

Upgrade of MAD-X for HL-LHC project and FCC studies

ICAP'18

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Overview



- MAD-X = Methodical Accelerator Design 10
 - delivered on Linux, MacOSX and Windows (32 & 64 bits).
 - standalone all-in-one application (no dependency), open source, CERN copyright.
 - 3-4 releases / year, built and tested every night (~ 500 test suites).
 - support, website, e-groups, git repository, issue tracker (~600 tickets).
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- Single particle beam code.
 - motion of particles in 5D-6D phase space under external fields.
 - design, optics, tracking, matching, slicing, orbit steering, orbit correction, orbit measures, fields and alignment errors, aperture offsets, aperture margins, emittance equilibrium, frozen space charge, radiation, survey, plots, etc...

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- Scripting language.
 - sequences, commands, variables, macros, tables, files, and more...
 - special constructs: `if-else`, `while`, `macro`, `line`, `table`.
 - deferred expressions (`bx := sqrt(betx);`).
 - global workspace.



Overview (cont.)



- MAD-X is an all-in-one application for
 - machine design (reference @ CERN).
 - optics and tracking calculation.
 - validation, tolerances, margins, studies.
 - optimisations (e.g. from measurements).
 - legacy physics for large machines (e.g. LHC).
 - PTC for smaller machines, 6D (also 4D, 5D, 56D), complex topology, high orders.
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- CERN has ~8 millions lines of MAD-X scripts.
 - +95% are clones or generated (history, variants, layout dump).
- Used world wide.
 - community is about 500 users.
 - used in many projects (often ignored by team).
 - colliders, boosters, storage rings, linacs, gantries, transfer lines, FFAG, racetracks, ...



Scripting language



- Weak string “parsing”, still good enough for complex macros (=input generator).

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```
sortable(tblname, colname, coldir) : macro = { ! optimized shellsort
    sortable._g = 1 ; sortable.count = 0 ;
    sortable._n = table(tblname, tablelength)+1 ;
    while (sortable._g <= sortable._n+1) {
        sortable._g = sortable._g * 3 + 1 ;
    }
    sortable._g = floor(sortable._g / 3) ;
    while ( sortable._g > 0 ) {
        sortable._i = sortable._g ;
        while (sortable._i < sortable._n) {
            setvars table = tblname, row = sortable._i ; sortable._t = colname*coldir ;
            fill table = tblname, row = sortable._n ;
            sortable._j = sortable._i - sortable._g ; sortable._f = 1 ;
            while (sortable._j > 0 && sortable._f > 0) {
                setvars table = tblname, row = sortable._j ; sortable._v = colname*coldir ;
                if (sortable._v < sortable._t) {
                    sortable.count = sortable.count + 1 ;
                    fill table = tblname, row = sortable._j + sortable._g ;
                    sortable._j = sortable._j - sortable._g ;
                } else { sortable._f = 0 ; }
            }
            sortable.count = sortable.count + 1 ;
            setvars table = tblname, row = sortable._n ;
            fill table = tblname, row = sortable._j + sortable._g ;
            sortable._i = sortable._i + 1 ;
        }
        sortable._g = floor(sortable._g / 3) ;
    }
    shrink table = tblname ;
}
```

Functions (op), commands, factories (elem)

- 👉 Functions (and unary/binary operators) return a number
 - ➡ unary operator: $(-x)$
 - ➡ binary operators: $x+y$, $x-y$, $x*y$, x/y , x^y (power)
 - ➡ relational operators: $x==y$, $x<=y$, $x>=y$, $x<y$, $x>y$, $x<>y$
 - ➡ logical operators: $lexpr \&& lexpr$ (and), $lexpr \mid\mid lexpr$ (or)
 - ➡ **abs**(x), **sqrt**(x), **exp**(x), **log**(x), **log10**(x)
 - ➡ **sin**(x), **cos**(x), **tan**(x), **asin**(x), **acos**(x), **atan**(x),
sinh(x), **cosh**(x), **tanh**(x)
 - ➡ **erf**(x), **erfc**(x) (error functions)
 - ➡ **round**(x), **floor**(x), **ceil**(x), **frac**(x) (integral and fractional parts)
 - ➡ **ranf()**, **gauss()**, **tgauss**(x) (random number generators)

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- ➡ relational
 - ➡ **exit**, **quit**, **stop** stop execution
- ➡ logical op
 - ➡ **call** runs (load) scripts, **return** returns from current script
- ➡ **abs**(x), **sqrt**(x)
 - ➡ **exec** expands macros in current script (defined with **macro**)
- ➡ **sin**(x), **cos**(x)
 - ➡ **system** runs shell commands (platform & shell specific)
- ➡ **sinh**(x), **cosh**(x)
 - ➡ **value**, **show**, **help** query global environment
- ➡ **erf**(x), **erfc**(x)
 - ➡ **option**, **title**, **set** setup global environment
- ➡ **round**(x), **ceil**(x), **floor**(x)
 - ➡ **beam** setups physics environment
- ➡ **ranf()**, **gauss()**, **rand()**, **randn()**
 - ➡ **use**, **select** setup specific environment (commands, modules)
 - ➡ **assign** sets output file for echoing
 - ➡ **print**, **printf** print raw text and formated text
 - ➡ **copyfile**, **renamefile**, **removefile** portable files manipulation
 - ➡ **plot**, **setplot**, **resplot** plotting facilities

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- ➡ `round(x), trunc(x)`
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- ➡ `call` runs (load) scripts, `return` returns from current script
- ➡ `exec` expands macros in current script (defined with `macro`)
- ➡ `system` runs shell commands (platform & shell specific)
 - ➡ `sequence` : sequence
 - ➡ `marker`: marker
 - ➡ `drift` : drift
 - ➡ `sbend, rbend, quadrupole, sextupole, octupole, multipole, solenoid, dipedge, rfmultipole`: magnets
 - ➡ `rfcavity, twcavity, crabcavity, rfmultipole`: cavities
 - ➡ `kicker, hkicker, vkicker, tkicker`: correctors
 - ➡ `ecollimator, rcollimator`: collimators*
 - ➡ `monitor, hmonitor, vmonitor, blmonitor`: monitors
 - ➡ `instrument, placeholder`: placeholders
 - ➡ `srotation, yrotation`: rotations
 - ➡ `elseparator, beambeam, matrix`: others

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 - ➡ `angle(k0l), k1, [k2]`
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Survey, Track, Twiss, Match



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 - can track thousand particles (no hard limits, ***OMP parallelization***). NEW
 - synchrotron radiation (***distribution and quantum***). NEW
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- MATCH: global non-linear optimiser
 - handle multiple sequences and beta blocks.
 - local/global constraints, ranges, equality or inequality.
 - direct (run TWISS) or indirect (run macro, PTC_TWISS).
 - can match at any positions inside elements. NEW

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- ```

match use_macro, ... ;
... vary statements ...
macro1: macro = { ... madx statements ... }
constraint expr= "lhs1 < | = | > rhs1" ;
constraint expr= "lhs2 < | = | > rhs2" ;
... constraint statements ...
macro2: macro = { ... madx statements ... }
... constraint statements ...
macro3: macro = { ... madx statements ... }
... constraint statements ...
... methods statements ...
endmatch ;

```

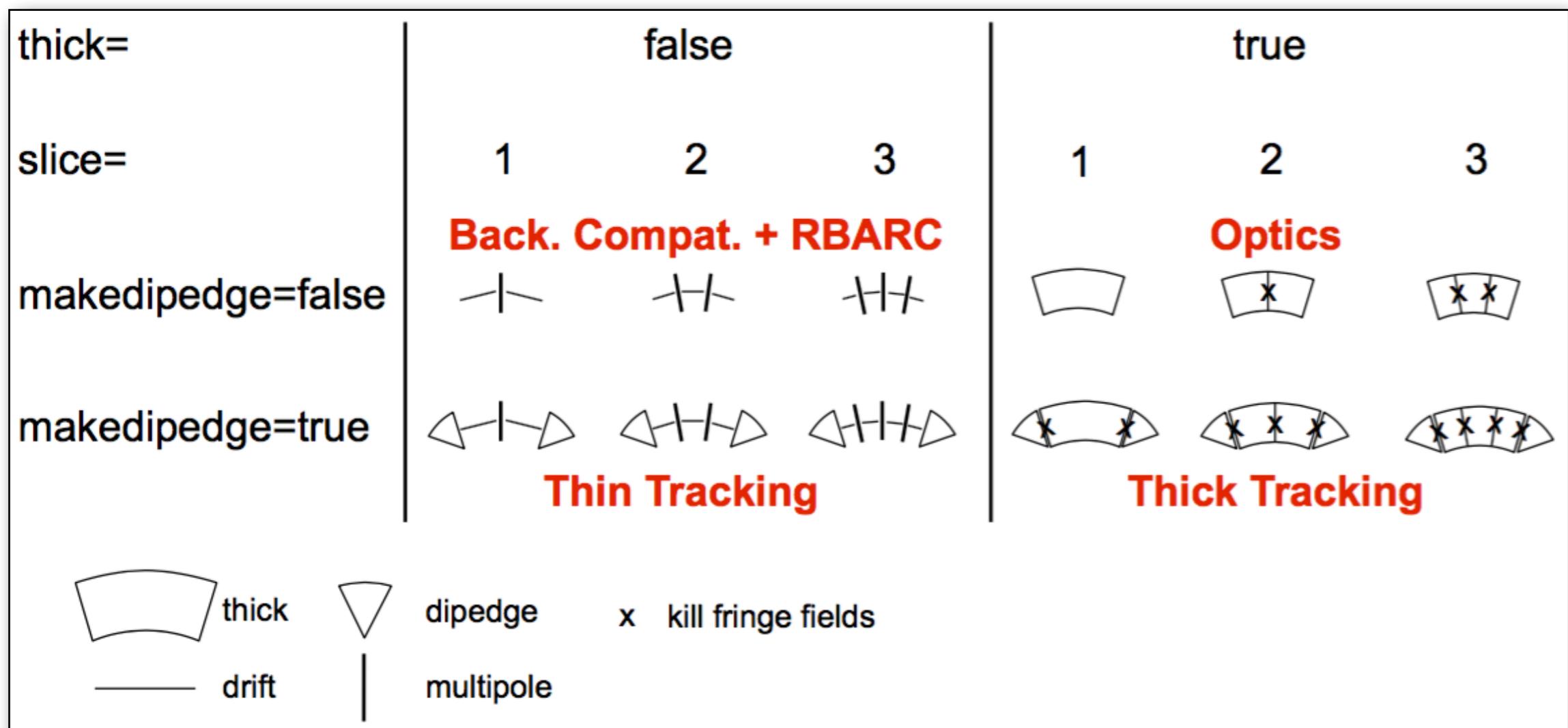


# Makethin improvement



- Convert thick sequence into thin sequence (in place).
  - new module (2015-) in C++ by H. Burkhardt.
  - generalise TEAPOT symplectic integration scheme for  $n > 4$  slices to minimise beta beating using better interpolation (i.e. than SIMPLE) of thick quadrupole.
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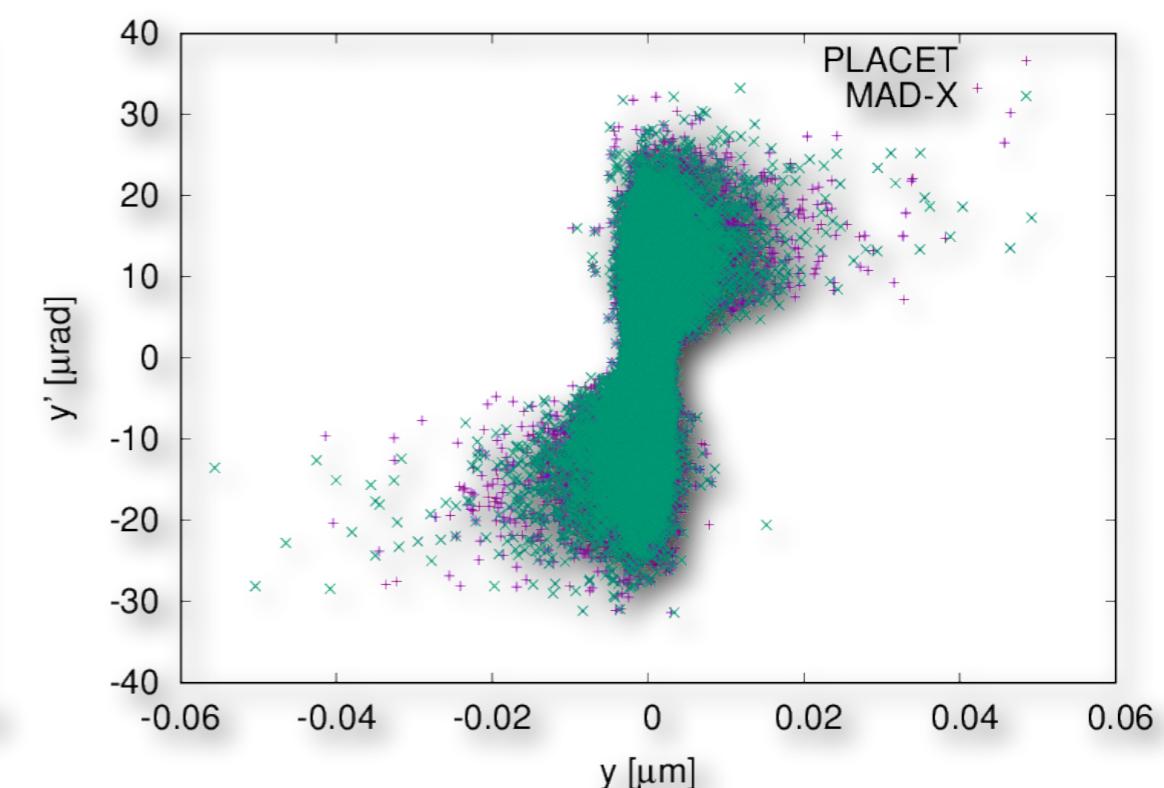
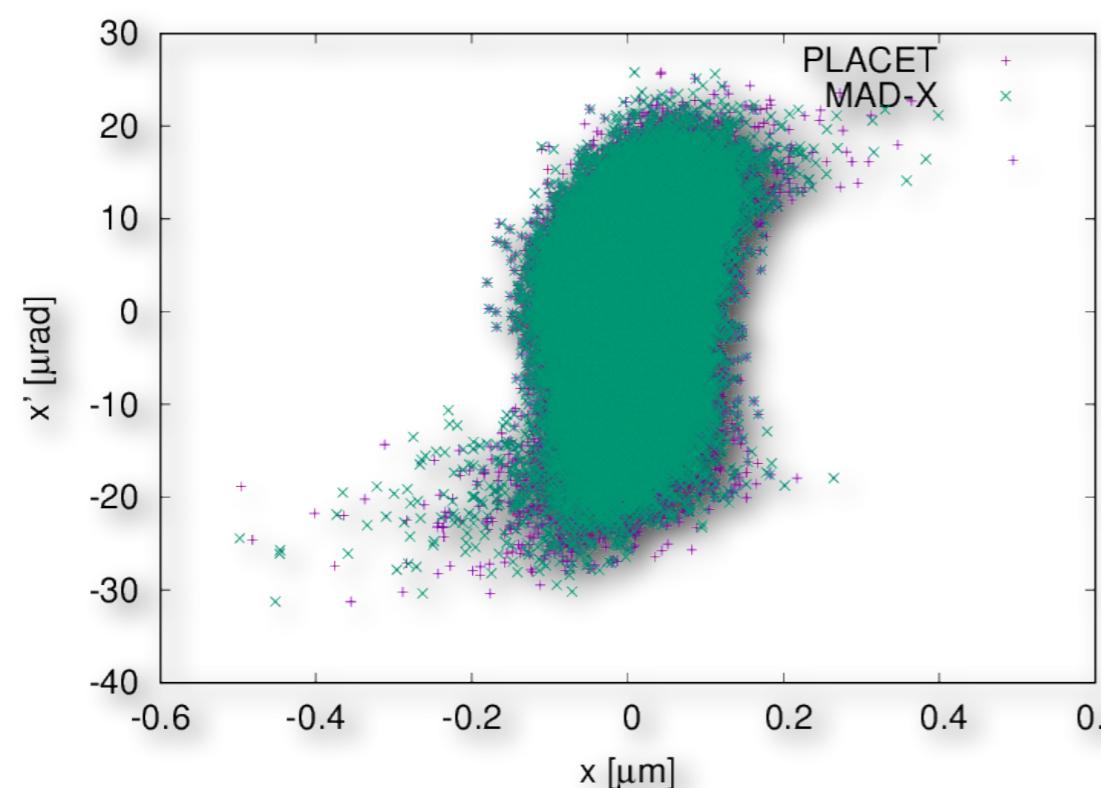
# Synchrotron radiation effects



- Fixed and improved in Twiss, Track and Emit modules.
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- 4 levels of activation
  - *no radiation*, corresponding to the usual Hamiltonian dynamics.
  - *deterministic radiation*, all particles radiate as a single particle on the closed-orbit.
  - *deterministic radiation*, with full dependence on canonical coordinates to generate natural radiation damping (Track and Emit only).  
Preferred method for dynamic aperture calculations in high-energy lepton rings.
  - *Individual quantum excitation* with stochastic photon emissions, provides particle distributions and equilibrium emittances (Track only).

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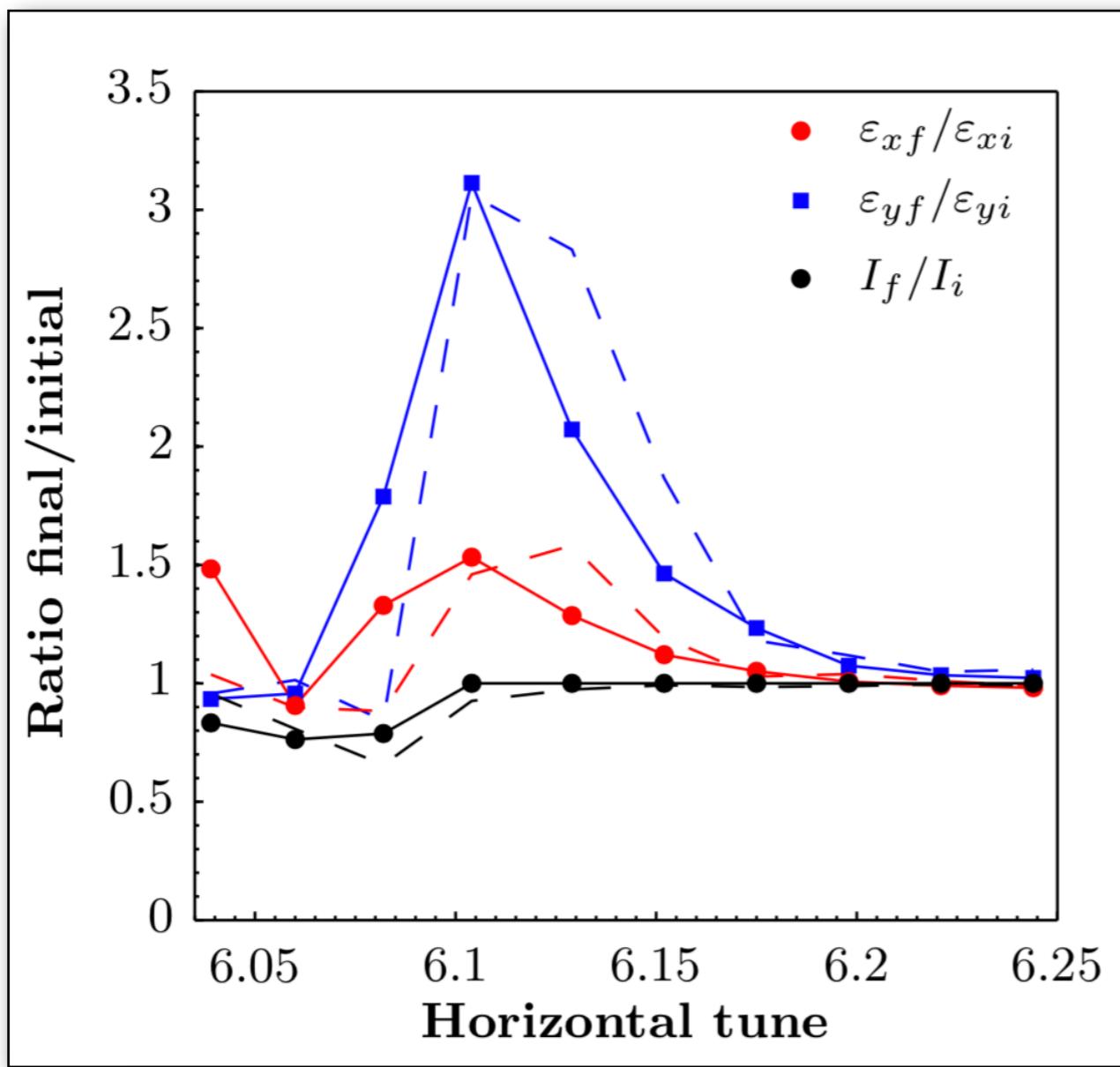


# Frozen space charge



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  - new separate Fortran module.
  - simplify support and extension without changing the model.
  - could become a new separate MAD-X command in the future.

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Emittance growth and beam intensity as computed with MAD-X (adaptive mode). Experimental data are shown with dashed lines.



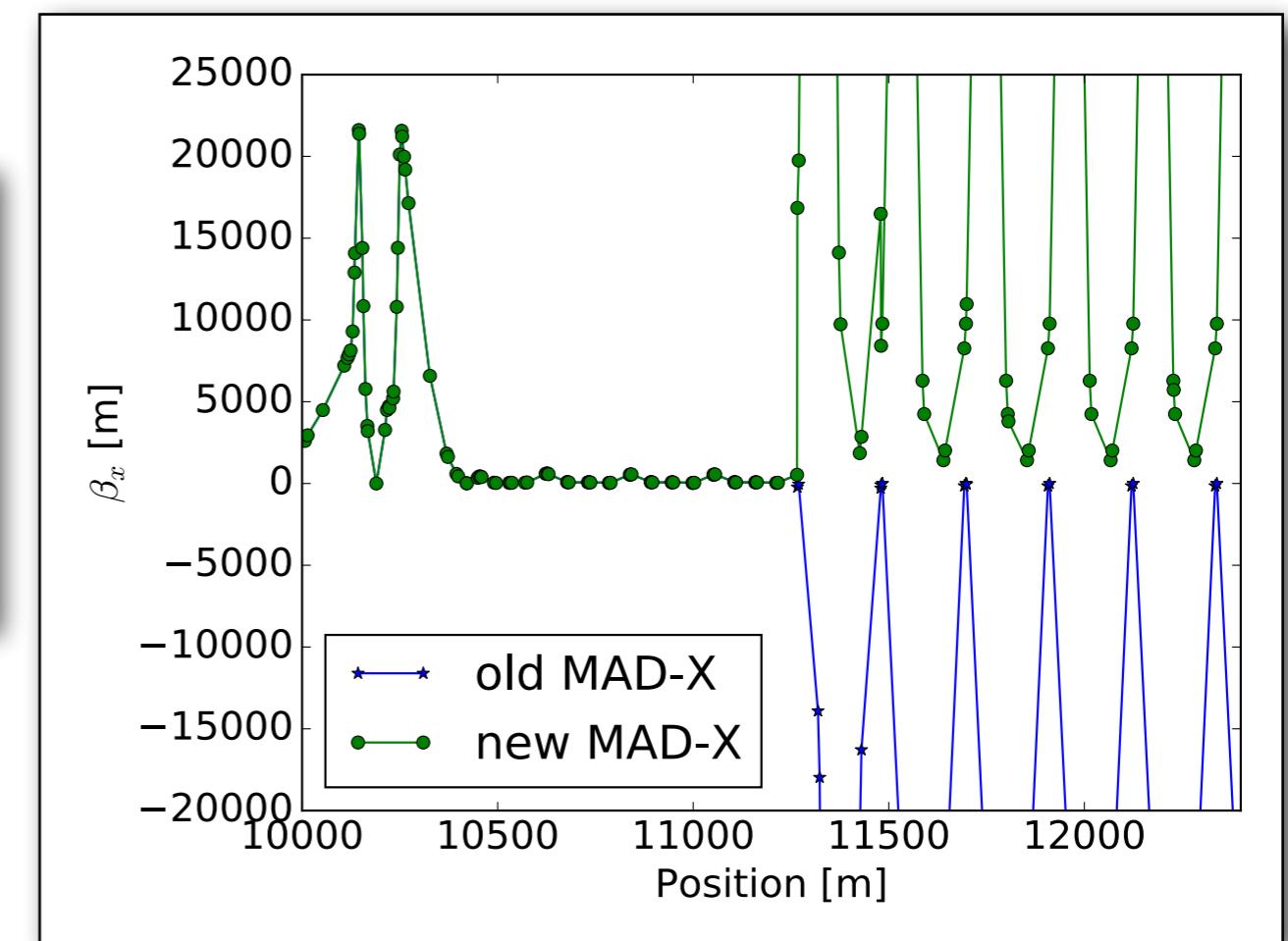
# Linear coupling review



- Review triggered by negative beta functions in the presence of strong coupling
  - occurred for a single seed leading to strong misalignment in HL-LHC studies.
  - tried two alternate implementations without success (Sagan & Rubin, Talman).
  - added the flip mode to the implementation (fixed the problem).
  - added a couple missing checks to validate assumptions.
  - review the theory and the implementation from scratch (lost knowledge).
  - review fixed typos in the manual and restored the understanding of the method.

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Negative (blue) beta functions in HL-LHC studies, right after the skew quadrupole starting from IP (left). Coupling is so strong that optics is not stable. Fixed in recent released (green).





# Linear coupling review (initialisation)



- Restore the derivation of the equations, provide intermediate validity checks and more stable formula (see the links in refs for details).

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$$\vec{X}(s_2) = M \vec{X}(s_1),$$
$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} = R_M \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix} R_M^{-1} = R_M M_\perp R_M^{-1}.$$

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$$\vec{X}(s_2) = M \vec{X}(s_1),$$
$$M =$$

$$\text{eig}(M) = \text{eig}(M^{-1}) = \{(\lambda_i, \lambda_i^{-1}), i = 1..2\},$$
$$\text{eig}(M + \bar{M}) = \{\Lambda_i = \lambda_i + \lambda_i^{-1}, i = 1..2\},$$
$$\det(M + \bar{M} - \Lambda I) = (\text{tr } A - \Lambda)(\text{tr } D - \Lambda) - |C + \bar{B}| = 0,$$
$$\Lambda_{A,D} = \frac{1}{2}(\text{tr } A + \text{tr } D) \pm \frac{1}{2} \text{sign}(\text{tr } A - \text{tr } D)\sqrt{\Delta}.$$

Eigenmodes

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*Eigenmodes*

$$\det \begin{pmatrix} (\text{tr } A - \Lambda_A)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_A)I \end{pmatrix} \begin{pmatrix} X \\ R_A X \end{pmatrix} = 0,$$

$$\begin{pmatrix} (\text{tr } A - \Lambda_D)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_D)I \end{pmatrix} \begin{pmatrix} R_D Y \\ Y \end{pmatrix} = 0,$$

$$R = - \left( \frac{1}{2}(\text{tr } A - \text{tr } D) + \frac{1}{2} \text{sign}(\text{tr } A - \text{tr } D) \sqrt{\Delta} \right)^{-1} (C + \bar{B}),$$

*Eigenvectors*

$$\text{with } R_A = -\bar{R}_D = R.$$

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*Eigenvectors*

$$R = - \begin{aligned} M_{\perp} &= R_M^{-1} M R_M = g^2 \bar{R}_M M R_M \\ &= g^2 \begin{pmatrix} I & -\bar{R} \\ R & I \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I & \bar{R} \\ -R & I \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}, \end{aligned}$$

*Normal form*

$$E = g^2 (A - \bar{R}C - (BR - \bar{R}DR)) = A - \bar{R}C = A - BR,$$

$$F = g^2 (D + RB + (C\bar{R} + RAR\bar{R})) = D + RB = D + C\bar{R}.$$

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$$\begin{aligned}
 & \vec{X}(s_2) = M \vec{X}(s_1), \\
 M = & \left[ \begin{array}{l} \text{eig}(M) = \text{eig}(M^{-1}) = \{(\lambda_i, \lambda_i^{-1}), i = 1..2\}, \\ \text{eig}(M + \bar{M}) = \{\Lambda_i = \lambda_i + \lambda_i^{-1}, i = 1..2\}, \\ \det \begin{pmatrix} (\text{tr } A - \Lambda_A)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_A)I \end{pmatrix} \begin{pmatrix} X \\ R_A X \end{pmatrix} = 0, \\ \det \begin{pmatrix} (\text{tr } A - \Lambda_D)I & B + \bar{C} \\ C + \bar{B} & (\text{tr } D - \Lambda_D)I \end{pmatrix} \begin{pmatrix} R_D Y \\ Y \end{pmatrix} = 0, \end{array} \right] \\
 \text{Eigenmodes} \\
 R = - & \left[ \begin{array}{l} M_{\perp} = R_M^{-1} M R_M = g^2 \bar{R}_M M R_M \\ = g^2 \begin{pmatrix} I & -\bar{R} \\ R & I \end{pmatrix} \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} I & \bar{R} \\ -R & I \end{pmatrix} = \begin{pmatrix} E & 0 \\ 0 & F \end{pmatrix}, \end{array} \right] \\
 \text{Eigenvectors} \\
 E = & \left[ \begin{array}{l} E = \begin{pmatrix} E_{1,1} & E_{1,2} \\ E_{2,1} & E_{2,2} \end{pmatrix} = \begin{pmatrix} \cos \mu_A + \alpha_A \sin \mu_A & \beta_A \sin \mu_A \\ -\gamma_A \sin \mu_A & \cos \mu_A - \alpha_A \sin \mu_A \end{pmatrix}, \\ F = \end{array} \right] \\
 \text{Normal form} \\
 \cos \mu_A = & \frac{1}{2} \text{tr } E, \quad \sin \mu_A = \text{sign}(E_{1,2}) \sqrt{-E_{1,2} E_{2,1} - \left( \frac{E_{1,1} - E_{2,2}}{2} \right)^2} \\
 \beta_A = & \frac{E_{1,2}}{\sin \mu_A}, \quad \gamma_A = -\frac{E_{2,1}}{\sin \mu_A}, \quad \alpha_A = \frac{E_{1,1} - E_{2,2}}{2 \sin \mu_A}, \\
 \text{Optical parameters} &
 \end{aligned}$$

# Linear coupling review (propagation)

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$$\begin{aligned}M_2 &= M_{12} M_1 M_{12}^{-1} = M_{12} \left( R_{M_1} M_{1\perp} R_{M_1}^{-1} \right) M_{12}^{-1}, \\M_{2\perp} &= R_{M_2}^{-1} M_2 R_{M_2} = \left( R_{M_2}^{-1} M_{12} R_{M_1} \right) M_{1\perp} \left( R_{M_1}^{-1} M_{12}^{-1} R_{M_2} \right) \\&= W_{12} M_{1\perp} W_{12}^{-1}.\end{aligned}$$

- Restore the derivation of the equations, provide intermediate validity checks and more stable formula (see the links in refs for details).

$$M_2 = M_{12} M_1 M_{12}^{-1} = M_{12} (R_{M_1} M_{1\perp} R_{M_1}^{-1}) M_{12}^{-1},$$

$$M_{2\perp} = R_{M_2}^{-1} M_2 R_{M_2} = (R_{M_2}^{-1} M_{12} R_{M_1}) M_{1\perp} (R_{M_1}^{-1} M_{12}^{-1} R_{M_2})$$

$$R_{M_2} W_{12} = M_{12} R_{M_1},$$

*Normal form*

$$g_2 \begin{pmatrix} I & \bar{R}_2 \\ -R_2 & I \end{pmatrix} \begin{pmatrix} E_{12} & 0 \\ 0 & F_{12} \end{pmatrix} = g_1 \begin{pmatrix} A_{12} & B_{12} \\ C_{12} & D_{12} \end{pmatrix} \begin{pmatrix} I & \bar{R}_1 \\ -R_1 & I \end{pmatrix},$$

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|                                                                                                                                                                           |                                                                                                                                                                                            |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| $g_2 \quad \begin{array}{l} E_{12} = g_{12}(A_{12} - B_{12}R_1) \\ F_{12} = g_{12}(D_{12} + C_{12}\bar{R}_1) \\ R_2 = -g_{12}(C_{12} - D_{12}R_1)E_{12}^{-1} \end{array}$ | $\quad \left  \begin{array}{l} E_{12} = g_{12}(B_{12} + A_{12}\bar{R}_1) \\ F_{12} = g_{12}(C_{12} - D_{12}R_1) \\ R_2 = -g_{12}(D_{12} + C_{12}\bar{R}_1)E_{12}^{-1} \end{array} \right.$ |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

*Flip mode*

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*Flip mode*

$$R_2 \quad E_2 = E_{12} E_1 \bar{E}_{12} / |E_{12}|$$

$$F_2 = F_{12} F_1 \bar{F}_{12} / |F_{12}|$$

$$E_2 = E_{12} F_1 \bar{E}_{12} / |E_{12}|$$

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*Solution*

$$R_2 = -(C_{12} - D_{12}R_1) \frac{\bar{E}_{12}}{|E_{12}|}$$

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$$R_2 \quad E_2 = E_{12} E_1 \bar{E}_{12} / |E_{12}|$$

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*F*<sub>2</sub>

$$T_2 = W_{12} T_1 W_{12}^{-1},$$

$$T^{(E,F)} = \begin{pmatrix} \beta & -\alpha \\ -\alpha & \gamma \end{pmatrix},$$

*Solution*

$$\alpha_2 = -((E_{2,1}\beta_1 - E_{2,2}\alpha_1)(E_{1,1}\beta_1 - E_{1,2}\alpha_1) + E_{1,2}E_{2,2})/(|E_{12}|\beta_1),$$

$$\beta_2 = ((E_{1,1}\beta_1 - E_{1,2}\alpha_1)^2 + E_{1,2}^2)/(|E_{12}|\beta_1),$$

$$\mu_2 = \mu_1 + \tan^{-1}(E_{1,2}, (E_{1,1}\beta_1 - E_{1,2}\alpha_1)),$$

$$\gamma_2 = (1 + \alpha_2^2)/\beta_2.$$

# Element specific extensions

- Patches added to SURVEY, TWISS and TRACK
  - s-rotation, x-rotation, y-rotation and translation elements.
  - used to change the reference frame.
  - used to misalign magnets in HL-LHC studies.
  - used to keep “flat” non-flat beam lines (gantry).

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*x-rotation tracking map*

$$x^f = x^i + \frac{y p_x^i \tan \theta}{p_z^i - p_y^i \tan \theta}$$

$$p_x^f = p_x^i$$

$$y^f = \frac{y^i}{\cos \theta - p_y^i \tan \theta / p_z^i}$$

$$p_y^f = p_y^i \cos \theta + p_z^i \sin \theta$$

$$t^f = t_i - \frac{y^i(1/\beta_0 + p_t) \tan \theta}{p_z^i - p_y^i \tan \theta}$$

$$p_z = \sqrt{1 + 2p_t/\beta_0 + p_t^2 - p_x^2 - p_y^2}$$

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- SELECT updated and used by TWISS and MATCH
  - extended with INTERPOLATE to specify points of interpolation (output) within elements.
  - matching constraints can refer to arbitrary positions.
  - still under testing (will be in end-of-the-year release)

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```
SELECT, FLAG=INTERPOLATE, RANGE=mq1, AT={0.5, 1};
MATCH, SEQUENCE=seq;
VARY, NAME=k1; # vary strength of quadrupole mq1
CONSTRAINT, RANGE=mq1, IINDEX=0, BETX=5;
LMDIF; # match betx at centre of mq1 varying k1
ENDMATCH;
```

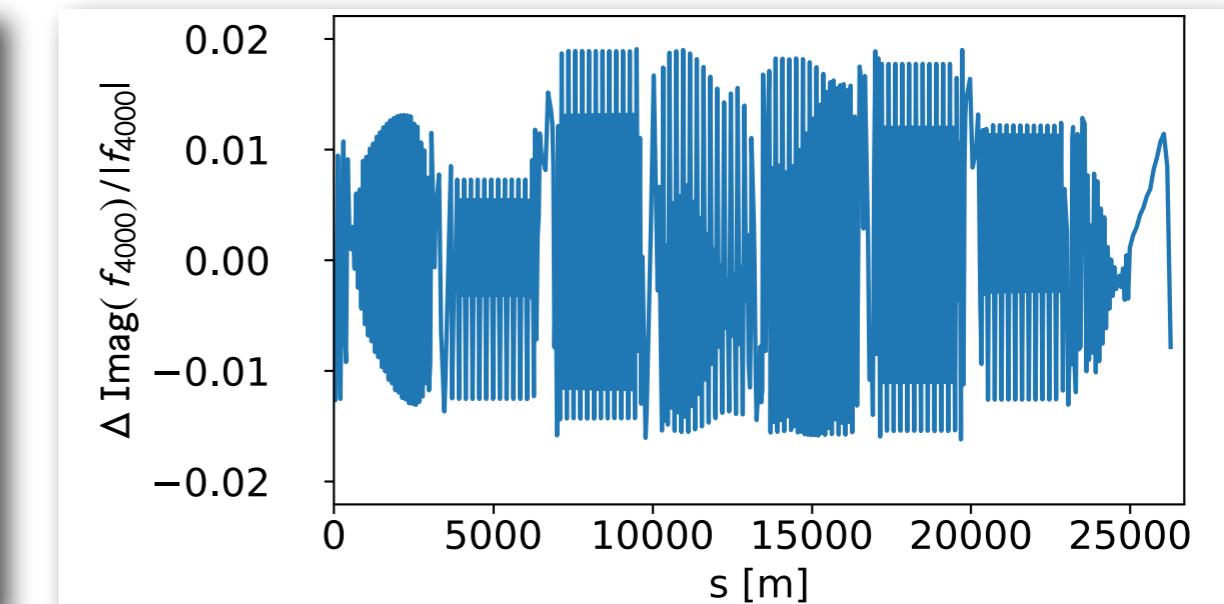
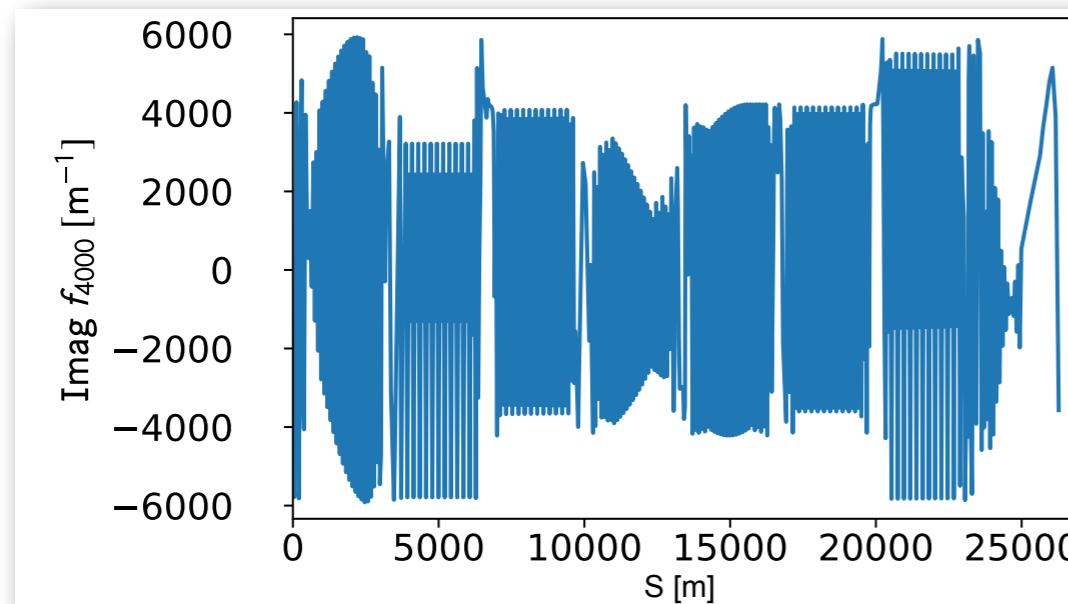


# MAD-X PTC extensions



- Interface between MAD-X and PTC/FPP embedded library is constantly improved.
  - follow-up of new releases of PTC/FPP from E. Forest for update in MAD-X.
  - synchrotron radiation effects connected to MAD-X, `PTC_TWISS` outputs damping times and equilibrium emittances now.
  - optimise the sector-bend maps with the exact Hamiltonian by automatic detection of maximum multipole order required, speed-up `PTC_TWISS` on LHC by factor x3.
  - added `RECLOSS` to record lost particles in table by `PTC_TRACK`.
  - added 6D closed orbit search with `TOTALPATH`, and correctly calculates the dependence of the beam momentum on RF frequency.
  - added `NORMAL` and `TRACKRDTS` options to `PTC_TWISS` to output the three tunes, dispersions, eigenvectors, RDTs (generating functions), Hamiltonian, and one-turn map to the new tables `NONLIN` and `TWISSRDT` and hence become available for `MATCHing`.

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# Acknowledgements

**Many thanks to**

MAD-X team: A. Latina, T. Persson P. Skowronski, *I. Shreyber, G. Roy.*

MAD-X contributors: H. Burkhardt, R. De Maria, F. Schmidt, T. Gläßle,  
and many others...

PTC/FPP author: E. Forest.

CERN BE/ABP: M. Giovannozzi, G. Arduini, P. Collier.



# MAD season 4 (advertisement)





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*And the story continue,*

# MAD season 4 (advertisement)

*And the story continue,*

MAD8, MAD9, MAD-X, ... MAD-NG

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**MAD Next Generation is coming soon!**



# Extra slides



# Polymorphic Tracking Code (in MAD-X)



- Developed by E. Forest (KEK) since the 90's (+2 books)
  - continuously updated in MAD-X since 2015 (following Etienne's releases), new connection from P. Skowronsky.
  - advanced Fortran 90 Library for beam dynamics, require to develop a Fortran program for your studies.

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- Principle: track high order maps using TPSA from Berz  
TPSA = Truncated Power Series Algebra.
  - provide true 3D geometry for lattice design (i.e. patches).
  - provide true 6D physics for beam dynamics (small machines),  
`icase = 4, 5, 56, 6` (resp. 4D, 5D dp, 6D wo-cav, 6D w-cav),  
`model = 1, 2, 3` (resp. D-K-D, M-K-M, D-M-K-M),  
`method = 2, 4, 6` (integrator order, Forest & Ruth, Yoshida scheme).
  - provide **high order normal forms** analysis (`ptc_normal`, `ptc_twiss`).
  - slow because of its complexity and underlying toolbox (FPP, TPSA).

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- Integrated into MAD-X by F. Schmidt, E. Forest et al. in 2002  
Directly available from MAD-X scripting language.  
Weak connection with only a subset of PTC...