High Gain FEL based on a Transverse Gradient Undulator

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Overview

- The transverse gradient undulator (TGU) is a concept for reducing the sensitivity of the FEL gain with respect to the energy spread of the electron beam.

- Previous studies include:

  low gain regime: T. Smith et. al., J. Appl. Phys. 50, 4580 (1979).
  N. Kroll et. al., IEEE Journal of Quan. Electro. QE-17, 1496 (1981).
  high gain regime (1-D theory and 3-D simulations): Z. Huang, Y. Ding and C. Schroeder, PRL 109, 204801 (2012)

- In this talk, we present a 3-D theory of a high-gain TGU-based FEL and discuss some numerical examples.

Introduction

- The FEL performance depends critically on the energy spread of the driving electron beam.

✓ Efficient lasing requires

$$\boxed{\frac{\sigma_\delta}{\rho} < 1}$$

Big energy spread results in a large spread of the resonant wavelength

$$\boxed{\rho = \left(\frac{K_0^2 JJ^2}{16\gamma_0^3\, k_u^2 \sigma_x \sigma_y} \frac{I}{I_A} \right)^{1/3}}$$

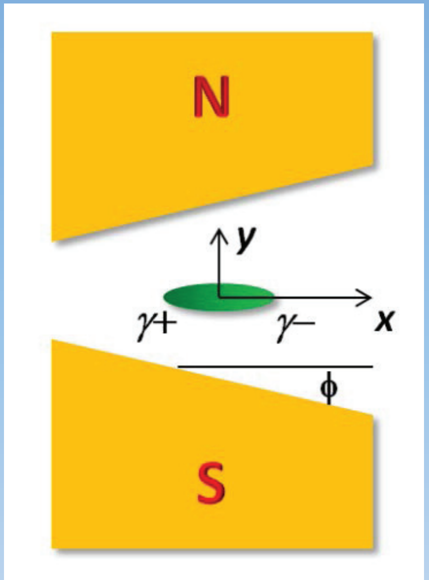
is the FEL parameter.

✓ For beams from plasma wakefield accelerators, it is hard to satisfy this condition (low emittance and high peak current but also large energy spread).

Transverse Gradient Undulator (TGU)

- ✓ Use a dispersive element to spread out the beam:

$$\delta = \frac{\gamma - \gamma_0}{\gamma_0} = \frac{x}{\eta} \rightarrow \gamma = \gamma_0(1 + x/\eta)$$



- ✓ Introduce a linear field gradient by canting the poles: $K = K_0(1 + ax)$

- ➢ If we select $a = \frac{2 + K_0^2}{\eta K_0^2}$

all particles in the beam satisfy the resonance condition $\lambda_r = \lambda_u \frac{1 + K^2/2}{2\gamma^2}$

Self-consistent 3-D theory

- FEL phase equation in the parallel beam limit

$$\theta' = \frac{d\theta}{dz} = 2k_u(\delta - \frac{x}{\eta})$$

(extra term due to the field gradient)

- Linearized, frequency-domain, *Vlasov-Maxwell equations*

$$\frac{\partial f_\nu}{\partial z} + i\nu\theta' f_\nu = -\kappa_1 \frac{\partial f_0}{\partial \delta} E_\nu e^{-i\Delta\nu k_u z}$$

$$\left(\frac{\partial}{\partial z} + \frac{\nabla_\perp^2}{2ik_r}\right) E_\nu = -\kappa_2 e^{i\Delta\nu k_u z} \int_{-\infty}^{\infty} d\delta f_\nu$$

$\Delta\nu = \nu - 1$

$\nu = \omega/\omega_r$

- The background distribution for the dispersed beam:

$$f_0 = \frac{N_b/l_b}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_\delta} \exp\left(-\frac{(x-\eta\delta)^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right) \exp\left(-\frac{\delta^2}{2\sigma_\delta^2}\right)$$

- ✓ We seek the guided FEL eigenmodes: $\boxed{E_\nu(\mathbf{x}, z) \propto A(\mathbf{x}) e^{i\mu z}}$
- ✓ This leads to an eigenmode equation

$$\boxed{\begin{aligned}\left(\mu - \frac{\nabla_\perp^2}{2k_r}\right) A(\mathbf{x}) &= -8\rho_T^3 k_u^3 A(\mathbf{x}) \exp\left(-\frac{x^2}{2\sigma_T^2} - \frac{y^2}{2\sigma_y^2}\right) \\ &\times \int_{-\infty}^{0} d\xi \xi e^{i(\mu - \Delta\nu k_u)\xi} e^{-2\sigma_{ef}^2 k_u^2 \xi^2} \exp(-2ik_u\xi \frac{\sigma_x^2}{\sigma_T^2}\frac{x}{\eta})\end{aligned}}$$

$\boxed{\sigma_T = (\sigma_x^2 + \eta^2\sigma_\delta^2)^{1/2}}$ total horizontal beam size

$\boxed{\rho_T = \rho(1 + \frac{\eta^2\sigma_\delta^2}{\sigma_x^2})^{-1/6}}$ attenuated FEL parameter and

$\boxed{\sigma_{ef} = \sigma_\delta(1 + \frac{\eta^2\sigma_\delta^2}{\sigma_x^2})^{-1/2}}$ effective energy spread. An efficient TGU requires $\eta\sigma_\delta/\sigma_x >> 1$, so $\sigma_{ef} \approx \sigma_x/\eta$

- ✓ Assuming a trial solution of the form

$$\boxed{A(\mathrm{x}) = \exp(-a_x x^2 + bx)\exp(-a_y y^2)}$$

we use a variational technique to obtain an approximation to the growth rate of the fundamental FEL eigenmode.

- ✓ The final dispersion relations can be expressed in a fully analytical form (allows for fast calculations).

- ✓ The eigenmode analysis can be extended in order to take into account emittance in the y-direction as well as the undulator natural focusing.

- ✓ However, we disregard natural focusing in the x-direction (~ $(\gamma\eta)^{-1}$, typically weak) and assume that a small net bending (~ B_0/γ) has been corrected.

Soft X-ray FEL with 1 GeV LPA beam

LETTERS

GeV electron beams from a centimetre-scale accelerator

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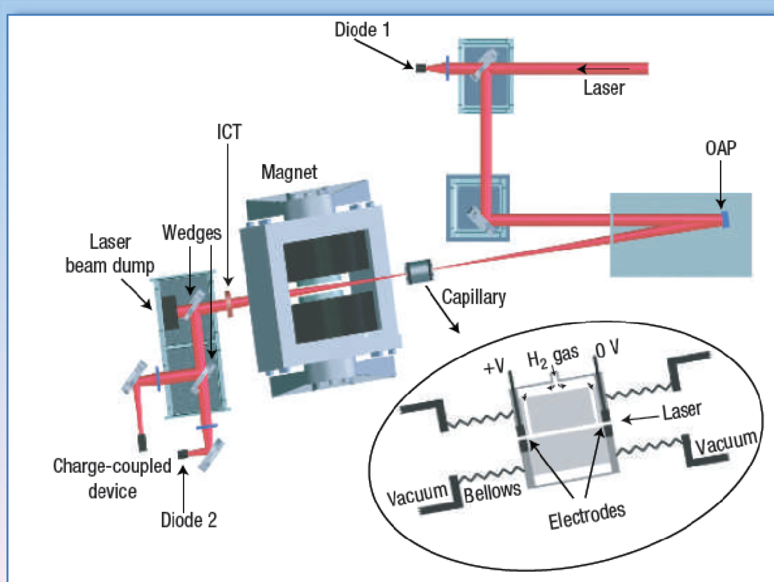
Nat. Phys. 2, 696 (2006)

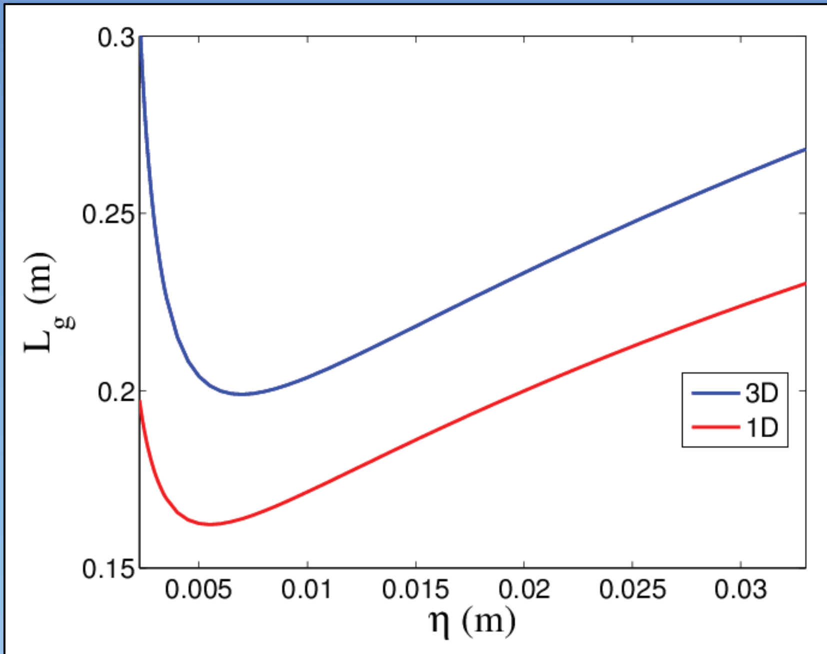
Laser-plasma accelerators (LPAs) have demonstrated the capability to produce e-beams in the GeV range.



- 1 GeV, λ_u = 1 cm, K_0 = 2

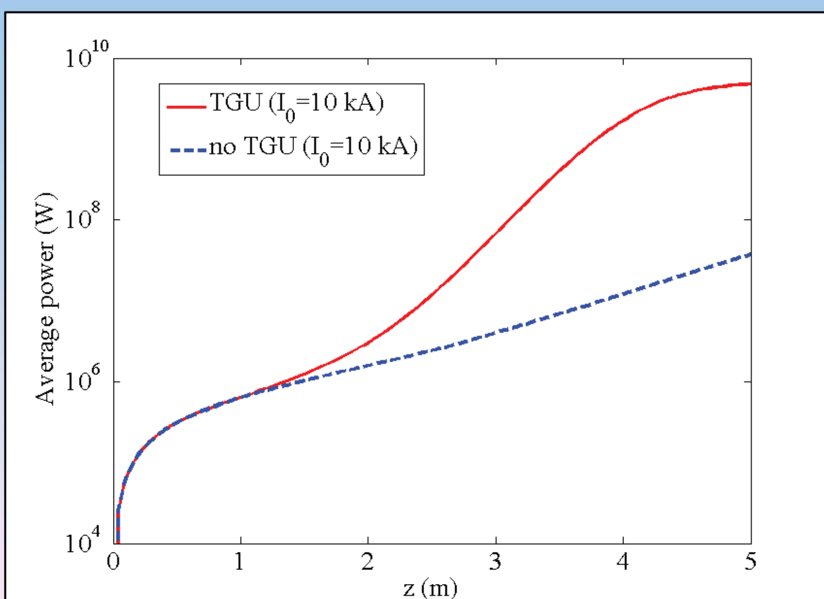
  radiation wavelength λ_r = 3.9 nm
- 10 kA peak current,
- 0.1 mm-mrad norm. emittance
- 1% energy spread

$\sigma_x = \sigma_y \sim 10\ \mu m$



- ✓ We calculate the frequency-optimized gain length as a function of dispersion using the parallel beam theory.
- ✓ Results are compared with the 1D formula

$$L_g \approx \frac{\lambda_u}{4\pi\sqrt{3}\rho_T}\left[1+\frac{\sigma_{ef}^2}{\rho_T^2}\right]$$



For a 1 cm dispersion

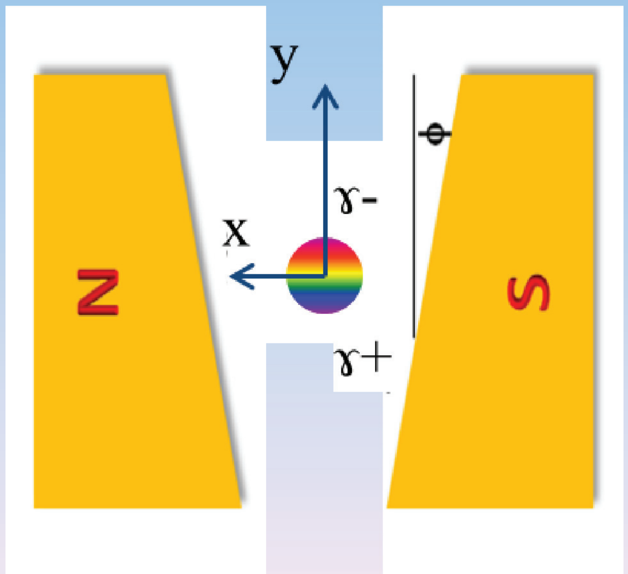
- ✓ e-beam size: 100 um x 10 um
- ✓ optimum gain length ~ 21 cm
- ✓ Saturation within 5 m

Soft X-ray FEL in an Ultimate Storage Ring

- A single pass, high-gain FEL based on an ultimate storage ring (USR) may be possible using a TGU.

✓ The undulator is placed in a bypass next to the ring.

✓ Rotate by 90º to take advantage of the very low vertical emittance.

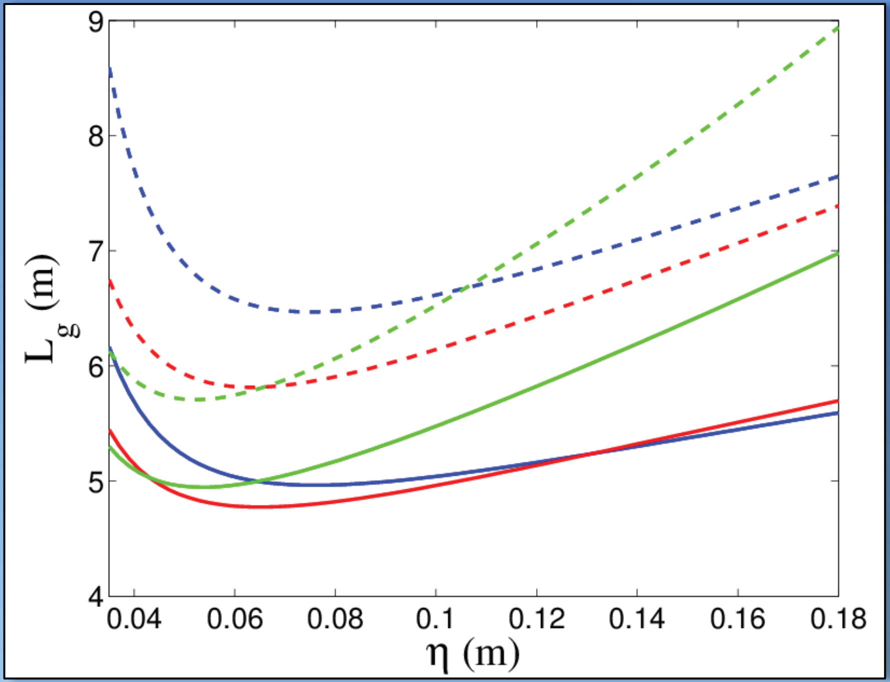


PEP-X as an example

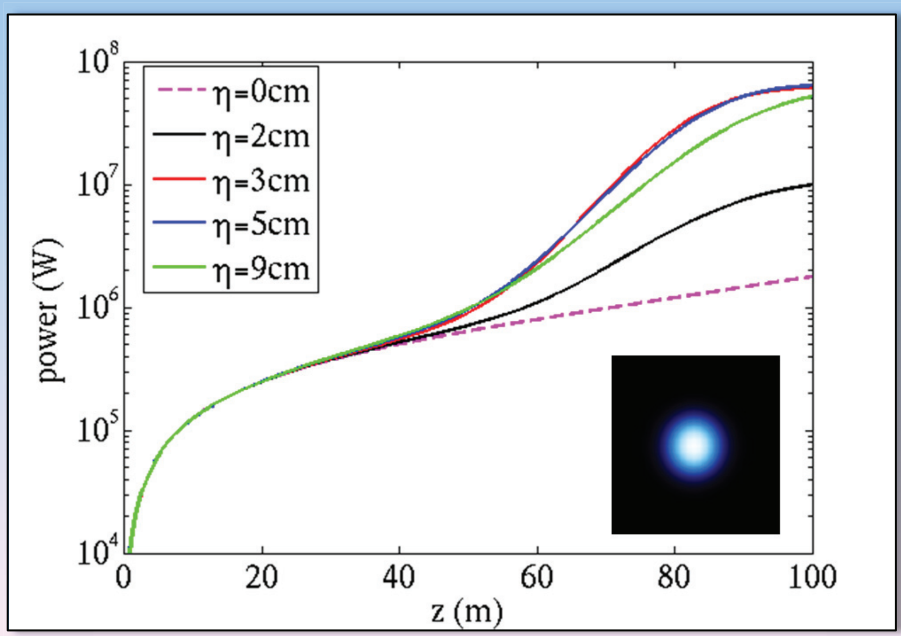
(Y. Ding et al. , IPAC-13)

- 4.5 GeV, λ_u = 2 cm, K_0 = 3.68
  radiation wavelength λ_r = 1 nm
- 200 A peak current
  0.0123/1.23 mm-mrad n.emittance
  0.15% energy spread

$\sigma_x \sim 10\ \mu m$, $\sigma_y \sim 40\ \mu m$



Gain length vs. dispersion for several values of the detuning ($\Delta\nu/(2\rho)$ = 0.0/-0.2/-0.4) from the parallel beam model (solid lines) and including emittance (dashed).



For a dispersion ~ 3-5 cm

- ✓ Saturation within 100 m.
- ✓ Roughly a round electron/radiation beam (better transverse coherence).

Summary

- A self-consistent 3D theory has been developed for a TGU-based, high-gain FEL.
- We use a variational method to determine the properties of the fundamental FEL mode.
- In the parallel beam limit, we derived fully analytical relations.
- This model has been be extended to include vertical emittance and natural focusing (relevant for USR-based TGU FELs).
- The theory agrees with simulation as well as other analytical results and can be used for optimization studies.