

Simulation of Laser Cooling of Heavy Ion Beams at High Intensities

L. Eidam, D. Winters, O. Boine-Frankenheim

Contents

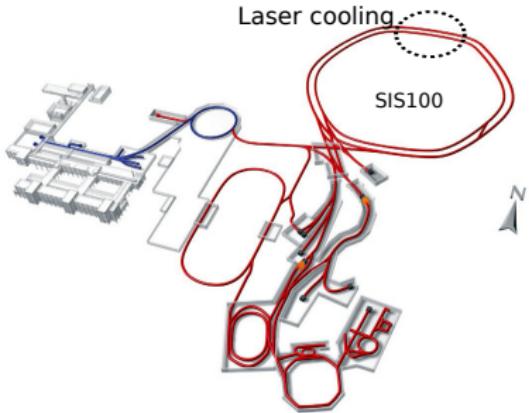


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- ▶ Motivation
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Motivation

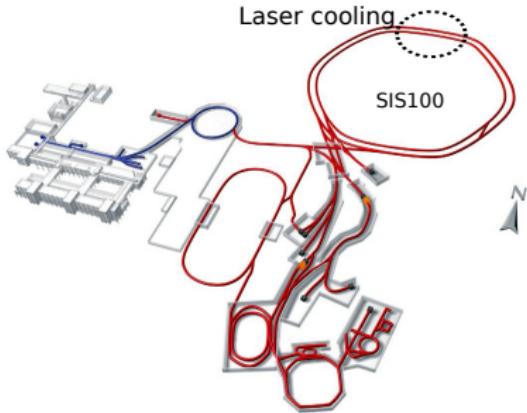
Laser Cooling at FAIR



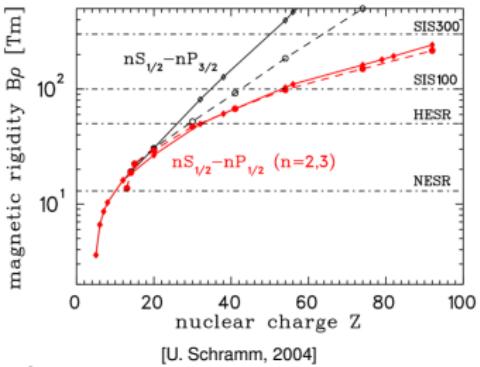
- ▶ first time of laser cooling at high beam energies
- ▶ high beam intensities available

Motivation

Laser Cooling at FAIR



Doppler shift: $\lambda' = \frac{\lambda_L}{\gamma(1-\beta)}$
for Li-like ions: ($\lambda_L = 257 \text{ nm}$)

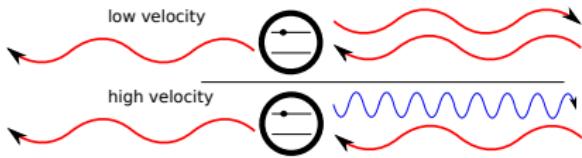


- ▶ first time of laser cooling at high beam energies
- ▶ high beam intensities available
- ▶ high magnetic rigidity allows cooling of heavy ions

Cooling at High Energies



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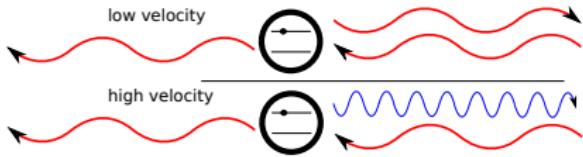


- ▶ $\langle \Delta p_{lab} \rangle = \frac{\hbar \omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta)$
- ▶ $\frac{\langle \Delta p_{lab} \rangle}{p}$ nearly constant

Cooling at High Energies

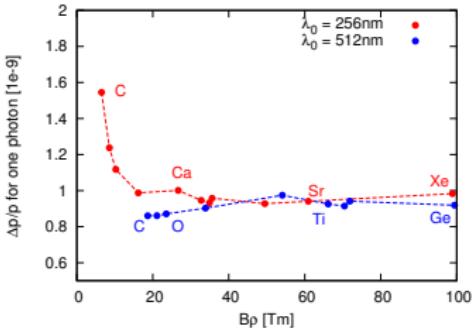


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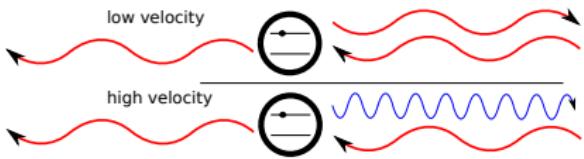
Li-like (2p1/2 - 2s1/2):



Cooling at High Energies

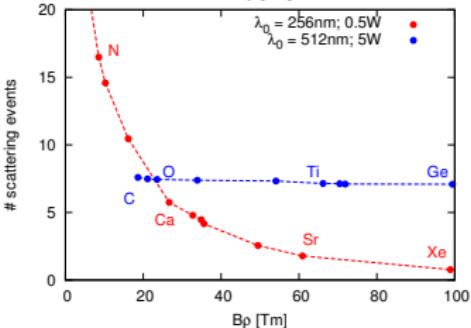
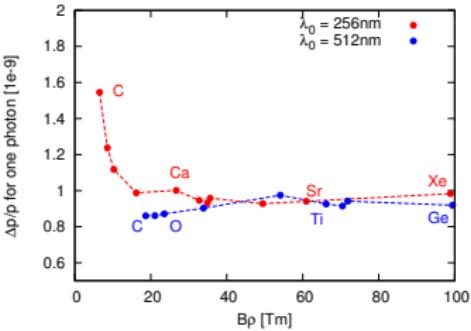


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- ▶ $\langle \Delta p_{lab} \rangle = \frac{\hbar \omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta)$
- ▶ $\frac{\langle \Delta p_{lab} \rangle}{p}$ nearly constant
- ▶ for low/high energies ions with similar cooling force exist
⇒ pick the right ion & transition
- ▶ adiabatic damping
⇒ no pre-cooling necessary

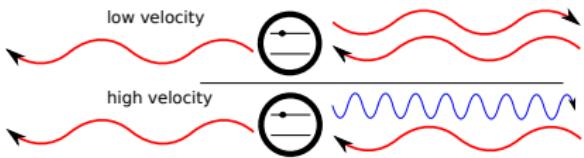
Li-like (2p1/2 - 2s1/2):



Cooling at High Energies



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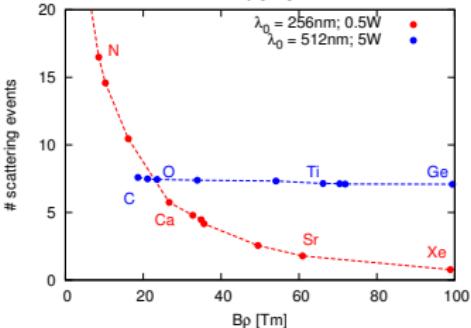
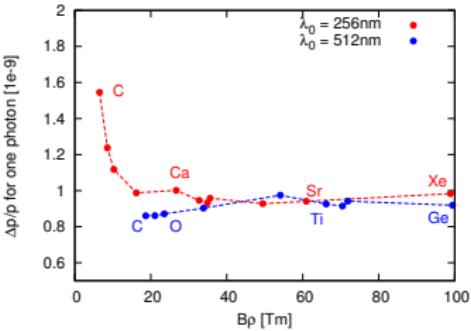


- ▶ $\langle \Delta p_{lab} \rangle = \frac{\hbar \omega_{lab}}{c_0} \cdot \gamma^2 (1 + \beta)$
- ▶ $\frac{\langle \Delta p_{lab} \rangle}{p}$ nearly constant
- ▶ for low/high energies ions with similar cooling force exist
⇒ pick the right ion & transition
- ▶ adiabatic damping
⇒ no pre-cooling necessary

▶ speed of synchrotron motion:

$$Q_s = -\frac{L\eta}{2\pi} \frac{\delta_0}{\hat{z}_0} \text{ with } \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

Li-like (2p1/2 - 2s1/2):



Contents



- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Simulation Tool

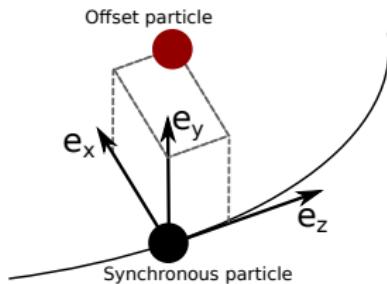


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Solve Vlasov-Fokker-Planck Equation:

$$\frac{\partial f}{\partial t} - \eta \beta c \delta \frac{\partial f}{\partial z} + \frac{qV(z,t)}{p_0} \frac{\partial f}{\partial \delta} = \left(\frac{\partial f}{\partial \delta} \right)_c$$

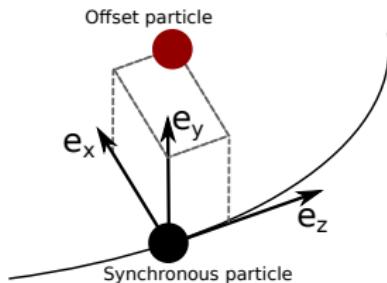
- ▶ use macro particles (PIC-code)
- ▶ macro particles : $N_{macro} \approx 10^5 - 10^7$
- ▶ discretization of time (T_{rev})



Solve Vlasov-Fokker-Planck Equation:

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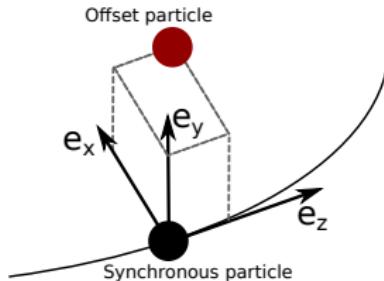
- ▶ use macro particles (PIC-code)
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- ▶ assume fast transverse motion is unaffected by cooling process
⇒ 1D longitudinal problem



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- ▶ assume fast transverse motion is unaffected by cooling process
⇒ 1D longitudinal problem



No direct coulomb interactions
⇒ results not exact at very low temperatures
⇒ concentrate on cooling process

Simulation Tool Modeling of Laser Force

simple two level system in electromagnetic mode \Rightarrow optical Bloch equations

$$\dot{\rho} = \frac{i}{\hbar} [\rho, H] - \frac{1}{\tau} \rho \quad \text{with} \quad H = \begin{pmatrix} \hbar\omega_e & (\hbar\frac{\Omega}{2})e^{-i\omega t} \\ (\hbar\frac{\Omega}{2})e^{i\omega t} & \hbar\omega_g \end{pmatrix}; \rho = \begin{pmatrix} \rho_{ee} & \rho_{eg} \\ \rho_{ge} & \rho_{gg} \end{pmatrix}$$

Simulation Tool

Modeling of Laser Force

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- ▶ spontaneous emission rate:

$$k(\delta) = \frac{1}{\tau} \cdot \rho_{ee}(\delta)$$

- ▶ cooling Force:

$$F_{Laser}(\delta) = \Delta p_{lab} \cdot k(\delta)$$

Simulation Tool

Modeling of Laser Force



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- ▶ if $U_j \leq \frac{\Delta t_0}{\tau} \rho_{ee}(\delta_j)$: $\Delta\delta = \Delta p_{lab}/p_0$
 U_j : uniform random number
- ▶ $\Delta p_{lab} = \frac{\hbar\omega_{lab}}{c_0} \cdot \gamma^2(1 + \beta) \cdot 2U_2$

Simulation Tool Modeling of Laser Force

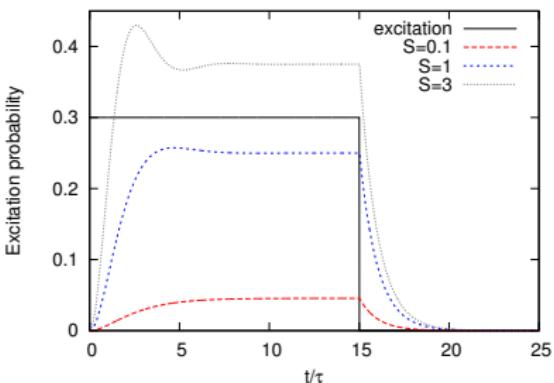


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$$\rho_{ee}^{Sat} = \frac{1}{2} \frac{S}{1+S+(2\Delta\nu\cdot\tau)^2}$$

Simulation Tool

Modeling of Laser Force



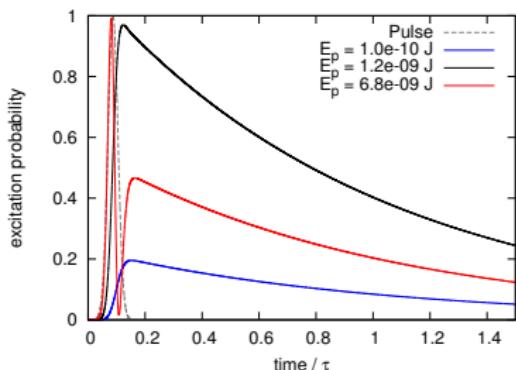
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pulsed laser:



Contents



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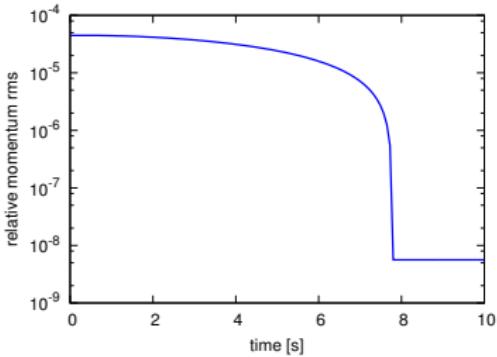
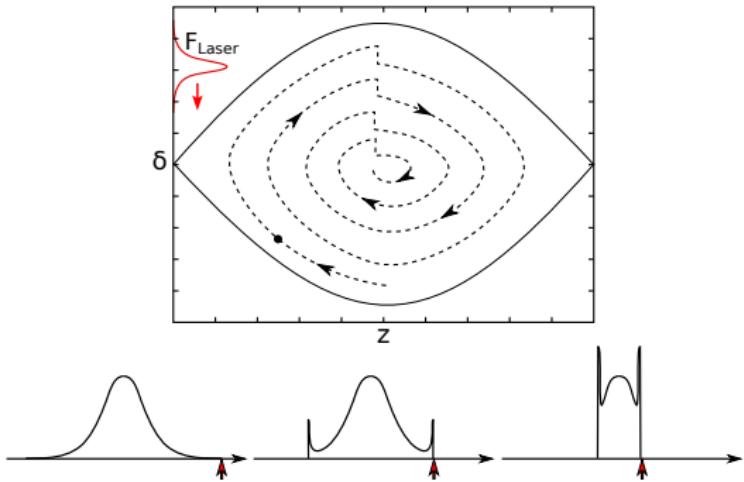
- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Cooling at Low Intensities

Cooling Process



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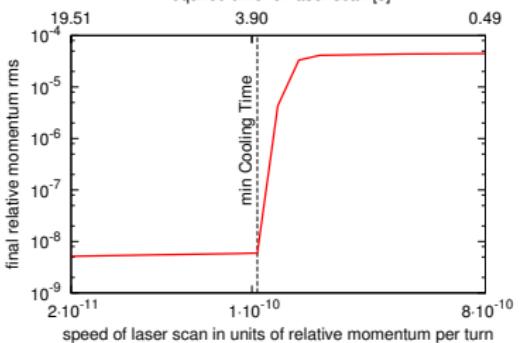
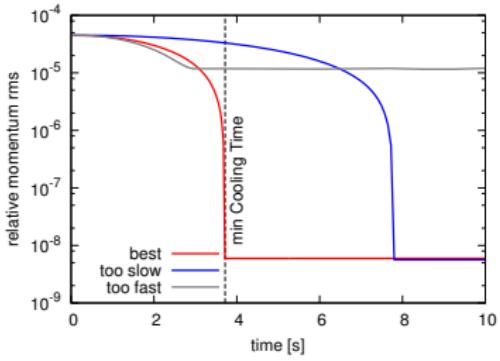
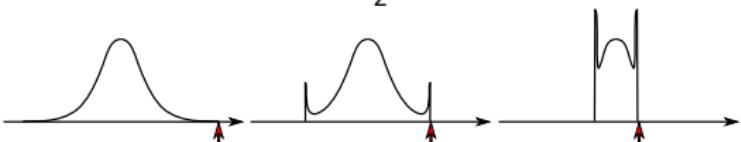
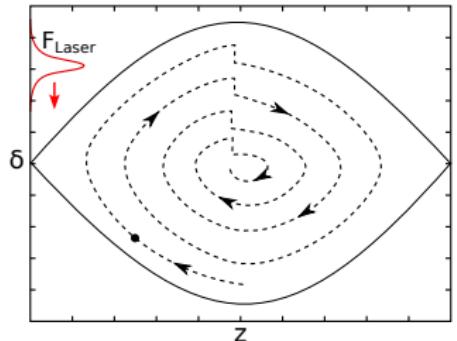


Cooling at Low Intensities

Cooling Process



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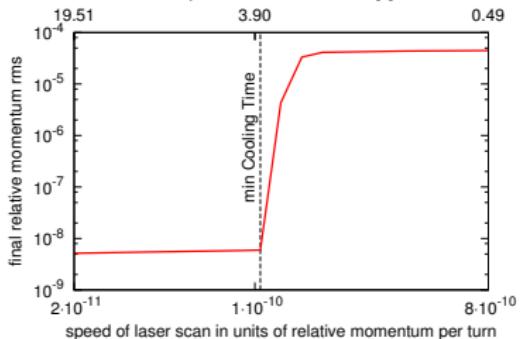
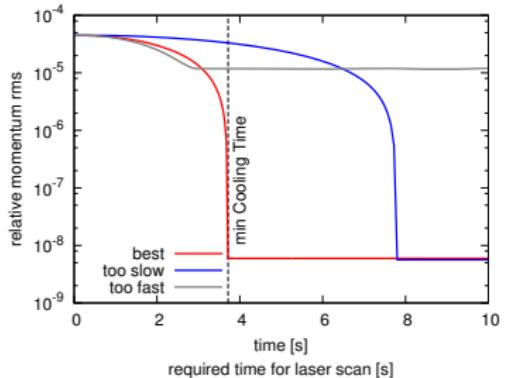
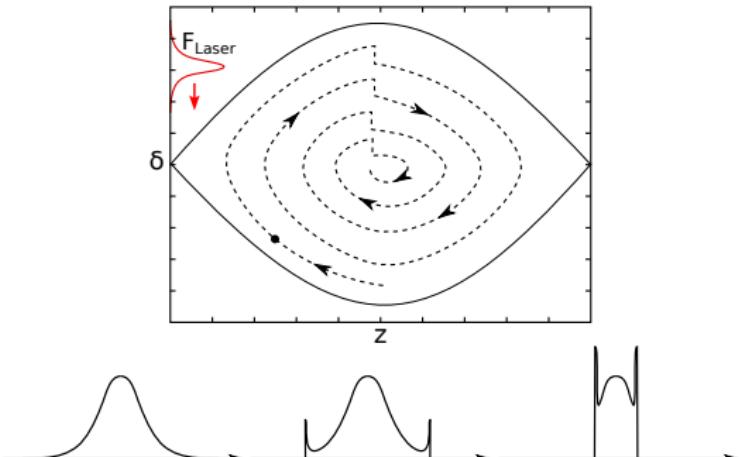


Cooling at Low Intensities

Cooling Process



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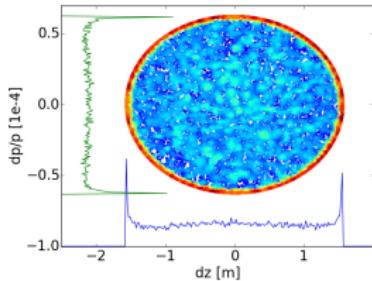
- ▶ influence of synchrotron frequency?
Synchrotron tune:

$$Q_s = \frac{-L\eta \cdot \hat{\delta}}{2\pi^2} \text{ with } \eta = \frac{1}{\gamma_t^2} - \frac{1}{\gamma^2}$$

Cooling at Low Intensities

Dependence of Synchrotron Tune

high Q_s ($\approx 10^{-3}$)



- ▶ verified in storage rings
- ▶ symmetric reduction of momentum spread and bunch length

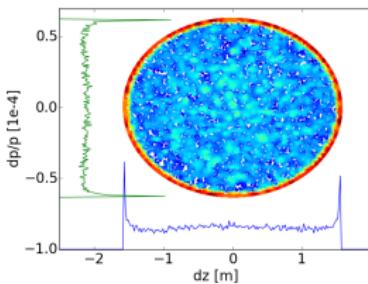
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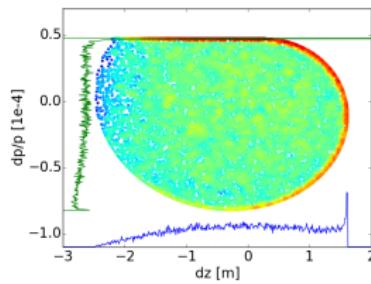


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high Q_s ($\approx 10^{-3}$)



medium Q_s ($\approx 10^{-4}$)



- ▶ verified in storage rings
- ▶ symmetric reduction of momentum spread and bunch length
- ▶ particles are captured by laser force
- ▶ non-symmetric

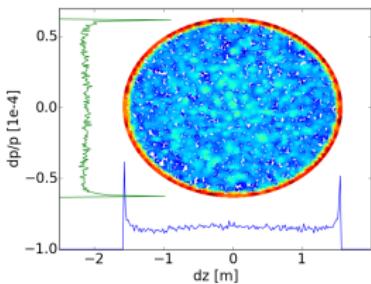
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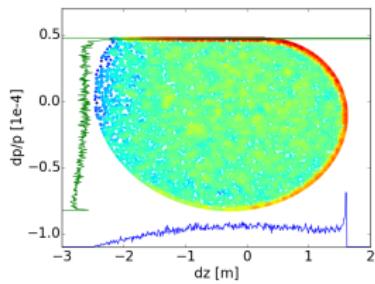


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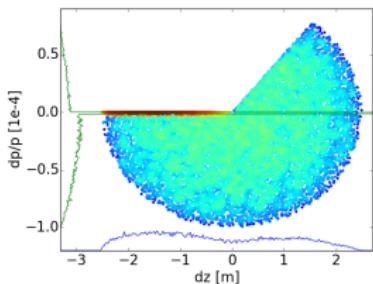
high Q_s ($\approx 10^{-3}$)



medium Q_s ($\approx 10^{-4}$)



low Q_s ($\approx 10^{-5}$)



- ▶ verified in storage rings
- ▶ symmetric reduction of momentum spread and bunch length

- ▶ particles are captured by laser force
- ▶ non-symmetric

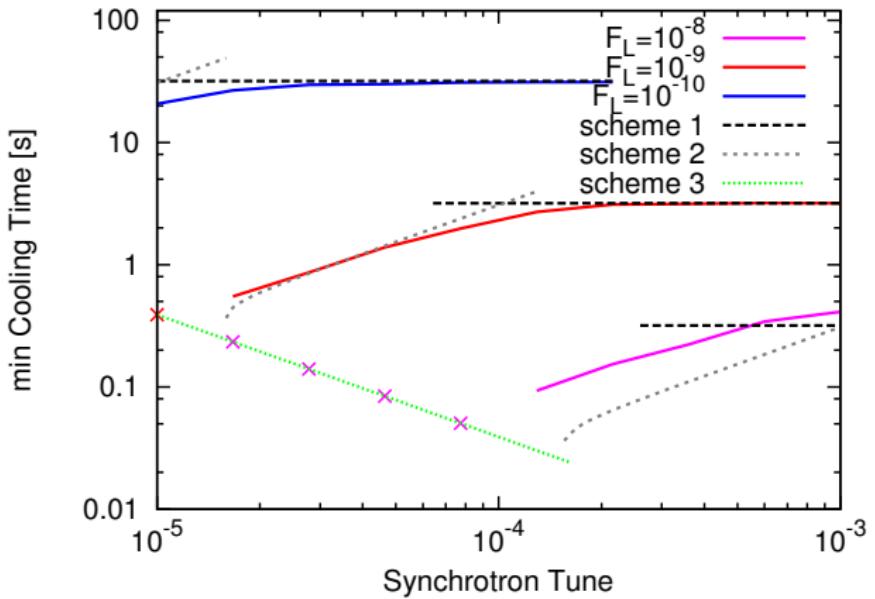
- ▶ laser position is fixed
- ▶ laser counteracts rf kick
- ▶ particles are rotated into laser force

Cooling at Low Intensities

Required Cooling Time



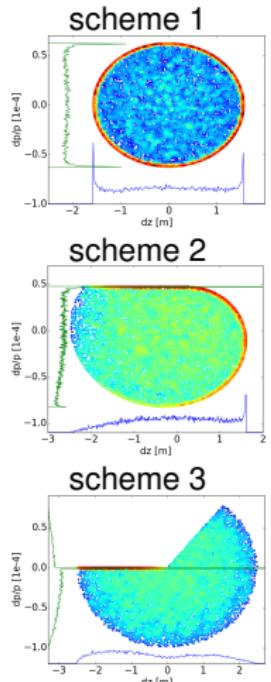
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$$\delta_{Laser} = 10^{-9}$$
$$T_{excitation} = 10\tau$$

$$\Delta_{FWHM} = 10^{-7}$$

$$\hat{\delta} = 10^{-4}$$
$$\hat{z} = 2.5 \text{ m}$$



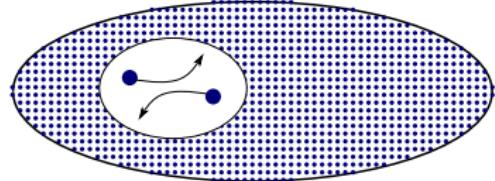
Contents



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- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

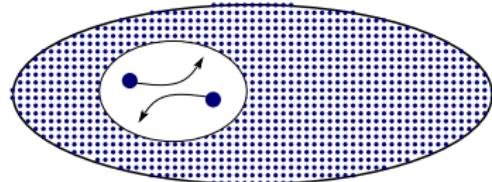
Intra Beam Scattering



- ▶ transverse oscillations heat longitudinal motion

Space Charge

Intra Beam Scattering

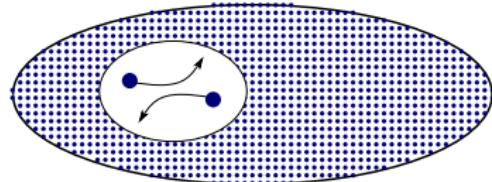


Space Charge

- ▶ transverse oscillations heat longitudinal motion
- ▶ described by diffusion in FPE

$$\delta_{IBS} = Q_j \cdot \sqrt{2D_{zz} \frac{L_{acc}}{\beta c_0}}$$

Intra Beam Scattering



Space Charge

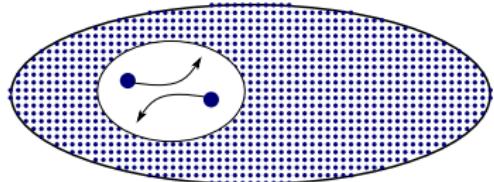
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- ▶ non-Gaussian distribution \Rightarrow local diffusion model (BETACOOL)

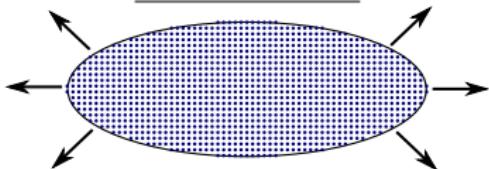
$$D_{zz} = \frac{e^4 Z^4 \Lambda \sqrt{2}}{128\pi m_i^2 \epsilon_0^2 c_0^3 \gamma^3 \beta^3 \langle \beta^{1/2} \rangle \epsilon_{\perp}^{3/2}} \cdot n$$

Intra Beam Scattering



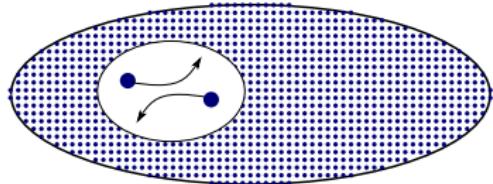
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Space Charge



- ▶ describes interaction of particle with electric field of the bunch in perfectly conducting pipe

Intra Beam Scattering



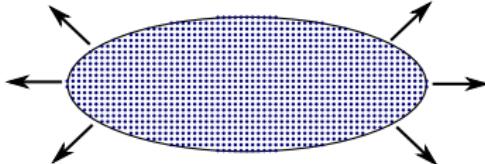
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Space Charge



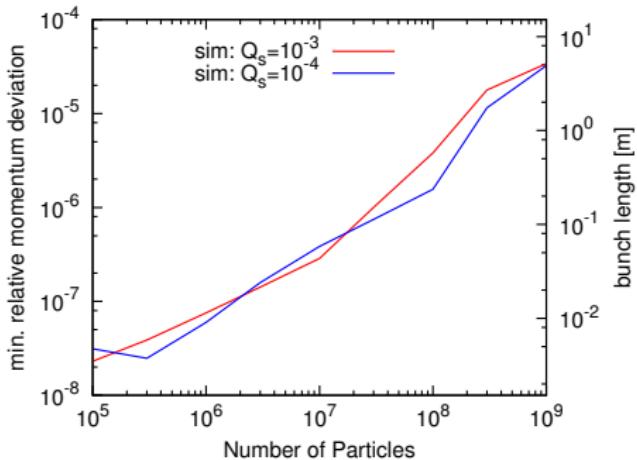
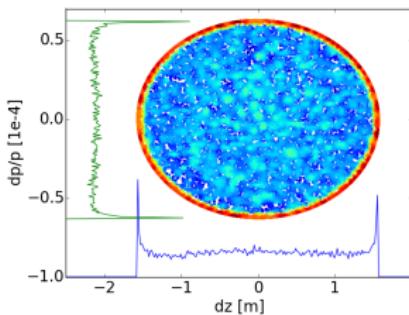
- ▶ describes interaction of particle with electric field of the bunch in perfectly conducting pipe
- ▶ space charge impedance:
 $Z_{sc} = -i \cdot f \frac{g L_{acc}}{2\epsilon_0 \beta^2 c^2 \gamma^2}$ with
 $g = 1 + 2 \log(b/a)$
- ▶ potential:
 $U_{sc}(z) = Z_{ion} \cdot q \cdot IFFT\{Z_{sc}(\omega) \cdot FFT\{\lambda(t)\}\}$

Intensity Effects Intra Beam Scattering (IBS)



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high synchrotron tune $Q_s \approx 10^{-3}$



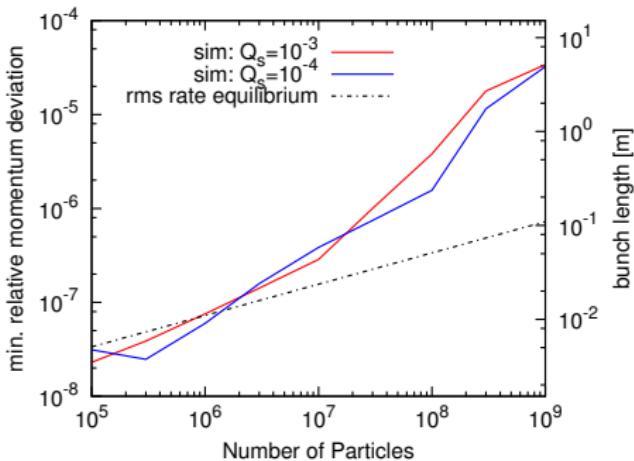
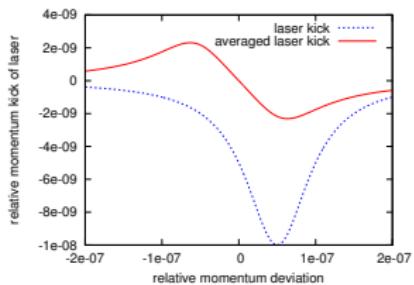
Intensity Effects Intra Beam Scattering (IBS)



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high synchrotron tune $Q_s \approx 10^{-3}$

rms rate equilibrium:



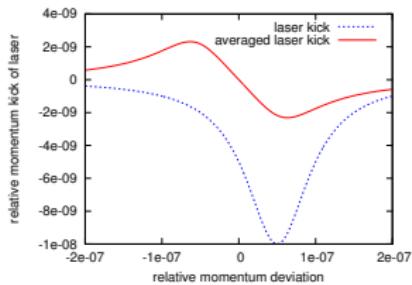
Intensity Effects Intra Beam Scattering (IBS)



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high synchrotron tune $Q_s \approx 10^{-3}$

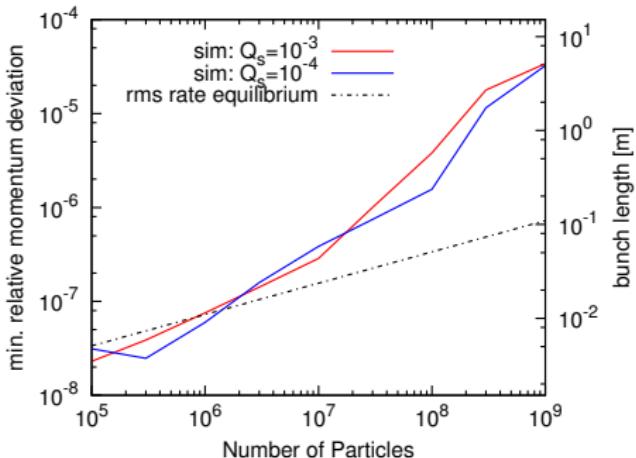
rms rate equilibrium:



$$\tau_{IBS}^{-1} + \tau_{Laser}^{-1} = 0$$

$$\tau_{Laser}^{-1} = \frac{1}{T_{rev}} \cdot \left. \frac{\partial \langle \Delta\delta \rangle_{syn}}{\partial \delta_{pos}} \right|_0$$

$$\langle \Delta\delta \rangle_{syn} = \frac{1}{\pi} \int_0^\pi \cos(\phi) \cdot \Delta\delta(\delta_{pos} \cdot \cos(\phi)) d\phi$$



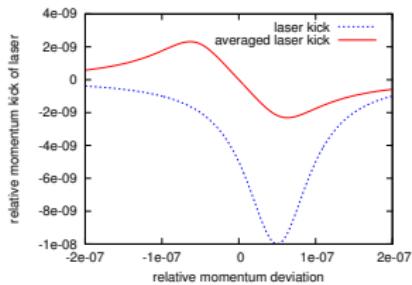
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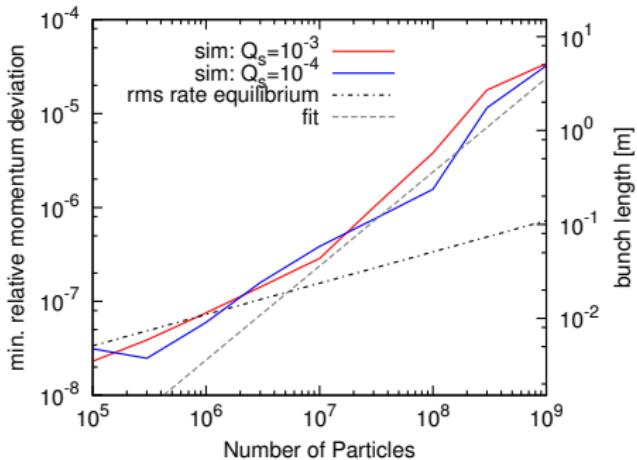
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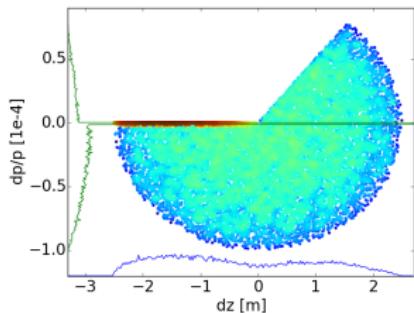
$$\text{fit: } \langle \delta_{IBS} \rangle = f(\delta_{laser}) = \text{const.}$$
$$D_{\parallel \max}^{IBS} \propto \frac{N}{L_{bunch}} \propto \frac{N}{\delta_{rms}}$$

Intensity Effects Intra Beam Scattering (IBS)

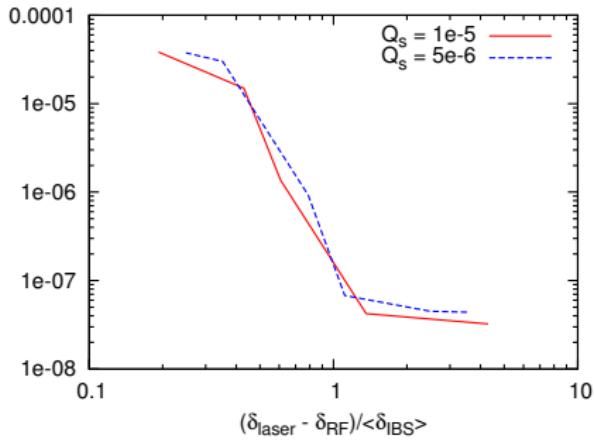


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low synchrotron tune $Q_s \approx 10^{-5}$



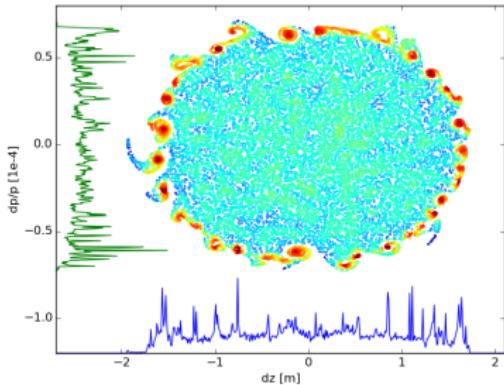
minimal relative momentum deviation



- ▶ limit: $\frac{\hat{\delta}_{laser} - \hat{\delta}_{rf}}{\langle \delta_{IBS} \rangle} > 1$ (changed $N_p \rightarrow \langle \delta_{IBS} \rangle$)
⇒ Barrier of laser has to be higher than IBS kicks
- ▶ longitudinal density increases only by factor 2

Intensity Effects Space Charge

high synchrotron tune $Q_s \approx 10^{-3}$

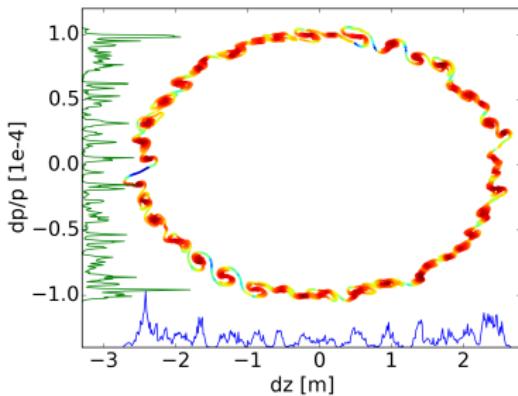
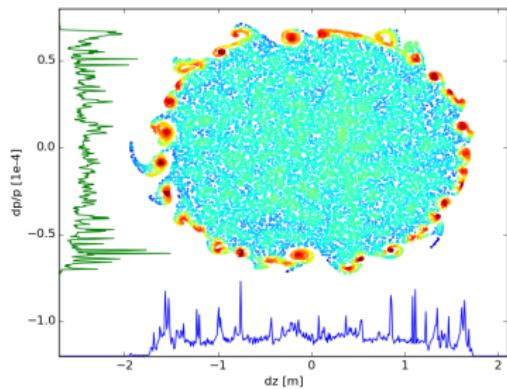


Intensity Effects Space Charge



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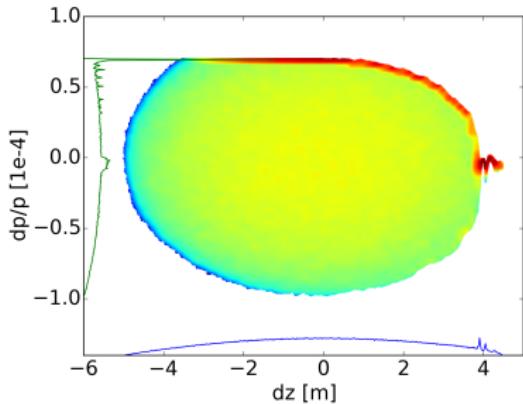


kind of two stream instability:

- ▶ perturbation in one stream produces bunching of second
 ⇒ bunching increases amplitude of perturbation
- ▶ in progress ⇒ comments are welcome

Intensity Effects Space Charge

medium synchrotron tune $Q_s \approx 10^{-4}$

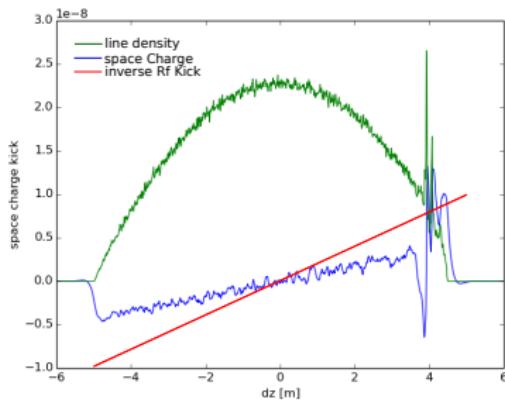
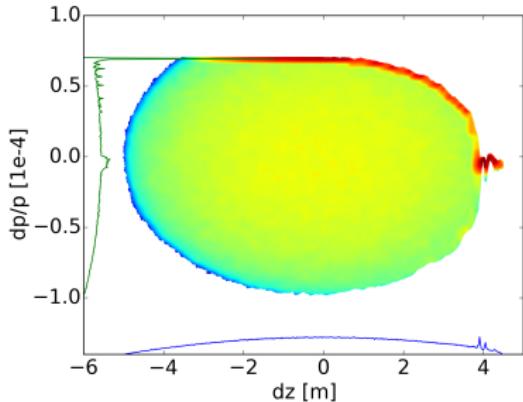


Intensity Effects Space Charge



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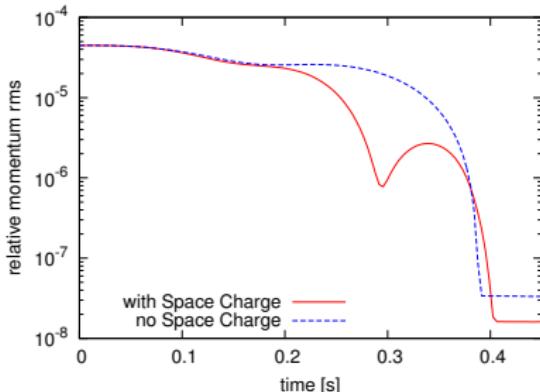
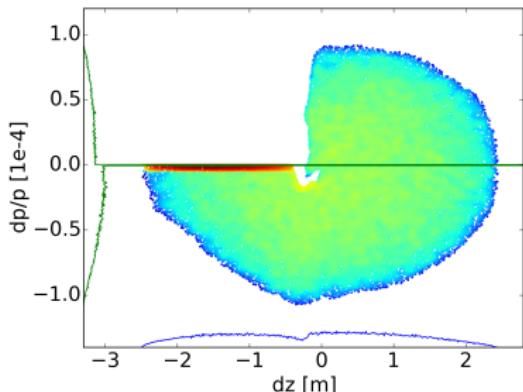
- ▶ space charge compensates Rf-kick
⇒ synchrotron oscillation is stopped

Intensity Effects Space Charge



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low synchrotron tune $Q_s \approx 10^{-5}$



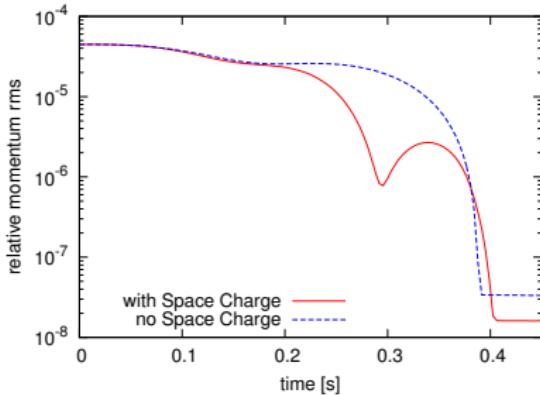
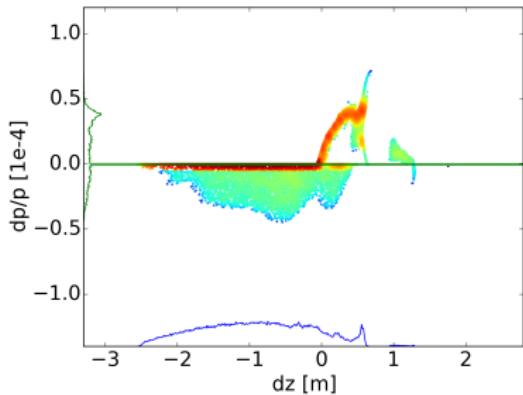
- ▶ space charge deforms bunch

Intensity Effects Space Charge



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low synchrotron tune $Q_s \approx 10^{-5}$



- ▶ space charge deforms bunch
- ▶ if deformation too strong \Rightarrow space charge exceeds laser force (not stable)

Contents



- ▶ Introduction
- ▶ Simulation Tool
- ▶ Cooling at Low Intensities
- ▶ Intensity Effects
- ▶ Conclusion and Outlook

Conclusion and Outlook



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- ▶ laser cooling looks promising at high beam energies
- ▶ cooling time and efficiency depend strongly on the way of laser scan
- ▶ IBS limits the beam intensity
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Thank you for your attention!