GLC – formas normais de Chomsky e de Greibach

- Uma gramática é dita "*normalizada*" quando todas as suas regras de derivação seguem as restrições impostas por um certo padrão (formas normais).
- Uma gramática livre de contexto $G = (V, \Sigma, P, S)$ está na **forma normal de Chomsky** se suas regras de derivação tem uma das seguintes formas:
  1. $S \to \varepsilon$,
  2. $A \to a$, ou
  3. $A \to BC$;

  onde $A \in V$, $B, C \in V - \{S\}$ e $a \in \Sigma$.
- Uma gramática livre de contexto $G = (V, \Sigma, P, S)$ está na **forma normal de Greibach** se suas regras de derivação têm uma das seguintes formas:
  1. $S \to \varepsilon$,
  2. $A \to a$, ou
  3. $A \to aB_1B_2 \dots B_n$;

  onde $A \in V$, $B_i \in V - \{S\}$, $i = 1, 2, \dots, n$, e $a \in \Sigma$.

Atenção: Embora as respostas dos exercícios sejam elaboradas com esforço e cuidado, e continuamente revisadas, algumas delas ainda estão incompletas ou podem conter erros que passaram despercebidos. Comentários ou correções específicas são bem-vindos, especialmente se forem relacionados a erros críticos!

$$\mathcal{L}_1 = \{w \in \Sigma^* = \{0,1\}^* \mid w = u1^{|u|_0},\ u \in \Sigma^*\}$$

$$\mathcal{L}_2 = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m u,\ |u|_0 \leqslant m,\ m \in \mathbb{N}^+,\ u \in \Sigma^*\}$$

$$\mathcal{L}_3 = \{w \in \Sigma^* = \{0,1\}^* \mid w = 1^5 u,\ 2 \cdot |w|_0 = 3 \cdot |w|_1,\ u \in \Sigma^*\}$$

$$\mathcal{L}_4 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u|_1 \geqslant |u|_0 + 4,\ u, v \in \Sigma^*\}$$

$$\mathcal{L}_5 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u| = |v|,\ |v|_1 \geqslant 1,\ u, v \in \Sigma^*\}$$

$$\mathcal{L}_6 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv,\ |u| \geqslant |v|,\ v = r1s,\ u, r, s \in \Sigma^*\}$$

- $G_6 = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AS \mid ASA \mid 01 \mid 11, \\ A \to 0 \mid 1 \end{array} \right\}$.
- $G_6^5 = (\{A, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to AS \mid ASA \mid 01 \mid 11, \\ S \to AS \mid ASA \mid 01 \mid 11, \\ A \to 0 \mid 1 \end{array} \right\}$.

- Gramática G_6^6 na forma normal de Chomsky:

$$\begin{aligned} G_6^6 &= (V_6, \Sigma, P_6, S_1) \\ &= (\{A, B, S, S_1, T_0, T_1, T_2\}, \{0,1\}, P_6, S_1), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to AS \mid T_2A \mid T_0T_1 \mid T_1T_1, \\ S \to AS \mid T_2A \mid T_0T_1 \mid T_1T_1, \\ A \to 0 \mid 1, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to AS \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_7 = \{w \in \Sigma^* = \{0,1\}^* \mid w = uv^Rv,\ u \in \Sigma^*,\ v \in \Sigma^+\}$

- $G_7 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AB, \\ A \to 0A \mid 1A \mid \varepsilon, \\ B \to 0B0 \mid 1B1 \mid 00 \mid 11 \end{array} \right\}$.
- $G_7^5 = (\{A, B, S\}, \{0,1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to AB \mid 0B0 \mid 1B1 \mid 00 \mid 11, \\ A \to 0A \mid 1A \mid 0 \mid 1, \\ B \to 0B0 \mid 1B1 \mid 00 \mid 11 \end{array} \right\}$.
- Gramática G_7^6 na forma normal de Chomsky:

$$\begin{aligned} G_7^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, S, T_0, T_1, T_2, T_2\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S \to AB \mid T_0T_2 \mid T_1T_3 \mid T_0T_0 \mid T_1T_1, \\ A \to T_0A \mid T_1A \mid 0 \mid 1, \\ B \to T_0T_2 \mid T_1T_3 \mid T_0T_0 \mid T_1T_1, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to BT_0, \\ T_3 \to BT_1 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_8 = \{w \in \Sigma^* = \{0,1\}^* \mid w = u0v,\ |w| = 2 \cdot k + 1,\ |u| = |v|,\ k \in \mathbb{N},\ u, v \in \Sigma^+\}$

- $G_8 = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to ASA \mid ABA, \\ A \to 0 \mid 1, \\ B \to 0 \end{array} \right\}$.

- $G_8^5 = (\{A, B, S, S_1\}, \{0, 1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to ASA \mid ABA, \\ S \to ASA \mid ABA, \\ A \to 0 \mid 1, \\ B \to 0 \end{array} \right\}$.
- Gramática G_8^6 na forma normal de Chomsky:

$$\begin{aligned} G_8^6 &= (V_6, \Sigma, P_6, S_1) \\ &= (\{A, B, S, S_1, T_0, T_1\}, \{0, 1\}, P_6, S_1), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to AT_0 \mid AT_1, \\ S \to AT_0 \mid AT_1, \\ A \to 0 \mid 1, \\ B \to 0, \\ T_0 \to SA, \\ T_1 \to BA \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_9 = \{w \in \Sigma^* = \{0,1\}^* \mid w = cuc,\ c \in \Sigma,\ u \in \Sigma^+,\ |w|_0 = |w|_1\}$

- $G_9 = (\{A, B, C, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 0A0 \mid 1B1, \\ A \to 0A1 \mid 1A0 \mid C11 \mid 1C1 \mid 11C, \\ B \to 0B1 \mid 1B0 \mid C00 \mid 0C0 \mid 00C, \\ C \to 0C1 \mid 1C0 \mid \varepsilon \end{array} \right\}$.
- $G_9^5 = (\{A, B, C, S\}, \{0, 1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to 0A0 \mid 1B1, \\ A \to 0A1 \mid 1A0 \mid C11 \mid 1C1 \mid 11C \mid 11, \\ B \to 0B1 \mid 1B0 \mid C00 \mid 0C0 \mid 00C \mid 00, \\ C \to 0C1 \mid 1C0 \mid 01 \mid 10 \end{array} \right\}$.

- Gramática G_9^6 na forma normal de Chomsky:

$$\begin{aligned} G_9^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, C, S, S_1, T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9\}, \{0,1\}, P_6, S), \text{ com} \end{aligned}$$

$$P_6 = \left\{ \begin{array}{l} S \to T_0T_2 \mid T_1T_5, \\ A \to T_0T_3 \mid T_1T_2 \mid T_7T_1 \mid T_1T_71 \mid T_1T_9 \mid T_1T_1, \\ B \to T_0T_5 \mid T_1T_4 \mid T_6T_0 \mid T_0T_6 \mid T_0T_8 \mid T_0T_0, \\ C \to T_0T_7 \mid T_1T_6 \mid T_0T_1 \mid T_1T_0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to AT_0, \\ T_3 \to AT_1, \\ T_4 \to BT_0, \\ T_5 \to BT_1, \\ T_6 \to CT_0, \\ T_7 \to CT_1, \\ T_8 \to T_0C, \\ T_9 \to T_1C \end{array} \right\}.$$

$\mathcal{L}_{10} = \{w \in \Sigma^* = \{0,1\}^* \mid |w| = 3 \cdot |w|_0\}$

- $G_{10} = (\{S\}, \{0,1\}, P, S)$, com $P = \{\ S \to S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \varepsilon\ \}$.
- $G_{10}^1 = (\{S, S_1\}, \{0,1\}, P_1, S_1)$, com $P_1 = \left\{ \begin{array}{l} S_1 \to S, \\ S \to S0S1S1S \mid S1S0S1S \mid S1S1S0S \mid \varepsilon \end{array} \right\}$.
- $G_{10}^5 = \ldots$ ☺
- Gramática G_{10}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{10}^6 &= (V_6, \Sigma, P_6, S) \\ &= \ldots \text{☺} \end{aligned}$$

$\mathcal{L}_{11} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_0 \neq |w|_1\}$

$\mathcal{L}_{12} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_0 = 2 \cdot |w|_1\}$

- $G_{12} = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to SS \mid A1A \mid 00S1 \mid 1S00 \mid \varepsilon, \\ A \to 0S \mid SA \end{array} \right\}$.

- $G_{12}^5 = (\{A, R, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{ll} S_1 \to & SS \mid A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100 \mid \varepsilon, \\ S \to & A1AR \mid 00S1R \mid 1S00R \mid 001R \mid 100R \mid \\ & A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100, \\ R \to & SR \mid \\ & A1AR \mid 00S1R \mid 1S00R \mid 001R \mid 100R \mid \\ & A1A \mid 00S1 \mid 1S00 \mid 001 \mid 100, \\ A \to & 0S \mid SA \mid 0 \end{array} \right\}$.

- Gramática G_{12}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{12}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, R, S, S_1, T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}\}, \{0,1\}, P_6, S), \text{ com} \end{aligned}$$

$$P_6 = \left\{ \begin{array}{ll} S_1 \to & SS \mid AT_5 \mid T_2T_3 \mid T_1T_4 \mid T_2T_1 \mid T_1T_2 \mid \varepsilon, \\ S \to & AT_{10} \mid T_2T_8 \mid T_1T_9 \mid T_2T_6 \mid T_1T_7 \mid \\ & AT_5 \mid T_2T_3 \mid T_1T_4 \mid T_2T_1 \mid T_1T_2, \\ R \to & SR \mid \\ & AT_{10} \mid T_2T_8 \mid T_1T_9 \mid T_2T_6 \mid T_1T_7 \mid \\ & AT_5 \mid T_2T_3 \mid T_1T_4 \mid T_2T_1 \mid T_1T_2, \\ A \to & T_0S \mid SA \mid 0, \\ T_0 \to & 0, \\ T_1 \to & 1, \\ T_2 \to & T_0T_0, \\ T_3 \to & ST_1, \\ T_4 \to & ST_2, \\ T_5 \to & T_1A, \\ T_6 \to & T_1R, \\ T_7 \to & T_2R, \\ T_8 \to & T_3R, \\ T_9 \to & T_4R, \\ T_{10} \to & T_5R \end{array} \right\}.$$

$\mathcal{L}_{13} = \{w \in \Sigma^* = \{0,1\}^* \mid |w|_{101} = |w|_{010}\}$

- $G_{13} = (\{S\}, \{0,1\}, P, S)$, com $P = \{\ S \to SS \mid 0S1 \mid 10\ \}$.

- $G_{13}^5 = (\{R, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{ll} S_1 \to & SS \mid 0S1 \mid 10, \\ S \to & 0S1R \mid 10R \mid 0S1 \mid 10, \\ R \to & SR \mid 0S1R \mid 10R \mid 0S1 \mid 10 \end{array} \right\}$.

- Gramática G_{13}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{13}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{R, S, S_1, T_0, T_1, T_2, T_3, T_4\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to SS \mid T_0 S T_1 \mid T_1 T_0, \\ S \to T_0 T_4 \mid T_2 R \mid T_0 T_3 \mid T_1 T_0, \\ R \to SR \mid T_0 T_4 \mid T_2 R \mid T_0 T_3 \mid T_1 T_0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_1 T_0, \\ T_3 \to S T_1, \\ T_4 \to T_3 R \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{14} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n,\ m \neq n \text{ e } 2 \cdot m \neq n,\ m, n \in \mathbb{N}\}$

$\mathcal{L}_{15} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^n,\ 3 \cdot m \leqslant n \leqslant 5 \cdot m,\ m, n \in \mathbb{N}\}$

$\mathcal{L}_{16} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n 0^n,\ n \in \mathbb{N}\}$

$\mathcal{L}_{17} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^m 0^n,\ n \geqslant 2 \cdot m,\ m, n \in \mathbb{N}\}$

- $G_{17} = (\{S\}, \{0,1\}, P, S)$, com $P = \{\ S \to 01S00 \mid S0 \mid \varepsilon\ \}$.
- $G_{17}^5 = (\{R, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 01S00 \mid S0 \mid 0100 \mid 0 \mid \varepsilon, \\ S \to 01S00R \mid 0100R \mid 0R \mid 01S00 \mid 0100 \mid 0, \\ R \to 0R \mid 0 \end{array} \right\}$.
- Gramática G_{17}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{17}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{R, S, S_1, T_0, T_1, T_2, T_3, T_4, T_5, T_6\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to T_3 T_4 \mid S T_0 \mid T_3 T_2 \mid 0 \mid \varepsilon, \\ S \to T_3 T_6 \mid T_3 T_5 \mid T_0 R \mid T_3 T_4 \mid T_3 T_2 \mid 0, \\ R \to T_0 R \mid 0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0 T_0, \\ T_3 \to T_0 T_1, \\ T_4 \to S T_2, \\ T_5 \to T_2 R, \\ T_6 \to T_4 R \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{18} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 110(10)^n 0^{n-1},\ n \in \mathbb{N}\}$

- $G_{18} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 110A, \\ A \to 10B, \\ B \to 10B0 \mid \varepsilon \end{array} \right\}$.
- $G_{18}^5 = (\{A, B, S\}, \{0, 1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to 110A, \\ A \to 10B \mid 10, \\ B \to 10B0 \mid 100 \end{array} \right\}$.
- Gramática G_{18}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{18}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, S, T_0, T_1, T_2, T_3, T_4, T_5\}, \{0, 1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S \to T_1T_4, \\ A \to T_3B \mid T_1T_0, \\ B \to T_3T_5 \mid T_3T_0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_0, \\ T_3 \to T_1T_0, \\ T_4 \to T_3A, \\ T_5 \to BT_0 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{19} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^m 0^n,\ m, n \in \mathbb{N}\}$

- $G_{19} = (\{A, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to S0 \mid A, \\ A \to 0A1 \mid \varepsilon \end{array} \right\}$.
- $G_{19}^5 = (\{A, R, S, S_1\}, \{0, 1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to S0 \mid 0 \mid \varepsilon, \\ S \to 0A1R \mid 01R \mid 0R \mid 0A1 \mid 01 \mid 0, \\ R \to 0R \mid 0, \\ A \to 0A1 \mid 01 \end{array} \right\}$.

- Gramática G_{19}^{6} na forma normal de Chomsky:

$$
\begin{aligned}
G_{19}^{6} &= (V_6, \Sigma, P_6, S) \\
&= (\{A, R, S, S_1, T_0, T_1, T_2, T_3, T_4\}, \{0,1\}, P_6, S), \text{ com} \\
P_6 &= \left\{ \begin{array}{l}
S_1 \to ST_0 \mid 0 \mid \varepsilon, \\
S \to T_0T_4 \mid T_0T_3 \mid T_0R \mid T_0T_2 \mid T_0T_1 \mid 0, \\
R \to T_0R \mid 0, \\
A \to T_0T_2 \mid T_0T_1, \\
T_0 \to 0, \\
T_1 \to 1, \\
T_2 \to AT_1, \\
T_3 \to T_1R, \\
T_4 \to T_2R
\end{array} \right\}.
\end{aligned}
$$

$\mathcal{L}_{20} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m+q \leqslant n,\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{21} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ n \neq m+q,\ m,n,q \in \mathbb{N}\}$

- $G_{21} = (\{A, B, C, D, E, F, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to A \mid B, \\ A \to CED, \\ B \to FCDF \mid FCD \mid CDF, \\ C \to 0C1 \mid \varepsilon, \\ D \to 1D0 \mid \varepsilon, \\ E \to 1E \mid 1, \\ F \to 0F \mid 0 \end{array} \right\}$.

- $G_{21}^{5} = (\{C, D, E, F, S\}, \{0,1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to CED \mid ED \mid CE \mid 1E \mid 1 \mid \\ \qquad FCDF \mid FCD \mid CDF \mid \\ \qquad FDF \mid FD \mid DF \mid \\ \qquad CD \mid 1D0 \mid 10, \\ C \to 0C1 \mid 01, \\ D \to 1D0 \mid 10, \\ E \to 1E \mid 1, \\ F \to 0F \mid 0 \end{array} \right\}$.

- Gramática G_{21}^6 na forma normal de Chomsky:

$$
\begin{aligned}
G_{21}^6 &= (V_6, \Sigma, P_6, S) \\
&= (\{C, D, E, F, S, T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7\}, \{0,1\}, P_6, S), \text{ com} \\
P_6 &= \left\{ \begin{array}{l}
S \to CT_2 \mid ED \mid CE \mid T_1E \mid 1 \mid \\
\qquad FT_3 \mid FT_5 \mid CT_4 \mid \\
\qquad FT_4 \mid FD \mid DF \mid \\
\qquad CD \mid T_1T_7 \mid T_1T_0, \\
C \to T_0T_6 \mid T_0T_1, \\
D \to T_1T_7 \mid T_1T_0, \\
E \to T_1E \mid 1, \\
F \to T_0F \mid 0 \\
T_0 \to 0, \\
T_1 \to 1, \\
T_2 \to ED, \\
T_3 \to CT_4, \\
T_4 \to DF, \\
T_5 \to CD, \\
T_6 \to CT_1, \\
T_7 \to DT_0
\end{array} \right\}.
\end{aligned}
$$

$\mathcal{L}_{22} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m \neq q,\ m,n,q \in \mathbb{N}\}$

- $G_{22} = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 0S0 \mid A0 \mid 0B, \\ A \to 1A0 \mid 1A \mid A0 \mid \varepsilon, \\ B \to 0B1 \mid B1 \mid 0B \mid \varepsilon \end{array} \right\}$.

- $G_{22}^5 = (\{A, B, R_1, R_2, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 0S0 \mid A0 \mid 0B \mid 0, \\ S \to 0S0 \mid A0 \mid 0B \mid 0, \\ A \to 1A0R_1 \mid 1AR_1 \mid 10R_1 \mid 1R_1 \mid 0R_1 \mid \\ \qquad 1A0 \mid 1A \mid 10 \mid 1 \mid 0, \\ R_1 \to 0R_1 \mid 0, \\ B \to 0B1R_2 \mid 0BR_2 \mid 01R_2 \mid 1R_2 \mid 0R_2 \mid \\ \qquad 0B1 \mid 0B \mid 01 \mid 1 \mid 0, \\ R_2 \to 1R_2 \mid 1 \end{array} \right\}$.

- Gramática G_{22}^6 na forma normal de Chomsky:

$$
\begin{aligned}
G_{22}^6 &= (V_6, \Sigma, P_6, S) \\
&= (\{A, B, R_1, R_2, S, S_1, T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8\}, \{0,1\}, P_6, S), \text{ com} \\
P_6 = \Bigg\{ & \begin{array}{l}
S_1 \to T_0T_2 \mid AT_0 \mid T_0B \mid 0, \\
S \to T_0T_2 \mid AT_0 \mid T_0B \mid 0, \\
A \to T_1T_6 \mid T_1T_4 \mid T_1T_5 \mid T_1R_1 \mid T_0R_1 \mid \\
\qquad T_1T_3 \mid T_1A \mid T_1T_0 \mid 1 \mid 0, \\
R_1 \to 0R_1 \mid 0, \\
B \to T_7T_8 \mid T_7R_2 \mid T_0T_8 \mid T_1R_2 \mid 0R_2 \mid \\
\qquad T_7T_1 \mid T_0B \mid T_0T_1 \mid 1 \mid 0, \\
R_2 \to T_1R_2 \mid 1, \\
T_0 \to 0, \\
T_1 \to 1, \\
T_2 \to ST_0, \\
T_3 \to AT_0, \\
T_4 \to AR_1, \\
T_5 \to T_0R_1, \\
T_6 \to T_3R_1, \\
T_7 \to T_0B, \\
T_8 \to T_1R_2
\end{array} \Bigg\}.
\end{aligned}
$$

$\mathcal{L}_{23} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ q = 2 \cdot (m+n),\ m,n,q \in \mathbb{N}\}$

- $G_{23} = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 0S00 \mid A, \\ A \to 1A00 \mid \varepsilon \end{array} \right\}$.

- $G_{23}^5 = (\{A, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 0S00 \mid 1A00 \mid 100 \mid 000 \mid \varepsilon, \\ S \to 0S00 \mid 1A00 \mid 100 \mid 000, \\ A \to 1A00 \mid 100 \end{array} \right\}$.

- Gramática G_{23}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{23}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, S, S_1, T_0, T_1, T_2, T_3, T_4\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to T_3T_2 \mid T_4T_2 \mid T_1T_2 \mid T_0T_2 \mid \varepsilon, \\ S \to T_3T_2 \mid T_4T_2 \mid T_1T_2 \mid T_0T_2, \\ A \to T_4T_2 \mid T_1T_2, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_0, \\ T_3 \to T_0S, \\ T_4 \to T_1A \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{24} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m > 5,\ n > 3,\ q \leqslant n,\ m, n, q \in \mathbb{N}\}$

- $G_{24} = (\{A, B, C, D, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AB, \\ A \to 0A \mid 000000, \\ B \to 1B0 \mid 1B \mid CD, \\ C \to 1111, \\ D \to \varepsilon \mid 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}$.

- $G_{24}^5 = (\{A, B, C, D, S\}, \{0,1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to AB, \\ A \to 0A \mid 000000, \\ B \to 1B0 \mid 1B \mid CD \mid 1111, \\ C \to 1111, \\ D \to 0 \mid 00 \mid 000 \mid 0000 \end{array} \right\}$.

- Gramática G_{24}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{24}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, C, D, S, T_0, T_1, T_2, T_3, T_4\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S \to AB, \\ A \to T_0A \mid T_2T_3, \\ B \to T_1T_5 \mid T_1B \mid CD \mid T_4T_4, \\ C \to T_4T_4, \\ D \to 0 \mid T_0T_0 \mid T_0T_2 \mid T_2T_2, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_0, \\ T_3 \to T_2T_2, \\ T_4 \to T_1T_1, \\ T_5 \to B0 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{25} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m \leqslant 2 \cdot n \text{ ou } n \leqslant 3 \cdot q,\ m,n,q \in \mathbb{N}\}$

- $G_{25} = (\{A, B, C, D, E, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AC \mid CD, \\ A \to BBA1 \mid \varepsilon, \\ B \to 0 \mid \varepsilon, \\ C \to 0C \mid 0, \\ D \to EEED0 \mid \varepsilon, \\ E \to 1 \mid \varepsilon \end{array} \right\}$.
- $G_{25}^5 = (\{A, B, C, D, E, R_1, R_2, S\}, \{0, 1\}, P_5, S)$, com

$$P_5 = \left\{ \begin{array}{l} S \to AC \mid CD \mid 0C \mid 0, \\ A \to BBA1R_1 \mid BA1R_1 \mid BB1R_1 \mid B1R_1 \mid 1R_1, \\ \qquad BBA1 \mid BA1 \mid BB1 \mid B1 \mid 1, \\ R_1 \to 1R_1 \mid 1, \\ B \to 0, \\ C \to 0C \mid 0, \\ D \to EEED0R_2 \mid EED0R_2 \mid ED0R_2 \mid \\ \qquad EEE0R_2 \mid EE0R_2 \mid E0R_2 \mid 0R_2, \\ \qquad EEED0 \mid EED0 \mid ED0 \mid \\ \qquad EEE0 \mid EE0 \mid E0 \mid 0, \\ R_2 \to 0R_2 \mid 0, \\ E \to 1 \end{array} \right\}.$$

- Gramática G_{25}^{6} na forma normal de Chomsky:

$$
\begin{aligned}
G_{25}^{6} &= (V_6, \Sigma, P_6, S) \\
&= (\{A, B, C, D, E, R_1, R_2, S, T_0, T_1, T_2, T_3, T_4, T_5, T_6, T_7, T_8, T_9, T_{10}, T_{11}\}, \{0,1\}, P_6, S), \text{ com}
\end{aligned}
$$

$$
P_6 = \left\{
\begin{array}{l}
S \to AC \mid CD \mid T_0C \mid 0, \\
A \to T_2T_5 \mid BT_5 \mid T_2T_3 \mid BT_3 \mid T_1R_1, \\
\qquad T_2T_4 \mid BT_4 \mid T_2T_1 \mid BT_1 \mid 1, \\
R_1 \to T_1R_1 \mid 1, \\
B \to 0, \\
C \to T_0C \mid 0, \\
D \to T_{11}T_7 \mid T_9T_7 \mid T_8T_7 \mid \\
\qquad T_{10}T_7 \mid T_6T_7 \mid ET_7 \mid T_0R_2, \\
\qquad T_{11}T_0 \mid T_9T_0 \mid T_8T_0 \mid \\
\qquad T_{10}T_0 \mid T_6T_0 \mid ET_0 \mid 0, \\
R_2 \to T_0R_2 \mid 0, \\
E \to 1, \\
T_0 \to 0, \\
T_1 \to 1, \\
T_2 \to BB, \\
T_3 \to T_1R_1, \\
T_4 \to AT_1, \\
T_5 \to AT_3, \\
T_6 \to EE, \\
T_7 \to T_0R_2, \\
T_8 \to ED, \\
T_9 \to T_6D, \\
T_{10} \to T_6E, \\
T_{11} \to T_6T_8
\end{array}
\right\}.
$$

$\mathcal{L}_{26} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q,\ m = 1 \Rightarrow n = q,\ m, n, q \in \mathbb{N}\}$

- $G_{26} = (\{A, B, C, S\}, \{0,1\}, P, S)$, com $P = \left\{
\begin{array}{l}
S \to 0A \mid B \mid 00C, \\
A \to 1A0 \mid \varepsilon, \\
B \to 1B \mid B0 \mid \varepsilon, \\
C \to 0C \mid B
\end{array}
\right\}$.

- $G_{26}^5 = (\{A, B, C, R, S\}, \{0, 1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to 0A \mid 1B \mid B0 \mid 1 \mid 00C \mid 00 \mid 0 \mid \varepsilon, \\ A \to 1A0 \mid 10, \\ B \to 1BR \mid 1R \mid 0R \mid 1B \mid 1 \mid 0, \\ R \to 0R \mid 0, \\ C \to 0C \mid 1B \mid B0 \mid 1 \mid 0 \end{array} \right\}$.
- Gramática G_{26}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{26}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, C, R, S, T_0, T_1, T_2, T_3, T_4\}, \{0, 1\}, P_6, S), \text{ com} \end{aligned}$$

$$P_6 = \left\{ \begin{array}{l} S \to T_0A \mid T_1B \mid BT_0 \mid 1 \mid T_2C \mid T_0T_0 \mid 0 \mid \varepsilon, \\ A \to T_1T_3 \mid T_1T_0, \\ B \to T_1T_4 \mid T_1R \mid T_0R \mid T_1B \mid 1 \mid 0, \\ R \to T_0R \mid 0, \\ C \to T_0C \mid T_1B \mid BT_0 \mid 1 \mid 0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_0, \\ T_3 \to AT_0, \\ T_4 \to BR \end{array} \right\}.$$

$\mathcal{L}_{27} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m 1^{m+n} 0^n,\ m+n > 0,\ m, n \in \mathbb{N}\}$

- $G_{27} = (\{A, B, S\}, \{0, 1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AB \mid A \mid B, \\ A \to 0A1 \mid 01, \\ B \to 1B0 \mid 10 \end{array} \right\}$.
- $G_{27}^5 = (\{A, B, S\}, \{0, 1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to AB \mid 0A1 \mid 01 \mid 1B0 \mid 10, \\ A \to 0A1 \mid 01, \\ B \to 1B0 \mid 10 \end{array} \right\}$.

- Gramática G_{27}^{6} na forma normal de Chomsky:

$$\begin{aligned} G_{27}^{6} &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, S, T_0, T_1, T_2, T_3\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S \to AB \mid T_0T_2 \mid T_0T_1 \mid T_1T_3 \mid T_1T_0, \\ A \to T_0T_2 \mid T_0T_1, \\ B \to T_1T_3 \mid T_1T_0, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to AT_1, \\ T_3 \to BT_0, \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{28} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^{m-n},\ m > n,\ m, n \in \mathbb{N}\}$

$\mathcal{L}_{29} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^r1^s,\ m = 2 \cdot s,\ n = r,\quad m, n, r, s \in \mathbb{N}\}$

$\mathcal{L}_{30} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n01^{m+1},\ m, n \in \mathbb{N}\}$

- $G_{30} = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 0S1 \mid A, \\ A \to 1A \mid 01 \end{array} \right\}$.
- $G_{30}^{5} = (\{A, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 0S1 \mid 1A \mid 01, \\ S \to 0S1 \mid 1A \mid 01, \\ A \to 1A \mid 01 \end{array} \right\}$.
- Gramática G_{30}^{6} na forma normal de Chomsky:

$$\begin{aligned} G_{30}^{6} &= (V_6, \Sigma, P_6, S) \\ &= (\{A, S, S_1, T_0, T_1, T_2, T_3\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to T_0T_2 \mid T_1A \mid T_0T_1, \\ S \to T_0T_2 \mid T_1A \mid T_0T_1, \\ A \to T_1A \mid T_0T_1, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to ST_1 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{31} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n0^m(01)^n,\ m < 3,\ m, n \in \mathbb{N}\}$

- $G_{31} = (\{A, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 01S01 \mid A, \\ A \to 0 \mid 00 \mid \varepsilon \end{array} \right\}$.

- $G_{31}^5 = (\{S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 01S01 \mid 0 \mid 00 \mid 0101 \mid \varepsilon, \\ S \to 01S01 \mid 0 \mid 00 \mid 0101 \end{array} \right\}$.
- Gramática G_{31}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{31}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{S, S_1, T_0, T_1, T_2, T_3\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to T_2T_3 \mid 0 \mid T_0T_0 \mid T_2T_2 \mid \varepsilon, \\ S \to T_2T_3 \mid 0 \mid T_0T_0 \mid T_2T_2, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_1, \\ T_3 \to ST_2 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{32} = \{w \in \Sigma^* = \{0,1\}^* \mid w = (01)^n(01^{m_n})^n,\ m_n, n \in \mathbb{N}^+\}$

- $G_{32} = (\{A, B, S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to 01SA \mid 01A \\ A \to 0B \\ B \to 1B \mid 1 \end{array} \right\}$.
- $G_{32}^5 = (\{A, S, S_1\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S_1 \to 01SA \mid 01A \\ S \to 01SA \mid 01A \\ A \to 0B \\ B \to 1B \mid 1 \end{array} \right\}$.
- Gramática G_{32}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{32}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A, B, S, S_1, T_0, T_1, T_2, T_3, T_4\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S_1 \to T_2T_3 \mid T_0T_4 \\ S \to T_2T_3 \mid T_0T_4 \\ A \to T_0B \\ B \to T_1B \mid 1, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_0T_1, \\ T_3 \to SA, \\ T_4 \to T_1A \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{33} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 1^m(01)^n(10)^n,\ m \geqslant 4,\ m,n \in \mathbb{N}^+\}$

- $G_{33} = (\{A,B,S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to AB, \\ A \to A1 \mid 1111, \\ B \to 01B10 \mid 0110 \end{array} \right\}$.
- $G_{33}^5 = (\{A,B,R,S\}, \{0,1\}, P_5, S_1)$, com $P_5 = \left\{ \begin{array}{l} S \to AB, \\ A \to 1111R \mid 1111, \\ R \to 1R \mid 1, \\ B \to 01B10 \mid 0110 \end{array} \right\}$.
- Gramática G_{33}^6 na forma normal de Chomsky:

$$\begin{aligned} G_{33}^6 &= (V_6, \Sigma, P_6, S) \\ &= (\{A,B,S,T_0,T_1,T_2,T_3,T_4,T_5,T_6\}, \{0,1\}, P_6, S), \text{ com} \\ P_6 &= \left\{ \begin{array}{l} S \to AB, \\ A \to T_2T_3 \mid T_2T_2, \\ R \to T_1R \mid 1, \\ B \to T_4T_6 \mid T_4T_5, \\ T_0 \to 0, \\ T_1 \to 1, \\ T_2 \to T_1T_1, \\ T_3 \to T_2R, \\ T_4 \to T_0T_1, \\ T_5 \to T_1T_0, \\ T_6 \to BT_5 \end{array} \right\}. \end{aligned}$$

$\mathcal{L}_{34} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m10^n10^q \text{ ou } w = 0^n10^{2n},\ m,n,q \in \mathbb{N}\}$

$\mathcal{L}_{35} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^n10^{2n} \text{ ou } w = 1^n01^{3n},\ n \in \mathbb{N}\}$

- $G_{35} = (\{A,B,S\}, \{0,1\}, P, S)$, com $P = \left\{ \begin{array}{l} S \to A \mid B, \\ A \to 0A00 \mid 1, \\ B \to 1B111 \mid 0 \end{array} \right\}$.
- $G_{35}^5 = (\{A,B,S\}, \{0,1\}, P_5, S)$, com $P_5 = \left\{ \begin{array}{l} S \to 0A00 \mid 1 \mid 1B111 \mid 0, \\ A \to 0A00 \mid 1, \\ B \to 1B111 \mid 0 \end{array} \right\}$.

- Gramática G_{35}^{6} na forma normal de Chomsky:

$$
\begin{aligned}
G_{35}^{6} &= (V_6, \Sigma, P_6, S) \\
&= (\{A, B, S, T_0, T_1, T_2, T_3, T_4, T_5, T_6\}, \{0,1\}, P_6, S), \text{ com} \\
P_6 &= \left\{ \begin{array}{l}
S \to T_3T_2 \mid 1 \mid T_4T_5 \mid 0, \\
A \to T_3T_2 \mid 1, \\
B \to T_4T_5 \mid 0, \\
T_0 \to 0, \\
T_1 \to 1, \\
T_2 \to T_0T_0, \\
T_3 \to T_0A, \\
T_4 \to T_1B, \\
T_5 \to T_1T_6, \\
T_6 \to T_1T_1
\end{array} \right\}.
\end{aligned}
$$

$$\mathcal{L}_{36} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q1^r0^s1^t,\ m+q+r+t = n+s,\ m,n,q,r,s,t \in \mathbb{N}\}$$

$$\mathcal{L}_{37} = \{w \in \Sigma^* = \{0,1\}^* \mid w = 0^m1^n0^q1^r0^s1^t,\ m+q+r = n+s+t,\ m,n,q,r,s,t \in \mathbb{N}\}$$