



RF Basics and TM Cavities

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DC versus RF

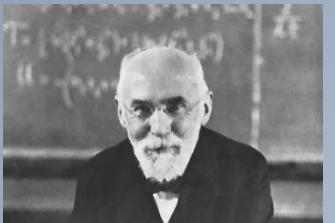
DC accelerator potential

$$\Delta W = q \int \vec{E} \cdot d\vec{s} = -q\Delta\Phi$$



RF accelerator





Hendrik A. Lorentz
1853 – 1928

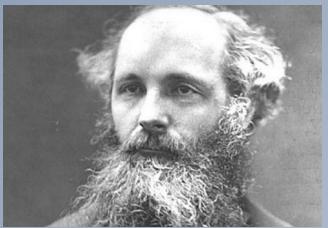
Lorentz force

- A charged particle moving with velocity $\vec{v} = \frac{\vec{p}}{m\gamma}$ through an electromagnetic field in vacuum experiences the Lorentz force $\frac{d\vec{p}}{dt} = q(\vec{E} + \vec{v} \times \vec{B})$.
- The total energy of this particle is $W = \sqrt{(mc^2)^2 + (pc)^2} = \gamma mc^2$, the kinetic energy is $W_{kin} = mc^2(\gamma - 1)$.
- The role of acceleration is to increase W .
- Change of W (by differentiation):

$$W dW = c^2 \vec{p} \cdot d\vec{p} = qc^2 \vec{p} \cdot (\vec{E} + \vec{v} \times \vec{B}) dt = qc^2 \vec{p} \cdot \vec{E} dt$$
$$dW = q \vec{v} \cdot \vec{E} dt$$

Note: **Only the electric field can change the particle energy!**





James Clerk Maxwell
1831 – 1879

Maxwell's equations (in vacuum)

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = \mu_0 \vec{J} \quad \nabla \cdot \vec{B} = 0$$
$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = \mu_0 c^2 \rho$$

1. Why not DC?

DC ($\frac{\partial}{\partial t} \equiv 0$): $\nabla \times \vec{E} = 0$, which is solved by $\vec{E} = -\nabla \Phi$

Limit: If you want to gain 1 MeV, you need a potential of 1 MV!



2. Circular machine: DC acceleration impossible since $\oint \vec{E} \cdot d\vec{s} = 0$

With time-varying fields:

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t} \vec{B}, \quad \oint \vec{E} \cdot d\vec{s} = - \iint \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}.$$



World's highest energy
Van-de-Graaff Generator
on Daresbury Lab campus



Maxwell's equations in vacuum (continued)

Source-free:

$$\nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial}{\partial t} \vec{E} = 0 \quad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} + \frac{\partial}{\partial t} \vec{B} = 0 \quad \nabla \cdot \vec{E} = 0$$

curl (rot, $\nabla \times$) of 3rd equation and $\frac{\partial}{\partial t}$ of 1st equation:

$$\nabla \times \nabla \times \vec{E} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0.$$

Using the vector identity $\nabla \times \nabla \times \vec{E} = \nabla \nabla \cdot \vec{E} - \Delta \vec{E}$ and the 4th Maxwell equation, this yields:

$$\Delta \vec{E} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \vec{E} = 0,$$

i.e. the 4-dimensional Laplace equation.

e.g. $f(x \pm ct)$

From waveguide to cavity

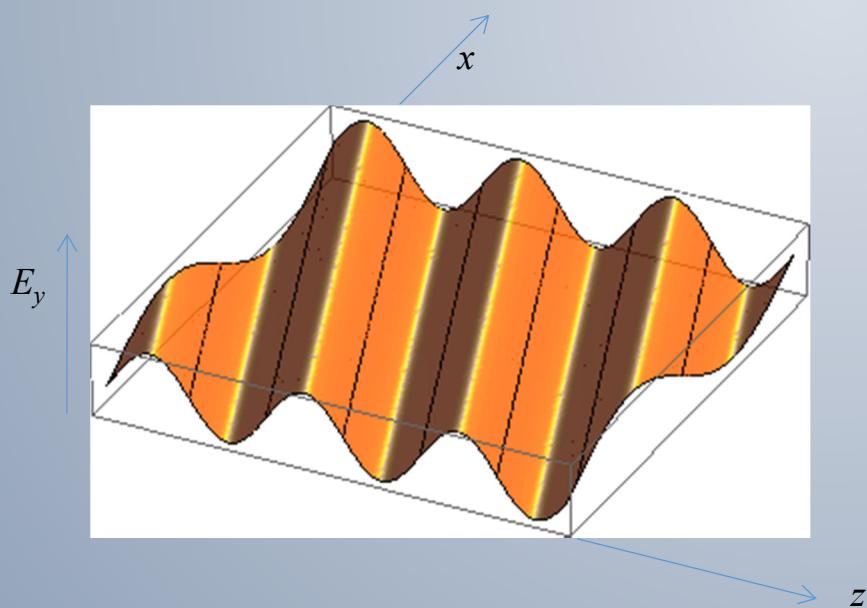


Homogeneous plane wave

$$\vec{E} \propto \vec{u}_y \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{B} \propto \vec{u}_x \cos(\omega t - \vec{k} \cdot \vec{r})$$

$$\vec{k} \cdot \vec{r} = \frac{\omega}{c} (z \cos \varphi + x \sin \varphi)$$



Wave vector \vec{k} :

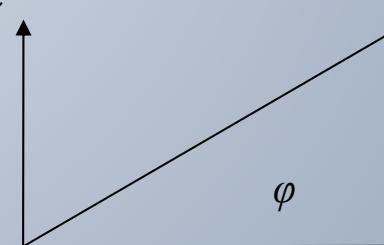
the direction of \vec{k} is the direction of propagation,

the length of \vec{k} is the phase shift per unit length.

\vec{k} behaves like a vector.

$$k_{\perp} = \frac{\omega_c}{c}$$

$$k = \frac{\omega}{c}$$

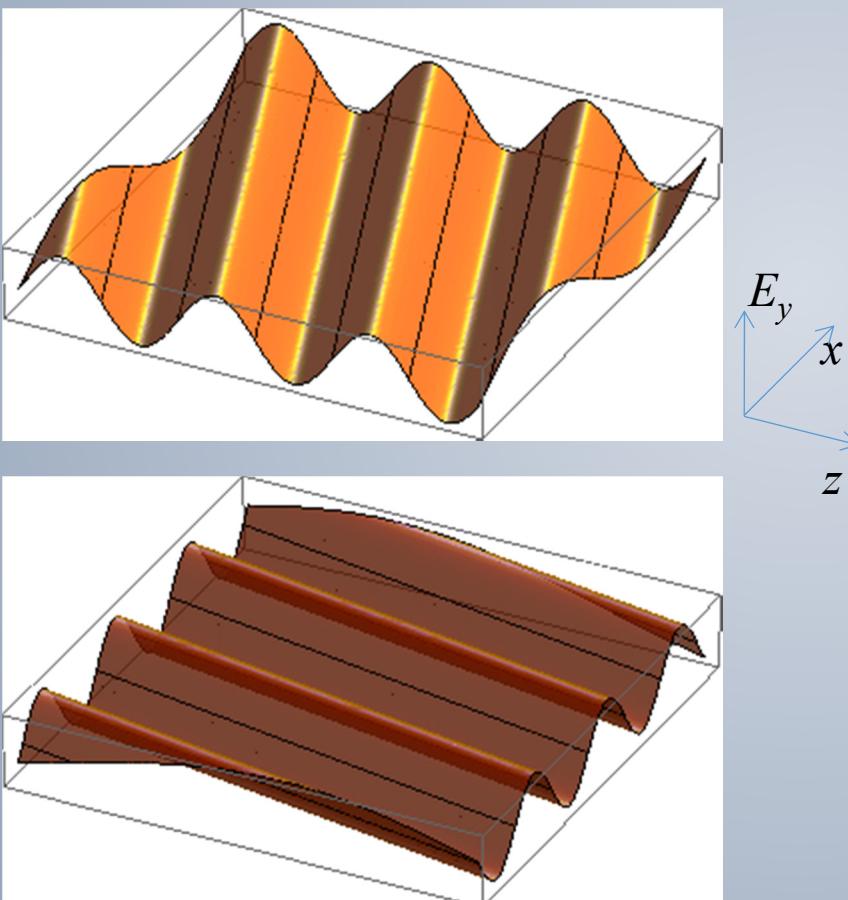


$$k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$



Wave length, phase velocity

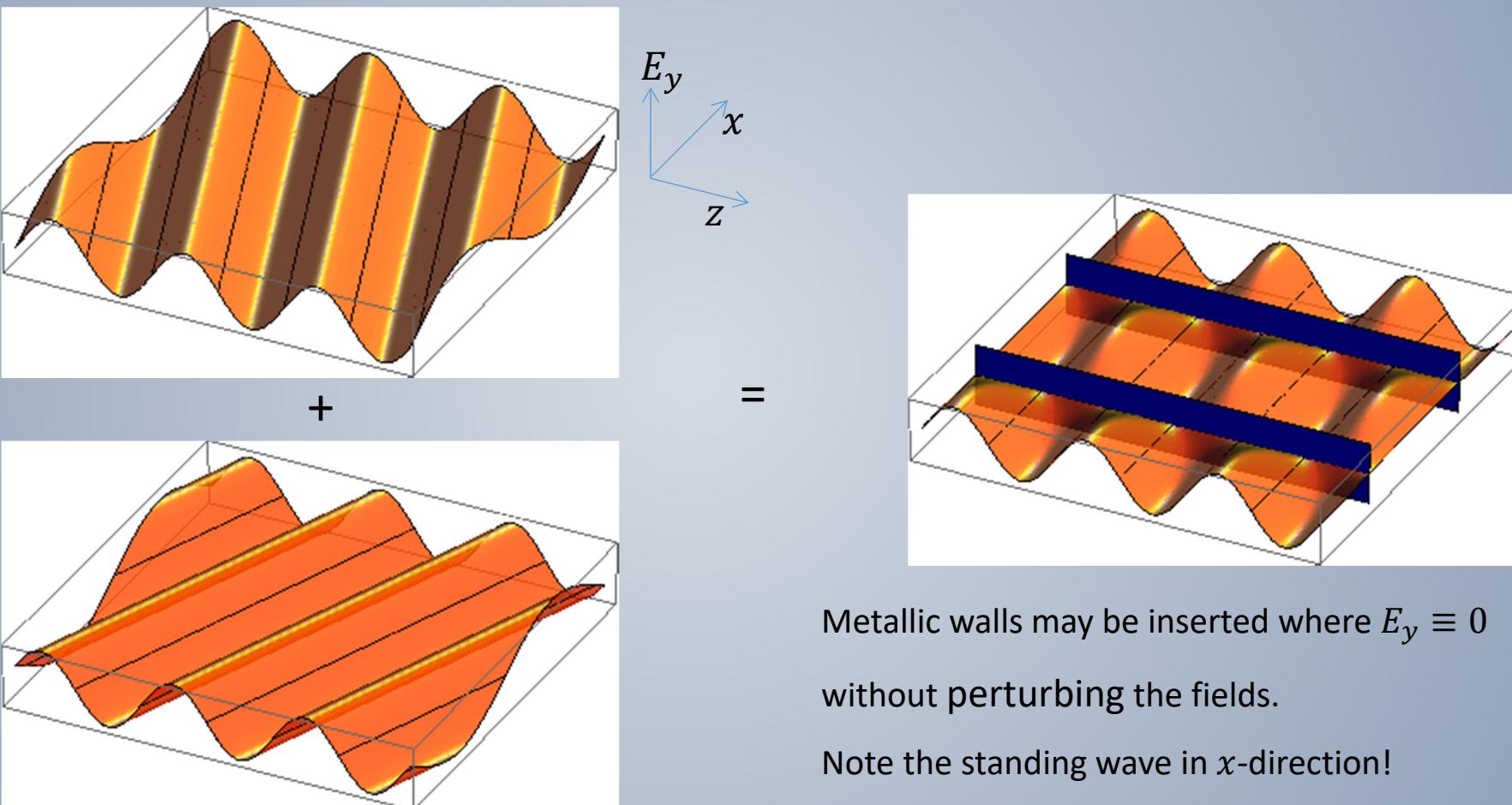
- The components of \vec{k} are related to the wavelength in the direction of that component as $\lambda_z = \frac{2\pi}{k_z}$ etc. , to the phase velocity as $v_{\varphi,z} = \frac{\omega}{k_z} = f\lambda_z$.



$$k_{\perp} = \frac{\omega_c}{c}$$
$$k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$
$$k = \frac{\omega}{c}$$
$$k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$



Superposition of 2 homogeneous plane waves



Metallic walls may be inserted where $E_y \equiv 0$
without perturbing the fields.
Note the standing wave in x -direction!

This way one gets a hollow rectangular waveguide.



Rectangular waveguide

Fundamental (TE_{10} or H_{10}) mode
in a standard rectangular waveguide.

Example 1: "S-band": 2.6 GHz ... 3.95 GHz,

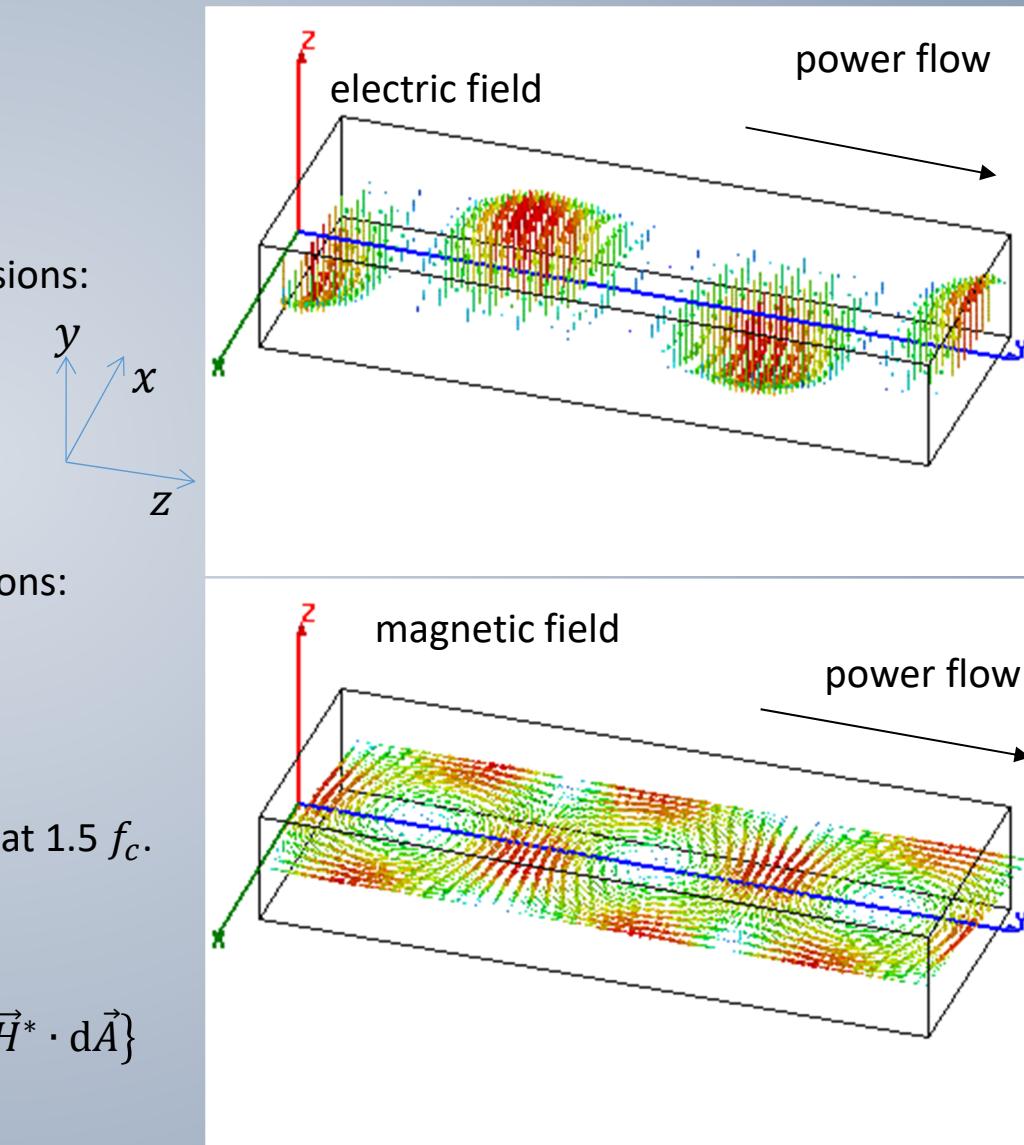
Waveguide type WR284 (2.84" wide), dimensions:
72.14 mm x 34.04 mm.
cut-off: $f_c = 2.078$ GHz.

Example 2: "L-band" : 1.13 GHz ... 1.73 GHz,

Waveguide type WR650 (6.5" wide), dimensions:
165.1 mm x 82.55 mm.
cut-off: $f_c = 0.908$ GHz.

Both these pictures correspond to operation at $1.5 f_c$.

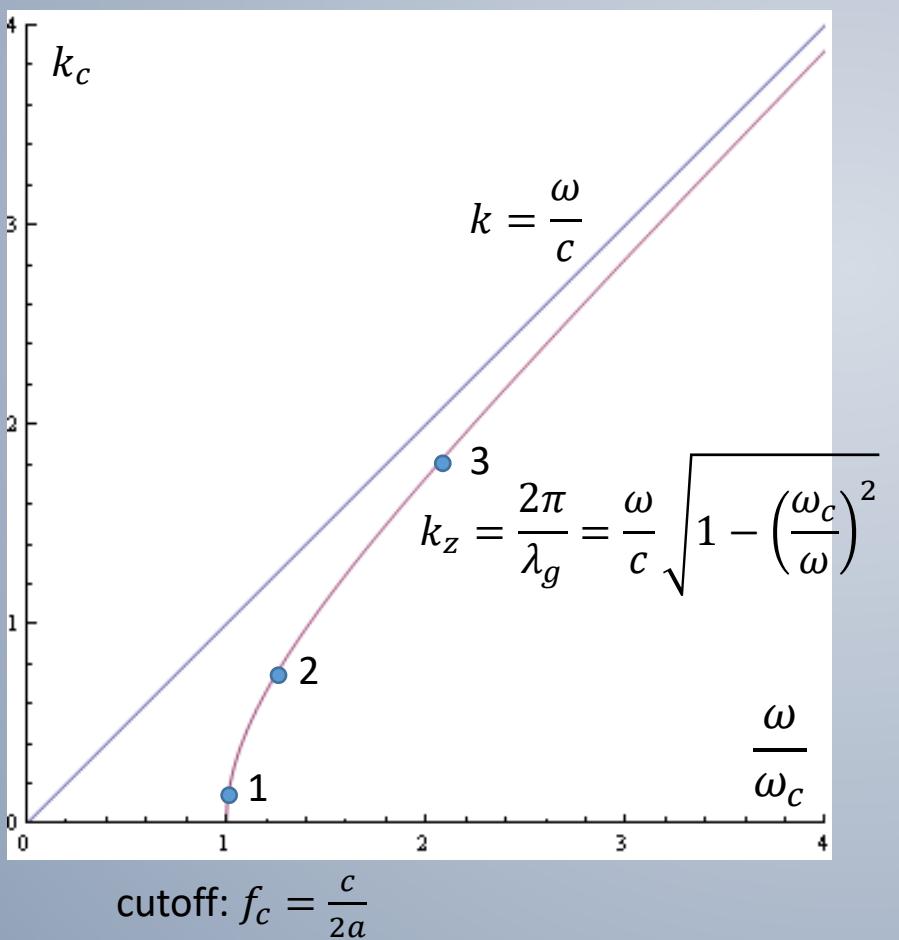
$$\text{power flow: } \frac{1}{2} \text{Re} \left\{ \iint \vec{E} \times \vec{H}^* \cdot d\vec{A} \right\}$$





Waveguide dispersion

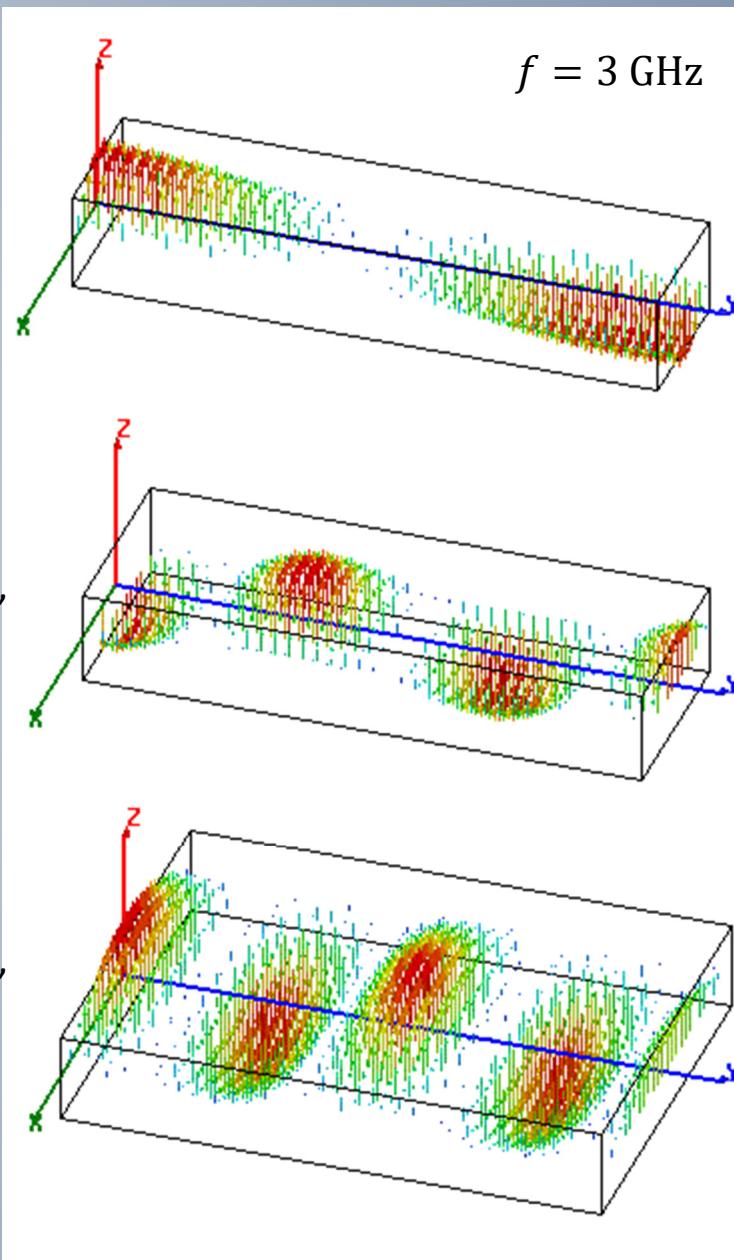
What happens with different waveguide dimensions (different width a)?



1:
 $a = 52 \text{ mm}$,
 $f/f_c = 1.04$

2:
 $a = 72.14 \text{ mm}$,
 $f/f_c = 1.44$

3:
 $a = 144.3 \text{ mm}$,
 $f/f_c = 2.88$



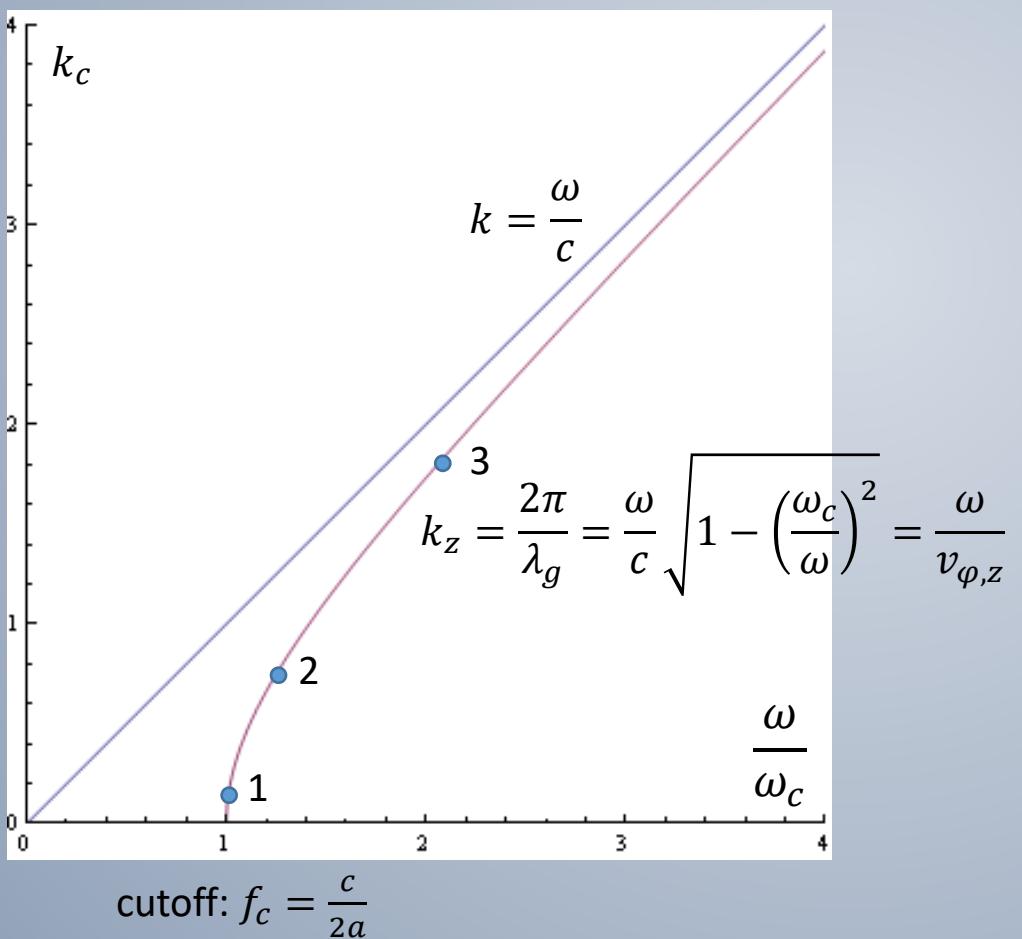


Phase velocity $v_{\varphi,z}$

The phase velocity $v_{\varphi,z}$ is the speed at which the crest (or zero-crossing) travels in z -direction.

Note on the 3 animations on the right that, at constant f , $v_{\varphi,z} \propto \lambda_g$. Note also that at $f = f_c$, $v_{\varphi,z} = \infty$!

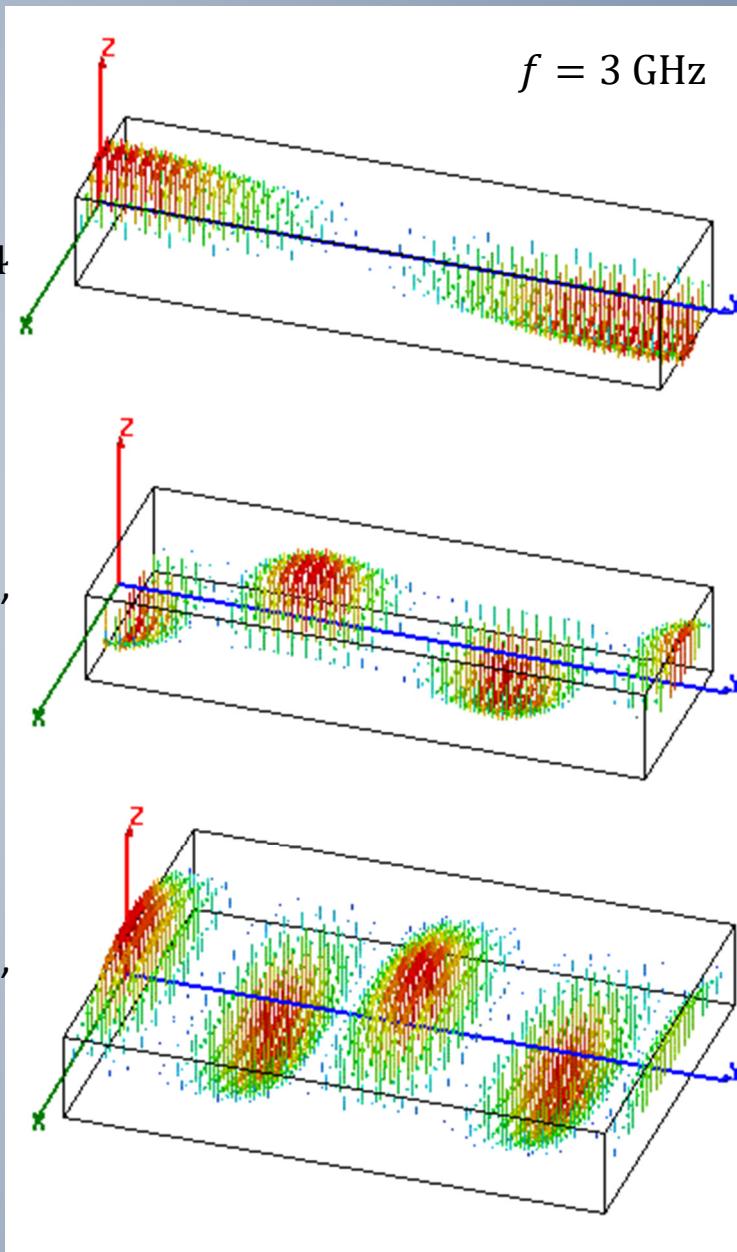
With $\nu \rightarrow \infty$, $v_{\varphi,z} \rightarrow c$!



1:
 $a = 52$ mm,
 $f/f_c = 1.04$

2:
 $a = 72.14$ mm,
 $f/f_c = 1.44$

3:
 $a = 144.3$ mm,
 $f/f_c = 2.88$





Summary waveguide dispersion and phase velocity:

In a **general** cylindrical waveguide:

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - k_{\perp}^2} = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$$

Propagation in z -direction: $\propto e^{j(\omega t - k_z z)}$

$$Z_0 = \frac{\omega\mu}{k_z} \text{ for TE, } Z_0 = \frac{k_z}{\omega\varepsilon} \text{ for TE modes}$$

$$k_z = \frac{2\pi}{\lambda_g}$$

Example: TE10-mode in a rectangular waveguide of width a :

$$k_{\perp} = \frac{\pi}{a}$$

$$k_z = \sqrt{\left(\frac{\omega}{c}\right)^2 - \left(\frac{\pi}{a}\right)^2}$$

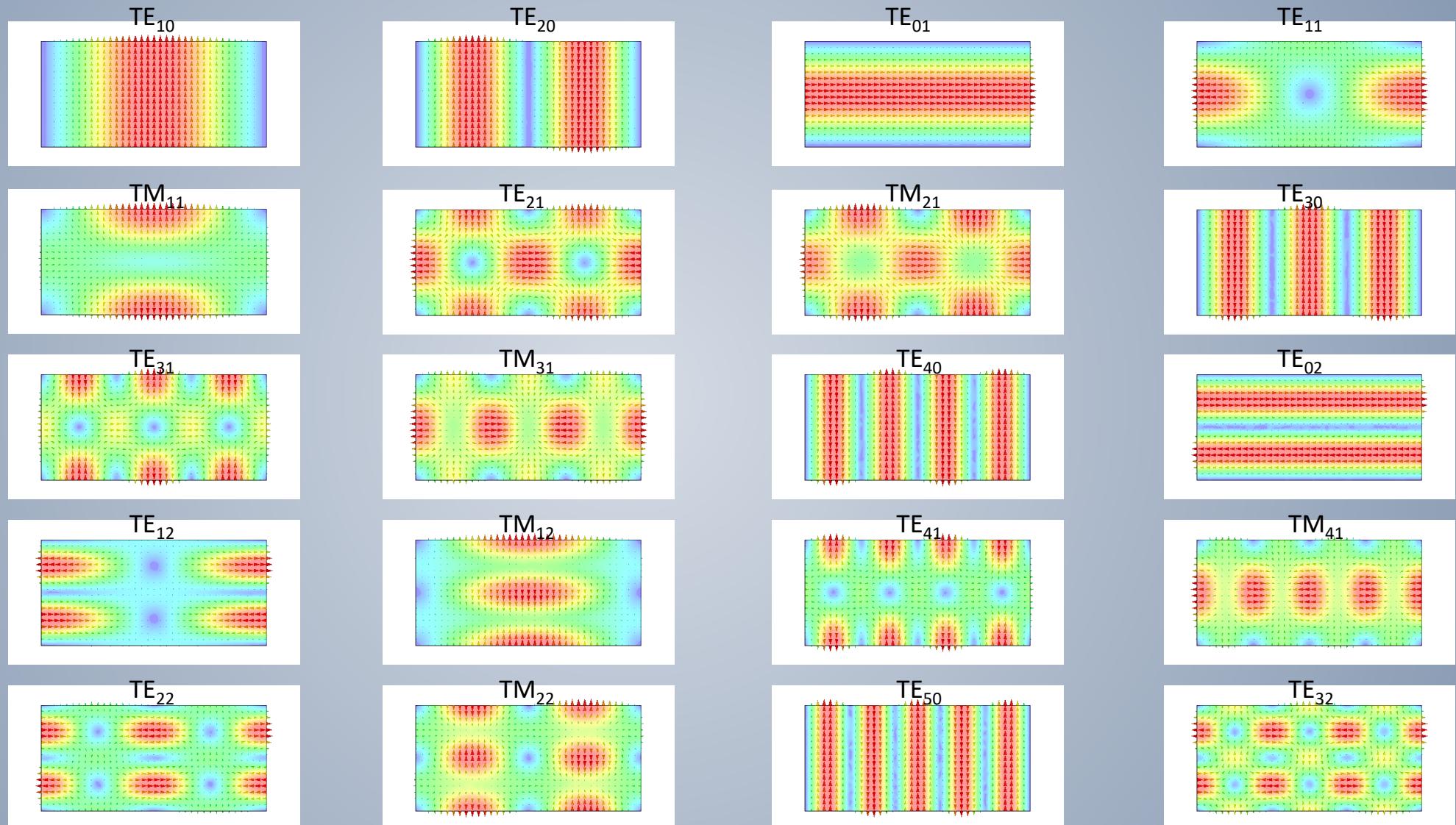
$$Z_0 = \frac{\omega\mu}{k_z}$$

$$\lambda_{\text{cutoff}} = 2a.$$

In a hollow waveguide: phase velocity $v_{\varphi} > c$, group velocity $v_{gr} < c$, $v_{gr} \cdot v_{\varphi} = c^2$.



Rectangular waveguide modes

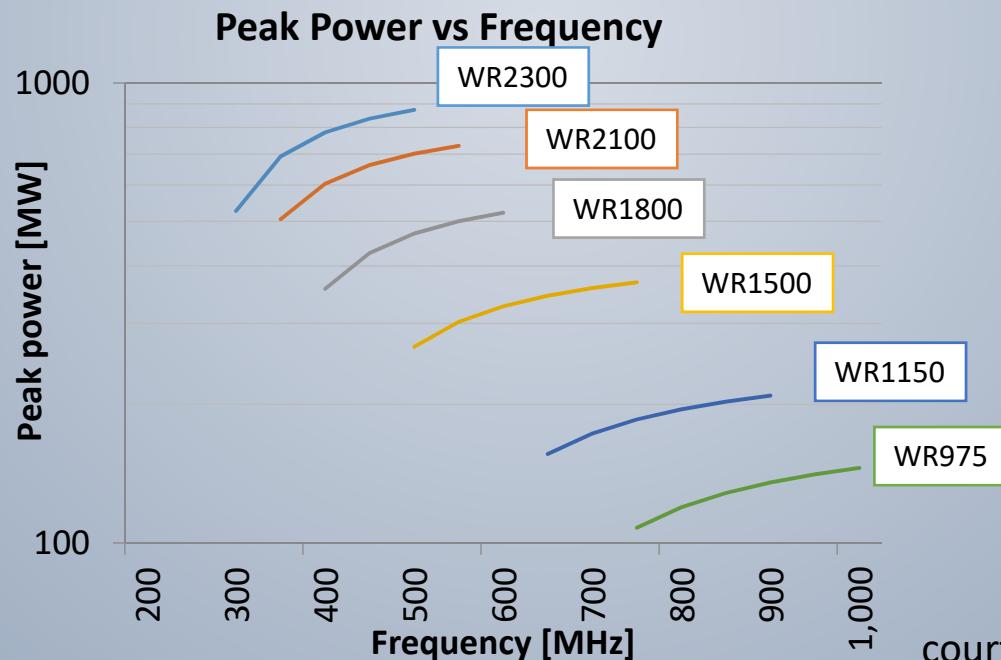
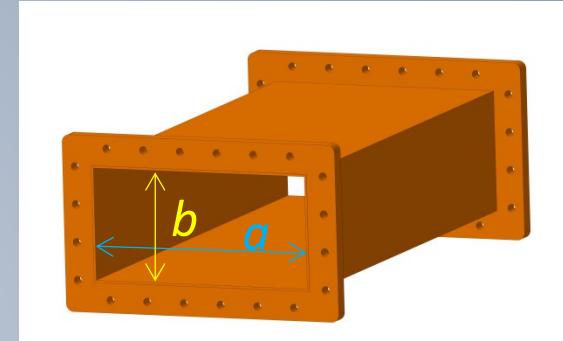


plotted: *E*-field

Some more standard rectangular Waveguides



Waveguide name			Recommended frequency band of operation (GHz)	Cutoff frequency of lowest order mode (GHz)	Cutoff frequency of next mode (GHz)	Inner dimensions of waveguide opening (inch)
EIA	RCSC	IEC				
WR2300	WG0.0	R3	0.32 — 0.45	0.257	0.513	23.0 × 11.5
WR1150	WG3	R8	0.63 — 0.97	0.513	1.026	11.50 × 5.75
WR340	WG9A	R26	2.2 — 3.3	1.736	3.471	3.40 × 1.70
WR75	WG17	R120	10 — 15	7.869	15.737	0.75 × 0.375
WR10	WG27	R900	75 — 110	59.015	118.03	0.10 × 0.05
WR3	WG32	R2600	220 — 330	173.571	347.143	0.034 × 0.017

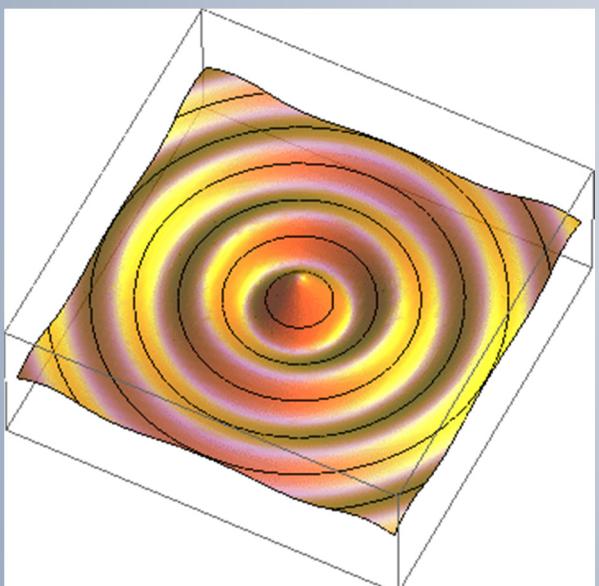


courtesy: Eric Montesinos/CERN

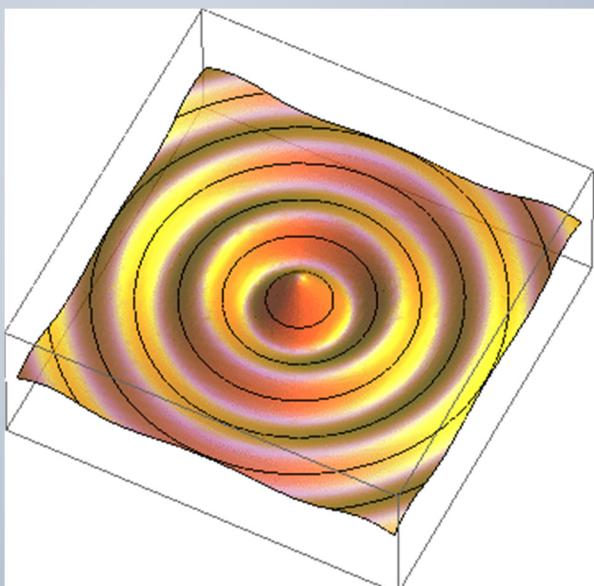


Radial waves

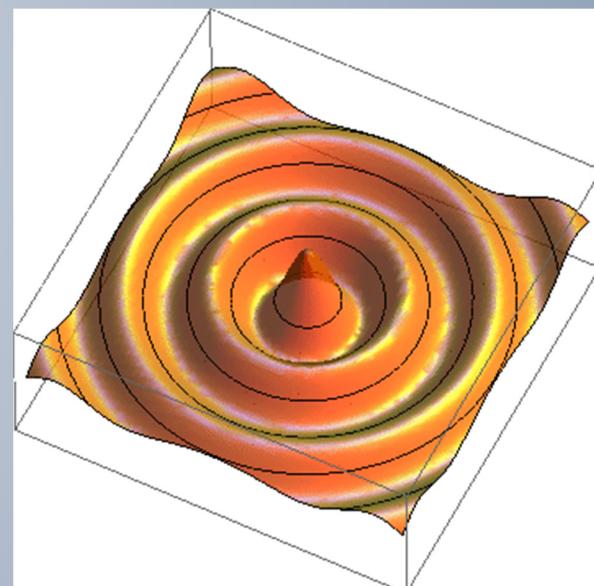
Also radial waves may be interpreted as superposition of plane waves.
The superposition of an outward and an inward radial wave can result
in the field of a round hollow waveguide.



$$E_z \propto H_n^{(2)}(k_\rho \rho) \cos(n\varphi)$$



$$E_z \propto H_n^{(1)}(k_\rho \rho) \cos(n\varphi)$$



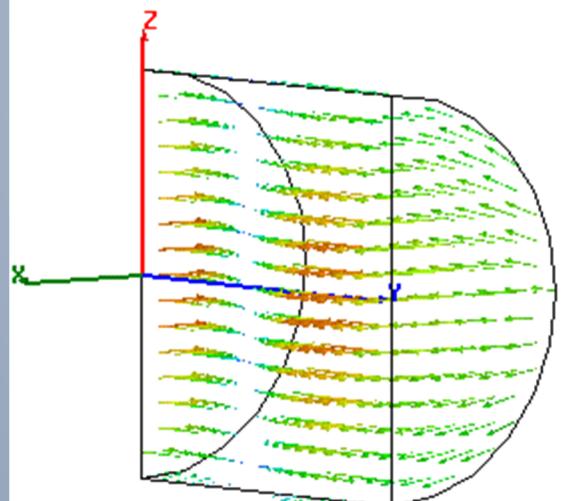
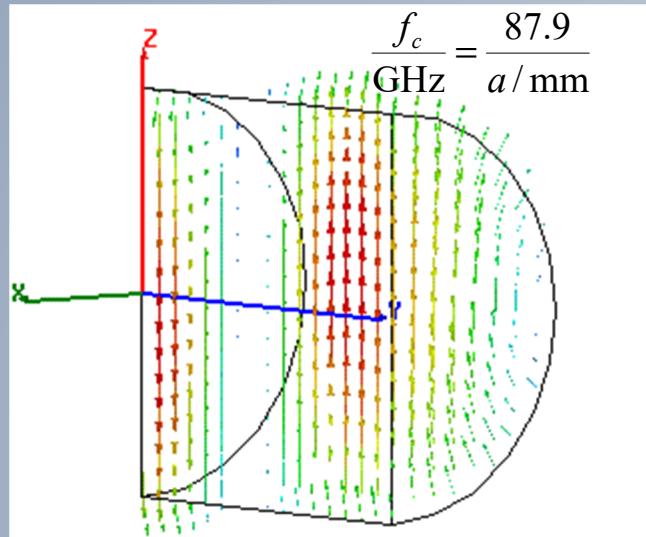
$$E_z \propto J_n(k_\rho \rho) \cos(n\varphi)$$

Round waveguide

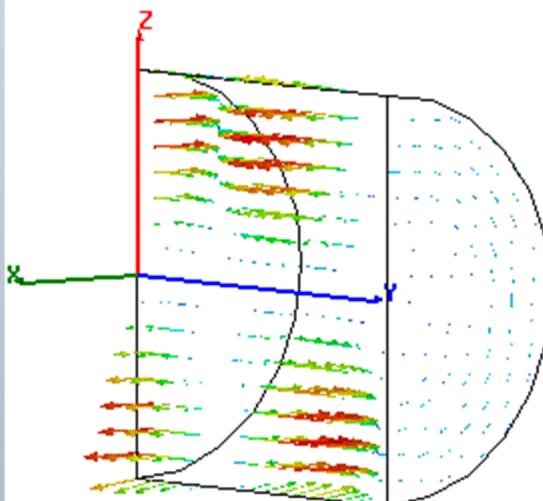
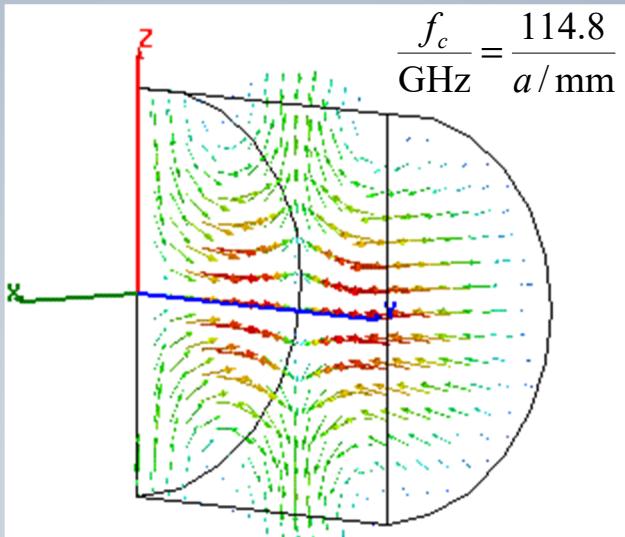
$$f/f_c = 1.44$$



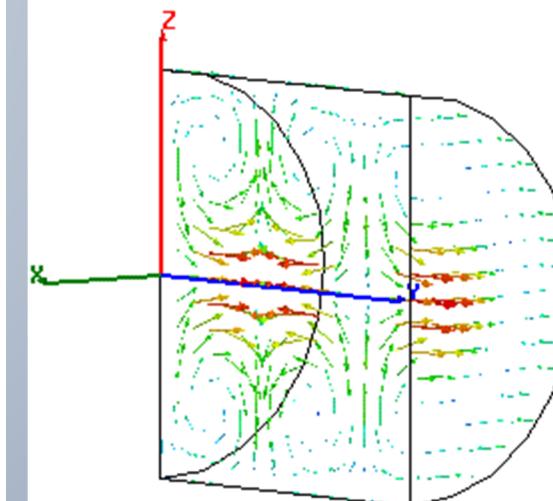
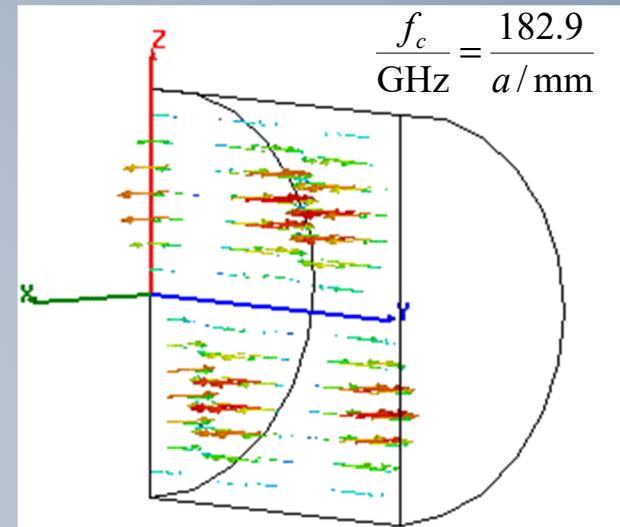
TE₁₁ – fundamental



TM₀₁ – axial E-field

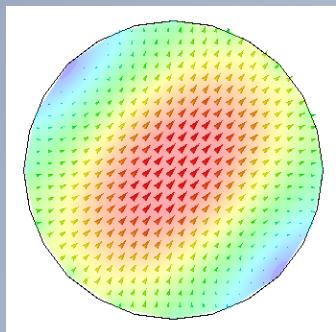


TE₀₁ – low loss mode

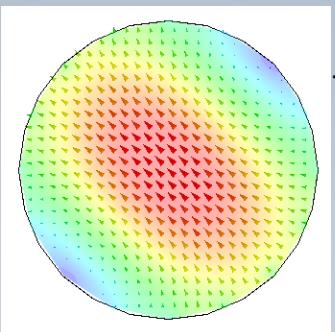




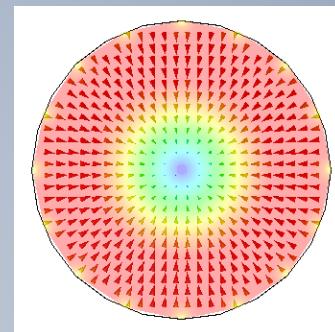
Circular waveguide modes



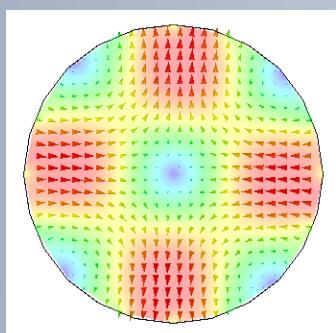
TE₁₁
fundamental



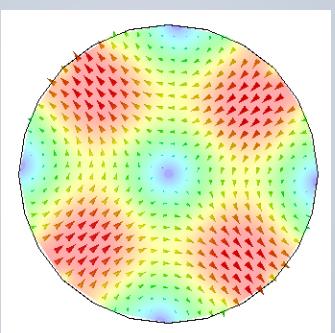
TE₁₁



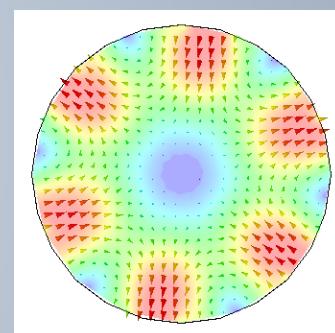
TM₀₁
axial *E*-field



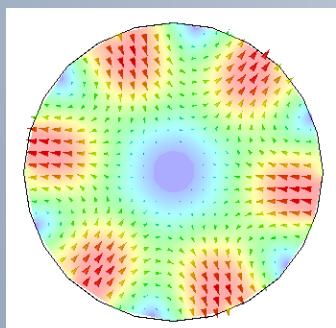
TE₂₁



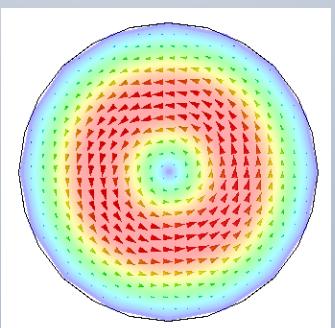
TE₂₁



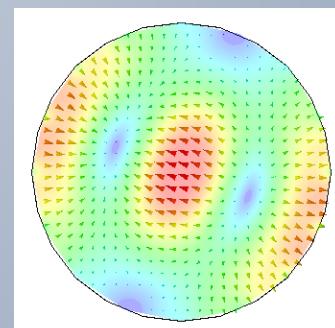
TE₃₁



TE₃₁



TE₀₁
low loss



TM₁₁

plotted: *E*-field



General waveguide equations:

Transverse wave equation (membrane equation): $\Delta T + \left(\frac{\omega_c}{c}\right)^2 T = 0$.

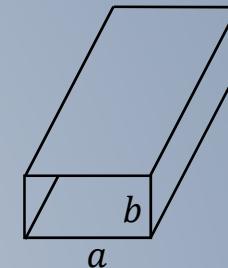
	TE (or H-) modes	TM (or E-) modes
Boundary condition:	$\vec{n} \cdot \nabla T = 0$	$T = 0$
longitudinal wave equations (transmission line equations):	$\frac{dU(z)}{dz} + jk_z Z_0 I(z) = 0$ $\frac{dI(z)}{dz} + \frac{jk_z}{Z_0} U(z) = 0$	
Propagation constant:		$k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$
Characteristic impedance:	$Z_0 = \frac{\omega\mu}{k_z}$	$Z_0 = \frac{k_z}{\omega\varepsilon}$
Ortho-normal eigenvectors:	$\vec{e} = \vec{u}_z \times \nabla T$	$\vec{e} = -\nabla T$
Transverse fields:		$\vec{E} = U(z) \vec{e}$ $\vec{H} = I(z) \vec{u}_z \times \vec{e}$
Longitudinal fields:	$H_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T}{j\omega\mu} U(z)$	$E_z = \left(\frac{\omega_c}{\omega}\right)^2 \frac{T}{j\omega\varepsilon} I(z)$



Special cases: rectangular and round waveguide

Rectangular waveguide: transverse eigenfunctions

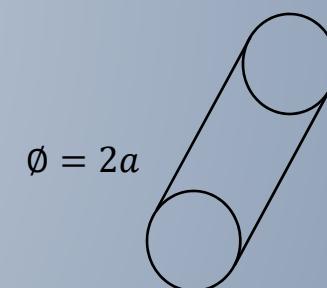
TE (H-) modes:	$T_{mn}^{(H)} = \frac{1}{\pi} \sqrt{\frac{a b \varepsilon_m \varepsilon_n}{(mb)^2 + (na)^2}} \cos\left(\frac{m\pi}{a}x\right) \cos\left(\frac{n\pi}{b}y\right)$
TM (E-) modes:	$T_{mn}^{(E)} = \frac{2}{\pi} \sqrt{\frac{a b}{(mb)^2 + (na)^2}} \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right)$



$$\frac{\omega_c}{c} = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

Round waveguide: transverse eigenfunctions

TE (H-) modes:	$T_{mn}^{(H)} = \sqrt{\frac{\varepsilon_m}{\pi(\chi'_{mn}^2 - m^2)}} \frac{J_m\left(\chi'_{mn} \frac{\rho}{a}\right)}{J_m(\chi'_{mn})} \begin{cases} \cos(m\varphi) \\ \sin(m\varphi) \end{cases}$
TM (E-) modes:	$T_{mn}^{(E)} = \sqrt{\frac{\varepsilon_m}{\pi}} \frac{J_m\left(\chi_{mn} \frac{\rho}{a}\right)}{J_{m-1}(\chi_{mn})} \begin{cases} \sin(m\varphi) \\ \cos(m\varphi) \end{cases}$

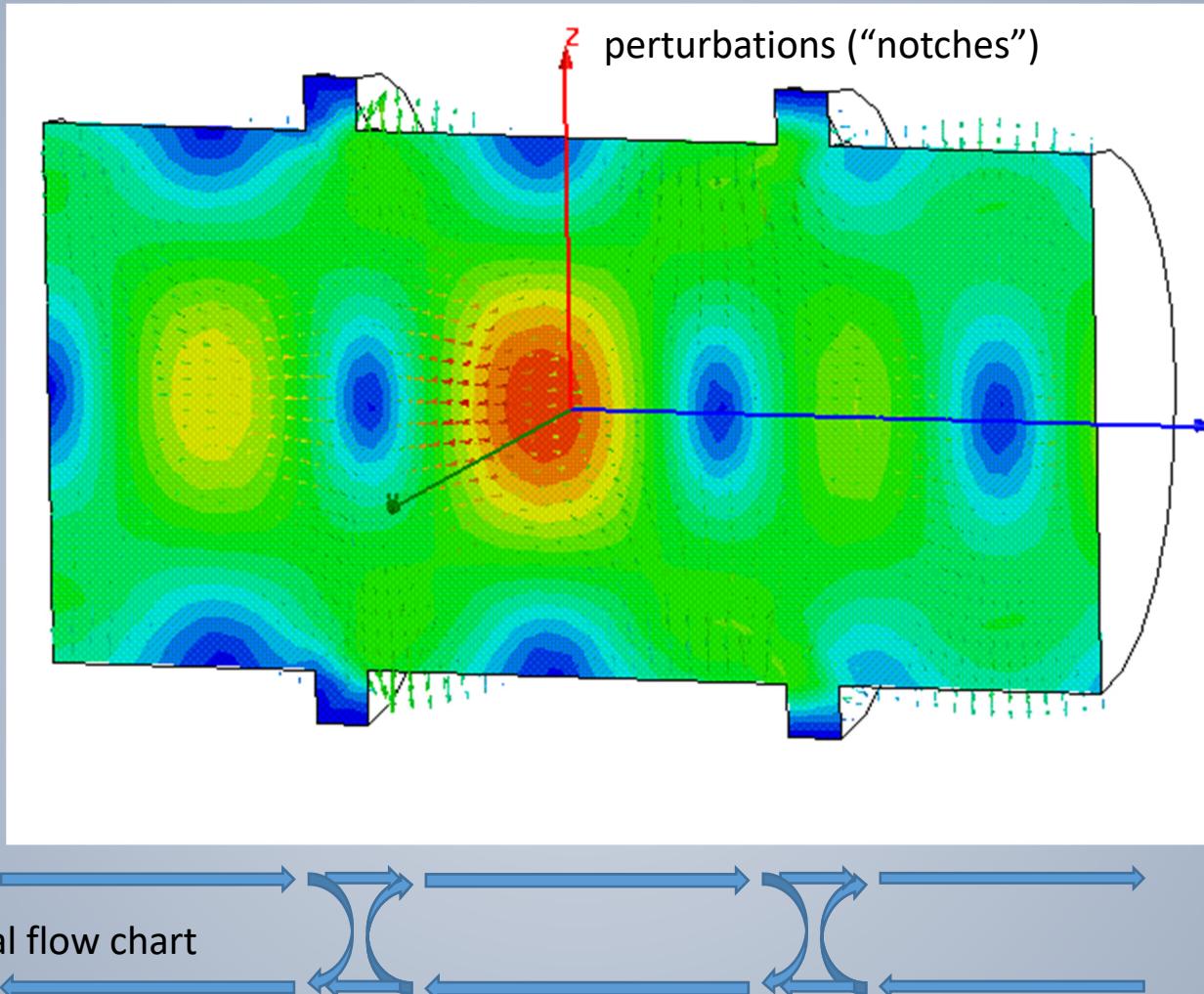


$$\frac{\omega_c}{c} = \frac{\chi_{mn}}{a}$$

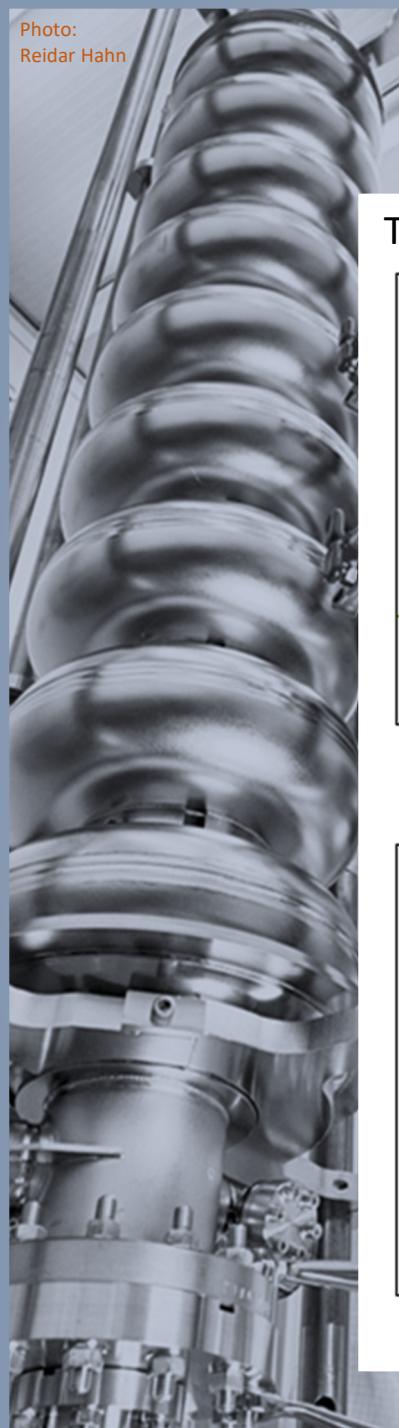
where in both cases $\varepsilon_i = \begin{cases} 1 & \text{if } i = 0 \\ 2 & \text{if } i \neq 0 \end{cases}$



Waveguide perturbed by notches

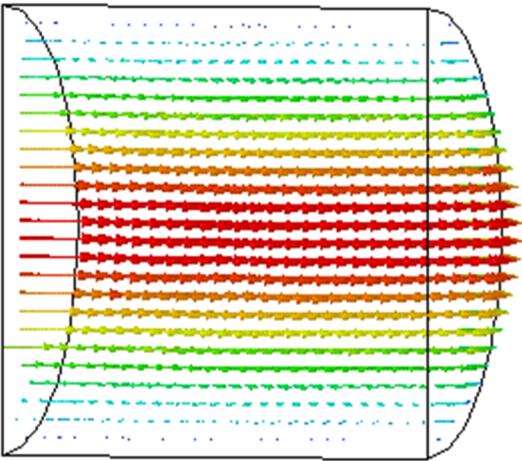


Reflections from notches lead to a superimposed standing wave pattern.
“Trapped mode”

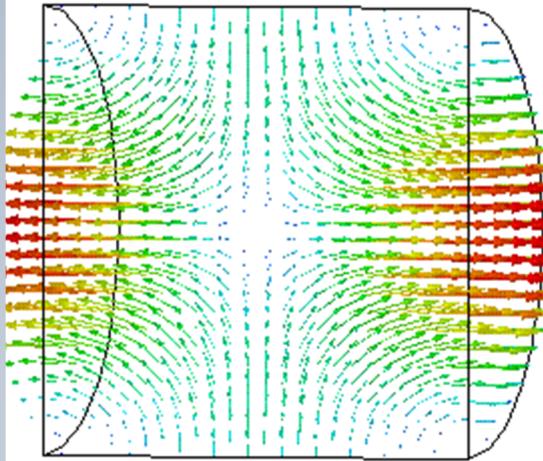


Short-circuited waveguide

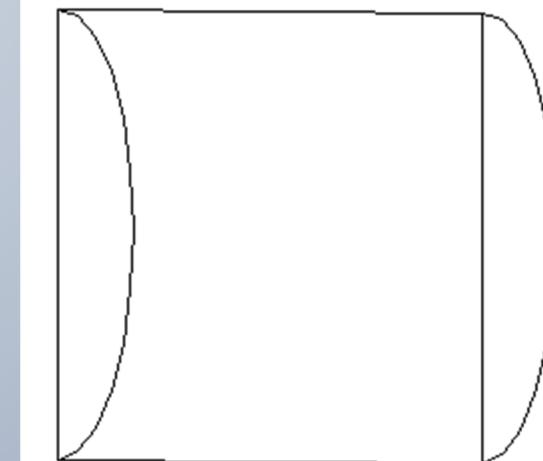
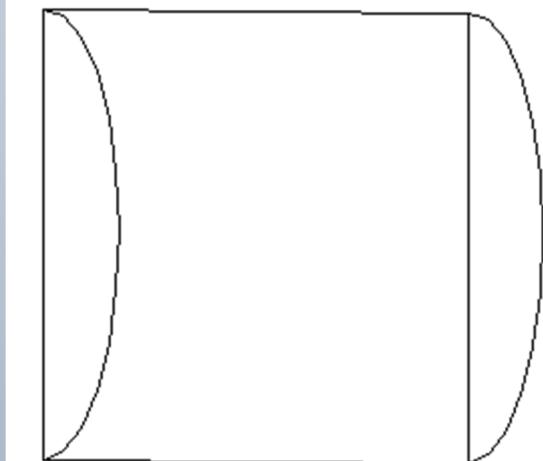
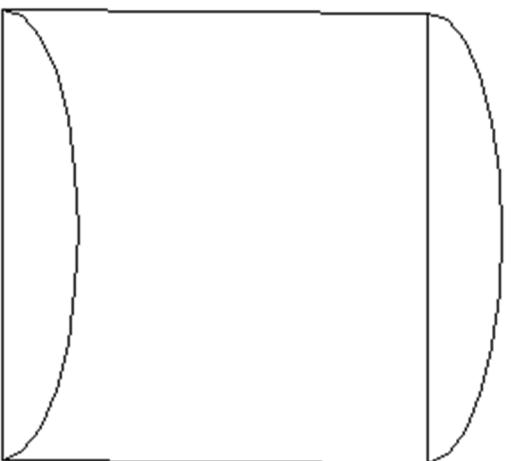
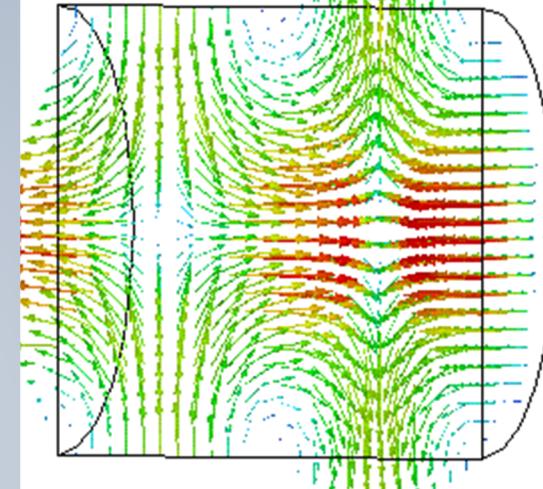
TM_{010} (no axial dependence)



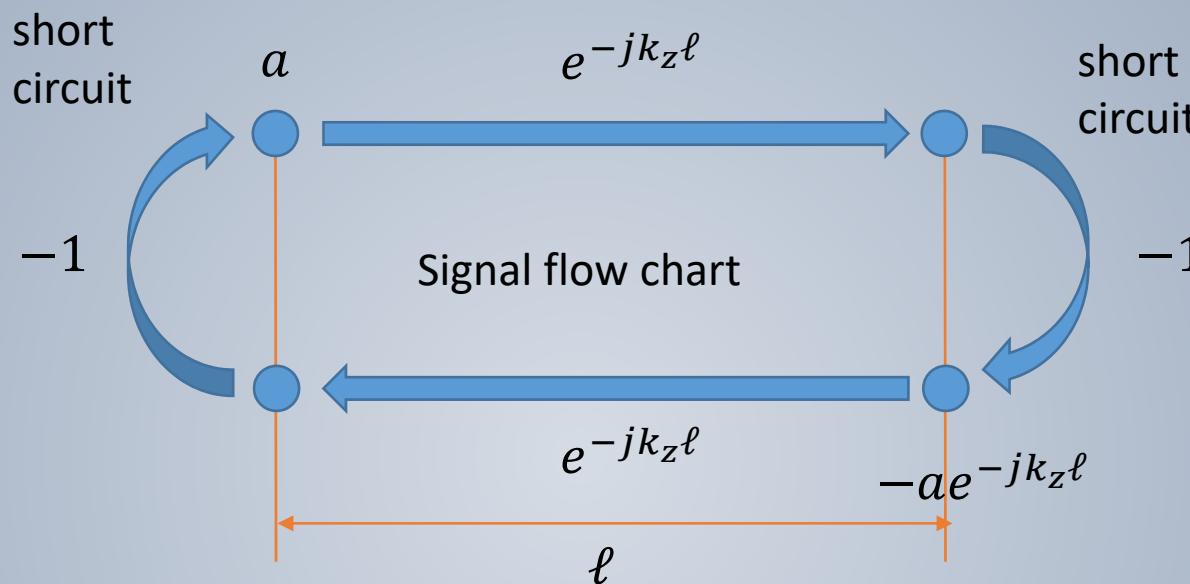
TM_{011}



TM_{012}



Single WG mode between two shorts



Eigenvalue equation for field amplitude a :

$$a = e^{-jk_z 2\ell} a$$

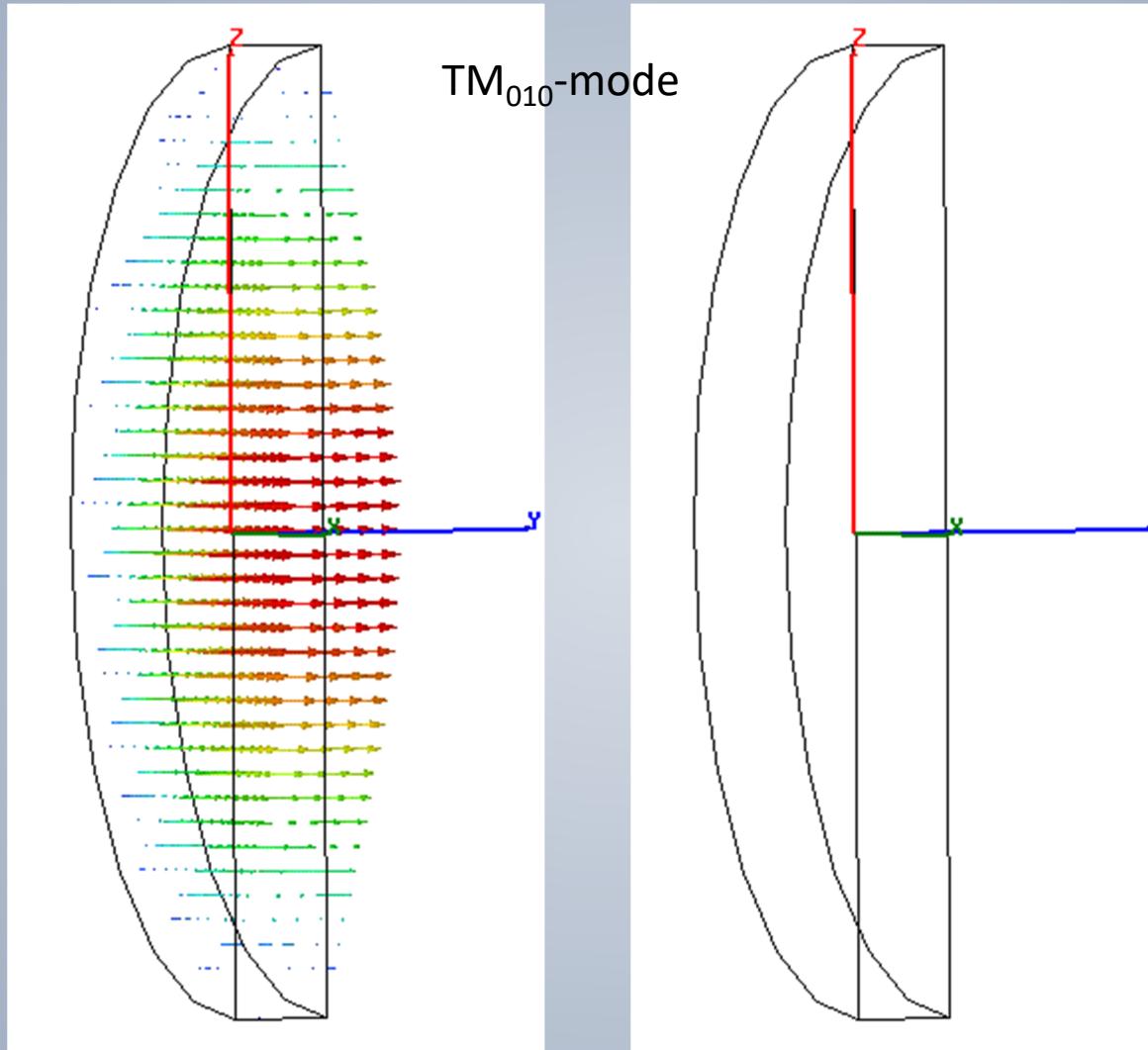
Non-vanishing solutions exist for $2k_z \ell = 2\pi m$:

With $k_z = \frac{\omega}{c} \sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}$, this becomes $f_0^2 = f_c^2 + \left(c \frac{m}{2\ell}\right)^2$.



Simple pillbox

(only 1/2 shown)



electric field (purely axial)

magnetic field (purely azimuthal)



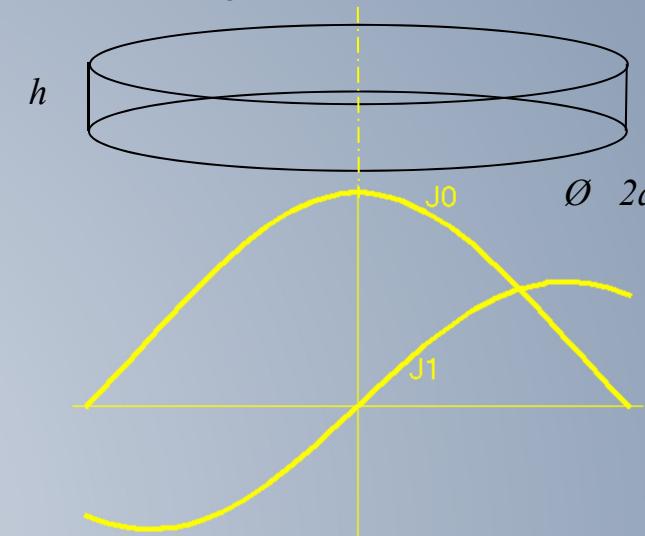
Pillbox cavity field (w/o beam tube)

$$T(\rho, \varphi) = \sqrt{\frac{1}{\pi} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{\chi_{01} J_1\left(\frac{\chi_{01}}{a}\right)}} \quad \text{with } \chi_{01} = 2.40483 \dots$$

The only non-vanishing field components :

$$E_z = \frac{1}{j\omega\varepsilon} \frac{\chi_{01}}{a} \sqrt{\frac{1}{\pi} \frac{J_0\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}}$$

$$B_\varphi = \mu_0 \sqrt{\frac{1}{\pi} \frac{J_1\left(\frac{\chi_{01}\rho}{a}\right)}{a J_1\left(\frac{\chi_{01}}{a}\right)}}$$



$$\omega_0|_{\text{pillbox}} = \frac{\chi_{01}c}{a}, \quad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} = 377 \Omega$$

$$Q|_{\text{pillbox}} = \frac{\sqrt{2a\eta\sigma\chi_{01}}}{2\left(1 + \frac{a}{h}\right)}$$

$$\left.\frac{R}{Q}\right|_{\text{pillbox}} = \frac{4\eta}{\chi_{01}^3 \pi J_1^2(\chi_{01})} \frac{\sin^2\left(\frac{\chi_{01}}{2} \frac{h}{a}\right)}{h/a}$$

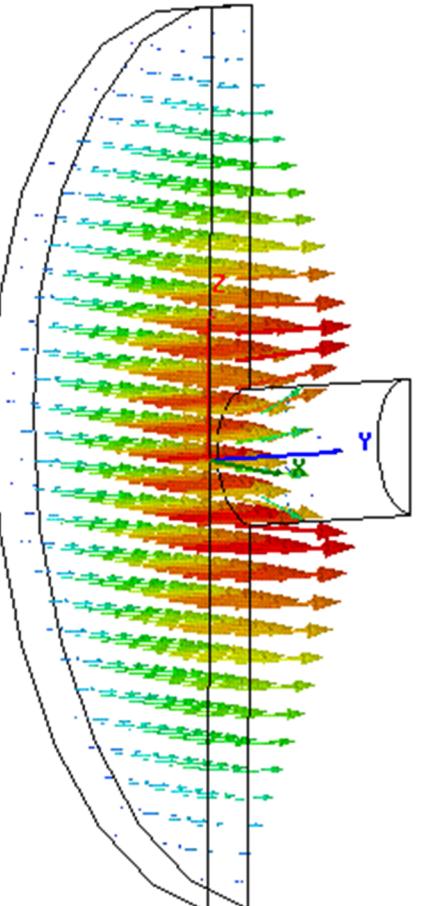


Pillbox with beam pipe

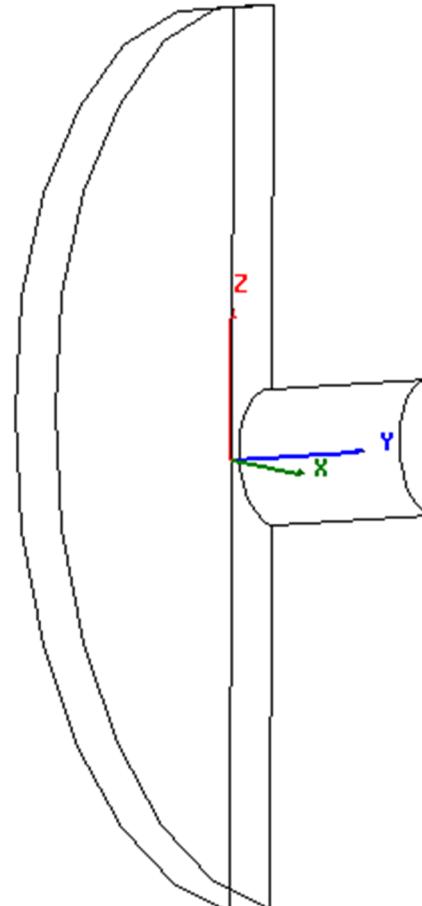
TM₀₁₀-mode

(only 1/4 shown)

One needs a hole for the beam passage – circular waveguide below cutoff



electric field

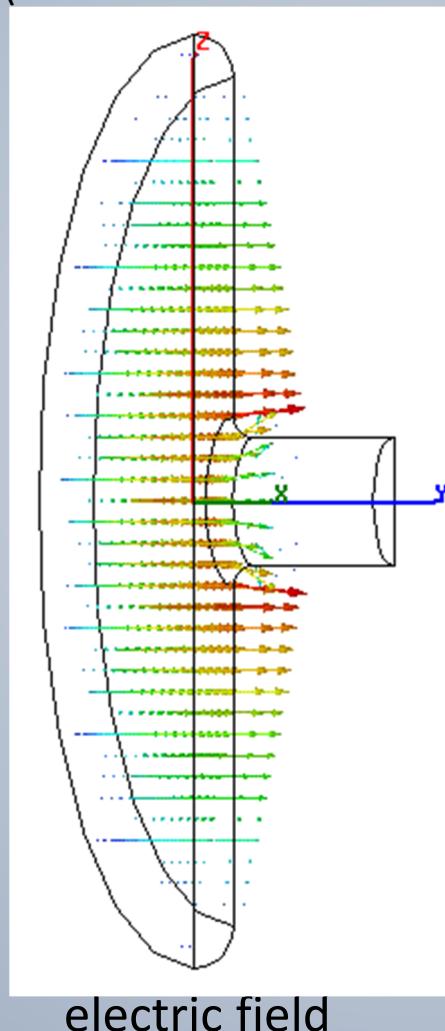


magnetic field



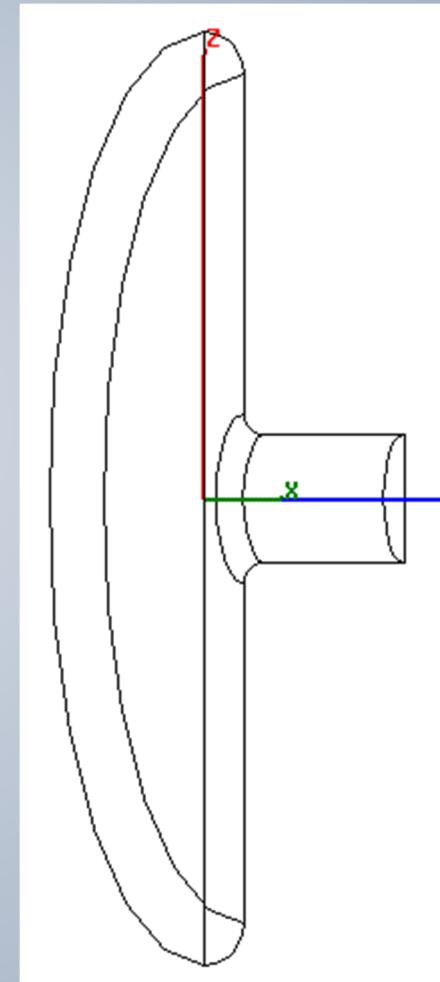
A more practical pillbox cavity

Rounding of sharp edges (to reduce field enhancement!)



electric field

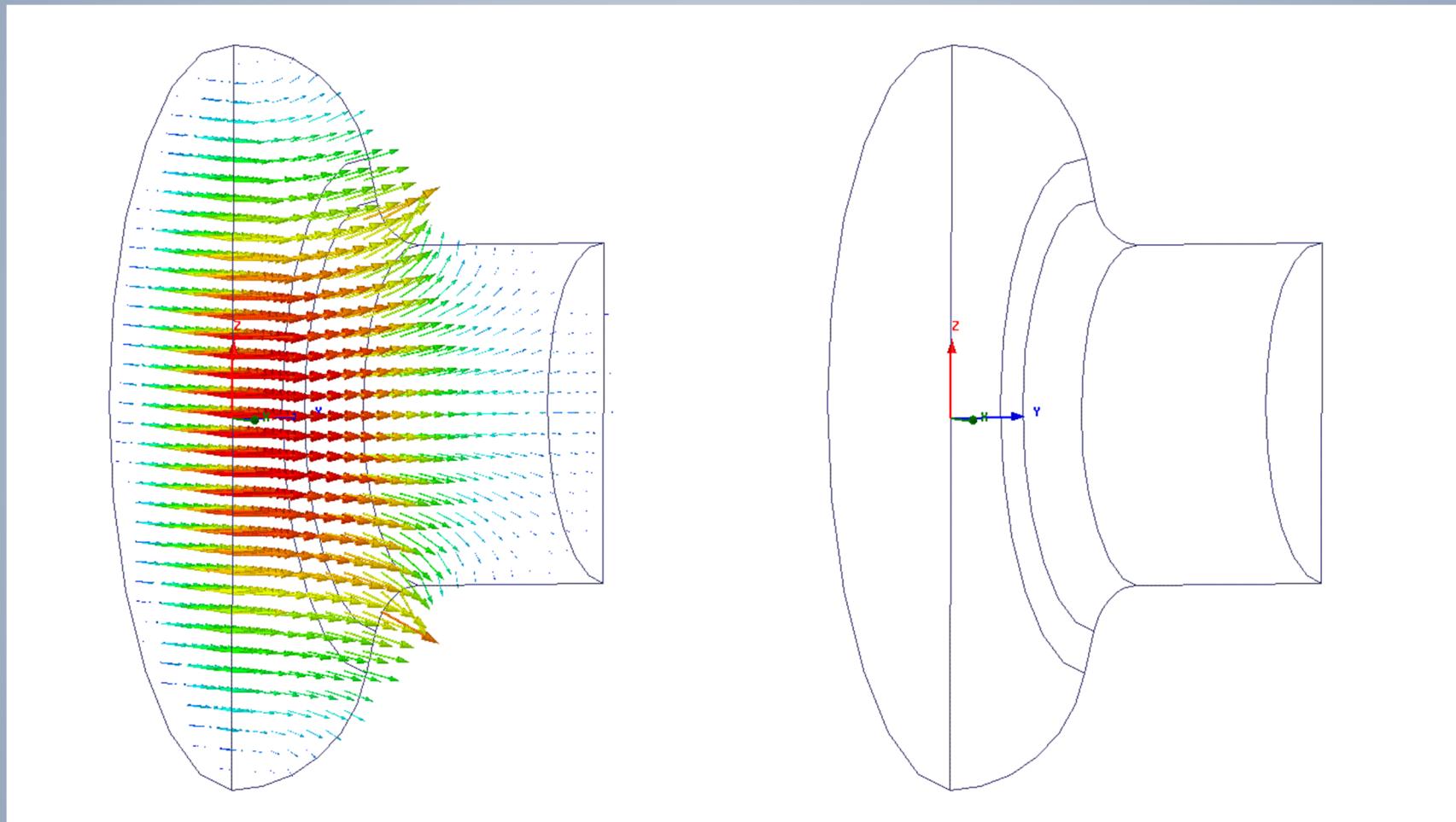
TM₀₁₀-mode (only 1/4 shown)



magnetic field

A (real) elliptical cavity

TM₀₁₀-mode (only 1/4 shown)



Choice of frequency

- Size:
 - Linear dimensions scale as f^{-1} , volume as f^{-3} .
 - amount of material, mass, stiffness, tolerances, ...
 - Outer radius of elliptical cavity $\sim 0.45 \lambda$.
- Beam interaction:
 - r/Q increases with f – but also for HOMs!
 - short bunches are easier with higher f .
- Technology:
 - superconducting: BCS resistance $\propto f^2$.
 - Power sources available?
 - Max. accelerating voltage?



Characterizing a cavity

Acceleration voltage and R/Q

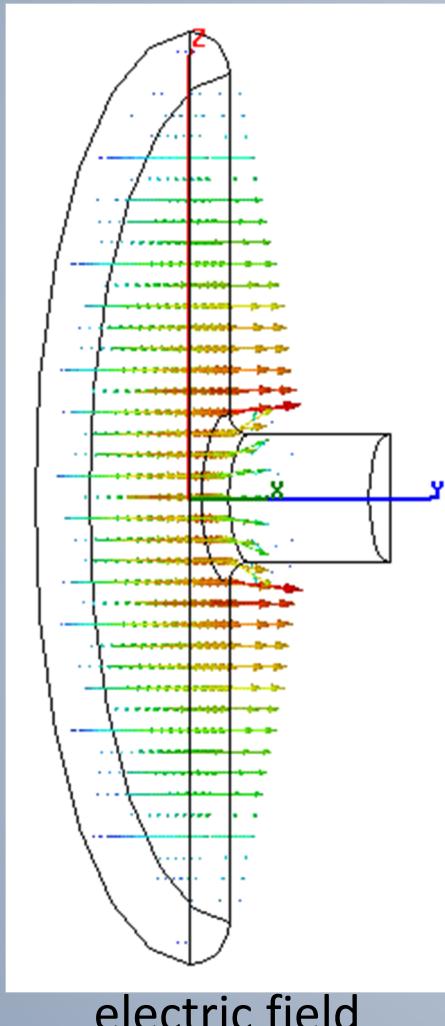
- I define

$$V_{acc} = \int_{-\infty}^{\infty} E_z e^{j\frac{\omega}{\beta c}z} dz .$$

- The exponential factor accounts for the variation of the field while particles with velocity βc are traversing the cavity gap.
- With this definition, V_{acc} is generally complex – this becomes important with more than one gap (cell).
- For the time being we are only interested in $|V_{acc}|$.
- The square of the acceleration voltage $|V_{acc}|^2$ is proportional to the stored energy W ; the proportionality constant defines the quantity called “R-upon-Q”:

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W} .$$

› **Attention – different definitions are used in literature!**





Transit time factor

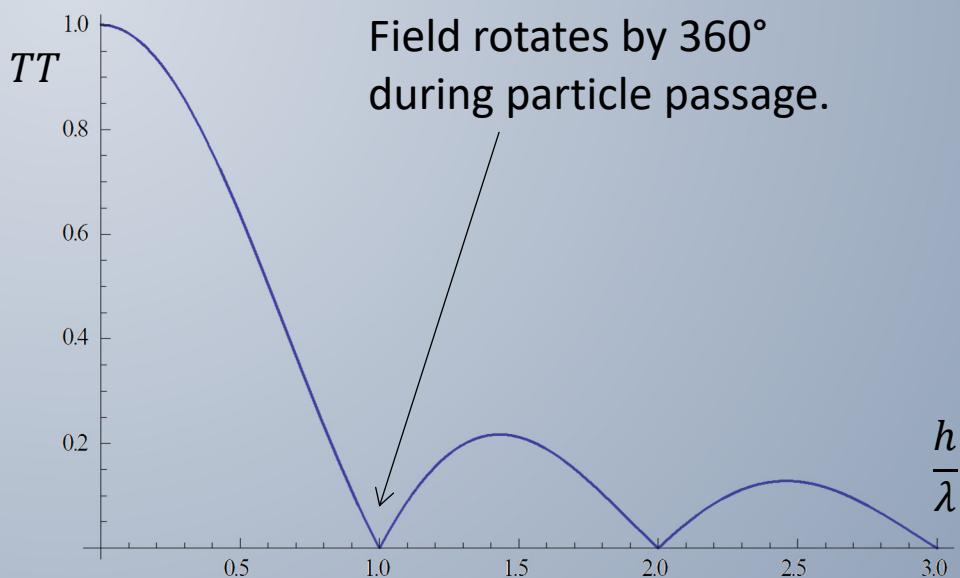
- The transit time factor is the ratio of the acceleration voltage to the (non-physical) voltage a particle with infinite velocity would see:

$$TT = \frac{|V_{acc}|}{\left| \int E_z dz \right|} = \frac{\left| \int E_z e^{j \frac{\omega}{\beta c} z} dz \right|}{\left| \int E_z dz \right|}.$$

- The transit time factor of an ideal pillbox cavity (no axial field dependence) of height (gap length) h is:

$$TT = \frac{\sin\left(\frac{\chi_{01}h}{2a}\right)}{\frac{\chi_{01}h}{2a}}$$

(remember: $\omega_0 = \frac{2\pi c}{\lambda} = \frac{\chi_{01}c}{a}$)





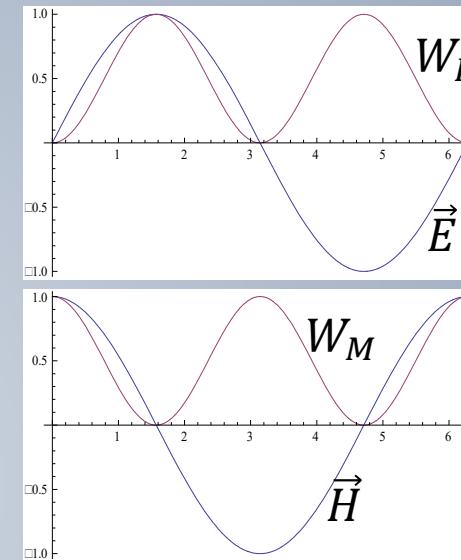
Stored energy

- The energy stored in the electric field is

$$W_E = \iiint_{\text{cavity}} \frac{\epsilon}{2} |\vec{E}|^2 dV.$$

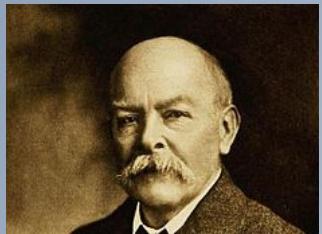
- The energy stored in the magnetic field is

$$W_M = \iiint_{\text{cavity}} \frac{\mu}{2} |\vec{H}|^2 dV.$$



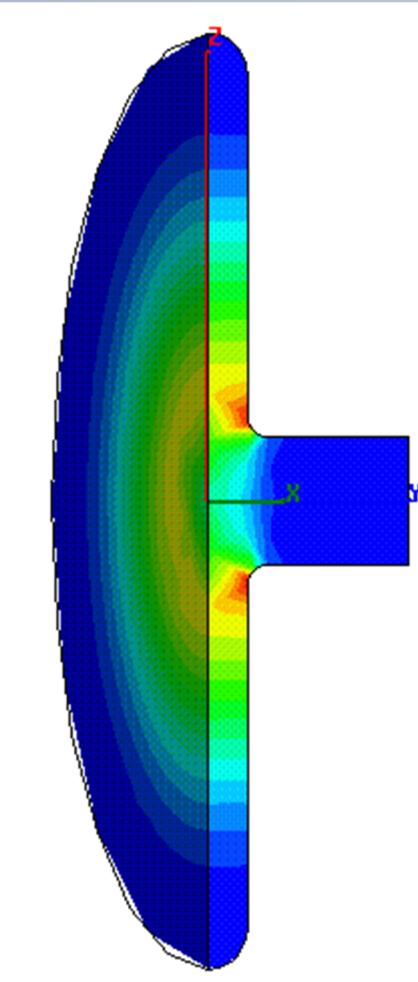
- Since \vec{E} and \vec{H} are 90° out of phase, the stored energy continuously swaps from electric energy to magnetic energy.
- On average, electric and magnetic energy must be equal.
- In steady state, the Poynting vector $\Re\{\vec{E} \times \vec{H}^*\}$ describes this energy flux.
- In steady state, the total energy stored (constant) is

$$W = \iiint_{\text{cavity}} \left(\frac{\epsilon}{2} |\vec{E}|^2 + \frac{\mu}{2} |\vec{H}|^2 \right) dV.$$

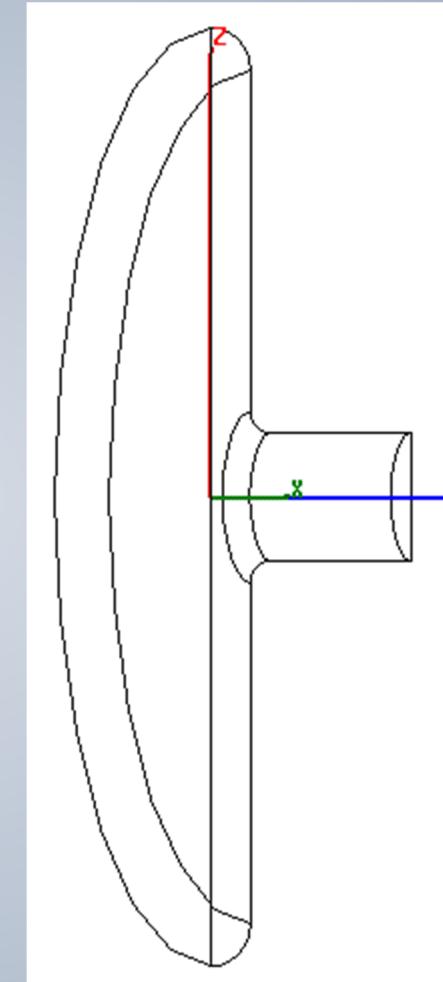


John Henry Poynting
1852 – 1914

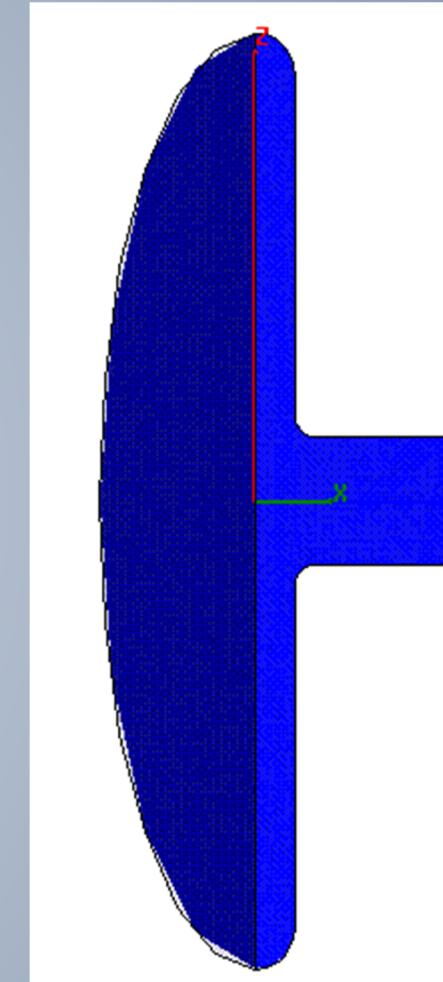
Stored energy and Poynting vector



electric field energy



Poynting vector



magnetic field energy



Wall losses & Q_0

- The losses P_{loss} are proportional to the stored energy W .
- The tangential \vec{H} on the surface is linked to a surface current $\vec{J}_A = \vec{n} \times \vec{H}$ (flowing in the skin depth δ).
- This surface current \vec{J}_A sees a surface resistance R_s , resulting in a local power density $R_s |H_t|^2$ flowing into the wall.
- R_s is related to skin depth δ as $\delta\sigma R_s = 1$.
 - Cu at 300 K has $\sigma \approx 5.8 \cdot 10^7 \text{ S/m}$, leading to $R_s \approx 8 \text{ m}\Omega$ at 1 GHz, scaling with $\sqrt{\omega}$.
 - Nb at 2 K has a typical $R_s \approx 10 \text{ n}\Omega$ at 1 GHz, scaling with ω^2 .
- The total wall losses result from $P_{\text{loss}} = \iint_{\text{wall}} R_s |H_t|^2 dA$.
- The cavity Q_0 (caused by wall losses) is defined as $Q_0 = \frac{\omega_0 W}{P_{\text{loss}}}$.
- Typical Q_0 values:
 - Cu at 300 K (normal-conducting): $\mathcal{O}(10^3 \dots 10^5)$, should improve at cryogenic T by a factor RRR .
 - Nb at 2 K (superconducting): $\mathcal{O}(10^9 \dots 10^{11})$ improves only by a factor ≈ 10 !



Shunt impedance

- Also the power loss P_{loss} is also proportional to the square of the acceleration voltage $|V_{\text{acc}}|^2$; the proportionality constant defines the “shunt impedance”

$$R = \frac{|V_{\text{acc}}|^2}{2 P_{\text{loss}}}.$$

- › **Attention, also here different definitions are used!**
 - Traditionally, the shunt impedance is the quantity to optimize in order to minimize the power required for a given gap voltage.
 - Now the previously introduced term “ R -upon- Q ” makes sense:

$$\left(\frac{R}{Q}\right) = R/Q$$



Geometric factor

- With

$$Q_0 = \frac{\omega_0 W}{\iint_{wall} R_s |H_t|^2 dA},$$

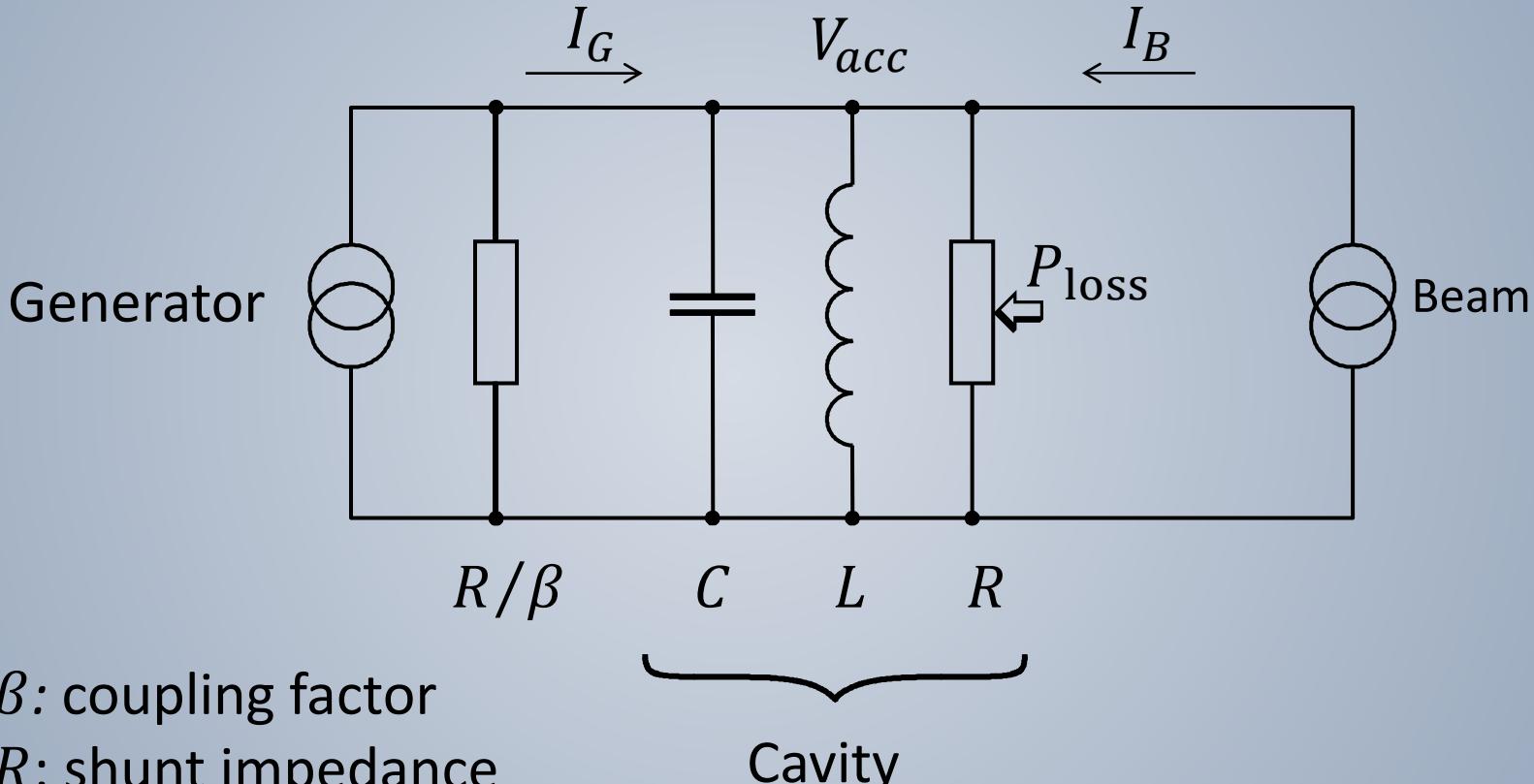
and assuming an average surface resistance R_s , one can introduce the “geometric factor” G as

$$G = Q_0 \cdot R_s = \frac{\omega_0 W}{\iint_{wall} |H_t|^2 dA}.$$

- G has dimension Ohm, depends only on the cavity geometry (as the name suggests) and typically is $\mathcal{O}(100 \Omega)$.
- Note that $R_s \cdot R = G \cdot (R/Q)$ (dimension Ω^2 , purely geometric)
- G is only used for SC cavities.

Cavity resonator – equivalent circuit

Simplification: single mode



β : coupling factor

R : shunt impedance

$$\sqrt{L/C} = \frac{R}{Q}: R\text{-upon-}Q$$



Power coupling - Loaded Q

- Note that the generator inner impedance also loads the cavity – for very large Q_0 more than the cavity wall losses.
- To calculate the loaded Q (Q_L), losses have to be added:

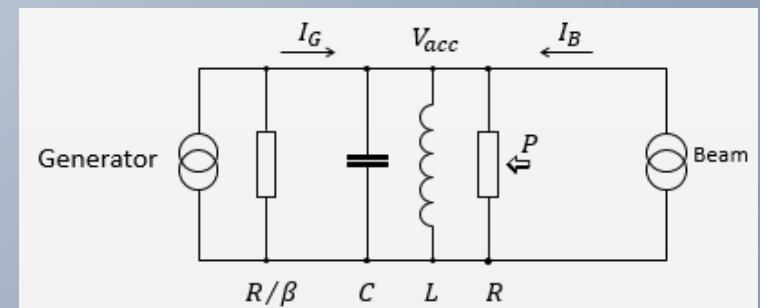
$$\frac{1}{Q_L} = \frac{P_{\text{loss}} + P_{\text{ext}} + \dots}{\omega_0 W} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} + \frac{1}{\dots}.$$

- The coupling factor β is the ratio $P_{\text{ext}}/P_{\text{loss}}$.
- With β , the loaded Q can be written

$$Q_L = \frac{Q_0}{1 + \beta}.$$

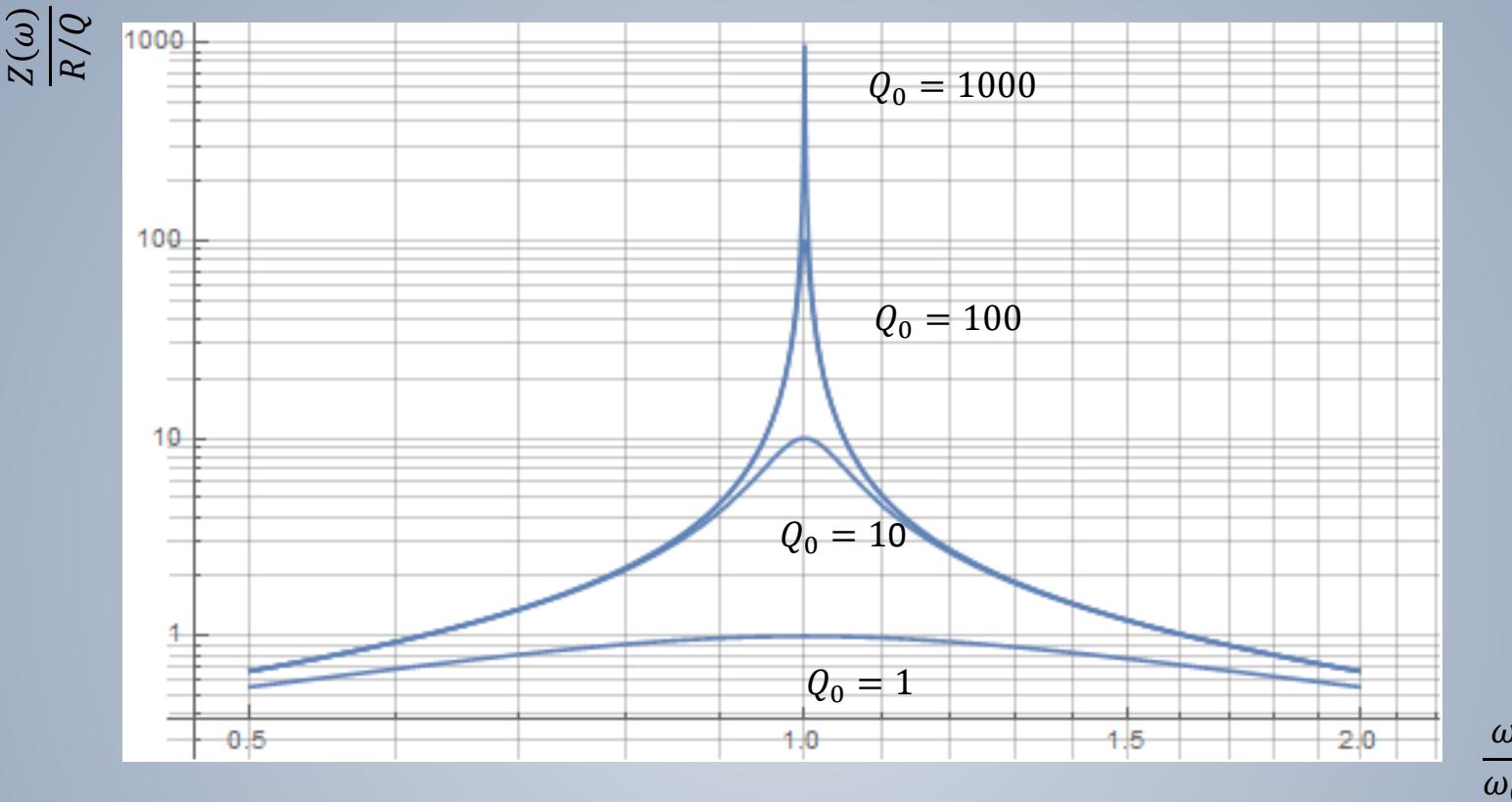
- For NC cavities, often $\beta = 1$ is chosen (power amplifier matched to empty cavity); for SC cavities, $\beta = \mathcal{O}(10^4 \dots 10^6)$.

More on how to measure the different Q 's in T. Powers' lecture!





Resonance

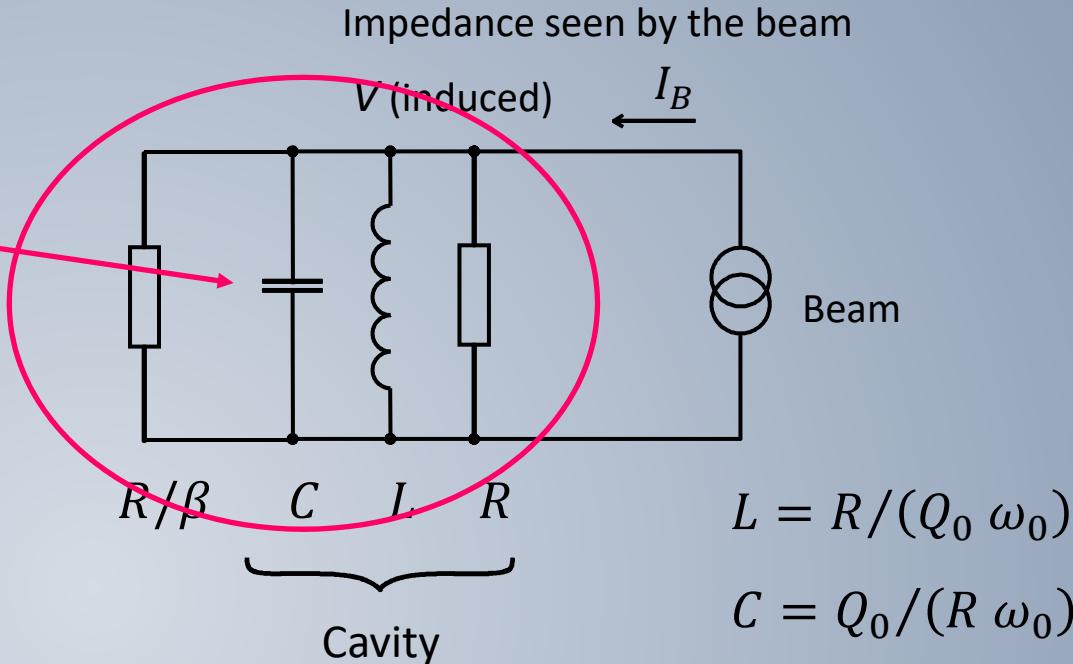


- While a high Q_0 results in small wall losses, so less power is needed for the same voltage.
- On the other hand the bandwidth becomes very narrow.
- Note: a 1 GHz cavity with a Q_0 of 10^{10} has a natural bandwidth of 0.1 Hz!
- ... to make this manageable, Q_{ext} is chosen much smaller!

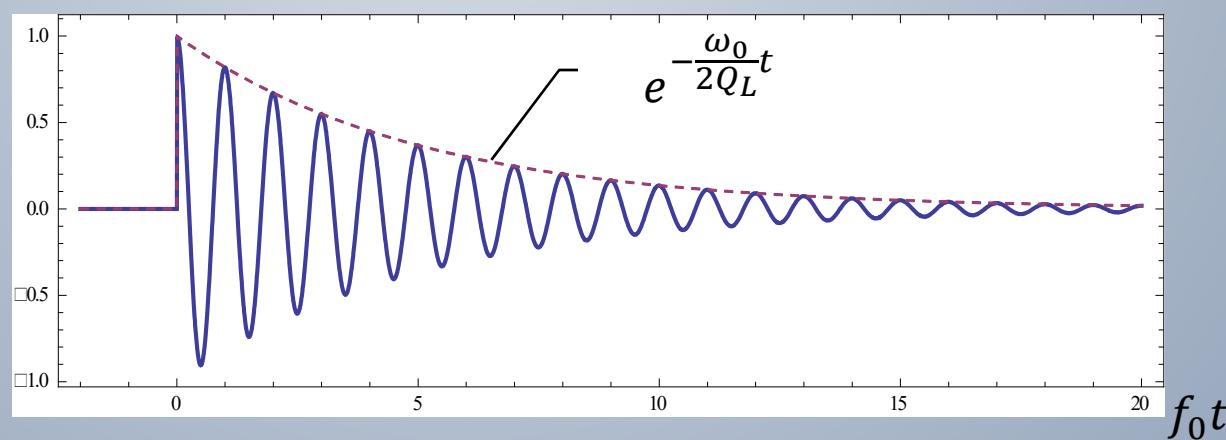


Loss factor

$$k_{\text{loss}} = \frac{\omega_0}{2} \left(\frac{R}{Q} \right) = \frac{|V_{\text{acc}}|^2}{4 W} = \frac{1}{2C}$$



$$\frac{V_{\text{acc}}}{2 k_{\text{loss}} q}$$





Summary: relations V_{acc} , W and P_{loss}

Attention – different definitions are used in literature !

V_{acc}

Accelerating voltage

$$\frac{R}{Q} = \frac{|V_{acc}|^2}{2\omega_0 W}$$

$$k_{loss} = \frac{\omega_0 R}{2 Q} = \frac{|V_{acc}|^2}{4W}$$

$$R = \frac{|V_{acc}|^2}{2P_{loss}} = \frac{R}{Q} Q_0$$

W

Energy stored

P_{loss}

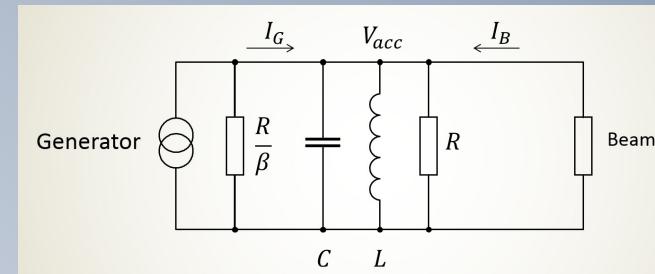
wall losses

$$Q_0 = \frac{\omega_0 W}{P_{loss}}$$



Beam loading

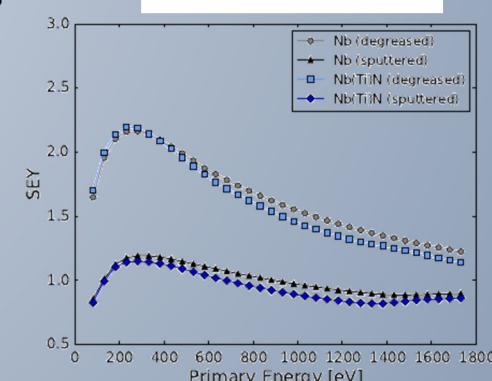
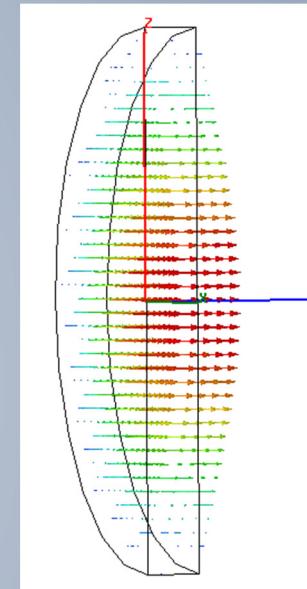
- The beam current “loads” the cavity, in the equivalent circuit this appears as an impedance in parallel to the shunt impedance.
- If the generator is matched to the unloaded cavity ($\beta = 1$), beam loading will (normally) cause the accelerating voltage to decrease.
- The power absorbed by the beam is $-\frac{1}{2} \Re\{V_{acc} I_B^*\}$.
- For high energy transfer efficiency RF → beam, the beam loading must be high!
- For SC cavities (very large β), the generator is typically matched to the beam impedance!
- Variation in the beam current leads to **transient beam loading**, which requires special care!
- Often the “impedance” the beam presents is strongly reactive – this leads to a detuning of the cavity.





Multipactor

- The words “multipactor”, “to multipact” and “multipacting” are artificially composed of “multiple” “impact”.
- Multipactor describes a resonant RF phenomenon in vacuum:
 - Consider a free electron in a simple cavity – it gets accelerated by the electric field towards the wall;
 - when it impacts the wall, secondary electrons will be emitted, described by the secondary emission yield (SEY);
 - in certain impact energy ranges, more than one electron is emitted for one electron impacting! So the number of electrons can increase;
 - when the time for an electron from emission to impact takes exactly $\frac{1}{2}$ of the RF period, resonance occurs – with the $SEY > 1$, this leads to an avalanche increase of electrons, effectively taking all RF power at this field level, depleting the stored energy and limiting the field!
- For this simple “2-point MP”, this resonance condition is reached at $\frac{1}{4\pi} \frac{e}{m} V = (fd)^2$ or $\frac{V}{112 \text{ V}} = \left(\frac{f}{\text{MHz}} \frac{d}{\text{m}} \right)^2$. There exist other resonant bands.

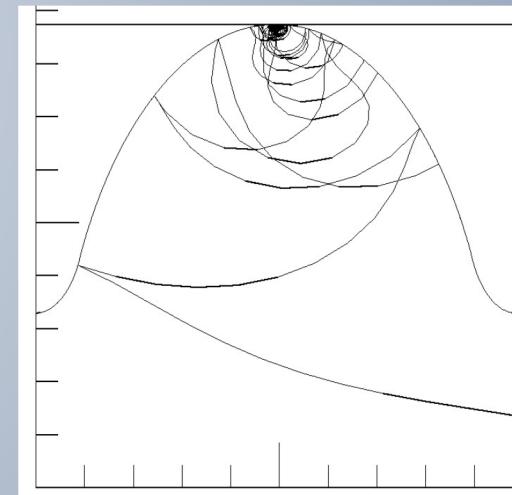


courtesy: Sarah Aull/CERN



Multipactor (contd.)

- Unfortunately, good metallic conductors (Cu, Ag, Nb) all have $SEY > 1$!
- 1-point MP occurs when the electron impact where they were emitted
- Electron trajectories can be complex since both \vec{E} and \vec{B} influence them; computer simulations allow to determine the MP bands (barriers)
- To reduce or suppress MP, a combination of the following may be considered:
 - Use materials with low SEY
 - Optimize the shape of your cavity (\rightarrow elliptical cavity)
 - Conditioning (surface altered by exposure to RF fields)
 - Coating (Ti, TiN, NEG, amorphous C ...)
 - Clearing electrode (for a superimposed DC electric field)
 - Rough surfaces



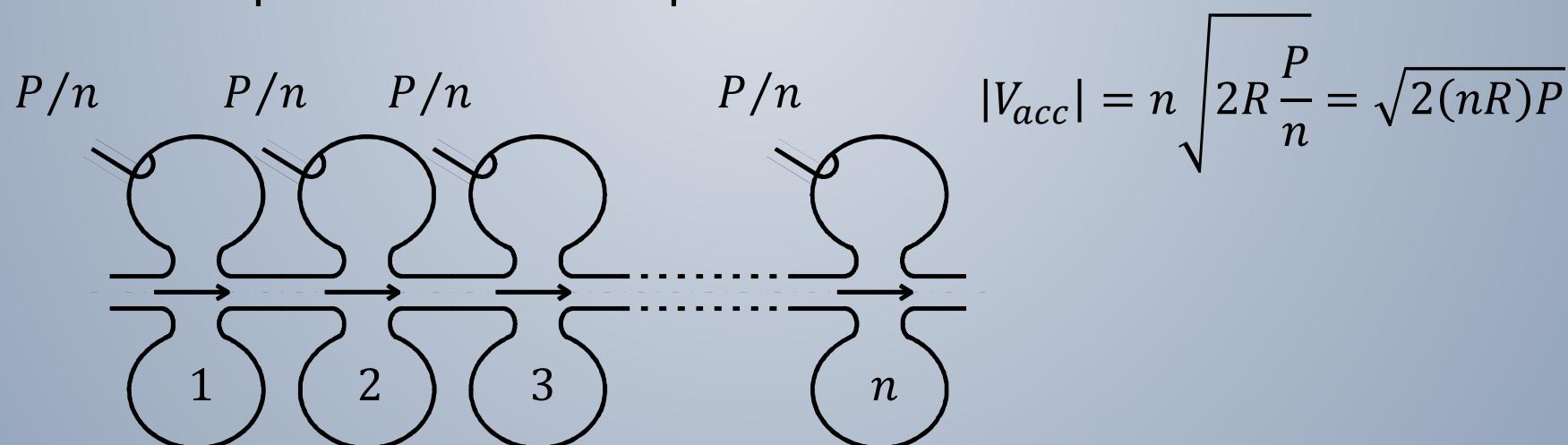
Approx. end of 1st hour

Many gaps



What do you gain with many gaps?

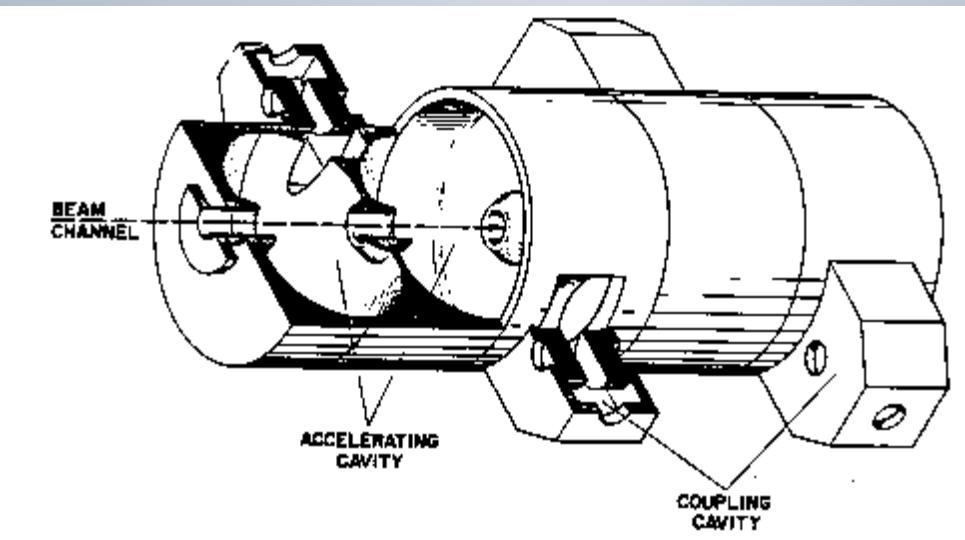
- The R/Q of a single gap cavity is limited to some 100Ω .
Now consider to distribute the available power to n identical cavities: each will receive P/n , thus produce an accelerating voltage of $\sqrt{2RP/n}$. (Attention: phase important!)
The total accelerating voltage thus increased, equivalent to a total equivalent shunt impedance of nR .





Standing wave multi-cell cavity

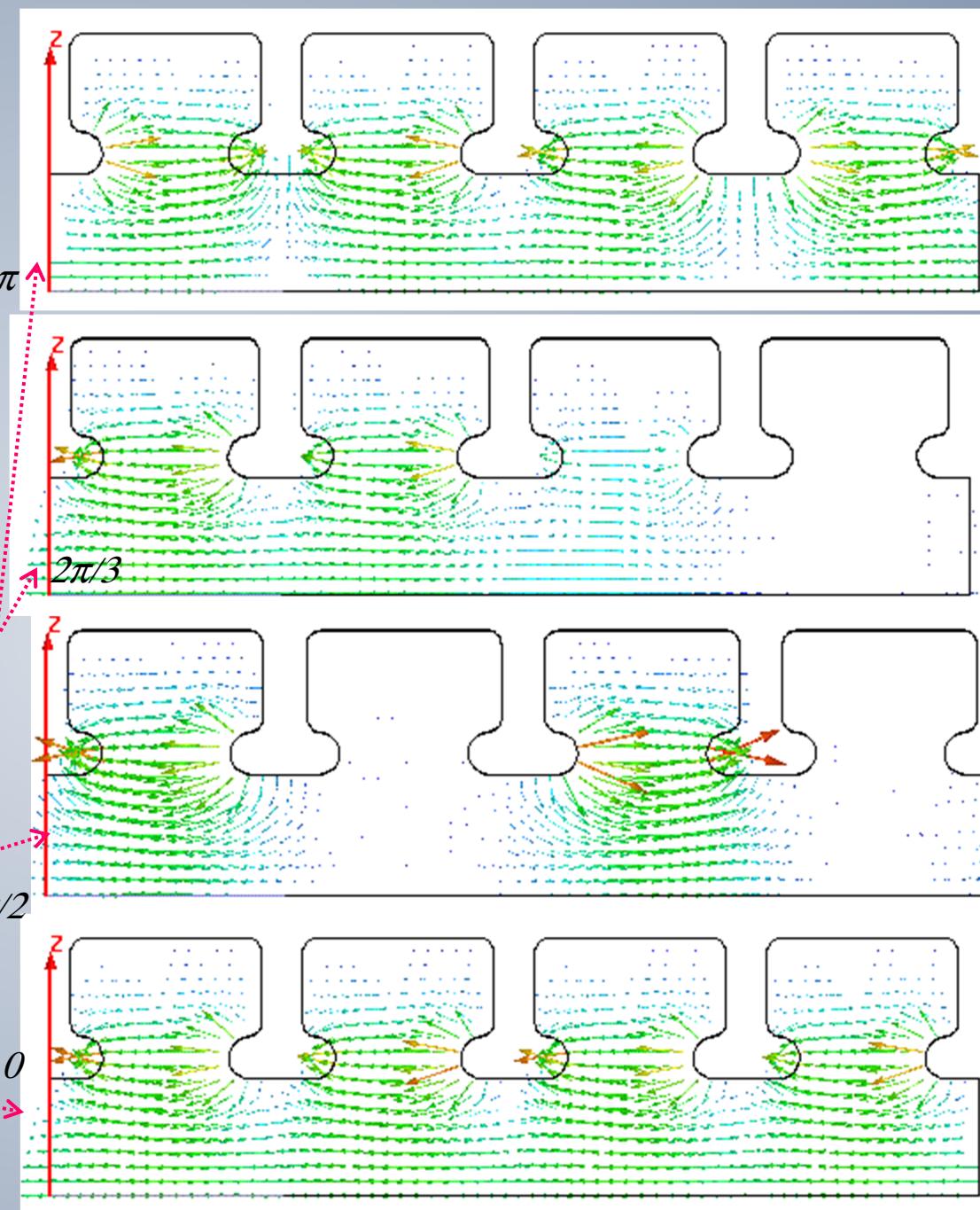
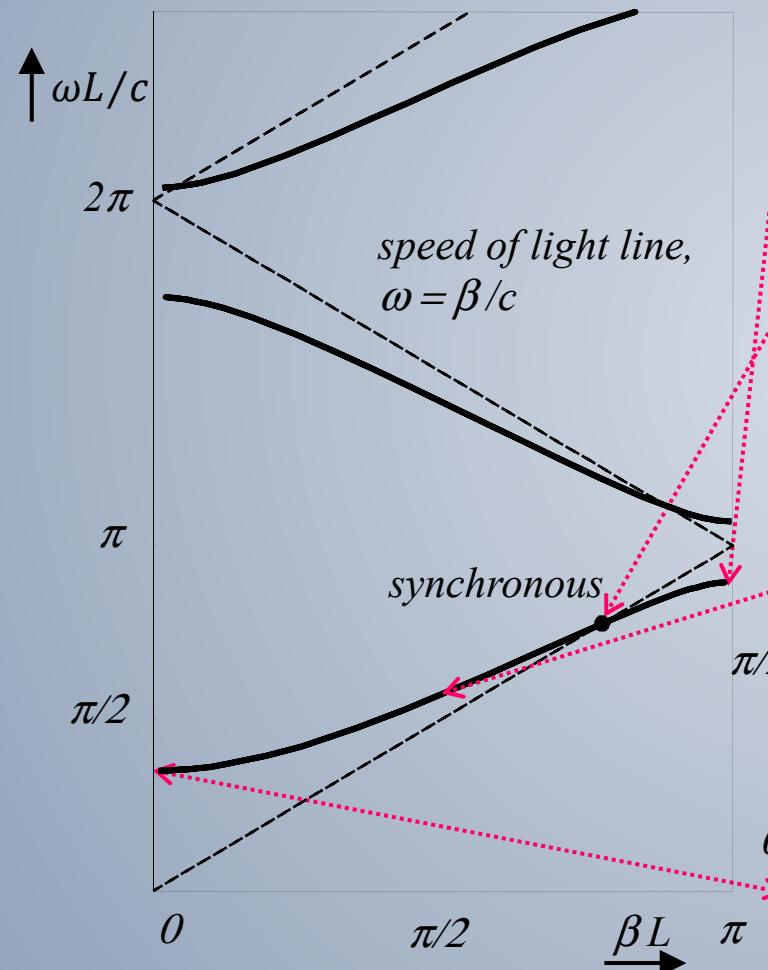
- Instead of distributing the power from the amplifier, one might as well couple the cavities, such that the power automatically distributes, or have a cavity with many gaps (e.g. drift tube linac).
- Coupled cavity accelerating structure (side coupled)



- The phase relation between gaps is important!



Brillouin diagram; Travelling wave structure

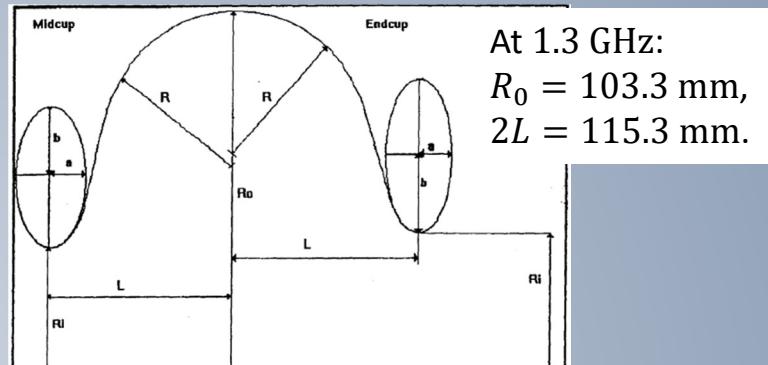




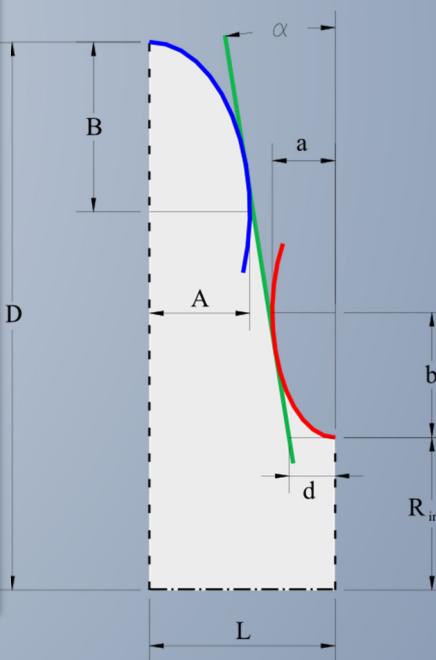
The elliptical cavity

- The elliptical shape was found as optimum compromise between
 - maximum gradient ($E_{acc}/E_{surface}$)
 - suppression of multipactor
 - mode purity
 - machinability
- A multi-cell elliptical cavity is typically operated in π -mode, i.e. cell length is exactly $\beta\lambda/2$.
- It has become de facto standard, used for ions and leptons! E.g.:
 - ILC/X-FEL: 1.3 GHz, 9-cell cavity
 - SNS (805 MHz)
 - SPL/ESS (704 MHz)
 - LHC (400 MHz)

*): <http://accelconf.web.cern.ch/AccelConf/SRF93/papers/srf93g01.pdf>



D. Proch, 1993 *)



Elliptical cavity – the *de facto* standard for SRF



FERMI 3.9 GHz



S-DALINAC 3 GHz



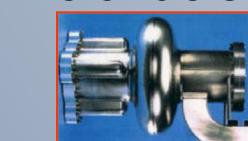
CEBAF 1.5 GHz



HEPL 1.3 GHz



CESR 0.5 GHz



KEK-B 0.5 GHz

TESLA/ILC 1.3 GHz



SNS $\beta = 0.61, 0.81, 0.805$ GHz



HERA 0.5 GHz

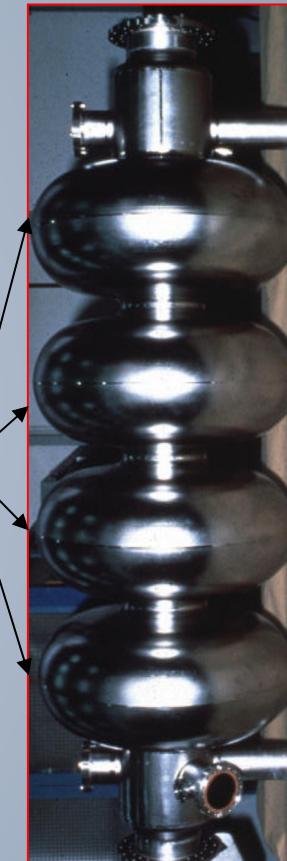


TRISTAN 0.5 GHz



cells

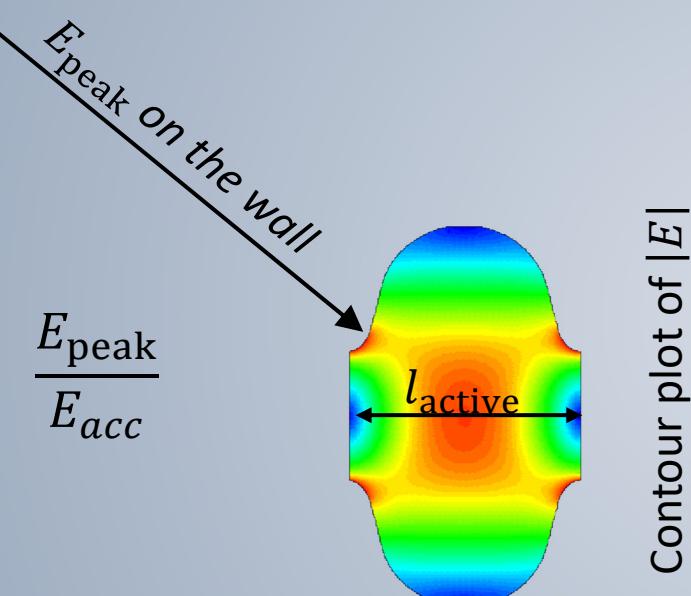
LEP 0.352 GHz





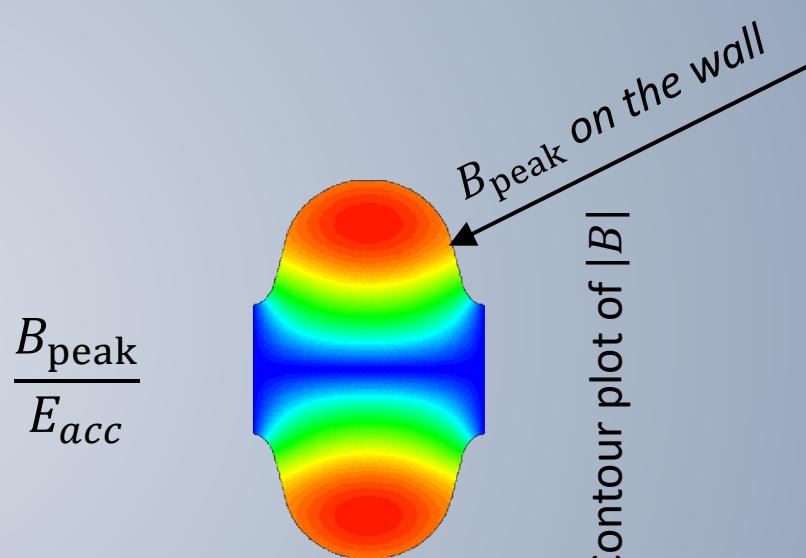
Practical RF parameters 1

- Average accelerating gradient: $E_{acc} = \frac{\sqrt{\omega W(R/Q)}}{l_{active}}$



Contour plot of $|E|$

The ratio shows sensitivity of the shape to the **field emission** of electrons.



Contour plot of $|B|$

The ratio shows limit in E_{acc} due to the breakdown of superconductivity (**quench**, Nb: ≈ 190 mT).

courtesy: Jacek Sekutowicz/DESY

Practical RF parameters 2

$$G \cdot (R/Q)$$

- Both G and R/Q are purely geometric parameters.
- Like the shunt impedance R , the product $G \cdot (R/Q)$ is a measure of the power loss for given acceleration voltage V_{acc} and surface resistance R_s .

$$P_{\text{loss}} = \frac{|V_{acc}|^2 R_s}{2 G \cdot (R/Q)}$$

Minimize R_s :

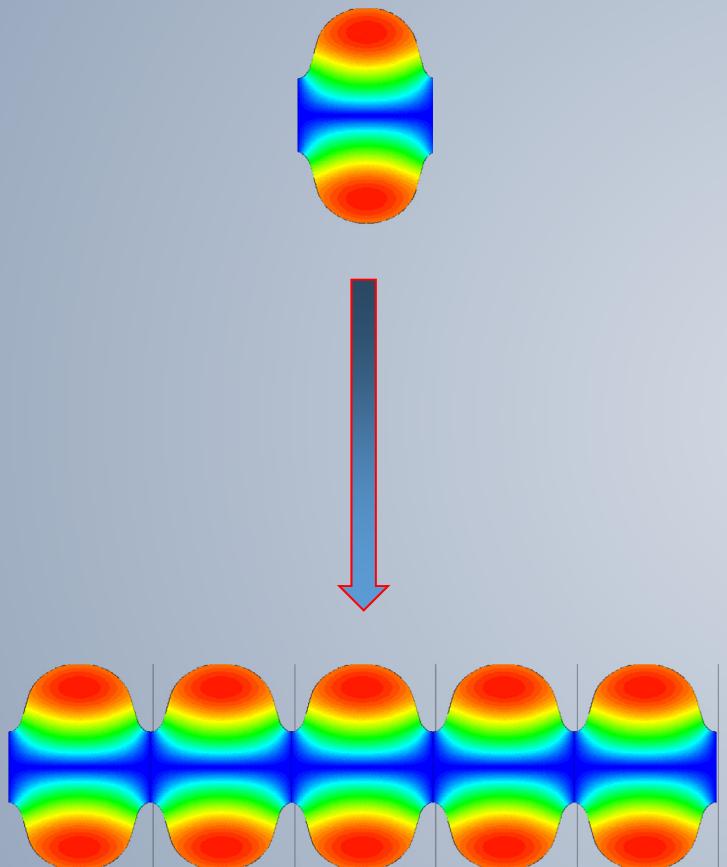
operation at lower T ,
better surface cleanliness,
lower residual resistance

Optimize geometry maximizing $G \cdot (R/Q)$.

courtesy: Jacek Sekutowicz/DESY



Single-cell versus multi-cell cavities



- Advantages of single-cell cavities:
 - It is easier to manage HOM damping
 - There is no field flatness problem.
 - Input coupler transfers less power
 - They are easy for cleaning and preparation

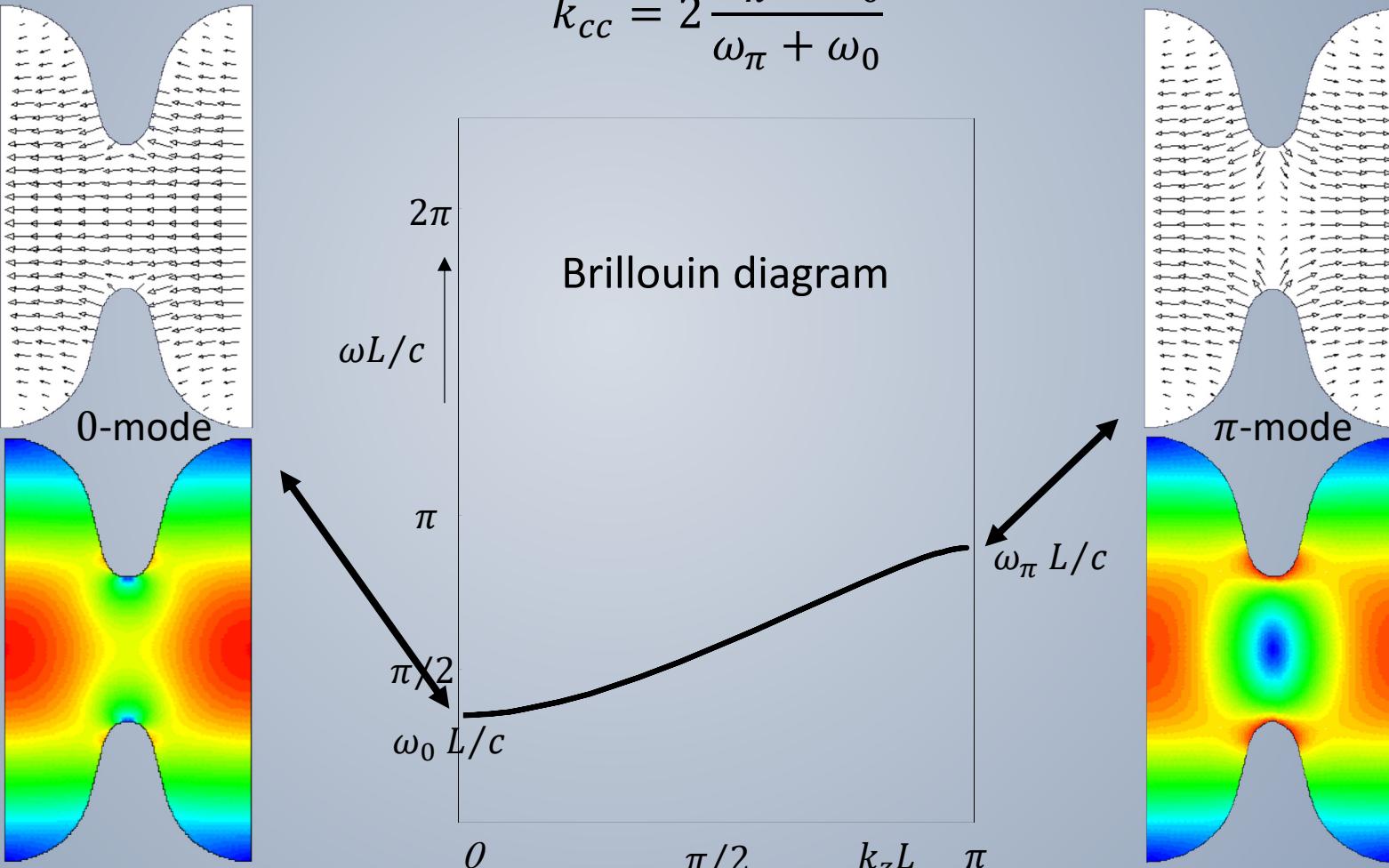
- Advantages of multi-cell cavities:
 - much more acceleration per meter!
 - better use of the power ($R \rightarrow n R$)
 - more cost-effective for most applications

courtesy: Jacek Sekutovicz/DESY

Practical RF parameters 3

- Cell-to-cell coupling k_{cc} will determine the width of the passbands in multi-cell cavities.

$$k_{cc} = 2 \frac{\omega_\pi - \omega_0}{\omega_\pi + \omega_0}$$

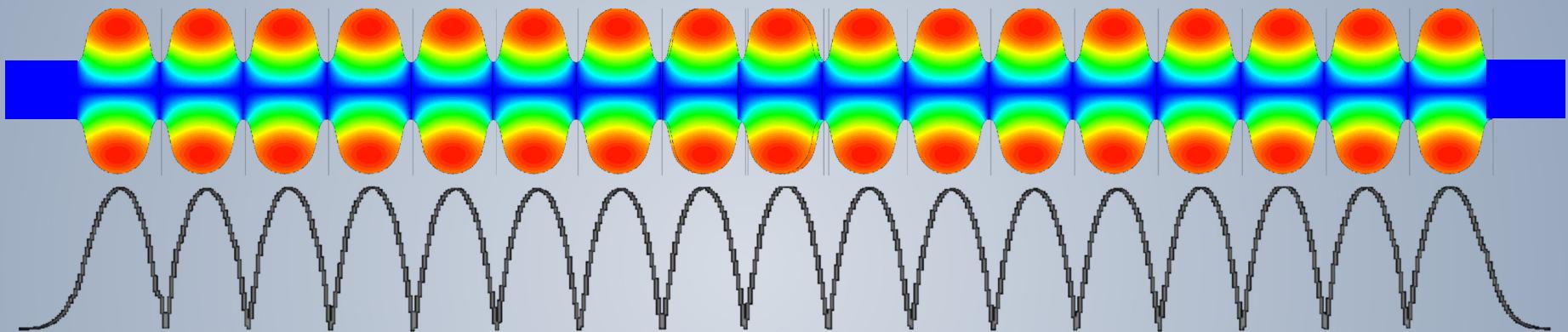


courtesy: Jacek Sekutovicz/DESY



Field flatness

- Field amplitude variation from cell to cell in a multi-cell structure
- Should be small for maximum acceleration.



- Field flatness sensitivity factor a_{ff} for a structure made of N cells:

$$\frac{\Delta A_i}{A_i} = a_{ff} \frac{\Delta f_i}{f_i}$$

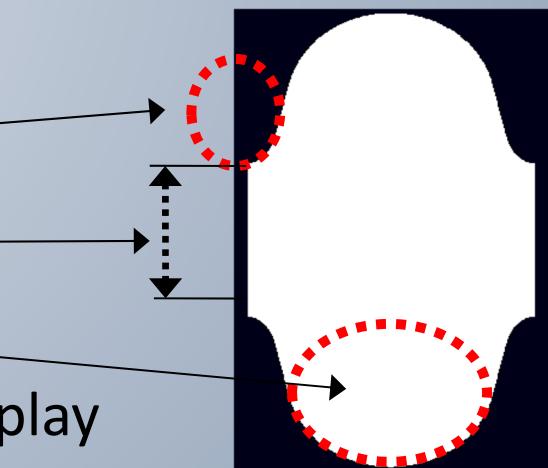
a_{ff} is related to the cell-to-cell coupling as $a_{ff} = \frac{N^2}{k_{cc}}$ and describes the sensitivity of the field flatness on the errors in individual cells.

courtesy: Jacek Sekutowicz/DESY



Criteria for Cavity Design (1)

- Here: Inner cells of multi-cell structures
- Parameters for optimization:
 - Fundamental mode: $\frac{R}{Q}, G, \frac{E_{\text{peak}}}{E_{\text{acc}}}, \frac{B_{\text{peak}}}{E_{\text{acc}}}, k_{cc}$.
 - Higher order modes: k_{\perp}, k_z .
- The elliptical cavity design has distinct advantages:
 - easy to clean (rinse)
 - little susceptible to MP – can be conditioned ...
- Geometric parameters for optimization:
 - iris ellipse half axes: a, b :
 - iris aperture radius: r_i ,
 - equator ellipse half axes: A, B
- Problem: 7 parameters to optimize, only 5 to play with – some compromise has to be found!



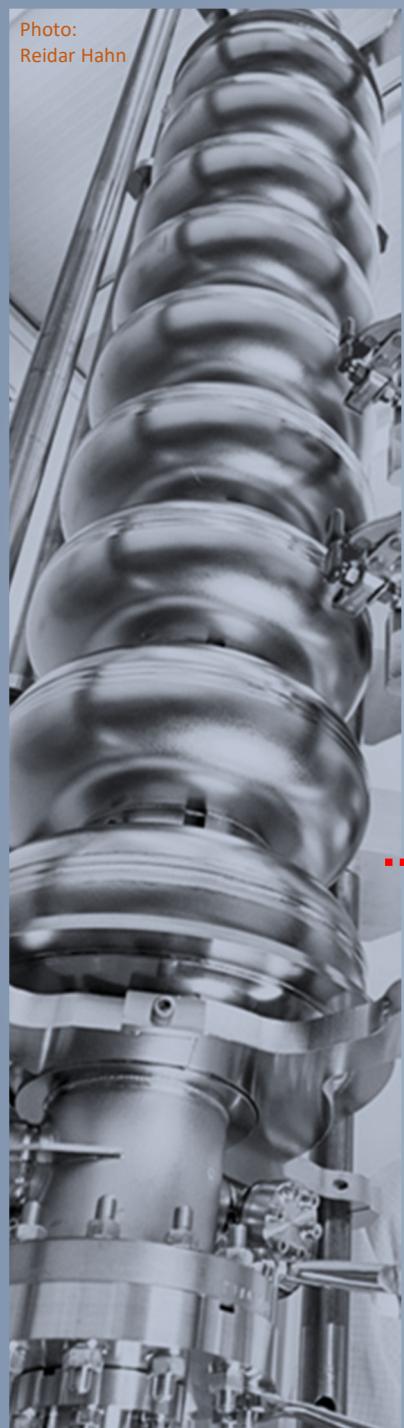
courtesy: Jacek Sekutovicz/DESY

Criteria for Cavity Design (2)

Criterion	RF parameter	Improves if	examples
high gradient operation	E_{peak}/E_{acc} B_{peak}/E_{acc}	r_i 	TESLA, CEBAF 12 GeV HG
low cryogenic losses	$\frac{R}{Q} \cdot G$	r_i 	CEBAF LL
High I_{beam}	k_{\perp}, k_z	r_i 	B-factory RHIC cooling LHeC

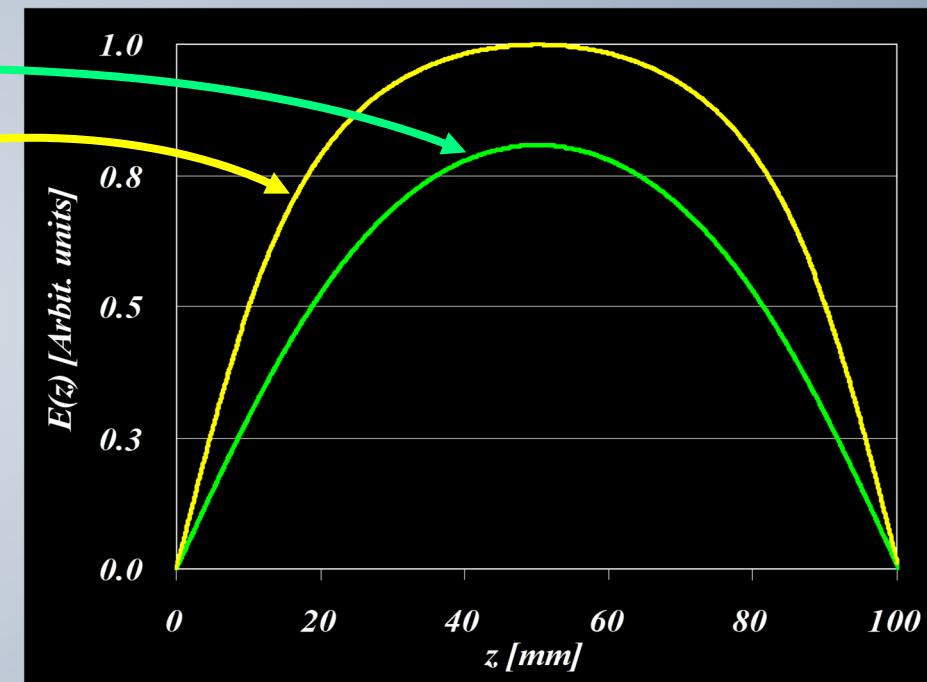
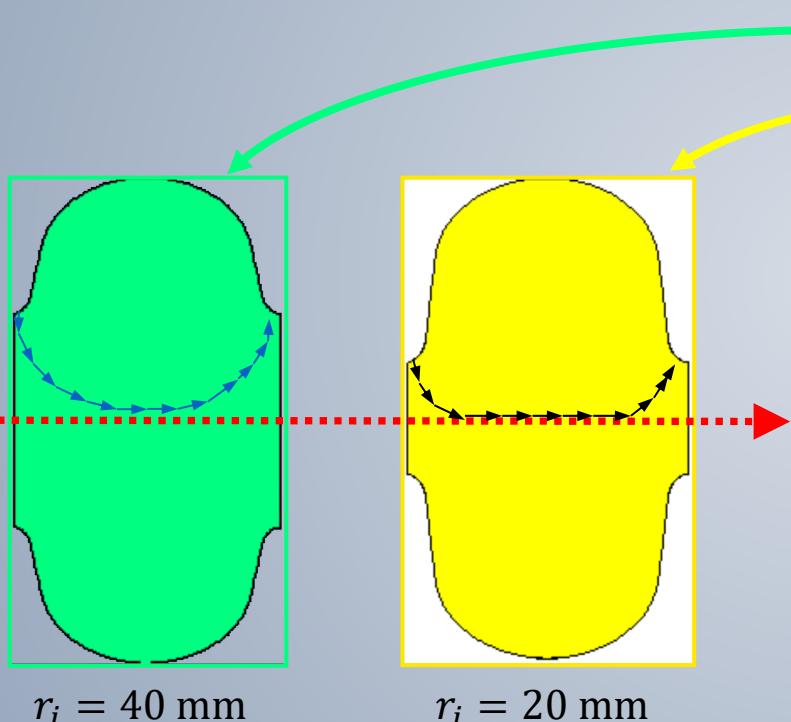
We see here that r_i is a very “powerful” variable to trim the RF-parameters of a cavity.
Of course it has to fit the aperture required for the beam!

courtesy: Jacek Sekutovicz/DESY



Effect of r_i

- Smaller r_i allows to concentrate E_z where it is needed for acceleration

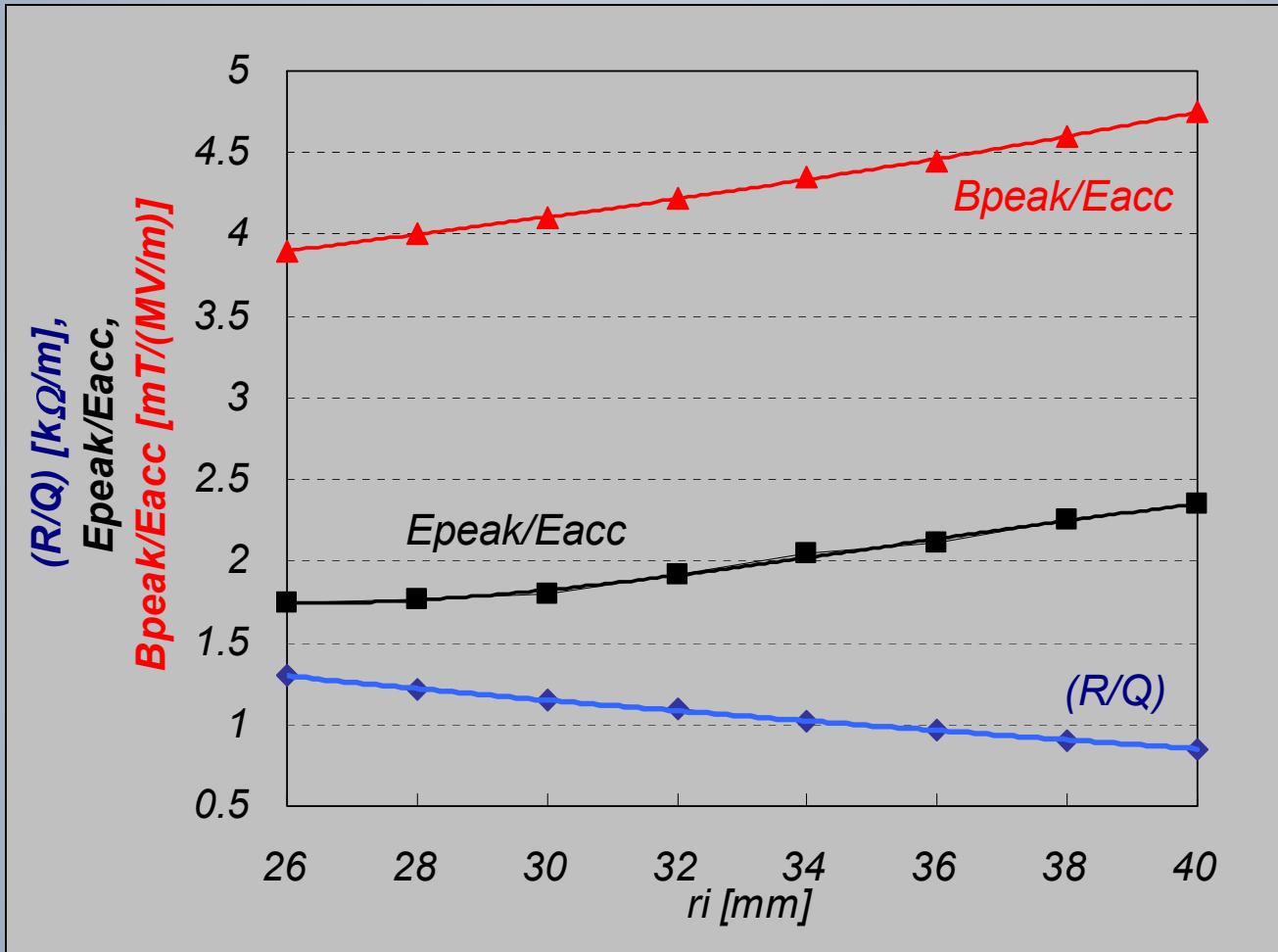


$E_z(z)$ for small and big iris radius

courtesy: Jacek Sekutovicz/DESY



Example: cell optimization at 1.5 GHz

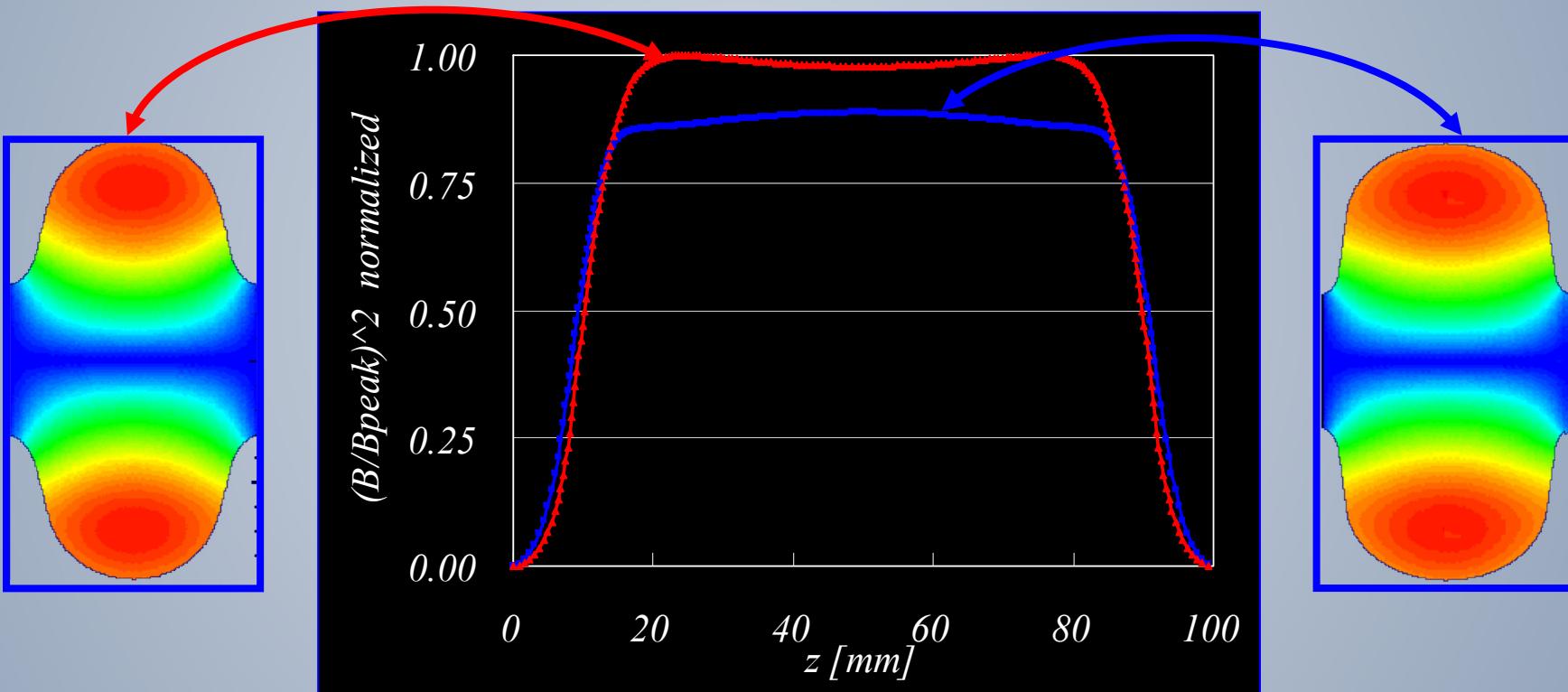


A. Mosnier, E. Haebel, SRF Workshop 1991



Equator shape optimization

- B_{peak}/E_{acc} (and G) change when changing the equator shape.

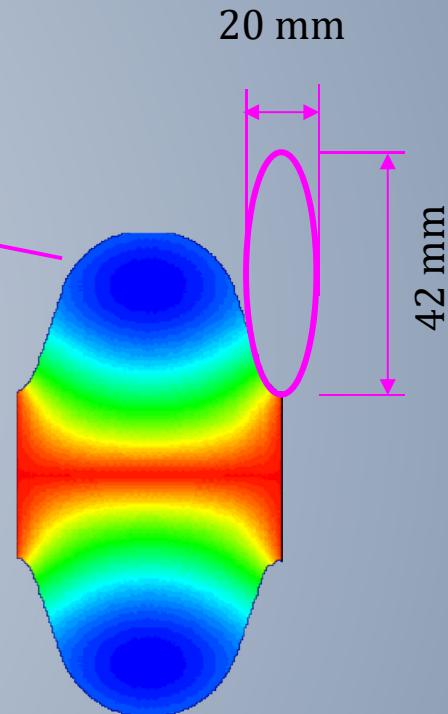
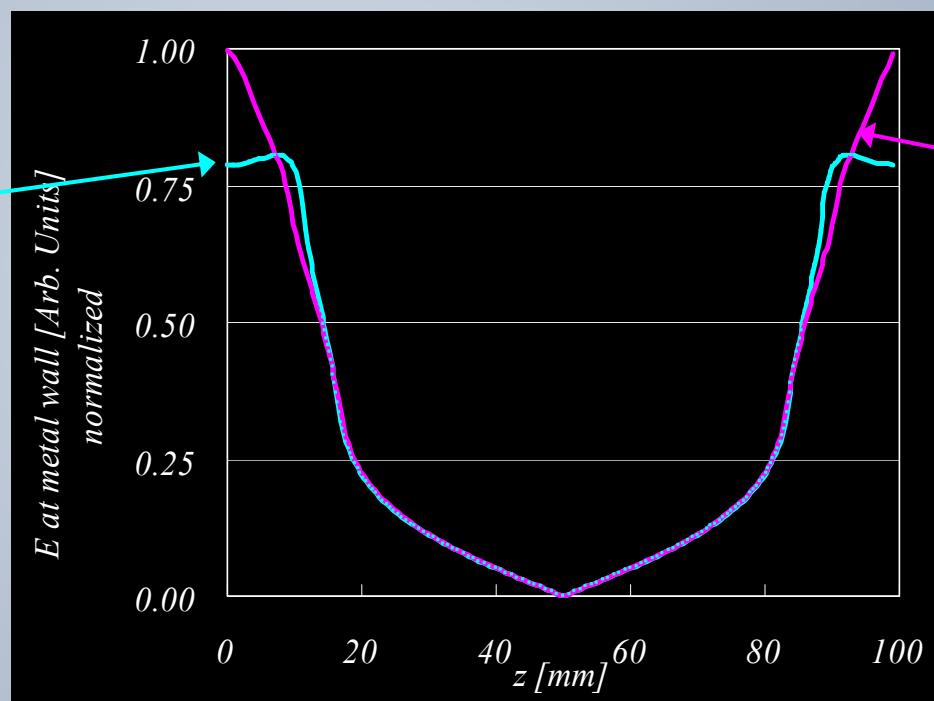
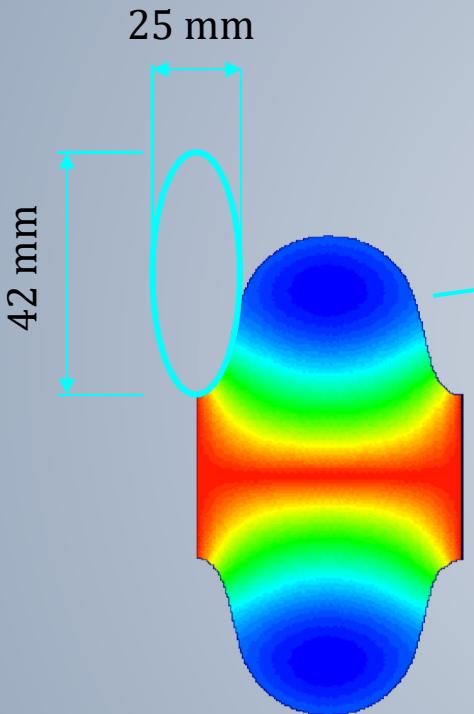


courtesy: Jacek Sekutovicz/DESY



Iris shape optimization

- E_{peak}/E_{acc} changes with the iris shape



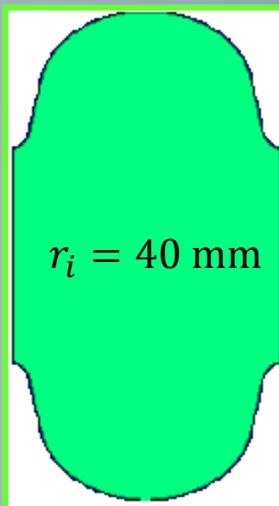
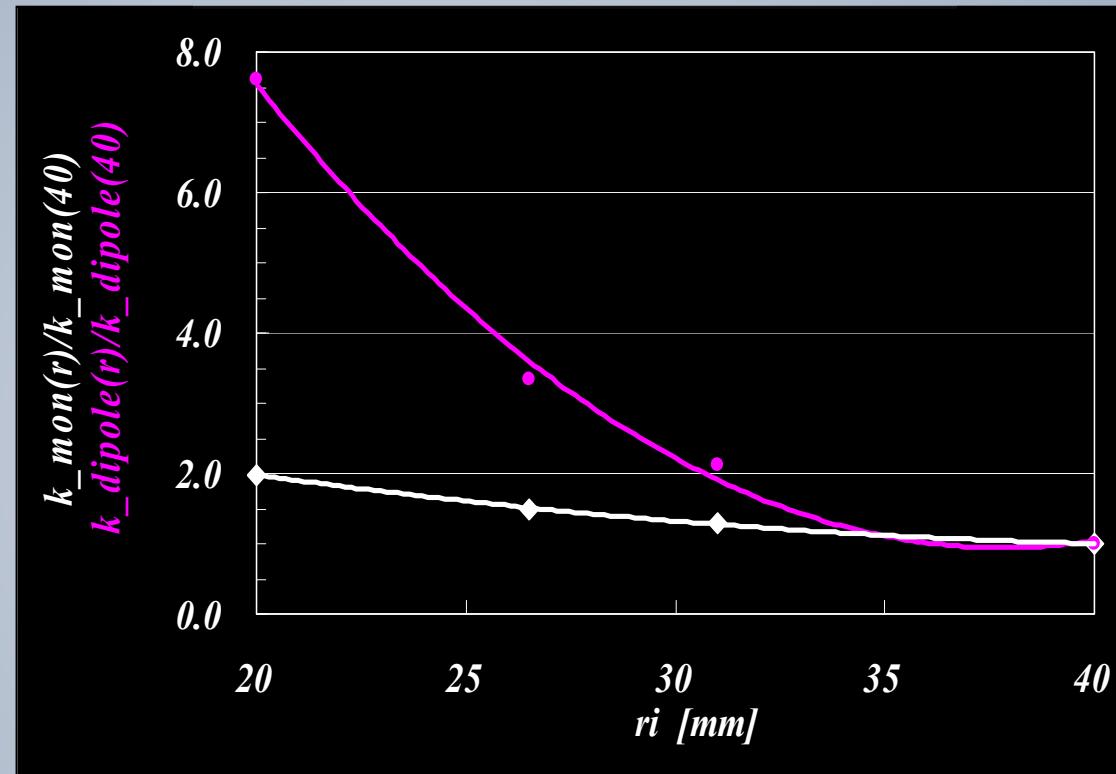
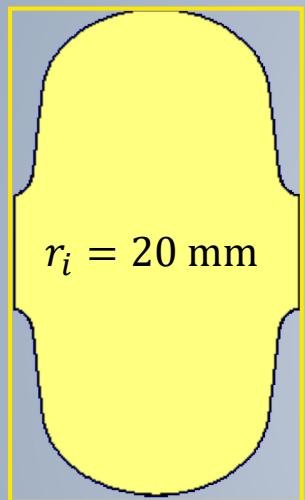
Both cells have the same: f_0 , R/Q , and r_i .

courtesy: Jacek Sekutovicz/DESY



Minimizing HOM excitation

HOMs loss factors ($k_{\text{loss},\perp}$, k_{loss})



$$R/Q = 152 \Omega$$

$$B_{\text{peak}}/E_{\text{acc}} = 3.5 \text{ mT/(MV/m)}$$

$$E_{\text{peak}}/E_{\text{acc}} = 1.9$$

$$R/Q = 86 \Omega$$

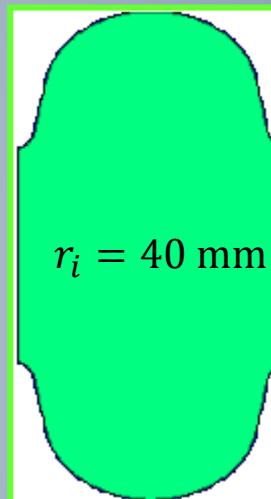
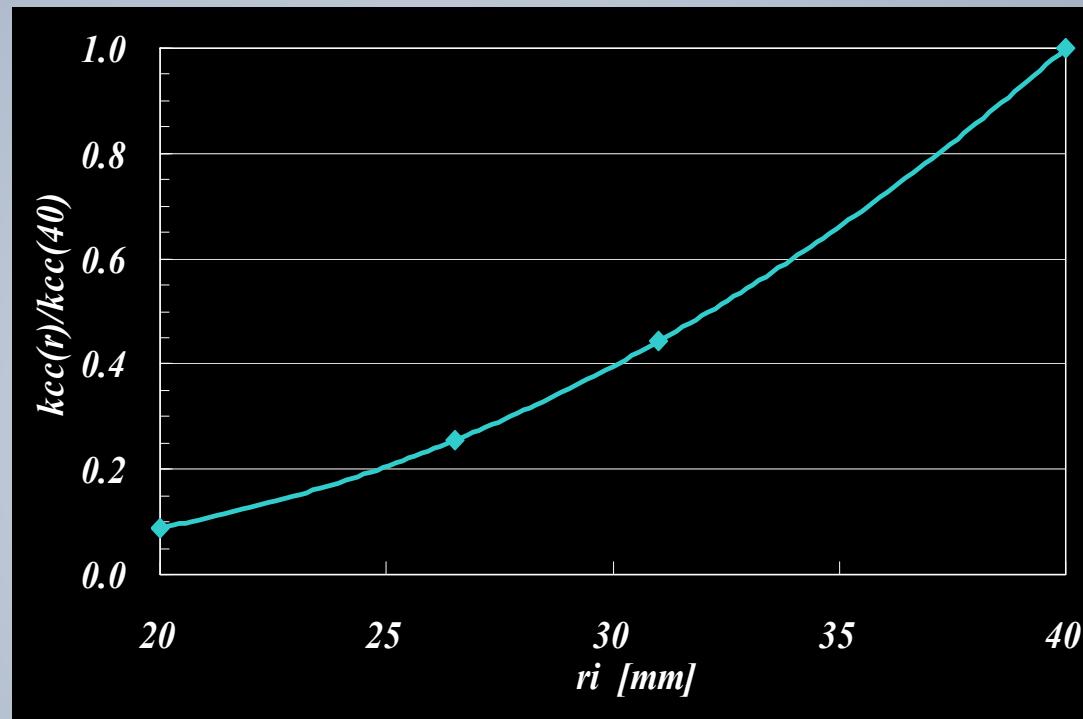
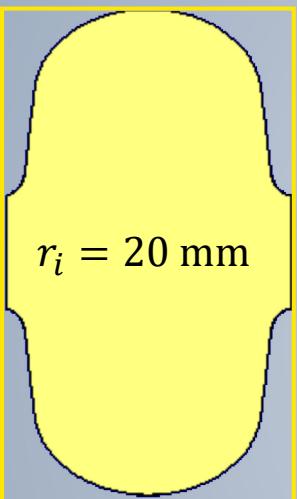
$$B_{\text{peak}}/E_{\text{acc}} = 4.6 \text{ mT/(MV/m)}$$

$$E_{\text{peak}}/E_{\text{acc}} = 3.2$$

courtesy: Jacek Sekutowicz/DESY



Cell-to-cell coupling k_{cc}



$$R/Q = 152 \Omega$$

$$B_{\text{peak}}/E_{\text{acc}} = 3.5 \text{ mT/(MV/m)}$$

$$E_{\text{peak}}/E_{\text{acc}} = 1.9$$

$$R/Q = 86 \Omega$$

$$B_{\text{peak}}/E_{\text{acc}} = 4.6 \text{ mT/(MV/m)}$$

$$E_{\text{peak}}/E_{\text{acc}} = 3.2$$

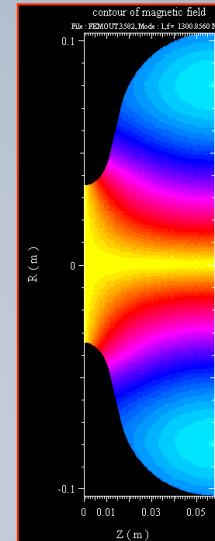
courtesy: Jacek Sekutowicz/DESY



Scaling the frequency



$\times 2 =$



f_π	[MHz]	2600
R/Q	[Ω]	57
r/Q	[Ω/m]	2000
G	[Ω]	271

$$r/Q = (R/Q)/l \propto f$$

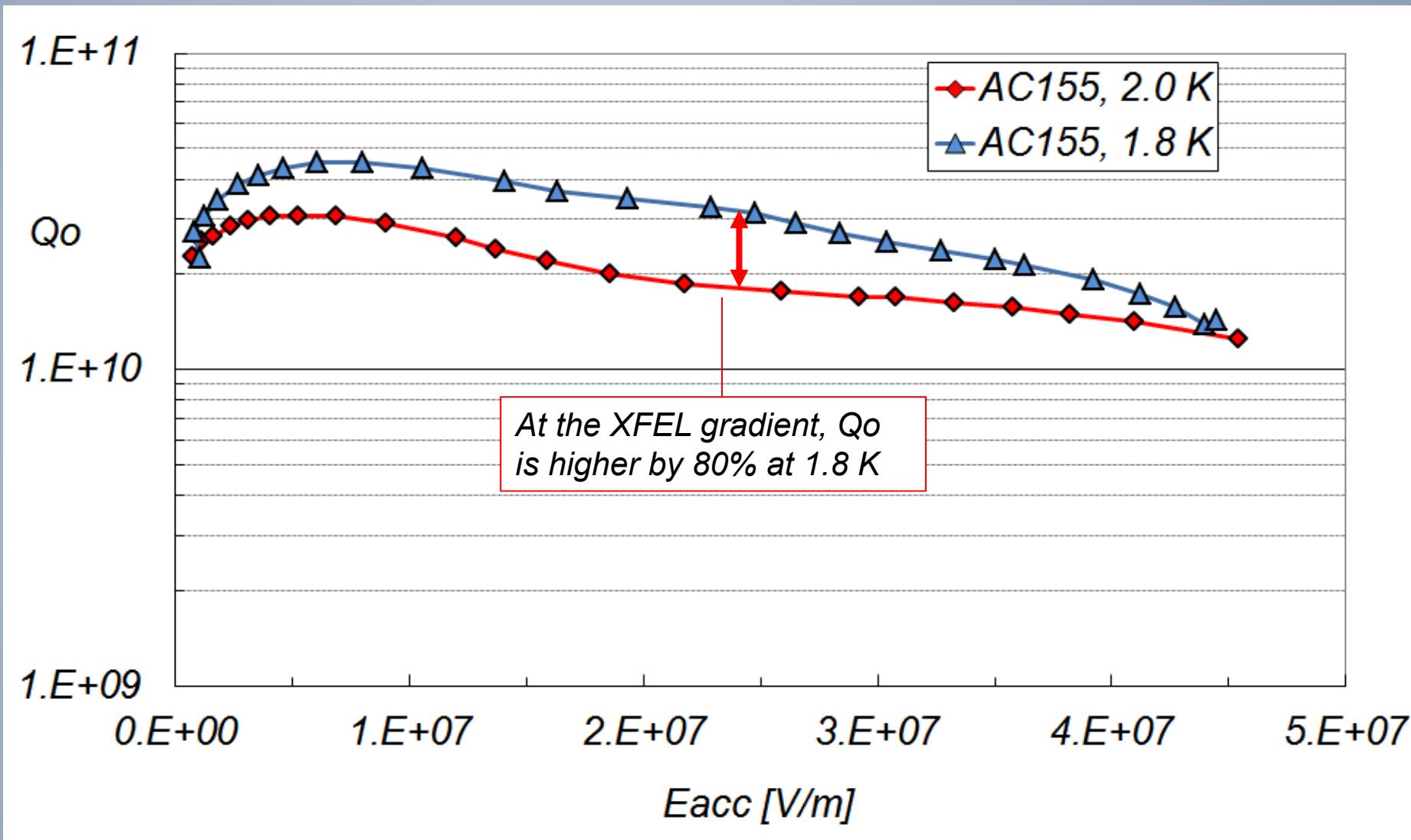
f_π	[MHz]	1300
R/Q	[Ω]	57
r/Q	[Ω/m]	1000
G	[Ω]	271

$$(or (R/Q)/\lambda = \text{const})$$

courtesy: Jacek Sekutowicz/DESY



Operating temperature

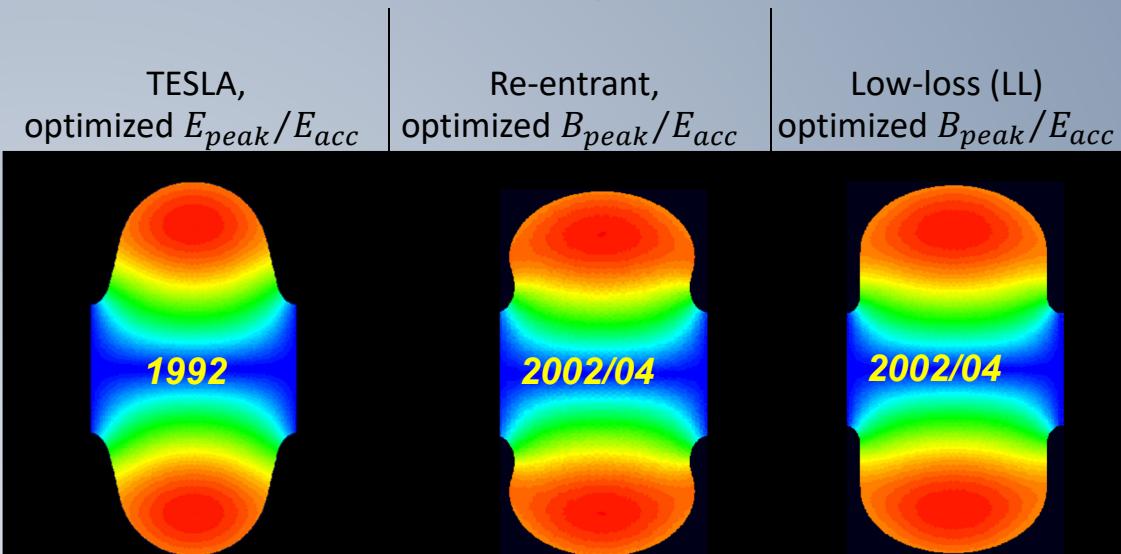


courtesy: Jacek Sekutowicz/DESY



Historic evolution of inner cell geometry

Example: ILC

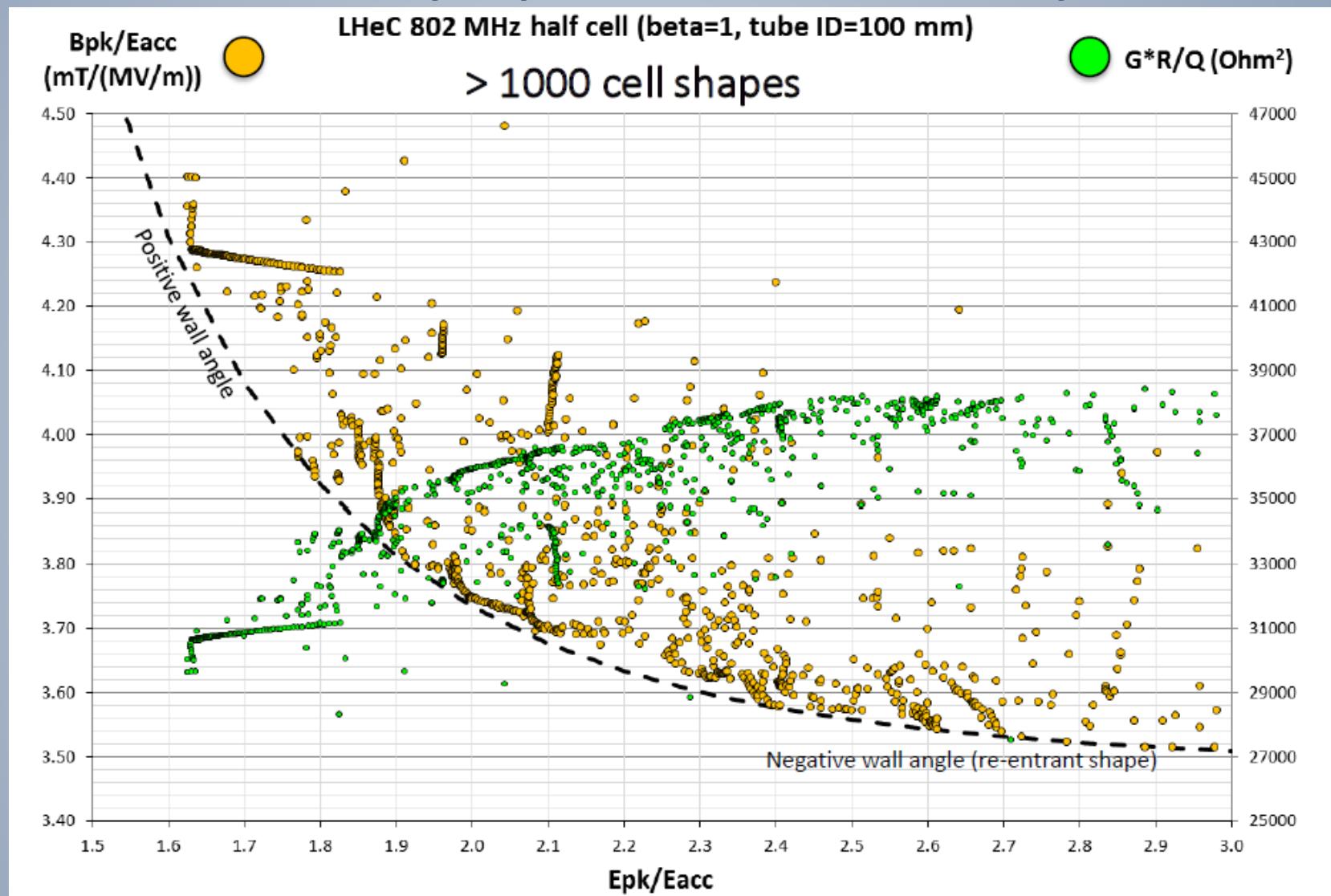


r_i	[mm]	35	30	30
k_{cc}	[%]	1.9	1.56	1.52
E_{peak}/E_{acc}	-	1.98	2.30	2.36
B_{peak}/E_{acc}	[mT/(MV/m)]	4.15	3.57	3.61
R/Q	[Ω]	113.8	135	133.7
G	[Ω]	271	284.3	284
$R/Q \cdot G$	[$\Omega^*\Omega$]	30840	38380	37970
$k_{loss,\perp} (\sigma_z = 1 \text{ mm})$	[V/pC/cm ²]	0.23	0.38	0.38
$k_{loss} (\sigma_z = 1 \text{ mm})$	[V/pC]	1.46	1.75	1.72

courtesy: Jacek Sekutowicz/DESY



Cavity optimization example

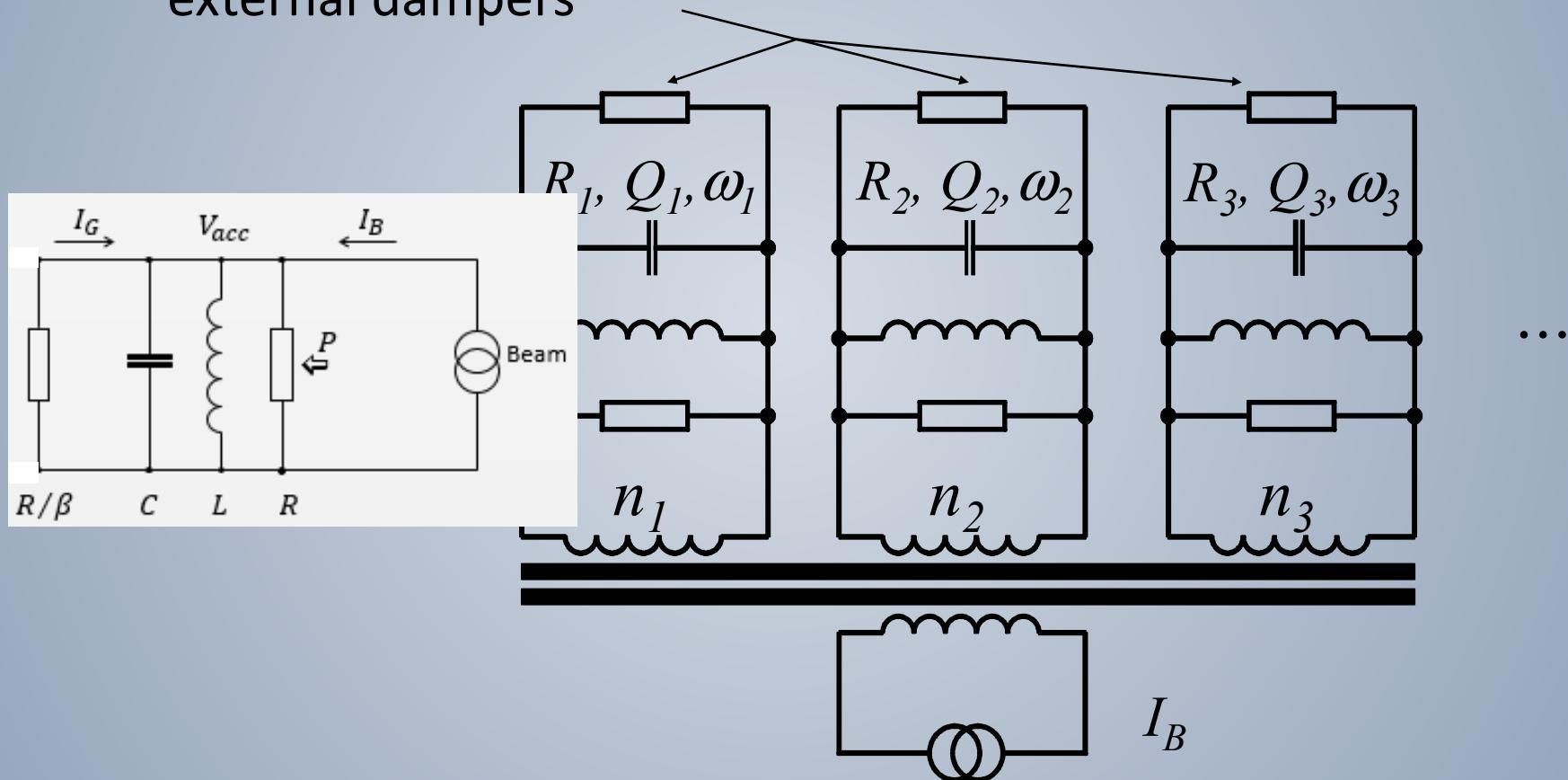


courtesy: Frank Marhauser/JLAB



Higher order modes

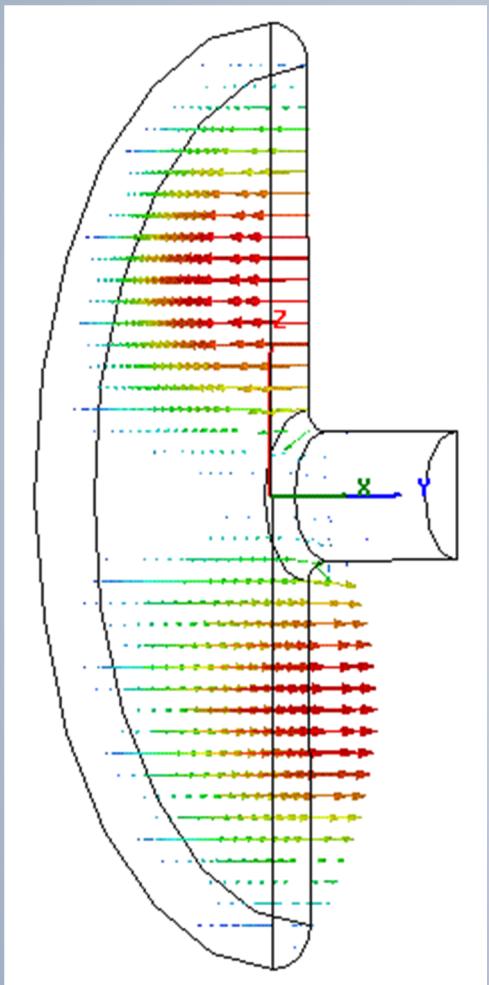
external dampers



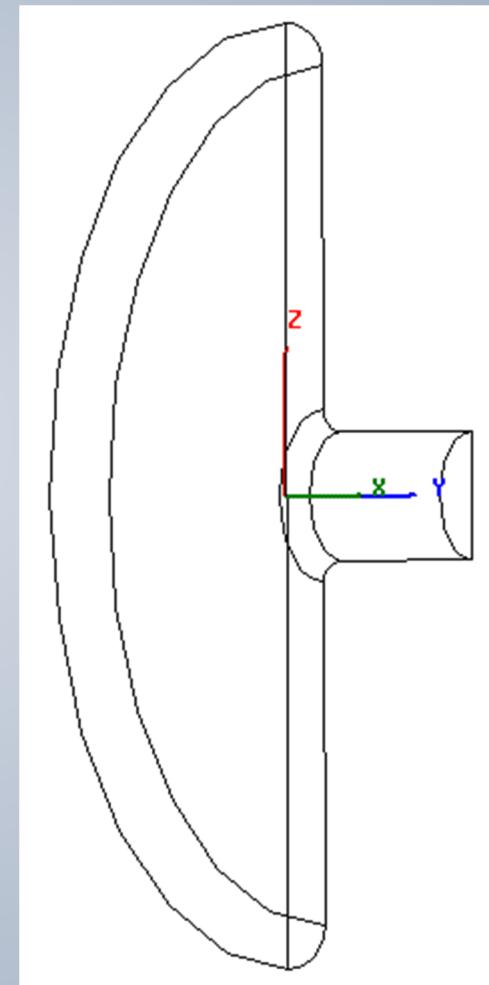


Pillbox: 1st dipole mode

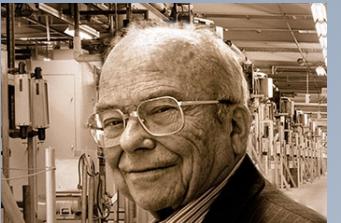
TM₁₁₀-mode (only 1/4 shown)



electric field



magnetic field



Wolfgang Panofsky
1919 – 2007

Panofsky-Wenzel theorem

For particles moving virtually at $v = c$, the integrated transverse force (kick) can be determined from the transverse variation of the integrated longitudinal force!

$$j \frac{\omega}{c} \vec{F}_\perp = \nabla_\perp F_z$$

Pure TE modes: No net transverse force!

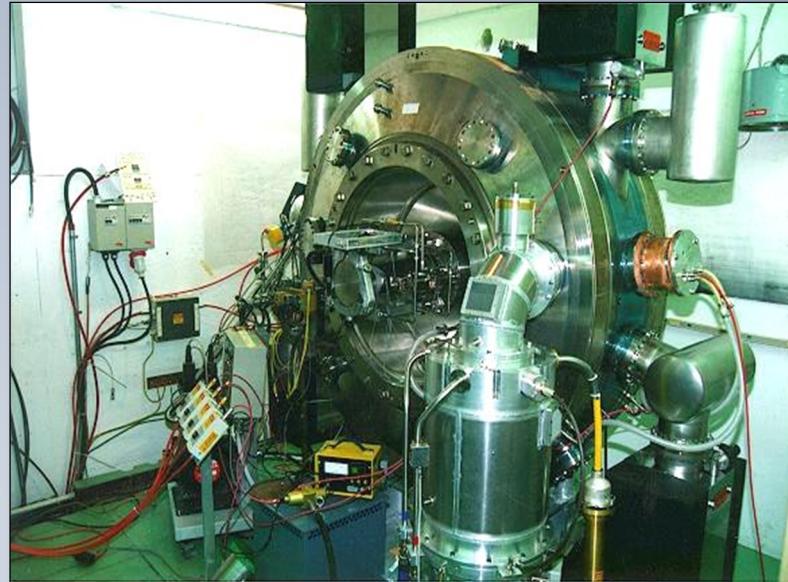
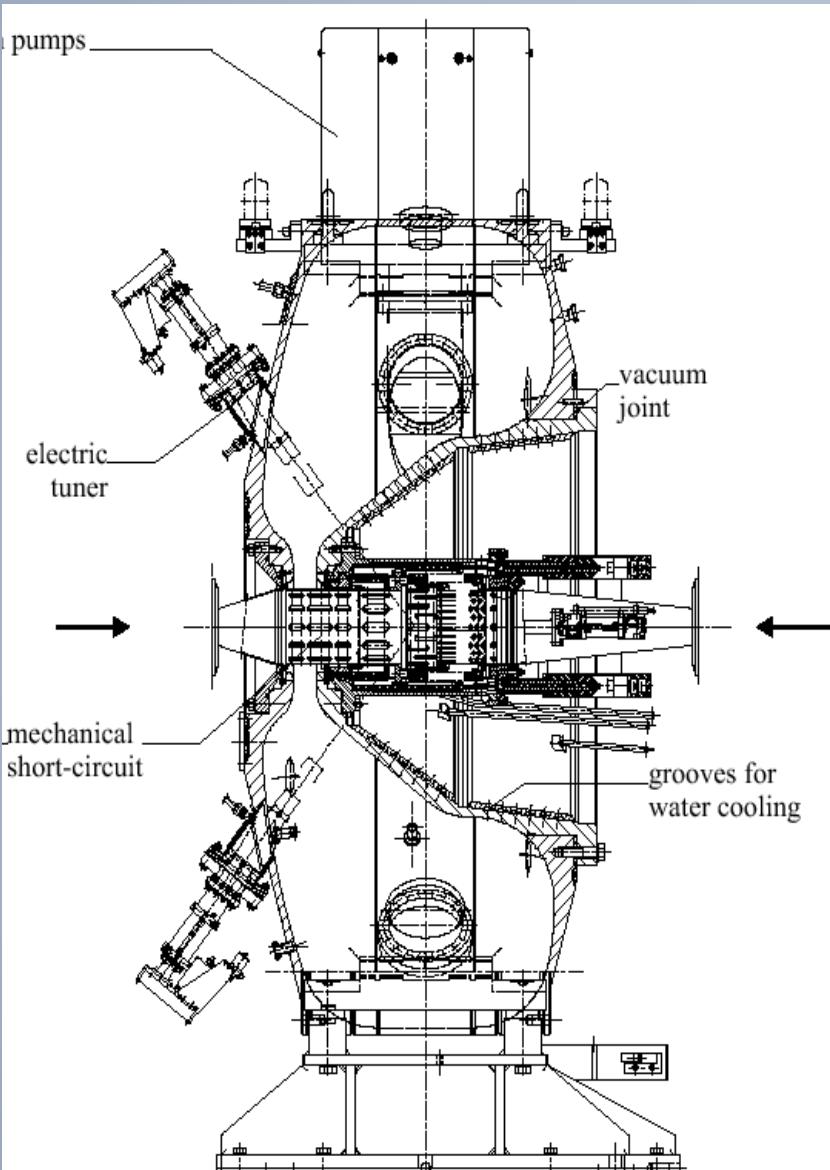
Transverse modes are characterized by

- the transverse impedance in ω -domain
- the transverse loss factor (kick factor) in t -domain!

W.K.H. Panofsky, W.A. Wenzel: "Some Considerations Concerning the Transverse Deflection of Charged Particles in Radio-Frequency Fields", RSI 27, 1957]



CERN/PS 80 MHz cavity (for LHC)



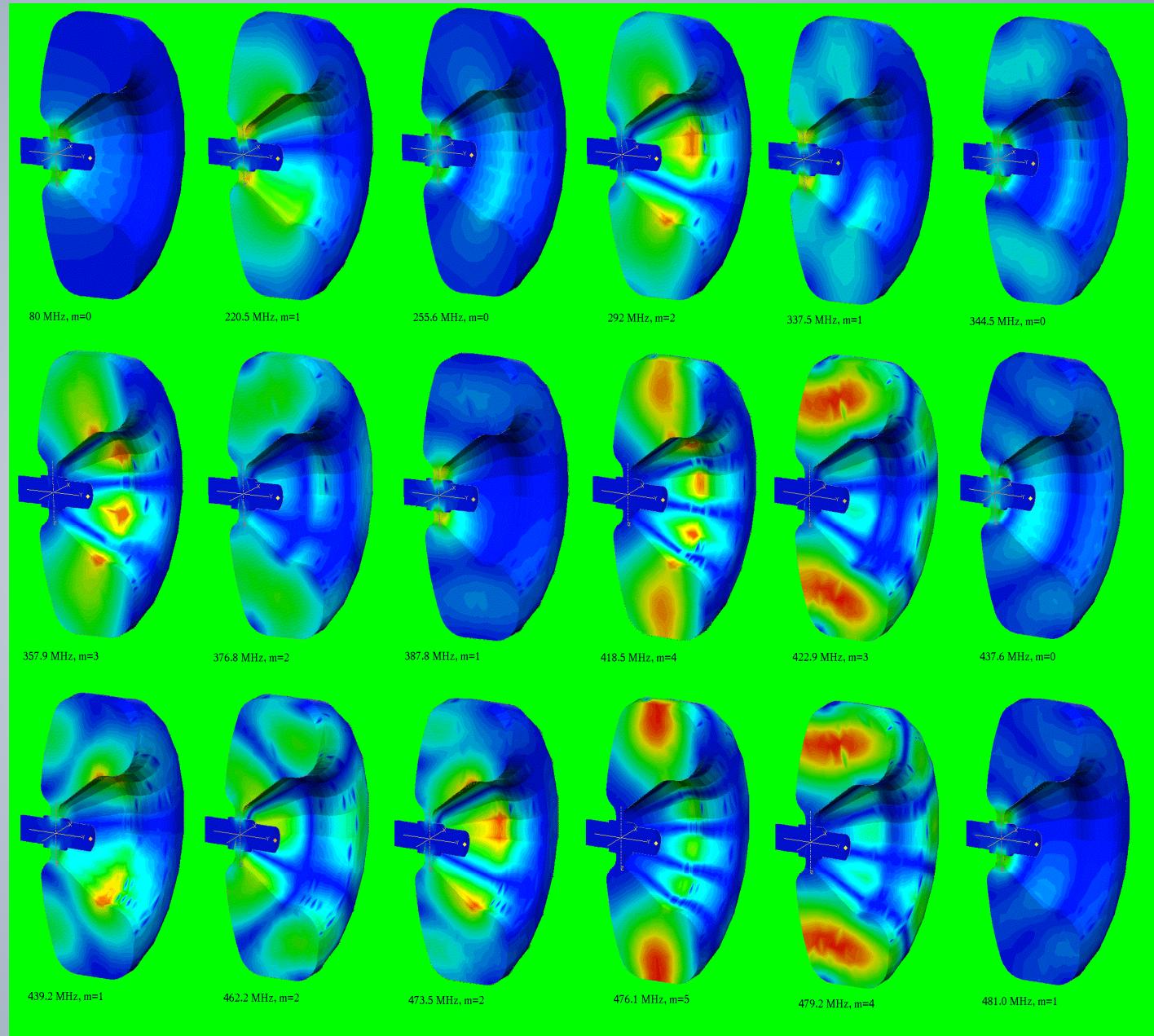
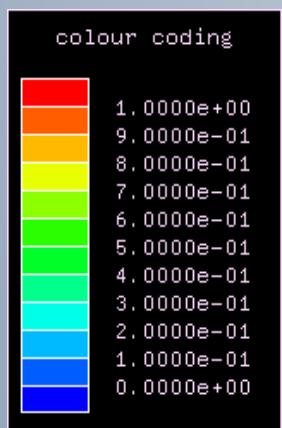


Higher order modes

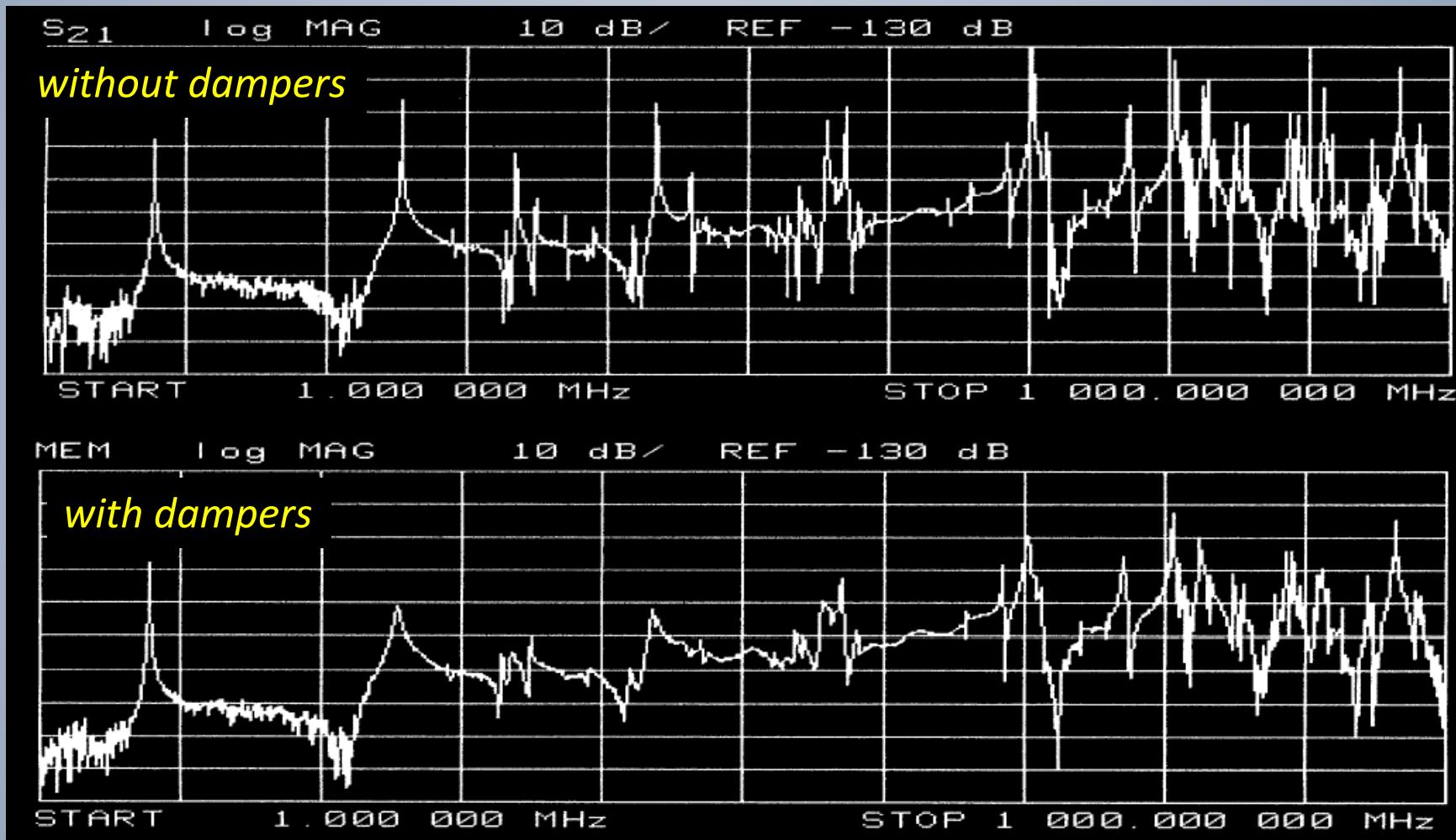
Example shown:
80 MHz cavity PS
for LHC.

Color-coded:

$$|\vec{E}|$$

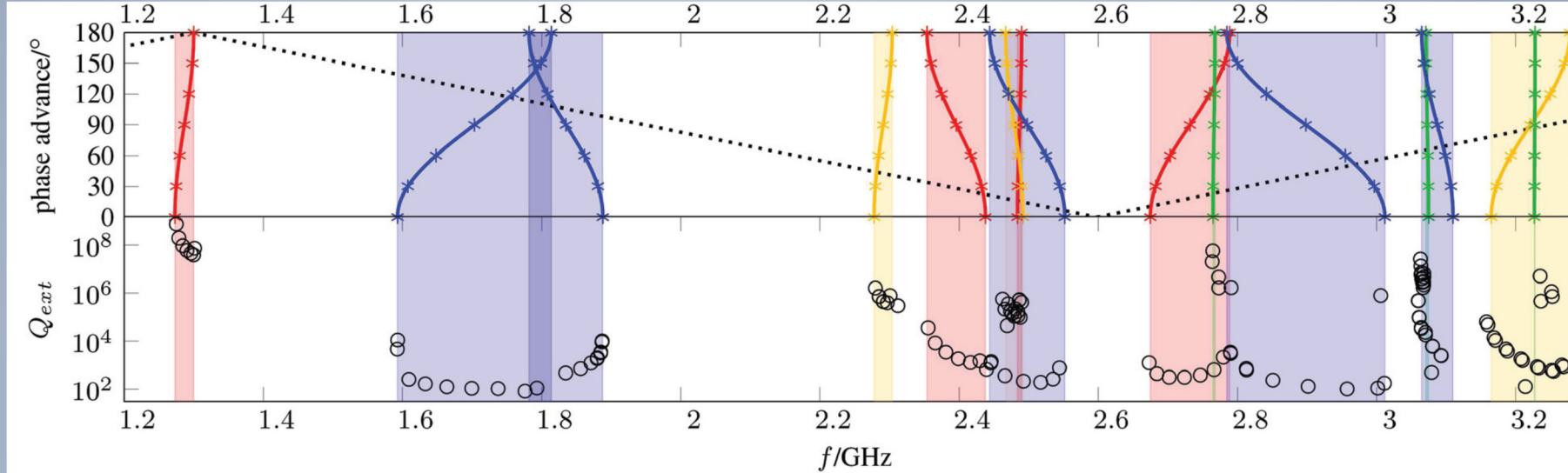


Higher order modes (measured spectrum)



7-cell 1.3 GHz structure for

bERLinPro

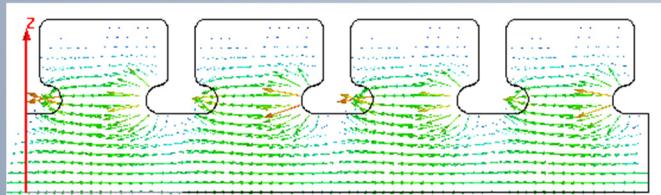


Band diagram (top) and Q-factors (bottom)

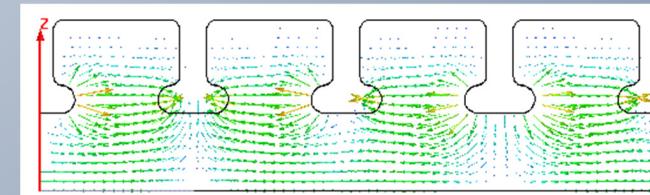
Galek et al.: IPAC2013

Reminder:

0-mode

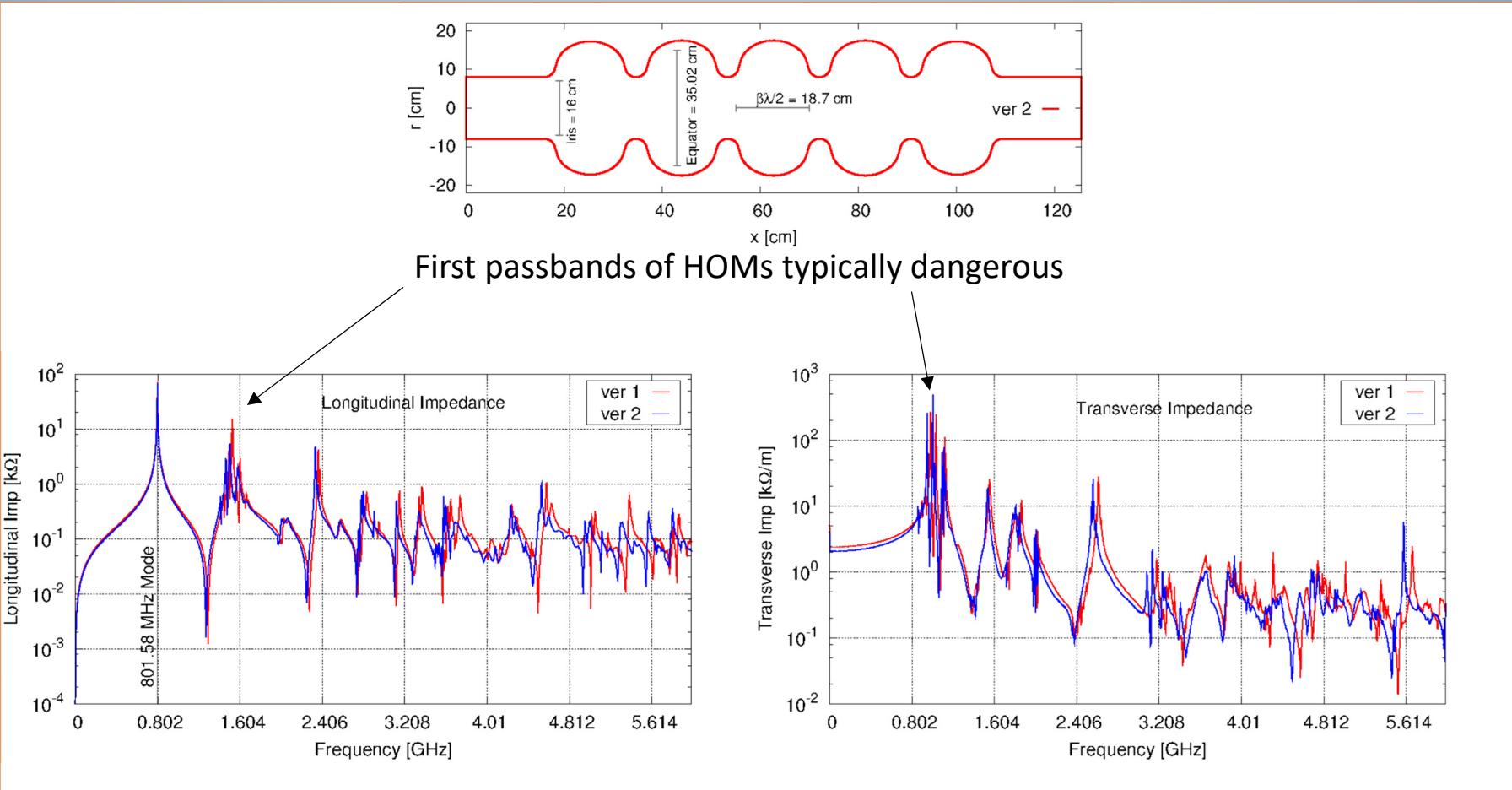


π -mode





HOMs: Example 5-cell cavity

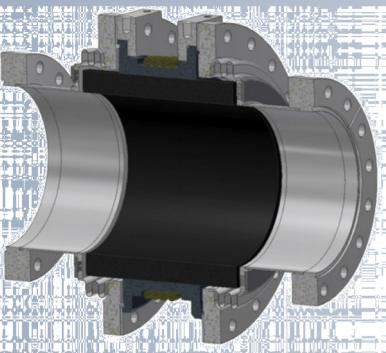


courtesy: Rama Calaga/CERN

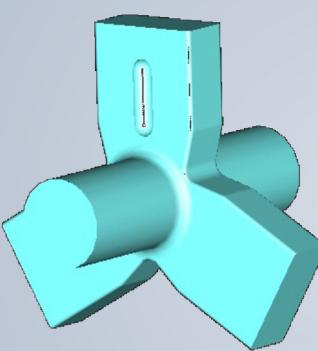


HOM dampers

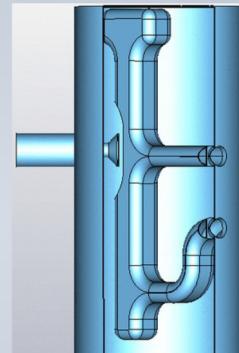
- Ferrite absorbers: broadband damper, room temperature
- Waveguides: better suited for higher frequencies (size!)
- Notch filters: narrow-band; target specific mode



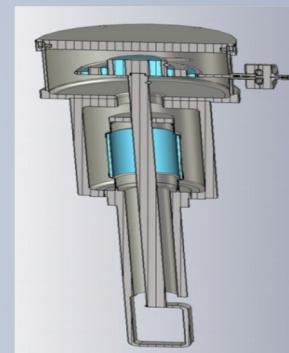
ferrite absorber



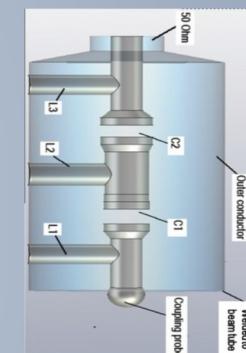
waveguides



notch filter



bandpass filter



double notch

- Multi-cell cavities require broadband dampers

More details in E. Kako's lecture

Tuners



Small boundary perturbation

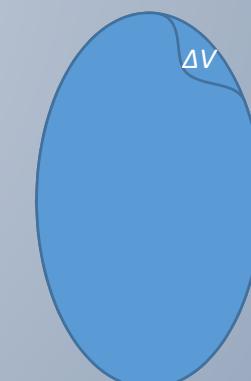
- Perturbation calculation is used to understand the basics for cavity tuning – it is used to analyse the sensitivity to (small) surface geometry perturbations.
 - This is relevant to understand the effect of fabrication tolerances.
 - Intentional surface deformation or introduced obstacles can be used to tune the cavity.
- The basic idea of the perturbation theory is use a known solution (in this case the unperturbed cavity) and assume that the deviation from it is only small. We just used this to calculate the losses (assuming H_t would be that without losses).
- The result of this calculation leads to a convenient expression for the (de)tuning:



unperturbed: ω_0

$$\frac{\omega - \omega_0}{\omega} = \frac{\iiint_{\Delta V} (\mu_0 |H_0|^2 - \epsilon |E_0|^2) dV}{\iiint_V (\mu_0 |H_0|^2 + \epsilon |E_0|^2) dV}$$

Slater's Theorem



perturbed: ω

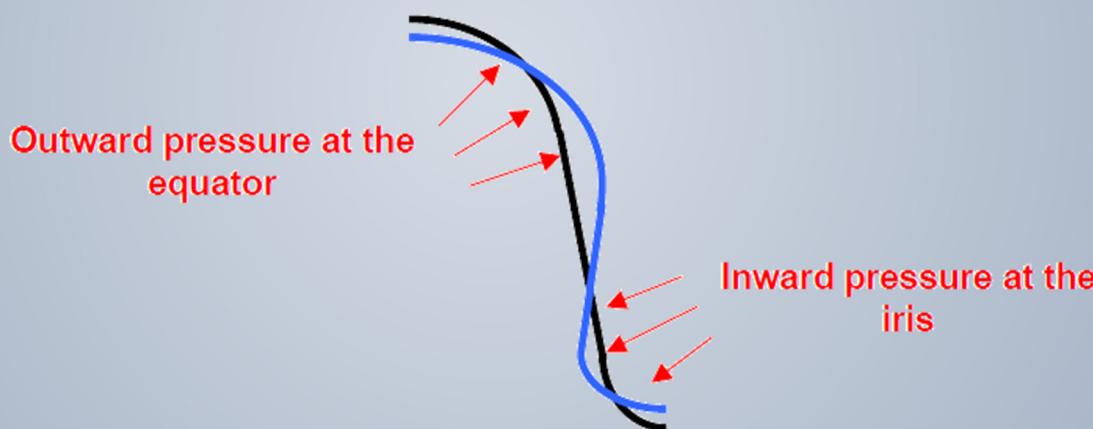


John C. Slater
1900 – 1976



Lorentz force detuning (“LFD”)

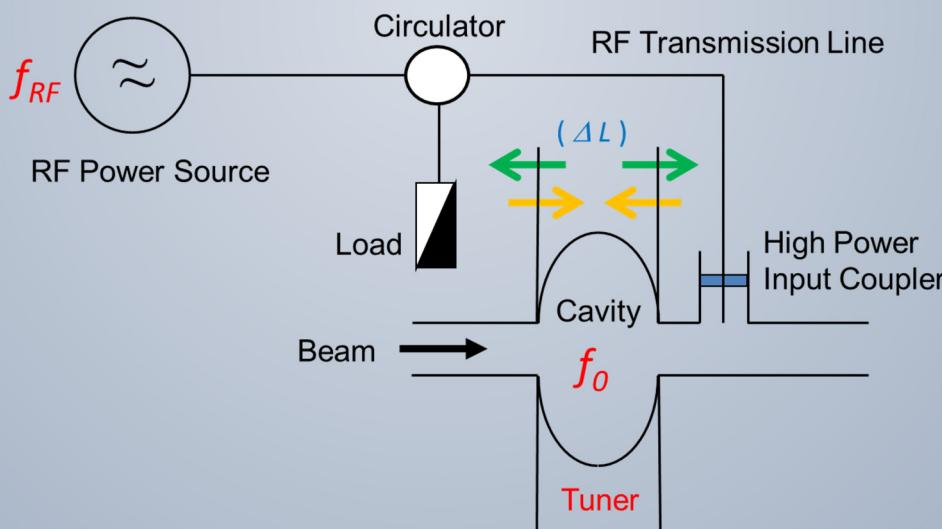
- The presence of electromagnetic fields inside the cavity lead to a mechanical pressure on the cavity.
- Radiation pressure: $P = \frac{1}{4} (\mu_0 |H|^2 - \epsilon_0 |E|^2)$
- Deformation of the cavity shape:



- Frequency shift: $\Delta f = K L |E_{acc}|^2$; typical: $K L \approx -(1 \dots 10) \text{Hz}/\left(\frac{\text{MV}}{\text{m}}\right)^2$
- This requires good stiffness – and the possibility to tune rapidly!

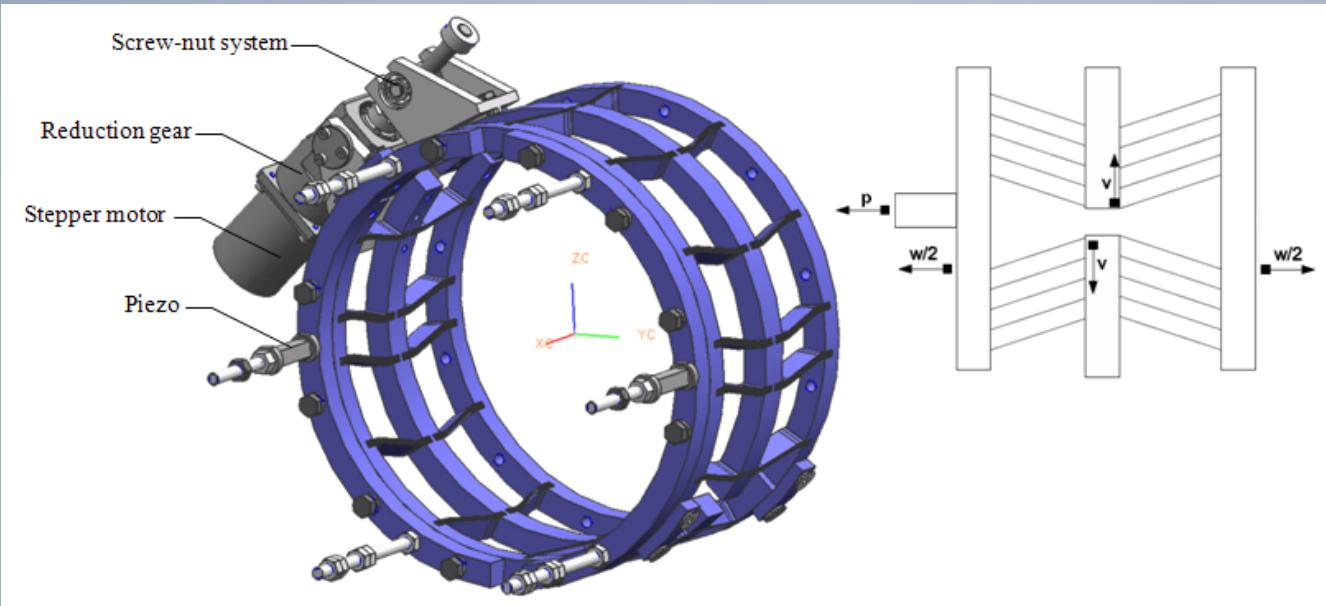
Tuner principle

- Slow tuners:
 - compensate for mechanical tolerances,
 - realized with stepper motor drives
- Fast tuners:
 - compensate Lorentz-force detuning and reactive beam loading
 - realized with piezo crystal (lead zirconate titanate – PZT)
- Tuning of SC cavities is often realized by deforming the cavity:



courtesy: Eiji Kako/KEK

Blade tuner

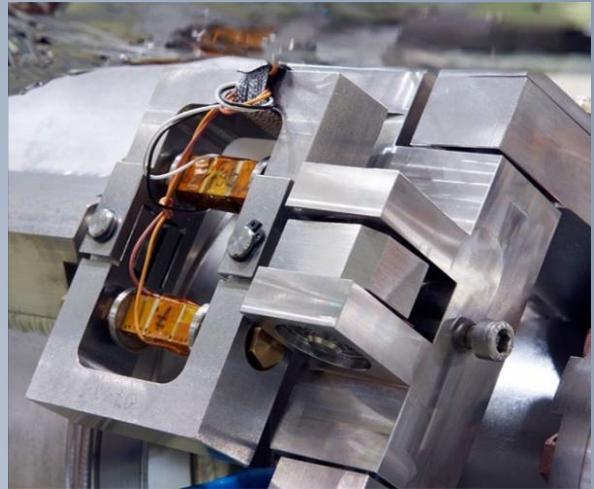
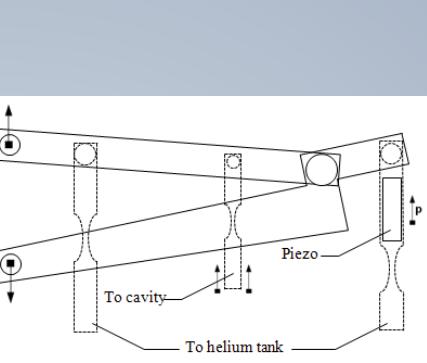
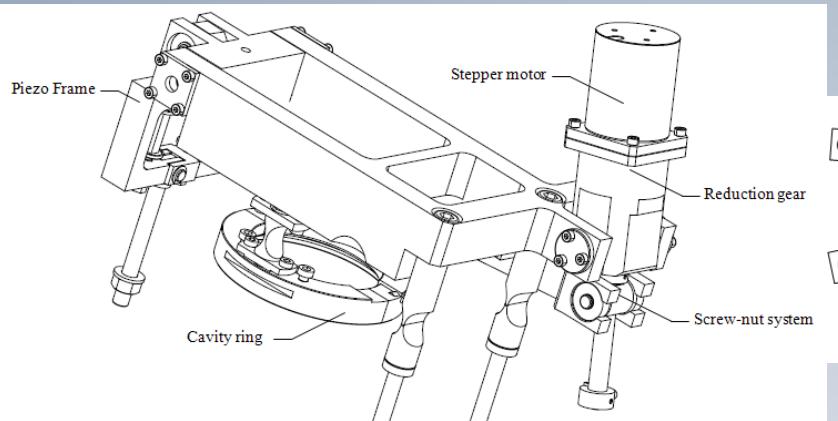


- Developed by INFN Milano
- Azimuthal motion transferred to longitudinal strain
- Zero backlash
- CuBe threaded shaft used for a screw nut system
- Stepping motor and gear combination driver
- Two piezo actuators for fast action
- All components in cold location

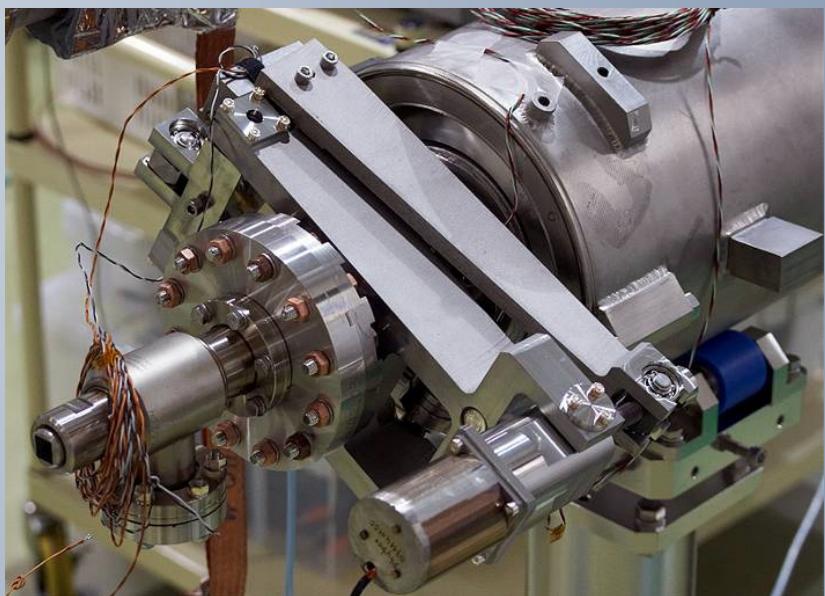


courtesy: Eiji Kako/KEK

“Saclay” lever-arm tuner

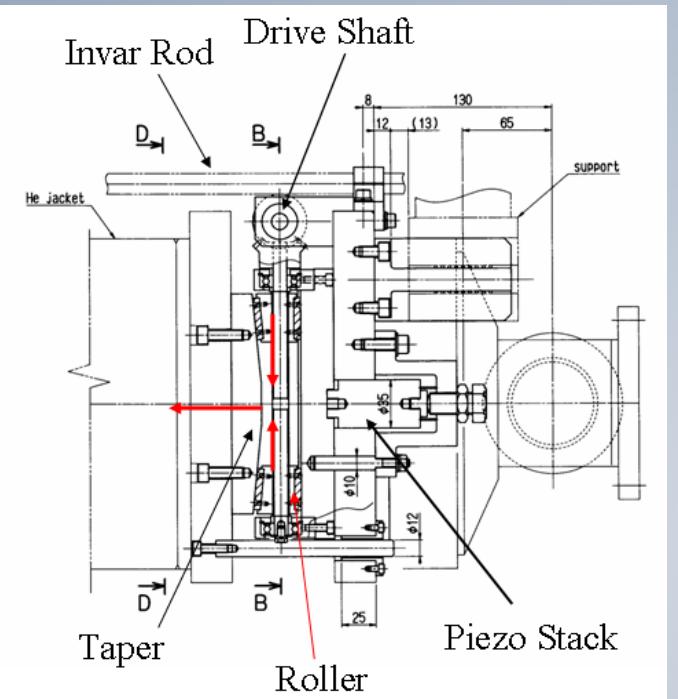


- Developed by DESY based on the Saclay design
- Double lever system (leverage 1.25)
- Cold stepping motor and gear combination
- Screw nut system
- Two piezo actuators for fast action in a preloaded frame
- All components in cold location

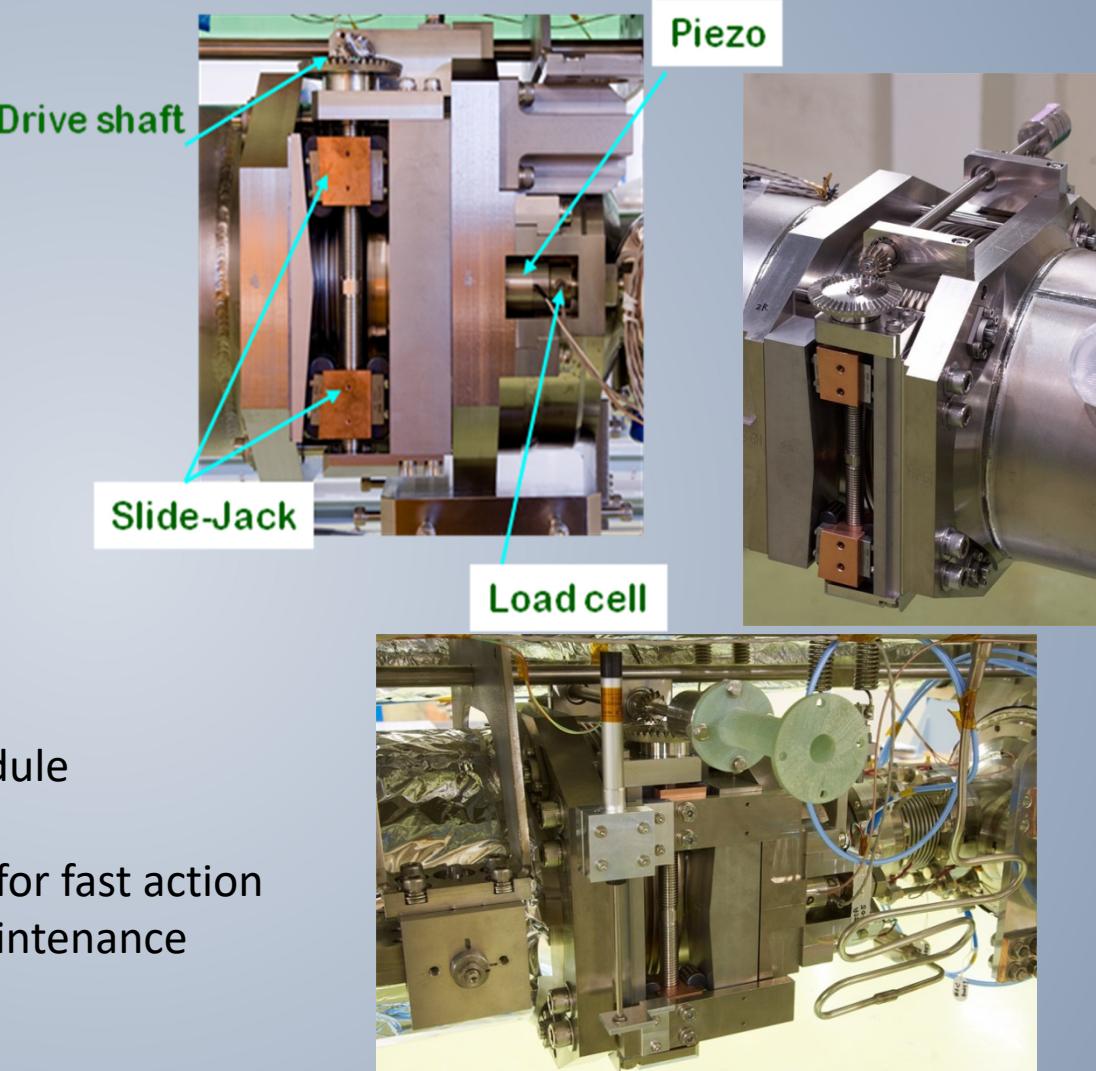


courtesy: Eiji Kako/KEK

Slide-jack tuner



- Developed by KEK for STF cryomodule
- Slide-jack mechanism
- Single high voltage piezo actuator for fast action
- Warm stepping motor for easy maintenance
- Access port for replacing piezo



courtesy: Eiji Kako/KEK

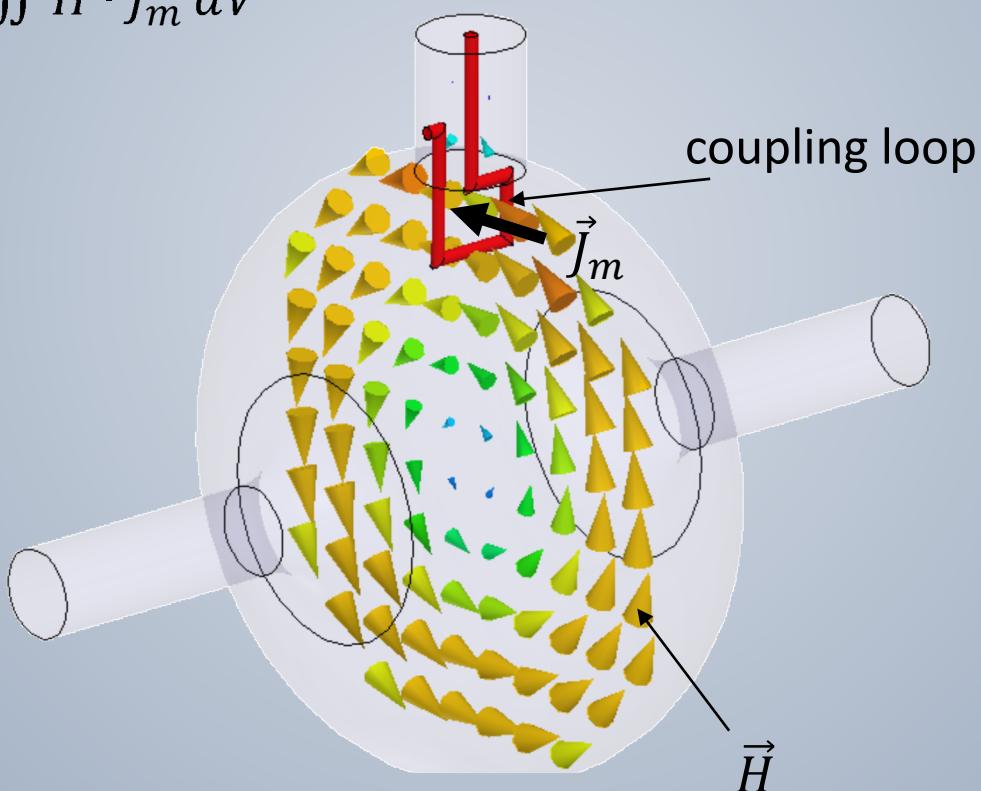
FPC

(Fundamental Power Coupler)



Magnetic (loop) coupling

- The magnetic field of the cavity main mode is intercepted by a coupling loop
- The coupling can be adjusted by changing the size or the orientation of the loop.
- Coupling: $\propto \iiint \vec{H} \cdot \vec{J}_m dV$

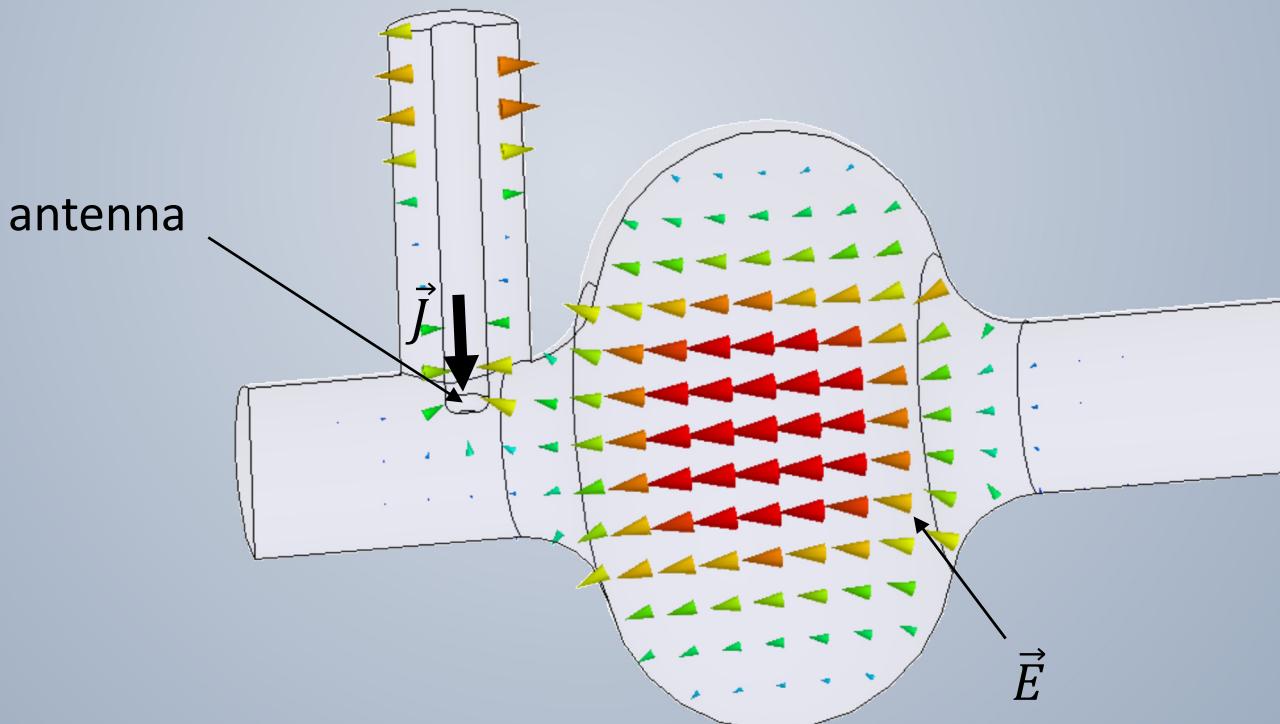


courtesy: David Alesini/INFN



Electric (antenna) coupling

- The inner conductor of the coaxial feeder line ends in an antenna penetrating into the electric field of the cavity.
- The coupling can be adjusted by varying the penetration.
- Coupling $\propto \iiint \vec{E} \cdot \vec{J} dV$

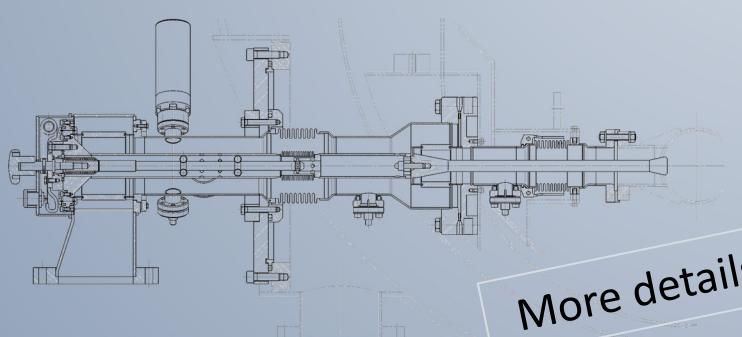


courtesy: David Alesini/INFN

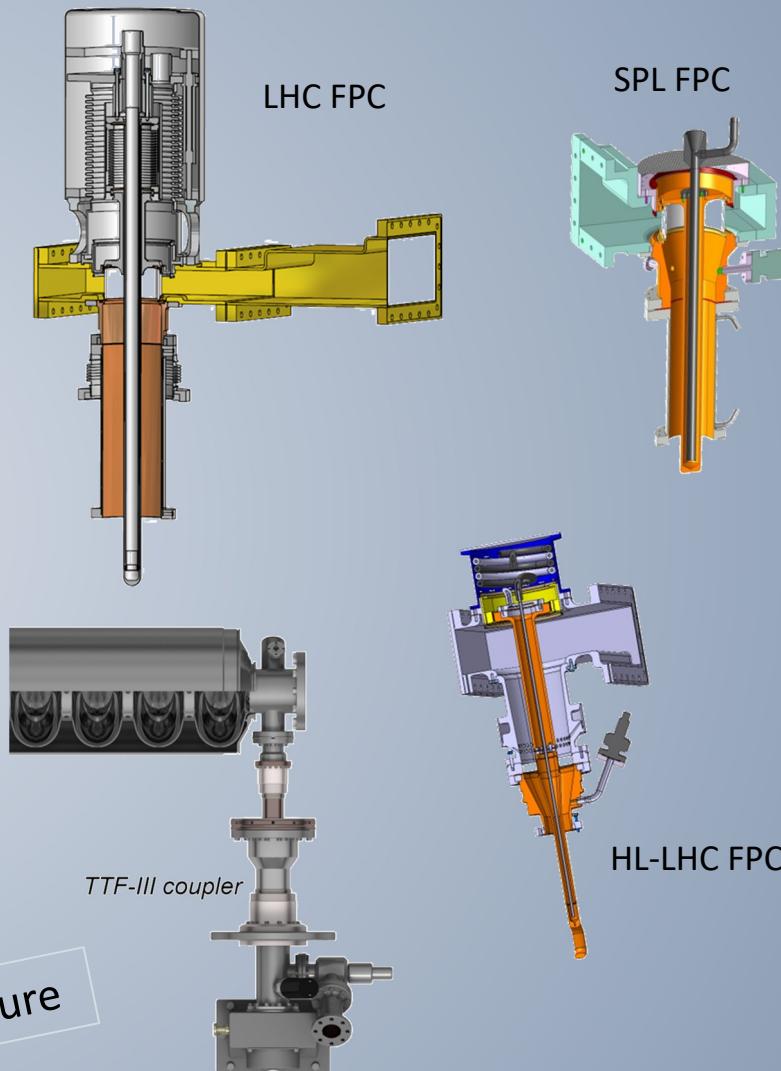


Fundamental Power Coupler – FPC

- The **Fundamental Power Coupler** is the connecting part between the RF transmission line and the RF cavity
- It is a specific piece of transmission line that also has to provide the cavity vacuum barrier.
- FPCs are amongst the most critical parts of the RF cavity system in an accelerator!
- A good RF design, a good mechanical design and a high quality fabrication are essential for an efficient and reliable operation.



More details in E. Kako's lecture



courtesy: Eric Montesinos/CERN

Cavity Fabrication techniques

Materials

- Selection criteria
 - Electrical conductivity
 - Secondary emission yield (SEY)
 - mechanical stiffness/hardness (at cryogenic T)
 - thermal conductivity, thermal expansion
 - machining & joining techniques
 - vacuum tight, low outgassing rate
 - creep resistance,
 - magnetic permeability
 - radiation hardness
 - fatigue stress
 - ...
- Important: specify what you need and what you can measure/control at reception!



Niobium

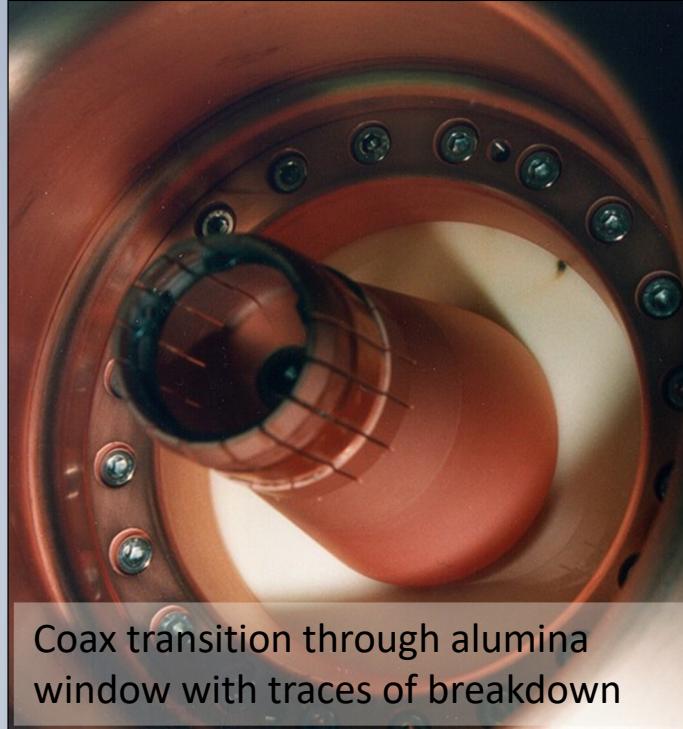
- Magnets use NbTi or Nb₃Sn, SC Cavities use mainly Nb!
- Pure Nb has a high critical magnetic field ($H_c = 200$ mT)
- Nb has a high transition temperature ($T_c = 9.3$ K).
- It is chemically inert (surface covered by oxide layer)
- It can be machined and deep-drawn
- It is available as bulk and sheet material in any size, fabricated by forging and rolling
- Large grain sizes (often favoured) obtained by e-beam melting (EBM).
- Instead of bulk or sheet Nb, Nb can also be coated (e.g. by sputtering) on Cu – CERN has built the LEP, LHC and ISOLDE cavities with this technique. Advantages: thermal stability, material cost, optimisation of R_{BCS} possible; disadvantages: difficult technology to master, lower Q_0 .



Nb ingot after EBM
at Heraeus (D)

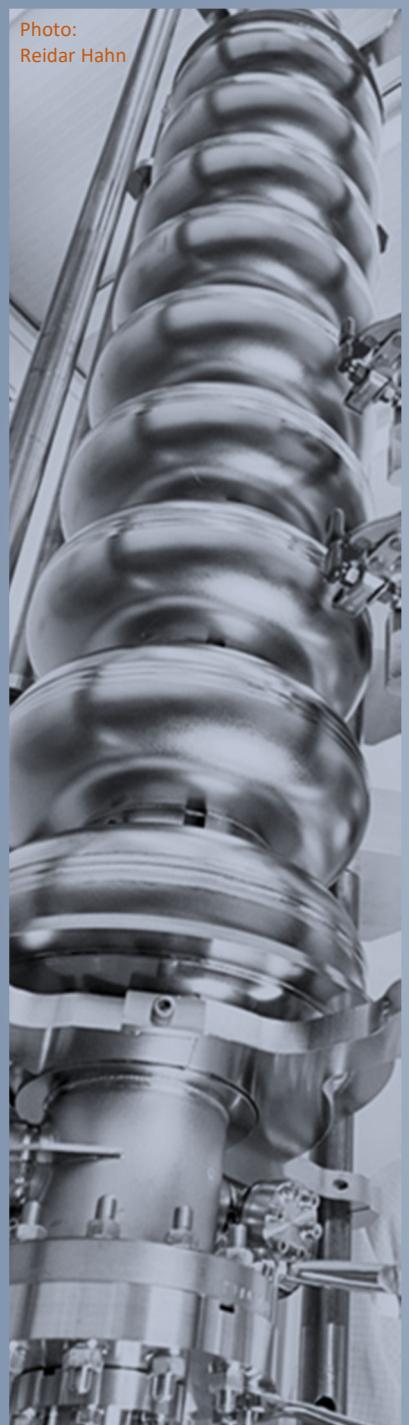
Dielectrics

- Aluminium oxide, Al_2O_3 , aka Alumina, is often used in vacuum/RF windows and vacuum feed-throughs.
- Isostatically pressed Al_2O_3 is used for leak-tightness (LEP power couplers, e.g.).
- To reduce multipactor, it is often coated with titanium or titanium nitride (TiN).
- Other window materials: Sapphire, BeO, quartz, diamond ...)
- Silicon carbide (SiC), C-loaded AlN have been effectively used as RF absorbers inside vacuum.
- Different ferrites are used as absorbers and to use their variable magnetic permeability.



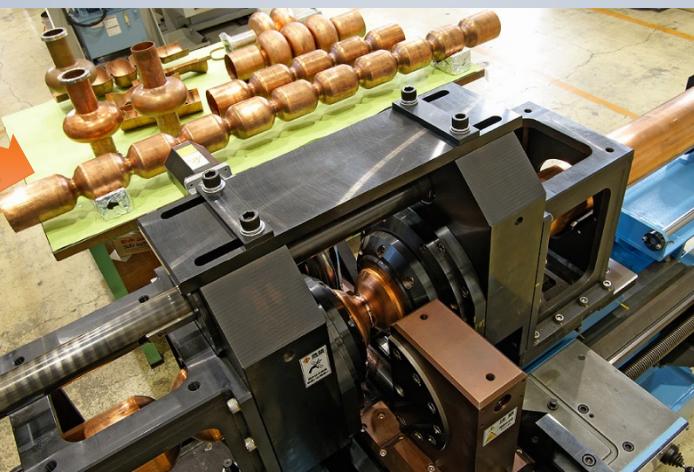
Fabrication of Nb sheets at





Other forming techniques

- Forging, pressing, deep drawing, ...
- Spinning, planing, rolling, ...
- lapping, polishing, electropolishing, ...
- Electro-forming, hydroforming, explosion forming
- Powder metal techniques, additive manufacturing (3-D printing)
- Sputtering
- Backward extrusion
- Necking



Necking machine at KEK for fabrication of multi-cell seamless cavities

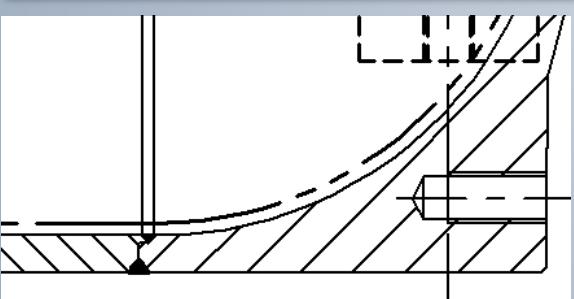


back-extruded Nb tubes for E-XFEL

courtesy: Waldemar Singer/DESY

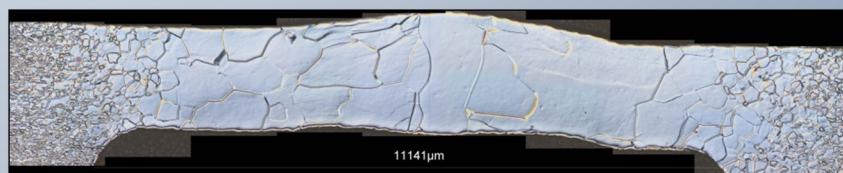
Joining techniques

TIG welding



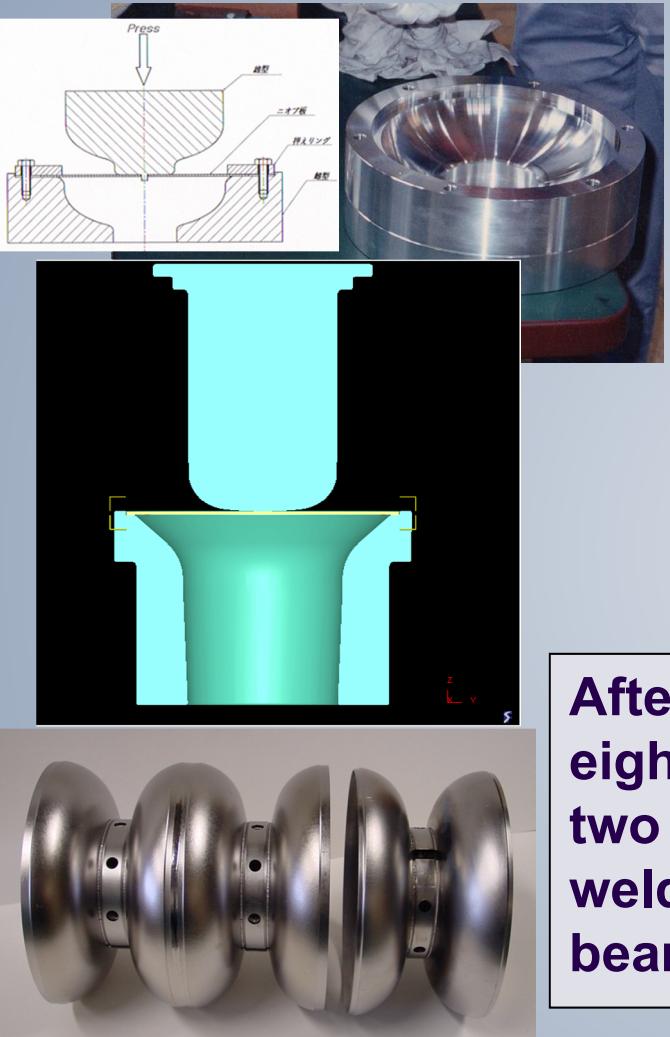
double joint:
inner for vacuum,
outer for mechanical stability

Electron beam (EB) welding



Microstructure of the EB welding area
grain size (50 ÷ 2000) μm

Established cavity fabrication technique



Half cells are shaped by deep drawing.

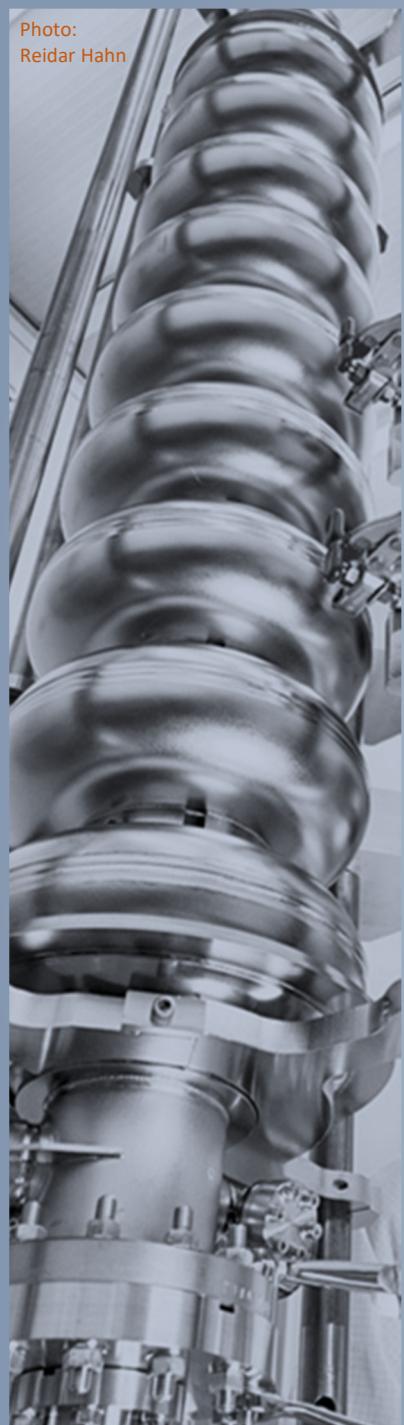
Dumb bells are assembled by electron beam welding.

After proper cleaning eight dumb bells and two end group sections welded by electron beam together

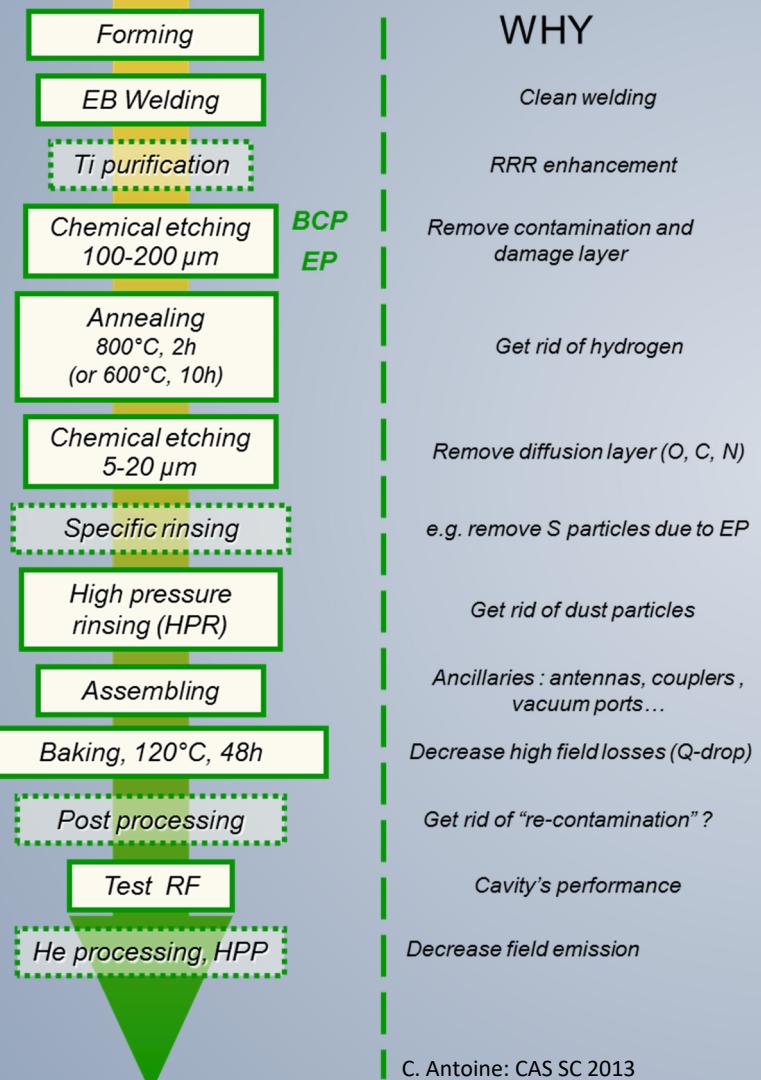
Important: clean conditions on all steps shape accuracy, preparation and EB welding



courtesy: Waldemar Singer/DESY



Advances in SCRF Technology



C. Antoine: CAS SC 2013



Photos: Rongli Geng



More in tomorrow's lectures by L.
Popielarski, A. Grasselino and C. Antoine



Thank you very much for your attention!