

System Identification of superconducting cavity parameters with recursive least-squares algorithms

Volker Ziemann, Jefferson Lab
Tor Lofnes, FREIA, Uppsala University

LLRF workshop, Newport News, VA

Disclaimer: presentation based on previously published work

PHYSICAL REVIEW ACCELERATORS AND BEAMS **26**, 112003 (2023)

Simulations of real-time system identification for superconducting cavities
with a recursive least-squares algorithm

Volker Ziemann^{*}
Uppsala University, 75120 Uppsala, Sweden

PHYSICAL REVIEW ACCELERATORS AND BEAMS **27**, 032001 (2024)

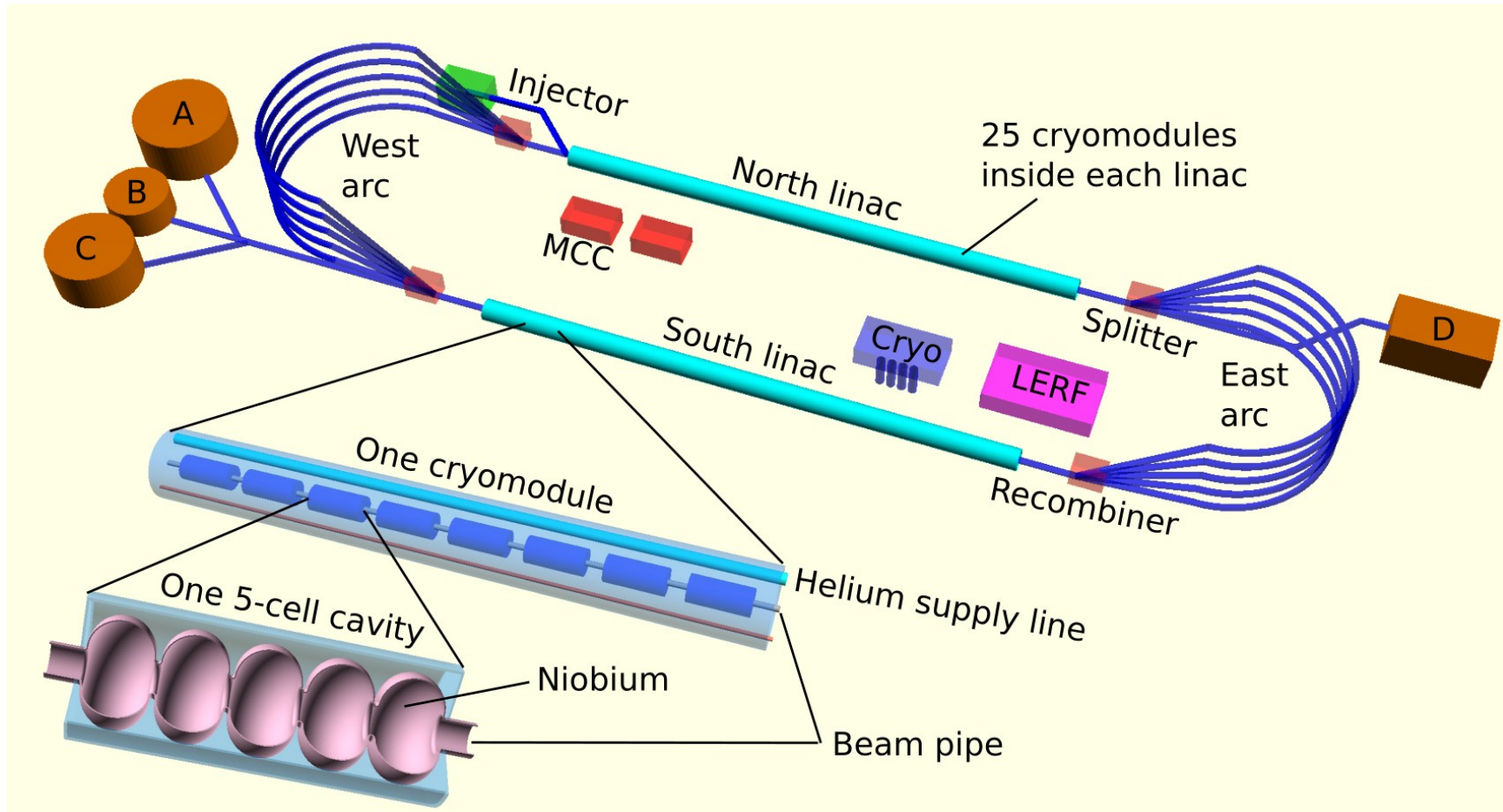
New method to measure the unloaded quality factor
of superconducting cavities

Volker Ziemann^{*}
Uppsala University, 75120 Uppsala, Sweden

System identification

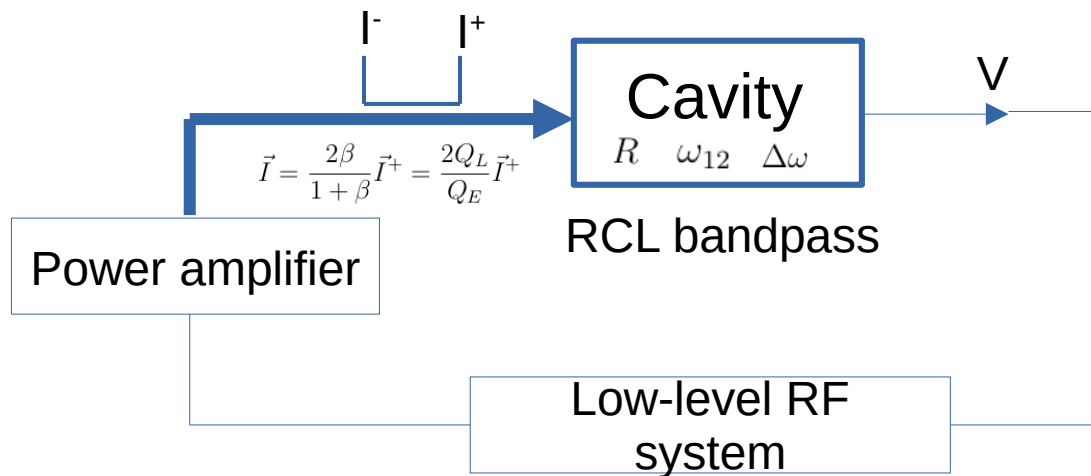
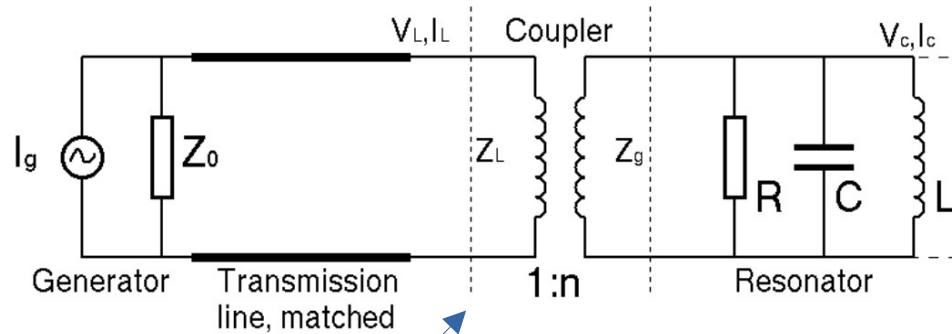
- Conceptually simplest case:
 - Determine system a in $y_i = a x_i$
 - from input x_i and output y_i
 - Solution: $a = \sum_i x_i y_i / \sum_i x_i^2$
- Only a little more tricky if a is a matrix and x and y are vectors

System Identification of Superconducting Cavities



SysId: determine detuning $\Delta\omega$ and bandwidth ω_{12} , maybe Q_0

Cavity model



Dynamical system of envelopes
real and imaginary, I and Q

$$\begin{pmatrix} \frac{dV_r}{dt} \\ \frac{dV_i}{dt} \end{pmatrix} = \begin{pmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{pmatrix} \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} \omega_{12}R & 0 \\ 0 & \omega_{12}R \end{pmatrix} \begin{pmatrix} I_r \\ I_i \end{pmatrix}$$

Express I through forward current I^+

$$\begin{pmatrix} \frac{dV_r}{dt} \\ \frac{dV_i}{dt} \end{pmatrix} = \begin{pmatrix} -\omega_{12} & -\Delta\omega \\ \Delta\omega & -\omega_{12} \end{pmatrix} \begin{pmatrix} V_r \\ V_i \end{pmatrix} + \begin{pmatrix} \omega_E R & 0 \\ 0 & \omega_E R \end{pmatrix} \begin{pmatrix} I_r^+ \\ I_i^+ \end{pmatrix}$$

$$\omega_{12} = \hat{\omega}/2Q_L$$

$$\omega_E = \hat{\omega}/Q_E$$

Discretize with time step Δt

$$\vec{V}'_{t+1} = (1 + F) \vec{V}'_t + B \vec{I}_t^+ \quad \text{with} \quad F = \begin{pmatrix} -\omega_{12}\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\omega_{12}\Delta t \end{pmatrix}$$

$B = \omega_E \Delta t R$

Isolate the known and the unknown

$$\vec{y}_{t+1} = \vec{V}'_{t+1} - \vec{V}'_t - B \vec{I}_t^+ = F \vec{V}'_t$$

Least squares

Rewrite the mixture of unknowns and known voltages to separate the unknowns

$$\vec{y}_{t+1} = \vec{V}'_{t+1} - \vec{V}'_t - B\vec{I}_t^+ = F\vec{V}'_t$$

$$\vec{y}_{t+1} = G_t \vec{q}$$

$$F\vec{V}'_t = \begin{pmatrix} -\omega_{12}\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\omega_{12}\Delta t \end{pmatrix} \begin{pmatrix} V'_r \\ V'_i \end{pmatrix}_t = \begin{pmatrix} -V'_r & -V'_i \\ -V'_i & V'_r \end{pmatrix}_t \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix} = G_t \vec{q}$$

Stack multiple copies, one for each time step

$$\begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix} = U_T \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix} \quad \text{with} \quad U_T = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_T \end{pmatrix}$$

Solve in the least-squares sense with the Moore-Penrose pseudo inverse

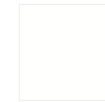
$$\vec{q}_T = \begin{pmatrix} \omega_{12}\Delta t \\ \Delta\omega\Delta t \end{pmatrix}_T = (U_T^\top U_T)^{-1} U_T^\top \begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix}$$

Tedious to calculate $(U_T^\top U_T)^{-1} U_T^\top$ for quickly growing U_T

Recursive least squares

Introduce $P_T^{-1} = U_T^\top U_T$, the inverse of empirical covariance matrix.

$$\begin{aligned} P_{T+1}^{-1} &= U_{T+1}^\top U_{T+1} \\ &= p_0 \mathbf{1} + G_1^\top G_1 + G_2^\top G_2 + \cdots + G_T^\top G_T + G_{T+1}^\top G_{T+1} \\ &= P_T^{-1} + G_{T+1}^\top G_{T+1}. \end{aligned}$$



$$G_t^\top G_t = (V_r'^2 + V_i'^2)_t \mathbf{1} = \vec{V}_t'^2 \mathbf{1}$$

Obtain the updated quantities p_{T+1} and q_{T+1} at the next time step by absorbing the new information from V' and y .

$$\begin{aligned} p_{T+1} &= \left[\frac{1}{1 + p_T \vec{V}_T'^2} \right] p_T & \vec{q}_{T+1} &= p_{T+1} (G_1^\top \vec{y}_2 + G_2^\top \vec{y}_3 + \cdots + G_T^\top \vec{y}_{T+1} + G_{T+1}^\top \vec{y}_{T+2}) \\ & & &= \left[\frac{1}{1 + p_T \vec{V}_T'^2} \right] p_T \left(\sum_{t=1}^T G_t^\top \vec{y}_{t+1} + G_{T+1}^\top \vec{y}_{T+2} \right) \\ & & &= \left[\frac{1}{1 + p_T \vec{V}_T'^2} \right] (\vec{q}_T + p_T G_{T+1}^\top \vec{y}_{T+2}). \end{aligned}$$

Forgetting factor α

for time-varying systems

- Time horizon N_f and $\alpha = 1 - 1/N_f$ “discount factor”

$$P_{T+1}^{-1} = \alpha P_T^{-1} + G_{T+1}^T G_{T+1}$$

- Update equations for p_T and \vec{q}_T from one time-step to the next

$$p_{T+1} = \left[\frac{1}{\alpha + p_T \vec{V}_T'^2} \right] p_T$$

New y_{T+2} and G_{T+1} come from new currents I and voltages V'

$$\vec{q}_{T+1} = \left[\frac{1}{\alpha + p_T \vec{V}_T'^2} \right] (\alpha \vec{q}_T + p_T \hat{G}_{T+1}^T \vec{y}_{T+2})$$

Simulations

Cavity: 1 GHz, $Q_L=5 \times 10^5$,
 $f_s=10$ MHz, $N_f=100$,
 $\sigma_p=10^{-4} V_{\max}$, $\sigma_m=10^{-3} V_{\max}$
Step in ω_{12}
2 kHz oscillation of $\Delta\omega$

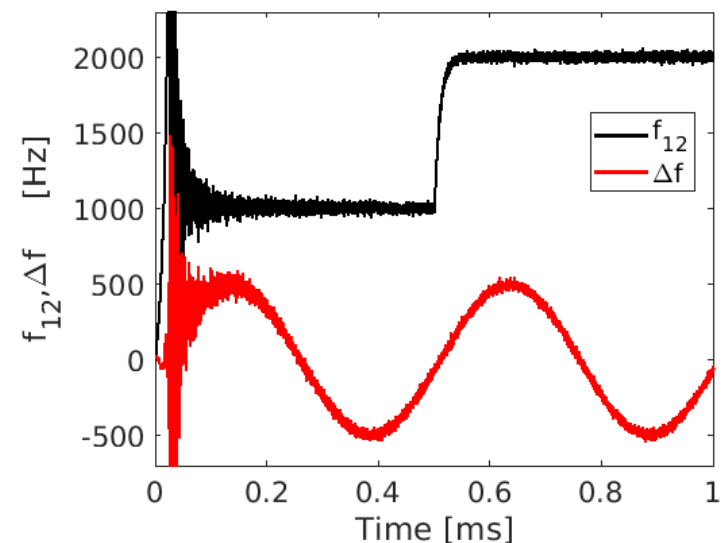
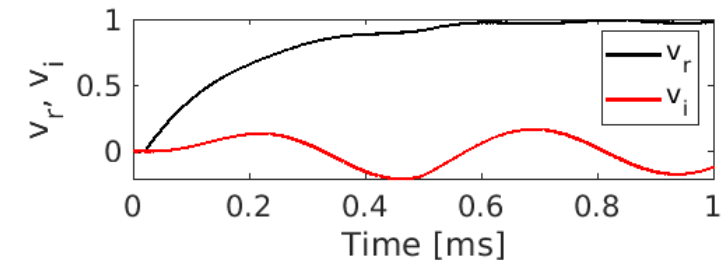
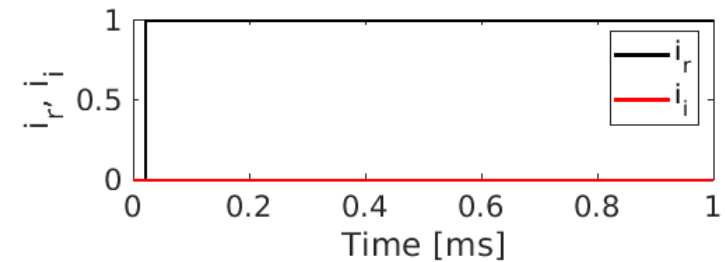
Current step

- Cavity starts charging
- observe wiggle after 0.5 ms
- Q-phase oscillates

Reconstructed parameters

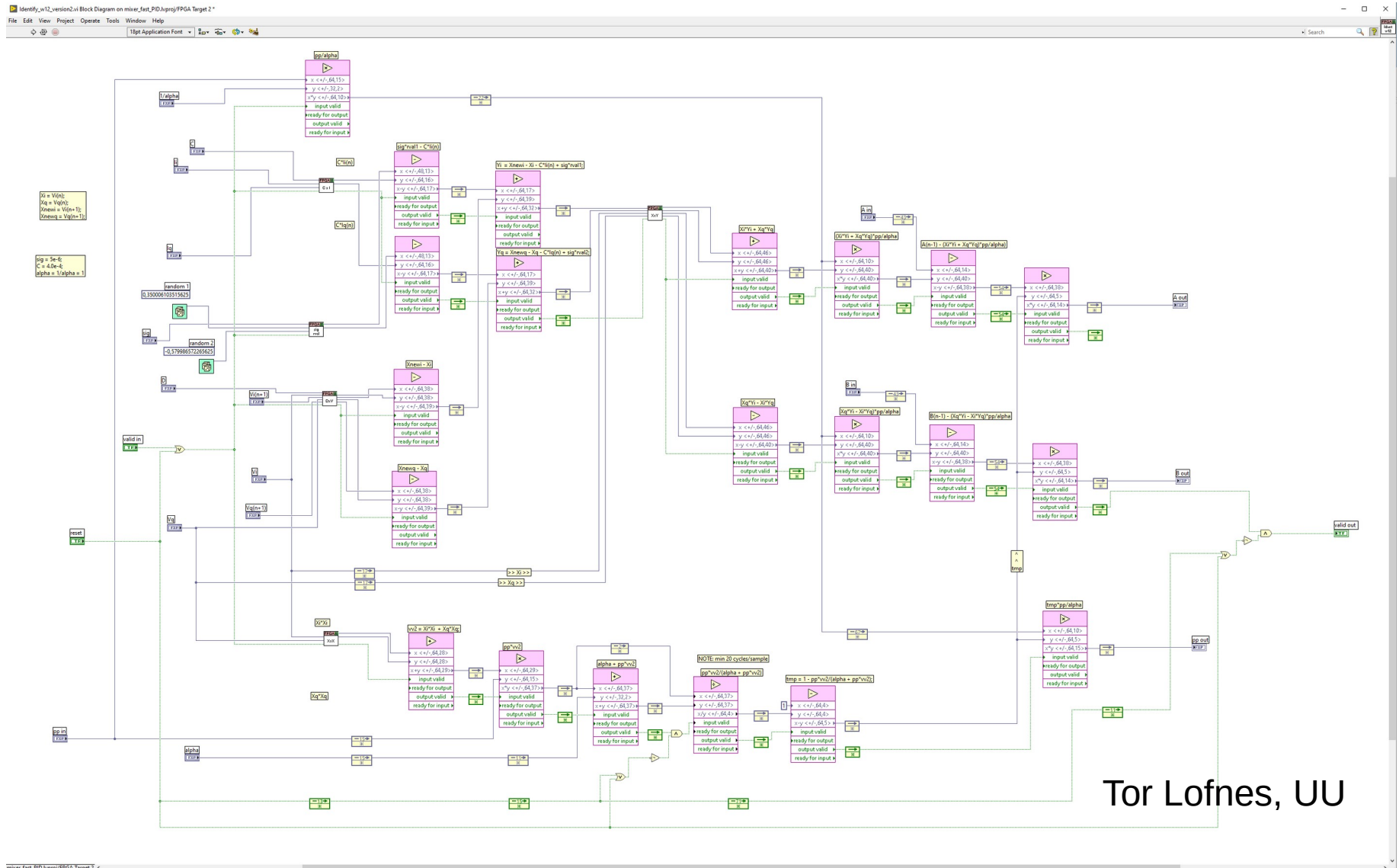
- Bandwidth ω_{12} doubles
- Frequency $\Delta\omega$ varies (2 kHz)

Behavior is reconstructed faithfully

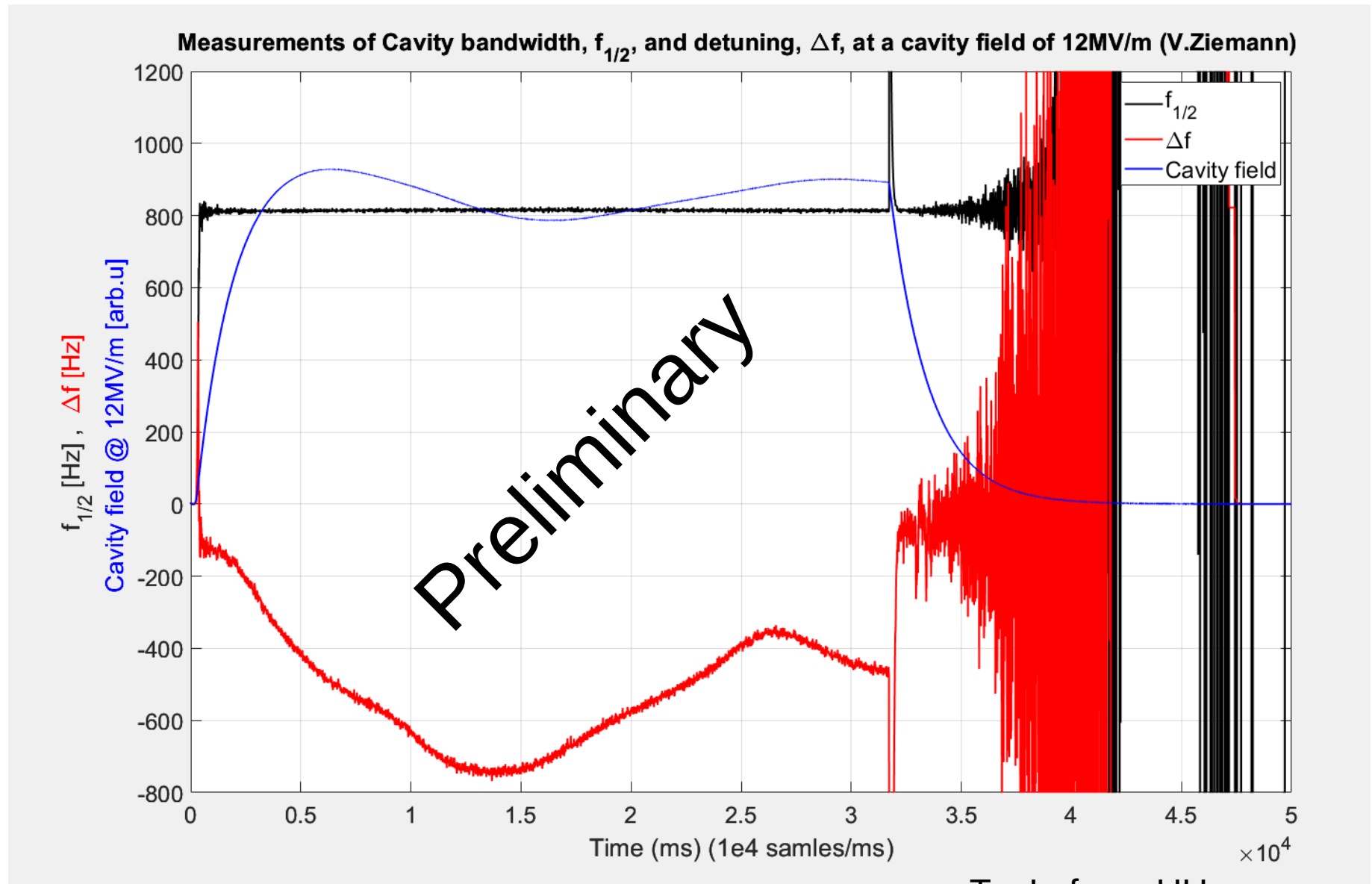


Implementation on FPGA

at FREIA at Uppsala University



ESS spoke cavity



Tor Lofnes, UU

Measuring the unloaded Q_0

Hunting the little discrepancy between Q_E and Q_L

$$\begin{aligned}\omega_E &= \hat{\omega}/Q_E \\ \omega_0 &= \hat{\omega}/Q'_0 \\ 1/Q'_0 &= (1/Q_0 + 1/Q_t)\end{aligned}$$

$$\vec{y}_{t+1} = \vec{V}'_{t+1} - \vec{V}'_t = F\vec{V}'_t + \omega_E\Delta t R\vec{I}_t^+ \quad \text{with} \quad F = \begin{pmatrix} -\frac{1}{2}(\omega_E + \omega_0)\Delta t & -\Delta\omega\Delta t \\ \Delta\omega\Delta t & -\frac{1}{2}(\omega_E + \omega_0)\Delta t \end{pmatrix}$$

Rewrite the right-hand side

$$\begin{aligned}F\vec{V}'_t + \omega_E\Delta t R\vec{I}_t^+ &= \omega_E\Delta t \begin{pmatrix} -\frac{1}{2}V'_r + RI_r^+ \\ -\frac{1}{2}V'_i + RI_i^+ \end{pmatrix}_t \\ &\quad + \Delta\omega\Delta t \begin{pmatrix} -V'_i \\ V'_r \end{pmatrix}_t + \omega_0\Delta t \begin{pmatrix} -\frac{1}{2}V'_r \\ -\frac{1}{2}V'_i \end{pmatrix}_t \\ &= \underbrace{\begin{pmatrix} -\frac{1}{2}V'_r + RI_r^+ & -V'_i & -\frac{1}{2}V'_r \\ -\frac{1}{2}V'_i + RI_i^+ & V'_r & -\frac{1}{2}V'_i \end{pmatrix}_t}_{G_t} \underbrace{\begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix}_t}_{\vec{q}}\end{aligned}$$

Stack consecutive measurements on top of each other

→ least-squares problem

$$\begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix} = U_T \begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix} \quad \text{with} \quad U_T = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_T \end{pmatrix}$$

Solve by Moore-Penrose

$$\vec{q}_T = \begin{pmatrix} \omega_E\Delta t \\ \Delta\omega\Delta t \\ \omega_0\Delta t \end{pmatrix}_T = (U_T^\top U_T)^{-1} U_T^\top \begin{pmatrix} \vec{y}_2 \\ \vec{y}_3 \\ \vdots \\ \vec{y}_{T+1} \end{pmatrix}$$

$$\vec{y}_{t+1} = G_t \vec{q}$$

Recursive ...

Technical difficulty: need to invert P_{T+1}^{-1} if new term is added

$$\begin{aligned} P_{T+1}^{-1} &= U_{T+1}^\top U_{T+1} \\ &= p_0 \mathbf{1} + G_1^\top G_1 + G_2^\top G_2 + \cdots + G_T^\top G_T + G_{T+1}^\top G_{T+1} \\ &= P_T^{-1} + G_{T+1}^\top G_{T+1}. \end{aligned} \quad (12)$$

Woodbury matrix identity to the rescue

$$(A + VW^\top)^{-1} = A^{-1} - A^{-1}V(\mathbf{1} + W^\top A^{-1}V)^{-1}W^\top A^{-1}$$

$$P_{T+1} = [\mathbf{1} - P_T G_{T+1}^\top (\mathbf{1} + G_{T+1} P_T G_{T+1}^\top)^{-1} G_{T+1}] P_T$$

Substitutions

$$A^{-1} = P_T$$

$$V = G_{T+1}^\top$$

$$W^\top = G_{T+1}$$

With forgetting factor α

$$P_{T+1} = \frac{1}{\alpha} [\mathbf{1} - P_T G_{T+1}^\top (\alpha + G_{T+1} P_T G_{T+1}^\top)^{-1} G_{T+1}] P_T$$

$$\begin{aligned} \vec{q}_{T+1} &= [\mathbf{1} - P_T G_{T+1}^\top (\alpha \mathbf{1} + G_{T+1} P_T G_{T+1}^\top)^{-1} G_{T+1}] \\ &\quad \times \left(\vec{q}_T + \frac{1}{\alpha} P_T G_{T+1}^\top \vec{y}_{T+2} \right). \end{aligned}$$

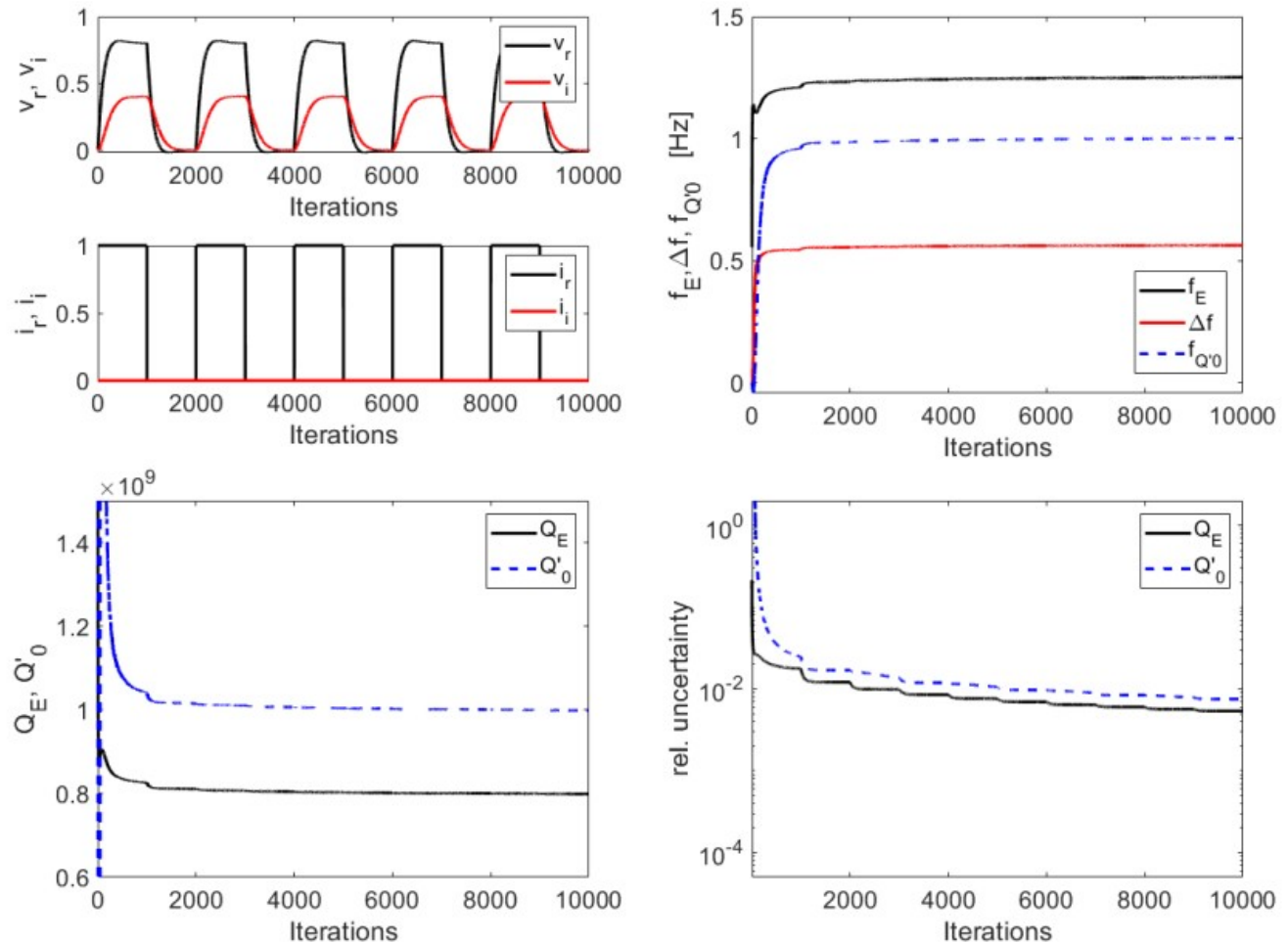
Critically coupled cavity

$$f=1\text{GHz}, Q_0=10^9, Q_E=0.8\times 10^9, N_f=\infty$$

Algorithm does not work
in steady-state, where the
equations are degenerate
→ pulsed

Track small changes:
→ need larger sample
time $\Delta t \sim \text{ms}$
→ 10 seconds real time

Fairly robust against small
errors in shunt resistance R



Over-coupled cavity

$$Q_0=10^9, Q_E=10^6$$

Sampling time $f_s=0.1$ ms
six seconds real time

Also works with random
errors in percent range

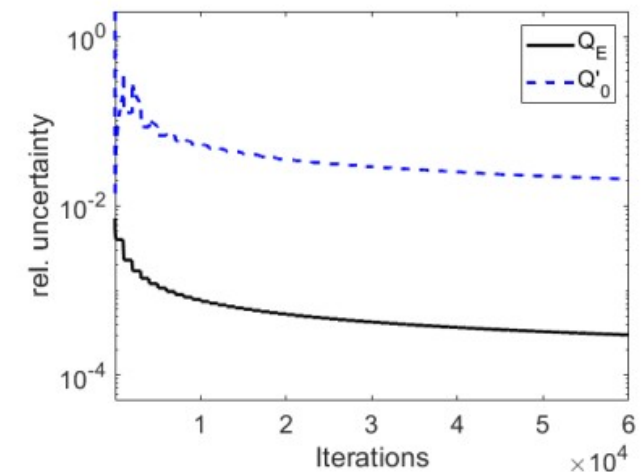
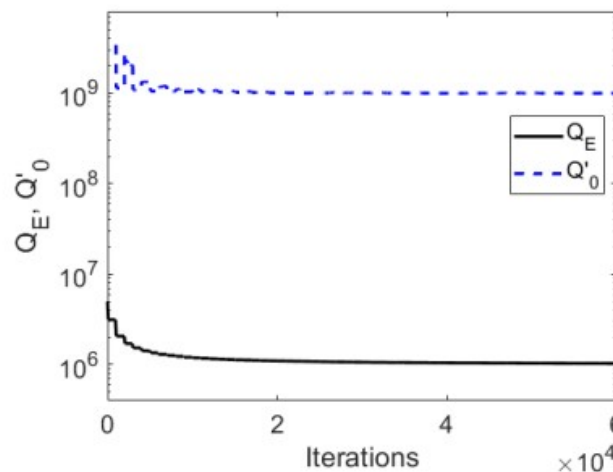
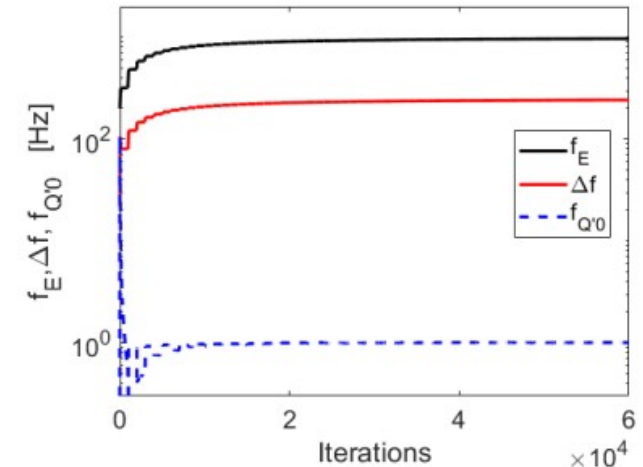
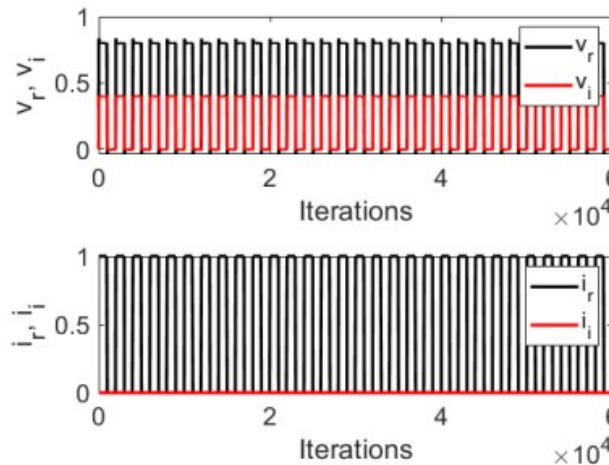
But!!!

Extreme sensitivity on
systematic error of R

Use RLS calibration of R
In steady-state

$$V'_t = R (I_t^+ + I_t^-)$$

Use RLS to continuously
update R for many
iterations



Conclusions

- Recursive least-squares algorithms are a powerful tool to continuously improve estimates in real time
 - Computational effort per time step is moderate
 - Colleague in Uppsala has a working system running close to 10 MHz