

# SIMULTANEOUS COMPENSATION OF PHASE AND AMPLITUDE DEPENDENT GEOMETRICAL RESONANCES USING OCTUPOLES\*

F. Plassard<sup>†</sup>, Y. Hidaka, Y. Li, T. Shaftan, V. Smaluk, G. Wang  
Brookhaven National Laboratory, New York, USA

## Abstract

As the new generation of light sources are pushing toward diffraction limited storage rings with ultra-low emittance beams, nonlinear beam dynamics become increasingly difficult to control. It is a common practice for modern designs to use a sextupole scheme that allows simultaneous correction of natural chromaticity and energy independent, or geometrical, sextupolar resonances. However, the remaining higher order terms arising from the cross talks of the sextupole families set a strong limitation on the achievable dynamic aperture. This paper presents a simulation-based recipe to use octupoles together with this sextupole scheme to provide simultaneous self-compensation of linear amplitude dependent tune shift together with phase-dependent octupolar and higher order geometrical resonant driving terms. The correction method was built based on observations made on a simple FODO model, then applied to a realistic low emittance lattice, designed in the framework of the upgrade of the National Synchrotron Light Source II (NSLS-II).

## STUDY GOAL

The sextupole scheme required for chromaticity correction open the box to a wide panel of nonlinear aberrations or resonant driving terms (RDTs) that consequently impact the stability of the particles at large amplitudes. The cross-talks between sextupoles RDTs generate amplitude dependent tune shift (ADTS), second and higher order phase-dependent terms affecting the overall efficiency of the machine, mainly characterized by its dynamic and momentum aperture. Large dynamic aperture (DA), defining the area in the phase-space within which the particle motion stays stable, is essential for efficient beam injection.

Lattice solutions with optimized phase-advance, forming an identity transformation matrix  $-\mathcal{I}_{x,y}$  between chromatic sextupole pairs, allowing self-cancellation of sextupole-like geometrical terms [1–3], have been extensively applied in high energy colliders but also in several upgrades to 4<sup>th</sup> generation light sources [4–8]. The remaining higher order RDTs and especially the ADTS, are usually minimized via additional sextupoles in zero-dispersion region, called harmonic sextupoles. Due to the complex mechanism governing second order perturbation on the beam dynamics, the harmonic sextupoles are optimized by means of numerical tools and high performance computers to explore and optimize the parameter space. A different way of correcting second order effects of the sextupoles is to use octupole magnets. Octupoles have been studied for their efficient cancellation

of linear amplitude detuning terms [9–11]. However, while correcting the linear ADTS, octupoles generate additional nonlinear perturbations that can rapidly limit the machine performance. Notably,  $s$ -dependent octupolar resonances can add up with the ones produced to the second order by the sextupoles [12, 13]. In addition, cross-talks among sextupole and octupole RDTs, produce important higher order terms, contributing to the more chaotic behavior of the particles. Minimizing those additional effects with octupoles, is the main goal of this study.

Our nonlinear scheme uses octupole triplets, powered to cancel linear ADTS, for which their location is optimized w.r.t the sextupole pairs, separated by a  $-\mathcal{I}_{x,y}$  matrix, to produce octupole-like RDTs that systematically counteract the ones generated to the second order by the sextupoles. As a domino effect, the resulting higher order geometrical RDTs, simulated up to the dodecapole order, are consistently minimized at the optimal octupole location. The effectiveness of the scheme has been demonstrated on a simple FODO model and on a realistic lattice option for the NSLS-II upgrade, for which good agreement with the model predictions have been observed, leading to a significant increase of the on-momentum DA, without the intervention of numerical optimization tools.

## OBSERVATIONS ON FODO MODEL

In order to build a correction method applicable to most modern light source designs, a toy model, simulating the  $-\mathcal{I}_{x,y}$  sextupole pairs configuration, was used. A FODO lattice is created and two pairs of sextupoles are inserted at the peaks of  $\beta_{x,y}$  and are separated by a horizontal and vertical phase-advances  $\Delta\mu_{x,y}^S = (2n + 1)\pi$ . The betatron amplitude, phases and magnet strength conditions, form an exact  $-\mathcal{I}_{x,y}$  transformation between the sextupole pairs. Three octupoles are then placed close to each other at locations of high  $\frac{\beta_x}{\beta_y}$ , high  $\frac{\beta_y}{\beta_x}$  and  $\frac{\beta_x}{\beta_y} \approx 1$ , in order to correct, with minimal strength, for the linear anharmonicities  $\alpha_{xx} = \frac{\partial v_x}{2J_x}$ ,  $\alpha_{yy} = \frac{\partial v_y}{2J_y}$  and  $\alpha_{xy} = \frac{\partial v_x}{2J_y} = \frac{\partial v_y}{2J_x}$ , respectively. The strength of the octupole triplet is calculated analytically, as described in [9, 10], by solving the linear system as in Eq. (1):

$$\vec{k}_{\text{oct}} = \mathcal{U}_{\text{oct}}^{-1} \cdot \vec{\alpha}, \quad (1)$$

where  $\vec{k}_{\text{oct}}$  is the vector strength of the 3 octupoles,  $\vec{\alpha}$  contains the three direct and cross terms of the linear ADTS that were calculated here using the tracking code PTC [14], and  $\mathcal{U}_{\text{oct}}$  is

**MC5: Beam Dynamics and EM Fields**

**D02 Non-linear Single Particle Dynamics**

\* Work supported by U.S. DOE under Contract No. DE-SC0012704.

<sup>†</sup> fplassard@bnl.gov

the  $3 \times 3$  matrix defined from the lattice optical functions as:

$$\mathcal{U}_{\text{oct}} = \frac{1}{8\pi} \begin{pmatrix} \frac{1}{2}\beta_x^2|_{\text{oct}_1} & \frac{1}{2}\beta_x^2|_{\text{oct}_2} & \frac{1}{2}\beta_x^2|_{\text{oct}_3} \\ \frac{1}{2}\beta_y^2|_{\text{oct}_1} & \frac{1}{2}\beta_y^2|_{\text{oct}_2} & \frac{1}{2}\beta_y^2|_{\text{oct}_3} \\ -\beta_x\beta_y|_{\text{oct}_1} & -\beta_x\beta_y|_{\text{oct}_2} & -\beta_x\beta_y|_{\text{oct}_3} \end{pmatrix}. \quad (2)$$

The impact of the octupole triplet location is observed for various different positions in the model, illustrated in Fig. 1, with the same  $\beta$ -function conditions and thus with the same strength  $k_{\text{oct}}$  for an exact cancellation of the anharmonicities. For each location, all RDTs are re-calculated at the end of the optics sequence. This scan shows the impact of octupole triplet positions, when powered for the same correction of linear ADTS, on the amplitude of the RDTs. The scan has been applied for various conditions of  $\Delta\mu_{x,y}^S$ , e.g.,  $(\pi, \pi)$ ,  $(3\pi, 3\pi)$ ,  $(3\pi, \pi)$ , etc. The results in Fig. 2 show the sum of all geometrical s-dependent hamiltonian RDTs (as defined in [15, 16]) of order  $n = 4$ , on the left side, and all dodecapole geometrical RDTs ( $n = 6$ ), on the right side, calculated at each position of the octupole triplet along the FODO lattice. It is clear from the different scans performed, that particular positions of the triplet, w.r.t to the sextupoles, present significant compensation of these terms. Figure 2 also shows that the level of dodecapole RDTs, that include 2<sup>nd</sup> order ADTS, is always increased by several order of magnitude, except for these optimal octupole positions. Similar compensation are observed for decapole RDTs ( $n = 5$ ). The pattern that emerge for every FODO model simulated, and only if  $\Delta\mu_{x,y}^S = (2n+1)\pi$ , is that the compensation appears when the octupole triplet is located at

$$\Delta\mu_{x,y}^{O-S} = m \times \Delta\mu_{x,y}^S, \quad (3)$$

where  $m$  is an integer. When these conditions are met, all the geometrical RDTs of order  $n = 4$  generated to the first order by the octupoles ( $h_{jklm|\text{Oct}}$ ) and to the second order by the sextupoles ( $h_{jklm|\text{Sext}}$ ) always counteract each other, i.e., their real and imaginary parts have opposite signs and

$$\frac{\text{Im}(h_{jklm|\text{Sext}})}{\text{Re}(h_{jklm|\text{Sext}})} = \frac{\text{Im}(h_{jklm|\text{Oct}})}{\text{Re}(h_{jklm|\text{Oct}})}. \quad (4)$$

This ensure a systematic partial compensation of all the phase-dependent geometrical RDTs of order  $n = 4$ , simultaneously with the cancellation of the linear ADTS coefficients. In addition, the scheme ensure minimal strength of higher order terms, providing the conditions for a quasi full geometrical correction of the system.

## SCHEME APPLIED ON LATTICE

When designing low emittance lattices, constraining the phase-advances between octupoles and sextupoles to  $\Delta\mu_{x,y}^{O-S} = m \times \Delta\mu_{x,y}^S$  with  $m \neq 0$ , is in practice troublesome. One simple optimal location for the triplet, is in phase with the chromatic sextupoles ( $m = 0$ ). However, the octupoles will be inevitably placed in dispersion region and will therefore impact nonlinear chromaticity. As a proof-of-principle, the octupole scheme has been tested on one of

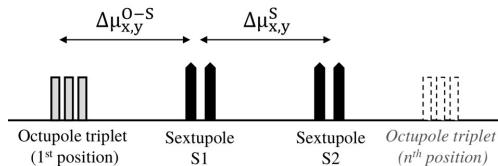


Figure 1: Schematic of the octupole triplet position scan.  $\Delta\mu_{x,y}^{O-S}$  is the phase-advance between the octupole triplet and the first sextupole.  $\Delta\mu_{x,y}^S$  is the phase advance between the sextupoles and is equal to  $(2n+1)\pi$ .

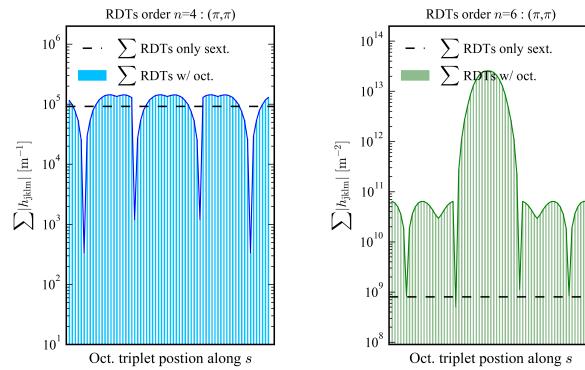


Figure 2: Sum of geometrical octupole (order  $n = 4$ ) (left) and dodecapole (order  $n = 6$ ) RDTs, as function of the position of the octupole triplet (vertical lines). The  $h_{2200}$ ,  $h_{0022}$  and  $h_{1111}$  terms, driving linear ADTS are not included in the sum. The position scan has been applied for  $\Delta\mu_{x,y}^S = \pi, \pi$ . The value of the RDT sum drops drastically at the sextupole location ( $m = 0$ ) and at  $\Delta\mu_{x,y}^{O-S} = m \times \Delta\mu_{x,y}^S$ . The black dashed line shows the sum value without octupoles.

the lattice options optimized for the upgrade of the NSLS-II. The cell design and optical functions are shown in Fig. 3. This preliminary option provides an equilibrium horizontal emittance of  $\epsilon_x = 25.3$  pm at a beam energy of 3 GeV, while fitting the tunnel of the existing NSLS-II ring and meeting all the optical parameter constraints required for the upgrade. The low emittance is achieved primarily thanks to the use of the novel *Complex Bend* (CB) magnet, which consists of a single element with conventional electromagnet dipole poles of same field polarity, superposed with strong focusing and defocusing quadrupole field generated by permanent magnets. A detailed technical description of this CB solution is given in [17, 18]. The 3 families of octupoles (see Fig. 3), are placed in phase with the sextupoles ( $m = 0$ ). Their integrated strengths are calculated from Eq. (1), giving  $\vec{k}_{\text{oct}} = (-2.88 \times 10^3, -8.50 \times 10^3, 3.95 \times 10^3) [\text{m}^{-3}]$ .

As predicted by the model, the optimal location of the octupole w.r.t the sextupole pairs, not only cancel for the linear ADTS but also minimize all geometrical phase-dependent octupole RDTs along with higher order terms. The evolution of the sum of these terms as function of the triplet strength applied  $w_{\text{oct}} \cdot k_{\text{oct}}$ , is shown in Fig. 4. The on-momentum DA rises with  $w_{\text{oct}}$  up to 7 times the original DA area.

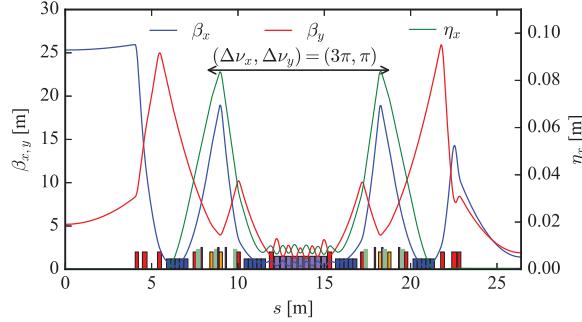


Figure 3: Twiss functions along one cell of a lattice option for the upgrade of NSLS-II, based on Complex Bend technology. The CB magnet is colored in purple, the sextupoles in green and the 3 octupole families in pink.

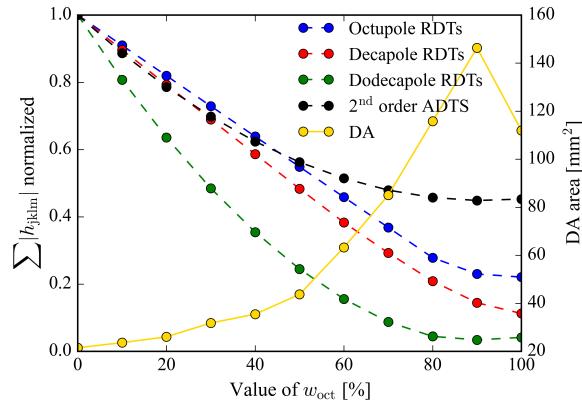


Figure 4: Evolution of the sum of the geometrical phase-dependent RDTs and the 6 2<sup>nd</sup> order ADTS terms, normalized to their value without octupoles, as function of the strength delivered by the octupole triplet  $w_{\text{oct}} k_{\text{oct}}$ , where  $k_{\text{oct}}$  is calculated from Eq. (1) and  $w_{\text{oct}}$  is a weight factor.

Content from this work must maintain attribution to the author(s), title of the work, publisher, and DOI

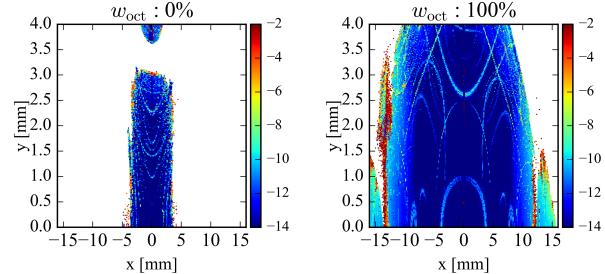


Figure 5: Dynamic aperture of the lattice without (left) and with (right) octupoles. The on-momentum particles were tracked for 1024 turns. The colorbar shows the tune diffusion calculated as  $D = \log_{10}(\Delta\nu_x^2 + \Delta\nu_y^2)$ .

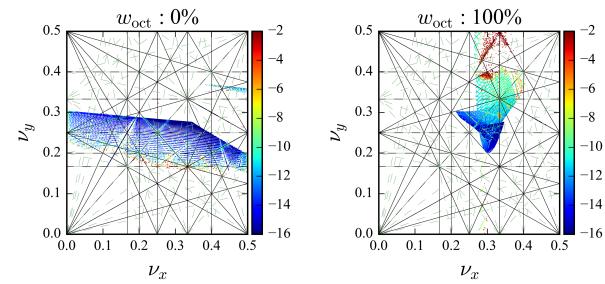


Figure 6: Tune shift comparison without octupoles for a DA area of  $21 \text{ mm}^2$  (left) and with octupole triplet scheme for a DA area of  $112 \text{ mm}^2$  (right).

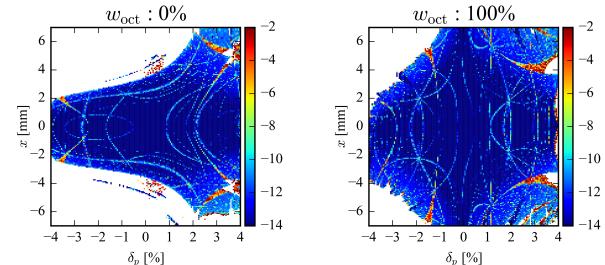


Figure 7: Momentum acceptance. No RF and synchrotron radiation included.

## CONCLUSIONS

The nonlinear scheme proposed here, is an efficient and robust way of minimizing locally, by-design, all geometrical RDTs, from the sextupole to the dodecapole order. The on-momentum DA can easily be tuned from the only knob  $w_{\text{oct}}$ , independently of the value of the corrected natural chromaticity. While the option of positioning the triplet in dispersion region tends to degrade the momentum aperture, the knowledge of an optimal sextupole/octupole configuration for a quasi full geometrical compensation, will be beneficial in future light source nonlinear optimization.

## REFERENCES

- [1] K. L. Brown and R. V. Serfranckx, “Optics modules for circular accelerator design”, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 258, pp. 0168-9002, 1987. doi:10.1016/0168-9002(87)90932-6
- [2] K. Oide and K. Haruyo, “Dynamic aperture of electron storage rings with noninterleaved sextupoles”, *Phys. Rev. E*, vol. 47, pp. 2010-2018, Mar. 1993. doi:10.1103/PhysRevE.47.2010
- [3] R. Bartolini, “Design and Optimization Strategies of Non-linear Dynamics in Diffraction-limited Synchrotron Light Sources”, in *Proc. 7th Int. Particle Accelerator Conf. (IPAC'16)*, Busan, Korea, May 2016, pp. 33–38. doi:10.18429/JACoW-IPAC2016-MOZA02
- [4] L. Farvacque *et al.*, “EBS Storage Ring Technical Report”, ESRF, Grenoble, France, Sep. 2018.
- [5] K. Bane *et al.*, “A Design Report of the Baseline for PEP-X: an Ultra-Low Emittance Storage Ring”, SLAC National Accelerator Laboratory, California, USA, Rep. SLAC-PUB-13999, Jun. 2010.
- [6] B. Singh, R. Bartolini, J. Bengtsson, H. Ghasem, and A. Streun, “Lattice Options for DIAMOND-II”, in *Proc. IPAC'18*, Vancouver, BC, Canada, Apr. 2018, pp. 4050–4053, doi:10.18429/JACoW-IPAC2018-THPMF009
- [7] S. C. Leemann, “Lattice Design for the MAXIV Storage Rings”, MAX IV Laboratory, Lund, Sweden, 2016.
- [8] A. Loulergue *et al.*, “Baseline Lattice for the Upgrade of SOLEIL”, in *Proc. 9th Int. Particle Accelerator Conf. (IPAC'18)*, Vancouver, Canada, Apr.-May 2018, pp. 4726–4729. doi:10.18429/JACoW-IPAC2018-THPML034
- [9] O. Brüning and J-P. Koutchouk, “Feasibility study for compensating the anharmonicities in LEP with additional octupoles”, Geneva, Switzerland, Rep. CERN-SL-97-011-AP, Feb. 1997.
- [10] S. C. Leemann and A. Streun, “Perspectives for future light source lattices incorporating yet uncommon magnets”, *Phys. Rev. ST Accel. Beams*, vol. 14, p. 030701, Mar. 2011. doi:10.1103/PhysRevSTAB.14.030701
- [11] J. Ögren and V. Ziemann, “Compensating amplitude-dependent tune-shift without driving fourth-order resonances”, *Nuclear Instruments and Methods in Physics Research Section A: Accelerators, Spectrometers, Detectors and Associated Equipment*, vol. 869, pp. 0168-9002, Jun. 2017. doi:10.1016/j.nima.2017.06.047
- [12] K. Soutome and H. Tanaka, “Higher-order formulas of amplitude-dependent tune shift caused by a sextupole magnetic field distribution”, *Phys. Rev. Accel. Beams*, vol. 20, p. 064001, Jun. 2017. doi:10.1103/PhysRevAccelBeams.20.064001
- [13] C. Wang, “Explicit formulas for 2nd-order driving terms due to sextupoles and chromatic effects of quadrupoles”, Argonne National Laboratory, Illinois, USA, Rep. ANL/APS/LS-330, Apr. 2012.
- [14] F. Schmidt, E. Forest, and E. McIntosh, “Introduction to the polymorphic tracking code: Fibre bundles, polymorphic Taylor types and Exact tracking”, CERN, Geneva, Switzerland, Rep. CERN-SL-2002-044-AP, Jul. 2002.
- [15] J. Bengtsson, “The Sextupole Scheme for the Swiss Light Source SLS”, Paul Scherrer Institut, Villigen, Switzerland, Rep. SLS Note 9/97, Mar. 1997.
- [16] A. Franchi *et al.*, “First simultaneous measurement of sextupolar and octupolar resonance driving terms in a circular accelerator from turn-by-turn beam position monitor data”, *Phys. Rev. Accel. Beams*, vol. 17, p. 074001, Jul. 2014. doi:10.1103/PhysRevSTAB.17.074001
- [17] T. Shaftan *et al.*, “Complex bend: Strong-focusing magnet for low-emittance synchrotrons”, *Phys. Rev. Accel. Beams*, vol. 21, p. 100703, Oct. 2018. doi:10.1103/PhysRevAccelBeams.21.100703
- [18] T. Shaftan *et al.*, “Complex bend. II. A new optics solution”, *Phys. Rev. Accel. Beams*, vol. 22, p. 110703, Nov. 2019. doi:10.1103/PhysRevAccelBeams.22.110703
- [19] M. Borland, “elegant: A Flexible SDDS-Compliant Code for Accelerator Simulation”, Argonne National Laboratory, Illinois, USA, Rep. LS-287, Aug. 2000.