

# Revisiting the Longitudinal 90° Limit for Superconducting Linear Accelerators

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# Overview

## □ Introduction

- The longitudinal „90° structure resonance stopband“
- The sum „envelope instability“
- Irregular periodic lattices
- Discussion
- Conclusions

Acknowledgments: O. Boine-Frankenheim, J. Struckmeier, Y. Yuan

# “Accepted criteria” for lattice design in high intensity accelerators

1. Keep **zero current phase advance per cell below  $90^\circ$**  for transverse and longitudinal to avoid structure resonance/parametric instability
2. Smooth (adiabatic) changes in transverse and longitudinal focusing
3. Avoid transverse-longitudinal emittance transfer via space charge resonance
4. Provide good matching between lattice transitions to avoid halo

1, 3 and 4 are resonant processes

# Overview on discussion of 90° stopband

- Was of concern as “envelope Instability” from envelope equations in 1970’s (*Lambertson et al., 1977, Reiser and Struckmeier, 1984*)
- some early experimental evidence in Berkeley coasting beam channel experiment (*M. Tiefenbach et al., 1985*)
- transverse 90° stopband confirmed first time experimentally in a Linac – GSI-UNILAC (*L. Groening et al., PRL 2009*)
- no experiment on longitudinal 90° mode!
- taken for granted and 90° applied to linac design transversely **and** longitudinally
- we found in 2017 (PRL) that longitudinally 90° limitation in some cases unnecessary and re-visiting is appropriate!

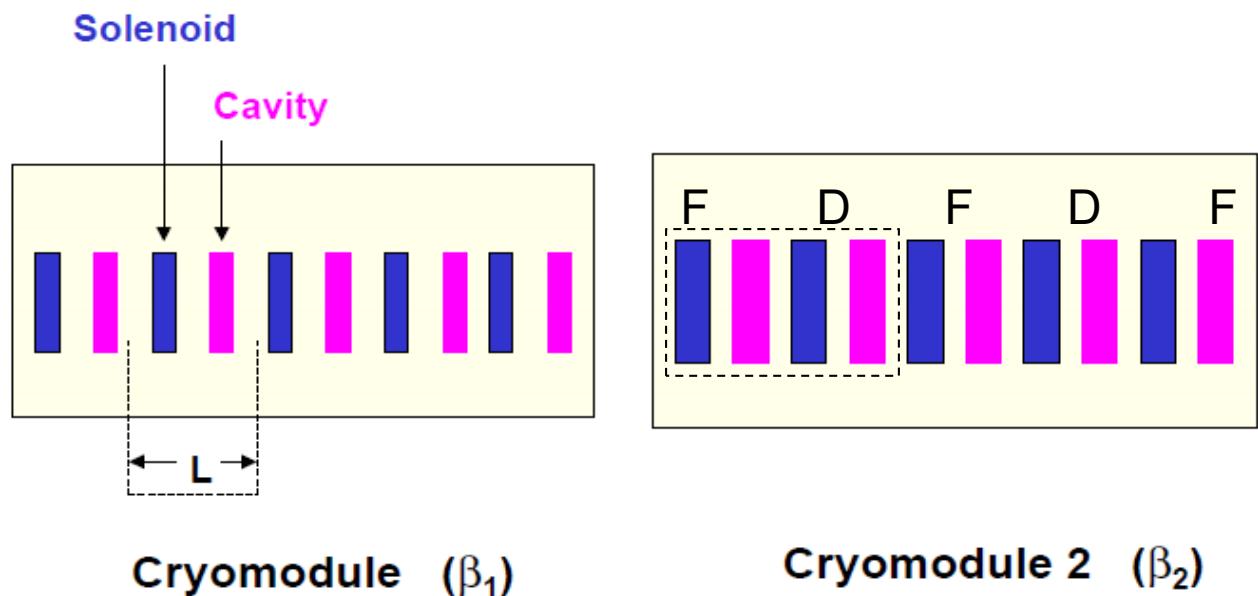
# Tom Wangler- discussion of 2002

## - ADS proposal -

Wangler:

- longitudinal 90° stop-band can limit the accelerating gradient at low velocities
- → shorten focusing period by SR- rather than FRDR-
- → higher accelerating gradients possible

Example of two cryomodules: Cryomodules are short FODO lattices with different focusing periods. Each period consists of one cavity and one solenoid.



source:

Longitudinal Beam-Dynamics  
Constraint on Accelerating Gradient

T.P.Wangler, Los Alamos National Laboratory  
and K.R.Crandall, TechSource

Workshop on Advanced Design of Spoke Resonators  
Los Alamos, NM

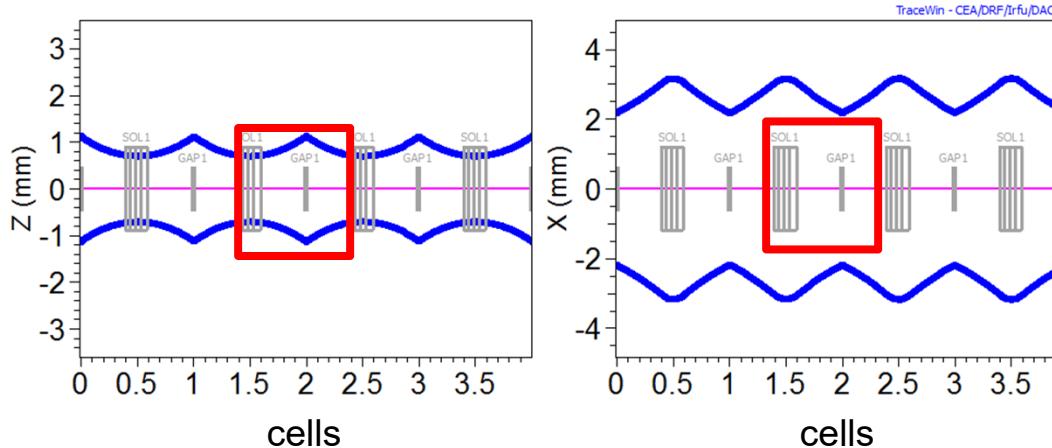
October 7-8, 2002

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# 1<sup>st</sup> lattice example: idealized solenoid + RF gap “toy lattice” - for simplicity

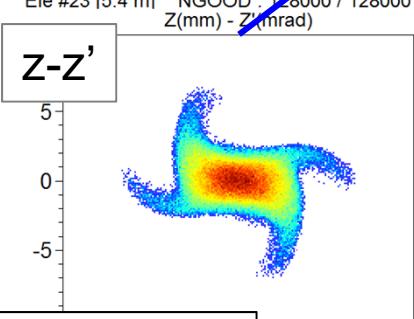
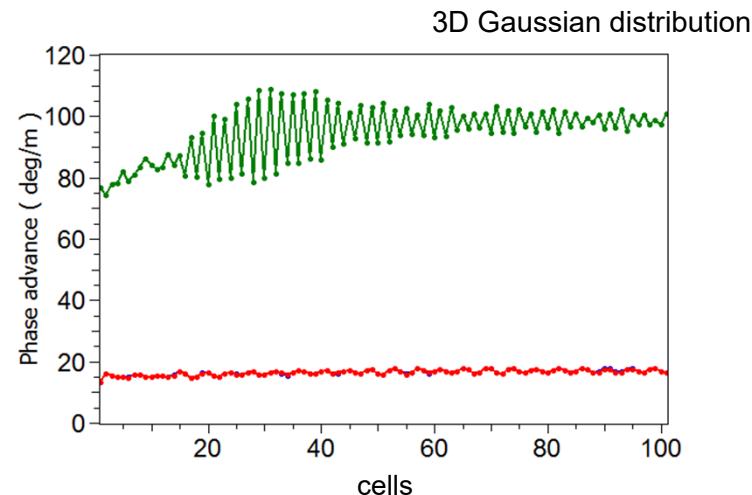
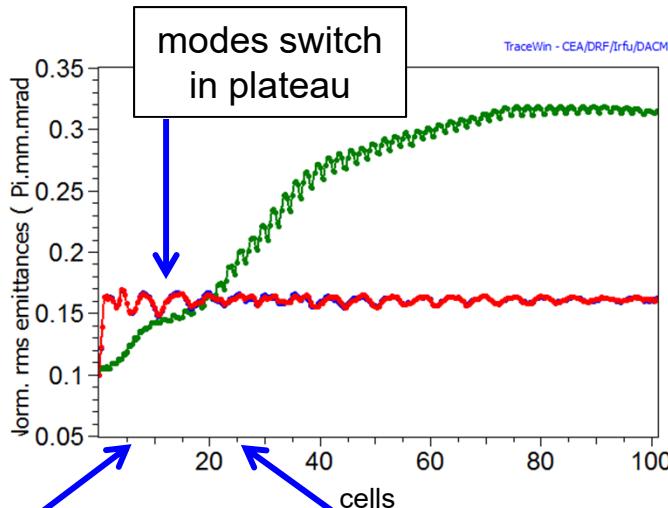
TRACEWIN -simulations



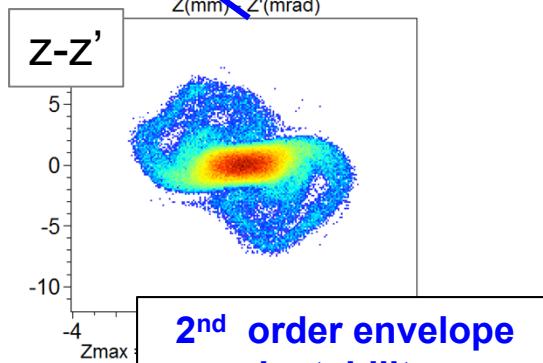
- periodic cell: solenoid + RF gap (no acceleration)
- 3D Gaussian bunches
- strong 90° effect - “as expected”
  - transversely similar to FODO
  - longitudinally ?

# Evidence of longitudinal 4<sup>th</sup> order structure resonance + envelope instability

$k_{0z}=120^0$   $k_z=76^0$



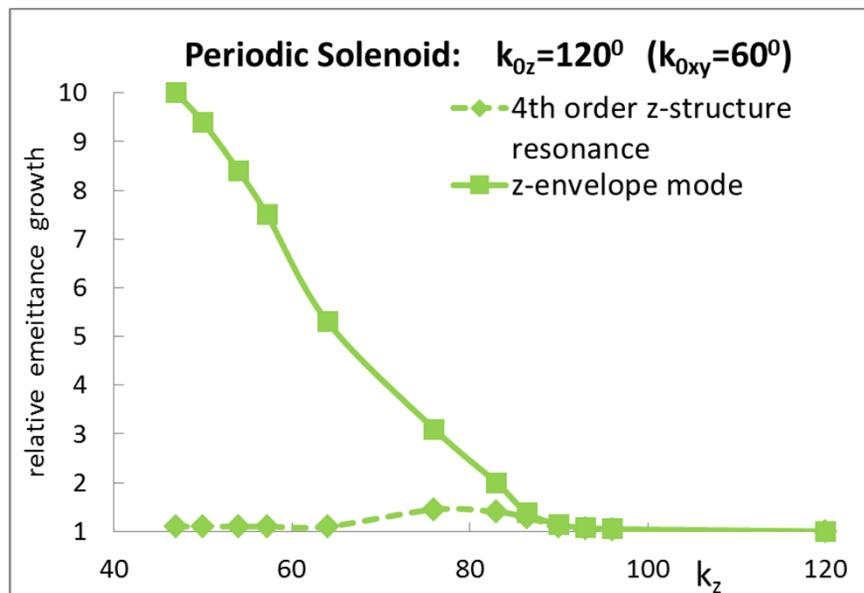
4<sup>th</sup> order structure resonance  
 $4k_z \sim 360^0$   
→ compare with transverse UNILAC experiment



2<sup>nd</sup> order envelope instability  
 $2k_z \sim 180^0$

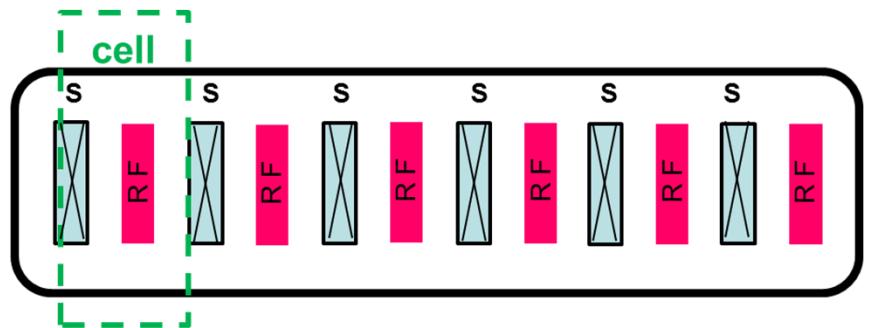
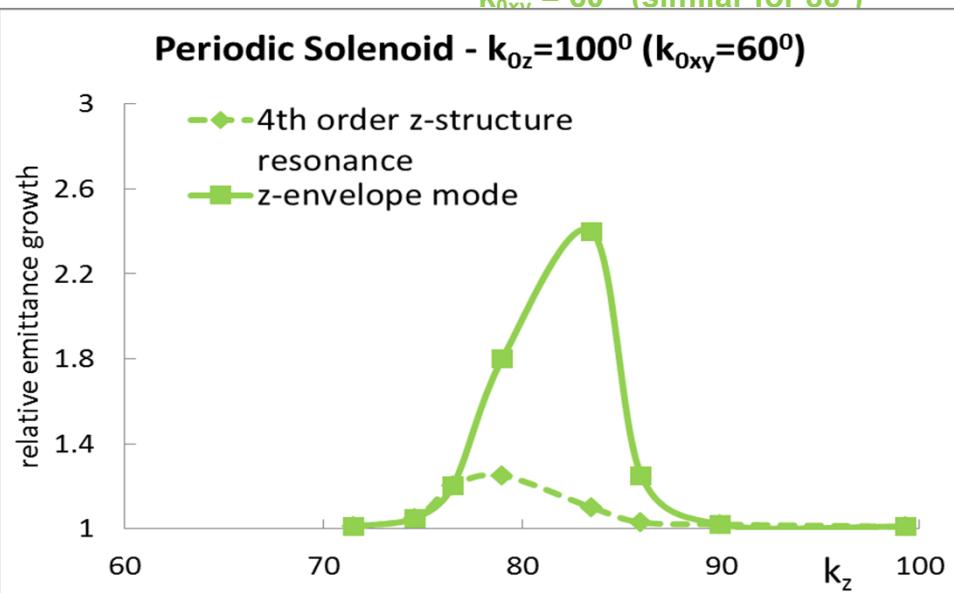
longitudinal envelope instability will occur earlier if larger mismatch

# Periodic solenoid lattice 90° longitudinal stopband confirming serious effect beyond certain intensity

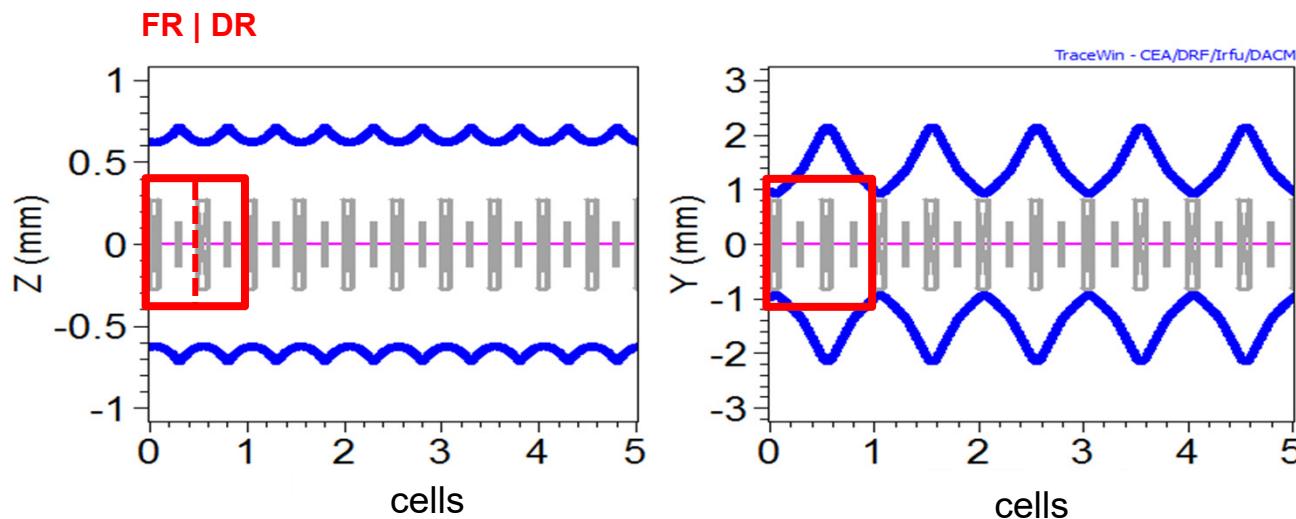


initial 4<sup>th</sup> order  
followed by env-  
instability

no emittance  
growth  
is it useable???  
probably not  
as SR cells short!



## 2<sup>nd</sup> lattice example: FODO + RF gap

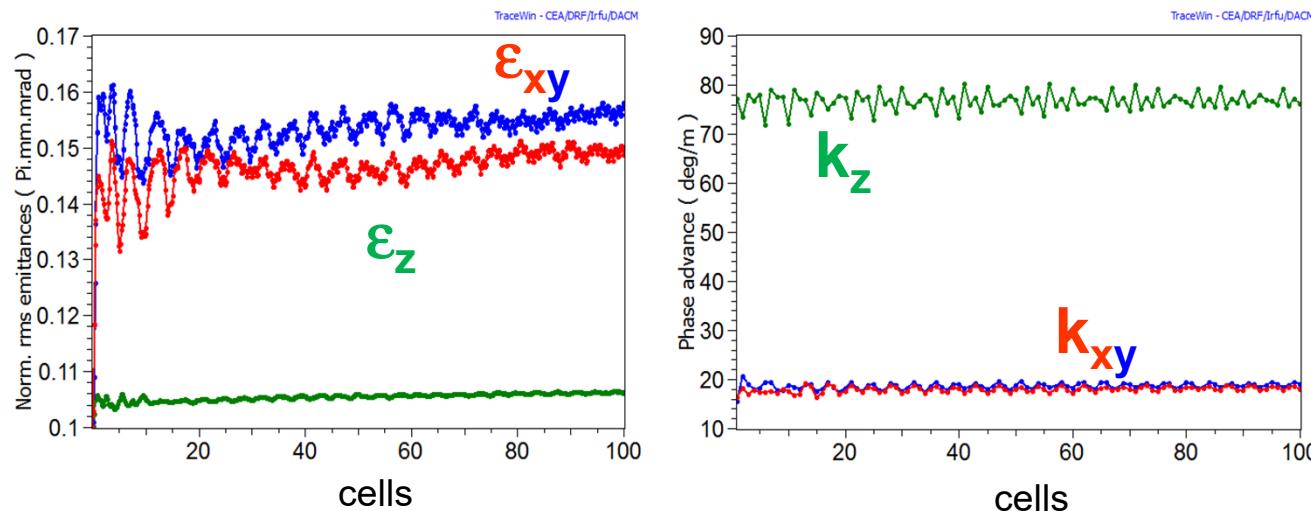


space charge forces:

- transverse force in FR half-cell different from DR half-cell
- in an exact sense longitudinal period same as transverse due to space charge coupling
- coupling to longitudinal in practice very small

# → Simulation for $k_{0z}=120^\circ$ $k_z=76^\circ$ ( $k_{0x}=60^\circ$ )

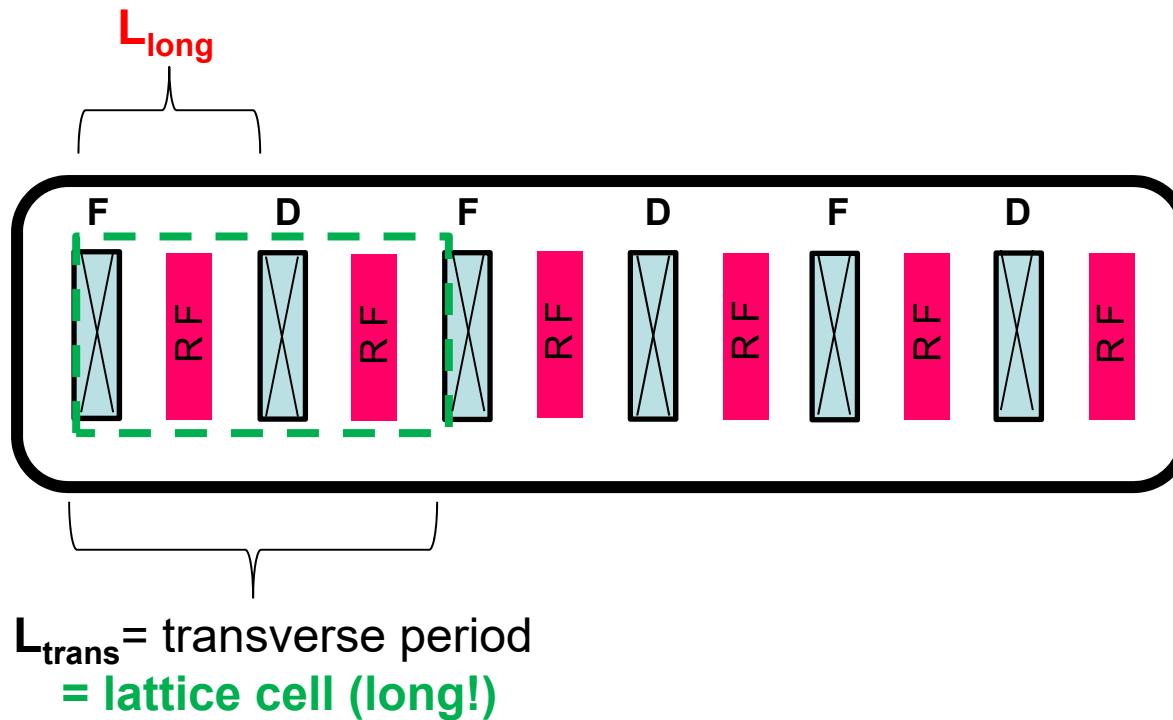
3D Gaussian distribution



**found no resonant effect (only initial nonlinear field energy jump)**

- contrary to same  $k_{0z}$ ,  $k_z$  in periodic solenoid case
- apparently longitudinal space charge force in FR half-cell can be assumed nearly identical to that in DR half-cell → “identical” cells

→ In FODO + RF longitudinal 90° stopband  
absent as long as  $k_{0z} < 180^\circ$



- ✓ allows choice of  $k_{0z}$  above 90°
- ✓ more design flexibility
- ✓ unless other sources of emittance degradation

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# “Sum envelope instability” – a new mode of some concern

O. Boine-Frankenheim, I. Hofmann and J. Struckmeier, POP 2016  
I. Hofmann and O. Boine-Frankenheim , PRL 2017  
Y. Yan et al, PRAB 2018

For **split tunes**

$$k_{0xy} < 90^\circ \text{ and } k_{0z} > 90^\circ$$

(both defined on transverse focusing period)

a “sum envelope instability” was found to exist provided that

$$k_{0xy} + k_{0z} > 180^\circ$$

**smooth approximation criterion for center of stopband of sum envelope instability:**

$$k_{0xy} + k_{0z} = 180^\circ + \Delta k_{coh,sum}$$

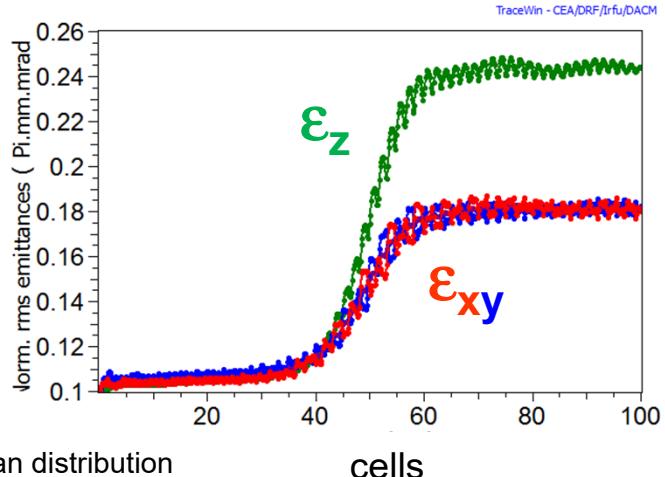
(single envelope instability  $k_{0xyz} = 90^\circ + \Delta k_{coh}$ )

- not to be confused with “sum resonance”  $k_{0x} + k_{0y} = 360^\circ$  by skew quads, which is a single particle resonance!

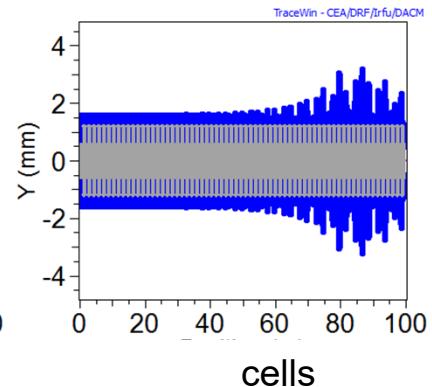
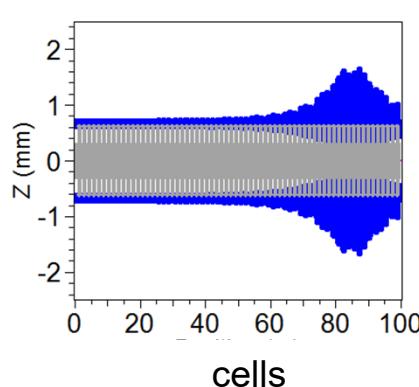
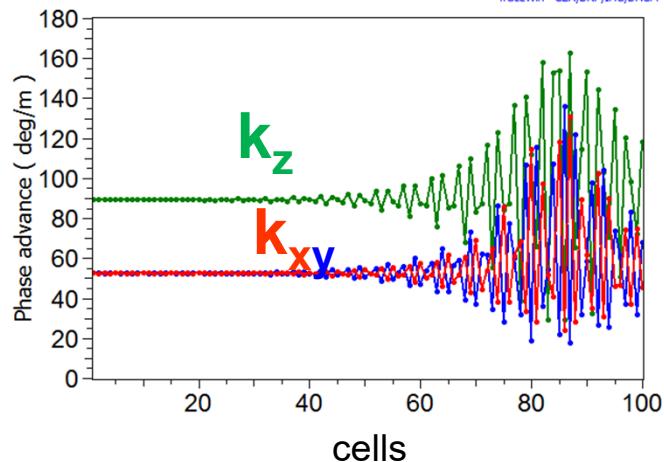
# Sum envelope instability criterion:

$$k_{0z} + k_{0xy} = 180^\circ + \Delta k_{\text{coherent}}$$

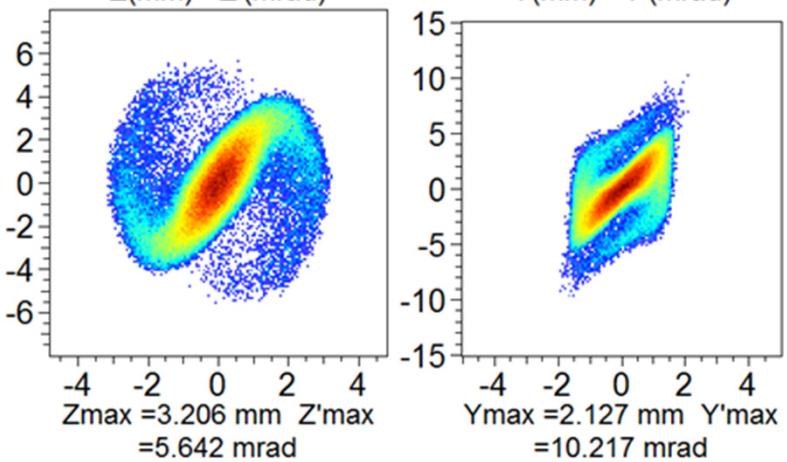
$$k_{0z}=120^\circ \quad (k_z=92^\circ) \quad \text{and} \quad k_{0xy}=90^\circ$$



3D Gaussian distribution

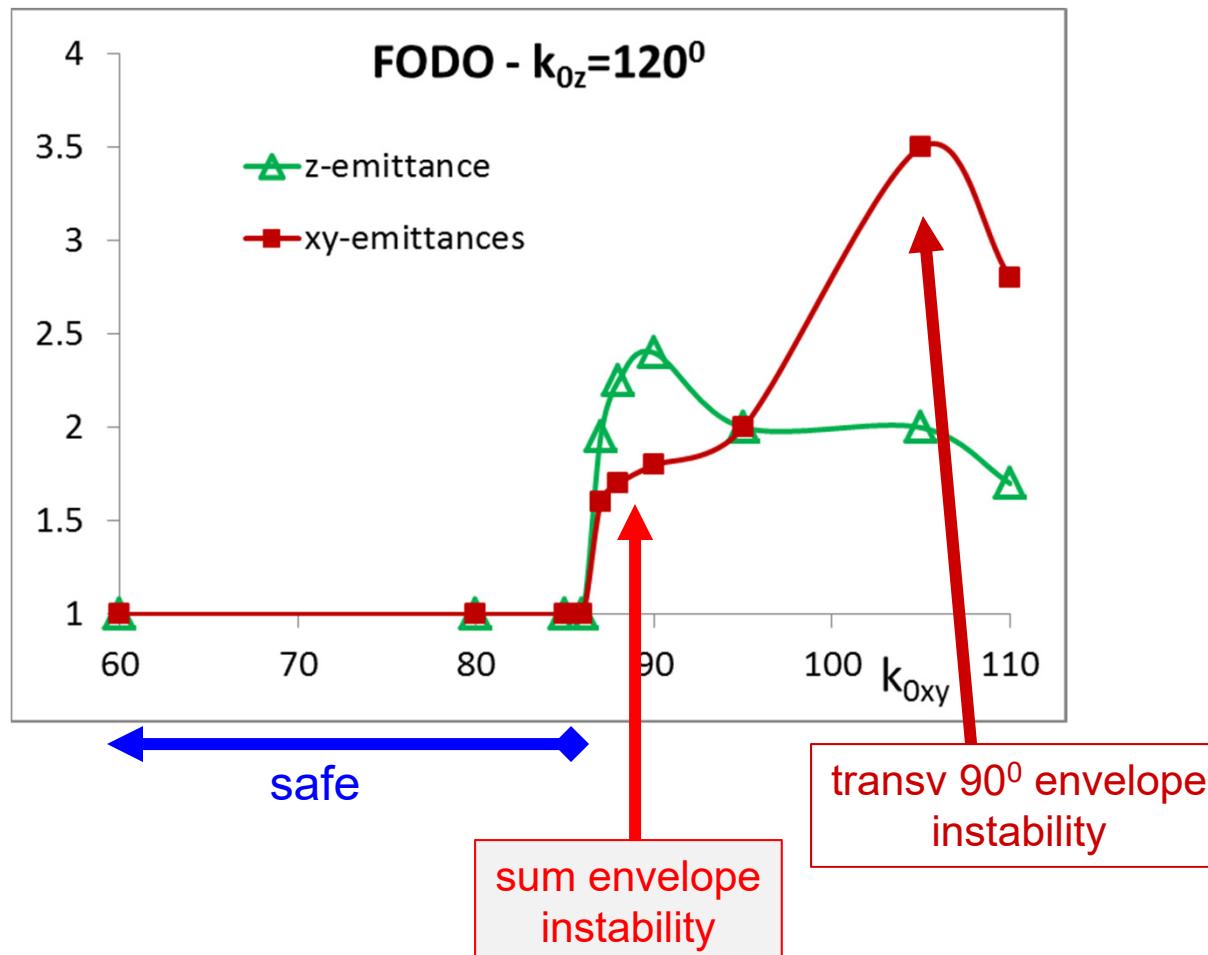


Ele #402 [50.1 m] NGOOD : 128000 / 128000  
Z(mm) - Z'(mrad) Y(mm) - Y'(mrad)



# Combined stopbands in xy and z

$k_{0z} + k_{0xy} = 180^\circ + \Delta k_{coh}$  for stopband center  
here:  $\Delta k_{coh} \sim \Delta k_{incoh,z} \sim 30^\circ$



# Summary

## for avoiding the 90° and sum mode

Periodic solenoid channel SR-SR-...

- $k_{0xyz} < 90^\circ$  safe
- no sum mode ( $k_{0z} + k_{0xy} < 180^\circ$ )

- Periodic quadrupole channel FRDR-FRDR-...
- $k_{0xy} < 90^\circ$  safe
- $k_{0z} > 90^\circ$  also ok, provided that:
- sum mode condition  $k_{0xy} + k_{0z} = 180^\circ + \Delta k_{coh}$  is avoided

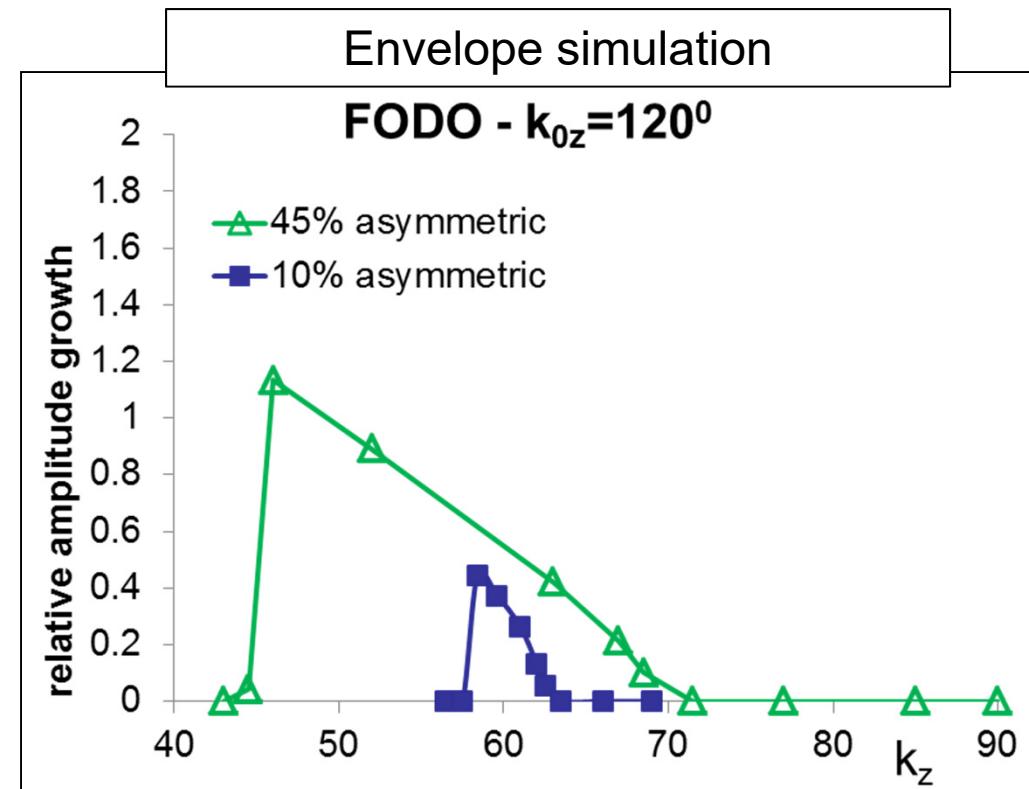
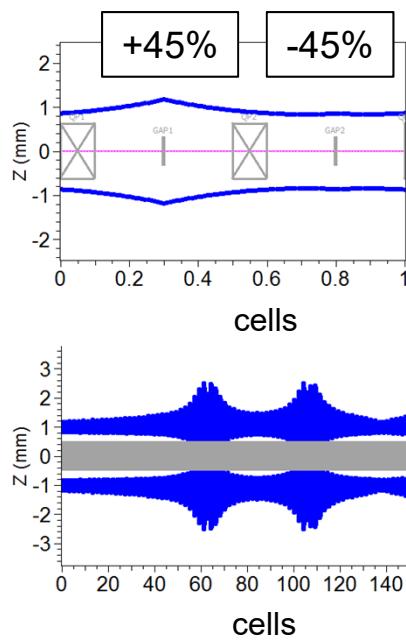
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# Linacs often **not** strictly periodic

## test case: breaking RF gap strength symmetry

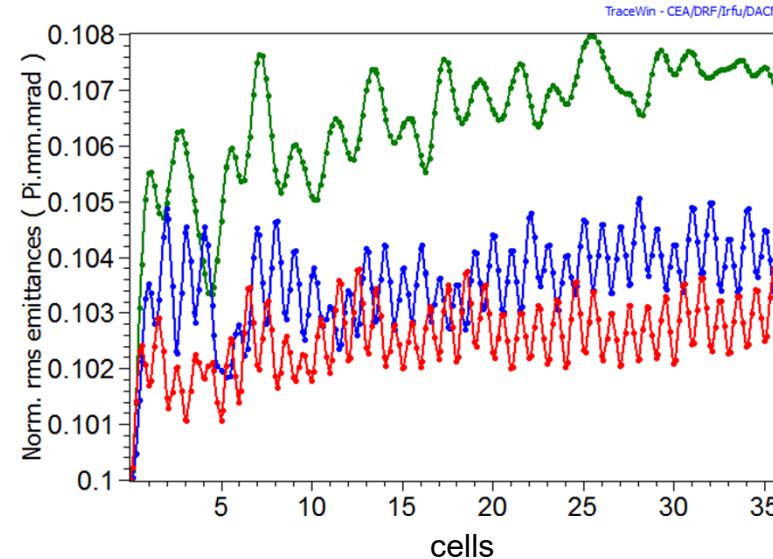
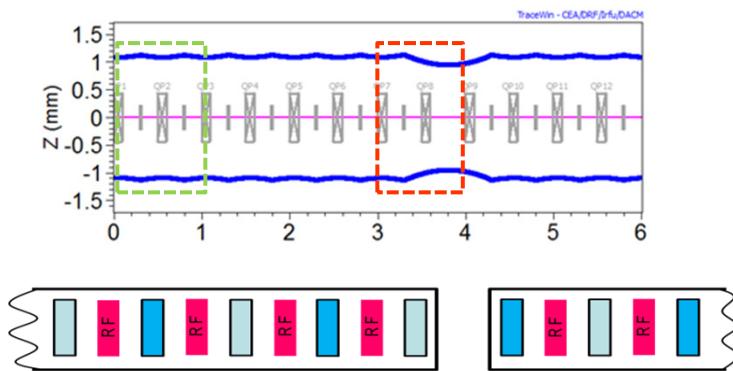
asymmetric gap voltage: 2→1 cell



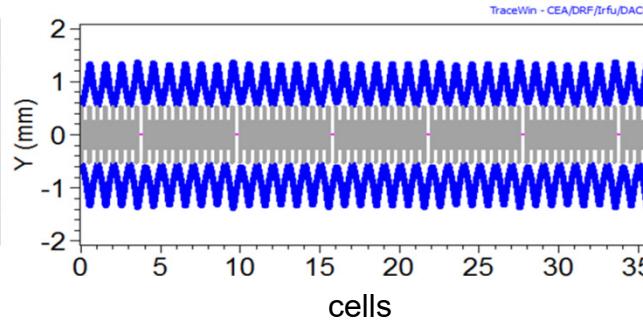
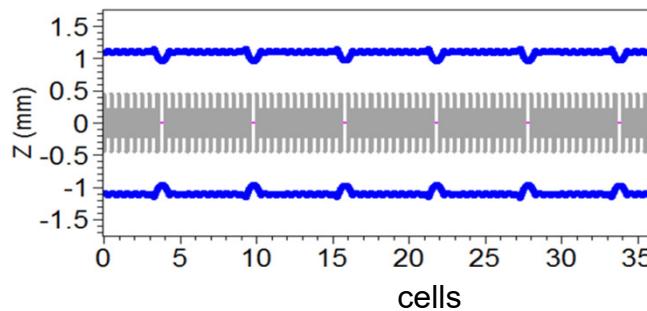
width of stopband  $\sim$  asymmetry  
→ small asymmetry ignorable

# Periodic interruptions – from tank to tank

“toy” example: missing every 6<sup>th</sup> gap (between tanks)



The missing 6<sup>th</sup> gap generates a space charge harmonic with a period over 2 cells equivalent to a phase advance  $2k_{0z} = 120^\circ$  - but no evidence for envelope instability

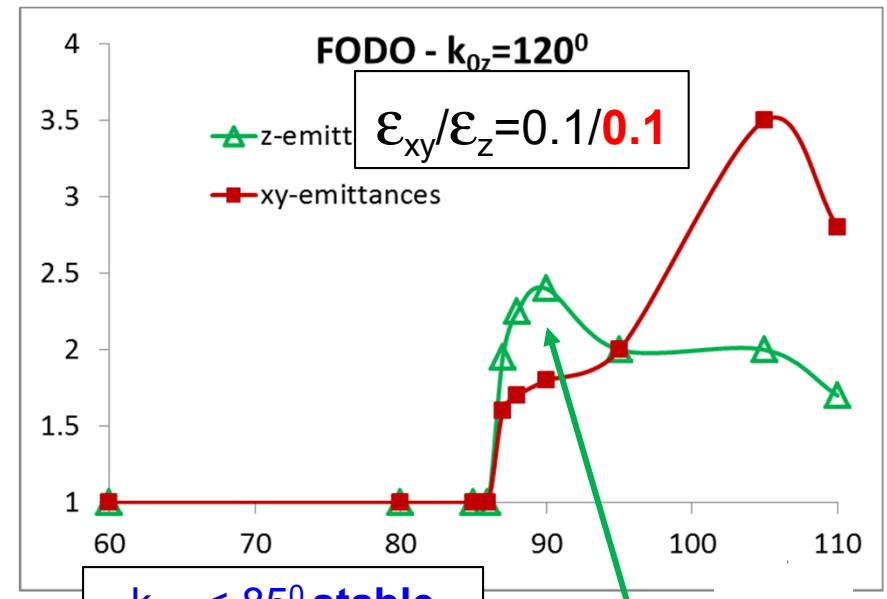
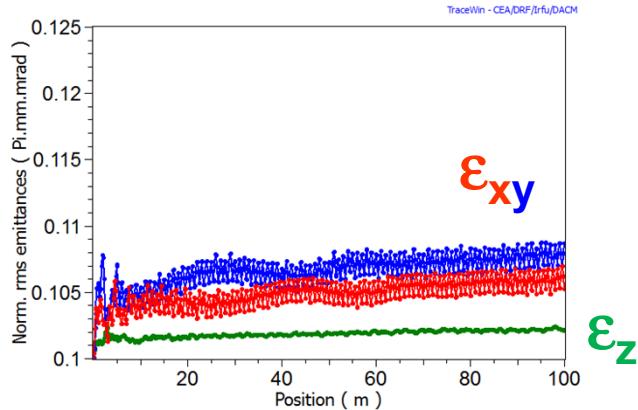
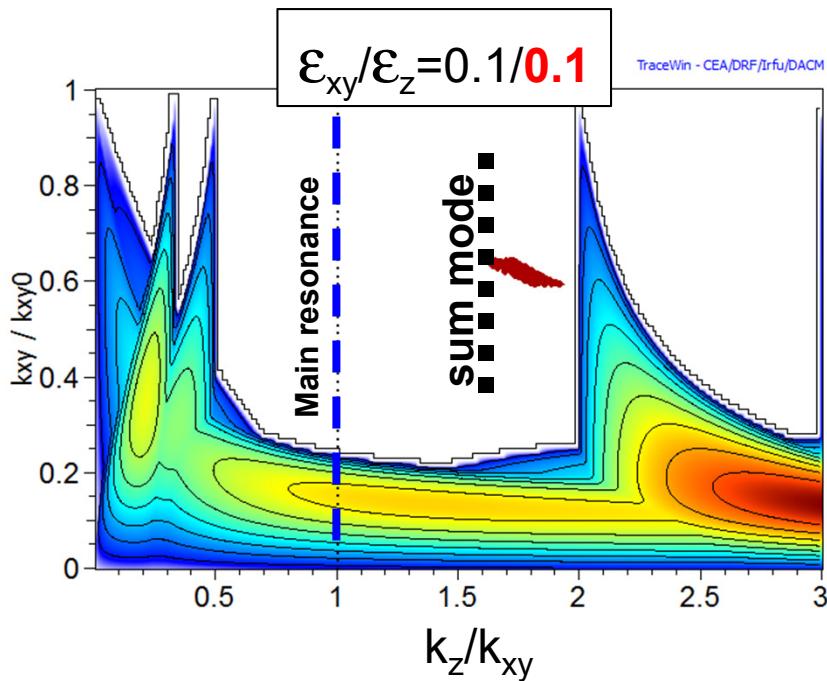


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- ❑ **Discussion**
- ❑ Conclusions

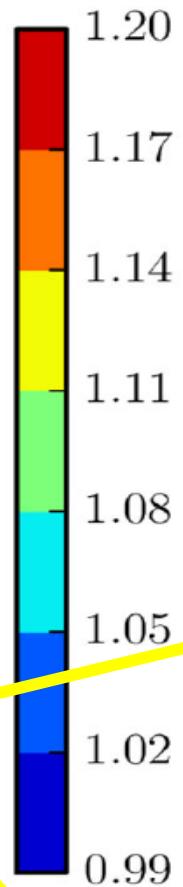
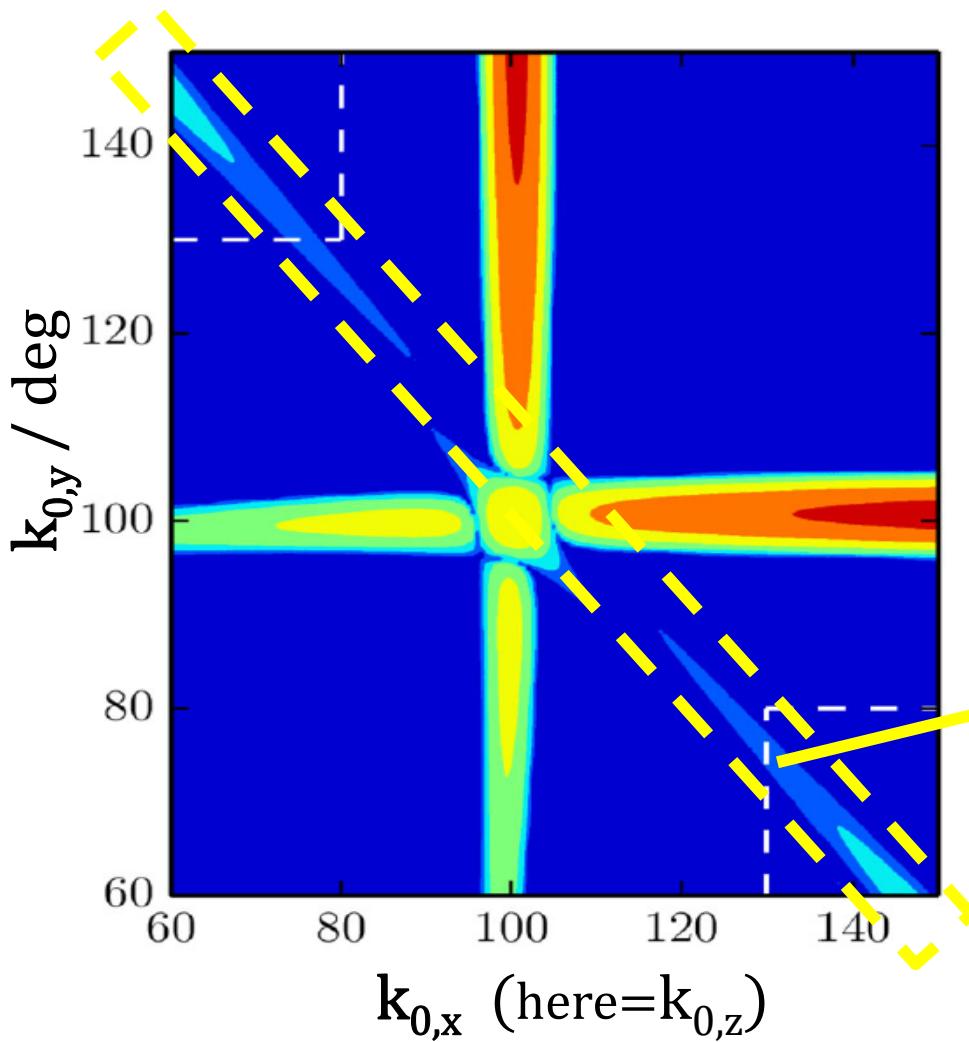
# Interference with “stability chart”

enable operation far to the right of “main resonance”



$k_{oz} = 120^0 \quad k_z = 100^0$   
 $k_{oxy} = 85^0 \quad k_{xy} = 54^0$

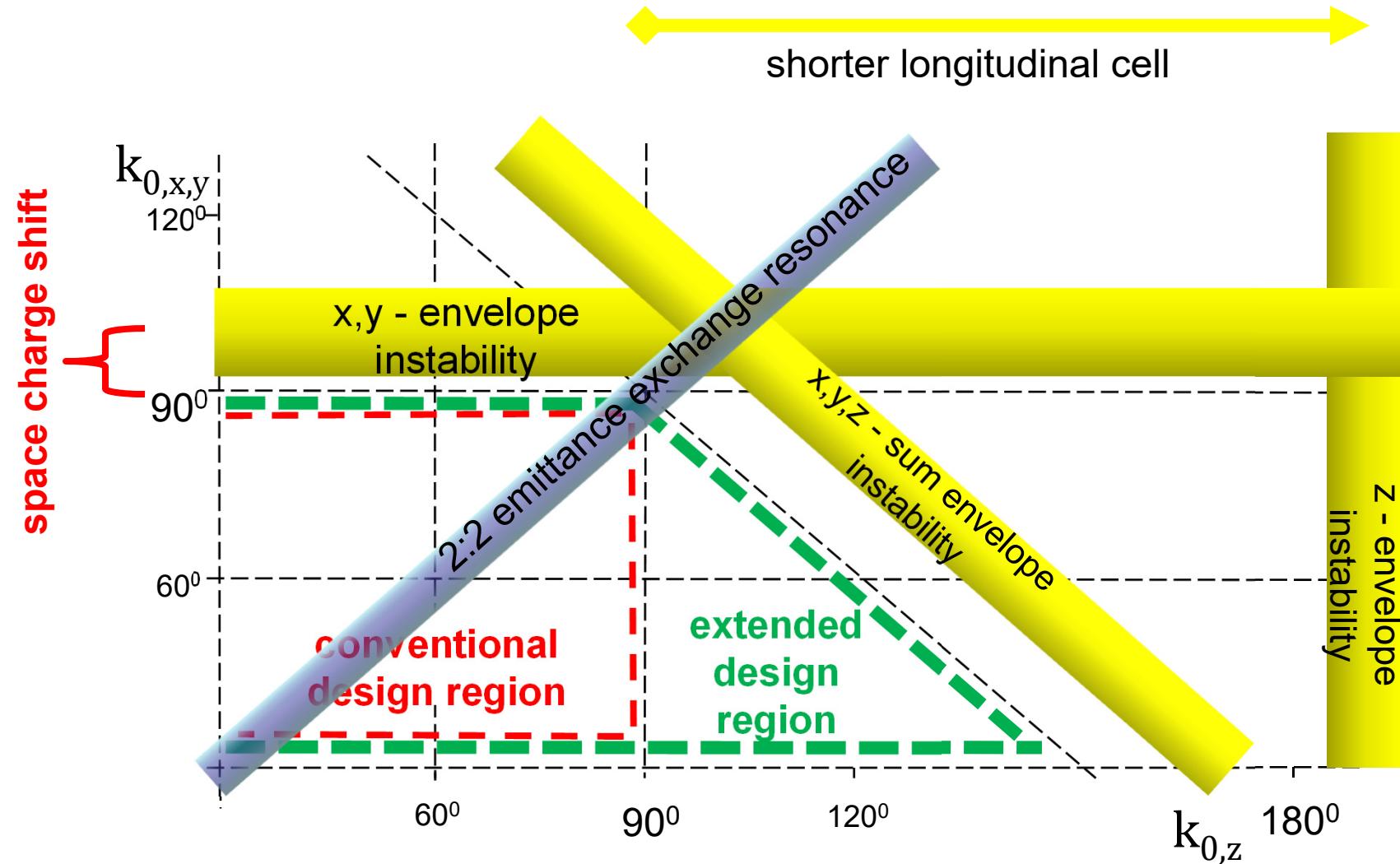
# Scan of envelope instabilities in x - y (similar in xy – z)



linearized envelope equation  
**growth rates** in transverse  
FODO channel with  
**symmetric** focusing in x-y

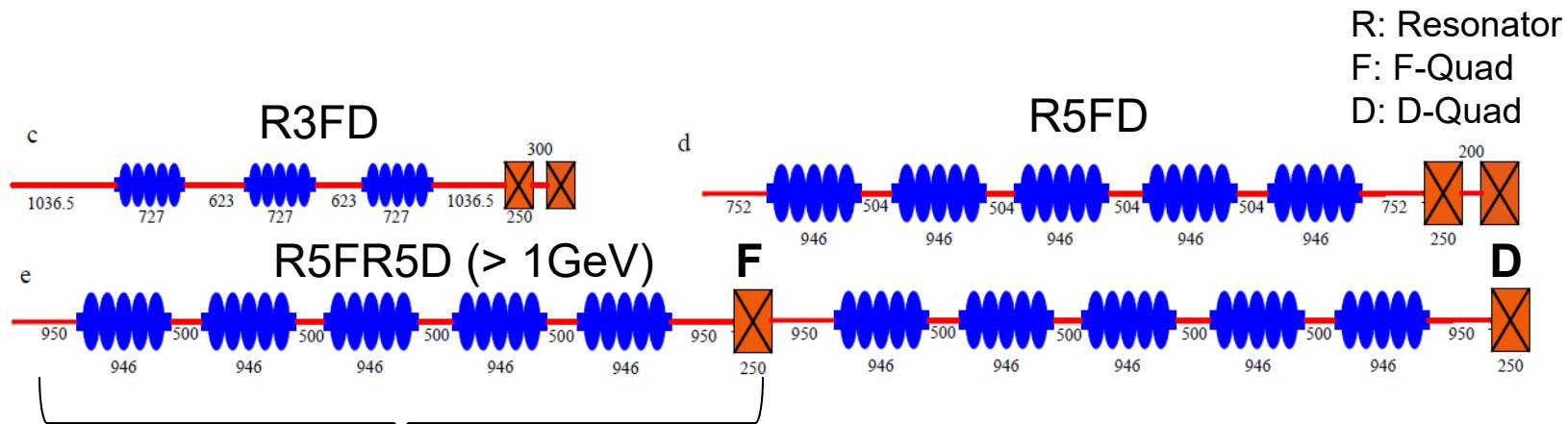
source: Yao-Shuo Yan et al., PRAB 2107

# 3D chart of structure resonances



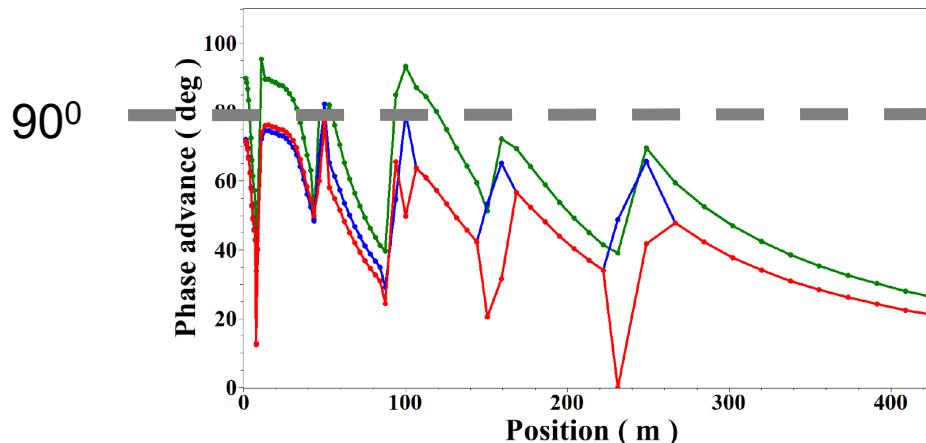
# Application example

Discussion of CW s.c. linac C-Neutrino Driver (J-Y. Tang)



1 longitudinal period, where “effective”  $k_{0z} < 45^\circ$   
- could be chosen larger!

## R5FR5D ( $> 1\text{GeV}$ )



# Conclusions

- Retrieved interplay of fourth order and envelope instability in longitudinal plane for Solenoid + RF – similar to transverse 90°
- Demonstrated that in FODO+RF longitudinal period effectively halved and absence of 90° stopband
- Allows longitudinal phase advance (per focusing cell) above 90°
- Watch additional constraint for  $k_{0z} > 90^0$ : “sum envelope instability”
- No need to replace FODO by shorter solenoid cells
- Added design flexibility in high gradient superconducting linacs
- Expect that full linac studies including high acceleration gradients also shows mitigated 90° effects in transverse plane
- → conventional “under-90-degrees” (longitudinally) is an over-emphasized criterion – needs to be relaxed in number of cases!

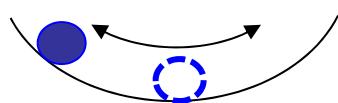


Thank you for your attention!

# Resonance – Instability

- 2 distinct sources of emittance growth -

periodic kick



## resonant excitation

- single particle resonances
- coherent resonances
- in linacs only **structure** resonances
- **here:** fourth order, driven by space charge



## instability

- parametric resonance
- **here:** envelope instabilities

Beam potential from lattice **and** self-consistent electric field

# Sum mode activated by increased $\epsilon_z$ but only “delayed” growth, if beam well-matched

$k_{oz} = 120^0$

