Monte Carlo Simulation of Slope Failure Probability

Application User Manual

1 Introduction

The following report outlines the user manual that will guide how to use the web application tool for assessing the probability of the failure of an infinite slope. Additionally, a case study is included, describing a slope stability assessment of the hillside in the north-western area of Bath, UK. This illustrates a typical analysis that can be performed using the developed tool.

2 Web Application User Manual

2.1 Overview

Monte Carlo simulation algorithm to estimate the failure probability of any given infinite slope was developed using R programming language. Further, it was deployed as an interactive and user-friendly web application via the shinyapps environment. The application can be accessed online via the following link:

https://iurietarlev.shinyapps.io/MonteCarloSimulations_InfiniteSlopeStability/

In the meantime, if required, full source code behind the application can be found here:

https://github.com/iurietarlev/MonteCarloSimulations_InfiniteSlopeStability

It has to be noted that application has been run and tested via the Google Chrome web browser and, thus, it is recommended to use this browser for optimal performance.

2.2 Formulation of Infinite Slope Stability

One of the techniques to perform translation slope stability analysis is the infinite slope method. Here, it is assumed that the slope is infinitely long and will fail along the planar surface. The Factor of Safety (FOS) of the slope can be determined using the equation shown below, which is derived from the equilibrium of applied and resisting forces. The slope is considered to be unstable when $FOS \leq 1$, meaning that the driving forces equal or exceed the resisting ones.

$$FOS = \frac{c' + (\gamma - m\gamma_w) \cdot H \cdot \cos^2(\beta) \cdot \tan(\phi')}{\gamma \cdot H \cdot \sin(\beta) \cdot \cos(\beta)} \tag{1}$$

Table 1: Factor of safety variables

Symbol	Unit	Description
γ	kN/m^3	Unit weight of soil
γ_w	kN/m^3	Unit weight of water
β	$\circ$	Average slope angle
ϕ'	$\circ$	Angle of shearing resistance of the soil
c'	kN/m^2	Cohesion of soil
H	m	Slope height
m	N/A	Saturation depth ratio (H_{GWT}/H)

2.3 Monte Carlo Method

Since most of the parameters in the slope stability equation cannot be accurately represented with a single value, it is necessary to make uncertainty considerations. The primary reason for this is that the laboratory and field measurements may have a range of readings and, if the deviation from the mean is measurably large, a single value may not be deemed as representative.

Hence, parameters with a significant level of uncertainty can be defined in terms of statistical distribution. Taking mean and standard deviation parameters of a set of readings, distribution type can be fitted based on a number of goodness-of-fit tests. One of the most intuitive techniques is graphical assessment: distribution is plotted on a probability paper with modified x-axis in order to linearise the distribution curve. Hence, it may be determined if data fits the distribution well by visually assessing how close observation points align with the theoretical line.

Combining distributions of a multivariate system is a complex task and, hence, Monte Carlo simulation method is typically used to tackle this problem. By generating random values for each input variable based on the distribution type, these are substituted into the equation to output FOS value. Repeating this process for a specified number of times, each time generating a random combination (or realisation) of variables is called Monte Carlo simulation. Consequently, this allows obtaining a final distribution for the value of FOS under a given uncertainty of input parameters. By dividing it into bins, the total number of realisations which result in the value FOS falling below 1 can be counted to estimate the total probability of slope failure:

$$P_{failure} = \frac{COUNTIF(FOS \leq 1)}{COUNT(FOS)} \tag{2}$$

2.4 Application User Interface

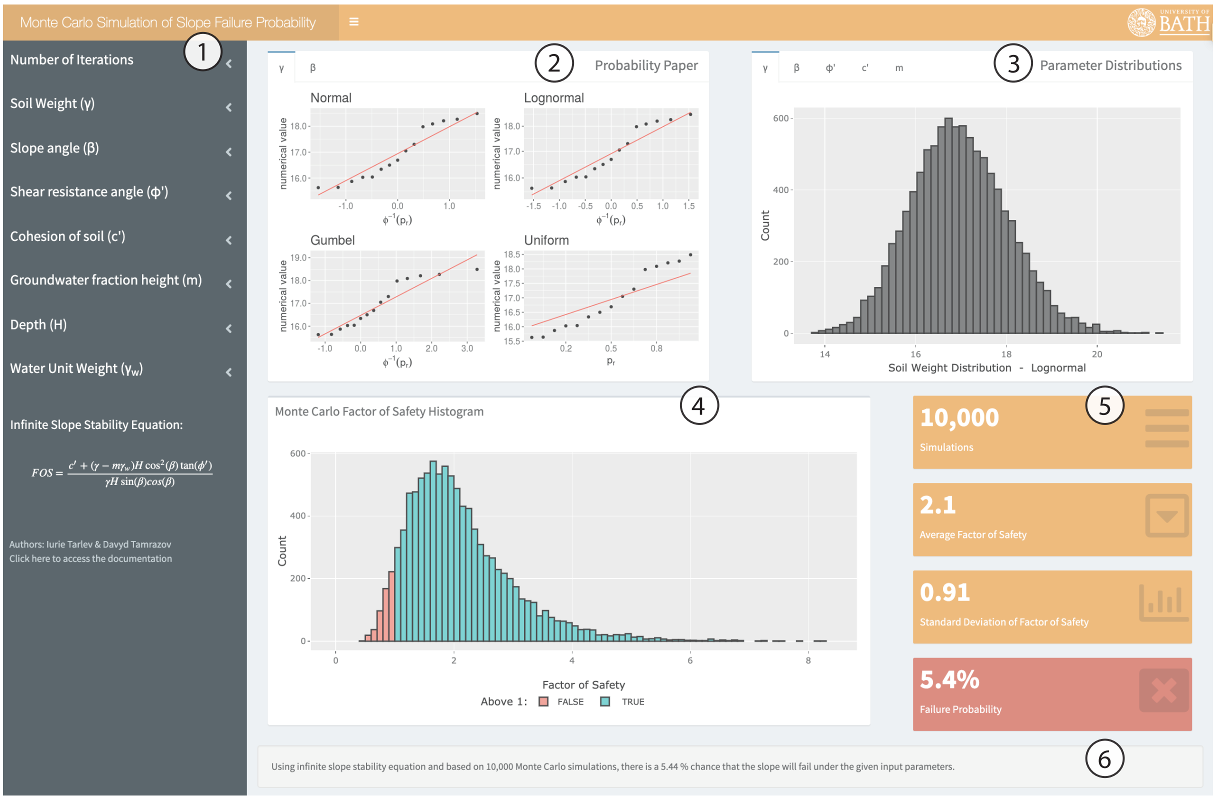

Figure 1: Monte Carlo Simulation of Slope Failure Probability - Application Layout

Figure 1 illustrates the layout of the application with important fields of the interface labelled from 1 to 6. These are described further in more detail.

The sidebar (**Label 1**) serves as an input area, where slope stability equation parameters, as well as the number of realisations for Monte Carlo simulation, can be specified. Each parameter has an expandable field with a number of various input options to make the application more flexible. FOS equation variables can be defined by the user in three different ways, as shown on the example of soil weight in Figure 2.

First, the value of the input parameter may be a constant, meaning that no distribution is required. Alternatively, if a certain level of uncertainty is associated with the parameter value, a distribution type needs to be identified. This can be done manually by specifying mean and standard deviation of a dataset and choosing the expected distribution type to which it will be fitted to.

On the other hand, if the expected distribution is unknown, "CSV File" can be selected to input the whole dataset as a CSV file. It has to be noted that the CSV file has to be in the format containing one single column of values with no headings.

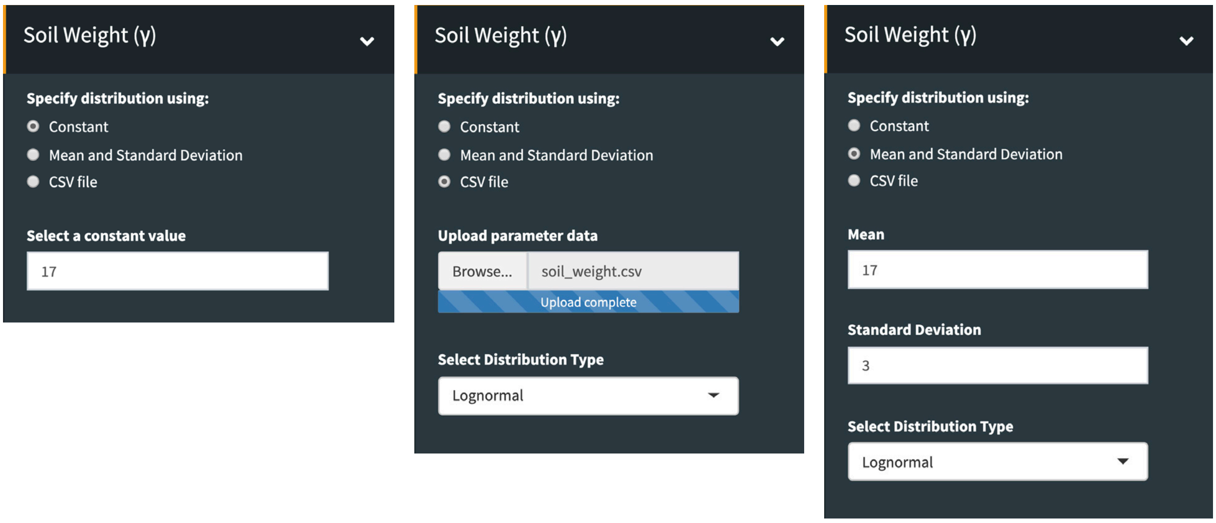

Figure 2: Input options – Soil Weight (γ) example

Once the CSV file has been uploaded, the user can choose the distribution type by assessing goodness-of-fit of the dataset to that particular type. This can be done with the visual comparison of the data plotted on probability paper and theoretical distribution line, both displayed in a separate box (**Label 2**). For each input parameter that has the "CSV File" option selected, a new tab will appear in the Probability Paper box showing probability plots for each of the distributions.

The choice of four different distribution types is provided, namely, Normal, Log-Normal, Gumbel, and Uniform. These can be specified in the drop-down menu called "Select Distribution Type" in the input box for each parameter (see Figure 2). Mentioned distribution types were chosen because, generally, they are sufficient to accurately describe most of the soil / slope parameters.

For every parameter of non-constant value, a distribution histogram will be generated and shown in the "Parameter Distribution" window (**Label 3**), for a visual examination of a specified distribution. This plot varies with different parameters or datasets inputted in the relevant box in the sidebar as well as on the number of iterations (or realisations) specified.

Water unit weight and the number of iterations variables do not have the same input fields in the drop-down menu as the rest of the parameters. Instead, for practical reasons, these are set up as sliders and numeric input boxes to facilitate user input of a constant value within a limited range.

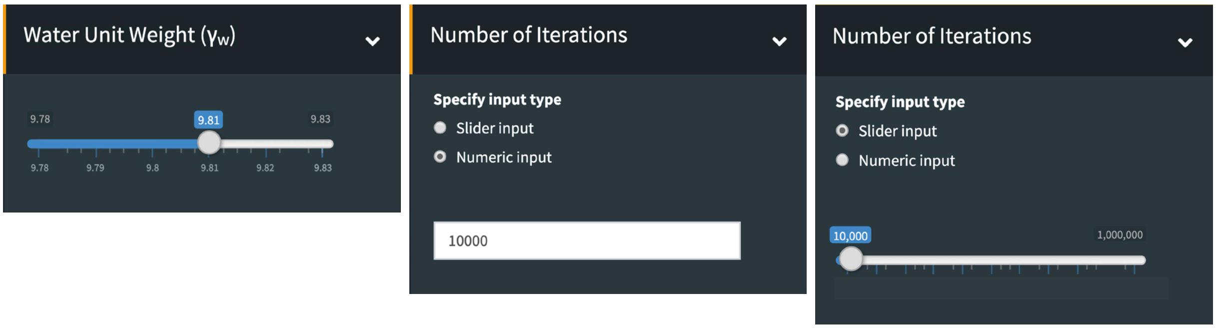

Figure 3: Number of iterations and water unit weight (γ_w) drop-down menu input fields

After all of the input fields have been filled in, results from the Monte Carlo simulation are plotted as a histogram (**Label 4**). To be noted, results will not be generated unless all input parameters in the sidebar have been specified.

The produced histogram shows counts of randomly calculated FOS values in each bin. Here, bin width is set to 0.1 in order to balance accuracy and efficiency of the calculation process. Overall, the FOS plot represents a combined distribution of all variables in the infinite slope stability equation obtained via the Monte Carlo simulation. Histogram bands highlighted in red identify FOS values that are below or equal to 1 and, thus, represent the failure of the slope.

For the purpose of clarity of the results, the key parameters of the obtained distribution and the Monte Carlo simulation specifications are displayed in the boxes to the right from the plot (**Label 5**). These include the number of realisations as well as mean and standard deviation of the distribution of the calculated FOS values. Also, the box in red identifies the percentage probability of the failure of the slope under specified parameters to convert probabilistic theory terms in a more user-friendly format.

Finally, results retrieved from the Monte Carlo simulation are briefly reiterated again in the box under the plot (**Label 6**), allowing the user to obtain a succinct summary of the simulation and its results.

3 Slope stability assessment in north-western Bath

In this section a probabilistic investigation of the Factor of Safety and the associated probability of slope failure is performed on a hillside in the north-western Bath, UK. Location of the following case study is circled on the map in Figure 4.

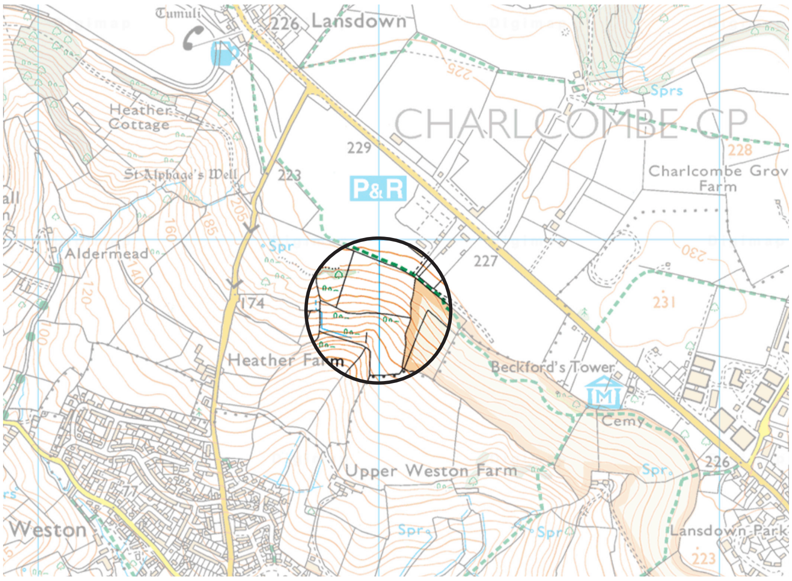

Figure 4: Location of the slope under investigation

In the problem specification, values or distribution parameters of the majority of the infinite slope stability variables have been provided. As an exception, soil unit weight was given as a CSV file with 15 field measurements. In addition, the slope angle was estimated using approximate measurements of the gradient from the contour map in Figure 4 and stored in a CSV file for further manipulations.

Both CSV files have been inputted in the developed web application and using the probability paper plots, analysed for the goodness-of-fit of four distribution types. From the visual inspection, it was determined that, although both Log-normal and Gumbel distributions fit soil weight and slope angle data equally well, specifying a Log-normal distribution for both parameters, as a rule, yields more conservative results (i.e. higher failure probability). Hence, the full summary of all input variables and parameters of the specified distribution types that were used calculating the probability of failure of the given slope is outlined in Table 2.

Table 2: Variables input values

Symbol	Unit	Mean	Standard Deviation	Distribution
γ	kN/m^3	16.9	1.04	Log-normal
β	$\circ$	16.5	5.80	Log-normal
ϕ'	$\circ$	22.5	4.00	Uniform
c'	kN/m^2	20.0	10.0	Log-normal
m	m	0.50	$1/\sqrt{12}$	Uniform
H	m	9.20	NA	N/A
γ_w	kN/m^3	9.20	NA	N/A

The slope stability model was assessed using different values for the number of simulations, resulting in vastly different failure probabilities. Nevertheless, as the number of realisation was set to beyond 1000, the final probability started to stabilise and did not deviate considerably from approximately 14%, as shown in Figure 5.

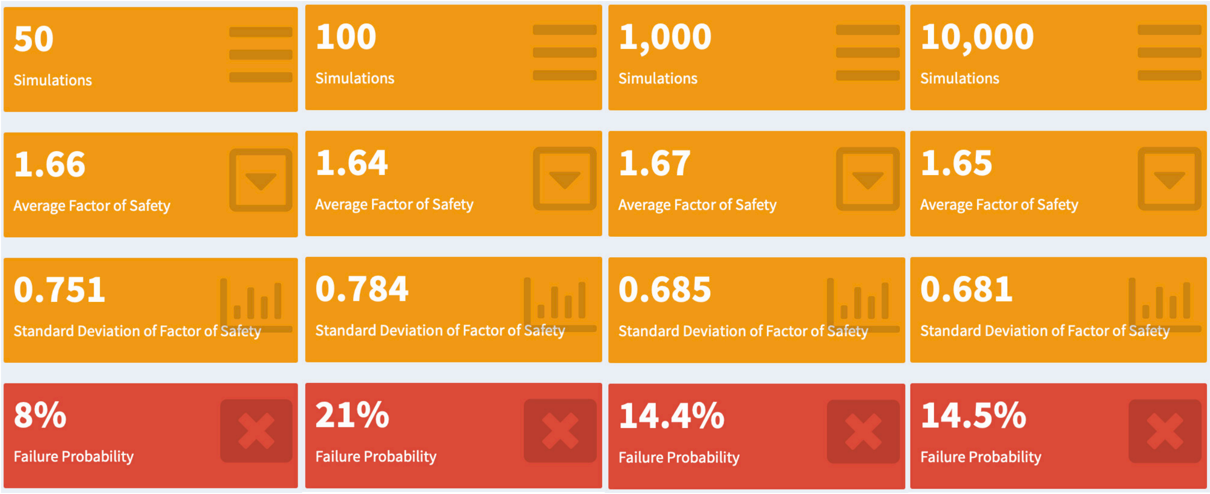

Figure 5: Increasing number of simulations - failure probability comparison

Depending on the problem size, industry standards generally set the number of iterations to around 10,000 in order to minimise computational requirements. However, with a relatively non-complex problem like the infinite slope stability equation, more comprehensive Monte Carlo simulations can be performed. As such, the number of realisations was specified as 100,000 in this case, as computational expenses even with such a large number of simulations were still comparatively low. Meanwhile, the increased accuracy of the performed analysis allowed to marginally reduce the predicted failure probability from that achieved with 1,000 and even 10,000 simulations.

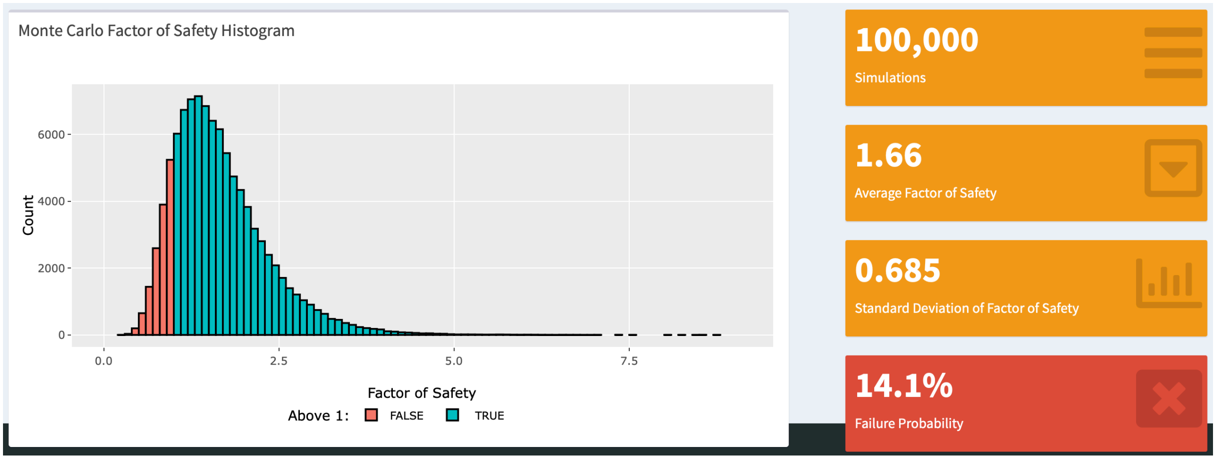

Figure 6: Factor of Safety probabilistic analysis results

As a result, based on 100,000 Monte Carlo simulations of the infinite slope stability analysis, there is a 14.1 % chance that the slope will fail under the given input parameters (see Figure 6). Given the considerable spread of the possible values of the slope and soil characteristics, it is consistent that the obtained failure probability appears to be significant. Hence, even for a geotechnical project which is always associated with a high level of uncertainty, this slope would likely be deemed as unsafe.