

| $N, k \setminus n$ | $n = 25$  | $n = 100$   | $n = 1,000$   | $n = 10,000$  | $n = 100,000$   |
|--------------------|---|---|---|---|---|
| $N = 10, k = 8$    | $\sigma_{min}^2 =$<br>0.0085000<br>$\sigma^2 =$<br>0.0093824<br>$\sigma =$<br>0.0968630<br><i>bias</i> =<br>0.0311427 | $\sigma_{min}^2 =$<br>0.0115000<br>$\sigma^2 =$<br>0.0120411<br>$\sigma =$<br>0.1097321<br><i>bias</i> =<br>0.0164967 | $\sigma_{min}^2 =$<br>0.0124000<br>$\sigma^2 =$<br>0.0132953<br>$\sigma =$<br>0.1153054<br><i>bias</i> =<br>0.0157728 | $\sigma_{min}^2 =$<br>0.0124900<br>$\sigma^2 =$<br>0.0130691<br>$\sigma =$<br>0.1143201<br><i>bias</i> =<br>0.0137832 | $\sigma_{min}^2 =$<br>0.0124990<br>$\sigma^2 =$<br>0.0129597<br>$\sigma =$<br>0.1138406<br><i>bias</i> =<br>0.0117763 |
| $N = 10, k = 20$   | $\sigma_{min}^2 =$<br>0.0010000<br>$\sigma^2 =$<br>0.0011546<br>$\sigma =$<br>0.0339799<br><i>bias</i> =<br>0.0235270 | $\sigma_{min}^2 =$<br>0.0040000<br>$\sigma^2 =$<br>0.0040888<br>$\sigma =$<br>0.0639438<br><i>bias</i> =<br>0.0085761 | $\sigma_{min}^2 =$<br>0.0049000<br>$\sigma^2 =$<br>0.0051264<br>$\sigma =$<br>0.0715989<br><i>bias</i> =<br>0.0054482 | $\sigma_{min}^2 =$<br>0.0049900<br>$\sigma^2 =$<br>0.0051315<br>$\sigma =$<br>0.0716342<br><i>bias</i> =<br>0.0056440 | $\sigma_{min}^2 =$<br>0.0049990<br>$\sigma^2 =$<br>0.0050839<br>$\sigma =$<br>0.0713017<br><i>bias</i> =<br>0.0053531 |
| $N = 25, k = 8$    | $\sigma_{min}^2 =$<br>0.0034000<br>$\sigma^2 =$<br>0.0035917<br>$\sigma =$<br>0.0599312<br><i>bias</i> =<br>0.0243128 | $\sigma_{min}^2 =$<br>0.0046000<br>$\sigma^2 =$<br>0.0047326<br>$\sigma =$<br>0.0687940<br><i>bias</i> =<br>0.0094167 | $\sigma_{min}^2 =$<br>0.0049600<br>$\sigma^2 =$<br>0.0051705<br>$\sigma =$<br>0.0719064<br><i>bias</i> =<br>0.0053594 | $\sigma_{min}^2 =$<br>0.0049960<br>$\sigma^2 =$<br>0.0051424<br>$\sigma =$<br>0.0717106<br><i>bias</i> =<br>0.0052871 | $\sigma_{min}^2 =$<br>0.0049996<br>$\sigma^2 =$<br>0.0050815<br>$\sigma =$<br>0.0712848<br><i>bias</i> =<br>0.0043617 |
| $N = 25, k = 20$   | $\sigma_{min}^2 =$<br>0.0004000<br>$\sigma^2 =$<br>0.0004593<br>$\sigma =$<br>0.0214317<br><i>bias</i> =<br>0.0216920 | $\sigma_{min}^2 =$<br>0.0016000<br>$\sigma^2 =$<br>0.0016225<br>$\sigma =$<br>0.0402800<br><i>bias</i> =<br>0.0072931 | $\sigma_{min}^2 =$<br>0.0019600<br>$\sigma^2 =$<br>0.0019559<br>$\sigma =$<br>0.0442256<br><i>bias</i> =<br>0.0021189 | $\sigma_{min}^2 =$<br>0.0019960<br>$\sigma^2 =$<br>0.0020773<br>$\sigma =$<br>0.0455779<br><i>bias</i> =<br>0.0023715 | $\sigma_{min}^2 =$<br>0.0019996<br>$\sigma^2 =$<br>0.0020285<br>$\sigma =$<br>0.0450394<br><i>bias</i> =<br>0.0021467 |
| $N = 50, k = 8$    | $\sigma_{min}^2 =$<br>0.0017000<br>$\sigma^2 =$<br>0.0018190<br>$\sigma =$<br>0.0426496<br><i>bias</i> =<br>0.0227596 | $\sigma_{min}^2 =$<br>0.0023000<br>$\sigma^2 =$<br>0.0023500<br>$\sigma =$<br>0.0484771<br><i>bias</i> =<br>0.0080022 | $\sigma_{min}^2 =$<br>0.0024800<br>$\sigma^2 =$<br>0.0025156<br>$\sigma =$<br>0.0501555<br><i>bias</i> =<br>0.0034917 | $\sigma_{min}^2 =$<br>0.0024980<br>$\sigma^2 =$<br>0.0025394<br>$\sigma =$<br>0.0503928<br><i>bias</i> =<br>0.0025221 | $\sigma_{min}^2 =$<br>0.0024998<br>$\sigma^2 =$<br>0.0025245<br>$\sigma =$<br>0.0502445<br><i>bias</i> =<br>0.0018834 |
|                    | $\sigma_{min}^2 =$<br>0.0002000<br>$\sigma^2 =$   | $\sigma_{min}^2 =$<br>0.0008000<br>$\sigma^2 =$   | $\sigma_{min}^2 =$<br>0.0009800<br>$\sigma^2 =$   | $\sigma_{min}^2 =$<br>0.0009980<br>$\sigma^2 =$   | $\sigma_{min}^2 =$<br>0.0009998<br>$\sigma^2 =$   |

|                       |                    |                    |                    |                    |                    |
|-----------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| $N =$<br>$50, k = 20$ | $\sigma_{min}^2 =$ | $\sigma_{min}^2 =$ | $\sigma_{min}^2 =$ | $\sigma_{min}^2 =$ | $\sigma_{min}^2 =$ |
|                       | 0.0002000          | 0.0008000          | 0.0009800          | 0.0009980          | 0.0009998          |
|                       | $\sigma^2 =$       | $\sigma^2 =$       | $\sigma^2 =$       | $\sigma^2 =$       | $\sigma^2 =$       |
|                       | 0.0002201          | 0.0008131          | 0.0009943          | 0.0010184          | 0.0010138          |
|                       | $\sigma =$         | $\sigma =$         | $\sigma =$         | $\sigma =$         | $\sigma =$         |
|                       | 0.0148343          | 0.0285147          | 0.0315330          | 0.0319125          | 0.0318400          |
|                       | $bias =$           | $bias =$           | $bias =$           | $bias =$           | $bias =$           |
|                       | 0.0211769          | 0.0054468          | 0.0013239          | 0.0010915          | 0.0006501          |

$$\hat{n} = \frac{k}{1-\exp(\overline{L}_u)} \overline{L}_u = \frac{1}{N} \sum_{i=1}^N \ln(1 - x_i)$$

$$\sigma_{min}^2 = \text{Var} \left( \frac{\hat{n}}{n} \right)_{\min} \approx \frac{1}{N} \left( \frac{1}{k} - \frac{1}{n} \right)$$

$$N_{samples} = 10,000$$