

# Challenges in Simulating Beam Dynamics of Dielectric Laser Acceleration



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Bundesministerium  
für Bildung  
und Forschung

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Stanford University,  
Stanford, CA 94305, USA



- Introduction to Dielectric Laser Acceleration (DLA)
  - Experimental demonstrations
  - The ACHIP collaboration
- Beam dynamics: tracking with *DLAtrack6D*
- Stabilization and scalability of DLA
- Current and future experiments

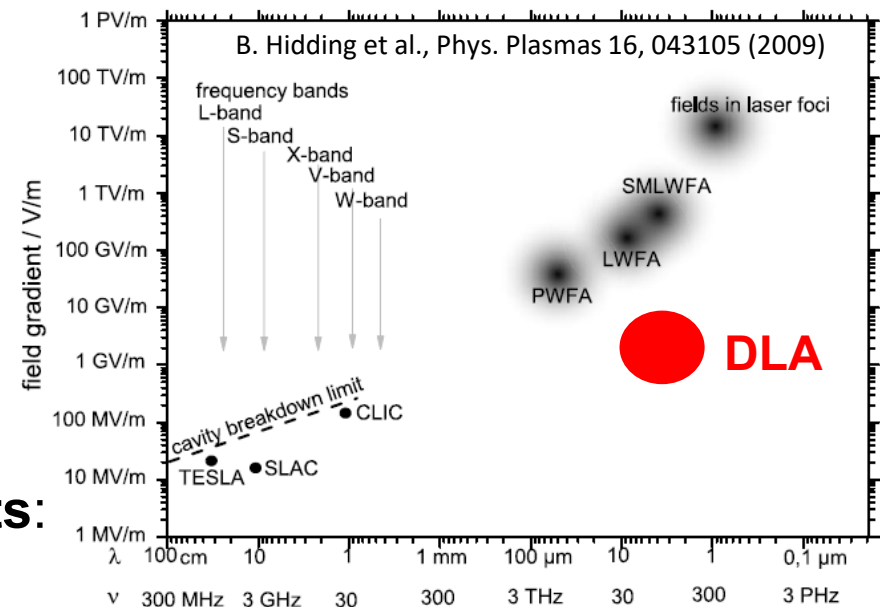
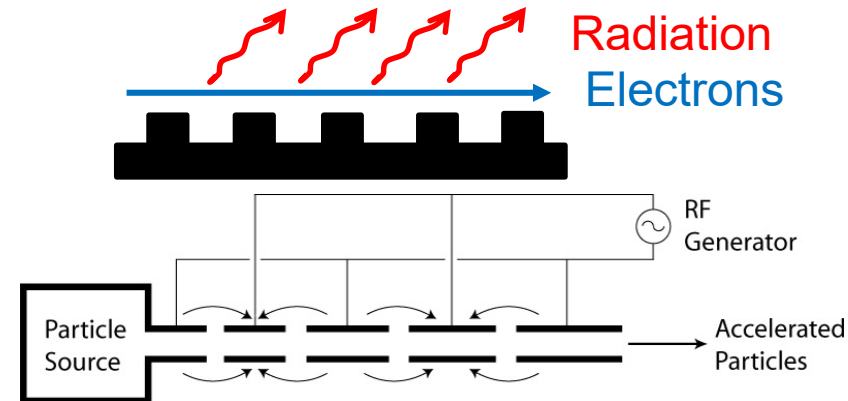
# Dielectric Laser Accelerator (DLA) principle

- Idea is quite old  
→ **Inverse** effects
  - Smith-Purcell (grating radiation)
  - Cherenkov (electrons superluminal in material)

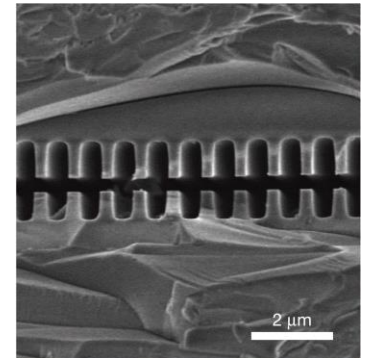
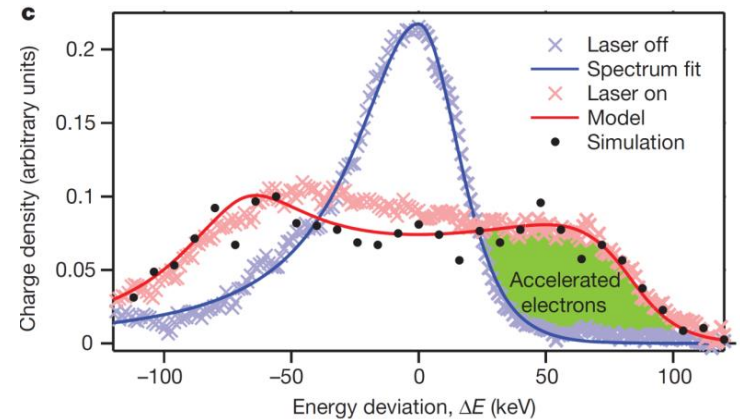
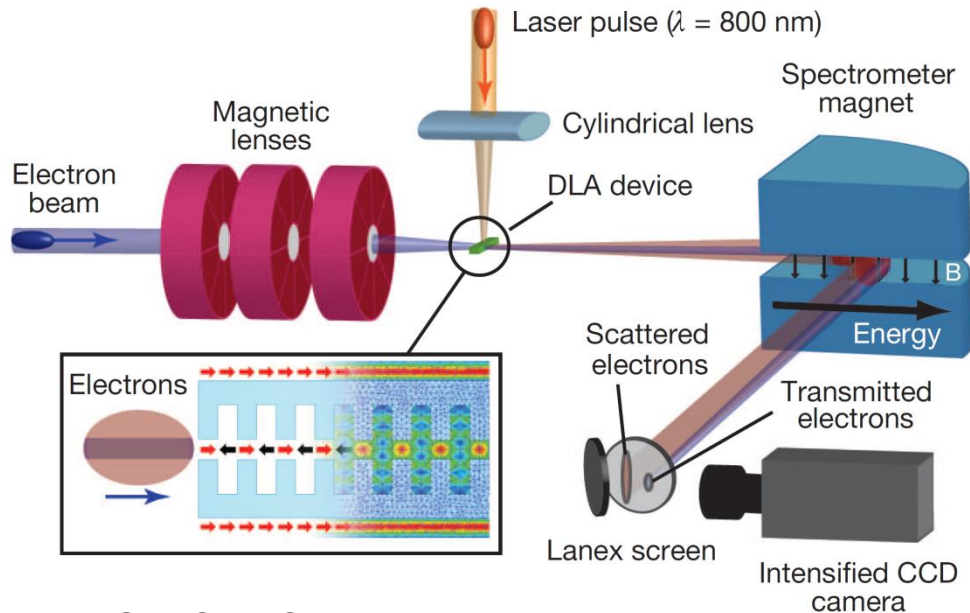
- Same principle as **Wideroe-Linac** (Non- resonant)

- Dielectrics** can withstand fields up to **10GV/m**
- Gradients larger than 1GeV/m** (limited by breakdown)

- Recent **technological improvements**:  
Laser pulses, micro-fabrication



# Experiments with relativistic e-beams



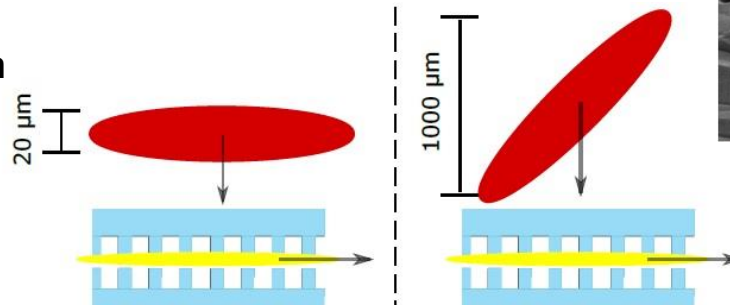
## 60 MeV SLAC NLCTA beam:

- E. Peralta et al. *Nature* 2013, **300 MeV/m**

- K. Wooton et al.: increased to **690 MeV/m**  
(*Optics Letters* 2016)

## 6 MeV UCLA Pegasus beam:

D. Cesar et al. *Nat Comm Phys.* 2018,  
**850 MeV/m** with pulse front tilt





# Accelerator on a Chip Intl. Program (ACHIP)

- funded by the Moore Foundation



“Small really is beautiful”, *the economist*, 2013



GORDON AND BETTY  
**MOORE**  
FOUNDATION

**“Make a chip that provides acceleration”**



FRIEDRICH-ALEXANDER  
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TECHNISCHE  
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DARMSTADT



2015



2017



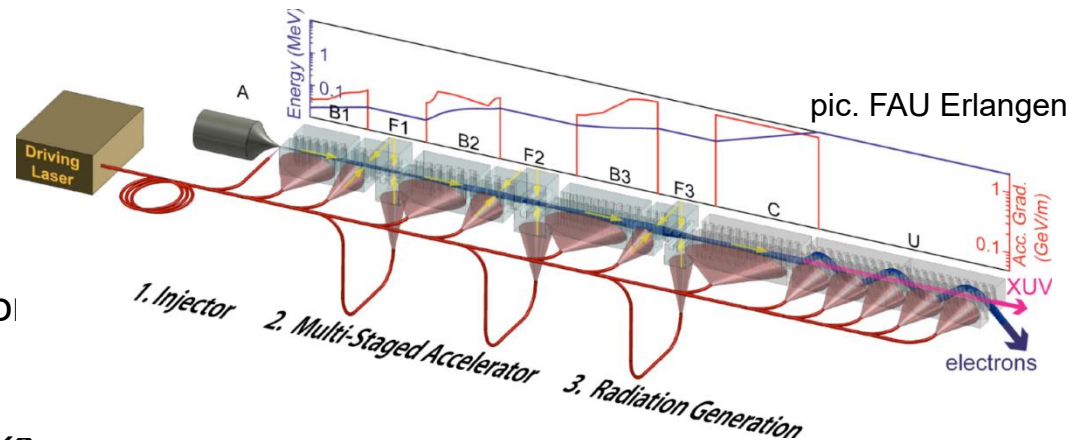
2018



# Working groups:

...to work towards our dream laser accelerator

- Injectors
  - Electron sources
  - Sub-relativistic DLA
- Relativistic Acceleration
  - Large scale integration (Accelerator)
- Lasers and Laser Coupling
  - On-chip laser delivery tree-networks
- **Simulations and Beam Dynamics**
  - Beam dynamics schemes design
  - Large and small scale computation
  - Intensity effects
- Radiation Generation and Applications
- Integration (fit everything in a shoe-box)



**Goal #1:**  
**60keV → 1MeV**  
**on-chip accelerator**

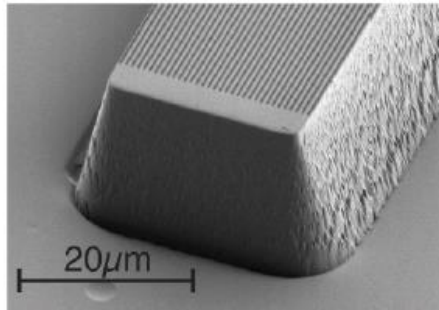
# Sub-relativistic accelerators

## FAU Erlangen:

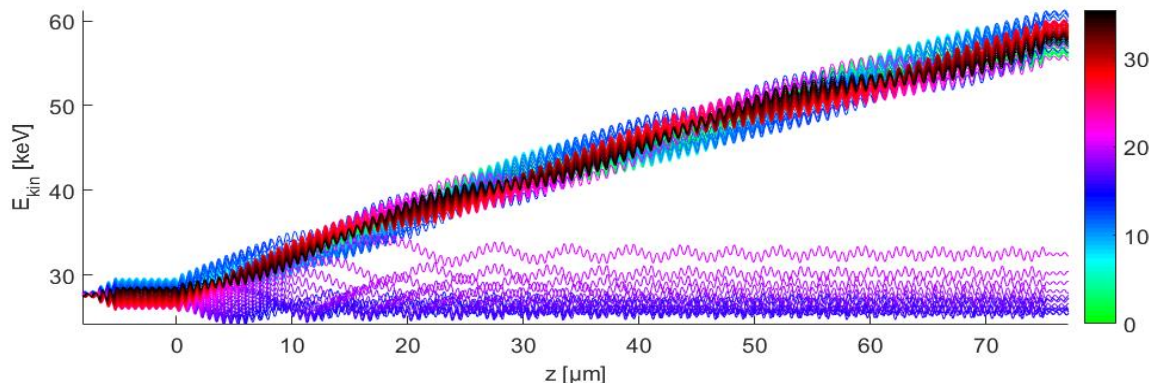
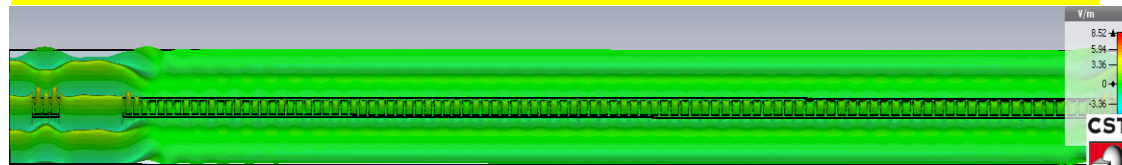
28 keV electrons  
( $v/c=0.32$ )

**25 MeV/m** gradient  
Single grating

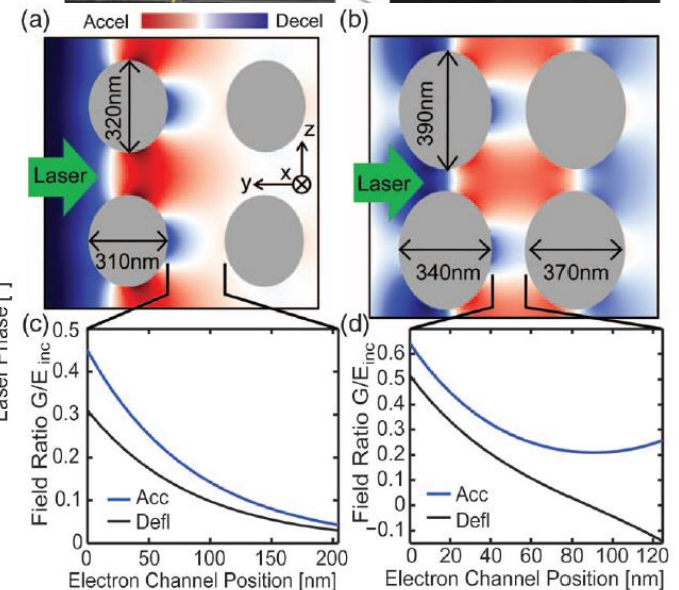
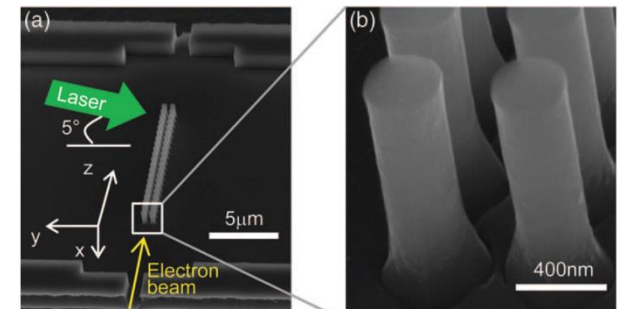
Breuer et. al.  
PRL 2013



Inherent problem: **dephasing** due to velocity increase  
Solution: **chirped grating**



**Stanford: 370 MeV/m @ ~90 keV**  
K. Leedle et al. Optics Letters, 2015

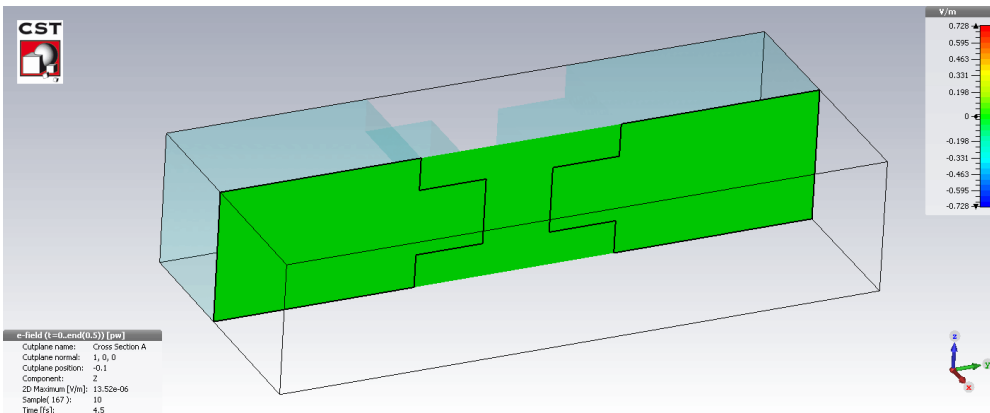


U Niedermayer et al. J. Phys.: Conf. Ser. **874** 012041 (2017).

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# Fields in a single DLA cell



Simulation not very challenging

- FD / FI TD
- FD FD
- FE FD

$$\begin{aligned}
 \Delta W(x, y; s) &= q \int_{-\lambda_g/2}^{\lambda_g/2} E_z(x, y, z; t = (z + s)/v) dz \\
 &= q \int_{-\lambda_g/2}^{\lambda_g/2} \Re\{E_z(x, y, z) e^{i\omega(z+s)/v}\} dz
 \end{aligned}$$

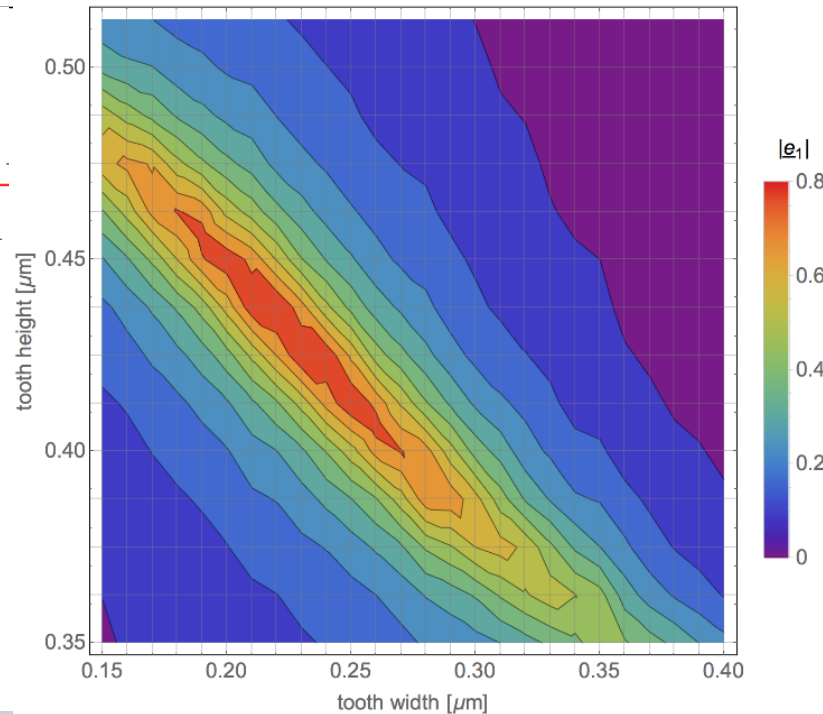
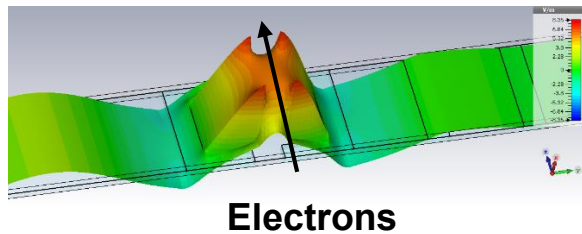
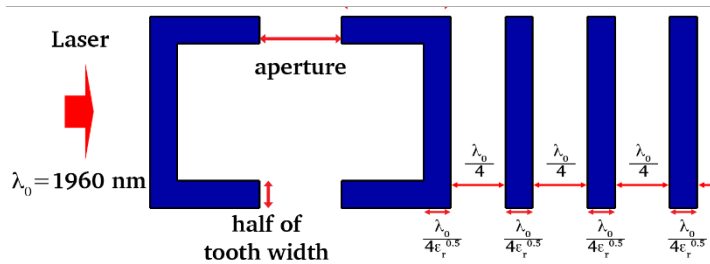
$$\lambda_g = m\beta\lambda_0$$

$$= q\lambda_g \Re\left\{e^{2\pi i \frac{s}{\beta\lambda_0}} \underline{e}_m(x, y)\right\}$$

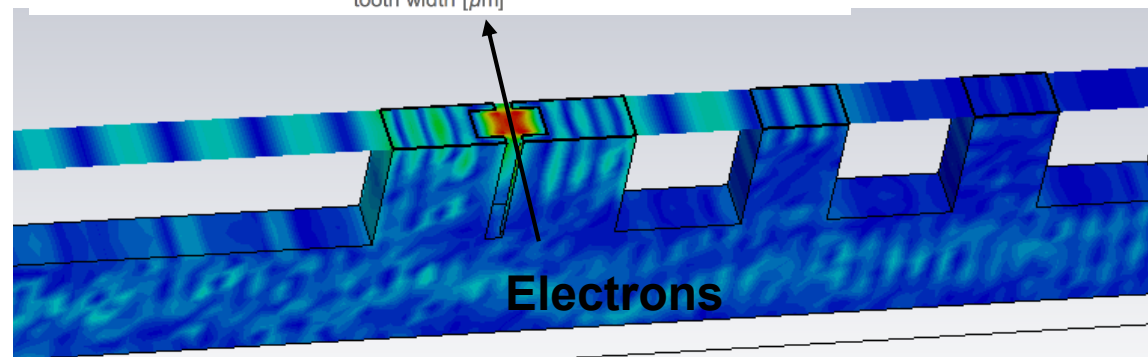
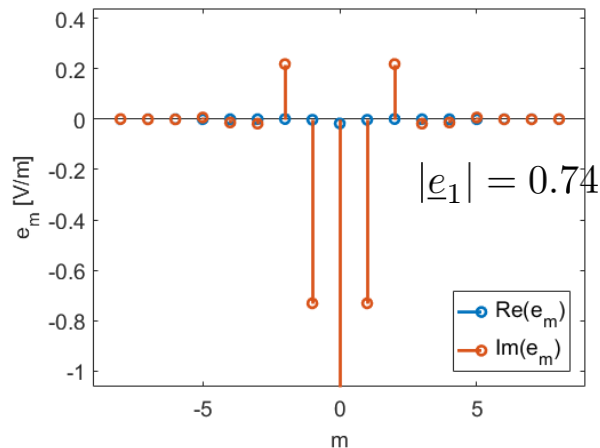
$$\underline{E}_z(x, y, z) = \sum_m \underline{e}_m e^{-im \frac{2\pi}{\lambda_g} z}$$

$$\Delta W_{\max} = q\lambda_g |\underline{e}_m|$$

# Single DLA cell optimization process

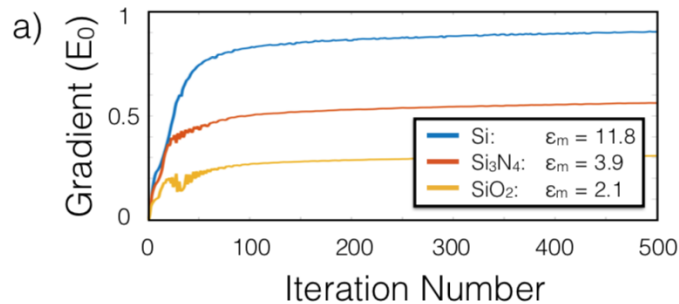


T. Egenolf,  
TU Darmstadt

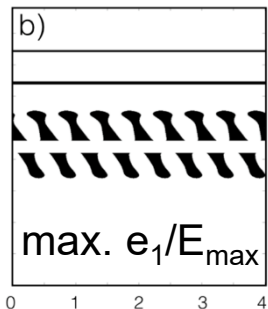
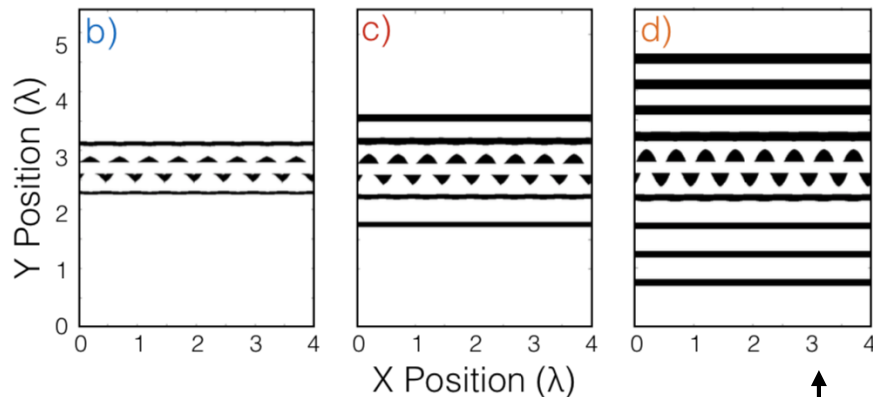


# Advanced Optimization Schemes

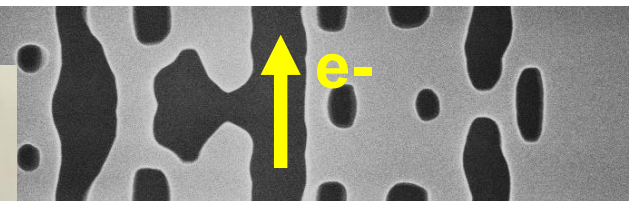
## Inverse Design by Adjoint Variable Method (AVM)



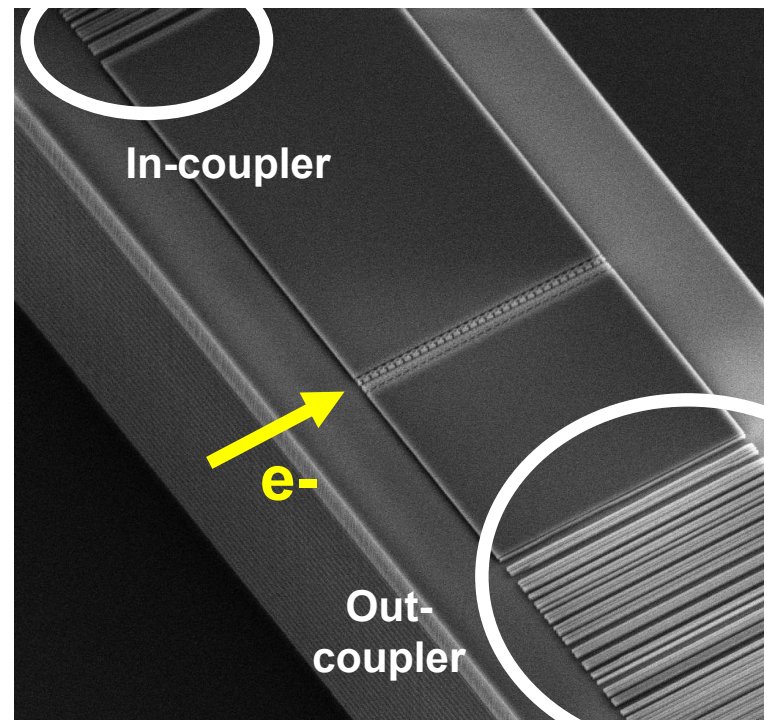
T. Hughes et al.  
Opt. Expr.  
21, 19 (2017)



T Hughes,  
N. Sapra,  
Ginzton Lab,  
Stanford



N. Sapra et al.  
<https://arxiv.org/abs/1808.07630>



# Bottom-Up design of DLA structures: *DLAtrack6D*

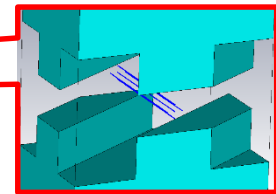
- One kick per grating cell (**numerically lightweight**)
- **Symplectic code** → No artificial emittance increase
- Kicks by resonant Fourier coefficient

$$\underline{e}_1(x, y) = \underline{e}_1(0, 0) \cosh(ik_y y) e^{ik_x x}$$

- Transverse kick by Panofsky-Wenzel theorem

$$\Delta \vec{p}_\perp(x, y; s) = -\nabla_\perp \int ds \Delta p_\parallel(x, y; s) = \frac{q\lambda_g\lambda_0}{2\pi c} \Im \left\{ e^{2\pi i \frac{s}{\lambda_g}} \nabla_\perp \underline{e}_1 \right\}$$

- Can be applied to laterally coupled structures
  - Subrelativistic structures / Relativistic structures
  - Tilted grating structures
  - Alternating phase / Spatial Harmonic focusing structures





# DLAtrack6D

→ One kick per grating period



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$$x' = \frac{p_x}{p_{z0}} , \quad \Delta x' = \frac{\Delta p_x(x, y, \varphi)}{p_{z0}} , \quad y' = \frac{p_y}{p_{z0}} , \quad \Delta y' = \frac{\Delta p_y(x, y, \varphi)}{p_{z0}}$$

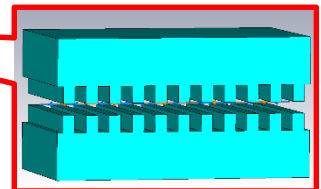
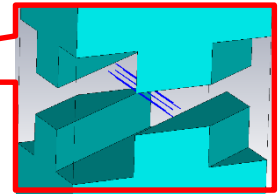
$$\varphi = 2\pi \frac{s}{\lambda_{gz}} , \quad \delta = \frac{W - W_0}{W_0} , \quad \Delta\delta = \frac{\Delta W(x, y, \varphi) - \Delta W(0, 0, \varphi_s)}{W_0}$$

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ \varphi \\ \delta \end{pmatrix}^{(n+1)} = \begin{pmatrix} x \\ Ax' + \Delta x'(x, y, \varphi) \\ y \\ Ay' + \Delta y'(x, y, \varphi) \\ \varphi \\ \delta + \Delta\delta(x, y, \varphi; \varphi_s) \end{pmatrix}^{(n)} + \begin{pmatrix} \lambda_{gz} x'(x, y, \varphi) \\ 0 \\ \lambda_{gz} y'(x, y, \varphi) \\ 0 \\ -\frac{2\pi}{\beta^2 \gamma^2} \delta(x, y, \varphi) \\ 0 \end{pmatrix}^{(n+1)}$$

$$\Delta x' = -\frac{q\lambda_0}{p_{z0}c} \tan(\alpha) \cosh(ik_y y) \Re \left\{ \underline{e}_1 e^{i\varphi + i\frac{2\pi x}{\lambda_{gx}}} \right\}$$

$$\Delta y' = \frac{-ik_y \lambda_0^2 q \beta}{2\pi p_{z0} c} \sinh(ik_y y) \Im \left\{ \underline{e}_1 e^{i\varphi + i\frac{2\pi x}{\lambda_{gx}}} \right\}$$

$$\Delta\delta = \frac{q\lambda_{gz}}{\gamma m_e c^2} \Re \left\{ \underline{e}_1 \left( \cosh(ik_y y) e^{i\varphi + i\frac{2\pi x}{\lambda_{gx}}} - e^{i\varphi_s} \right) \right\}$$



U. Niedermayer et al. Phys. Rev. Accel. Beams **20**, 111302 (2017)

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# Stabilization and scalability of DLA

- Magnetic focusing is too weak to counteract acceleration defocusing (see e.g. Ody et al. NIM A 865, 75-83 (2017))
- Spatial harmonic focusing in travelling wave structures
  - see B. Naranjo et al., PRL 109, 1 (2012) → Galaxie DLA
  - Shows only stability, not what emittance fits into the tiny aperture
- Focusing in laterally coupled (standing wave) structures
  - Programmable Spatial Light Modulator (SLM)
  - Small drift sections → Alternating Phase Focusing (APF)

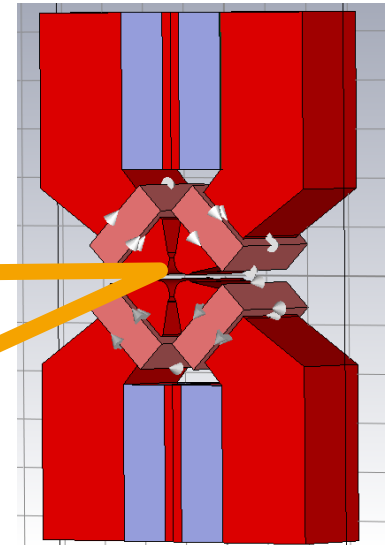
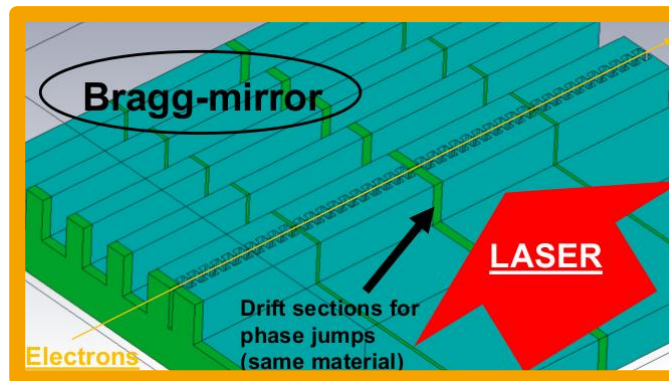
U. Niedermayer et al. accepted for publication in PRL. (see arXiv 1806.07287)

# Alternating Phase Focusing (APF)

- Originally invented for conventional RF-ion-linacs in the 1950' , but outperformed by RFQ
- “Earnshaw’s theorem” forbids simultaneous focusing in all directions
- We cannot make 3D structures on the  $\mu$ -chip scale
- APF lattice is 2D: Possible to fabricate by lithographic methods



Constant focusing in  $z$   
Alternating in  $x$  and  $y$



Constant focusing in  $x$   
Alternating in  $z$  and  $y$



# Hamiltonian (dual pillar structure)

Read off directly from symplectic tracking scheme:

$$\dot{y} = \frac{p_y}{m_e \gamma}$$

$$\dot{p}_y = -q e_1 \frac{\lambda_{gz}}{2\pi} \frac{\omega}{\beta \gamma c} \sinh\left(\frac{\omega y}{\beta \gamma c}\right) \sin\left(\frac{2\pi s}{\lambda_{gz}}\right)$$

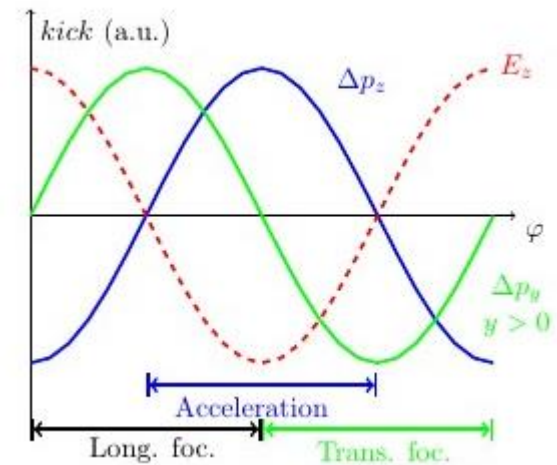
$$\dot{s} = \frac{\Delta p_z}{m_e \gamma^3}$$

$$\dot{\Delta p_z} = q e_1 \left[ \cosh\left(\frac{\omega y}{\beta \gamma c}\right) \cos\left(\frac{2\pi s}{\lambda_{gz}}\right) - \cos \varphi_s \right].$$

$$H = \frac{1}{2m_e \gamma} (p_x^2 + p_y^2 + (\Delta p_z / \gamma)^2) + V$$

Panofsky-Wenzel:  $\vec{F} = -\nabla' V$

$$V = q e_1 \left[ \frac{\lambda_{gz}}{2\pi} \cosh\left(\frac{\omega y}{\beta \gamma c}\right) \sin\left(\frac{2\pi s}{\lambda_{gz}}\right) - s \cos \varphi_s \right]$$



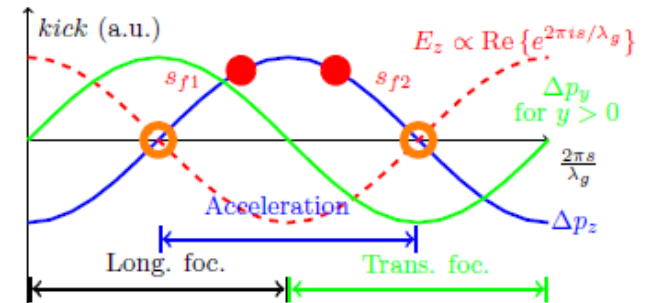
“Earnshaw’s theorem”

# Design of an Alternating Phase Focusing Accelerator Chip

## ▪ Time dependent potential

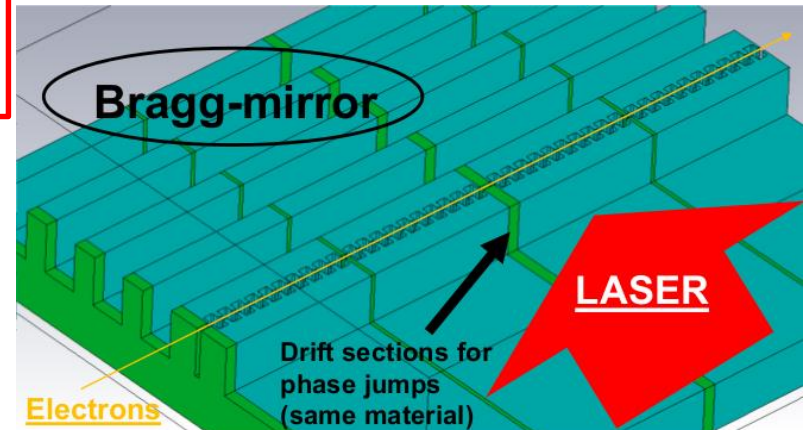
$$V = q\Im\left\{e_1\left[\frac{\lambda_g}{2\pi}\cosh\left(\frac{\omega y}{\beta\gamma c}\right)e^{2\pi i s/\lambda_g} - i s e^{i\varphi_s}\right]\right\}$$

$$\varphi_s^{(n)} = \varphi_0 - \arg(e_1)^{(n)} \quad \varphi_0 = \arccos\left[\frac{m_e c^2}{q\lambda_0} \frac{\gamma^3}{|e_1|} \frac{\Delta\lambda_g}{\lambda_0}\right]$$

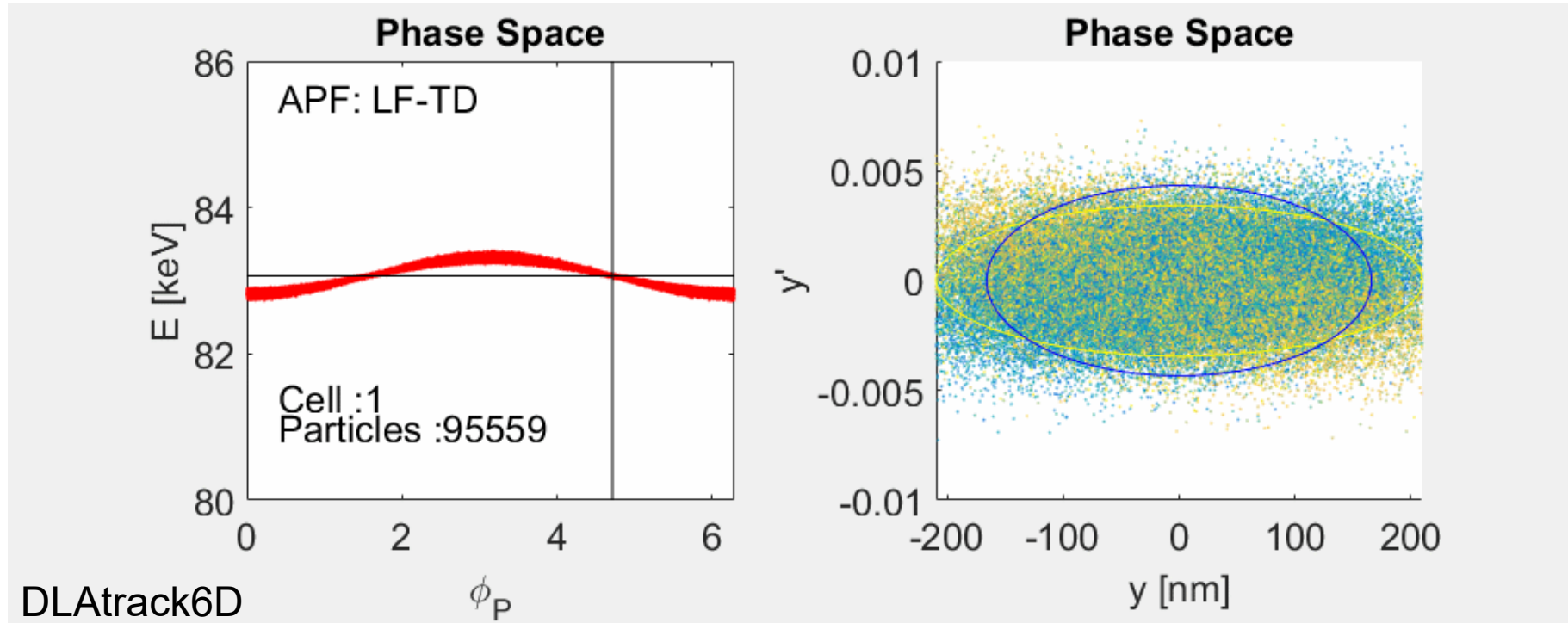


$$\begin{aligned}
 V(x, y, s = s_{f1} + \Delta s) &= -V(x, y, s = s_{f2} + \Delta s) \\
 &= \frac{q|e_1|\lambda_g}{2\pi} \left[ \frac{1}{2} \left( \frac{\omega y}{\beta\gamma c} \right)^2 - \frac{1}{2} \left( \frac{2\pi}{\lambda_g} \Delta s \right)^2 \right] \sin(\varphi_0)
 \end{aligned}$$

$$\begin{aligned}
 y'' + Ky &= 0 \\
 \Delta s'' - K\Delta s &= 0 \\
 K &= \frac{|q|\omega e_1}{(m_e\beta^3\gamma^3c^3)} \sin(\varphi_s) \\
 \arg(e_1)(P) &= \begin{cases} 0, P \text{ odd} \\ 2\varphi_0, P \text{ even} \end{cases}
 \end{aligned}$$



# Unbunched transversely matched



$$\varepsilon(y, y') = \hat{\gamma}y^2 + 2\hat{\alpha}yy' + \hat{\beta}y'^2$$

Longitudinal Courant-Snyder Invariant:  $\varepsilon_L(\Delta s, \Delta s') = \hat{\gamma}_L \Delta s^2 + 2\hat{\alpha}_L \Delta s \Delta s' + \hat{\beta}_L \Delta s'^2$

U. Niedermayer et al. accepted for publication in PRL. (see arXiv 1806.07287)

# Design of a lattice incl. acceleration

Ramp:

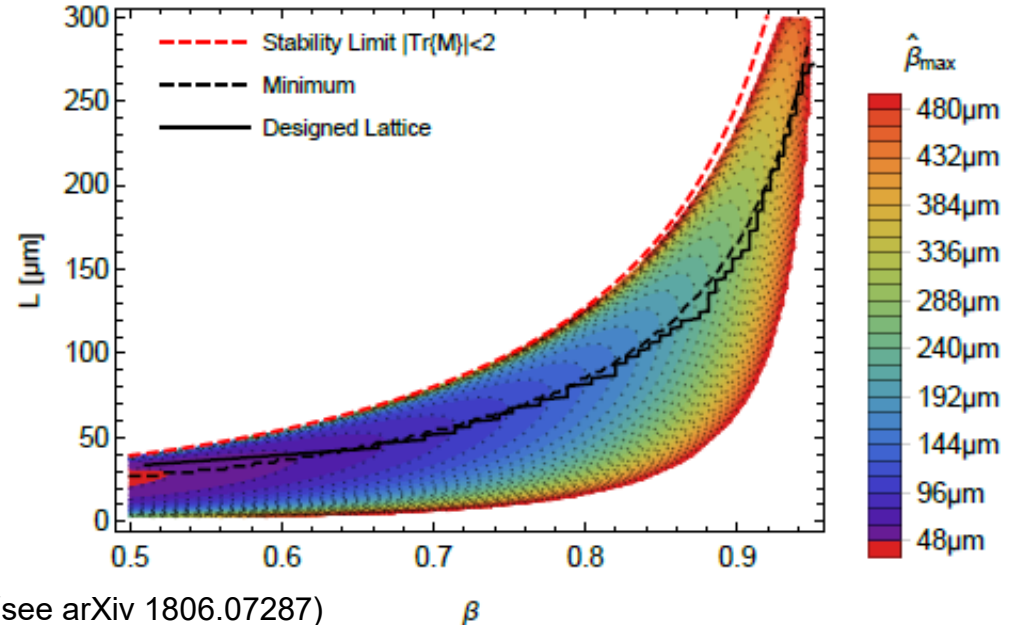
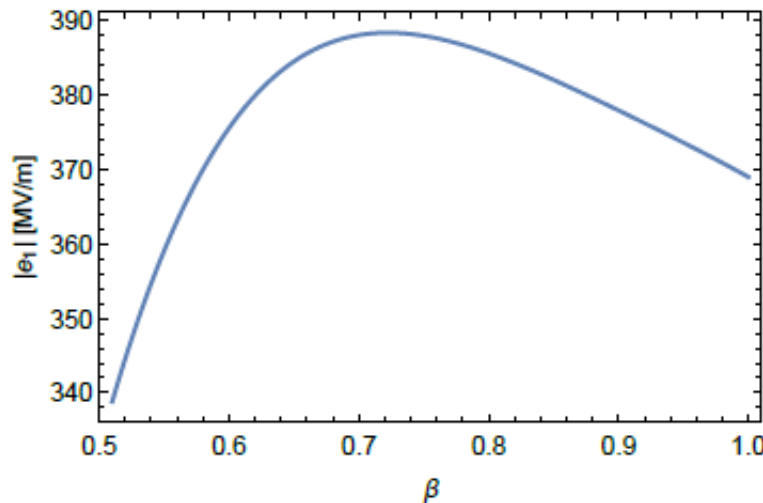
$$W_{kin}(N) = W_{kin,0} + q \sum_{n=1}^N \lambda_g^{(n)} \Re\{e_1^{(n)} e^{i\varphi_s^{(n)}}\}$$

Length chirp according to  $\lambda_g = \beta \lambda_0$ :

$$\frac{\lambda_g^{(n+1)} - \lambda_g^{(n)}}{\lambda_0} = \beta^{(n+1)} - \beta^{(n)} = \frac{q \lambda_0 \Re\{e_1^{(n)} e^{i\varphi_s^{(n)}}\}}{m_e c^2 \gamma^{(n)3}}$$

Optimization such that

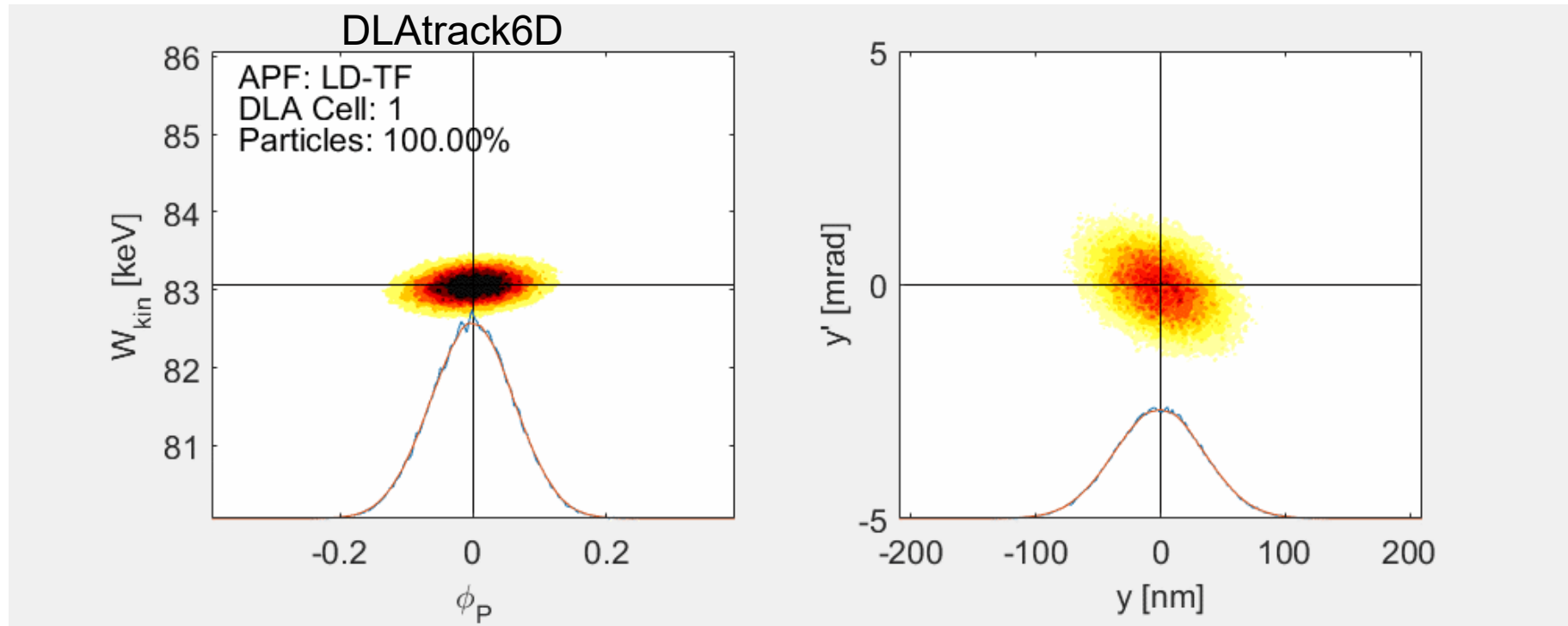
$$\arg(e_1)^{(n)} \neq f(\lambda_g^{(n)})$$



U. Niedermayer et al. accepted for publication in PRL. (see arXiv 1806.07287)

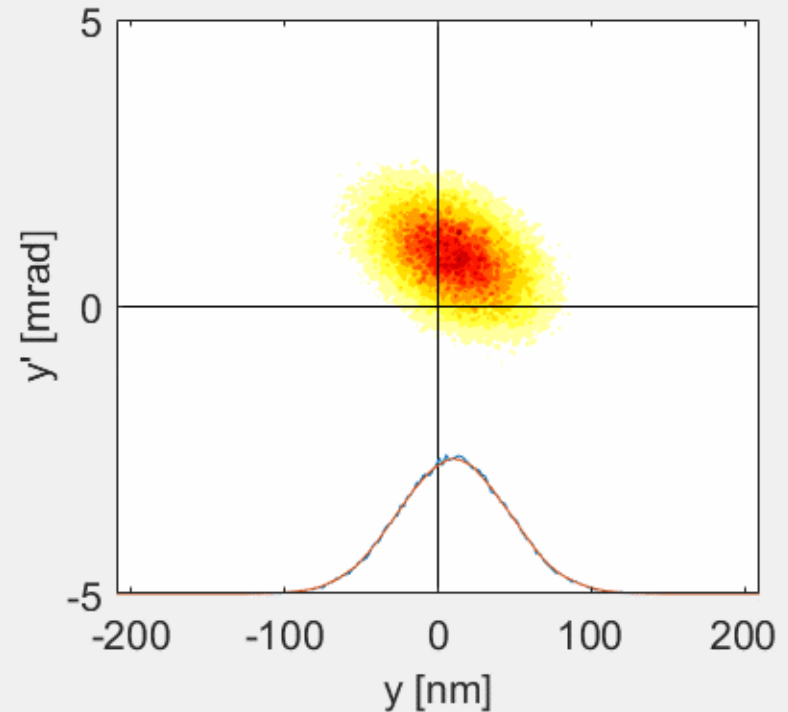
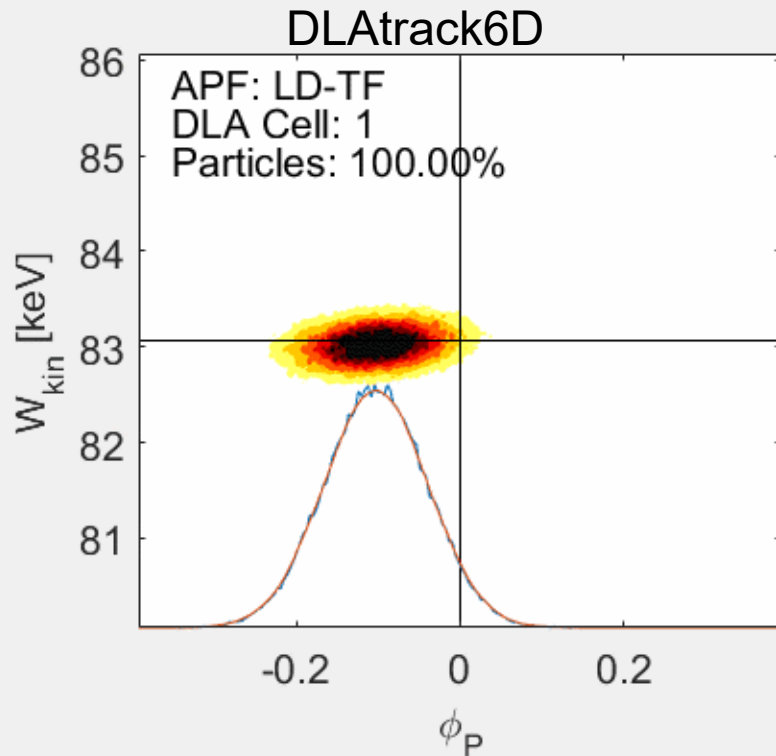


# The ideal ACHIP scenario



93% transmission at  $\varepsilon = 0.025$  nm

# Misalignment example



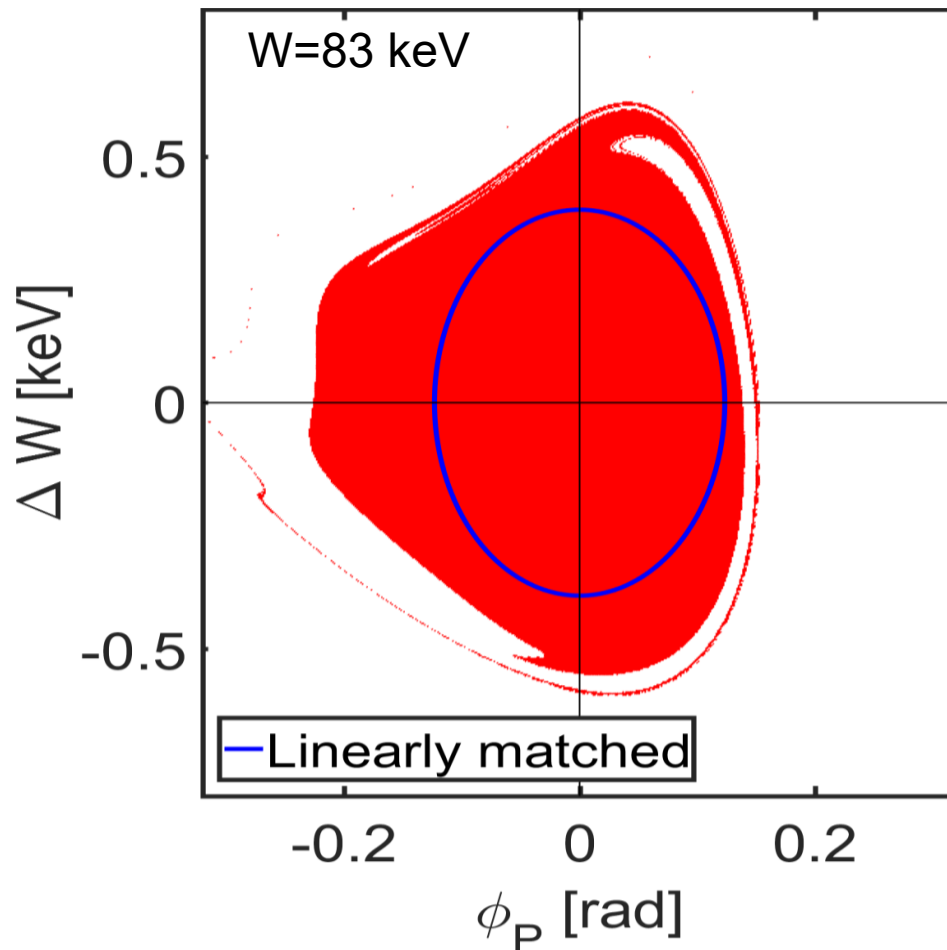
66% transmission at  $\varepsilon = 0.025$  nm

**Initial mismatch:**

- $y = 10$  nm
- $\Phi_P = 0.1$  rad

$y' = 1$  mrad  
 $\Delta W = 0$

# Initial phase space



**Longitudinal matching condition:**

$$\frac{\sigma_\varphi}{2\pi} = \frac{\hat{\beta}_L}{\beta^3 \gamma^3 \lambda_0} \frac{\sigma_{\Delta W}}{m_e c^2}$$

$$\hat{\alpha}_L = 0$$

Bunch length (numerically) :

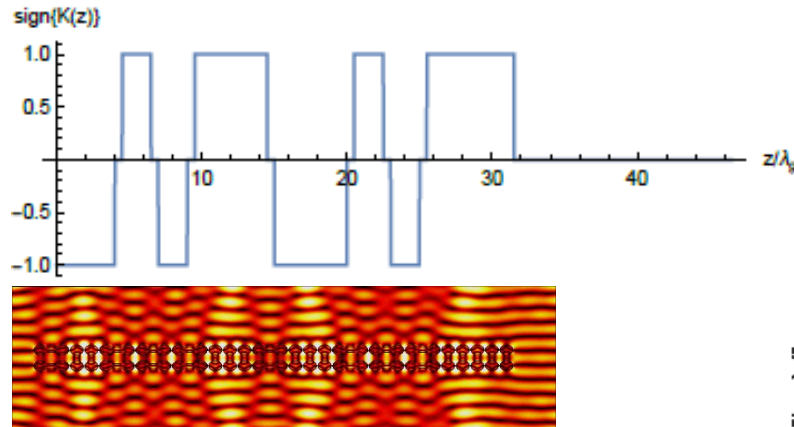
$$4\sigma_z = 40 \text{ nm}$$

**Transverse matching condition:**

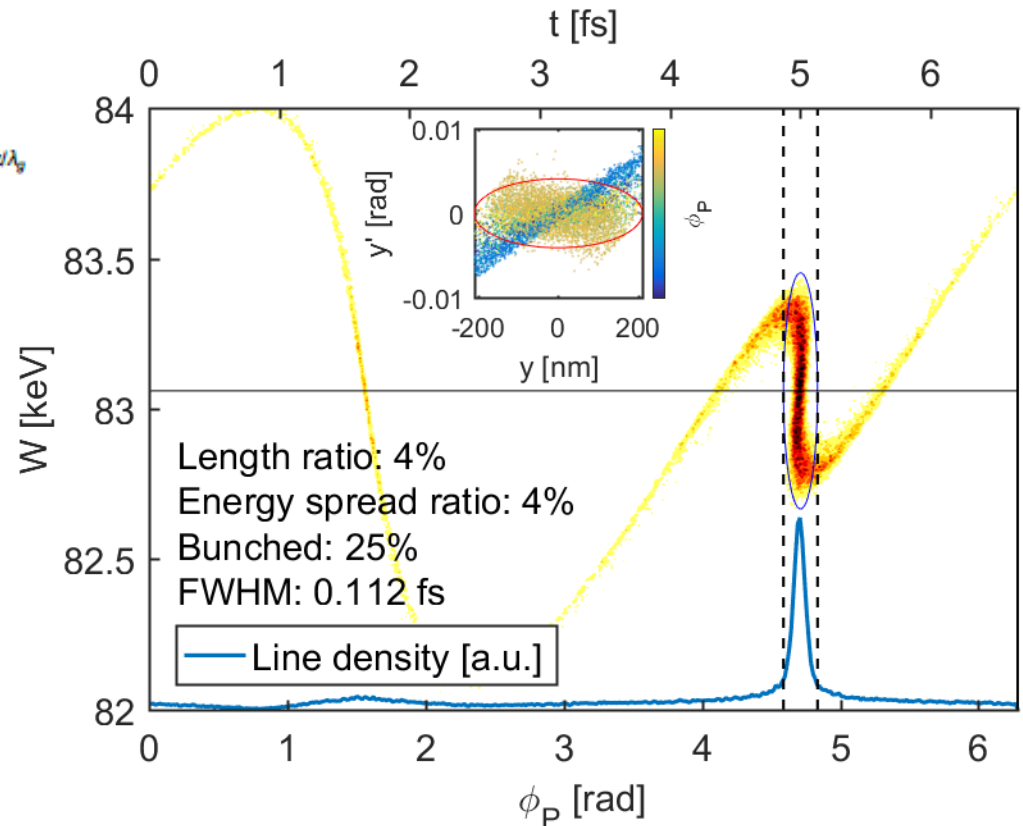
$$\sigma_y = \sqrt{\hat{\beta} \varepsilon}$$

$$\hat{\alpha} = 0 \Rightarrow \sigma_{y'} = \sqrt{\varepsilon / \hat{\beta}}$$

# Bunching



- **~0.1 fs bunches**
- Low (matched) energy spread
- Transverse confinement

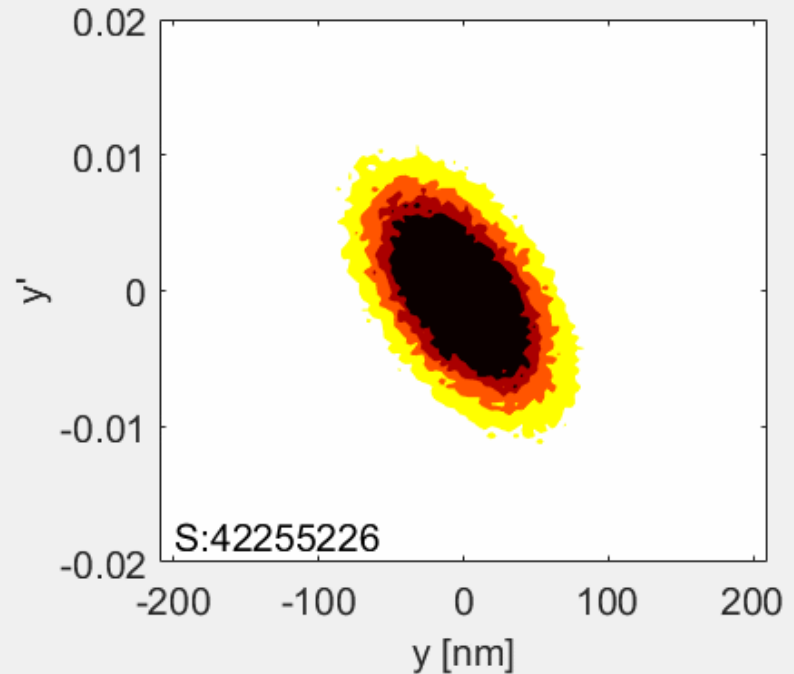
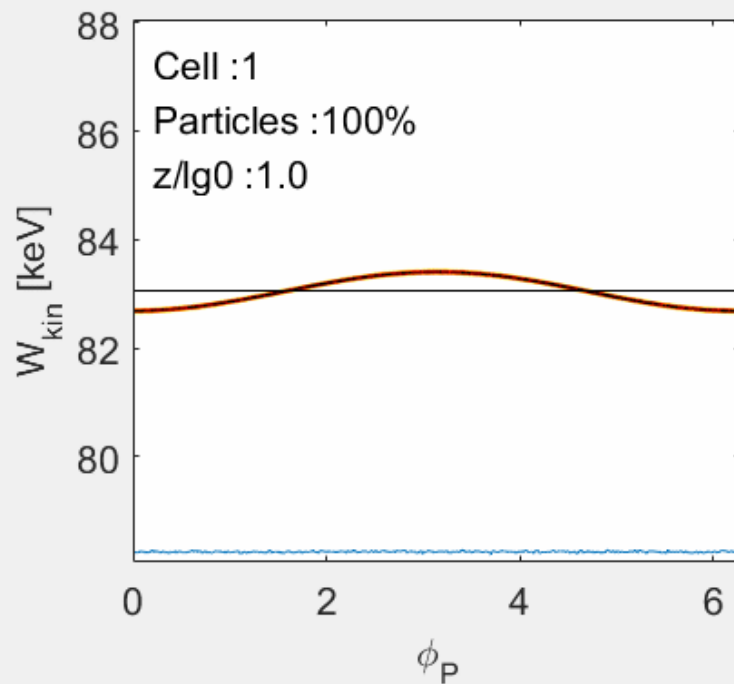


$$\mathbf{M}_b = \mathbf{M}_o^{30} \mathbf{M}_f^6 \mathbf{M}_o \mathbf{M}_d^2 \mathbf{M}_o \mathbf{M}_f^2 \mathbf{M}_o \mathbf{M}_d^5 \mathbf{M}_o \mathbf{M}_f^5 \mathbf{M}_o \mathbf{M}_d^2 \mathbf{M}_o \mathbf{M}_f^2 \mathbf{M}_o \mathbf{M}_d^4$$

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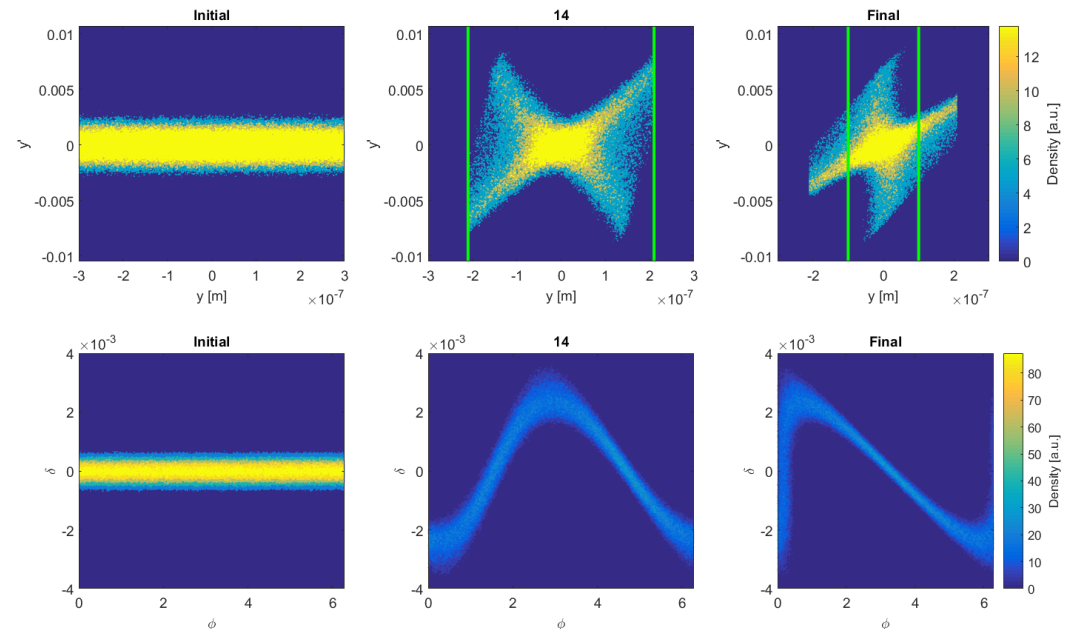
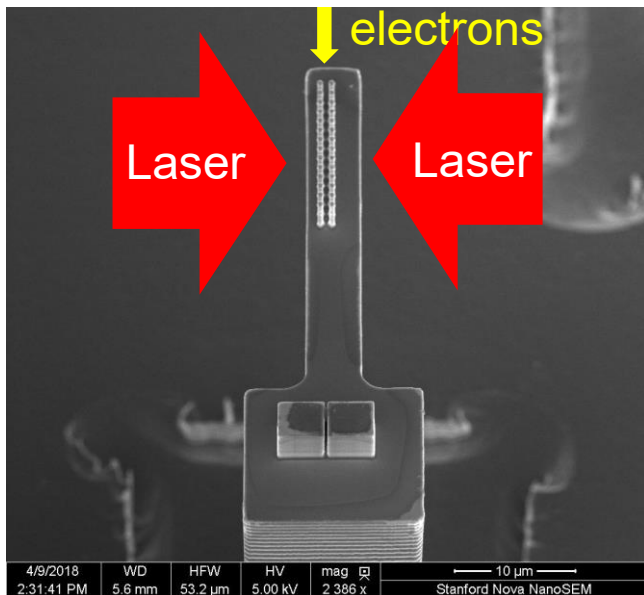


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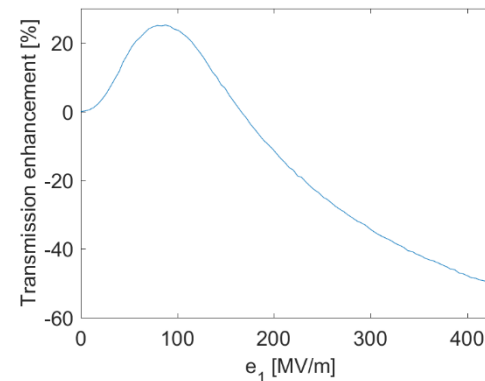
# Single stage focusing experiment with aperture



Data has been taken, publication almost ready



Dylan Black,  
Stanford



U. N.  
DLAtrack6D  
simulation

*preliminary*

# Buncher / streaker experiment at Stanford

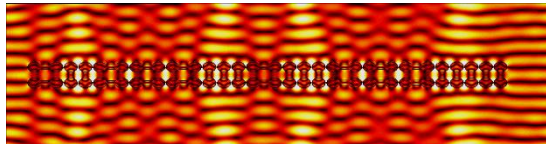
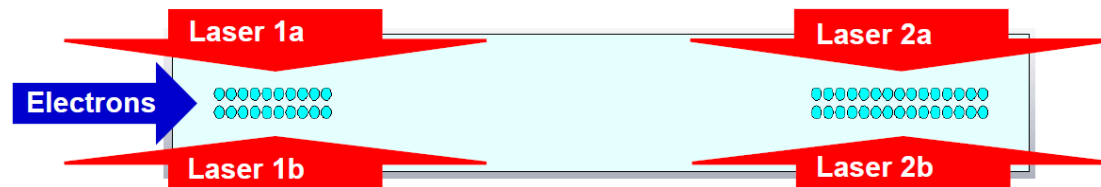
Symmetric DLA in out of phase mode:

$$\underline{e}_1(x, y) = \underline{e}_1(0, 0) \sinh(ik_y y) e^{ik_x x}$$

$$\Delta y' = \frac{-ik_y \lambda_0^2 q \beta}{2\pi p_{z0} c} \cosh(ik_y y) \Im \{ \underline{e}_1 e^{i\varphi} \}$$

$$\Delta \delta = \frac{q \lambda_{gz}}{\gamma m_e c^2} \sinh(ik_y y) \Re \{ \underline{e}_1 e^{i\varphi} \}$$

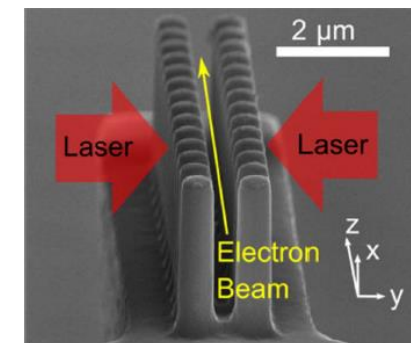
4 lasers allow bunching and coherent streaking:



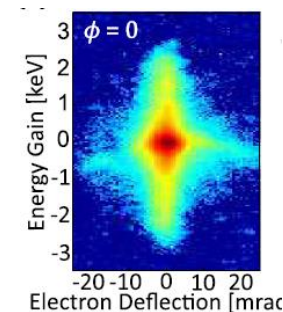
**APF buncher:**

- Small energy spread
- coherent acceleration
- **All 4 quadrants coherent!**

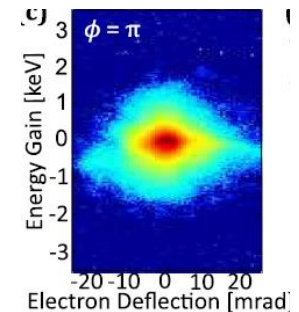
2 lasers: “incoherent”  
Acceleration/streaking



**Acceleration**

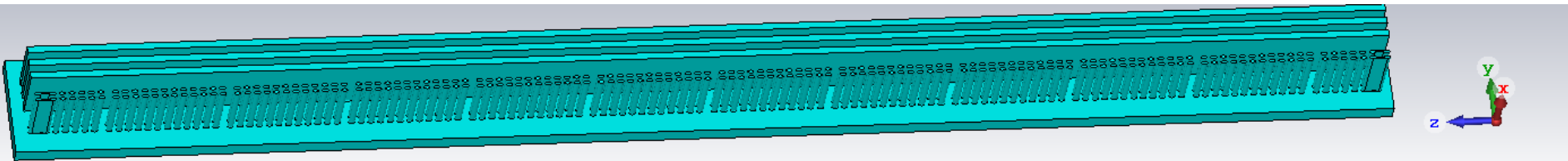


**Streaking**

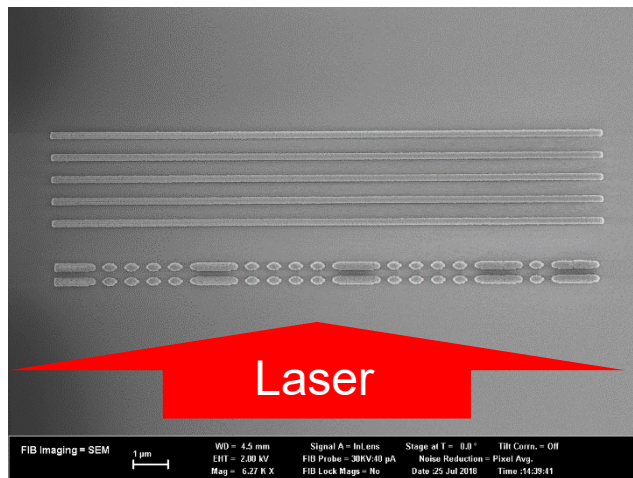


K. Leedle et al.  
Optics Letters, 2018

# Transport Experiment at FAU Erlangen



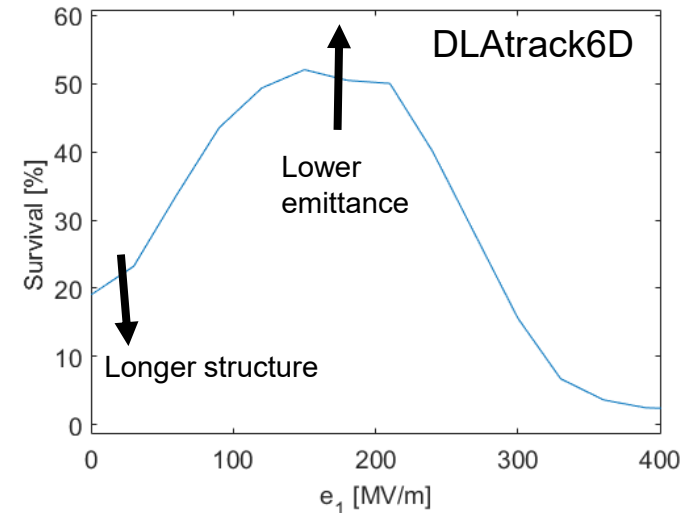
- Fully scalable (length independent transmission)
- Limited by x-Rayleigh length 800 $\mu$ m
- Transversely matched (min betafct.  $\sim 20\mu$ m @  $E_L=0.8$ GV/m)



$e_1/E_L \sim 0.25$   
(incl. DBR)

Geo. Emittance 0.3nm  
 $E_{kin}=26.47$ keV

**Multistage focusing:  
(Laser on/off contrast)**



P. Yousefi, J. Illmer:  
Fabrication test. Final fab ongoing.

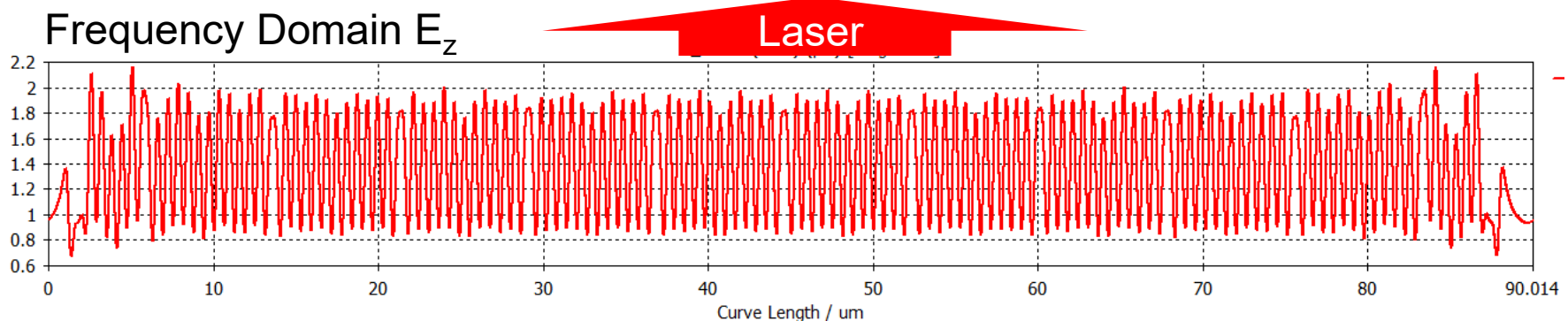
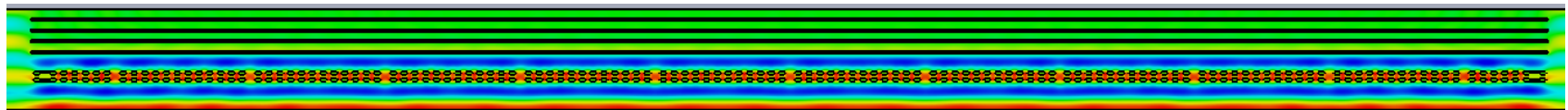
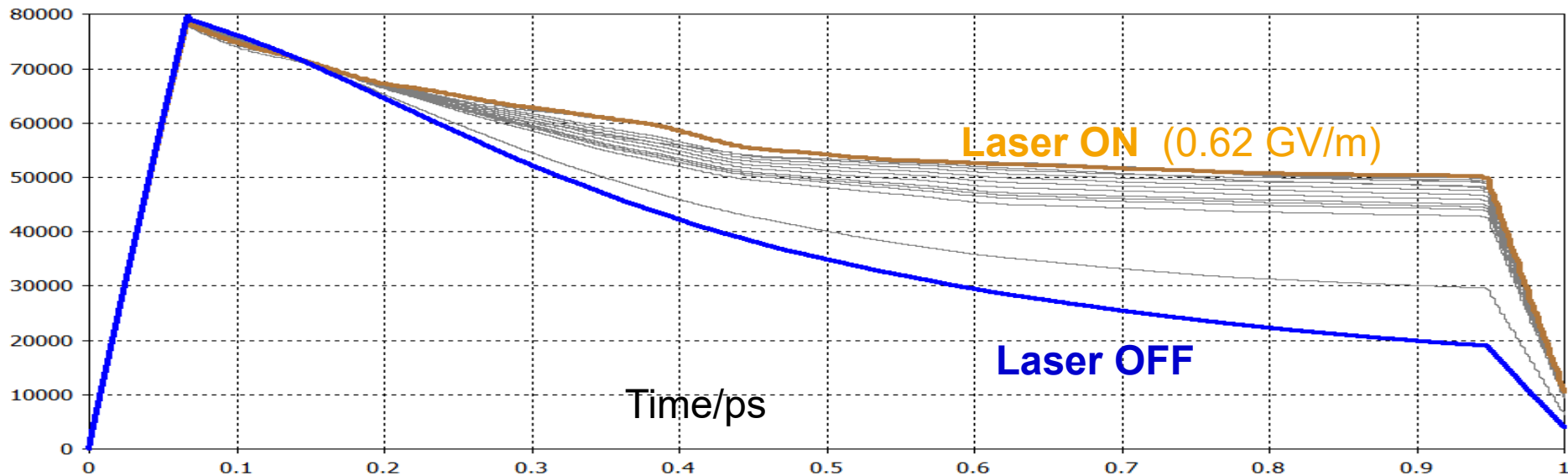


# PIC simulation (CST Particle Studio)

100,000 electrons with random phase (uniform dist.)



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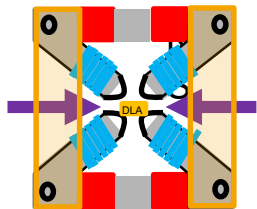


# Experiments: Shoebox at Stanford

(a slightly more conservative design)

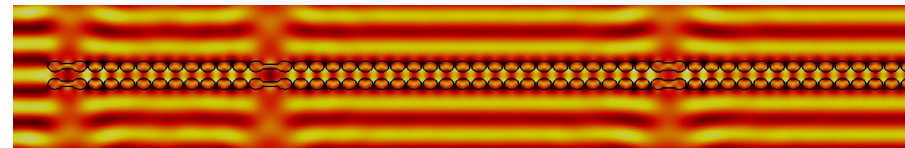
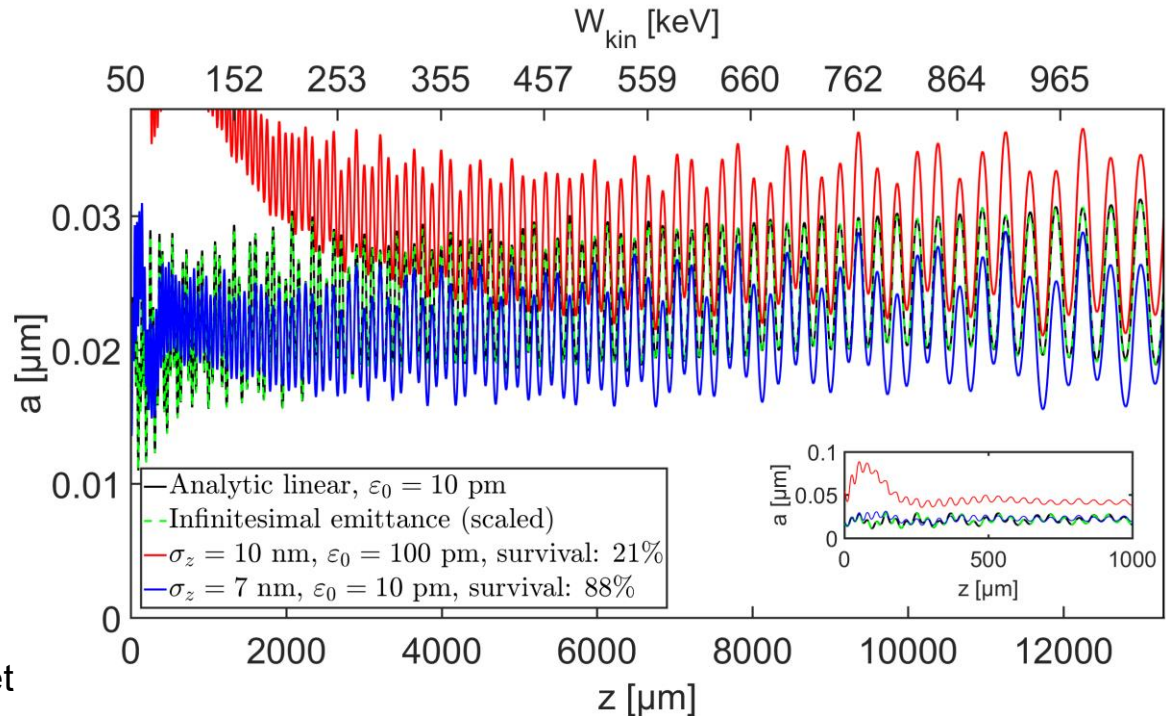
Chirped APF dual pillar  
grating for acceleration  
**50keV  $\rightarrow$  1MeV**

- Laser 300 MV/m (2 sides)
- Av. gradient 73 MeV/m
- Length 1.3 cm
- Emittance requirement:  
**<0.1nm (geo. @ 50keV)**
- Bunch length: **< 40nm**



Quadrupole magnet  
for focusing in the  
invariant direction

$$B'_{\text{quad}} = 1 \text{ kT/m} \ll B'_{\text{APF}}^{\text{equiv.}} = 5 \text{ MT/m}$$

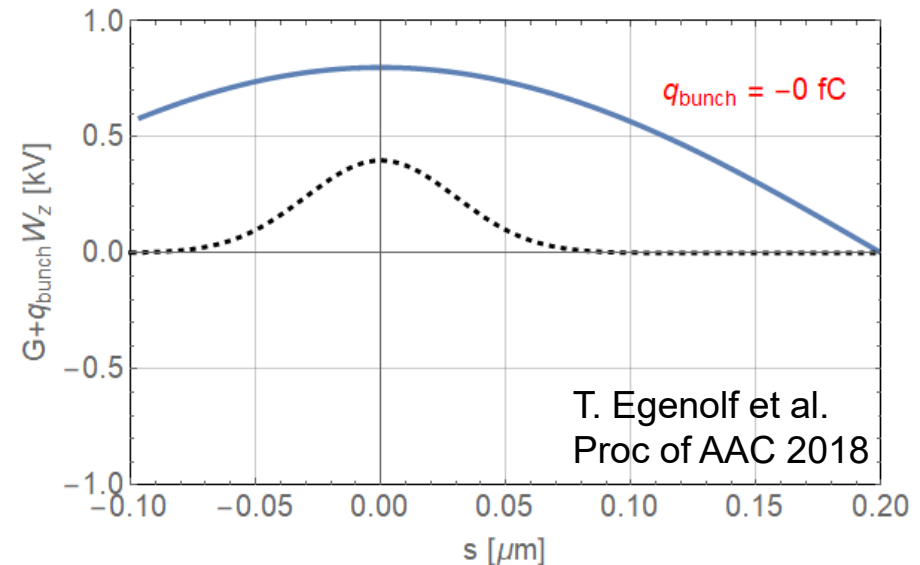
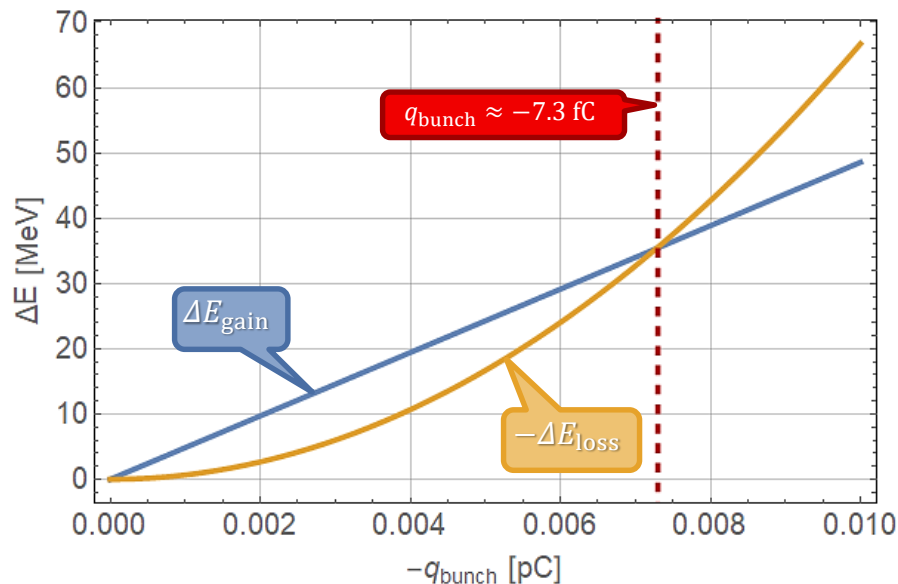


# Summary

- Simplified simulations with DLAttrack6D in place
- APF beam dynamics scheme works in theory
  - Overcomes resonant acceleration defocusing problem
  - Can also used for bunching in the attosecond range
- Field flatness needs to be tuned (optimized)
- Quadrupole magnet is being developed

# Outlook and Wish List

- Plans are to include wake field kicks in DLAttrack6D



**A moving window 3D track/PIC code would be nice!**  
(incl. dielectrics, pulse front tilted laser, open BC)

**Thank you for your attention!**

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# The End

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# Questions?