

Beam dynamics simulations for the FAIR SIS100 synchrotron

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FAIR main facts

a new facility for research with anti-protons and heavy-ions.



Primary ions: Protons-Uranium

Max. energy: 100 Tm

Max. beam intensity on target: $10^{11}/s$ (15 kW)

Beam intensity/quality limitations:

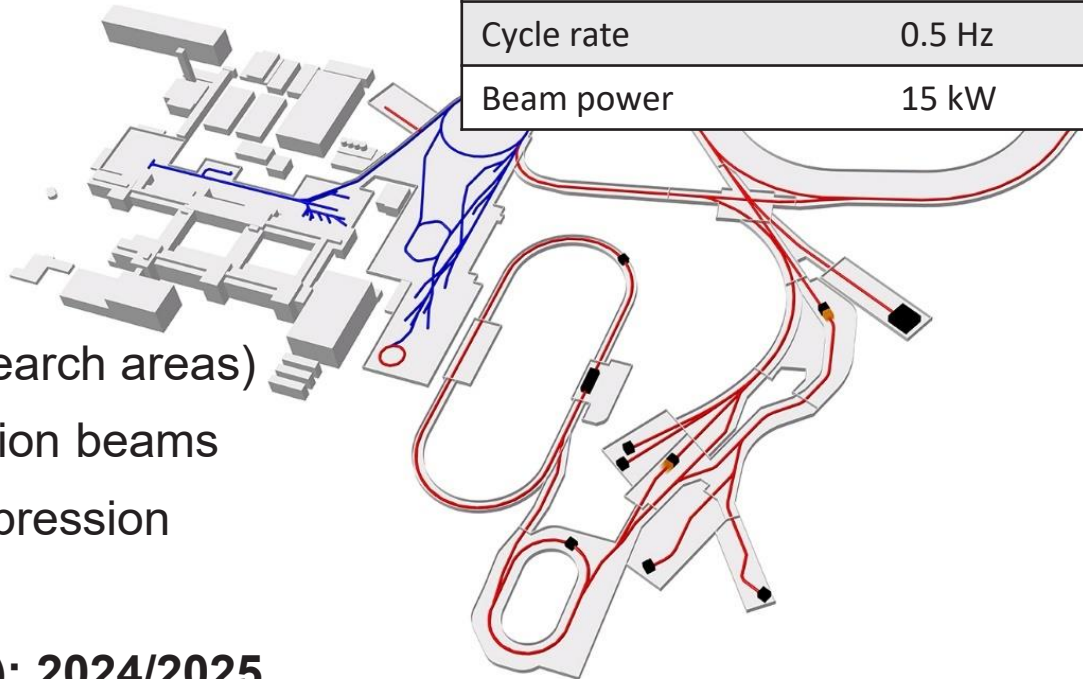
- Sources, injection (Liouville), cycle times
- space charge / resonances
- lifetime
- activation

Unique features:

- Parallel operation (serves 4 research areas)
- Intense and high-energy heavy-ion beams
- Slow extraction and bunch compression
- Storage rings and beam cooling

Start/end commissioning (day-1): 2024/2025

SIS-100	
Reference primary ion	U^{28+}
Reference energy	1.5 GeV/u
Ions per cycle	$3E11$
Bunch length	60 ns
Cycle rate	0.5 Hz
Beam power	15 kW



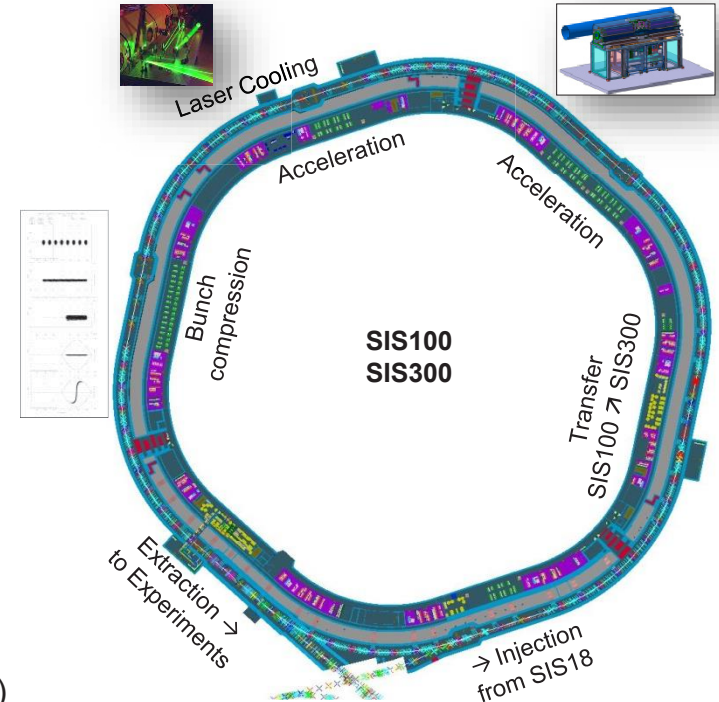


<https://youtu.be/wSN7jloV5nM>

The SIS100 synchrotron



Aerial photo of the construction site taken on May 25, 2014 (photo: Jan Schäfer for FAIR) (2014, before start of construction)

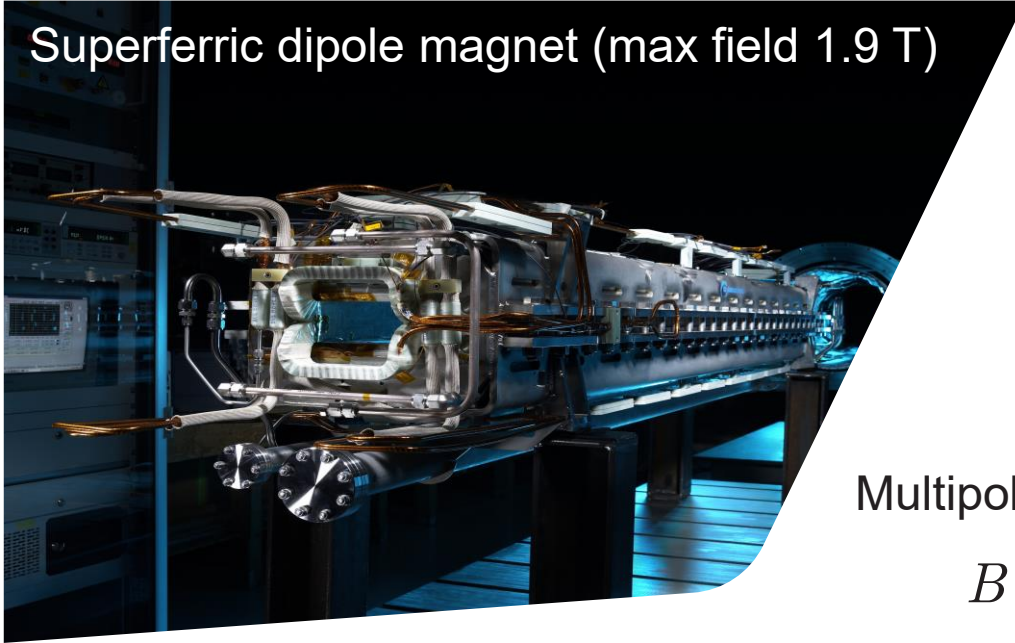


Images courtesy of M. Konradt / J. Falenski

- Circumference: 1 km
- Rigidity: 100 Tm
- Fast ramping superferric 'nuclotron' magnets (4 T/s)
- Cycle rates of up to 1 Hz (1 s accumulation after injection)
- Slow extraction (over seconds) or fast extraction (single compressed bunches)

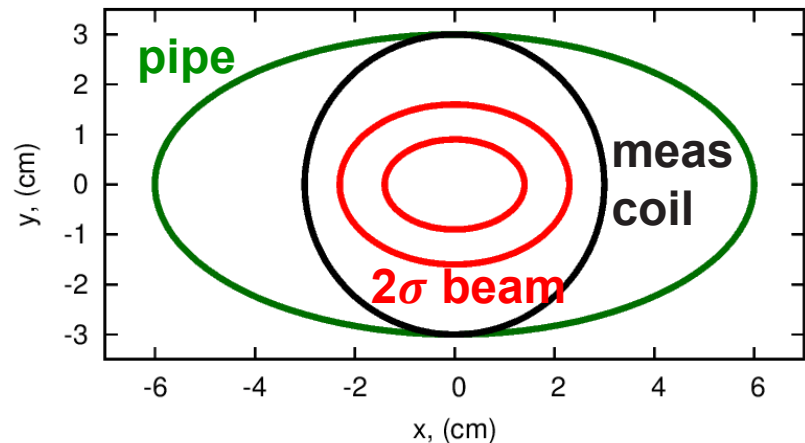
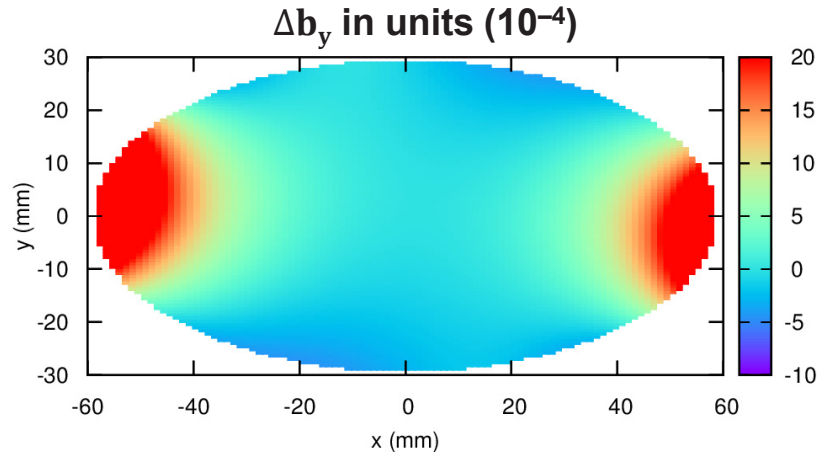
SIS100 Machine model and simulations

Superferric dipole magnet (max field 1.9 T)



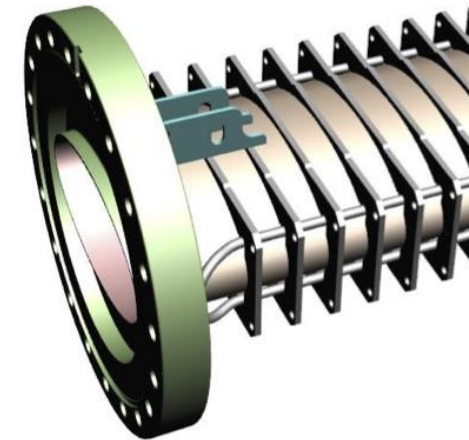
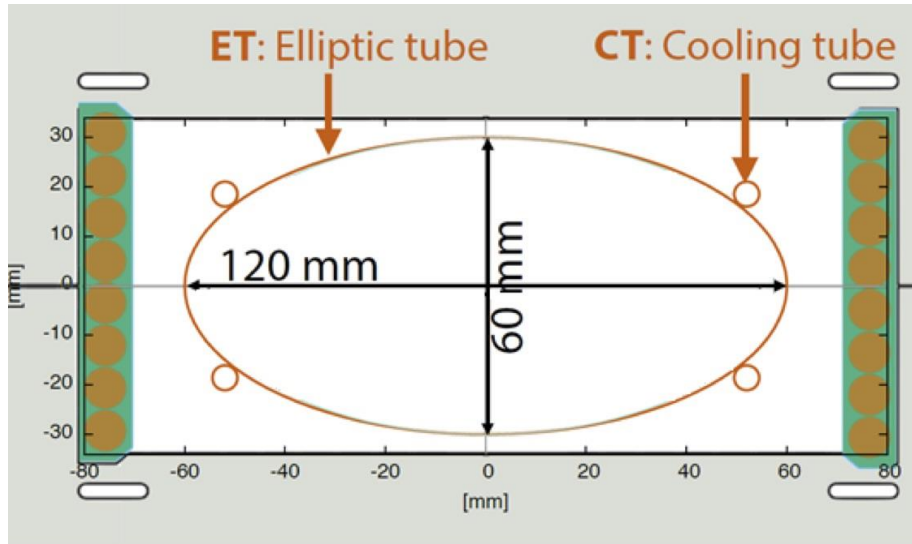
Multipoles from measurements with rotating coils

$$B(z) = B_y + iB_x = \sum_{n=1} C_n \left(\frac{z}{R_c} \right)^{n-1}$$



Dynamic effects: Eddy currents and beam pipe

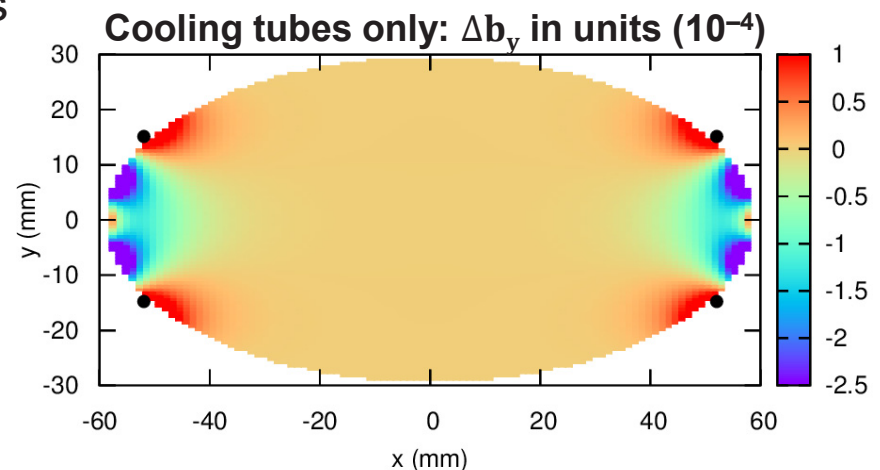
Cooling pipes to keep pipe wall at 10 K



Magnet field distortion due to cooling pipes during fast ramping (4 T/s)

The thin (0.3 mm) stainless steel pipe is also the main transverse impedance source for beam instabilities !

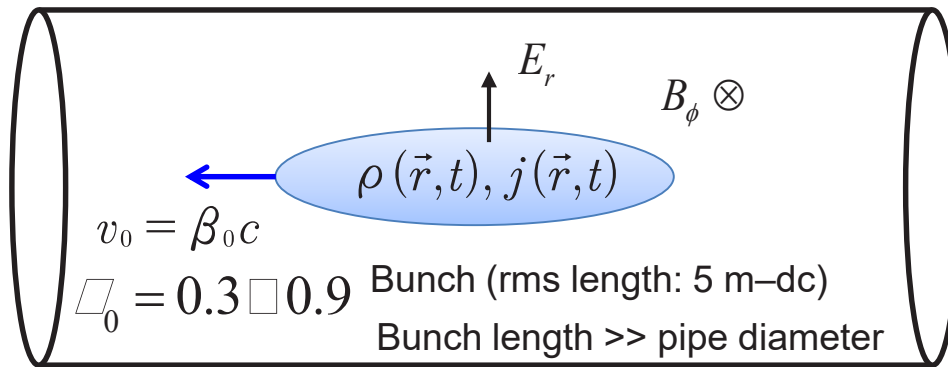
Impedance/instability/damping simulation studies are not part of this talk.



(Transverse) space charge force in SIS100

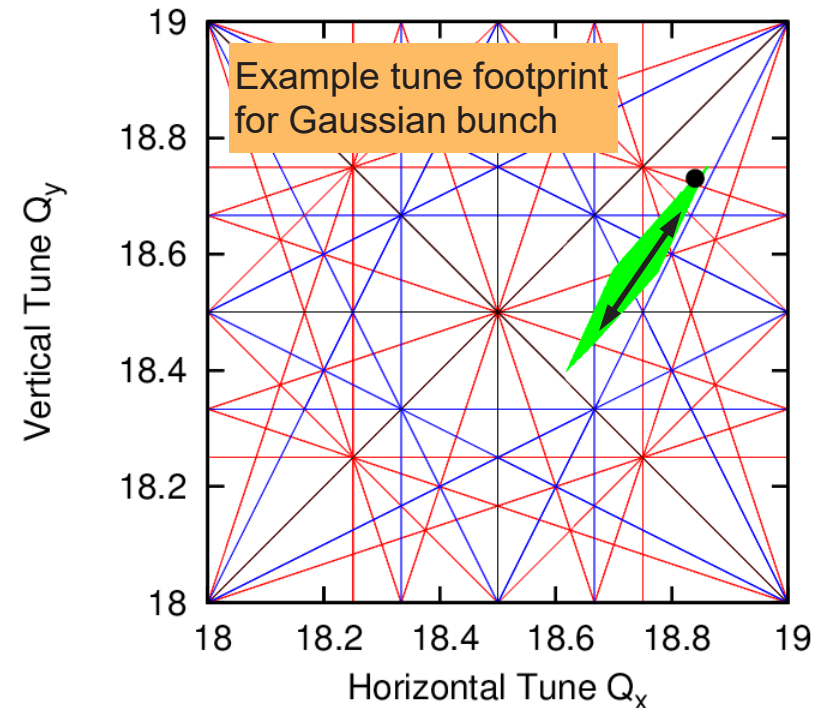
$$\epsilon_0 \nabla \cdot \vec{E} = \rho$$

(in the rest system of the beam)



The transverse space charge force is one of the main intensity limiting effect in in the FAIR synchrotrons !

Space charge tune shift: $\Delta Q_y^{sc} \propto -\frac{q^2}{m} \frac{N}{B_f} \frac{4}{\epsilon \beta_0^2 \gamma_0^3}$



Space charge tune shifts in SIS100: 0.2 – 0.4 (> 0.5 during bunch compression)

Time scales: 1000-10⁶ turns (1 ms - 1 s)

Tolerable emittance growth < 10 %, Beam loss (a few %)

Simulation challenge: Control numerical errors/emittance growth ! Performance !!!!

Particle Tracking Codes used for SIS100

Elegant (M. Borland)

- **3D static nonlinear space charge kicks**
- Elegant for parallel tracking
- For flexible, also for longitudinal tracking
- Script input

V. Kornilov (2018)

py-orbit

(A.Shishlo, S.Cousineau, J.Holmes, S.Appel)

<http://sourceforge.net/projects/py-orbit/>

- Teapot tracking
- **3D static space charge kicks**
- **2D/2.5D self-consistent space charge**
- MPI
- C++ sources / Python interface

Y. Yuan, O. Boine-F., I. Hofmann, PRAB 2018

pyPATRIC:

- 3D particle tracking with **self-consistent 2.5D space charge solvers**
- MADX maps, arbitrary rf bucket forms
- Automatized parameter scans.
- **python/numpy implementation**
- Optional: **gridless space charge solvers**

O. Boine-Frankenheimer, W. Stem, NIMA 2018

MAD-X (L. Deniau, F. Schmidt, et al.)

<https://github.com/MethodicalAcceleratorDesign/MAD-X>

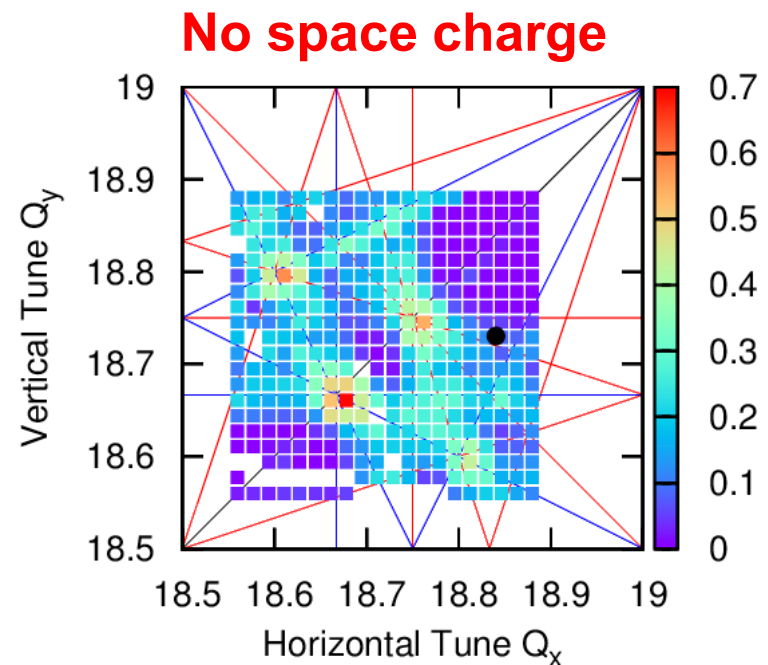
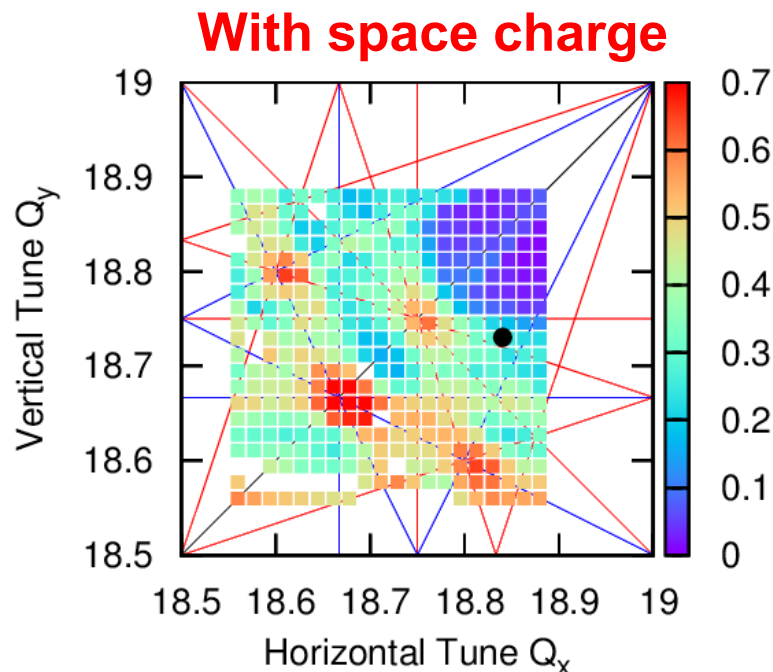
- Thin lens / PTC tracking
- **3D static space charge kicks**
- Fortran/C sources, Script input

V. Chetvertkova (2018)

Example: Tracking with static 3D space charge

- U^{28+} bunch at the injection energy (200 MeV/u) in SIS100
- Space-charge tune shifts: vertical $\Delta Q_{sc}=0.3$, horizontal $\Delta Q_{sc}=0.2$
- **Elegant:** 3D static “frozen” nonlinear space-charge kicks
- Field errors in the main dipole magnets and in the main quadrupole magnets.
- Beam loss after 20k turns (130ms)
- Lines: black (2nd order, quadrupole), blue (3rd order, sextupole), red (4th order, octupole)

V. Kornilov (2018)



Self-consistent (grid-based) tracking: PIC for beams

The present “production code” for beam quality/loss predictions.

$$\begin{array}{l}
 \begin{pmatrix} x_j \\ x'_j \\ y_j \\ y'_j \end{pmatrix}_{n+1} \overset{\substack{\text{integration step} \\ \text{(4D, for simplicity)}}}{=} \underset{\substack{\mathcal{M}(s_n, s_{n+1}) \\ \text{(map/kicks from} \\ \text{PTC or Teapot)}}}{\mathcal{M}(s_n, s_{n+1})} \begin{pmatrix} x_j \\ x'_j + \Delta x'_j \\ y_j \\ y'_j + \Delta y'_j \end{pmatrix}_n
 \end{array}
 \quad
 \begin{array}{l}
 \rho(x, y, s) = Q' \sum_j^M S(x - x_j) \\
 \text{(favorite interpolation scheme)} \\
 \epsilon_0 \nabla \cdot \mathbf{E} = \rho(x, y, s) \\
 \text{(favorite Poisson solver)} \\
 x'' = \frac{qE_x}{m\beta_0^2 c^2 \gamma_0^3} \\
 \text{(space charge kicks)}
 \end{array}
 \quad
 \begin{array}{l}
 Q' = \frac{Q}{L} \\
 \text{(macro particle} \\
 \text{charge)}
 \end{array}$$

Artificial emittance growth depending on the ratio of real N to macro-particles M

$$\text{Growth rate: } \nu \propto \frac{N^2}{M} \quad \text{For example: Boine-Frankenheim, Hofmann, Struckmeier, Appel, NIM A (2015)}$$

Because of noise and performance issues „frozen“ or „adaptive“ sc kicks are used.

However, this might be justified only for weak space charge and Gaussian distributions !
(Example: Bunch compression with strong space charge in SIS100)

(Fast) gridless space charge solvers

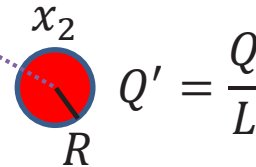
$$F_1 = \sum_{j=1}^M F_{12}$$

(sum over all macroparticles)



$$F(x_1) = \frac{qQ'(x_1 - x_2)}{2\pi\epsilon_0\gamma^2 |x_1 - x_2|^2}$$

Direct particle-macroparticle force



Potential: $\phi(x_1) = \frac{Q'}{2\pi\epsilon_0\gamma^2} \sum_{j=1}^M \ln |x_2 - x_1|$

Smoothed „cloud“ macroparticles:

$$F_{12} = \begin{cases} \frac{qQ'(x_2 - x_1)}{2\pi\epsilon_0\gamma^2 |x_2 - x_1|^2} & |x_2 - x_1| > R \\ \frac{qQ'}{2\pi\epsilon_0\gamma^2 R^2} (x_2 - x_1) & |x_2 - x_1| \leq R \end{cases}$$



Advantages:

- Underlying (multi-particle) Hamiltonian
- No ‚grid heating‘, but (smooth) ‚collisions‘
- Controlled noise smoothing (shapes) !
- Cylindrical pipe with image charges.
- Fast Multipole Method

Disadvantages:

- Complex pipe boundaries

Greengard and Rokhlin, *A Fast Algorithm for Particle Simulations*, J. Comput. Phys. (1997)

Zhang and Berz, *The fast multipole method in the differential algebra framework*, Nucl. Instr. Meth. A (2011)

FMMLIB2D: Gimbutas and Greengard, *Simple FMM Libraries for Electrostatics ...*, Comm. Comput. Phys. (2015)

Tests: (Single-particle) Symplecticity

R. Ruth, A Canonical Integration Technique, 1983; E. Forrest, Geometric Integration for Particle Accelerators, 2006

$$\mathbf{x}_2 = M_{1,2}(\mathbf{x}_1)$$

Mapping of particle coordinates
from position "1" to position "2"

$$M^T S M = S$$

Symplecticity condition
M: Jacobian or transport matrix

$$S_{2D} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Symplectic matrix

Symplectic error [2]: $\|M^T S M - S\| = \eta$

$$\begin{pmatrix} x_j \\ x'_j \\ y_j \\ y'_j \end{pmatrix}_{n+1} = M(s_n, s_{n+1}) \begin{pmatrix} x_j \\ x'_j + \Delta x'_j \\ y_j \\ y'_j + \Delta y'_j \end{pmatrix}_n$$

(sector map without space charge)

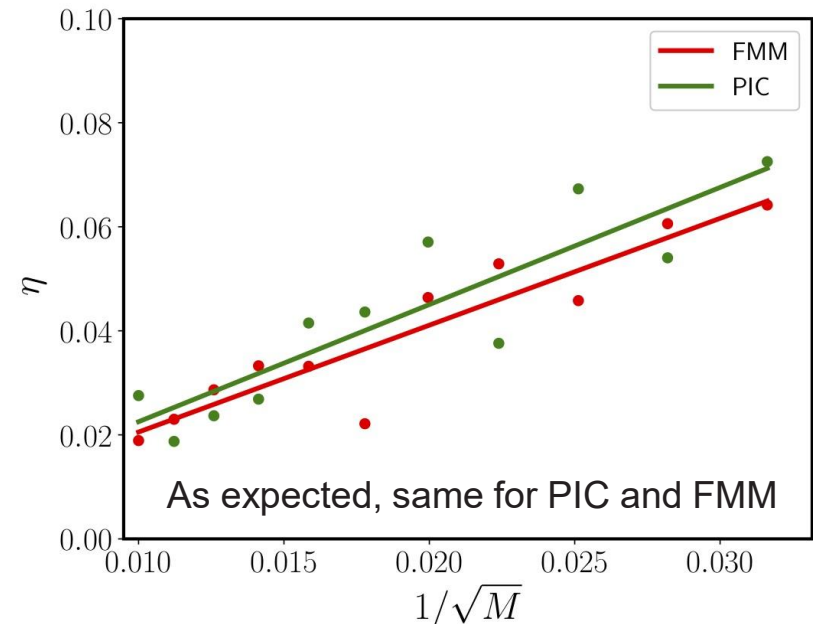
$$x_j^{n+1} = \sum_{i=0}^6 M_{j,i} x_i^n \quad j=1,6, \quad n=N, N+5 \Rightarrow M_j$$

Reconstruction of the individual particle
transport matrix M for one cell with space charge [1]

[1] A. Luccio, N. D'Imperio,
Eigenvalues of the One-Turn matrix, BNL (2003)

[2] M. Titze, ICFA-HB 2016

Symplectic error for FODO (PIC vs. FMM)



How to test multi-particle
symplecticity in a tracking code ?

Conclusions

- The FAIR SIS100 construction is progressing ! The focus of beam dynamics simulations is now on the characterization of the magnets and the identification of optimum parameter windows for high-intensity operation.
- Also for the purpose of benchmarking, several codes are employed, with different tracking implementations and space charge models/solvers.
- Self-consistent space charge solvers are required for realistic predictions ! At present they are employed only for short-term simulations ($< 10k$ turns) because of performance/noise issues.
- Gridless space charge solvers based on the Fast Multipole Method are very promising in terms of flexibility and performance for 2.5D or 3D tracking with self-consistent space charge.
- Gridless solvers are „closer“ to a Hamiltonian multi-particle system for which fluctuations follow the „well-known“ IBS theory. Therefore the numerical errors might be easier to predict and to control (cloud shapes).
- More difficult to add complex beam pipe geometries for image contributions !