Heavy-ion Cyclotron Gymnastics and their Beam Dynamics Issues

Contents

- History
- Uses of heavy-ion cyclotrons
- Basic features of cyclotrons
- Intensity-limiting factors of heavy-ion cyclotrons
- Space charge effects in cyclotrons
  "from Gordon to neighboring turn effects"

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The Beginning

Livingston compiled the earliest heavy ion accelerators in his review article “A survey of methods of accelerating heavy ions”.

Livingston, 5th Int. Cyclotron Conf., 423.

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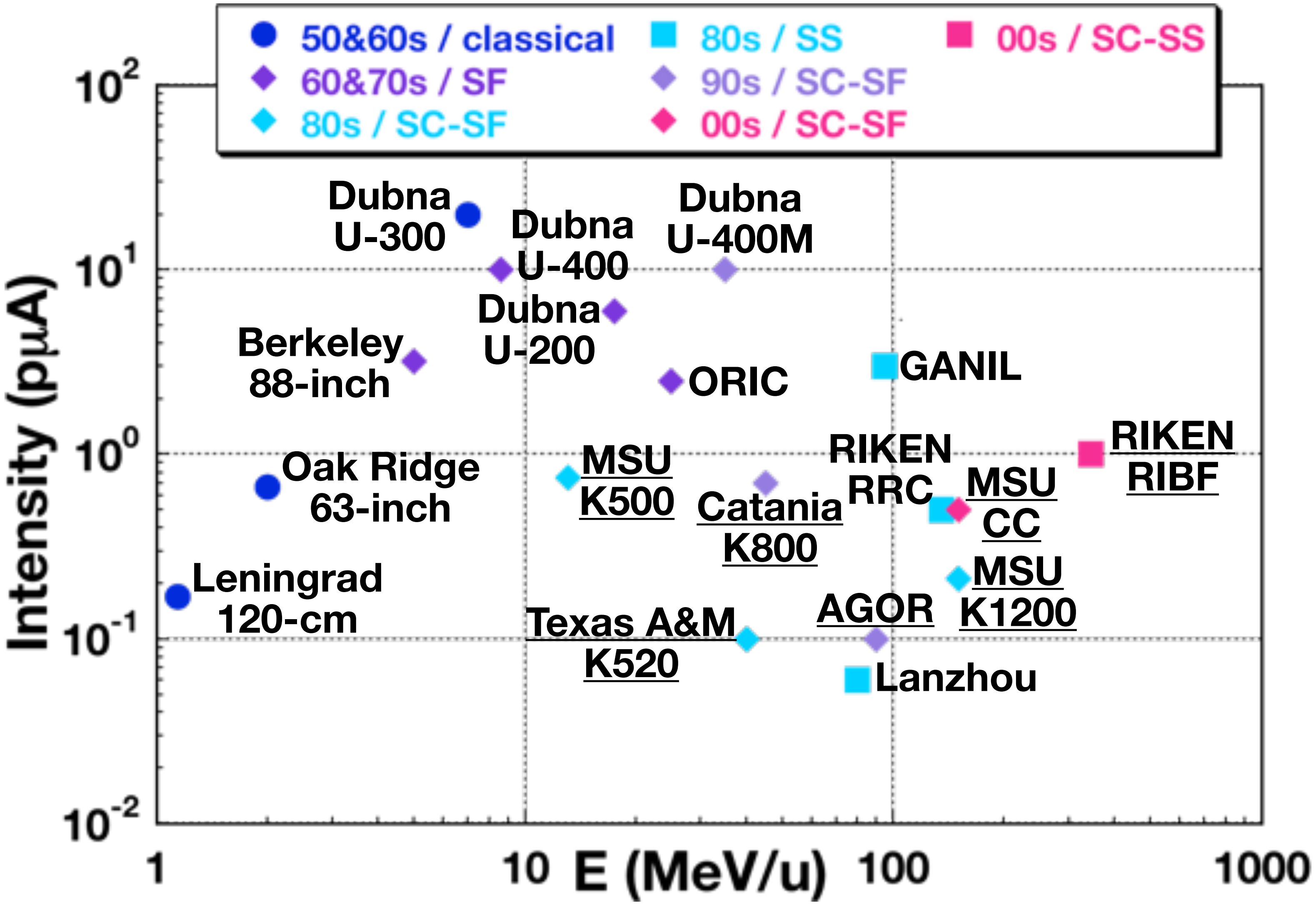
Table 1. EARLIEST HEAVY ION ACCELERATORS

Date	*Machine*	*Location*	*Typical particle*	*Energy*	*Extracted beam*
1940	37-in. Cyclotron	Berkeley	$^{12}C^{2+,6+}$	50 MeV	8/s*
1950	60-in. Cyclotron	Berkeley	$^{12}C^{2+,6+}$	100 MeV	10^5/s
1953	225-cm Cyclotron	Stockholm	$^{12}C^{2+,4+}$	130 MeV	10^{11}/s*
1953	63-in. Cyclotron	Oak Ridge	$^{14}N^{3+}$	28 MeV	2μA
1953	156-cm Cyclotron	Birmingham	$^{12}C^{2+,6+}$	120 MeV	
1955	180-cm Cyclotron	Saclay	$^{12}C^{2+,6+}$		
1956	120-cm Cyclotron	Leningrad	$^{14}N^{3+}$	16 MeV	0·5 μA

* Internal beam

Developments of Heavy-ion Cyclotrons

Heavy-ion cyclotrons have pioneered the energy frontier of CW accelerators.



classical : classical cyclotron
SF : Sector Focusing cyclotron
SS : Separate Sector cyclotron

Underline : superconducting magnet

Oak Ridge 63-inch, Reningrad 120-cm, Dubna U-300 : data from Livingston's review
(Proc. Int. Cyclo. Conf. (1969) CYC69E01)

Others from "List of Cyclotrons"
(compiled in Cyclotrons and their Applications 2004, edited by Y. Yano and A. Goto)

Various Uses of Heavy-ion Cyclotrons

Nuclear Physics

Synthesis of superheavy elements @ JINR FLNR

Physics of unstable nuclei → GANIL, MSU, RIKEN, RCNP, HIRFL, Catania, JINR etc.

→ Beam intensity becomes a crucial issue for heavy-ion cyclotrons

Material Science

Material modification

Analysis (PIXE, RBS etc.)

→ *for example, see A. Denker et al., NIM B 240 (2005) 61.*

Medical Science, Biology, Agriculture

Cancer therapy → IBA C400 *(Y. Jongen et al., NIM A 624 (2010) 47.)*

Isotope production

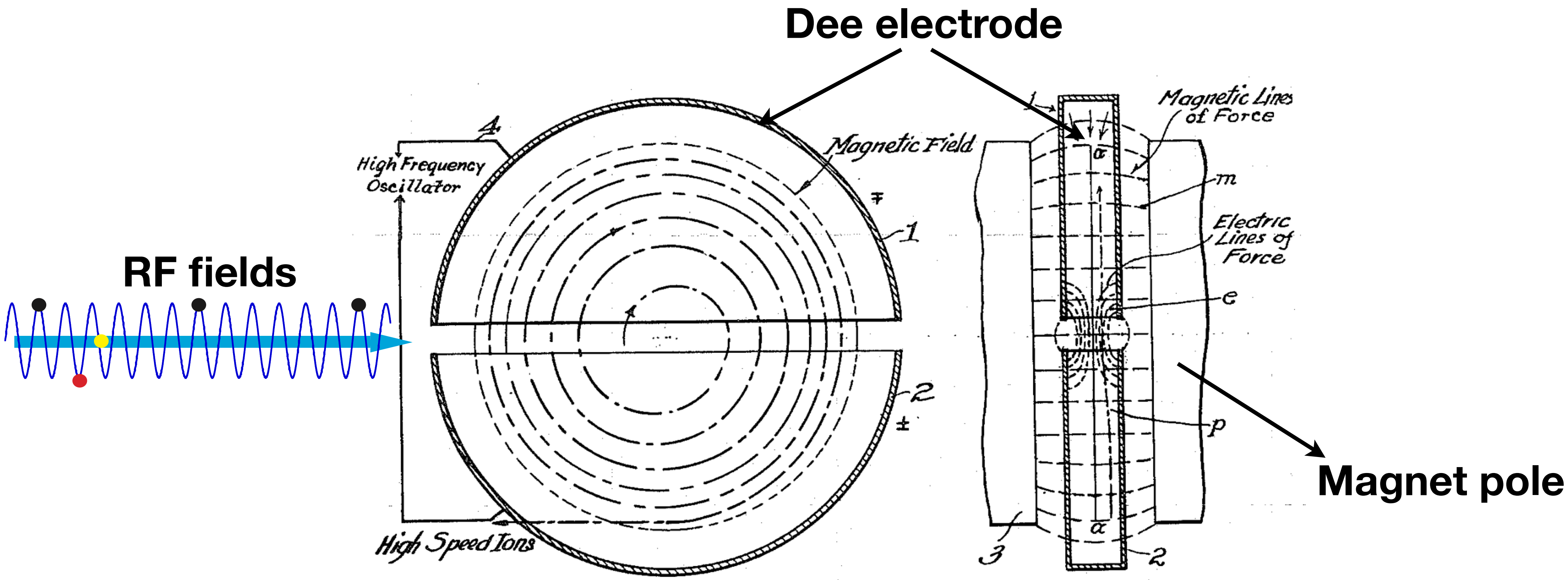
Heavy-ion induced mutation → RIKEN, JAERI Takasaki

Microbeam → JAERI Takasaki

Useful information is available, for example, from

- *L. M. Onishchenko, "Cyclotrons: A Survey", Phys. of Part. Nuclei 39, No.6 pp 950-979, (2008).*
- *A. Goto, "Review of High-power Cyclotrons for Heavy-ion Beams", 19th Int. Cyclotron Conf. (2010) 9.*

Classical Cyclotron



E. O. Lawrence and N. E. Edlefsen, Science, 72 (1930) 376.

Approximate isochronism and vertical focusing

$$\nu_z^2 = -n, \quad n : \text{field index}$$

$$B = \gamma B_0 \Leftrightarrow \nu_z^2 = -\beta^2\gamma^2$$

Energy limit → E_{max} ~ 12 MeV with V_{acc} = 50 kV

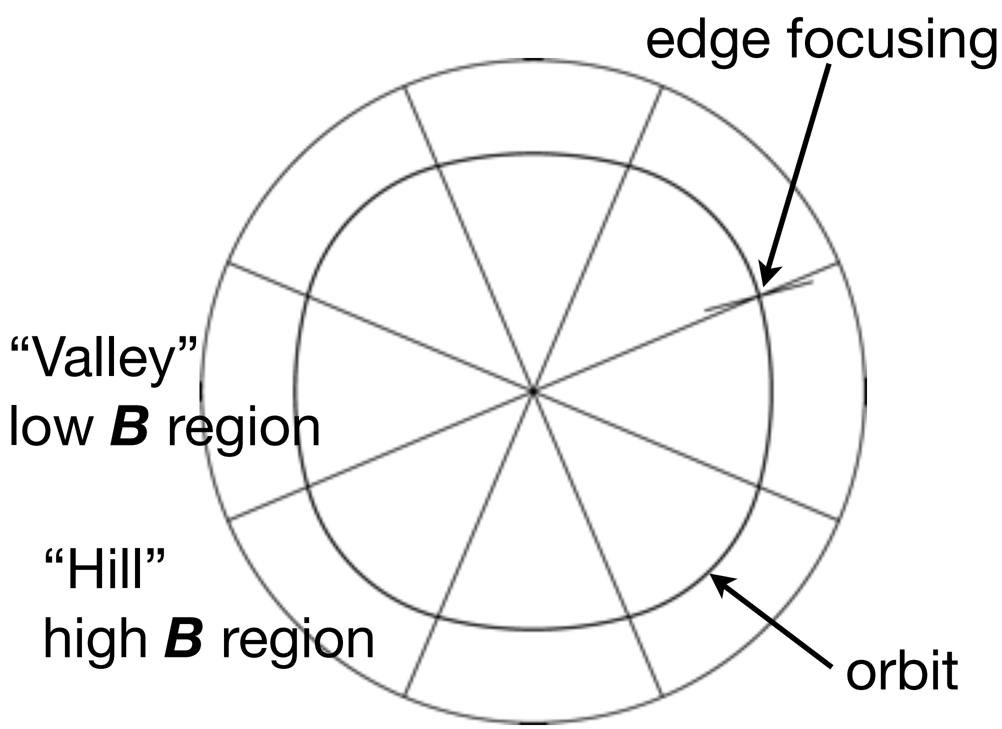
H. A. Bethe and M. E. Rose, Phys. Rev. 52 (1937) 1254.

Sector Focusing Cyclotrons

Magnetic fields to obtain vertical focusing and rigorous isochronism simultaneously

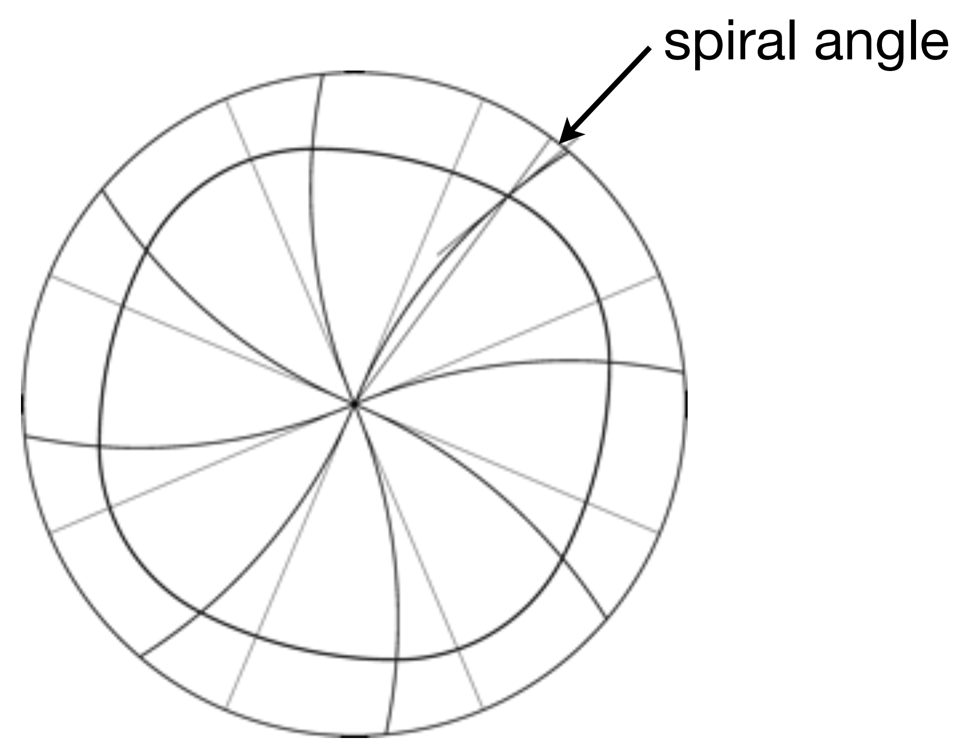
Azimuthally Varying Field (AVF)

L. H. Thomas, Phys. Rev. 54 (1938) 580.



Introduction of a spiral angle

D. W. Kerst et al., Phys. Rev. 98 (1955) 1153(A).



Normal conducting

U-400, U-400m (JINR FLNR)

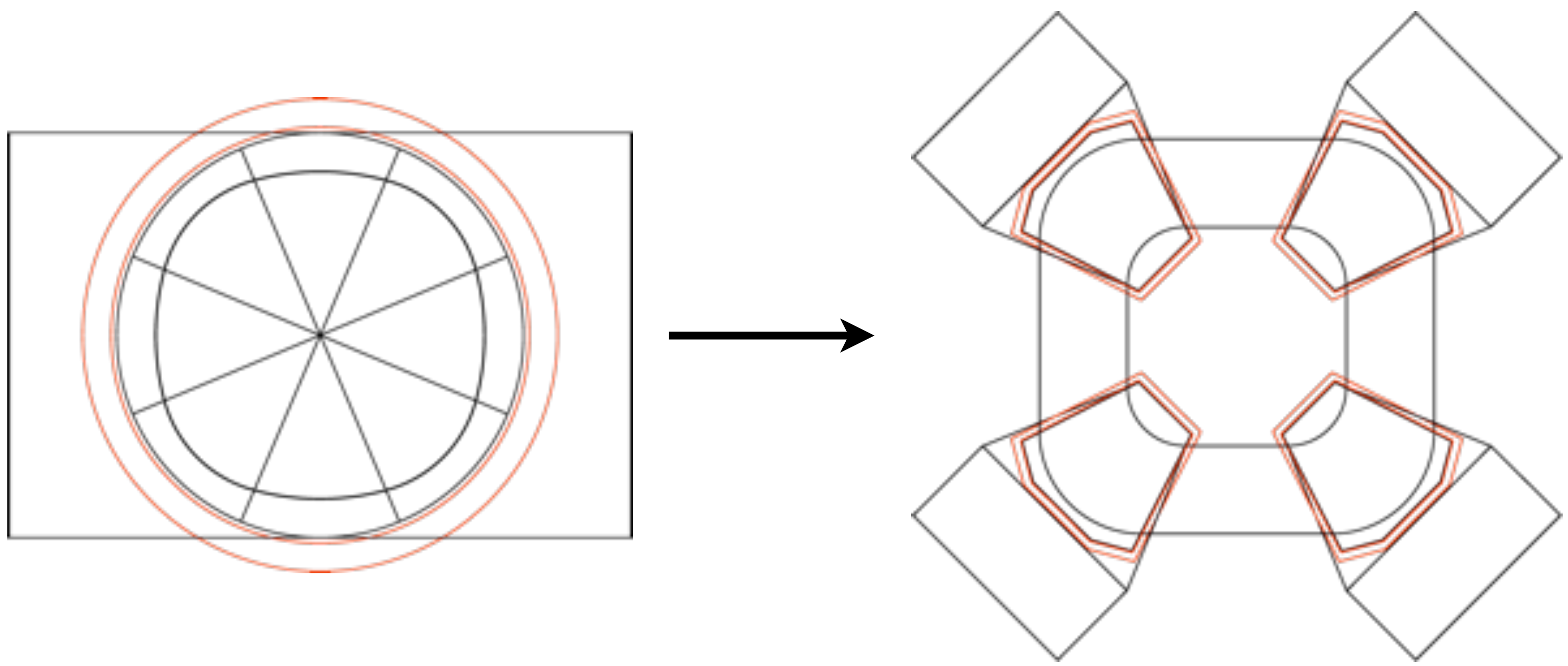
Super conducting

K500 & K1200 (MSU/NSCL), K800 (Catania), K500 (Texas A&M), AGOR (K600)

Separate Sector Cyclotrons

(Ring Cyclotron, Open Sector Cyclotron)

One magnet → piecewise magnets (sector magnets).



Stronger vertical focusing (high flutter)

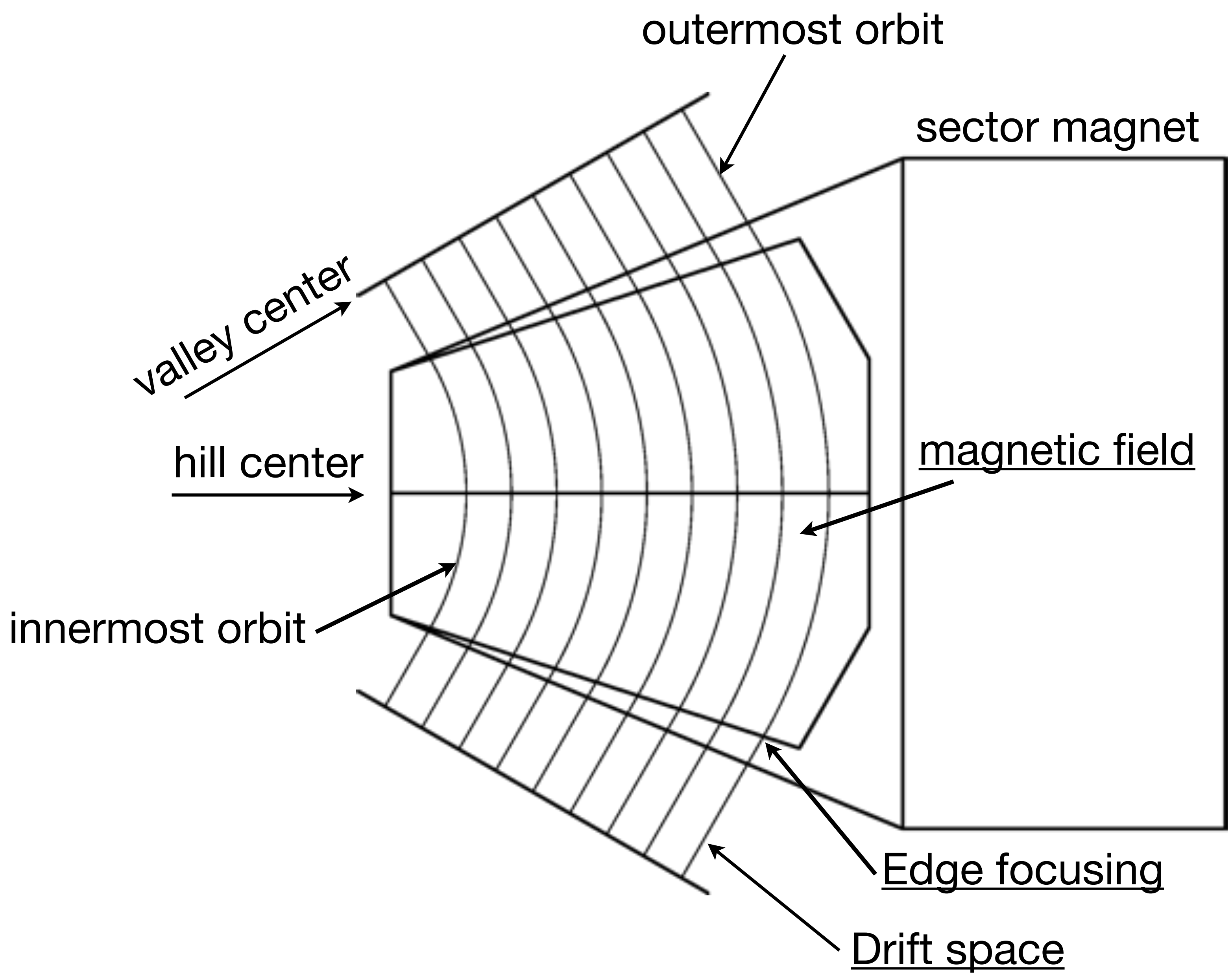
High-voltage resonators can be installed between sector magnets

H. A. Willax, Proc. Int. Conf. Sector Focused Cyclotron and Meson Factores (1963) 386.
V. P. Dmitrievsky, "Relativistic Cyclotron with Space Variation of Magnetic Field", Doctoral Dissertation Manuscript in mathematical Physics, Dubna, 1961.

CSS1&CSS2 (GANIL), CSS (IMP-Lanzhou), K400 (RCNP),
RRC, fRC, IRC, SRC (RIBF)

Beam Optics in Cyclotrons

Optics is determined for each equilibrium orbit.
Optics depends on energy of a particle.



Tune

L. Smith and A. A. Garren, UCRL-8598. (1959).
K. R. Symon et al., Phys. Rev. 103 (1956) 1837.

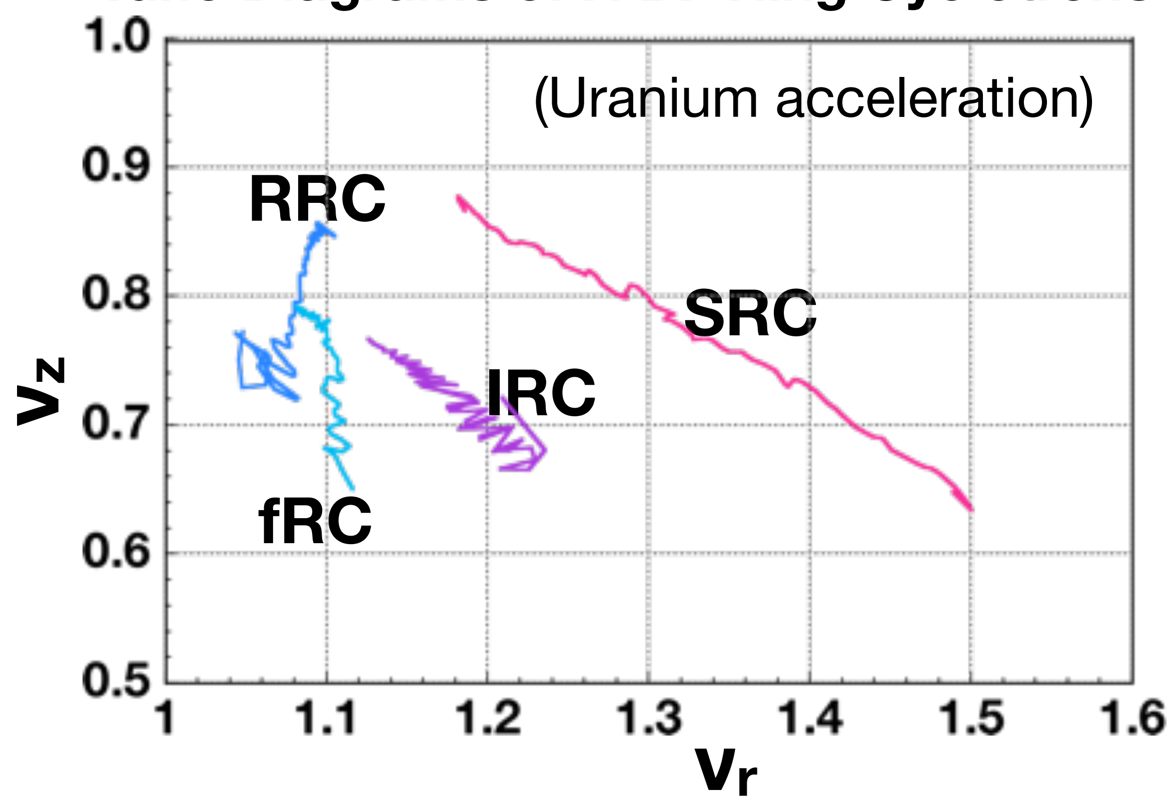
$$\nu_r^2 = 1 + n + O(N^{-2})$$

$$\nu_z^2 = -n + \frac{1}{2} f^2 (1 + 2tan^2\phi) + O(N^{-2})$$

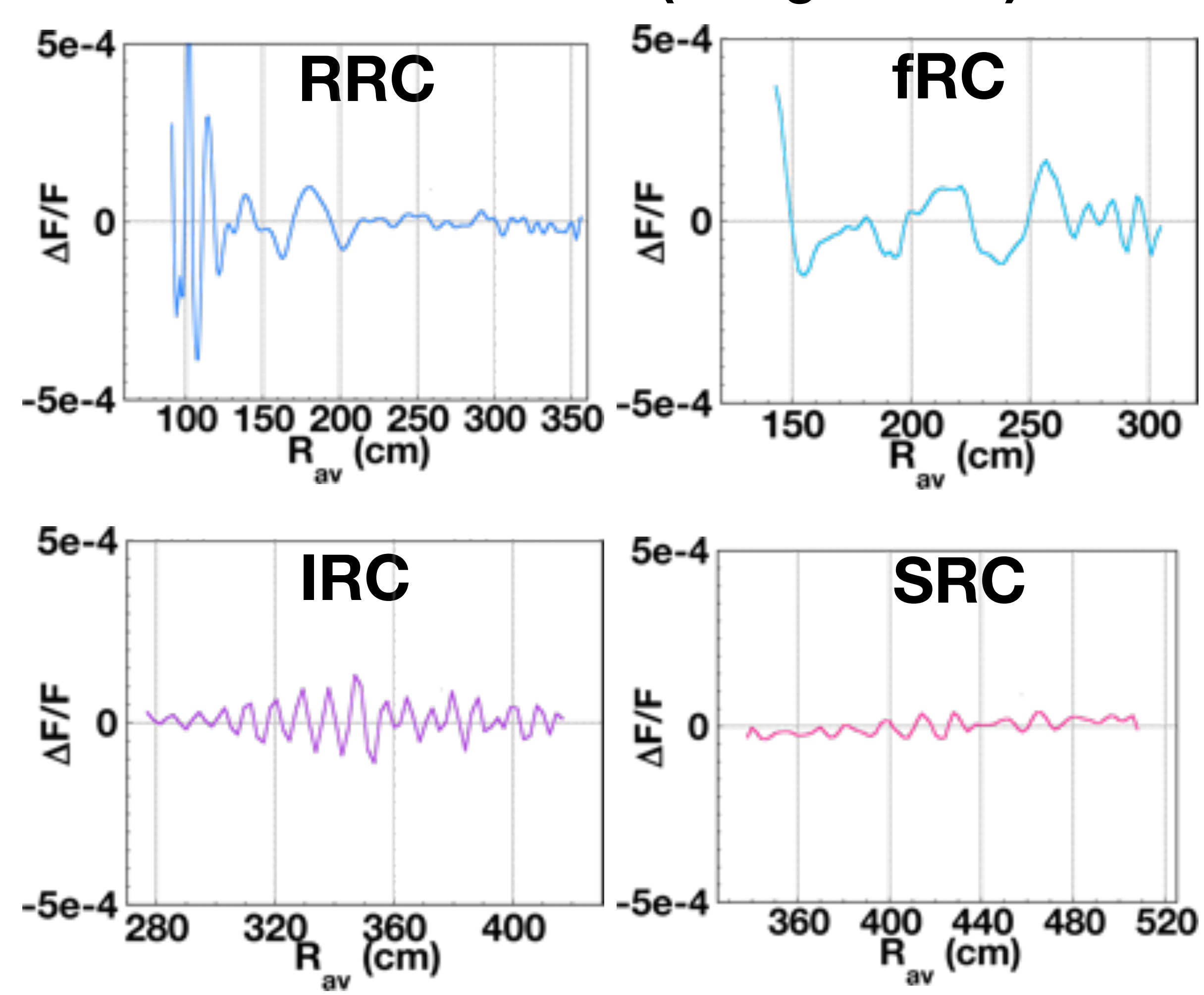
$$B(r,\theta) = \sum_N a_N(r)\cos(N\theta) + b_N(r)\sin(N\theta)$$

$$f^2 = \sum_N a_N^2 + b_N^2,\ \ \phi : \text{spiral angle}$$

Tune Diagrams of RIBF Ring Cyclotrons

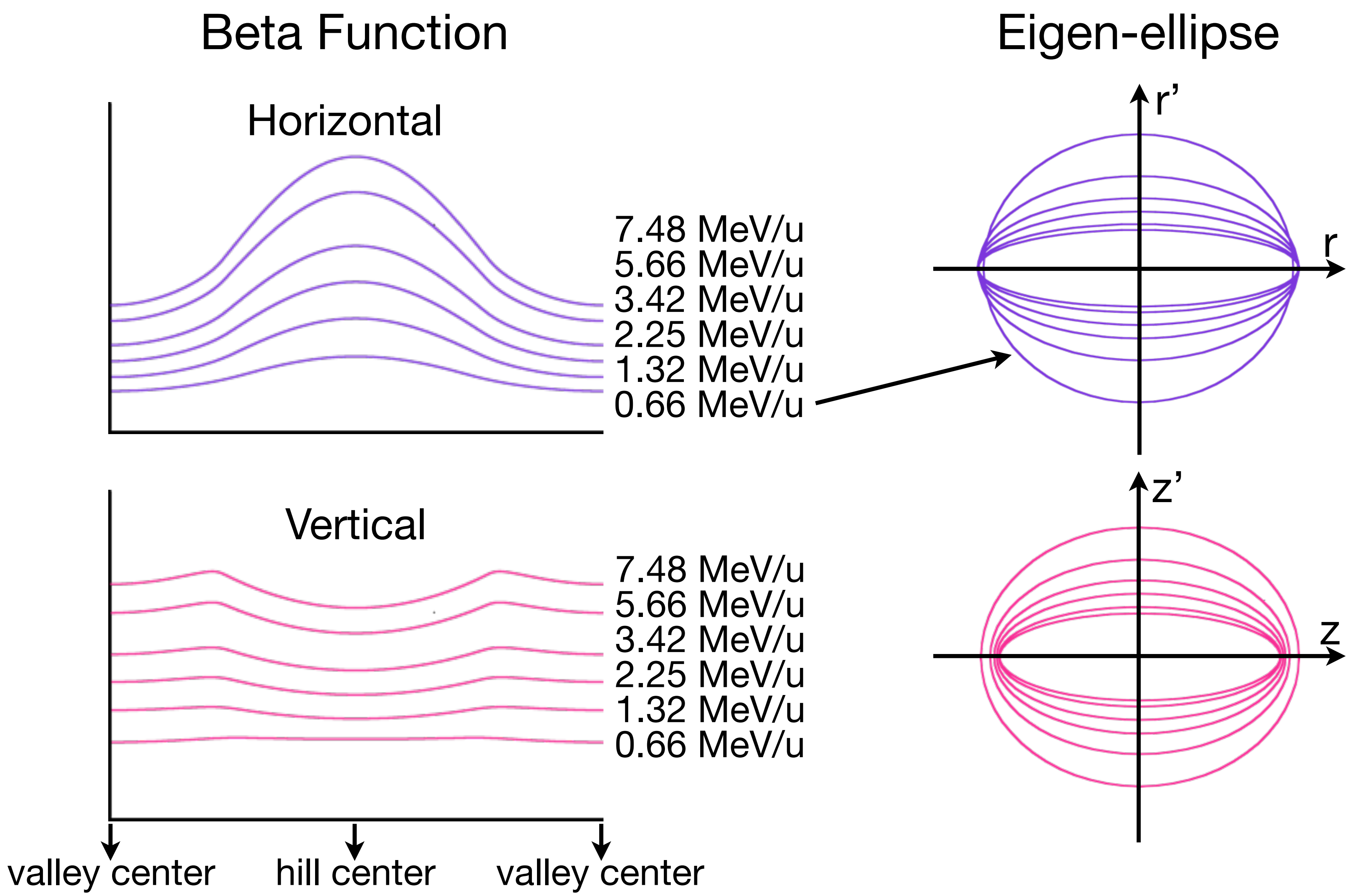


Isochronism (design value)



Beta Function and Eigen-Ellipse

$^{238}U^{35+}$ 18.25 MHz @ RRC

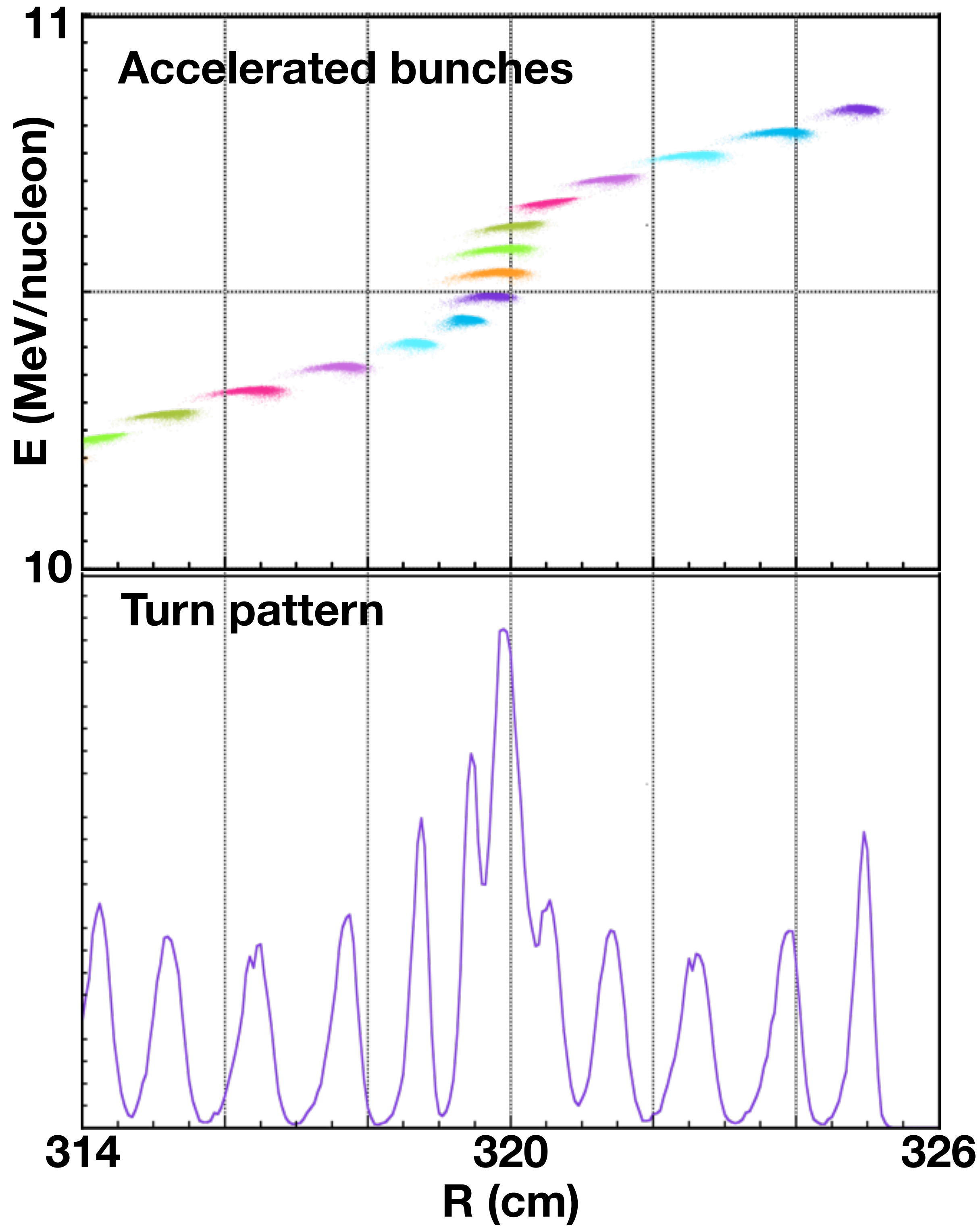


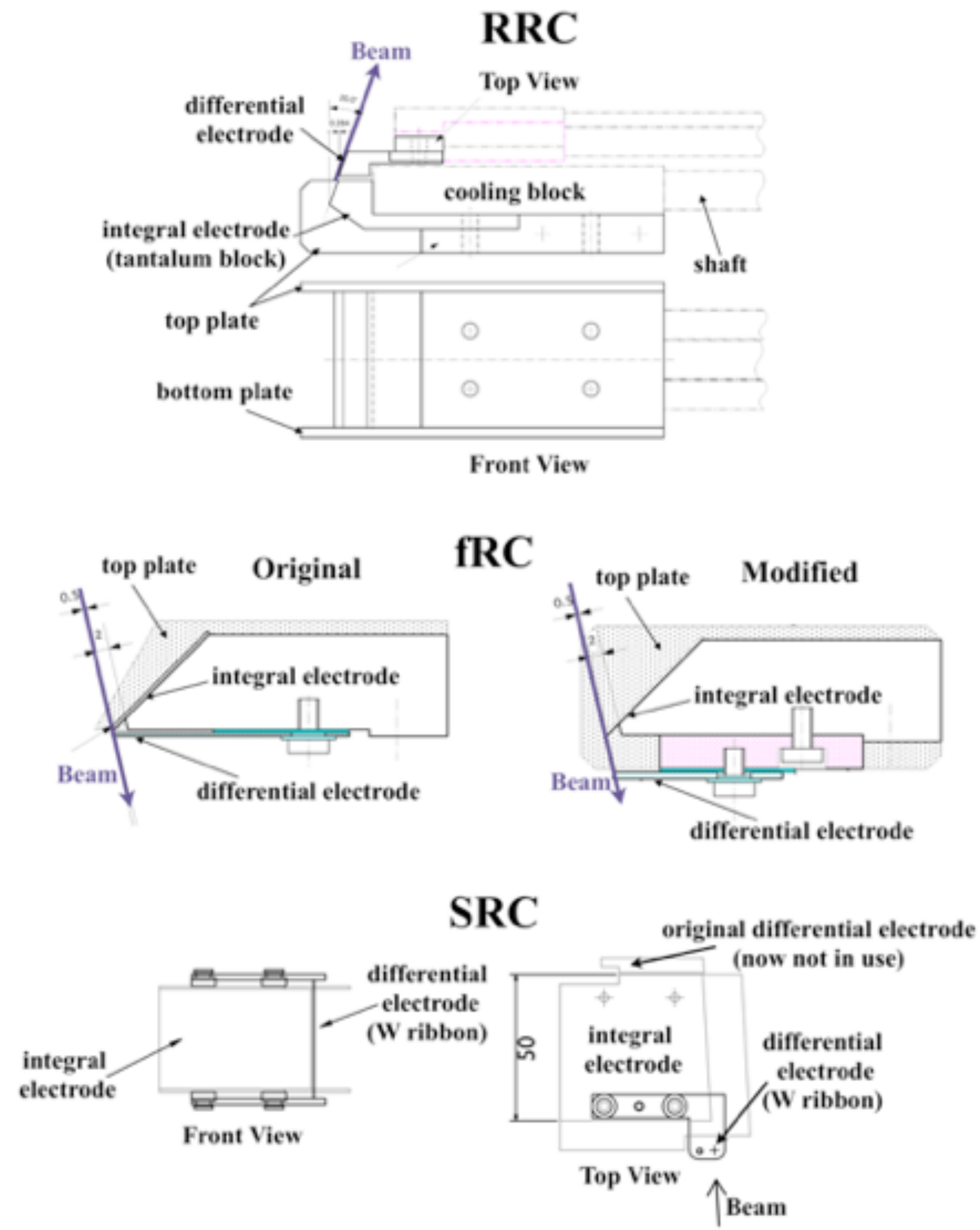
$$\hat{\beta} \propto R, \;\; R : \text{orbital radius}$$

$$\sigma \sim \sigma_0/\gamma, \;\; \sigma_0 : \text{const.}$$

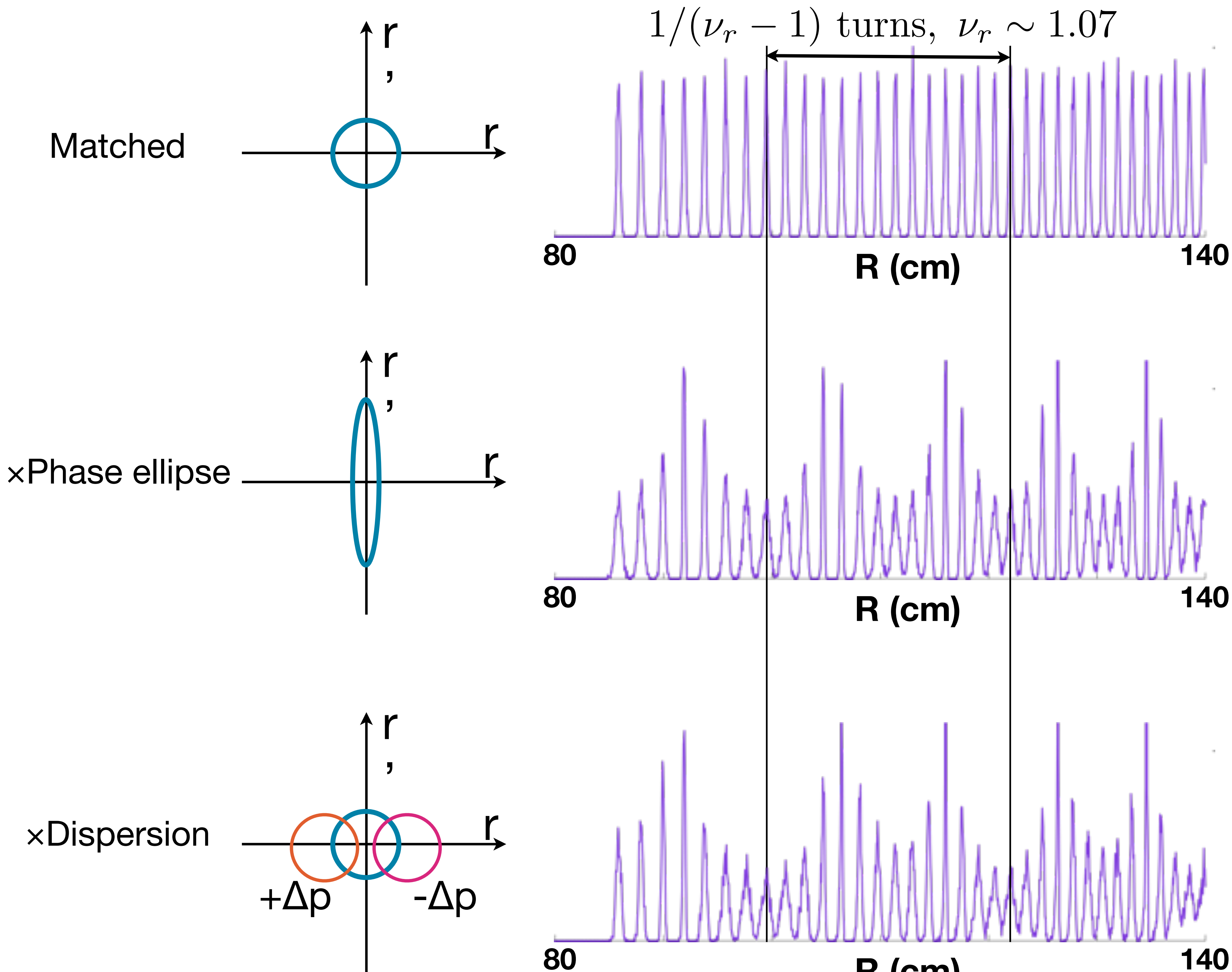
Charge Density Distribution (Turn Pattern)

A good observable to understand beam behavior in a cyclotron





Example of Turn Pattern : Mismatched Injection



Intensity Limiting Factors of Heavy-ion Cyclotrons

Three kinds of intensity-limiting factors of heavy-ion cyclotrons

J. Stetson et al., 19th Int. Cyclotron Conf. (2010) 27.

Source-limited

medium-heavy elements, especially metal species (MSU, RIBF)

Stripper-limited

Very heavy ions like uranium (MSU, RIKEN)

Beam-power limited → All the limits caused by unacceptable beam losses

Beam loss especially at extraction → strongly relates to beam dynamics

The first two strongly depends on an acceleration scheme used in each facility.
(Charge-state stripping at a higher energy is a better choice.)

Example of Source-Limited Case

NSCL/MSU K500+K1200

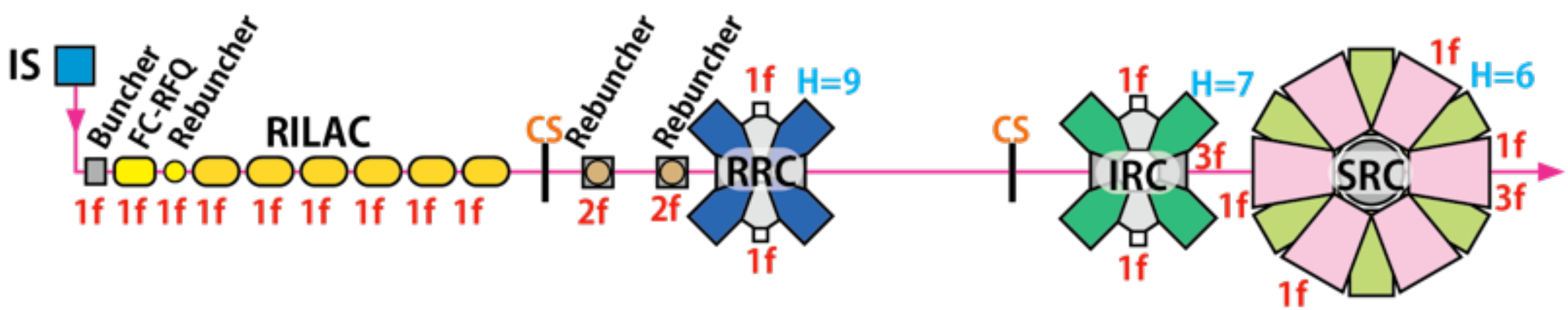
	E (MeV/u)	I (pnA)
58	160	40
76	120	3
112	120	10

J. Stetson et al., 19th Int. Cyclotron Conf. (2010) 27.

Variable Energy Mode at RIBF

	E (MeV/u)	I (pnA)
48	345	415
70	345	123

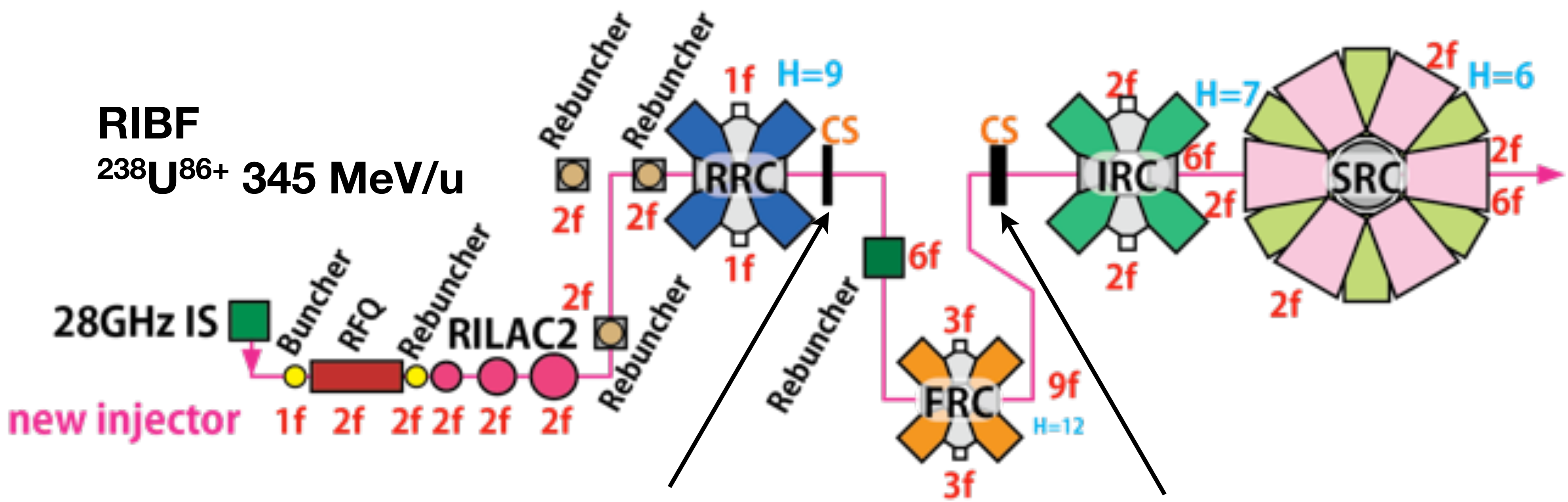
Acceleration Scheme of RIBF Variable Energy Mode



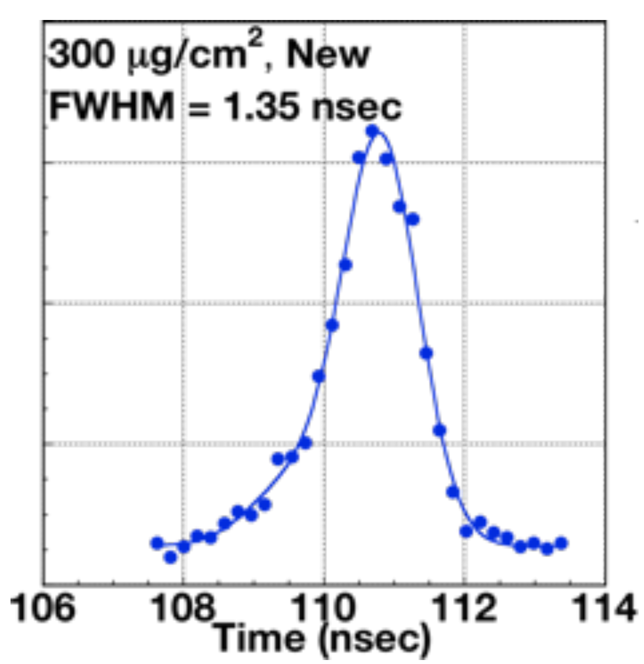
Usage of cutting-edge ion source technologies (28-GHz SC-ECR etc.) will improve the present situation.

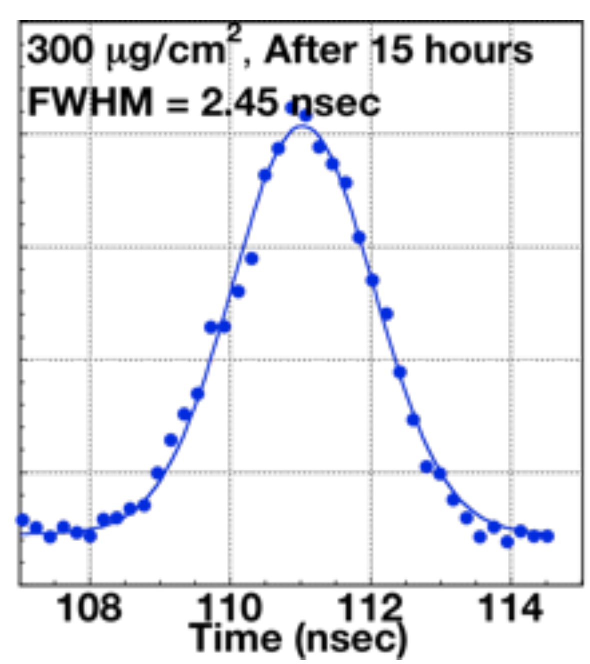
Example of Stripper-Limited Case

NSCL/MSU K500+K1200 : $^{238}U^{30+}$ 7.7 MeV/u 20 pnA → τ_{foil} = < a few minutes



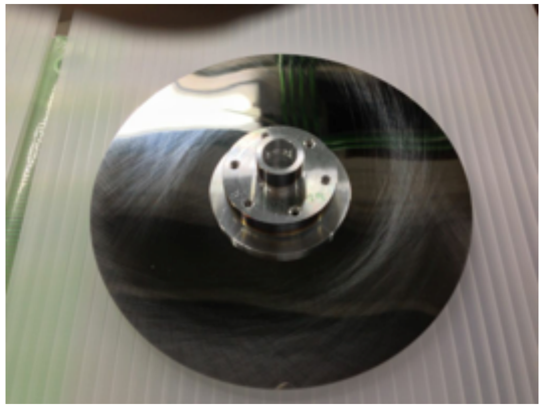
First-stage stripper (0.3 mg/cm², @11 MeV/nucleon)
τ_{foil} ~ 12 hours for a 15-pnA beam

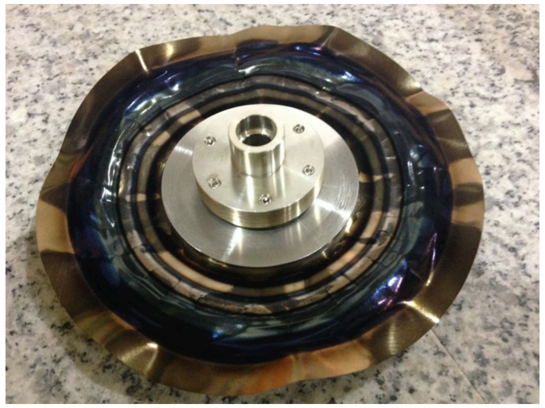




Second-stage Stripper
16 ~ 20 mg/cm², 50 MeV/nucleon

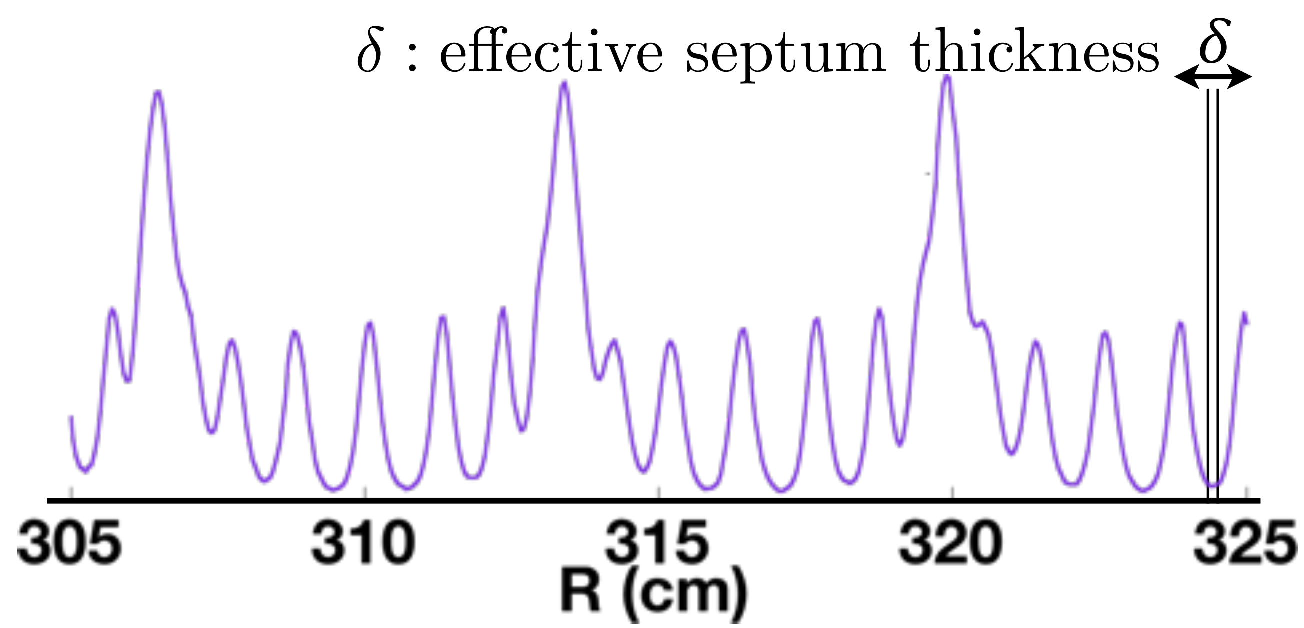
Rotating-beryllium-disk stripper
< 150 pnA





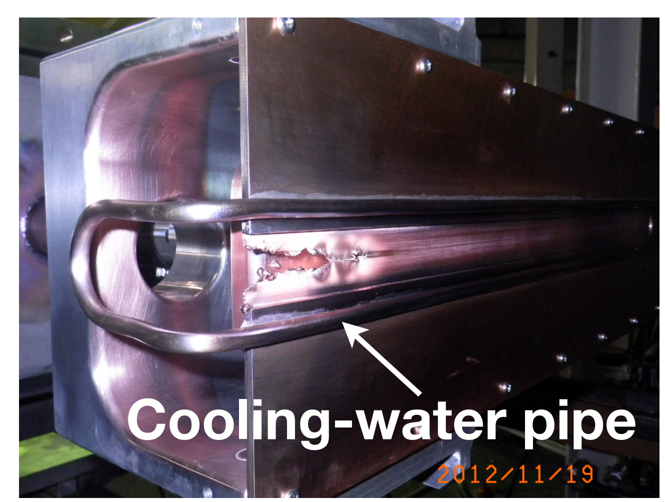
Beam-power Limited Case : Importance of Clean Extraction

Many heavy-ion cyclotrons use an electrostatic deflector as the first-stage of beam extraction.



An allowable heat load is 0.3 ~ 0.5 kW. (our experience)

Damaged RRC-EDC (Nov. 19, 2012)
The water-cooled septum electrode of the Electric Deflection Channel (EDC) of RRC was broken because of an unacceptable beam loss.



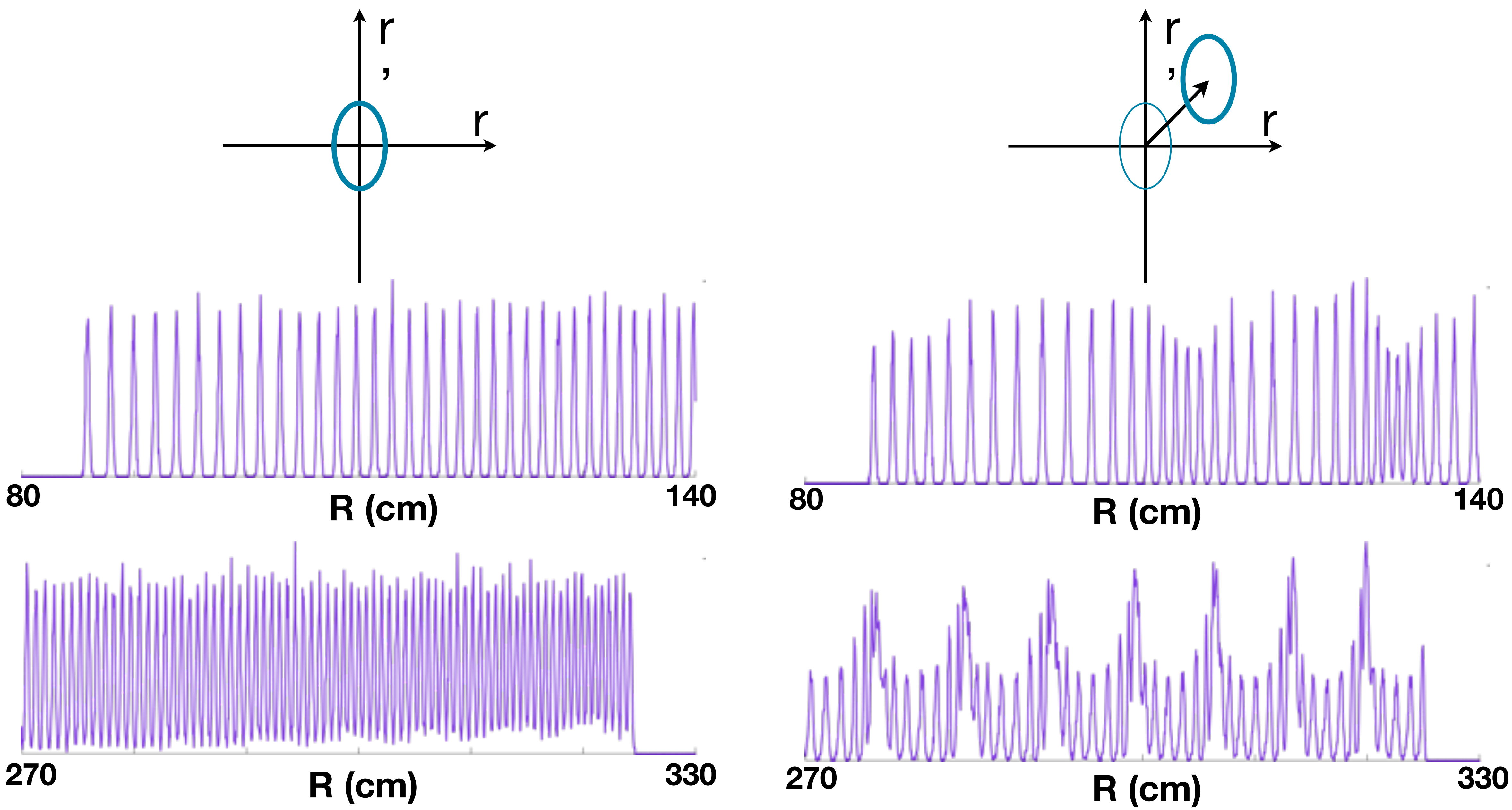
How to obtain Clean Extraction

J. M. van Nieuwland, " Extraction of Particles from a Compact Isochronous Cyclotron", Doctoral Thesis, Technische Universiteit Eindhoven (1972).
W. Joho "Extaction of a 50-MeV Proton Beam from the SIN Ring Cyclotron", SIN report TM-11-8 (1970)

Use coherent radial motion
- Off-centering (precessional) acceleration
- Regenerative acceleration
- Beam orbit expansion
  - *V. P. Dmitrievsky et al., Proc. 19th Cyclo. Conf. Caen, France (1981) 505.*

For long bunches
- Flat-topping acceleration
- Radially increasing RF fields (Bunch compression effect)
  - *W. Joho, Part. Accel. 6 (1974) 41.*

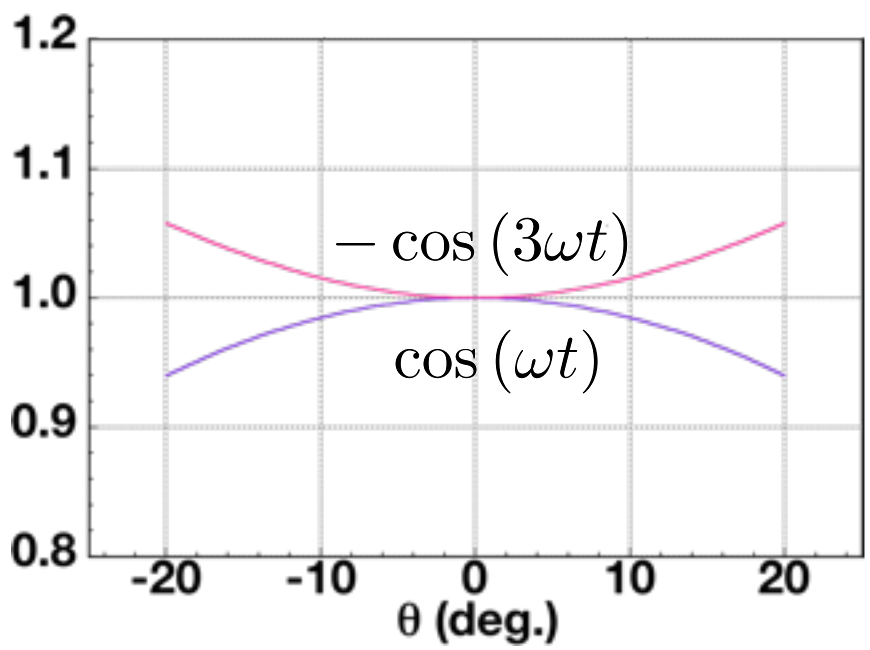
Off-centering Acceleration

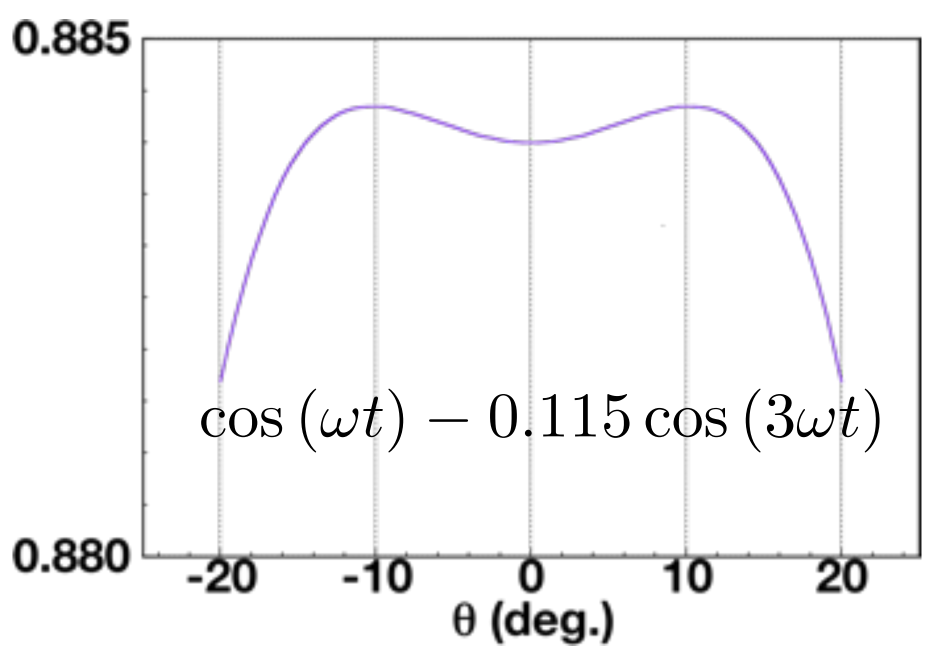


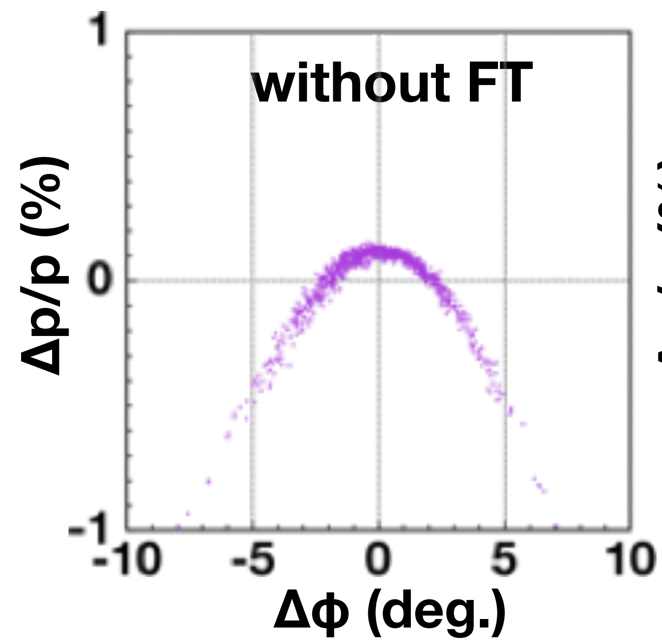
Off centering acceleration is routinely used to obtain large turn separations.

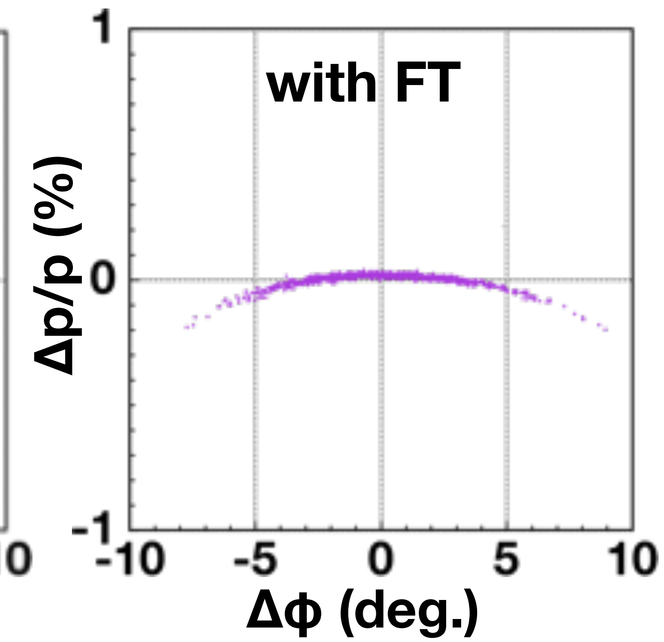
Flat-topping Acceleration

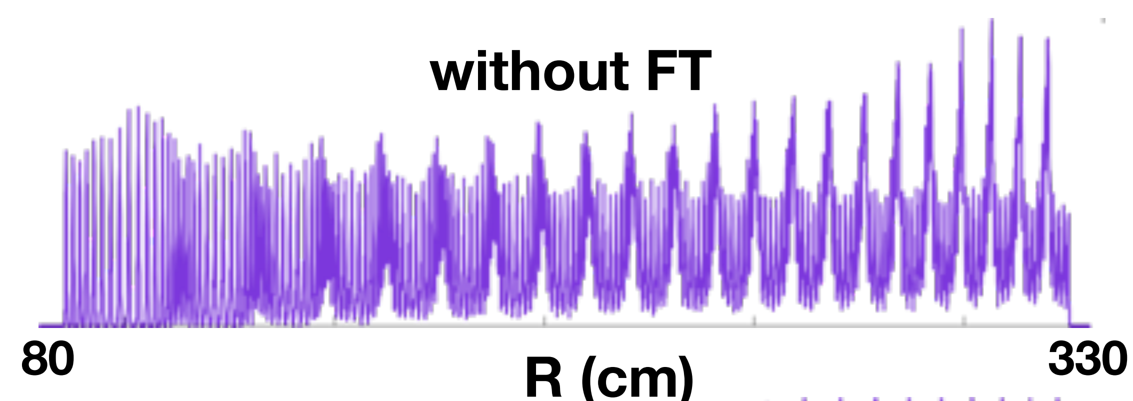
$$\cos(\omega t) - N^{-2}\cos(N\omega t) = 1 - N^{-2} + O((\omega t)^4)$$

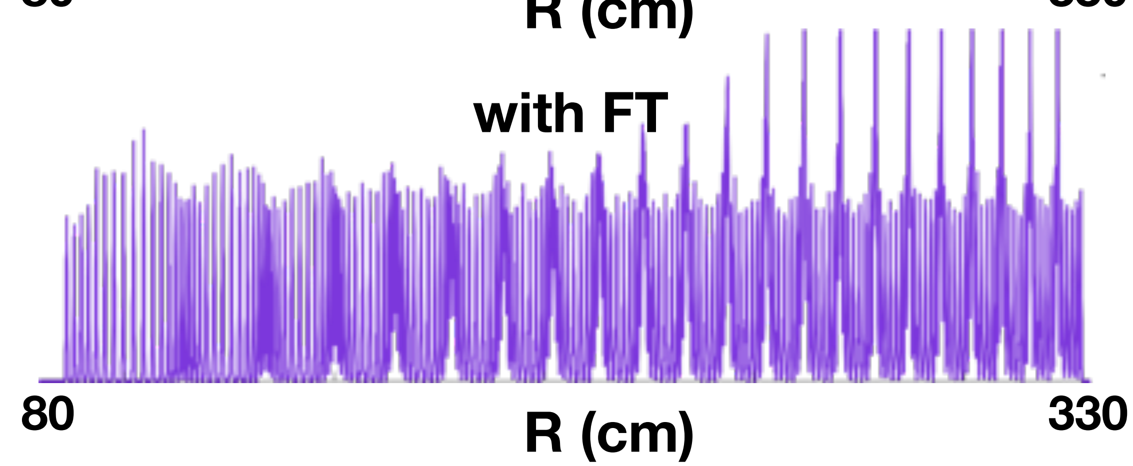












Phase Compression Effect

A radially increasing RF field induces bunch compression.

W. Joho, Particle Accelerators 6 (1974) 41.

A time-dependent magnetic field exist for $E(r) \neq \text{const.}$

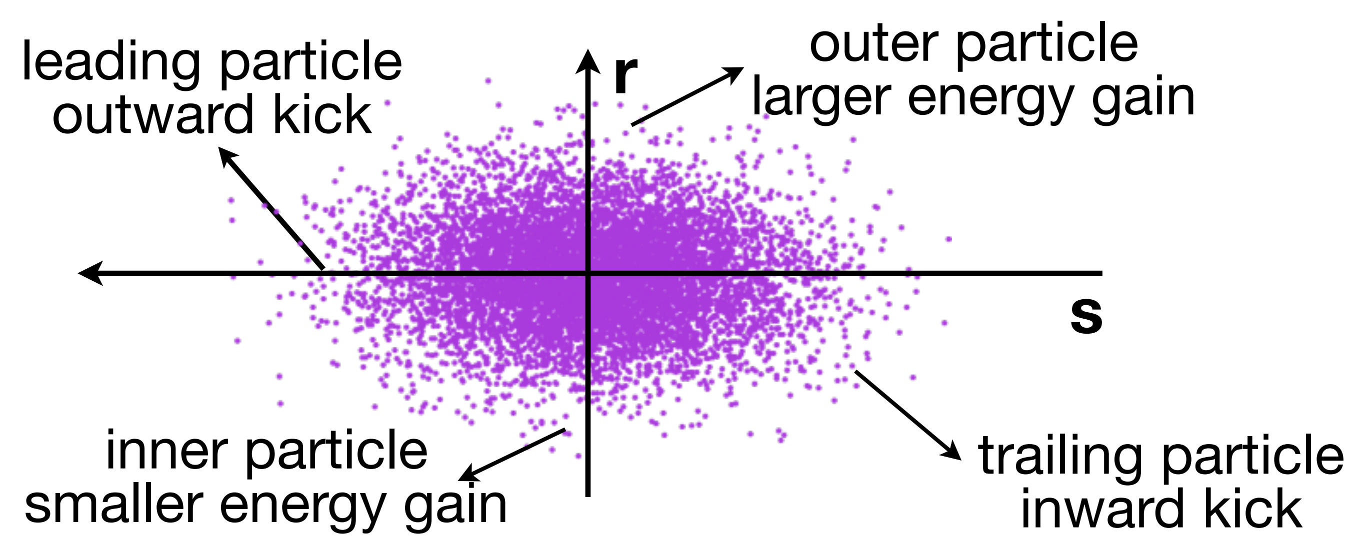
$$\text{rot}\vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

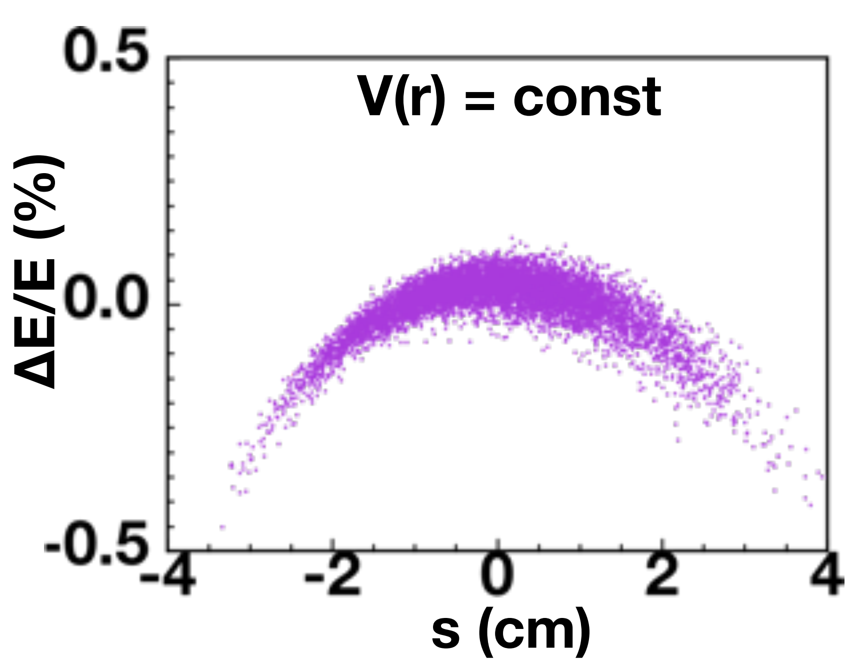
Conserved quantities

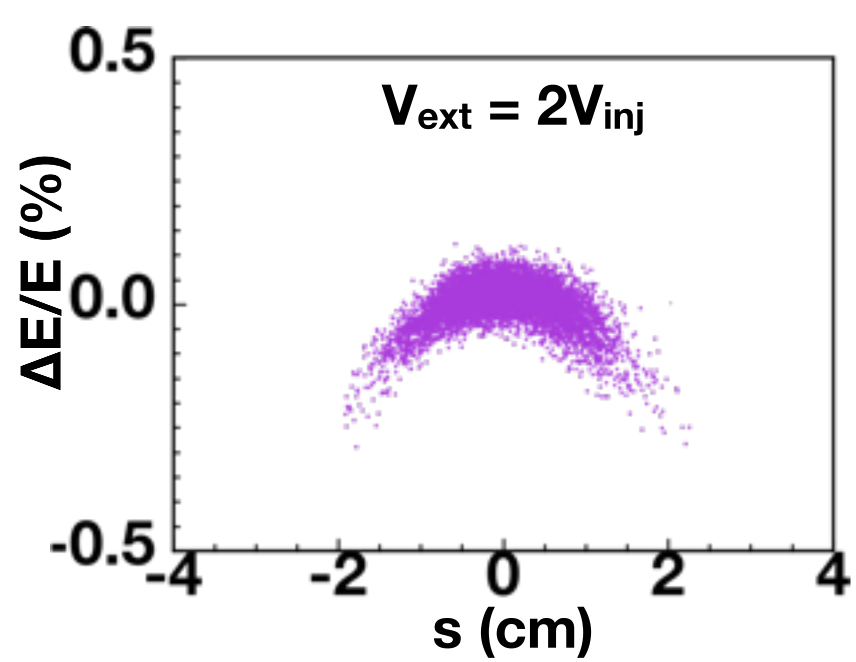
$$V(r)\sin\phi(r) = \text{const.}$$

V : acceleration voltage

ϕ : phase width of the beam







Space Charge Effect - Overall Vortex Motion

Space charge forces induce “vortex motion” via a longitudinal-horizontal coupling.

M. M. Gordon, 5th Int. Cyclotron Conf. (1969) CYC69D04.

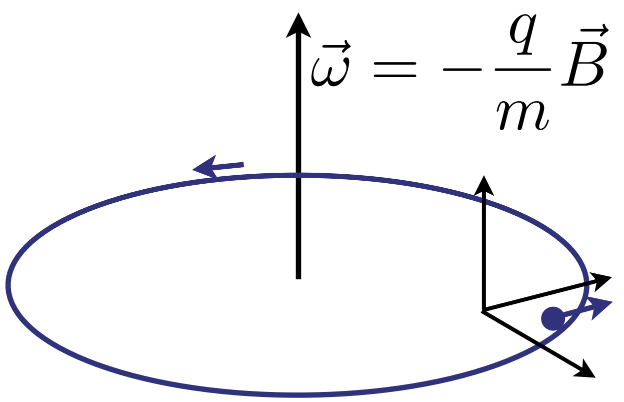
Start with $\vec{B}:\mathrm{const},\ \gamma\sim 1,\ \vec{E}:\text{space charge field}$

Equation of motion in the rotating frame

$$\frac{d\vec{v}}{dt}+\vec{\omega}\times\vec{v}=\frac{q}{m}\vec{E}$$

Steady state

$$\vec{\omega}\times\vec{v}\sim\frac{q}{m}\vec{E}$$



beam frame (x, y, z)

Particles move along equipotential surfaces in the radial-longitudinal plane.

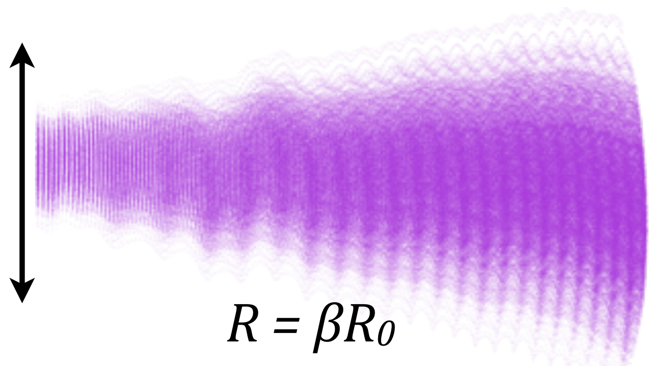
Overall and Local Vortex Motions

Overall vortex motion

(in case of insufficient separation of turns)

$l = R\Delta\theta$

$\Delta\theta$: phase width, constant

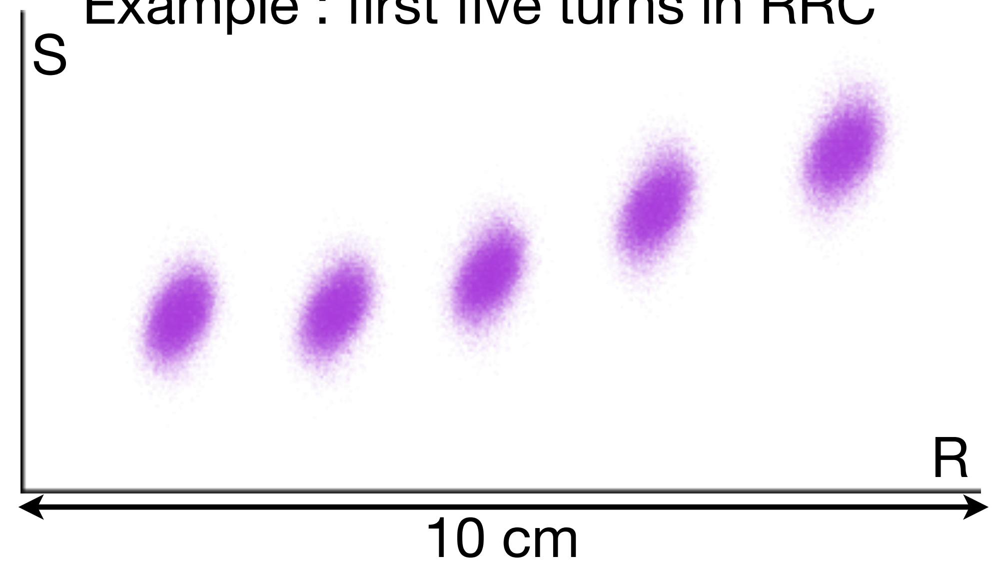


Dangerous because equipotential surfaces and particle motion extends over many turns.

Local vortex motion

(in case of separated turns)

Example : first five turns in RRC



Vortex motion is limited in each bunch.

A “round beam”

Manifestation of Local Vortex Motion : A Round Beam

A “round beam” was discovered and confirmed by
a series of studies for PSI Injector 2.

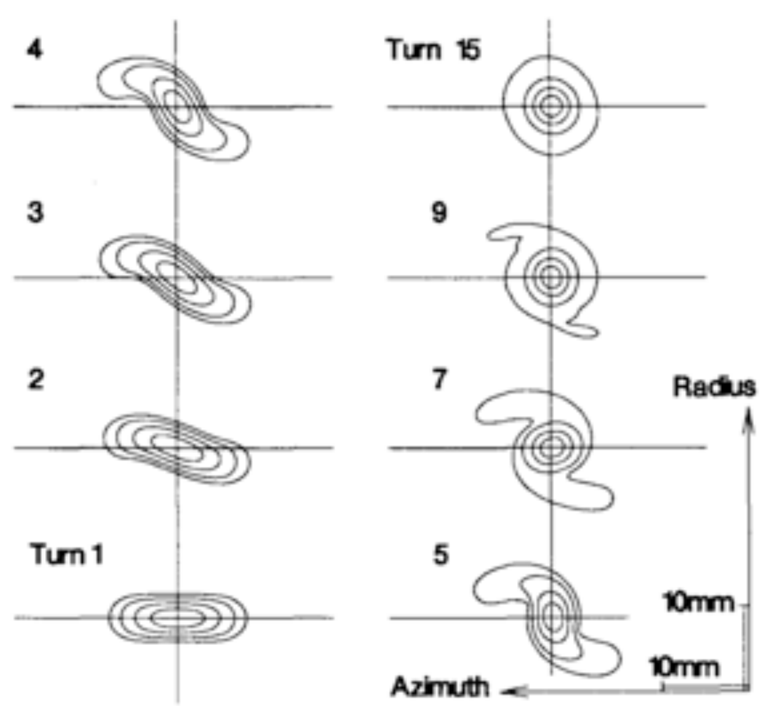
Operating experience

- *vortex motion... leads to a compact ball of charges....*[(1)]”
- *“best settings for high intensity beams in the injector 2 need no flattop voltage*[(2)]”

(1) Th. Stammbach, 13th Int. Cyclotron Conf. (1992) 28.
(2) S. Adam, 14th Int. Cyclotron Conf. (1995) 446.

Theoretical Studies

S. Adam, 14th Int. Cyclotron Conf. (1995) 446.



Direct Measurement

R. Dölling, DIPAC2001 (2001) 111.

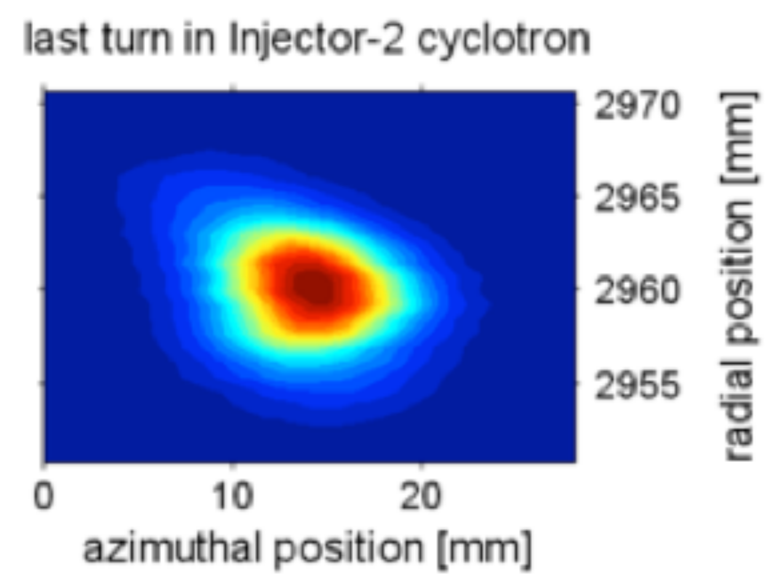


Figure 4 of “Production of a 1.3 MW Proton Beam at PSI”, M. Seidel, Proc. IPAC’10 (2010) tuyra03.

Theoretical Analysis for Local Vortex Motion

A “round beam” is stable!

Kleeven : Moment Analysis of Space Charge Effects

W. J. G. M. Kleeven, Doctoral Thesis, Technische Universtiteit Eindhoven (1988).

Smooth approximation for magnetic fields & linearization of space charge forces

$$\frac{d}{d\tau}\langle \hat{x^2} \rangle = 2\langle \hat{x}\hat{p_x} \rangle + \nu_x \langle \hat{x}\hat{s} \rangle$$

$$\frac{d}{d\tau}\langle \hat{x}\hat{p_x} \rangle = \langle \hat{p_x^2} \rangle - \frac{1}{4}(\nu_x^2 - 4a)\langle \hat{x^2} \rangle + \frac{1}{2}\nu_x(\langle \hat{x}\hat{p_s} \rangle + \langle \hat{s}\hat{p_x} \rangle) + d\langle \hat{x}\hat{s} \rangle$$

$$\vdots$$

Stationary solution symmetric for radial and longitudinal directions exists.

Bertrand and Ricaud :Linear Analysis and Non-linear PIC Simulation

P. Bertrand , Ch. Ricaud, Proc. 16th Int. Cyclotron Conf. (2001) 423.

Linear analysis → conditions required for a stationary distribution
3D space charge simulation → a stable round beam exists.

Bertrand and Ricaud : Linear Analysis

A solvable model with uniform $\vec{B} = (0, 0, B_z)$

A beam bunch is assumed to be a uniformly charged sphere. $\mathrm{charge} = Q, \mathrm{radius} = r_0$

Equation of Motion

$$m\ddot{x} = qB_z\dot{y} + qk(x - x_0)$$
$$m\ddot{y} = -qB_z\dot{x} + qk(y - y_0)$$

$$k = \frac{Q}{4\pi\epsilon_0 r_0^3}$$

(x_0, y_0) : bunch center

Introduce a complex variable

$$z = x + iy$$
$$\ddot{z} = i\omega\dot{z} + \lambda(z - z_0)$$

$$\lambda = \frac{qk}{m}$$

Solution

$$z - z_0 = A\exp(i\omega\nu_+ t) + B\exp(i\omega\nu_- t)$$

Eigen modes

$$\omega^2\nu_\pm^2 - \omega^2\nu_\pm + \lambda = 0$$

$$\nu_\pm = \frac{1 \pm \sqrt{1 - 4\lambda/\omega^2}}{2} \to \nu_- \sim \frac{\lambda}{\omega^2} = \frac{Q}{4\pi\epsilon_0 (m/q) r_0^3 \omega^2}$$

Intensity Limit (limit of stable vortex motion)

$$4\lambda/\omega^2 = 1 \to Q_{\max} = \pi\epsilon_0 \frac{m}{q}\omega^2 r_0^3$$

Bertrand and Ricaud: Single Particle Motion

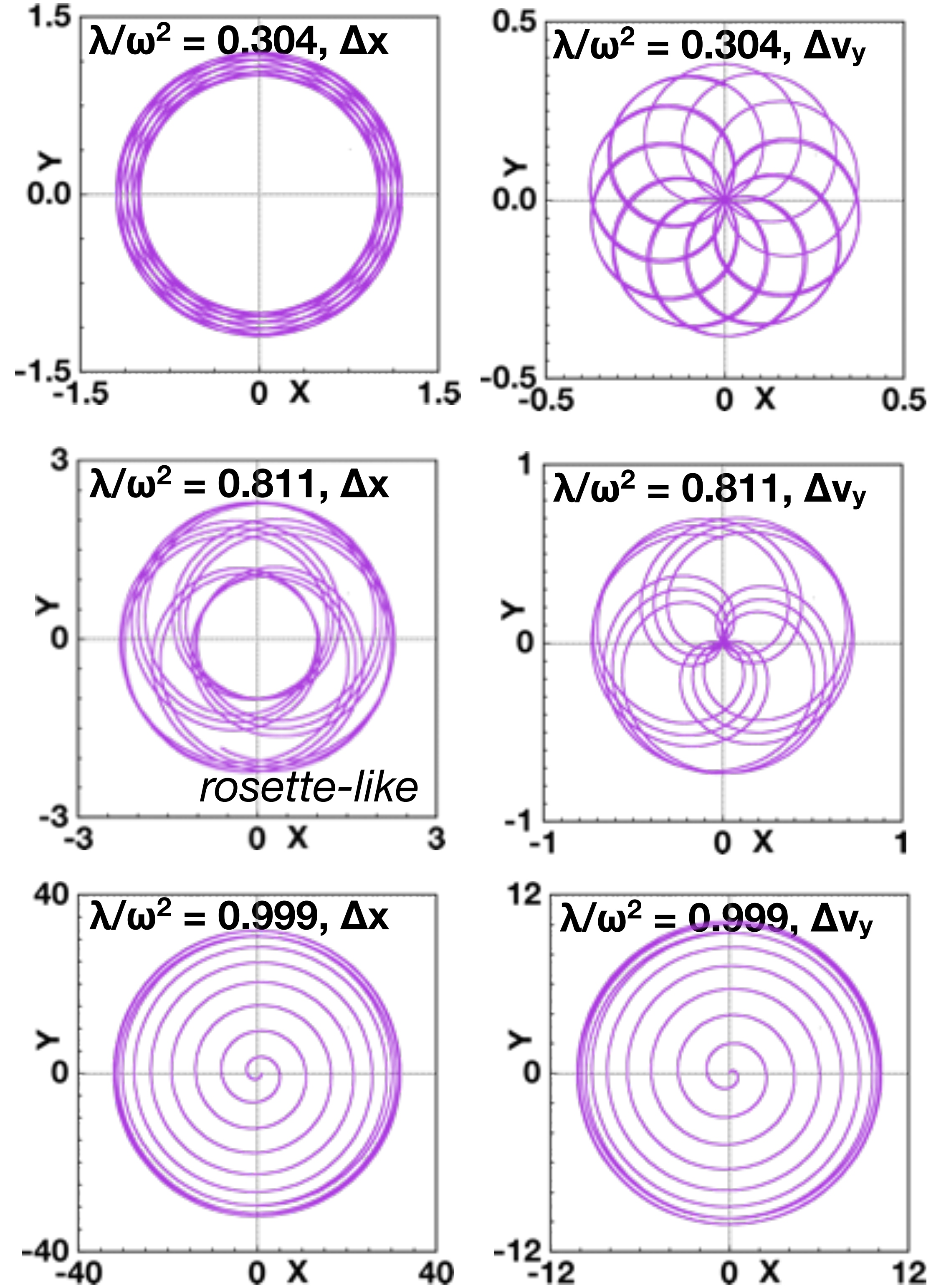
Solution of the model

$$\ddot{z} = i\omega\dot{z} + \lambda(z - z_0)$$
$$z - z_0 = A\exp(i\omega\nu_+ t) + B\exp(i\omega\nu_- t)$$
$$z = x + iy \qquad \lambda = \frac{qk}{m}$$

Zero-current Limit

$$\nu_+ = 1 \;\; \rightarrow \text{betatron motion}$$
$$\nu_- = 0 \;\; \rightarrow \text{radius change for } \Delta\text{p} \neq 0$$

Single particle motion in the rotating frame



Bertrand and Ricaud : Stationary Distribution

Stationary distribution is analytically obtained in this model.

Transfer Matrix

$$\mathbf{T} = (T_{ij})$$

$$T_{ij} : \text{function of } \cos(\nu_{\pm}\omega t),\ \sin(\nu_{\pm}\omega t)$$

Beam Matrix

$$\mathbf{V} = (\Delta x, \Delta y, \Delta p_x/p_0, \Delta p_y/p_0)$$

$$\sigma_0 =^t \mathbf{V}\mathbf{V}$$

Condition of stationary state

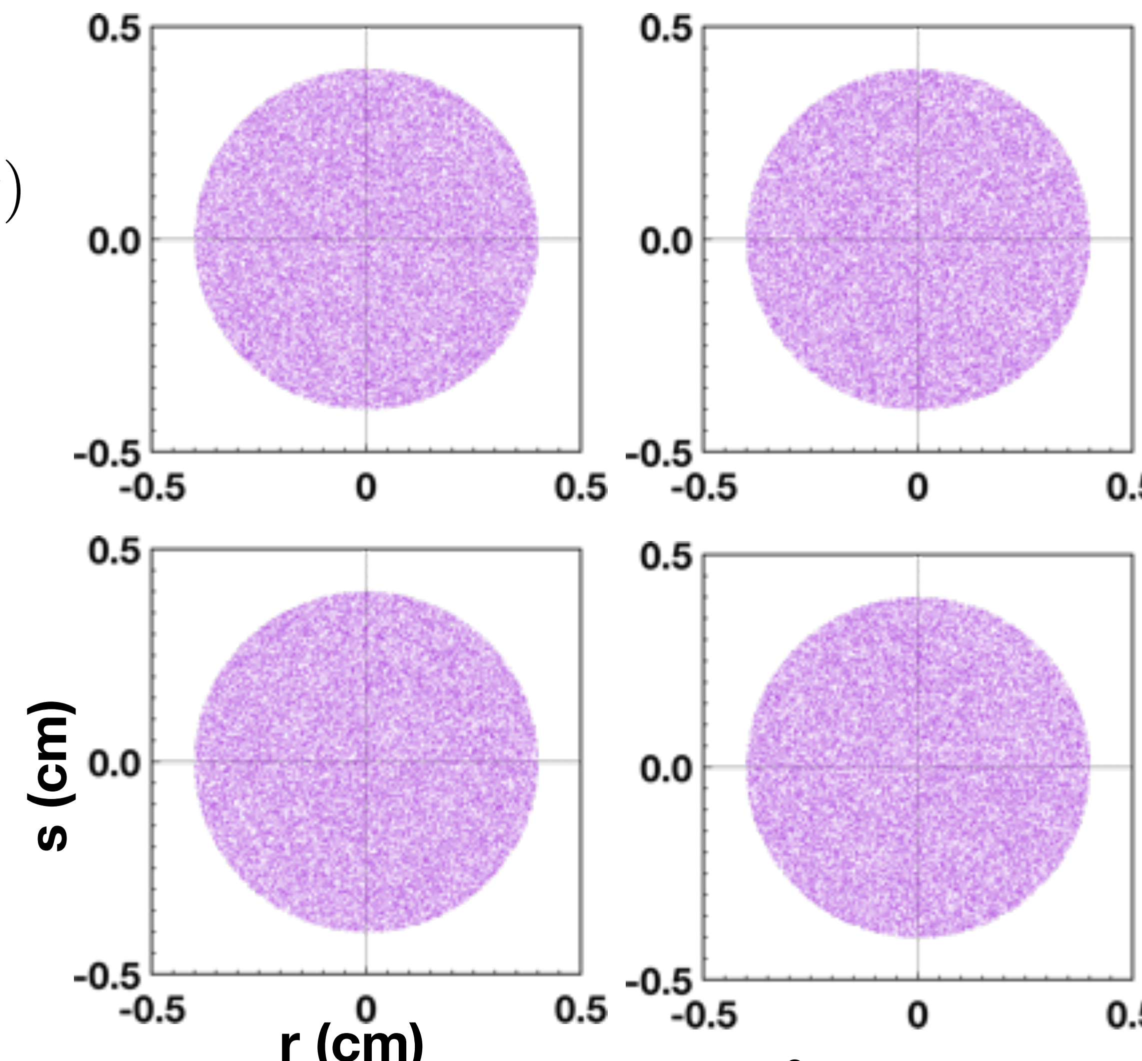
$$\sigma_0 = \mathbf{T}\sigma_0^t\mathbf{T}$$

Stationary state

$$\sigma_0 = \begin{pmatrix} \langle r^2\rangle \mathbf{I} & b\mathbf{J} \\ b\mathbf{J} & \frac{1-u}{4R^2}\langle r^2\rangle \mathbf{I} \end{pmatrix}$$

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \qquad \mathbf{J} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Particle distribution in co-moving frame



$\lambda/\omega^2 = 0.816$
Radius = 0.8 m
ω = 19.3 MHz
Q = 3.5×10^7 e
Beam size = 4 mm

Particle Motion outside a Beam Core

All the particles are bound.

C. Chasman and A. J. Baltz, NIM 219 (1984) 279.

Assumption : flat B, non-relativistic

$\rho, \phi \sim \text{polar coordinates in the rotating frame}$

$u = \rho/\rho_0$

$\Omega^2 = e^2 Qq/m\rho_0^3$

Equation of motion in rotating frame

$$\ddot{u} - u(\dot{\phi}^2 + \dot{\phi}\omega) = \Omega^2/u^2$$

$$u\ddot{\phi} + 2\dot{u}\dot{\phi} + \dot{u}\omega = 0$$

Approximation introduced

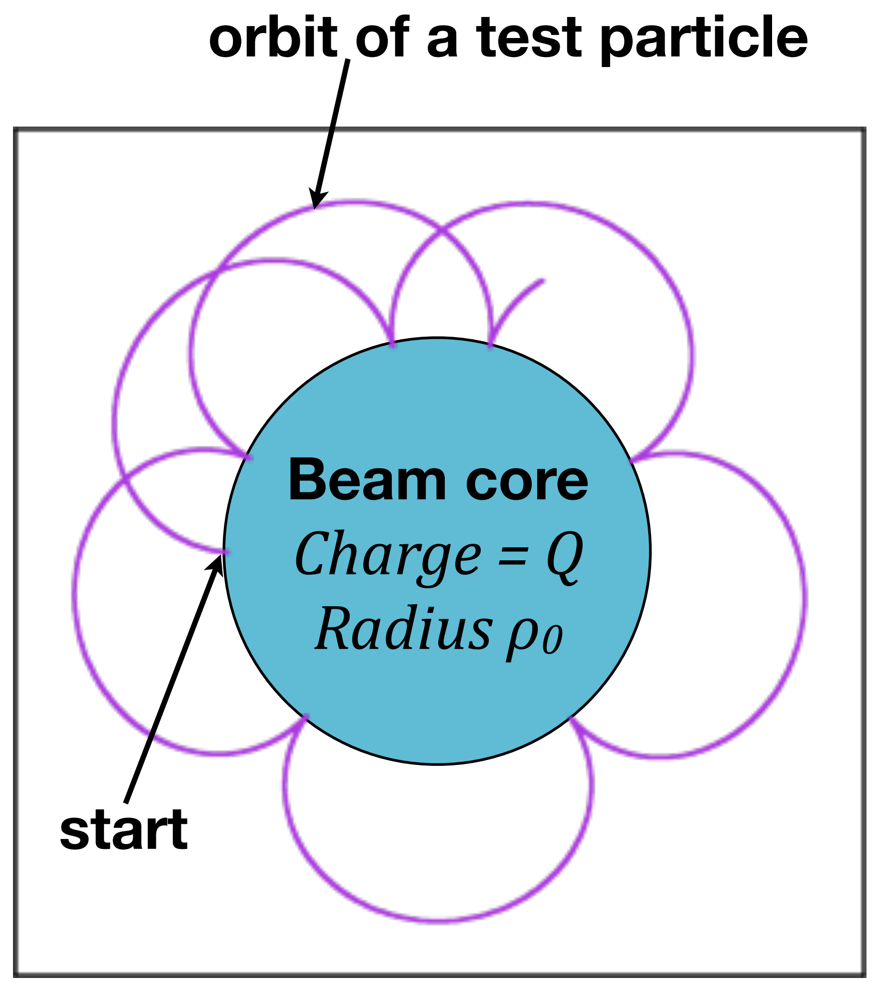
$$u = 1 + \epsilon \rightarrow 1/u^2 \sim 1 - \alpha\epsilon$$

Solution with a special initial condition

$$\epsilon = (\Omega/k)^2(1 - \cos kt)$$

$$\phi = \frac{\alpha\omega}{2}(\Omega/k)^2\left(-t + \frac{\sin kt}{k}\right) + \phi_0$$

$$k = \sqrt{\alpha(\omega^2/2 + \Omega^2)}$$



Chasman and Baltz also derived an exact analytic solution using elliptic integrals.

Space Charge Simulation

Nonlinear effects
Strongly modulated magnetic fields
Acceleration

→ Fully 3D space charge simulations are necessary, for example, OPAL
(A. Adelmann et al., Proc. ICAP2009 (2009) 107.

A classical Particle-in-Cell (PIC) based on the particle-mesh method

Magnetic Field :	3D map from 3D simulation (TOSCA)
RF Fields :	approximated by a relatively simple function but include phase compression effect
Integration :	Runge-Kutta, 6th order
Space charge force →	electric field in co-moving frame only
Poisson solver :	3D DFT
Computation box :	a rectangular parallelepiped
Boundary condition :	periodic for longitudinal* & transverse
	$\phi = 0$ at both boundaries in vertical direction
Number of test particles :	0.1M ~ 1M

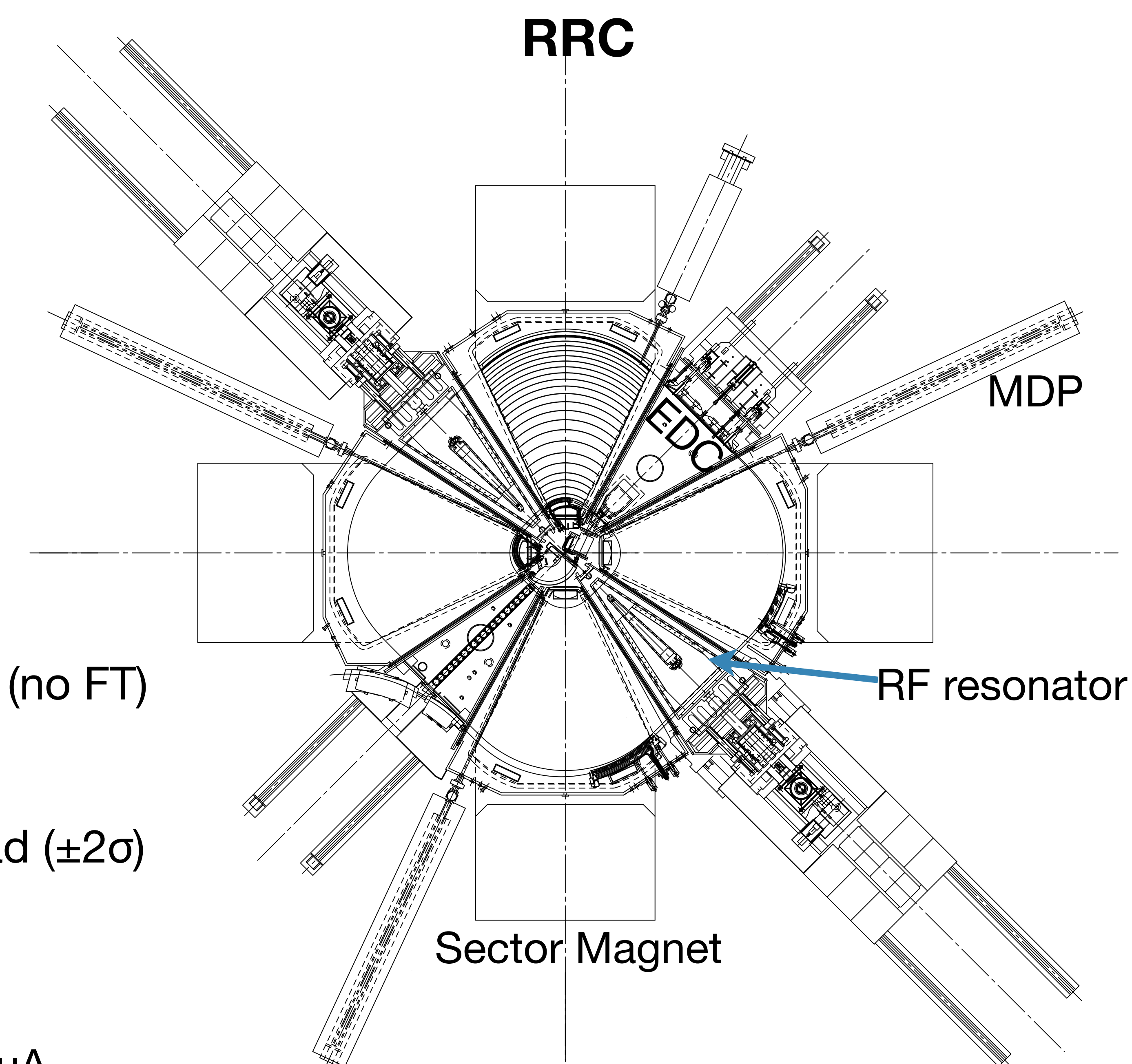
**a sufficiently long grid is used, hence virtually open.*

Simulation Conditions

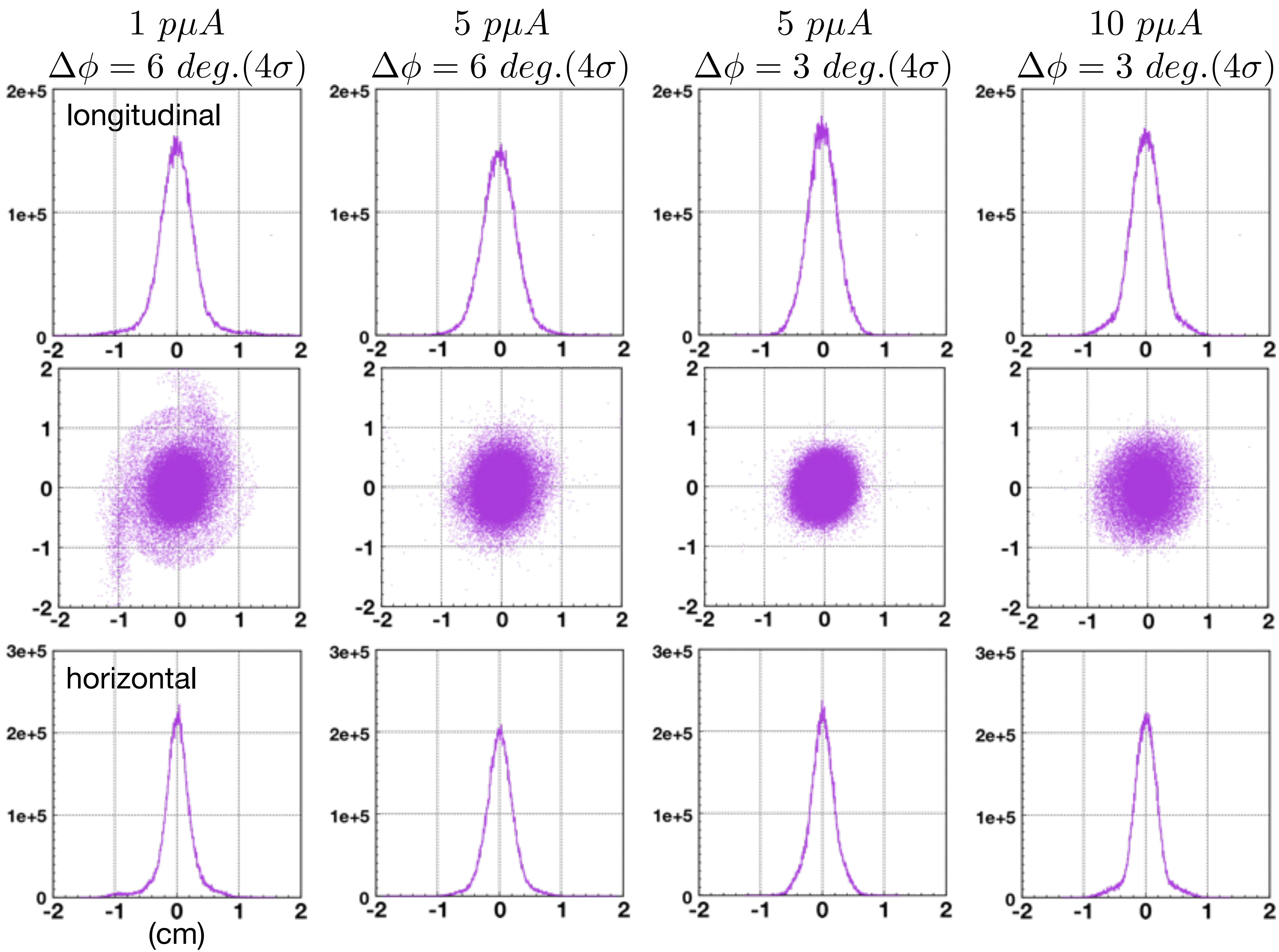
RIKEN Ring Cyclotron
(4 sector SS cyclotron)

R_{inj}	89 cm
R_{ext}	356 cm
K	540 MeV
Ion	$^{238}U^{35+}$
Injection energy	0.67 MeV/u
Extraction energy	10.8 MeV/u
RF frequency	18.25 MHz
Harmonic number	9
Acceleration voltage	0.28 MV/turn (no FT)
Bean intensity*	1 ~ 10 pμA
Emittance#	10 π mm mrad (±2σ)
Momentum spread#	±0.2% (±2σ)
Phase width#	±3 deg. (±2σ)

*Present operating intensity : ~1.5 pμA
*$4\lambda\omega^{-2} = 1$ for 4.5 pμA at injection
at injection



Example of PIC simualtion : Beam Quality at Extraction



Difficulty of Heavy-ion Cyclotrons

Separated turns are not easily obtained in many heavy-ion cyclotrons.

Average turn separation in cyclotrons

$$\Delta R = \frac{qeV_{acc}}{mc^2}\frac{\gamma}{(\gamma^2-1)\nu_r^2}R \longleftarrow \frac{dp}{p} = (1+\frac{R}{B}\frac{dB}{dR})\frac{dR}{R} = \nu_r^2\frac{dR}{R}$$

V_{acc} : acceleration voltage per turn
ΔR : turn separation

$\underline{\text{high } V_{acc} \text{ \& low } \vec{B}}$ is not satisfied for heavy-ion cyclotrons.

	PSI Injector 2	PSI Ring	MSU K1200	RIKEN RRC	RIKEN SRC
B	0.33 ~ 0.36	0.58 ~ 0.78	3.0 ~ 5.3	0.9	1.5
V	1	3	0.5	0.28*	2
R	3.3	4.45	1.03	3.56	5.36

** in case of 18.25 MHz operation*

Space Charge Simulation including Neighboring Turns

Essential for heavy-ion cyclotrons because of their small turn separations.
Pioneered by Yang et al. using OPAL-cyclo.

J. J. Yang et al., Phys. Rev. ST Accel. Beams 13 (2010) 064201.

BEAM DYNAMICS IN HIGH INTENSITY . . . Phys. Rev. ST Accel. Beams **13,** 064201 (2010)

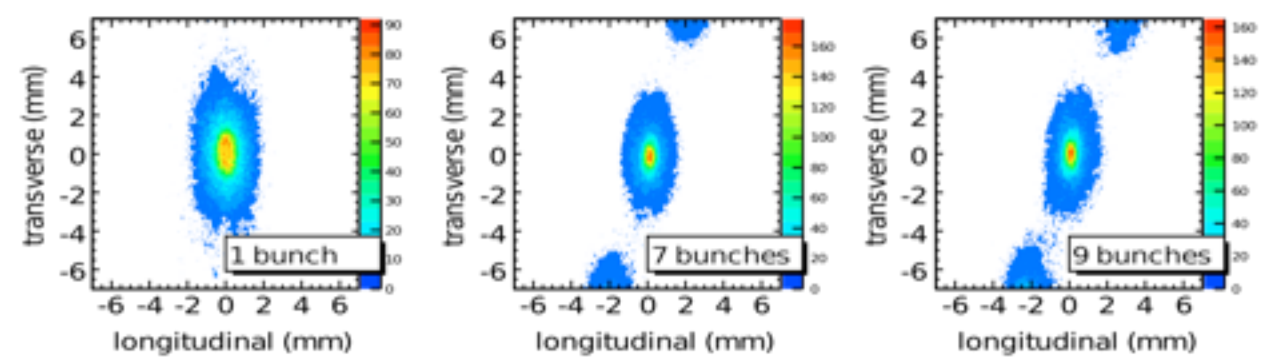


FIG. 11. (Color) Top view of 1 mA bunch distributions at the turn 130 in the local frame $\mathbf{S}_{local}$ at the 112° azimuthal position of turn 130 in the PSI ring cyclotron. The results are obtained from single bunch (left), seven bunches (middle), and nine bunches (right) simulations, respectively.

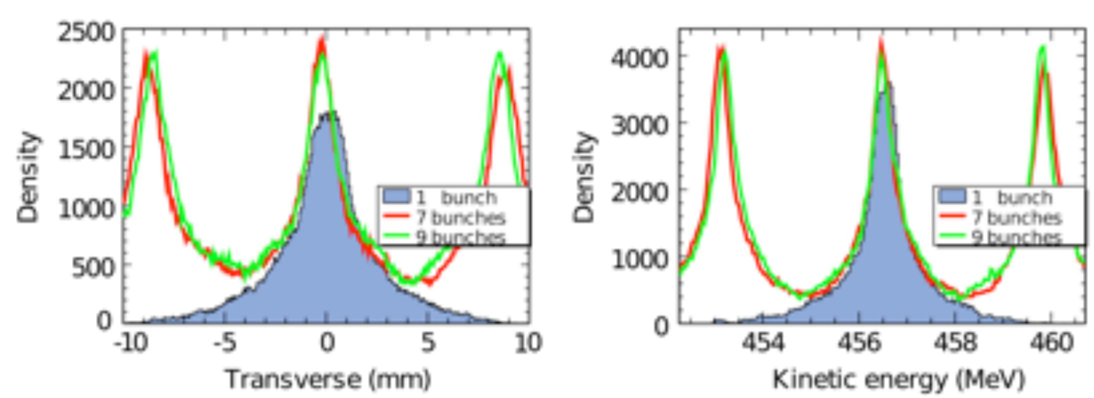


FIG. 12. (Color) Comparison of the histograms along the transversal direction in the local frame $\mathbf{S}_{local}$ (left) and the energy spectra (right) of 1 mA beam at the 112° azimuthal position of turn 130 in the PSI ring cyclotron.

Up to 9 bunches : “viewed as converged”

Reduction of the transverse beam profile and the energy spread

Results of Multi-bunch Simulation

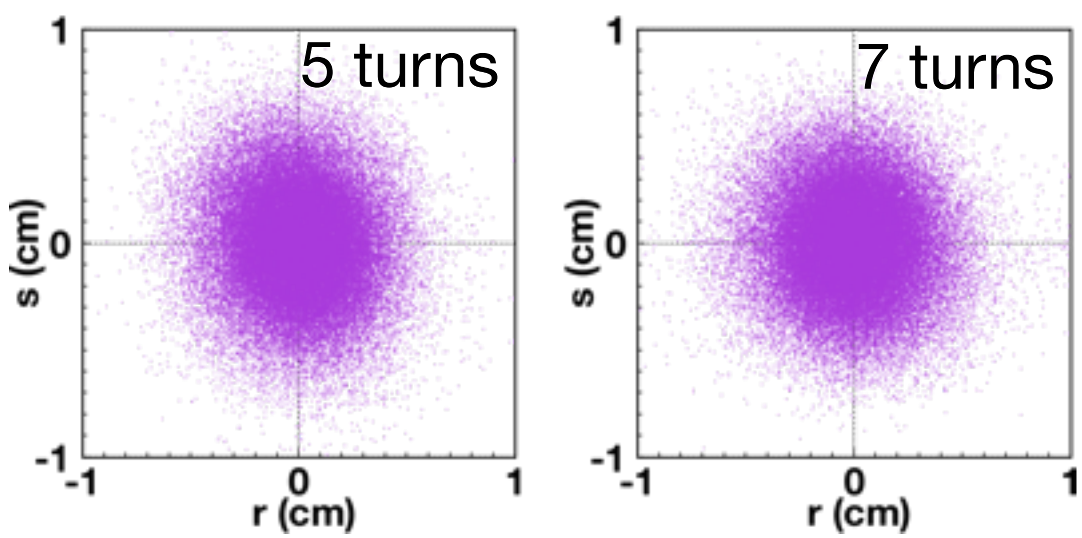
10.8A-MeV uranium beam at RRC

V = 0.28 MV/turn at 18.25 MHz
(present)

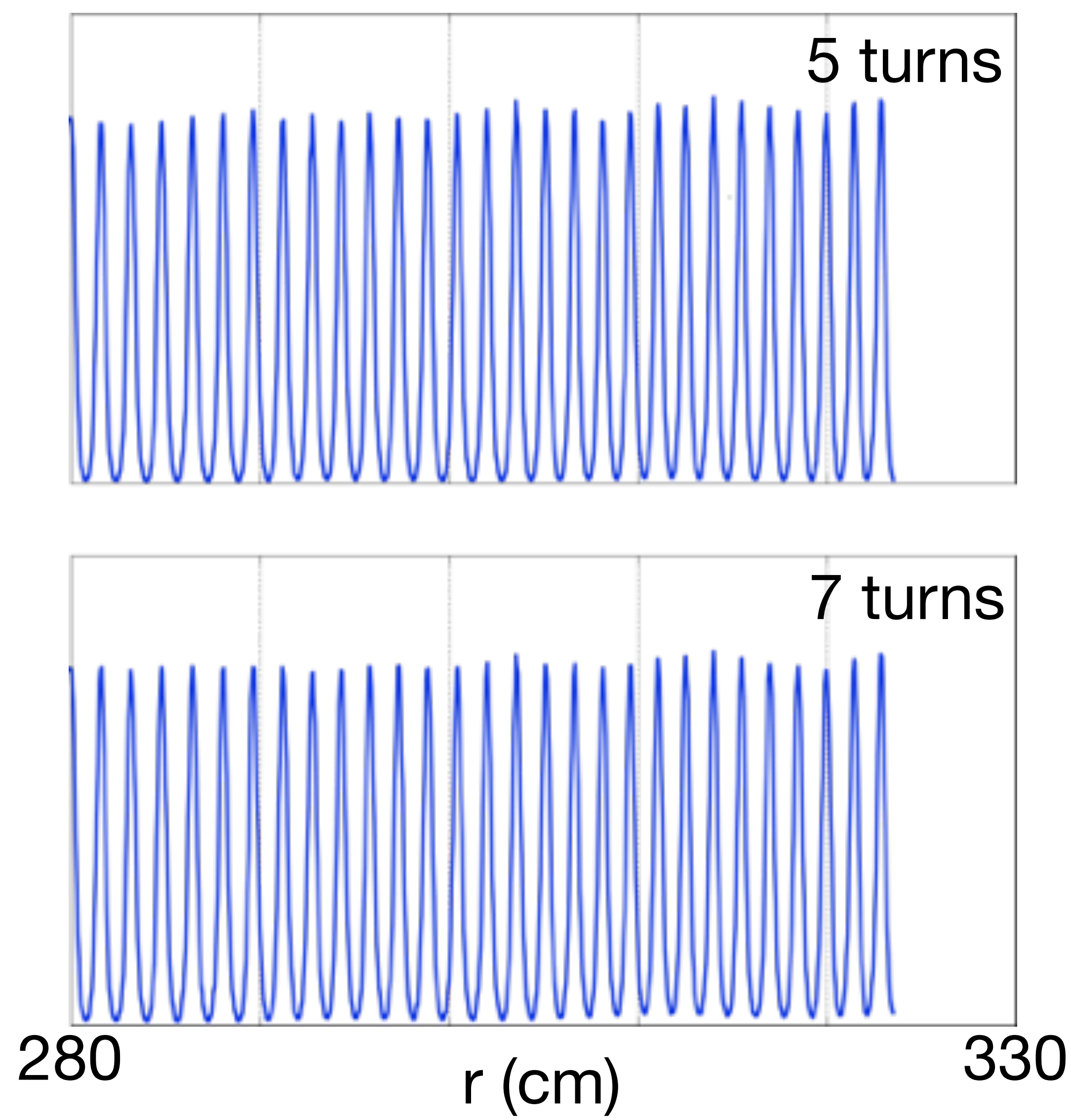
↕

V = 0.48 MV/turn at 24.33 MHz
(planned)

Distribution at extraction (2 pμA)



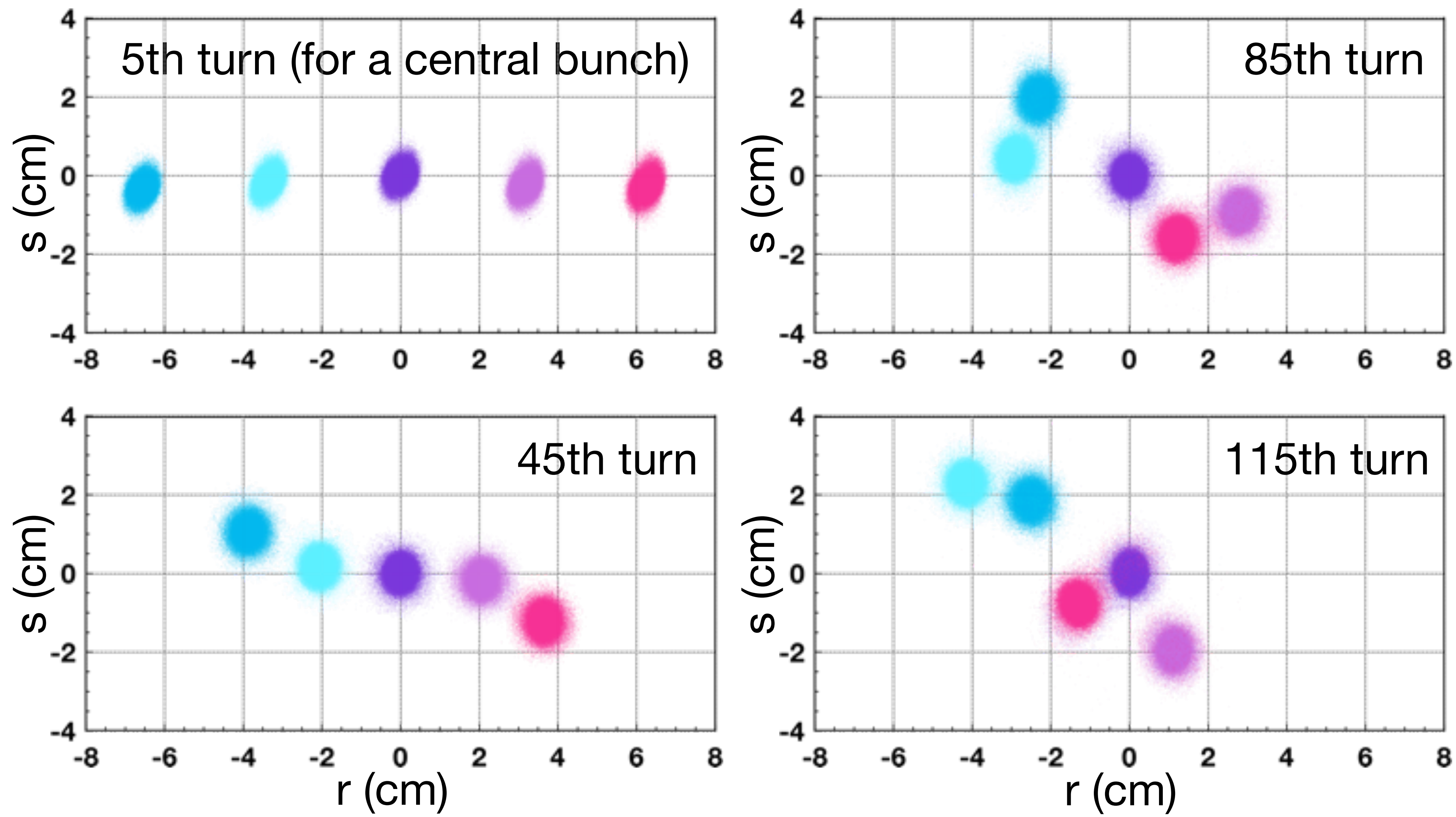
Turn pattern (2 pμA)



Simulation results fluctuate depending on the number of turns included.

Simulation Artifact Caused by a Insufficient Number Turns

5 pμA case



The outermost bunch moves backward and is decelerated by inner bunches.

Summary

Heavy-ion cyclotrons have pioneered the energy frontier of CW accelerators.

Beam intensity becomes a crucial issue in heavy-ion cyclotrons.

Problems such as charge-state strippers for very heavy ions and beam losses at extraction are not yet solved.

To obtain higher intensity beams, upgrades of acceleration voltages are important because all the currently working heavy-ion cyclotrons do not have sufficient turn separations.

Deeper understanding of the space charge effect, especially effects caused by neighboring turns is necessary.

Energy upgrade to 1-GeV/u region is a possible challenge for future heavy-ion cyclotrons.

Backup

Toy Model vs PIC Simulation

Toy model : 1 bunch = 1 macro particle

Motion of bunch centers relative to the central bunch