Mechanical Engineering Study Program
Faculty of Mechanical and Aerospace Engineering
Institut Teknologi Bandung
WF2202 Partial Differential Equations and Numerical Methods
HW#3, due date: Sunday, 30 March 2025 at 11.59pm in the Edunex

1. Consider an elastic string of length L whose ends are held fixed. The string is set in motion with no initial velocity from an initial position $u(x,0) = f(x)$. In this problem, carry out the following steps. Let $L = 10$ and $a = 1$ in parts (b) and (c).

$$f(x) = \begin{cases} \dfrac{4x}{L}, & 0 \le x \le \dfrac{L}{4}, \\ 1, & \dfrac{L}{4} \le x \le \dfrac{3L}{4}, \\ \dfrac{4(L-x)}{L}, & \dfrac{3L}{4} \le x \le L \end{cases}$$

   a. Find the displacement $u(x,t)$ for the given initial position $f(x)$.
   b. Plot $u(x,t)$ versus x for $0 \le x \le 10$ and for several values of t between $t = 0$ and $t = 20$.
   c. Plot $u(x,t)$ versus t for $0 \le t \le 20$ and for several values of x.
   d. Describe the motion of the string in a few sentences.

2. Consider an elastic string of length L whose ends are held fixed. The string is set in motion from its equilibrium position with an initial velocity $u_t(x,0) = g(x)$. In this problem, carry out the following steps. Let $L = 10$ and $a = 1$ in parts (b) and (c).

$$g(x) = \begin{cases} \dfrac{4x}{L}, & 0 \le x \le \dfrac{L}{4}, \\ 1, & \dfrac{L}{4} \le x \le \dfrac{3L}{4}, \\ \dfrac{4(L-x)}{L}, & \dfrac{3L}{4} \le x \le L \end{cases}$$

   a. Find the displacement $u(x,t)$ for the given g(x).
   b. Plot $u(x,t)$ versus x for $0 \le x \le 10$ and for several values of t between $t = 0$ and $t = 20$.
   c. Plot $u(x,t)$ versus t for $0 \le t \le 20$ and for several values of x.
   d. Describe the motion of the string in a few sentences.

3. Consider an elastic string of length L. The end $x = 0$ is held fixed, while the end $x = L$ is free; thus the boundary conditions are $u(0,t) = 0$ and $u_x(L,t) = 0$ The string is set in motion with no initial velocity from the initial position $u(x,0) = f(x)$, where

$$f(x) = \begin{cases} 0, & 0 \le x \le \dfrac{L}{2} - 1, \\ 1, & \dfrac{L}{2} - 1 \le x \le \dfrac{L}{2} + 1, \\ 0, & \dfrac{L}{2} + 1 \le x \le L \end{cases} \qquad (assume\ L > 2)$$

a. Find the displacement $u(x,t)$.
b. With $L = 10$ and $a = 1$, plot u versus x for $0 \leq x \leq 10$ and for several values of t. Pay particular attention to values of t between 3 and 7. Observe how the initial disturbance is reflected at each end of the string.
c. With $L = 10$ and $a = 1$, plot u versus t for several values of x.
d. Describe the motion of the string in a few sentences.