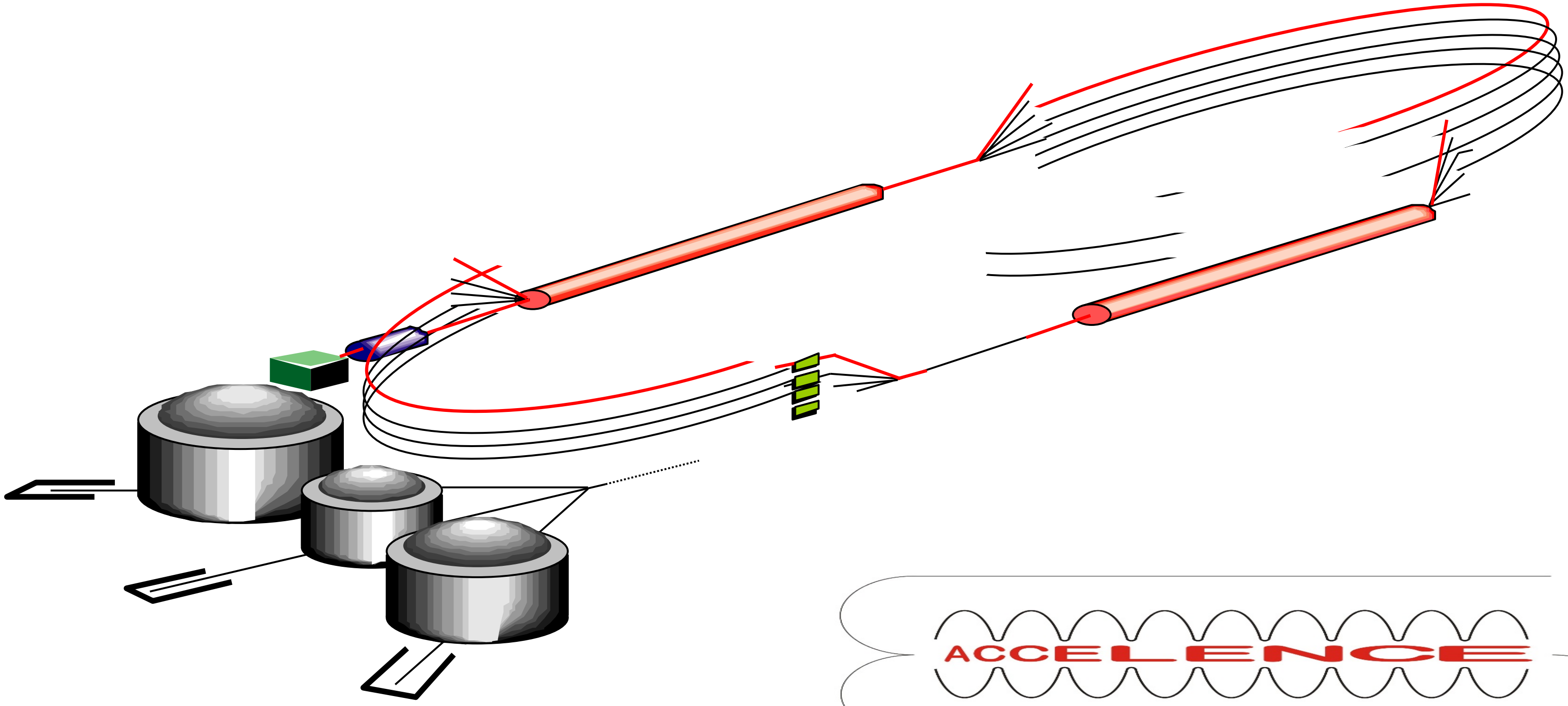
Study of Microbuncing Instability in MESA



Gefördert durch die DFG im Rahmen des GRK 2128

Outline

- Introduction and Motivation
  - MESA
  - Motivation
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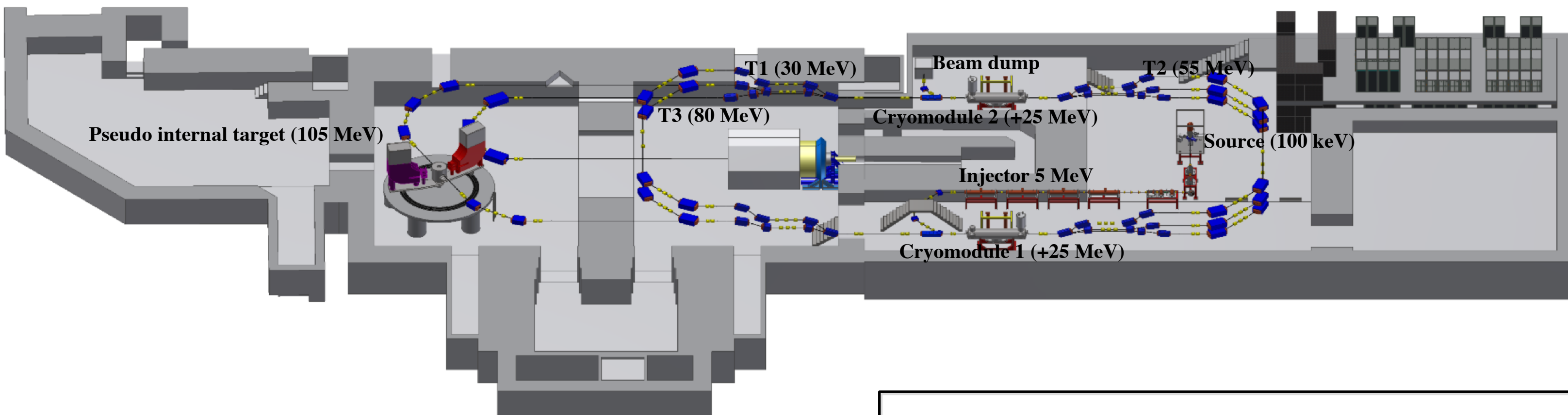
- Theoretical formulation
  - Longitudinal Space Charge and Impedance Model
  - Beam Envelopes with SC
- Simulation results
- Conclusion and Future Work

Mainz Energy-Recovering Superconducting Accelerator (MESA)



- A normal conducting injector linac with extraction ener
- Two SRF linac modules with 25 MeV energy gain.
- Four spreader sections for vertically separating and rec
- Three 180^0 arcs for beam circulations.
- Two chicanes for injection and extraction of the beam.
- Internal experiment arc for 180^0 phase inversion.

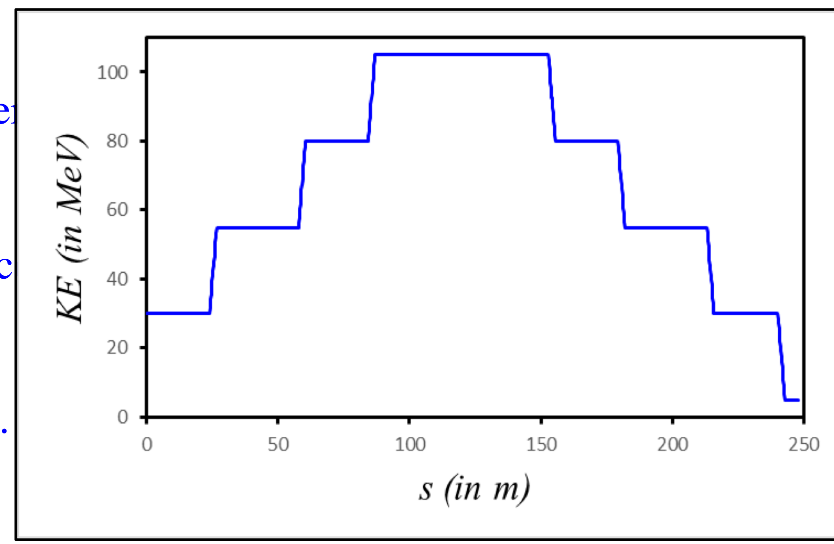
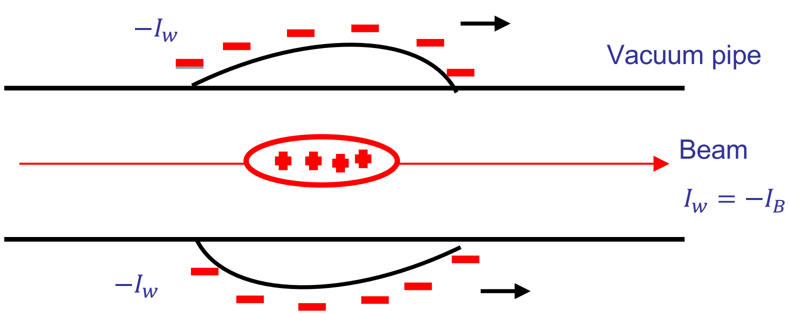


Photo: D.Simon

Motivation

- In an ERL, the beam quality preservance is a great concern during recirculation .
- A beam becomes unstable when a moment $(eg.\ \sigma_x, \sigma_y, Dx, etc.)$ of its distribution exibits an exponential growth and resulting into emittance increase and energy spread.
- For high intensity beams space charge reduces the focusing forces and for a non-periodic structure like MESA it‘s a big challenge to match the beam envelopes.
- To avoid numerical noise from tracking codes. It‘s better to develop an alternative model for optics optamization and MBI gain analysis.

Longitudinal Beam Instabilities



I_w wall current due to circulating bunch .
Vacuum pipe is not smooth, I_w sees an IMPEDANCE.
Impedance, $Z = Z_r + Z_i$
Induced voltage, $V \sim I_w Z = -I_B Z$

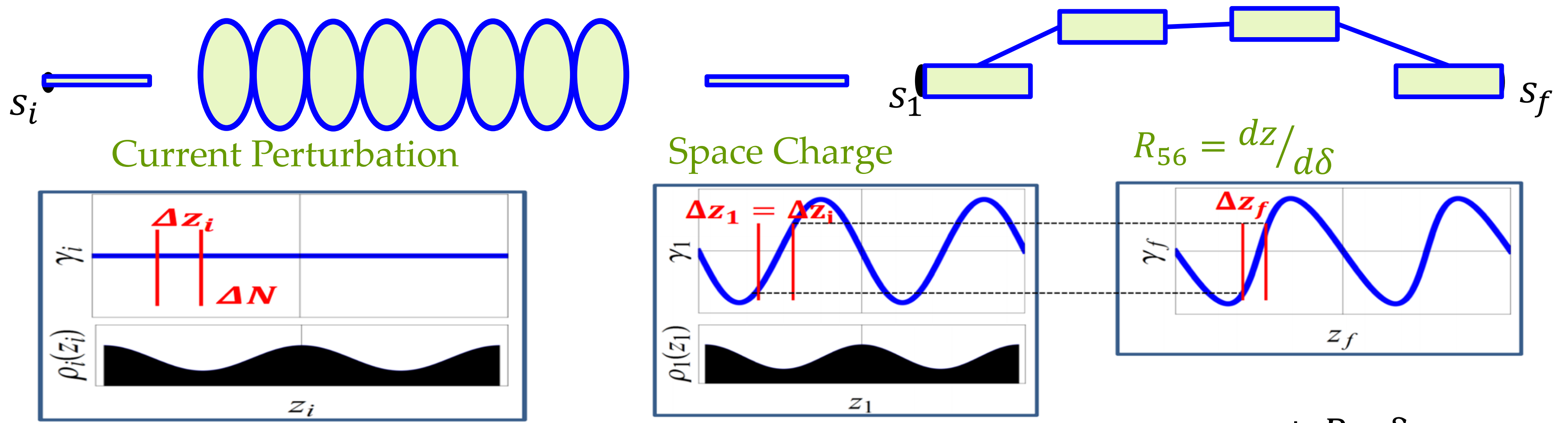
V may act back on the beam ➡ Instability is intensity dependent

General Scheme to investigate instabilities

1. Start with a nominal particle distribution (i.e. longitudinal position, density)
2. Compute fields and induced wall currents with small perturbation of this nominal distribution, and determine forces acting back on them.
3. Calculate change of distribution due to forces.

If Initial Small Perturbation → Increased? Instability
→ Decreased? Stability

Analytical MBI Gain



There are ΔN *particles in interval* Δz_i

$$z_1 = z_i$$

$$z_f = z_i + R_{56}\delta_1$$

There ΔN *particles are in shorter interval* $\Delta z_f < \Delta z_i$

Linear gain, $G = \left|\frac{\Delta\widehat{I_f}}{\Delta\widehat{I_i}}\right| = 4\pi\frac{I_0}{I_A} L_s \frac{|Z(k)|}{Z_0} k|R_{56}|$, Where, $I_A = 17\ kA$ is Alfven Current.

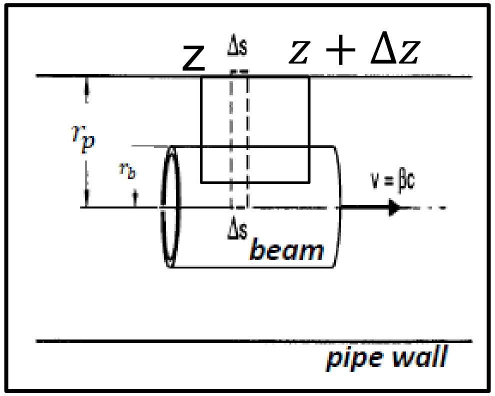
Longitudinal Space Charge

For a beam with cylindrical charge density of radius r_b (ultra-relativistic approximation)

$$\boxed{E_z(r,z) \approx -\frac{2qN}{4\pi\varepsilon_0\gamma^2}\frac{d\lambda(z)}{dz}\left(log\frac{r_p}{r_b}+\frac{{r_b}^2-r^2}{2{r_b}^2}\right)}$$

Space-charge suppression at high energy

Field is proportional to derivative of bunch profile



Impedance due to LSC is, $Z(k) = \frac{iZ_0}{\pi r_b \gamma}\left[1-\frac{kr_b}{\gamma}K_1\left(\frac{kr_b}{\gamma}\right)\right]$

Where, $r_b = 1.7(\sigma_x+\sigma_x)/2$ is beam radius

Envelope eq's with space charge

General equation of motion with space charge, $\frac{d^2x}{ds^2} + \kappa(s)x = \frac{2K}{a_x(a_x+a_y)}x$

Quadrupole focusing

Space charge defocusing

Space Charge perveance: $K = \frac{eI}{2\pi\varepsilon_0 m\gamma^3\beta^3c^3}$

$$x(s) = \bar{x} + \delta D_x \qquad p_x(s) = \bar{p}_x + \delta D'_x$$

$$\sigma_x = \sqrt{\langle\bar{x}^2\rangle} \quad \sigma_y = Y = \sqrt{\langle\bar{y}^2\rangle}, \quad X = \sqrt{\langle x^2\rangle} = \sqrt{\langle{\sigma_x}^2 + {\sigma_\delta}^2{D_x}^2\rangle}$$

RMS beam envelope equations :

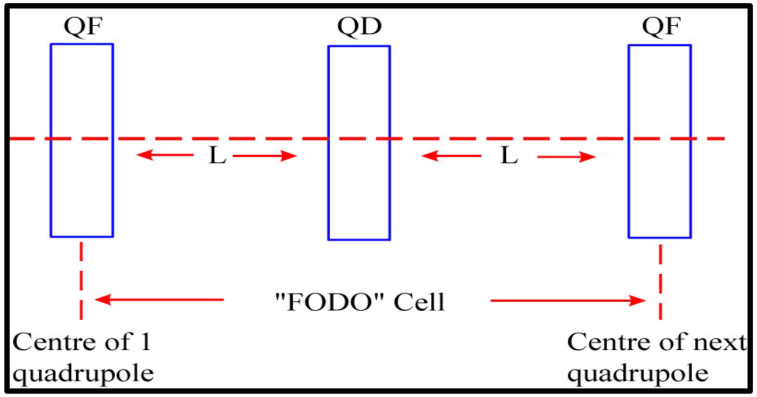
$$\frac{d^2\sigma_x}{ds^2} + \left(\kappa_x(s) - \frac{K}{2X(X+Y)}\right)\sigma_x - \frac{{\epsilon_x}^2}{{\sigma_x}^3} = 0$$

$$\frac{d^2\sigma_y}{ds^2} + \left(\kappa_y(s) - \frac{K}{2X(X+Y)}\right)\sigma_y - \frac{{\epsilon_y}^2}{{\sigma_y}^3} = 0,$$

$$\frac{d^2x}{ds^2} + \left(k(s) - \frac{K}{{r_b}^2}\right)x = 0$$

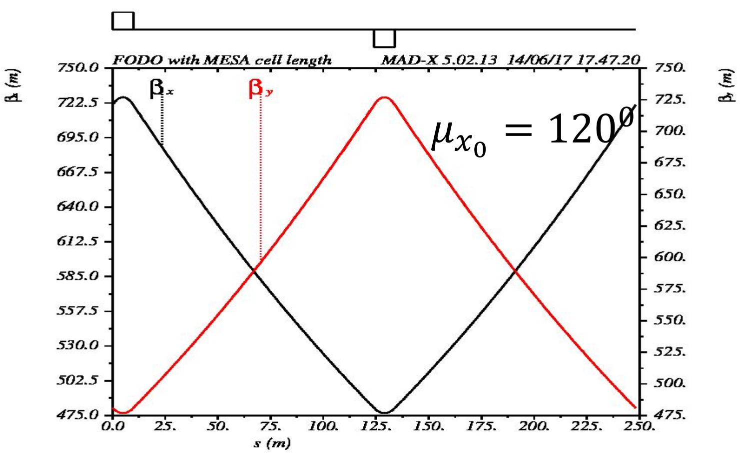
$$\beta_x, \beta_y \longrightarrow r_b = \sqrt{\beta_x\varepsilon_x}$$

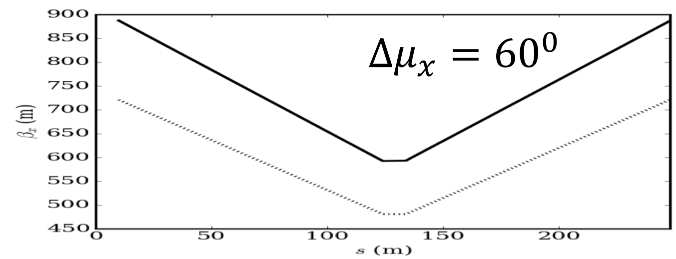
Matched beam in a FODO Cell

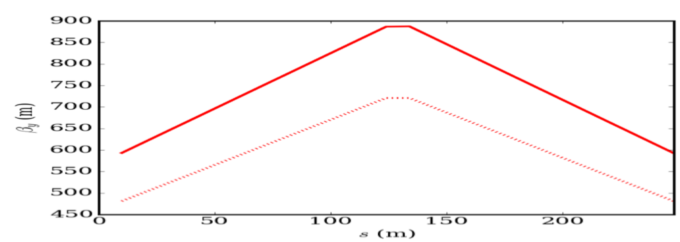


phase advance shift due to space charge:

$$\Delta\mu_x = \mu_x - \mu_{x0} \qquad \mu_x = \int_s^{s+L} \frac{ds}{\hat{\beta}_x(s)}$$







Beam envelopes increased due to SC

Numerical transformation of the lattice functions with SC

$$B(s) = \varepsilon^2 \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix}$$

(Beam matrix)

$$B_{s_1} = M.B_{s_0}.M^T$$

(Transformation of beam matrix)

$$\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}_{s_1} = M.\begin{pmatrix} D \\ D' \\ 1 \end{pmatrix}_{s_0}$$

(Transformation of dispersion matrix)

Matched lattice functions (periodic system): $(\alpha, \beta, \gamma, D)_s = (\alpha, \beta, \gamma, D)_{s+L}$

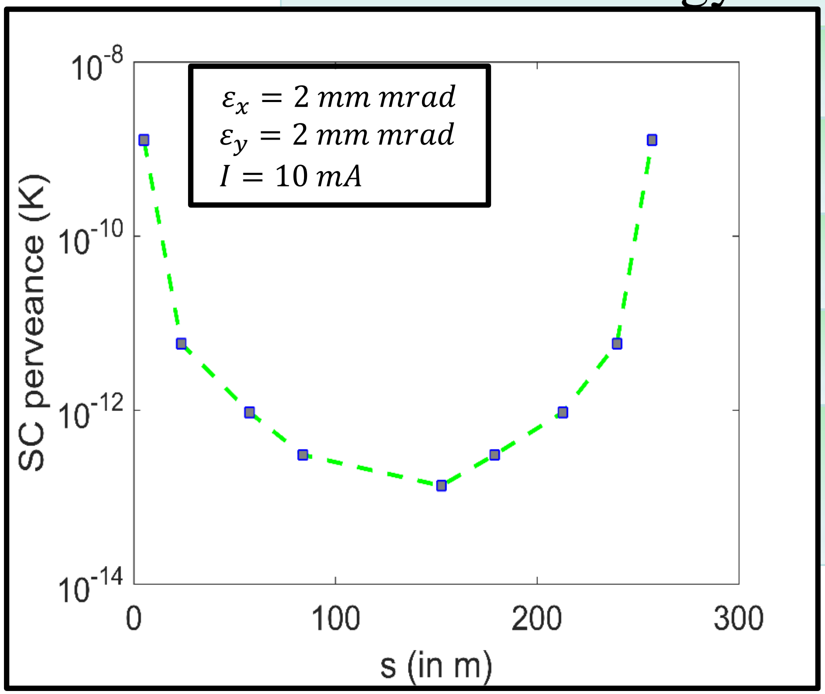
Transport matrix with linear space charge (kick): $M(s_0, s_0 + \Delta s) = M_{\Delta s/_2} M^{sc}_{\Delta s} M_{\Delta s/_2}$

$$\boldsymbol{M}^{sc}_{\Delta s} = \begin{pmatrix} 1 & 0 \\ f_{sc}\Delta s & 1 \end{pmatrix} \qquad f_x = \frac{K}{a_x(a_x + a_y)} \qquad {a_x}^2 = \beta_x \varepsilon_x + (D\delta)^2,\ {a_y}^2 = \beta_y \varepsilon_y$$

Matched Solution: Shooting scheme with iterations

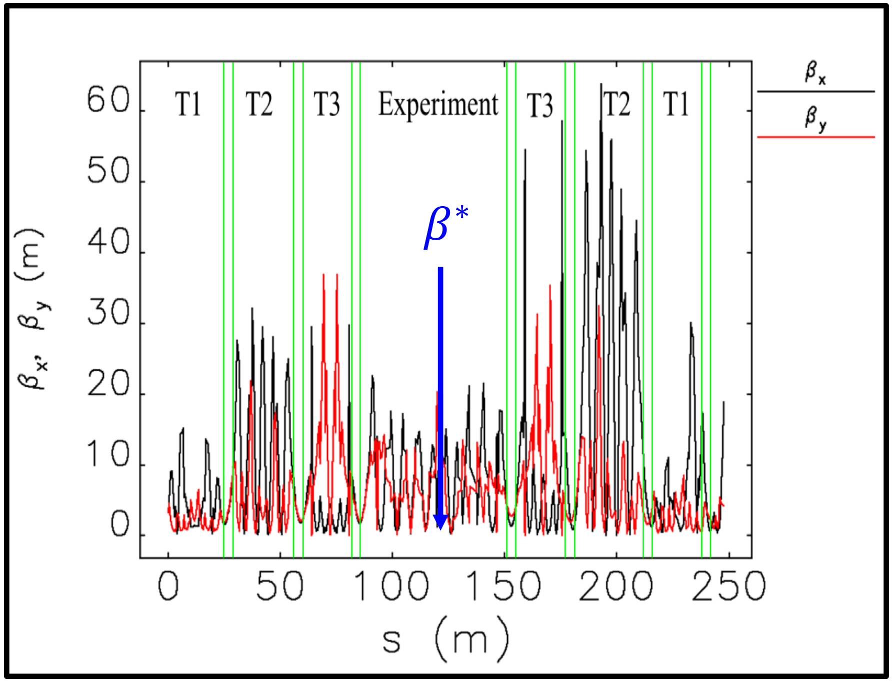
Beam Parameters for MESA

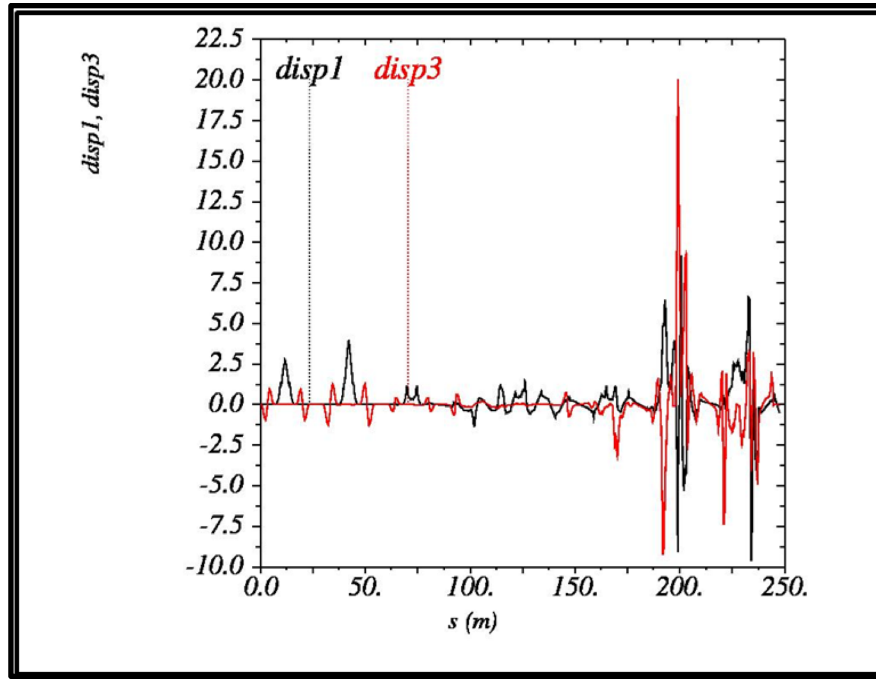
Beam Energy	105 MeV
	1300 MHz, CW
	10 mA
	7.7 pC
	1.281 mm
	< 1 mm mrad



SC affect is maximum at injection and extraction at lower energy 30 MeV

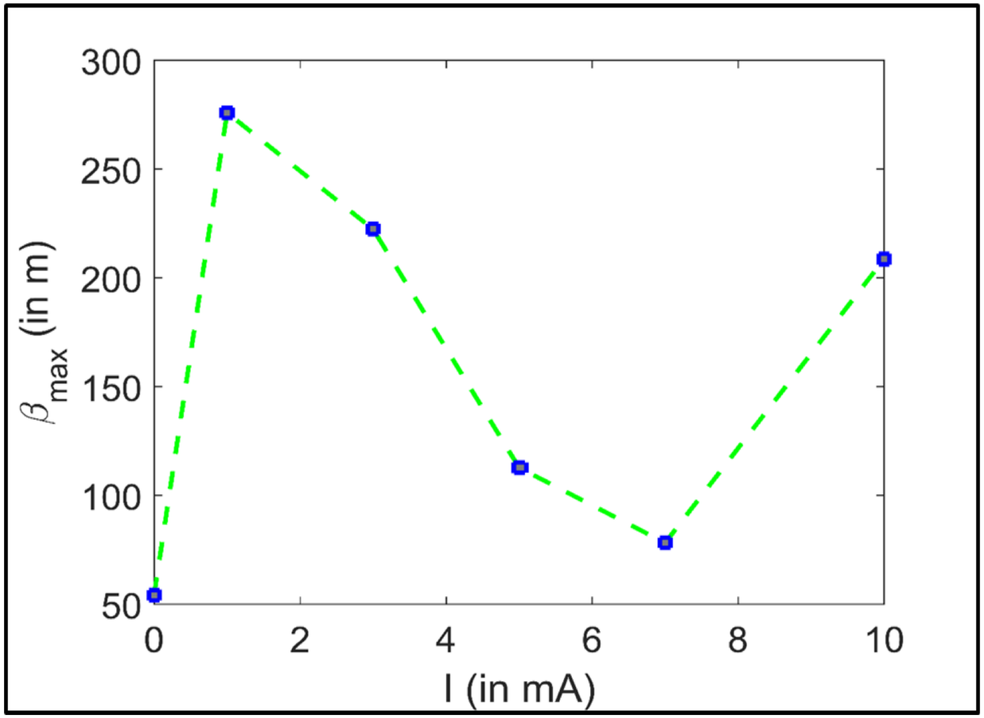
MESA beam envelopes with SC (I)





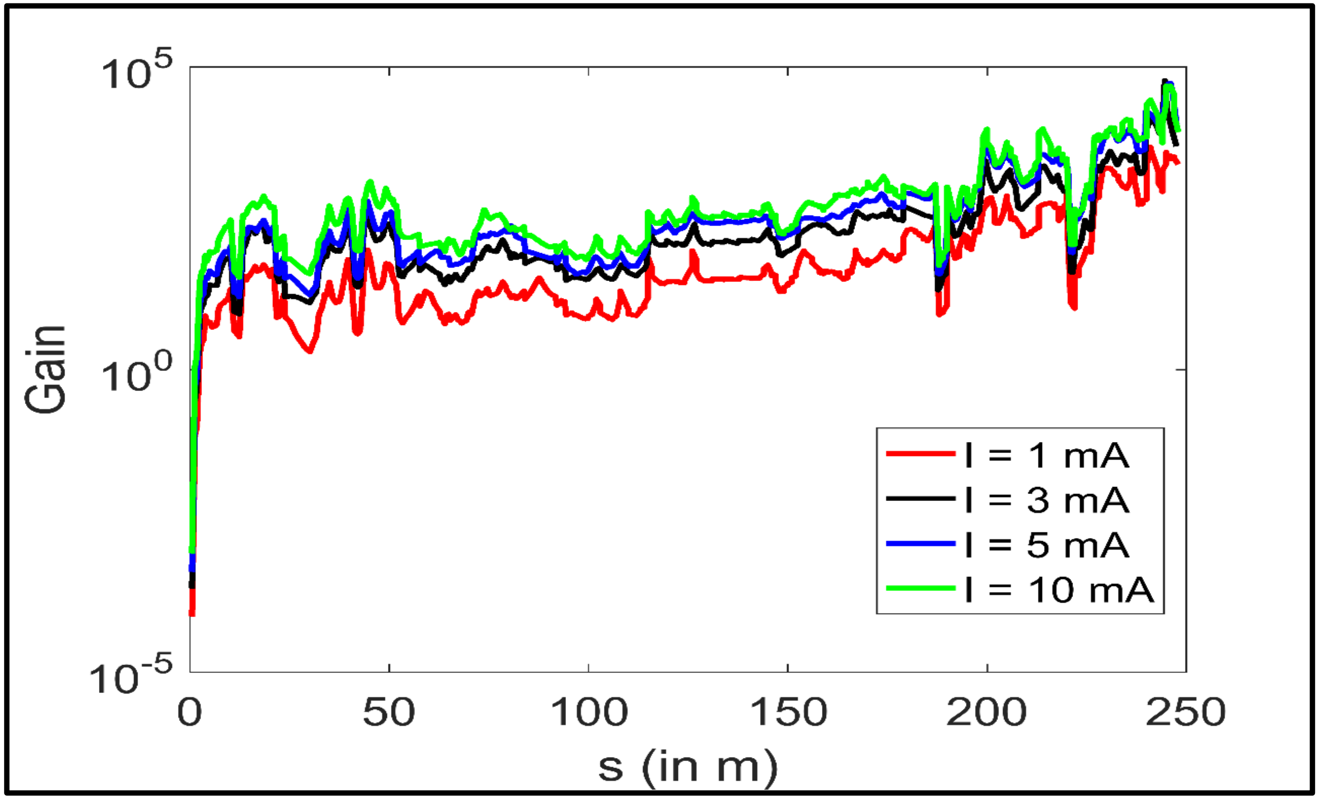
Defocusing due to Space Charge

MESA beam envelopes with SC(II)



Lattice is not robust against small changes. There is a strong mismatch of envelopes with SC.

MBI Gain at modulation wavelength 30 μm



Conclusion

- This lattice is not reliable for Energy Recovery. Symmetric lattice is required for energy recovery.
- Matching of envelopes with SC is a challenge for non-periodic lattice like MESA.
- Fixed beta function for fixed internal experiment is a challenge.

Future Plans:

- Optimization of arcs with SC consideration.
- Dispersion study with SC.
- Benchmarking with particle tracking codes.

Many thanks to

Oliver Boine-Frankenheim, Daniel Simon, Kurt Aulenbacher

Thank you for your attention!