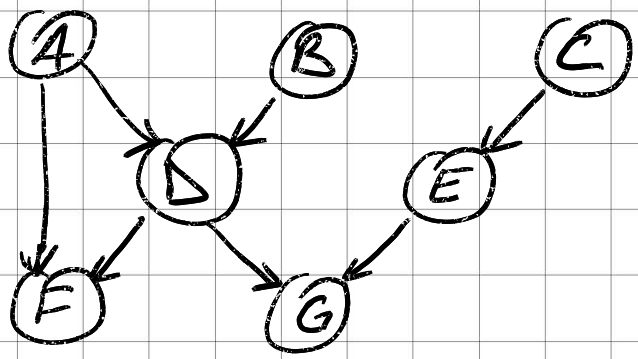
# ELIMINAREA VARIABILELOR
## ~ EXEMPLU ~

- Fie rețeaua Bayesiană




definită de următoarele tabele de probabilități:

$p(A) = 0.3$

$\phi_A$:

| A | |
|---|---|
| 0 | 0.7 |
| 1 | 0.3 |

$p(B) = 0.5$

$\phi_B$:

| B | |
|---|---|
| 0 | 0.5 |
| 1 | 0.5 |

$p(C) = 0.6$

$\phi_C$ :

| C | |
|---|---|
| 0 | 0.4 |
| 1 | 0.6 |

| A | B | P(D \| A, B) |
|---|---|---|
| 0 | 0 | .25 |
| 0 | 1 | .3 |
| 1 | 0 | .4 |
| 1 | 1 | .8 |

$\phi_D$ :

| A | B | D | |
|---|---|---|---|
| 0 | 0 | 0 | .75 |
| 0 | 0 | 1 | .25 |
| 0 | 1 | 0 | .7 |
| 0 | 1 | 1 | .3 |
| 1 | 0 | 0 | .6 |
| 1 | 0 | 1 | .4 |
| 1 | 1 | 0 | .2 |
| 1 | 1 | 1 | .8 |

| C | P(E \| C) |
|---|---|
| 0 | 0.75 |
| 1 | 0.25 |

$\phi_E$ :

| C | E | |
|---|---|---|
| 0 | 0 | 0.25 |
| 0 | 1 | 0.75 |
| 1 | 0 | 0.75 |
| 1 | 1 | 0.25 |

| A | D | P(F \| A, D) |
|---|---|---|
| 0 | 0 | .4 |
| 0 | 1 | .6 |
| 1 | 0 | .3 |
| 1 | 1 | .2 |

$\phi_F$:

| A | D | F | |
|---|---|---|---|
| 0 | 0 | 0 | .6 |
| 0 | 0 | 1 | .4 |
| 0 | 1 | 0 | .4 |
| 0 | 1 | 1 | .6 |
| 1 | 0 | 0 | .7 |
| 1 | 0 | 1 | .3 |
| 1 | 1 | 0 | .8 |
| 1 | 1 | 1 | .2 |

| D | E | P(G \| D, E) |
|---|---|---|
| 0 | 0 | .9 |
| 0 | 1 | .8 |
| 1 | 0 | .5 |
| 1 | 1 | .6 |

$\phi_G$:

| D | E | G | |
|---|---|---|---|
| 0 | 0 | 0 | .1 |
| 0 | 0 | 1 | .9 |
| 0 | 1 | 0 | .2 |
| 0 | 1 | 1 | .8 |
| 1 | 0 | 0 | .5 |
| 1 | 0 | 1 | .5 |
| 1 | 1 | 0 | .4 |
| 1 | 1 | 1 | .6 |

- Ne propunem să calculăm

$$P(C, F \mid G = 0)$$

- Vom aplica eliminarea succesivă a variabilelor pentru a calcula:

$\phi_{CFG}$ astfel:

1. "Conditionăm" factorii cu $G = 0$
2. Eliminăm pe rând variabilele

$$E, B, A, D$$

3. Înmulțim factorii rămași
4. $$P(C, F \mid G = 0) = \frac{\phi_{CFG}(C, F, G = 0)}{\sum_{C,F} \phi_{CFG}}$$

1. Un singur factor contine variabila G: $\phi_G$

$\phi_G \rightarrow \phi_{G0}$

$\phi_{G0}$:

| $\Delta$ | E | G | |
|---|---|---|---|
| 0 | 0 | 0 | 0.1 |
| 0 | 1 | 0 | 0.2 |
| 1 | 0 | 0 | 0.5 |
| 1 | 1 | 0 | 0.4 |

## 2 Eliminăm variabilele

### 2.1 Eliminăm E

$$w_E = \phi_E \cdot \phi_{G0}$$

| C | D | E | G | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $0.25 \cdot 0.1 = 0.025$ * |
| 0 | 0 | 1 | 0 | $0.75 \cdot 0.2 = 0.15$ * |
| 0 | 1 | 0 | 0 | $0.25 \cdot 0.5 = 0.125$ |
| 0 | 1 | 1 | 0 | $0.75 \cdot 0.4 = 0.3$ |
| 1 | 0 | 0 | 0 | $0.75 \cdot 0.1 = 0.075$ |
| 1 | 0 | 1 | 0 | $0.25 \cdot 0.2 = 0.05$ |
| 1 | 1 | 0 | 0 | $0.75 \cdot 0.5 = 0.375$ |
| 1 | 1 | 1 | 0 | $0.25 \cdot 0.4 = 0.1$ |

$$\phi_{CDG} = \sum_E w_E =$$

| C | D | G | |
|---|---|---|---|
| 0 | 0 | 0 | 0.175 |
| 0 | 1 | 0 | 0.425 |
| 1 | 0 | 0 | 0.125 |
| 1 | 1 | 0 | 0.475 |

## (2.2) Eliminate B

$$\omega_B = \phi_B \cdot \phi_D$$

| A | B | D | | |
|---|---|---|---|---|
| 0 | 0 | 0 | .5 · .75 | = 0.375 |
| 0 | 0 | 1 | .5 · .25 | = 0.125 |
| 0 | 1 | 0 | .5 · .7 | = 0.350 |
| 0 | 1 | 1 | .5 · .3 | = 0.15 |
| 1 | 0 | 0 | .5 · .6 | = 0.3 |
| 1 | 0 | 1 | .5 · .4 | = 0.2 |
| 1 | 1 | 0 | .5 · .2 | = 0.1 |
| 1 | 1 | 1 | .5 · .8 | = 0.4 |

$$\phi_{AD} = \sum_B \omega_B :$$

| A | D | |
|---|---|---|
| 0 | 0 | 0.725 |
| 0 | 1 | 0.275 |
| 1 | 0 | 0.4 |
| 1 | 1 | 0.6 |

## (2.3) Eliminăm A

$$\omega_A = \phi_A \cdot \phi_{AD} \cdot \phi_F$$

| A | D | E | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0.7 | × 0.725 | × 0.6 | = 0.3045 | |
| 0 | 0 | 1 | 0.7 | × 0.725 | × 0.4 | = 0.203 | |
| 0 | 1 | 0 | 0.7 | × 0.275 | × 0.4 | = 0.077 | |
| 0 | 1 | 1 | 0.7 | × 0.275 | × 0.6 | = 0.1155 | ← |
| 1 | 0 | 0 | 0.3 | × 0.4 | × 0.7 | = 0.084 | |
| 1 | 0 | 1 | 0.3 | × 0.4 | × 0.3 | = 0.036 | |
| 1 | 1 | 0 | 0.3 | × 0.6 | × 0.8 | = 0.144 | |
| 1 | 1 | 1 | 0.3 | × 0.6 | × 0.2 | = 0.036 | ← |

$$\phi_{DF} = \sum_A \omega_A :$$

| D | E | |
|---|---|---|
| 0 | 0 | 0.3885 |
| 0 | 1 | 0.239 |
| 1 | 0 | 0.221 |
| 1 | 1 | 0.1515 ← |

Factori rămași $\phi_C$, $\phi_{DF}$, $\phi_{CDG}$

## (2.4) Eliminar variable $D$

$$\omega_D = \phi_{CDG} \cdot \phi_{DF}$$

| C | D | F | G | |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | $0.175 \cdot 0.3885$ |
| 0 | 0 | 1 | 0 | $0.175 \cdot 0.239$ |
| 0 | 1 | 0 | 0 | $0.425 \cdot 0.221$ |
| 0 | 1 | 1 | 0 | $0.425 \cdot 0.1515$ |
| 1 | 0 | 0 | 0 | $0.125 \cdot 0.3885$ |
| 1 | 0 | 1 | 0 | $0.125 \cdot 0.239$ |
| 1 | 1 | 0 | 0 | $0.475 \cdot 0.221$ |
| 1 | 1 | 1 | 0 | $0.475 \cdot 0.1515$ |

$$\phi'_{CFG} = \sum_D \omega_D =$$

| C | F | G | |
|---|---|---|---|
| 0 | 0 | 0 | 0.1619125 |
| 0 | 1 | 0 | 0.1062125 |
| 1 | 0 | 0 | 0.1535375 |
| 1 | 1 | 0 | 0.1018375 |

3 Înmulțim cei doi factori

rămași: $\phi_{CFG} = \phi_C \cdot \phi'_{CFG}$

| C F G | |
|---|---|
| 0 0 0 | $0.161925 \cdot 0.4 = 0.06477$ |
| 0 1 0 | $0.1062125 \cdot 0.4 = 0.042485$ |
| 1 0 0 | $0.1535375 \cdot 0.6 = 0.0921225$ |
| 1 1 0 | $0.1018375 \cdot 0.6 = 0.0611025$ |

4 $P(C=1, F=0 \mid G=0)$

$$= \frac{0.0921225}{0.26048} \simeq 0.3537$$

TB, 2017