Estimation of longitudinal profiles of ion bunches in the LHC using Schottky-based diagnostics

K. Łasocha[1] and D. Alves

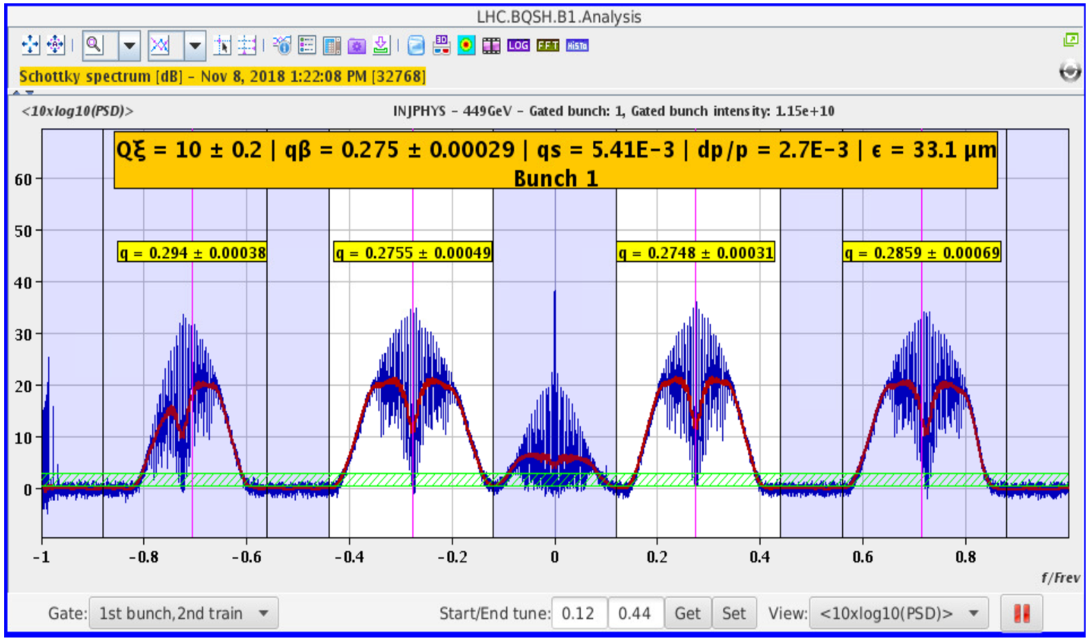
CERN, Geneva, Switzerland
[1]also at Institute of Physics, Jagiellonian University, Kraków, Poland

Paper ID: MOCO03

Schottky-based diagnostics

Traditionally used for:

- Average momentum
- Momentum spread
- Tune
- Chromaticity
- Intensity
- Emittance



Another application:

- Longitudinal bunch profiles

Motivation

What we want to achieve:

- Better understanding of the LHC Schottky spectra
  - Be able to simulate Schottky spectra
- Explore other capabilities of LHC Schottky monitors
  - Estimation of longitudinal bunch profiles

Overview

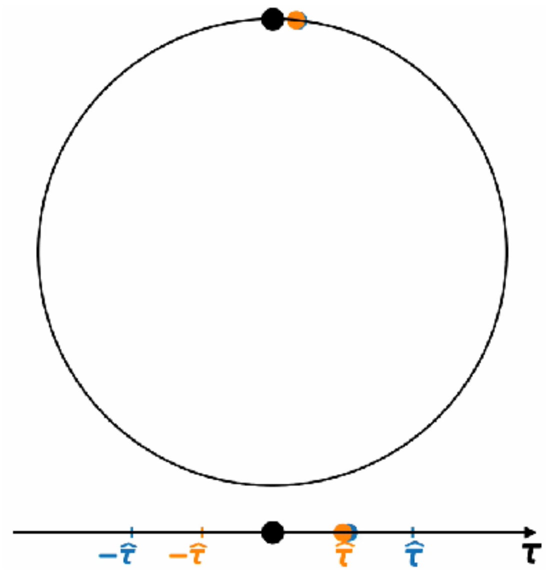
- Longitudinal bunch profile
- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- Results
- Conclusions and outlook

Overview

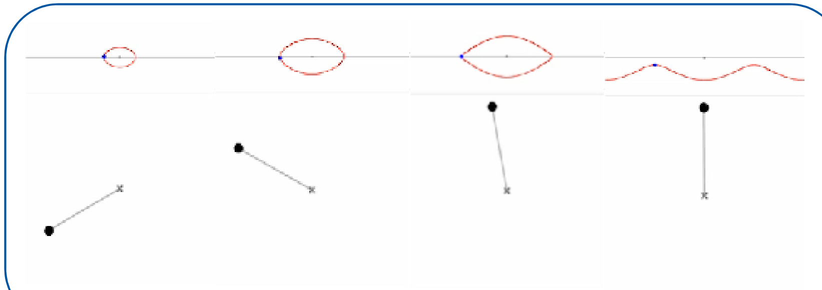
- Longitudinal bunch profile
- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- Results
- Conclusions and outlook

Synchrotron motion – brief review

$$\tau_i = \hat{\tau}_i \sin(\omega_{s_i} t + \phi_{s_i})$$ [Boussard1986]



$$\omega_{s_i} \simeq \left(1 - \frac{(h\omega_0\hat{\tau}_i)^2}{16}\right)\omega_{s_0}$$

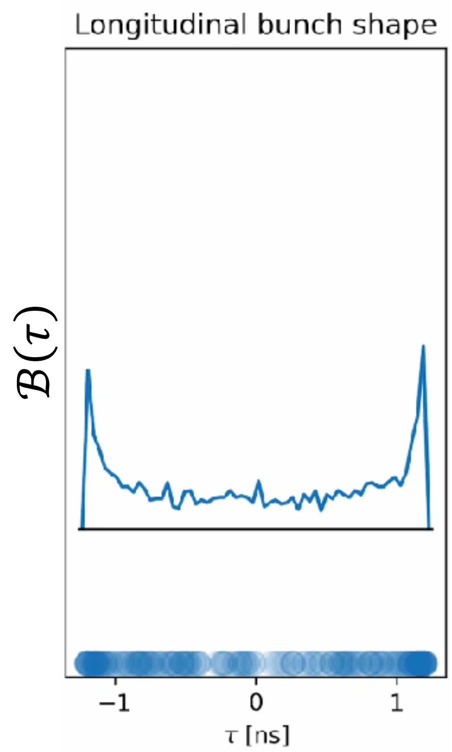


https://en.wikipedia.org/wiki/Pendulum_(mathematics)#Examples

- Each particle performs simple harmonic motion
- Synchrotron frequency, ω_{s_i}, depends on synchrotron amplitude, $\hat{\tau}_i$

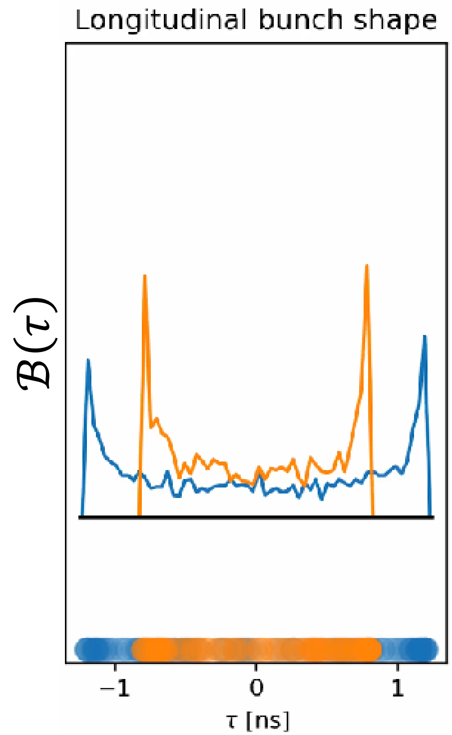
Synchrotron motion – single particle distribution

$$\underbrace{g_\tau(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}}_{\text{Single particle distribution}}$$



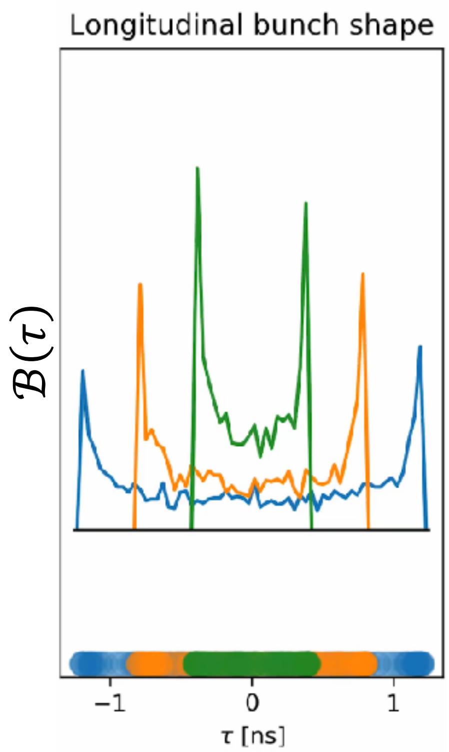
Synchrotron motion – single particle distribution

$$\underbrace{g_\tau(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}}_{\text{Single particle distribution}}$$



Synchrotron motion – single particle distribution

$$\underbrace{g_\tau(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}}_{\text{Single particle distribution}}$$



Longitudinal bunch profile

Without intra-bunch coherent motion:

$$\underbrace{\mathcal{B}(\tau) = \int_{|\tau|}^{\infty} \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}} g_{\hat{\tau}}(\hat{\tau}) d\hat{\tau}}_{\text{Longitudinal bunch profile}}$$

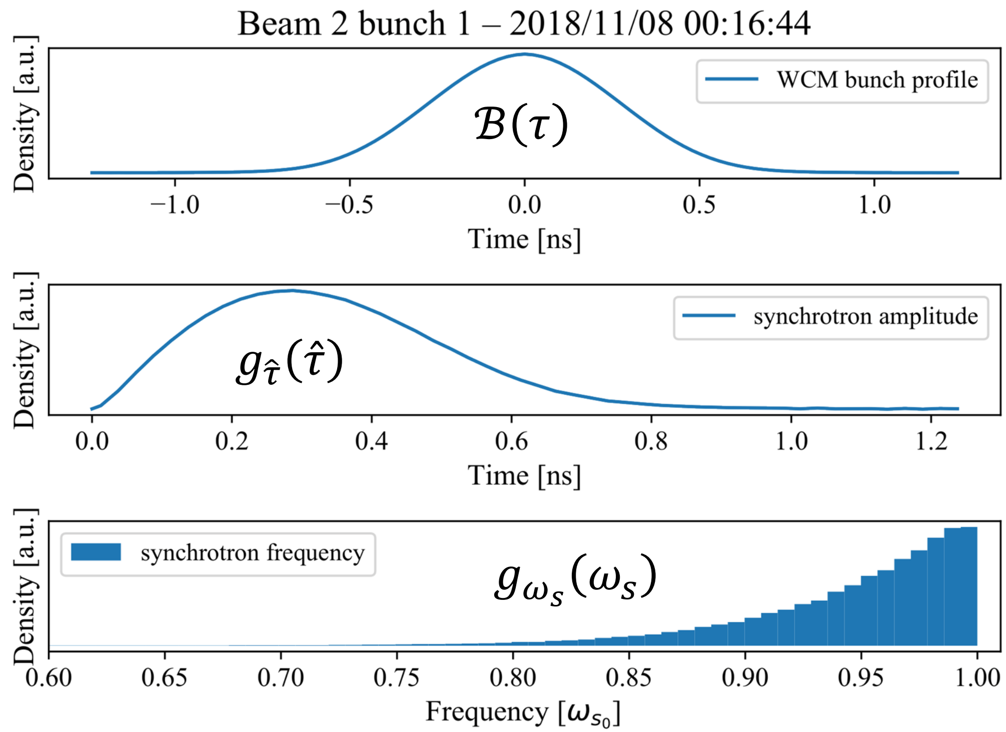
$g_{\hat{\tau}}(\hat{\tau})$ - Amplitude distribution

$$\underbrace{g_{\tau}(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}}_{\text{Single particle distribution}}$$

Equivalence of distributions

$$\boxed{\mathcal{B}(\tau)} = \int_{|\tau|}^{\infty} \frac{\boxed{g_{\hat{\tau}}(\hat{\tau})}}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}\, d\hat{\tau}$$

$$\omega_s \simeq \left(1 - \frac{(h\omega_0\hat{\tau})^2}{16}\right)\omega_{s_0}$$



Overview

- Longitudinal bunch profile
- **LHC Schottky system**
- Longitudinal Schottky spectrum in the LHC
- Results
- Conclusions and outlook

LHC Schottky systems

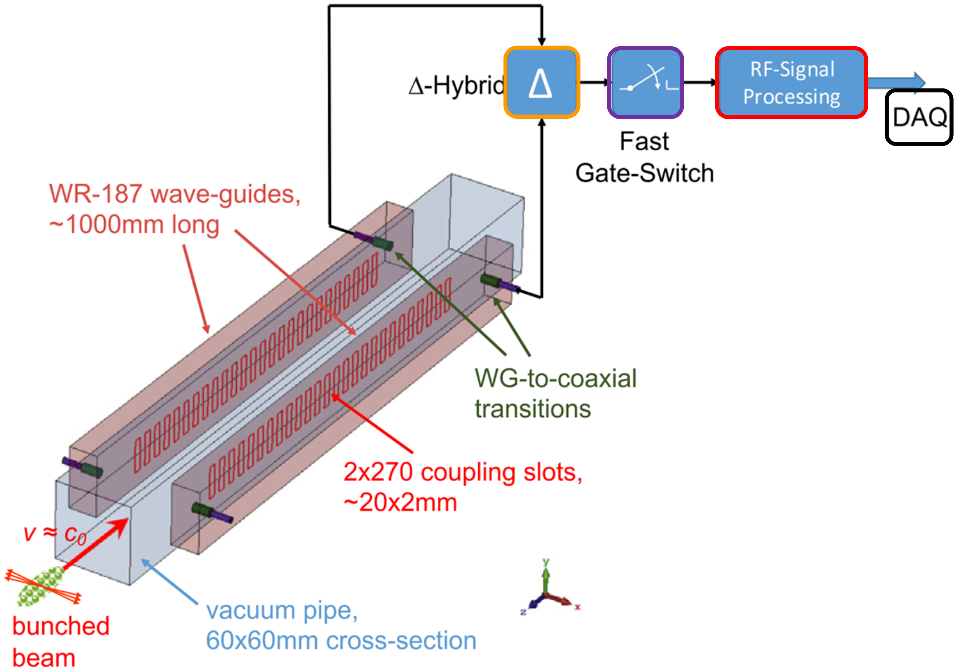


Figure courtesy of M. Betz et al, NIM 2017

- 4 transverse (Δ) systems (H/V, B1/B2)
- Longitudinal (Σ) signal not fully suppressed

- Flexible bunch gating

- Down-mix 4.8GHz→11kHz
- Final BW≈15kHz

- ADC + FFT
  - Fs = 44kHz
  - FFT len ~ 1.5s
  - Averaging ~ 1.5min
  - Update rate = 1Hz

Overview

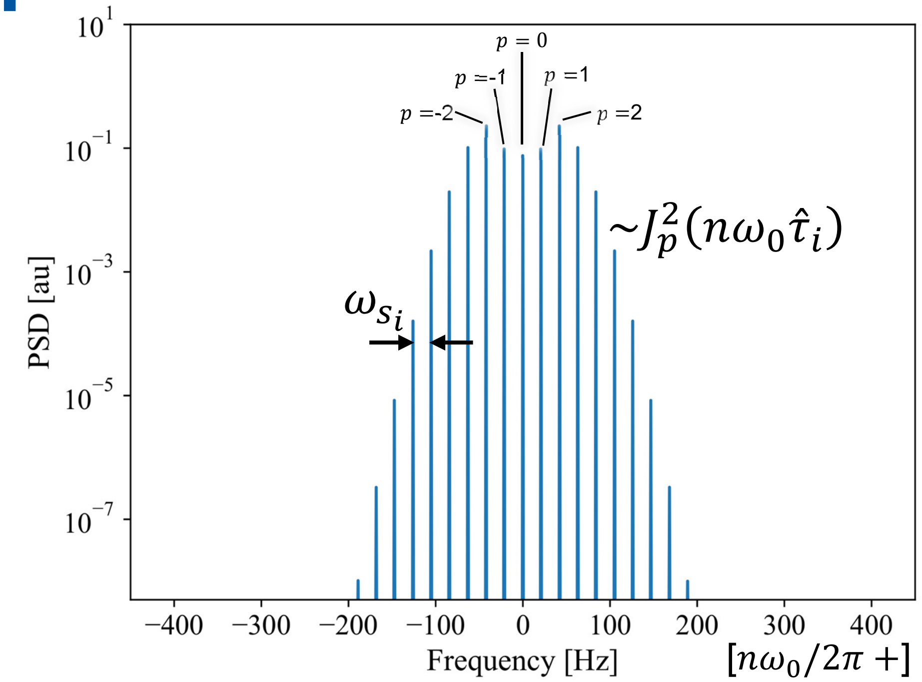
- Longitudinal bunch profile
- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- Results
- Conclusions and outlook

Single particle longitudinal Schottky spectrum

Single particle contribution
to sum (Σ) signal

$$s_i(t) = \sum_{p=-\infty}^{\infty} j^p J_p(n\omega_0\hat{\tau}_i) e^{j\left[\left(n\omega_0 + p\omega_{s_i}\right)t + p\phi_{s_i}\right]}$$

[Boussard1986]



Longitudinal Schottky spectrum

Total Σ signal: $s(t) = \sum_{i=1}^{N} s_i(t)$

Instantaneous Schottky spectrum is not deterministic!

- depends on the instantaneous phases → random

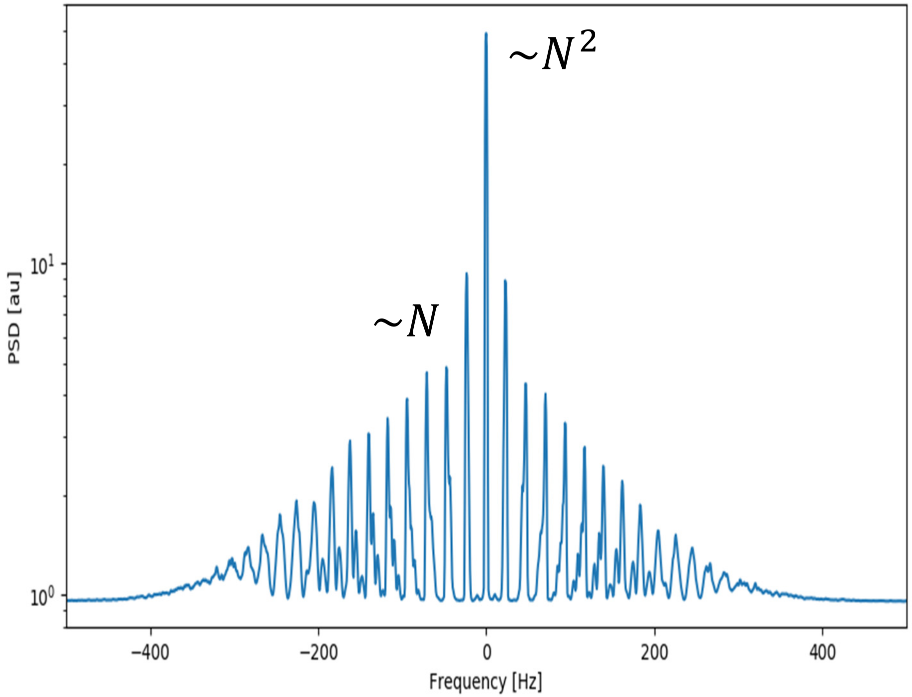
… so we average …

No intra-bunch coherent motion

Synch amp pdf

Single particle spectrum

$$\boxed{\langle P(\omega)\rangle \sim N \langle P_i(\omega, \hat{\tau}_i)\rangle} \sim N \int_0^{\infty} \boxed{g_{\hat{\tau}}(\hat{\tau})} \boxed{P(\omega, \hat{\tau})} d\hat{\tau}$$

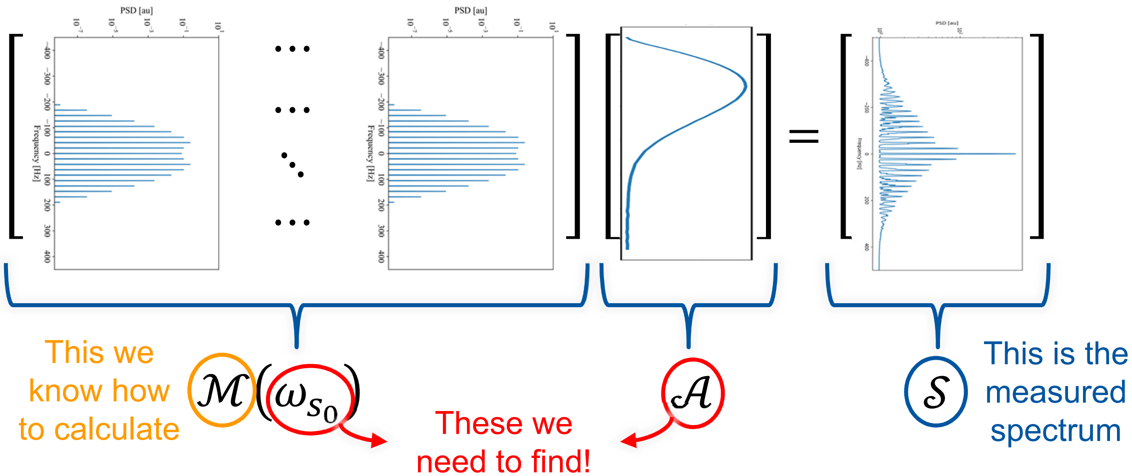


Matrix formalism

$$\underbrace{\begin{bmatrix} P(\omega_1,\hat{\tau}_1) & \cdots & P(\omega_1,\hat{\tau}_n) \\ P(\omega_2,\hat{\tau}_1) & \cdots & P(\omega_2,\hat{\tau}_n) \\ \vdots & \ddots & \vdots \\ P(\omega_m,\hat{\tau}_1) & \cdots & P(\omega_m,\hat{\tau}_n) \end{bmatrix}}_{\mathcal{M}(\omega_{s_0})} \underbrace{\begin{bmatrix} g_{\hat{\tau}}(\hat{\tau}_1) \\ g_{\hat{\tau}}(\hat{\tau}_2) \\ \vdots \\ g_{\hat{\tau}}(\hat{\tau}_n) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} \langle P(\omega_1) \rangle \\ \langle P(\omega_2) \rangle \\ \vdots \\ \langle P(\omega_m) \rangle \end{bmatrix}}_{\mathcal{S}}$$

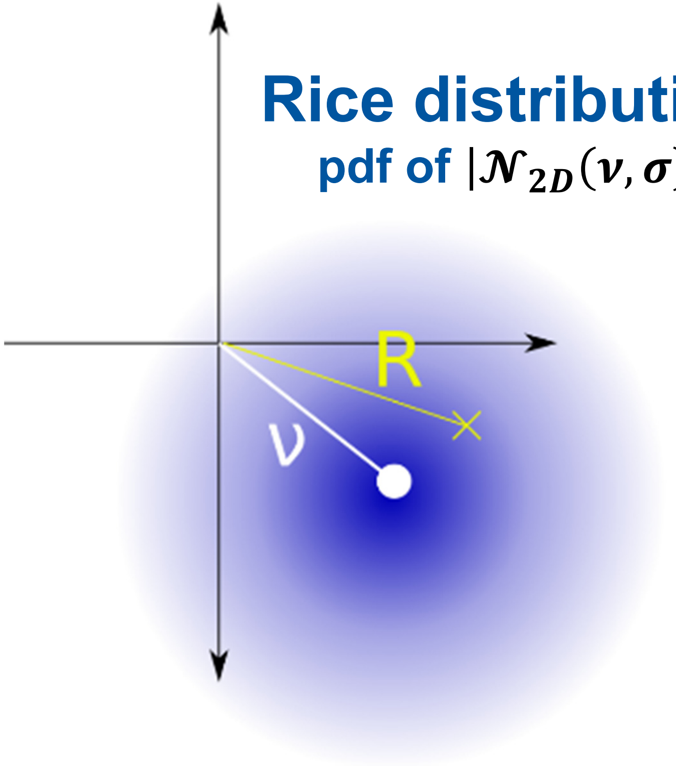
- $\mathcal{M}$ – cols are the single particle spectra
- $\mathcal{A}$ – synchrotron amplitude distribution
- $\mathcal{S}$ – averaged power spectral density

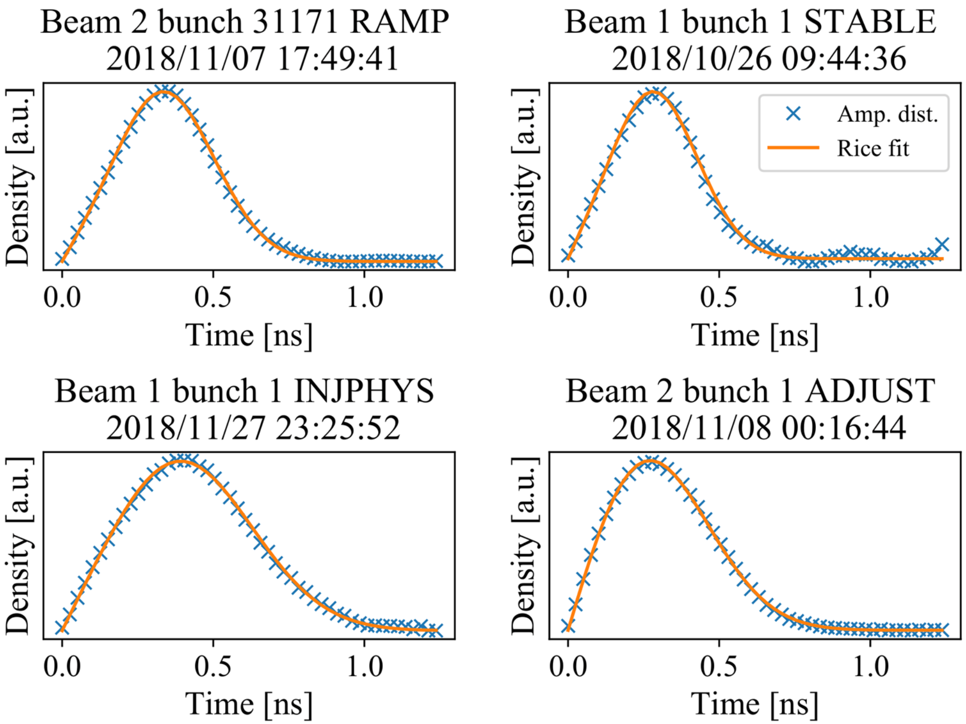
Matrix formalism



- $\mathcal{M}$ – cols are the single particle spectra
- $\mathcal{A}$ – synchrotron amplitude distribution
- $\mathcal{S}$ – averaged power spectral density

Amplitude distribution in the LHC





Overview

- Longitudinal bunch profile
- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- **Results**
- Conclusions and outlook

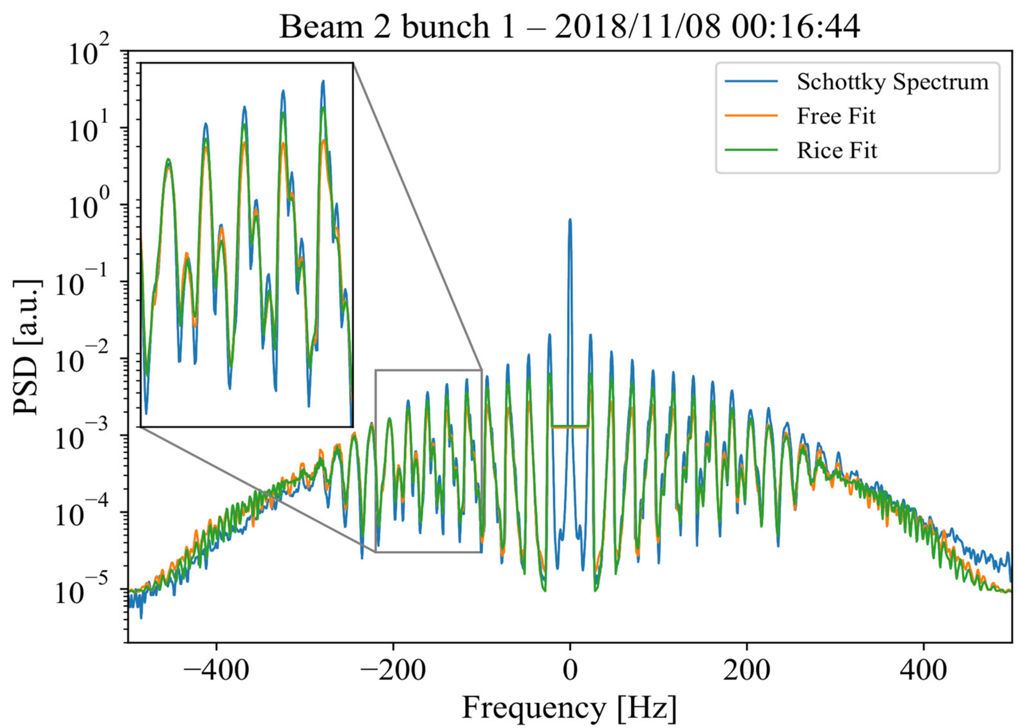
Fitting the spectrum

Minimise cost function:

$$C(\omega_{s_0}, \mathcal{A}) = \left|\log[\mathcal{M}(\omega_{s_0})\mathcal{A}] - \log[\mathcal{S}]\right|^2$$

Beam parameters:

- Pb^{82+}
- E = 6.37TeV
- $I_{beam} = 9.4 \times 10^{10}$ charges
- $I_{gate} = 5 \times 10^{9}$ charges



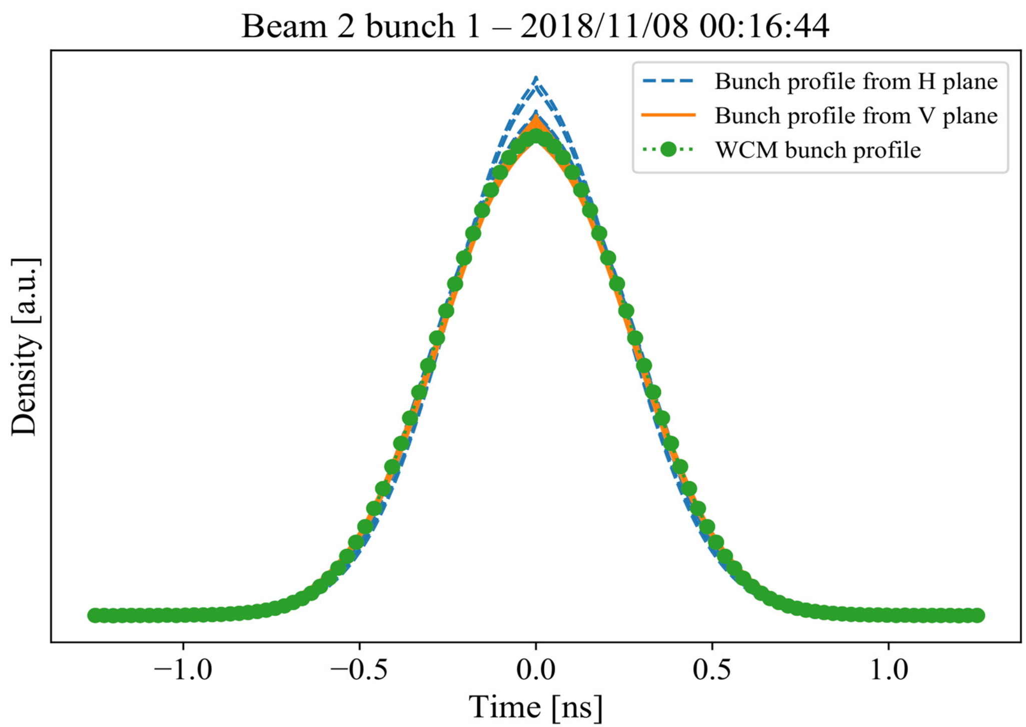
Estimated bunch profiles

Minimise cost function:

$$C(\omega_{s_0}, \mathcal{A}) = \left|\log[\mathcal{M}(\omega_{s_0})\mathcal{A}] - \log[\mathcal{S}]\right|^2$$

Beam parameters:

- Pb^{82+}
- E = 6.37TeV
- $I_{beam} = 9.4 \times 10^{10}$ charges
- $I_{gate} = 5 \times 10^{9}$ charges



Overview

- Longitudinal bunch profile
- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- Results
- Conclusions and outlook

Conclusions

- Fitted spectrum similar to experimental one
- Profiles from H&V channels are consistent
- Profiles match WCM measurements
- Reconstruction of longitudinal profile from narrowband measurements
- Developed mathematical framework for simulating Schottky spectra

Outlook

- Important step towards a better understanding of the LHC Schottky spectra
  - Mathematical framework can be adapted to transverse signals (e.g. estimate chromaticity and tune)
  - Obtained bunch profiles can be compared with WCMs and serve as a validation of the estimation of other parameters
- Extend mathematical framework to include the treatment of coherent motion
  - Can be applied e.g. during injection oscillations

ACKNOWLEDGEMENTS

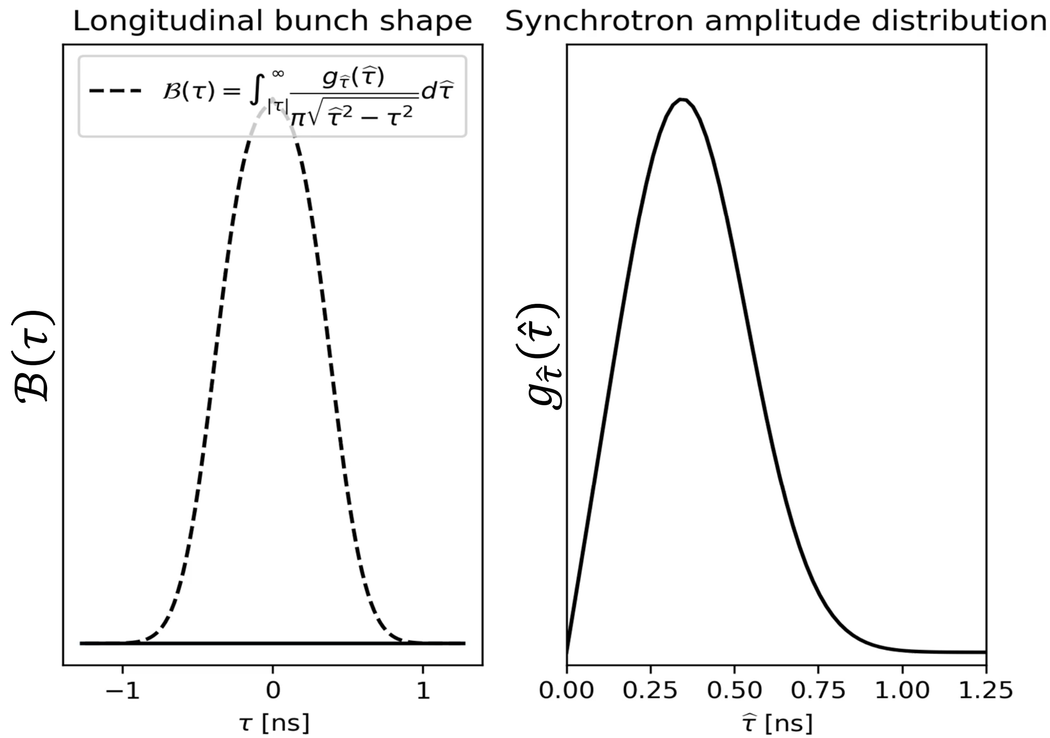
- T. Argyropoulos
- O. R. Jones
- T. Lefèvre
- T. Levens
- O. Marqversen
- M. Wendt

Thank you for your attention!

SPARE SLIDES

Typical LHC bunch profile

$$\mathcal{B}(\tau) = \int_{|\tau|}^{\infty} \frac{g_{\hat{\tau}}(\hat{\tau})}{\pi\sqrt{\hat{\tau}^2 - \tau^2}} d\hat{\tau}$$

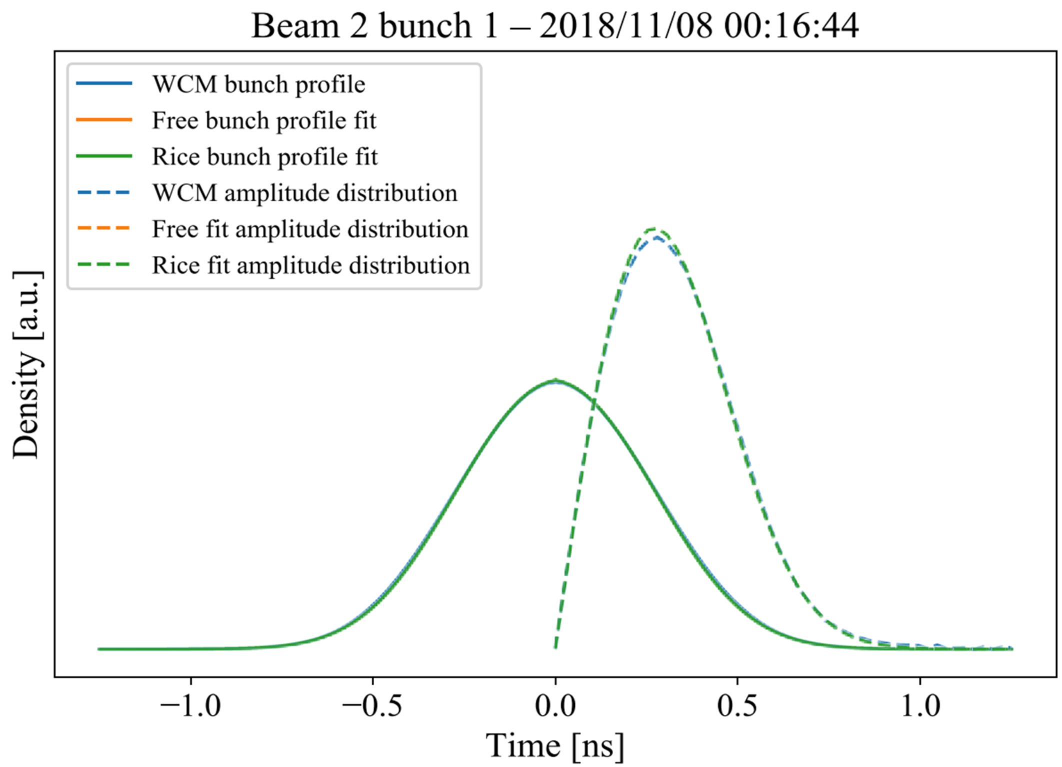


Optimisation process

Minimise cost function:

$$C(\omega_{s_0}, \mathcal{A}) = \left|\log[\mathcal{M}(\omega_{s_0})\mathcal{A}] - \log[\mathcal{S}]\right|^2$$

Optimization process uses a *differential evolution* algorithm for global minimisation

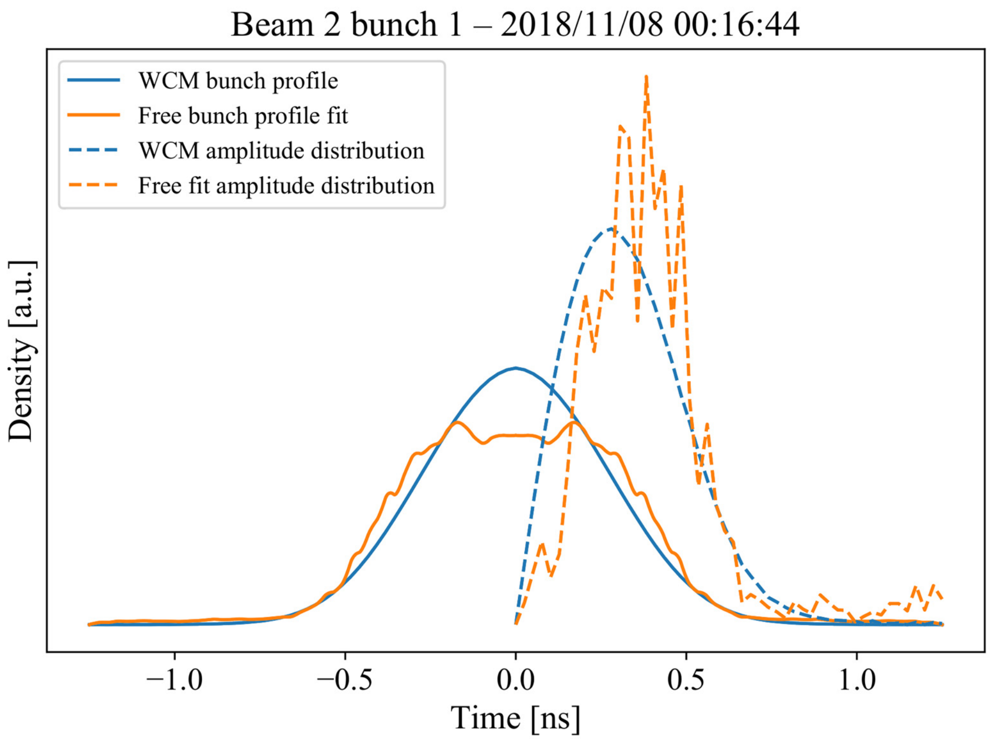


Free fit optimisation process

Minimise cost function:

$$C(\omega_{s_0}, \mathcal{A}) = \left|\log[\mathcal{M}(\omega_{s_0})\mathcal{A}] - \log[\mathcal{S}]\right|^2$$

Optimization process uses a *differential evolution* algorithm for global minimisation

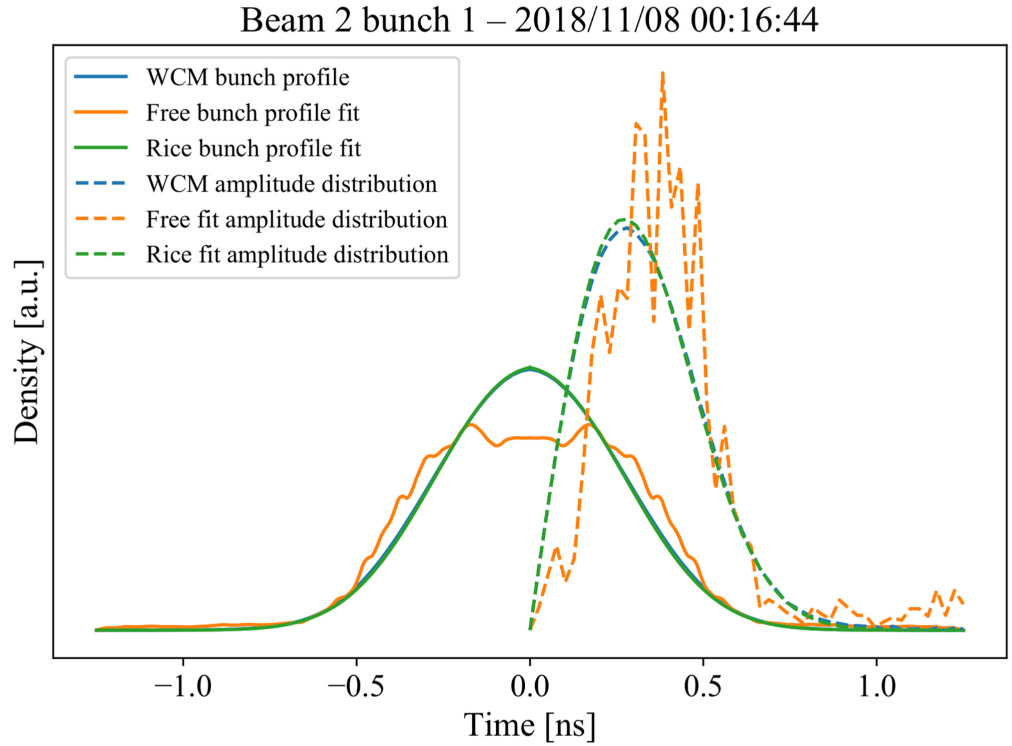


Optimisation process

Minimise cost function:

$$C(\omega_{s_0}, \mathcal{A}) = \left|\log[\mathcal{M}(\omega_{s_0})\mathcal{A}] - \log[\mathcal{S}]\right|^2$$

Optimization process uses a *differential evolution* algorithm for global minimisation



Matrix formalism

$$\underbrace{\begin{bmatrix} P(\omega_1, \widehat{\tau_1}) & \cdots & P(\omega_1, \widehat{\tau_n}) \\ P(\omega_2, \widehat{\tau_1}) & \cdots & P(\omega_2, \widehat{\tau_n}) \\ \vdots & \ddots & \vdots \\ P(\omega_m, \widehat{\tau_1}) & \cdots & P(\omega_n, \widehat{\tau_n}) \end{bmatrix}}_{\mathcal{M}} \cdot \underbrace{\begin{bmatrix} g(\widehat{\tau_1}) \\ g(\widehat{\tau_2}) \\ \vdots \\ g(\widehat{\tau_n}) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} \langle P(\omega_1) \rangle \\ \langle P(\omega_2) \rangle \\ \vdots \\ \langle P(\omega_m) \rangle \end{bmatrix}}_{\mathcal{S}}$$

$\mathcal{M}(:, i)$ - spectrum of single particle with amplitude τ_i

$\mathcal{A}$ - synchrotron amplitude distribution

$\mathcal{S}$ - cumulative spectrum

Matrix formalism

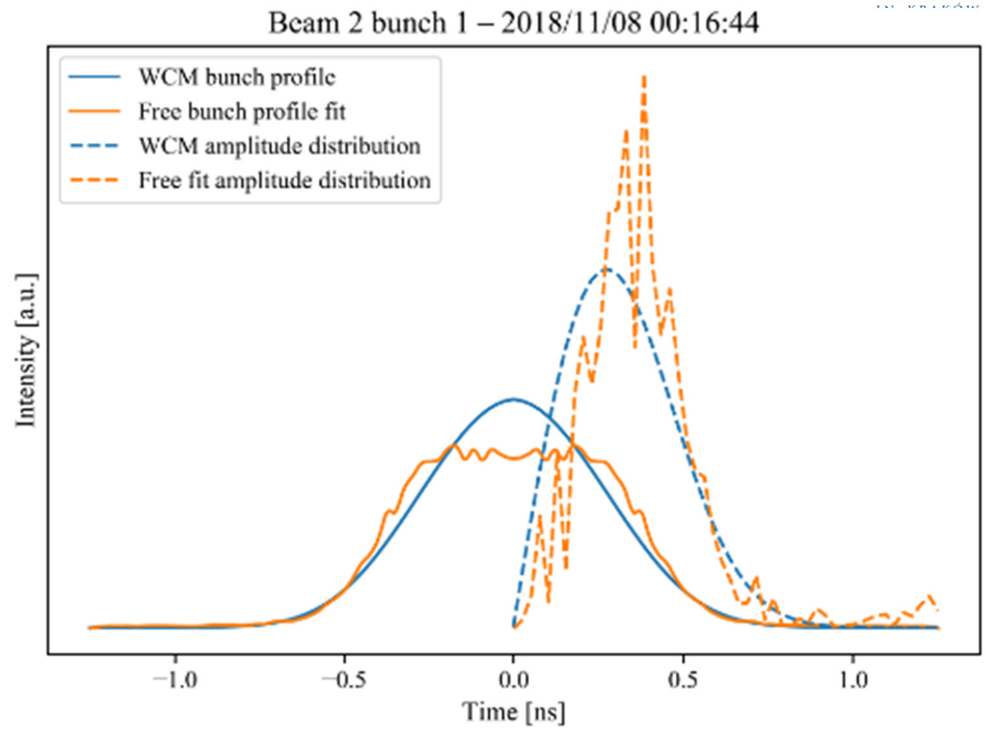
$$\underbrace{\begin{bmatrix} P(\omega_1, \widehat{\tau_1}) & \cdots & P(\omega_1, \widehat{\tau_n}) \\ P(\omega_2, \widehat{\tau_1}) & \cdots & P(\omega_2, \widehat{\tau_n}) \\ \vdots & \ddots & \vdots \\ P(\omega_m, \widehat{\tau_1}) & \cdots & P(\omega_m, \widehat{\tau_n}) \end{bmatrix}}_{\mathcal{M}} \cdot \underbrace{\begin{bmatrix} g(\widehat{\tau_1}) \\ g(\widehat{\tau_2}) \\ \vdots \\ g(\widehat{\tau_n}) \end{bmatrix}}_{\mathcal{A}} = \underbrace{\begin{bmatrix} \langle P(\omega_1) \rangle \\ \langle P(\omega_2) \rangle \\ \vdots \\ \langle P(\omega_m) \rangle \end{bmatrix}}_{\mathcal{S}}$$

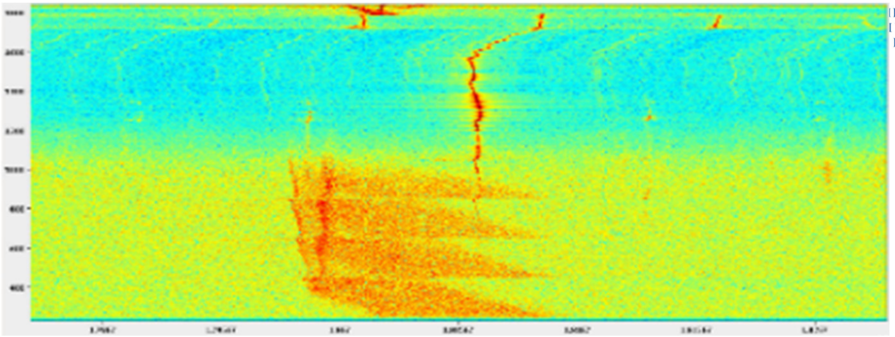
Precisely:

$$\mathcal{M} = \mathcal{M}\left(\omega_{s_0}\right)$$

Overview

- LHC Schottky system
- Longitudinal Schottky spectrum in the LHC
- Longitudinal bunch profile
- Results
- Conclusions and outlook


Beam 2 bunch 1 – 2018/11/08 00:16:44
WCM bunch profile
Free bunch profile fit
WCM amplitude distribution
Free fit amplitude distribution
Intensity [a.u.]
−1.0
−0.5
0.0
0.5
1.0
Time [ns]



$$\begin{bmatrix} P(\omega_1,\hat{\tau}_1) & \cdots & P(\omega_1,\hat{\tau}_n) \\ P(\omega_2,\hat{\tau}_1) & \cdots & P(\omega_2,\hat{\tau}_n) \\ \vdots & \ddots & \vdots \\ P(\omega_m,\hat{\tau}_1) & \cdots & P(\omega_m,\hat{\tau}_n) \end{bmatrix} \begin{bmatrix} g_{\hat{\tau}}(\hat{\tau}_1) \\ g_{\hat{\tau}}(\hat{\tau}_2) \\ \vdots \\ g_{\hat{\tau}}(\hat{\tau}_n) \end{bmatrix} = \begin{bmatrix} \langle P(\omega_1)\rangle \\ \langle P(\omega_2)\rangle \\ \vdots \\ \langle P(\omega_m)\rangle \end{bmatrix}$$

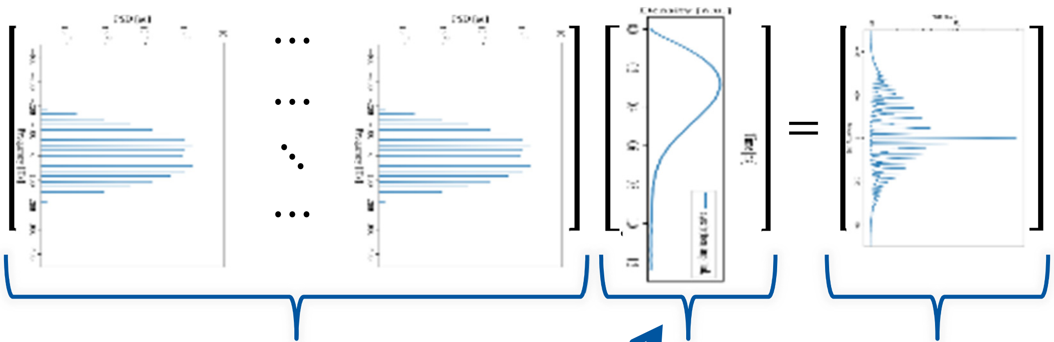
Notation

- τ = longitudinal time difference w.r.t. synchronous particle,
- $\hat{\tau}$ = time amplitude of synchrotron oscillations,
- ϕ_s = initial synchrotron phase,
- ω_{s_0} = nominal synchrotron frequency,
- ω_s = synchrotron frequency.

Assumptions

- Simple harmonic motion for synchrotron oscillations (% of RF bucket~2.5ns)
  - $\tau = \tau(\hat{\tau}, \phi_s) = \hat{\tau}\cos(\omega_s t + \phi_s)$ [Boussard1986]
- No coherent intra-bunch motion

Matrix formalism



$\mathcal{M}(\omega_{s_0})$ $\mathcal{A}$ $\mathcal{S}$

This we need to precompute

This we need to find

This we know from Schottky Monitor

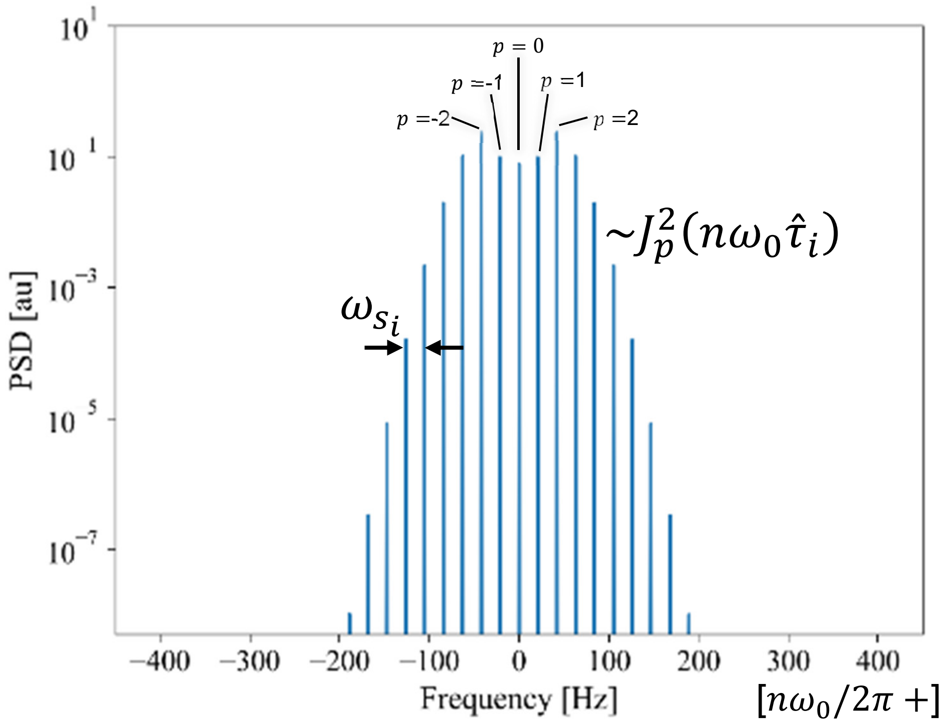
Single particle Schottky spectrum

Single particle contribution
to sum (Σ) signal
[Boussard1986]

$$s_i(t) = \sum_{p=-\infty}^{\infty} j^p J_p(n\omega_0\hat{\tau}_i) e^{j\left[\left(n\omega_0 + p\omega_{s_i}\right)t + p\phi_{s_i}\right]}$$

As ω_{s_i} and $\hat{\tau}_i$ are related, we have that:

$$P_i(\omega) = P_i(\omega, \hat{\tau}_i)$$



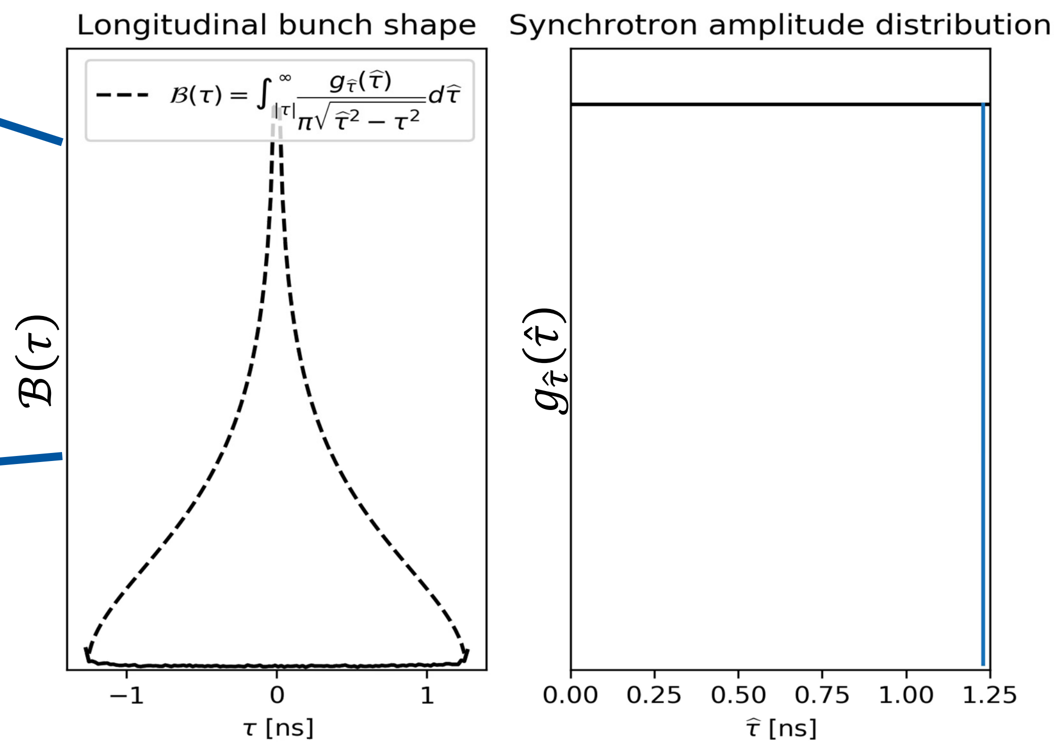
Bunch profile numerical example

$$g_\tau(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}$$

Single particle distribution

$$\mathcal{B}(\tau) = \int_{|\tau|}^{\infty} \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}} g_{\hat{\tau}}(\hat{\tau}) d\hat{\tau}$$

Longitudinal bunch profile

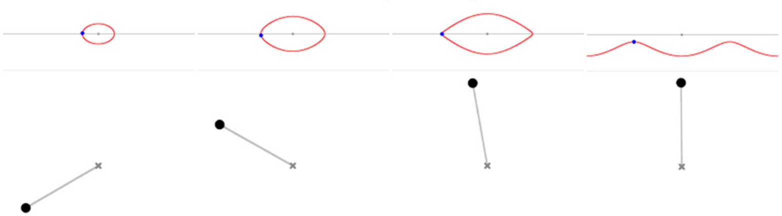


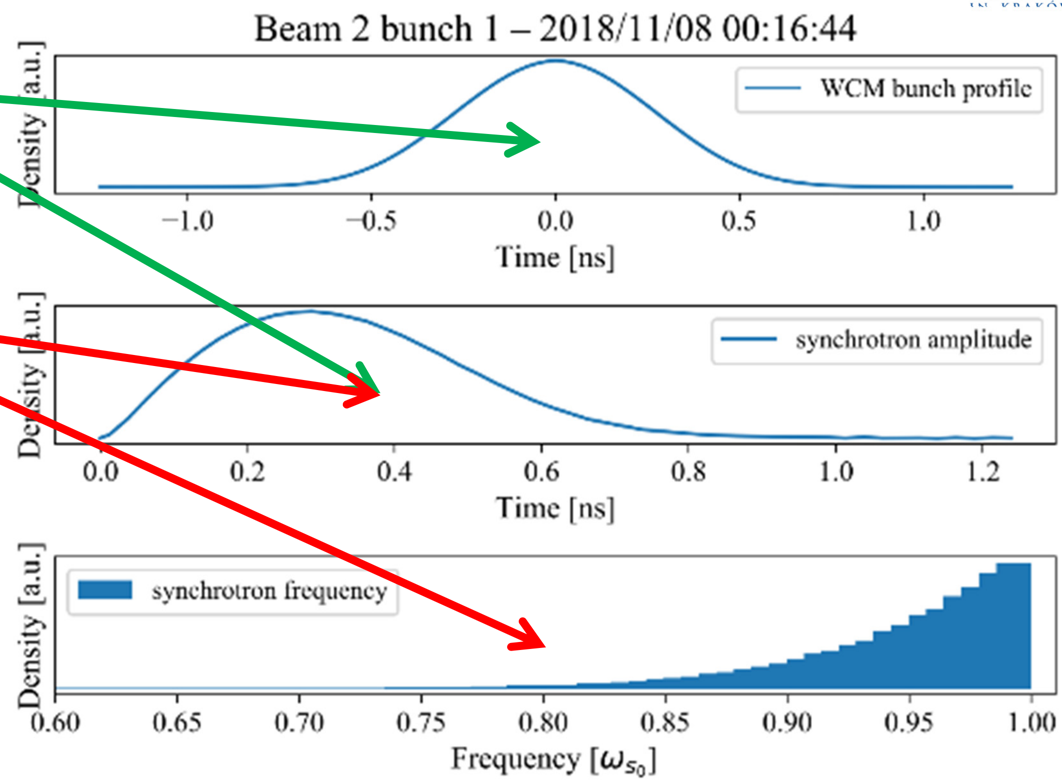
Equivalence of distributions

$$\mathcal{B}(\tau) = \int_{|\tau|}^{\infty} \frac{g_{\hat{\tau}}(\hat{\tau})}{\pi\sqrt{\hat{\tau}^2 - \tau^2}} d\hat{\tau}$$

$$\omega_s = \frac{\pi}{2K\left[\sin\left(\frac{h\omega_0\hat{\tau}}{2}\right)\right]}\omega_{s_0}$$

From the theory of pendulum:

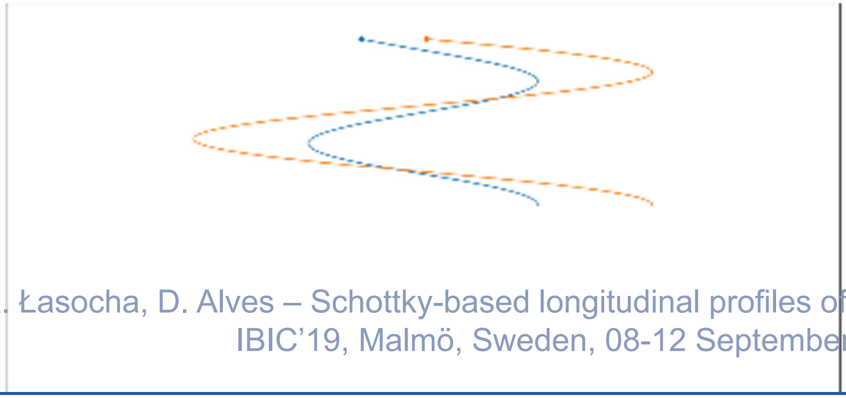




Synchrotron motion

Harmonic oscillator with amplitude dependent frequency

$$\tau_i = \hat{\tau}_i \cos(\omega_{s_i} t + \phi_{s_i}) \qquad \text{[Boussard1986]}$$



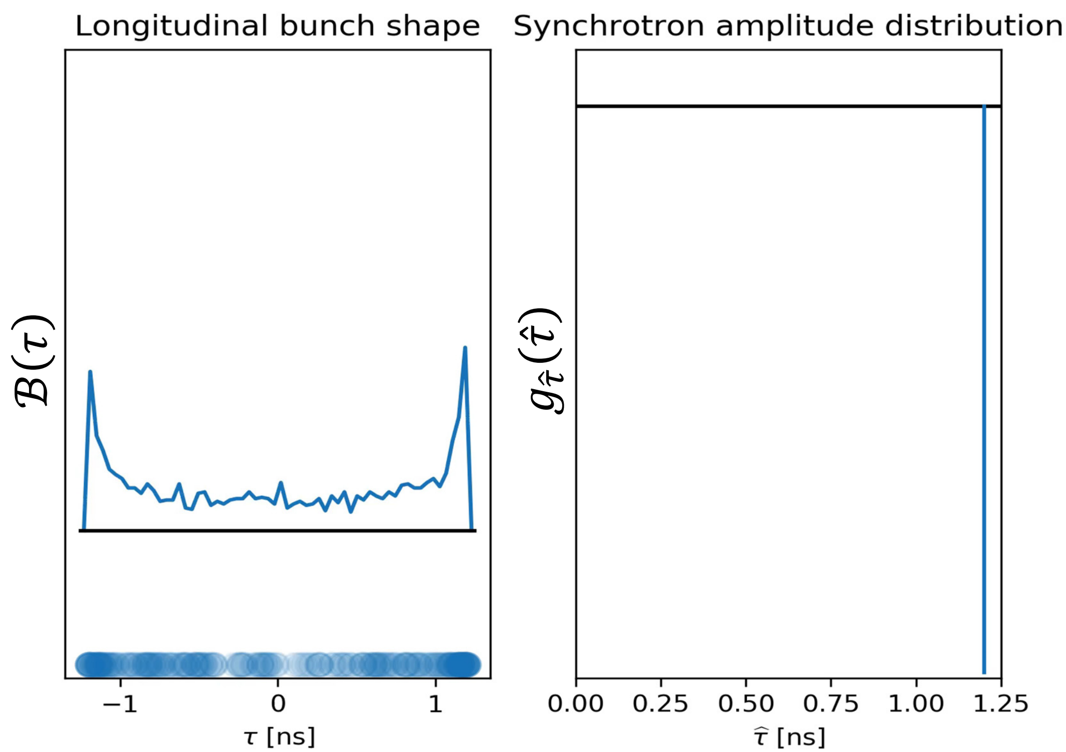
Longitudinal profile

How to relate the distribution of synchrotron amplitudes with the longitudinal bunch profile …

$$g_{\hat{\tau}}(\hat{\tau}) \leftrightarrow \mathcal{B}(\tau)$$

Synchrotron motion – single particle distribution

$$\underbrace{g_\tau(\tau|\hat{\tau}) = \frac{1}{\pi\sqrt{\hat{\tau}^2 - \tau^2}}}_{\text{Single particle distribution}}$$



Longitudinal Schottky spectrum

Total pick-up signal: $s(t) = \sum_{i=1}^{N} s_i(t)$

Instantaneous Schottky spectrum is not deterministic!

- depends on ϕ_s
- changes with time

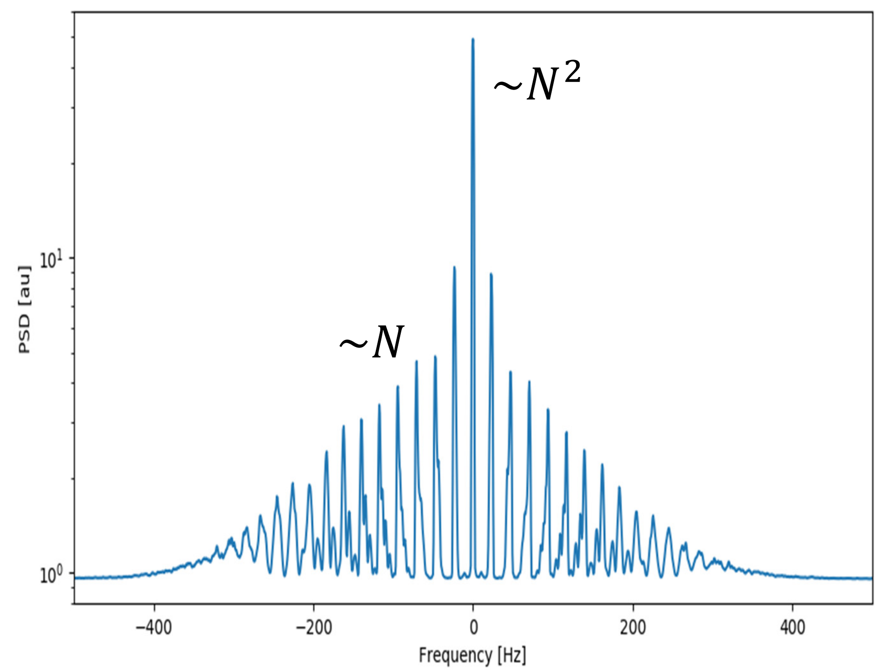
… so we average …

No intra-bunch coherent motion

Synch amp pdf

Single particle spectrum

$$\boxed{\langle P(\omega)\rangle \propto N\langle P_i(\omega,\hat{\tau}_i)\rangle} \sim N\sum_{\hat{\tau}} \boxed{g_{\hat{\tau}}(\hat{\tau})}\,\boxed{P(\omega,\hat{\tau})}$$



$$\omega_{s_i} = \frac{\pi}{2K\left[\sin\left(\frac{h\omega_0\hat{\tau}_i}{2}\right)\right]}\omega_{s_0}$$

$$g_{\phi_s}(\phi_s) = \mathcal{U}[-\pi,\pi]$$
$$g_{\phi_s,\hat{\tau}}(\phi_s,\hat{\tau}) = g_{\phi_s}(\phi_s)g_{\hat{\tau}}(\hat{\tau})$$

ACKNOWLEDGEMENTS

- T. Argyropoulos
- O. R. Jones
- T. Lefèvre
- T. Levens
- O. Marqversen
- M. Wendt

REFERENCES

[Boussard1986]

Outlook

- Important step towards understanding the Schottky spectra
  - Mathematical framework can be adapted to transverse signals (e.g. estimate chromaticity and tune)
  - Obtained bunch profiles can be compared with WCMs and serve as a validation of the estimation of other parameters
- Mathematical treatment of coherent motion
  - Evolution of bunch profiles @ injection
- Apply method to proton bunches (higher intensities and shorter bunch lengths)