Gramática Livre de Contexto – GLC

- Quádrupla $G = (V, \Sigma, P, S)$, onde:

  V : Conjunto finito de variáveis,
  - $V = \{A, B, \ldots, S, \ldots\}$;

  Σ : Conjunto finito de símbolos terminais ($\Sigma \cap V = \varnothing$);
  - $\Sigma = \{0, 1, \ldots\}$, ou
  - $\Sigma = \{a, b, c, \ldots\}$ ou $\ldots$;

  P : Conjunto finito de regras de derivação ($A \to w \in P \Rightarrow A \to w \equiv (A, w) \in V \times (V \cup \Sigma)^*$);

  S : Variável inicial ($S \in V$).

$\mathcal{L}_1 = \{w \in \Sigma^* = \{0,1\}^* \mid w = u1^{|u|_0},\ u \in \Sigma^*\}$

- $G_1 = (\{S\}, \{0,1\}, P, S)$, com $P = \left\{\ S \to 0S1 \mid 1S \mid \varepsilon\ \right\}$.

$\mathcal{L}_2 = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m u,\ |u|_0 \leqslant m,\ m \in \mathbb{N}^+,\ u \in \Sigma^*\}$

- $G_2 = (\{S\}, \{0,1\}, P, S)$, com $P = \left\{\ S \to 0S0 \mid 0S \mid S1 \mid 0\ \right\}$.

$\mathcal{L}_3 = \{w \in \Sigma^* = \{0,1\}^* \mid w = 1^5 u,\ 2 \cdot |w|_0 = 3 \cdot |w|_1,\ u \in \Sigma^*\}$

- $G_3 = (\{A, B, C, D, E, F, S\}, \{0,1\}, P, S)$, com

$$P = \left\{ \begin{array}{ll} S \to AB, & \\ A \to 11111, & \\ B \to C0^9 \mid 0C0^8 \mid 0^2C0^7 \mid 0^3C0^6 \mid 0^4C0^5 \mid 0^5C0^4 \mid 0^6C0^3 \mid 0^7C0^2 \mid 0^8CB0 \mid 0^9C, & \\ C \to D00 \mid 0D0 \mid 00D, & \\ \quad\ \ E01 \mid 0E1 \mid 01E, & \\ \quad\ \ E10 \mid 1E0 \mid 10E, & \\ \quad\ \ F11 \mid 1F1 \mid 11F \mid 1, & \\ D \to C011 \mid 0C11 \mid 01C1 \mid 011C, & \\ \quad\ \ C101 \mid 1C01 \mid 10C1 \mid 101C, & \\ \quad\ \ C110 \mid 1C10 \mid 11C0 \mid 110C, & \\ E \to C001 \mid 0C01 \mid 00C1 \mid 001C, & \\ \quad\ \ C010 \mid 0C10 \mid 01C0 \mid 010C, & \\ \quad\ \ C100 \mid 1C00 \mid 10C0 \mid 100C, & \\ F \to C000 \mid 0C00 \mid 00C0 \mid 000C & \end{array} \right\}.$$

$\mathcal{L}_4 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u|_1 \geqslant |u|_0 + 4,\ u, v \in \Sigma^*\}$

- $G_4 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to A1A1A1A1AB, \\ A \to 0A1 \mid 1A0 \mid AA \mid 1A \mid A1 \mid \varepsilon, \\ B \to 0B \mid 1B \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_5 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u| = |v|,\ |v|_1 \geqslant 1,\ u, v \in \Sigma^*\}$

- $G_5 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A1 \mid 1A1 \mid 0S0 \mid 1S0, \\ A \rightarrow BAB \mid \varepsilon, \\ B \rightarrow 0 \mid 1 \end{array} \right\}$.

$\mathcal{L}_6 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u| \geqslant |v|,\ v = r1s,\ u, r, s \in \Sigma^*\}$

- $G_6 = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S0 \mid 0S1 \mid 1S0 \mid 1S1 \mid 0S \mid 1S \mid 0A1 \mid 1A1, \\ A \rightarrow 0A0 \mid 0A1 \mid 1A0 \mid 1A1 \mid 0A \mid 1A \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_7 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv^Rv,\ u \in \Sigma^*,\ v \in \Sigma^+\}$

$\mathcal{L}_8 = \{w \in \Sigma^* = \{0,1\}^* \mid w = u0v,\ |w| = 2 \cdot k + 1,\ |u| = |v|,\ k \in \mathbb{N},\ u, v \in \Sigma^+\}$

- $G_8 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow ASA \mid ABA, \\ A \rightarrow 0 \mid 1, \\ B \rightarrow 0 \end{array} \right\}$.

$\mathcal{L}_9 = \{w \in \Sigma^* = \{0,1\}^* \mid w = cuc,\ c \in \Sigma,\ u \in \Sigma^+,\ |w|_0 = |w|_1\}$

- $G_9 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0A0 \mid 1B1, \\ A \rightarrow 0A1 \mid 1A0 \mid C11 \mid 1C1 \mid 11C \mid 11, \\ B \rightarrow 0B1 \mid 1B0 \mid C00 \mid 0C0 \mid 00C \mid 00, \\ C \rightarrow 0C1 \mid 1C0 \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_{10} = \{w \in \Sigma^* = \{0,1\}^* \mid |w| = 3 \cdot |w|_0\}$

$\mathcal{L}_{11} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_0 \neq |w|_1\}$

$\mathcal{L}_{12} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_0 = 2 \cdot |w|_1\}$

$\mathcal{L}_{13} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_{101} = |w|_{010}\}$

$\mathcal{L}_{14} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n,\ m \neq n \text{ e } 2 \cdot m \neq n,\ m, n \in \mathbb{N}\}$

- $G_{14} = (\{A, B, C, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow A \mid 00B111 \mid C, \\ A \rightarrow 0A1 \mid 0A \mid 0, \\ B \rightarrow 0B1 \mid 0B11 \mid \varepsilon, \\ C \rightarrow 0C11 \mid C1 \mid 1 \end{array} \right\}$.

- $\mathcal{L}_{14} = L_A \cup L_B \cup L_C = \{0^m1^n \mid m > n\} \cup \{0^m1^n \mid m < n < 2m\} \cup \{0^m1^n \mid n > 2m\}$.

$\mathcal{L}_{15} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n,\ 3\cdot m \leqslant n \leqslant 5\cdot m,\ m,n \in \mathbb{N}\}$

$\mathcal{L}_{16} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n0^n,\ n \in \mathbb{N}\}$

- $G_{16} = (\{S\}, \{0,1\}, P, S)$, com $P = \left\{\ S \to 01S0 \mid \varepsilon\ \right\}$.

$\mathcal{L}_{17} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^m0^n,\ n \geqslant 2\cdot m,\ m,n \in \mathbb{N}\}$

$\mathcal{L}_{18} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 110(10)^n0^{n-1},\ m,n \in \mathbb{N}\}$

$\mathcal{L}_{19} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^m0^n,\ m,n \in \mathbb{N}\}$

- $G_{19} = (\{S,A\}, \{0,1\}, P, S)$, com $P = \left\{\begin{array}{l} S \to S0 \mid A, \\ A \to 0A1 \mid \varepsilon \end{array}\right\}$.

$\mathcal{L}_{20} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m+q \leqslant n,\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{21} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ n \neq m+q,\ m,n,q \in \mathbb{N}\}$

- $G_{21} = (\{A,B,C,S\}, \{0,1\}, P, S)$, com $P = \left\{\begin{array}{l} S \to A \mid B, \\ A \to CED, \\ B \to FCDF \mid FCD \mid CDF, \\ C \to 0C1 \mid \varepsilon \\ D \to 1D0 \mid \varepsilon \\ E \to 1E \mid 1, \\ F \to 0F \mid 0, \end{array}\right\}$.

- $\mathcal{L}_{21} = L_A \cup L_B = \{0^m1^n0^q \mid m+q < n,\ m,n,q \in \mathbb{N}\} \cup \{0^m1^n0^q \mid n < m+q,\ m,n,q \in \mathbb{N}\}$.
- $L_A = \{0^m1^n0^q \mid m+q < n,\ m,n,q \in \mathbb{N}\} \equiv \{0^m1^m1^r1^q0^q \mid r \geqslant 1,\ m,n,q,r \in \mathbb{N}\}$.
- $L_{B^1} = \{0^m1^n0^q \mid m,q > n,\ m,n,q \in \mathbb{N}\} \equiv \{0^{m-n_1}1^{n_1}1^{n_2}0^{q-n_2} \mid m,q > n_1+n_2,\ m,n_1,n_2,q \in \mathbb{N}\}$.
- $L_{B^2} = \{0^m1^n0^q \mid m > q,\ q < n,\ m+q > n,\ m,n,q \in \mathbb{N}\} \equiv \{0^{m'}0^{n'}1^{n'}1^q0^q \mid m' = m-n',\ n' = n-q,\ m',n',q \in \mathbb{N}\}$.
- $L_{B^3} = \{0^m1^n0^q \mid m < n,\ m < q,\ m+q > n,\ m,n,q \in \mathbb{N}\} \equiv \{0^m0^m1^{n'}1^{n'}0^{q'} \mid q' = q-n',\ n' = n-m,\ m,n',q' \in \mathbb{N}\}$.

$\mathcal{L}_{22} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m \neq q,\ m,n,q \in \mathbb{N}\}$

- $G_{22} = (\{S, A, B\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S0 \mid A0 \mid 0B, \\ A \rightarrow 1A0 \mid 1A \mid A0 \mid \varepsilon, \\ B \rightarrow 0B1 \mid B1 \mid 0B \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_{23} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ q = 2 \cdot (m+n),\ m,n,q \in \mathbb{N}\}$

- $G_{23} = (\{S, A, B\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S00 \mid A, \\ A \rightarrow 1A00 \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_{24} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m > 5,\ n > 3,\ q \leqslant n,\ m,n,q \in \mathbb{N}\}$

- $G_{24} = (\{S, A, B, C, D\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB, \\ A \rightarrow 0A \mid 000000, \\ B \rightarrow 1B0 \mid 1B \mid CD, \\ C \rightarrow 1111, \\ D \rightarrow \varepsilon \mid 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}$.

$\mathcal{L}_{25} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m \leqslant 2 \cdot n$ ou $n \leqslant 3 \cdot q,\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{26} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m = 1 \Rightarrow n = q,\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{27} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^{m+n}0^n,\ m + n > 0,\ m,n \in \mathbb{N}\}$

- $G_{27} = (\{S, A, B\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow AB \mid A \mid B, \\ A \rightarrow 0A1 \mid 01, \\ B \rightarrow 1B0 \mid 10, \end{array} \right\}$.

$\mathcal{L}_{28} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^{m-n},\ m > n,\ m,n \in \mathbb{N}\}$

- $G_{28} = (\{S, A\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 0S0 \mid 0A0, \\ A \rightarrow 0A1 \mid \varepsilon \end{array} \right\}$.

- $0^m1^n0^{m-n}$ $(m > n,\ m,n \in \mathbb{N})$ é equivalente a $0^k0^n1^n0^k$, pois $k = m - n \Rightarrow m = k + n$, para $k > 0$.

$\mathcal{L}_{29} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^r1^s,\ m = 2 \cdot s,\ n = r,\ \ m,n,r,s \in \mathbb{N}\}$

- $G_{29} = (\{S, A\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \rightarrow 00S1 \mid A, \\ A \rightarrow 1A0 \mid \varepsilon \end{array} \right\}$.

$\mathcal{L}_{30} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n01^{m+1},\ m,n \in \mathbb{N}\}$

$\mathcal{L}_{31} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n0^m(01)^n,\ m < 3,\ m,n \in \mathbb{N}\}$

$\mathcal{L}_{32} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n(01^{m_n})^n,\ m_n,n \in \mathbb{N}^+\}$

$\mathcal{L}_{33} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 1^m(01)^n(10)^n,\ m \geqslant 4,\ m,n \in \mathbb{N}^+\}$

- $G_{33} = (\{S,A,B\},\{0,1\},P,S)$, com $P = \left\{\begin{array}{l} S \rightarrow AB, \\ A \rightarrow A1 \mid 1111, \\ B \rightarrow 01B10 \mid 0110 \end{array}\right\}$.

$\mathcal{L}_{34} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m10^n10^q$ ou $w = 0^n10^{2n},\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{35} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^n10^{2n}$ ou $w = 1^n01^{3n},\ n \in \mathbb{N}\}$

- $G_{35} = (\{S,A,B\},\{0,1\},P,S)$, com $P = \left\{\begin{array}{l} S \rightarrow A \mid B, \\ A \rightarrow 0A00 \mid 1, \\ B \rightarrow 1B111 \mid 0 \end{array}\right\}$.

$\mathcal{L}_{36} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ m,n,q,r,s,t \in \mathbb{N}\}$

- $G_{36} = (\{S,A,B\},\{0,1\},P,S)$, com $P = \left\{\begin{array}{l} S \rightarrow ABBA \mid ABCA \mid DA \mid AE \mid AFBA, \\ A \rightarrow 0A1 \mid \varepsilon, \\ B \rightarrow 1B0 \mid \varepsilon, \\ C \rightarrow 0C0 \mid 0B0, \\ D \rightarrow 0D0 \mid 0ABA0, \\ E \rightarrow 1E1 \mid 1BBA1, \\ F \rightarrow 1F1 \mid 1B1 \end{array}\right\}$.

- $\mathcal{L}_{36} = L^1 \cup L^2 \cup L^3 \cup L^4 \cup L^5$.
- $L^1 = \{0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ n = m+q,\ s = r+t,\ m,n,q,r,s,t \in \mathbb{N}\} \equiv \{0^m1^m1^q0^q1^r0^s0^t1^t \mid m,n,q,r \in \mathbb{N}\}$.
- $L^2 = \{0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ n < m+q,\ m < q,\ s > r+t,\ m,n,q,r,s,t \in \mathbb{N}\} \equiv \{0^m1^m1^{q-x}0^{q-x}0^x1^r0^r0^x0^t1^t \mid m,n,q,r,x \in \mathbb{N},\ x > 0\}$.
  - $m+q+r+t = n+s$, $n < m+q$, $m < q$ e $s > r+t \Rightarrow n+x = m+q$ e $s = x+r+t$, ou seja, $n = m+(q-x)$ e $s = r+x+t$.
- $L^3 = \{0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ n < m+q,\ m > q,\ s > r+t,\ m,n,q,r,s,t \in \mathbb{N}\} \equiv \{0^x0^{m-x}1^{m-x}1^q0^q1^r0^r0^x0^t1^t \mid m,n,q,r,x \in \mathbb{N},\ x > 0\}$.
  - $m+q+r+t = n+s$, $n < m+q$, $m > q$ e $s > r+t \Rightarrow n+x = m+q$ e $s = x+r+t$, ou seja, $n = (m-x)+q$ e $s = r+x+t$.

- $L^4 = \{0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ n > m+q,\ s < r+t,\ r < t,\ m,n,q,r,s,t \in \mathbb{N}\} \equiv \{0^m1^m1^x1^q0^q1^r0^r0^x0^{t-x}1^{t-x}1^x \mid m,n,q,r,x \in \mathbb{N},\ x > 0\}$.
  - $m+q+r+t = n+s$, $n > m+q$, $s < r+t$ e $r < t \Rightarrow n = x+m+q$ e $s+x = r+t$, ou seja, $n = m+x+q$ e $s = r+(t-x)$.
- $L^5 = \{0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ n > m+q,\ s < r+t,\ r < t,\ m,n,q,r,s,t \in \mathbb{N}\} \equiv \{0^m1^m1^x1^q0^q1^x1^{r-x}0^{r-x}0^t1^t \mid m,n,q,r,x \in \mathbb{N},\ x > 0\}$.
  - $m+q+r+t = n+s$, $n > m+q$, $s < r+t$ e $r > t \Rightarrow n = x+m+q$ e $s+x = r+t$, ou seja, $n = m+x+q$ e $s = (r-x)+t$.

$\boldsymbol{\mathcal{L}_{37} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q1^r0^s1^t,\ m+q+r = n+s+t,\ m,n,q,r,s,t \in \mathbb{N}\}}$