

Autômato com Pilha (PDA – Pushdown Automaton)

• Sextupla $P = \langle \Sigma, \Gamma, S, s_0, \delta, F \rangle$, onde:

 Σ : alfabeto de entrada, Γ : alfabeto da pilha,

 $S \neq \emptyset$: conjunto finito de estados,

 $s_0 \in S$: estado inicial,

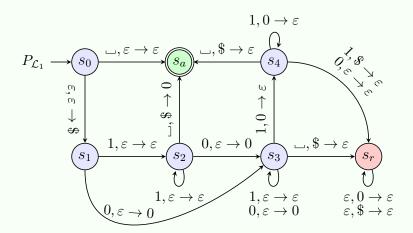
 $\delta: S \times (\Sigma \cup \{\varepsilon\}) \times (\Gamma \cup \{\varepsilon\}) \to \mathcal{P}(S \times (\Gamma \cup \{\varepsilon\}))$

: função de transição de estados, e

 $F \subseteq S$: conjunto de estados finais (ou de aceitação).

Atenção: Embora as respostas dos exercícios sejam elaboradas com esforço e cuidado, e continuamente revisadas, algumas delas ainda estão incompletas ou podem conter erros que passaram despercebidos. Comentários ou correções específicas são bem-vindos, especialmente se forem relacionados a erros críticos!

$\mathcal{L}_1 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = u1^{|u|_0}, \ u \in \Sigma^* \}$

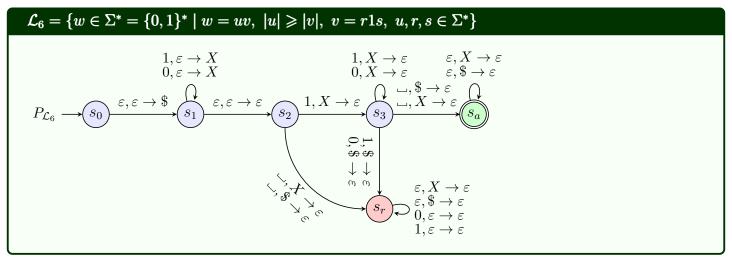


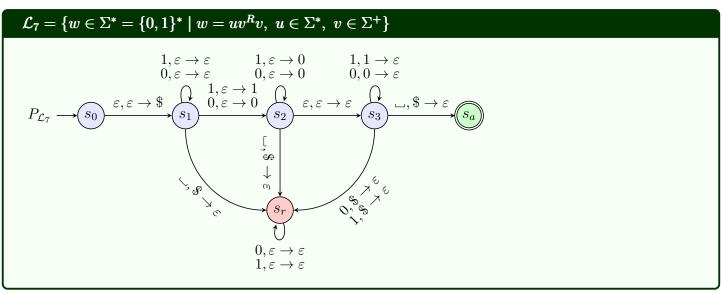
$$\mathcal{L}_2 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m u, \ |u|_0 \leqslant m, \ m \in \mathbb{N}^+, \ u \in \Sigma^* \}$$

$$\mathcal{L}_3 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 1^5 u, \ 2 \cdot |w|_0 = 3 \cdot |w|_1, \ u \in \Sigma^* \}$$

$\mathcal{L}_{4} = \{ w \in \Sigma^{*} = \{0,1\}^{*} \mid w = uv, \mid u \mid_{1} \geqslant |u|_{0} + 4, u,v \in \Sigma^{*} \}$ $\downarrow 0, \varepsilon \to 0 \qquad \downarrow 0, \varepsilon \to \varepsilon \qquad \downarrow 0, \varepsilon \to 0, \varepsilon \to \varepsilon \qquad \downarrow 0, \varepsilon \to \varepsilon \rightarrow \varepsilon \qquad \downarrow 0, \varepsilon \to 0, \varepsilon \to \varepsilon \qquad \downarrow 0, \varepsilon \to \varepsilon \rightarrow \varepsilon \rightarrow 0, \varepsilon \to \varepsilon \rightarrow 0, \varepsilon \to 0, \varepsilon$

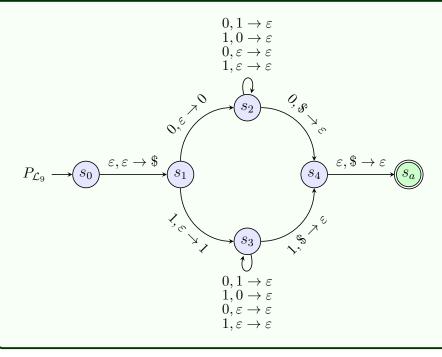
$$\mathcal{L}_5 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = uv, \ |u| = |v|, \ |v|_1 \geqslant 1, \ u, v \in \Sigma^* \}$$





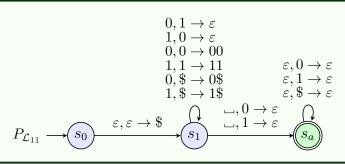
$\mathcal{L}_8 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = u0v, \ |w| = 2 \cdot k + 1, \ |u| = |v|, \ k \in \mathbb{N}, \ u, v \in \Sigma^+ \}$

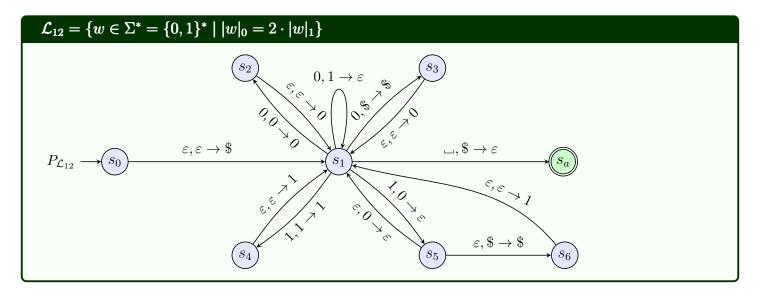
$\mathcal{L}_9 = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = cuc, \ c \in \Sigma, \ u \in \Sigma^+, \ |w|_0 = |w|_1 \}$



$\mathcal{L}_{10} = \{ w \in \Sigma^* = \{0, 1\}^* \mid |w| = 3 \cdot |w|_0 \}$

$\mathcal{L}_{11} = \{ w \in \Sigma^* = \{0, 1\}^* \mid |w|_0 \neq |w|_1 \}$



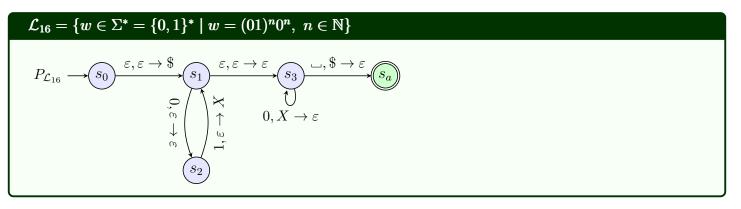


$$\mathcal{L}_{13} = \{ w \in \Sigma^* = \{0, 1\}^* \mid |w|_{101} = |w|_{010} \}$$

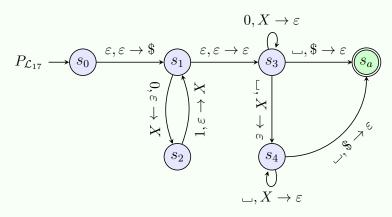
$$\mathcal{L}_{14} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n, \ m \neq n \ \mathbf{e} \ 2 \cdot m \neq n, \ m, n \in \mathbb{N} \}$$

$$\mathcal{L}_{15} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n, \ 3 \cdot m \leqslant n \leqslant 5 \cdot m, \ m, n \in \mathbb{N} \}$$

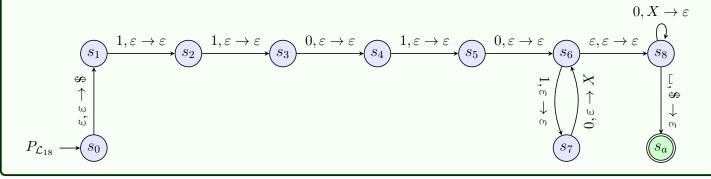
$$P_{\mathcal{L}_{15}} \longrightarrow \underbrace{ s_0 }_{0, \varepsilon \to 0} \underbrace{ s_1 }_{0, \varepsilon \to 0} \underbrace{ s_2 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to 0} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_3 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_4 }_{0, \varepsilon \to 0} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_5 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_5 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_5 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_5 }_{0, \varepsilon \to \varepsilon} \underbrace{ 1, \varepsilon \to \varepsilon }_{0, \varepsilon \to \varepsilon} \underbrace{ s_5 }_{0, \varepsilon \to \varepsilon} \underbrace$$



$\mathcal{L}_{17} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = (01)^m 0^n, \ n \geqslant 2 \cdot m, \ m, n \in \mathbb{N} \}$



$\mathcal{L}_{18} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 110(10)^n 0^{n-1}, \ n \in \mathbb{N} \}$



$$\mathcal{L}_{19} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^m 0^n, \ m, n \in \mathbb{N} \}$$

$$P_{\mathcal{L}_{19}} \xrightarrow{\delta_0} \underbrace{s, \varepsilon \to \$} \underbrace{s_1} \underbrace{s, \varepsilon \to \varepsilon} \underbrace{s_2} \underbrace{s_2} \underbrace{s, \varepsilon \to \varepsilon} \underbrace{s_3} \underbrace{s_a} \underbrace{s_a}$$

$\mathcal{L}_{20} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, \ m + q \leqslant n, \ m, n, q \in \mathbb{N} \}$

$$0, \varepsilon \to 0 \qquad 1, 0 \to \varepsilon \qquad 1, \varepsilon \to \varepsilon \qquad 1, \varepsilon \to 1 \qquad 0, 1 \to \varepsilon$$

$$P_{\mathcal{L}_{20}} \longrightarrow \underbrace{s_0} \qquad \underbrace{s, \varepsilon \to \varepsilon} \qquad \underbrace{s_1} \qquad \underbrace{s, \varepsilon \to \varepsilon} \qquad \underbrace{s_2} \qquad \underbrace{s_3} \qquad \underbrace{s, \varepsilon \to \varepsilon} \qquad \underbrace{s_4} \qquad \underbrace{s, \varepsilon \to \varepsilon} \qquad \underbrace{s_5} \qquad \underbrace{s_5} \qquad \underbrace{s_5} \qquad \underbrace{s_6} \qquad$$

• Se $m + q \le n$, então m + p + s = n, para $s \ge 0$. Logo, $0^m 1^n 0^p = (0^m 1^m)(1^s)(1^p 0^p)$.

$$\mathcal{L}_{21} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, \ n \neq m + q, \ m, n, q \in \mathbb{N} \}$$

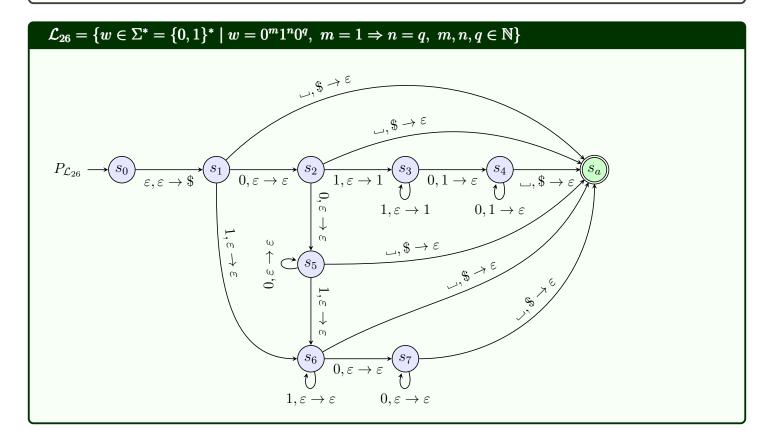
$$\mathcal{L}_{22} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, \ m \neq q, \ m, n, q \in \mathbb{N} \}$$



$\mathcal{L}_{23} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^q, \ q = 2 \cdot (m+n), \ m, n, q \in \mathbb{N} \}$

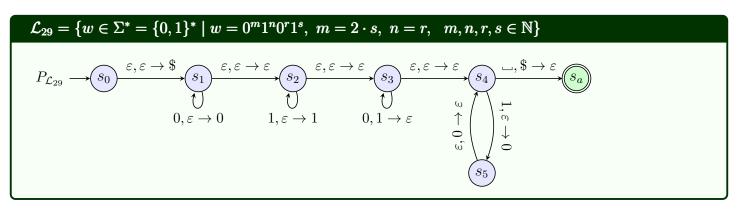
$\mathcal{L}_{24} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n 0^q, \ m > 5, \ n > 3, \ q \leqslant n, \ m, n, q \in \mathbb{N} \}$ $P_{\mathcal{L}_{24}} \longrightarrow \underbrace{s_0} \xrightarrow{\varepsilon, \varepsilon \to \$} \underbrace{s_1} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_2} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_3} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_4} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_5} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_6} \xrightarrow{s_6} \underbrace{s_7} \xrightarrow{s_7} \underbrace{s_7} \underbrace{s_7} \xrightarrow{s_7} \underbrace{s_7} \underbrace{s_7} \xrightarrow{s_7} \underbrace{s_7} \underbrace{$

$\mathcal{L}_{25} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n 0^q, \ m \leqslant 2 \cdot n \text{ ou } n \leqslant 3 \cdot q, \ m, n, q \in \mathbb{N} \}$



$\mathcal{L}_{27} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^{m+n} 0^n, \ m+n > 0, \ m, n \in \mathbb{N} \}$ $0, \varepsilon \to 0 \qquad 1, 0 \to \varepsilon$ $0, \varepsilon \to 0 \qquad 1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$ $0, \varepsilon \to 0 \qquad 1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$ $0, 0 \to 0 \qquad 1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$ $1, 0 \to \varepsilon$

$$\mathcal{L}_{28} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 1^n 0^{m-n}, \ m > n, \ m, n \in \mathbb{N} \}$$



$$\mathcal{L}_{30} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n 0 1^{m+1}, \ m, n \in \mathbb{N} \}$$

$$0, \varepsilon \to 0 \qquad 1, \varepsilon \to \varepsilon \qquad 1, \varepsilon \to 0$$

$$P_{\mathcal{L}_{30}} \xrightarrow{\varepsilon, \varepsilon \to \$} \underbrace{s_1} \xrightarrow{\varepsilon, \varepsilon \to \varepsilon} \underbrace{s_2} \xrightarrow{0, \varepsilon \to \varepsilon} \underbrace{s_3} \xrightarrow{1, \varepsilon \to \varepsilon} \underbrace{s_4} \xrightarrow{\sqcup, \$ \to \varepsilon} \underbrace{s_a}$$

$$\mathcal{L}_{31} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = (01)^n 0^m (01)^n, \ m < 3, \ m, n \in \mathbb{N} \}$$

$$\mathcal{L}_{32} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = (01)^n (01^{m_n})^n, \ m_n, n \in \mathbb{N}^+ \}$$

$$\mathcal{L}_{33} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 1^m (01)^n (10)^n, \ m \geqslant 4, \ m, n \in \mathbb{N}^+ \}$$

$$\mathcal{L}_{34} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^m 10^n 10^q \text{ ou } w = 0^n 10^{2n}, \ m, n, q \in \mathbb{N} \}$$

$$0, \varepsilon \to 0 \qquad 0, \varepsilon \to \varepsilon$$

$$s_2 \qquad 1, \varepsilon \to \varepsilon \qquad s_4$$

$$\downarrow \qquad \qquad \downarrow \qquad$$

$$\mathcal{L}_{35} = \{ w \in \Sigma^* = \{0, 1\}^* \mid w = 0^n 10^{2n} \text{ ou } w = 1^n 01^{3n}, \ n \in \mathbb{N} \}$$

$$\mathcal{L}_{36} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n 0^q 1^r 0^s 1^t, \ m+q+r+t = n+s, \ m,n,q,r,s,t \in \mathbb{N} \}$$

$$\mathcal{L}_{37} = \{ w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n 0^q 1^r 0^s 1^t, \ m+q+r = n+s+t, \ m,n,q,r,s,t \in \mathbb{N} \}$$