

| $N, k \setminus n$ | $n = 25$ | $n = 100$ | $n = 1,000$ | $n = 10,000$ | $n = 100,000$ |
|--------------------|--|--|--|--|--|
| $N = 10, k = 8$ | $\sigma_{min}^2 = 0.00850$ $\sigma^2 = 0.00938$ $\sigma = 0.09686$ $bias = 0.03114$ | $\sigma_{min}^2 = 0.01150$ $\sigma^2 = 0.01204$ $\sigma = 0.10973$ $bias = 0.01650$ | $\sigma_{min}^2 = 0.01240$ $\sigma^2 = 0.01330$ $\sigma = 0.11531$ $bias = 0.01577$ | $\sigma_{min}^2 = 0.01249$ $\sigma^2 = 0.01307$ $\sigma = 0.11432$ $bias = 0.01378$ | $\sigma_{min}^2 = 0.01250$ $\sigma^2 = 0.01296$ $\sigma = 0.11384$ $bias = 0.01178$ |
| $N = 10, k = 20$ | $\sigma_{min}^2 = 0.00100$ $\sigma^2 = 0.00115$ $\sigma = 0.03398$ $bias = 0.02352$ | $\sigma_{min}^2 = 0.00400$ $\sigma^2 = 0.00409$ $\sigma = 0.06394$ $bias = 0.00858$ | $\sigma_{min}^2 = 0.00490$ $\sigma^2 = 0.00513$ $\sigma = 0.07160$ $bias = 0.00545$ | $\sigma_{min}^2 = 0.00499$ $\sigma^2 = 0.00513$ $\sigma = 0.07163$ $bias = 0.00564$ | $\sigma_{min}^2 = 0.00500$ $\sigma^2 = 0.00508$ $\sigma = 0.07130$ $bias = 0.00535$ |
| $N = 25, k = 8$ | $\sigma_{min}^2 = 0.00340$ $\sigma^2 = 0.00359$ $\sigma = 0.05993$ $bias = 0.02431$ | $\sigma_{min}^2 = 0.00460$ $\sigma^2 = 0.00473$ $\sigma = 0.06879$ $bias = 0.00942$ | $\sigma_{min}^2 = 0.00496$ $\sigma^2 = 0.00517$ $\sigma = 0.07191$ $bias = 0.00536$ | $\sigma_{min}^2 = 0.00500$ $\sigma^2 = 0.00514$ $\sigma = 0.07171$ $bias = 0.00529$ | $\sigma_{min}^2 = 0.00500$ $\sigma^2 = 0.00508$ $\sigma = 0.07128$ $bias = 0.00436$ |
| $N = 25, k = 20$ | $\sigma_{min}^2 = 0.00040$ $\sigma^2 = 0.00046$ $\sigma = 0.02143$ $bias = 0.02169$ | $\sigma_{min}^2 = 0.00160$ $\sigma^2 = 0.00162$ $\sigma = 0.04028$ $bias = 0.00729$ | $\sigma_{min}^2 = 0.00196$ $\sigma^2 = 0.00196$ $\sigma = 0.04423$ $bias = 0.00212$ | $\sigma_{min}^2 = 0.00200$ $\sigma^2 = 0.00208$ $\sigma = 0.04558$ $bias = 0.00237$ | $\sigma_{min}^2 = 0.00200$ $\sigma^2 = 0.00203$ $\sigma = 0.04504$ $bias = 0.00215$ |
| $N = 50, k = 8$ | $\sigma_{min}^2 = 0.00170$ $\sigma^2 = 0.00182$ $\sigma = 0.04265$ $bias = 0.02276$ | $\sigma_{min}^2 = 0.00230$ $\sigma^2 = 0.00235$ $\sigma = 0.04848$ $bias = 0.00800$ | $\sigma_{min}^2 = 0.00248$ $\sigma^2 = 0.00252$ $\sigma = 0.05016$ $bias = 0.00349$ | $\sigma_{min}^2 = 0.00250$ $\sigma^2 = 0.00254$ $\sigma = 0.05039$ $bias = 0.00252$ | $\sigma_{min}^2 = 0.00250$ $\sigma^2 = 0.00252$ $\sigma = 0.05024$ $bias = 0.00188$ |
| $N = 50, k = 20$ | $\sigma_{min}^2 = 0.00020$ $\sigma^2 = 0.00022$ $\sigma = 0.01483$ $bias = 0.02118$ | $\sigma_{min}^2 = 0.00080$ $\sigma^2 = 0.00081$ $\sigma = 0.02851$ $bias = 0.00545$ | $\sigma_{min}^2 = 0.00098$ $\sigma^2 = 0.00099$ $\sigma = 0.03153$ $bias = 0.00132$ | $\sigma_{min}^2 = 0.00100$ $\sigma^2 = 0.00102$ $\sigma = 0.03191$ $bias = 0.00109$ | $\sigma_{min}^2 = 0.00100$ $\sigma^2 = 0.00101$ $\sigma = 0.03184$ $bias = 0.00065$ |

$$\hat{n} = \frac{k}{1 - \exp(-\bar{L}_u)}, \text{ where } \bar{L}_u = \frac{1}{N} \sum_{i=1}^N \ln(1 - x_i)$$

$$\sigma_{min}^2 = \text{Var}(\hat{n})_{min} \approx \frac{1}{N} \left(\frac{1}{k} - \frac{1}{n} \right)$$