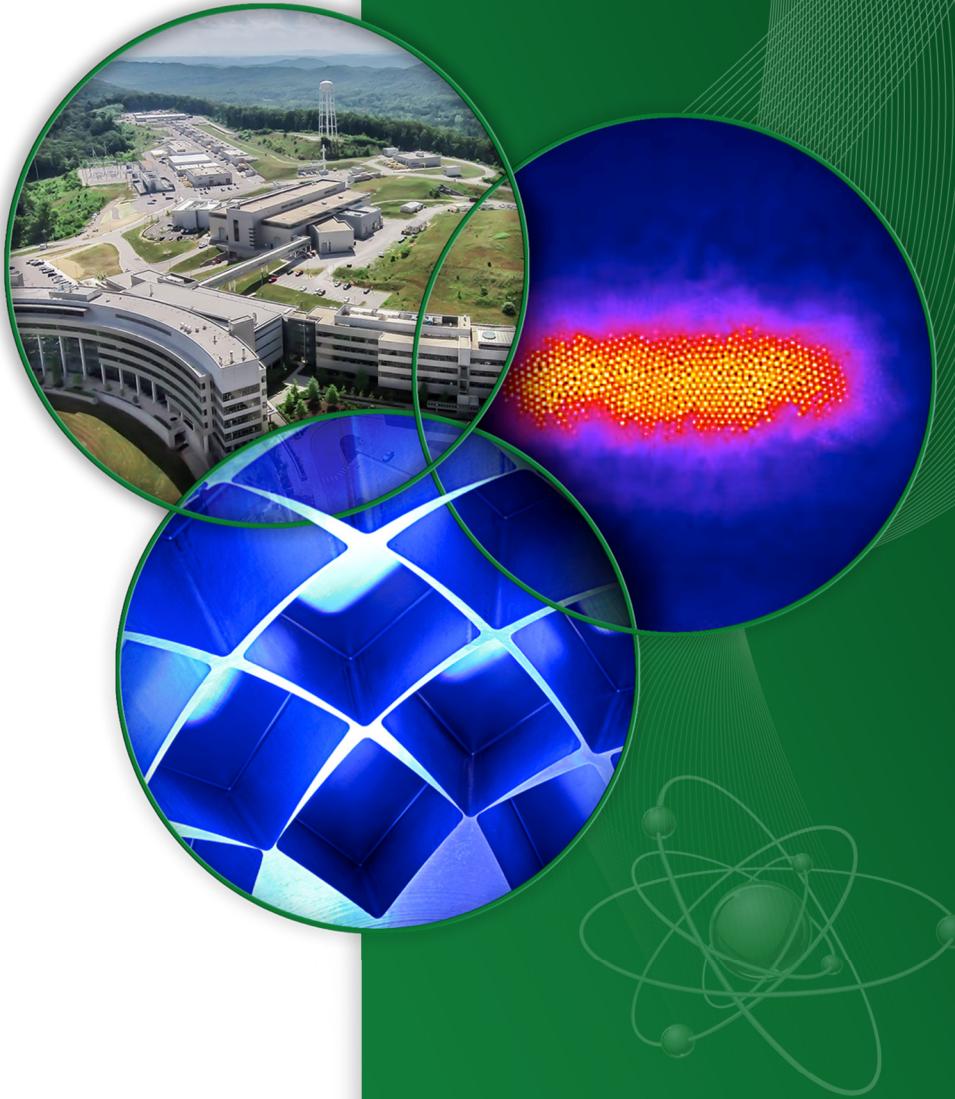


4-D and 6-D Emittance Determination for Hadron LINACs

A. Aleksandrov

Oak Ridge National Laboratory,
USA



ORNL is managed by UT-Battelle
for the US Department of Energy

Outline

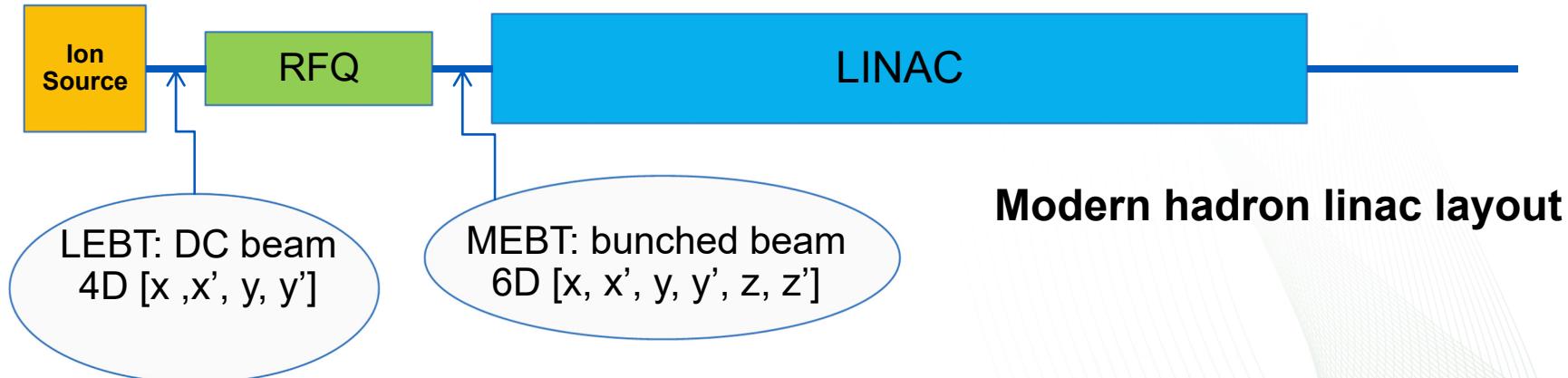
- Introduction
 - Why, what, definitions, etc.
- Example of 4D RMS emittance diagnostics at GSI
- Set up for 6D phase space scan at SNS

Why do we need to know 4D or 6D emittance?

- Generic answer is to provide input data for computer simulation of beam dynamics

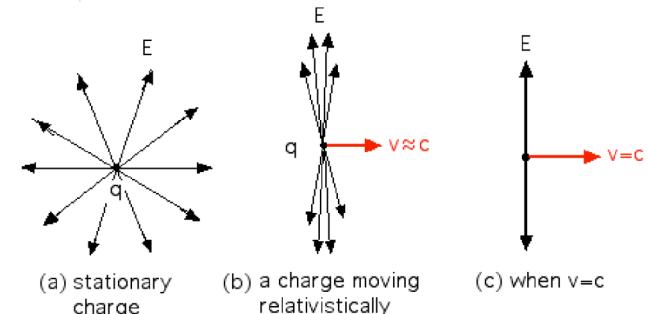
What is specific about RF linacs?

- Linac is single pass system → initial conditions in large degree define particles dynamics
- Beam is bunched and bunch is short → 6D phase space



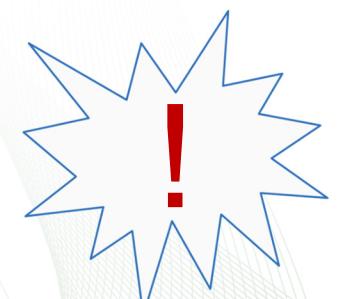
What is specific about hadron RF linacs?

- Non-relativistic energy ($\gamma=1- 2$)
 - Large un-normalized emittance (mm range beam size)
 - Significant space charge
 - Weak synchrotron radiation
 - Weak EM field contraction
- Particles interaction with materials
 - Large power deposition volume density
 - Material sputtering
 - Neutron production and activation

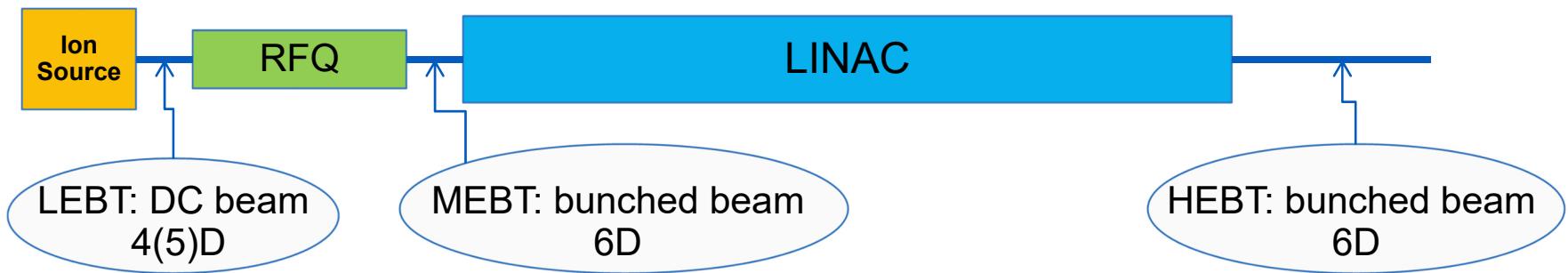


There is significant difference even within hadrons family:

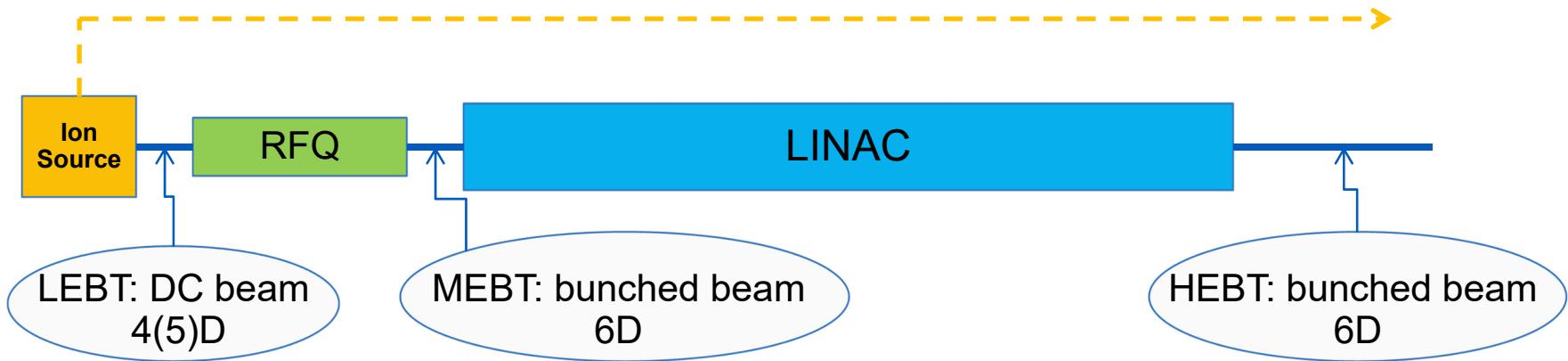
- Protons
- H-
- Light ions
- Heavy ions



Initial parameters for beam dynamics simulation

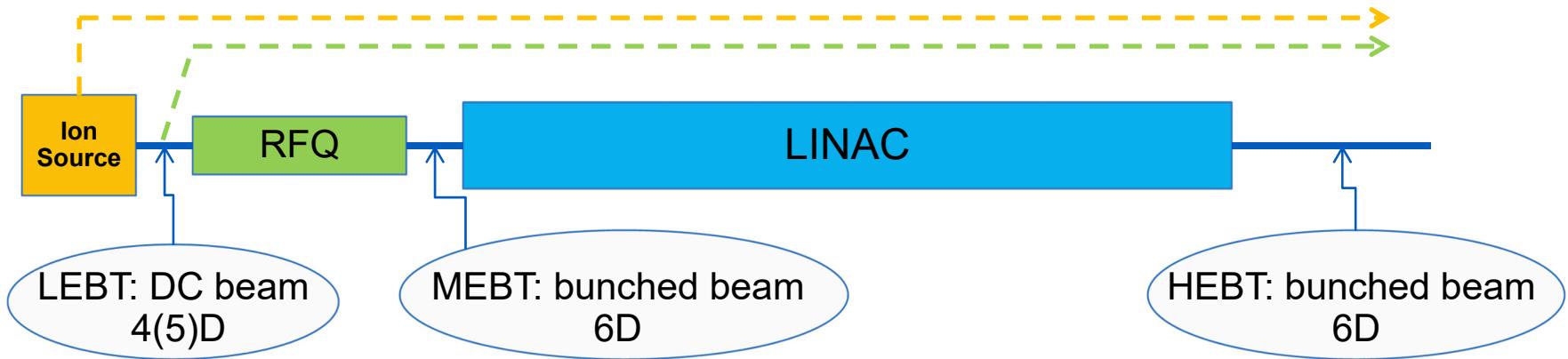


Initial parameters for beam dynamics simulation



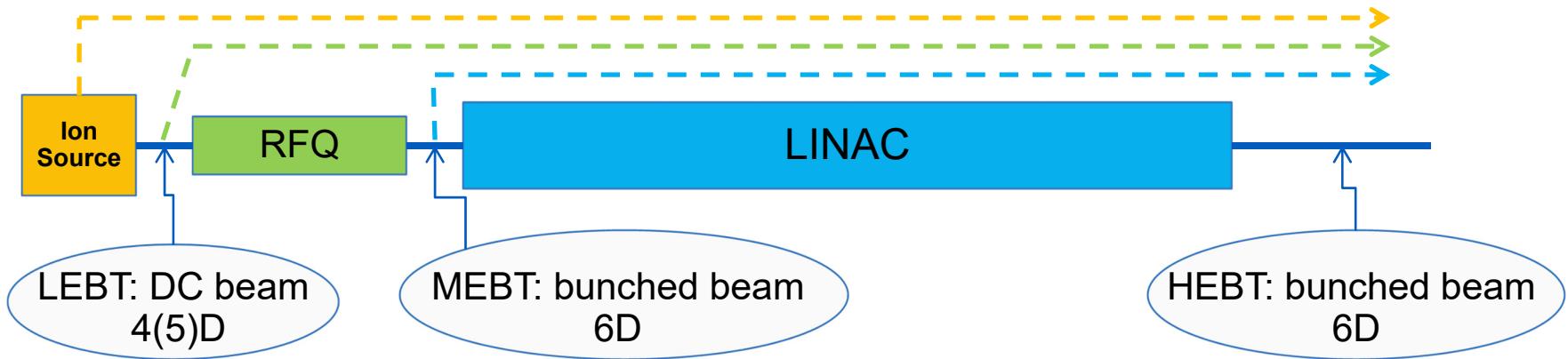
- End-to-end simulation starting from ion source plasma surface
 - No measurements involved. No comments

Initial parameters for beam dynamics simulation



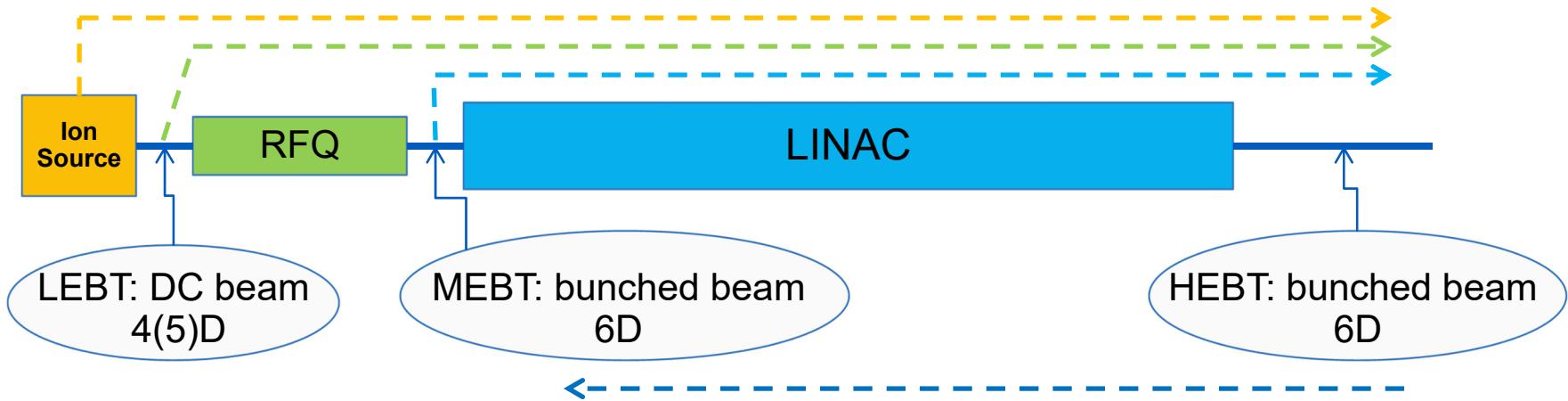
- End-to-end simulation starting from ion source plasma surface
 - No measurements involved. No comments
- Measure 4D distribution at ion source exit
 - Beam dynamics in RFQ is the most challenging part of linac simulation: strong space charge, many cells, no diagnostics

Initial parameters for beam dynamics simulation



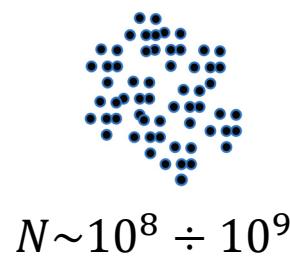
- End-to-end simulation starting from ion source plasma surface
 - No measurements involved. No comments
- Measure 4D distribution at ion source exit
 - Beam dynamics in RFQ is the most challenging part of linac simulation: strong space charge, many cells, no diagnostics
- Measure 6D distribution at RFQ exit
 - The most challenging from beam instrumentation point of view

Initial parameters for beam dynamics simulation



- End-to-end simulation starting from ion source plasma surface
 - No measurements involved. No comments
- Measure 4D distribution at ion source exit
 - Beam dynamics in RFQ is the most challenging part of linac simulation: strong space charge, many cells, no diagnostics
- Measure 6D distribution at RFQ exit
 - The most challenging from beam instrumentation point of view

Bunch representation



particle #	coordinates
1	x, x', y, y', z, z'
2	x, x', y, y', z, z'
.	.
N	x, x', y, y', z, z'

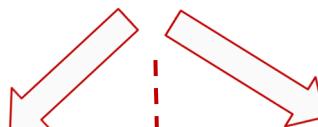
6N numbers

impossible to measure individual particles positions
need to simplify representation

σ -matrix formalism

x	x'	y	y'	z	z'
$\langle xx \rangle$	$\langle xx' \rangle$	$\langle xy \rangle$	$\langle xy' \rangle$	$\langle xz \rangle$	$\langle xz' \rangle$
$\langle x'x \rangle$	$\langle x'x' \rangle$	$\langle x'y \rangle$	$\langle x'y' \rangle$	$\langle x'z \rangle$	$\langle x'z' \rangle$
$\langle yx \rangle$	$\langle yx' \rangle$	$\langle yy \rangle$	$\langle yy' \rangle$	$\langle yz \rangle$	$\langle yz' \rangle$
$\langle y'x \rangle$	$\langle y'x' \rangle$	$\langle y'y \rangle$	$\langle y'y' \rangle$	$\langle y'z \rangle$	$\langle y'z' \rangle$
$\langle zx \rangle$	$\langle zx' \rangle$	$\langle zy \rangle$	$\langle zy' \rangle$	$\langle zz \rangle$	$\langle zz' \rangle$
$\langle z'x \rangle$	$\langle z'x' \rangle$	$\langle z'y \rangle$	$\langle z'y' \rangle$	$\langle z'z \rangle$	$\langle z'z' \rangle$

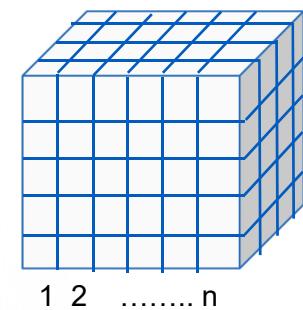
Statistical RMS representation requires
21 numbers for 6D
10 numbers for 4D



distribution function

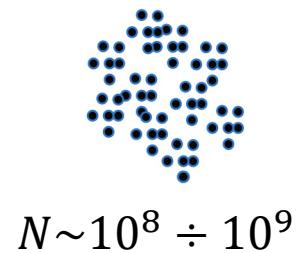
number of particles per bin in phase space

$$f(x, x', y, y', z, z') = \frac{N(x \pm \Delta, x' \pm \Delta, y \pm \Delta, y' \pm \Delta, z \pm \Delta, z' \pm \Delta)}{N_{total}}$$



Distribution function representation requires
 n^6 numbers for 6D
 n^4 numbers for 4D

Bunch representation



particle #	coordinates	
1	x, x', y, y', z, z'	
2	x, x', y, y', z, z'	
.	.	
N	x, x', y, y', z, z'	
		$6N$ numbers

impossible to measure individual particles positions
need to simplify representation

σ -matrix formalism

	x	x'	y	y'
x	$\langle xx \rangle$	$\langle xx' \rangle$	$\langle xy \rangle$	$\langle x y' \rangle$
x'	$\langle x'x \rangle$	$\langle x'x' \rangle$	$\langle x'y \rangle$	
y	$\langle yx \rangle$	$\langle yx' \rangle$	$\langle yy \rangle$	
y'	$\langle y'x \rangle$	$\langle y'x' \rangle$	$\langle y'y \rangle$	
z	$\langle zx \rangle$	$\langle zx' \rangle$	$\langle zy \rangle$	$\langle zy' \rangle$
z'	$\langle z'x \rangle$	$\langle z'x' \rangle$	$\langle z'y \rangle$	$\langle z'y' \rangle$

Statistical RMS representation requires

21 numbers for 6D

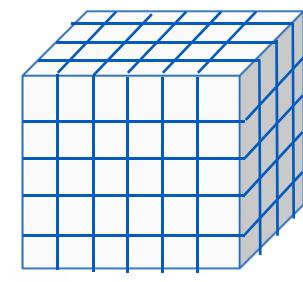
10 numbers for 4D

Both representations
are measurable and
usable to initiate
simulation

Distribution function

particles per
space

$$N_{total} \cdot (x \pm \Delta, y \pm \Delta, z \pm \Delta, z' \pm \Delta)$$



Distribution function representation requires

n^6 numbers for 6D

n^4 numbers for 4D

σ -matrix vs. distribution function

Measured **σ -matrix** can be used directly as input for RMS envelope tracking codes:

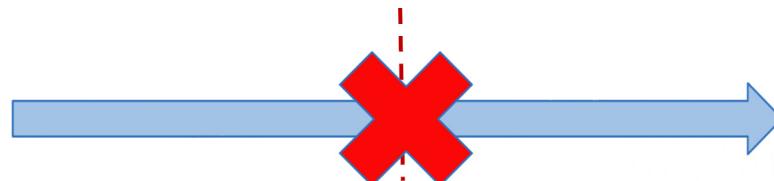
- Simulate dynamics of beam core only (RMS bunch size)
- Linear motion only
- Cannot predict beam loss

Measured **distribution function** can be used to generate particles as input for Particle-In-Cell (PIC) tracking codes:

- Simulate dynamics of beam core, tails and halo (track individual particles)
- Non-linear motion in realistic e/m fields
- Should be capable of predicting beam loss



Known distribution function is sufficient for calculating σ matrix



Known σ matrix is not sufficient for calculating distribution function

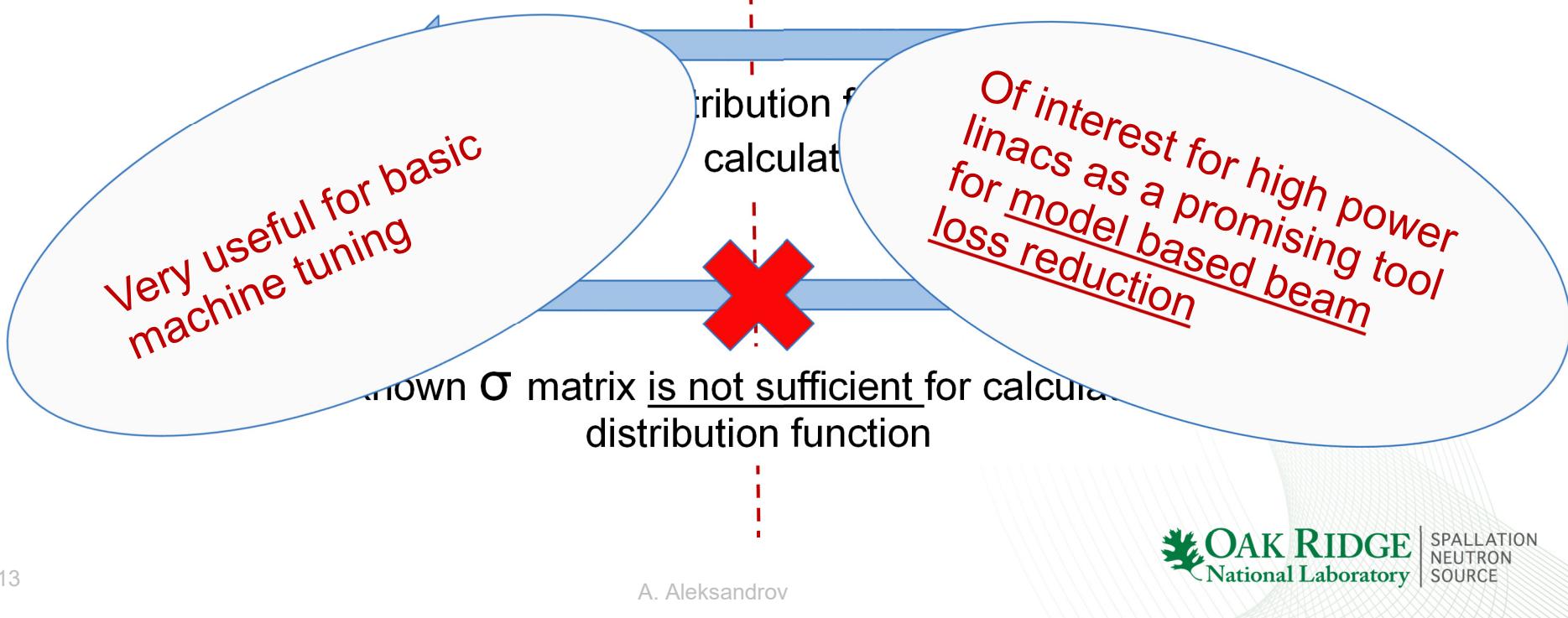
σ -matrix vs. distribution function

Measured **σ -matrix** can be used directly as input for RMS envelope tracking codes:

- Simulate dynamics of beam core only (RMS bunch size)
- Linear motion only
- Cannot predict beam loss

Measured **distribution function** can be used to generate particles as input for Particle-In-Cell (PIC) tracking codes:

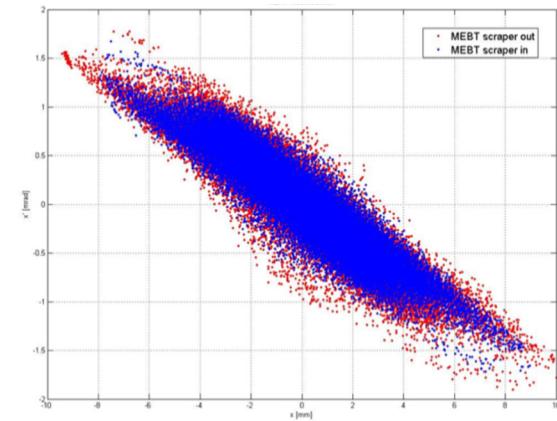
- Simulate dynamics of beam core, tails and halo (track individual particles)
- Non-linear motion in realistic e/m fields
- Should be capable of predicting beam loss



True high dimensional distribution functions

$f_6(x, x', y, y', z, z')$ - true 6D distribution function
as defined earlier

$f_2(x, x'); f_2(y, y'); f_2(z, z')$ →
easily measurable
2D projections of f_6
on x, y, z planes



$$f_{3*2}(x, x', y, y', z, z') = f_2(x, x') \cdot f_2(y, y') \cdot f_2(z, z')$$

↑
Sometimes is called 6D erroneously but

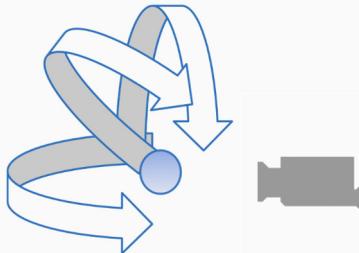
$$f_{3*2}(x, x', y, y', z, z') \neq f_6(x, x', y, y', z, z')$$

except for special case of no any correlations between degrees of freedom

zero correlation terms in 6D Σ -matrix do not guarantee absence of higher order correlations in f_6

How to measure emittance?

Reconstruction from lower dimensional projections



Rotate object
or detector

σ -matrix

exact solution in absence of space charge if number of projections:

- >3 for 2D
- >10 for 4D
- >21 for 6D

well established in 2D ('quad scan' technique)

well established in 4D (example in next section)

??? for 6D

distribution function

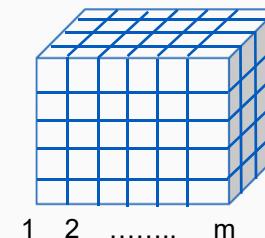
phase space tomography
provides approximate solution

well established in 2D

algorithm proposed for 4D

??? for 6D

direct phase space sampling



distribution function

well established in 2D (slit-slit; slit-grid etc. scan)

well established in 4D (pepper pot)

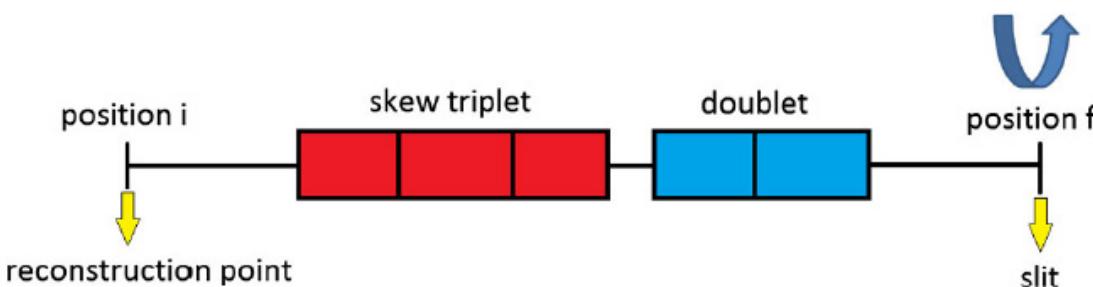
to be demonstrated in 6D (last part of this talk)

Measuring RMS in 4D example: ROSE at GSI

PHYSICAL REVIEW ACCELERATORS AND BEAMS 19, 072802 (2016)

'Rotating system for four-dimensional transverse rms-emittance measurements'

C. Xiao, M. Maier, X. N. Du, P. Gerhard, L. Groening, S. Mickat, and H. Vormann



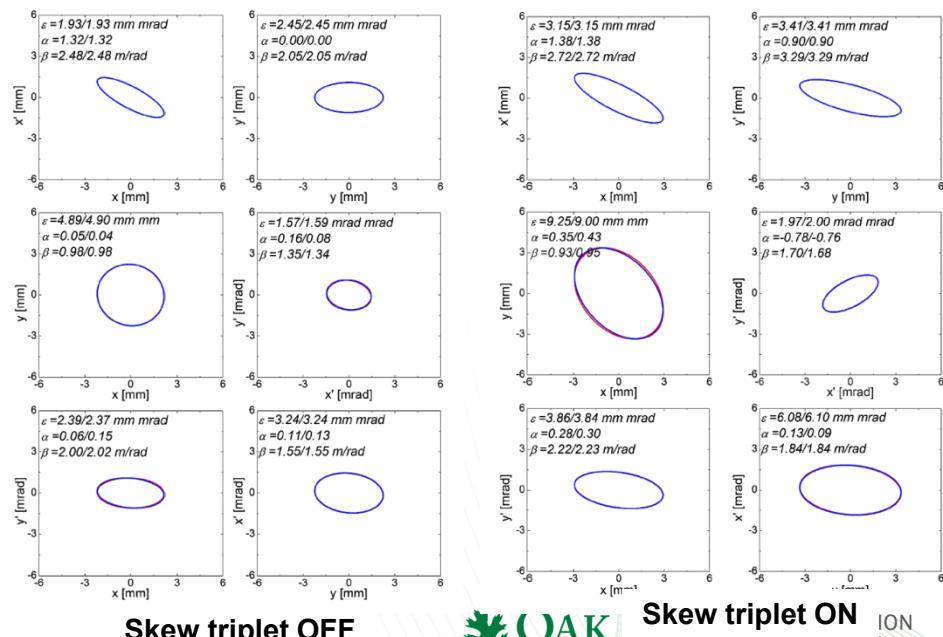
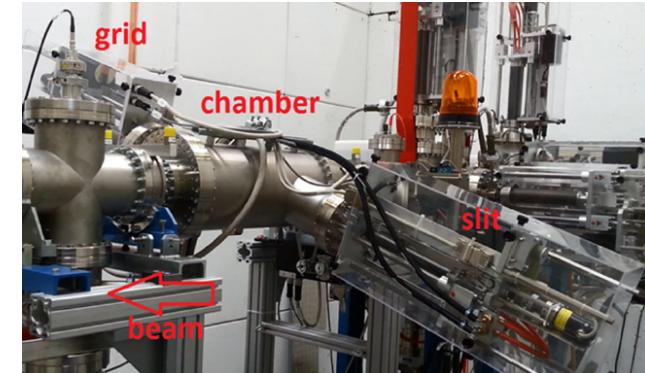
$$\langle xy \rangle_f^{a,b} = m_{11}^{a,b} m_{33}^{a,b} \langle xy \rangle_i + m_{11}^{a,b} m_{34}^{a,b} \langle xy' \rangle_i \\ + m_{12}^{a,b} m_{33}^{a,b} \langle x'y \rangle_i + m_{12}^{a,b} m_{34}^{a,b} \langle x'y' \rangle_i,$$

$$\langle xx' \rangle_\theta^{a,b} = \cos^2 \theta \langle xx' \rangle_f^{a,b} + \sin \theta \cos \theta \langle xy' \rangle_f^{a,b} \\ + \sin \theta \cos \theta \langle x'y \rangle_f^{a,b} + \sin^2 \theta \langle yy' \rangle_f^{a,b},$$

$$\begin{bmatrix} \langle xy \rangle_i \\ \langle xy' \rangle_i \\ \langle x'y \rangle_i \\ \langle x'y' \rangle_i \end{bmatrix} = (\Gamma^T \Gamma)^{-1} \Gamma^T \Lambda,$$

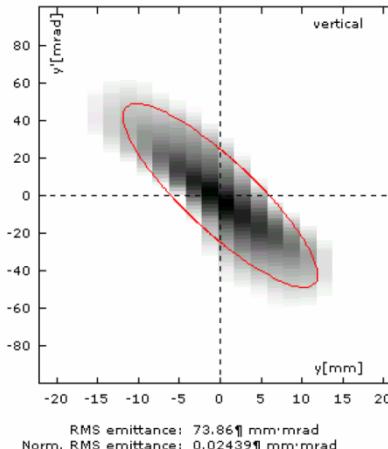
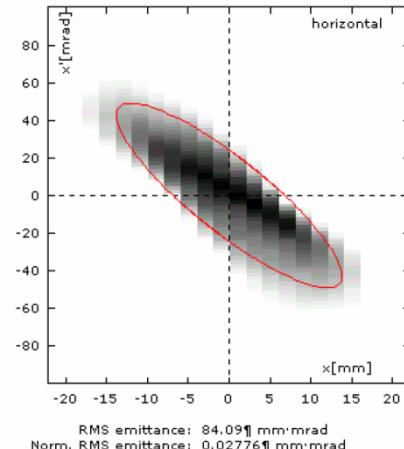
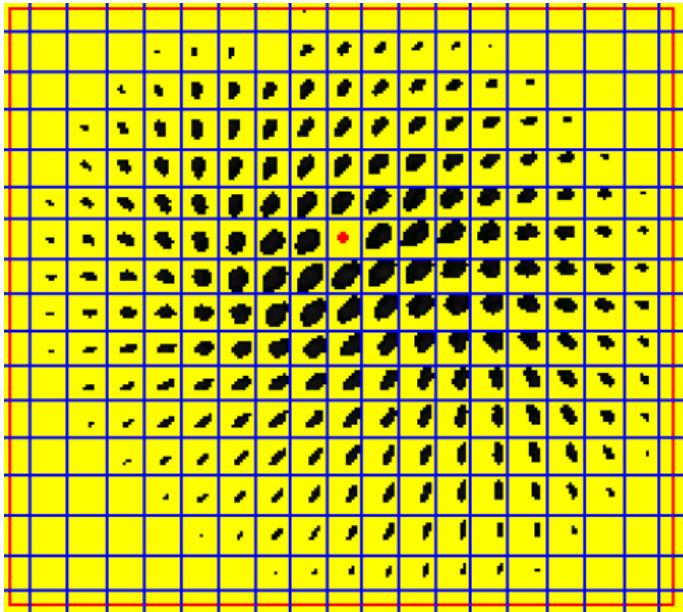
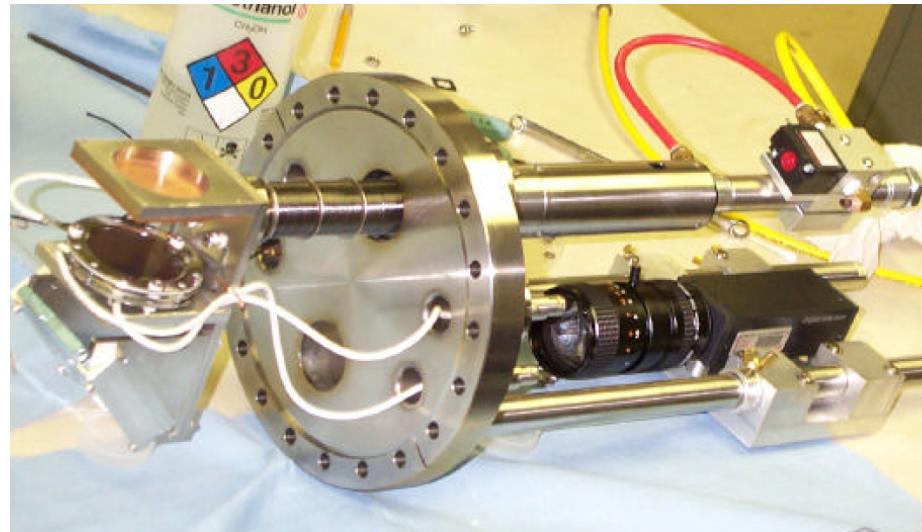
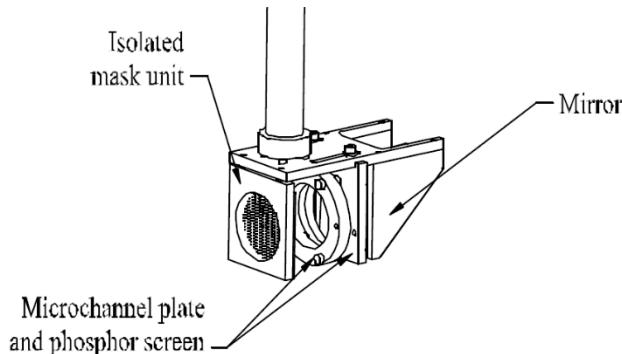
Contains known transport matrix elements

Contains measured 2-nd moments



Measuring 4D distribution function. Pepper pot.

BNL C-AD Technote C-A/AP/#244 A. Pikin, A. Kponou, J. Ritter, V. Zajic

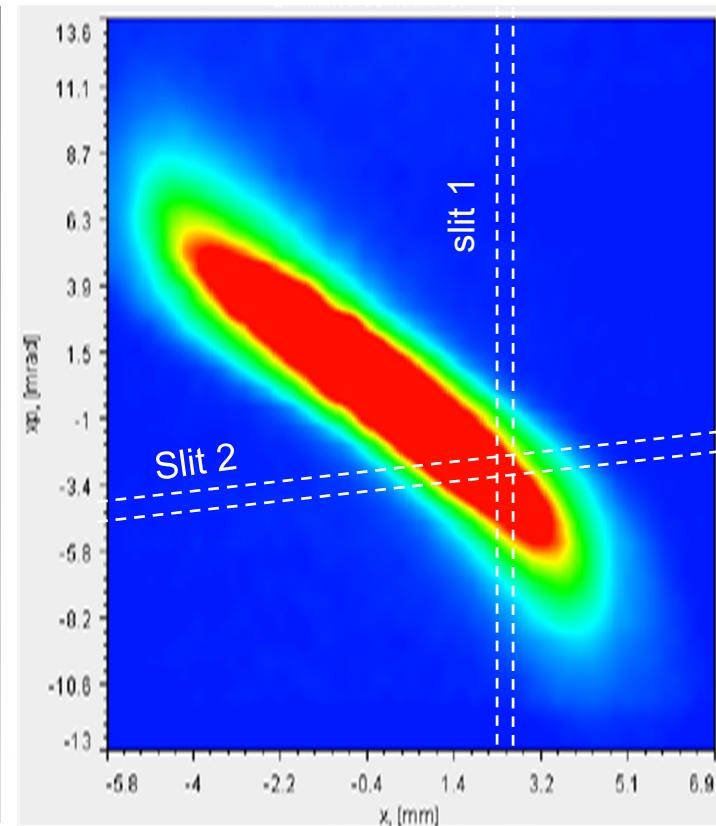
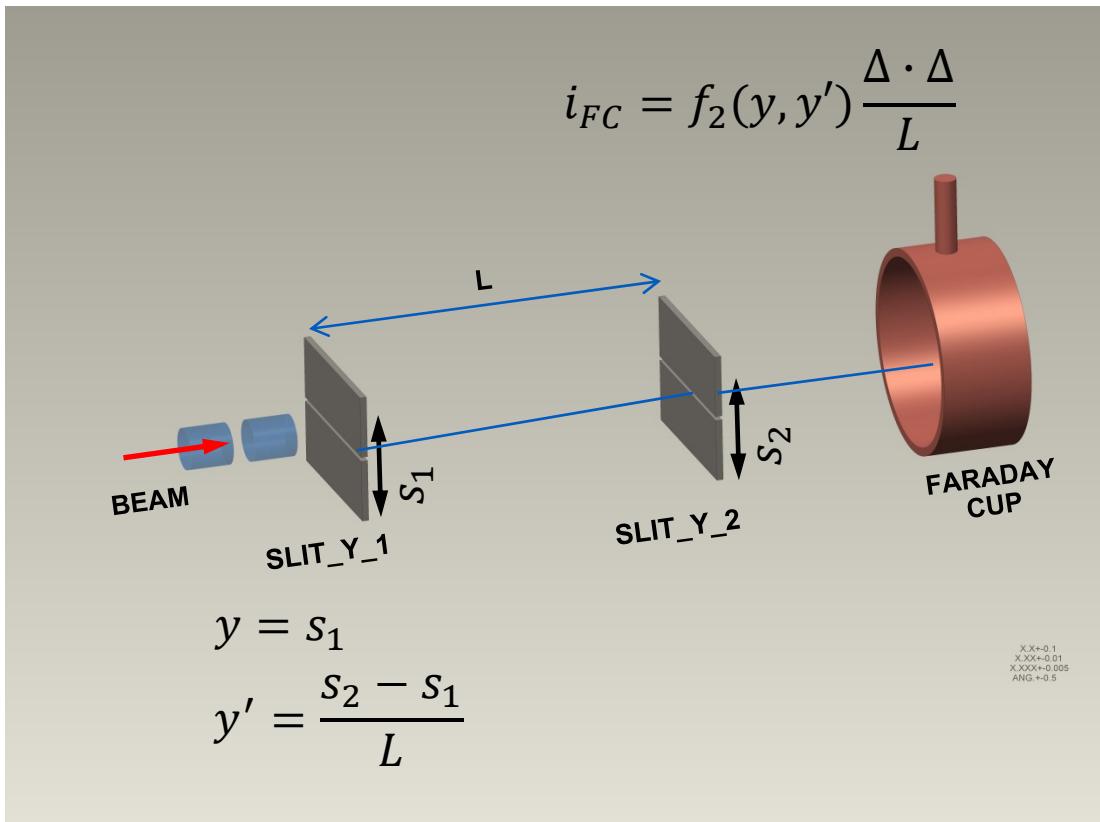


4D emittance measurement
techniques are well established

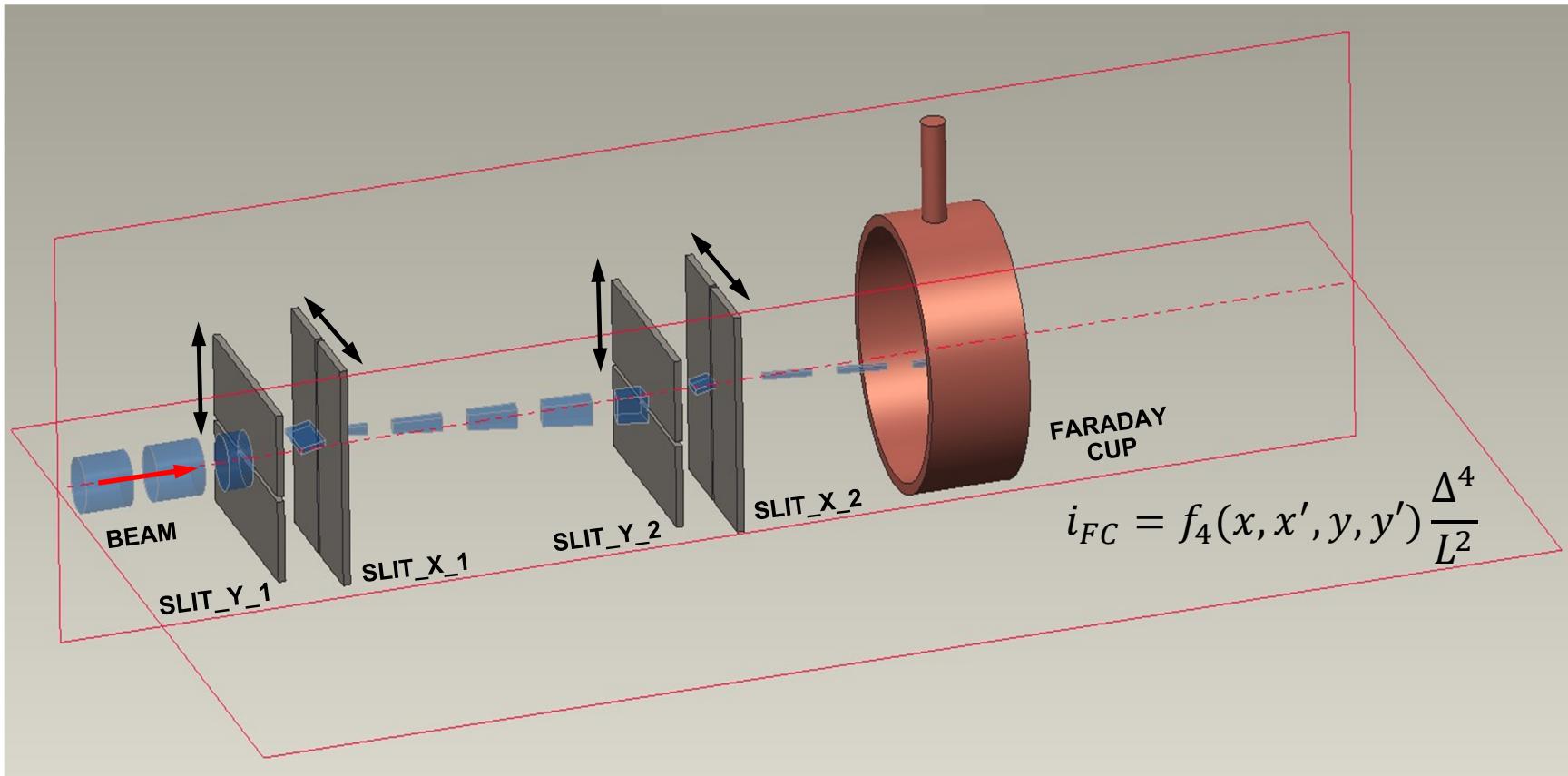
Can we measure 6D emittance?

Preferably, the distribution
function ?

2D distribution measurement (emittance) using slit-slit technique



4D distribution measurement using four slits arrangement

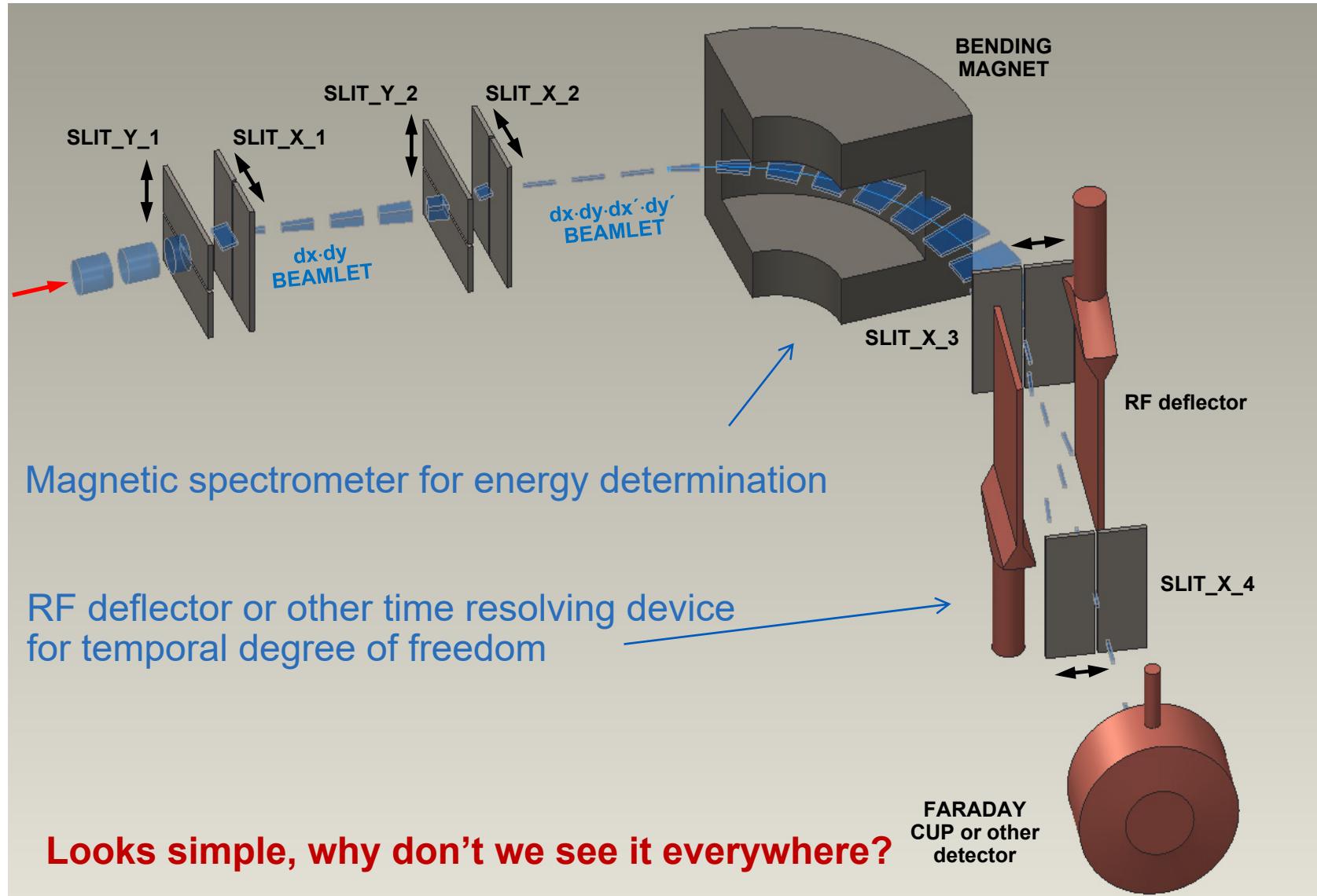


$$x = s_{x1}$$

$$y = s_{y1}$$

$$x' = \frac{s_{x2} - s_{x1}}{L} \quad y' = \frac{s_{y2} - s_{y1}}{L}$$

6D distribution measurement arrangement



“Curse of dimensionality” problem:

What looks simple in low-dimension problem can become
ridiculously difficult in higher dimensions

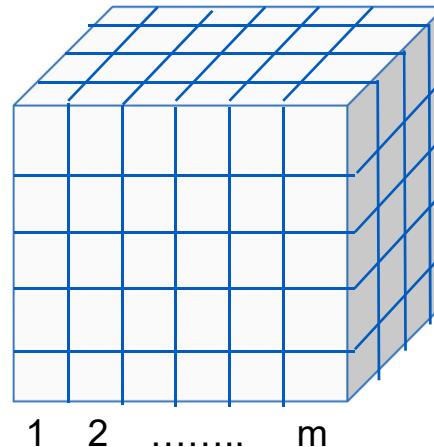
- High-dimensional spaces have very large volume: $V \sim a^D$
 - Large scan time
 - Low charge density
 - Large data sets

Scan time estimate

$$N_{bins} = m^D$$

dimensionality
Total number of bins
Number of steps per degree of freedom

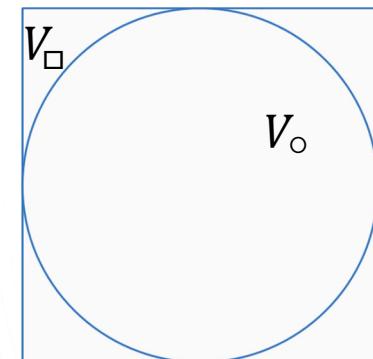
For $m = 10, D = 6 \quad N_{bins} = 10^6$



Total scan time at $1 \frac{\text{step}}{\text{sec}}$: $T_{total} = 10^6 \text{ sec} = 280 \text{ hours}$

Total scan time at $10 \frac{\text{step}}{\text{sec}}$: $T_{total} = 10^5 \text{ sec} = 28 \text{ hours}$

$$\frac{V_o}{V_\square} = \frac{\pi^{D/2}}{\Gamma(D/2 + 1)2^D} = \begin{cases} .79; & D = 2 \\ .52; & D = 3 \\ .081; & D = 6 \end{cases}$$



Tens of hours total scan time

Signal strength estimate

$$i = I_0 \cdot \frac{\exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y^2}{2\sigma_y^2} - \frac{y'^2}{2\sigma_{y'}^2} - \frac{w^2}{2\sigma_w^2} - \frac{\varphi^2}{2\sigma_\varphi^2}\right)}{8\pi^3} \frac{\Delta_x}{\sigma_x} \frac{\Delta_{x'}}{\sigma_{x'}} \frac{\Delta_y}{\sigma_y} \frac{\Delta_{y'}}{\sigma_{y'}} \frac{\Delta_w}{\sigma_w} \frac{\Delta_\varphi}{\sigma_\varphi} \approx \frac{\exp(\dots)}{8\pi^3} (\Delta/\sigma)^6$$

For $\Delta/\sigma \approx .2$ current after all 6 slits $i \approx I_0 \cdot 2.6 \cdot 10^{-7} \cdot \exp(\dots)$

Number of particles in $I_0 \approx 32 \text{ mA}$, $\tau \approx 50 \mu\text{s}$ beam pulse is $N_0 \approx 10^{13}$

Number of particles after 6 slits: $N_{FC} \approx 2.6 \cdot 10^6$ at the distribution center $r = 0$

$N_{FC} \approx 1.6 \cdot 10^6$ at $r = 2\sigma$

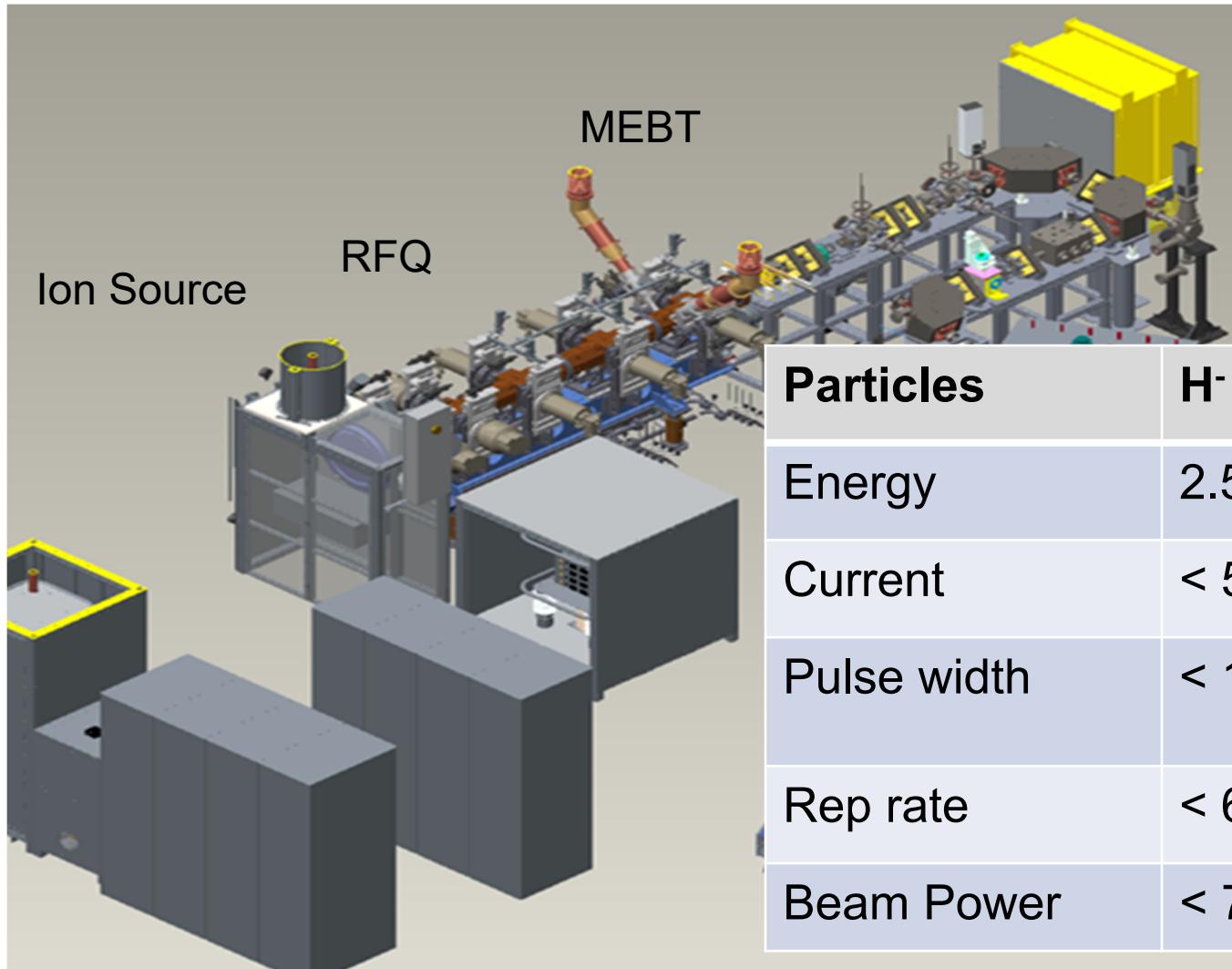
$N_{FC} \approx 2.9 \cdot 10^4$ at $r = 3\sigma$

$N_{FC} \approx 9.7$ at $r = 5\sigma$

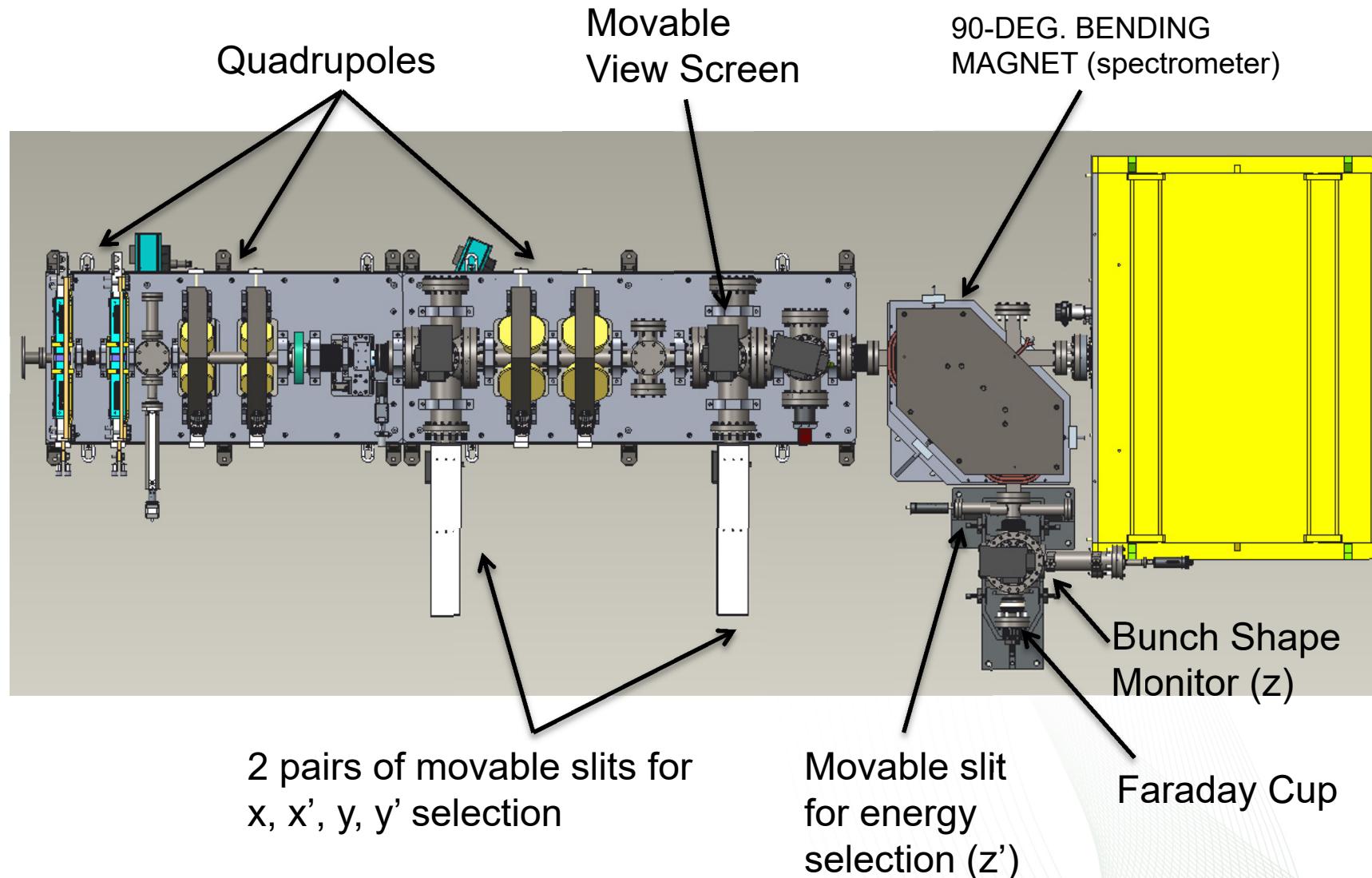
Many hours of beam time allocated
for single measurement
is big challenge for any large scale
accelerator facility

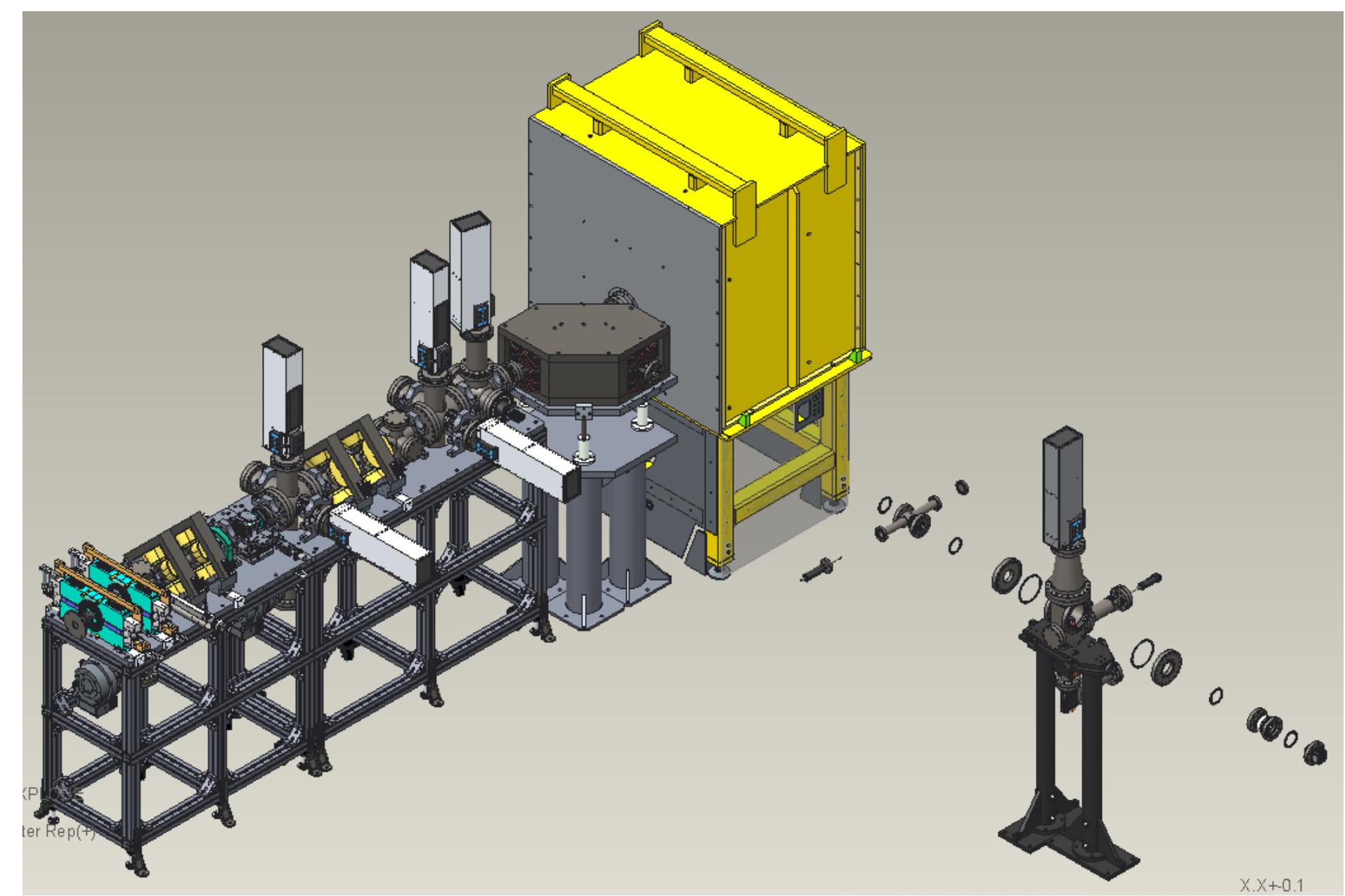
Only feasible at a dedicated facility

SNS Beam Test Facility (BTF)



SNS BTF set up for 6D phase space measurement

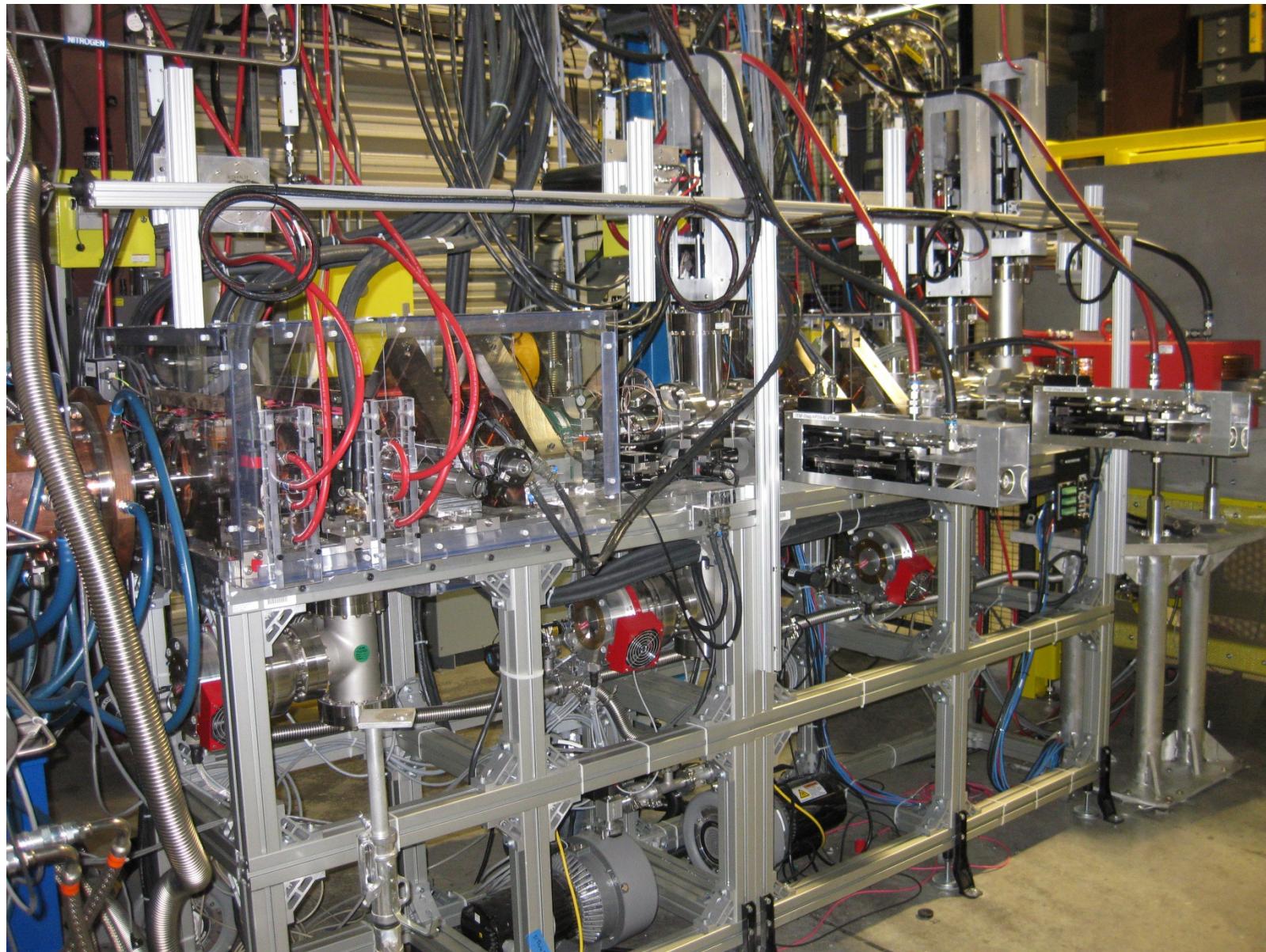




X.X+0.1

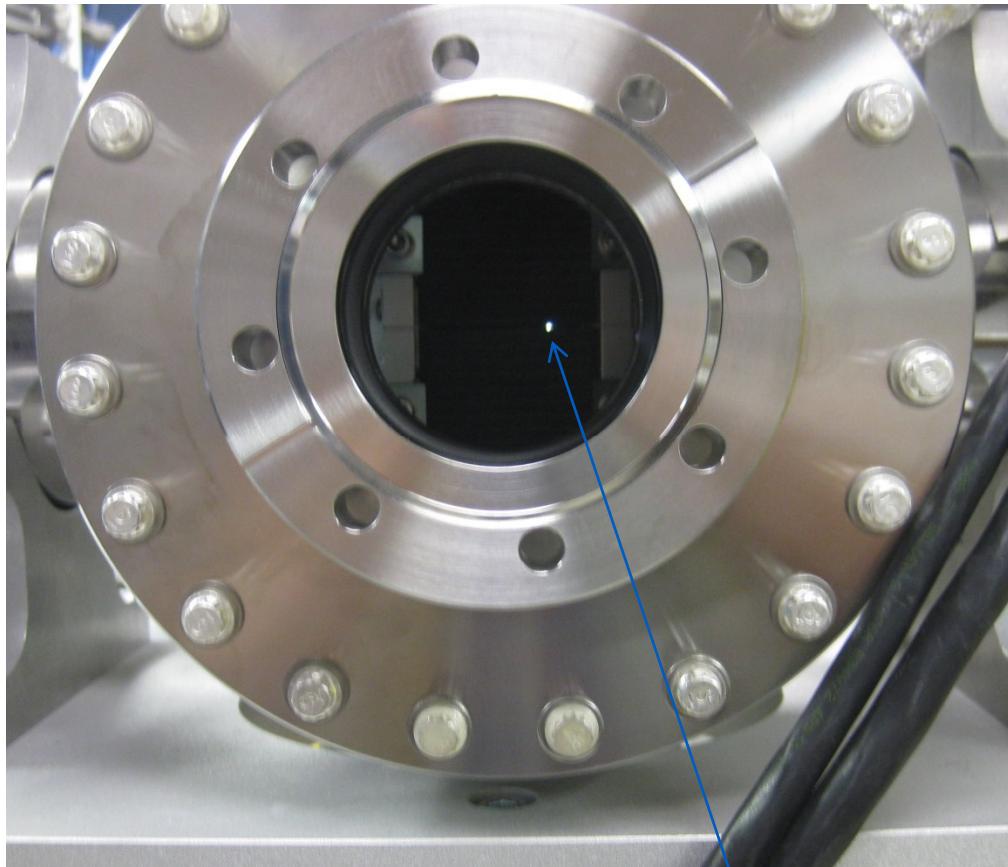
A. Aleksandrov

BTF MEBT

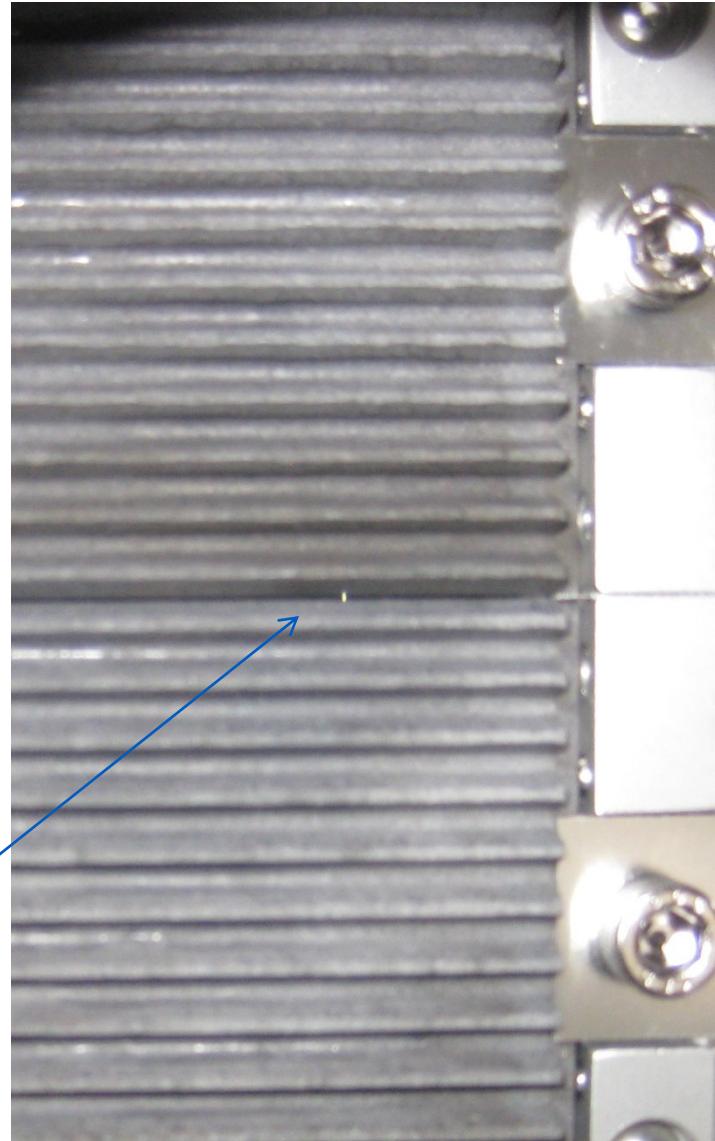


A. Aleksandrov

X-Y Slits arrangement



200-by-200 μm aperture



A. Aleksandrov

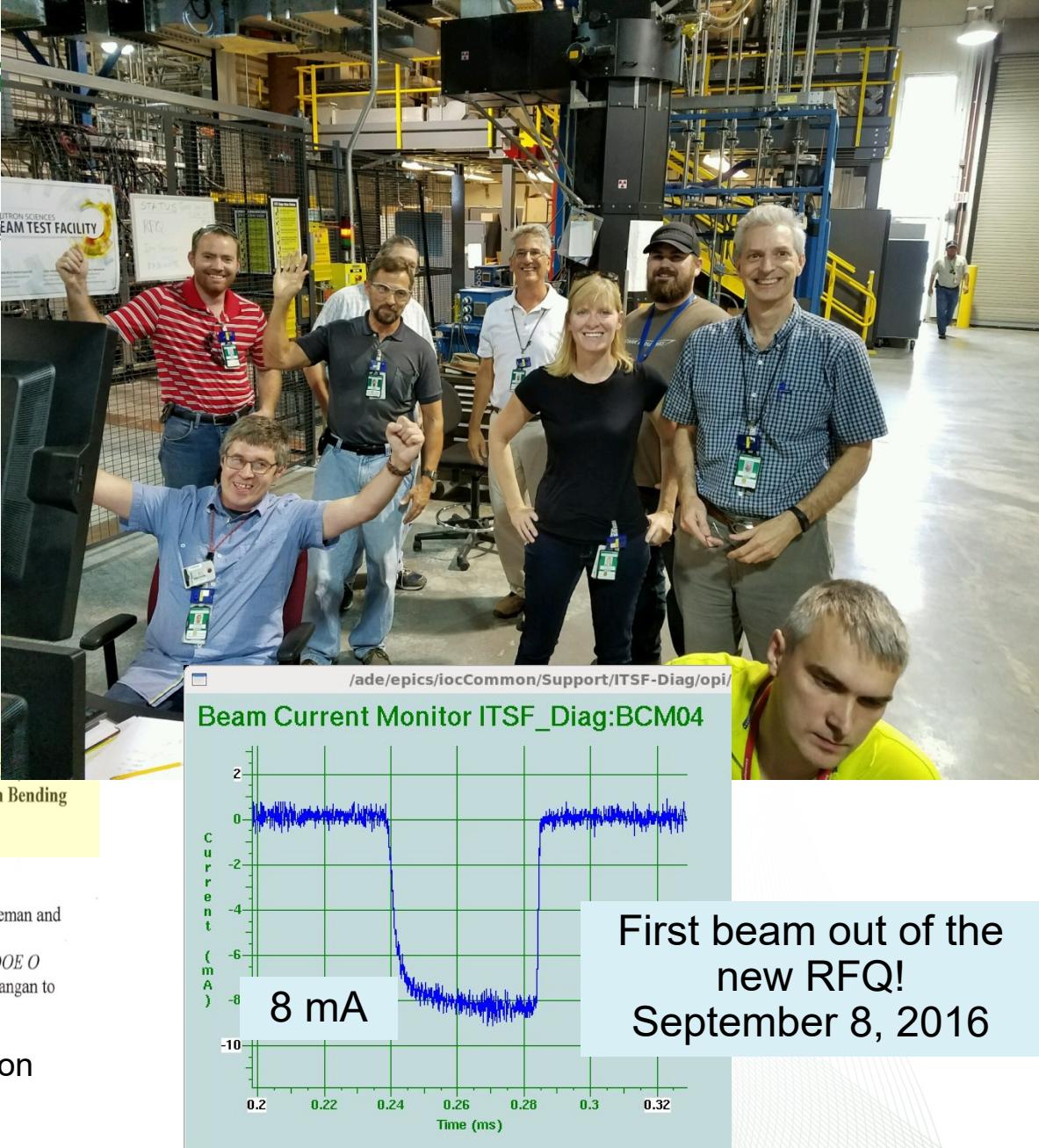
~~4D scan results~~ First



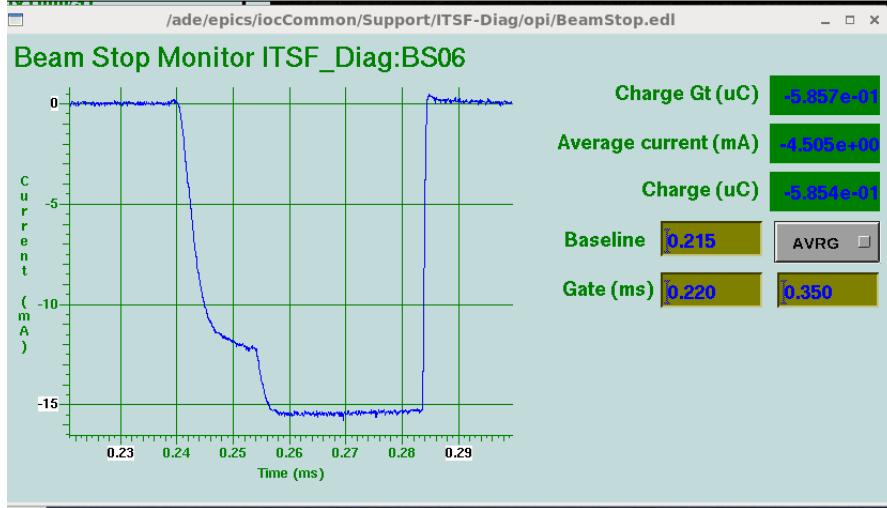
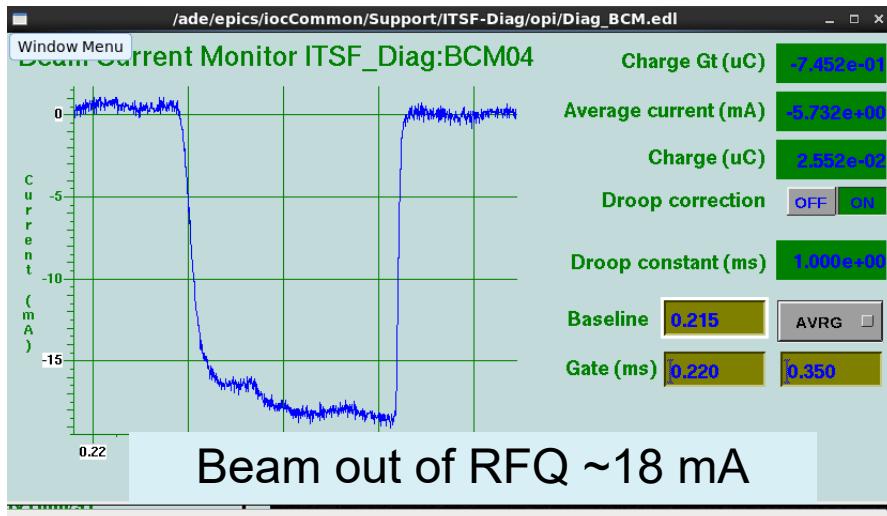
Date: September 6, 2016
Ref: NSCD-RAD-16-0001-R00
To: A. V. Aleksandrov
M. E. Middendorf
G. D. Johns
Cc: G. W. Dodson
S. M. Cousineau
D. E. Paul
M. J. Baumgartner
M. S. Champion
S. Kim
K. S. White
L. A. Longcoy
K. L. Mahoney
From: K. W. Jones
Subject: Authorization for Integrated Operation of the Beam Test Facility (BTF) Testing, RFQ Commissioning and Initial Physics Measurements with Bending Magnet Disabled
References:

1. "Safety Analysis for SNS Beam Test Facility," A. Aleksandrov, G. Dodson, D. Freeman and K. Jones, SNS-102030103-ES0059, July 18, 2016.
2. "Contract DE-AC05-00OR22725, Request for Exemption from the Provisions of DOE O 420.2C for the Spallation Neutron Source Beam Test Facility," Letter from Paul Langan to Johnny O. Moore, dated July 19, 2016.

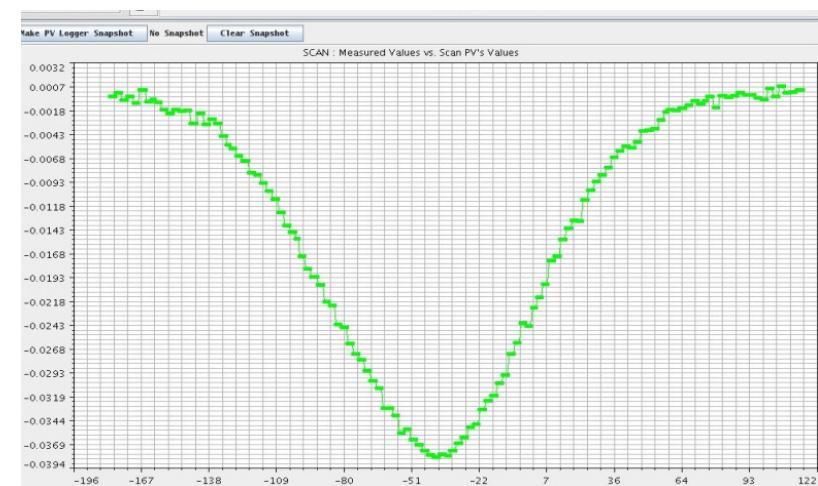
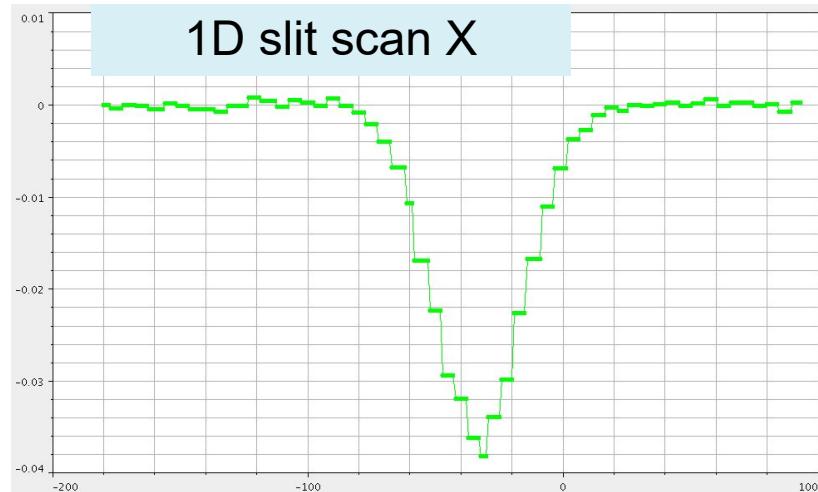
Received authorization to run the facility on September 6th 2016



September 9, 2016

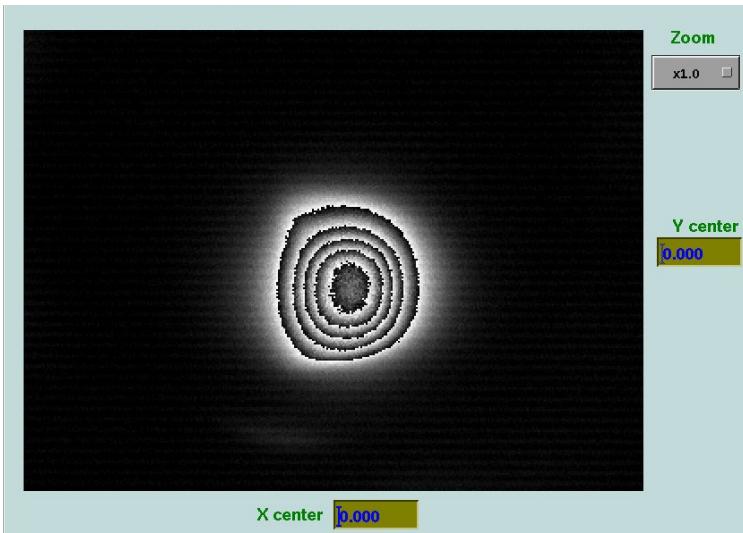


Beam to beam dump \sim 17.5 mA



1D slit scan Y

September 9, 2016



View screen image with 2 slits
inserted (1 horizontal, 1 vertical)



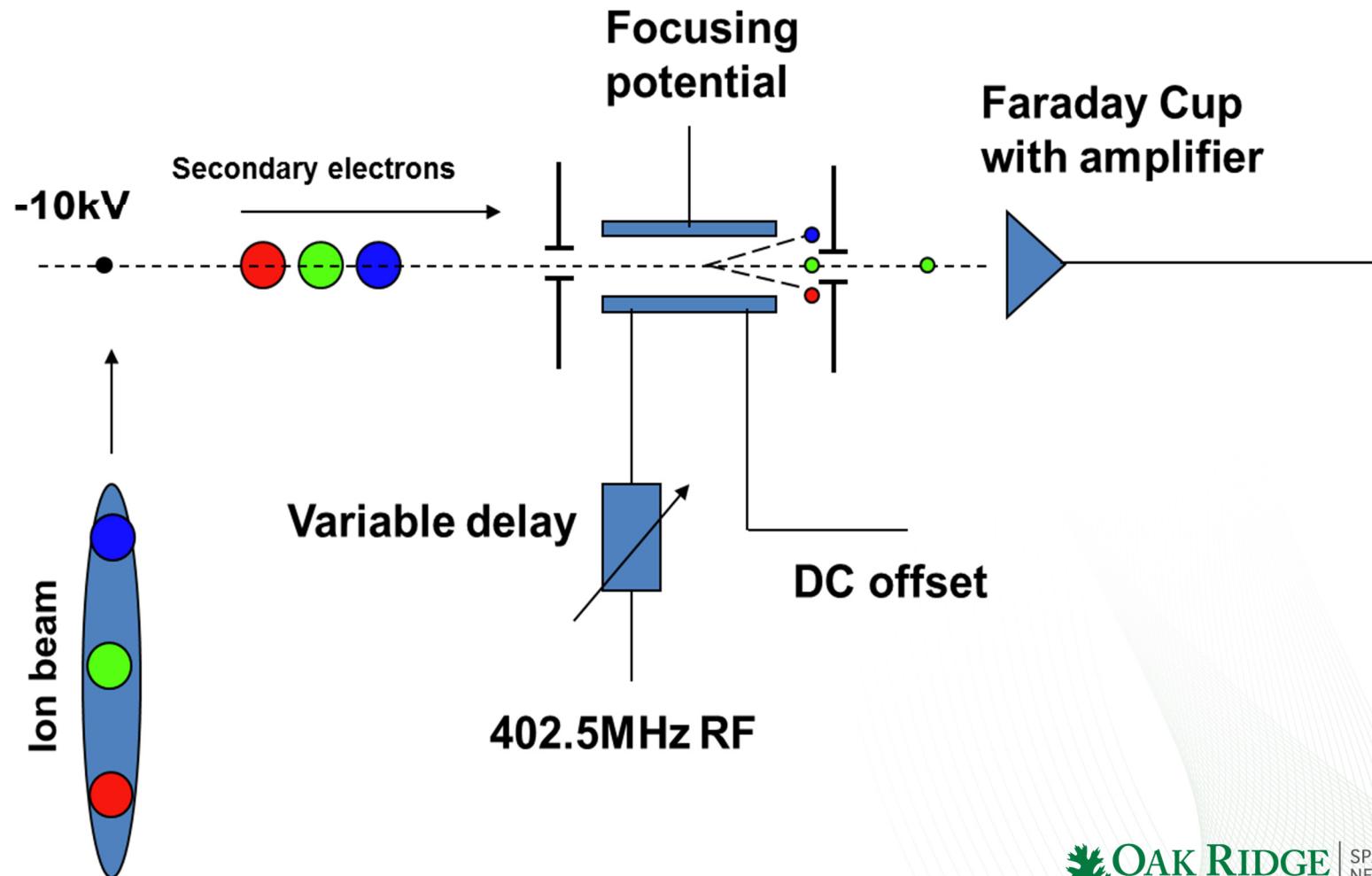
View screen image with 4 slits
inserted (2 horizontal, 2 vertical)

Equipment for 4D scan commissioned but we did not have time for full scan

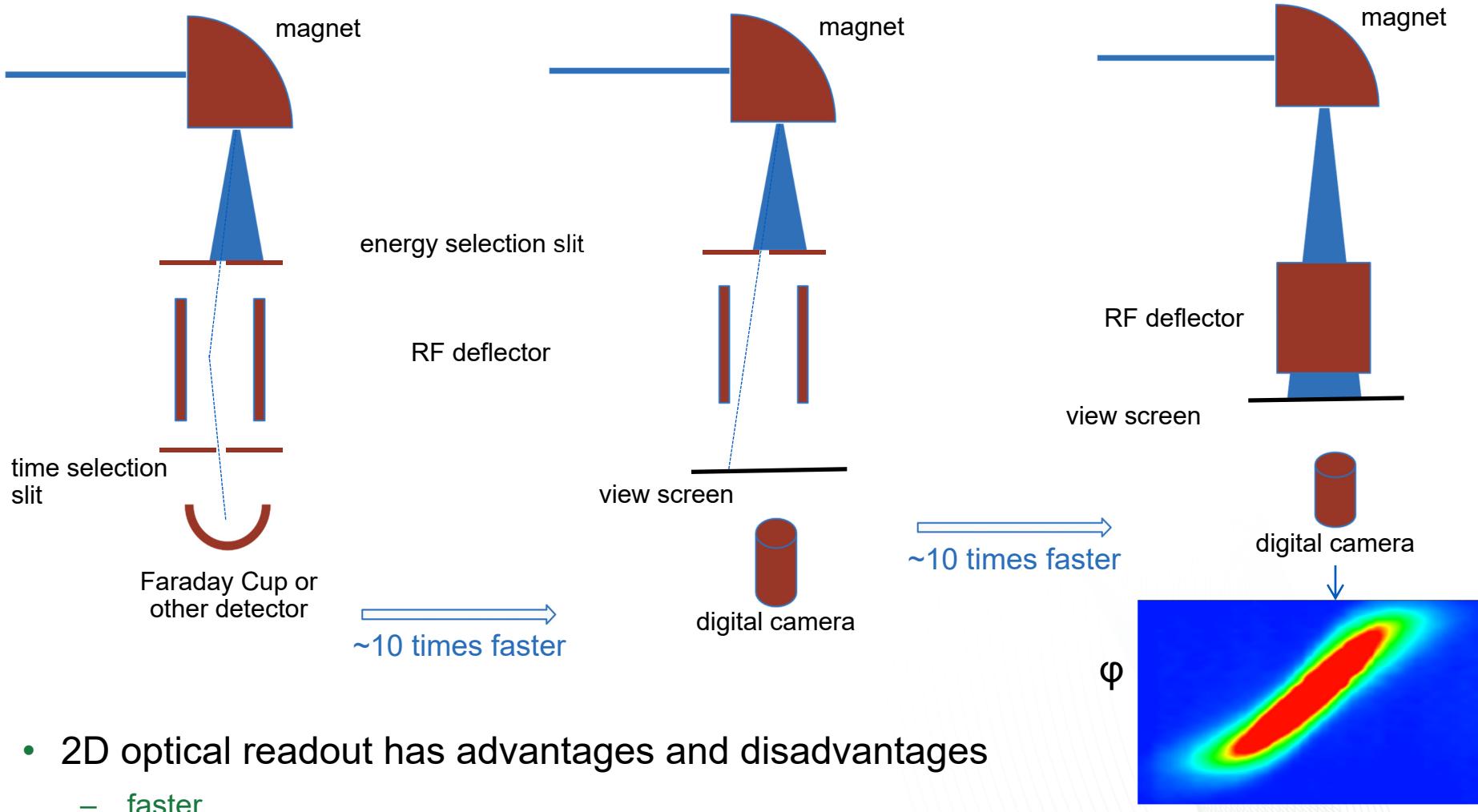
Longitudinal plane is next step

Bunch Shape Monitor principle of operation

Deflecting 2.5MeV proton beam directly with an RF cavity is expensive therefore we use Beam Shape Monitor aka “Feschenko monitor”

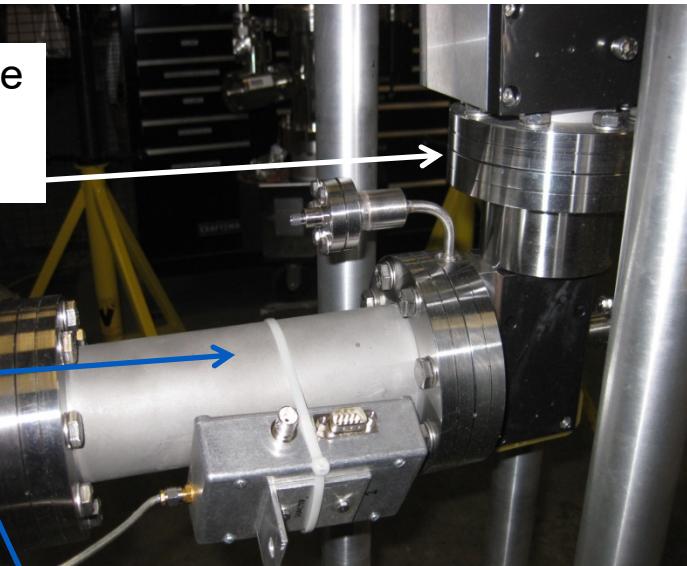


Speeding up scan in z-z' plane

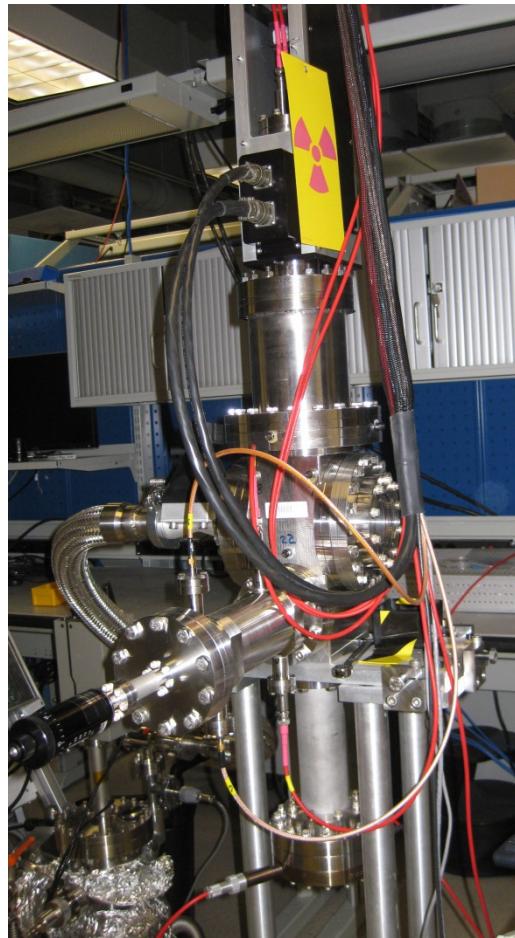
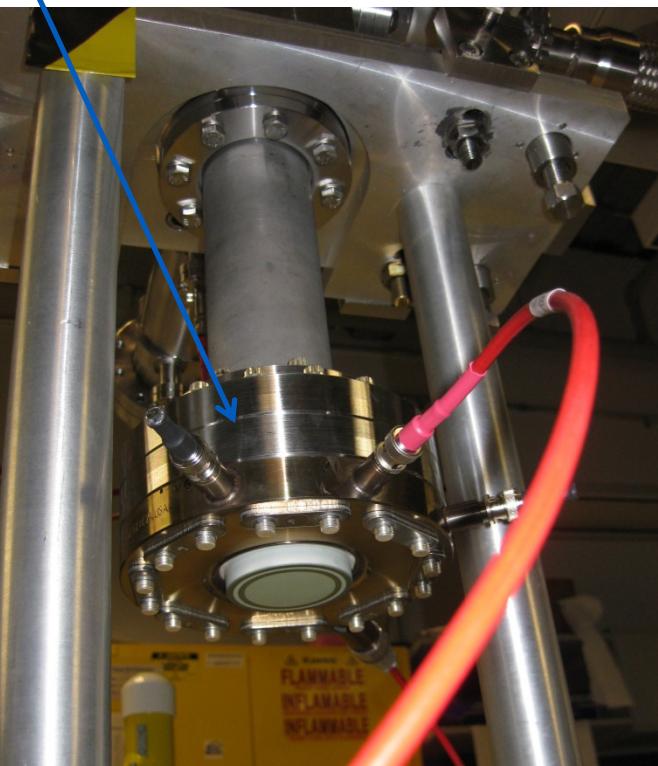


- 2D optical readout has advantages and disadvantages
 - faster
 - smaller dynamic range of charge-to-light-to-charge conversion

Slit mounted on this flange
to be replaced
with view screen



SEM and Faraday Cup
to be replaced
with digital camera

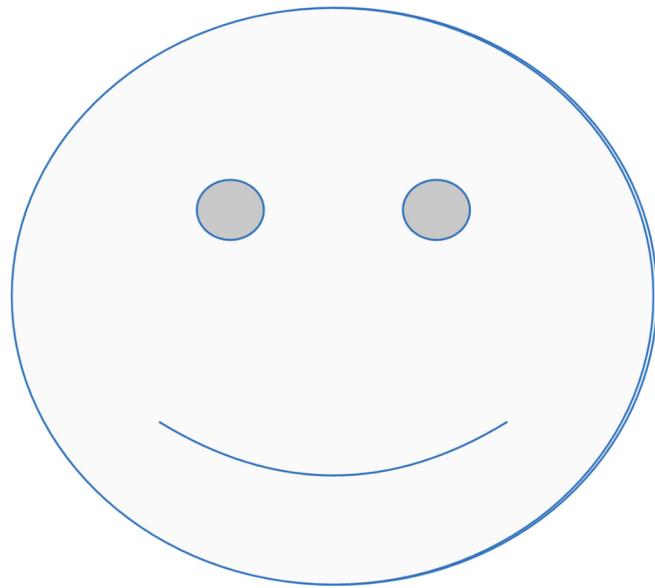


6D scan results



not there yet

6D scan results

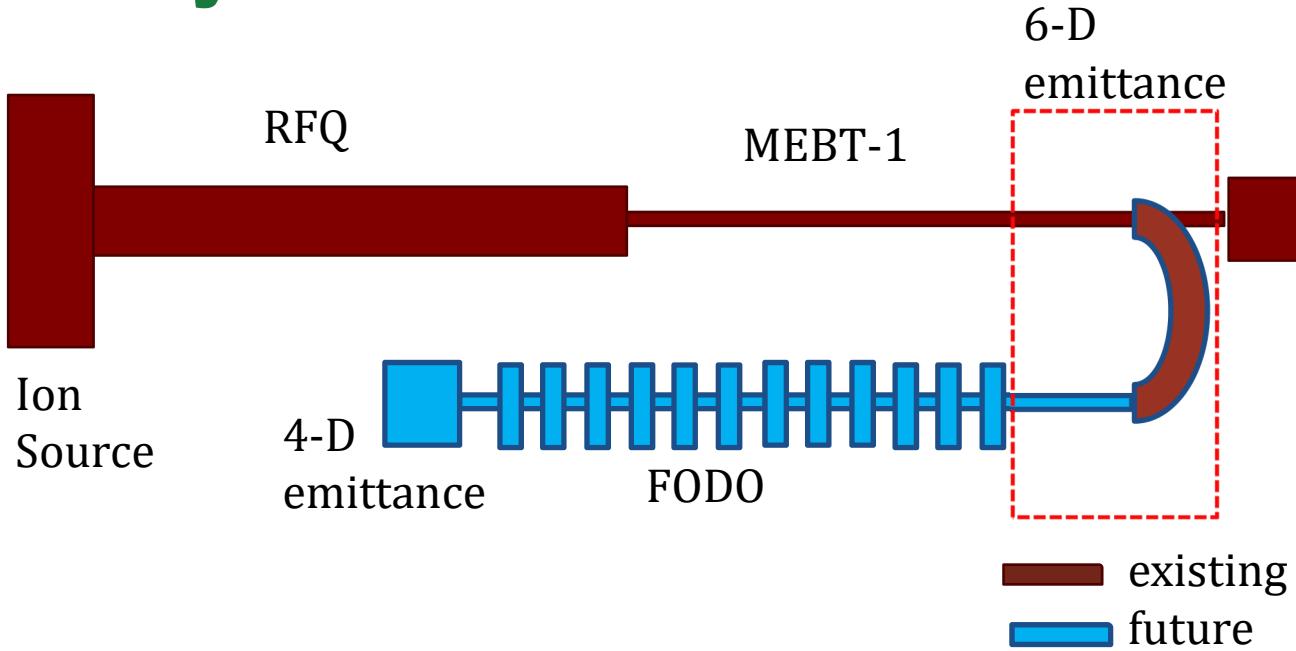


but will be soon

Near Term Research Goals

- Optimize 6D phase space measuring system for maximum resolution and dynamic range
- Develop algorithm for generating particle distributions for loading to PIC codes
- Search for high-dimensional correlations in the measured distribution
- Develop and verify methods of generating 6D distributions from low-dimensional projections
- Repeat LEDA beam dynamics experiment with newly developed diagnostics

Experimental study of halo formation in high intensity beam



- Experiment highlights:
 - Direct 6D phase space measurement.
 - Study halo formation in FODO structure.
 - Benchmark simulation codes

Thank you for your attention!