

Acceleration and Transportation of Multiple Ion Species at EBIS-Based Preinjector

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Acknowledgements



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Outline of Talk

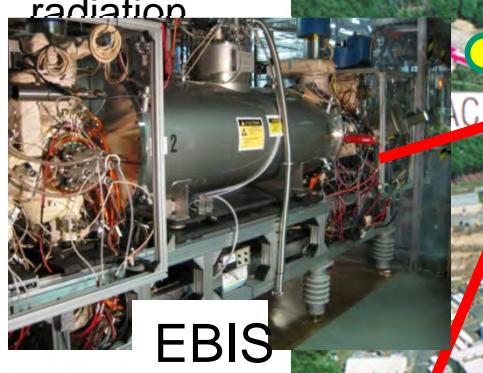


- Brookhaven Accelerator Complex
- Overview of the EBIS Pre-injector
- Electron Beam Ion Source (EBIS)
- Why Multiple Ion Species
- Acceleration and Transportation of multiple ion species
- Conclusions

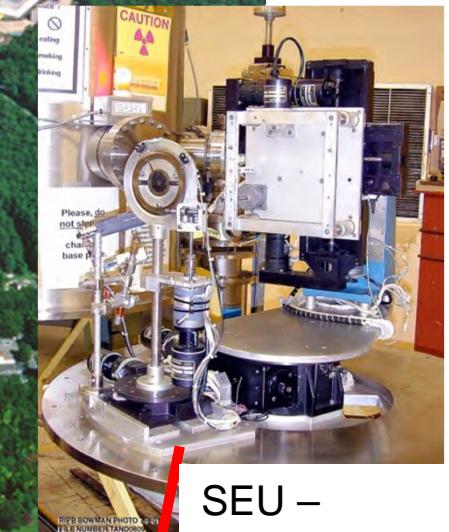
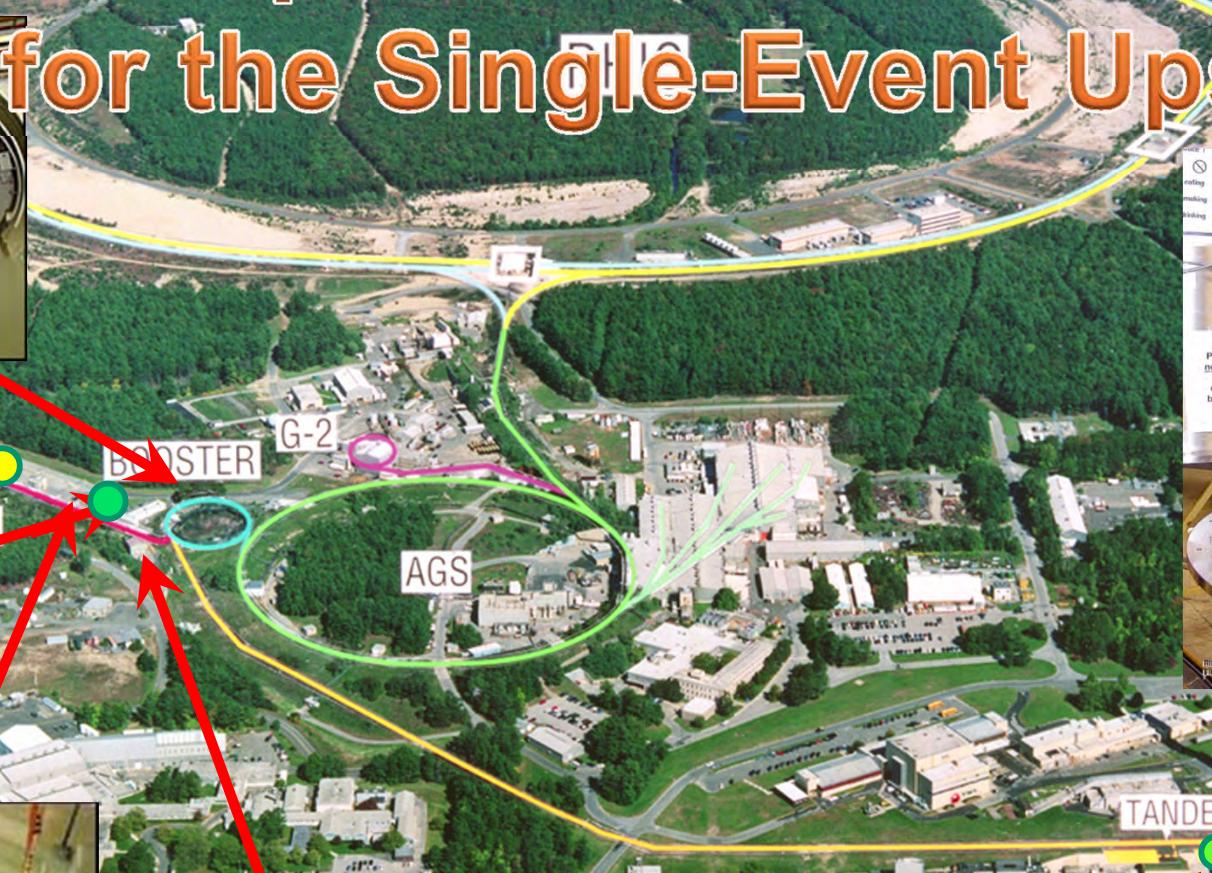
He Many (or all) of these can be running simultaneously!

Heavy ions for the Single-Event Upset Facility

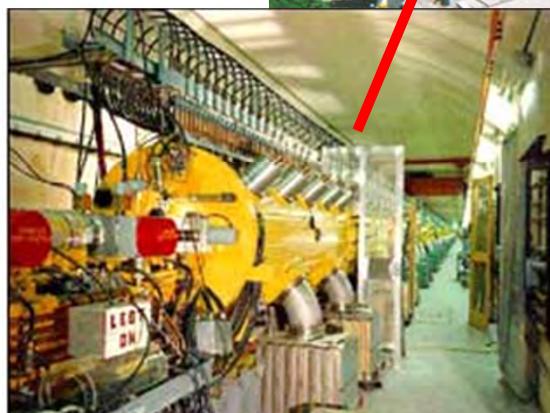
NSRL – biological effects of space radiation



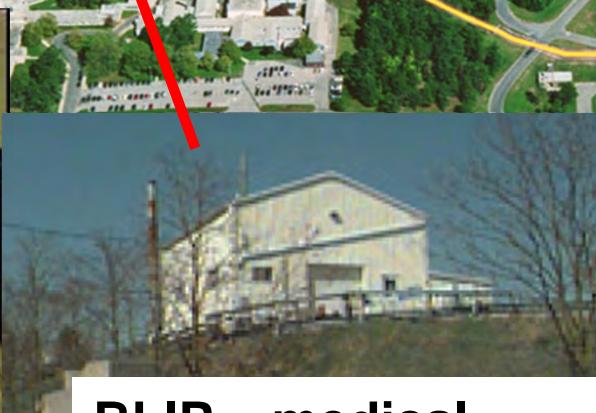
EBIS



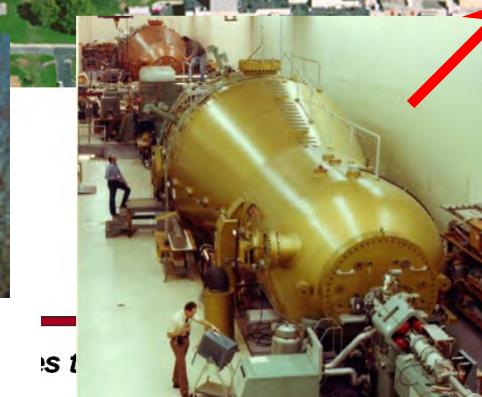
SEU – Radiation effects on electronics



200 MeV Linac



BLIP – medical isotopes



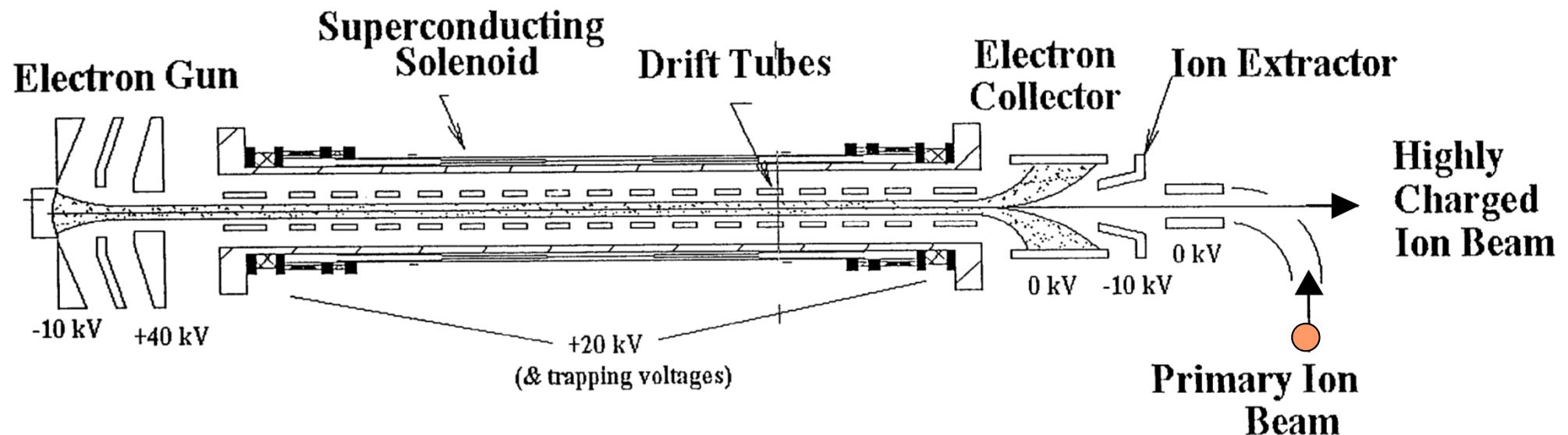
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Two 14 MV Tandem Van de Graaffs

Motivation

- Increased flexibility to handle the simultaneous needs of RHIC and NASA (fast switching between species ~ 1s)
- Capability to provide ions not presently available, such as noble gas ions (for NASA), uranium (RHIC).
- Simpler technology, robust, more modern
- Elimination of two stripping stages and an 860 m long transport line, leading to improved performance

Principle of EBIS

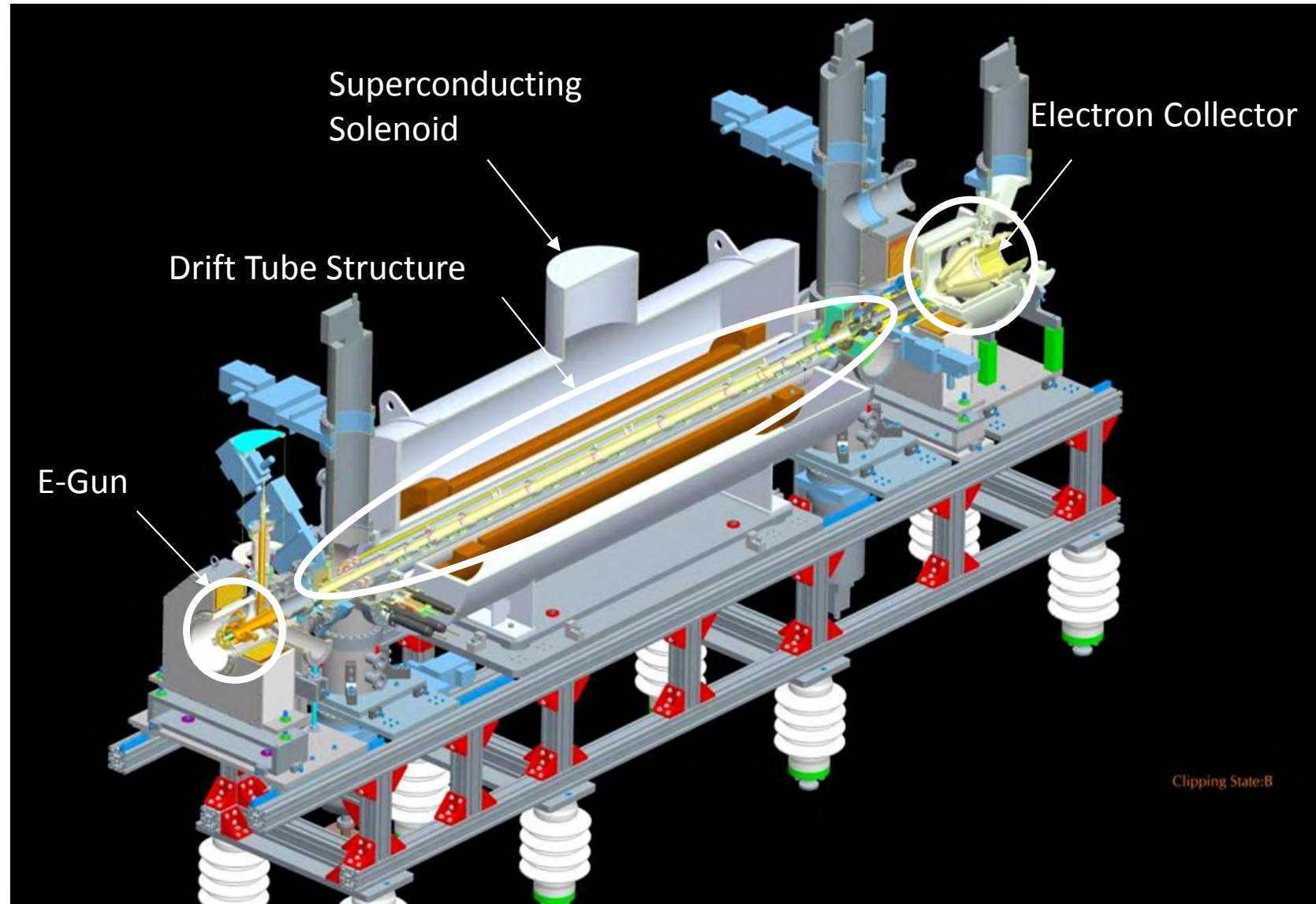


Radial trapping of ions by the space charge of the electron beam.
Axial trapping by applied electrostatic potentials at ends of trap.

- The total charge of ions extracted per pulse is $\sim (0.5 - 0.8) \times (\# \text{ electrons in the trap})$
- Ion output per pulse is proportional to the trap length and electron current.
- Ion charge state increases with increasing confinement time.
- Output current pulse is \sim independent of species or charge state!

Electron Beam Ion Source (EBIS)

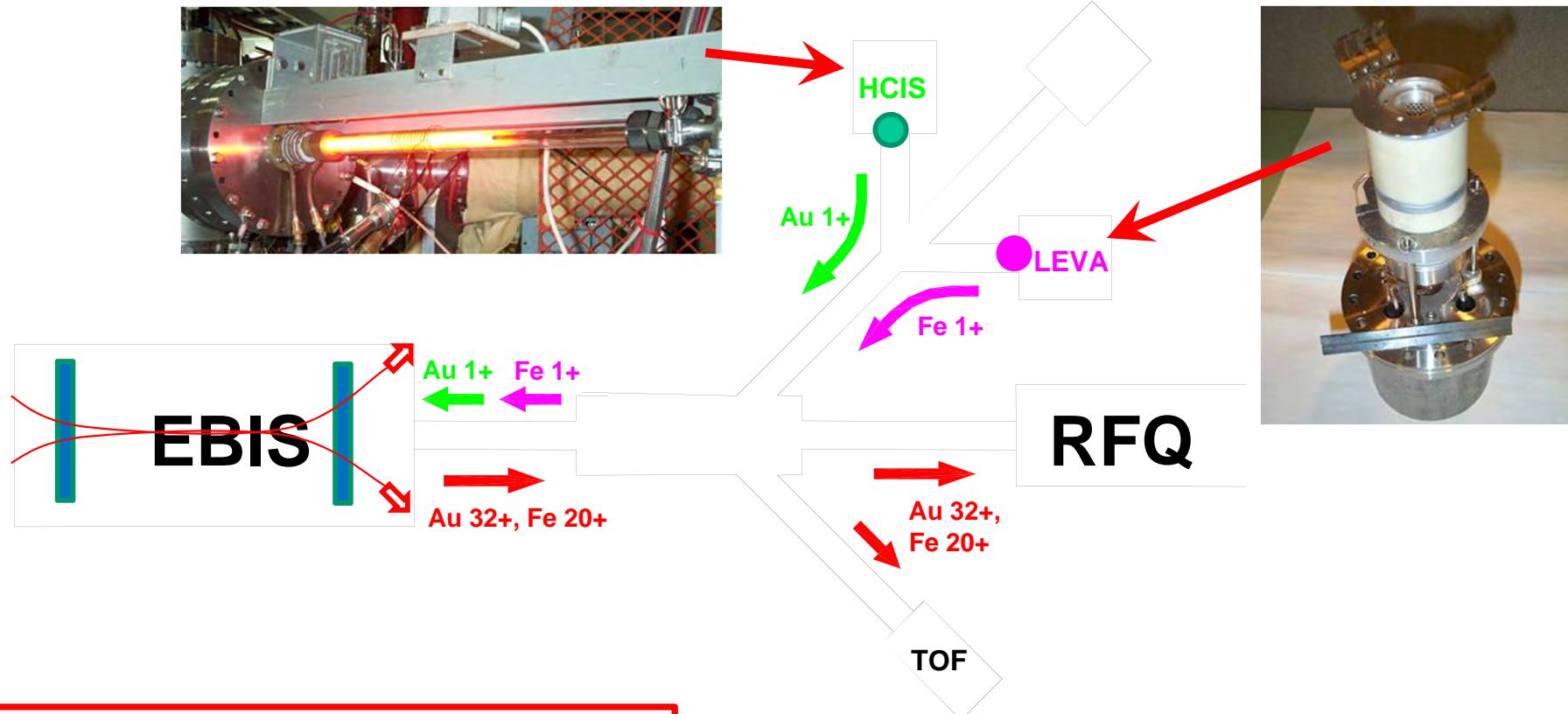
High charge state heavy ions produced by trapping and stepwise ionizing in an electron beam



BNL: 10 A electron beam; 5T magnet; 1.5 m long trap region

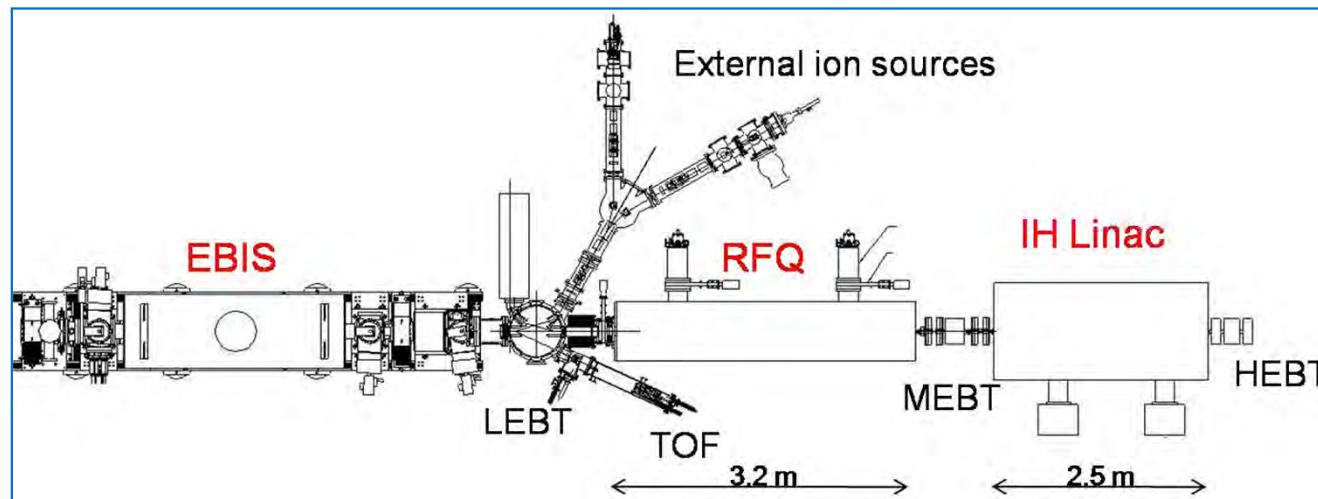
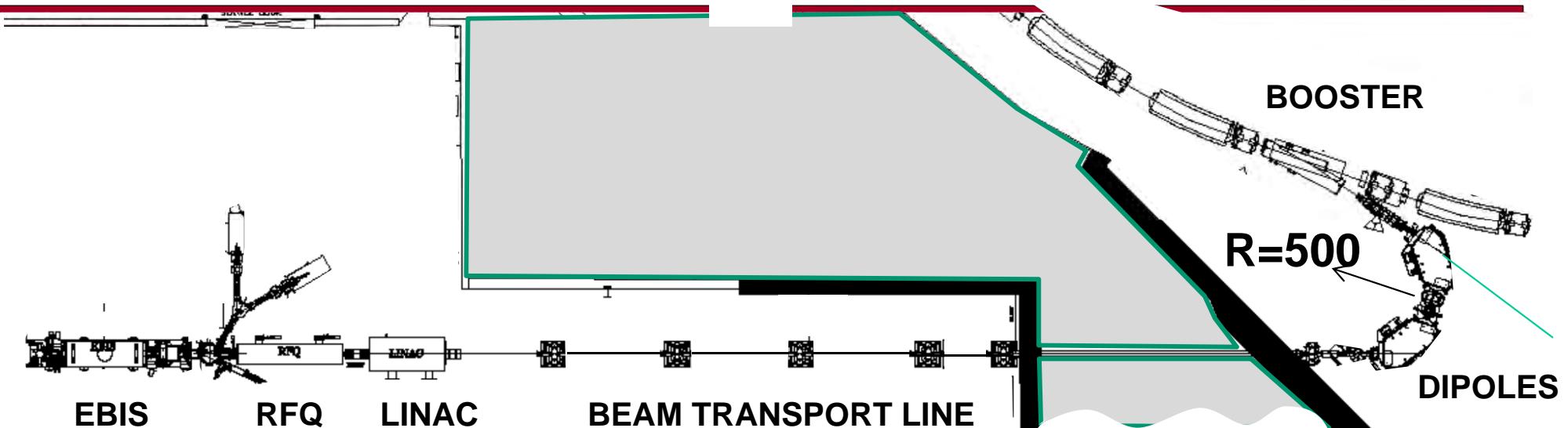
Ion Injection and Extraction from the EBIS

External ion injection provides most ion species.



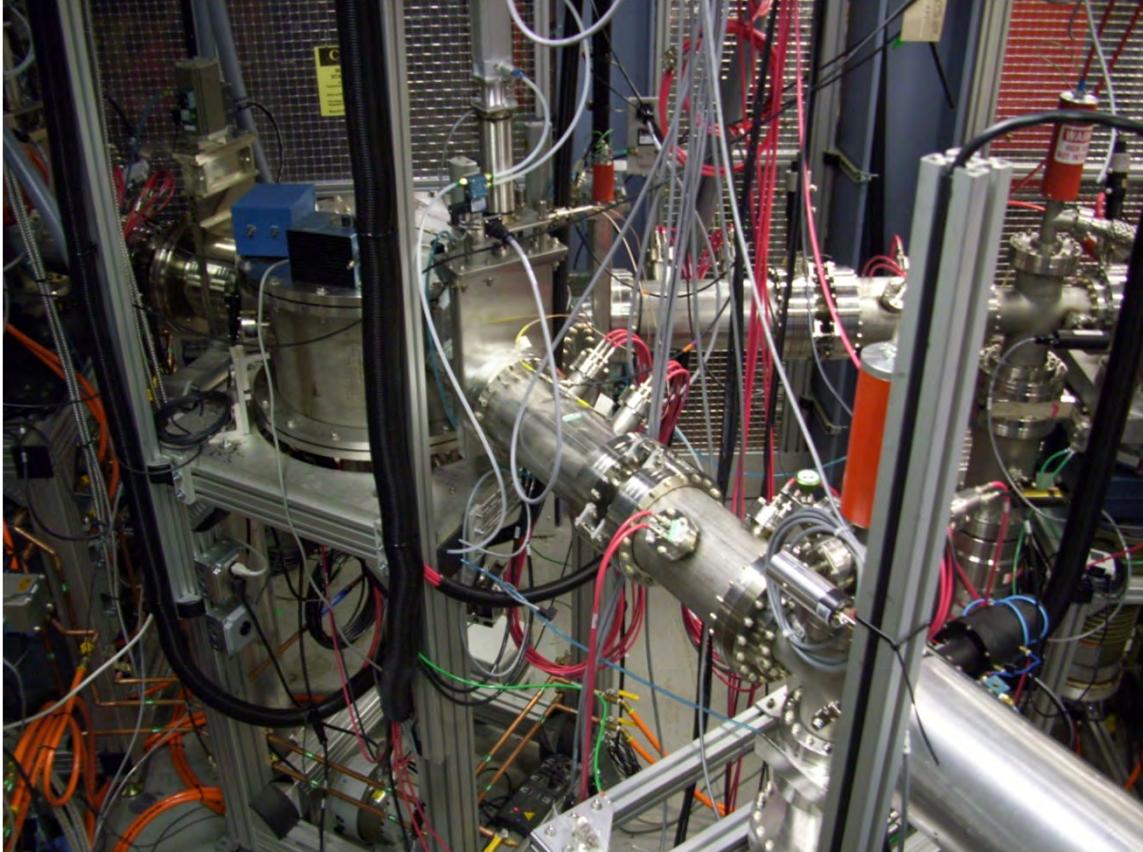
One can change species and charge state
on a pulse to pulse basis

EBIS Preinjector in lower equipment bay of the 200 MeV Linac



| | |
|------------------------|------------------------|
| Ions | He - U |
| Q / m | $\geq 1/6$ |
| Current | > 1.5 emA (10 μ S) |
| Pulse length | 10-40 μ s |
| Rep rate | 5 Hz |
| Output energy | 2 MeV / u |
| $\Delta P/P$ | <0.05% |
| Time to switch species | 1 second |

1+ ion injection lines



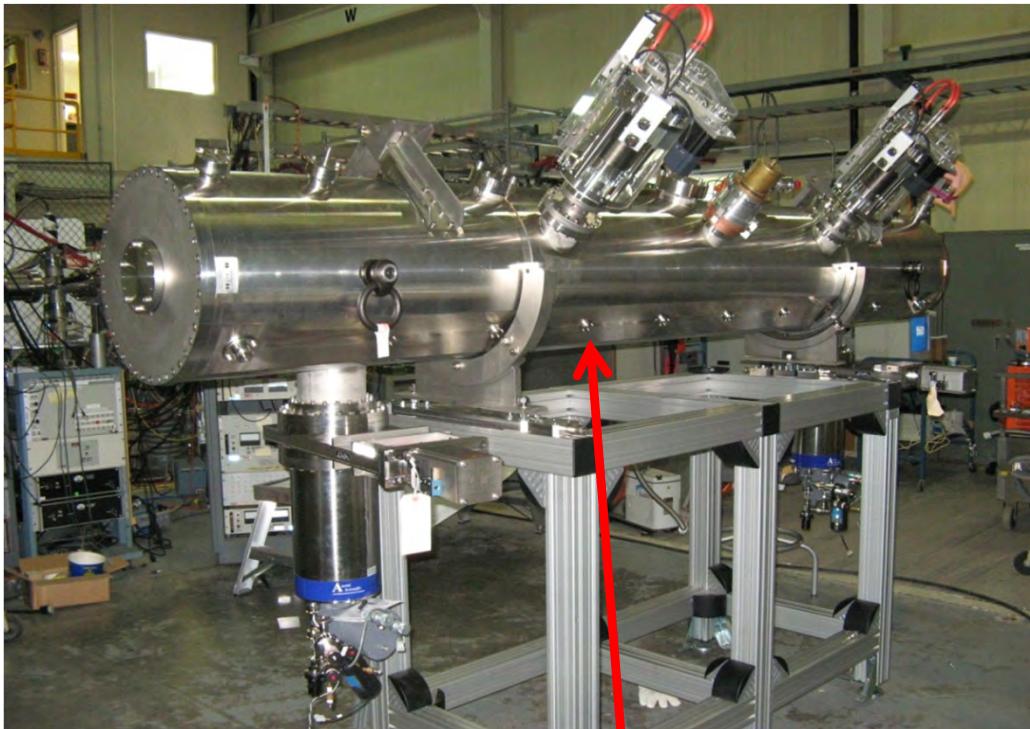
Low Energy Vacuum Arc
Source (I. Brown);



Hollow Cathode Ion Source
based on design used on Saclay
EBIS.



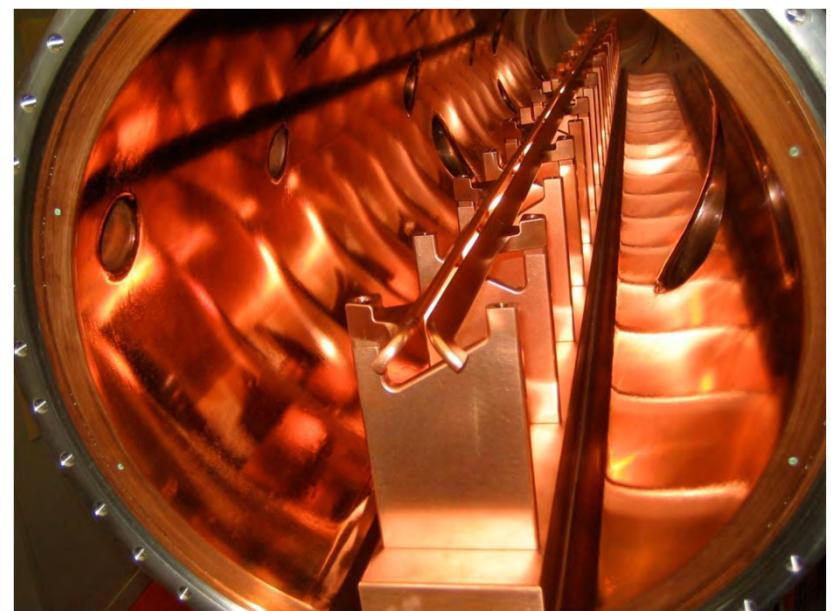
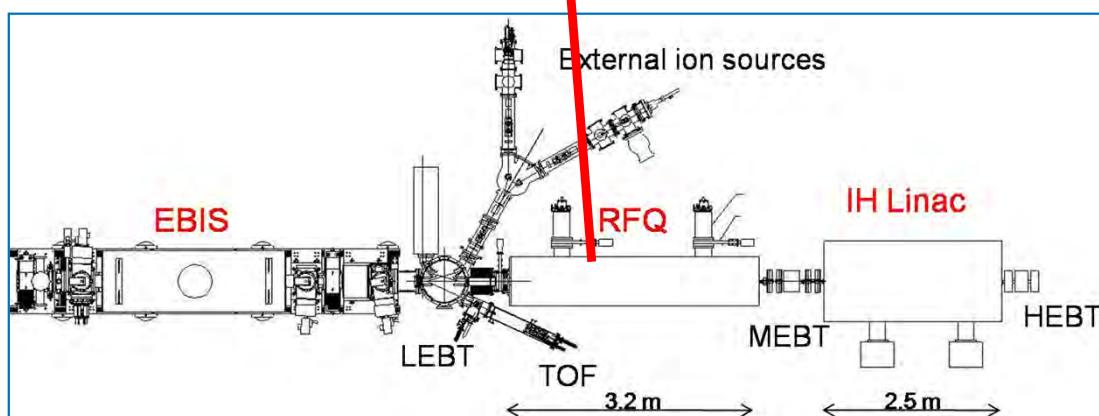
RFQ from IAP, Frankfurt (A. Schempp)



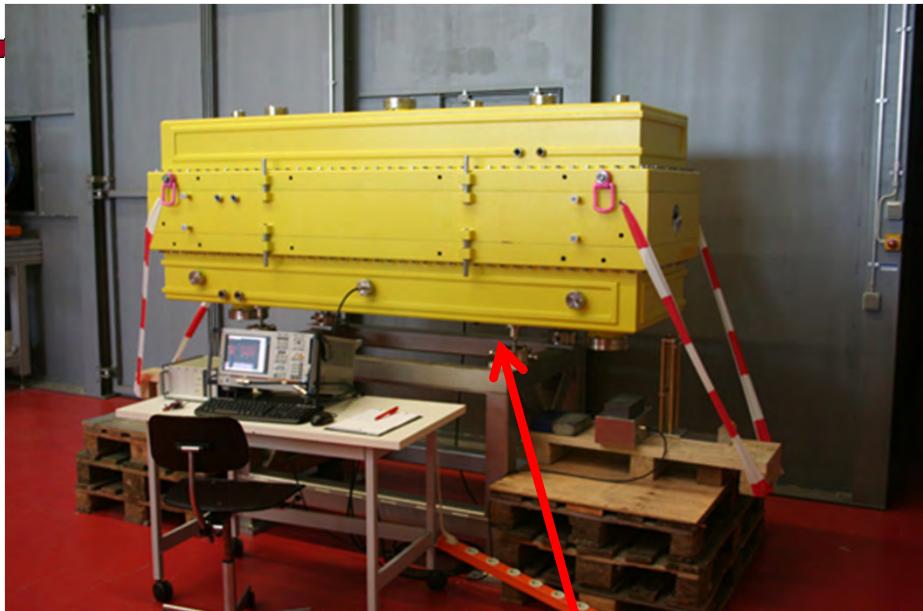
100 MHz

Accelerates the beam from 17 keV/u to 300 keV/u

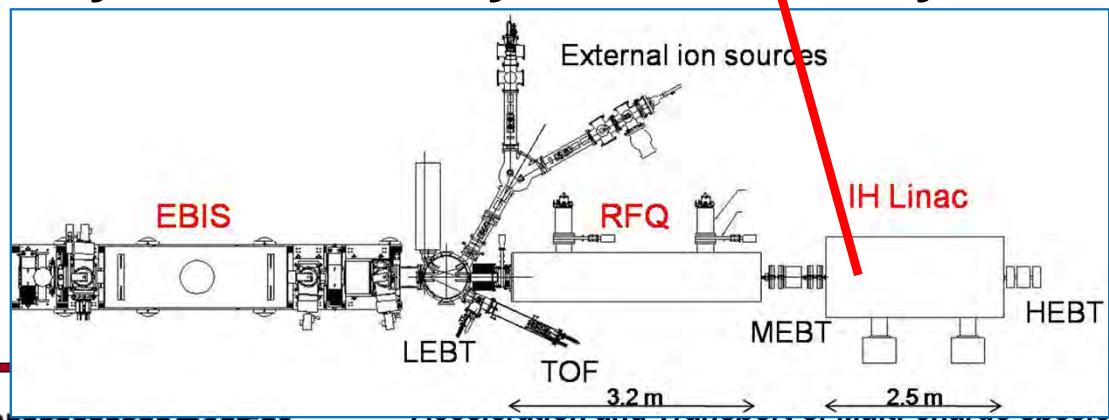
Fabricated by NTG



Linac from IAP, Frankfurt (U. Ratzinger)



100 MHz
Accelerates the beam from 300 keV/u
to 2 MeV/u
Cavity Fabricated by PINK; Quad by Bruker



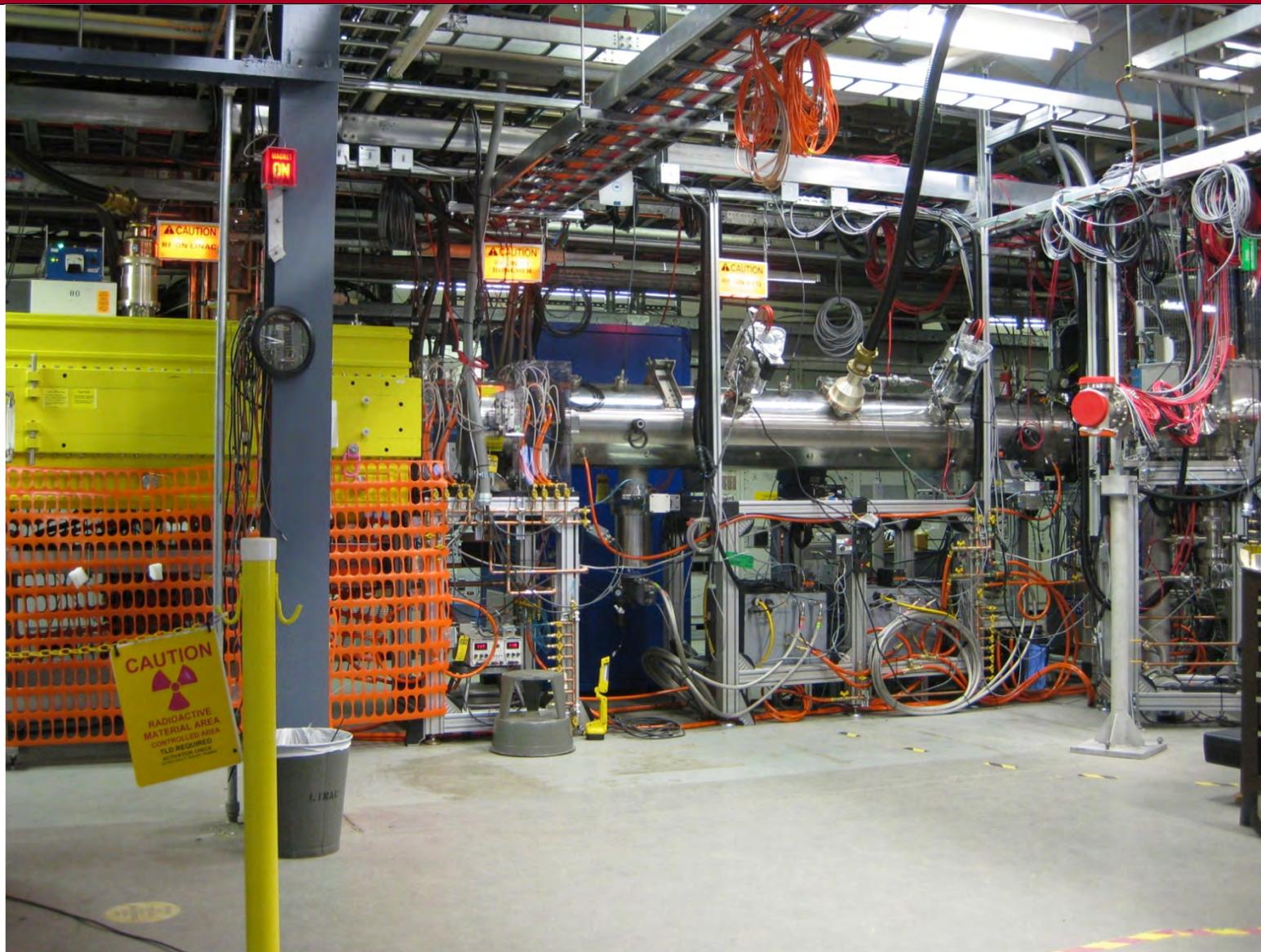
D. Raparia

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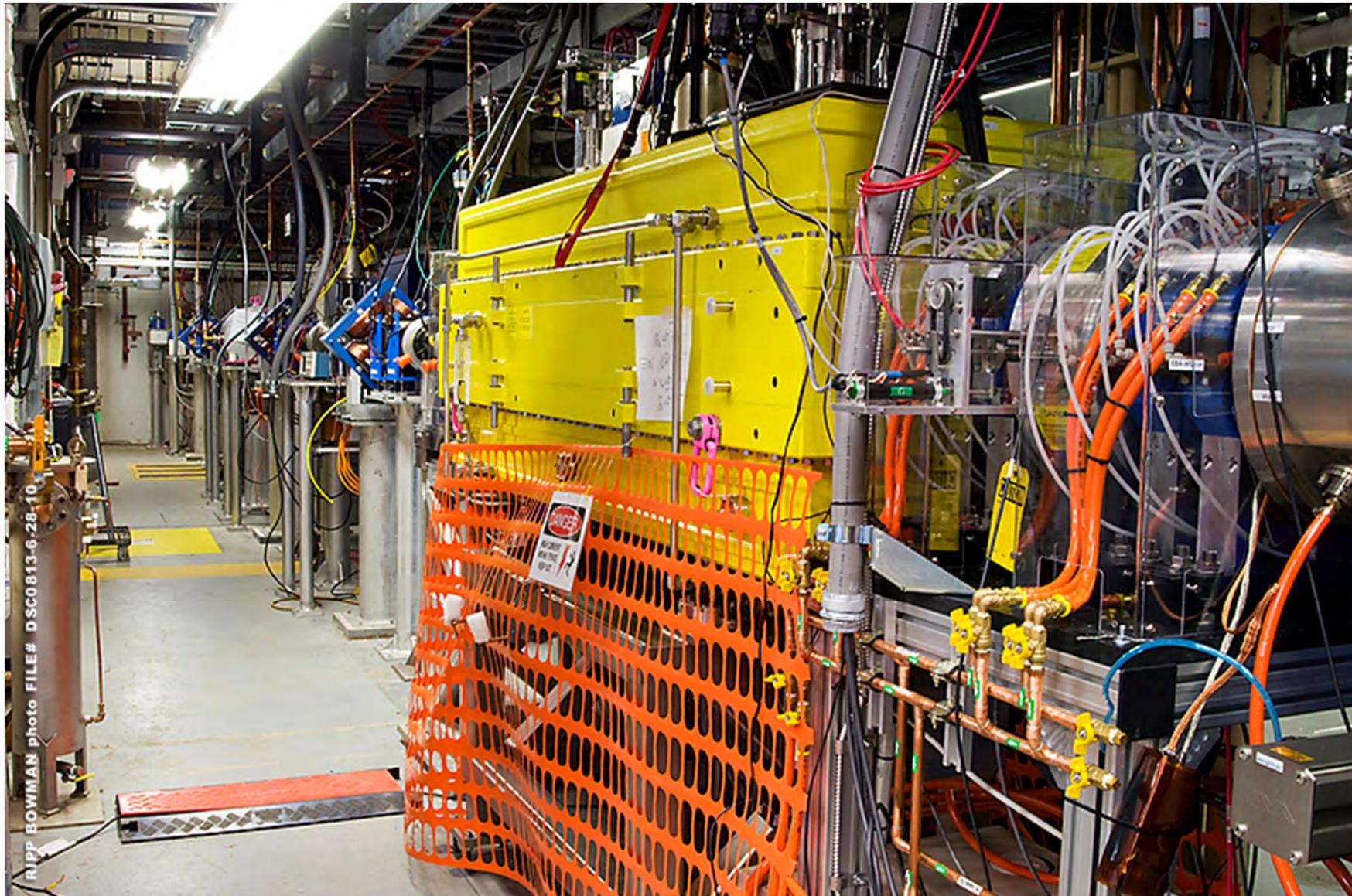


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RFQ, MEBT, and Linac



Linac and EBIS-to-Booster transport

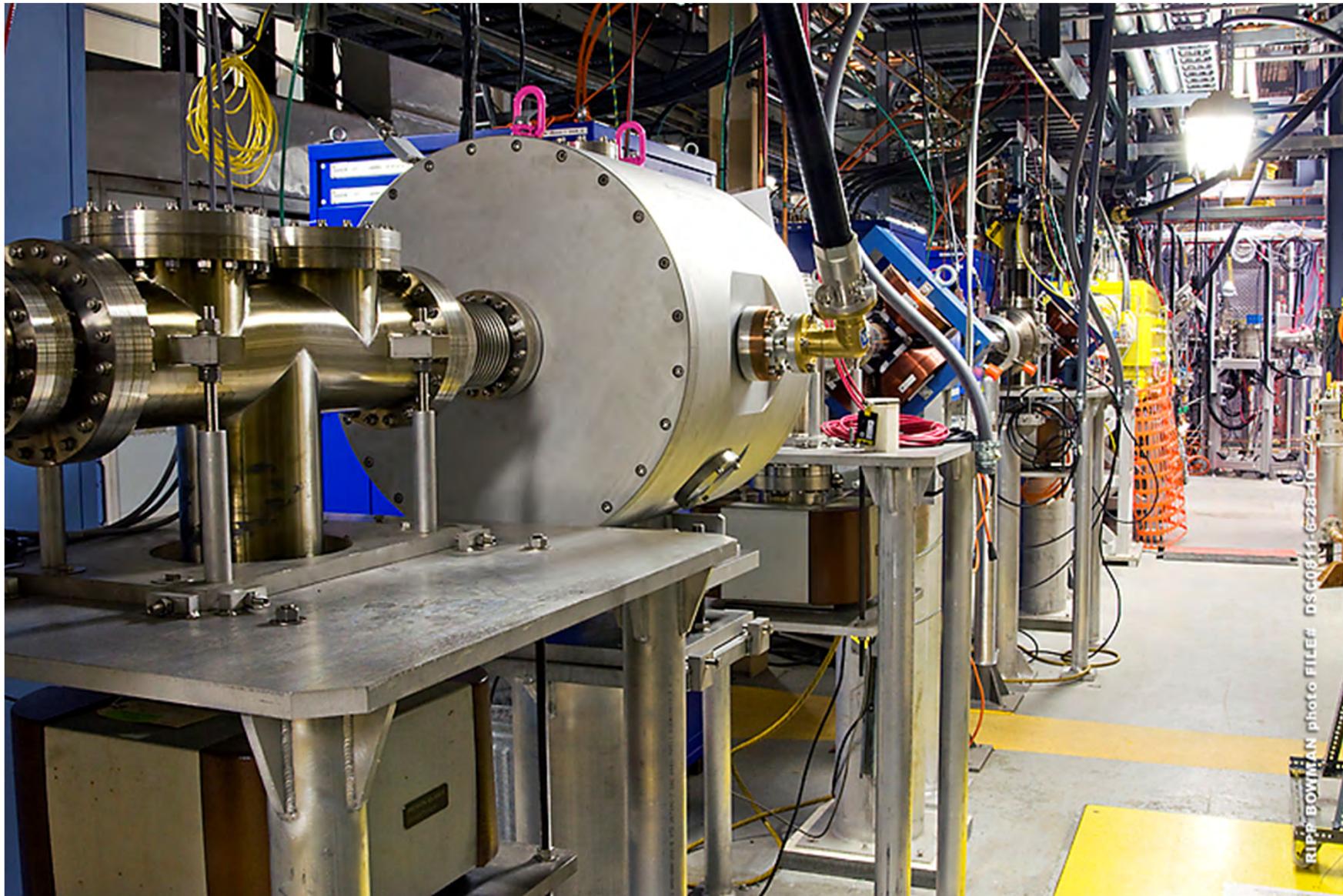


Acceleration and Transport of Multi-charge species trough EBIS Preinjector

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Buncher C-2 in transport line

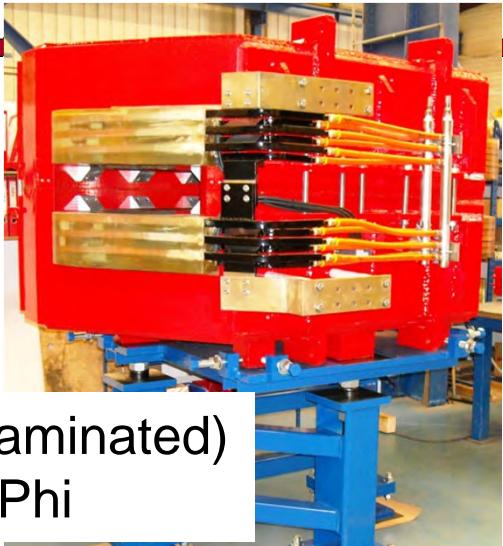


Acceleration and Transport of Multi-charge species trough EBIS Preinjector

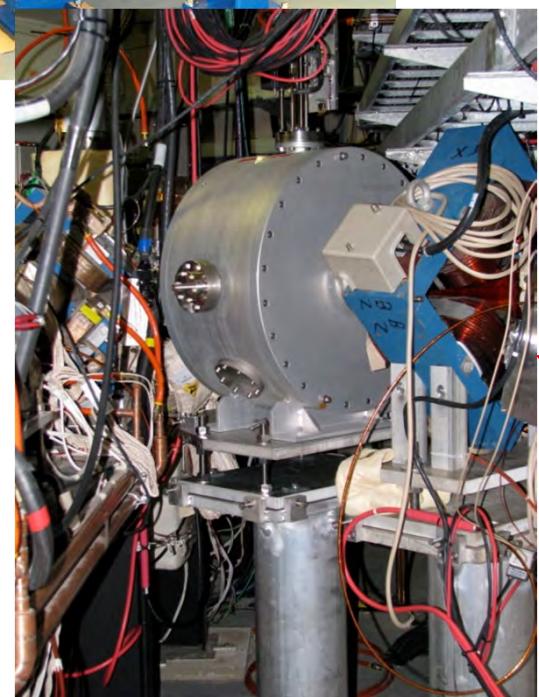
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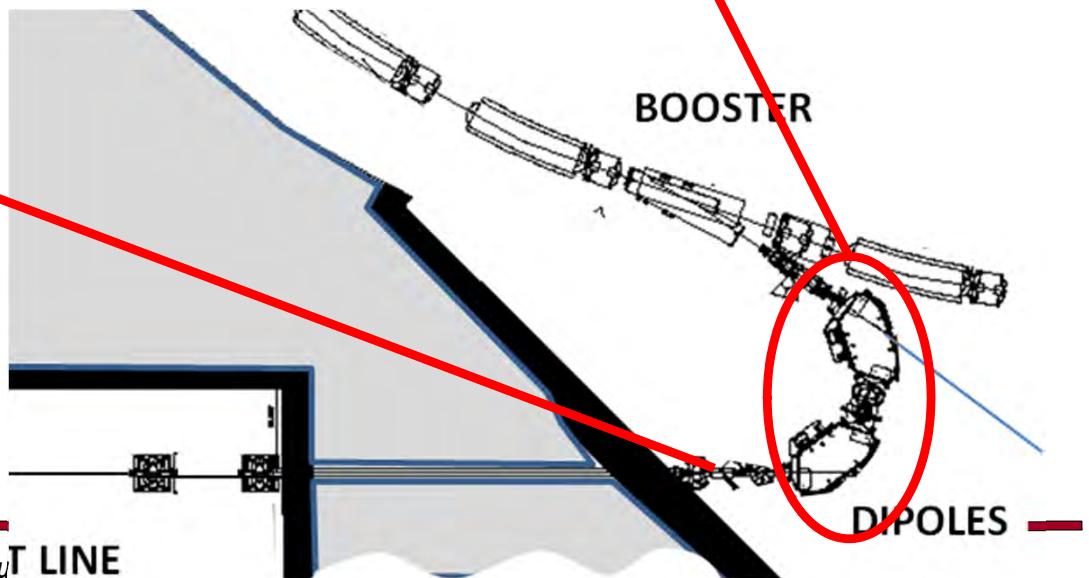
Beamline in Booster



Dipole (laminated)
- SigmaPhi



Buncher



Acceleration and Transport of Mu^T LINE

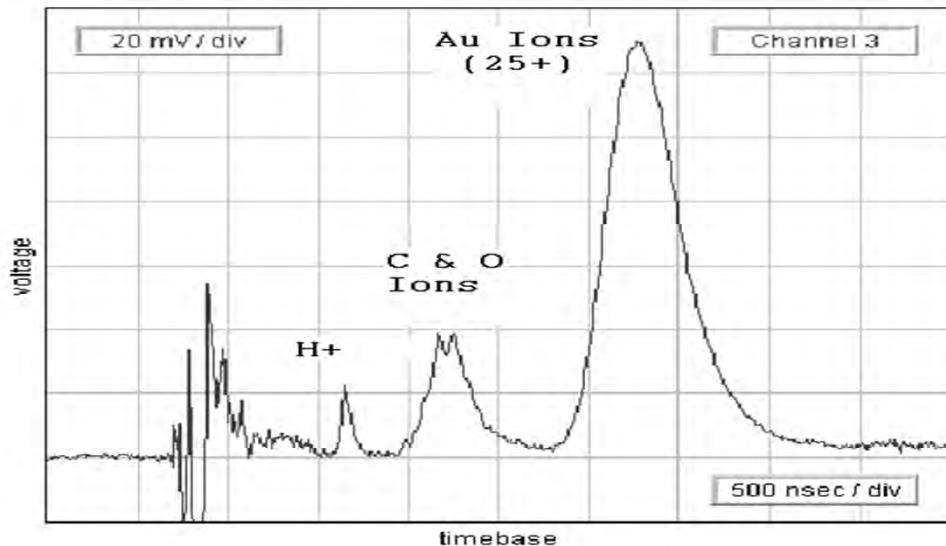
D. Raparia

Ion Beam Delivered to NSRL and RHIC

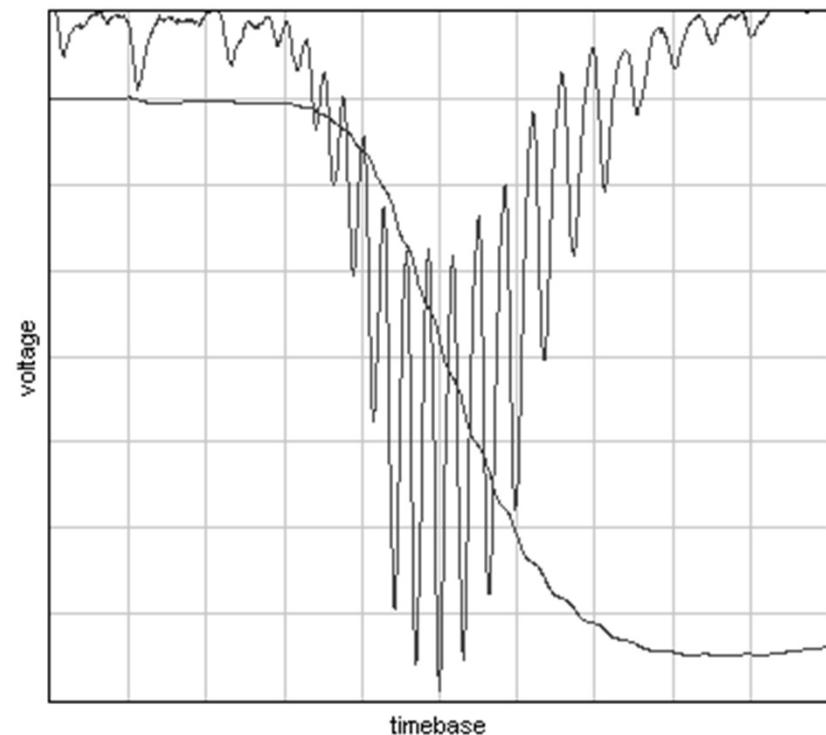


| IONS | Q/m | delivered to | |
|-------------------------|--------|--------------|-------|
| $^3\text{He}^2$ | 0.6667 | AGS | |
| $^4\text{He}^2$ | 0.5000 | NSRL | |
| $^{12}\text{C}^6$ | 0.5000 | NSRL | |
| $^{16}\text{O}^8$ | 0.5000 | NSRL | |
| $^4\text{He}^1$ | 0.2500 | NSRL | |
| $^{20}\text{Ne}^{+5}$ | 0.2500 | NSRL | |
| $^{40}\text{Ar}^{+10}$ | 0.2500 | NSRL | |
| $^{63}\text{Cu}^{11}$ | 0.1746 | RHIC | 4.8e9 |
| $^{40}\text{Ar}^{+7}$ | 0.1750 | AGS | |
| $^{48}\text{Ti}^{+9}$ | 0.1875 | NSRL | |
| $^{56}\text{Fe}^{+10}$ | 0.1786 | NSRL | |
| $^{84}\text{Kr}^{+20}$ | 0.2439 | NSRL | |
| $^{131}\text{Xe}^{+30}$ | 0.2290 | NSRL | |
| $^{181}\text{Ta}^{+40}$ | 0.2210 | NSRL | |
| $^{197}\text{Au}^{+32}$ | 0.1624 | RHIC, NSRL | 1.6e9 |
| $^{238}\text{U}^{+39}$ | 0.1638 | RHIC | 1.3e9 |

Why Multiple Ion Species

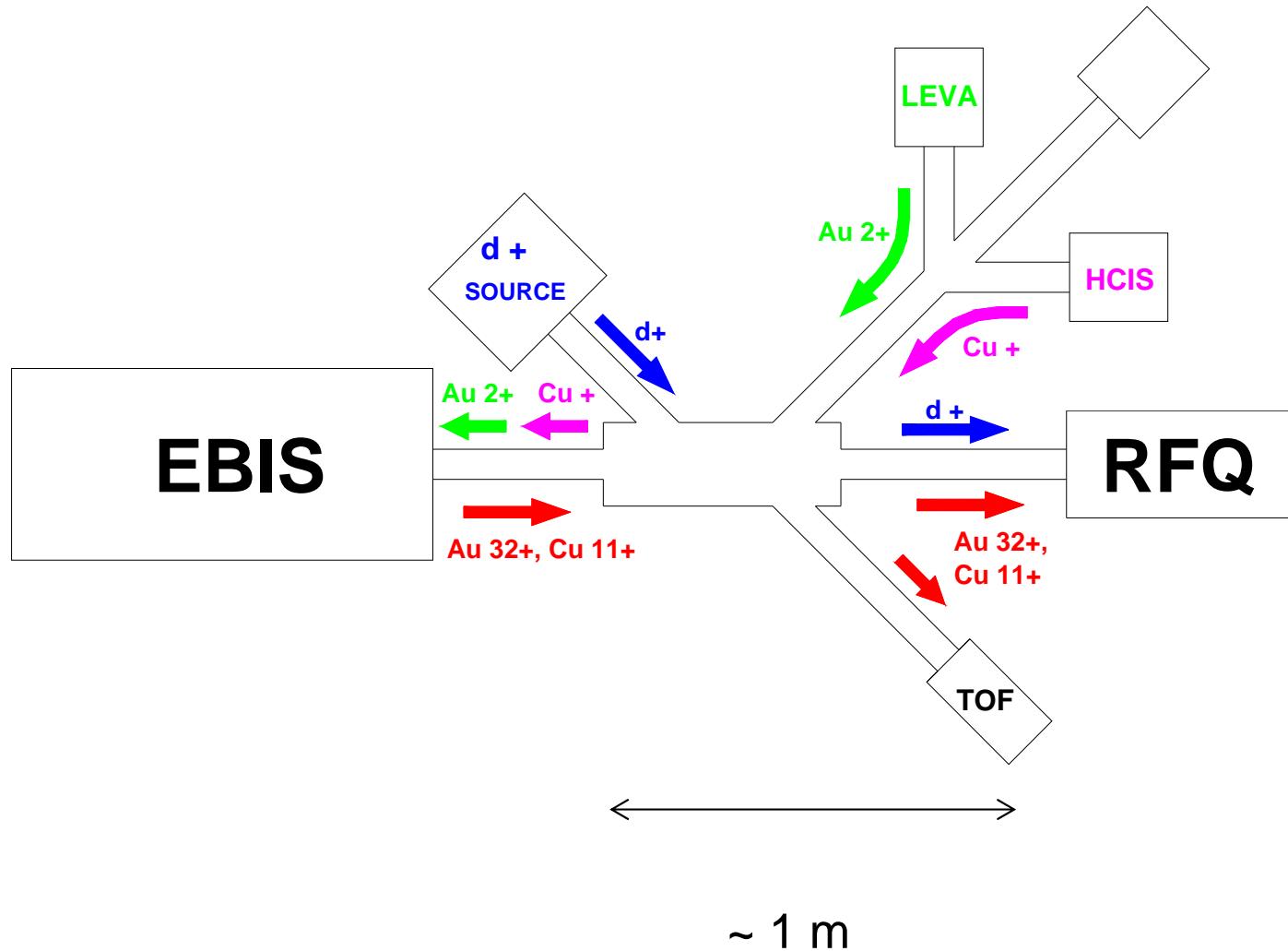


- Full ion beam sampled and collected on Faraday Cup
- $I_e = 7\text{A}$;
- 10 ms confinement
- Au = 83%; C&O = 15%; H = 2%



Au time-of-flight charge state distribution; Peak at Au 31+
 $I(e)=7.6 \text{ A}$; 65ms confinement

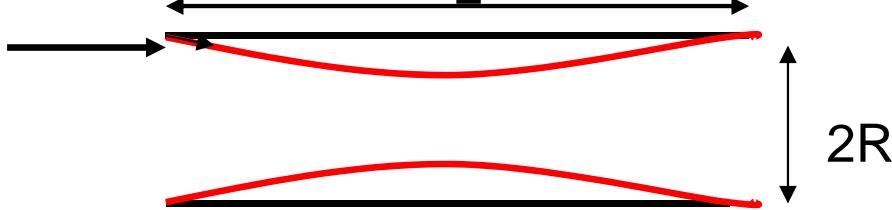
Drift space needed for injection in LEBT



LEBT Length

- Distance needed between two lenses for external ion injection (L) ~ 1 m.
- Beam radius (R) ~ 2.0 cm, Lenses (solenoid) aperture radius 5.0 cm
- Maximum beam current for SC dominated drift (L), maximum beam radius (R) and initial slope $R'_0 = -0.92$ given by

$$I_{\max}(A) = 1.166 * \frac{mc^2}{(30 * q)} * \beta^3 \gamma^3 (R/L)^2$$

$R'_0 = -0.92$ 

$$\begin{aligned}
&= 20 \text{ mA for } \text{Au}^{+32} \\
&= 6 \text{ mA for D} \\
&= 5 \text{ mA for } {}_3^3\text{He}^{+2}
\end{aligned}$$

Alternative LEBT: Charge Separation before RFQ

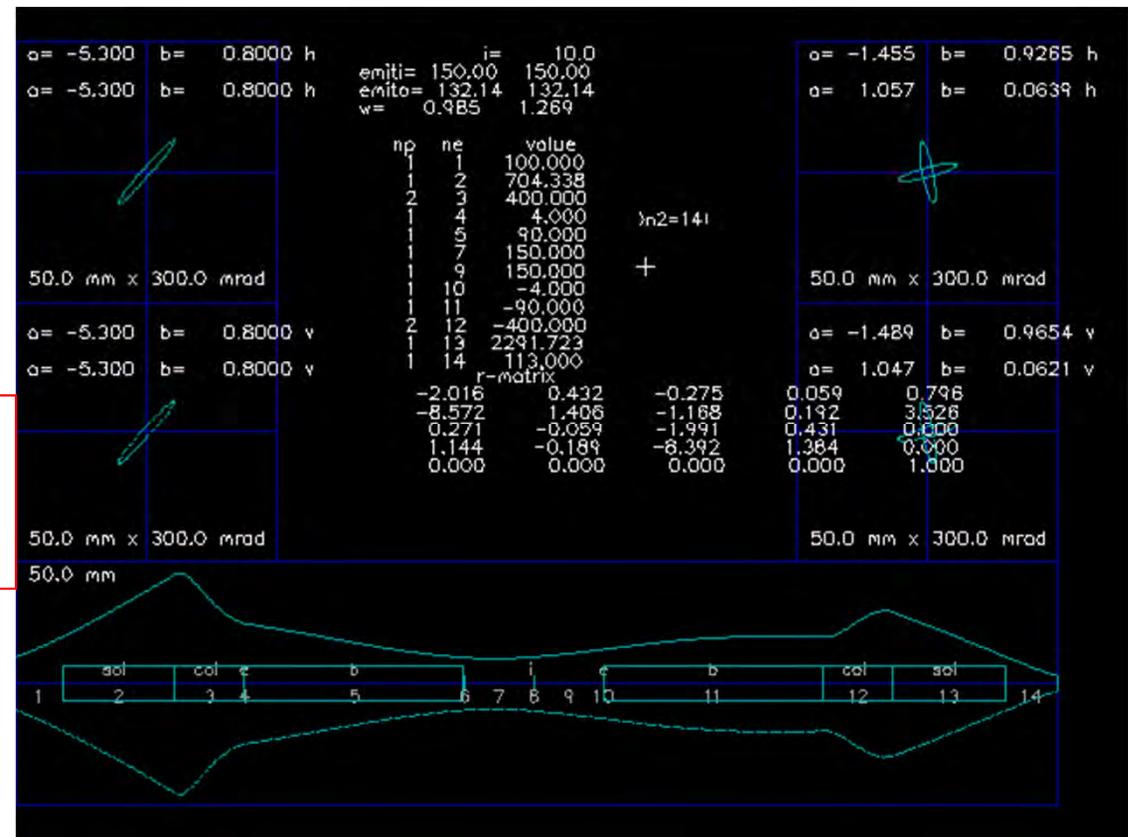
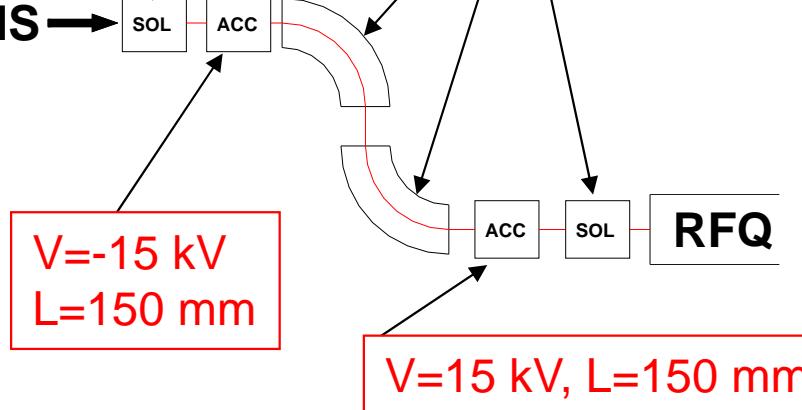


- Dispersion 0.8 meter
- Sufficient for charge separation
- Magnets has to float to 40 kV
- Unnecessary complication
- Cost

90° bend with gradient ($n=0.50$)
 $p=300$ mm, edge angle=4°
 Gap 100 mm

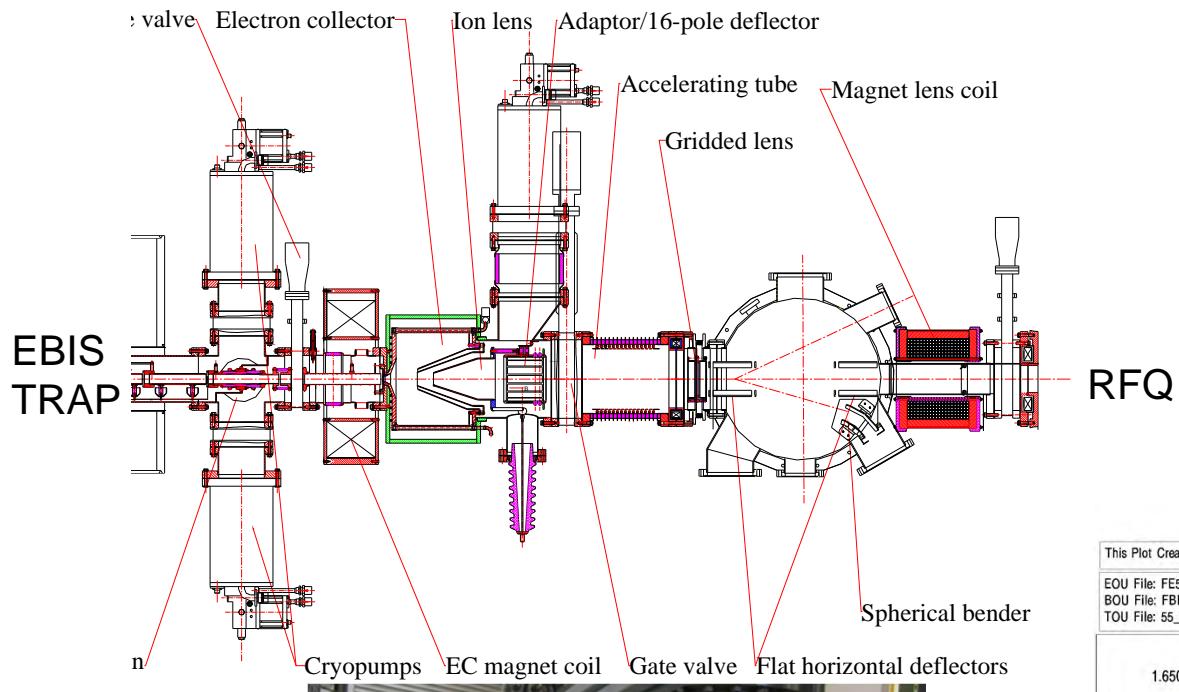
L=241 mm, B=8 KG

EBIS →

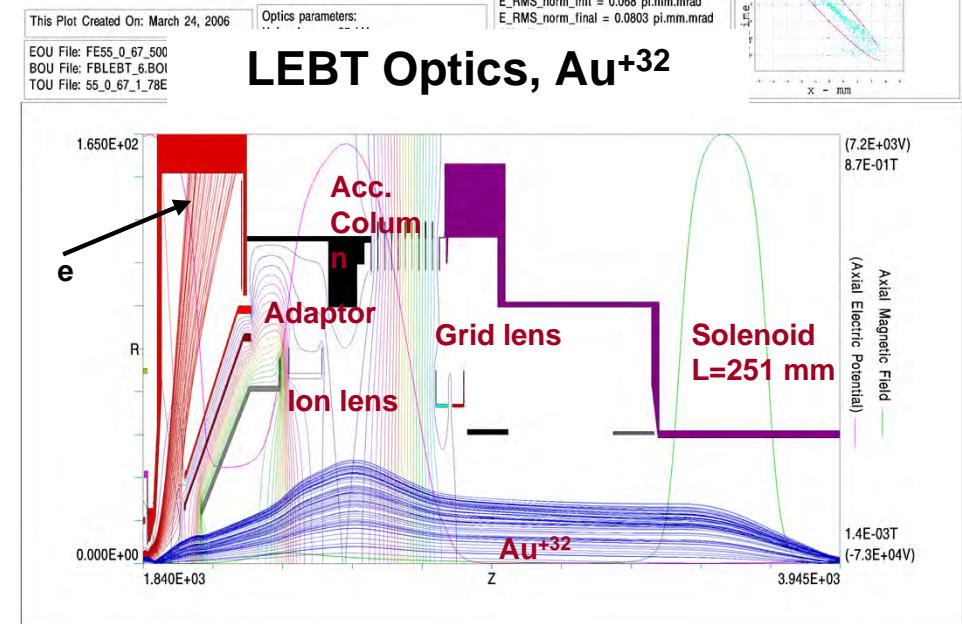
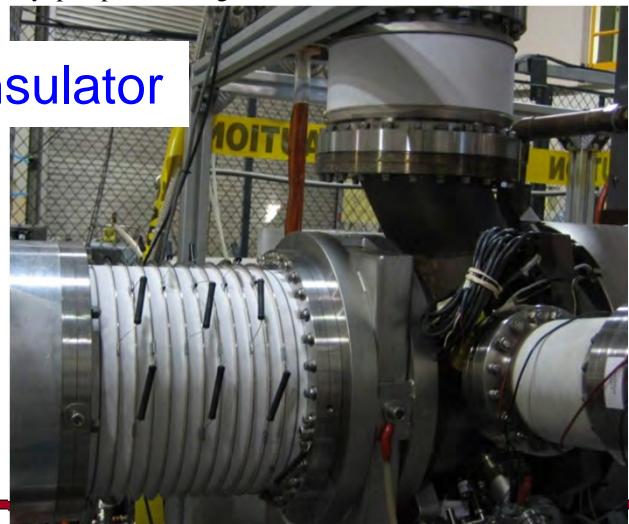


Two 90 degree gradient magnet with 4° edge angle, 10 cm gap, Float at 40 kV for ${}^3\text{He}^{+2}$

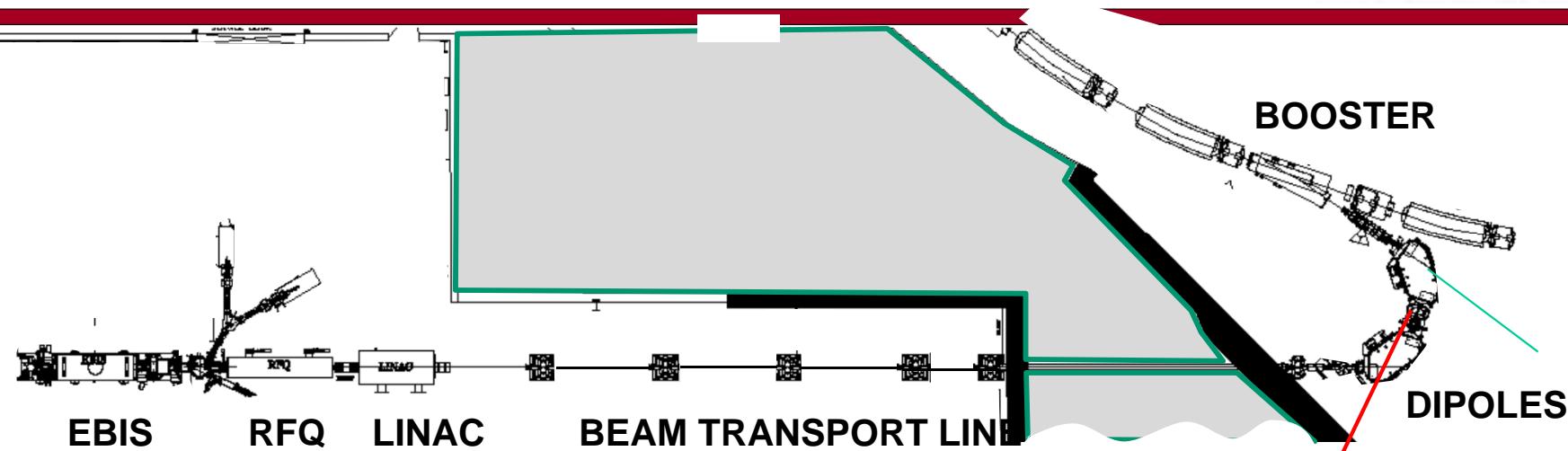
Matching to the RFQ



100 kV insulator

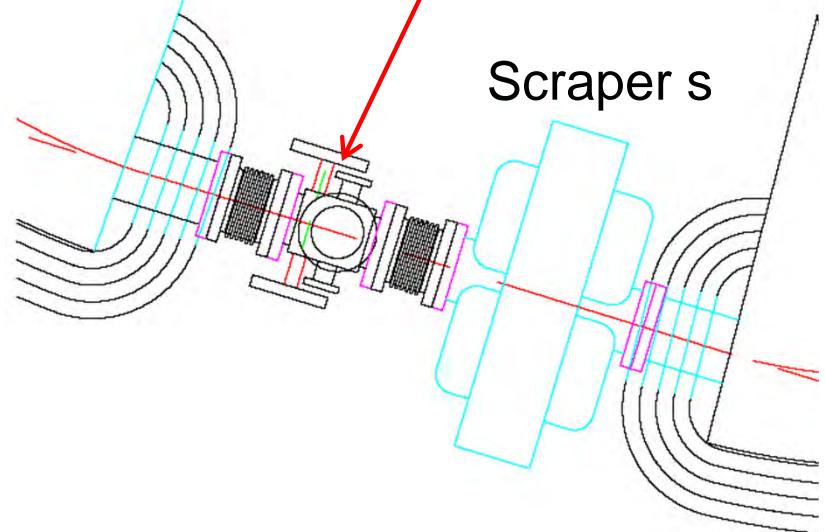


Charge Discrimination



At this location

- $R_{16}=1.1$ m
- $2r_0=2.2$ mm
- Resolution = 500 @2MeV/u
- 310@0.3 MeV/u
- Efficiency =100%



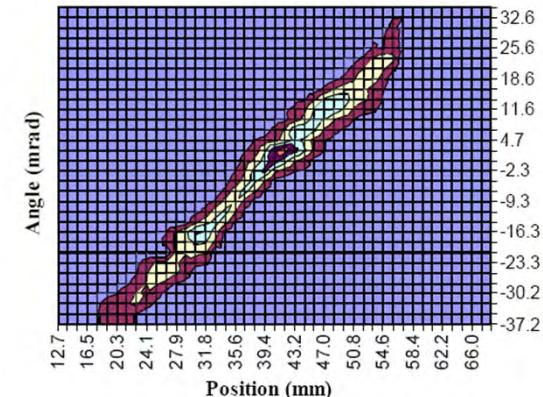
Emittance

Measured emittance of a 1.7 ma Au beam

We have assumed emittance scales as Q/m² for different ion Species

$$\varepsilon(n) = 0.16 * r^2 * B_z * (Q/M) \text{ m-rad (100% of beam)}$$

=> He⁺² emittance will ~3 times of Au⁺³²



$$\varepsilon(n, \text{rms}) = 0.1 \pi \text{ mm mrad.}$$

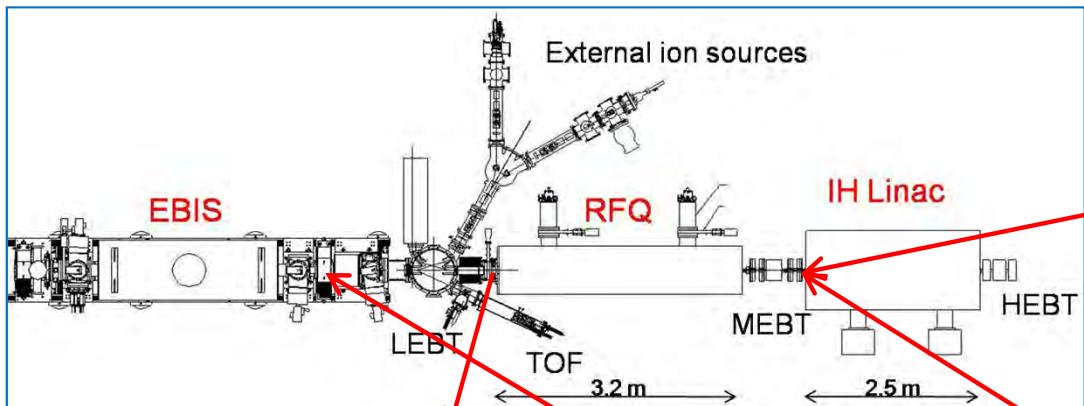
Emittance Budget for the EBIS-Injector

| | Vertical $\pi \text{ mm mrad (N, 95\%)}$ | Horizontal $\pi \text{ mm mrad (N, 95\%)}$ |
|----------|---|---|
| Booster* | 4.5 | 10.0 |
| Linac | 1.4 | 1.4 |
| Source | 0.7 | 0.7 |

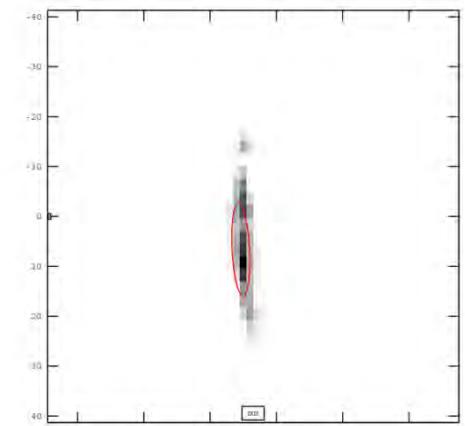
* mismatch injection

Note: Booster acceptance (VXH) 4.5 X 15 $\pi \text{ mm mrad (N, 95\%)}$

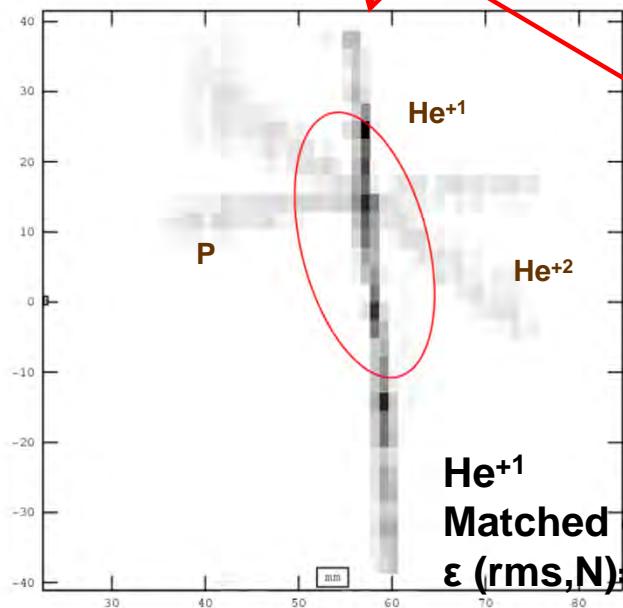
The LEBT and MEBT beam lines were each first commissioned with temporary diagnostics, before moving the RFQ, and then the linac, into place.



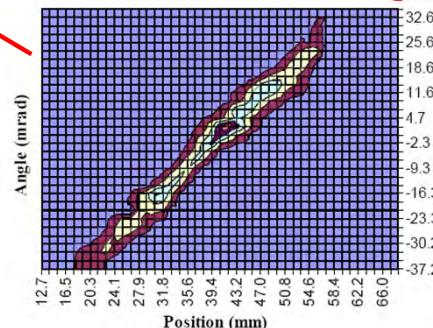
MEBT



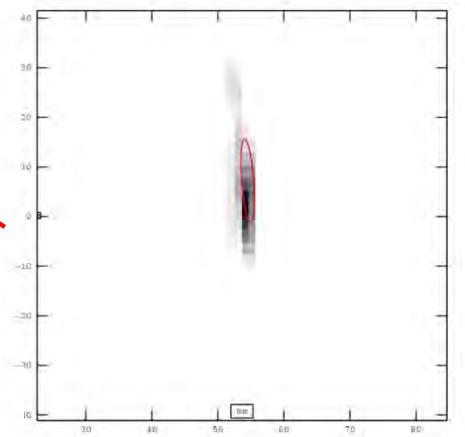
Helium: matched condition into LINAC
 $\epsilon(n, rms) = 0.13 \pi mm mrad$



LEBT



Gold: $\epsilon(n, rms) = 0.1 \pi mm mrad$.
 Matched condition into the RFQ
 $\epsilon(rms, N) = 0.09 \pi mm mrad$



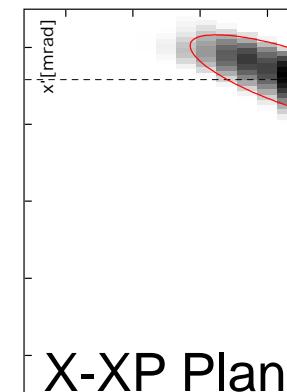
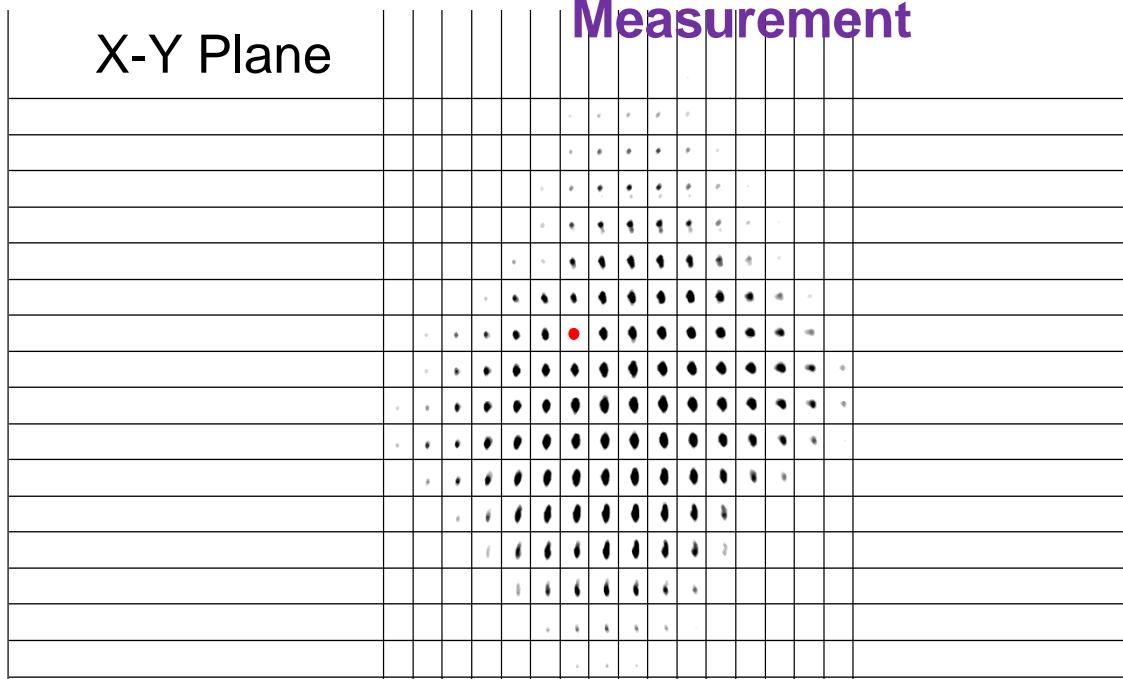
Gold: Matched conditions into linac
 $\epsilon(n, rms) = 0.19 \pi mm mrad$

Image of the Beam (U) after linac

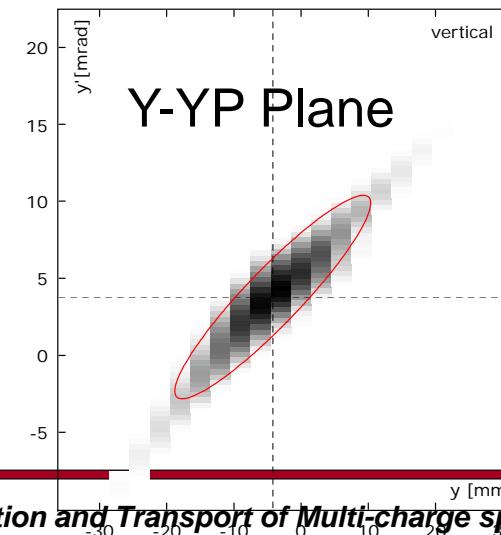


X-Y Plane

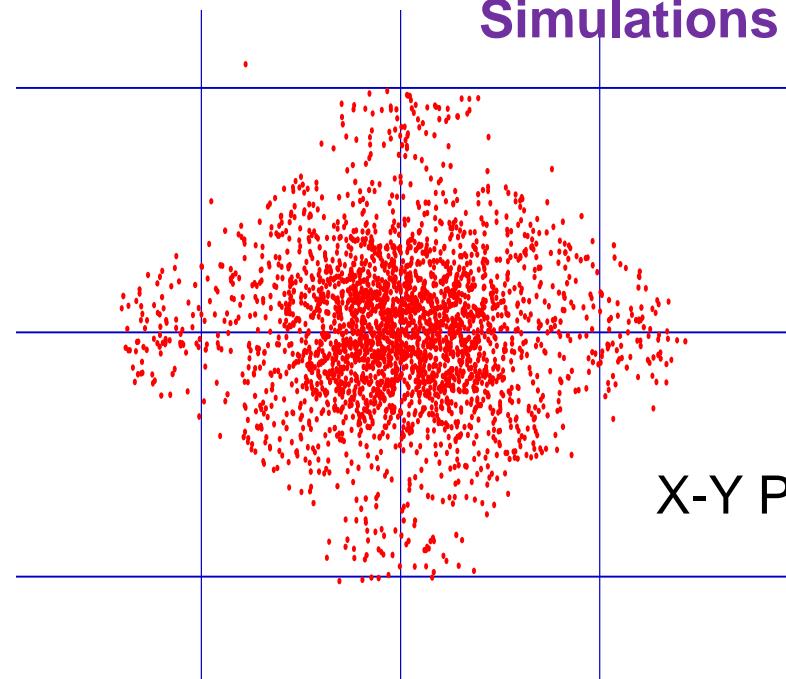
Measurement



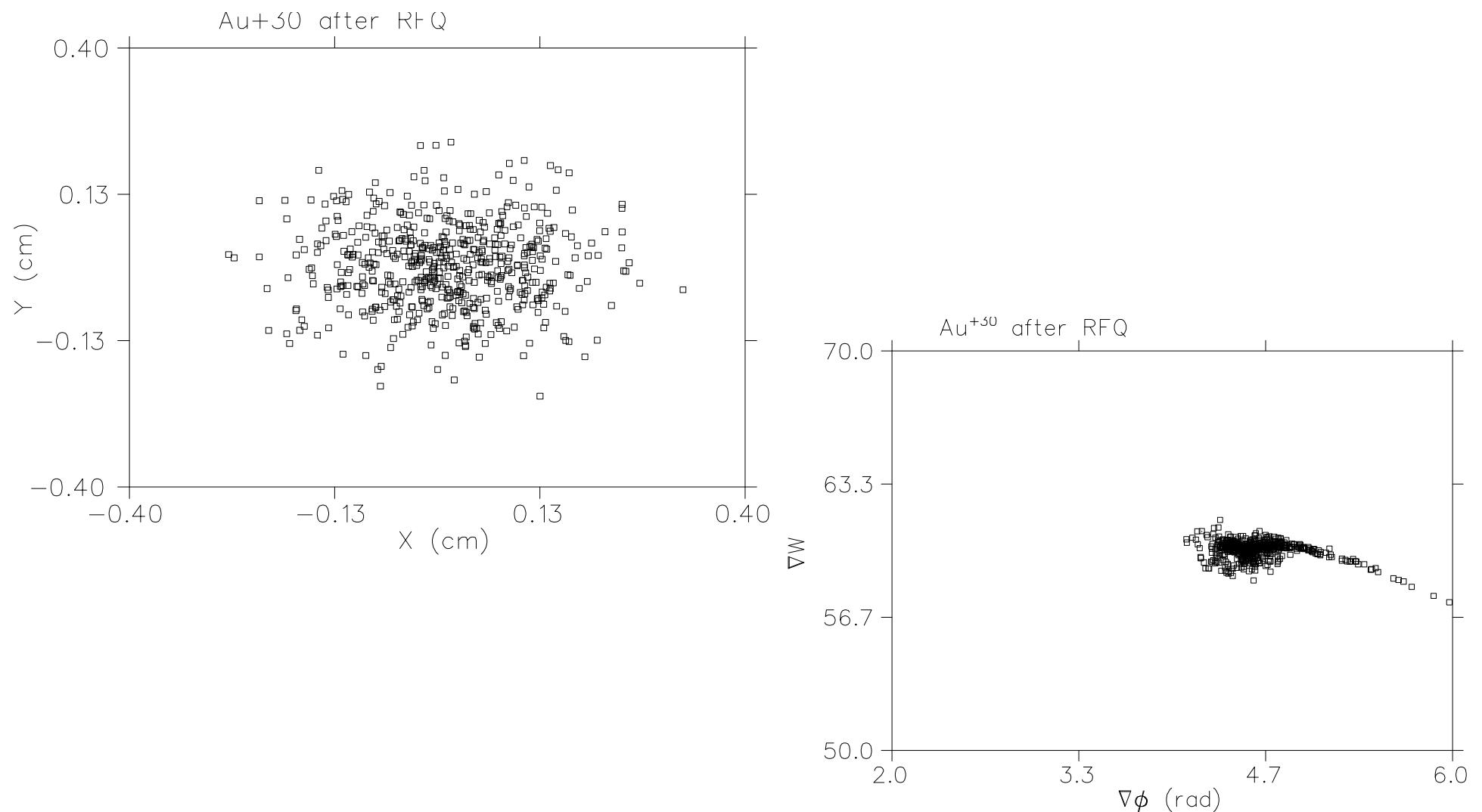
Y-YP Plane



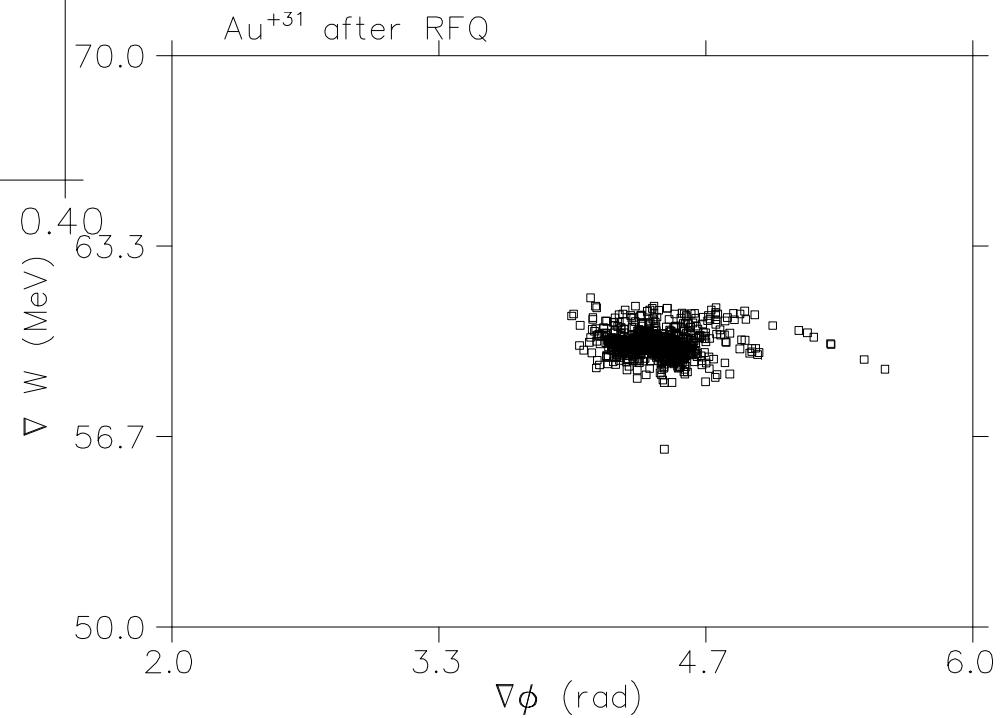
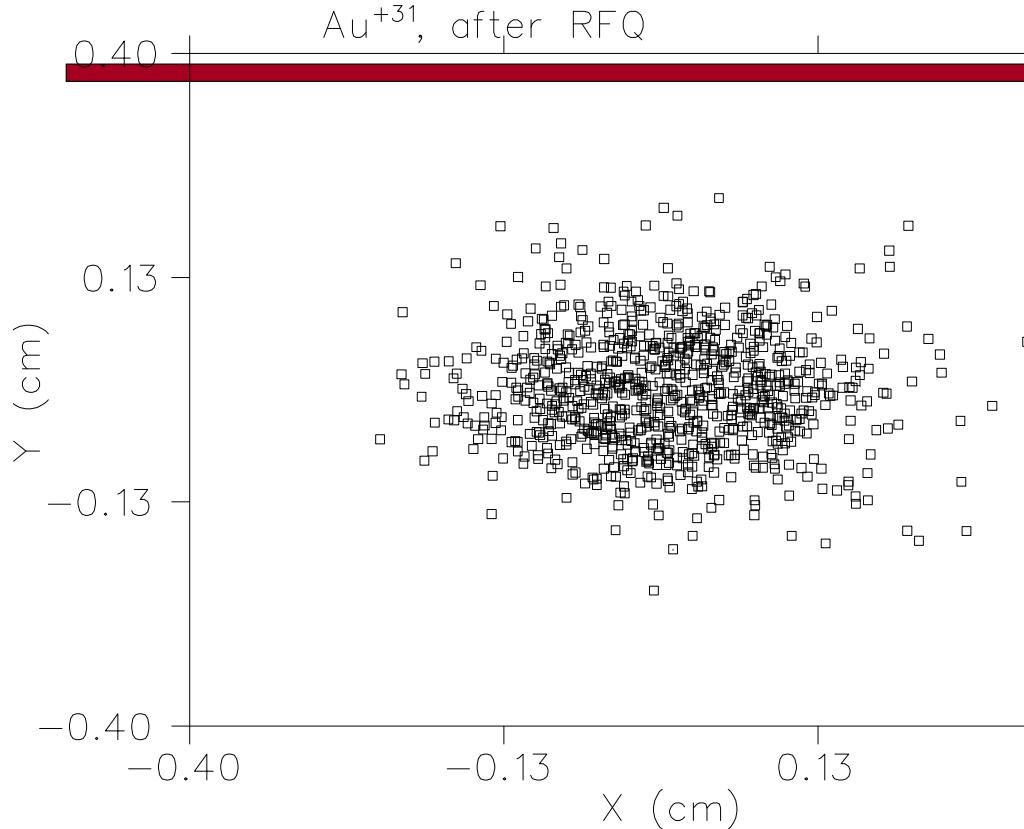
Simulations



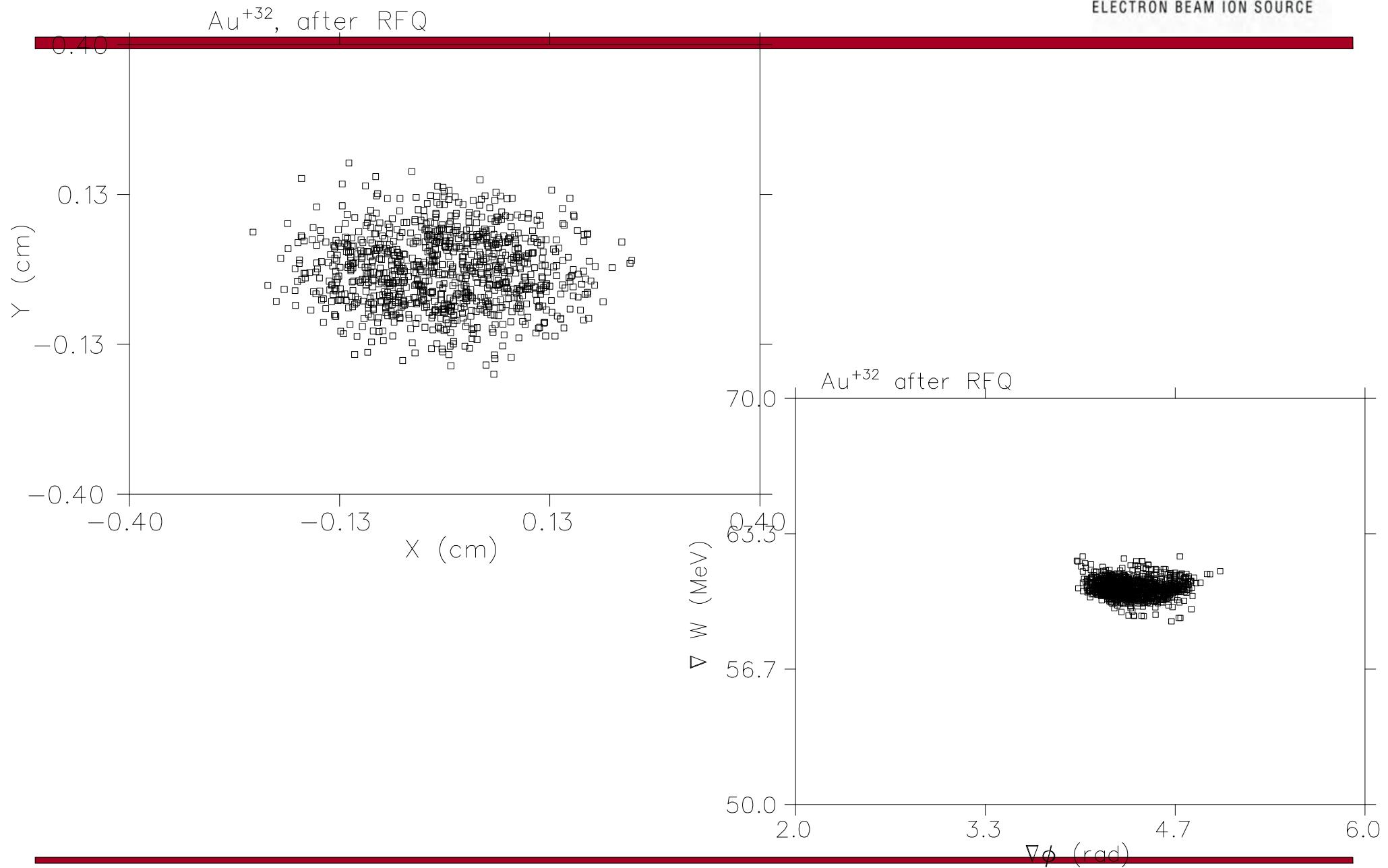
X-Y Plane



Au 31



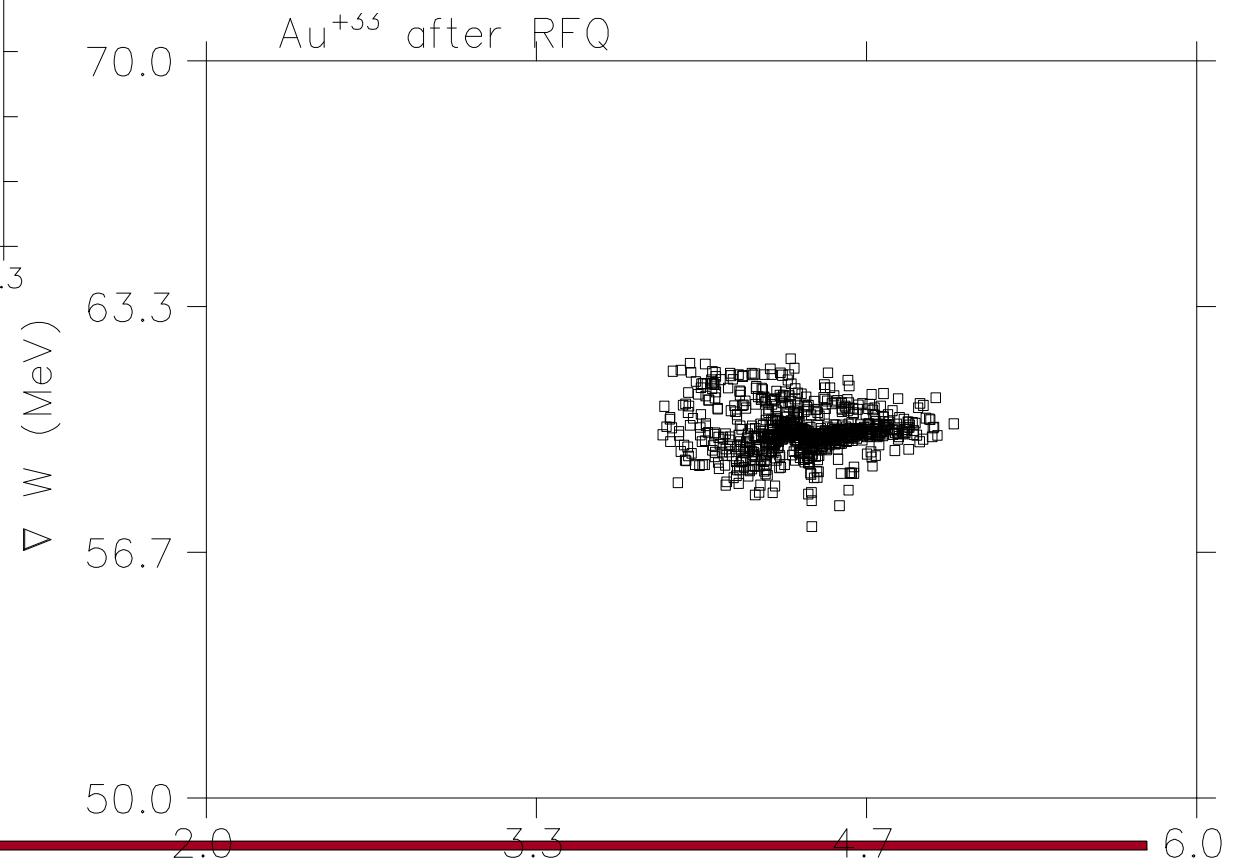
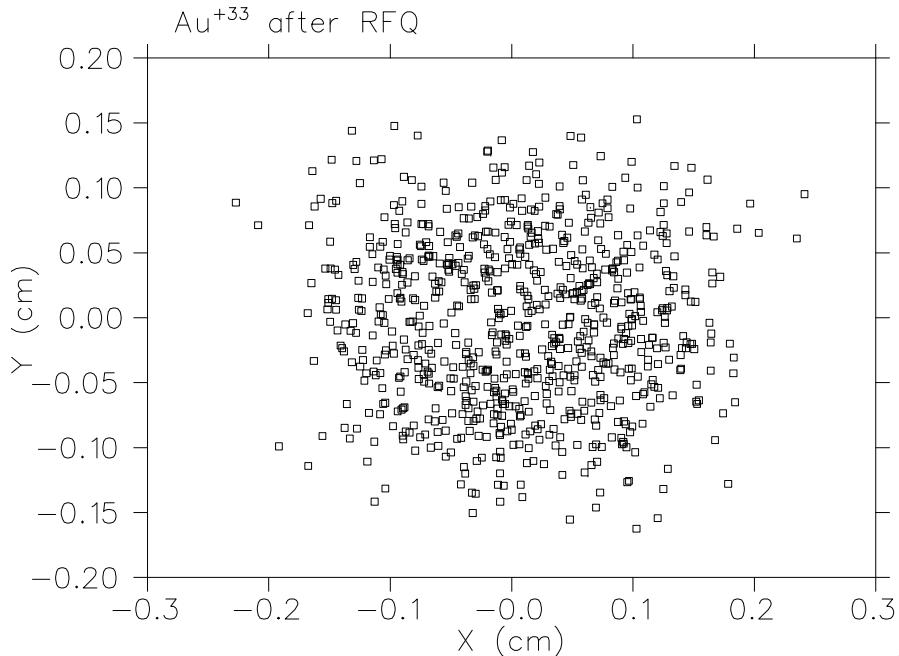
Au+32



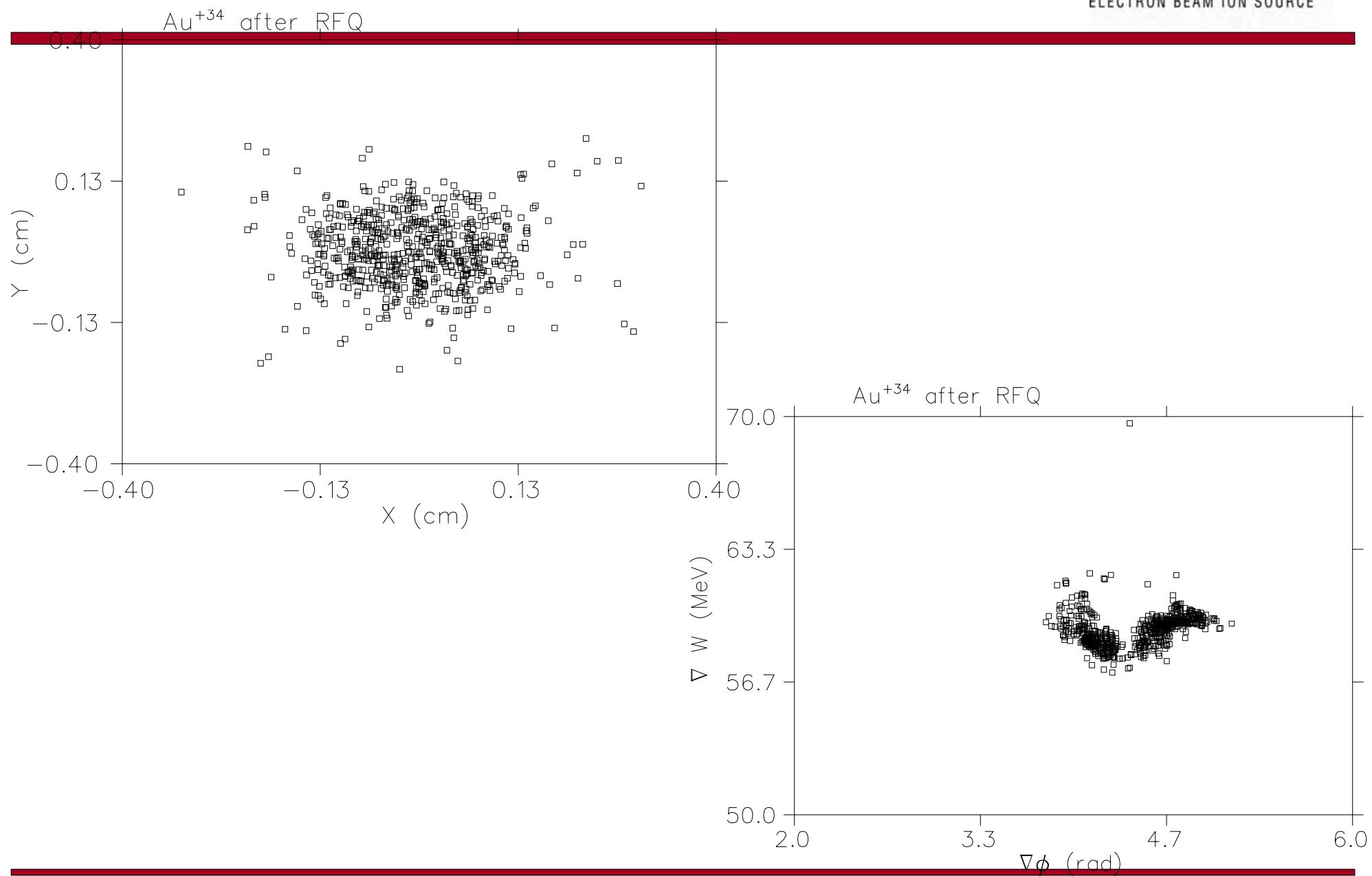
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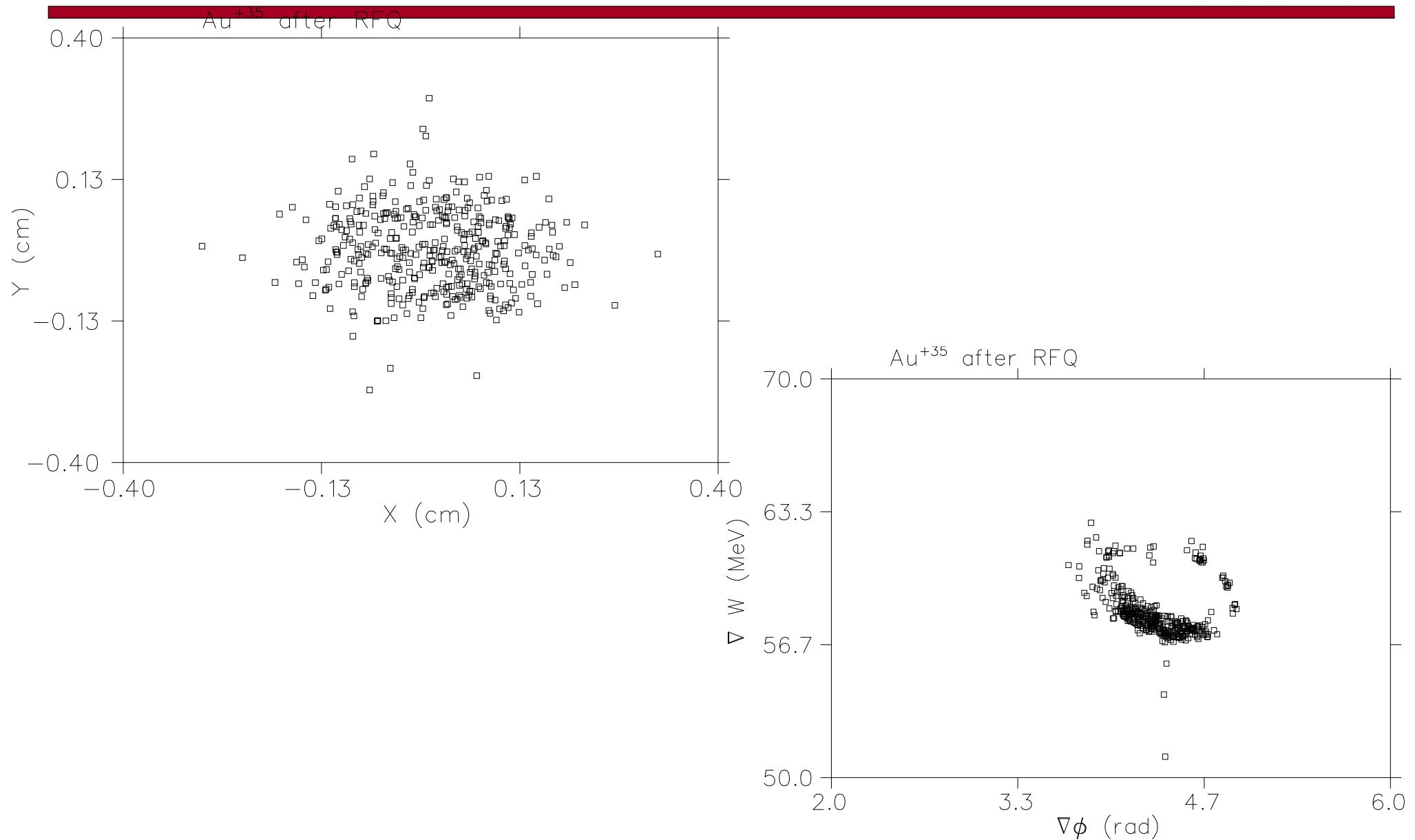
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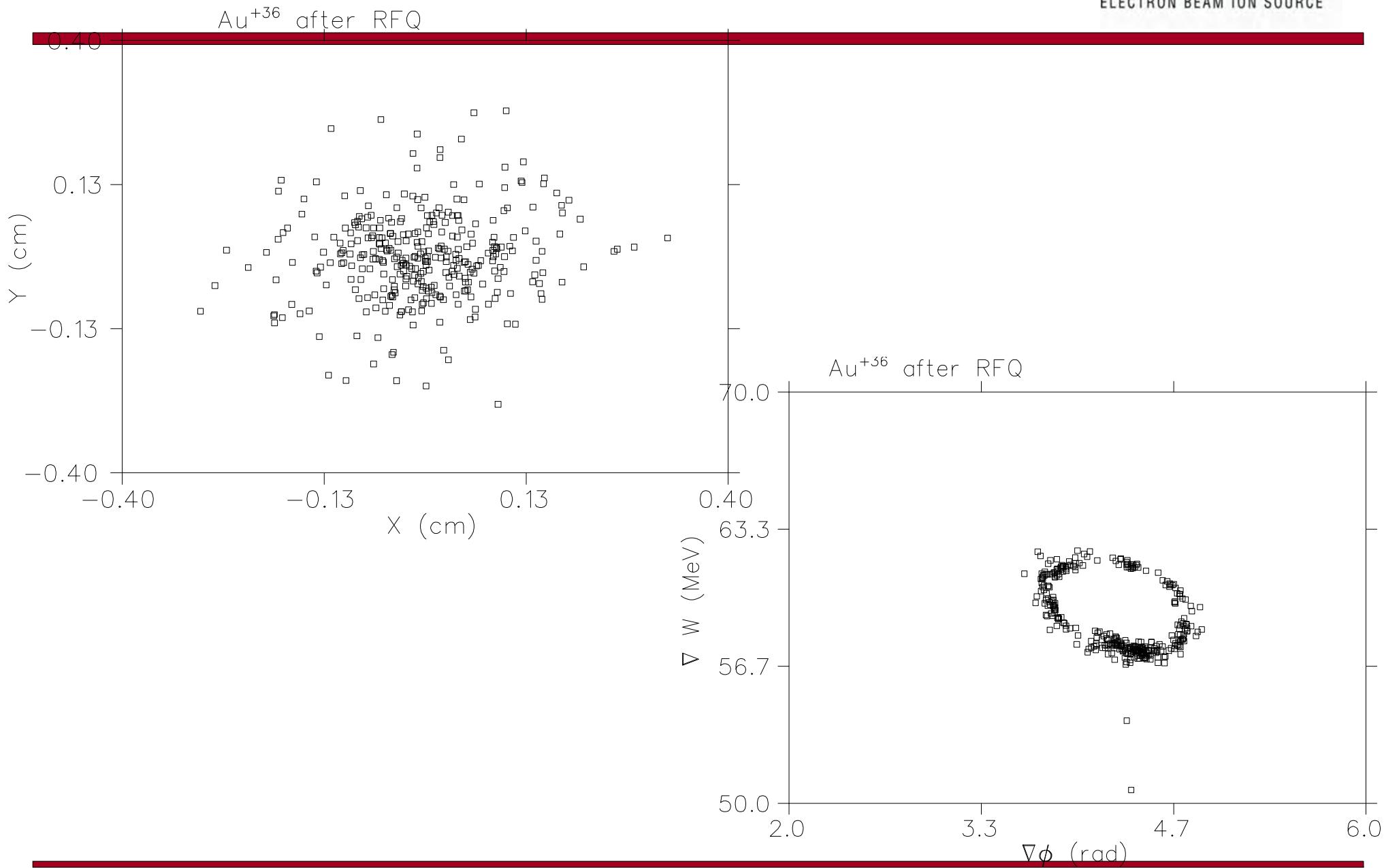


Au +34

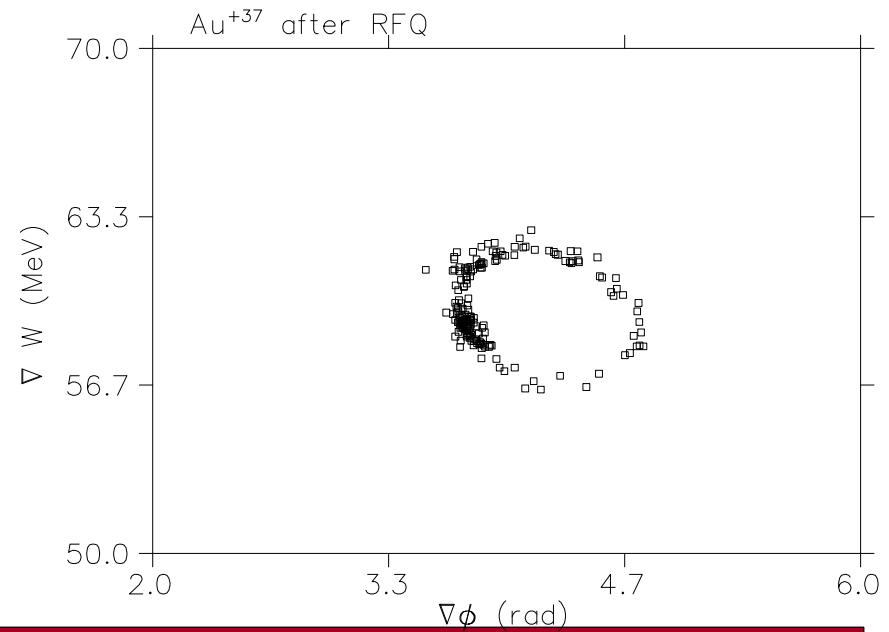
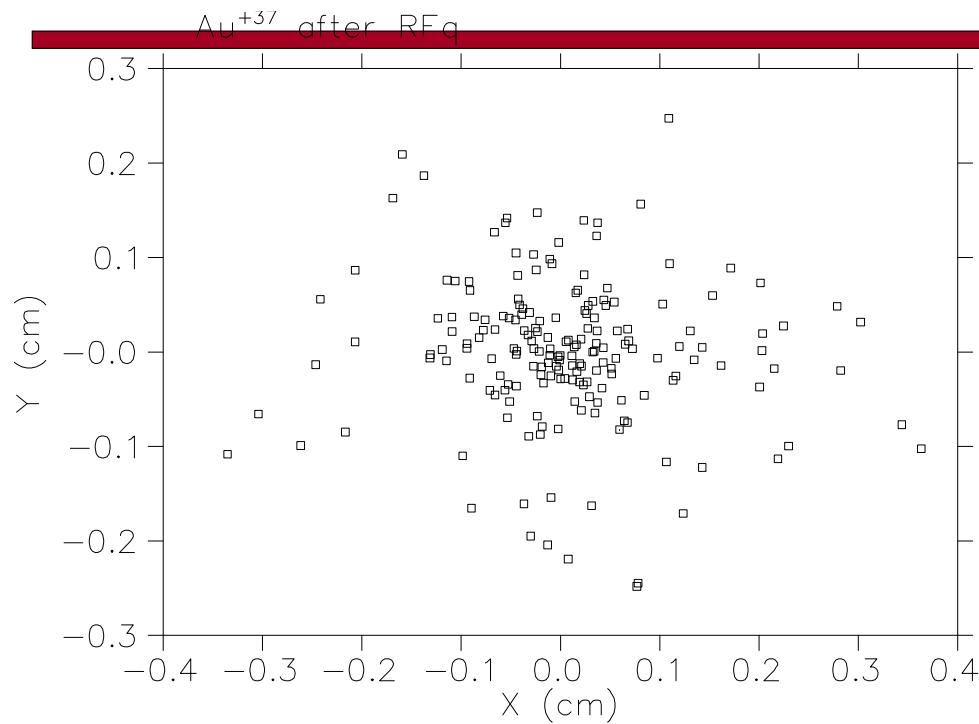




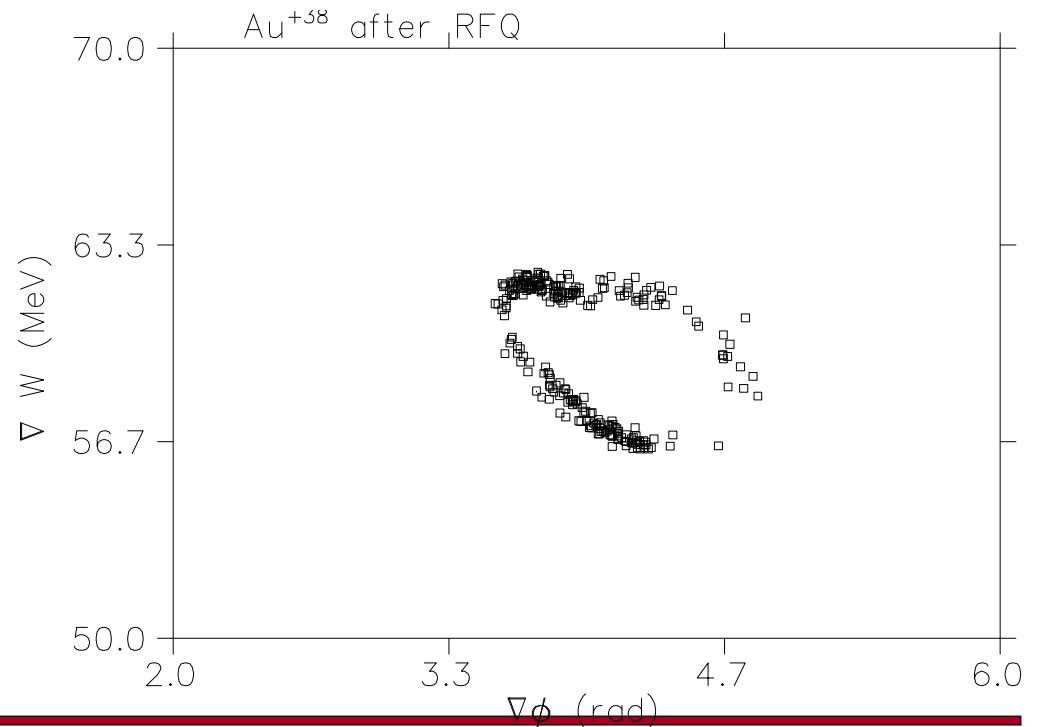
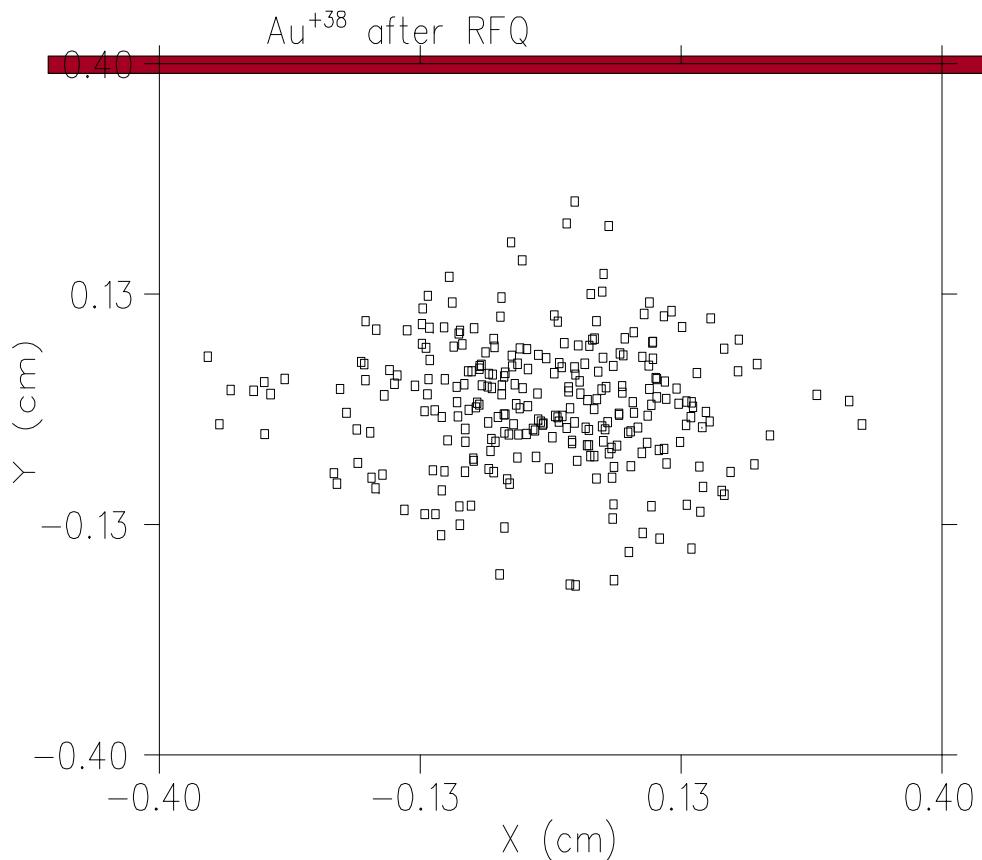
Au +36

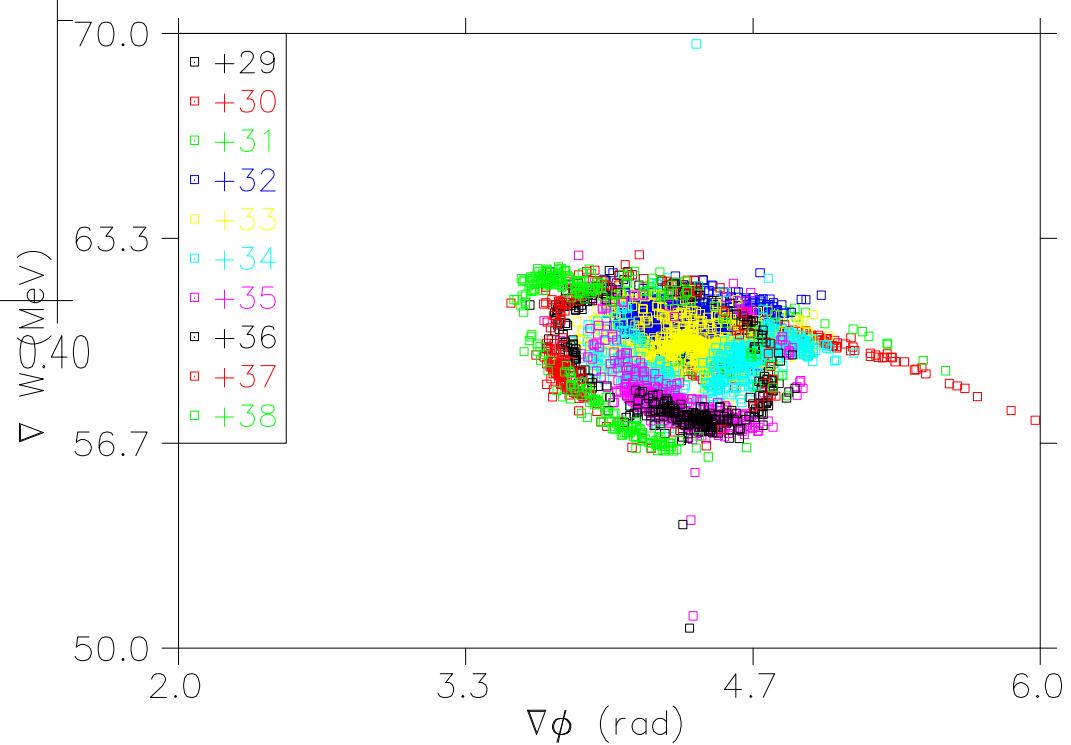
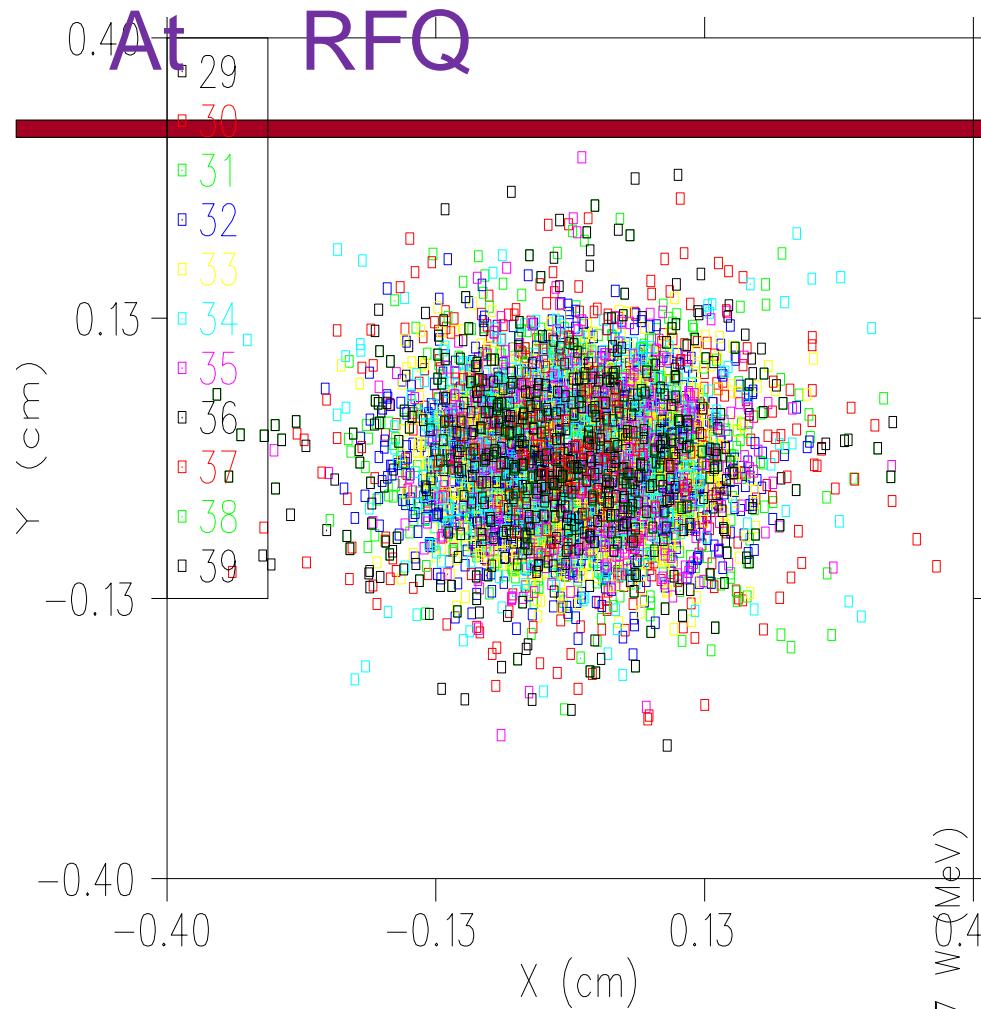


Au +37

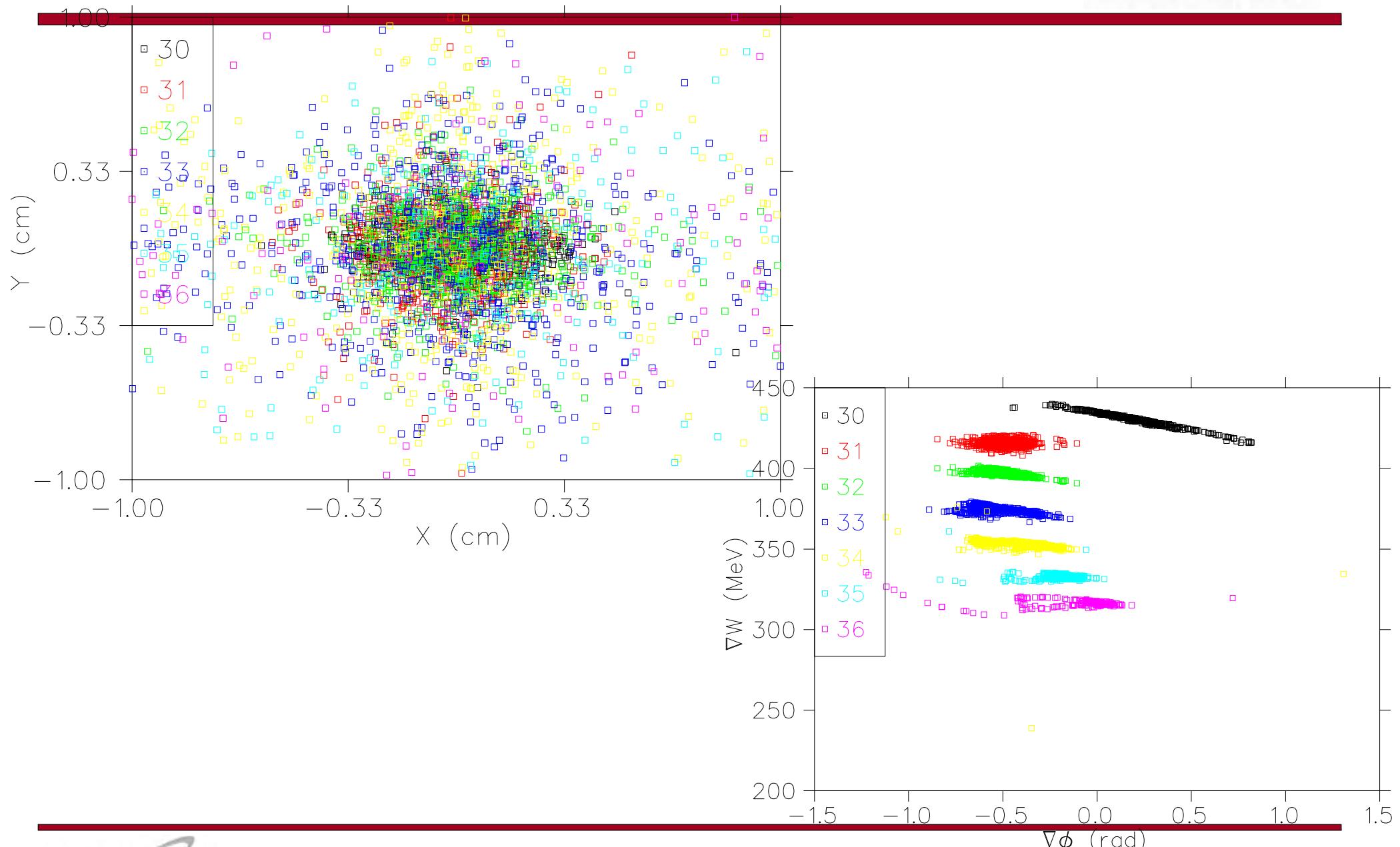


Au +38

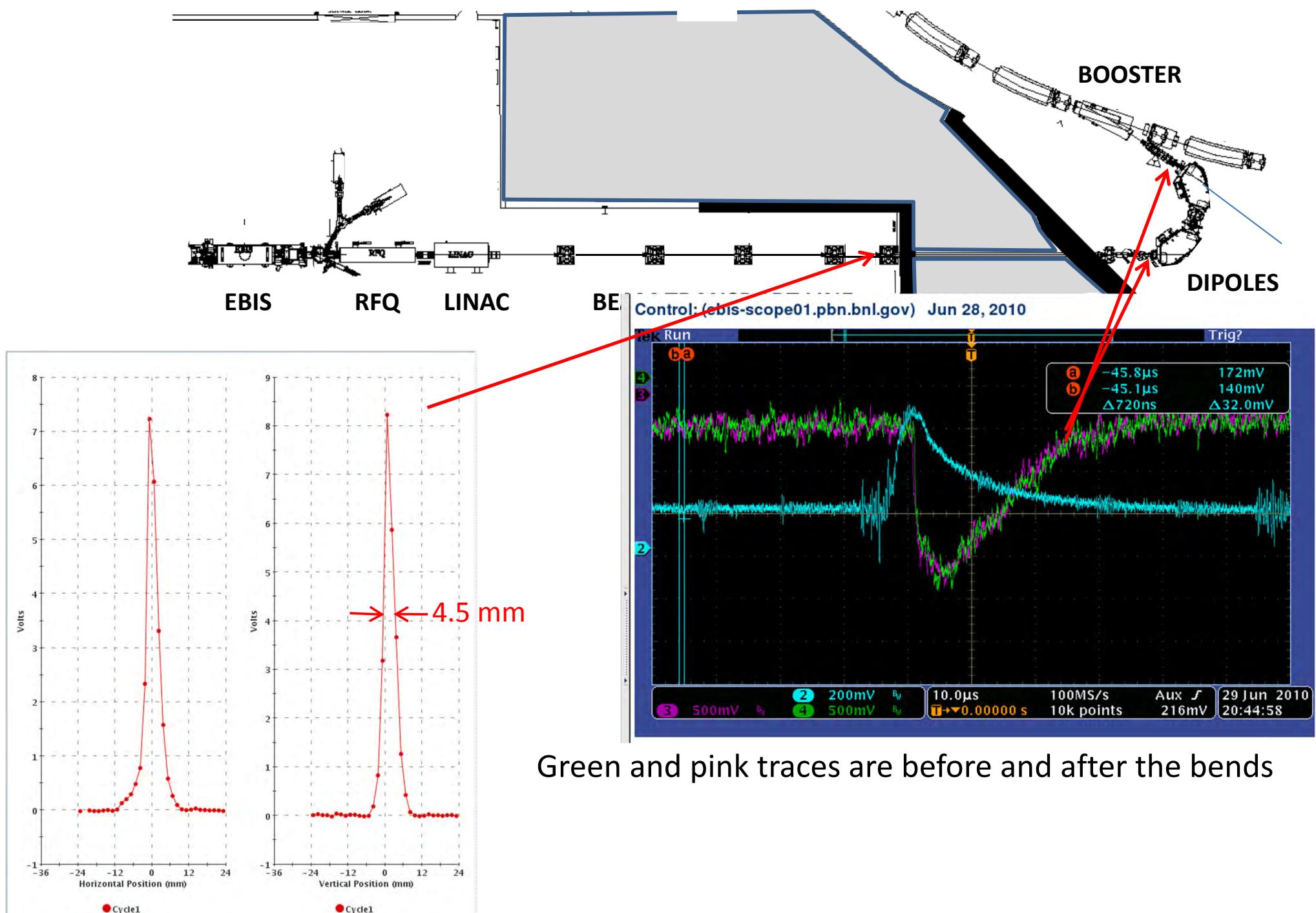




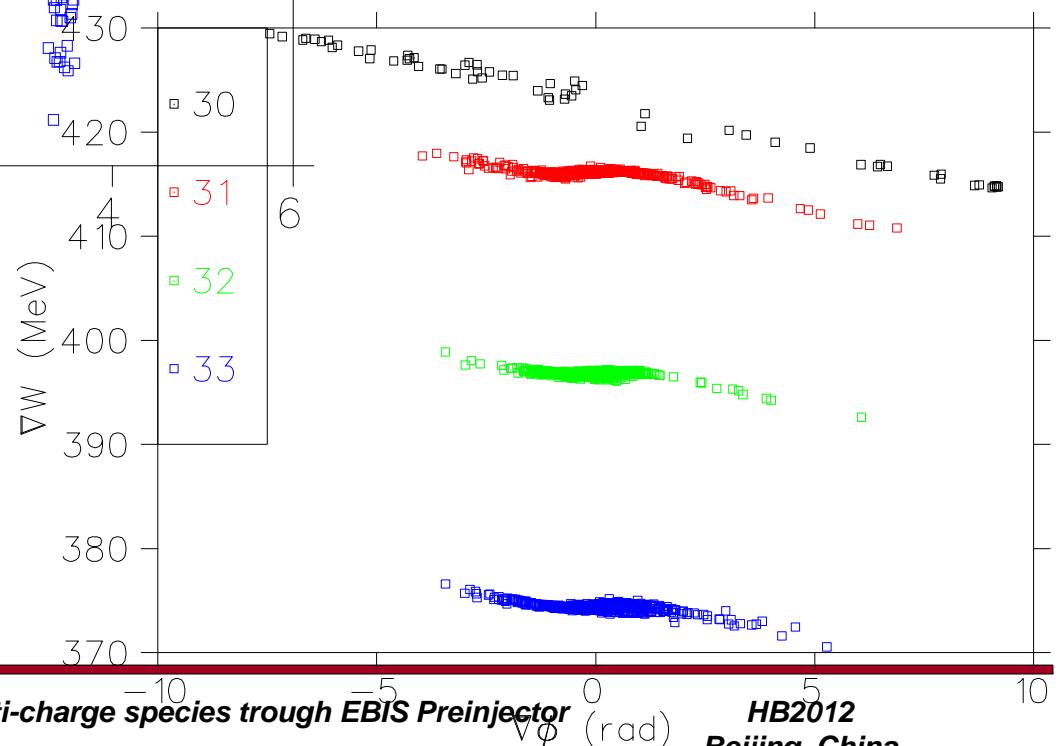
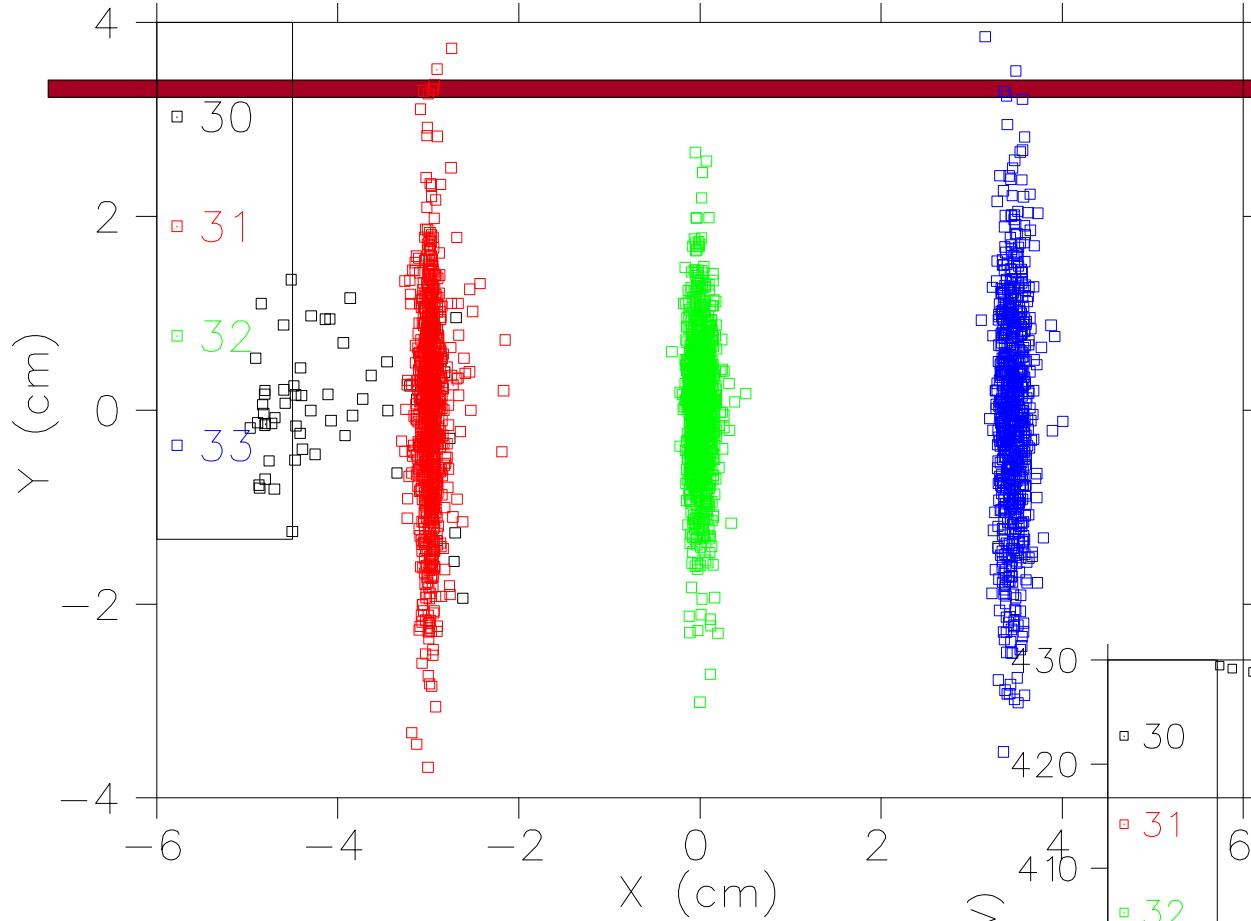
At Linac



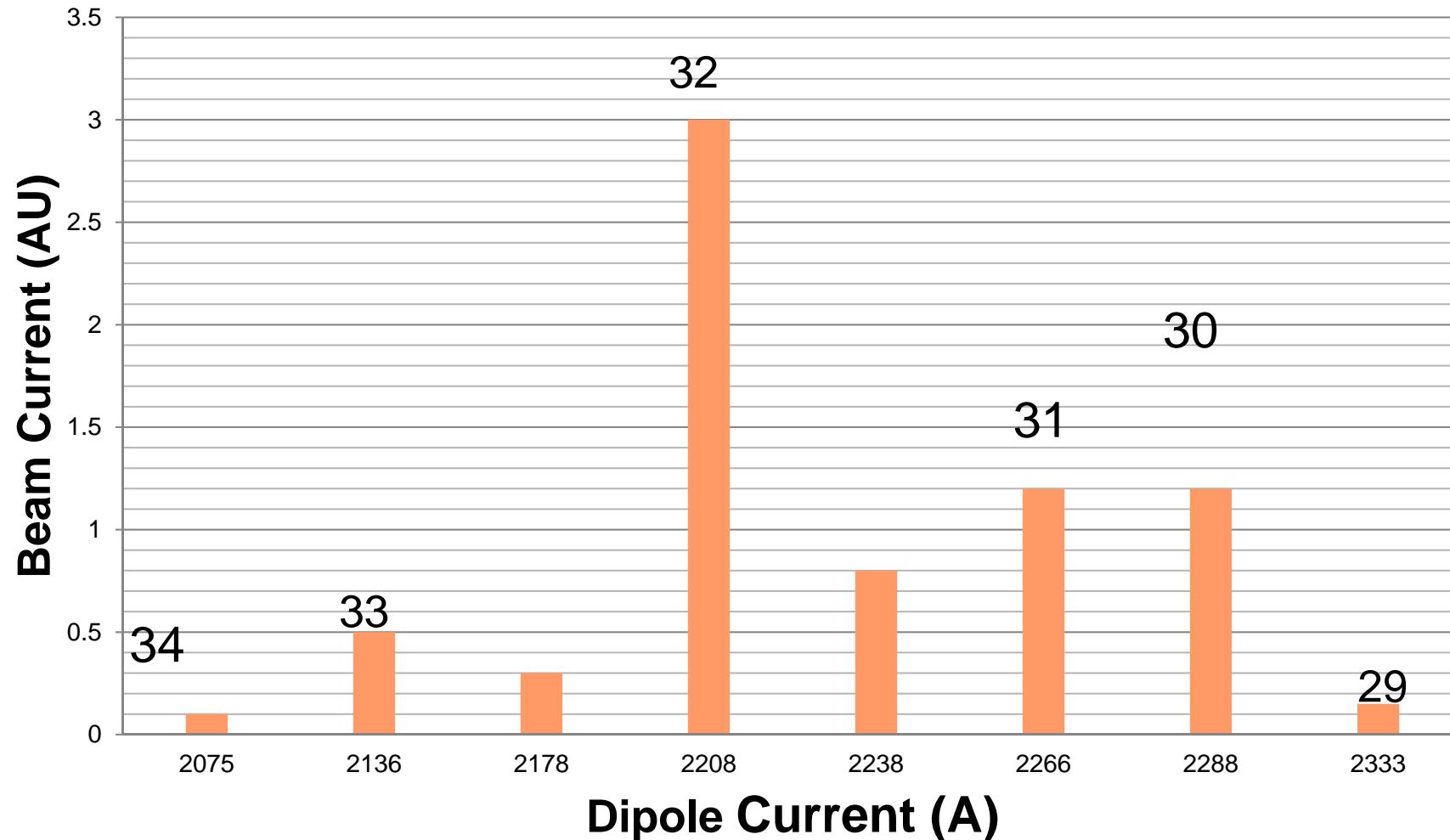
He^{1+} – first beam from EBIS transported to Booster



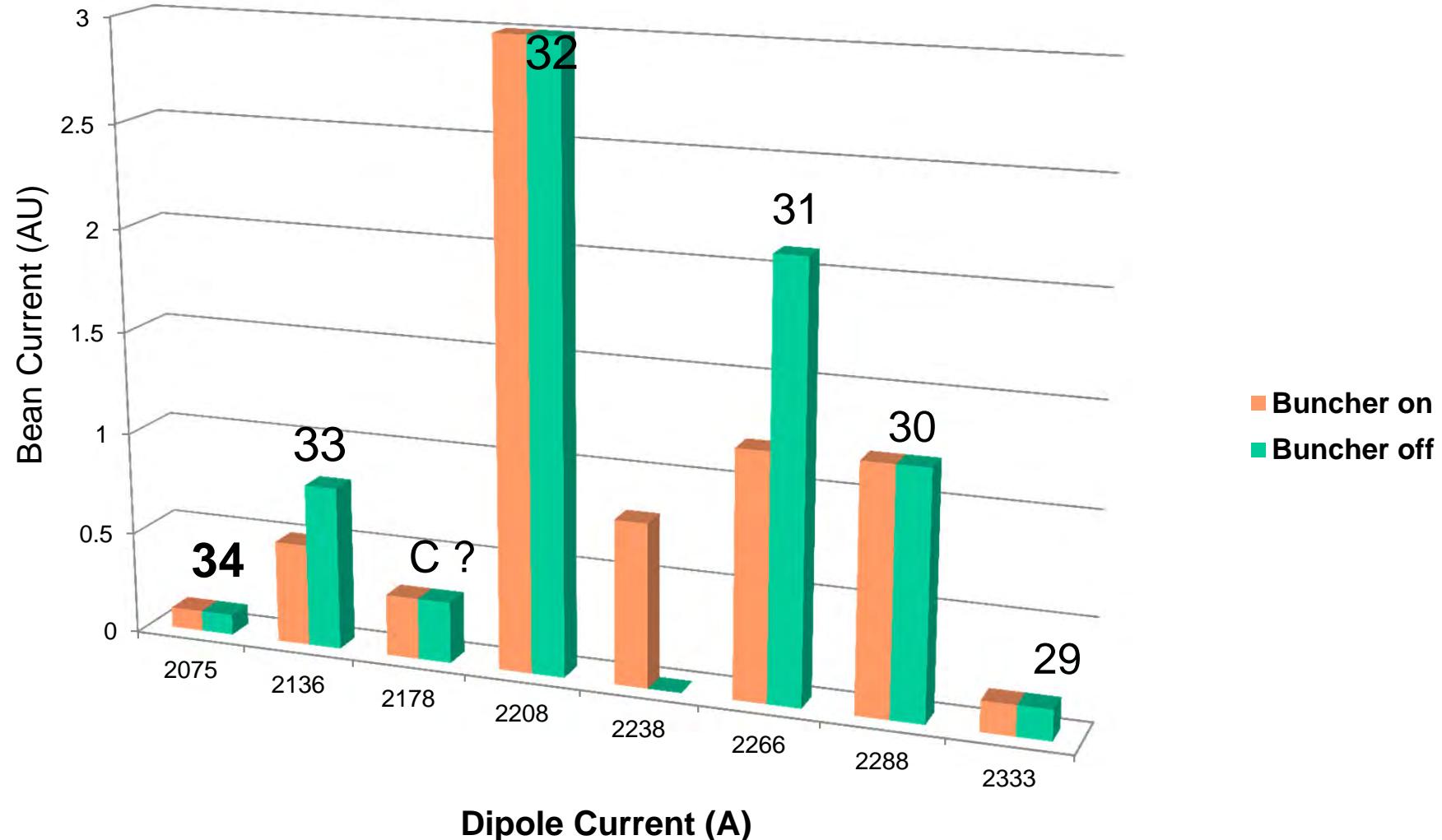
Middle of the Bend



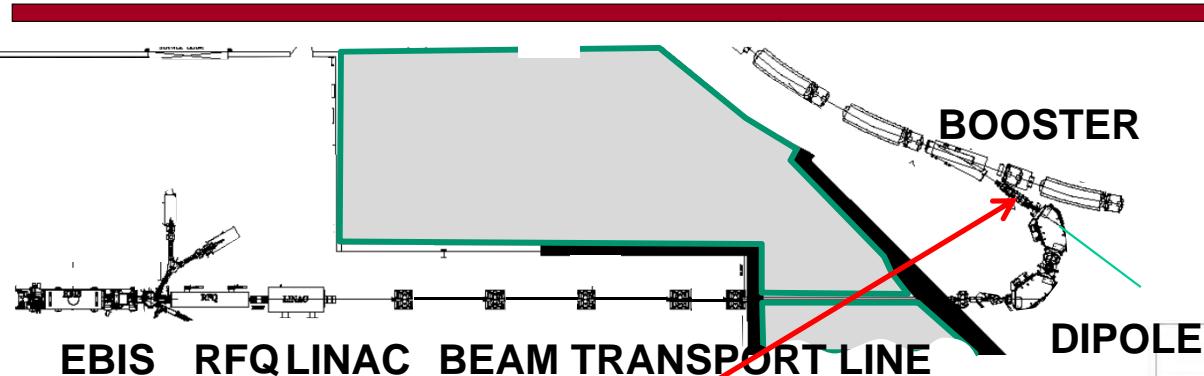
Gold at Middle of Bend



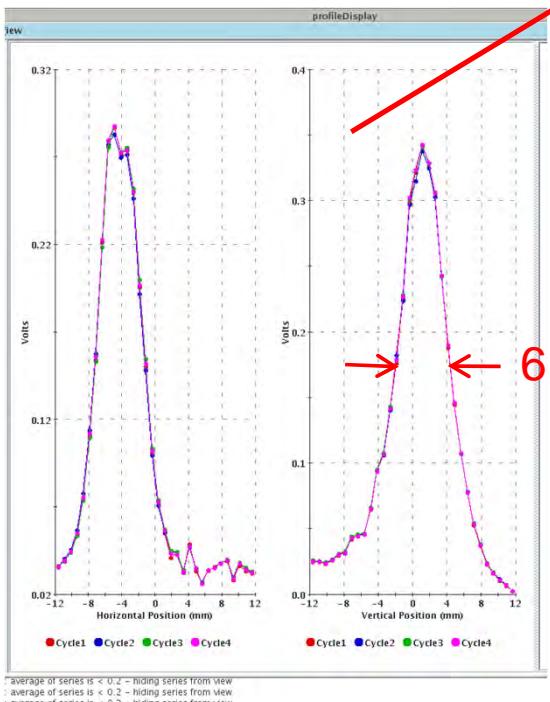
Gold at Middle of Bend: Buncher On/Off



EBIS Beam circulating in the Booster

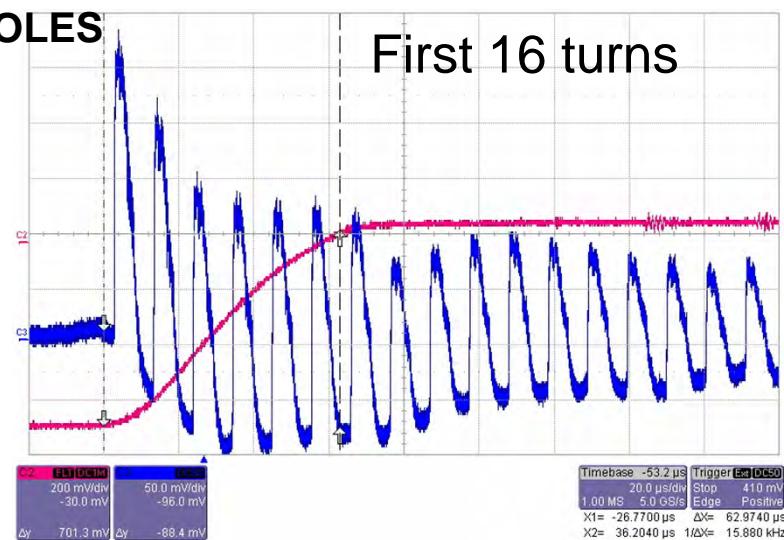


He⁺ beam from EBIS
circulating in the Booster



Profiles right
before the Booster
injection

6 mm



From the measure of the beam debunching time in Booster
 $\Delta p/p = 2.0 \times 10^{-4} \pm 1.0 \times 10^{-4}$
(Requirement $\leq 5.0 \times 10^{-4}$)

EBIS beams transported to middle of bend (2 MeV/u)



| | | |
|------------------|-------------|---------------------------------|
| He^{+1} | Conf. Time | 2.5 ms |
| | RFQ Power | 40 kW |
| | Linac Power | 76 kW |
| | Pulse width | <20 μs |
| | Intensity | 7×10^{10} ions / pulse |

Transmission (helium):

| | | |
|---------------------------------|------------------|-----------------------|
| RFQ Input FC | 1.4 mA (18.5 nC) | |
| RFQ Output FC | 1.4 mA (15.0 nC) | T=100%,(T=81% charge) |
| XF14 (after linac) | 1.1 mA | T=79% |
| XF 89 (before bend) | 1.1 mA | T=100% |
| XF108 (after both bends) | 1.1 mA | T=100% |

Gold Transmission to booster injection

| Charge | | EBIS# (Measu) | RFQ | Linac | Bend | B INPUT | |
|------------|------|---------------|----------|----------|------|-------------|----------|
| 26 | 1.2 | 0.033622 | | 0 | 0 | 0 | 0 |
| 27 | 7.5 | 0.044829 | | 0 | 0 | 0 | 0 |
| 28 | 20 | 0.064041 | | 0 | 0 | 0 | 0 |
| 29 | 45 | 0.092859 | 0.000464 | | 0 | 0 | 0 |
| 30 | 80 | 0.108229 | 0.060933 | 0.044206 | | 0.005698116 | 0 |
| 31 | 111 | 0.120077 | 0.107589 | 0.106748 | | 0.105307397 | 0 |
| 32 | 124 | 0.128082 | 0.123727 | 0.123727 | | 0.123599103 | 0.123599 |
| 33 | 111 | 0.120077 | 0.096182 | 0.091499 | | 0.085134486 | 0 |
| 34 | 80.5 | 0.112072 | 0.07139 | 0.062872 | | 0 | 0 |
| 35 | 49 | 0.076049 | 0.028975 | 0.021598 | | 0 | 0 |
| 36 | 25.5 | 0.048031 | 0.015754 | 0.008357 | | 0 | 0 |
| 37 | 11.5 | 0.03202 | 0.005487 | 0 | | 0 | 0 |
| 38 | 4.5 | 0.020013 | 0.003162 | 0 | | 0 | 0 |
| | | 1 | 0.513662 | 0.459008 | | 0.123599 | 0.123599 |
| Sim (nC) | | 54.5 | 27.9945 | 25.01659 | | 6.7362 | 6.7362 |
| Measu (nC) | | 54.5 | 29.6 | 16.2 | | 7.4 | 5.8 |
| Ratio | | 1.000 | 1.05735 | 0.647 | | 1.10 | 0.86 |

Electron current 7.5 A

Design

0.81*.128*54.5

=5.7 nC

Achieved

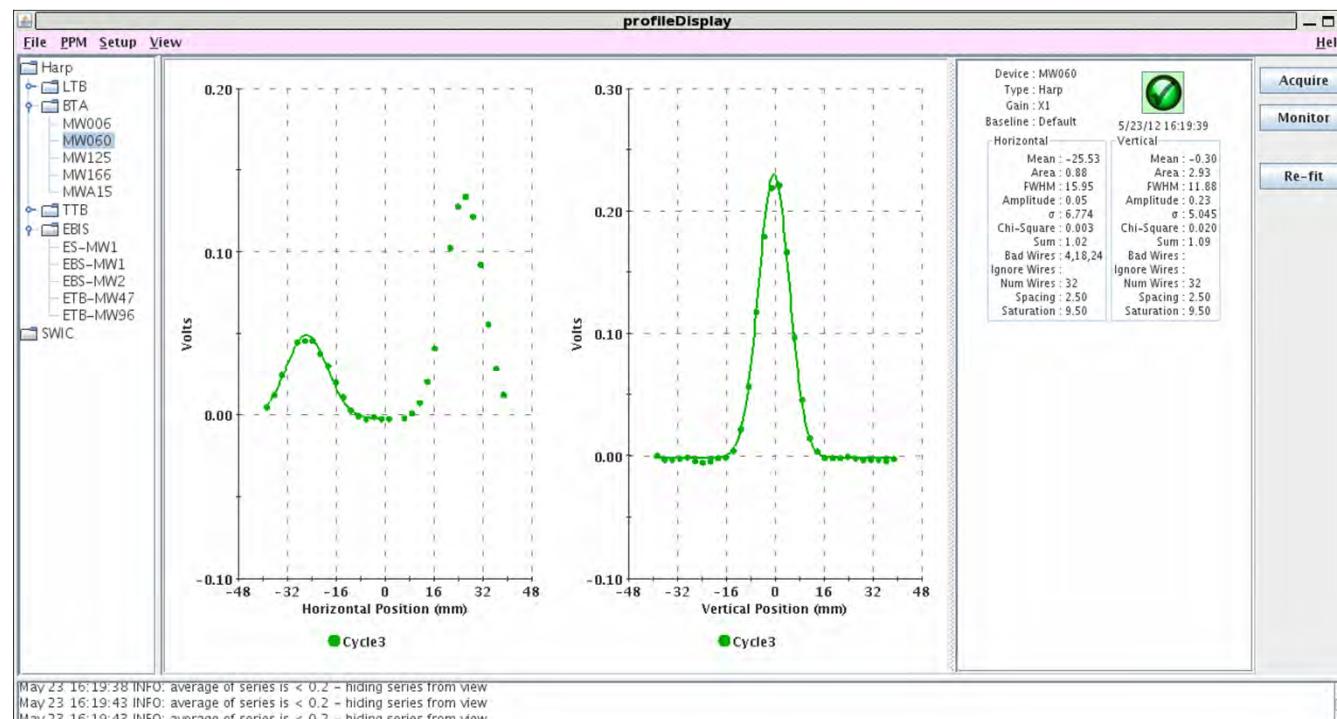
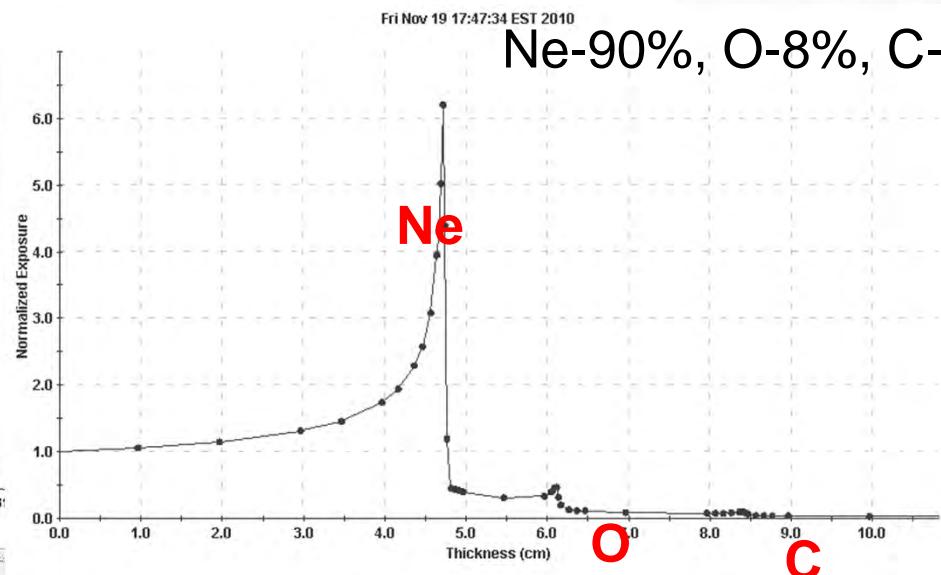
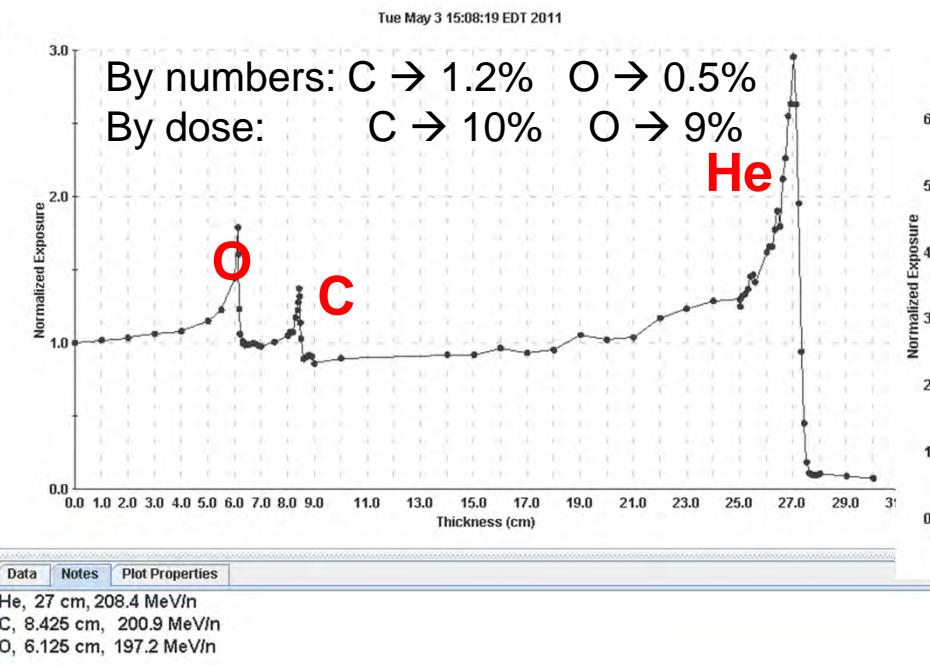
5.8 nC

Ratio

5.8/5.7

=1.02

Multiple ion Species seen after Booster



Beam profile in BTA
~ 60% Cu⁺¹¹
~ 40% Ar⁺⁷

EBIS Preinjector

HB2012
Beijing, China
September 17-21, 2012

Tuning procedure with Multiple Charge Species (PLAN vs Real)



We have studies the tuning procedure for the linac. Summary as follows:

Set all the quad to calculated values and bunchers off

•LEBT

- With TOF maximize beam current for desire charge state (NOT practiced)
- Maximize beam current after RFQ to find the match into RFQ

•RFQ

- Since RFQ will be commissioned about 1.5 years earlier than linac, (NOT done) we will set up analyzing magnet to set the correct amplitude (e. g Au^{+32})

•IH Linac

Maximizing the beam current for desire charge state using Booster to set phase and amplitude of IH Linac

•MEBT

- Maximizing current after IH Linac, by tuning quads and buncher (using Booster)

•HEBT

- Minimize the energy spread by tuning buncher amplitude and looking beam size with profile monitor at high dispersion at the bend (using Booster)

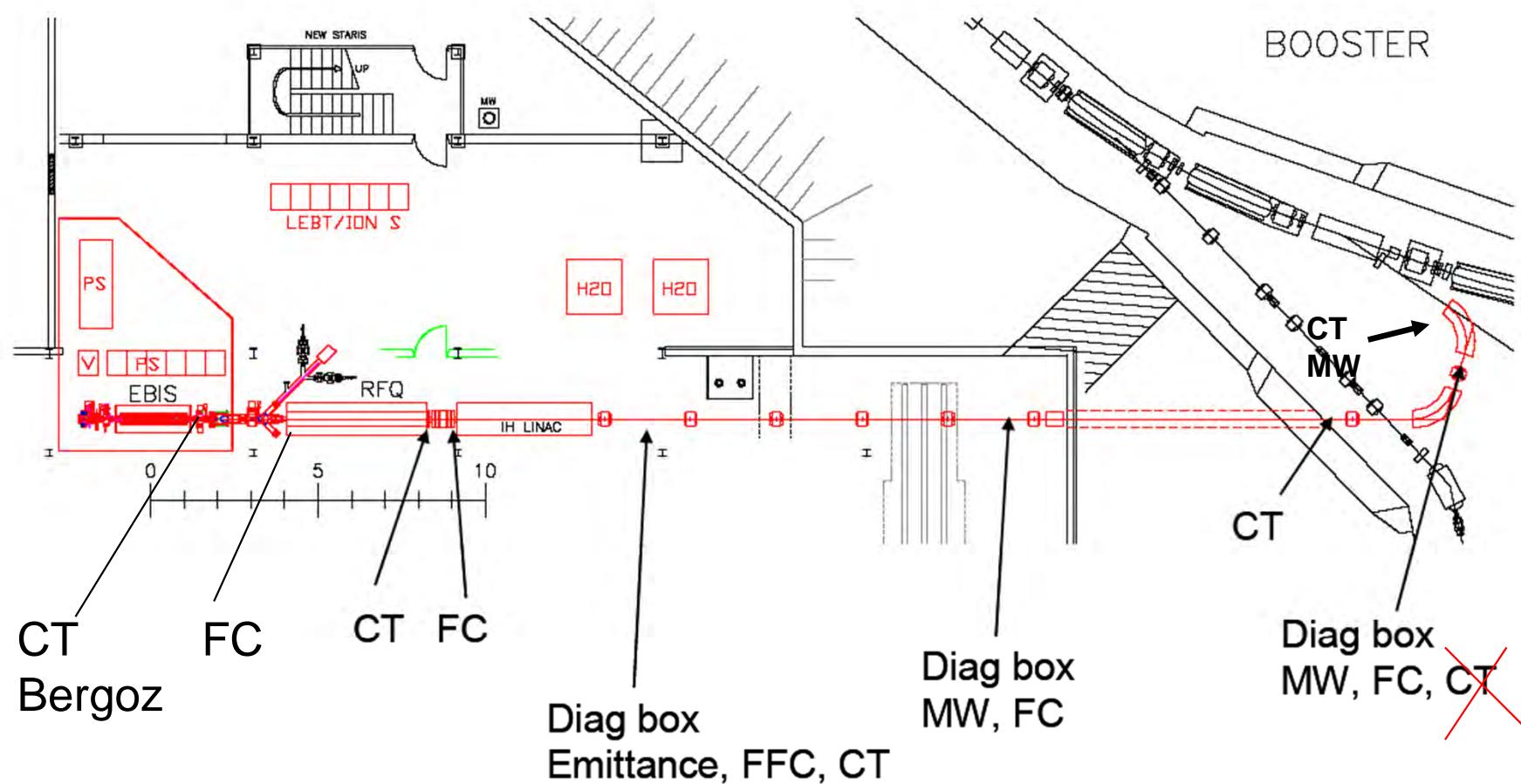
-Verify bend magnet setting with Booster

Conclusions



- EBIS pre-injector provided Au and Cu beam for RHIC Run 12 and tens ions species to NSRL
- Acceleration and transport of multi ion species seen in the EBIS pre-injector and Booster
- EBIS pre-injector is very stable, reliable and reproducible

Diagnostics



CT=current transformer; FC= Faraday cup; FFC=fast Faraday cup; MW=multiwire.

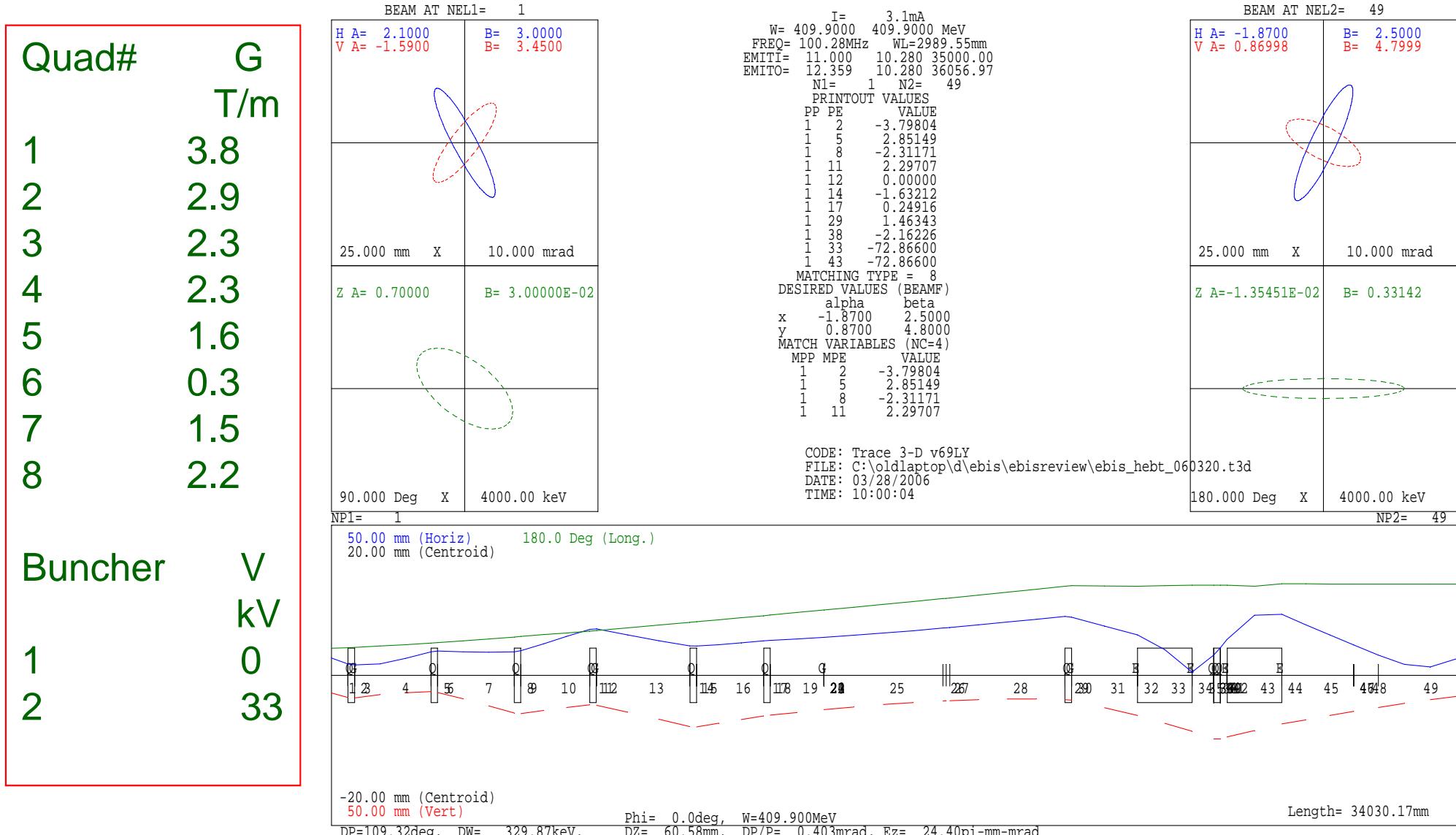
- Exit of EBIS: CT (Bergoz), depends on the voltage on nearby electrostatic grid lens
- RFQ Input: FC, diameter 13 mm, can collect +/- 30% delQ/Q
- RFQ Output: CT, lot of noise due to RF of RFQ, and pulse quadrupole, NOT USED
- MEBT Output: FC, Not biased, electron suppression due to magnetic field of quad
- Linac Output: CT (XF14), sensitive to EBIS voltage configuration
 - FFC: due to multiple charge, not much used
- Middle of HEBT: FC47, diameter 30mm, not much used due to its size , beam diameter is much bigger
 - MW47: Used to determine emittance with multiple charge species

Diagnostics (cont.)

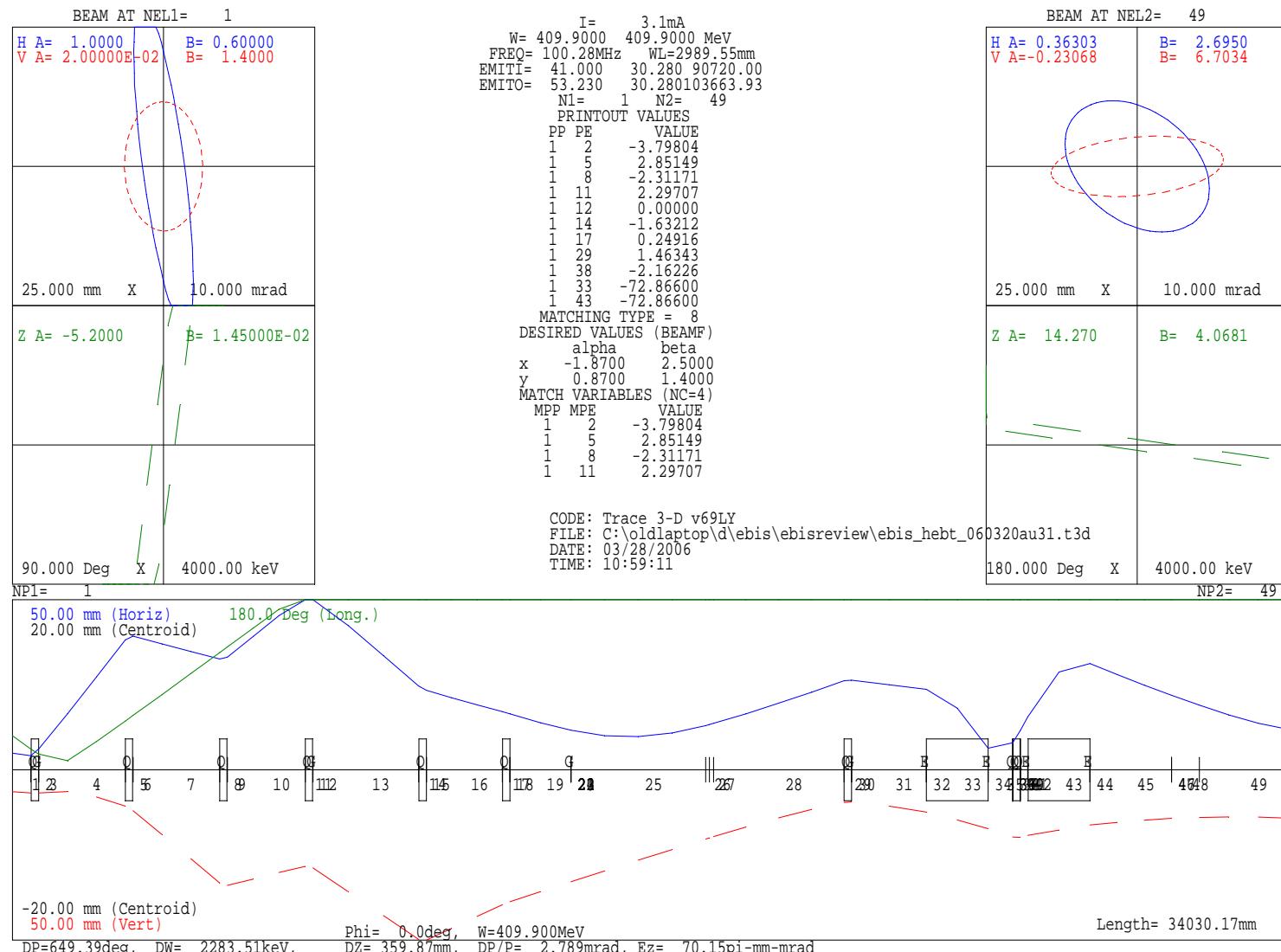


- Before the bend: CT, (XF89)
- Middle of the bend: FC96, 30 mm diameter, sensitive to ion species since biased is limited to few kV, read higher for lower q/m not used in transmission calculation
MW96: very useful, scanning magnetic field see all the ion species,
scrapers, Very important to stop other charge
- Input of booster: CT (XF108), very important to determine injection efficiencies
- Looking for diagnostics to distinguish charge states

Optics of the Transport line to the Booster

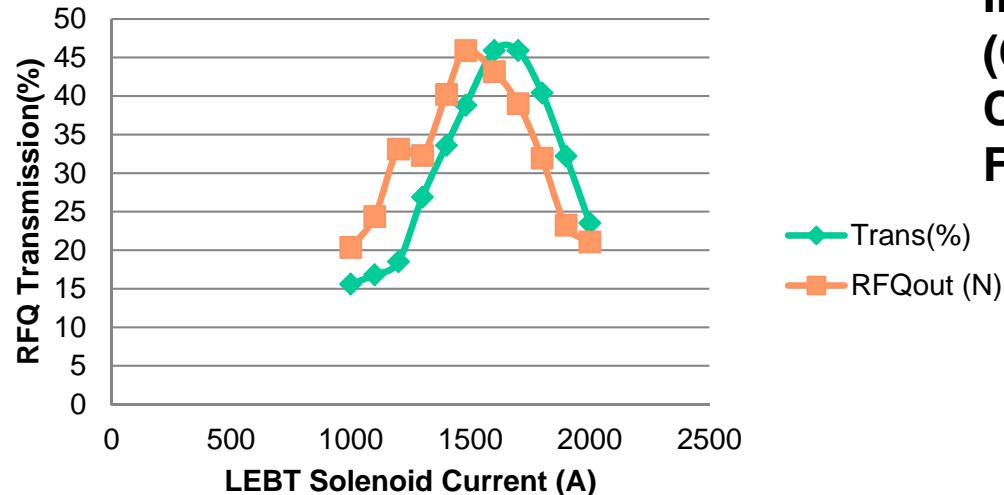


Beam Envelop for Au⁺³¹



Transmission of Au ions

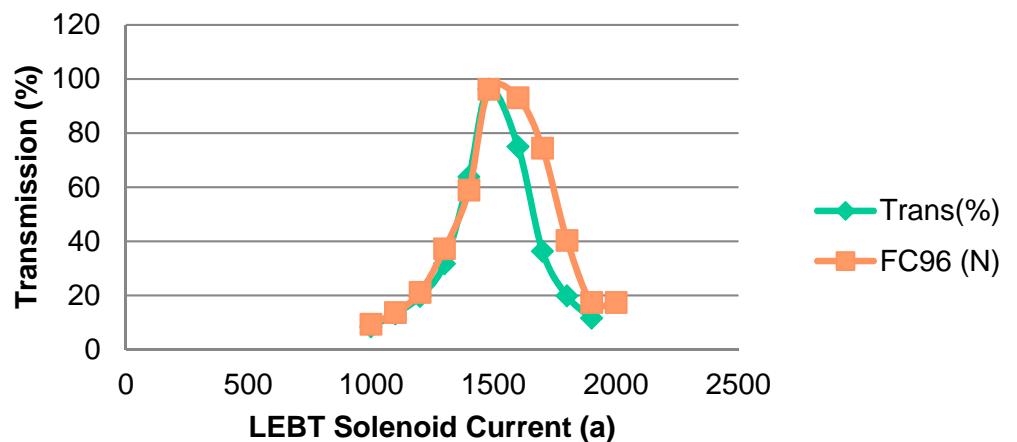
RFQ Trans vs LEBT Solenoid



Simulation done using TRACK
Input included Au^{32} and $\Delta p/p = \pm 10\%$
(C^2 , N^3 , O^3 , Ne^{3-4}) at EBIS voltage of 104 kV.
Current was measured at end of MEBT by Faraday cup

TRACK, PARMILA, and LORAS codes were used in simulations to transport Au^{+32} ions to the middle of the bend. Current was measured by Faraday cup (FC96) in the middle of the bend.

Au^{+32} Trans to Middle of the Bend vs LEBT Solenoid

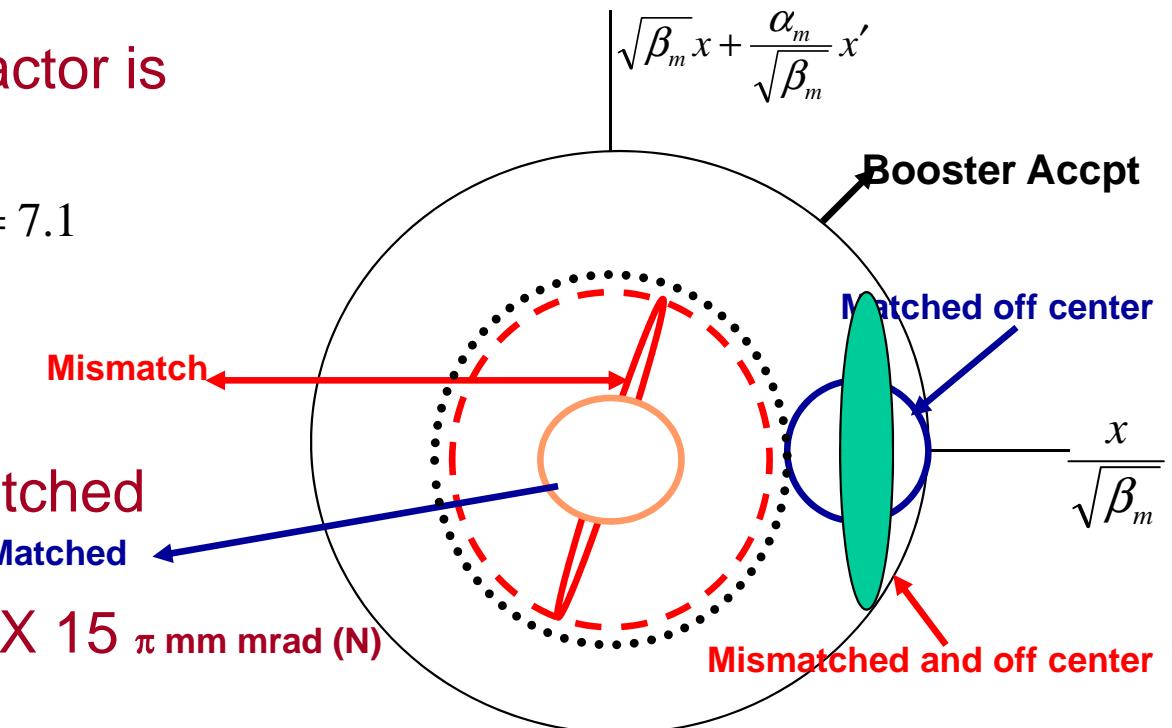


Inflector Aperture

- Inflector gap is 17 mm and 2.2 meter long (**BNL #159, Gardner**)
=> Geometric acceptance $29 \pi \text{ mm mrad}$, $\alpha_H = -1.87$, $\beta_H = 2.5\text{m}$
Infector acceptance $1.9 \pi \text{ mm mr (N)}$
- Lattice function at exit of inflector are $\alpha_H = -1.72$, $\beta_H = 11\text{m}$, $\eta_x = -2.4 \text{ m}$
- Full beam size $2a = 2\sqrt{\varepsilon_{90} \beta_H + (\eta_x \frac{\Delta p}{p})^2}$ =22.1 mm for matched injection
- Mismatch emittance dilution factor is

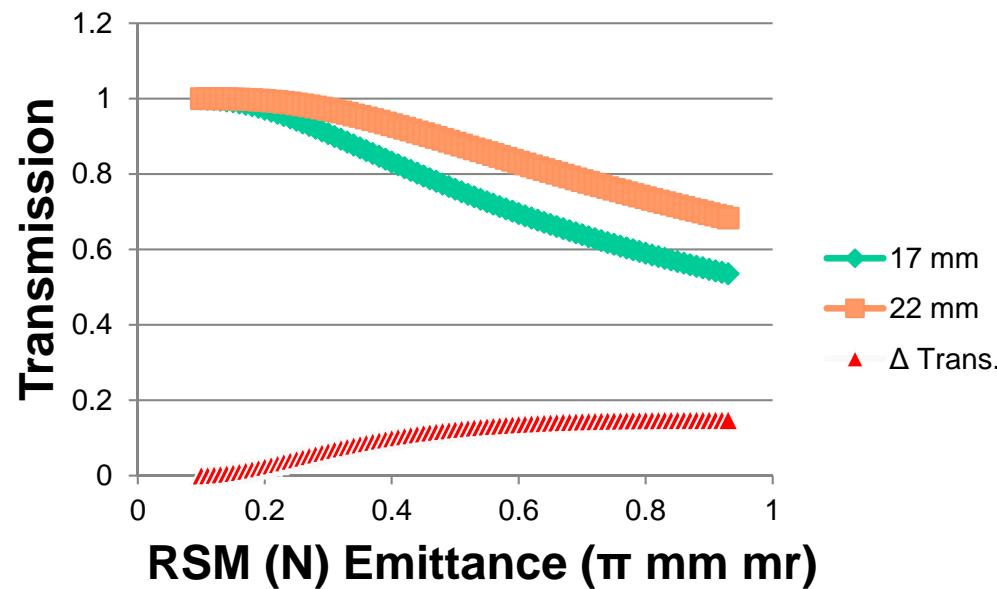
$$\frac{\varepsilon_i}{\varepsilon_m} = \frac{\sigma_i^2}{\sigma_m^2} = \frac{1}{2}(\beta_m \gamma_i + \beta_i \gamma_m - 2\alpha_i \alpha_m) = 7.1$$

- Linac emittance of 0.7(norm)
will result in $5 \pi \text{ mm mrad}$
- R=15, 3.31 matched. 2.6 mismatched
- Incoherent tune shift 0.08
- Booster acceptance (VXH) $4.5 \times 15 \pi \text{ mm mrad (N)}$



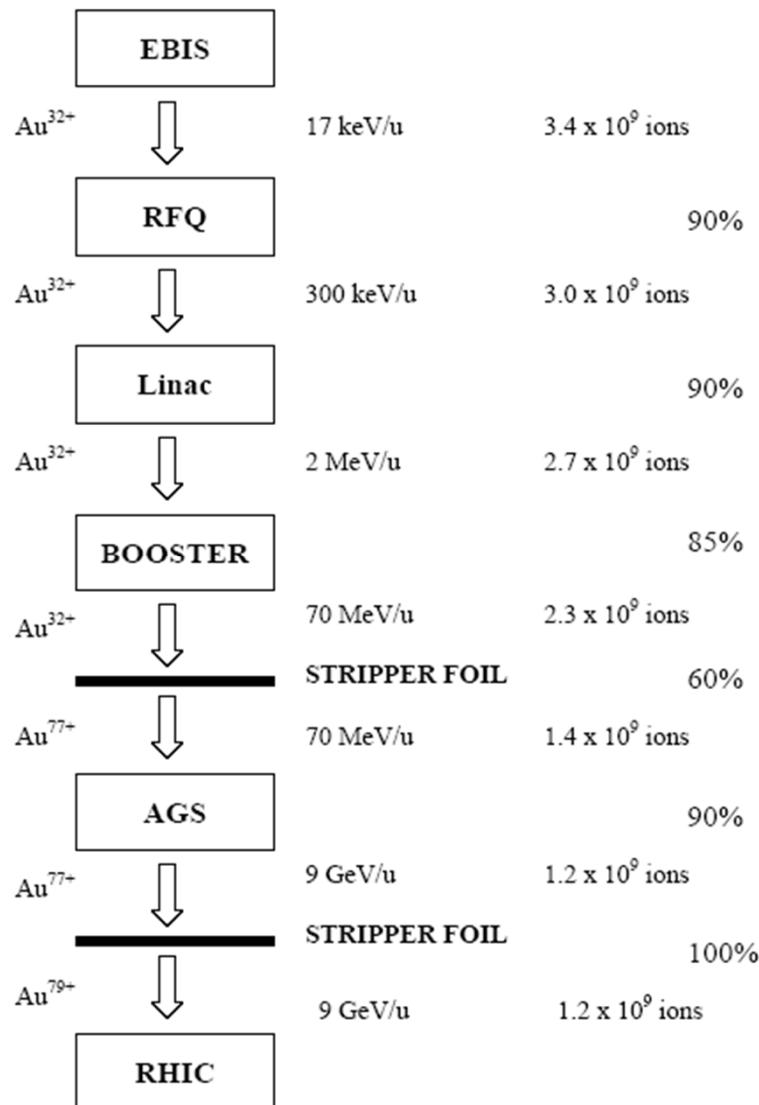
Emittance at Inflector

Last December inflector aperture was increase to 22 mm from 17 mm
The transmission was increase about 85% from 70% percent



=> Emittance (rms ,N) about 0.55 ($\pi \text{ mm mr}$)

Ion/Pulse and Efficiencies for Au⁺³²

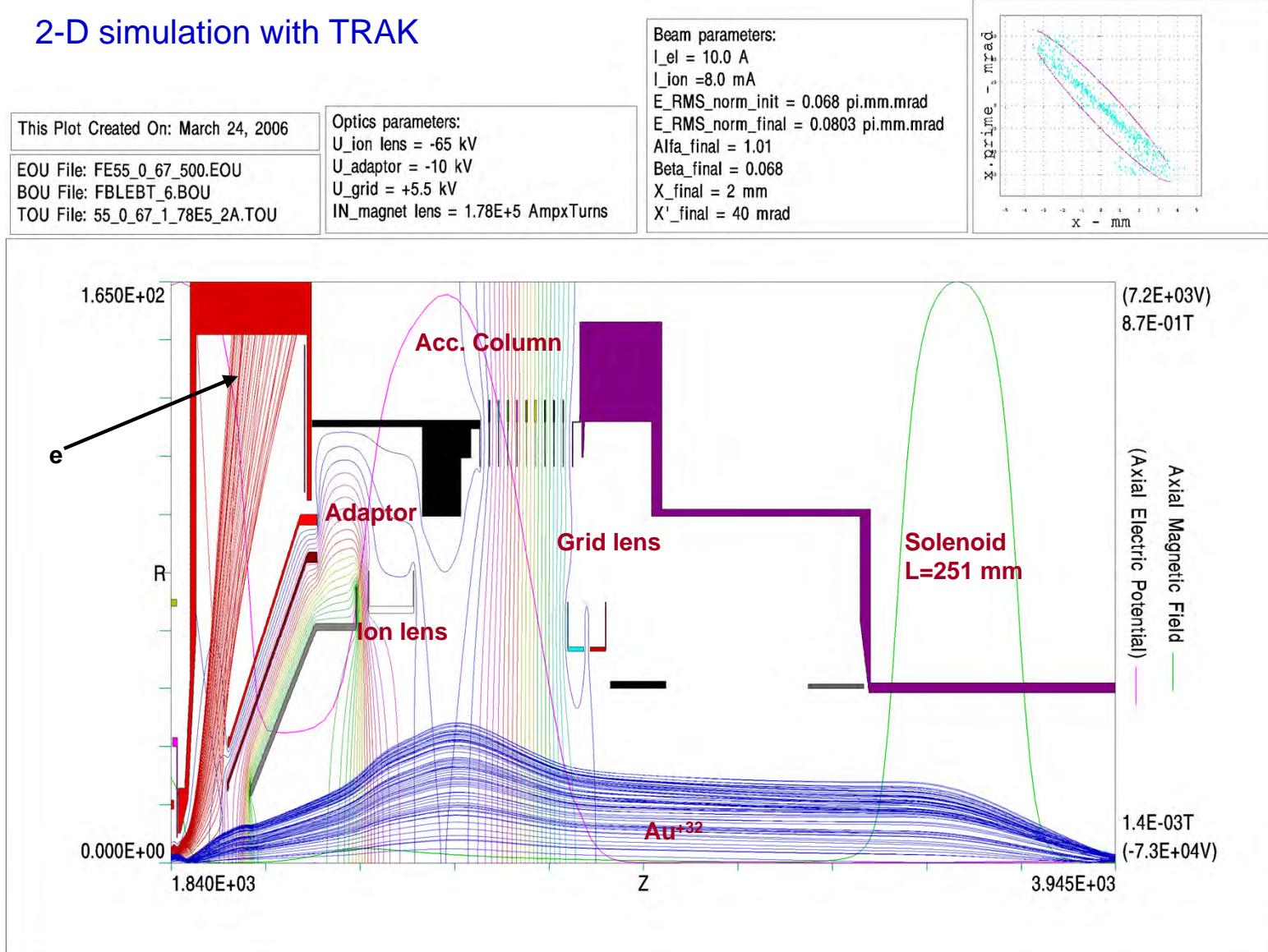


End-to-End Simulations (LEBT, Au⁺³²)

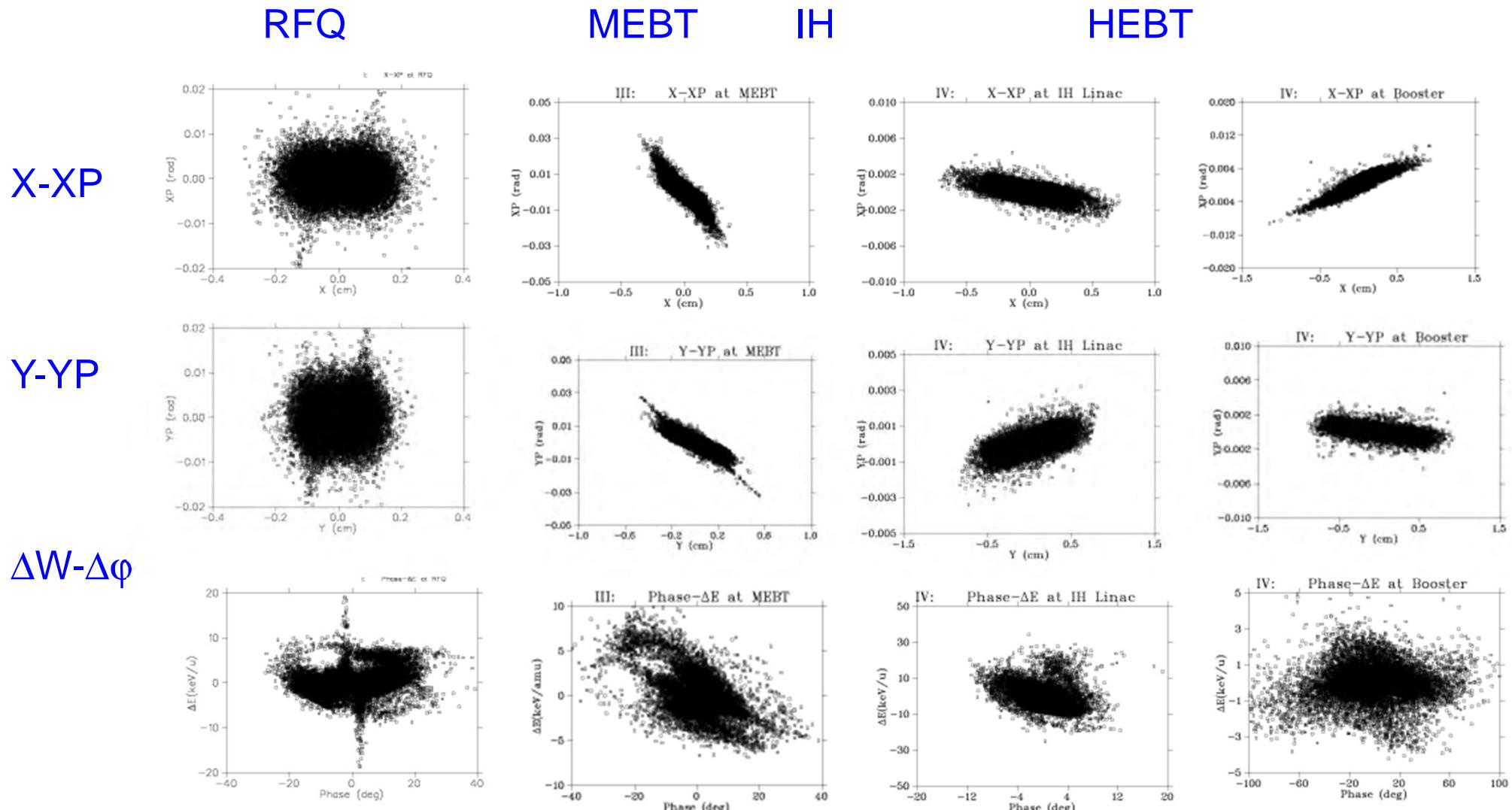


Pikin

2-D simulation with TRAK

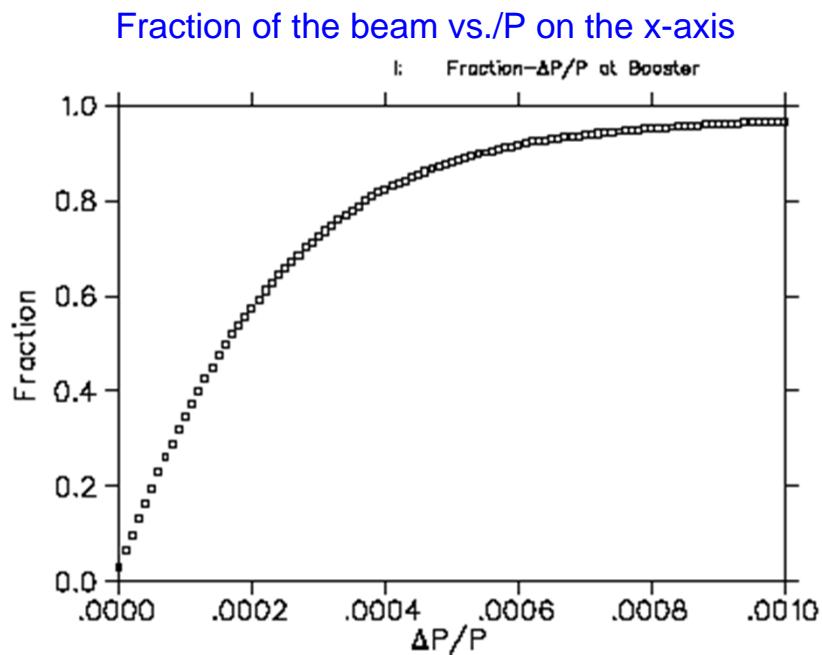


End-to-end Simulations



End-to-End Simulations

Transmission is defined with respect to the EBIS source



| Location | ϵ (N, RMS) | |
|----------|------------------------------|--------|
| LEBT | $E_T(\pi \text{ mm mr})$ | 0.085 |
| RFQ | $E_x (\pi \text{ mm mr})$ | 0.086 |
| | $E_y (\pi \text{ mm mr})$ | 0.092 |
| | $E_z(\pi \text{ ns-keV/u})$ | 0.0658 |
| | Transmission | 0.987 |
| MEBT | $E_x(\pi \text{ mm mr})$ | 0.096 |
| | $E_y (\pi \text{ mm mr})$ | 0.090 |
| | $E_z(\pi \text{ ns-keV/u})$ | 1.425 |
| | Transmission | 0.982 |
| IH-DTL | $E_x (\pi \text{ mm mr})$ | 0.097 |
| | $E_Y (\pi \text{ mm mr})$ | 0.096 |
| | $E_Z (\pi \text{ ns-keV/u})$ | 0.779 |
| | Transmission | 0.966 |
| HEBT | $E_x (\pi \text{ mm mr})$ | 0.146 |
| | $E_y (\pi \text{ mm mr})$ | 0.122 |
| | T. $(\Delta p/P=0.1\%)$ | 0.960 |
| | T. $(\Delta p/P=0.05\%)$ | 0.896 |

End-to-End Simulations with error tolerances



- Ion extractor $\pm 1\%$, Ion lens $\pm 0.5\%$, Adapter $\pm 1\%$, Gridded lens $\pm 0.2\%$, Platform $\pm 0.5\%$, Solenoid $\pm 0.2\%$
- Quad Alignment ± 0.1 mm (MEBT, IH-DTL, HEBT)
- Quad Strength $\pm 0.1\%$ (MEBT, IH-DTL, HEBT)
- Phase and Amplitude ± 0.5 deg, $\pm 0.5\%$ (RFQ, IH-DTL, Bunchers)
- Dipole Field Uniformity ± 0.1 % (x-y plane)
- Dipole Alignment ± 0.1 mm
- Harp 0.2 mm

End-to-End Simulations with Errors



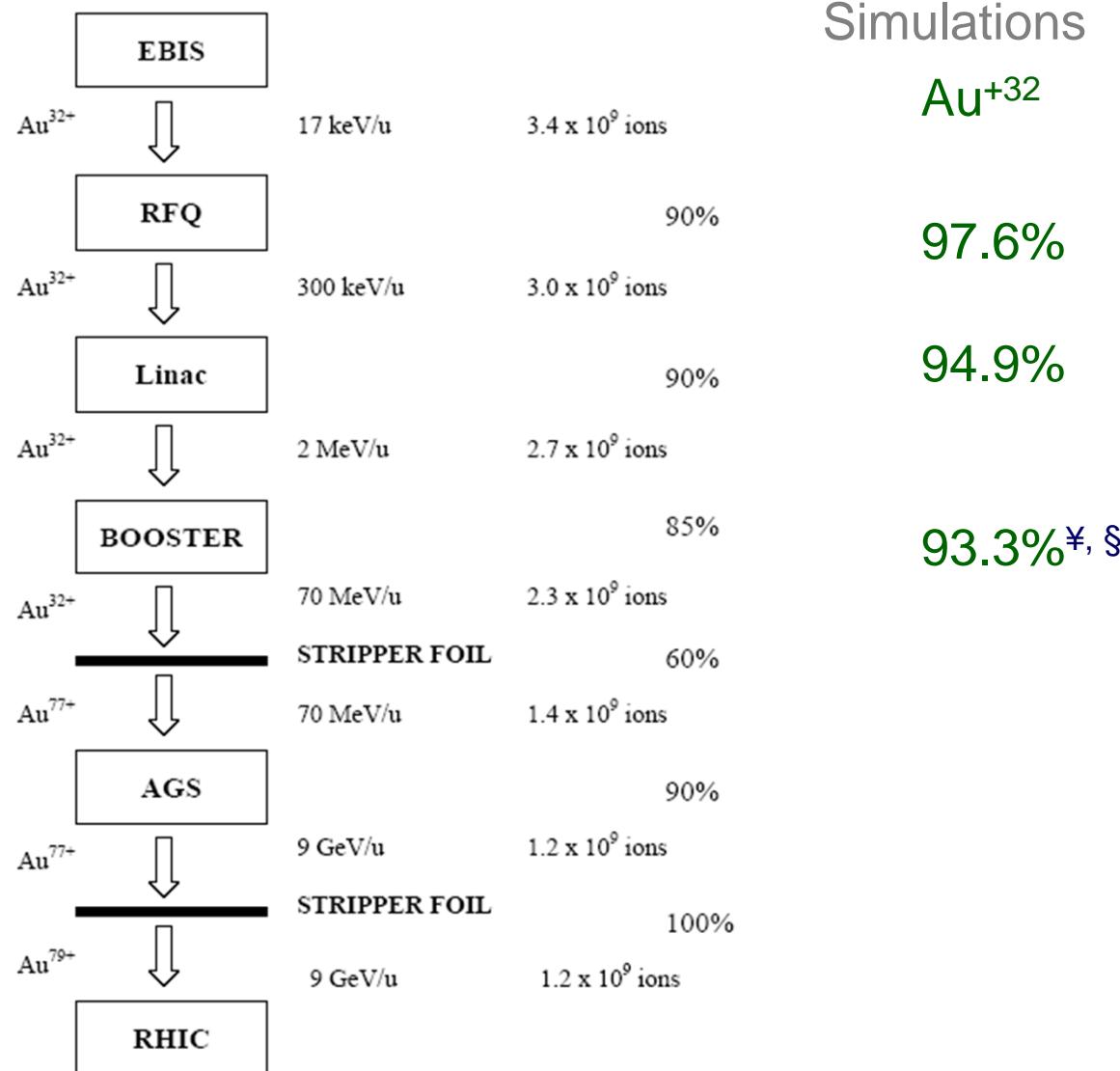
- 20 random seeds were used
- All the errors were uniformly distributed
- Simulation started at EBIS with 1000 micro-particles (TRAK 2D)
- At RFQ $r-r'$ converted to $x-x'$ and $y-y'$ with >10000 micro-particles
- Alignment errors were corrected with automated steering scheme

End-to-End Simulations with Errors

- Transmission with respect to the EBIS Source
- Transmission for HEBT defines as particles with $\Delta p/p < \pm 0.05\%$

| Location | Transmission (%) | ε (RMS, N) π mm mrad | ε (RMS, N) π mm mrad | ε (RMS) π ns-keV/u | MMF (x/y) |
|-------------------------|------------------|---|---|---------------------------------------|--------------------------|
| LEBT Average (STD) | 100 (0.00) | 0.0927 (0.001) | 0.0897 (0.003) | 0.1390 (0.002) | 0.05/0.25 (0.04/0.02) |
| RFQ Average (STD) | 97.6 (0.75) | 0.1223 (0.020) | 0.1160 (0.009) | 0.8500 (0.010) | 0.04/0.06 (0.03/0.03) |
| MEBT Average (STD) | 96.7 (0.79) | 0.1159 (0.004) | 0.1102 (0.006) | 0.8043 (0.079) | 0.55/0.46 (0.12/0.12) |
| IH-DTL Average (STD) | 92.6 (1.19) | 0.1286 (0.008) | 0.1380 (0.012) | 1.4513 (0.328) | 0.38/0.57 (0.09/0.12) |
| HEBT Average (STD) | 86.4 (1.22) | 0.1670 (0.008) | 0.1480 (0.011) | 0.8078 (0.054) | 0.03/0.08 (0.04/0.18) |

Ion/Pulse and Efficiencies for Au⁺³²



Performance Requirements of the EBIS



| | |
|--|--|
| Species | He to U |
| Output (single charge state) | $\geq 1.1 \times 10^{11}$ charges |
| Intensity (examples) | $3.4 \times 10^9 \text{ Au}^{32+} / \text{pulse}$ (1.7 mA) $5 \times 10^9 \text{ Fe}^{20+} / \text{pulse}$ (1.6 mA) $6.3 \times 10^{10} \text{ He}^{1+} / \text{pulse}$ (2.0 mA) |
| Q/m | ≥ 0.16, depending on ion species |
| Repetition rate | 5 Hz |
| Pulse width | 10 - 40 μs |
| Switching time between species | 1 second |
| Output emittance (Au^{32+}) | $< 0.18 \pi \text{ mm mrad, norm, rms}$ |
| Output energy | 17 keV/amu |
| $\Delta P/P$ | <0.05 % |

Space Charge in LEBT

Current 10 mA

| | Z | A | Q | Q/m | vext | EPS(un) | Perv(gen) | ET | SCT | Ratio | Debye Len |
|-----|----|-----|----|------|-------|---------|-----------|--------|--------|----------|-----------|
| P | 1 | 1 | 1 | 1.00 | 16.2 | 67 | 0.00296 | 0.0019 | 0.1971 | 104.5375 | 0.000436 |
| He3 | 2 | 3 | 2 | 0.67 | 24.4 | 67 | 0.00197 | 0.0019 | 0.1314 | 69.69164 | 0.000534 |
| D | 1 | 2 | 1 | 0.50 | 32.5 | 67 | 0.00148 | 0.0019 | 0.0985 | 52.26873 | 0.000616 |
| Si | 14 | 28 | 12 | 0.43 | 37.9 | 67 | 0.00127 | 0.0019 | 0.0845 | 44.80177 | 0.000665 |
| Au | 79 | 197 | 32 | 0.16 | 100.0 | 67 | 0.00048 | 0.0019 | 0.0320 | 16.9807 | 0.001081 |

Measurement for 1.7 mA Au⁺²⁵ (n,rms) = 0.125 π mm mrad @ 20keV
 Calculated(n,rms) = 0.125 π mm mrad

Envelope equation

$$R'' + k_0^2 R - \frac{(4\mathcal{E}_{rms})^2}{R^3} - \frac{K}{R} = 0$$

Gen. Perv. (K) = QI/(2 π ε₀ m c³ β³ γ³)

Debye Length(λD) = 2 ε²rms/K = (1/8)(R/λD)²

Energy = 17 keV/u
 $\beta=0.006017$
 $\gamma=1.000018$
 $R= 15 \text{ mm}$