Electron Cooler Introduced Perturbations on Ion Beam

中国科学院近代物理研究所
Institute of Modern Physics, Chinese Academy of Sciences

中国科学院大学
University of Chinese Academy of Sciences

Youjin Yuan[1,2] and Peng Li[1,2]

[1]Institute of Modern Physics, CAS, 730000, Lanzhou, China

[2]School of Nuclear Science and Technology, University of CAS, 100049, Beijing

- General requirements of e-cooling
  - ✓ Parallel and similar velocity of e-beam and ion-beam
  - ✓ Lower electron beam temperature
  - ✓ Adequate cooling force (n_e , L_e)
  - ✓ Neglectable or compensable influence on ion beam
- List of perturbations and their compensations (to 1st order)
  - ✓ Main solenoid ➔ coupling and focusing 🡐 solenoid pair
  - ✓ Toroids ➔ transversal bending 🡐 correction dipoles
  - ✓ **Space charge and current of e-beam ➔ focusing 🡐 ring Quadrupoles**

Paper focus

- The perturbation of e-beam may help to understand the following problems:
  - ✓ Why high e-beam current not be used to cool low energy beam?
    - Large tune shift and tune shift change during acceleration
  - ✓ Low ac/deceleration efficiency.
    - Change of tune shift and matching condition
  - ✓ Effects induced by the pulsed electron beam cooling.
- Electron cooler perturbations (solenoid and e-beam) in Hamiltonian representations:

$$H = \underbrace{\frac{1}{2}\left(1+\frac{P_\tau}{\beta_0}\right)\left[\left(p_x+\frac{1}{2}k_s y\right)^2+\left(p_y-\frac{1}{2}k_s x\right)^2\right]}_{\text{Solenoid}} + \underbrace{k_0 x}_{\text{Dipole}} + \underbrace{\frac{k_1(x^2+y^2)}{2}}_{\text{e-beam}} \cdots$$

The perturbation of e-beam field

- Electron-beam space charge ➔ e-field focusing, acc. and deacc. at ends

  $\vec{E}(\vec{r}) = -\frac{n\,e}{2\varepsilon_0}\vec{r}$, $K_E = \frac{n\,e}{2\varepsilon_0\,\beta c\,B\rho}$ (main)

  (cooling is also from the E-field, with decreasing emittance)
- Electron-beam current ➔ magnet field defocusing

  $\vec{B}(\vec{r}) = \frac{\mu_0}{2\pi r^2}\vec{I}\times\vec{r} = \frac{\mu_0 n\,e\,c}{2\,r}\vec{\beta}\times\vec{r}$,

  $K_B = -\frac{\mu_0 n\,e\,\beta c}{2\,B\rho} = -\beta^2 K_E$
- Total electromagnetic field:

  $k = K_E + K_B = (1-\beta^2)\frac{n\,e}{2\varepsilon_0\,\beta c\,B\rho}$ for ion beam
- End and bending parts of e-beam field: higher order and short length, *neglected*

Transport Matrix of Independent Components

Solenoid:

$$R_{so} = \begin{pmatrix} \cos^2(k_sL) & \sin(2k_sL)/2k_s & \sin(2k_sL)/2 & \sin^2(k_sL)/k_s & 0 & 0 \\ -k_s\sin(2k_sL)/2 & \cos^2(k_sL) & -k_s\sin^2(k_sL) & \sin(2k_sL)/2 & 0 & 0 \\ -\sin(2k_sL)/2 & -\sin^2(k_sL)/k_s & \cos^2(k_sL) & \sin(2k_sL)/2k_s & 0 & 0 \\ k_s\sin^2(k_sL) & -\sin(2k_sL)/2 & -k_s\sin(2k_sL)/2 & \cos^2(k_sL) & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L/\gamma^2 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

L is the length of solenoid or e-beam $\quad k_s = \frac{Bs}{2B\rho} \quad k = \frac{n\,e(1-\beta^2)}{2\varepsilon_0\,\beta c\,B\rho}$

Electron beam field:

$$R_{ef} = \begin{bmatrix} \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}L) & 0 & 0 \\ -\sqrt{k}\sin(\sqrt{k}L) & \cos(\sqrt{k}L) & 0 & 0 \\ 0 & 0 & \cos(\sqrt{k}L) & \frac{1}{\sqrt{k}}\sin(\sqrt{k}L) \\ 0 & 0 & -\sqrt{k}\cdot\sin(\sqrt{k}L) & \cos(\sqrt{k}L) \end{bmatrix}$$

Drift:

$$R_f = \begin{bmatrix} 1 & L & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transport Matrix of e-cooler

- The combined transport matrix in the electron cooler should be analyzed.
- Lie algebraic method was used to find the polynomial expression of each matrix elements. Here, $f = -L\cdot H = -L\cdot\left[\frac{(x'+ksy)^2+(y'+ksx)^2}{2} + k\frac{x^2+y^2}{2}\right]$.

Lie operator

$$[f,g] = \sum_{k=1}^{3}\left(\frac{\partial f}{\partial q_k}\frac{\partial g}{\partial p_k} - \frac{\partial f}{\partial p_k}\frac{\partial g}{\partial q_k}\right) \qquad :f:g = [f,g]$$

and

Lie transformation

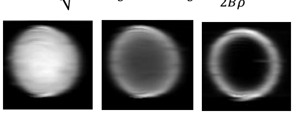
$$e^{:f:}g = \sum_{k=0}^{\infty}\frac{:f:^k}{k!}g.$$

- Then we found the transport matrix of **solenoid + e-beam field** as:

$$R = \begin{bmatrix} \cos(KL)\cos(k_sL) & \sin(KL)\cos(k_sL)/K & \cos(KL)\sin(k_sL) & \sin(Kz)\sin(k_sz)/K \\ -K\sin(KL)\cos(k_sL) & \cos(KL)\cos(k_sL) & -K\sin(KL)\sin(k_sL) & \cos(KL)\sin(k_sL) \\ -\cos(KL)\sin(k_sL) & \sin(Kz)\sin(k_sz)/K & \cos(KL)\cos(k_sL) & \sin(KL)\cos(k_sL)/K \\ K\sin(KL)\sin(k_sL) & -\cos(KL)\sin(k_sL) & -K\sin(KL)\cos(k_sL) & \cos(KL)\cos(k_sL) \end{bmatrix}$$

$$= \underbrace{\begin{pmatrix} \cos(KL) & \sin(KL)/K & 0 & 0 \\ -K\sin(KL) & \cos(KL) & 0 & 0 \\ 0 & 0 & \cos(KL) & \sin(KL)/K \\ 0 & 0 & -K\sin(KL) & \cos(KL) \end{pmatrix}}_{\text{Focusing}} \underbrace{\begin{pmatrix} \cos(k_sL) & 0 & \sin(k_sL) & 0 \\ 0 & \cos(k_sL) & 0 & \sin(k_sL) \\ -\sin(k_sL) & 0 & \cos(k_sL) & 0 \\ 0 & -\sin(k_sL) & 0) & \cos(k_sL) \end{pmatrix}}_{\text{Rotation}}$$

Where: $K = \sqrt{k + {k_s}^2}$, $\quad k_s = \frac{Bs}{2B\rho}$, $\quad k = k_E + k_B = \frac{n\,e(1-\beta^2)}{2\varepsilon_0\,\beta c\,B\rho}$



Hint: a hollow e-beam is preferred to reduce the perturbation, if cooling was sufficient in this case.

The Applications of the Transport Matrix

- The perturbation of e-beam may introduce additional tune shift and change the acceptance. The tune shift and acceptance change continuously during ac/deceleration or “mismatched” pulse cooling. This help to understand the following problems:

1. Why high e-beam current not be used to cool low energy beam?
   Example, CSRm 395G, L_e=2.56m/ r_e=3cm / I_e=0.33A / $\langle\beta_x\rangle$=10m
   Case 1:$^{238}U^{30+}$-1.0 MeV/u ➔k=0.03/m^2, ${k_s}^2$=0.0003/m^2,$\Delta Q_{x,y}$=0.061
   Case 2:$^{12}C^{6+}$-7.0 MeV/u ➔k=0.0064/m^2,${k_s}^2$=0.0007/m^2,$\Delta Q_{x,y}$=0.013

k	Tune X	Tune Y	Max βx (m)	Max βy (m)
0	3.630	2.624	14.99	30.16
0.01	3.679	2.695	16.95	53.89
0.02	3.709	2.736	19.53	66.81
0.03	3.735	2.773	22.15	81.35

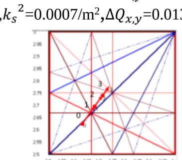


Figure 2. Tune shift induced by Solenoid and e-beam field

2. Instability and fast decay effects induced by the pulsed electron beam cooling. (L.J.Mao et al. COOL2007). This ‘naturally’ explained the experimental results.
3. Low de/acceleration efficiency. Example: Deceleration of Ni^{28+} from 400MeV/u to 4MeV/u at ESR/GSI. 1/6 efficiency to theory one. e-beam field may limit the efficiency. (Frank Herfurth, report, 2017)

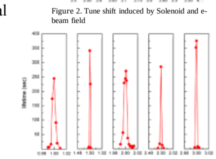


Figure 3: The dependence of lifetime on a synchronization between ion and electron pulses.

Summary

The combined transport matrix of solenoid + e-beam field was deduced by Lie Algebraic method for the 1st order study convenience.
The perturbation of e-beam field of electron cooler on ion beam is ***not*** neglectable, especially for low energy ion beam cooling, pulsed e-beam cooling and especially pulsed e-beam used before De/acceleration.

yuanyj@impcas.ac.cn