



Gramática Livre de Contexto – GLC

- Sejam o alfabeto Σ , a linguagem livre de contexto $\mathcal{L} \subset \Sigma^*$ e a gramática $G = (V, \Sigma, P, S)$, tal que $\mathcal{L} = \mathcal{L}(G)$. Logo, existem gramáticas:
 - $G^1 = (V_1, \Sigma, P_1, S_1)$ tal que $\mathcal{L}(G) = \mathcal{L}(G^1)$ e se $A \rightarrow w \in P_1$, então $A \in V_1$ e $w \in ((V_1 - \{S_1\}) \cup \Sigma)^*$.
 - $G^2 = (V_2, \Sigma, P_2, S_2)$ tal que $\mathcal{L}(G) = \mathcal{L}(G^2)$ e $(A \rightarrow \varepsilon) \in P_2$ se, e somente se, $\varepsilon \in \mathcal{L}(G)$ e $A = S_2$.
 - $G^3 = (V_3, \Sigma, P_3, S_3)$ tal que $\mathcal{L}(G) = \mathcal{L}(G^3)$ e $(A \rightarrow B) \notin P_3$, para $A, B \in V_3$.
 - $G^4 = (V_4, \Sigma, P_4, S_4)$ tal que $\mathcal{L}(G) = \mathcal{L}(G^4)$ e $(A \rightarrow Aw) \notin P_4$, para $A, B \in V_4$ e $w \in (V_4 \cup \Sigma)^*$.
 - $G^5 = (V_5, \Sigma, P_5, S_5)$ tal que $\mathcal{L}(G) = \mathcal{L}(G^5)$ e, para todo $x \in (V_5 \cup \Sigma_5)$, $S_5 \xRightarrow{*} uxv \xRightarrow{*} w$, com $u, v \in (V_5 \cup \Sigma_5)^*$ e $w \in \Sigma_5^*$.

Atenção: Embora as respostas dos exercícios sejam elaboradas com esforço e cuidado, e continuamente revisadas, algumas delas ainda estão incompletas ou podem conter erros que passaram despercebidos. Comentários ou correções específicas são bem-vindos, especialmente se forem relacionados a erros críticos!

$$\mathcal{L}_1 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = u1^{|u|_0}, u \in \Sigma^*\}$$

$$\mathcal{L}_2 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m u, |u|_0 \leq m, m \in \mathbb{N}^+, u \in \Sigma^*\}$$

- $G_2 = (\{S\}, \{0, 1\}, P, S)$, com $P = \{ S \rightarrow 0S0 \mid 0S \mid S1 \mid 0 \}$.
- $G_2^1 = (\{S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 0S0 \mid 0S \mid S1 \mid 0 \end{array} \right\}$.
- $G_2^2 = G_2^1$.
- $G_2^3 = (\{S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 0S0 \mid 0S \mid S1 \mid 0, \\ S \rightarrow 0S0 \mid 0S \mid S1 \mid 0 \end{array} \right\}$.
- $G_2^4 = (\{R, S, S_1\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow 0S0 \mid 0S \mid S1 \mid 0, \\ S \rightarrow 0S0R \mid 0SR \mid 0R \mid 0S0 \mid 0S \mid 0, \\ R \rightarrow 1R \mid 1 \end{array} \right\}$.
- $G_2^5 = G_2^4$.

$$\mathcal{L}_3 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 1^5 u, 2 \cdot |w|_0 = 3 \cdot |w|_1, u \in \Sigma^*\}$$

$$\mathcal{L}_4 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = uv, |u|_1 \geq |u|_0 + 4, u, v \in \Sigma^*\}$$

$$\mathcal{L}_5 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = uv, |u| = |v|, |v|_1 \geq 1, u, v \in \Sigma^*\}$$

$$\mathcal{L}_6 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv, |u| \geq |v|, v = r1s, u, r, s \in \Sigma^*\}$$

- $G_6 = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AS \mid ASA \mid 01 \mid 11, \\ A \rightarrow 0 \mid 1 \end{array} \right\}$.
- $G_6^1 = (\{A, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow AS \mid ASA \mid 01 \mid 11, \\ A \rightarrow 0 \mid 1 \end{array} \right\}$.
- $G_6^2 = G_6^1$.
- $G_6^3 = (\{A, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow AS \mid ASA \mid 01 \mid 11, \\ S \rightarrow AS \mid ASA \mid 01 \mid 11, \\ A \rightarrow 0 \mid 1 \end{array} \right\}$.
- $G_6^5 = G_6^4 = G_6^3$.

$$\mathcal{L}_7 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv^Rv, u \in \Sigma^*, v \in \Sigma^+\}$$

- $G_7 = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 0A \mid 1A \mid \varepsilon, \\ B \rightarrow 0B0 \mid 1B1 \mid 00 \mid 11 \end{array} \right\}$.
- $G_7^1 = G_7$.
- $G_7^2 = (\{A, B, S\}, \{0, 1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow AB \mid B, \\ A \rightarrow 0A \mid 1A \mid 0 \mid 1, \\ B \rightarrow 0B0 \mid 1B1 \mid 00 \mid 11 \end{array} \right\}$.
- $G_7^3 = (\{A, B, S\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S \rightarrow AB \mid 0B0 \mid 1B1 \mid 00 \mid 11, \\ A \rightarrow 0A \mid 1A \mid 0 \mid 1, \\ B \rightarrow 0B0 \mid 1B1 \mid 00 \mid 11 \end{array} \right\}$.
- $G_7^5 = G_7^4 = G_7^3$.

$$\mathcal{L}_8 = \{w \in \Sigma^* = \{0,1\}^* \mid w = u0v, |w| = 2 \cdot k + 1, |u| = |v|, k \in \mathbb{N}, u, v \in \Sigma^+\}$$

- $G_8 = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow ASA \mid ABA, \\ A \rightarrow 0 \mid 1, \\ B \rightarrow 0 \end{array} \right\}$.
- $G_8^1 = (\{A, B, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow ASA \mid ABA, \\ A \rightarrow 0 \mid 1, \\ B \rightarrow 0 \end{array} \right\}$.
- $G_8^2 = G_8^1$.



- $G_8^3 = (\{A, B, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow ASA \mid ABA, \\ S \rightarrow ASA \mid ABA, \\ A \rightarrow 0 \mid 1, \\ B \rightarrow 0 \end{array} \right\}$.
- $G_8^5 = G_8^4 = G_8^3$.

$$\mathcal{L}_9 = \{w \in \Sigma^* = \{0, 1\}^* \mid w = cuc, c \in \Sigma, u \in \Sigma^+, |w|_0 = |w|_1\}$$

- $G_9 = (\{A, B, C, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A0 \mid 1B1, \\ A \rightarrow 0A1 \mid 1A0 \mid C11 \mid 1C1 \mid 11C, \\ B \rightarrow 0B1 \mid 1B0 \mid C00 \mid 0C0 \mid 00C, \\ C \rightarrow 0C1 \mid 1C0 \mid \varepsilon \end{array} \right\}$.
- $G_9^1 = G_9$.
- $G_9^2 = (\{A, B, C, S\}, \{0, 1\}, P_2, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A0 \mid 1B1, \\ A \rightarrow 0A1 \mid 1A0 \mid C11 \mid 1C1 \mid 11C \mid 11, \\ B \rightarrow 0B1 \mid 1B0 \mid C00 \mid 0C0 \mid 00C \mid 00, \\ C \rightarrow 0C1 \mid 1C0 \mid 01 \mid 10 \end{array} \right\}$.
- $G_9^5 = G_9^4 = G_9^3 = G_9^2$.

$$\mathcal{L}_{10} = \{w \in \Sigma^* = \{0, 1\}^* \mid |w| = 3 \cdot |w|_0\}$$

- $G_{10} = (\{S\}, \{0, 1\}, P, S)$, com $P = \{ S \rightarrow S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \varepsilon \}$.
- $G_{10}^1 = (\{S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \varepsilon \end{array} \right\}$.



$$\bullet G_{10}^2 = (\{S, S_1\}, \{0, 1\}, P_2, S_1), \text{ com } P_2 = \left\{ \begin{array}{l} S_1 \rightarrow \varepsilon, \\ S \rightarrow S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \\ 0S1S1S \mid 1S0S1S \mid 1S1S0S \mid \\ S01S1S \mid S10S1S \mid S11S0S \mid \\ S0S11S \mid S1S01S \mid S1S10S \mid \\ S0S1S1 \mid S1S0S1 \mid S1S1S0 \mid \\ 01S1S \mid 10S1S \mid 11S0S \mid \\ 0S11S \mid 1S01S \mid 1S10S \mid \\ 0S1S1 \mid 1S0S1 \mid 1S1S0 \mid \\ S011S \mid S101S \mid S110S \mid \\ S01S1 \mid S10S1 \mid S11S0 \mid \\ S0S11 \mid S1S01 \mid S1S10 \mid \\ 011S \mid 101S \mid 110S \mid \\ 01S1 \mid 10S1 \mid 11S0 \mid \\ 0S11 \mid 1S01 \mid 1S10 \mid \\ S011 \mid S101 \mid S110 \mid \\ 011 \mid 101 \mid 110 \end{array} \right\}.$$



$$\bullet G_{10}^3 = (\{S, S_1\}, \{0, 1\}, P_3, S_1), \text{ com } P_3 = \left\{ \begin{array}{l} S_1 \rightarrow S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \\ 0S1S1S \mid 1S0S1S \mid 1S1S0S \mid \\ S01S1S \mid S10S1S \mid S11S0S \mid \\ S0S11S \mid S1S01S \mid S1S10S \mid \\ S0S1S1 \mid S1S0S1 \mid S1S1S0 \mid \\ 01S1S \mid 10S1S \mid 11S0S \mid \\ 0S11S \mid 1S01S \mid 1S10S \mid \\ 0S1S1 \mid 1S0S1 \mid 1S1S0 \mid \\ S011S \mid S101S \mid S110S \mid \\ S01S1 \mid S10S1 \mid S11S0 \mid \\ S0S11 \mid S1S01 \mid S1S10 \mid \\ 011S \mid 101S \mid 110S \mid \\ 01S1 \mid 10S1 \mid 11S0 \mid \\ 0S11 \mid 1S01 \mid 1S10 \mid \\ S011 \mid S101 \mid S110 \mid \\ 011 \mid 101 \mid 110 \mid \varepsilon, \\ S \rightarrow S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \\ 0S1S1S \mid 1S0S1S \mid 1S1S0S \mid \\ S01S1S \mid S10S1S \mid S11S0S \mid \\ S0S11S \mid S1S01S \mid S1S10S \mid \\ S0S1S1 \mid S1S0S1 \mid S1S1S0 \mid \\ 01S1S \mid 10S1S \mid 11S0S \mid \\ 0S11S \mid 1S01S \mid 1S10S \mid \\ 0S1S1 \mid 1S0S1 \mid 1S1S0 \mid \\ S011S \mid S101S \mid S110S \mid \\ S01S1 \mid S10S1 \mid S11S0 \mid \\ S0S11 \mid S1S01 \mid S1S10 \mid \\ 011S \mid 101S \mid 110S \mid \\ 01S1 \mid 10S1 \mid 11S0 \mid \\ 0S11 \mid 1S01 \mid 1S10 \mid \\ S011 \mid S101 \mid S110 \mid \\ 011 \mid 101 \mid 110 \end{array} \right\}.$$

$$\bullet G_{10}^4 = \dots \odot$$

$$\bullet G_{10}^5 = \dots \odot$$

$$\mathcal{L}_{11} = \{w \in \Sigma^* = \{0, 1\}^* \mid |w|_0 \neq |w|_1\}$$



$$\mathcal{L}_{12} = \{w \in \Sigma^* = \{0, 1\}^* \mid |w|_0 = 2 \cdot |w|_1\}$$

- $G_{12} = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid \varepsilon, \\ A \rightarrow 0S \mid SA \end{array} \right\}$.
- $G_{12}^1 = (\{A, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid \varepsilon, \\ A \rightarrow 0S \mid SA \end{array} \right\}$.
- $G_{12}^2 = (\{S, S_1\}, \{0, 1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S \mid \varepsilon, \\ S \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid S \mid 001 \mid 100, \\ A \rightarrow 0S \mid SA \mid 0 \mid A \end{array} \right\}$.
- $G_{12}^3 = (\{S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100 \mid \varepsilon, \\ S \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100, \\ A \rightarrow 0S \mid SA \mid 0 \end{array} \right\}$.
- $G_{12}^4 = (\{R, S, S_1\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow SS \mid A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100 \mid \varepsilon, \\ S \rightarrow A1AR \mid 00S1R \mid 1S00R \mid 001R \mid 100R \mid \\ \quad A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100, \\ R \rightarrow SR \mid \\ \quad A1AR \mid 00S1R \mid 1S00R \mid 001R \mid 100R \mid \\ \quad A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100, \\ A \rightarrow 0S \mid SA \mid 0 \end{array} \right\}$.
- $G_{12}^5 = G_{12}^4$.

$$\mathcal{L}_{13} = \{w \in \Sigma^* = \{0, 1\}^* \mid |w|_{101} = |w|_{010}\}$$

- $G_{13} = (\{S\}, \{0, 1\}, P, S)$, com $P = \{ S \rightarrow SS \mid 0S1 \mid 10 \}$.
- $G_{13}^1 = (\{S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow SS \mid 0S1 \mid 10 \end{array} \right\}$.
- $G_{13}^2 = G_{13}^1$.
- $G_{13}^3 = (\{S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow SS \mid 0S1 \mid 10, \\ S \rightarrow SS \mid 0S1 \mid 10 \end{array} \right\}$.
- $G_{13}^4 = (\{R, S, S_1\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow SS \mid 0S1 \mid 10, \\ S \rightarrow 0S1R \mid 10R \mid 0S1 \mid 10, \\ R \rightarrow SR \mid 0S1R \mid 10R \mid 0S1 \mid 10 \end{array} \right\}$.
- $G_{13}^5 = G_{13}^4$.

$$\mathcal{L}_{14} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n, m \neq n \text{ e } 2 \cdot m \neq n, m, n \in \mathbb{N}\}$$



$$\mathcal{L}_{15} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n, 3 \cdot m \leq n \leq 5 \cdot m, m, n \in \mathbb{N}\}$$

$$\mathcal{L}_{16} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n 0^n, n \in \mathbb{N}\}$$

$$\mathcal{L}_{17} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^m 0^n, n \geq 2 \cdot m, m, n \in \mathbb{N}\}$$

- $G_{17} = (\{S\}, \{0,1\}, P, S)$, com $P = \{ S \rightarrow 01S00 \mid S0 \mid \varepsilon \}$.
- $G_{17}^1 = (\{S, S_1\}, \{0,1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 01S00 \mid S0 \mid \varepsilon \end{array} \right\}$.
- $G_{17}^2 = (\{S, S_1\}, \{0,1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S \mid \varepsilon, \\ S \rightarrow 01S00 \mid S0 \mid 0100 \mid 0 \end{array} \right\}$.
- $G_{17}^3 = (\{S, S_1\}, \{0,1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 01S00 \mid S0 \mid 0100 \mid 0 \mid \varepsilon, \\ S \rightarrow 01S00 \mid S0 \mid 0100 \mid 0 \end{array} \right\}$.
- $G_{17}^4 = (\{R, S, S_1\}, \{0,1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow 01S00 \mid S0 \mid 0100 \mid 0 \mid \varepsilon, \\ S \rightarrow 01S00R \mid 0100R \mid 0R \mid 01S00 \mid 0100 \mid 0, \\ R \rightarrow 0R \mid 0 \end{array} \right\}$.
- $G_{17}^5 = G_{17}^4$.

$$\mathcal{L}_{18} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 110(10)^n 0^{n-1}, n \in \mathbb{N}\}$$

- $G_{18} = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 110A, \\ A \rightarrow 10B, \\ B \rightarrow 10B0 \mid \varepsilon \end{array} \right\}$.
- $G_{18}^1 = G_{18}$.
- $G_{18}^2 = (\{A, B, S\}, \{0,1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow 110A, \\ A \rightarrow 10B \mid 10, \\ B \rightarrow 10B0 \mid 100 \end{array} \right\}$.
- $G_{18}^5 = G_{18}^4 = G_{18}^3 = G_{18}^2$.

$$\mathcal{L}_{19} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^m 0^n, m, n \in \mathbb{N}\}$$

- $G_{19} = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow S0 \mid A, \\ A \rightarrow 0A1 \mid \varepsilon \end{array} \right\}$.
- $G_{19}^1 = (\{A, S, S_1\}, \{0,1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow S0 \mid A, \\ A \rightarrow 0A1 \mid \varepsilon \end{array} \right\}$.



- $G_{19}^2 = (\{A, S, S_1\}, \{0, 1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S \mid \varepsilon, \\ S \rightarrow S0 \mid A \mid 0, \\ A \rightarrow 0A1 \mid 01 \end{array} \right\}$.
- $G_{19}^3 = (\{A, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow S0 \mid 0A1 \mid 01 \mid 0 \mid \varepsilon, \\ S \rightarrow S0 \mid 0A1 \mid 01 \mid 0, \\ A \rightarrow 0A1 \mid 01 \end{array} \right\}$.
- $G_{19}^4 = (\{A, R, S, S_1\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow S0 \mid 0 \mid \varepsilon, \\ S \rightarrow 0A1R \mid 01R \mid 0R \mid 0A1 \mid 01 \mid 0, \\ R \rightarrow 0R \mid 0, \\ A \rightarrow 0A1 \mid 01 \end{array} \right\}$.
- $G_{19}^5 = G_{19}^4$.

$$\mathcal{L}_{20} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, m + q \leq n, m, n, q \in \mathbb{N}\}$$

$$\mathcal{L}_{21} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, n \neq m + q, m, n, q \in \mathbb{N}\}$$

- $G_{21} = (\{A, B, C, D, E, F, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow A \mid B, \\ A \rightarrow CED, \\ B \rightarrow FCDF \mid FCD \mid CDF, \\ C \rightarrow 0C1 \mid \varepsilon, \\ D \rightarrow 1D0 \mid \varepsilon, \\ E \rightarrow 1E \mid 1, \\ F \rightarrow 0F \mid 0 \end{array} \right\}$.
- $G_{21}^1 = G_{21}$.
- $G_{21}^2 = (\{A, B, C, D, E, F, S\}, \{0, 1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow A \mid B, \\ A \rightarrow CED \mid ED \mid CE \mid E, \\ B \rightarrow FCDF \mid FCD \mid CDF \mid \\ \quad FDF \mid FD \mid DF \mid \\ \quad CD \mid D, \\ C \rightarrow 0C1 \mid 01, \\ D \rightarrow 1D0 \mid 10, \\ E \rightarrow 1E \mid 1, \\ F \rightarrow 0F \mid 0 \end{array} \right\}$.

$$\bullet G_{21}^3 = (\{A, B, C, D, E, F, S\}, \{0, 1\}, P_3, S), \text{ com } P_3 = \left\{ \begin{array}{l} S \rightarrow CED \mid ED \mid CE \mid 1E \mid 1 \mid \\ \quad FCDF \mid FCD \mid CDF \mid \\ \quad FDF \mid FD \mid DF \mid \\ \quad CD \mid 1D0 \mid 10, \\ A \rightarrow CED \mid ED \mid CE \mid 1E \mid 1, \\ B \rightarrow FCDF \mid FCD \mid CDF \mid \\ \quad FDF \mid FD \mid DF \mid \\ \quad CD \mid 1D0 \mid 10, \\ C \rightarrow 0C1 \mid 01, \\ D \rightarrow 1D0 \mid 10, \\ E \rightarrow 1E \mid 1, \\ F \rightarrow 0F \mid 0 \end{array} \right\}.$$

$$\bullet G_{21}^4 = G_{21}^3.$$

$$\bullet G_{21}^5 = (\{C, D, E, F, S\}, \{0, 1\}, P_5, S), \text{ com } P_5 = \left\{ \begin{array}{l} S \rightarrow CED \mid ED \mid CE \mid 1E \mid 1 \mid \\ \quad FCDF \mid FCD \mid CDF \mid \\ \quad FDF \mid FD \mid DF \mid \\ \quad CD \mid 1D0 \mid 10, \\ C \rightarrow 0C1 \mid 01, \\ D \rightarrow 1D0 \mid 10, \\ E \rightarrow 1E \mid 1, \\ F \rightarrow 0F \mid 0 \end{array} \right\}.$$

$$\mathcal{L}_{22} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, m \neq q, m, n, q \in \mathbb{N}\}$$

$$\bullet G_{22} = (\{A, B, S\}, \{0, 1\}, P, S), \text{ com } P = \left\{ \begin{array}{l} S \rightarrow 0S0 \mid A0 \mid 0B, \\ A \rightarrow 1A0 \mid 1A \mid A0 \mid \varepsilon, \\ B \rightarrow 0B1 \mid B1 \mid 0B \mid \varepsilon \end{array} \right\}.$$

$$\bullet G_{22}^1 = (\{A, B, S, S_1\}, \{0, 1\}, P_1, S_1), \text{ com } P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 0S0 \mid A0 \mid 0B, \\ A \rightarrow 1A0 \mid 1A \mid A0 \mid \varepsilon, \\ B \rightarrow 0B1 \mid B1 \mid 0B \mid \varepsilon \end{array} \right\}.$$

$$\bullet G_{22}^2 = (\{A, B, S, S_1\}, \{0, 1\}, P_2, S_1), \text{ com } P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 0S0 \mid A0 \mid 0B \mid 0, \\ A \rightarrow 1A0 \mid 1A \mid A0 \mid 10 \mid 1 \mid 0, \\ B \rightarrow 0B1 \mid B1 \mid 0B \mid 01 \mid 1 \mid 0 \end{array} \right\}.$$



- $G_{22}^3 = (\{A, B, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 0S0 \mid A0 \mid 0B \mid 0, \\ S \rightarrow 0S0 \mid A0 \mid 0B \mid 0, \\ A \rightarrow 1A0 \mid 1A \mid A0 \mid 10 \mid 1 \mid 0, \\ B \rightarrow 0B1 \mid B1 \mid 0B \mid 01 \mid 1 \mid 0 \end{array} \right\}$.
- $G_{22}^4 = (\{A, B, R_1, R_2, S, S_1\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S_1 \rightarrow 0S0 \mid A0 \mid 0B \mid 0, \\ S \rightarrow 0S0 \mid A0 \mid 0B \mid 0, \\ A \rightarrow 1A0R_1 \mid 1AR_1 \mid 10R_1 \mid 1R_1 \mid 0R_1 \mid \\ \quad 1A0 \mid 1A \mid 10 \mid 1 \mid 0, \\ R_1 \rightarrow 0R_1 \mid 0, \\ B \rightarrow 0B1R_2 \mid 0BR_2 \mid 01R_2 \mid 1R_2 \mid 0R_2 \mid \\ \quad 0B1 \mid 0B \mid 01 \mid 1 \mid 0, \\ R_2 \rightarrow 1R_2 \mid 1 \end{array} \right\}$.
- $G_{22}^5 = G_{22}^4$.

$$\mathcal{L}_{23} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, q = 2 \cdot (m + n), m, n, q \in \mathbb{N}\}$$

- $G_{23} = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S00 \mid A, \\ A \rightarrow 1A00 \mid \varepsilon \end{array} \right\}$.
- $G_{23}^1 = (\{A, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 0S00 \mid A, \\ A \rightarrow 1A00 \mid \varepsilon \end{array} \right\}$.
- $G_{23}^2 = (\{A, S, S_1\}, \{0, 1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S \mid \varepsilon, \\ S \rightarrow 0S00 \mid A \mid 000, \\ A \rightarrow 1A00 \mid 100 \end{array} \right\}$.
- $G_{23}^3 = (\{A, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 0S00 \mid 1A00 \mid 100 \mid 000 \mid \varepsilon, \\ S \rightarrow 0S00 \mid 1A00 \mid 100 \mid 000, \\ A \rightarrow 1A00 \mid 100 \end{array} \right\}$.
- $G_{23}^5 = G_{23}^4 = G_{23}^3$.

$$\mathcal{L}_{24} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, m > 5, n > 3, q \leq n, m, n, q \in \mathbb{N}\}$$

- $G_{24} = (\{A, B, C, D, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 0A \mid 000000, \\ B \rightarrow 1B0 \mid 1B \mid CD, \\ C \rightarrow 1111, \\ D \rightarrow \varepsilon \mid 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}$.
- $G_{24}^1 = G_{24}$.



- $G_{24}^2 = (\{A, B, C, D, S\}, \{0, 1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 0A \mid 000000, \\ B \rightarrow 1B0 \mid 1B \mid CD \mid C, \\ C \rightarrow 1111, \\ D \rightarrow 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}.$
- $G_{24}^3 = (\{A, B, C, D, S\}, \{0, 1\}, P_3, S)$, com $P_3 = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 0A \mid 000000, \\ B \rightarrow 1B0 \mid 1B \mid CD \mid 1111, \\ C \rightarrow 1111, \\ D \rightarrow 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}.$
- $G_{24}^5 = G_{24}^4 = G_{24}^3.$

$$\mathcal{L}_{25} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, m \leq 2 \cdot n \text{ ou } n \leq 3 \cdot q, m, n, q \in \mathbb{N}\}$$

- $G_{25} = (\{A, B, C, D, E, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AC \mid CD, \\ A \rightarrow BBA1 \mid \varepsilon, \\ B \rightarrow 0 \mid \varepsilon, \\ C \rightarrow 0C \mid 0, \\ D \rightarrow EEED0 \mid \varepsilon, \\ E \rightarrow 1 \mid \varepsilon \end{array} \right\}.$
- $G_{25}^1 = G_{25}.$
- $G_{25}^2 = (\{A, B, C, D, E, S\}, \{0, 1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow AC \mid CD \mid C, \\ A \rightarrow BBA1 \mid BA1 \mid A1 \mid BB1 \mid B1 \mid 1, \\ B \rightarrow 0, \\ C \rightarrow 0C \mid 0, \\ D \rightarrow EEED0 \mid EED0 \mid ED0 \mid D0 \mid \\ \quad EEE0 \mid EE0 \mid E0 \mid 0, \\ E \rightarrow 1 \end{array} \right\}.$
- $G_{25}^3 = (\{A, B, C, D, E, S\}, \{0, 1\}, P_3, S)$, com $P_3 = \left\{ \begin{array}{l} S \rightarrow AC \mid CD \mid 0C \mid 0, \\ A \rightarrow BBA1 \mid BA1 \mid A1 \mid BB1 \mid B1 \mid 1, \\ B \rightarrow 0, \\ C \rightarrow 0C \mid 0, \\ D \rightarrow EEED0 \mid EED0 \mid ED0 \mid D0 \mid \\ \quad EEE0 \mid EE0 \mid E0 \mid 0, \\ E \rightarrow 1 \end{array} \right\}.$



- $G_{25}^4 = (\{A, B, C, D, E, R_1, R_2, S\}, \{0, 1\}, P_4, S)$, com

$$P_4 = \left\{ \begin{array}{l} S \rightarrow AC \mid CD \mid 0C \mid 0, \\ A \rightarrow BBA1R_1 \mid BA1R_1 \mid BB1R_1 \mid B1R_1 \mid 1R_1, \\ \quad BBA1 \mid BA1 \mid BB1 \mid B1 \mid 1, \\ R_1 \rightarrow 1R_1 \mid 1, \\ B \rightarrow 0, \\ C \rightarrow 0C \mid 0, \\ D \rightarrow EEED0R_2 \mid EED0R_2 \mid ED0R_2 \mid \\ \quad EEE0R_2 \mid EE0R_2 \mid E0R_2 \mid 0R_2, \\ \quad EEED0 \mid EED0 \mid ED0 \mid \\ \quad EEE0 \mid EE0 \mid E0 \mid 0, \\ R_2 \rightarrow 0R_2 \mid 0, \\ E \rightarrow 1 \end{array} \right\}.$$

- $G_{25}^5 = G_{25}^4$.

$$\mathcal{L}_{26} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, m = 1 \Rightarrow n = q, m, n, q \in \mathbb{N}\}$$

- $G_{26} = (\{A, B, C, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A \mid B \mid 00C, \\ A \rightarrow 1A0 \mid \varepsilon, \\ B \rightarrow 1B \mid B0 \mid \varepsilon, \\ C \rightarrow 0C \mid B \end{array} \right\}.$

- $G_{26}^1 = G_{26}$.

- $G_{26}^2 = (\{A, B, C, S\}, \{0, 1\}, P_2, S)$, com $P_2 = \left\{ \begin{array}{l} S \rightarrow 0A \mid B \mid 00C \mid 00 \mid 0 \mid \varepsilon, \\ A \rightarrow 1A0 \mid 10, \\ B \rightarrow 1B \mid B0 \mid 1 \mid 0, \\ C \rightarrow 0C \mid B \mid 0 \end{array} \right\}.$

- $G_{26}^3 = (\{A, B, C, S\}, \{0, 1\}, P_3, S)$, com $P_3 = \left\{ \begin{array}{l} S \rightarrow 0A \mid 1B \mid B0 \mid 1 \mid 00C \mid 00 \mid 0 \mid \varepsilon, \\ A \rightarrow 1A0 \mid 10, \\ B \rightarrow 1B \mid B0 \mid 1 \mid 0, \\ C \rightarrow 0C \mid 1B \mid B0 \mid 1 \mid 0 \mid 0 \end{array} \right\}.$

- $G_{26}^4 = (\{A, B, C, R, S\}, \{0, 1\}, P_4, S)$, com $P_4 = \left\{ \begin{array}{l} S \rightarrow 0A \mid 1B \mid B0 \mid 1 \mid 00C \mid 00 \mid 0 \mid \varepsilon, \\ A \rightarrow 1A0 \mid 10, \\ B \rightarrow 1BR \mid 1R \mid 0R \mid 1B \mid 1 \mid 0, \\ R \rightarrow 0R \mid 0, \\ C \rightarrow 0C \mid 1B \mid B0 \mid 1 \mid 0 \mid 0 \end{array} \right\}.$



- $G_{26}^5 = G_{26}^4$.

$$\mathcal{L}_{27} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^{m+n} 0^n, m + n > 0, m, n \in \mathbb{N}\}$$

- $G_{27} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB \mid A \mid B, \\ A \rightarrow 0A1 \mid 01, \\ B \rightarrow 1B0 \mid 10, \end{array} \right\}$.
- $G_{27}^2 = G_{27}^1 = G_{27}$.
- $G_{27}^3 = (\{A, B, S\}, \{0, 1\}, P_3, S)$, com $P_3 = \left\{ \begin{array}{l} S \rightarrow AB \mid 0A1 \mid 01 \mid 1B0 \mid 10, \\ A \rightarrow 0A1 \mid 01, \\ B \rightarrow 1B0 \mid 10, \end{array} \right\}$.
- $G_{27}^5 = G_{27}^4 = G_{27}^3$.

$$\mathcal{L}_{28} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^{m-n}, m > n, m, n \in \mathbb{N}\}$$

$$\mathcal{L}_{29} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^r 1^s, m = 2 \cdot s, n = r, m, n, r, s \in \mathbb{N}\}$$

$$\mathcal{L}_{30} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 01^{m+1}, m, n \in \mathbb{N}\}$$

- $G_{30} = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S1 \mid A, \\ A \rightarrow 1A \mid 01, \end{array} \right\}$.
- $G_{30}^1 = (\{A, S, S_1\}, \{0, 1\}, P, S_1)$, com $P = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 0S1 \mid A, \\ A \rightarrow 1A \mid 01, \end{array} \right\}$.
- $G_{30}^2 = G_{30}^1$.
- $G_{30}^3 = (\{A, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 0S1 \mid 1A \mid 01, \\ S \rightarrow 0S1 \mid 1A \mid 01, \\ A \rightarrow 1A \mid 01, \end{array} \right\}$.
- $G_{30}^5 = G_{30}^4 = G_{30}^3$.

$$\mathcal{L}_{31} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = (01)^n 0^m (01)^n, m < 3, m, n \in \mathbb{N}\}$$

- $G_{31} = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 01S01 \mid A, \\ A \rightarrow 0 \mid 00 \mid \varepsilon \end{array} \right\}$.



- $G_{31}^1 = (\{A, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 01S01 \mid A, \\ A \rightarrow 0 \mid 00 \mid \varepsilon \end{array} \right\}$.
- $G_{31}^2 = (\{A, S, S_1\}, \{0, 1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow S \mid \varepsilon, \\ S \rightarrow 01S01 \mid A \mid 0101, \\ A \rightarrow 0 \mid 00 \end{array} \right\}$.
- $G_{31}^3 = (\{A, S, S_1\}, \{0, 1\}, P_3, S_1)$, com $P_3 = \left\{ \begin{array}{l} S_1 \rightarrow 01S01 \mid 0 \mid 00 \mid 0101 \mid \varepsilon, \\ S \rightarrow 01S01 \mid 0 \mid 00 \mid 0101, \\ A \rightarrow 0 \mid 00 \end{array} \right\}$.
- $G_{31}^4 = G_{31}^3$.
- $G_{31}^5 = (\{S, S_1\}, \{0, 1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \rightarrow 01S01 \mid 0 \mid 00 \mid 0101 \mid \varepsilon, \\ S \rightarrow 01S01 \mid 0 \mid 00 \mid 0101 \end{array} \right\}$.

$$\mathcal{L}_{32} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = (01)^n(01^{m_n})^n, m_n, n \in \mathbb{N}^+\}$$

- $G_{32} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 01SA \mid 01A \\ A \rightarrow 0B \\ B \rightarrow 1B \mid 1 \end{array} \right\}$.
- $G_{32}^1 = (\{A, S, S_1\}, \{0, 1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \rightarrow S, \\ S \rightarrow 01SA \mid 01A \\ A \rightarrow 0B \\ B \rightarrow 1B \mid 1 \end{array} \right\}$.
- $G_{32}^2 = (\{A, S, S_1\}, \{0, 1\}, P_2, S_1)$, com $P_2 = \left\{ \begin{array}{l} S_1 \rightarrow 01SA \mid 01A \\ S \rightarrow 01SA \mid 01A \\ A \rightarrow 0B \\ B \rightarrow 1B \mid 1 \end{array} \right\}$.
- $G_{32}^5 = G_{32}^4 = G_{32}^3 = G_{32}^2$.

$$\mathcal{L}_{33} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 1^m(01)^n(10)^n, m \geq 4, m, n \in \mathbb{N}^+\}$$

- $G_{33} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow A1 \mid 1111, \\ B \rightarrow 01B10 \mid 0110 \end{array} \right\}$.
- $G_{33}^3 = G_{33}^2 = G_{33}^1 = G_{33}$.



- $G_{33}^4 = (\{A, B, R, S\}, \{0, 1\}, P_4, S_1)$, com $P_4 = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 1111R \mid 1111, \\ R \rightarrow 1R \mid 1, \\ B \rightarrow 01B10 \mid 0110 \end{array} \right\}$.
- $G_{33}^5 = G_{33}^4$.

$$\mathcal{L}_{34} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 10^n 10^q \text{ ou } w = 0^n 10^{2n}, m, n, q \in \mathbb{N}\}$$

$$\mathcal{L}_{35} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^n 10^{2n} \text{ ou } w = 1^n 01^{3n}, n \in \mathbb{N}\}$$

- $G_{35} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow A \mid B, \\ A \rightarrow 0A00 \mid 1, \\ B \rightarrow 1B111 \mid 0 \end{array} \right\}$.
- $G_{35}^2 = G_{35}^1 = G_{35}$.
- $G_{35}^3 = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A00 \mid 1 \mid 1B111 \mid 0, \\ A \rightarrow 0A00 \mid 1, \\ B \rightarrow 1B111 \mid 0 \end{array} \right\}$.
- $G_{35}^5 = G_{35}^4 = G_{35}^3$.

$$\mathcal{L}_{36} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q 1^r 0^s 1^t, m + q + r + t = n + s, m, n, q, r, s, t \in \mathbb{N}\}$$

$$\mathcal{L}_{37} = \{w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q 1^r 0^s 1^t, m + q + r = n + s + t, m, n, q, r, s, t \in \mathbb{N}\}$$