

# Nonlinear Optics for Suppression of Halo Formation in Space Charge Dominated Beams

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# Effect of Beam Halo in Linacs

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**Beam halo is a small fraction of particles (1% – 10%) which lies outside of the beam core and results in radio-activation and degradation of accelerator components.**

**Modern accelerator projects using high-intensity beams require keeping the beam losses at the level 1 Watt / m or less to avoid activation of the accelerator.**

## **Sources of Halo Formation in Linacs**

- 1. Mismatch of the beam with accelerator structure**
- 2. Transverse-longitudinal coupling in RF field**
- 3. Misalignments of accelerator channel components**
- 4. Aberrations and nonlinearities of focusing elements**
- 5. Beam energy tails from un-captured particles**
- 6. Particle scattering on residual gas, intra-beam stripping**
- 7. Non-linear space-charge forces of the beam**

# Emittance Growth and Halo Formation of a Non-Uniform Beam in FODO Quadrupole Channel

Injection of a continuous non-uniform beam in a focusing channel with linear field results in

(a) uniformity of beam core

(b) beam emittance growth

(c) halo formation

Example:

Beam energy      50 keV

Beam current     20 mA

Beam emittance   0.05  $\pi$  cm mrad

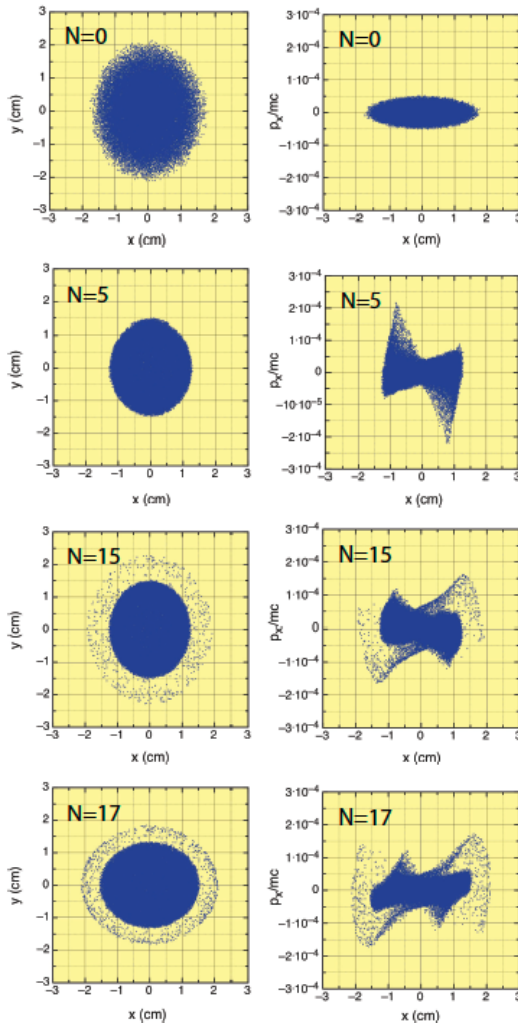
FODO period     15 cm

Lens length      5 cm

Quadrupole field gradient 0.0428 T/cm

Tune depression  $\sigma/\sigma_0 = 0.1$

(Numbers indicate focusing period)



# Self-Consistent Beam Equilibrium in Focusing Channel

Self-consistent problem:

Vlasov's Equation

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \vec{x}} \frac{d\vec{x}}{dt} + \frac{\partial f}{\partial \vec{P}} \frac{d\vec{P}}{dt} = 0$$

Poisson's Equation

$$\Delta U_b = -\frac{\rho}{\epsilon_o}$$

(Phys. Rev. E, Vol. 53, 1996, p. 5358)

Example: Beam with Gaussian distribution function

$$f = f_o \exp\left(-2 \frac{x^2 + y^2}{R^2} - 2 \frac{p_x^2 + p_y^2}{p_o^2}\right)$$

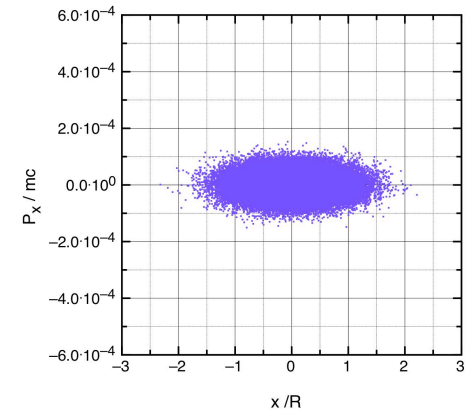
Total field  $E_{tot} = -\frac{mc^2}{q} \frac{1}{\gamma} \frac{\epsilon^2}{R^4} r$

Space-charge field

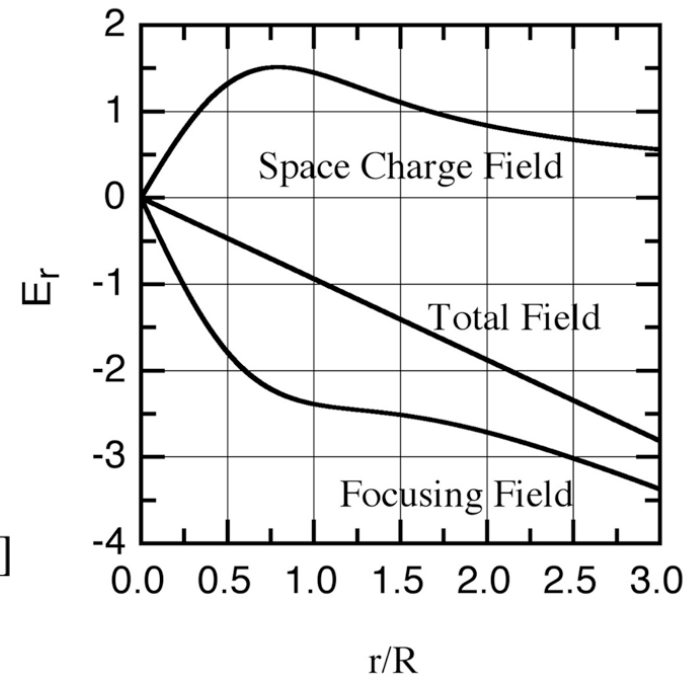
$$E_b = -\frac{\partial U_b}{\partial r} = \frac{I}{2\pi \epsilon_o \beta c} \frac{1}{r} [1 - \exp(-2 \frac{r^2}{R^2})]$$

Required focusing field

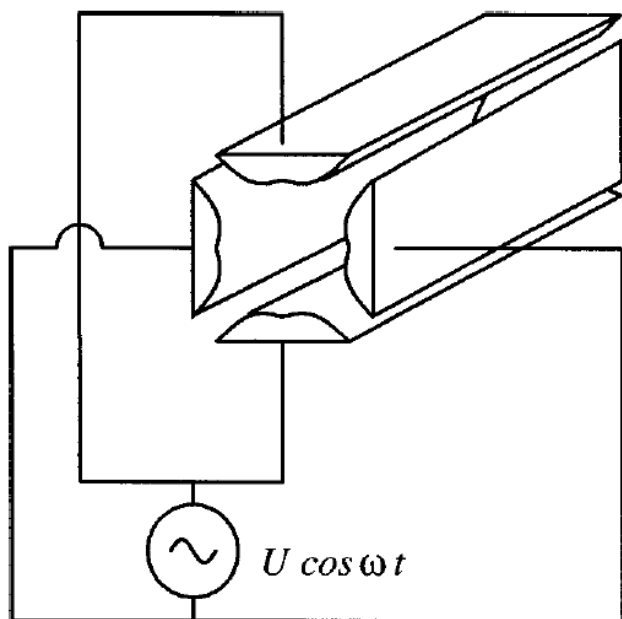
$$E_{ext} = -\frac{mc^2}{q R \gamma} \left[ \frac{\epsilon^2 r}{R^3} + 2 \frac{I}{I_c \beta \gamma} \frac{R}{r} (1 - \exp(-2 \frac{r^2}{R^2})) \right]$$



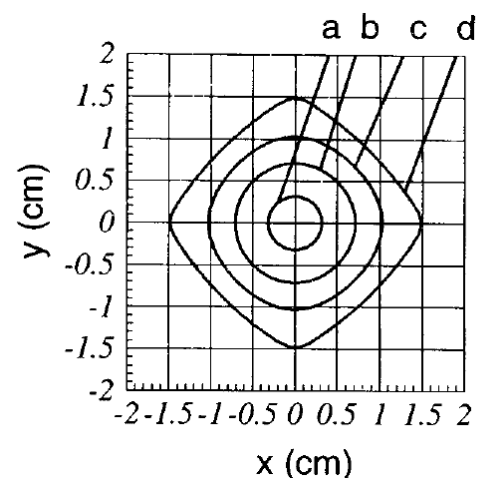
$$U_{ext} = U_{total} - \frac{U_b}{\gamma^2}$$



# Quadrupole-Duodecapole Focusing Structure



Proposed four vane quadrupole structure with a duodecapole field component (EPAC96, p.1236)



Lines of equal values of the function  $C = \frac{r^2}{2} + \zeta r^6 \cos 4\theta + \frac{\zeta^2}{2} r^{10}$   
for  $\zeta = -0.03$ : (a)  $C = 0.05$ , (b)  $C = 0.25$ , (c)  $C = 0.5$ , and (d)  $C = 0.85$

**Potential of the uniform four vanes structure:**

$$U(r, \theta, t) = \left( \frac{G_2}{2} r^2 \cos 2\theta + \frac{G_6}{6} r^6 \cos 6\theta \right) \sin \omega t$$

**Effective (time-independent) potential:**

$$U_{\text{eff}}(\vec{r}) = \frac{q}{4m\gamma} \frac{E^2(\vec{r})}{\omega^2}$$

$$U_{\text{eff}}(r, \theta) = \frac{mc^2}{q} \frac{\sigma_o^2}{2} \left( \frac{r}{\lambda} \right)^2 \left[ 1 + 2\eta \left( \frac{r}{R} \right)^4 \cos 4\theta + \eta^2 \left( \frac{r}{R} \right)^8 \right] \quad \eta = \frac{G_6}{G_2} R^4$$

# Space-Charge Density of the Matched Beam

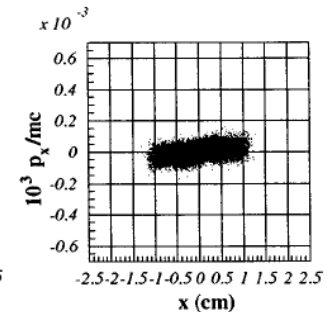
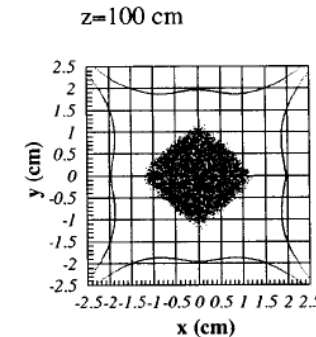
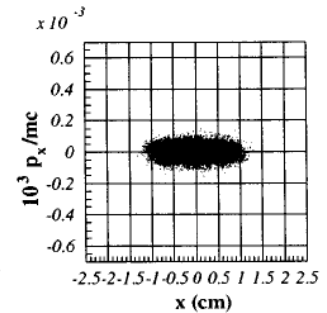
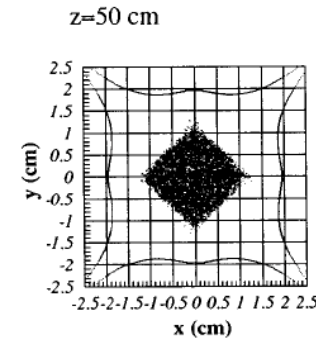
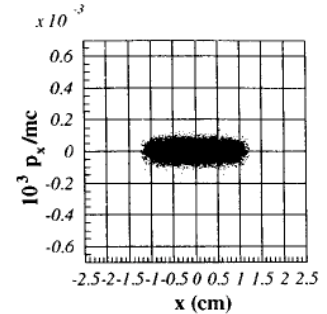
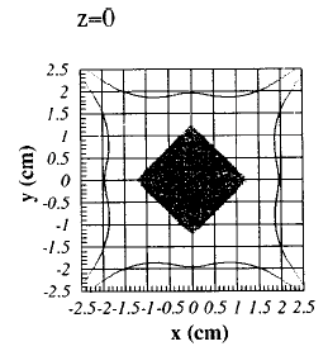
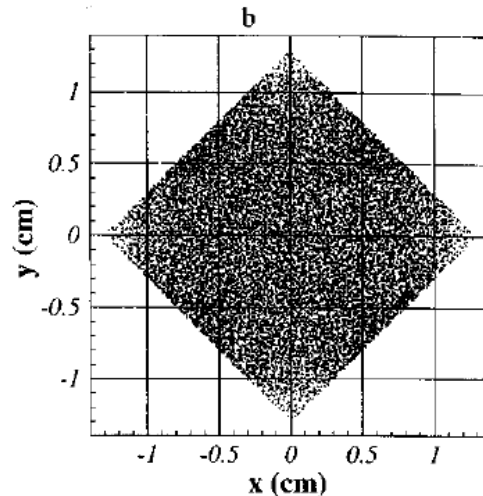
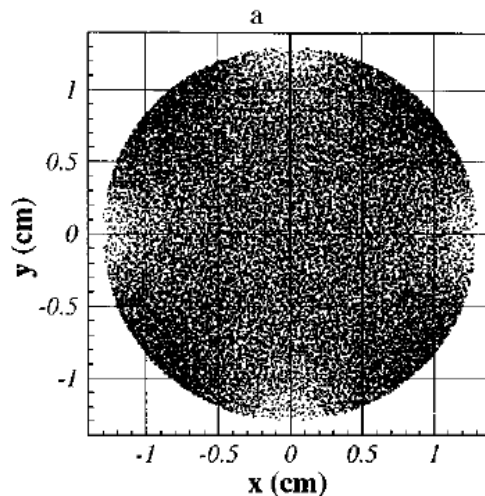
Self-consistent space  
charge potential of the  
matched beam

(Phys. Rev. E, Vol. 57, 1998,  
p. 6020)

$$U_b = - \frac{\gamma^2}{1 + \left( \frac{\beta\gamma}{2} \frac{I_c}{I} \frac{R^2}{\epsilon^2} \right)} U_{eff}$$

Space charge density

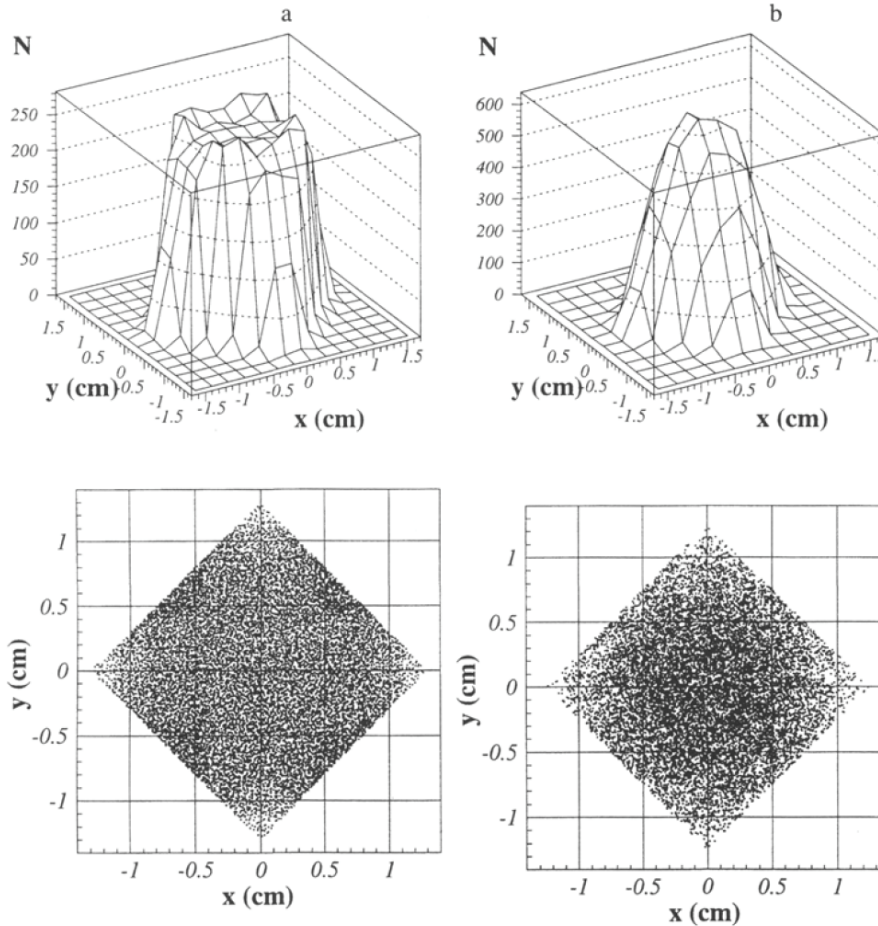
$$\rho_b = \rho_o (1 + 10\zeta r^4 \cos 4\theta + 25\zeta^2 r^8)$$



Dynamics of 150 keV, 100 mA,  $0.06 \pi$  cm mrad  
proton beam in a structure with  $G_2 = 48$  kV/cm<sup>2</sup>  
and  $G_6 = -1.3$  kV/cm<sup>6</sup>.



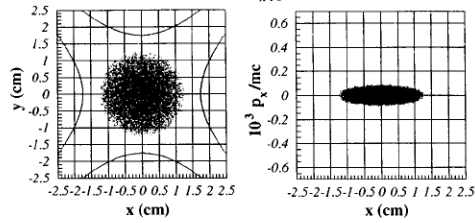
# Matched and Realistic Truncated Beam Distributions



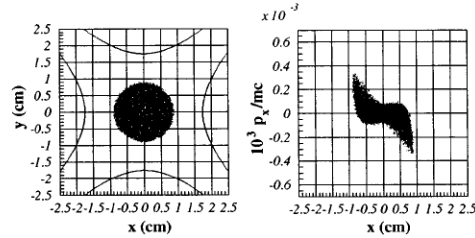
(a) Self-consistent particle distribution  $\rho_b = \rho_o(1 + 10\zeta r^4 \cos 4\theta + 25\zeta^2 r^8)$  of the matched beam in quadruple-duodecapole channel with parameter  $\zeta = -0.03$  and (b) beam with distribution  $\rho_b = \rho_o[1 - (r/R)^2]^2$  truncated along equipotential lines of effective focusing field.

$z = 0$

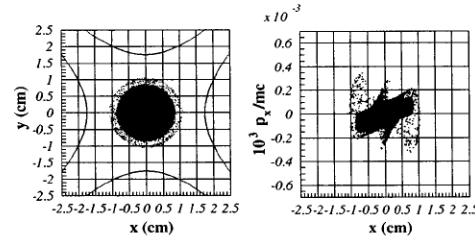
# Quadrupole channel



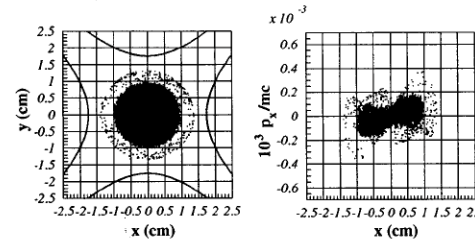
$z = 30$  cm



$z = 180$  cm

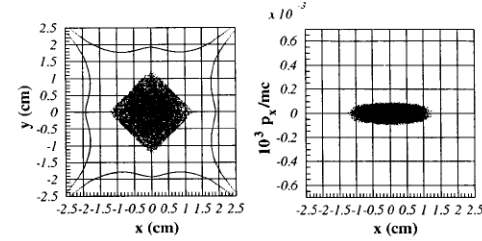


$z = 324$  cm

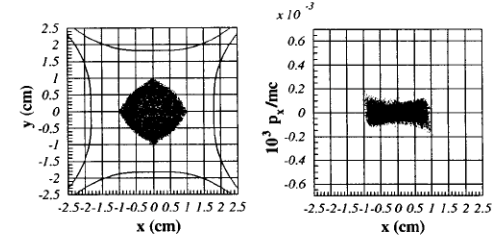


# Quadrupole-duodecapole channel

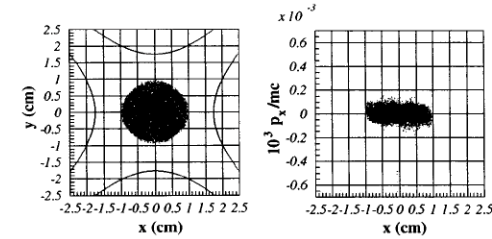
$z = 0$



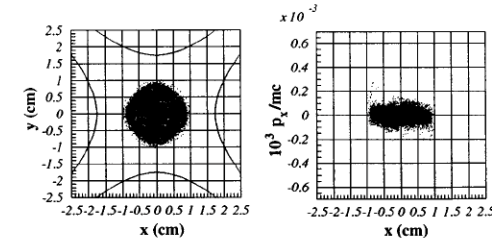
$z = 50$  cm



$z = 100$  cm



$z = 324$  cm

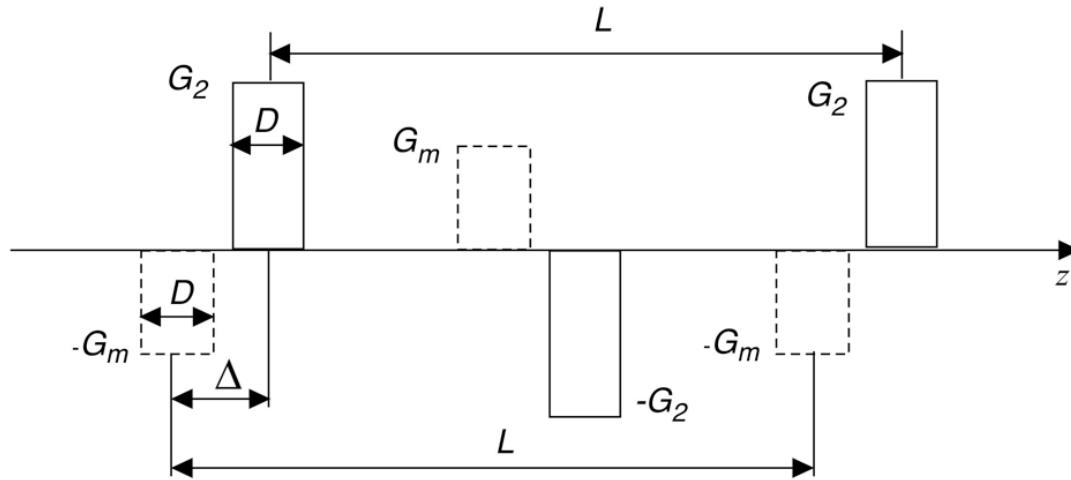


**Dynamics of 150 keV, 100 mA,  $0.06 \pi$  cm mrad proton beam in a structure with  $G_2 = 48$  kV/cm<sup>2</sup>.**

**Adiabatic matching of 150 keV, 100 mA,  $0.06 \pi$  cm mrad proton beam in a structure with  $G_2 = 48$  kV/cm<sup>2</sup>,  $G_6 = -1.9$  kV/cm<sup>6</sup>.**



# Combined FODO Structure with Arbitrary Multipoles\*



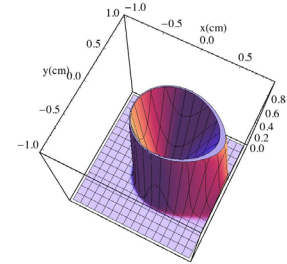
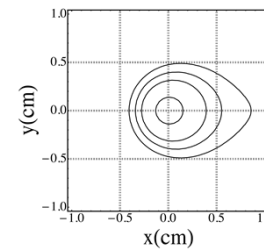
Combined FODO structure with quadrupoles  $G_2$  and multipoles  $G_m$  lenses.

Effective potential:

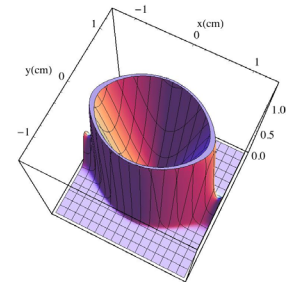
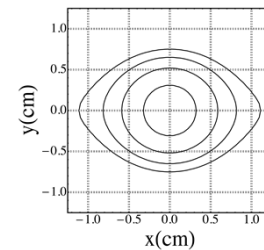
$$U_{eff} = \left(\frac{\sigma_o \beta c}{L}\right)^2 \left[ \frac{r^2}{2} + f \zeta r^m \cos(m-2)\theta + \zeta^2 \frac{r^{2(m-1)}}{2} \right]$$

\*Y.Batygin, A.Scheinker, TUPWA064, IPAC13

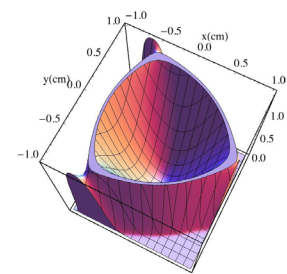
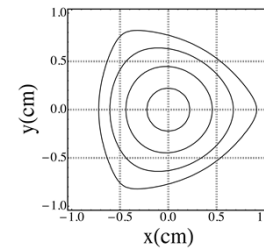
$m = 3$  Quadrupoles + Sextupoles



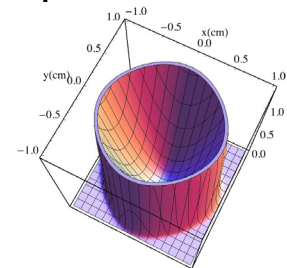
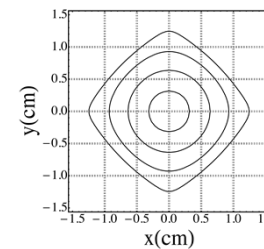
$m = 4$  Quadrupoles + Octupoles



$m = 5$  Quadrupoles + Decapoles



$m = 6$  Quadrupoles + Duodecapoles



# FODO Quadrupole-Duodecapole Channel \*

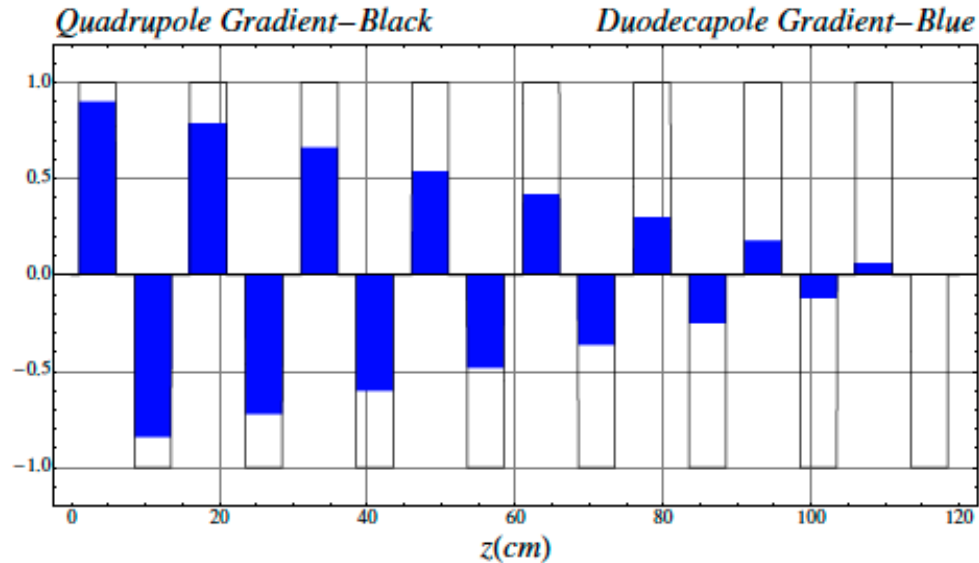
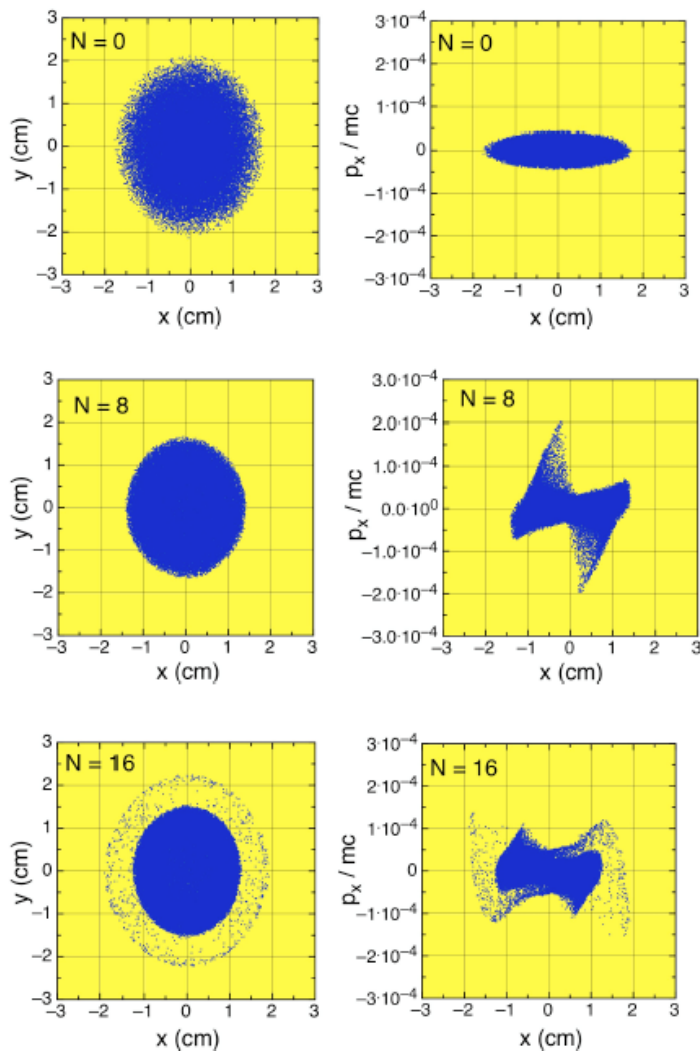


Figure 3: FODO quadrupole-duodecapole channel with combined lenses with the period of  $L = 15$  cm, lens length of  $D = 5$  cm, and adiabatic decline of duodecapole component to zero over a distance of 7 periods.

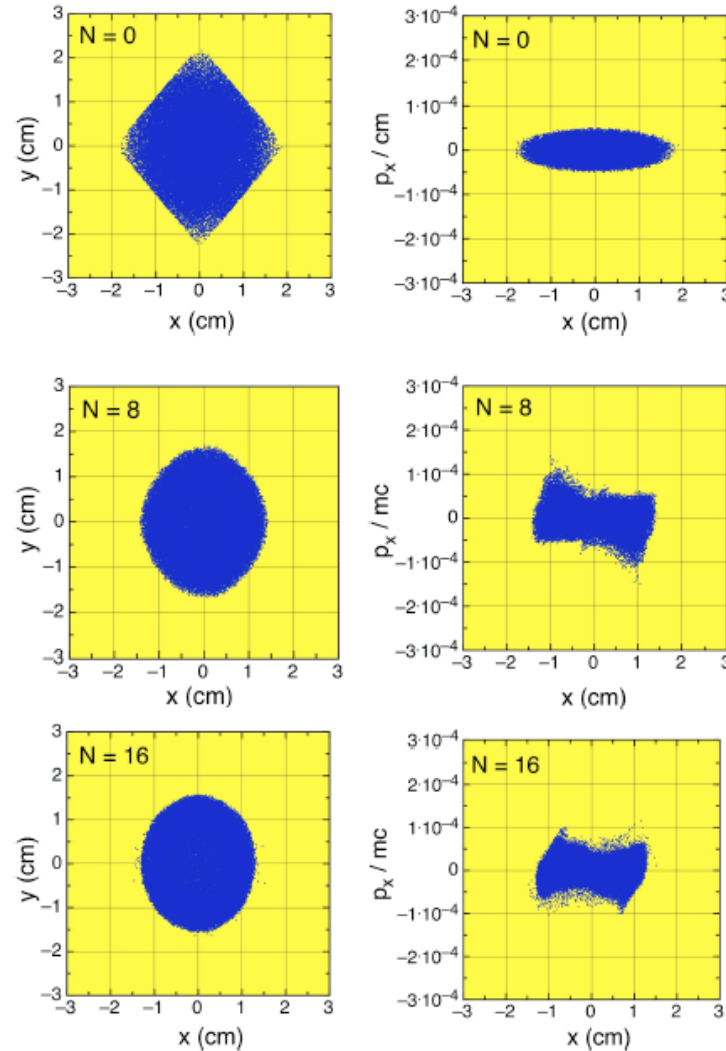
\*Y.Batygin, A.Scheinker, WEPPR039, IPAC12

## Quadrupole Channel



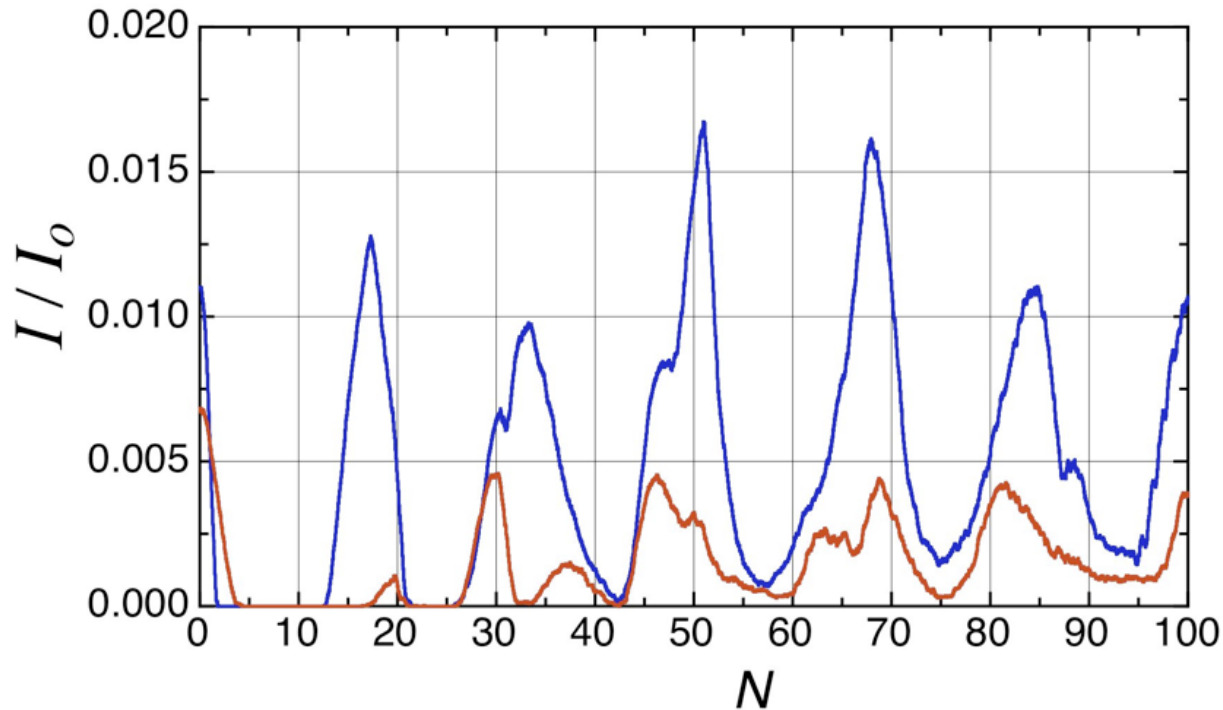
**Energy** 35 keV  
**Current** 11.7 mA  
**Emittance**  $0.05 \pi$  cm mrad  
**Quadrupole**  $G_2 = 0.03579$  T/cm

## Quadrupole-Duodecapole Channel



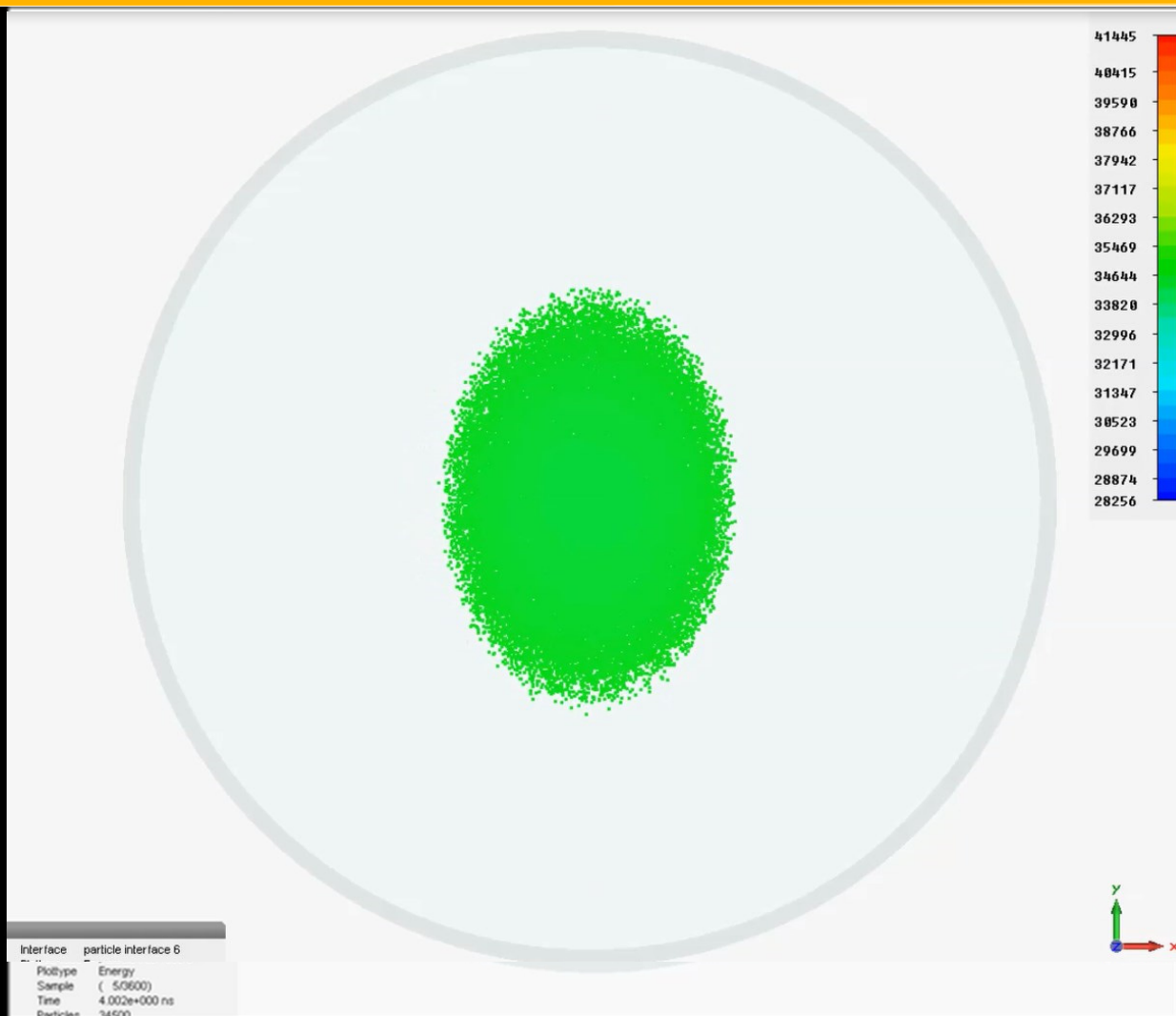
**Quadrupole**  $G_2 = 0.03579$  T/cm  
**Duodecapole**  $G_6 = -1.76e-04$  T/cm<sup>5</sup>  
**Numbers indicate FODO periods**

# Suppression of Beam Halo



Fraction of particles outside the beam core  
 $2.5\sqrt{\langle x^2 \rangle} \times 2.5\sqrt{\langle y^2 \rangle}$  as a function of FODO  
periods: (blue) quadrupole channel, (red) quadrupole-  
duodecapole channel.

# CST Particle Studio Simulation of Halo Formation in Quadrupole Channel



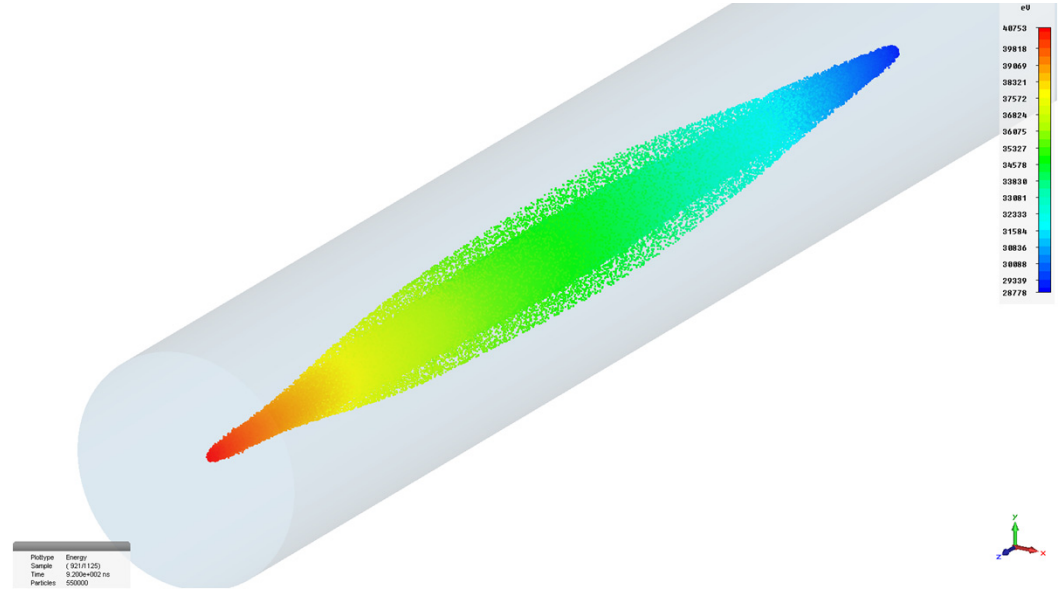
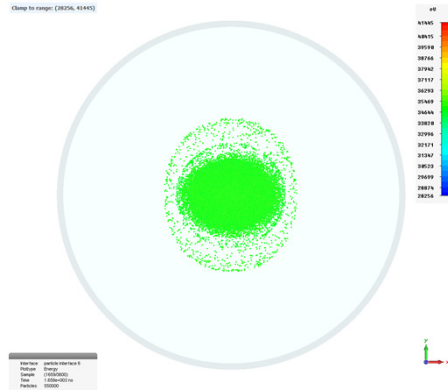
# CST Particle Studio Simulation of Halo Suppression in Quadrupole-Duodecapole Channel



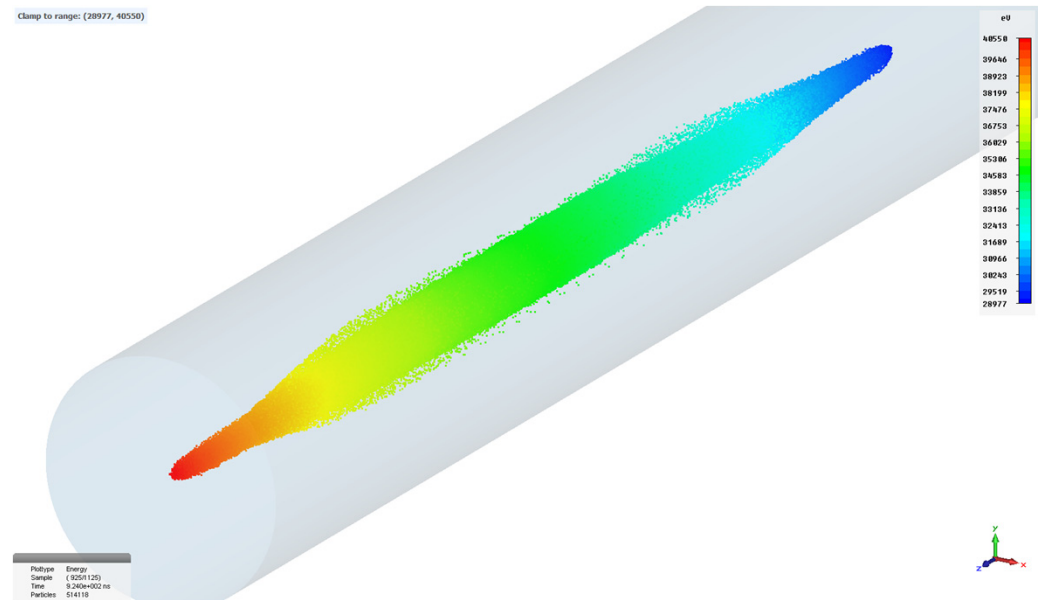
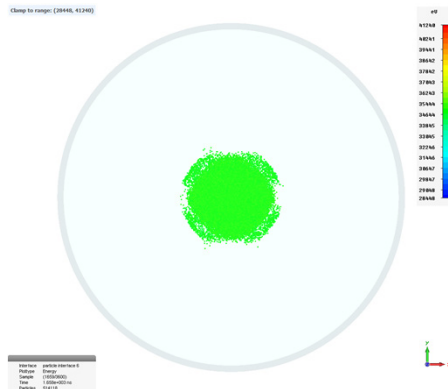


# Final Particle Distributions in Focusing Channels

## Quadrupole Channel



## Quadrupole-Duodecapole Channel



# Summary

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- 1. Beam emittance growth and halo formation due to free-energy excess in high-brightness beams are unavoidable in linear focusing channel.**
- 2. To prevent beam emittance growth and halo formation, focusing fields have to be a nonlinear function of radius.**
- 3. Quadrupole-duodecapole focusing structure is an effective way to suppress beam halo formation.**