Artificial noise in PIC codes and consequences on long term tracking

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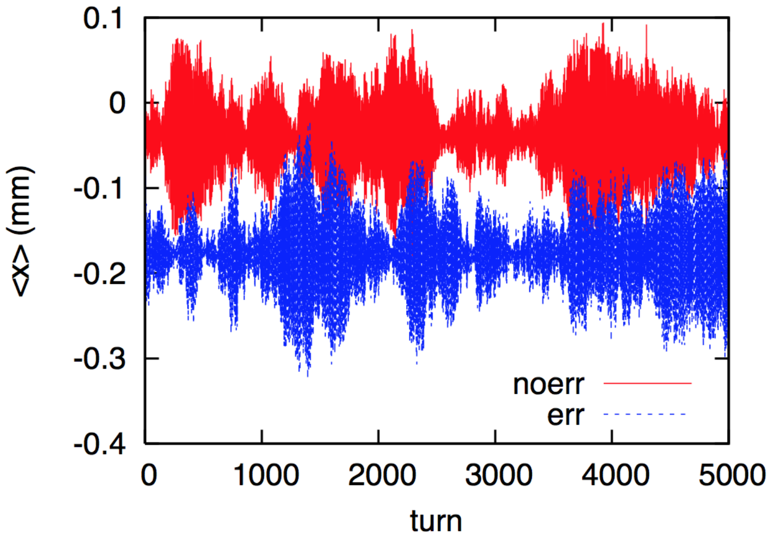
HB2014

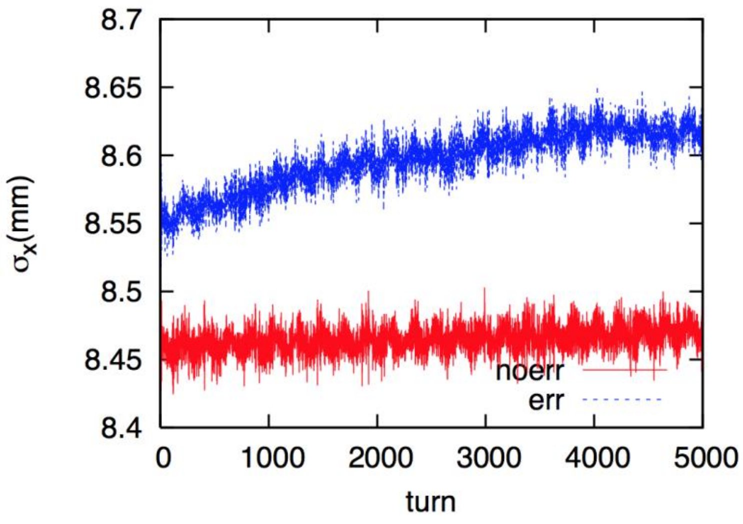
Thanks to H. Harada, S. Igarashi, A. Molodozhentsev, Y. Sato

What is noise in PICs

Example np=200,000 J-PARC MR using code "SCTR".

- Dipole position +-0.05mm~0.5%σ
- Beam size 8.5mm+-0.02mm, ~0.2%σ
- Beam shape





Experiences in Beam-beam interactions

- Fluctuation of collision offset due to crab cavity phase noise.

$$\delta x = \frac{c\theta_c}{2\omega_{RF}}\delta\varphi$$

θ_c: crossing angle, $\delta\varphi$: phase error, ω_{RF}:RF freq.

- Noise in bunch-by-bunch feedback system.
- Damping due to the feedback system

$$\Delta x_n = \left(1-\frac{1}{\tau}\right)\Delta x_{n-1} + \delta x\hat{r}$$

τ: feedback damping time   r:random number

δx: excitation noise (crab or feedback itself)

$$\Delta x^2 = \langle \Delta x_\infty^2 \rangle = \frac{\tau\delta x^2}{2} \qquad \langle \Delta x_\ell \Delta x_{\ell+n} \rangle = \Delta x^2 e^{-|n|/\tau}$$

Emittance growth due to noise

- Effective Hamiltonian due to round beam collision

$$U(x) = \frac{N_p r_p}{\gamma_p} \int_0^\infty \frac{1 - e^{-x^2/(2\sigma_r^2+q)}}{2\sigma_r^2 + q} dq$$

- Potential with offset

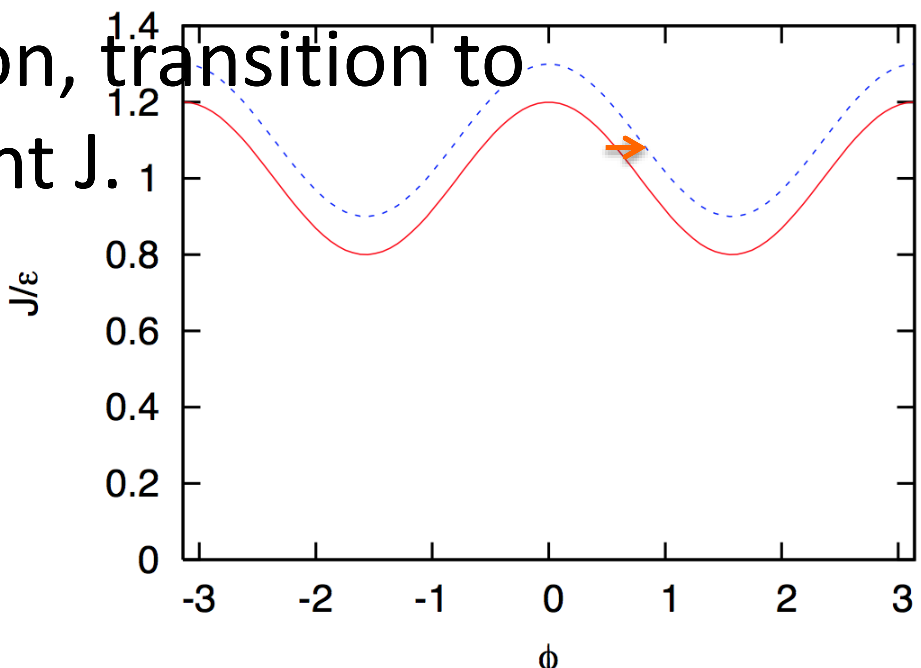
$$U(x + \Delta x) = U(x) + U'(x)\Delta x$$

- Change of J per collision, transition to trajectory with different J.

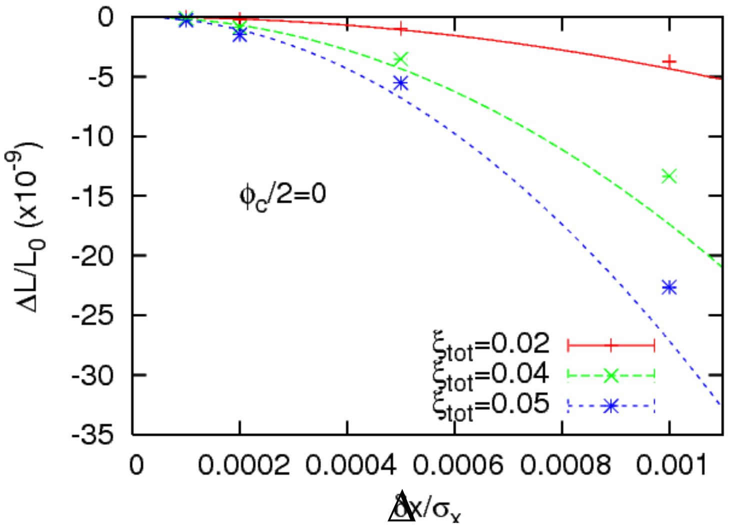
$$\Delta J = -\frac{\partial U}{\partial \psi}$$

$$\Delta L/L = \left(\xi \frac{\Delta x}{\sigma_r}\right)^2 \times 21.7$$

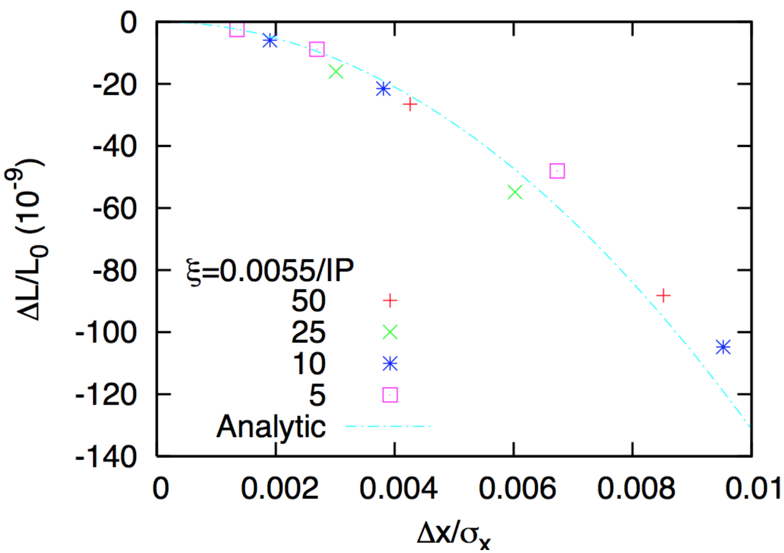
per IP

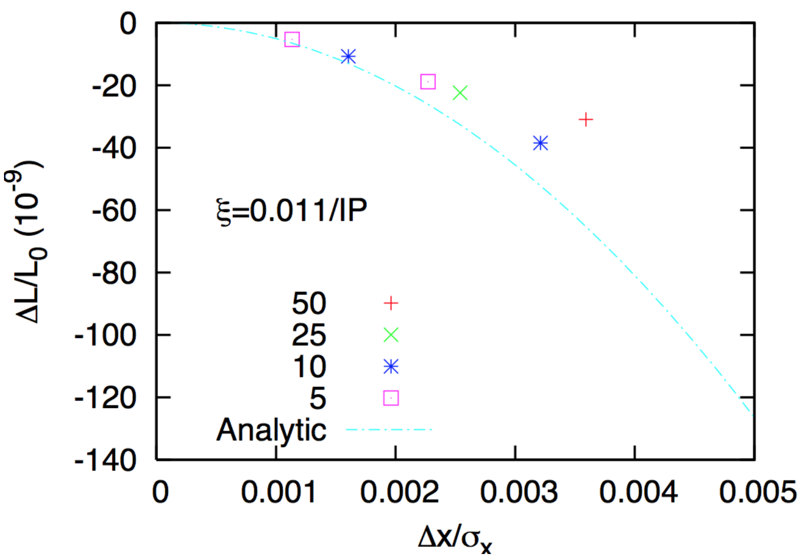


Luminosity degradation due to noise in LHC



- Frozen (W.S.) and nonfrozen (S.S.) models give good agreement with the formula.
- Nonfrozen model does not have accuracy for small noise $\Delta x/\sigma<0.001$ ($Np=10^6$).



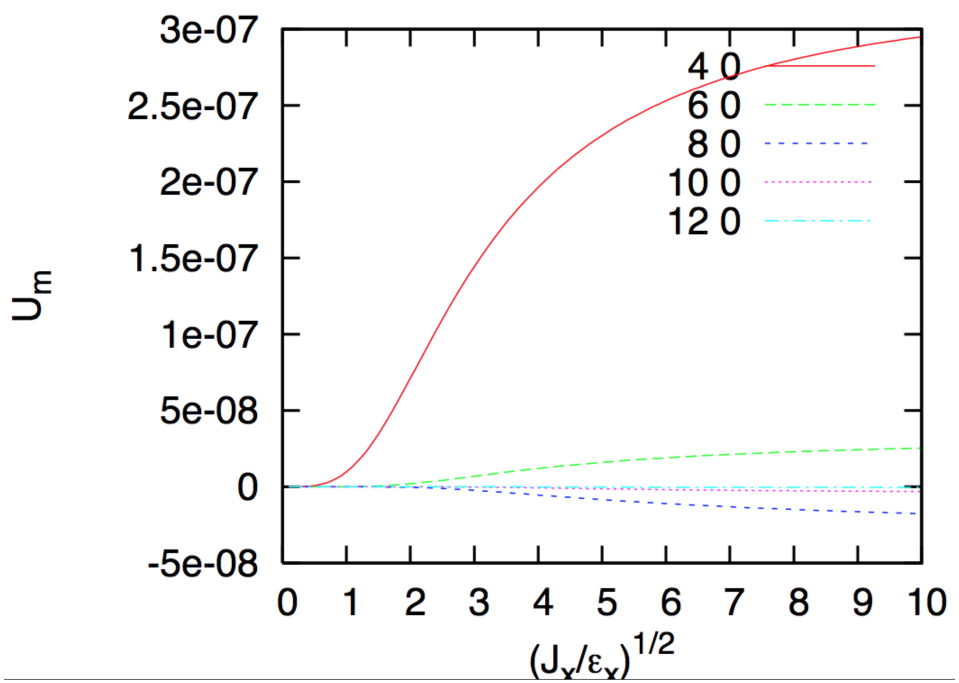


Application to space charge force

- Integrate along the beam line.

$$U_m(J_x, J_y) = \frac{\lambda_p r_p}{\beta^2\gamma^3} \oint ds \int_0^\infty \frac{du}{\sqrt{2\sigma_x^2+u}\sqrt{2\sigma_x^2+u}}$$

$$\left[\delta_{m_x 0}\delta_{m_y 0} - \exp(w_x - w_y)(-1)^{(m_x+x_y)/2} I_{m_x/2}(w_x) I_{m_y/2}(w_y) e^{-im_x\varphi_x - im_y\varphi_y}\right].$$



Real noise in J-PARC MR

- Tune modulation due to AC ripple
- $\Delta\nu_{xy}$~+-0.01, 50Hz

$$\Delta J = -\frac{\partial U}{\partial \psi}\Delta\psi \qquad J(N) - J(0) = -\sum_{i=0}^{N}\frac{\partial U}{\partial \psi}\Delta\psi_i$$

$$\langle \Delta J^2 \rangle \equiv \frac{\langle (J(N) - J(0))^2 \rangle}{N} = \frac{1}{N}\sum_{i=0}^{N}\sum_{j=0}^{N}\frac{\partial U_i}{\partial \psi}\frac{\partial U_j}{\partial \psi}\langle \Delta\psi_i \Delta\psi_j \rangle$$

- The summation is divided into (1)simple sum, which gives N, and (2) correlation of $\Delta\psi$.

$$\langle \Delta J^2 \rangle = \sum_{n=-\infty}^{\infty}\frac{\partial U_\ell}{\partial \psi}\frac{\partial U_{\ell+n}}{\partial \psi}\langle \Delta\psi_\ell \Delta\psi_{\ell+n} \rangle$$

Diffusion due to the tune modulation in J-PARC MR

- Tune modulation with correlation (50Hz, τ_c=4000 turns)

$$\langle \Delta\psi_\ell \Delta\psi_{\ell+n} \rangle = 4\pi^2 \Delta\nu^2 e^{-|n|/\tau_c}$$

$$\langle \Delta J^2 \rangle = \sum_{n=-\infty}^{\infty} \frac{\partial U_\ell}{\partial \psi} \frac{\partial U_{\ell+n}}{\partial \psi} \langle \Delta\psi_\ell \Delta\psi_{\ell+n} \rangle \qquad U = \sum_{k=0}^{\infty} U_k \cos 2k\psi$$

$$= 8\pi^2 \Delta\nu^2 \sum_{k=0}^{\infty} k^2 U_k^2 \sum_{n=0}^{\infty} e^{-|n|/\tau_s} \cos 4\pi k n \nu$$

$$= 8\pi^2 \Delta\nu^2 \sum_{k=0}^{\infty} k^2 U_k^2 \frac{\sinh 1/\tau_c}{\cosh 1/\tau_c - \cos 4\pi k\nu}$$

$\sqrt{\langle \Delta J^2 \rangle} \sim 5.6 \times 10^{-10} m$ at $J \sim 4\varepsilon = 40 \times 10^{-6} m$

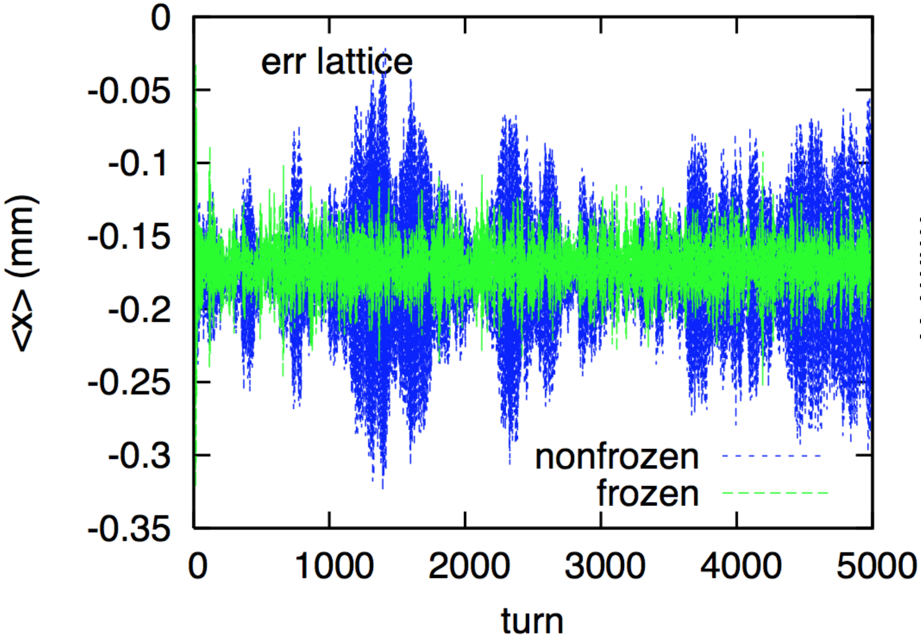
- Other fluctuations by ripple, β, η,…

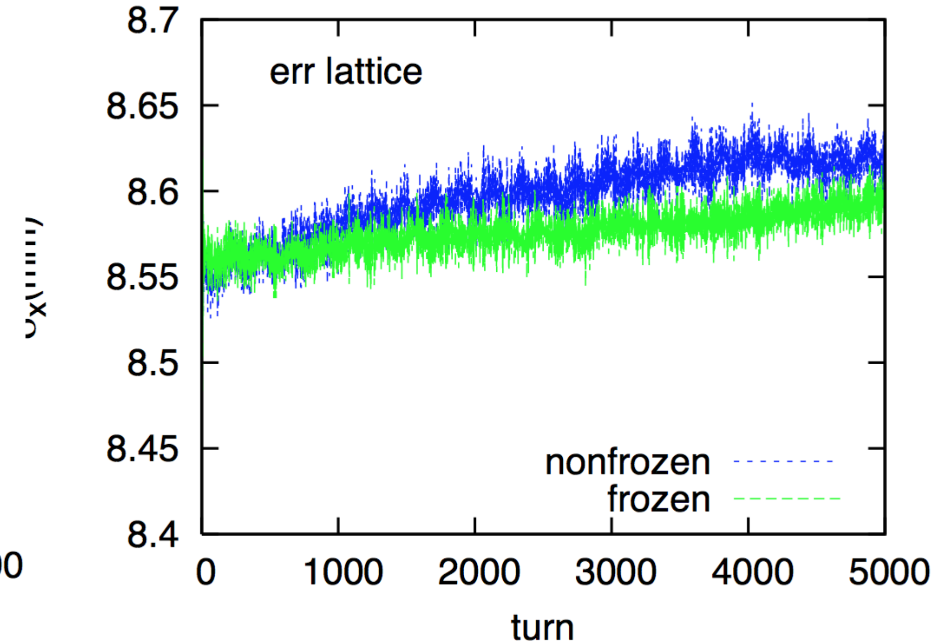
Frozen model in SCTR code

- Frozen mode has used to simulate acceleration of J-PARC MR (PAC07(THPAN040), HB2010(WEO1A04).
- Potential is calculated every 1-2 m and stored all spline data (~GB). It can be revised arbitrary time.
- Injection at flat energy 3 GeV, 120ms (25,000 turns) and acceleration 3-30GeV in 1.1 sec (+210,000 turns) in J-PARC MR.
- Potential is revised every 50 turns for J-PARC full acceleration simulation, since the beam size shrinks 0.3% (statistical level) in 50 turns due to acceleration.

Frozen model

- Dipole motion and beam size evolution

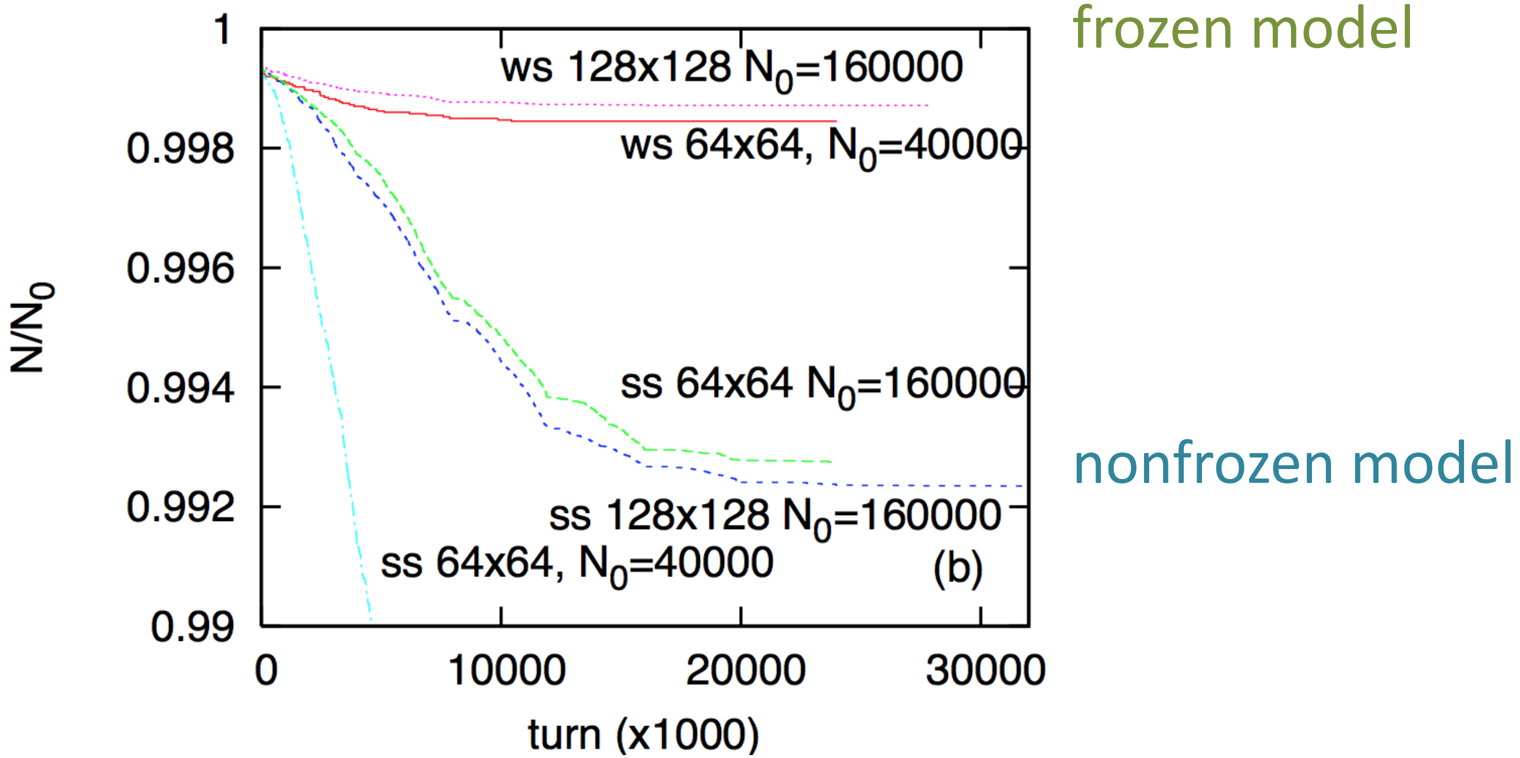




- Smooth noise in dipole motion.
- Amplitude in nonfrozen model is real or artifact?
- Some difference in beam size evolution. No artificial/real quadrupole motion.

Comparison of frozen model and turn-by-turn PIC (PAC07, (THPAN040))

- Beam loss is independent of statistics and mesh size in frozen model.
- Beam loss strongly depends on statistics in nonfrozen model.

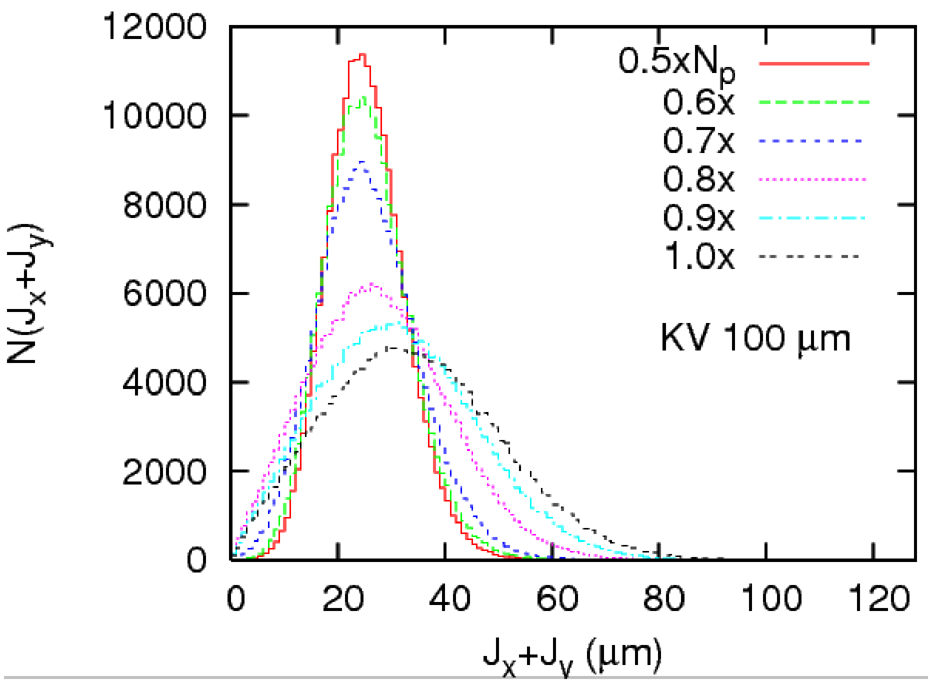


Long term tracking in J-PARC MR

- Initial KV $J_x+J_y=100\mu m$@400MeV in RCS

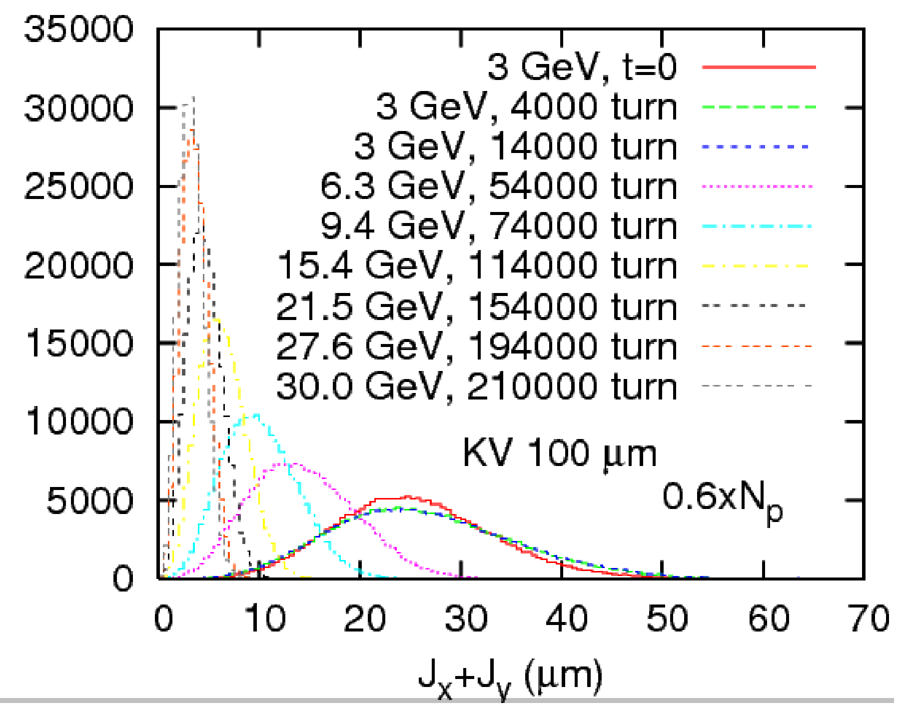
RCS extraction 3GeV

$\beta\gamma=1/4$, $J_x+J_y=25mm$



MR acceleration

HB2010(WEO1A04)

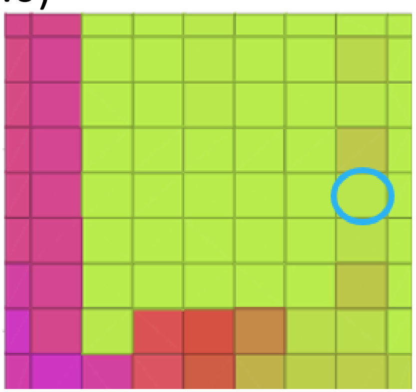


Recent studies on J-PARC MR
Choice of operating point

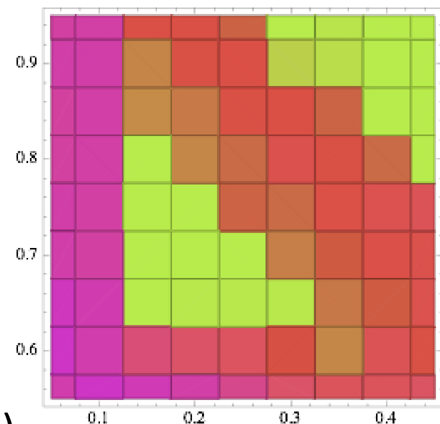
- Present operating point (22.4,20.75).
- Beam loss map in tune space.

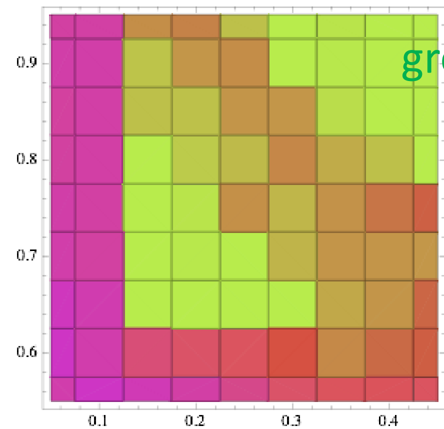
No error | with error | x-y cpupling corr.

(22.0,21.0)



(22.0,20.5) (22.5,20.5)





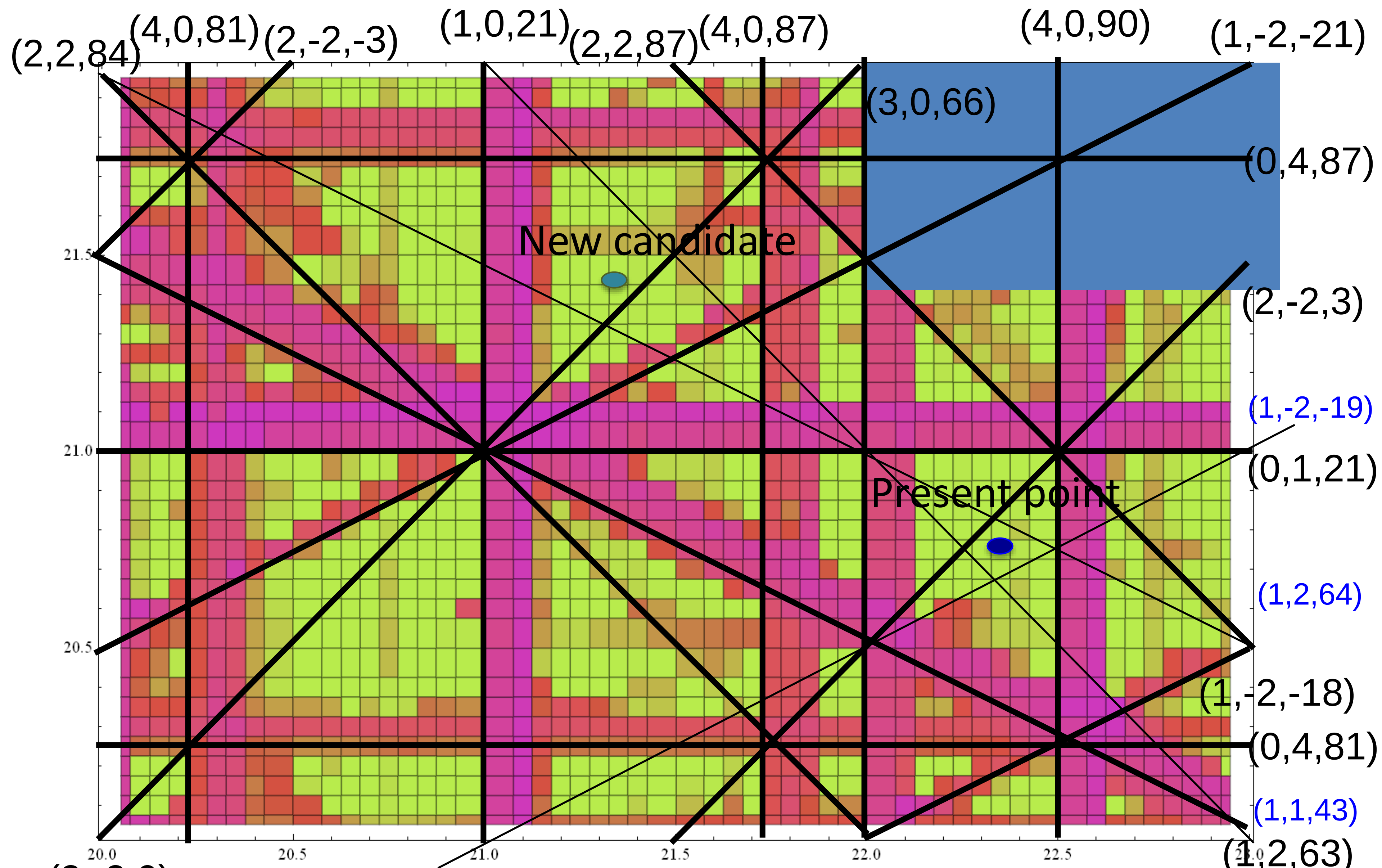
green: better

green: $\Delta N/N_0 < 10^{-4}$ @5000 turn

- Good tune area, but degrade with error.
- Even coupling correction, some degradation remains.

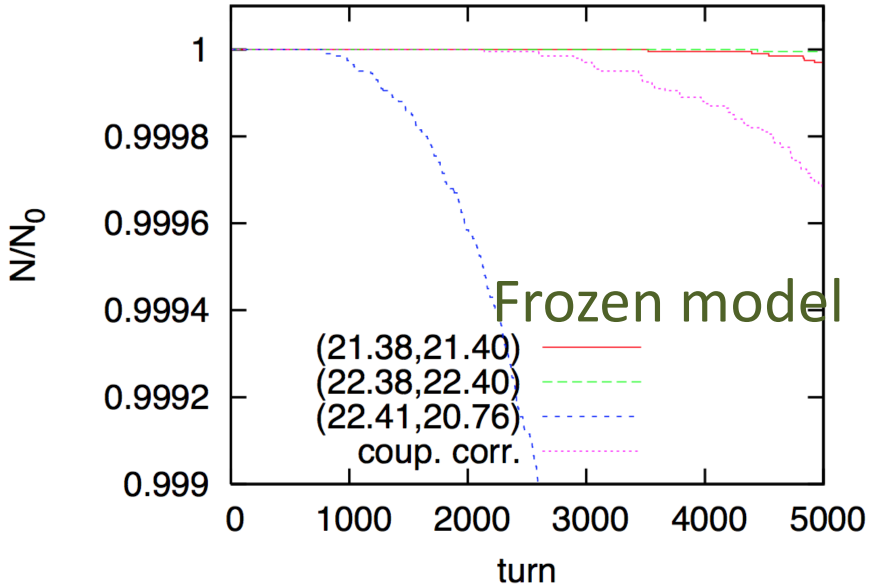
Initial tune shift: 0.16@Np=2.5x10^{13}, B=0.25

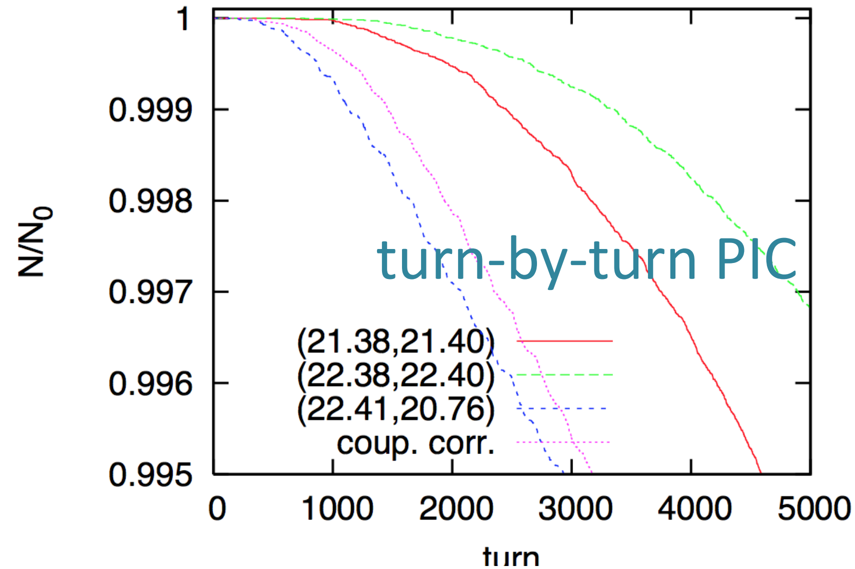
Beam loss due to space charge; green: better



green: $\Delta N/N_0 < 10^{-4}$ @5000 turn

Beam loss at new operating points





- *.38,*.40 is better than the present operating point, 22.41,20.76 even coupling correction.
- Why ν_x~ν_y? Better integrability due to angular momentum conservation.
- Trying 21.38,21.40 and 22.38,22.40 in Nov. 2014
- Difference is seen between frozen and nonfrozen.

Space charge induced resonances and their widths

- Frozen model simulates particle motion under the nonlinear space charge potential.
- Emittance growth and beam loss are caused by diffusion around resonances.

$$U(J_x, \phi_x, J_y, \phi_y) = \sum_{m_x, m_y = 0}^{\infty} U_{m_x, m_y} \cos(m_x \phi_x + m_y \phi_y)$$

$$U_{m_x,m_y}(J_x, J_y) = \frac{\lambda_p r_p}{\beta^2 \gamma^3} \oint ds \int_0^\infty \frac{du}{\sqrt{2\sigma_x^2 + u}\sqrt{2\sigma_x^2 + u}}$$

$$\left[\delta_{m_x 0}\delta_{m_y 0} - \exp(w_x - w_y)(-1)^{(m_x + x_y)/2} I_{m_x/2}(w_x) I_{m_y/2}(w_y) e^{-i m_x \varphi_x - i m_y \varphi_y}\right].$$

Tune shift and slope

- Tune shift

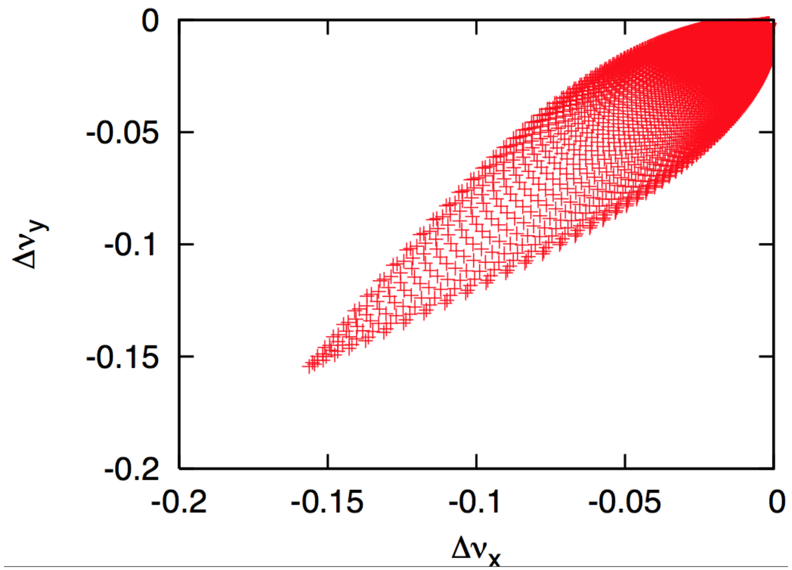
$$2\pi\Delta\nu_x = -\frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds \frac{\beta_x}{\sigma_x^2}\int_0^\infty \frac{d\eta}{(2+\eta)^{3/2}(2r_{yx}+\eta)^{1/2}}\left[e^{-w_x-w_y}(I_0(w_x)-I_1(w_x))I_0(w_y)\right]$$

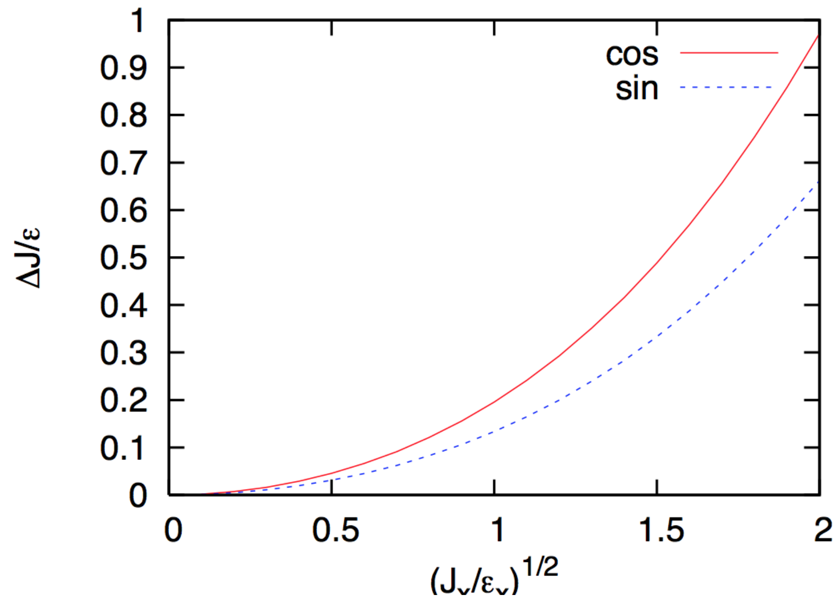
$$2\pi\Delta\nu_y = -\frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds \frac{\beta_x}{\sigma_x^2}\int_0^\infty \frac{d\eta}{(2+\eta)^{1/2}(2r_{yx}+\eta)^{3/2}}\left[e^{-w_x-w_y}I_0(w_x)(I_0(w_y)-I_1(w_y))\right]$$

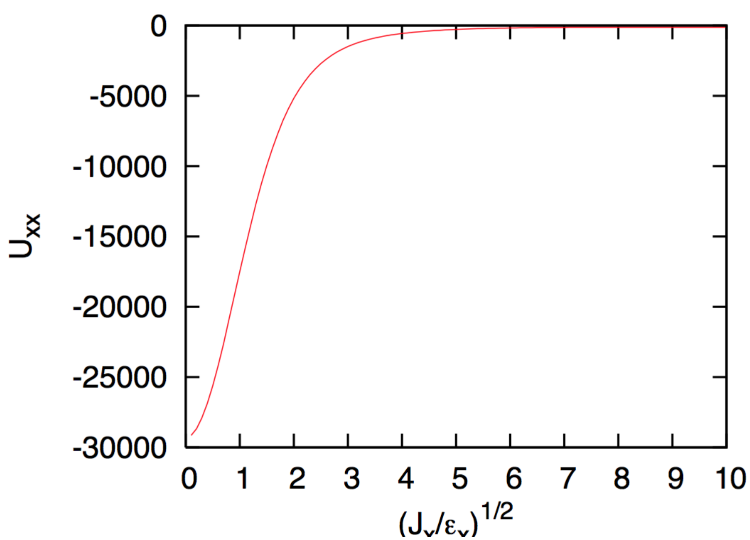
- Tune slope

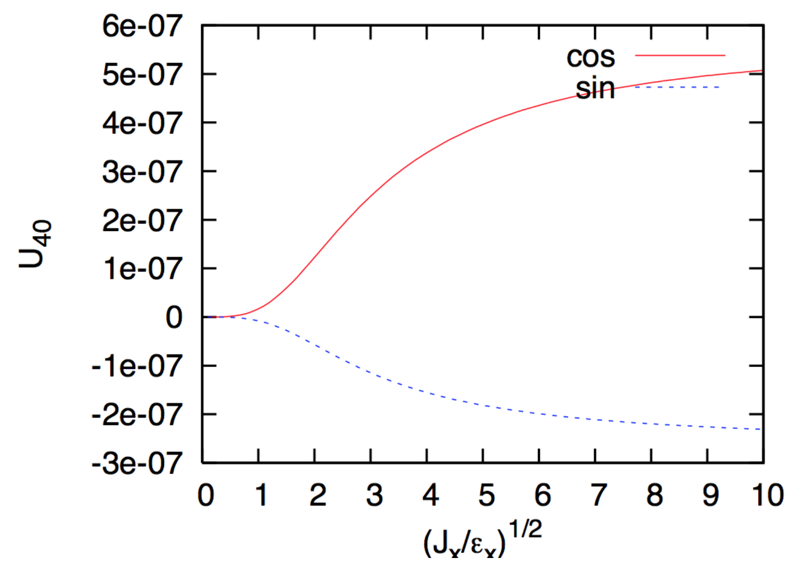
$$\begin{aligned}\frac{\partial^2 U_{00}}{\partial J_x^2} &= -2\pi\frac{\partial \nu_x}{\partial J_x}\\ &= \frac{\lambda_p r_p}{\beta^2\gamma^3}\oint ds \frac{\beta_x^2}{\sigma_x^4}\int_0^\infty \frac{d\eta}{(2+\eta)^{5/2}(2r_{yx}+\eta)^{1/2}}\left[e^{-w_x-w_y}\left\{\frac{3}{2}I_0(w_x)-2I_1(w_x)+\frac{1}{2}I_2(w_x)\right\}I_0(w_y)\right]\end{aligned}$$

Tune slope and resonance width





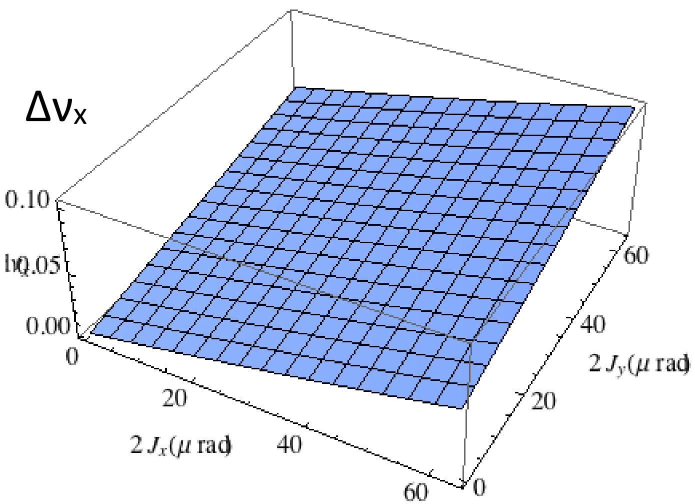


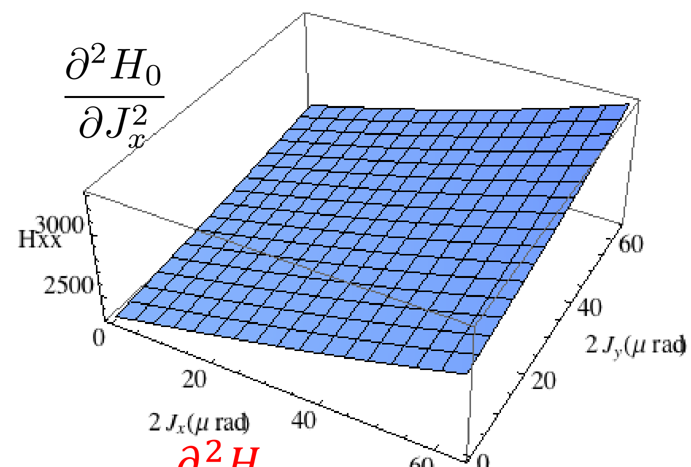


Lattice induced tune spread and resonances

- Expand one turn map by 12-th order

$$
\begin{aligned}
H_{00}(\boldsymbol{J}) \quad = \quad & 3.43103\times10^{14}J_x^6 + 7.36914\times10^{14}J_x^5J_y + 7.17029\times10^{11}J_x^5 + 2.34124\times10^{15}J_x^4J_y^2 \\
+ \quad & 1.70991\times10^{12}J_x^4J_y + 1.43961\times10^{8}J_x^4 + 4.48931\times10^{15}J_x^3J_y^3 + 2.20917\times10^{12}J_x^3J_y^2 \\
+ \quad & 2.50211\times10^{8}J_x^3J_y + 613899.J_x^3 + 3.33998\times10^{15}J_x^2J_y^4 + 1.79716\times10^{12}J_x^2J_y^3 \\
+ \quad & 7.07531\times10^{8}J_x^2J_y^2 + 809323.J_x^2J_y + 1095.71J_x^2 + 7.58773\times10^{14}J_xJ_y^5 \\
+ \quad & 5.7438\times10^{11}J_xJ_y^4 + 4.55828\times10^{8}J_xJ_y^3 + 650655.J_xJ_y^2 + 2096.06J_xJ_y \\
+ \quad & 4.11283\times10^{13}J_y^6 + 4.00294\times10^{10}J_y^5 + 5.3027\times10^{7}J_y^4 + 79924.4J_y^3 + 1106.98J_y^2
\end{aligned}
$$





$$\frac{\partial^2 H_{00}}{\partial J_x^2} = 1000\sim3000 \approx U_{00}(3\sigma)$$

shot 40722

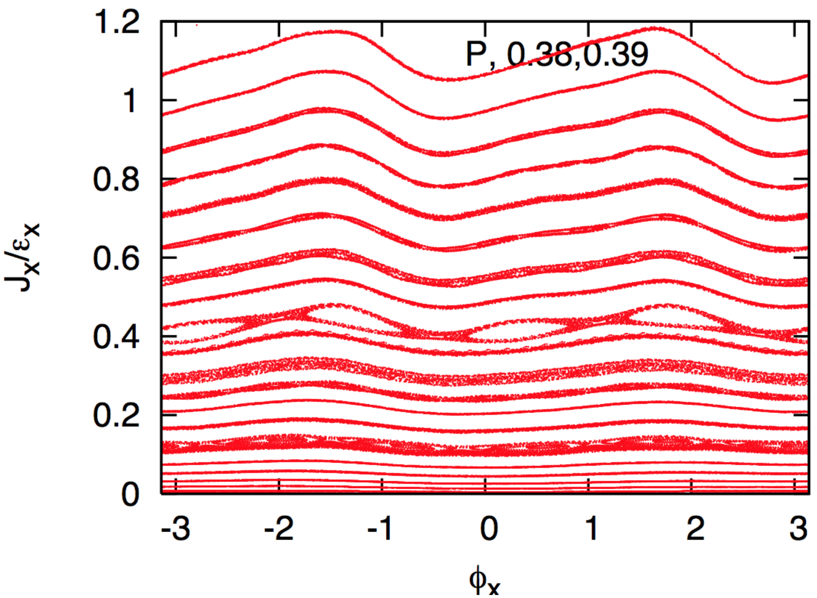
mx	my	Jx	Jy	|G|	B0	B	BR
1	0	3.6E-05	0.0E+00		4.84E-08	1.88E-07	1.86E-07
2	0	3.6E-05	0.0E+00		2.47E-08	4.55E-08	4.66E-08
1	1	1.8E-05	1.8E-05		1.28E-25	1.67E-26	4.01E-09
0	2	0.0E+00	3.6E-05		5.55E-09	3.91E-09	2.69E-09
3	0	3.6E-05	0.0E+00		5.46E-08	1.29E-07	1.32E-07
2	1	1.8E-05	1.8E-05		2.09E-25	1.42E-26	1.42E-07
2	-1	1.8E-05	1.8E-05		2.16E-25	4.52E-27	7.96E-08
1	2	1.8E-05	1.8E-05		4.66E-08	1.78E-07	1.83E-07
1	-2	1.8E-05	1.8E-05		1.48E-07	2.72E-07	2.72E-07
0	3	0.0E+00	3.6E-05		1.42E-25	1.59E-26	1.10E-07
4	0	3.6E-05	0.0E+00		2.50E-07	2.51E-07	2.51E-07
3	1	1.8E-05	1.8E-05		1.93E-26	2.52E-27	6.80E-09
3	-1	1.8E-05	1.8E-05		1.61E-26	4.97E-27	7.04E-10
2	2	1.8E-05	1.8E-05		2.49E-08	5.90E-09	5.58E-09
2	-2	1.8E-05	1.8E-05		1.27E-08	8.40E-09	8.03E-09
1	3	1.8E-05	1.8E-05		2.52E-26	5.66E-27	3.56E-09
1	-3	1.8E-05	1.8E-05		1.63E-26	1.10E-26	8.42E-10
0	4	0.0E+00	3.6E-05		1.20E-08	1.45E-08	1.42E-08

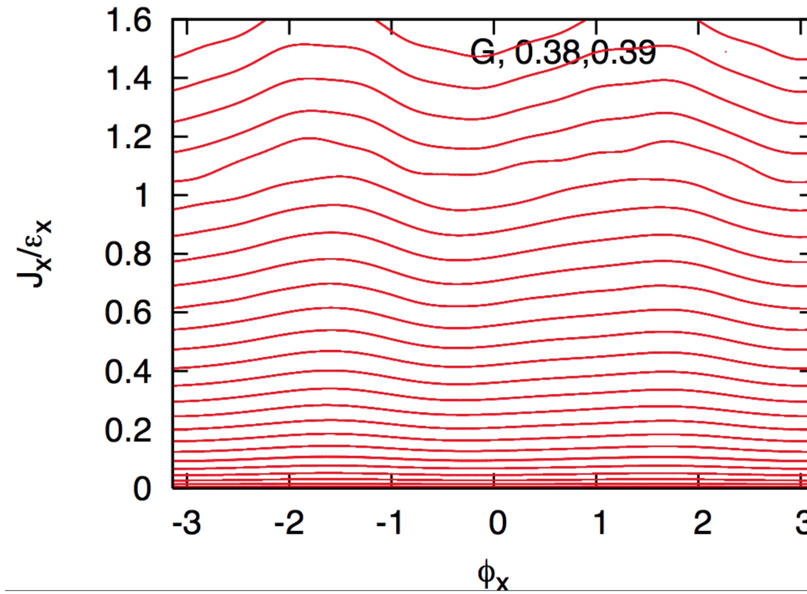
mx	my	Jx	Jy	|G|	B0	B	BR
5	0	3.6E-05	0.0E+00		4.03E-09	3.07E-09	3.08E-09
4	1	1.8E-05	1.8E-05		7.11E-27	1.22E-28	6.63E-10
4	-1	1.8E-05	1.8E-05		7.09E-27	1.79E-28	1.63E-10
3	2	1.8E-05	1.8E-05		1.21E-09	1.62E-09	1.68E-09
3	-2	1.8E-05	1.8E-05		1.16E-09	2.20E-09	2.24E-09
2	3	1.8E-05	1.8E-05		2.69E-27	6.82E-28	3.93E-10
2	-3	1.8E-05	1.8E-05		1.08E-27	1.59E-27	2.35E-10
1	4	1.8E-05	1.8E-05		7.77E-11	5.31E-10	5.55E-10
1	-4	1.8E-05	1.8E-05		2.09E-10	3.49E-10	3.70E-10
0	5	0.0E+00	3.6E-05		1.37E-26	5.48E-27	9.37E-11
6	0	3.6E-05	0.0E+00		2.24E-09	1.65E-09	1.64E-09
5	1	1.8E-05	1.8E-05		2.74E-28	4.68E-29	6.53E-11
5	-1	1.8E-05	1.8E-05		2.35E-28	8.57E-29	2.08E-11
4	2	1.8E-05	1.8E-05		1.49E-10	1.63E-10	1.66E-10
4	-2	1.8E-05	1.8E-05		4.80E-11	1.06E-10	1.07E-10
3	3	1.8E-05	1.8E-05		5.55E-28	1.78E-28	8.94E-11
3	-3	1.8E-05	1.8E-05		7.28E-28	1.02E-28	1.96E-11
2	4	1.8E-05	1.8E-05		6.74E-11	8.74E-11	8.47E-11
2	-4	1.8E-05	1.8E-05		3.62E-11	1.22E-11	9.02E-12
1	5	1.8E-05	1.8E-05		1.16E-27	4.70E-28	5.52E-12
1	-5	1.8E-05	1.8E-05		5.13E-28	9.87E-28	8.16E-12
0	6	0.0E+00	3.6E-05		6.07E-13	1.52E-11	9.43E-12

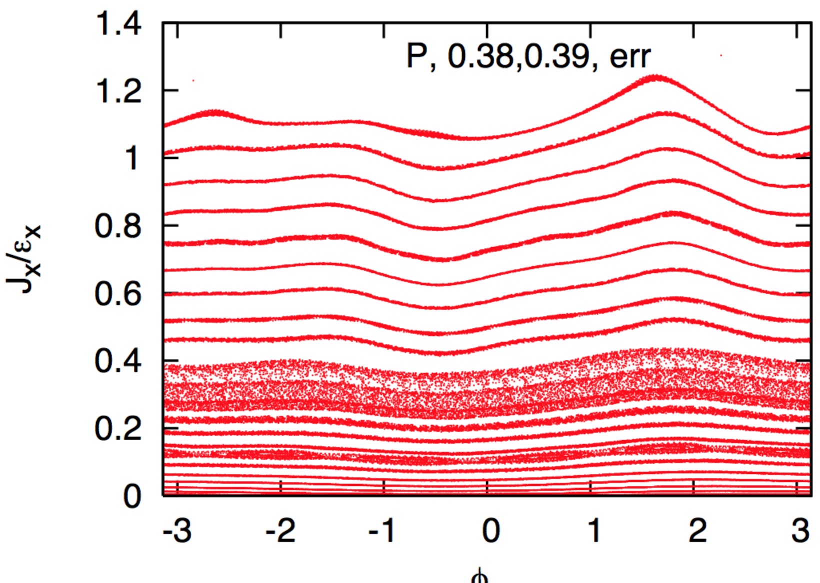
Resonances seen in frozen model

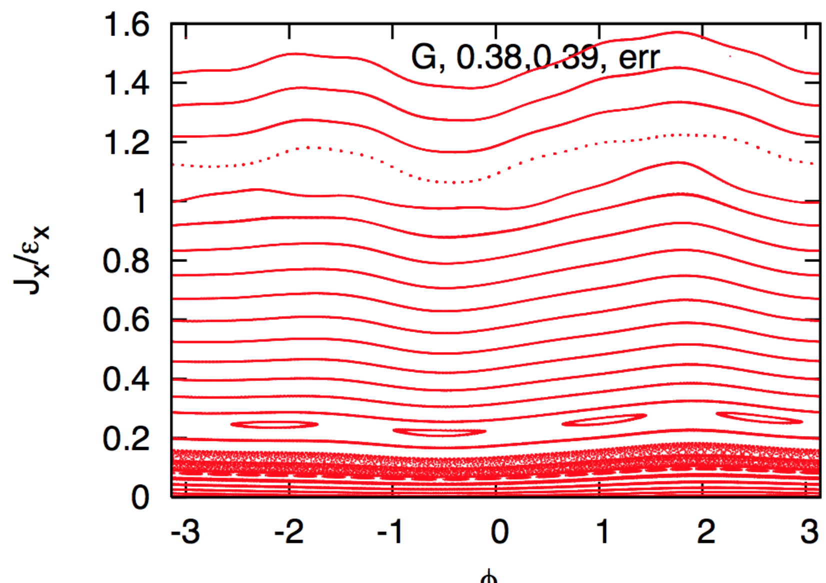
- Tune spread and resonances for combined space charge and lattice nonlinearity.
- Potential is fixed at first turn with parabolic or Gaussian.
- Particles generated an transverse interval are tracked in the fixed potential.
- No synchrotron motion to see resonances easily.

- Parabolic   Gaussian



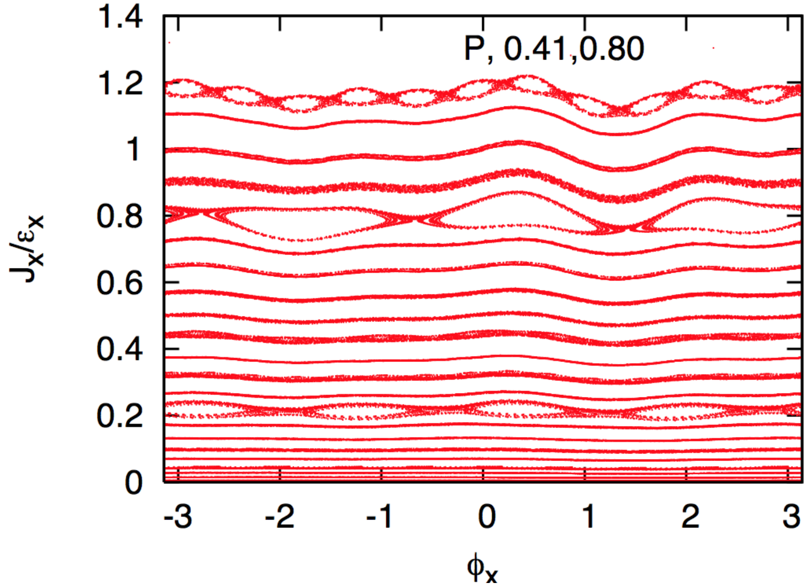






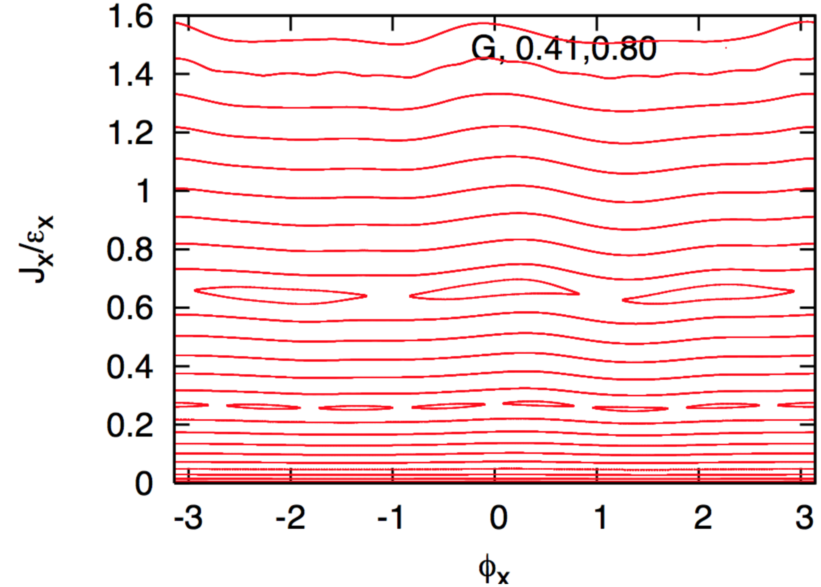
Parabolic

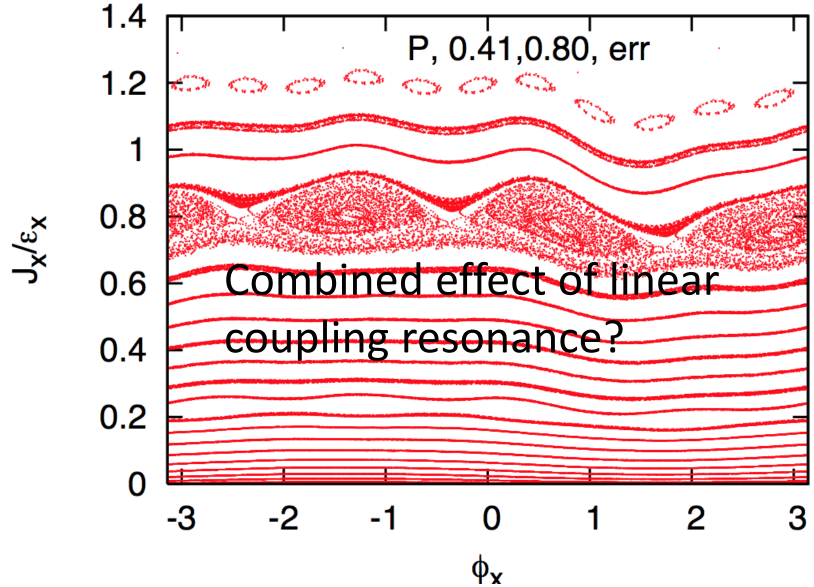
J/ε=1 is max amp.

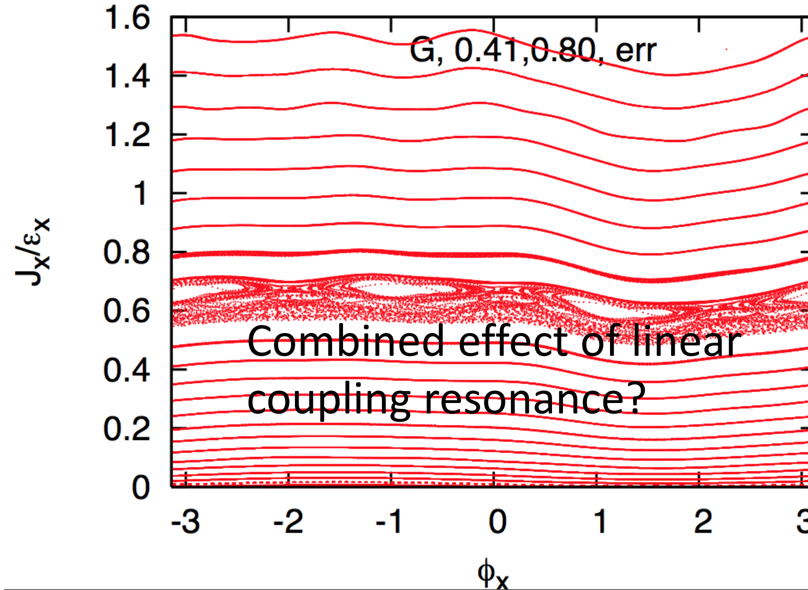


Gaussian

J/ε=1 is 2σ amp.







Resonance width and emittance growth

- Toy model reproducing a given resonance width

$$H = \mu J + \left(J + \frac{e^{-2aJ}}{2a} \right) + bJ\cos m\phi$$

- Corresponding symplectic map

$$J_{n+1} = \frac{J_n}{1 - mb\sin m\phi_n}$$

$$\phi_{n+1} = \phi_n + \mu + (1 - e^{-2aJ_{n+1}}) + b\cos m\phi_n$$

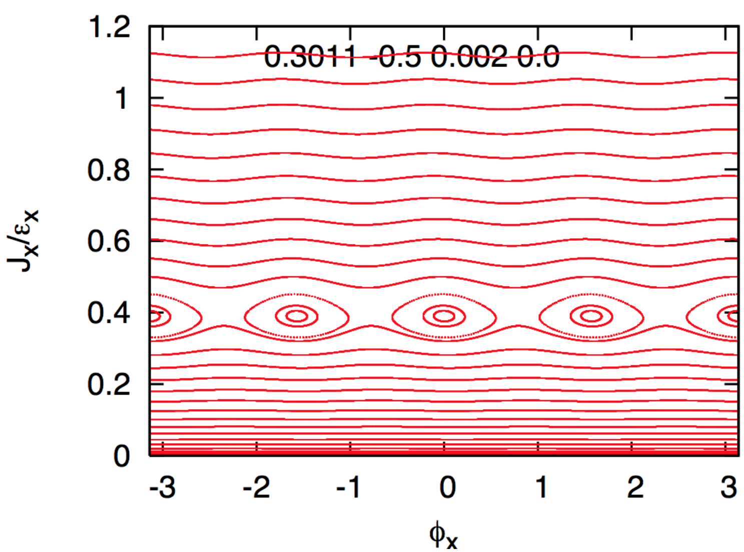
Resonance width in the toy model

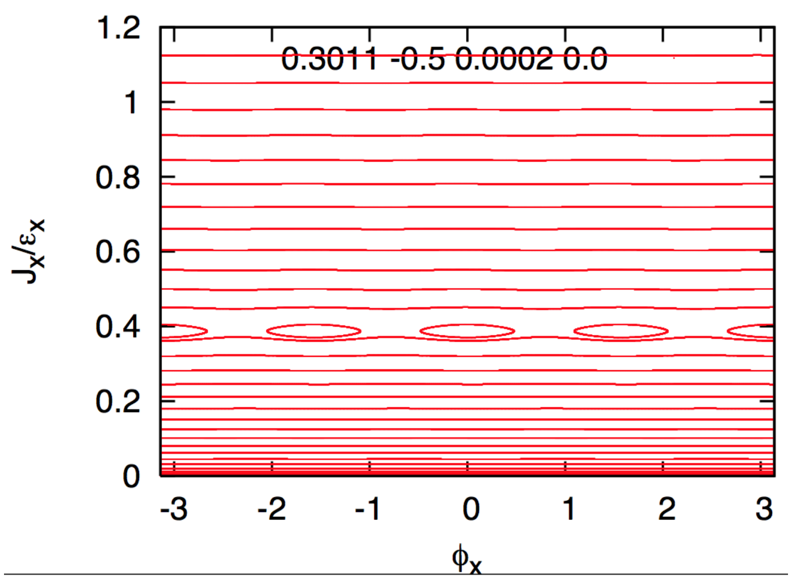
- Formula

$$\Delta J = \sqrt{\frac{2J_R b}{a e^{-2aJ_R}}} \quad \text{Half width}$$

a=0.5,

b=0.002, Δ*J*=0.07 *b*=0.0002, Δ*J*=0.02





Synchrotron motion

- Emittance growth due to a single resonance is not strong.
- Modulation of synchrotron oscillation makes worse the emittance growth drastically.

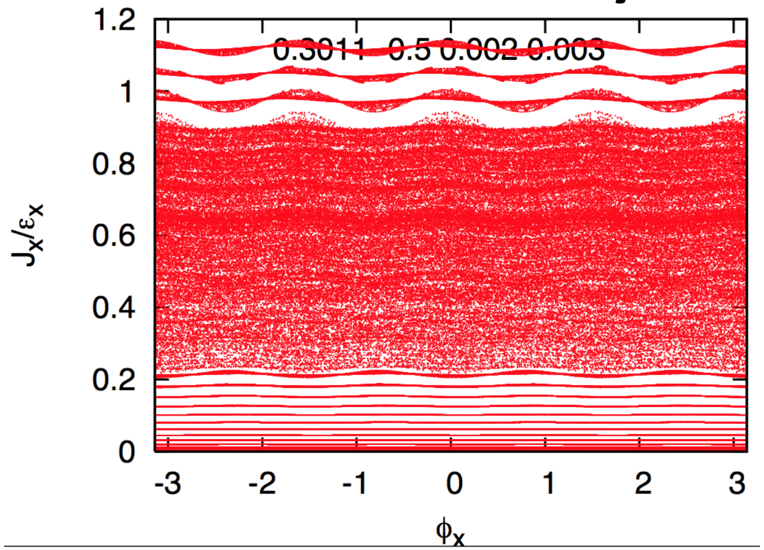
$$H = \mu J - \left(J + \frac{e^{-2aJ}}{2a} \right) + bJ \cos m\varphi$$

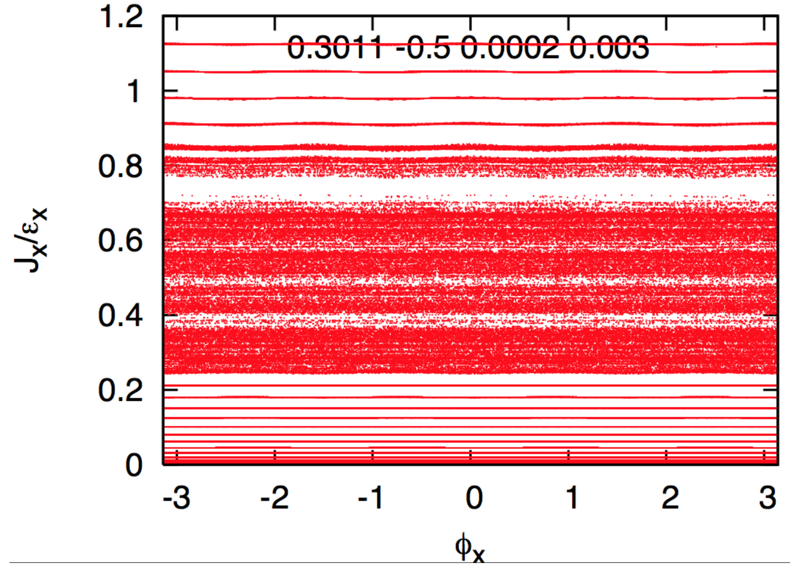
- Time dependence in *a* is assumed to take into account synchrotron motion effectively

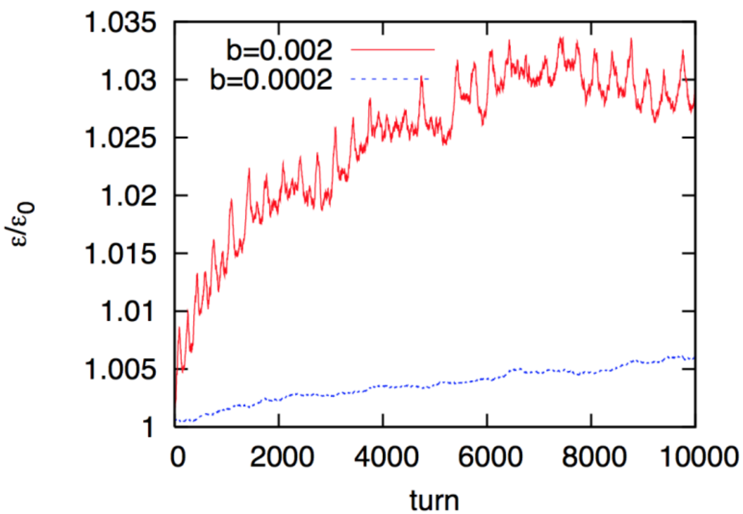
$$a = \frac{a_0}{2}(1 + \cos 2\pi\nu_s n)$$

- Resonant amplitude, which increases for small a, disagree space charge force. This model should be improved.

Enhanced diffusion due to the effective synchrotron oscillation







Emittance growth

Summary

- Noise of PIC simulation
- Collision offset noise in colliders, which is similar situation, is real issue.
- Noise of power supply ripple in J-PARC can be treated similar way.
- To avoid artificial PIC noise, frozen model is used as cross check of (ordinary) nonfrozen model.
- The frozen model is limited to study incoherent emittance growth.
- Tools to study incoherent resonance effects is being prepared.
- Synchrotron motion is important to understand emittance growth combined with the resonances.