

Intense Emission of Smith-Purcell Radiation at the Fundamental Frequency from a Grating Equipped with Sidewalls

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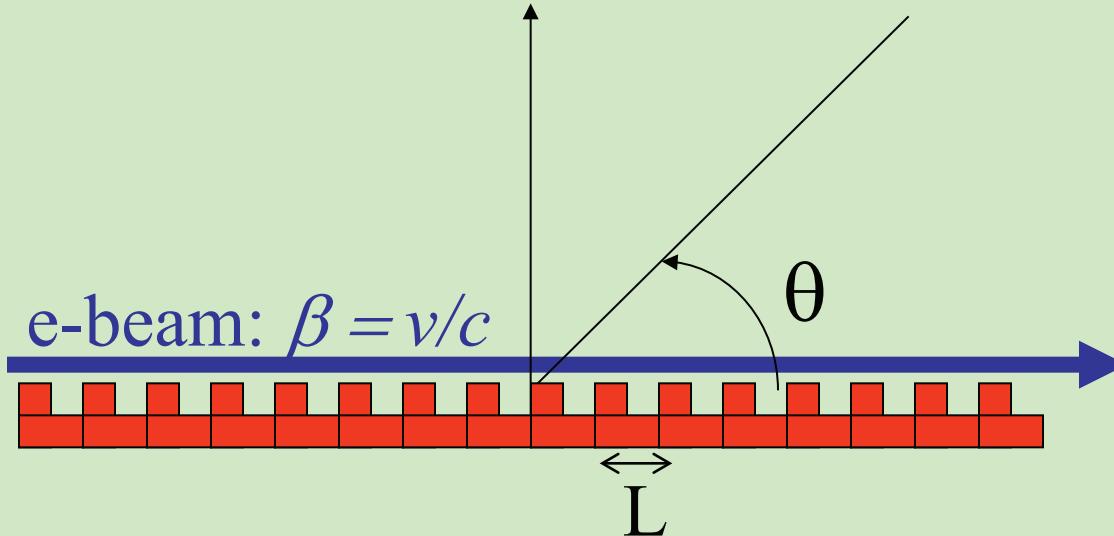
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SUMMARY

1. Coherent Smith-Purcell radiation (CSP)
2. Model of Andrews, Brau and collaborators (A&B)
3. 2D Experiment at CESTA (wide, flat, and intense beam, 2nd harmonic)
4. 3D Theory for a grating with sidewalls
5. 3D Experiment at CESTA (high power on the fundamental)

SMITH-PURCELL RADIATION

S. J. Smith and E. M. Purcell, Phys. Rev. **92**, 1069 (1953)



Incoherent Smith-Purcell radiation (*first observed in 1953*):

- Diffraction grating
- The observed wavelengths satisfy the following relation:

$$\lambda = \frac{L}{n} \left(\frac{1}{\beta} - \cos \theta \right)$$

Where: L is the grating period

n is the order of diffraction

β is the normalized beam velocity

All glasses except the vitreous silica showed a strong radiation independence resonance at $\epsilon < 3.00 \pm 0.20$ and $\Delta\epsilon$ is presumably due to spin paramagnetic impurities.

Measurement of the optical densities of a series of gamma-irradiated samples of lime glass revealed that the paramagnetic resonance amplitude varied linearly with optical density. Likewise, with annealing at 200°C., the signal amplitudes decreased proportionately with annealing. For the gamma-ray excitation, the dependence of spin concentration on total radiation showed a saturation characteristic with an initial density of over by 20% per spin channel.

Further studies on the processes involved will be published more completely in the near future.

J. H. Shaffer and J. A. Sauer, *Trans. Am. Inst. Min. Metall. Eng.*, 204, 921 (1955).

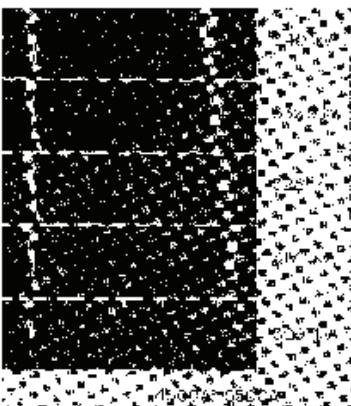


FIG. 1. Bright region of the light emitted from the grating surface at 300 kV. The bright region is about 4 mm in diameter. The electron current density at the right edge was recorded over a period of 1.5 sec. The concentric rings around the central spot are due to the interference of the light emitted from the grating. The distance between the concentric rings is about 1.5 mm, and $\theta = 15^\circ$ is the angle between the direction of motion of the electrons in the light cone. $k = 1.67$ indicates, as in a typical optical grating, that electrons of energy around 300 keV used, the light emitted forward should lie in the visible spectrum. As $k = 1.67$, we assume that the surface charge density decreasing as the distance d is equal to the total charge oscillating with an amplitude $\delta/2$, we find that in the forward direction the radiation intensity (assumed constant) should amount to 40×10^{-12} erg/cm² sec. In about 40 sec a current of 1 μampere of electron path. On the 300 kV. electron gun lies within perhaps $\delta/2$ of the center will the surface charges be so well localized. Nevertheless, with a reasonable electron current density over the surface actually emitted a current of light should be produced. In addition, in the second dimension, the total radiation per unit of grating surface, in millilumens, should be about four times the electron current density parallel to the surface in amperes.

We have tried the experiment using a beam crossover-coupled Van de Graaff generator and a carbon accelerating tube. A 20-microampere beam, focused electrostatically and magnetically to a diameter of 0.13 mm and divergence less than 0.004 radian, is subject to deflection effects to overcome so as to get at speed 10 cm/sec. The 40-sec time interval of the grating, taken from a forward position 10 or 20 degrees off the beam appears as a sharp, continuous colored line on the surface of the grating. The color of the light changes with angle of observation, our beam being too small to be separated. The light is entirely polarized with the electric vector perpendicular to the grating. The effect of varying δ can be investigated by rotating the grating in its own plane (as suggested by K. H. Ladd) and however the color changes with position.

The spectrum of the light has been recorded at low dispersion by photographing the beam through a low dispersion grating. Light from the electron gun is collected by a collecting lens, where an arc shaped aperture restricts the cones of angles θ , passed through the anchoring grating and focused by a lens into a 10-mm square \times 10-mm. cell. With this arrangement only one point on the line source is in good focus. Figure 1 shows a sequence of such photographs in which only the electron velocity was varied. With the necessary care still retains the improvements of wavelength and voltage, the predicted dependence of

on the current density $\delta - 1$ anticipated by evidence by the spectrograph. On spectrograms taken at $\theta = 30^\circ$, a distinct band, the fundamental wavelength is detectable. Diffracted harmonics appear at approximately 1.67 times $\lambda/30^\circ$, where λ is radiation of one third, fourth, fifth, and sixth, with the fundamental wavelength.

Although many details remain to be studied, we believe these observations establish the reality of the effect and suggest that it may have interesting applications.

Recipient of a General Electric Research in Applied Physics for 1952-53.
G. A. and J. F. Bell Laboratories, Holmdel, New Jersey.

Derivation and Renormalization of the Tamm-Dancoff Equations*

BRUNO COMPTON
Institute of Nuclear Studies, Cornell University, Ithaca, New York
(Received August 13, 1953)

In this note we show that (1) the derivation of Tamm-Dancoff (T.D.) equations for two nucleons as well as for mesons-nucleons are valid, (2) the equation of Bogoliubov's (B.B.) equations can be modified to a π wavefunction, (3) the renormalizations in the T.D. method can be achieved in a consistent manner without leaving the difficulties presented by the ring of T.D. and B.B.

We introduce the spinor function $\psi(12)$ by the relations

$$\psi(12) = \int S_{\mu\nu}(12) p_1(1'2) p_2(1'2) \int S_{\mu\nu}(1'2) p_1(1'2) d^4 z, \quad (1)$$

where $S_{\mu\nu}(12)$ is a γ_5 R.F.S. wave function for two nucleons. By using (1) in the R.F.E. of B.B. and we obtain

$$[i(\partial_{\mu} + \beta A_{\mu}) + (\bar{\psi}_1 \gamma_{\mu} - \beta \delta) \psi(12)]$$

$$= - \int \psi(12) \partial_{\mu} (\bar{\psi}_1 \gamma_{\mu} - \beta \delta) \psi(12). \quad (2)$$

S. J. Smith &
E. M. Purcell
Phys. Rev. 92
1069 (1953)

Van de Graaff, 300 kV

5 μ A

$L = 1.67 \mu\text{m}$

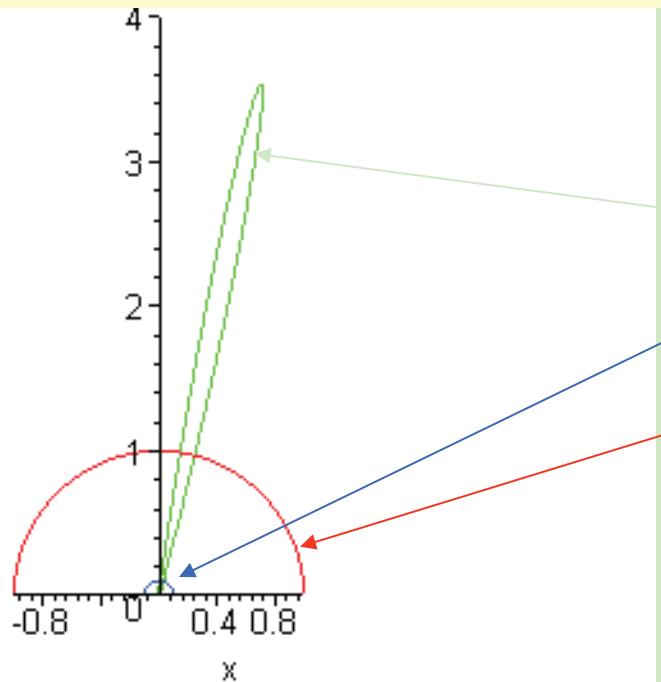
$\lambda: 440\text{-}550 \text{ nm}$

$\theta: 0\text{-}20^\circ$

COHERENT SMITH-PURCELL RADIATION (CSP)

CSP Radiation has two senses:

- Intra-bunch: bunch size $\ll \lambda$, Intensity $\propto N_e^2 \forall \theta$
- Inter-bunch (time interval T between successive bunches = $n\lambda_{SP}/c$, n integer) \Rightarrow Intensity is enhanced at certain angles



Radiation patterns for
Inter-bunch coherent
Incoherent S-P
Intra-bunch coherent

SMITH-PURCELL RADIATION SINCE 2004

Dispersion relation proposed by Andrews and Brau for 2D lamellar gratings:

H. L. Andrews and C. A. Brau, Phys. Rev. ST Accel. Beams **7**, 070701 (2004).

Evanescence Floquet wave above grating, uniform plasma, velocity = v.

$$B_z(x, y, t) = \sum_p B_p \exp(i(k + pK)x - \alpha_p y - \omega t)$$

$$(k + pK)^2 - \alpha_p^2 = (\omega/c)^2 \quad 0 < k < K = 2\pi/L$$

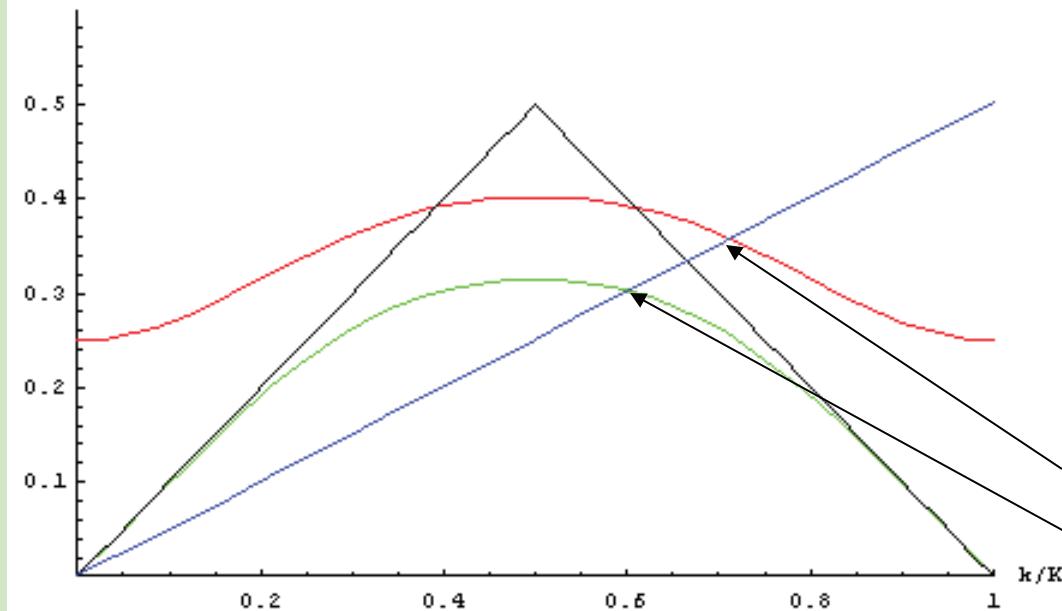
They established the 2D dispersion relation between k and ω .

We established the 3D dispersion relation among k , ω , and q , the transverse wave-number.

J. T. Donohue and J. Gardelle, Phys. Rev. ST Accel. Beams **14**, 060709 (2011).

See also B. D. McVey *et al*, IEEE Trans. Microwave Theory Tech. **42**, 995 (1994).

$$\omega/cK = L/\lambda$$



2D —————
 3D —————
 Beam —————
 light —————
 SP allowed
 SP forbidden

$$\omega_{3D}(k, q) = \sqrt{(\omega_{2D}(k))^2 + (cq)^2}$$

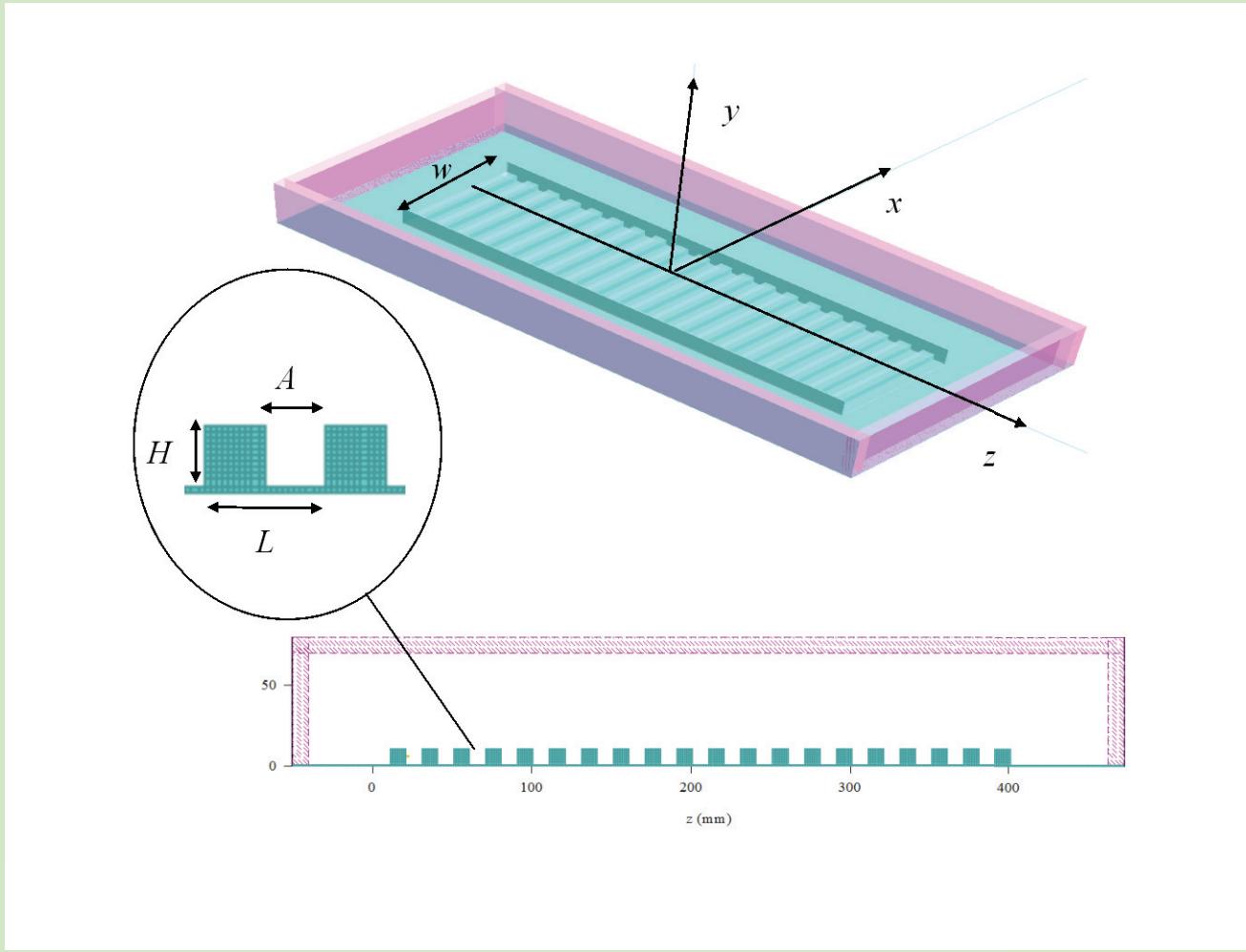
$$\omega_{3D}(k) = \sqrt{(\omega_{2D}(k))^2 + \left(\frac{c\pi}{W}\right)^2}$$

W = width between sidewalls

2D :
only Harmonics can be
radiated

3D :
may radiate Fundamental

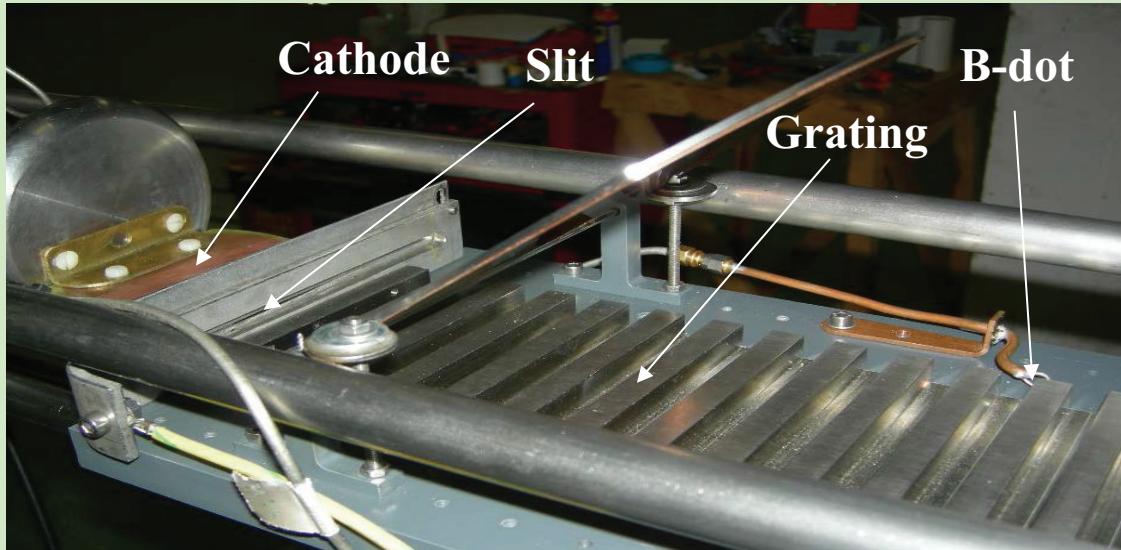
Typically
Bunching_{Fundamental} >> Bunching_{Harmonic}



Geometry of our grating with sidewalls:

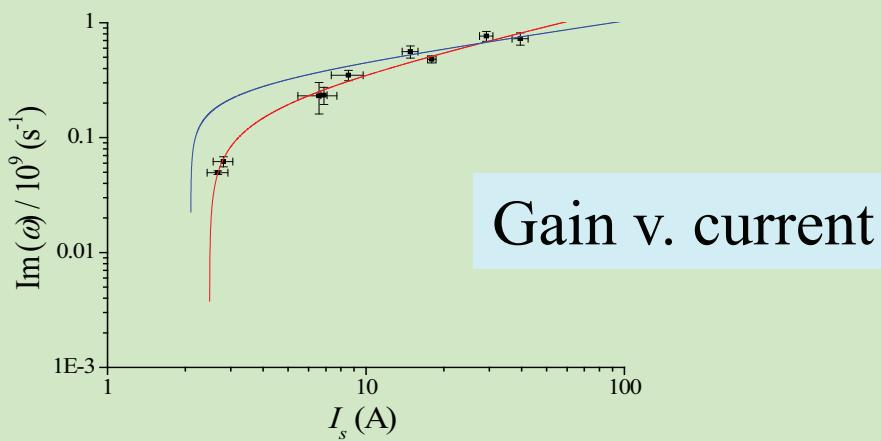
$$L = 2 \text{ cm}, A = H = 1 \text{ cm}, w = 4 \text{ cm}$$

THE 2D EXPERIMENT AT CEA/CESTA FEL 2010



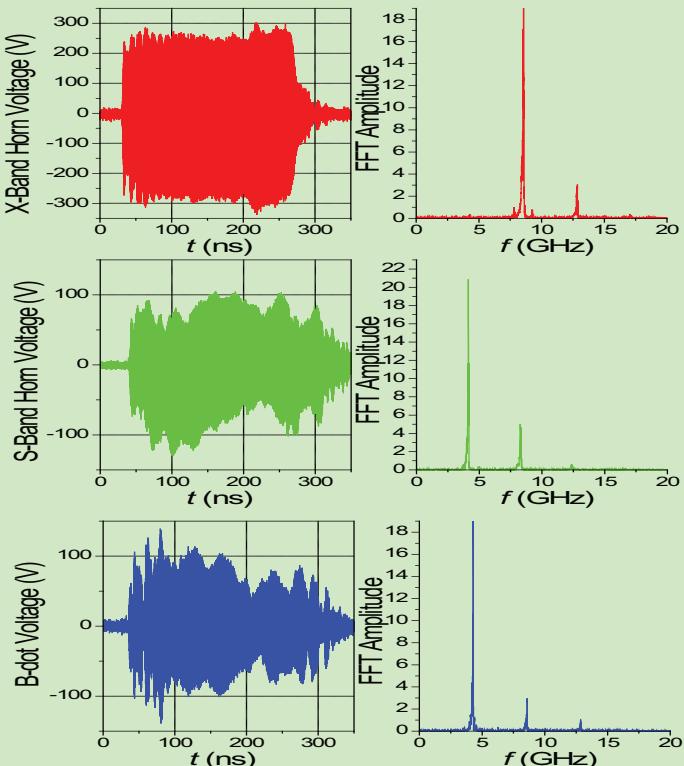
Experimental Parameters

Parameters	Value
Beam kinetic energy	95 keV
Peak current	0 - 280 A
Pulse duration	300 ns
Beam thickness	0.01-2 mm
Beam-grating distance	3 mm
Grating period	2 cm
Grating groove depth	1 cm
Grating groove width	1 cm
Grating width	10 cm
Number of periods	20
External magnetic field	0.3-0.5 T



Efficiency
= 0.1 %

2-D EXPERIMENTAL RESULTS



Wave forms on a
12 GHz BW oscilloscope

Time signals and their FFTs:

X-band horn placed outside vacuum chamber.

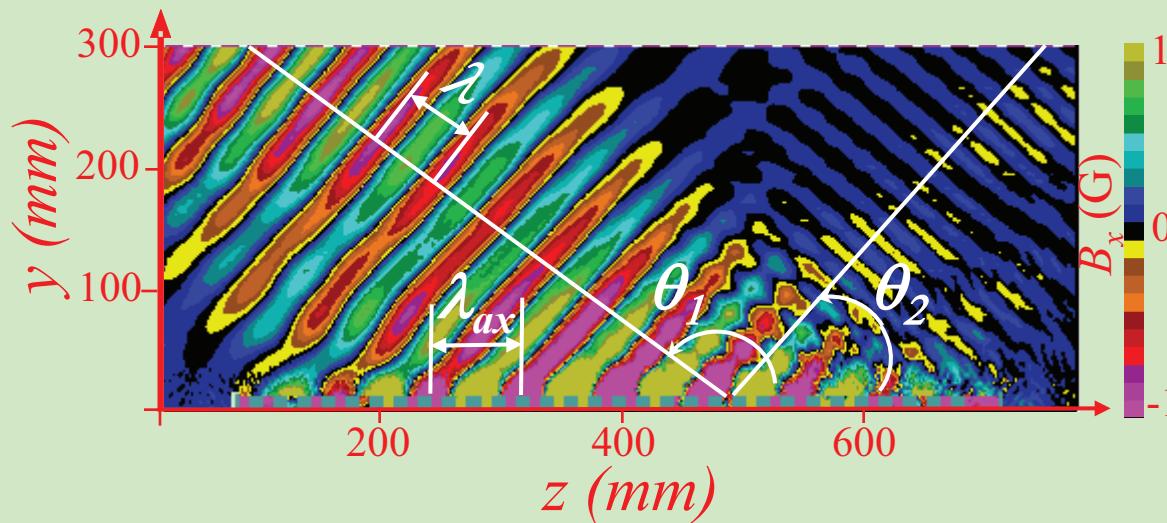
Magnetic field (parallel to groove) as measured with the B-dot probe at a groove end.

S-band horn to detect evanescent wave.

MAGIC 3D SIMULATION

Contours of B_x in the median y - z plane at fixed time

The fundamental is radiated at θ_1 (140 °)
The 2nd harmonic is radiated at θ_2 (60 °)

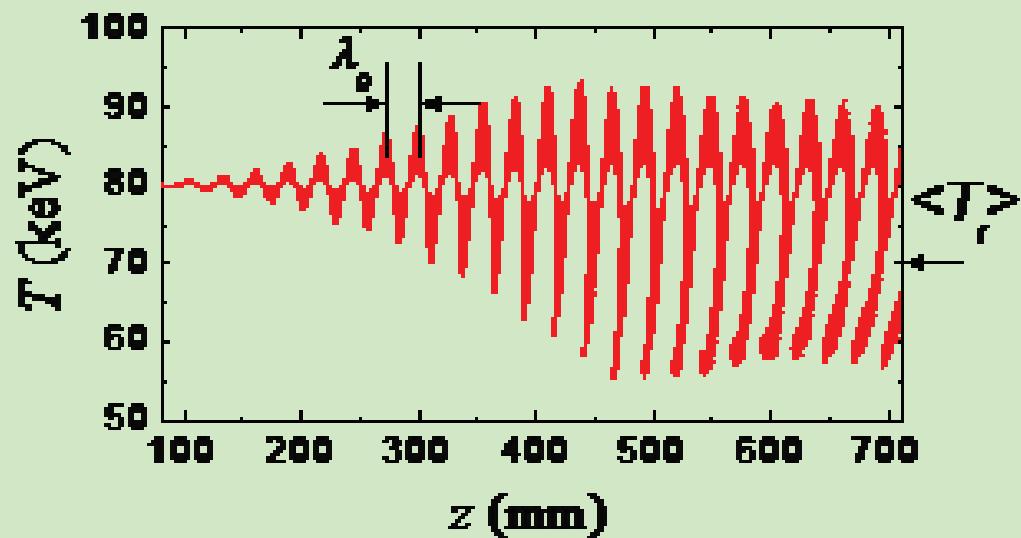
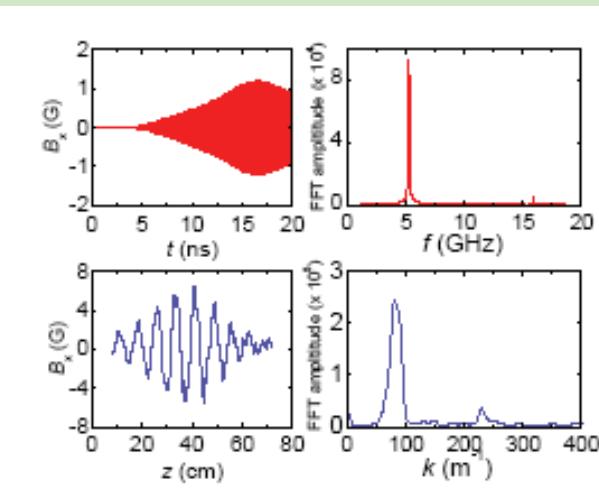


3D simulation with « free-space » on boundaries.
Reflections are tolerable.

MAGIC 3D SIMULATION

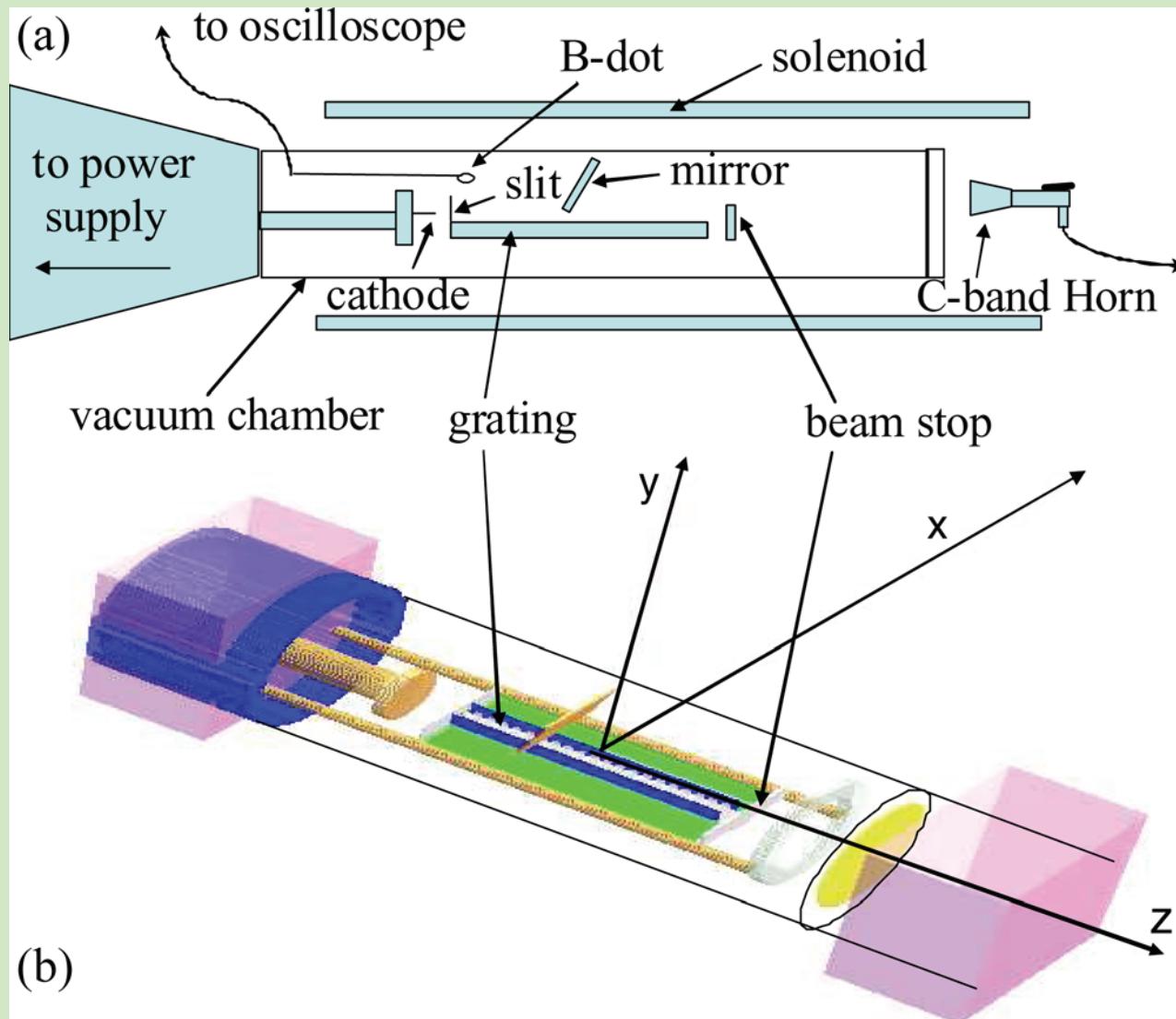
$$\begin{array}{ll} f = 5.2 \text{ GHz} & \lambda = 5.8 \text{ cm} \\ k = 230 \text{ m}^{-1} & \lambda_0 = 2.8 \text{ cm} \\ k \cdot K = -84 \text{ m}^{-1} & \lambda_{-1} = 7.6 \text{ cm} \end{array}$$

Mean energy loss 10 keV at 5 A:
50 kW emitted on fundamental
CSP efficacité = 12.5%



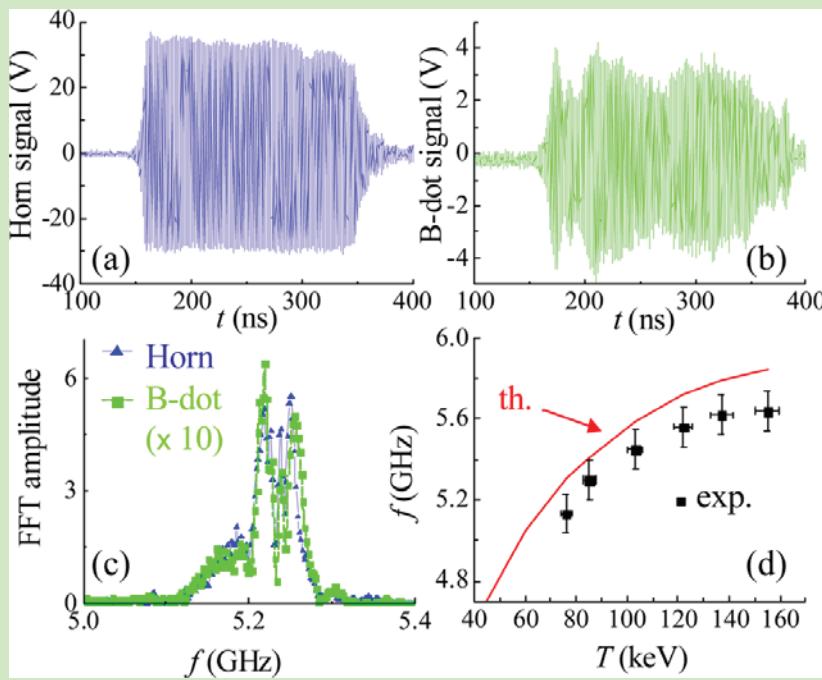
Simulation predicts high efficiency for the Fundamental

3-D CESTA Experiment: Set-up and Simulation volume

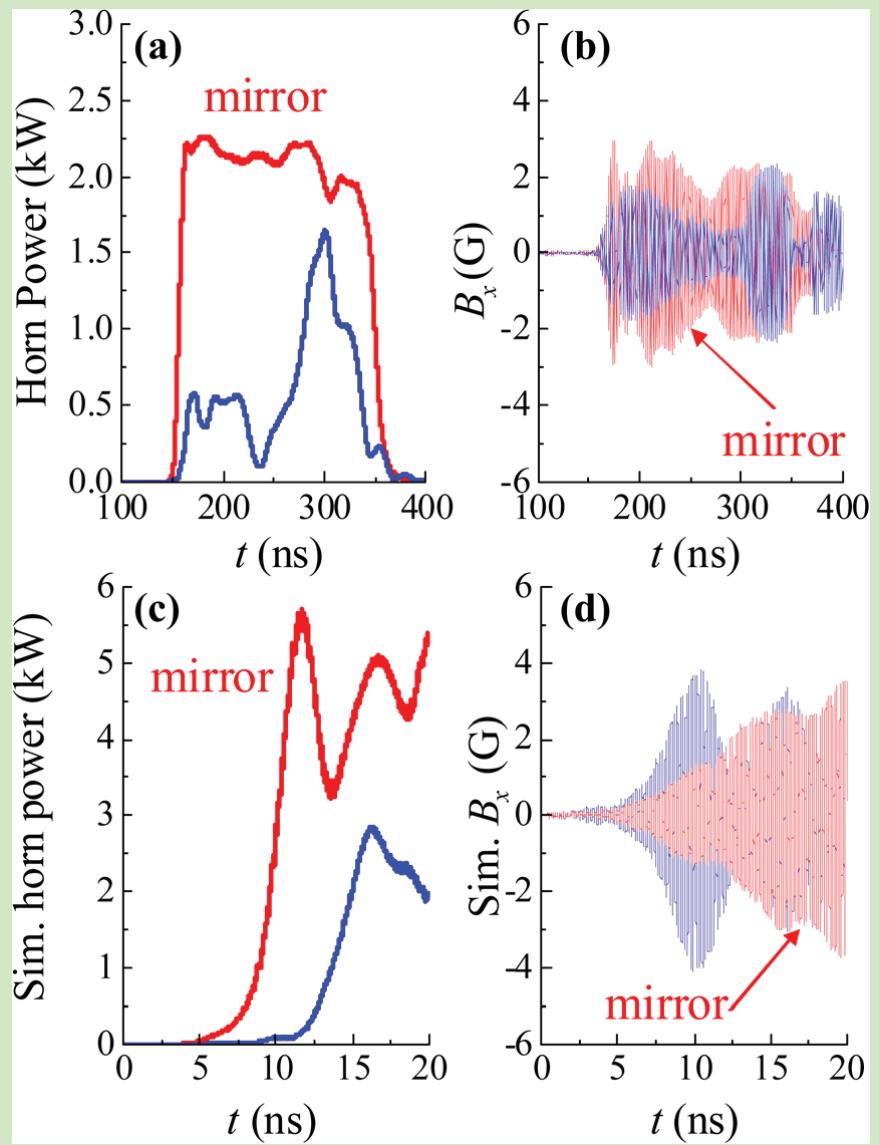


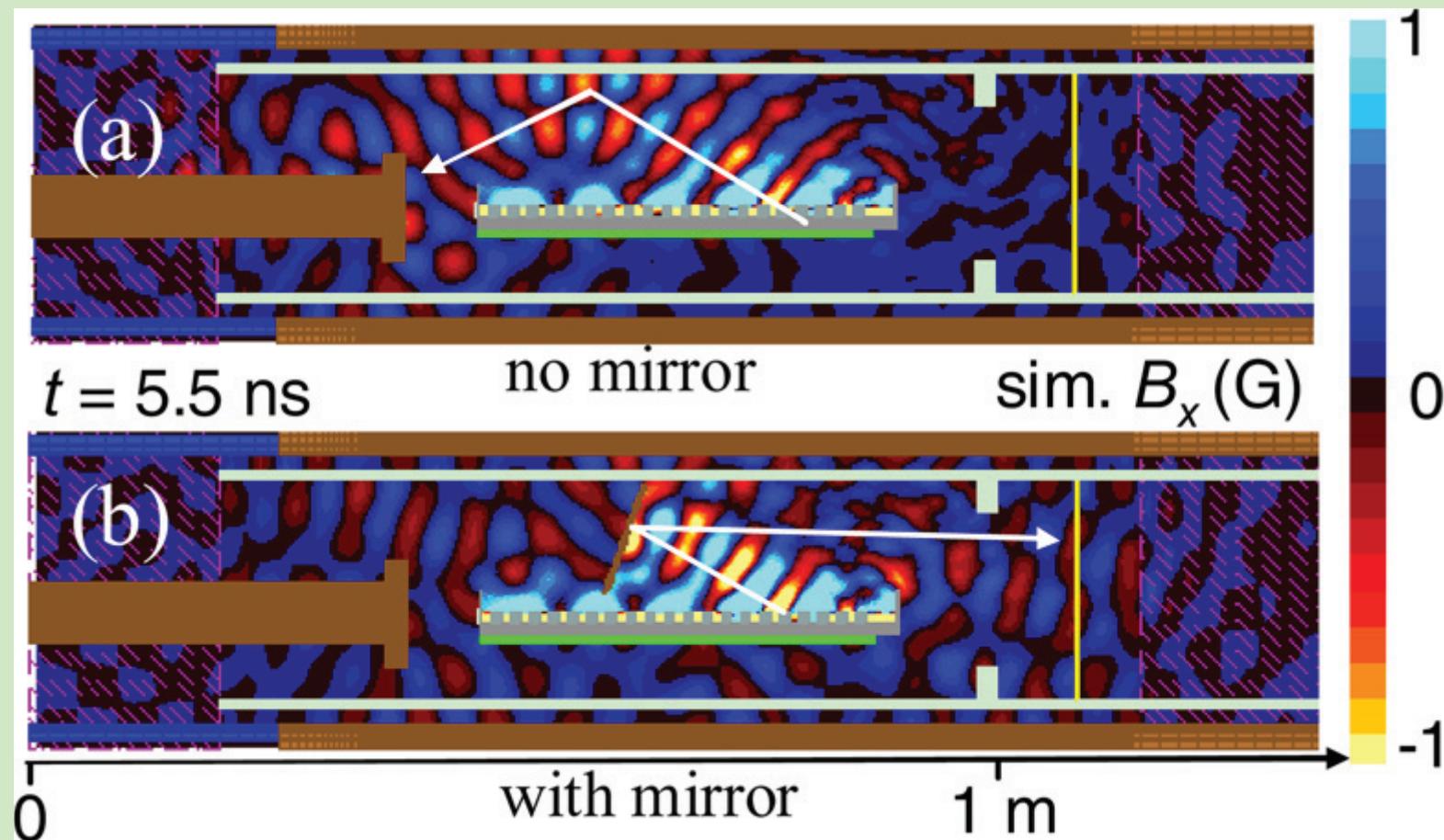
J. Gardelle, P. Modin and J.T. Donohue, "Observation of Copious Emission at the Fundamental Frequency by a Smith-Purcell Free-Electron Laser with Sidewalls", Appl. Phys. Lett. **100**, 131103 (2012),

3-D CESTA Experiment: Signals and Power.



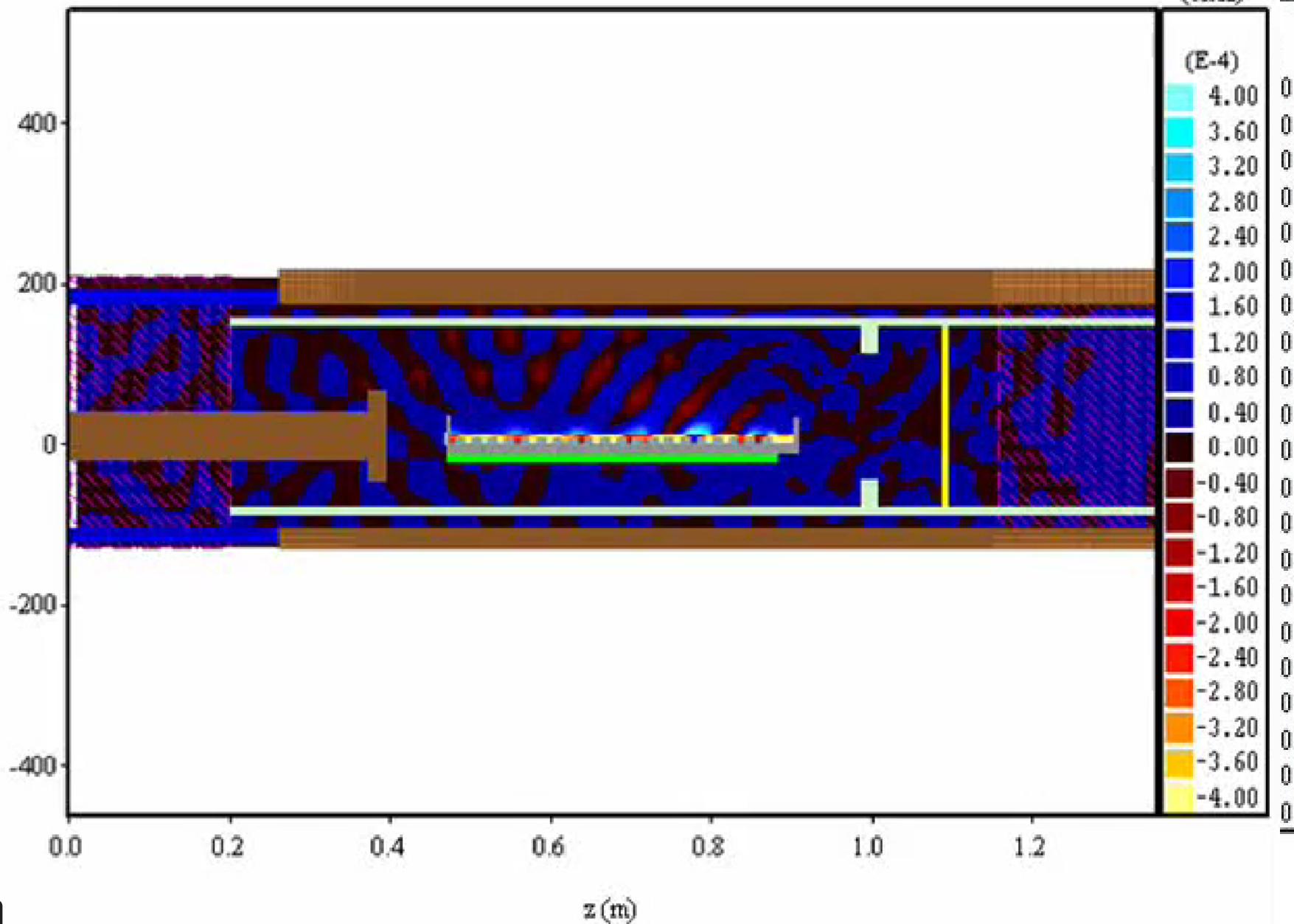
Oscilloscope signals,
FFT and energy
dependence of f .





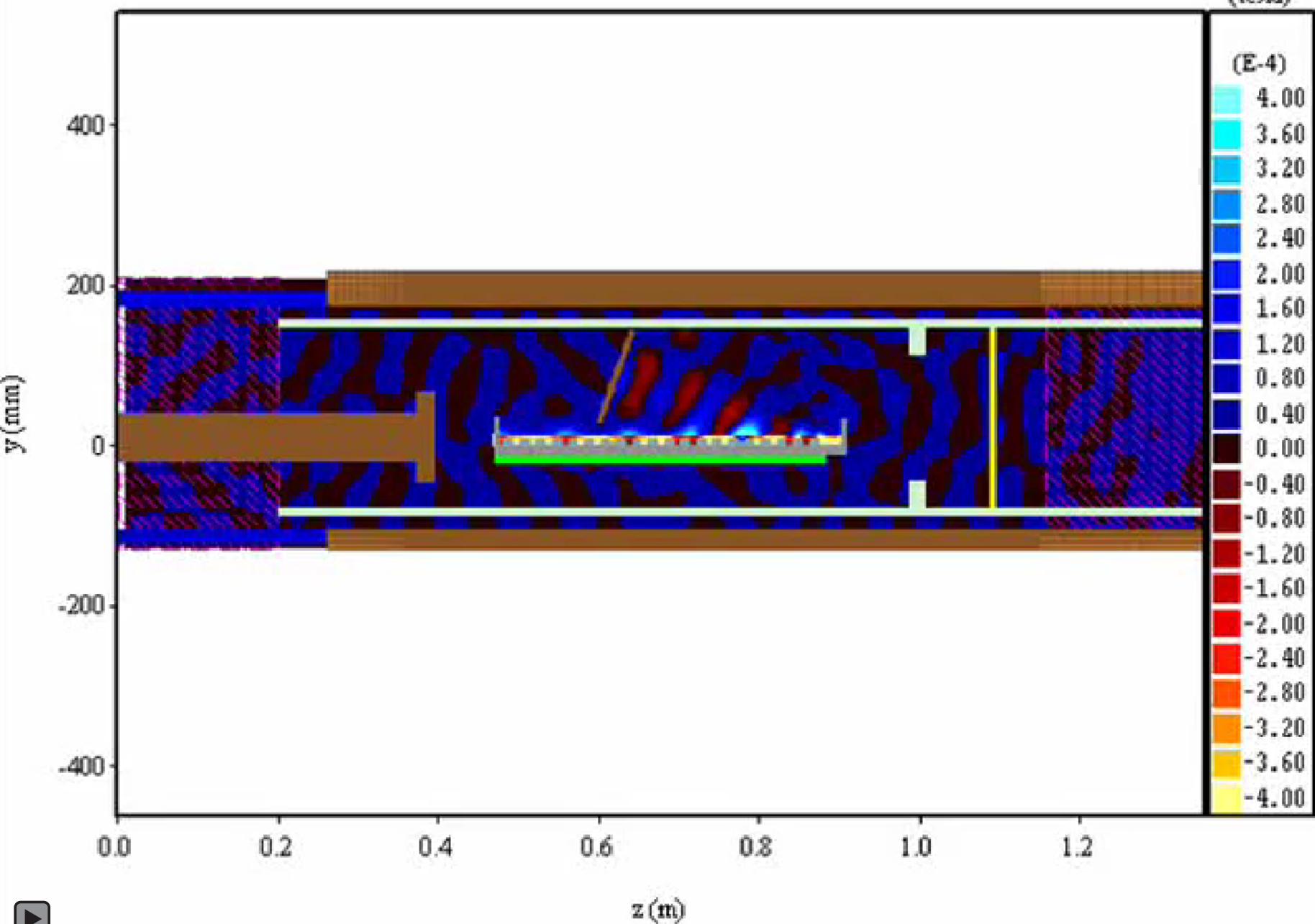
Bx at ZOOM @ 5.000 ns

(tesla)



Bx at ZOOM @ 5.000 ns

(tesla)



Conclusion

By a proper choice of grating width (between sidewalls), and beam energy, we can produce intense SP radiation at the fundamental frequency of the evanescent wave.