.

However, there are quantitative differences between simulation and this method. The reasons may be:

- Chromaticity and dispersion are not considered in the calculation
- Radiation damping is not considered
- Cross-wake force is a linear force with respect to x. It only consider the linear part of beam-beam force

Thank you for your attention!