XP Community

Graphs, theory and applications

Lennon Alves Dias Follow 5 min read · Nov 18, 2019

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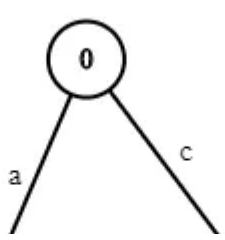
Graph theory* is an old subject, but with many applications in our daily lives. It was introduced in the 18th century by the Swiss mathematician Leonhard Euler, who used graphs to solve the problem we know as **The Seven Bridges of Königsberg** .

> ** Branch of mathematics that studies the relationships between objects in a given set.*

🤔 After all, what are graphs?

It is a structure composed of a (non-empty) set of points **(vertices)** and a set of lines that connect these points **(edges)** .



Graph with 3 vertices {0, 1, 2} and 3 edges {a, b, c}

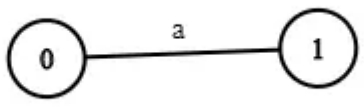
> *Formal definition: A graph **G = (V(G), E(G))** is a mathematical structure composed of two sets:*
> ***V(G)** , a set of elements that are called **vertices** ,*
> ***E(G)** , a set of pairs of elements of V(G), each pair of which is called an **edge***

It's time to simplify things! But not without first, some definitions about graphs:

- Two vertices connected by an edge are said to be **adjacent** .

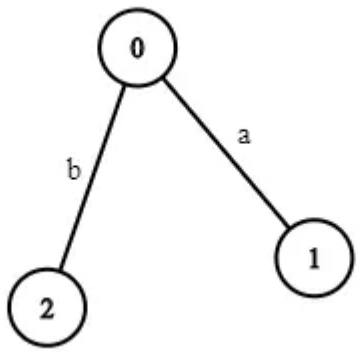
Vertices 0 and 1 are adjacent

- An edge connecting two vertices is said **to be incident** to each of the vertices.



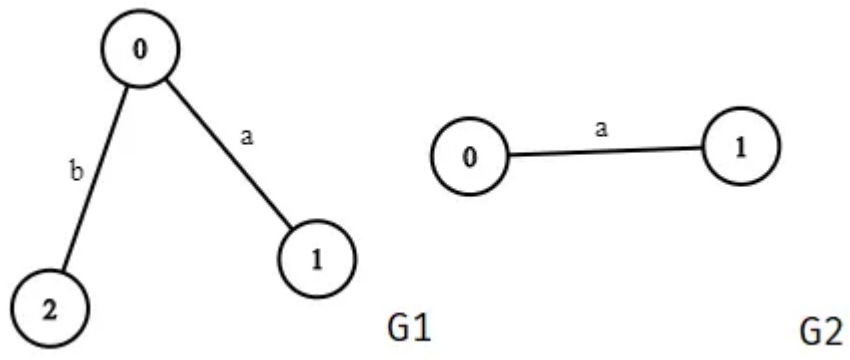
Edge 'a' is incident to 0 and 1

- The number of edges incident on a vertex is called the **degree of that vertex** . The **maximum degree** of the graph is the largest of the degrees of the vertices (Δ(G)), consequently the **minimum degree** (δ(G)) is the smallest of the degrees of the vertices.



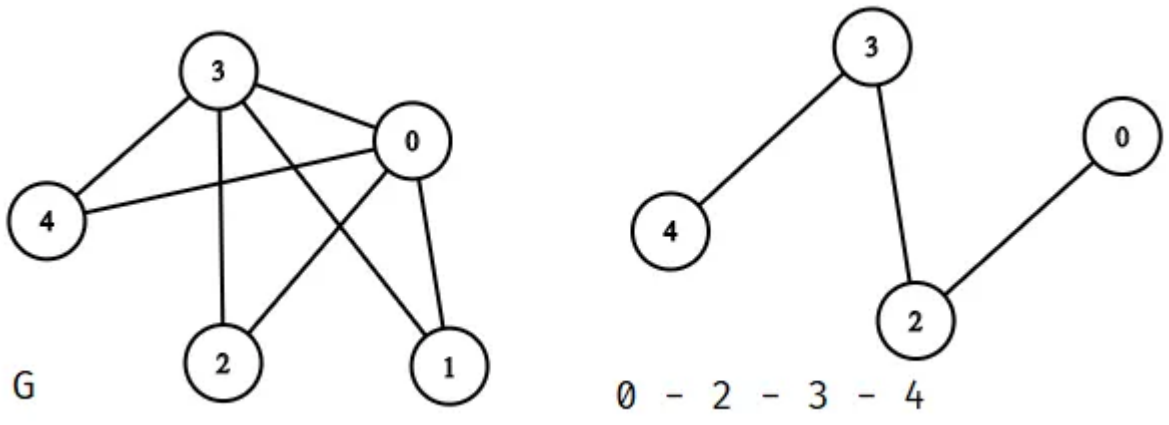
Vertex 0 has two incident edges, therefore degree 2. Vertices 1 and 2 have one incident edge, therefore, both are degree 1. Thus, the maximum degree of the graph is 2 and the minimum degree is 1.

- The subset of edges and vertices associated with them is called **a subgraph** of the original graph.



The set V(G) = {0, 1} E(G) = {a} found in G2 is a subset of V(G) = {0, 1, 2} E(G) = {a, b} found in G1, therefore, it can be said that G2 is a subgraph of G1

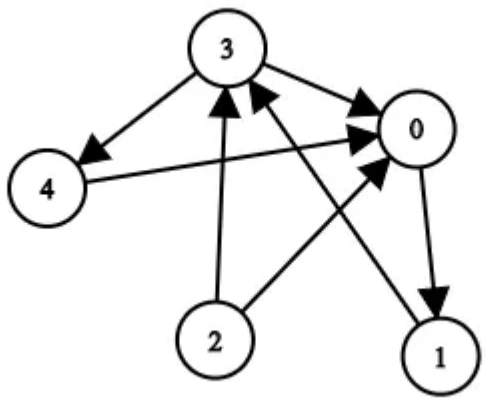
- A sequence of vertices in which successive vertices are connected by edges of the graph is called a **path** .



The vertices 0–2–3–4 represented (figure on the right) form a path found in the graph G (on the left)

In addition to the above definitions, other concepts of graphs are that: The set of vertices can be infinite, in which case it is called an **infinite graph** . When there is more than one edge between the same pair of vertices, we have **multiple edges** , and when we have an edge defined by a pair of non-distinct vertices (that is, an edge that connects at the same node), it is called a **loop** .

There are also graphs that have orientation on the edges, known as **oriented graphs** or **digraphs** or **directed graphs** .



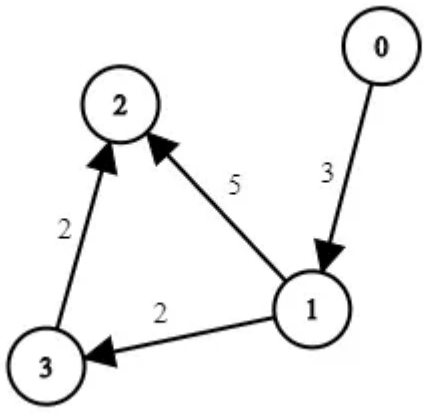
Graph with orientation on edges. G = (V(G),E(G)), V(G) = {0, 1, 2, 3, 4}, E(G) = {(0, 1), (1, 3), (2, 0), (2, 3), (3, 0), (3, 4), (4, 0)}

For directed graphs, the in-degree of a vertex is the number of edges that reach it and is denoted by `d-(v)` . The out-degree is the number of edges that leave the vertex towards others, where the denotation is `d+(v)` .

When the in-degree is equal to zero, the vertex is called a **source** , and when the out-degree is equal to zero, it is called a **half-way point** or **sink** .

By removing the orientation of the edges of a graph, an underlying graph results .

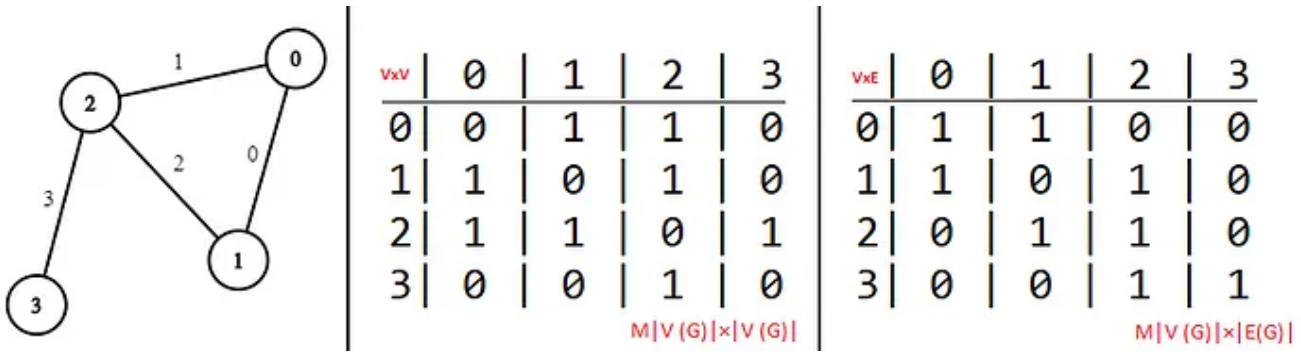
In addition to orientation, it is possible to assign cost to the vertex, to the edge or to both. The graph that receives these values are the **weighted graphs** .



Graph with cost on its edges

For graph representation, the most common forms are: **adjacency matrix** (M|V (G)|×|V (G)|), where if there is an edge between and otherwise, and the **incidence matrix** (M|V (G)|×|E(G)|), where if is one of the vertices of the edge

`.mij = 1 vivj  mij = 1  vi  ej`



Graph with adjacency matrix and incidence matrix

The concepts presented are the **basic** definitions of graphs, where we introduce the theory in order to explain some problems that we can model using this technique. From this point on, I invite you to explore the vast literature with additional concepts and definitions.

🤸 Let's simplify things? ✍️ Let me draw!

Graphs have an easy-to-understand graphical representation, which can translate more complex problems into trivial visualizations. To illustrate the concepts presented in this article, we will use examples that can be seen in our daily lives.

👩‍🏫Graph examples

(object = nodes, relationship = edges)

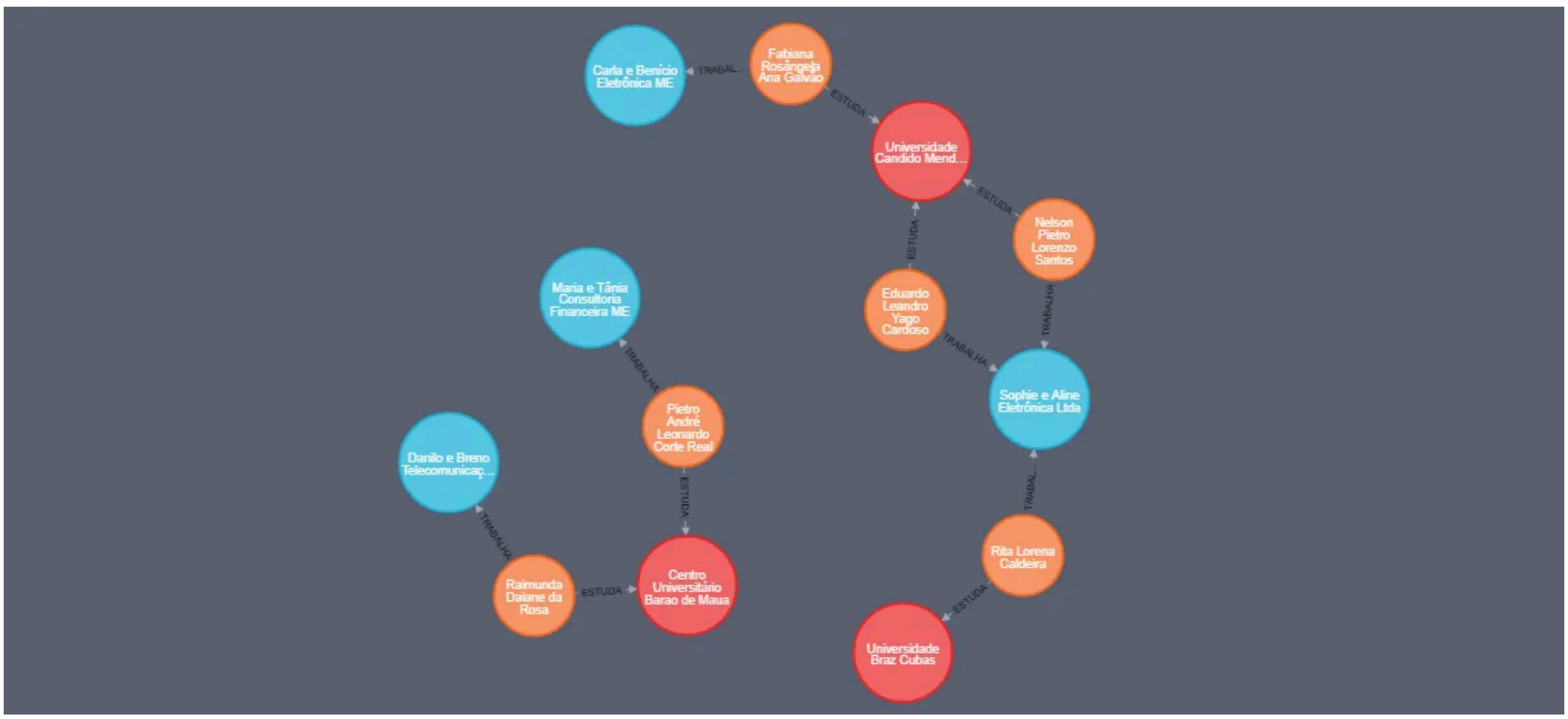
- **Air transport** (*Object* : cities, *Relationship* : commercial flight between two cities)
- **Actors and films** (*Object* : actors, *Relationship* : actors acted in the same film)
- **Web** (*Object* : web pages, *Relationship* : link from one page to another)
- **School schedule** (*Object* : teachers and subjects, *Relationship* : subject taught by the teacher)

- **Couples in a relationship** (*Object* : guys and girls, *Relationship* : mutual interest in dating)
- **Power grid robustness** (*Object* : transmission towers, *Relationship* : lines between towers)

👥 Relationship Network

A company decides to hold an event to present and sell its new portfolio, with several lectures, round tables and stands with the greatest experts in the market. In addition, the event also seeks to connect people in this area, since there are very specific branches such as: exact, biological and human sciences.

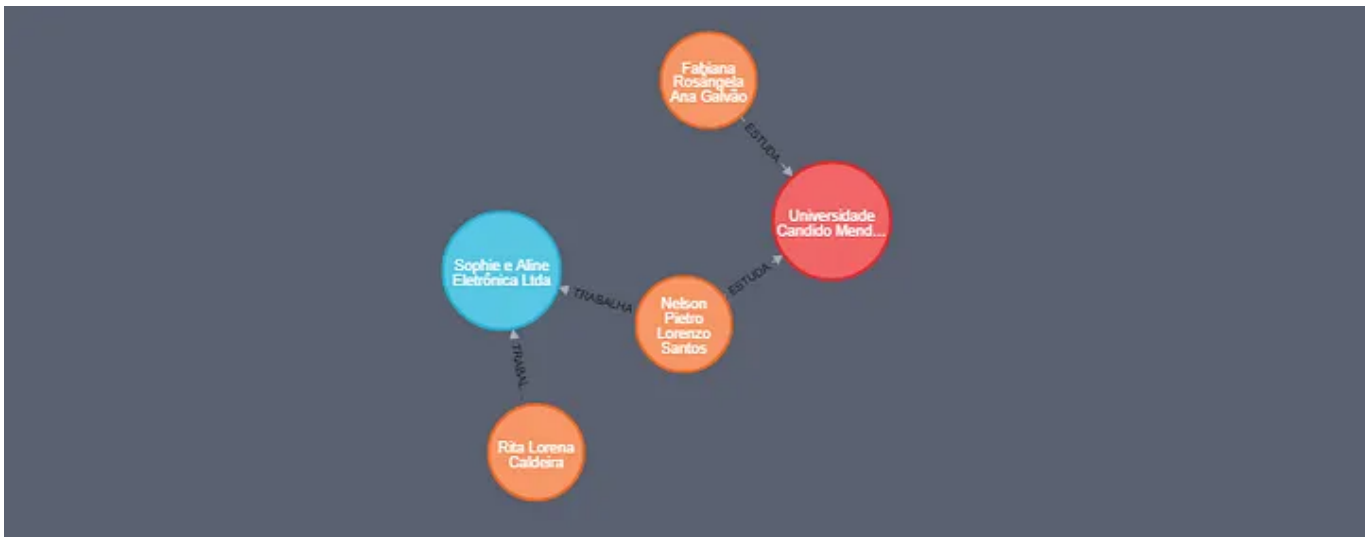
Each person fills in their name, date of birth, email, city, state and current position. Companies are registered with their name, city, state and area of activity. Universities have their name, state and city registered.



Representation with participants (orange), universities (red) and companies (blue)

Rita works in the humanities field, but she is very curious about exact sciences and has learned that Fabiana is a great expert in the area. With this interest, Rita wants to know how she could be introduced to Fabiana so they can have a chat.

Problem: What is the shortest path between Rita and Fabiana?



Representation of the path between Rita and Fabiana

The answer to this problem could be quite costly depending on its modeling, but it is clear that with the use of graphs it becomes a trivial problem. In the example we use few objects, but the same resolution is scalable to **N people, N companies** and **N universities** .

Resolution: Rita works at the same company as Nelson, who studies at the same university as Fabiana.

📃 References

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- Graphs and their applications, Fabiana Nascimento Santos Cavalcante and Severino Domingos da Silva (PUC/RS)
- Applications of Mathematics: Social Networks, Games, Engineering, Profº Fábio Protti (IC-UFF/RJ)

Bridges of Konigsberg | Graph Theory | Count

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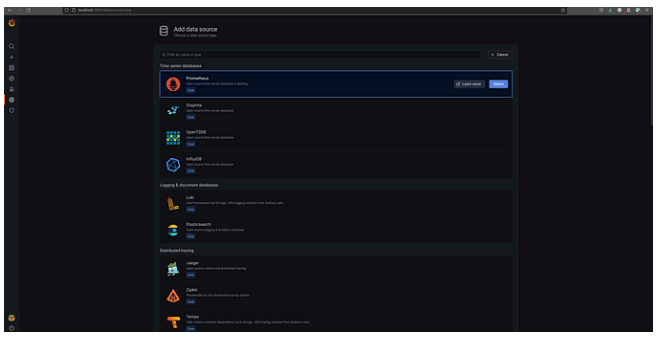

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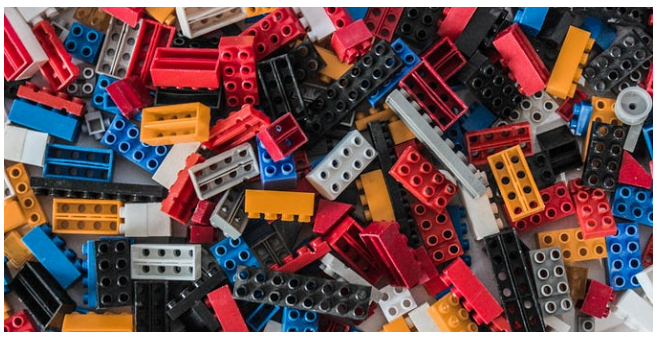

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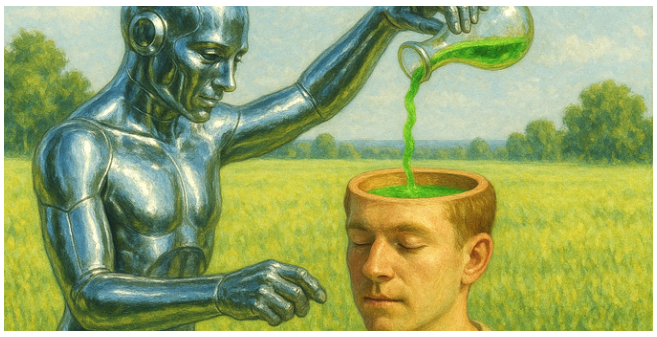

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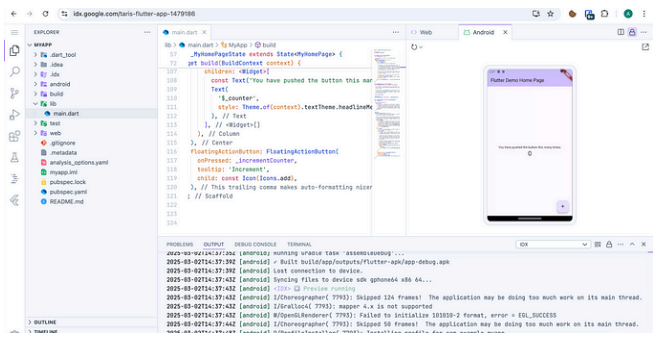

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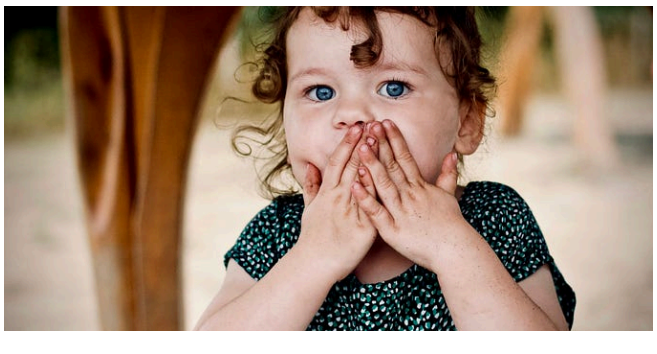

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