



# *Brightness and Coherence of Synchrotron Radiation and Free Electron Lasers*

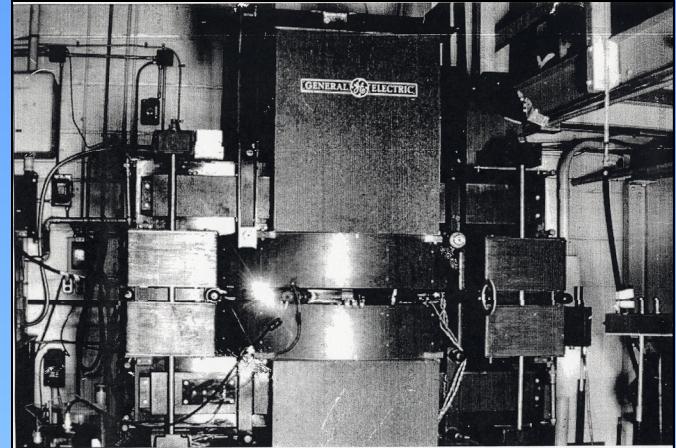
*Zhirong Huang*

*SLAC, Stanford University*

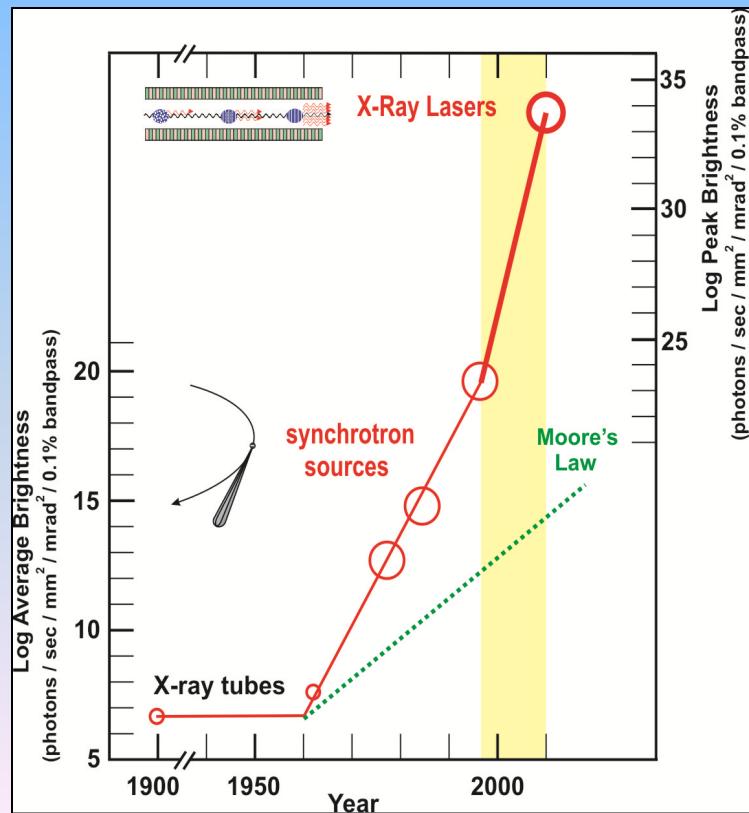
*May 13, 2013*

# Introduction

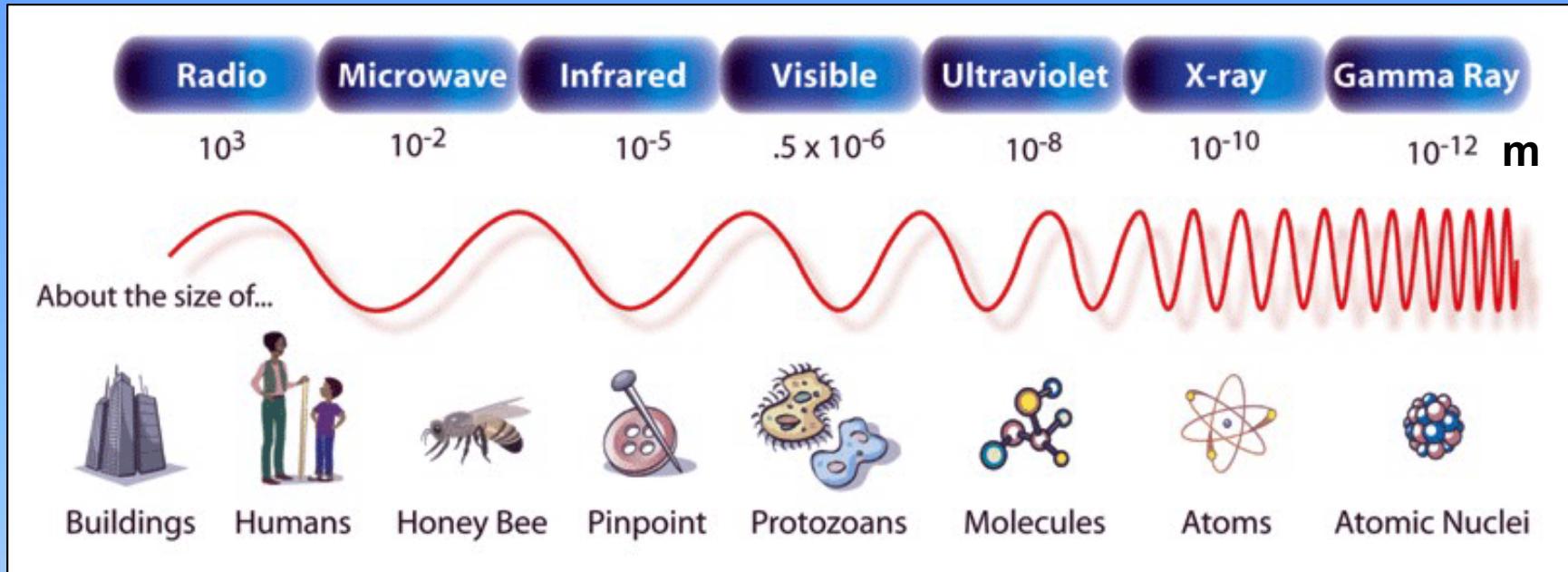
- GE synchrotron (1946) opened a new era of accelerator-based light sources.



- These light sources have evolved rapidly over four generations.
- The first three-generations are based on synchrotron radiation.
- The forth-generation light source is a game-changer based on FELs.
- The dramatic improvement of brightness and coherence over 60 years easily outran Moore's law.



# Bright X-ray Vision



# Bright X-ray Vision

## Wavelength

10nm      1nm       $0.1\text{nm}=1\text{\AA}$

Soft X-rays

Hard X-rays

100eV      1keV      10keV

## Photon Energy

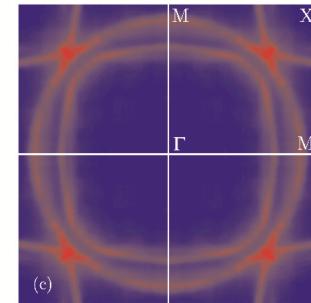
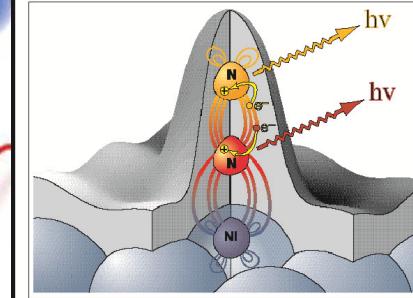
Visible

$5 \times 10^{-6}$

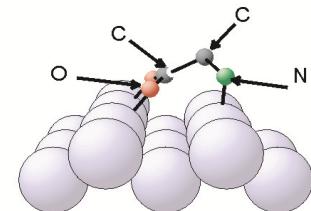


protozoans

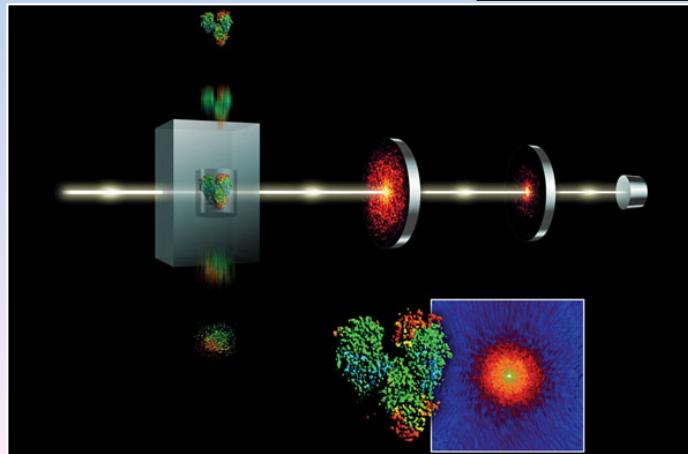
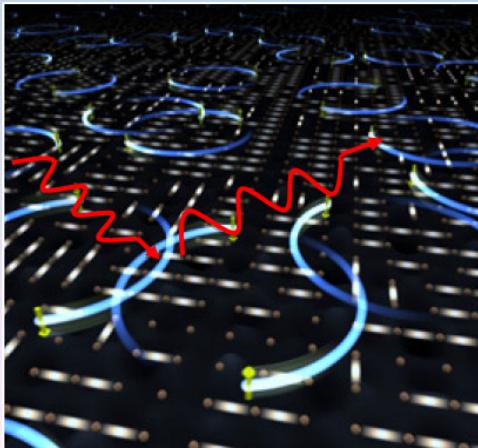
## Where are the electrons?



## Where are the atoms?

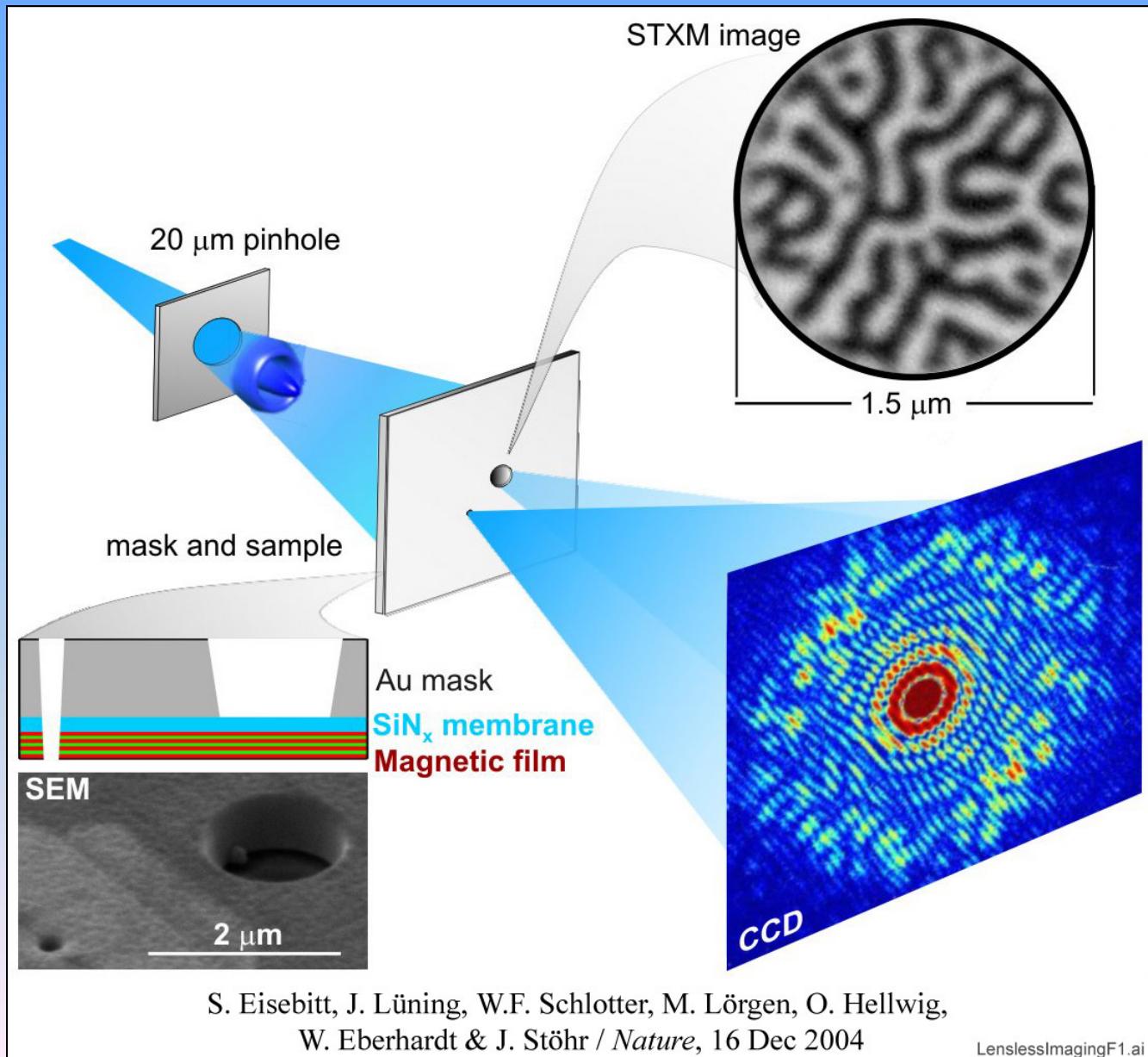
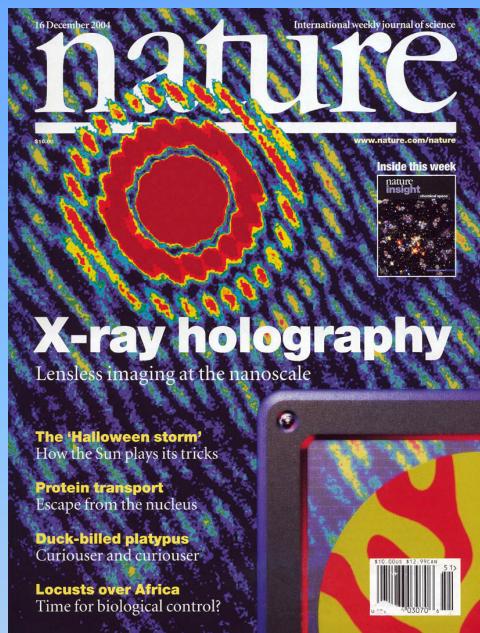


*Brighter sources, better vision*



# Coherence Wanted

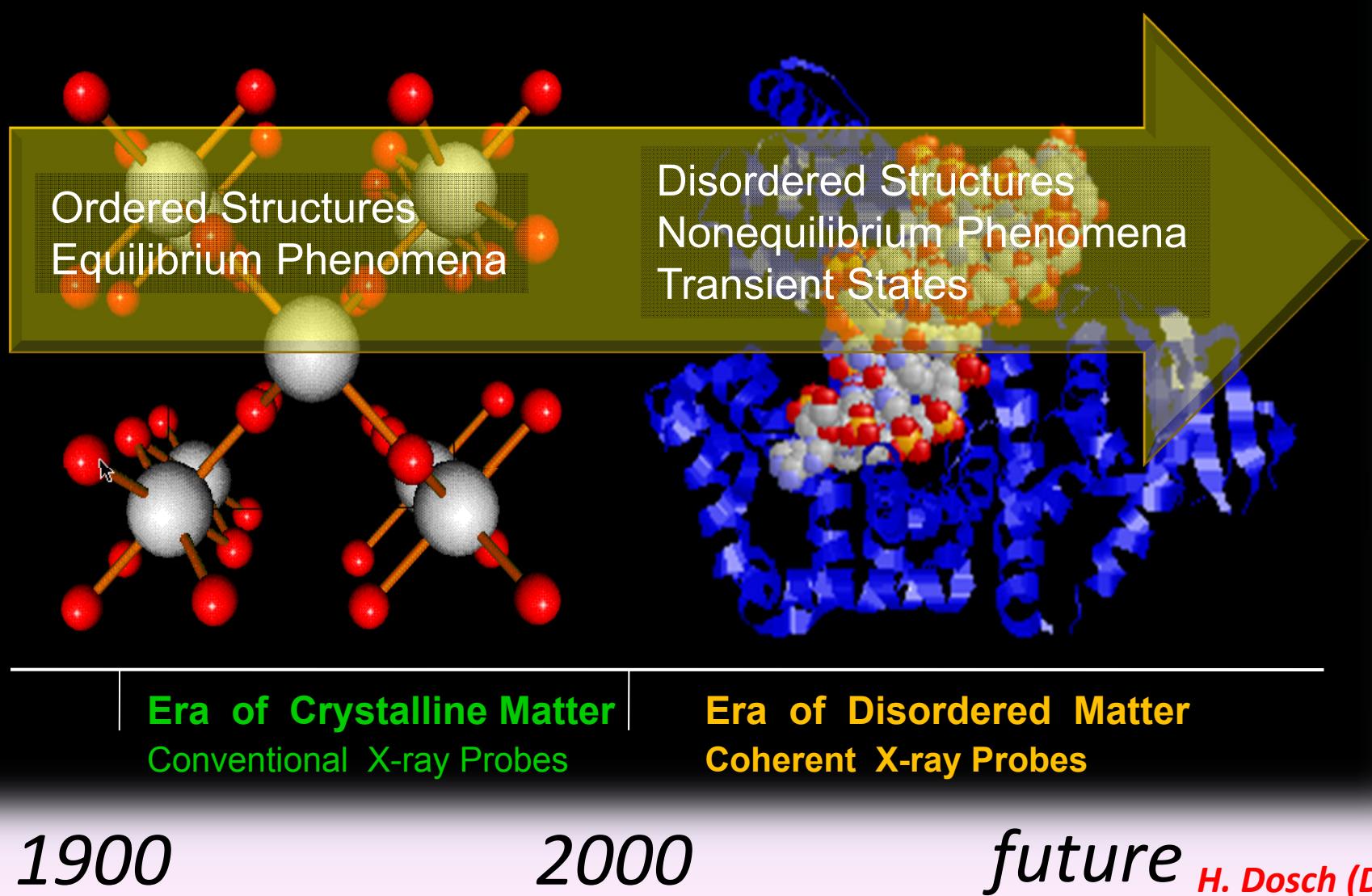
## Lensless imaging of magnetic nanostructures by x-ray holography



S. Eisebitt, J. Lüning, W.F. Schlötter, M. Lörgen, O. Hellwig,  
W. Eberhardt & J. Stöhr / *Nature*, 16 Dec 2004

LenslessImagingF1.ai

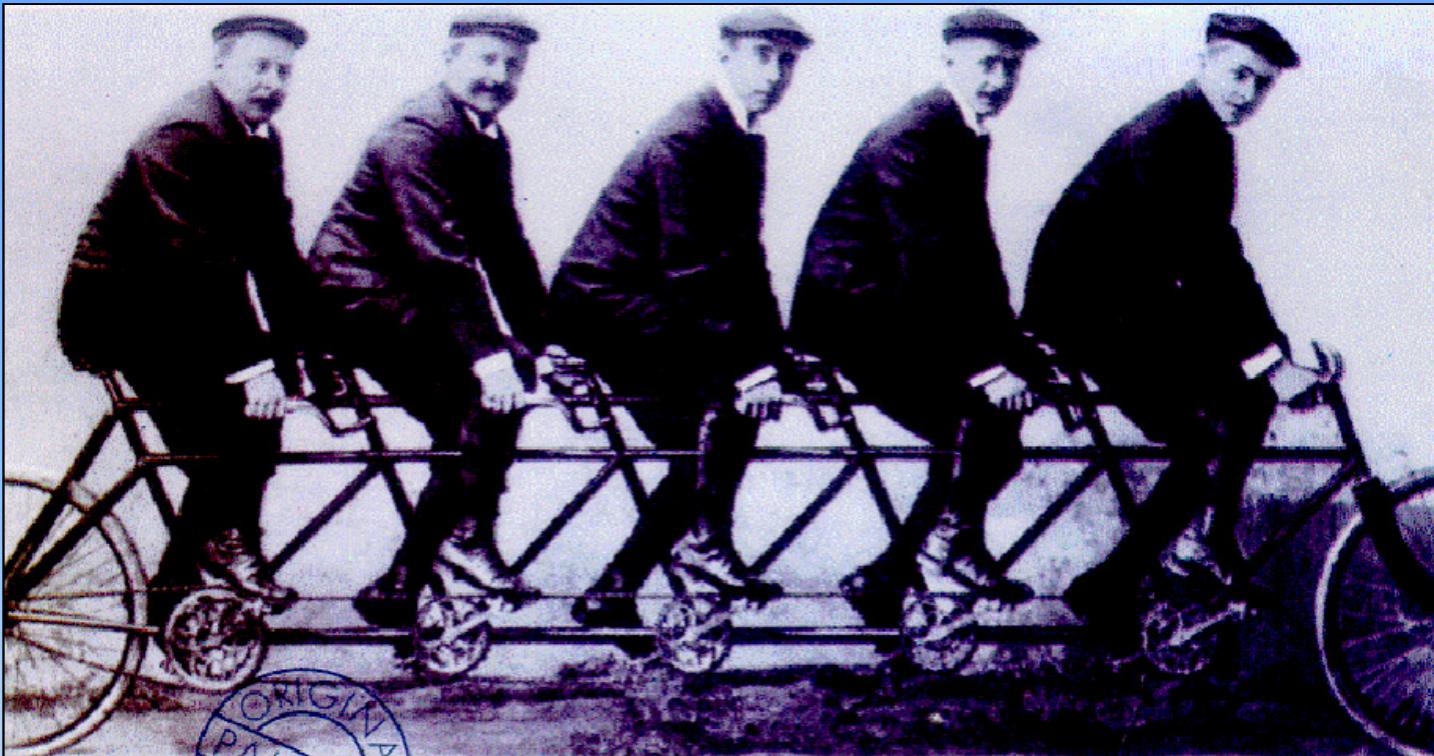
# *Future Role of FELs and Advanced Sources*



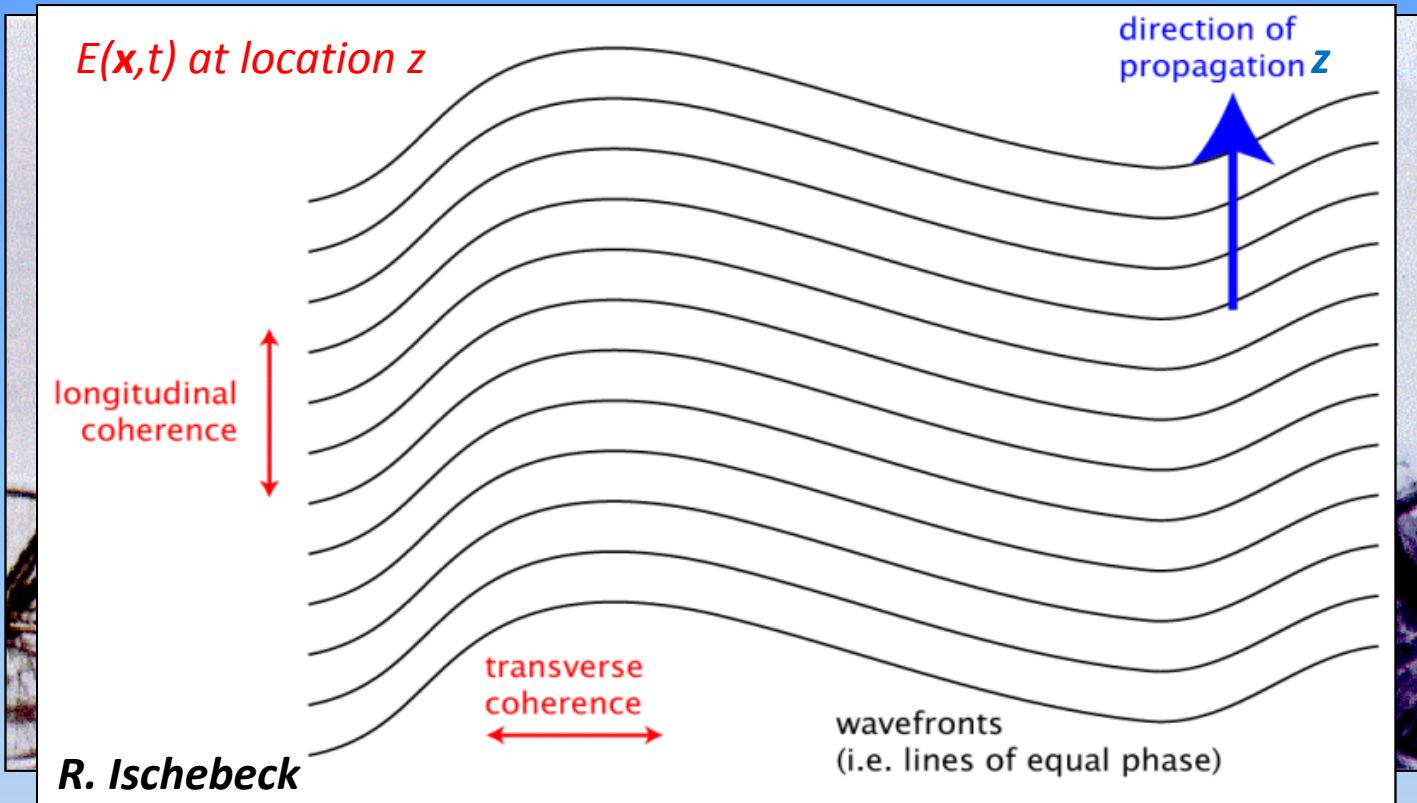
# *Outline*

- *Introduction*
- *Coherence*
- *Brightness*
- *Undulator Radiation*
- *Free Electron Lasers*
- *Summary*

# *What is coherence?*



# What is coherence?



## Complex degree of coherence

$$\gamma(\mathbf{x}_1, \mathbf{x}_2, \tau) = \frac{\langle E(\mathbf{x}_1, t) E^*(\mathbf{x}_2, t + \tau) \rangle}{\sqrt{\langle |E(\mathbf{x}_1, t)|^2 \rangle \langle |E(\mathbf{x}_2, t + \tau)|^2 \rangle}}$$

$\gamma(\mathbf{x}_1, \mathbf{x}_2, 0)$  describes the transverse coherence,  
 $\gamma(0, 0, \tau)$  characterizes the temporal coherence.

# Temporal (Longitudinal) Coherence

- Coherence time is determined by measuring the path length difference over which fringes can be observed in a Michelson interferometer.

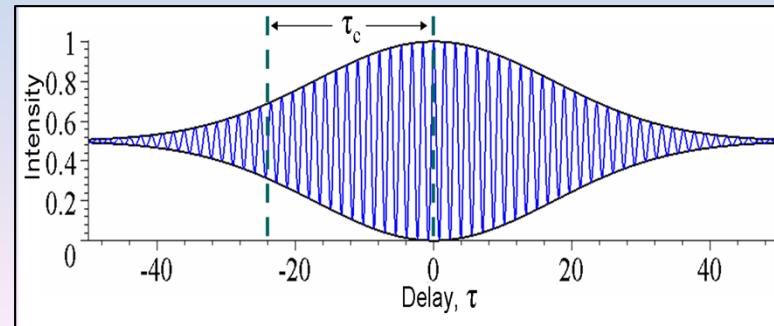
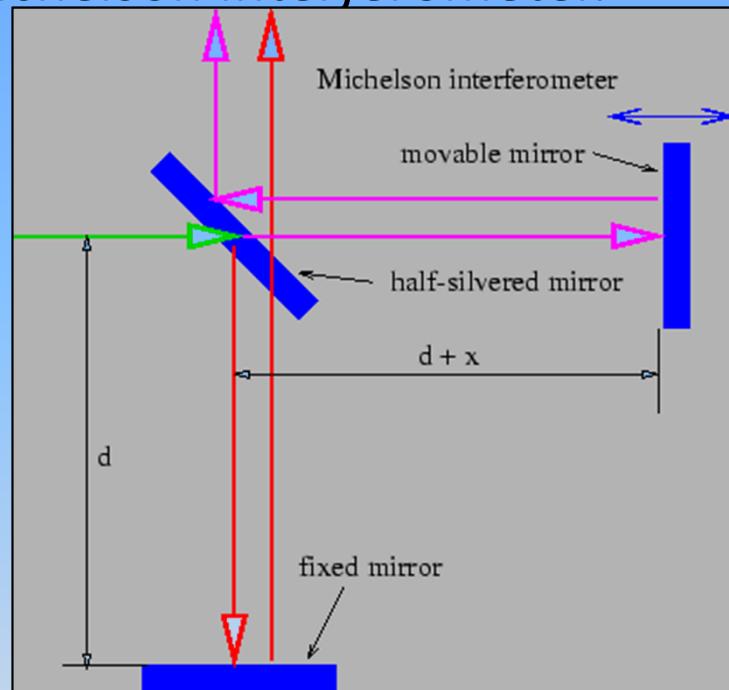
$$\tau_c = \int_{-\infty}^{\infty} d\tau |\gamma(\tau)|^2$$

- Temporal coherence function and the radiation spectrum forms a Fourier pair

$$\gamma(\tau) = \frac{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2 e^{-i\omega\tau}}{\int_{-\infty}^{\infty} d\omega |E(\omega)|^2}$$

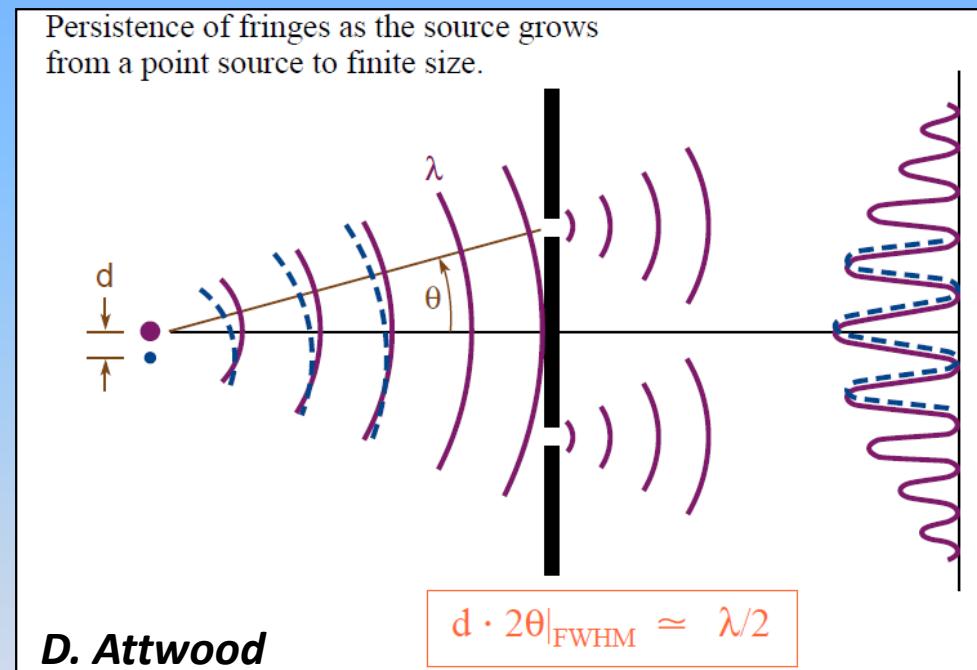
- For a Gaussian radiation spectrum,

$$\tau_c = \frac{\sqrt{\pi}}{\sigma_\omega}$$



# Transverse (Spatial) Coherence

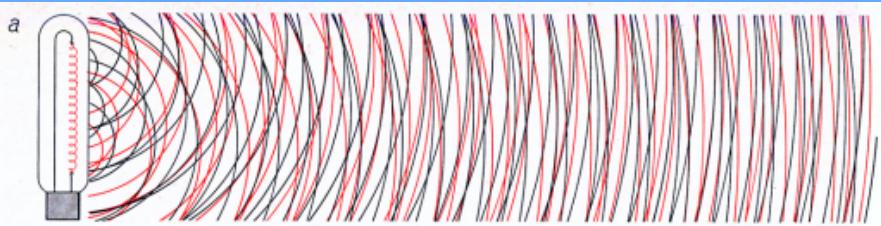
- Transverse coherence can be measured via the interference pattern in Young's double slit experiment.
- Near the center of screen, fringe visibility is described by  $\gamma(\mathbf{x}_1, \mathbf{x}_2, 0)$ .



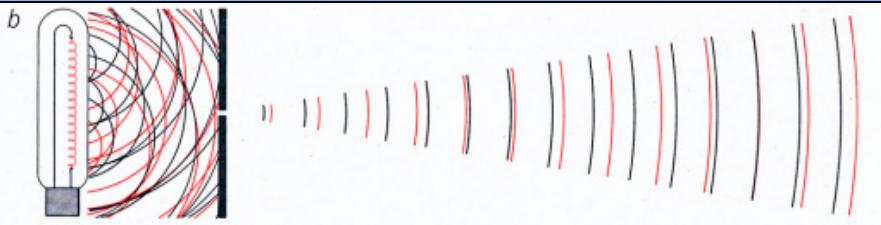
- Degree of transverse coherence (coherence fraction):

$$\zeta = \frac{\iint |\gamma(\mathbf{x}_1, \mathbf{x}_2, 0)|^2 I(\mathbf{x}_1) I(\mathbf{x}_2) d\mathbf{x}_1 d\mathbf{x}_2}{\int I(\mathbf{x}_1) d\mathbf{x}_1 \int I(\mathbf{x}_2) d\mathbf{x}_2}$$

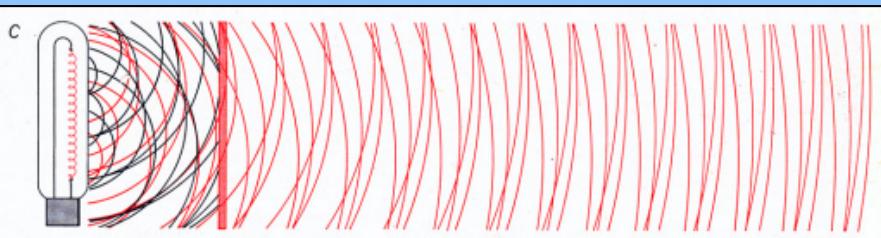
# *Light Bulb vs. Laser*



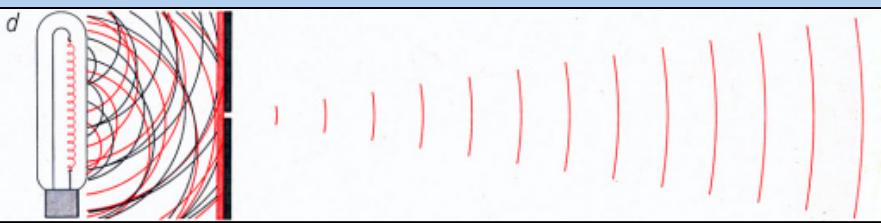
*Radiation emitted from light bulb is chaotic.*



*Pinhole can be used to obtain spatial coherence.*



*Monochromator can be used to obtain temporal coherence.*

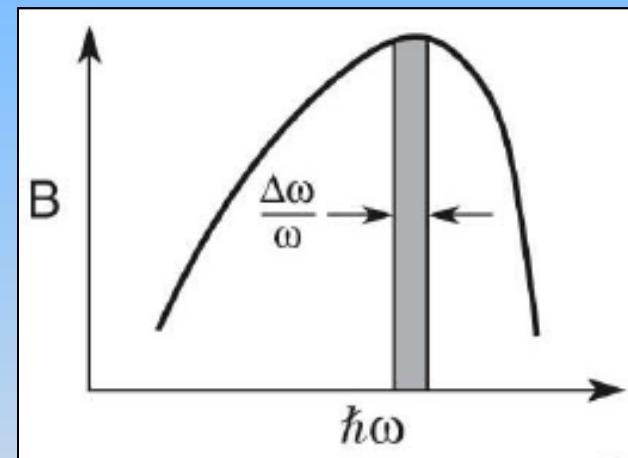
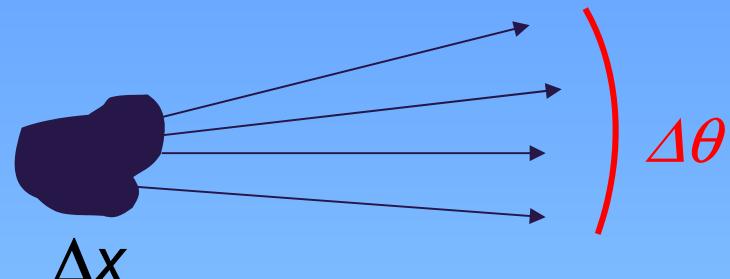


*Pinhole and Monochromator can be combined for coherence.*



*Laser light is spatially and temporally coherent.*

# Brightness



$$B = \frac{\text{Photons in unit spectral range in unit time}}{(\text{source size} \times \text{divergence})^2}$$

Peak  
Average

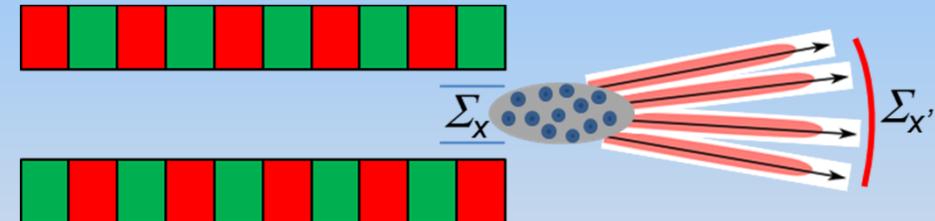
Units: photons/s/mm<sup>2</sup>/mrad<sup>2</sup>/0.1%BW

# Brightness via Wigner Function

- Spectral brightness defined via Wigner function, which is Fourier transformation of the transverse correlation function (K.J. Kim, 1986).

$$B(\mathbf{x}, \phi; z) = \frac{d\omega}{\hbar\omega} \frac{\omega^2 \varepsilon_0}{\pi c T} \int d\xi e^{ik\xi \cdot \phi} \langle E(\mathbf{x} + \frac{1}{2}\xi; z) E^*(\mathbf{x} - \frac{1}{2}\xi; z) \rangle$$

- Brightness is conserved in a perfect optical system: cannot increase brightness once the source is born.
- Brightness convolution theorem

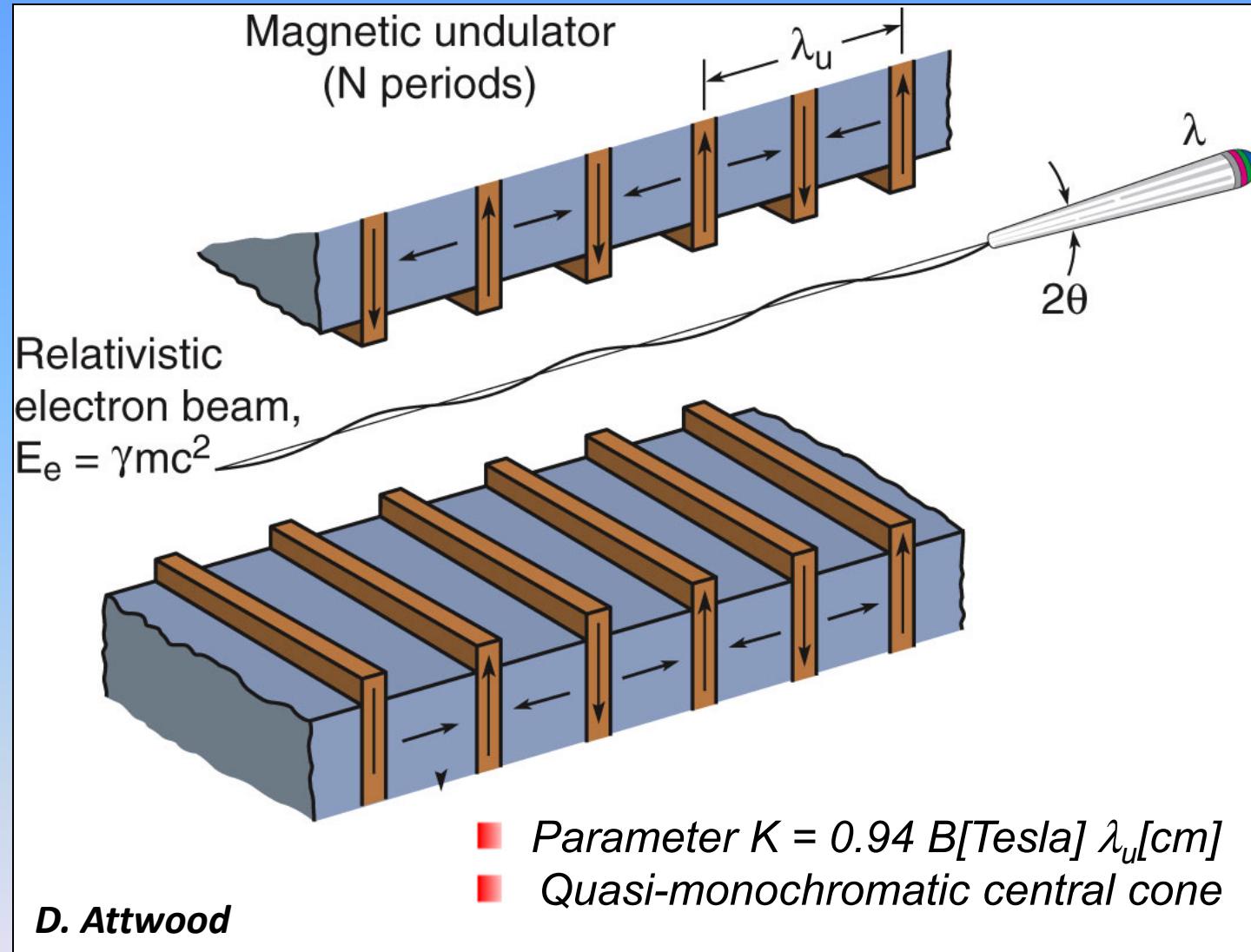


$$B(\mathbf{x}, \phi, z) = N_e \int d\mathbf{x}_j d\mathbf{x}'_j B_j(\mathbf{x} - \mathbf{x}_j, \phi - \mathbf{x}'_j, z) f(\mathbf{x}_j, \mathbf{x}'_j, z)$$

single electron rad. brightness

electron distribution function

# Undulator Radiation



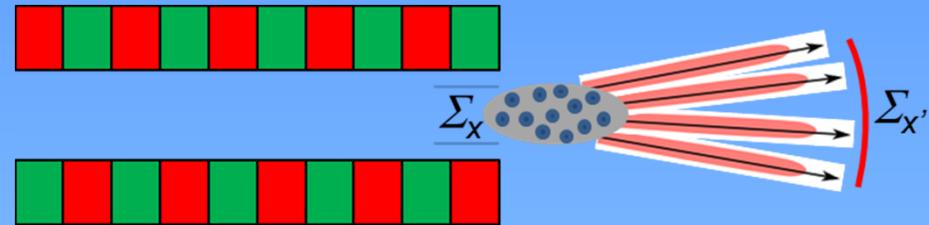
$$\lambda = \frac{\lambda_u}{2\gamma^2} \left( 1 + \frac{K^2}{2} \right)$$

Under Gaussian approximation,  
central radiation cone has

$$\sigma_{r'} = \sqrt{\frac{\lambda}{2L_u}}, \quad \sigma_r = \frac{\sqrt{2\lambda L_u}}{4\pi}$$

# Undulator Radiation Brightness

- Brightness convolution theorem



$$B(\mathbf{x}, \phi) = \frac{N_e F_1(\omega)}{(2\pi)^2 \Sigma_x \Sigma_y \Sigma_{x'} \Sigma_{y'}} \exp\left(-\frac{x^2}{2\Sigma_x^2} - \frac{y^2}{2\Sigma_y^2} - \frac{\phi_x^2}{2\Sigma_{x'}^2} - \frac{\phi_y^2}{2\Sigma_{y'}^2}\right)$$

$$\Sigma_{x,y}^2 \equiv \sigma_{x,y}^2 + \sigma_r^2, \quad \Sigma_{x',y'}^2 \equiv \sigma_{x',y'}^2 + \sigma_{r'}^2$$

- Emittance dominated regime

$$\varepsilon_{x,y} = \sigma_{x,y} \sigma_{x',y'} \gg \frac{\lambda}{4\pi}$$

$$B(\mathbf{0}, \mathbf{0}) = \frac{N_e F_1(\omega)}{(2\pi)^2 \sigma_x \sigma_{x'} \sigma_y \sigma_{y'}} = \frac{F(\omega)}{(2\pi)^2 \varepsilon_x \varepsilon_y}$$

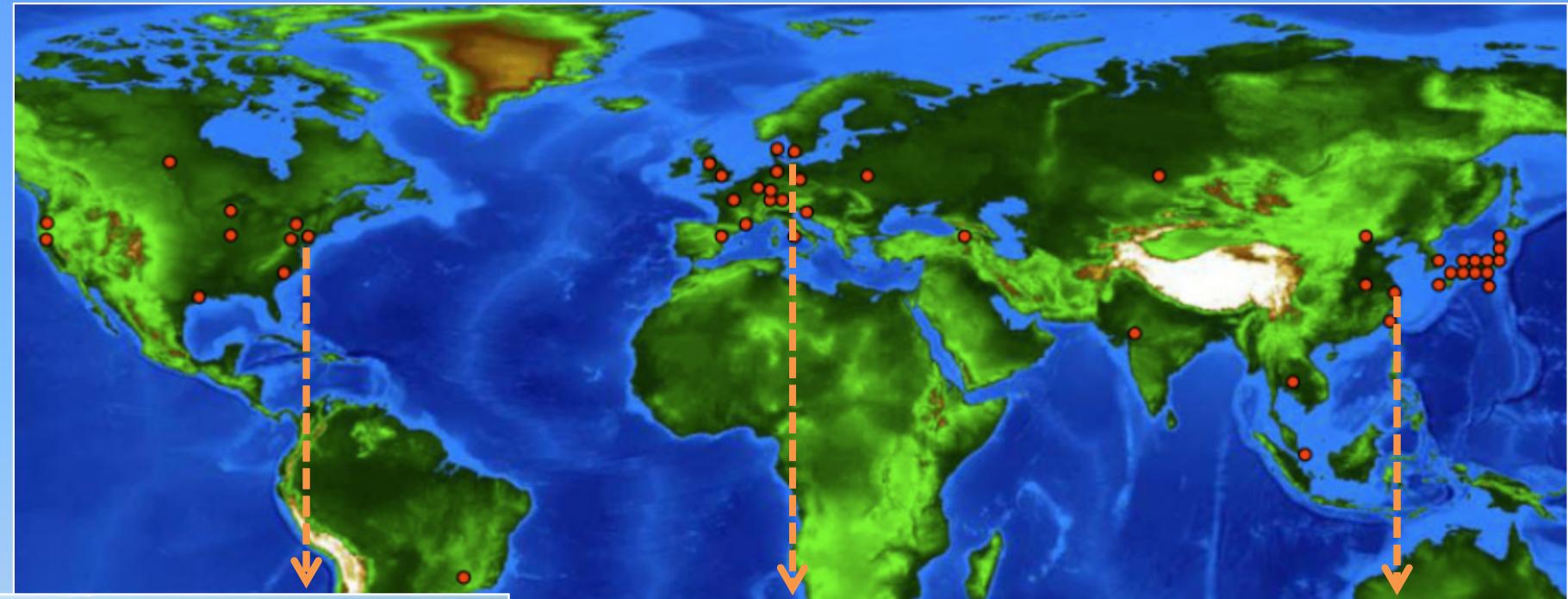
- Radiation dominated regime

$$\varepsilon_{x,y} \ll \frac{\lambda}{4\pi}$$

$$B(\mathbf{0}, \mathbf{0}) = \frac{F(\omega)}{(2\pi)^2 \sigma_r^2 \sigma_{r'}^2} = \frac{F(\omega)}{(\lambda/2)^2}$$

**Diffraction limit**

# *Synchrotron Radiation Facilities*



**NSLS-II (2014)**



**MAX-IV (2016)**

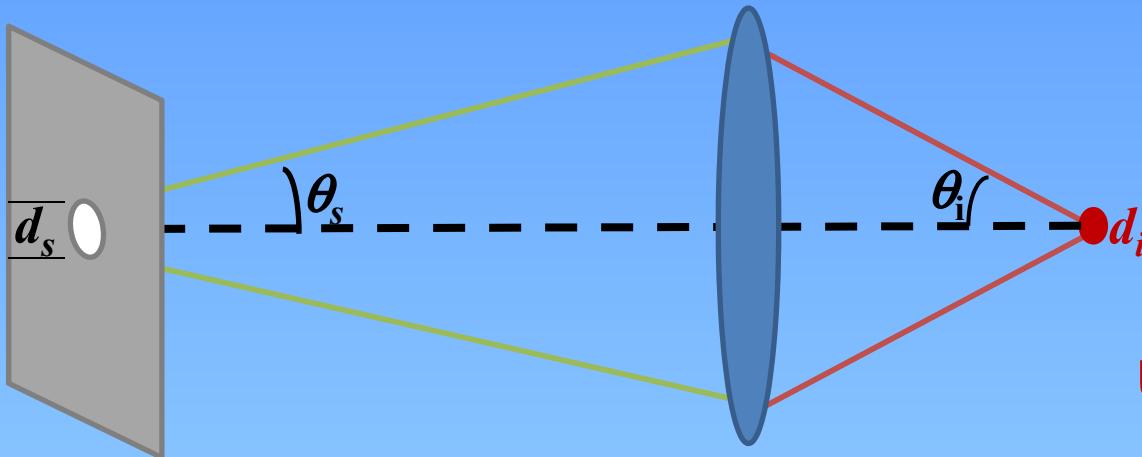


**SSRF (2009)**



- State-of-art storage rings have **pulse duration  $\sim 10 \text{ ps}$ , emittance  $\sim 1 \text{ nm}$** .
- Diffraction-limited storage rings and energy recovery linacs with **emittance  $\sim 10 \text{ pm}$**  are under active R&D.

# *Diffraction Limit*



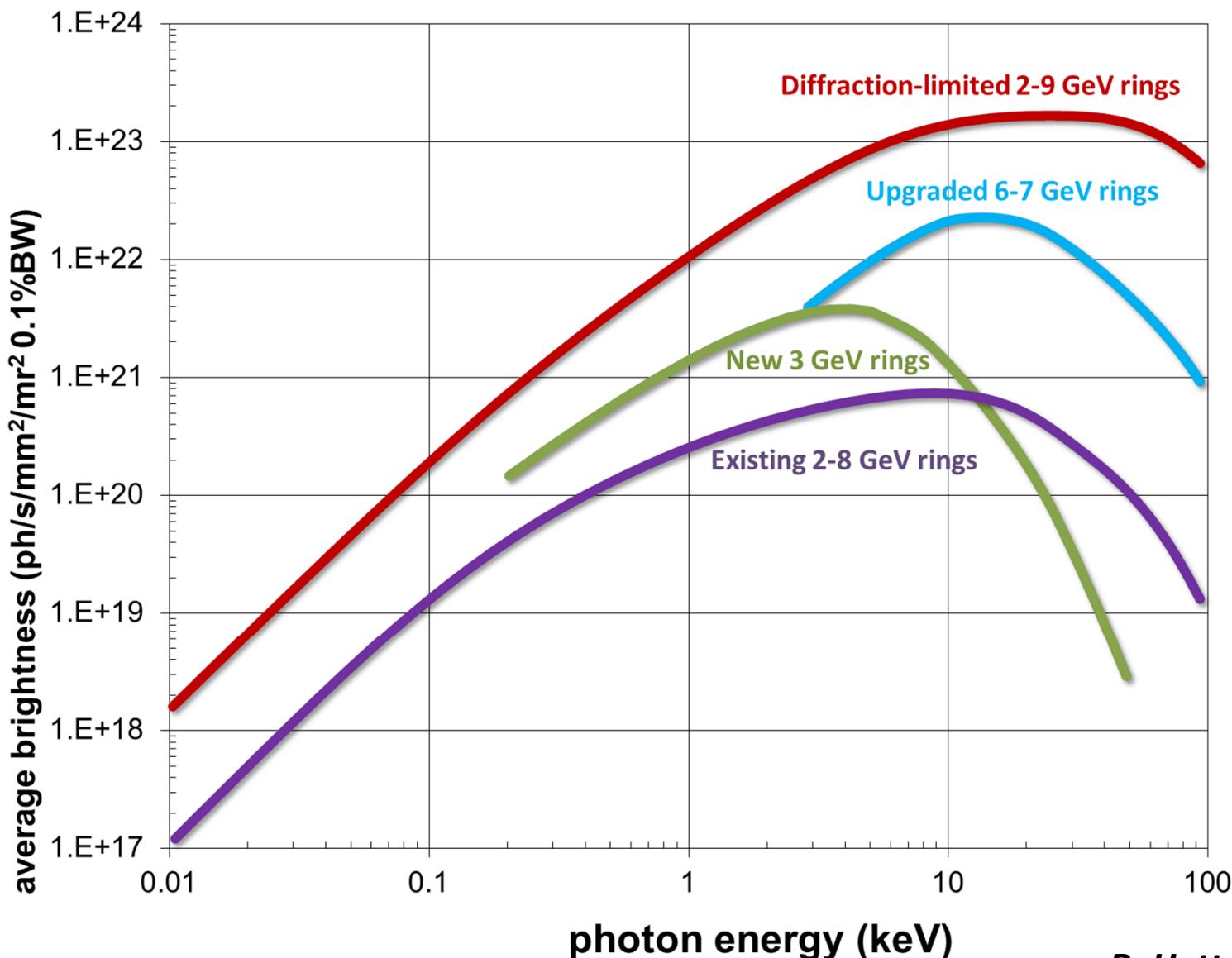
$$\mathcal{E}_x \sim \mathcal{E}_y \sim \frac{\lambda_r}{4\pi}$$

**Ultimate spatial resolution**

- Perfect optical system has  $d_s \theta_s = d_i \theta_i$   
 $\theta_i$  is the numerical aperture of focusing system
- Reducing pinhole size until  $d_s \theta_s \sim \lambda/2$   
since  $d_i \sim \lambda/(2\theta_i)$  reaches diffraction limit.
- A even smaller pinhole does not reduce the image size but only hurts the photon flux
- **Diffraction limited source does not require a pinhole and provide the most coherent flux**

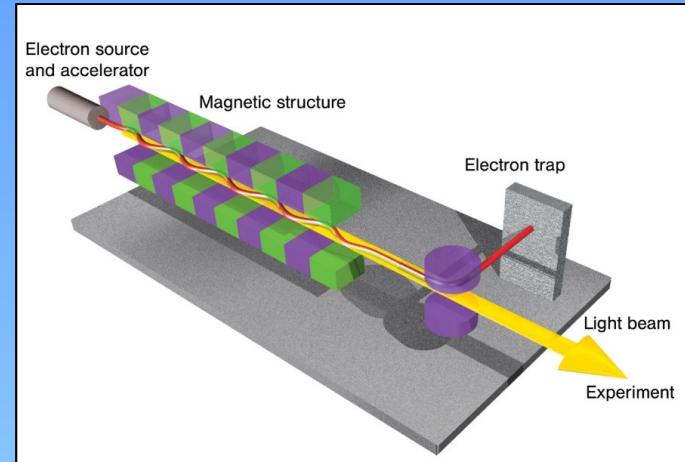
# Storage Ring Spectral Brightness

Brightness Envelopes (4-5 m IDs)

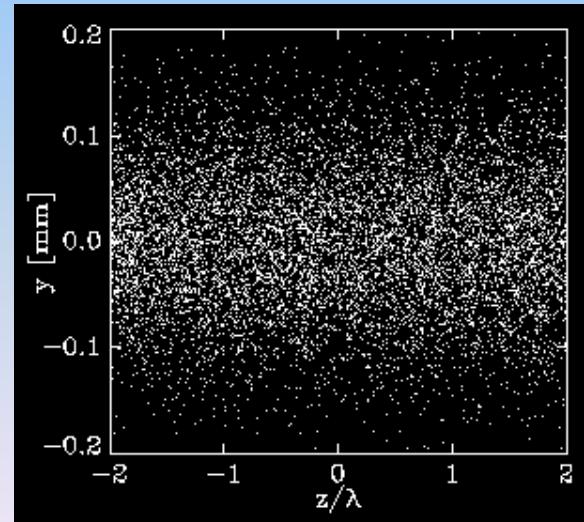
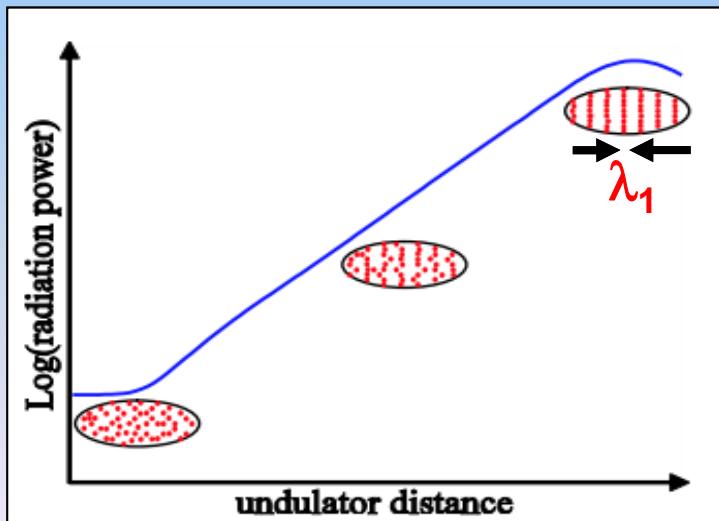


# Free Electron Laser (FEL)

- Resonant interaction of electrons with EM radiation in an undulator<sup>^</sup>
- Coherent radiation intensity  $\propto N^2$  due to beam microbunching  
(N: # of  $e^-$  involved  $\sim 10^6$  to  $10^9$ )



- At x-ray wavelengths, use **Self-Amplified Spontaneous Emission\*** (**a wonderful instability!**) to reach high peak power



S. Reiche

\* Kondratenko, Saldin, Part. Accel., 1980

\* Bonifacio, Pellegrini, Narducci, Opt. Com., 1984

# X-ray FELs

- FEL power grows exponentially with gain length

$$P = P_0 \exp\left(\frac{z}{L_G}\right)$$

$$L_G = \frac{\lambda_u}{4\pi\sqrt{3}\rho}$$

$\rho$  is the FEL efficiency parameter  $\sim 10^{-3}$  for x-ray FELs

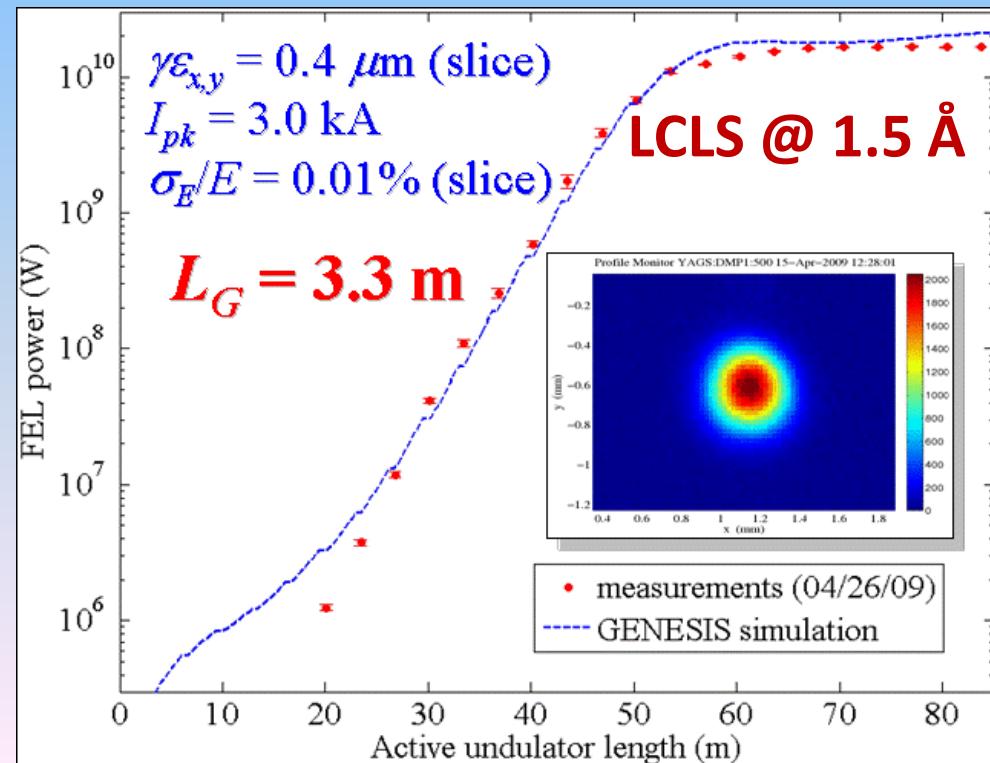
- Exponential gain process selects a Gaussian-like transverse mode with excellent transverse coherence

- Mode size and divergence

$$\sigma_r \approx \sqrt{\sigma_x \sigma_D}$$

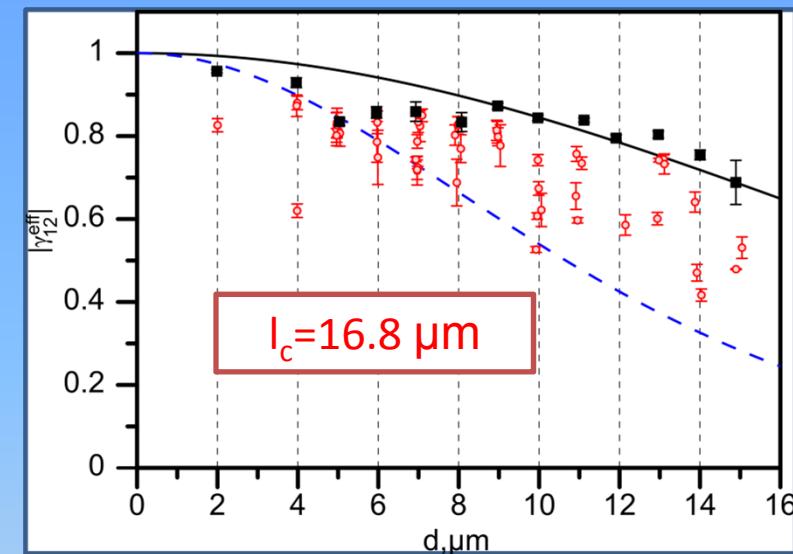
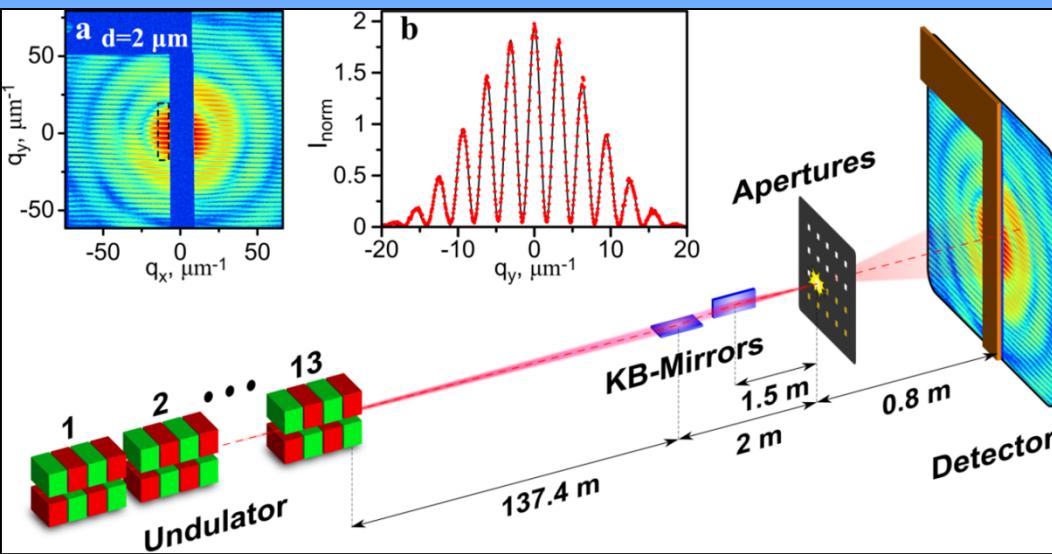
$$\sigma_{r'} \approx \frac{\lambda/(4\pi)}{\sigma_r}$$

$$\sigma_D = \sqrt{L_G \lambda / (4\pi)}$$



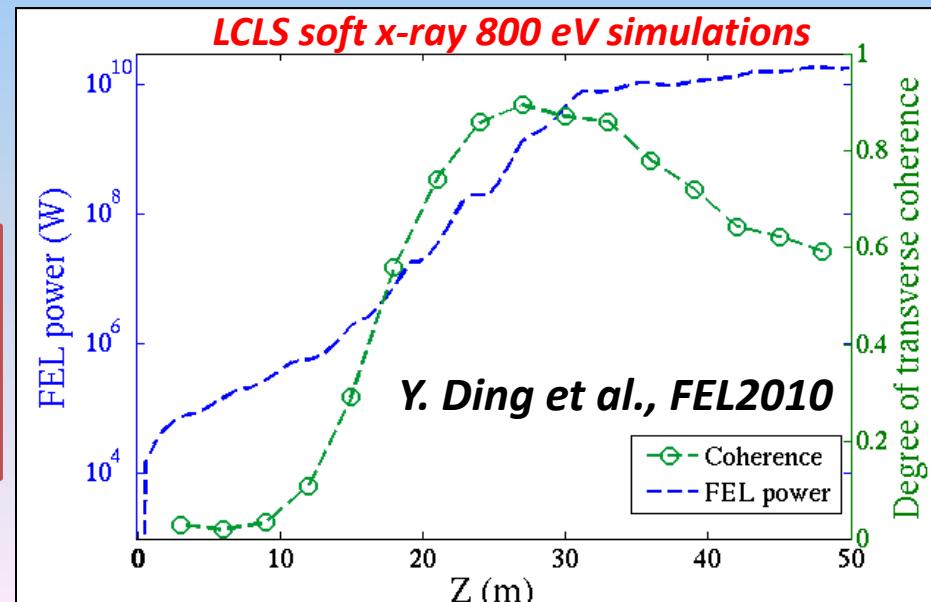
# Measured LCLS Transverse Coherence

LCLS SXR at 780 eV

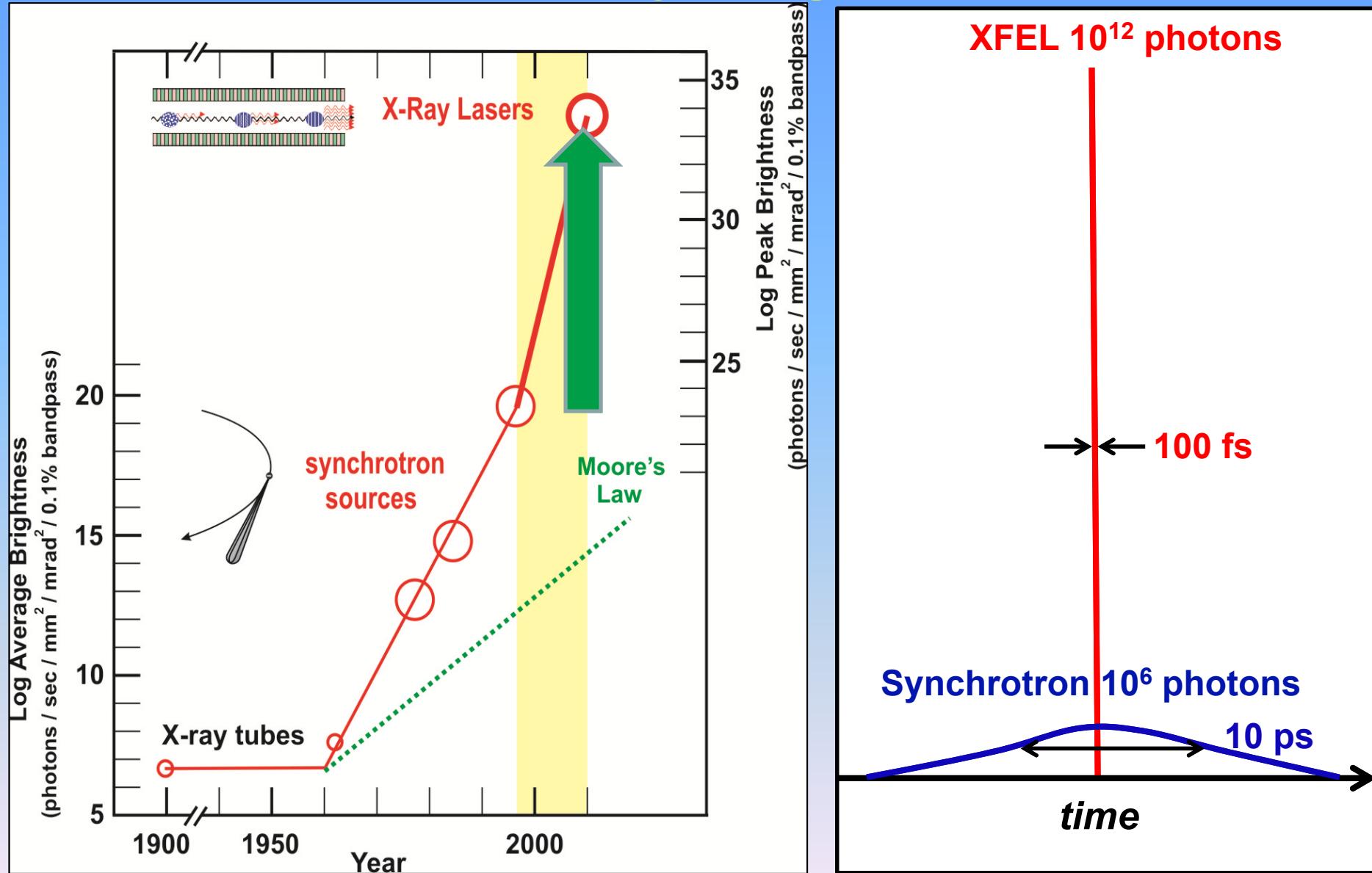


Beam size (FWHM) 17  $\mu\text{m}$   
Trans. Coherence length 16.8  $\mu\text{m}$   
**Vertical degree of coherence: ~75 %**  
**Global degree of coherence: ~56%**

Vartanyants et al. PRL 107, 144801 (2011)

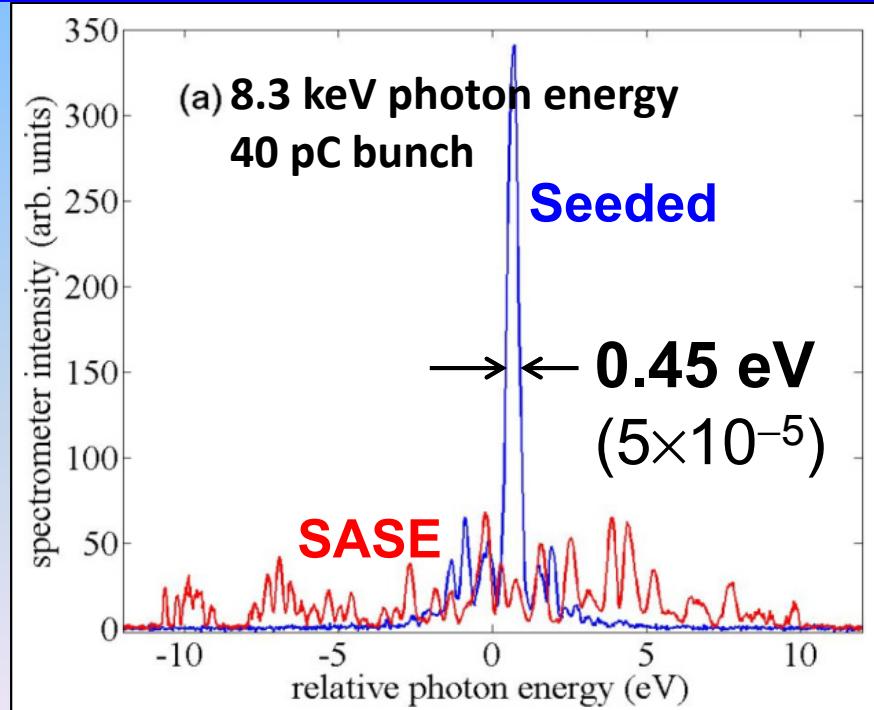
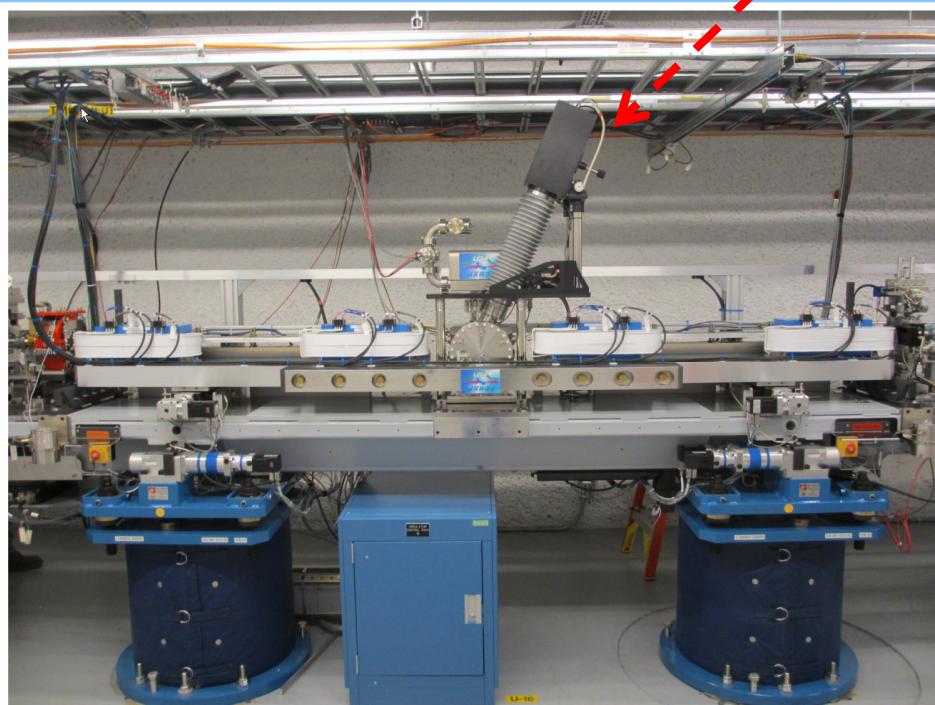
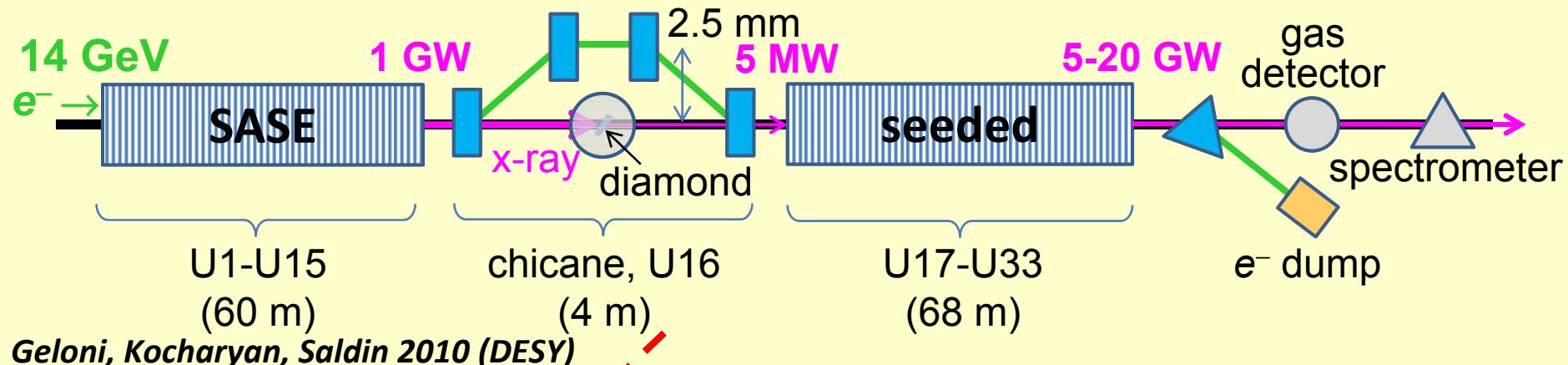


# XFELs are Extremely Bright and Ultrafast



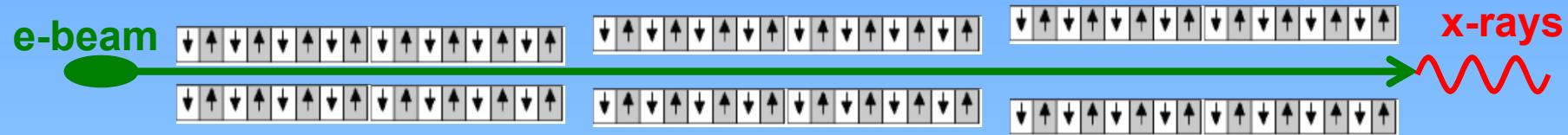
**Note: synchrotron sources are much higher rep. rate than XFELs**

# Seeding to Improve Temporal Coherence

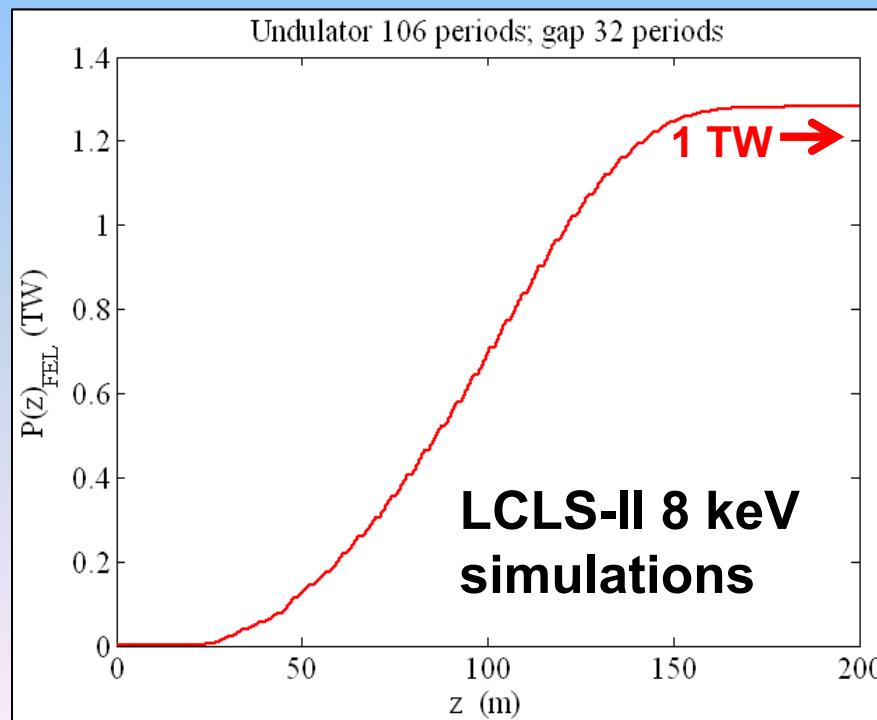


# *Terawatt FEL with Tapered Undulator*

- FEL power saturates due to significant E-loss
- Tapered undulator keeps FEL resonance and increase power



- *Taper works well for a seeded FEL. Seeded TW FEL increases peak brightness over SASE by another two orders of magnitude!*

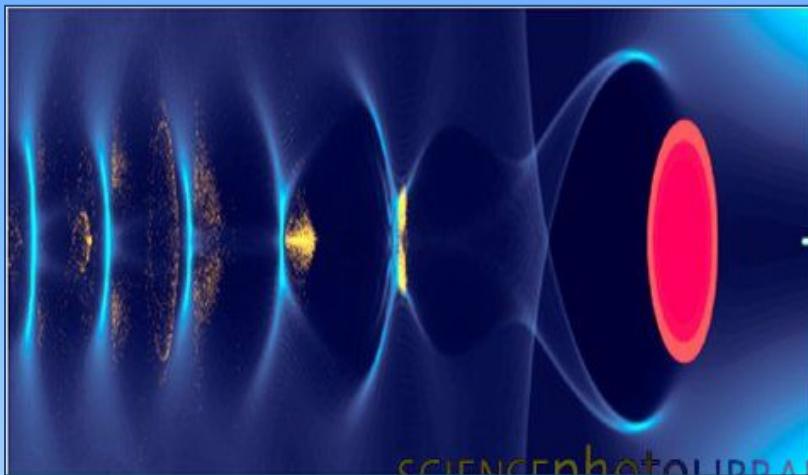


J. Wu et al., FEL2011

Y. Jiao et al., PRSTAB 2012.

# Compact X-Ray FELs

Laser Plasma Accelerator (LPA) or  
Beam-driven Plasma Accelerator

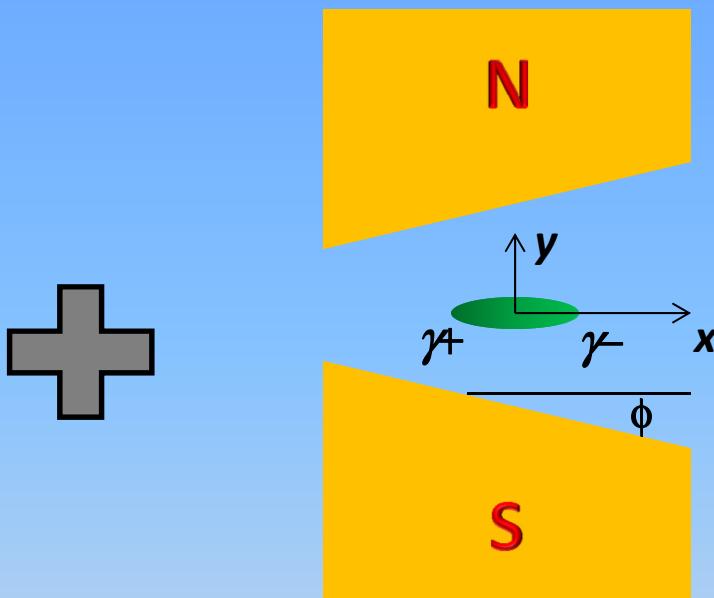


## LPA beam parameters\*

- Beam energy (0.5-1 GeV)
- Norm. emittance ( $\sim 0.1 \mu\text{m}$ )
- Peak current (3-10 kA)
- **Energy spread (1-2%)**

- W. Leemans, et. al., Nat. Phys. (2006).
- S. Kneip et al., Phys. Rev. Lett. (2009).
- J. S. Liu et al., Phys. Rev. Lett. (2011).

Transverse Gradient Undulator (TGU)\*



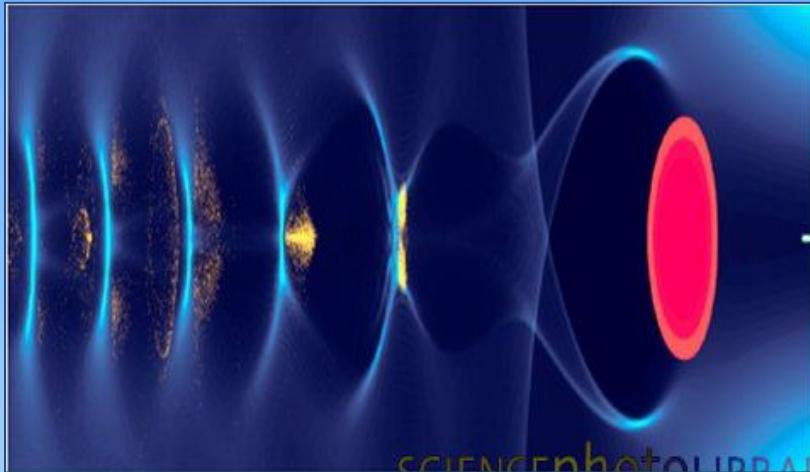
- FEL resonance can be kept for a large energy spread by TGU + Dispersion

$$\lambda_r = \frac{\lambda_u}{2\gamma(x)^2} \left( 1 + \frac{K(x)^2}{2} \right)$$

- T. Smith et al., J. Appl. Phys. 1979
- Z. Huang et al., Phys. Rev. Lett., 2012

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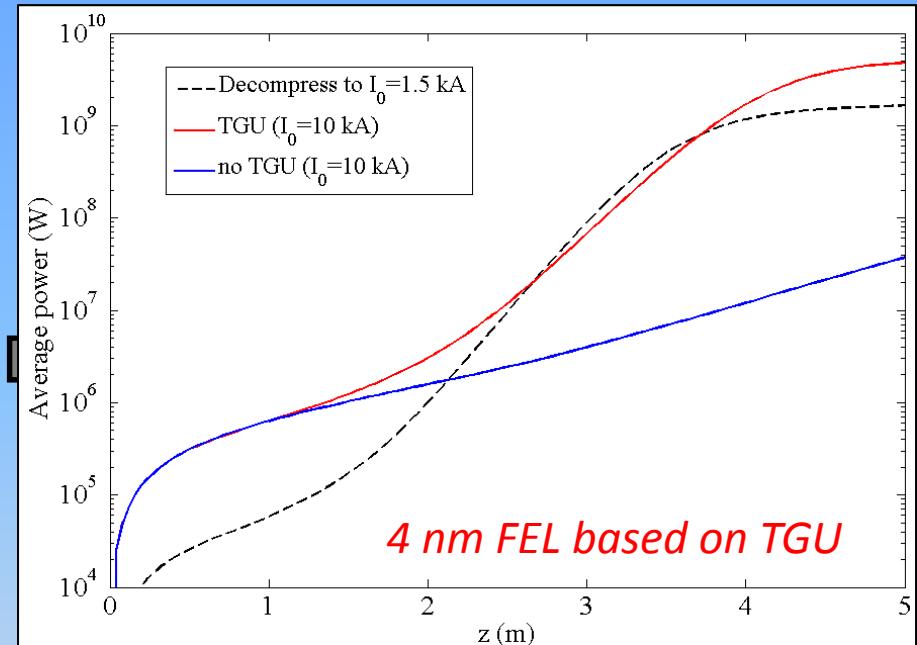


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## Transverse Gradient Undulator (TGU)\*



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- T. Smith et al., J. Appl. Phys. 1979
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# *Summary*

- *Despite spectacular successes in synchrotron radiation and FELs, the quest for brightness and coherence continues, with no sign of slowing down.*
- *Future light source development includes diffraction-limited light sources, high-peak and average power FELs, compact coherent sources and many more possibilities.*
- *The future of synchrotron radiation and FELs is as bright as Shanghai's skyline!*



*Thanks for your attention,* 谢谢！